Efficient transient simulation of transmission lines

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Abstract – **The paper focuses on revealing the salient structural aspects of a new transmission-line model with a view to exploiting them for gains in efficiency and accuracy. The new transmission-line model has as its basis the Telegraphers Equations but the manner of solution is what distinguishes the new approach from existing transmission-line simulation techniques. The technique is based on identifying natural modes of oscillation on the transmission line. The result is a model structure which can be tailored to the accuracy requirements of a simulation and which is amenable to tuning to fit measured admittance data.**

I. INTRODUCTION

With the ever-increasing interconnection densities and clock frequencies in modern digital circuits, interconnect effects such as ringing, reflection, delay and distortion can lead to serious degradation of transmitted signals. As a result, significant effort has been invested in developing accurate and efficient algorithms for including the distributed behaviour of interconnects in circuit simulators such as SPICE[1]. For example, [2-8] are just some of the many contributions to the field of research. However, for the most part, the models developed have been based on the travelling-wave solution of the Telegraphers Equations. In [9-11], the author presented an approach which differs from the travelling-wave solution. It is centred on modes of oscillation on the transmission line as opposed to modes of wave propagation. The advantage of this approach, as seen in [9-11], is that the resultant model structure is such that it allows ease of conversion to the time domain. In this paper, the structure of the model is examined and clarified with a view to obtaining increased gains in efficiency and accuracy. In particular, model reduction is possible through the concept of modal elimination. In addition, the technique is such that it is amenable to tuning to fit measured frequency-domain admittance results.

II. MATHEMATICAL MODEL

Consider a multiconductor transmission line. Assuming TEM mode of propagation, the behaviour of the transmission line is described in the frequency domain by the Telegraphers Equations:

$$\frac{d\boldsymbol{V}(x)}{dx} = -\boldsymbol{Z}(x)\boldsymbol{I}(x) \tag{1}$$

$$\frac{d\boldsymbol{I}(x)}{dx} = -\boldsymbol{Y}(x)\boldsymbol{V}(x) \tag{2}$$

where $\boldsymbol{V}(x)$ and $\boldsymbol{I}(x)$ are vectors of dimension equal to the number of conductors.

Eqns. (1) and (2) are first integrated to yield:

$$\boldsymbol{V}(x) = \boldsymbol{V}(l) + \int_x^l \boldsymbol{Z}(\tau)\boldsymbol{I}(\tau)d\tau \tag{3}$$

$$\boldsymbol{I}(x) = \boldsymbol{I}(l) + \int_x^l \boldsymbol{Y}(\tau)\boldsymbol{V}(\tau)d\tau \tag{4}$$

where l is the length of the transmission circuit.

The solution to (3) and (4) is obtained on a discrete basis owing to the absence of an analytical solution. The transmission line is therefore divided into K sections where each section is represented by an exact equivalent-π network as shown in Fig. 1.

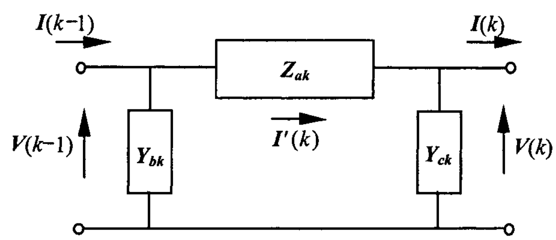

Fig. 1 Multi-terminal exact equivalent-π representation of kth section

Eqns. (3) and (4) may now be rewritten as

$$\boldsymbol{V}(k) = \boldsymbol{V}(K) + \sum_{j=k+1}^{K} \boldsymbol{Z}_{aj}\boldsymbol{I}'(j) \tag{5}$$

$$\boldsymbol{I}'(k) = \boldsymbol{I}'(K+1) + \sum_{j=k}^{K} (\boldsymbol{Y}_{b,j+1} + \boldsymbol{Y}_{cj})\boldsymbol{V}(j) \tag{6}$$

Eqns. (5) and (6) are rearranged into the following form:

$$\begin{bmatrix} \boldsymbol{I_B} \\ \boldsymbol{V'} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_B} \\ \boldsymbol{V'} \end{bmatrix} \tag{7}$$

where

$$V_B = \begin{bmatrix} V_S \\ V_R \end{bmatrix} \quad I_B = \begin{bmatrix} I_S \\ -I_R \end{bmatrix} \quad V' = \begin{bmatrix} V(1) \\ V(2) \\ \vdots \\ V(K-1) \end{bmatrix}$$

The *ABCD* matrices are as defined in [9].

Eqn. (7) could be solved to give:

$$I_B = \{A + B(1-D)^{-1}C\}V_B \tag{8}$$

$$V' = (1-D)^{-1}CV_B \tag{9}$$

Instead, however, with a view to obtaining an efficient transmission-line model, the matrix $\boldsymbol{D}$ is diagonalised:

$$D = Q\alpha Q^{-1} \tag{10}$$

α is a diagonal matrix the elements of which comprise the eigenvalues of $\boldsymbol{D}$.

Upon substitution of (10), eqns. (8) and (9) become:

$$I_B = \{A + P\zeta g P_t\}V_B \quad (=Y_B\, V_B) \tag{11}$$

$$V' = QgP_tV_B \tag{12}$$

where

$$P = [p_1 \;\; p_2 \cdots p_n] = [Q^{-1}C]_t$$

$$g = (1-\alpha)^{-1} \quad \text{and} \quad \zeta_i = \frac{p_{it}x_i}{p_{it}p_i}$$

p_i is the ith column of $\boldsymbol{P}$. x_i is the ith column of $\boldsymbol{X}$ where

$$X = [x_1 \;\; x_2 \;\cdots\; x_n] = BQ$$

Eqns. (11) and (12) define the new transmission-line model which is shown in block diagram format in Fig. 2.

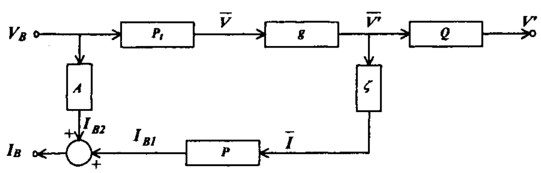

Fig. 2 Multiconductor transmission-line model

It is important to note that this model gives exactly the same results as the travelling-wave model in the frequency domain. There is no approximation involved as each section is represented by an exact-π network.

III. Model structure

The transmission line is represented, relative to its boundary terminals, by (11), the admittance equation. Since ζ and g are diagonal, this equation can be decomposed into the following form:

$$I_B = \left\{A + \sum_{i=1}^{n} \zeta_i g_i H_i\right\}V_B \tag{13}$$

where ζ_i and g_i are scalars and $H_i = p_i(p_i)_t$. H_i, for most practical cases, is independent of frequency.

Thus, it is seen that the admittance matrix of the transmission line is formed from the summation of n+1 terms. The crucial part of the model lies within the summation sign. The physical significance of the individual terms in this summation will now be outlined to indicate the possibilities for obtaining increased efficiencies in the modelling technique.

To clarify the structure of the new transmission-line model, the definitive case of a single-conductor lossless transmission line is examined. The essential nature of the individual terms is retained on inclusion of frequency-dependent losses. Details of the transmission line are given in the Appendix. The line is divided into K=4 sections each of length l_s.

Analysis as described in Section II is carried out. For a lossless line, the $\boldsymbol{D}$ matrix in (10) has the following form:

$$D = 4\sin^2\left(\frac{\omega l_s\sqrt{LC}}{2}\right)R \tag{14}$$

$$R_{ij} = \frac{(K-i)}{K}j \quad i \geq j$$

and

$$R_{ij} = \frac{(K-j)}{K}i \quad j > i$$

Since $\boldsymbol{R}$ is symmetrical, it can be diagonalised as:

$$R = Q\beta Q_t \tag{15}$$

where $\boldsymbol{Q}$ is the purely real matrix of eigenvectors of $\boldsymbol{R}$ and β is the diagonal matrix comprising the eigenvalues of $\boldsymbol{R}$.

Returning to eqn. (13), each element, g_i (known as a modal transfer function), may now be identified as:

$$g_i = \frac{1}{1 - 4\beta_i \sin^2\left(\frac{\omega l_s\sqrt{LC}}{2}\right)} \tag{16}$$

From (16), it follows that resonances occur at

$$\omega_i = \frac{2}{l_s\sqrt{LC}}\sin^{-1}\left(\frac{1}{2\sqrt{\beta_i}}\right) \tag{17}$$

Values for ω_i for the illustrative case are given in the Appendix. From the given values, it is clearly evident that ω_1 corresponds to the first short-circuit natural resonant frequency of the transmission line, i.e. $= 2\pi/(2l\sqrt{LC})$. ω_2 corresponds to the second short-circuit natural resonance and so on.

Hence, it is clearly evident that the given technique involves decomposing the solution of the Telegraphers Equations into a set of modal components each of which corresponds to a short-circuit natural frequency of the transmission line.

The first near-singularity in the amplitude spectrum of the elements of ζ defines a folding frequency, f_n, which is the bandwidth of the subsequent time-domain model (All frequency-dependent elements are accurately modelled up to this frequency). For a lossless transmission line, the folding frequency is given by:

$$f_n = \frac{1}{2\sqrt{LCl_s}} \tag{18}$$

Thus, the bandwidth of a given model is governed by the choice of section length.

IV. TIME-DOMAIN IMPLEMENTATION

Details of time-domain implementation are given in [9] where a coupled transmission line inclusive of frequency-dependent losses is modelled.

In summary, development of the time-domain model involves approximating the elements of the $\boldsymbol{g}$ matrix, the elements of the ζ matrix and the elements of the $\boldsymbol{A}$ matrix with low-order ARMA functions.

Fig 3 is a sample time-domain result taken from [9]. It shows the near-end voltages on a transmission line in response to the input of a digital pulse. As evidenced by this result, minimal accuracy is lost in the conversion process to the time domain.

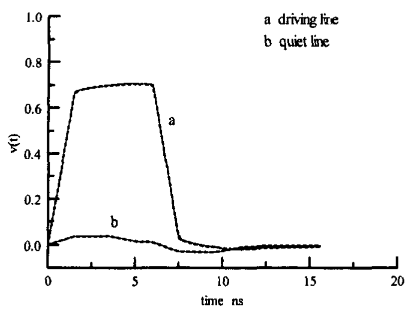

Fig. 3 Time-domain result – full model

V. MODAL DELETION

Having established that the new technique involves a superposition of the effects of individual natural resonant oscillations, it is possible to arrive at the conclusion that model reduction techniques can be applied. For example, if, in a particular simulation, the bandwidth required for an interconnect model is less than the resonant frequencies of the highest modes, then it seems reasonable to neglect these modes. Suppression of the j^{th} mode is achieved simply by neglecting the j^{th} term in the summation in (13). In matrix terms, this corresponds to deleting the j^{th} column of $\boldsymbol{P}$ (and the j^{th} row of $\boldsymbol{P}_t$), deleting the j^{th} column of $\boldsymbol{Q}$, and deleting the j^{th} rows and columns of the ζ and $\boldsymbol{g}$ matrices.

Fig. 4 shows the result corresponding to Fig. 3 with the two highest modes neglected (a saving of >10% in computational effort). Thus, the computational requirements of any given time-domain model can be tailored to match the performance requirements of a particular simulation.

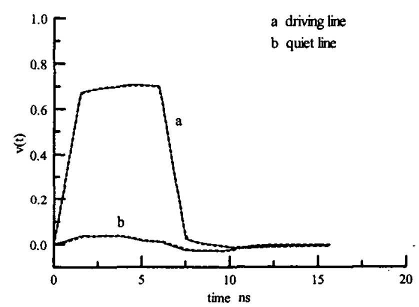

Fig. 4 Time- domain result – reduced model

VI. TUNING OF MODELS

The structure of the transmission-line model is such that it is readily amenable to tuning to fit measured admittance results. An initial model is developed based on an estimate of the transmission-line parameters. Owing to the structure of the new model, the regions of the frequency spectrum in which particular terms dominate are known. For example, the first term in the summation in (13) dominates in the region around the resonant frequency of the first mode. Hence, only the terms dominant in a frequency region need to be tuned in that region. The tuning process involves adjustment of the coefficients of the approximating functions used in the initial time-domain model for the elements of the ζ and $\boldsymbol{A}$ matrices and the systematic adjustment of the poles of the approximating ARMA transfer functions for the g_i.

Adjustment of the angle of the poles alters the location of the resonances while adjustment of the radius of the poles alters the damping. Standard optimisation procedures [12] can be used to fine-tune the process. For example, Fig. 5 shows an initial amplitude spectrum superimposed on a simulated measured result. Fig. 6 shows the result after tuning.

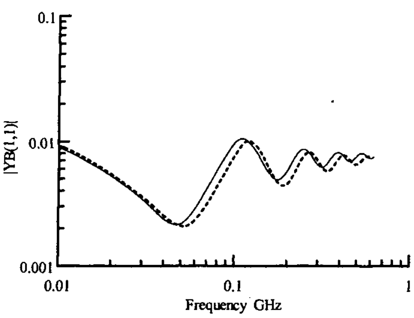

Fig. 5 Estimated amplitude spectrum superimposed on exact result

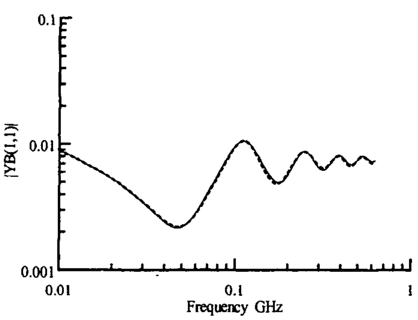

Fig. 6 Amplitude spectrum from tuned model superimposed on exact result

VII. Conclusions

The paper has emphasised the structural features of a new transmission-line model. In particular, it is shown that the technique is based on modal decomposition into natural resonances. The number of modes included in a simulation can be adjusted to match the accuracy requirements of a simulation. Furthermore, the resonant structure is particularly suited to tuning to fit measured admittance data.

References

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Appendix

Lossless transmission line parameters

L=539nH
C=39pF
Length=0.635m

Table A1: Resonant frequencies of transmission line

ω_i GHz
1.08
2.16
3.24