An Efficient Wavelet-based Nonlinear Circuit Simulation Technique with Model Order Reduction

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Abstract: This paper proposes further improvement to a novel method for the analysis and simulation of ICs recently proposed by the authors. The circuits are assumed to be subjected to input signals that have widely separated rates of variation, e.g. in communication systems an RF carrier modulated by a low-frequency information signal. The previously proposed technique enables the reuse of the results obtained using a lower-order accuracy model to calculate a response of higher-order accuracy model. In this paper, the efficiency of this method is further improved by using a nonlinear model order reduction technique. Results will highlight the efficiency of the proposed approach.

1. INTRODUCTION

Harmonic Balance [1]-[3] and Time-Domain Integration [4] are the two most widely employed circuit simulation techniques in circuit simulators for the analysis of high-frequency nonlinear circuits. However, since Harmonic Balance (HB) is most effective for periodic or quasi-periodic steady-state analysis of mildly nonlinear circuits and Time-Domain Integration (TDI) is only practical for baseband systems, they prove to be very limited when dealing with the systems that are subject to transient high-frequency signals or complex modulated RF carriers. As a result, several Envelope Transient Analysis approaches have recently been proposed whereby a mixed-mode technique is implemented [5]-[6]. The essence of these approaches is that the slowly varying envelope of a signal is treated by TDI and that HB treats the high-frequency carrier. However, existing techniques have limitations, for example, restrictions in the bandwidth of the excitation signal [5] and the limitations of HB with respect to strong nonlinearities. Roychowdhury in [7] proposes converting the differential-algebraic equations that describe the circuit to multi-time partial differential equations (MPDE), and applying time-domain methods directly to solve the resultant systems. Pedro [8] also employs a MPDE approach, but uses a combination of Harmonic Balance and Time-Domain Integration to solve the resultant system.

As microprocessor clock speeds continue to rise towards the 10 GHz range and the physical size of transistors has reached nanometre levels, accurate analysis and simulation of integrated circuits (IC) has become computationally expensive both in terms of time and computer resources. However, in order to ascertain the signal integrity of an IC, the designer needs to perform numerous simulations before settling on a final design. Any change in the requirements for the circuit design will necessitate the simulation process to restart from the beginning. The overall result is a prolonged design cycle that is economically unacceptable. Hence, there is a need for a simulation technique that enables

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the designer to obtain the circuit response with the desired accuracy and within a reasonable time-frame.

The technique presented in this paper is a variation and improvement of the approach presented by authors in [9]–[11]. A modification of the wavelet-based collocation approach proposed by Cai and Wang in [12] forms the core of the technique and unlike Christoffersen and Steer [13], the cubic spline wavelet basis is employed to solve the MPDE representation of the system rather than the original ODE representation.

The rest of the paper is organized as follows. Sections 2 and 3 briefly describe the method recently proposed by the authors [11] for obtaining a higher-degree accuracy approximation incorporating results from a lower-degree accuracy approximation. Section 4 further extends this contribution incorporating the nonlinear Model Order Reduction (MOR) technique to further improve the efficiency of the proposed method. Finally, simulation results obtained for sample circuits using the proposed technique are presented in Section 5.

2. EFFICIENT CALCULATION OF LOWER-DEGREE APPROXIMATION

Consider a signal $x(t)$ that is composed of a carrier modulated by an envelope where the envelope signal is assumed to be uncorrelated with the carrier. The signal may be represented in two independent time variables as $x(t)=\hat{x}(t_1,t_2)$ where t_1 relates to the low-frequency envelope and t_2 relates to the high-frequency carrier.

Now consider a general nonlinear circuit described by:

$$\dot{x}(t)=f(\mathbf{x}(t))+\boldsymbol{b}(t) \tag{1}$$

where $\boldsymbol{b}(t)$ is the excitation vector, $\boldsymbol{x}(t)$ are the state variables and f is a nonlinear function. The corresponding MPDE system can be written as shown in [7] as:

$$\frac{\partial\hat{x}}{\partial t_1}+\frac{\partial\hat{x}}{\partial t_2}=f(\hat{x}(t_1,t_2))+\hat{\boldsymbol{b}}(t_1,t_2) \tag{2}$$

This MPDE is then solved using a pseudo-wavelet collocation method derived from that proposed by Cai and Wang [12] and suggested by the authors in [10]. The technique involves approximating the unknown function $\hat{x}(t_1,t_2)$ with a wavelet series in the t_2 dimension, i.e.

$$\begin{aligned}\hat{x}_J(t_1,t_2)&=\bar{x}_{-1,-3}(t_1)\eta_1(t_2)+\bar{x}_{-1,-2}(t_1)\eta_2(t_2)+\bar{x}_{-1,-1}(t_1)\varphi_b(t_2)+\sum_{k=0}^{L-4}\bar{x}_{-1,k}(t_1)\varphi_k(t_2)+\\&+\bar{x}_{-1,L-3}(t_1)\varphi_b(L-t_2)+\sum_{j=0}^{J-1}\sum_{k=-1}^{n_j-2}\bar{x}_{j,k}(t_1)\psi_{jk}(t_2)+\bar{x}_{-1,L-2}(t_1)\eta_2(L-t_2)+\bar{x}_{-1,L-1}(t_1)\eta_1(L-t_2)\\&=\sum_{k=1}^{N}\bar{x}_k(t_1)\Psi_k(t_2)\end{aligned} \tag{3}$$

where t_2 is scaled such that $t_2\in[0, L]$, $L>4$. $\phi(t)$ and $\psi(t)$ are scaling and wavelet functions respectively and $\eta(t)$ are spline functions introduced to approximate boundary-nonhomogeneities as described in [12]. $\bar{x}(t_1)$ are the unknown coefficients which are a function of t_1 only. The total number of unknown coefficients is $N=2^J L+3$ where J determines the level of wavelet coefficients to be taken into account when approximating

$\hat{x}(t_1,t_2)$. For simplicity, from this point forward, $\Psi_k(t)$ shall be referred to as wavelets where it is understood that these comprise the scaling functions, $\varphi(t)$, the wavelet functions, $\psi(t)$ and the nonhomogeneity functions, $\eta(t)$.

As proposed by the authors in [9], (2) is then collocated in t_2 on collocation points depicted in [12] to result in a semidiscretised ordinary differential equation (ODE) system:

$$\boldsymbol{E}\frac{d\bar{\boldsymbol{x}}}{dt_1} = -\boldsymbol{D}\bar{\boldsymbol{x}} + \boldsymbol{f}_N(\bar{\boldsymbol{x}}) + \boldsymbol{b}_N. \tag{4}$$

$\bar{x}(t_1)$ is an N-dimensional column vector of the unknown wavelet coefficients at the collocation points in t_2 at a specific instant in t_1. Both $\boldsymbol{E}$ and $\boldsymbol{D}$ are constant N-dimensional square matrices whose columns comprise the values of the N wavelet functions, $\Psi_k(t_2)$, and the derivatives of the wavelet functions in (3) respectively, at the N collocation points. Both the $\boldsymbol{E}$ and $\boldsymbol{D}$ matrices are evaluated once at the outset of the algorithm. $\boldsymbol{f}_N$ and $\boldsymbol{b}_N$ are column vectors comprising the values of f and b at the collocation points.

In order to facilitate an efficient solution of the system of ODEs described by (4), the nonlinear MOR technique is used before actually employing an ODE solver. The particular model reduction strategy chosen is based on that presented by Gunupudi and Nakhla in [14] and its adaptation is fully described in [10]. Applying this MOR technique yields a new reduced equation system:

$$\hat{\boldsymbol{E}}\frac{d\hat{\bar{\boldsymbol{x}}}}{dt_1} = -\hat{\boldsymbol{D}}\hat{\bar{\boldsymbol{x}}} + \boldsymbol{Q}^T\boldsymbol{f}_N(\boldsymbol{Q}\hat{\bar{\boldsymbol{x}}}) + \hat{\boldsymbol{b}}_N \tag{5}$$

where $\hat{\boldsymbol{E}} = \boldsymbol{Q}^T\boldsymbol{E}\boldsymbol{Q}$, $\hat{\boldsymbol{D}} = \boldsymbol{Q}^T\boldsymbol{D}\boldsymbol{Q}$ and $\hat{\boldsymbol{b}}_N = \boldsymbol{Q}^T\boldsymbol{b}_N$. Matrix $\boldsymbol{Q}$ is obtained from orthogonal decomposition of a Krylov subspace formed for coefficients of an expansion of the vector of coefficients, $\bar{x}(t_1)$, in a Taylor series

This new system, (5), of dimension q may then be solved to determine $\hat{\bar{x}}$ over the entire time domain of interest. Once the q coefficients, $\hat{\bar{x}}$, have been determined, $\bar{x}(t_1)$ and consequently the circuit response may be obtained in one single post-processing step. It is shown [10] that the above solution process is significantly more efficient than solving directly for coefficients at each time step as was done in [9].

3. FORMATION OF AN APPROXIMATION OF HIGHER-DEGREE ACCURACY

Assume that a preliminary circuit response is obtained by applying the technique presented in Section 2. If now a response with a higher degree of accuracy is required, the wavelet series approximating the unknown function, $\hat{x}(t_1,t_2)$, can be expanded for another layer, i.e.

$$\hat{x}_{J_1}(t_1,t_2) = \sum_{k=1}^{N_1} \bar{x}_k(t_1)\Psi_k(t_2) \tag{6}$$

where $J_1=J+1$ and the total number of unknown coefficients is now $N_1 = 2^{J_1}L + 3$. The size of the ODE system to be solved is increased to $N_1 = 2^{J_1}L + 3 = 2^{J+1}L + 3$ and consequently the computational requirements for obtaining the required solution are also increased. Although the method proposed by the authors in Section 2 may be implemented from scratch to obtain the circuit response, it is shown [11] that, due to the multiresolution nature of wavelet expansions, it is possible to greatly increase the efficiency of the calculations by reusing already calculated values from a lower-degree accuracy (LDA) result to obtain the coefficients for a higher-degree accuracy (HDA) response.

After some simple mathematical transformations [11] the wavelet series approximating the unknown function, $\hat{x}(t_1,t_2)$, at level J_I can be written as:

$$\hat{x}_{J_1}(t_1,t_2)=\sum_{k=1}^{N}\bar{c}_k(t_1)\Psi_k(t_2)+\sum_{m=1}^{M}\bar{g}_m(t_1)\Psi_{N+m}(t_2) \tag{7}$$

where $\bar{c}_k$ denote already known values of wavelet coefficients and $\bar{g}_m$ are unknown wavelet coefficients on layer J_I. $M=N_1-N=2^J L$ is total number of unknown coefficients $\bar{g}_m$. If, now, the expression in (7), if collocated on the M collocation points of the added layer in t_2, and substituted into (2), the following ODE system is obtained [11]:

$$\boldsymbol{E}_I\frac{d\bar{\boldsymbol{g}}}{dt_I}=-\boldsymbol{D}_I\bar{\boldsymbol{g}}+\boldsymbol{F}_M(\bar{\boldsymbol{c}},\bar{\boldsymbol{g}})+\boldsymbol{B}_M \tag{8}$$

where $\bar{g}(t_I)$ is an M-dimensional column vector of the *unknown* wavelet coefficients of layer J_I. $\bar{c}(t_1)$ is an N-dimensional column vector of the *known* wavelet coefficients at the collocation points in t_2 at a specific instant in t_I and its entries are either already known directly or may be obtained as interpolated values for any time t_I. $\boldsymbol{E}_I$ and $\boldsymbol{D}_I$ are constant M-dimensional square matrices whose columns comprise the values of the M wavelet functions, $\Psi_k(t_2)$, and the derivatives of the wavelet functions evaluated at each of the M collocation points of the extra layer in t_2. $\boldsymbol{F}_M$ and $\boldsymbol{B}_M$ are column vectors whose entries are combinations of the values of f and b at the collocation points of level J_I multiplied with certain constant matrices [11]. All constant matrices are again evaluated only once at the outset of the algorithm.

4. EFFICIENT CALCULATION OF HIGHER-DEGREE APPROXIMATION

Equation (8) represents a $M\times M$ system of ODEs where the unknowns $\bar{g}$ may be readily determined using any commercially available technique. However, as the degree of accuracy is increased by one layer, the number of additional coefficients M is effectively doubled, i.e. the total number of unknowns grows as a power of two. This in turn can drastically slow down computation of higher-order accuracy circuit responses. Therefore, it is desirable to reduce the size of this $M\times M$ system of ODEs before solving it.

Indeed, if the structure of the equations (4) and (8) are compared it can be seen that they are exactly the same. Therefore, the same MOR technique used in Section 2 may readily be applied to the system (8) yielding a new reduced equation system:

$$\hat{\boldsymbol{E}}_1\frac{d\hat{\bar{\boldsymbol{g}}}}{dt_I}=-\hat{\boldsymbol{D}}_1\hat{\bar{\boldsymbol{g}}}+\boldsymbol{Q}_1^T\boldsymbol{F}_M(\boldsymbol{Q}_1\hat{\bar{\boldsymbol{g}}})+\hat{\boldsymbol{B}}_M \tag{9}$$

where $\hat{\boldsymbol{E}}_1=\boldsymbol{Q}_1^T\boldsymbol{E}_1\boldsymbol{Q}_1$, $\hat{\boldsymbol{D}}_1=\boldsymbol{Q}_1^T\boldsymbol{D}_1\boldsymbol{Q}_1$ and $\hat{\boldsymbol{B}}_M=\boldsymbol{Q}_1^T\boldsymbol{B}_M$. Again matrix $\boldsymbol{Q}_I$ is obtained from orthogonal decomposition of a Krylov subspace formed for coefficients of an expansion of the vector of coefficients, $\bar{g}(t_I)$, in a Taylor series

Thus instead of solving an M^{th} order system at each time-step to obtain the unknown state-variables, a reduced-order system of transformed coefficients is solved. The order of the reduced system q_1 is significantly less than M. Once the transformed coefficients are determined for the entire time range of interest, the additional M coefficients, $\overline{g}(t_1)$ and consequently, the value of the state variables and the output quantity $x(t)$ may be obtained in one single post-processing step. As a result, even more gains in computational efficiency are achieved as is confirmed for the sample system presented in the following section.

5. SAMPLE SYSTEM

The proposed technique has been employed for a diode rectifier circuit (Fig. 1) which is deliberately selected as it is strongly nonlinear in nature. This sample circuit is excited with the following source:

$$b(t) = sin(\frac{2\pi}{T_1}t)\,sin(\frac{2\pi}{T_2}t) \tag{10}$$

where T_1 corresponds to the envelope period and T_2 corresponds to the carrier period.

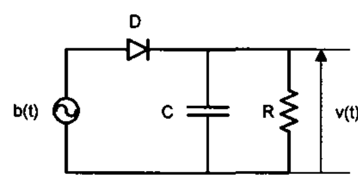

Figure 1. *Diode rectifier circuit*

Fig. 2 presents the output of the sample system involving the reuse of already calculated wavelet coefficients in lower-order accuracy model as proposed by authors in [11]. In this case, coefficients calculated at wavelet level $J=1$ are used to obtain results for HDA response ($J_1=2$). The collocation points range parameter, L, is set to $L=80$ in both instances. As reported, it took only 14% of the computing time to obtain the total HDA circuit response when compared to the time necessary to compute the total circuit response

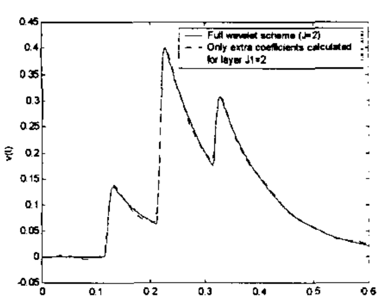

Figure 2. *Result from higher-degree technique after adding an extra layer ($J_1=2$)*

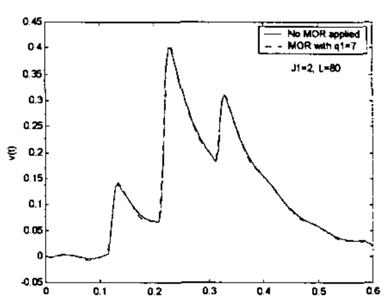

Figure 3. *Result from proposed new technique with MOR applied*

by simply restarting the full wavelet simulation scheme at the same order of accuracy ($J=2$).

Fig. 3 shows an output of the diode rectifier circuit using the enhanced technique proposed in this paper. Parameters $J_1=2$ and $L=80$ are the same as before and the system (8) is reduced to $q_1=7$. It took only 9% of overall computing time to obtain the complete solution, which represents an additional efficiency improvement of 5%. This additional gain in computational efficiency is due to the fact that reduced system (8) with only 7 unknowns is solved using a standard ODE solver and the values for all coefficients in the extra layer are obtained in single post-processing step involving only matrix multiplication.

6. CONCLUSIONS

In this paper a further improvement to the IC design technique previously proposed by the authors has been presented. The proposed technique involves three stages.

Initially, a particular order result for the circuit response is obtained using a multiresolution collocation scheme involving cubic spline wavelet decomposition. Secondly, a novel technique introduced by authors enables the reuse of the results acquired in the first stage to calculate a response of higher-order accuracy. Finally, the efficiency of this method is further improved by using a nonlinear model order reduction technique in the process of obtaining wavelet coefficients for a higher-degree approximation.

The results from highly nonlinear sample circuits indicate the efficiency and accuracy of the proposed approach.

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