

# Efficient simulation of interconnects in high-speed circuits

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**Abstract:** The paper presents an efficient approach for the simulation of interconnects in high-speed circuits based on measured data or simulated EM data. The approach involves forming an initial transmission-line model based on a resonant approach introduced by the authors and subsequently tuning this model to match the measured data. Following tuning, the model is converted to a state-space formulation. This then enables a standard model reduction routine such as the Lanczos process to be applied so as to gain further improvements from a computational efficiency viewpoint. The overall result is a highly efficient interconnect model based on measured data.

## 1. INTRODUCTION

With ever-increasing signal speeds and the ever-shrinking feature size of modern integrated circuits, accurate simulation of interconnect networks is essential for the reliable design of high-speed circuits for microwave, RF and mixed-signal applications. Interconnect effects include signal delay, distortion, ringing and reflection and can lead to logic signal corruption and degradation of system performance. Consequently, the transient simulation of transmission lines has received much attention over the past decade or so eg. [1-2] and many more. While, in the past, RC approximations were deemed adequate for predicting interconnect effects, the current signal speeds are such that distributed circuit behaviour must be taken into account when modelling interconnect networks. Furthermore, the complexity of the interconnect geometry is now such that there is a need for models based on measured data or data from full-wave EM simulators. Such data would typically consist of the scattering or  $y$ -parameters of the network in question.

Against this background, the present contribution is concerned with the development of an efficient approach for the simulation of interconnect networks based on measured or simulated terminal port data. The proposed method involves a two-stage approach. Initially,  $z$ -domain approximations of the given data are developed using a novel resonant model structure [3-5] and from these a state-space formulation is determined. The second stage involves the application of a model reduction strategy to obtain a reduced-order model that is both highly efficient and accurate. The paper presents an example to confirm the efficacy of proposed technique.

## 2. REVIEW OF FORMATION OF RESONANT MODEL

The behaviour of a transmission line, assuming TEM mode of propagation, is described by the Telegraphers Equations:

$$\frac{dV(x)}{dx} = -Z(x)I(x) \quad \frac{dI(x)}{dx} = -Y(x)V(x) \quad (1)$$

Integrating these equations and discretising as described in detail in [3] owing to the absence of an analytical solution yields:

$$V(k) = V(K) + \sum_{j=k+1}^K Z_{aj} I'(j) \quad I'(k) = I'(K+1) + \sum_{j=k}^K (Y_{b,j+1} + Y_{cj}) V(j) \quad (2)$$

Discretisation corresponds to dividing the transmission line into  $K$  multi-terminal  $\pi$  sections.

Eqn. 2 is rearranged into the following form:

$$\begin{bmatrix} I_B \\ V' \end{bmatrix} = \begin{bmatrix} A_1 + A_2 & B \\ C & D \end{bmatrix} \begin{bmatrix} V_B \\ V' \end{bmatrix} \quad \text{or} \quad I_B = \{A_1 + A_2 + B(1-D)^{-1}C\} V_B \quad \text{and} \quad V' = (1-D)^{-1} C V_B \quad (3)$$

where

$$V_B = \begin{bmatrix} V_S \\ V_R \end{bmatrix} \quad \text{and} \quad I_B = \begin{bmatrix} I_S \\ -I_R \end{bmatrix} \quad \text{are the boundary voltages and currents.}$$

$V' = [V(1) \ V(2) \ \dots \ V(K-1)]^*$  (\*=transpose) are the voltages at the internal nodes.

The  $A_1 A_2 B C D$  matrices are as defined in [3].

With a view to obtaining an efficient transmission-line model, the matrix  $D$  is diagonalised:

$$D = Q \alpha Q^{-1} \quad (4)$$

where  $\alpha$  is a diagonal matrix the elements of which comprise the eigenvalues of  $D$ .

Upon substitution of (4), eqn. 3 become:

$$I_B = \{A_1 + A_2 + P \zeta g P_i\} V_B \quad (= Y_B V_B) \quad V' = Q g P_i V_B \quad (5)$$

where

$$P = [p_1 \ p_2 \ \dots \ p_n] = [Q^{-1} C]_t \quad g = (1 - \alpha)^{-1} \quad \text{and} \quad \zeta_i = \frac{p_{ii} x_i}{p_{ii} p_i}$$

$p_i$  is the  $i$ th column of  $P$ .  $x_i$  is the  $i$ th column of  $X$  where  $X = [x_1 \ x_2 \ \dots \ x_n] = BQ$

Eqn. 5 defines the resonant transmission-line model.

Eqn. 5 may also be written as:

$$I_B = \left\{ A_1 + A_2 + \sum_{i=1}^n \zeta_i g_i H_i \right\} V_B \quad (6)$$

where  $\zeta_i$  and  $g_i$  are scalars and  $H_i = p_i (p_i)_t$ .  $H_i$ , for most practical cases, is independent of frequency. This particular format is suitable for the tuning procedure that follows.

For conversion to the time domain, it is necessary to examine the nature of the individual model components and to identify the frequency-dependent elements. The diagonal matrix  $g$  is the kernel of the transmission-line model. Its elements identify the *natural resonances or resonant modes* of the transmission line.

Each element of  $g$  is approximated with an ARMA model of the form:

$$g_k(z) = \frac{a_{1k} z^{-1} + a_{2k} z^{-2}}{1 + b_{1k} z^{-1} + b_{2k} z^{-2} + b_{3k} z^{-3}} \quad (7)$$

The elements of the diagonal matrix  $\zeta$  and the elements of the  $A (= A_1 + A_2)$  matrix are also frequency dependent and are also approximated using low-order  $z$ -transfer functions.

At the conclusion of the linear modelling procedure, the transmission-line model is in the following form:

$$\begin{bmatrix} I_S(z) \\ -I_R(z) \end{bmatrix} = [Y_B(z)] \begin{bmatrix} V_S(z) \\ V_R(z) \end{bmatrix} \quad (8)$$

This format translates directly to the time domain yielding:

$$\begin{bmatrix} i_S \\ -i_R \end{bmatrix}^t = \begin{bmatrix} y_{B11} & y_{B12} \\ y_{B21} & y_{B22} \end{bmatrix} \begin{bmatrix} v_S \\ v_R \end{bmatrix}^t + \begin{bmatrix} i_{his1} \\ i_{his2} \end{bmatrix}^t \quad (9)$$

where the elements of the  $y_B$  matrix are determined from the coefficients of the ARMA models and the history currents are dependent only on past values of the terminal voltages and currents.

### 3. TUNING TO MEASURED DATA

The structure of the transmission-line model described in Section 2 is such that it is readily amenable to tuning to fit measured admittance results. An initial model is developed based on a rough estimate of the transmission-line parameters (or simply an arbitrary estimate of the resonant model parameters). Owing to the structure of the new model, the regions of the frequency spectrum in which particular terms dominate are known. For example, the first term in the summation in (6) dominates in the region around the resonant frequency of the first mode. Hence, only the terms dominant in a frequency region need to be tuned in that region. The tuning process involves adjustment of the coefficients of the approximating functions used in the initial time-domain model for the elements of the  $\zeta$  and  $A_1$  and  $A_2$  matrices and the systematic adjustment of the poles of the approximating ARMA transfer functions for the  $g_i$ . Adjustment of the angle of the poles alters the location of the resonances while adjustment of the radius of the poles alters the damping. Standard optimisation procedures can be used to fine-tune the process. For example, Fig. 1a shows an initial amplitude spectrum superimposed on a simulated measured result. Fig. 1b shows the result after tuning.

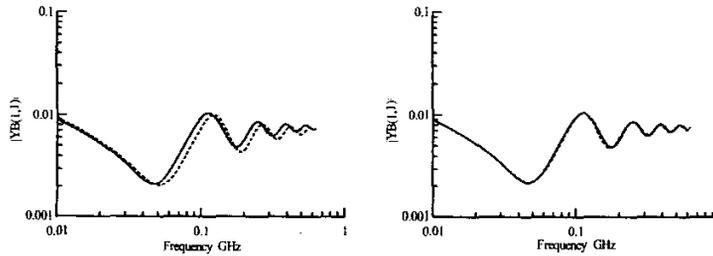


Fig. 1a Estimated amplitude spectrum superimposed on exact result

Fig. 1b Amplitude spectrum from tuned model superimposed on exact result

### 4. STATE-SPACE FORMULATION AND MODEL REDUCTION

While the individual ARMA models for each element in the tuned transmission-line model obtained as described in Section 3 are of low order, the overall order of the elements of  $Y_B(z)$  in eqn. 8 may be quite high. Consequently, this section suggests a strategy for reducing the order of the model significantly thereby obtaining significant gains in efficiency. The first step involves rearranging eqn. 9 into the form of a state-space representation. I.e.

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k) & F \in \mathfrak{R}^{n \times n}, x, G \in \mathfrak{R}^n, u \in \mathfrak{R} \\ y(k) &= Hx(k) & y \in \mathfrak{R}, H \in \mathfrak{R}^n \end{aligned} \quad (10)$$

The common techniques such as canonical controllability and canonical observability realisations for conversion to a state-space are abandoned for a different conversion

process for the purposes of avoiding problems that may arise if the  $F$  matrix in eqn. 10 is poorly scaled. The approach adopted is that described in [5].

Having formed a well-conditioned state-space realisation, the second step in forming a reduced-order transmission-line model is to apply a standard model reduction technique such as the Lanczos process.

## 5. RESULTS

Fig. 2a shows the results from a tuned transmission-line model for a typical interconnect system [6] superimposed on an exact frequency-domain transient result. Fig. 2b shows the corresponding result with the reduced-order model formed as described in Section 4. While major gains in efficiency have been achieved (order reduction of  $\sim 75\%$ ), the accuracy has not been compromised.

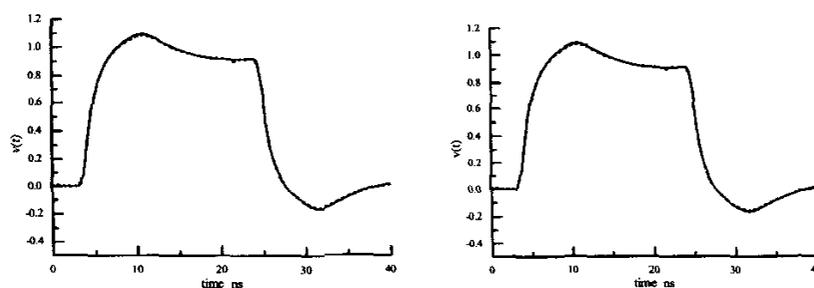


Fig. 2a Receiving-end voltage with full model

Fig. 2b Receiving-end voltage with reduced model

## 6. CONCLUSIONS

The paper has presented a strategy for modelling interconnects in high-speed circuits and systems. The strategy comprises two main parts. The first involves tuning an initial approximate resonant model to measured data using a standard tuning technique. The result of this process is an accurate transmission-line model but one which may have a very high order in terms of the overall  $z$ -domain rational functions approximating the  $y$  or  $s$  parameters. To overcome this, the second part of the strategy involves converting the given model to a state-space formulation and applying a standard model-reduction technique such as the Lanczos process. This greatly improves the model from an efficiency viewpoint. The overall result then is a highly efficient model for interconnects.

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