Interdependence between emerging and major Markets.

Adel Sharkasi, Heather Ruskin and Martin Crane
School of Computing, Dublin City University, Dublin, Ireland
Email: asharkasi, hruskin, mcrane@computing.dcu.ie
October 23, 2006

Abstract

In this paper, we investigate the price spillover effects among two developed markets, (the US and the UK), and two developing markets, (Irish and Portuguese), using a new testing method suggested by Lee (2002). We find that there are interrelationships between any two of the Irish, the UK and Portuguese markets and that the co-movements between the emerging markets and the US are statistically significant but weak. We also found that the US market is slightly influenced by the UK but not vice versa.

keywords: Simple Regression, Volatility and Wavelet Analysis.

1 Introduction

The relationships between international stock markets have been investigated in several articles, especially after “Black Monday”, (October 1987). These studies indicated that co-movements among stock markets have increased the possibilities for national markets to be influenced by the changes in international ones ([12],[9],[6],[7] and [13]).

The advantage of global portfolio diversification has been noted in the finance literature for some time. Several studies ([11], [14] and [2]) showed that it is useful to spread content internationally, rather than locally, as stocks in different markets are less correlated than those within the same market. Tang [16] investigated, for instance, Asian emerging and mature markets and reported that an increase in the correlation between worldwide stock markets may cause the reduction of some or all of the diversification benefits and this means that diversification benefits depend upon the degree of the relationships among different stock markets. Tang [17] found that the intertemporal stability of the correlation matrix is important in examining the ex-ante diversification benefits and stock market co-movements. The potential diversification effects have decreased and become less important due to increase in the international co-movement among stock markets, especially since the mid 1990’s ([15] and [16]).

More recently, Lee [10] developed a new testing technique based on the wavelet transform, in order to study the international transmission effects between three developed markets (the US, Germany and Japan) and two emerging markets in the MENA region, namely Egypt and Turkey. He documented that
innovation from the major markets affected the emerging markets but the that opposite was not true.

In addition, Bessler and Yang [3] employed an Error Correction Model and Directed Acyclic Graphs (DAG) to study the co-integration among nine major markets namely Japan, the US, the UK, France, Switzerland, Hong Kong, Germany, Canada and Australia. Their results showed that changes in the UK, Switzerland, Hong Kong, France and Germany influenced the US market, while the US market is affected by its own innovation as well. Moreover, Brooks and Negro [4] studied the relationship between market co-integration and the degree to which companies operate internationally. They considered three factors, (global, country-specific and industry-specific), and found that the importance of the international factor has increased since the 1980s while that of the country-specific factor has decreased.

Furthermore, Wongswan [18] found strong evidence of international transmission from the US and Japanese markets to Korean and Thai markets during the late 1990’s. Most recently, Antoniou et al. [1] applied a VAR-EGARCH model to study the relationships among three EU markets namely Germany, France and the UK and their results showed evidence of co-integration among those countries.

Our goal in this article is to study whether or not there is evidence of co-integration between four stock markets (Irish, Portuguese-as developing and the UK and the US-as mature). To examine this, we applied a testing method, (based on the wavelet transform), suggested by Lee [10].

The remainder of this paper is organized as follows: In Section 2, a brief description of the testing method is given. The data and empirical results are described in Section 3 and our conclusion is presented in the final section.

2 Brief Description of the Testing Method

With the increase in media coverage of world events and a corresponding increase in access by the wider public to this coverage, global transmissions of information can be expected to be completed within a short period of time. The wavelet analysis and, in particular, the discrete wavelet transform (DWT), is very useful (for more detail see [5]) in splitting data series into different frequency wavelet crystals and high-frequency components which explain the short-term movements in the series . A new testing method based on wavelet analysis was developed by Lee [10] and it can be described as follows:

- Reconstruct the returns series using the first and the second high-frequency wavelet crystals ($d_1$ & $d_2$) separately.

- Estimate the simple regression and reverse regression models between each two using three different scales:
  - The row daily returns.
  - The returns series rebuilt form $d_1$.
  - The returns series rebuilt form $d_1$ plus that rebuilt from $d_2$.

- Test the significant of regression coefficient (slope) and $R^2$.  
3 Data and Empirical Results

The data used in the following analysis consists of the daily prices of stock market indices for two emerging markets, namely Portuguese and Irish and two major markets, (the US and the UK), during the period from January 1st, 1993 to September 30th, 2003. We considered the indices ISEQ Overall, PSI20, FTSE All Share and S&P500 to be representative of the Irish, Portuguese, UK and US markets respectively.

As these markets use their local currencies for presenting the values of their indices, so we use the daily returns instead of using the daily prices, where the former equal the natural logarithm of the ratio between the closing price of index at time $t$ and that at time $t-1$. Some daily observations have been deleted because the markets we studied have different holidays and closing trading days, (as has been done by e.g. [10]).

Table 1: Descriptive statistics of the daily returns of the stock markets indices series.

<table>
<thead>
<tr>
<th>Measure</th>
<th>ISEQ</th>
<th>PSI20</th>
<th>FTSE</th>
<th>S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Observations</td>
<td>2556</td>
<td>2556</td>
<td>2556</td>
<td>2556</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00052</td>
<td>0.00029</td>
<td>0.00012</td>
<td>0.00033</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>0.0104</td>
<td>0.0109</td>
<td>0.0099</td>
<td>0.0111</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0757</td>
<td>-0.0959</td>
<td>-0.0515</td>
<td>-0.0704</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0584</td>
<td>0.0694</td>
<td>0.0509</td>
<td>0.0557</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.3580**</td>
<td>-0.5760**</td>
<td>-0.1820</td>
<td>-0.021</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.503**</td>
<td>6.849**</td>
<td>2.794**</td>
<td>3.077**</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>2203.63**</td>
<td>5109.643**</td>
<td>840.70**</td>
<td>1002.87**</td>
</tr>
</tbody>
</table>

Note:** denotes statistically significant at 1% level.

Table 1 represents the descriptive statistics of the stock market indices and shows that the sample means of all indices are positive. We test whether or not the skewness and kurtosis of all these series are different from zero. The results show that the returns series of ISEQ and PSI20 indices have significant negative skewness, but those of FTSE and S&P500 are not significantly different from zero. The returns of all indices are leptokurtic and the results of a normal test (Jarque-Bera) also show that all returns series can not be regarded as normally distributed.

Table 2: Percentages of energy by wavelet crystals for the daily returns of indices series.

<table>
<thead>
<tr>
<th>Wavelet Crystals</th>
<th>ISEQ</th>
<th>PSI20</th>
<th>FTSE</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_6$</td>
<td>0.028</td>
<td>0.039</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td>$d_6$</td>
<td>0.023</td>
<td>0.025</td>
<td>0.017</td>
<td>0.012</td>
</tr>
<tr>
<td>$d_5$</td>
<td>0.036</td>
<td>0.042</td>
<td>0.027</td>
<td>0.031</td>
</tr>
<tr>
<td>$d_4$</td>
<td>0.070</td>
<td>0.058</td>
<td>0.047</td>
<td>0.048</td>
</tr>
<tr>
<td>$d_3$</td>
<td>0.155</td>
<td>0.163</td>
<td>0.157</td>
<td>0.145</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.274</td>
<td>0.267</td>
<td>0.301</td>
<td>0.234</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.431</td>
<td>0.406</td>
<td>0.436</td>
<td>0.518</td>
</tr>
</tbody>
</table>

From Table 2, It can be seen that high-frequency components have more energy than low-frequency ones and this implies that the movements in all index
returns are caused by the short-term fluctuations. It also implies that the first “d_1” and the second “d_2” components of the wavelet transform account for more than 60% of the energy. This indicates that there are no long memory effects in the returns series of these indices.

In order to study the co-movements among those markets, firstly, we built simple regression models between each of the two European markets on the same trading day and similarly for each European market on the US market of the previous trading day. Secondly, we built a simple regression model of the US market on each European market on the same trading day and these models are estimated using the three different scales mentioned in Section 2. The results are given in Tables 3(A) to 3(F) for each case and clearly show that there are significant levels of inter-correlation between the Irish and UK markets and also between the Irish and Portuguese. However, the relationship between the Irish and US markets is weak. From Table 3 (D), (E) and (F), we can see that there is significant co-movement between Portuguese and UK markets and there are spillover effects from both Portuguese and UK markets on the US market but not vice versa.

Table 3: Regression Analysis between each pair of four stock markets using three different scales.

<table>
<thead>
<tr>
<th>Scales</th>
<th>( M_t^{IRL} ) on ( M_t^{UK} )</th>
<th>( M_t^{UK} ) on ( M_t^{IRL} )</th>
</tr>
</thead>
</table>
| Return | \( \begin{array}{ll}
\text{Constant} & 4.46E-04 \\
\text{Slope} & 0.592 \\
\text{R}^2 & 0.322
\end{array} \) \( \begin{array}{ll}
\text{Constant} & -1.58E-04 \\
\text{Slope} & 0.544 \\
\text{R}^2 & 0.322
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & -1.58E-04 \\
\text{Slope} & 0.322 \\
\text{R}^2 & 0.322
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.544 \\
\text{Slope} & 0.322 \\
\text{R}^2 & 0.322
\end{array} \) |
| Return.D1 | \( \begin{array}{ll}
\text{Constant} & -5.85E-07 \\
\text{Slope} & 0.509 \\
\text{R}^2 & 0.251
\end{array} \) \( \begin{array}{ll}
\text{Constant} & -1.06E-06 \\
\text{Slope} & 0.492 \\
\text{R}^2 & 0.251
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & -1.06E-06 \\
\text{Slope} & 0.251 \\
\text{R}^2 & 0.251
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.492 \\
\text{Slope} & 0.251 \\
\text{R}^2 & 0.251
\end{array} \) |
| Return.D1.2 | \( \begin{array}{ll}
\text{Constant} & 6.18E-08 \\
\text{Slope} & 0.552 \\
\text{R}^2 & 0.300
\end{array} \) \( \begin{array}{ll}
\text{Constant} & -3.31E-06 \\
\text{Slope} & 0.544 \\
\text{R}^2 & 0.300
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & -3.31E-06 \\
\text{Slope} & 0.300 \\
\text{R}^2 & 0.300
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.544 \\
\text{Slope} & 0.300 \\
\text{R}^2 & 0.300
\end{array} \) |

A: ISEQ Overall and FTSE

<table>
<thead>
<tr>
<th>Scales</th>
<th>( M_t^{IRL} ) on ( M_{t-1}^{US} )</th>
<th>( M_t^{US} ) on ( M_t^{IRL} )</th>
</tr>
</thead>
</table>
| Return | \( \begin{array}{ll}
\text{Constant} & 4.46E-04 \\
\text{Slope} & 0.356 \\
\text{R}^2 & 0.145
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 1.93E-04 \\
\text{Slope} & 0.258 \\
\text{R}^2 & 0.057
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & 1.93E-04 \\
\text{Slope} & 0.057 \\
\text{R}^2 & 0.057
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.258 \\
\text{Slope} & 0.057 \\
\text{R}^2 & 0.057
\end{array} \) |
| Return.D1 | \( \begin{array}{ll}
\text{Constant} & -1.94E-06 \\
\text{Slope} & 0.172 \\
\text{R}^2 & 0.039
\end{array} \) \( \begin{array}{ll}
\text{Constant} & -2.65E-06 \\
\text{Slope} & 0.066 \\
\text{R}^2 & 0.002
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & -2.65E-06 \\
\text{Slope} & 0.002 \\
\text{R}^2 & 0.002
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.066 \\
\text{Slope} & 0.002 \\
\text{R}^2 & 0.002
\end{array} \) |
| Return.D1.2 | \( \begin{array}{ll}
\text{Constant} & -3.41E-06 \\
\text{Slope} & 0.273 \\
\text{R}^2 & 0.092
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 1.26E-06 \\
\text{Slope} & 0.135 \\
\text{R}^2 & 0.019
\end{array} \) | \( \begin{array}{ll}
\text{Constant} & 1.26E-06 \\
\text{Slope} & 0.019 \\
\text{R}^2 & 0.019
\end{array} \) \( \begin{array}{ll}
\text{Constant} & 0.135 \\
\text{Slope} & 0.019 \\
\text{R}^2 & 0.019
\end{array} \) |

B: ISEQ Overall and S&P500
### Regression Table

<table>
<thead>
<tr>
<th>Scales</th>
<th>$M_{t}^{IRL}$ on $M_{t}^{P}$</th>
<th>$M_{t}^{P}$ on $M_{t}^{IRL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>Constant: 4.19E-04 (0.029)</td>
<td>Constant: 9.67E-05 (0.632)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.340 (0.000)</td>
<td>Slope: 0.378 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.128</td>
<td>$R^{2}$: 0.128</td>
</tr>
<tr>
<td>Return.D1</td>
<td>Constant: -1.28E-06 (0.992)</td>
<td>Constant: -6.94E-08 (1.000)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.352 (0.000)</td>
<td>Slope: 0.384 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.135</td>
<td>$R^{2}$: 0.135</td>
</tr>
<tr>
<td>Return.D1.2</td>
<td>Constant: -3.64E-06 (0.995)</td>
<td>Constant: 4.22E-06 (0.370)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.341 (0.000)</td>
<td>Slope: 0.370 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.126</td>
<td>$R^{2}$: 0.126</td>
</tr>
</tbody>
</table>

C: ISEQ Overall and PSI20

<table>
<thead>
<tr>
<th>Scales</th>
<th>$M_{t}^{P}$ on $M_{t}^{UK}$</th>
<th>$M_{t}^{UK}$ on $M_{t}^{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>Constant: 2.29E-04 (0.230)</td>
<td>Constant: -1.45E-06 (0.993)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.517 (0.000)</td>
<td>Slope: 0.428 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.221</td>
<td>$R^{2}$: 0.221</td>
</tr>
<tr>
<td>Return.D1</td>
<td>Constant: 2.84E-07 (0.998)</td>
<td>Constant: -1.51E-06 (0.989)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.516 (0.516)</td>
<td>Slope: 0.459 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.236</td>
<td>$R^{2}$: 0.237</td>
</tr>
<tr>
<td>Return.D1.2</td>
<td>Constant: 5.65E-06 (0.971)</td>
<td>Constant: -6.18E-06 (0.976)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.505 (0.000)</td>
<td>Slope: 0.458 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.231</td>
<td>$R^{2}$: 0.231</td>
</tr>
</tbody>
</table>

D: PSI20 and FTSE

<table>
<thead>
<tr>
<th>Scales</th>
<th>$M_{t}^{P}$ on $M_{t-1}^{US}$</th>
<th>$M_{t-1}^{US}$ on $M_{t}^{P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>Constant: 2.29E-04 (0.280)</td>
<td>Constant: 2.48E-04 (0.241)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.196 (0.000)</td>
<td>Slope: 0.266 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.040</td>
<td>$R^{2}$: 0.066</td>
</tr>
<tr>
<td>Return.D1</td>
<td>Constant: 2.88E-07 (0.996)</td>
<td>Constant: -2.62E-06 (0.987)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.516 (0.058)</td>
<td>Slope: 0.194 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.236</td>
<td>$R^{2}$: 0.028</td>
</tr>
<tr>
<td>Return.D1.2</td>
<td>Constant: 2.88E-06 (0.987)</td>
<td>Constant: 1.22E-07 (0.999)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.122 (0.000)</td>
<td>Slope: 0.228 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.017</td>
<td>$R^{2}$: 0.044</td>
</tr>
</tbody>
</table>

E: PSI20 and S&P500

<table>
<thead>
<tr>
<th>Scales</th>
<th>$M_{t}^{UK}$ on $M_{t-1}^{US}$</th>
<th>$M_{t-1}^{US}$ on $M_{t}^{UK}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>Constant: 3.49E-05 (0.852)</td>
<td>Constant: 2.69E-04 (0.179)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.272 (0.000)</td>
<td>Slope: 0.471 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.092</td>
<td>$R^{2}$: 0.177</td>
</tr>
<tr>
<td>Return.D1</td>
<td>Constant: -1.81E-06 (0.989)</td>
<td>Constant: -2.20E-06 (0.989)</td>
</tr>
<tr>
<td></td>
<td>Slope: 4.46E-03 (0.793)</td>
<td>Slope: 0.300 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.000</td>
<td>$R^{2}$: 0.060</td>
</tr>
<tr>
<td>Return.D1.2</td>
<td>Constant: -5.17E-06 (0.975)</td>
<td>Constant: 2.59E-06 (0.989)</td>
</tr>
<tr>
<td></td>
<td>Slope: 0.151 (0.000)</td>
<td>Slope: 0.368 (0.000)</td>
</tr>
<tr>
<td></td>
<td>$R^{2}$: 0.029</td>
<td>$R^{2}$: 0.106</td>
</tr>
</tbody>
</table>

F: FTSE and S&P 500

- P-values of t-tests are given in parentheses.
- Where subscript refers to the day in question and the superscript indicates the market (e.g. IRL, P are the Irish and Portuguese markets respectively).
- Return.D1 is an indicator of the returns series, reconstructed using the first wavelet crystal ($d_1$).
- Return.D1.2 is an indicator of the returns series, reconstructed using the first and the second wavelet crystals ($d_1$ & $d_2$).
Table 4: The Percentages of error variance of the market in the first column explained by innovation in the market in the first row.

<table>
<thead>
<tr>
<th>Market Explained</th>
<th>Days Ahead</th>
<th>Ireland</th>
<th>Portugal</th>
<th>The UK</th>
<th>The US</th>
<th>OM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland</td>
<td>5</td>
<td>5</td>
<td>0.51</td>
<td>77.54</td>
<td>18.40</td>
<td>3.54</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.59</td>
<td>0.83</td>
<td>76.61</td>
<td>18.83</td>
<td>3.73</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.83</td>
<td>76.52</td>
<td>18.83</td>
<td>3.82</td>
<td>3.82</td>
</tr>
<tr>
<td>Portugal</td>
<td>5</td>
<td>0.38</td>
<td>0.30</td>
<td>88.77</td>
<td>10.56</td>
<td>11.24</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.59</td>
<td>0.54</td>
<td>87.99</td>
<td>10.87</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.59</td>
<td>0.55</td>
<td>87.99</td>
<td>10.88</td>
<td>12.02</td>
</tr>
<tr>
<td>The UK</td>
<td>5</td>
<td>0.37</td>
<td>0.78</td>
<td>19.87</td>
<td>78.98</td>
<td>21.02</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.45</td>
<td>1.16</td>
<td>20.44</td>
<td>77.95</td>
<td>22.05</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.45</td>
<td>1.17</td>
<td>20.45</td>
<td>77.93</td>
<td>22.07</td>
</tr>
<tr>
<td>The US</td>
<td>5</td>
<td>0.37</td>
<td>0.78</td>
<td>19.87</td>
<td>78.98</td>
<td>21.02</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.45</td>
<td>1.16</td>
<td>20.44</td>
<td>77.95</td>
<td>22.05</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.45</td>
<td>1.17</td>
<td>20.45</td>
<td>77.93</td>
<td>22.07</td>
</tr>
</tbody>
</table>

Note: OM denotes the percentage of forecast error variance explained collectively by the other markets.

To compare our results with one of the common methods, we estimated the vector autoregressive (VAR) model of order 10 of the daily returns of these markets. The percentages of the decomposition of 5-day, 10-day and 15-day ahead forecasts of the returns series have been measured\(^1\). At 15 days ahead, for example, the results, given in Table 4, show that the most of the variance in these markets is explained by their own innovations and that the UK is the most influential market while the Irish is the most influenced market. The UK explains 26.30, 18.83 and 20.45 percent for Irish, Portuguese and the US respectively and the US explains 12.17, 3.82 and 10.88 percent of the variance of Irish, Portuguese and the UK respectively. We also found that the forecast error variance is very sensitive to the order of variables for orthogonalization and to the stability of these series and this suggests that the new technique, based on wavelet analysis, is more reliable than the VAR method.

4 Conclusion

Our objective in this paper has been to study the international transmission between four markets namely the Irish, Portuguese, UK and US. A new testing method suggested by Lee [10] has been applied to do so. Our results show that there are significant inter-correlations between each pair of Irish, Portuguese and UK markets separately. In addition, the indications are that the US has insignificant spillover effects from or on to the other markets. We can say that the emerging markets have significant spillover effects on each other but there is no co-integration between the major markets.

\(^1\)The orthogonalization is ordered as the UK, Portuguese, the US and Irish.
References


A Wavelet Analysis

The Wavelet Transform (WT) has been explained in some detail, (particularly in [5] and [10]) and the following offers a brief explanation only. The WT has two types of wavelets called father and mother wavelets, φ and ψ respectively, where \( \int \phi(t)dt = 1 \) and \( \int \psi(t)dt = 0 \). These can be computed using the following equations

\[
\phi(t) = \sqrt{2} \sum_k \ell_k \phi(2t - k) \quad (1)
\]

\[
\psi(t) = \sqrt{2} \sum_k h_k \phi(2t - k) \quad (2)
\]

The orthogonal wavelet series approximation to a given signal \( f(t) \) is defined by

\[
f(t) = \sum_k s_{J,k} \phi_{J,k}(t) + \sum_k d_{J,k} \psi_{J,k}(t) + \ldots + \sum_k d_{1,k} \psi_{1,k}(t) \quad (3)
\]

where \( J \) is the number of multiresolution levels, (or crystals), and \( k \) ranges from 1 to the number of coefficients in the specified components (or levels). The coefficient \( s_{J,k}, d_{J,k}, \ldots, d_{1,k} \) are the wavelet transform coefficients given by

\[
s_{J,k} = \int \phi_{J,k}(t)f(t)dt \quad (4)
\]

\[
d_{j,k} = \int \psi_{j,k}(t)f(t)dt \quad (j = 1, 2, \ldots, J) \quad (5)
\]

The discrete wavelet transform (DWT) computes the coefficient of the wavelet series approximation in Equation (3) for a discrete signal \( f_1, \ldots, f_n \) of finite extent. The DWT maps the vector \( f = (f_1, f_2, \ldots, f_n)' \) to a vector of \( n \) wavelet coefficients \( w = (w_1, w_2, \ldots, w_n)' \) which contains the “smooth” coefficient \( s_{J,k} \) and “detail” coefficients \( d_{J,k} [j = 1, 2, \ldots, J] \). The \( s_{J,k} \) describes the underlying smooth behaviour of the signal at coarse-scale \( 2^J \) while \( d_{J,k} \) describes the coarse-scale deviations from the smooth behaviour and the \( d_{J-1,k}, \ldots, d_{1,k} \) provide progressively finer-scale deviations from the smooth behaviour.

Acknowledgement: A.S would like to gratefully acknowledge the receipt of a grant from his government (Libya) in support of this research. Professors Gama and Duarte and J. A. Matos are thanked for facilitating access to the Portuguese data.