Multiscaled Cross-Correlation Dynamics in Financial Time Series

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The cross-correlation matrix between equities comprises multiple interactions between traders with varying strategies and time horizons. In this paper, we use the Maximum Overlap Discrete Wavelet Transform (MODWT) to calculate correlation matrices over different time scales and then explore the *eigenvalue spectrum* over sliding time windows. The dynamics of the eigenvalue spectrum at different scales provides insight into the interactions between the numerous constituents involved.

A study of the eigenvalue spectrum in its entirety provides further insight. On partitioning the eigenvalue time series, we show that negative index returns, (*drawdowns*), are associated with periods where the largest eigenvalue is greatest, while positive index returns, (*drawups*), are associated with periods where the largest eigenvalue is smallest. Furthermore, through the study of the small eigenvalues of the correlation matrix, we show that information about the correlation dynamics is visible at both ends of the eigenspectrum across all scales.

1. Introduction

In recent years, the equal-time cross-correlation matrix for a variety of multivariate data sets across different disciplines such as financial data [1–11], electroencephalographic (EEG) recordings [12, 21], magnetoencephalographic (MEG) recordings [13] and others, has been studied extensively. In particular, Random Matrix Theory (RMT) has been applied to filter the relevant information from the statistical fluctuations inherent in empirical cross-correlation matrices, constructed for various types of financial, [1–9], and MEG, [13], time series.

It has been suggested recently by several authors, [14–18] that there may, in fact, be some real correlation information hidden in the RMT defined part of the eigenvalue spectrum. A technique, involving the use of *power mapping* to identify and estimate the noise in financial correlation matrices was described, [14], allowing the suppression of those noise eigenvalues, to reveal different correlation structures buried underneath. The relationship, between the eigenvalue density c of the true correlation matrix, and that of an empirical correlation matrix C, was derived, [15, 16], to show that correlations can be measured in the random part of the spectrum. The bulk of the spectrum was shown to deviate from the Wishart RMT class through

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the use of a Kolmogorov test, [17], where the existence of factors such as an firm size, industry type and the overall market effect was due to collective influence of the assets. Additional evidence that the RMT fit is not perfect was provided, [18], where the dispersion of "noise" eigenvalues was shown to be inflated, indicating that the bulk of the eigenvalue spectrum contains correlations masked by measurement noise.

The behaviour of the largest eigenvalue of a cross-correlation matrix over small windows of time, was studied, [19], for the Dow Jones industrial average (DJIA) and DAX indices where evidence of a time-dependence between 'drawdowns'/'drawups' and an increase/decrease in the largest eigenvalue was obtained. The dynamics between the stocks of two different markets (DAX and DJIA) were then investigated, [20], revealing two distinct eigenvalues of the combined cross-correlation matrix, corresponding to each of the markets. By adjusting for time-zone delays, the two eigenvalues coincided, implying that one market leads the dynamics in the other.

Equal-time cross-correlation matrices have been used also, [21], to characterise dynamical changes in nonstationary multivariate time-series. As the synchronisation of k time series within an $M-$dimensional multivariate time series increases, a repulsion between eigenstates of the correlation matrix results, in which k levels participate. Through the use of artificially-created time series with pre-defined correlation dynamics, it was demonstrated that there exist situations, where the relative change in eigenvalues from the lower edge of the spectrum is greater than that for the large eigenvalues, implying that information drawn from the smaller eigenvalues is highly relevant.

This technique was subsequently applied to the dynamic analysis of the eigenvalue spectrum of the equal-time cross-correlation matrix of multivariate Epileptic Seizure time series, using sliding windows. The authors demonstrated that information about the correlation dynamics is visible in *both* the lower and upper eigenstates. A further study, [12], which investigated temporal dynamics of focal onset epileptic seizures[a]. It was shown that the zero-lag correlations between multichannel EEG signals tend to decrease during the first half of a seizure and increase gradually before the seizure ends. Information about cross correlations were found in the RMT bulk of eigenvalues, [22], with that extracted at the *lower* edge statistically *more significant* than that from the larger eigenvalues. Application of this technique to multichannel EEG data showed small eigenvalues to be more sensitive to detection of subtle changes in the brain dynamics than the largest.

Multiscale analysis of the correlations between time series has been applied to a number of problems in the physical sciences, [23, 24]. Specifically, wavelet multiscale analysis has been used by numerous authors to decompose economic and financial time series into orthogonal time-scale components of varying granularities. Gençay and co-workers have variously examined the scaling properties of foreign

[a]A focal onset or partial seizure occurs when the discharge starts in one area of the brain and then spreads over other areas.

exchange volatility, [25], and volatility models without intraday seasonalities, [26], using wavelet multiscaling techniques. The systematic risk in a Capital Asset Pricing Model was estimated over different granularities, [27], where it was shown that the return of a portfolio and its beta[b] becomes stronger as the scale increases for the S&P 500. This technique was then applied to the markets of various other countries, [28], with similar results also found. The dependence of stock return cross-correlations on the sampling time scale is known as the *Epps* effect [29].

The use of wavelet multiscale analysis has not been limited to pure financial data. The relationship between consumption and income was examined, [30], where wavelets were used to demonstrate that the marginal propensity to consume, for all goods and services, is about 0.94. It was also shown that, at longer timescales, interest rates and the consumption of durable and non-durable goods have an inverse relationship. The analysis of scaling laws in an economic context, [31, 32], has been approached for areas such as GDP, firm size and growth rate.

In this paper, we build upon the techiques applied to detect changes in the correlation structure of EEG data, [21], through the use of wavelet multiscale analysis. This allows us to explore the eigenvalue dynamics of the correlation matrix between the stocks of the Dow Jones Eurostoxx 50 at different granularities. An analysis of eigenvalue behaviour during drawdowns provides evidence of the sensitivity of small eigenvalues to subtle changes in interactions between traders, with different strategies and time horizons. Sections(2 - 3) describe the techniques and data used, Section (4) details the results obtained and conclusions are given in Section (5).

2. Methods

2.1. *Correlation dynamics*

The equal-time correlation matrix between time series of stock returns is calculated using a sliding time window where the number of stocks, N, is smaller than the window size T. Given time series of stock returns $R_i(t)$, $i = 1, \ldots, N$, we normalise the time series within each window as follows:

$$r_i(t) = \frac{R_i(t) - \widehat{R_i(t)}}{\sigma_i} \tag{1}$$

where σ_i is the standard deviation of stock $i = 1, \ldots, N$ and $\widehat{R_i}$ is the time average of R_i over a time window of size T. Then, the equal time cross-correlation matrix, expressed in terms of $r_i(t)$, is

$$C_{ij} \equiv \langle r_i(t)\, r_j(t) \rangle \tag{2}$$

The elements of C_{ij} are limited to the domain $-1 \leq C_{ij} \leq 1$, where $C_{ij} = 1$ defines perfect positive correlation, $C_{ij} = -1$ corresponds to perfect negative correlation

[b]The Beta of an asset is a measure of the volatility, or systematic risk, of a security or a portfolio in comparison to the market as a whole.

and $C_{ij} = 0$ corresponds to no correlation. In matrix notation, the correlation matrix is expressed as $\mathbf{C} = \frac{1}{T}\mathbf{R}\mathbf{R}^t$, where $\mathbf{R}$ is an $N \times T$ matrix with elements r_{it}.

The eigenvalues λ_i and eigenvectors $\hat{\mathbf{v}}_i$ of the correlation matrix $\mathbf{C}$ are found using $\mathbf{C}\hat{\mathbf{v}}_i = \lambda_i \hat{\mathbf{v}}_i$ and then ordered according to size, such that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$. Given that the sum of the diagonal elements of a matrix (the Trace) remains constant under linear transformation, $\sum_i \lambda_i$ must always equal the trace of the original correlation matrix. Hence, if some eigenvalues increase then others must decrease, to compensate, and vice versa (*Eigenvalue Repulsion*).

There are two limiting cases for the distribution of the eigenvalues [12, 21], with perfect correlation, $C_i \approx 1$, when the largest is maximised with value N (all others taking value zero). When each time series consists of random numbers with average correlation $C_i \approx 0$, the corresponding eigenvalues are distributed around 1, (where deviations are due to spurious random correlations). Between these two extremes, the eigenvalues at the lower end of the spectrum can be much smaller than λ_{max}. To study the dynamics of each of the eigenvalues using a sliding window, we normalise each eigenvalue in time using

$$\tilde{\lambda}_i(t) = \frac{\left(\lambda_i - \bar{\lambda}(\tau)\right)}{\sigma^{\lambda(\tau)}} \tag{3}$$

where $\bar{\lambda}(\tau)$ and $\sigma^{\lambda(\tau)}$ are the mean and standard deviation of the eigenvalues over a particular reference period, τ. This normalisation allows us to visually compare eigenvalues at both ends of the spectrum, even if their magnitudes are significantly different. The reference period used to calculate mean and standard deviation of the eigenvalue spectrum can be chosen to be a low volatility sub-period, (which helps to enhance the visibility of high volatility periods), or the full-time period studied.

2.2. *Wavelet multiscale analysis*

Wavelets provide an efficient means of studying the multiresolution properties of a signal, by decomposition into different time horizons or frequency components (Discrete Wavelet Transform, DWT), [23, 24]. The definitions of the two basic wavelet functions, the father ϕ and mother ψ wavelets are the functions:

$$\phi_{j,k}(t) = 2^{\frac{j}{2}} \phi\left(2^j t - k\right) \tag{4}$$

$$\psi_{j,k}(t) = 2^{\frac{j}{2}} \psi\left(2^j t - k\right) \tag{5}$$

where $j = 1, \ldots J$ in a J-level decomposition. The father wavelet integrates to 1 and reconstructs the longest time-scale component of the series, while the mother wavelet integrates to 0 and is used to describe the deviations from the trend. The wavelet representation of a discrete signal $f(t)$ in $L^2(R)$ is:

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\phi_{J,k}(t) + \ldots + \sum_k d_{1,k}\phi_{1,k}(t) \tag{6}$$

where J is the number of multiresolution levels (or *scales*) and k ranges from 1 to the number of coefficients in the specified level. The coefficients $s_{J,k}$ and $d_{J,k}$ are the smooth and detail component coefficients respectively and given by

$$s_{J,k} = \int \phi_{J,k} f(t)dt \tag{7}$$

$$d_{j,k} = \int \psi_{j,k} f(t)dt \quad (j = 1, \dots J) \tag{8}$$

Each of the coefficient sets $S_J, d_J, d_{J-1}, \dots d_1$ is called a *crystal*.

In this paper, we apply the MODWT, [23, 24], a linear filter that transforms a series into coefficients related to variations over a set of scales. Like the DWT it produces a set of time-dependent wavelet and scaling coefficients with basis vectors associated with a location t and a unitless scale $\tau_j = 2^{j-1}$ for each decomposition level $j = 1, \dots, J_0$. The MODWT, unlike the DWT, has a high level of redundancy, however, is nonorthogonal and can handle any sample size N. It retains downsampled[c] values at each level of the decomposition that would be discarded by the DWT. This reduces the tendency for larger errors at lower frequencies when calculating frequency dependent variance and correlations, (Section 2.3), as more data is available.

2.3. *Wavelet covariance and correlation*

The wavelet covariance between functions $f(t)$ and $g(t)$ is defined to be the covariance of the wavelet coefficients at a given scale. The *unbiased* estimator of the wavelet covariance at the j^{th} scale is given by

$$\nu_{fg}(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \tilde{D}_{j,t}^{f(t)} \tilde{D}_{j,t}^{g(t)} \tag{9}$$

where all the wavelet coefficients affected by the boundary are removed [23] and $M_j = N - L_j + 1$. The wavelet variance at a particular scale $\nu_f^2(\tau_j)$ is found similarly.

The MODWT estimate of the wavelet cross correlation between functions $f(t)$ and $g(t)$ may then be calculated using the wavelet covariance and the square root of the wavelet variance of the functions at each scale j. The MODWT estimator [24] of the wavelet correlation is then given by:

$$\rho_{fg}(\tau_j) = \frac{\nu_{fg}(\tau_j)}{\nu_f(\tau_j)\nu_g(\tau_j)} \tag{10}$$

3. Data

The data set comprises the 49 equities of the Dow Jones (DJ) Euro Stoxx 50 where full price data is available from May 1999 to August 2007, resulting in 2183 daily returns. The Dow Jones Euro Stoxx 50 is a stock index of Eurozone equities, designed to provide a blue-chip representation of supersector leaders in the Eurozone. As the

[c]Downsampling or decimation of the wavelet coefficients retains half of the number of coefficients that were retained at the previous scale and is applied in the DWT

number of stocks in the index is small, this allows us to calculate cross-correlation matrices for small time windows, without reducing the rank of the matrix.

4. Results

4.1. *Eigenvalue dynamics*

We analyse the dynamics of the cross-correlations between stocks, calculated at different time scales, through the use of the eigenvalue spectrum. We first consider, Fig. 1, the dynamics of the largest eigenvalue at each time scale, calculated using a sliding time window of 100 days. This window was chosen such that $Q = \frac{T}{N} = 2.04$, thus ensuring that the data would be close to non-stationary in each sliding window (Different values of Q were examined, [11], with little variation in the results). Fig. 1(a) shows the value of the DJ Euro Stoxx Index over the period studied. Fig. 1(b) displays the largest eigenvalue, calculated using the unfiltered (one day) time series data for different time windows. As shown, the largest eigenvalue is far from static, rising from a minimum of 7.5 to a maximum of 30.5 from early 2001 to late 2003 (coinciding with the bursting of the "tech" bubble). This corresponds to an increase in the influence of the "Market", with the behaviour of traders becoming more correlated. The next major increase occurred in early 2006, followed by a relatively quick decline until the beginning of the "Credit Crunch" in 2007. Similar to [19], we note an increase in the value of the largest eigenvalue during times of market stress, with lower values during more "normal" periods.

We next calculate, using the MODWT (Section 2.2), the value of the largest eigenvalue of the cross-correlation matrix over longer time horizons of 3, 6 and 11 days (Fig. 1(c−e)). Certain traders, (such as Hedge Fund managers), may have very short trading horizons while others, (such as Pension Fund managers), have much longer horizons. By looking at the value of the largest eigenvalue at different scales, we try to characterise the impact of these different trading horizons on the cross-correlation dynamics between large capitalisation stocks. In Fig. 1(c−e), we see that the main features found in the unfiltered data are preserved over longer time scales. However, certain features, such as the sizeable drop in the largest eigenvalue at the longest scale in late 2003, are not seen at shorter scales, but the aggregate impact for the unfiltered data is a moderate drop. Other features, such as the increase in 2006, are not preserved across all scales. The different features, found at various scales, suggest that the correlation matrix is made up of interactions between stocks, traded by investors with different time horizons. This has implications for risk management, as the correlation matrix used for input in a portfolio optimisation should depend on the investors time horizon.

The repulsion, between eigenstates of the cross-correlation matrix as the level of synchronisation between time series increases has been demonstrated for artificial and EEG time series, [21], and for financial data [11]. In Fig. 2, we compare the normalised largest eigenvalue with the average of the normalised 40 smallest eigenvalues over different time scales. The normalisation, (Eqn. 3), is carried out using

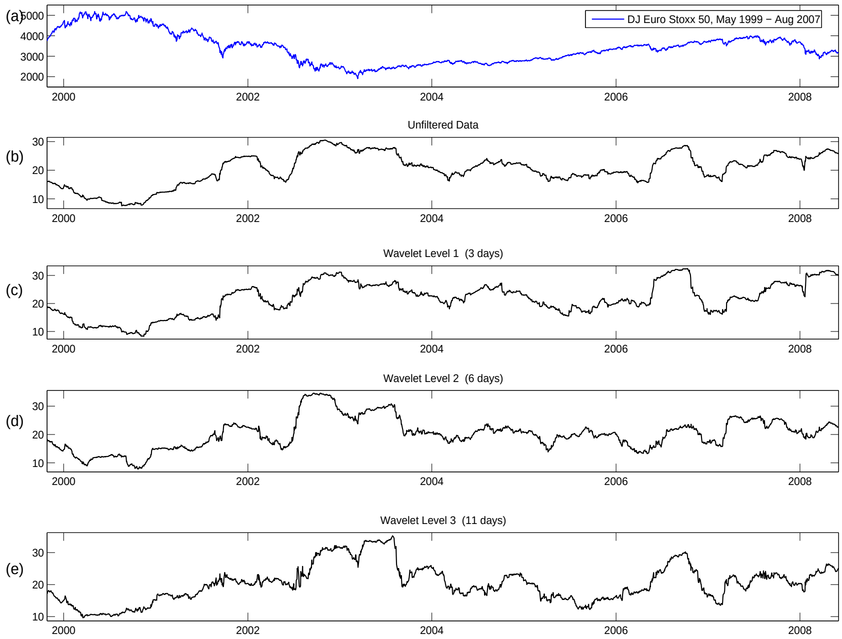

Fig. 1. (a) DJ Euro Stoxx Index (b) Largest eigenvalue dynamics original data (c) 3 day scale (d) 6 day scale (e) 11 day scale

the average and standard deviation over the entire period, to allow comparison of eigenvalues at both ends of the spectrum. The features found are in agreement with those identified for the largest eigenvalue previously. Additionally, the dynamics of the small eigenvalues are contrary to those of the largest eigenvalue across all scales, (eigenvalues are repulsed). The small eigenvalues are shown to decrease from 2001 to 2003 across all scales, contrary to findings for the largest eigenvalue.

Redistribution of the correlation structure across the eigenvalue spectrum can also be captured using *ratios of eigenvalues*. Fig. 3 shows the ratio of the largest eigenvalue to the second largest eigenvalue, (calculated using sliding windows of 100 days), for the original unfiltered data and for correlation matrices calculated with data corresponding to 3, 6 and 11 days. For unfiltered data, the ratio was found to gradually increase from 2000 to 2003, corresponding to greater importance of the largest eigenvalue, followed by a large decrease. At higher scales (6 and 11 days), the ratio was found to increase more abruptly from mid 2002 implying that long and short timescales capture different features of the correlation structure.

The distinct features and abrupt changes visible in the dynamics of eigenvalue

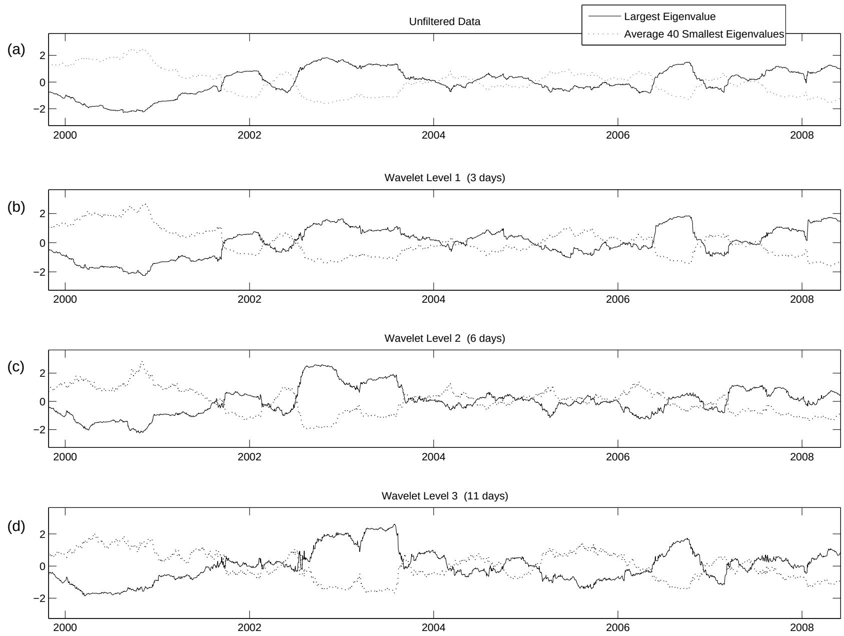

Fig. 2. (a) DJ Euro Stoxx Index (b) Eigenvalue Dynamics at 3 day scale (c) Eigenvalue Dynamics at 6 day scale (d) Eigenvalue Dynamics at 11 day scale

ratios, can be explained by the variation in the 2^{nd} largest eigenvalue over time. In the context of the analysis using deviating eigenvalues from RMT, [5], the information contained in the second largest eigenvalue may correspond to distinctly different *sectors* at the various scales. This has important implications for portfolio optimisation, where correlation between assets is a fundamental input.

4.2. *Drawdown analysis*

As indicated, (Section 1 and [11, 19]), drawdowns, (or periods of large negative returns), and drawups, (periods of large positive returns), tend to be accompanied by an increase in different eigenstates of the cross-correlation matrix. In this section, we attempt to characterise the market according to the relative size of the eigenvalues, as well as through the use of eigenvalue ratios.

The returns, correlation matrix and eigenvalue spectrum time series for overlapping windows of 100 days were calculated and normalised using the mean and standard deviation over the entire series (Eqn. 3). By representing normalised eigenvalues in terms of standard deviation units (SDU), we can partition the eigenvalues according to their magnitude. The average return of the index is shown in Ta-

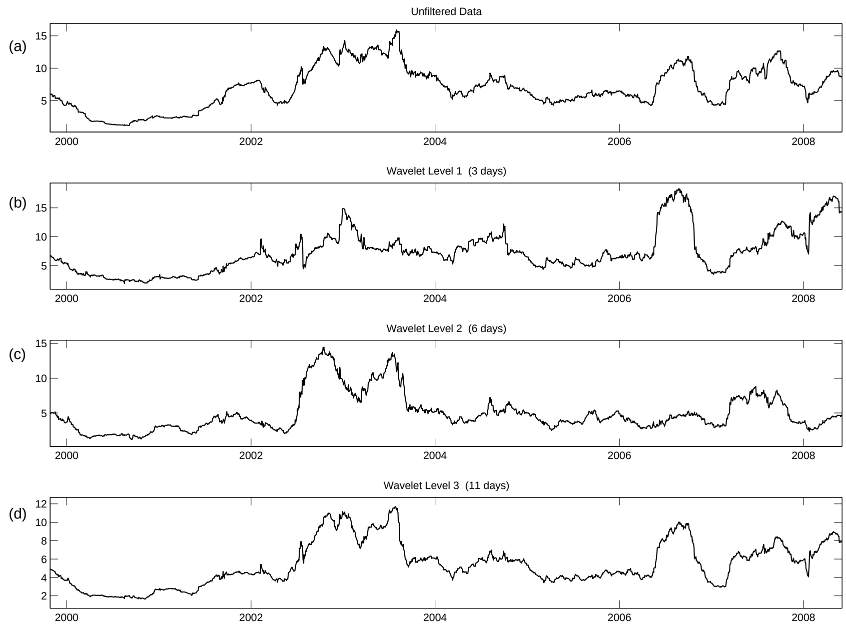

Fig. 3. Ratio largest eigenvalue to 2^{nd} largest eigenvalue for (a) original data (b) 3 day scale (c) 6 day scale (d) 11 day scale

ble 1, (during periods when the largest eigenvalue is ± 1 SDU), for both the largest eigenvalue and the average of the normalised 40 smallest eigenvalues.

Looking first at the original or unfiltered data, we see that when the largest eigenvalue is > 1 SDU, the average index return is found to be -9%. In contrast, when it is < -1 SDU, the average index return is 6.4%. This allows us to characterise the market into drawdowns and drawups by examining the relative size of the eigenvalues. For the average of the 40 smallest eigenvalues, the partition reflects more marked drawdowns and drawups but with opposite signs. This indicates that information on the interaction between traders, captured in the correlation dynamics, is visible in both the lower and upper eigenstates.

Table 1 also shows the average index returns for partitioned eigenvalues, calculated using data at the longer scales (3, 6 and 11 days). The results show that the characterisation of the largest eigenvalue, (well above average during drawdowns and well below during drawups) is *consistent across scales*. The opposite is found to occur for the small eigenvalues. The implication is that the correlation structure between stocks is less dependent on the trader's time horizon than the current state of the market.

Eigenvalues	*No. Std*	*Unfiltered*	*1*	*2*	*3*
Large	<-1	6.4%	2.5%	8.5%	10.4%
	>1	-9%	-10%	-9.3%	-7.6%
Average 40 Smallest	<-1	-9.6%	-9.8%	-12%	-8.4%
	>1	10%	9.6%	8.2%	8%
Ratio $\frac{Largest}{2^{nd} Largest}$	<-1	1.5%	0.4%	7.7%	6.4%
	>1	-6.2%	-7.2%	-9.0%	-7.2%
Ratio $\frac{Largest}{Sum40Smallest}$	<-1	3.2%	3.5%	-1%	6.9%
	>1	-5.6%	-4.7%	-5.9%	-3.8%

Table 1. Drawdown/Drawup analysis. Average Index Returns when various eigenvalue partitions in SDU are > 1 and < -1.

In Table 1, we also present the ratio of the largest eigenvalue to the second largest, (normalised over time as above). For the unfiltered data, the ratio is largest during periods of negative returns and smallest during periods of positive return, again implying that the market effect dominates during drawdowns. This is seen across all scales with larger absolute values of the returns found at larger scales. This again implies, that correlations calculated using longer time scales may be more sensitive to market behaviour. The ratio of the largest eigenvalue to the sum of the forty smallest eigenvalues is also shown in Table 1. This ratio is less sensitive than that shown in Figure 3, with values varying from 0.22 to 2.02. However, there is still evidence that negative returns occur during periods when the largest eigenvalue is much greater than the sum of the small eigenvalues. The reverse, though less marked, is also indicated.

5. Conclusions

The multiscale correlation structure of multivariate financial time series was studied by investigation of the eigenvalue spectrum of the equal-time cross-correlation matrix.

(1) Using the MODWT and a sliding window, the dynamics of the largest eigenvalue of the correlation matrix were examined and shown to be time dependent (Fig. 1). Similar dynamics were visible across all scales, but with particular features markedly apparent at certain scales. This suggests that the correlation matrix between stocks comprises interactions between stocks, traded by investors with different time horizons.
(2) Large and small eigenvalues demonstrated the expected repulsion across all scales (Fig. 2).
(3) A partition of the time normalised eigenvalues demonstrated quantitatively that the largest eigenvalue is greatest (smallest) during drawdowns (drawups). Small

eigenvalues were, in general, found to be more sensitive than large eigenvalues to the behaviour of the market, with this persisting across all scales. This suggests that underlying state of the market is more important to the correlation structure than the time horizons of different traders.

Clearly indicated by these results is a need to study the correlation structure of high-frequency stock data (to assess the influence on market crashes). In particular, high-frequency investigation will also reveal scaling behaviour of correlations (The Epps effect) and prove more effective in the prediction process. Frequency decomposition, through the use of wavelet energy at different scales, will enable the determination of Market drawdown (drawup) features.

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