Metallic Tube Type Energy Absorbers:
A Synopsis.

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Abstract.

This paper presents an overview of energy absorbers in the form of tubes in which the material used is predominantly mild steel and/or aluminium. A brief summary is also made of frusta type energy absorbers. The common modes of deformation such as lateral and axial compression, indentation and inversion are reviewed. Theoretical, numerical and experimental methods which help to understand the behaviour of such devices under various loading conditions are outlined. Although other forms of energy absorbing materials and structures exist such as composites and honeycombs, this is deemed outside the scope of this review. However, a brief description will be given on these materials. It is hoped that this work will provide a useful platform for researchers and design engineers to gain a useful insight into the progress made over the last few decades in the field of tube type energy absorbers.

Keywords: Energy absorber, tube, frusta, mild steel, aluminium, quasi-static, dynamic, ANSYS, LS-DYNA.

1 Introduction.

Due to advances in transport technology and increased spending money pertaining to this, there has been a noticeable increase in the number of transportation vehicles in society. This has resulted in a greater number of fatalities, casualties due to impact collisions of one form or another, not to mention the increase in the financial burden placed on society. Due to these associated increases, society has become more aware and concerned for the safety aspects of transportation. This has fuelled scientist/ engineers particularly in the last few decades to research and develop energy absorbers with an objective to attenuate the effect of impact on people and structures. It should be noted that energy absorbers are not only applicable to the transportation sector but also to other fields of engineering such nuclear reactors, oil-rigs and oil-tankers, crash barriers for roadsides, air-drop cargo etc.

In the early stages of engineering design, theoretical and experimental methods were the main techniques applied to analyse engineering structures and their response to different loading conditions. However, over the last decade due to the availability of increased computing power, FEA (Finite Element Analysis) as a form of computer simulation technique is being widely used in design and analysis across all branches of engineering to sole complex problems. This technique is proving to be very popular and is a powerful tool to understand the deformation mechanisms and responses of energy absorbers under impact loading. Application of this tool earlier on in the design / development stage can reduce the need to manufacture expensive prototypes for physical testing.

The objective of this paper is to provide an insight in the area of kinetic energy absorbers that are used to mitigate impact collisions. Various work developed by researchers and engineers over the last half century pertaining to metallic energy absorbers are highlighted. Such absorbers can appear in the form of circular tubes, square tubes, frusta, struts, honeycombs and sandwich plates. This work will highlight the research and findings mainly related to tubes and frusta's as impact energy absorbers.

2 Review of the Literature Available for the Study Energy Absorbers.

To acquire a sound understanding of impact phenomena in relation to transport vehicles and structures, one is required to have sound combined knowledge of related topics such as mechanics of materials, structural mechanics, impact dynamics and plasticity. Various sources of relevant literature are available which allow interested readers to gain a detailed insight in the area of energy absorbers; The International Journal of Mechanical Science which commenced publication in 1960 has many research papers related in the field of structural mechanics (buckling and deformation of metallic and composite structures under impact). The International Journal of Impact Engineering founded in 1983 is also a strong source of literature to study the response of structures and bodies subjected to dynamic loads arising from exposure to blast, collision or other impact events. A more recent journal, namely the International Journal of Crashworthiness was launched in 1996 which is devoted to the crash behaviour of vehicles, structures, materials and impact biomechanics. Relevant literature can also be found in the International Journal of Solids and Structures, International Journal of Pressure Vessels and Piping and the Journal of Thin-Walled Structures. Hardback manuscripts such as Crashworthiness of Vehicles edited by Johnson and Mamalis [1], Structural Impact and Crashworthiness edited by Morton [2], Structural Crashworthiness and Failure edited by Wierzbicki [3] provide worthwhile reading. Finally, Lu an Yu [4] published a manuscript which combined fundamental topics pertaining to energy absorbers in order to gain a sound understanding of these devices subjected to impact loading.

Zukas [5] provided two useful tables to describe the characteristics of the finite element computer code used to simulate high and low velocity impact of structures which are inherent in vehicle crashworthiness, transportation safety etc (See Table 1 and Table 2.). Ezra [6] detailed a useful overview on the assessment of energy absorbing devices for prospective use in the Aircraft industry. It was outlined that energy absorption can fall into three categories; 1) material deformation, 2) friction and 3) extrusion, in which the former is the topic of this paper. The performance of energy absorbers depends on their specific application and suitability. Various parameters such as specific energy, crush efficiency, energy efficiency are usually measured to describe the behaviour of these devices.

3 Analysis of Energy Absorbers.

The function of an energy absorber is to absorb kinetic energy upon impact and dissipate it in some other form of energy, ideally in an irreversible manner. Non-recoverable (inelastic) energy can exist in various forms such as plastic deformation, viscous energy and friction or fracture energy. Circular or square sectioned tubes are one of the most commonly used structural elements due to their prevalent occurrence and easy manufacturability. Circular tubes, for example, can dissipate elastic and inelastic energy through different modes of deformation, resulting in different energy absorption responses. Such methods of deformation include lateral compression, lateral indentation, axial crushing, tube splitting and tube inversion which is outlined in the following sections. It is important to study their energy absorption characteristics and mean crushing loads so as to determine their applicability to practical energy absorption situations. Such practical cases may consist of energy absorbers in the aircraft, automobile and spacecraft industries, nuclear reactors, steel silos and tanks for the safe transportation of solids and liquids.

3.1 The Multi Collisional Situation for Multi-Transport Bodies.

A transporting body which can be exposed to vehicular impact may be classified into five main categories of vehicle such as motor cars, aircraft/spacecraft, ships, locomotives and escalators/elevators. Such transporting bodies may be considered as a bounding envelope of a well defined outer shape containing personnel or cargo to be transported. To minimise damage to this bounding envelope and/or to the contents, ideally the relative velocity should be kept to a minimum. In a collision, it is this envelope which experiences the first collision, undergoing inadvertent plastic deformation along with some component fracture. The contents continue to move, due to Newton's first law of motion, while the bounding envelope is arrested and a short time after the impact collision, the contents will collide with the inner surface of the bounding envelope. This is termed a secondary collision. However the driver may be considered as an 'envelope and contents' in respect to protection of the human brain. Damage to the skull may be regarded as damage to the envelope and due to the semi-liquid nature of the brain and its ability to transmit stress waves, the brain contents may also undergo damage, this would be a third collision. This set of phenomena may be referred to as a three-collision situation. In conclusion, the outer surface of any moving body undergoes the primary collision and consecutively, the contents within sustain the secondary and subsequent collisions [1].

3.2 Lateral Compression / Flattening.

Several researchers have analysed analytically the compression of a tube between rigid platens and proposed a deformation mechanism to describe the lateral compression process. DeRuntz and Hodge [7] analysed the compression of a mild steel tube subjected to quasi-static lateral loading. A rigid perfectly plastic material model was used to predict the load deformation response. The geometrical component of stiffening was accounted for in the theoretical model. However the rate of increase was under-estimated, this was due to omission of the material strain hardening phenomena. The deformed contour of the tube consists of four circular arcs which maintain their original radius whilst plastic deformation occurs at the hinges only. Redwood [8] endeavoured to include the effects of material strain hardening which was omitted by DeRuntz. A rigid linear strain hardening material model as opposed to a rigid perfectly plastic model was used to predict the force and energy absorption response. The effect of strain hardening was further examined by Reid and Reddy [9]. The theoretical model produced by the authors is based on a rigid linear strain hardening material model and is the most accurate one to date. The authors improved the strain hardening prediction by replacing the localised hinges with an arc in which its length changes with deflection. Hence, this theoretical model accounted for both the geometric and material strain hardening effect. An important dimensionless parameter which was developed, governs the shape of the force-deflection curve. This parameter is defined as 'mR' and is a function of the yield stress in tension, the mean radius R of the tube, the strain hardening modulus E_p and the thickness t. According to Reid and Reddy, it may be possible to maximise the energy absorbing capacity by choosing appropriate tube dimensions such that the 'mR' value is minimised since this is a function of tube geometry.

Reddy [10] studied the phenomenon associated with the crushing of metal tubes between rigid plates. Aluminium and mild steel tubes were compressed laterally in an Instron machine. It was found that intermittently annealed tubes corresponded closely to the perfectly plastic theory. This was due to the fact that strain-hardening effects were gradually removed due to this annealing process. For the ‘as-received’ tubes and the ‘once’ annealed tube, a large discrepancy was found between those of experiment and the perfectly plastic theory. This was due to existence of strain hardening in both materials which is not accounted for in the theory.

Gupta et al [11] conducted a comprehensive experimental and computational investigation of circular metallic tubes subjected to quasi-static lateral loading. Specimens analysed consisted of both mild steel and aluminium tubes with different diameter to thickness ratios. Their corresponding force-deflection responses were obtained and examined in detail. An in depth description was provided on the deformation mechanism of a tube compressed between flat rigid platens. A quarter cross section of a typical tube was divided into zones to help describe the deformation mechanism as shown in Fig. 1.

Avallel [12] examined the strain field generated during the lateral compression of aluminium tubes and proceeded to verify the various theoretical models such as [7, 8 and 9], it was found that the latter accounted for all the main features observed experimentally, hence this model seems the most realistic in describing the actual behaviour of the tube compressed laterally both qualitatively and quantitatively.

Reddy and Reid [13] examined both theoretically and experimentally the quasi-static lateral compression of a tube constrained so that it’s horizontal diameter is prevented from increasing. This is a way of increasing the specific energy absorption capacity of the tube by introducing more plastic hinges into the structure. Also, the relationship between a single tube and a system of tubes with different configurations was investigated. It was found that the energy absorbed by a closed system (side constraints) is three times more than that of an open system (no constraints); however the maximum deflection of the former is less that that of an open system. Overall it can be concluded that the introduction of side constraints and creating a closed system is a feasible method of increasing its energy absorbing efficiency.

The compression of copper tubes between flat plates with R/t ratios from 1.5 to 7.5 was studied experimentally by Reid and Reddy [14]. It was found that the ‘Plastica’ theory developed by the authors could be used to predict the ring compression behaviour for a ring having an *R/t* value greater than 3.5. It was found that rings with an *R/t* ratio less than 3.5 did not correlate very well with the experiments, this was due to the fact that shear effects as opposed to bending effects was the dominant mode of deformation, and this is not accounted for in the aforementioned theory.

Reddy and Reid [15] proposed a method to calculate a more realistic force-deflection curve using a rigid linear work hardening material model. These tubes were also compressed laterally between rigid platens. It was suggested that an average value of strain hardening modulus could be used to calculate the parameter mR [9], therefore these two parameters would considered constant throughout the deflection range. However, it has been further proposed that if the variation of strain hardening modulus with strain is known, this could be used to update mR at each load step or load increment and thus obtaining a more realistic load-deflection characteristic. It was suggested that the method described above could be used as a basis for obtaining some of the material properties from a ring compression test.

A nested system analysed by Shrive et al [16] consisted of two concentric rings with a layer of smaller tubes between them, the axis of all tubes been parallel. Tack welding was used to attach the rings to the concentric tubes. It was found that increases in system stiffness, maximum load and energy absorption was apparent as the level of tack welding increased. From the impact loading experiment, it was found that full deformation did not occur but maximum opposing forces similar to the quasi-static case were observed.

A nested system in the form of orthogonal layers of aluminium and mild steel tubes under quasi - static lateral compression was investigated by Johnson et al [17]. Such an orthogonal layer consists of a row of tubes stacked upon each other with every second row rotated ninety degrees. The authors concluded that nested ductile tube systems play an important part in producing a *monotonic* load-deflection response. In addition to this, the systems which exhibit cracks after loading only induce oscillations into the response and do not produce catastrophic failure in the system as a whole.

Reid et al [18] investigated the role of system inertia of nested energy absorbers in the form of a line of rings subjected to impact. Dynamic test were carried out on a simple one-dimensional apparatus. This apparatus consisted of a horizontal base plate with two guide rails in which a sledge is attached and impinges the line of rings. The sledge is propelled by means of a cartridge gun which can achieve velocities in the range 30-120 m/s. Various systems of rings were experimentally tested, with changing parameters such as different materials, diameter to thickness ratios and the number of rings. They concluded that system inertia has the effect of controlling the time over which particular elements of the system deforms. Also, they concluded that the insertion of discrete masses in the form of small plates between the rings further increases the time in which the system deforms. This is seen as a more efficient use of the ring as a basic energy absorbing element.

The effect of strain rate on the dynamic lateral compression of tubes was examined by Reid and Reddy [19]. They endeavoured to find a relationship between the dynamic and quasi-static load-deformation characteristic of thin walled tubes compressed laterally between rigid platens. This was achieved by using an existing quasi-static large deflection theory developed by Reid and Reddy [9]. The theory was modified to include the effects of strain rate due to the loading rates applied. They found that the modified theory can give a good estimate of the dynamic load-deflection response and subsequently the energy absorbing capacity of mild steel and aluminium tubes can be estimated.

Nested systems in the form of a line of rings subjected to end impact loading were examined by Reid and Reddy [20]. The authors were principally concerned with identifying the main mechanism which controls the deformation of such systems. Upon experimentation, the main parameters were identified and varied, thereby leading to a suggestion for the construction of a mathematical model of the system. It was found that in low speed impact testing on tube systems, the effect of inertia was secondary; therefore the design of energy absorbing systems could be achieved provided that the material strain rate was taken into account. Reddy et al [21] described experiments in which a variety of one dimensional systems with free distal ends, as opposed to fixed ends, were subjected to lateral impact by a rigid projectile. An elastic–plastic structural shock wave theory, which employs a bilinear material model to describe the collapse behaviour of the rings, was used to analyse the deformation of typical ring chain systems.

Reid et al [22] experimentally analysed the energy absorbing capacity and collapse mechanism of braced metal tubes compressed under rigid platens. Their aim was to design an energy absorber to cope with both the ‘redirectional’ and ‘trapping’ of vehicles involved in side impacts. ‘Redirectional’ is a term used to describe where the vehicle has moved back onto its original line of travel after the collision has occurred. ‘Trapping’ involves catching the vehicle so as to prevent the probability of secondary conflicts occurring with oncoming traffic. They conducted initial investigations on small scale components to explore the possibility of achieving the desired response by introducing tension members across the diameters of the mild steel tubes. Both single and double braced tubes of various angles were analysed. They stated that the response of a braced tube, whether singly or doubly, is sensitive to the direction of loading, however, significant enhancement in the energy absorbing capacity of such systems can be achieved. Full scale testing was also carried out on double braced tubes used as a cluster in a modular crash cushion.

The finite element computer simulation of the lateral compression of aluminium tubes was conducted and analysed by Leu [23]. An elastic-plastic model based on the updated Lagrangian algorithm was employed to predict the buckling, punch load and deformed geometries of aluminium tubes. The static explicit approach was used as opposed to the implicit method so as to avoid convergence problems. A power law relationship was used to represent the stress-strain curve of the aluminium tubes. The effect of the strain hardening exponent, friction coefficient values, elastic modulus, thickness of tube were examined in relation to the deformation process and how these parameters affect the occurrence of buckling and punch load. It was concluded that such analysis of these parameters may help to understand the buckling mechanism of aluminium and clad tubes.

Gupta and Ray [24] studied the collapse of thin walled empty and filled square tubes under lateral loading compressed between rigid plates. Various sizes of tubes were analysed both in their 'as-received' and 'annealed' conditions. The filler material was in the form of polyurethane foam and Kail wood. A theoretical analysis was presented to compute the peak load and load-deflection responses. Excellent agreement was found between the predicted and actual measured values. As expected, the inclusion of a filler material increases the specific energy absorbing capacity. Moreover, the major advantage of using this material is that the post collapse load-deflection response increases without increasing the collapse load. Another advantage as reported by the author was that the stroke length increases due to the filler material which delayed the onset of locking until a later displacement was reached.

In view of how the effect of strain hardening becomes more important in impact situations due to high strain rates and large deformations, Sherbourne [25] analysed theoretically the compression of tubes under rigid platens using the 'Moving Hinge Method'. Although this theory is mainly applied using a rigid-perfectly plastic material model, the author attempted to include the effects of strain hardening into the model. It was found that the load-deflection results were in good agreement with experimental data. Also, in comparison to other theories such as 'Limit Analysis' by DeRuntz [7] and the 'Plastica Theory' by Reid [9] in predicting the collapse load, it was found that the 'Moving Hinge Method' allows for more manoeuvrability without compromising the accuracy of solution. Finally, it was noted that the 'Moving Hinge Method' can also be applied to other deformation modes such as axial compression of tubes. This appears to be an advantage of the theory since the other methods used are only applicable to tubes compressed laterally.

Wu and Carney [26] analytically analysed the initial collapse of braced elliptical tubes under later compression. Elliptical tubes provide a distinct advantage over their original circular tube counterparts in that their crush efficiency is greater. This automatically implies that the specific energy absorbing capacity of such devices increases, which is a desirable feature in the design of impact attenuation devices. Another method of increasing the specific energy absorbing capacity of these cylindrical or elliptical devices is through the inclusion of metallic braces or wire. These can be attached to the devices either horizontally or at an angle. This generates a larger collapse load and hence the specific energy absorbing capacity is increased. Three possible collapse mechanisms for braced circular tubes as established by Reid [22] have also been found to exist for braced elliptical tubes. These three cases are tubes with small bracing angles, tubes with horizontal bracing and tubes with large bracing angles. It was found that the initial collapse load of braced elliptical tubes was dependant on both the ratio of the ellipse axes b/a, and the bracing angle. ABAQUS, a finite element software was used to capture the deformation mechanisms and the magnitude of the initial collapse load of the devices. Fig. 2 shows the symmetric and asymmetric deformations for a 0^0 braced elliptical tube.

In a companion paper, Wu and Carney [27], presented the experimental results of braced elliptical tubes compressed laterally under rigid platens. This was to authenticate the numerical and theoretical results presented in their initial paper. It was found that for a ratio of b/a = 1 (Circular tubes), the initial collapse loads predicted from the EST method (Equivalent Structure Technique), ABAQUS and experiments were comparable, particularly for small-angle braced tubes. However, for large brace angles, the experimental result were below theoretical predictions and according to the authors, was due to the tubes being highly sensitive to geometrical imperfections. Finally, it was noted that the ABAQUS results were much lower than the results predicted by the EST method and this appears to be due to the omission of membrane stresses in the latter case. Fig. 3 shows the finite element model at two stages of displacement.

Similar to the above author’s work, Olabi et al [28] analysed both experimentally and numerically the dynamic lateral compression of elongated circular tubes (oblong tubes). The dynamic lateral crushing of mild steel (DIN 2393) nested tube systems was conducted using a ZWICK ROELL impact tester. The tests were performed with impact velocities ranging between 3 and 5 m/s, achieved using a fixed mass impinging onto the specimens under the influence of gravity as shown in Fig. 4. The various nested tube systems consisted of one standard and one optimised design. Their crushing behaviour and energy absorption capabilities were obtained and analysed. It was found that the optimised energy absorbers exhibited a more desirable force-deflection response than their standard counterparts due to a simple design modification which was incorporated in the optimised design. This mechanism proved to be a very favourable one for the reason that the resulting crushing force remained constant throughout the deflection once the collapse load has been reached. This is a desirable feature in the design of energy absorbers (See Fig. 5).

Morris et al [29] analysed the quasi-static lateral compression of nested tube systems between rigid platens both experimentally and numerically. These energy absorbers consisted of both two and three tube systems which were assembled ‘In Plane’ as shown in Fig. 6. Such a term describes two or more tubes of varying diameter being placed within each other and their axes being parallel. This type of energy absorber was compressed under flat rigid platens at a velocity of 3mm/min to ensure no dynamic effects were present. It was demonstrated how such a system is well suited to applications where space or volume restrictions are an important design consideration without compromising energy absorbing requirements.

Morris et al [30] also analysed the post collapse response of nested tube systems with side constraints (See Fig. 7). This work illustrated how the introduction of external constraints allows a greater volume of material within the structure to deform plastically in the post collapse stage of compression, thereby absorbing more energy. Nested systems consist of ‘short tubes’ of varying diameter placed within each other with an eccentric configuration. Both experimental and computational techniques were used to analyse the quasi-static response of such systems. Numerical results generated were found to be in good agreement with those of experiment. In a related work, Morris [31] modified the ‘In Plane’ system by rotating the central tube ninety degrees to define an ‘Out of Plane System’ as depicted in Fig. 8. In doing so, the force-deflection response changed from a non-monotonically increasing response to one that increased monotonically without sacrificing its energy absorbing capabilities. The quasi-static lateral compression was achieved using three different devices such as flat rigid platens, cylindrical and point-load indenters and their corresponding force-deflection response were compared.

Morris et al [32] also studied nested tubes crushed laterally between rigid platens at two different velocities. The first category of energy absorber consisted of an ‘In Plane’ system; the second device consisted of an ‘Out of Plane’ system. Material used was cold finished, drawn over mandrel (DIN 2393 ST 37-2) mild steel. The Cowper-Symonds relation was used to predict the dynamic yield stress of the rings and this was included in the finite element material model.

3.3 Lateral Indentation.

Sowerby et al [33] was one of the early authors to analyse the diametric compression of circular rings by point loads. An etching technique was used to reveal the specific shapes of the plastic hinges on the loaded ring. The collapse load was estimated using the slip-line field theory. This theory assumes a rigid-perfectly plastic material, quasi-static loading and plain strain deformation. The theory can provide analytical solutions to problems involving large deformations and velocities discontinuities; however, it is limited only to simple structures. The author discovered that the collapse load predicted on the basis of this theory correlated well with experimentally measured values.

The response of ‘square’ cross-sectioned tubes under both quasi-static and dynamic lateral loading was conducted by Jing and Barton [34]. Experimental tests consist of tubes of two different thicknesses loaded in either a fully clamped or simply supported condition. The authors conducted research into the collapse mechanism of these tubes and the relationship between energy absorption and tube deflection. In the dynamic cases, velocities of up to 6m/s were loaded upon the specimens. DYNA3D, the finite element code, was used to simulate the dynamic events. It was found that the mode of deformation for the thin-walled tube is more complex since a ‘wrinkle’ occurs which indicates local buckling. This occurs in both the quasi-static and dynamic cases. There was a tendency for the numerical code to under-predict the deformations of the fully clamped tube because complete constraint of the tube ends did not actually occur, instead, slippage occurred at the constrained ends. However, correlation between the numerical code and the actual test proved to be satisfactory, considering that the impact loading conditions and interaction between the impinging mass and specimen tend to be complex. Most importantly, the author briefed that little difference was found between the modes of deformation of tubes tested quasi - statically and dynamically and hence the energy absorption capacities could be predicted using quasi-static methods. However, the issue of strain rate sensitivity of the material in question to accurately capture the force-deflection response must be considered.

Thomas et al [35] identified three modes of deformation for a tube subjected to a quasi-static lateral load at its mid-span. The first mode was a pure crumpling phase in which the load increased sharply until a point was reached in which deflection of the base of the tube occurred. This load was defined as the maximum pure crumpling load. A bending and crumpling phase was identified as the second mode in which further crumpling of the tube combined with bending between the supports was observed. During this period of deformation, the force decreased slightly before increasing to its maximum. The final mode of deformation was structural collapse in which the maximum load was reached causing the tube to collapse followed by a drop in load. The drop in load was characterized by a large rotation of the tube ends about their supports.

In a companion paper [36], the authors experimentally observed the crushing of circular tubes by centrally opposed wedge shaped indenters. According to the authors, the purpose of this investigation was to facilitate the assessment of the energy absorbing capacity of tubes in an impact situation where the speed of deformation is not large enough to generate significant inertial forces. Upon experimentation, the author identified three modes of deformation; 1) Ring mode for short lengths ($L < 1.5D$), it was found that deformation was similar to that of a compressed ring and consist of hoop bending about the generators, 2) Transitional mode for medium lengths. In this case, the deformation starts as a quasi-static ring mode then proceeds into a mode involving 'increasing ovality' as the deformation continues, 3) Reversing Ovality for moderate to long length of tubes. In this mode, membrane stretching in the axial direction was observed and this behaviour predominates close to the indenters accompanied by axial bending of the generators.

Finally, in a concluding paper, [37] obtained further experimental results concerning the transverse loading of simply supported tubes. In this work, the authors examined the surface stresses generated by means of the brittle lacquer technique using strain gauges. By using this technique, the authors revealed that the use of simple plastic beam theory to predict bending failure cannot be justified. They discovered that unlike normal beam problems, the section of the tube over hanging the supports play an active part in the force-deflection response.

Zhao et al [38] examined the quasi-static compression of metallic thin walled rings with arc shaped supports. The author conducted an approximate analysis based on the 'Equivalent Structure Technique' (EST) which provides an upper bound solution and can be used to determine the collapse load of tubes under point loading. From experiments, they discovered that the structure deforms elastically followed by a softening stage. It was found from the theoretical analysis that the collapse load of the structure was accurately predicted, in addition to locating the positions of the plastic hinges.

Zhao and Fang [39] subjected metallic thin walled rings with arc shaped supports to a symmetrically concentrated impact load. Two supports of arc angle sixty and ninety degrees were used, in which it was discovered that the deformation was a five-hinge model. The arc angle played an important role in the final deformation of the ring, such that the smaller the arc angle at the support, the bottom of the rings would become slightly straightened. Upon analysis of the high-speed photos from the Cranz-Schardin camera system, the authors found that the final deformation always occurred during the first impact of the drop hammer and that any rebounds of the hammer did not affect the final shape of the rings. An approximate linear relationship was found between the drop height and the residual displacement between the impact face and the base of the rings. Finally, they noted that similar deformation mechanisms were found between the quasi-static and dynamic cases concluding that the impact velocity was low and hence strain rate sensitivity could be neglected.

Reid and Bell [40] identified that the role of strain hardening is significant in the post collapse response of rings loaded by opposing point loads. Application of the ‘Plastica’ theory by the authors in order to analyse the plastic regions gave an insight into the effects of strain hardening and that the particular solution observed indicates certain limitations on the earlier rigid perfectly theory [7] derived for the compression of rings between flat plates.

Lu [41] conducted a study of the crushing of mild steel tubes by two indenters, the tubes having four different diameters and lengths. The aim of this paper was to conduct experiments using various combinations of diameter D and length L of the tubes, to obtain the corresponding force-deflection curves and thereby develop some empirical relations. It should be noted however that the empirical relations derived are only valid within the range of tube parameters and loading conditions specified in the work.

Liu et al [42] analysed both experimentally and numerically the dynamic behaviour of ring systems subjected to pulse loading. The objective of the work was to understand the mechanism of elastic stress wave propagation and how this elastic energy is distributed throughout the ring system owing to the physical nature of such a system. The explicit version of the numerical code via LS-DYNA was used to simulate the pulse loaded ring systems. The experimental procedure was carried using a modified split Hopkinson pressure bar test system (SHPB). Fig. 9 illustrates such an apparatus with a finite element model of the system depicted in Fig. 10. It was found that the numerical results were in good agreement with those of experiment. The main conclusion of the work showed that energy redistribution is primarily affected by the thickness of the rings and not by the loading duration or the number of rings in the system.

Shim and Stronge [43] examined the post-collapse response of ductile, thin walled tubes compressed between cylindrical indenters. It was discovered that, depending on the radius of curvature of the indenters along with the degree of side constraints, the post collapse behaviour of laterally compressed tubes can either be stable (*monotonically*- increasing) or unstable (*monotonically*- decreasing). Post collapse stability increases as the curvature of the indenter becomes larger. It was found that side constrained tubes compressed between cylindrical indenters are sensitive to load imperfections. Small shear forces are generated which in addition to the normal applied forces, initiate an asymmetric mode of deformation, therefore exhibiting a post collapse response that is initially unstable.

Kardaras and Lu [44] used the finite element method for the investigation of large deformations of thin cylindrical tubes subjected to point loads about its mid-span. The objective of the paper was to produce a detailed report on the effect of large deflections, stationary and travelling plastic hinges, rigid body rotations of the generators, change in curvature and finally the dominant strains that occur such as membrane and bending strains. It was concluded that the finite element method can be a powerful tool in providing detailed results on the aforementioned parameters which otherwise would have been more difficult using experimental or analytical techniques.

A theoretical insight into the indentation of tubes under combined loading was provided by Wierzbicki and Suh [45]. The combined loading was in the form of lateral indentation, bending moments and axially applied forces. The most significant conclusion was that the resistance of the tube to local indentation depends strongly on the type of boundary conditions such as axially and rotationally restrained / unrestrained tubes. The theoretical solutions provided an accuracy between10%-20% in comparison with experimentally measured values.

Ghosh et al [46] analysed both theoretically and experimentally the deformation response of mild steel rings and short tubes of various thickness and lengths loaded centrally by opposed conically-headed cylindrical punchers. The mild steel specimens were tested in their ‘as-received’ and ‘annealed’ conditions. The aim of the work was to predict the collapse load of these short tubes as a function of yield stress, ultimate tensile strength and strain to failure. It was found that short rings of L/D ratio of up to 1.5 exhibited ‘knee-shaped’ response in the plastic stages of deformation. It is clear that the initial structural collapse load occurs somewhere along this ‘knee-shaped’ region. Interrupted annealed testing provided a very useful way of removing the strain-hardening of the short tubes and hence the initial collapse can be calculated for moderately deformed tubes. Finally, they concluded that the prediction of the increase in the horizontal diameter as loading proceeds becomes more accurate for tubes with decreasing L/D ratios. This was achieved by using a rigid-perfectly plastic material model and the concept of plastic hinges.

3.4 Axial Loading / Buckling.

Reid [47] provided an insight into the various modes of deformation that can be achieved from the axial compression of ductile material such as mild steel. Discussion was made on the axial splitting, inversion and buckling modes of deformation in which attention was given to the gross plastic deformation of the various specimens. Also, some reference was made to thin-walled tubes filled with polyurethane foam which can increase the specific energy absorption capacity of such a devise without compromising it weight.

Reid and Reddy [48] researched the quasi - static and dynamic crushing response of tapered sheet metal tubes of rectangular cross-section using mainly experimental techniques with the assistance of theoretical methods. It appears that tapered tubes offer the distinct advantage of absorbing off-axis/oblique loads which are commonly encountered in vehicular collisions. Good estimates of the mean crushing forces were estimated in both the quasi - static and dynamic cases. The authors concluded that tapered tubes as opposed to straight tubes are the preferred energy absorber since they are more efficient in absorbing energy from oblique impacts and are less likely to fail by global buckling.

A detailed experimental analysis was carried out on the quasi-static axial compression of thin-walled circular aluminium tubes by Guillow et al [49]. A classification chart was developed to characterise the various modes of deformation for D/t= 10 – 450. Empirical formulae were developed using the average force which was non-dimensionalised. The effect of filling aluminium tubes with polyurethane foam was also briefly investigated.

Wang and Lu [50] subjected a cylindrical shell to axial impact velocities in the region of 300 m/s and discovered a particular deformation mechanism termed the ‘Mushrooming’ effect, which caused the walls of the shell to thicken. It appears that the high end impact deformation mechanism generates a complex problem and the only feasible means of analysing such problems is by means of experimental testing and finite element analysis. Fig. 11 shows a typical cylindrical specimen subjected to high impact velocity. In general, three modes of deformation can occur; dynamic progressive folding for thin tubes under low impact, end mushrooming with folds formed away from the striking end at medium velocity and finally mushrooming and wrinkling for thick tubes at high velocity. The authors stated that direct correlation between actual and numerical results were difficult because no well-verified material model existed with the ability to capture the dynamic failure of these structures at high impact velocities. However, the current simulations which predicted the deformation mechanism at lower impact velocities offer valuable information to the designer since this information can be difficult to achieve through experimental testing alone.

Hsu and Jones [51] investigated the response of thin-walled circular stainless steel, mild steel and aluminium alloy tubes subjected to quasi-static and dynamic axial loading. Stainless steel was chosen to examine the effects of strain rate properties and strain hardening. Aluminium alloy 6063 T6 was chosen in order to assess the influence of inertia while the material's strain-rate sensitivity are negligible and the choice of mild steel in order to assess the outcome of strain-rate effects with negligible strain hardening. Comparisons were made on the performance of these three material types in terms energy absorbing efficiency and capacity. It was found that stainless steel tubes absorb the most energy per unit volume however; its dimensionless energy absorbing efficiency was the lowest of the three materials. The aluminium alloy was found to be the most efficient but absorbing the least energy per unit volume.

An experimental study on axially preloaded steel tubes subjected to lateral impacts was examined by Zeinoddini et al [52]. A common occurrence which involves steel tubes is the collision of supply ships with bracing members of offshore oil rigs. It was commented by the author that tubular structural members will be carrying their usual service loads before impact accidents occur. Therefore, it is very important to understand the effect of preload on a tubular member if the effect of impact damage is to be accurately measured. The work describes an experimental program in which axially preloaded tubes were subjected to lateral impact at their mid-span using a drop weight test rig as shown in Fig. 12. It was concluded that pre-loading has a substantial effect on the level of damage when combined with lateral impacts. It should be noted that only one influential parameter, the magnitude of axial preload, was adjusted during the testing. However, other parameters such as indenter shape, impact location, orientation of indenter and the residual stress level inherent within the tube can also have a marked effect on the response of such devices when subjected to lateral impact. Therefore, more work would be needed in this area in order to obtain a greater understanding of how the various parameters interact with each other and their effect on the output response.

Langseth et al [53] presented finite element simulations validated by experimental findings of quasi - statically and dynamically axially loaded square aluminium extrusions. Excellent correlations were achieved based on using isotropic elasticity, the Von Mises yield criterion, the associated flow rule and non-linear isotropic strain hardening. The uniaxial tensile test was used to obtain the yield stress and strain hardening characteristic of the aluminium tubing. In order to generate a symmetric folding mechanism, the authors incorporated a ‘trigger’ mechanism into the numerical model as shown in Fig. 13. Once the simulations were validated, a parametric study was used to study the effect of the impact velocity of the projectile on the response parameter, the mean load. It was shown that the response parameter was an increasing function with respect to an increase in impact velocity and the mass ratio of the specimen to the projectile had no effect on this parameter.

The axial collapse of aluminium alloy extruded polygon sections was analysed by Rossi et al [54] in order to develop an understanding of the post buckling behaviour of such systems subjected to impact loads. Their goal was to achieve this using the finite element software package LS-DYNA. The work was divided into two sections; firstly to validate the numerical results related to thin walled aluminium square tubes with existing published experimental data and secondly, to analyse the post-buckling deformation features such as symmetric and asymmetric behaviour. It was found that the numerical study can accurately predict both these modes of deformation, as with the prediction of the mean dynamic crushing force and permanent displacement which were within +/- 5% of actual values. Fig. 14 illustrates the final post buckling deformation state of a hexagonal sectioned model.

A numerical study was reported by Nagel and Thambiratnam [55] on the impact response and energy absorption of tapered thin-walled tubes. An advantage of such devices is that they are proficient at absorbing oblique as well as axial loads which is desirable in the design of energy absorbing devices. The main aim of the paper was to compare the energy absorption response of both the tapered and straight specimens subjected to quasi-static and dynamic loading conditions. Also, a detailed account was reported on the effect of inertia of both specimens. It was found that tapered tubes are less influenced by lateral effects than straight tubes. A factorial study was used to determine the influence of the input parameters such as thickness, angle of taper, impact velocity on the output response. It was concluded that the most important parameters that control the energy absorbing response are the taper angle and wall thickness. Fig. 15 and Fig. 16 illustrates a finite element model of a straight and taper energy absorber in its initial and final deformed conditions respectively.

In a companion paper by the same authors [56], attention was given to the behaviour of tapered tubes consisting of either a straight, double taper, triple taper and finally a frusta (four tapered sides). The aim of the work was to study the dynamic energy absorption response of these devices under impact loading conditions. The simulations showed that triple tapered tubes have the highest energy absorption capacity followed by straight tubes and frusta’s. However, it was realised that increasing the number of tapers decreases the specific energy absorption per unit mass. Therefore, it appears that straight tubes are the most efficient for absorbing energy when mass or weight is an important consideration. In general, this work was seen as outlining the advantages of using tapered tube devices that may be used in the transportation industry. An additional study was conducted by the same authors [57] on the energy absorption response of tapered thin-walled tubes. The primary outcome of the work was to gather important research information that will facilitate the design engineer in creating efficient tapered tube type energy absorbers.

In another work, Nagel and Thambiratnam [58] applied the concept of the tapered wall tube to enhance the energy absorption of an existing VFPS (Vehicle Frontal Protection Systems). Such a device acts to minimise the damage that may be caused due to animal strike. Since there is a gap between the chassis rail of the vehicle and the VFPS, it was decided that this space could be utilised to improve the energy absorption feature of the VFPS by introducing one of three possible energy absorption mechanisms. These three mechanisms consisted of the axial crushing of honeycomb, recoverable semi-rigid foams and tapered tubes. The outcome of using each mechanism and their suitability in the enhancement of the VFPS is to be published in a future paper.

The axial collapse of mild steel and aluminium tubes with cut-outs and in both their 'as-received' and 'annealed' conditions was analysed by Gupta and Gupta [59]. Tubes of various L/D and D/t ratios were compressed quasi - statically in an Instron machine. The cut-outs were in the form of holes. It was found that without the presence of holes, the mode of deformation is dependant on the initial state of work hardening and the ensuing annealing process. With the presence of holes, it was discovered that the peak load of such devices is reduced. It was concluded that employing such devices offers the advantage of much longer crush displacements before the onset of Euler buckling that may occur, which is usually associated with the crushing of tubes with large L/D ratios.

3.5 Axial Inversion.

Colokoglu and Reddy [60] analysed the strain-rate and inertial effects in the free inversion of mild steel circular tubes using experimental and theoretical techniques. As expected, the strain-rate sensitivity of mild steel was found to increase it's resistance to inversion. Inertial effects were found to play a part in both resisting and assisting the force required for the inversion process. Just prior to impact between the striker mass and the tube mass, it was realised that the acceleration of both masses acted in the same direction as the compression force thereby assisting the inversion process. After impact to initiate plastic deformation, a net acceleration between both masses occurred giving rise to a resisting force and hence the final force required to invert the tube was greater. The Cowper-Symonds relation was used to predict the dynamic flow stress of the material. It was found that the predicted values over-estimated the actual values, reason being that the values of the empirical constants D and q appeared to be too low for the strains that occur in the inversion process.

Webb et al [61] reported the simulation of quasi - static and dynamic axial inversion of tubes consisting of square or circular cross sections. A comparison of results between experimental and numerical was made. It was emphasised how the finite element method plays important role in the design of such energy absorbers and to assist in the planning of experimental programmes and therefore optimising the outcome of such procedures.

Kinkead [62] endeavoured to provide an improved correlation between the theory and experimental results for the external inversion of metallic circular tubes. An attempt was made to incorporate ‘engineering strain’ as opposed to ‘natural strain’ previously practiced in the current theory (Limit Analysis) in predicting the collapse load of inverted tubes. In doing so, a simpler technique was developed which was seen as acceptable since it introduces no large errors when correlated with experimental results. It was suggested by the author that the modified theory would help engineers to design energy absorbing devices based on the inversion process with more precision and assurance.

The plastic deformation mechanism of circular metallic tubes during quasi-static internal inversion was analysed by Reid and Harrigan [63]. Attention was given by the authors to a particular mode of deformation called tube nosing which appears not to have received much attention in the area of energy absorbers. Tube nosing is analogous to force internal inversion with the exception that a larger die radius is employed to encourage increasing hoop compression in the leading edge of the tube. Fig. 17 shows the deformed shapes produced by ABAQUS and that of experiment. Excellent agreement was found between the numerical code and experiment for the quasi-static internal inversion of mild steel tubes. Fig. 18 shows the force-deflection response of such a tube.

In similar work by Harrigan et al [64], the inertial effects due to dynamic loading was analysed in the internal inversion of tapered circular metal tubes and also on aluminium honeycomb material. It was found that for both cases, lateral inertial effects cause early load peaks to occur due to impact loading, the magnitude of which is governed by the material of the tube and the honeycomb wall. The aim of analysing the two different structures was to shed light on the role of inertia on the magnitude of force generated in these particular energy absorbing devices.

An experimental and theoretical analysis of an external inversion process was investigated by Miscow and Al-Qureshi [65]. The aim of the paper was to define a method to predict the dynamic inversion load based on quasi-static experimental data. Experiments were carried out on copper and brass tubes. Fig. 19 shows a typical schematic of a tube subjected to an external inversion process. It was noted by the authors that the predicted theoretical results such as collapse and dynamic mean load, impact velocity should not be taken as absolute values since other variables stemming from the dynamic testing have an influence on the estimated values. However despite this, it was found that a reasonable agreement existed between the theory and experiments.

3.6 Axial Splitting / Tearing.

The axial splitting of circular metallic tubes was conducted by Reddy and Reid [66] experimentally to examine the mode of deformation and it corresponding force-deflection response. The splitting of tubes can be seen as the intermediate between axial compression and axial inversion. The mean crushing force is somewhat lower, however high crush efficiencies in the order of 95% can be achieved. This type of device was analysed both quasi - statically and dynamically using experimental techniques. The splitting of tubes is a function of the die radius and frictional effects, therefore these parameters can be modified in order to achieve a desired rectangular shaped force-deflection which is ideal in the design of energy absorbers. It was noted by the authors that a combination of a rectangular force-deflection response plus the successful operation over a wide range of tube properties and geometries cannot be coexistent with axial inversion and axial compression (buckling modes).

Lu et al [67] sought to obtain experimentally the tearing energy required in a typical square tube splitting process. This provided a challenge for the authors since the tube splitting process involves a number energy dissipating mechanisms such as frictional, bending and tearing. It was discovered that, by ‘pre-cutting’ some corners to different lengths, the tearing energy could be determined. This was possible because the tearing energy per unit torn area can be related to the ultimate tensile stress of the material and the strain to fracture.

Jiang et al [68] scrutinised the size effects in the axial tearing of circular tubes during quasi-static and impact loading conditions. This was achieved by using the Buckingham Pi theorem and consequently 11 input and 8 output parameters were identified. The effect of strain hardening on the mild steel tube specimens under dynamic loading conditions was analysed. It was found that the material strain hardening effects do not comply with the geometrically scaling laws. Divergences of approximately 11% to 57% were observed between the measured and predicted values as a result of this incompliance. The author therefore endeavoured to develop a new scaling law which included the effects of strain hardening by means of a correction factor relating the dynamic yield stress of the prototype and the model. As a result of this solution, the corresponding deviations were reduced to a significant level. Consequently, this new scaling law can be seen as valuable for the design and safety assessment of larger scale energy absorbing devices.

Stronge et al [69] subjected a square tube to an axial force using a die causing the corners to split and curl outwards as deformation proceeds. For this mode of deformation, the energy absorbing mechanisms are fracture energy due to splitting, plastic deformation as large deformations ensue and frictional energy as the tube is passed over the mandrel. The theoretical advantage of using a square tube over any other cross section such as a circular tube stems from the fact that the specific energy dissipation processes can be separated analytically. This helps to accurately analyse the contribution and influence of each mechanism when a square tube is subjected to an axial splitting process.

Huang et al [70] detailed the study of energy absorption in splitting square metal tubes both theoretically and experimentally. The scope of the work was to analyse the role played by the different energy absorbing mechanisms in the splitting of these devices. Such mechanisms are bending, tearing and frictional energy and equations were presented for each mechanism. Good agreement was found between the actual observed values and those of theory. It was concluded that tubes which exhibit both splitting and curling behaviour may be used as efficient energy absorbing devices. Fig. 20 depicts a typical model of a square tube subjected against a pyramidal die.

In a companion paper by Huang et al [71], a detailed discussion using both experimental and theoretical techniques in the axial splitting and curling of circular metal tubes was presented. Mild steel and aluminium tubes were pressed axially on a series of dies, each with different semi-angles. An approximate analysis based on simplifying assumptions was successfully used to predict the number of propagated cracks, the curling radius and the applied force. Fig. 21 illustrates photographs of typical mild steel specimens after testing. As in previous work by the same authors, the three energy dissipating mechanisms such as bending, tearing and frictional were involved in the splitting process. Each mechanism was separated in order to clarify the role of each in such a splitting process which may be of benefit to the designer.

4 Brief Overview on the Analysis on Type I and Type II Structures.

Calladine and English [72] studied the strain rate and inertial effects on the collapse of two types of structures. Typically, the lateral or axial compression of rings/tubes can represent a Type I structure while axial loading of two steel plates clamped at either end represent a Type II structure. The authors revealed that the latter are sensitive to both strain rate and inertial effects; hence in the scale modelling of these structures, special care of these parameters must be taken into account. It was confirmed that inertial effects are sensitive only to the initial 'straightness' of the specimen i.e. the two plates clamped together, therefore the lateral compression of ring/tubes was not considered.

Zhang and Yu [73] provided a detailed discussion on the velocity sensitivity of a Type II structure using theoretical techniques. An attempt was made by the authors to provide a quantitative account of the effects of both strain-rate and inertia on such a structure and to provide a comparison with results obtained by Calladine [72].

Tam and Callidine [74] presented a thorough analysis on the response of Type II structures with respect to inertia and strain-rate effects. The work details the study by means of analytical and experimental methods. The aim of the authors was to remove the various limitations imposed on the work by Calladine [72] since their work was not entirely definitive. As a means of improving the phenomena behind the response of Type II structures, the authors endeavoured to introduce more variables such as changing the material and size of the specimens and consequently a more comprehensive theoretical analysis was provided using the concept of dimensional analysis.

Su et al [75] analysed theoretically the effects of inertia and elasticity on a Type II structure when subjected to impact loading. An elastic-perfectly plastic constitutive relation for the material was employed to predict the peak load which is important in the design of energy absorbers. The mathematical model consists of four compressible elastic-plastic rods connected by four elastic-plastic hinges as shown in Fig. 22 with Fig. 23 illustrating schematics of a Type I and Type II structure. It was identified that the dynamic behaviour of a type II structure is significantly different from its quasi-static counterpart even when the effect of strain-rate is neglected; thereby suggesting that inertia played an important role in this problem. In a companion paper [76], the effect of strain-rate was analysed with the aid of the Cowper-Symonds relation. It was found that strain-rate effects play an equally important role as inertial effects on the dynamic behaviour of Type II structures. The combined effects of inertia and strain-rate cause the peak load to be much higher than its quasi-static counterpart and the resulting displacement is much smaller. The authors observed the strain energy stored in the structure (strain-rate dependant) due to an increase in the yield stress which expands the range of elastic deformation, is notably larger than that of a structure which is made of a rate-independent material. Therefore when strain-rate effects are involved, elasticity plays a very important part in the structures response to impact loading.

5 Other Miscellaneous Topics.

The following sections gives a brief outline of other forms of energy absorbers such as a chain of rings, bumpers on cars and heavy vehicles and common roadside safety structures.

5.1 Plastic Extension of a Chain of Rings.

Gomes [77] analysed experimentally and in part analytically the plastic extension of a chain of aluminium rings due to an axial impact load. The test procedure involved the joining the rings together laterally and applying a mass to the bottom end of the chain. The chain was suspended from a carriage which could slide along two rigid vertical rails. Two stops were inserted on the vertical rails in order for the system to be arrested when released from a height. In doing so, the sudden arrest will cause a plastic deformation mechanism to occur on the ring system. A rigid-perfectly plastic material was assumed in the theory to describe the deformation process. Using this assumption, the total extension could be calculated, excluding the effects of strain-hardening and strain-rates, by equating the initial kinetic energy to the plastic work done in rotating the four quadrants of each ring. By excluding the dynamic effects in the simplified theory, an over-estimate of the actual deflection was observed and hence such strain-rate effects must be included if more accurate representation of the process is required.

5.2 Road Side Safety Structures.

Due to the high cost of full scale field testing and the advent of increased computing power, the finite element has been extensively used to analysis the response of vehicles impinging roadside barriers. It appears that finite element modelling can help designers achieve a better understanding of the behaviour of roadside safety structures and the vehicle involved and hence improve the crashworthiness of the energy absorber in question. Various authors [78, 79, 80, 81] have used the explicit version of the FE code to analyse various types of road side barriers and their interaction with the impinging vehicle.

The quasi-static and impact response of a scaled down W-Beams as a form of energy absorber was studied by Hui [82]. The W-beam guardrail is one of the most popular forms of impact attenuators located at roadsides to contain and to redirect out of control vehicles. Three end supporting conditions were employed to analyse their effect on the corresponding load-deflection responses. They consist of; 1) simply roller supported, 2) Axially constrained roller supported and 3) simply box supported. It was observed from the downscale experiments that the performance under the different end support conditions are affected by the following; 1) material strain hardening 2) structural softening due to cross-section distortion and 3) tensile force generated due to the axial constraints imposed on the guardrail. Finally, a simple mathematical model was developed to predict the characteristic load (collapse load). This model was in the form of a single degree of freedom spring-mass model. However, a dynamic enhancement factor needed to be incorporated into the model in order to account for the dynamic effects of loading such as inertia and strain rate effects of the material. In doing so, good agreement was found between the mathematical and experimental results.

5.3 Energy Absorption of Bumpers.

Johnson [83] reported an investigation into the energy dissipation of car bumpers under quasi-static lateral loads. By assuming a specified deflection of the bumper in question, the author was able to determine the impact velocity by equating the kinetic energy to the plastic work done on the bumper. It was found that most car bumpers have no substantial protective worth as energy absorbers in high impact collision. The same authors [84] noted the importance of attempting to improve the lorry frontal structure design in order to reduce the fatalities caused by under-run collisions between car and such Lorries. The aim of the paper was to examine the nature of car/lorry collisions. It was concluded that some advantages can be achieved by fitting a form of energy absorber to mitigate the effect of impact from an under run collision. However, the effectiveness depends on the collision speed of car and lorry and the mass ratio since the smaller mass will experience the greater velocity change.

Wasiowych [85] reported on an experimental investigation with an objective to reducing the injury associated with head-on collisions between cars and heavy vehicles. Full scale testing was performed using a prototype under ride-resisting barrier attached to the front of a heavy vehicle such as a lorry.

Energy absorption was achieved through plastic deformation of the thin-walled steel tubing via buckling and inversion. One of the successful achievements using this devise was the prevention of under run in frontal impacts by keeping the lorry in question away from the passenger compartment of the car.

6 Conclusion.

6.1 Advantages and Disadvantages of the Various forms of Deformation Mechanisms.

The energy absorbing capacity of laterally compressed tubes can be increased by encouraging the tube to collapse in an alternative mode which involves plastic hinges. Axial loaded tubes or tubes involving a splitting or inversion mode of deformation contain certain defects. For example, axially loaded tubes result in a force-deflection response which consists of a high peak collapse load followed by large fluctuations of force about a mean load. This fluctuation about the mean load can be as much as 50% of the collapse load. As a consequence, the impact loads transmitted to the protected structure will not be at its minimum magnitude and hence, the decelerating force tends to be high at this point. It is usually good practice to design an energy absorber such that the peak load is relatively close to its mean operating load. The deformation mode of a tube compressed laterally is one of bending and therefore the resulting response will smooth without any incidence of oscillations.

In relation to axially compressed systems, successful loading can only occur if the angle of load application is less than 15% to the longitudinal axis of the tube itself. In terms of transport applications, such behaviour is undesirable since the line of action of the kinetic force may be outside of this range of 15% [13]. It is possible to load tubular structures laterally over an angle greater than 15% since the tubes will still begin to deform for most of the deformation stroke before symmetry is lost. Tubes compressed axially can be unstable due to the fact that they often failed in an Euler type global buckling mode particularly in tubes with large L/D ratios. As a result, the energy absorbing capacity of such systems may be reduced considerably. Laterally loaded tubes are easier devices to build in comparison to tubes which are axially split or inverted (external or internally). This is due to the fact that special dies and mandrels must be manufactured to a high level of accuracy to ensure successful operation and desired output responses. A typical example of a die used for axial inversion of tubes is shown in Fig. 19.

Despite the certain drawbacks of axially compressed tubes, it appears that they receive more research attention in contrast to laterally compressed tubes. This is due to the fact that axially compressed tubes possess greater crush efficiency’s because of their inherently greater displacement stroke. In addition to this, the force magnitude of axially compressed tubes is several order so f magnitudes greater than tubes compressed laterally which is obviously a distinct advantage in terms of specific energy absorption capacity.

Successful inversion of tubes is only possible when the material is of certain ductility and its strain hardening is not significant. In addition to this, the global dimensions of the die radius must be within a compatible range in conjunction with suitable material properties in order to achieve the desired energy absorbing behaviour. Therefore it can be seen that axial inversion of ductile material is a complex process. For bending dominated systems, a wide choice of ductile material can employed in order to convert the kinetic energy into plastic work which can be achieved through relatively little design effort.

Selection of the appropriate deformation mechanism, choice of materials and type of tube to be employed will depend on the engineer's knowledge of the problem at hand and his ability to choose the best design in order to achieve the most desirable energy absorbing response.

6.2 Recent Advances and Trends for the Future in Energy Absorbing Devices.

In the transportation industry, there exists considerable motivation to reduce vehicle weight through the adoption of lightweight materials, such as aluminium and magnesium alloys while maintaining energy absorption and component integrity under crash conditions. This design requirement is currently being achieved through a new form of manufacturing process termed Hydroforming (or hydramolding). This process has become a popular method to produce complex structural shapes in recent years and is the most popular manufacturing process used in the automobile industry [86]. In tube hydroforming, a fluid pressure is applied internally to the tube causing it to deform and conform to the shape of a custom built die. Such a process results in more accurate component dimensions, reduction in weight, increased stiffness and strength and the production of unibody components. This leading edge technology results in an overall reduction in manufacturing costs and an improvement in energy absorption characteristics of components in contrast to conventional techniques such as drop forging and welding. Such hydroforming process can be easily employed to produce circular, square and frusta shaped energy absorbers. With the advent of increased computing power, the Finite Element Method is being employed as a powerful engineering tool to optimise the complex cross sectional design profiles of hydroformed unibodies in order to achieve minimal weight, maximal stiffness, strength and energy absorption characteristics.

New materials are also being developed by researchers that can find application to energy absorbing devices. Such materials include cellular materials in the form of honeycombs and foams. Honeycomb materials are widely used as the core structure in sandwich panels and are seen as good energy absorbing materials. They appear in different cross sectional shapes such as triangular, square, rhombic, circular and hexagonal forms. Their structure is essentially two dimensional and regular. The materials from which the honeycomb is made may consist of polymers, metals, ceramics and paper. Foams are a three dimensional form of the honeycomb and may be classified as an open cell (polyurethane) and closed cell (polyethylene) foam. More recently, metal foams have emerged as a new and novel class of material which has great potential for energy absorbing devices along with other applications. There are a number of processing routes available for the manufacture of metal foams, including bubbling gas through molten alloys, stirring a foaming agent through the molten alloy, consolidation of a metal powder with a particulate foaming agent. Typically, aluminium and nickel are commonly employed for application to metal foams. Metal foams have low density with good shear and fracture strength and are ideal for sandwich panel construction. Their exceptional ability to absorb large amounts of energy at almost constant force suggests applications ranging from automobile bumpers to escalator and elevators.

With respect to automobile crashworthiness and roadside safety, huge advances are being made using powerful computational techniques which enable researchers to simulate several different crash scenarios and predict vehicle and occupant response to incidents. Such foresight leads to more efficient research time and more effective gathering of data in order to assist in the design optimisation of transporting bodies. Public domain finite element models of vehicles, roadside hardware devices, occupants with various levels of detail and complexity are being developed in order to allow researchers and engineers worldwide to conduct transportation safety research for the benefit and safety of the public involved in transportation. One of the key challenges is to realistically capture the physical response of vehicle impact. To achieve this, the finite element models must include all geometry information, which can result over 1,000,000 elements. In addition, these models must include sophisticated nonlinear material properties external to the automobile, such as soil, concrete, wood, polyurethane, and other materials. Moreover, the simulation run times of these impacts are in the order of five hundred millisecond to one second. There is an increasing demand for improvement in the simulation and prediction of failure in materials and jointing systems in vehicles, this must be achieved if a return to prototype design and manufacture is to be avoided. The availability of high performance computers will help engineers to overcome these challenges. High performance computing forms an integral role in crashworthiness modelling and simulation, and will continue to do so for the foreseeable future, thus allowing researchers to continually improve the crashworthiness of transporting bodies.

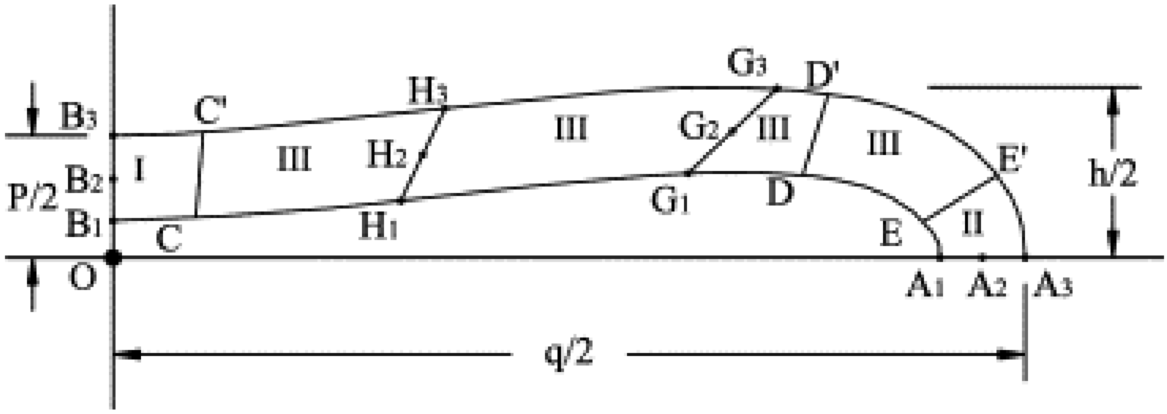


Fig. 1. Quarter cross-section of a typically deformed profile showing zones and points of interest [11].

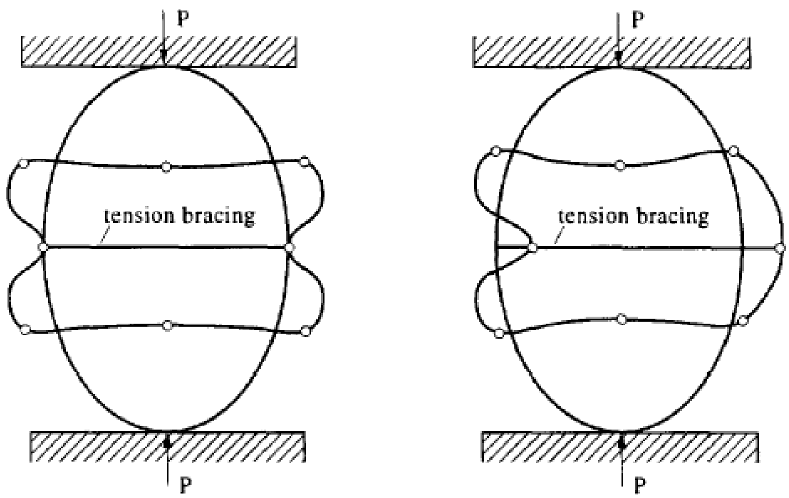


Fig. 2. Symmetric and asymmetric deformations for 0^0 bracing [27].

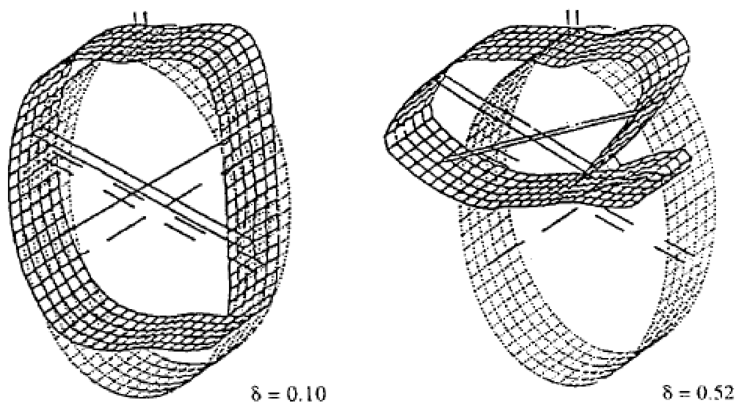


Fig. 3. Finite element mesh and two deformation stages for a 20^0 braced elliptical tube [27].

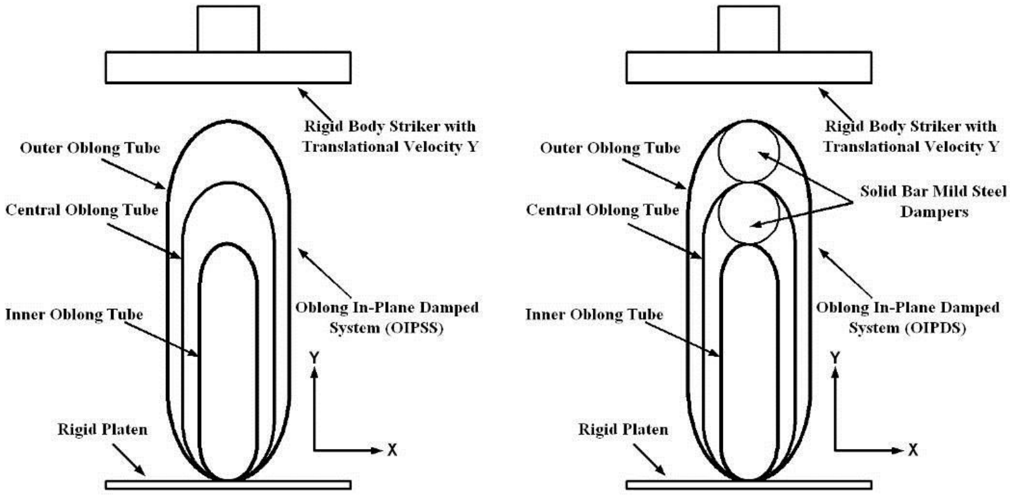
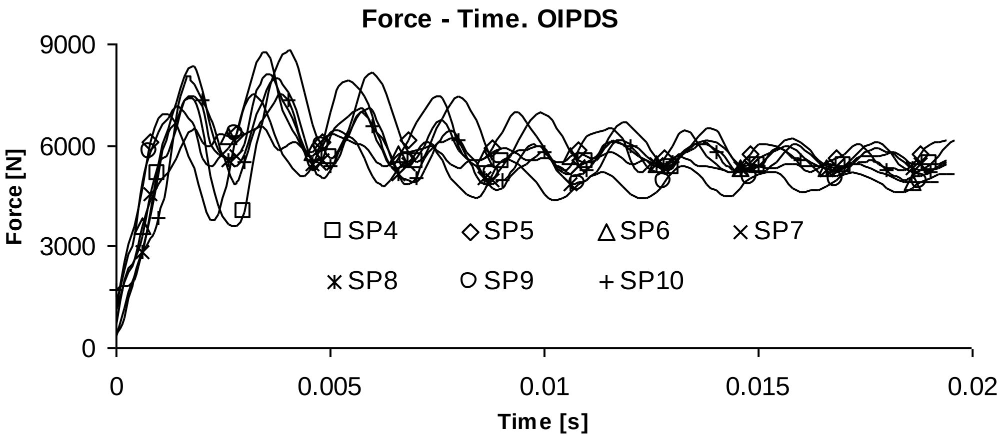
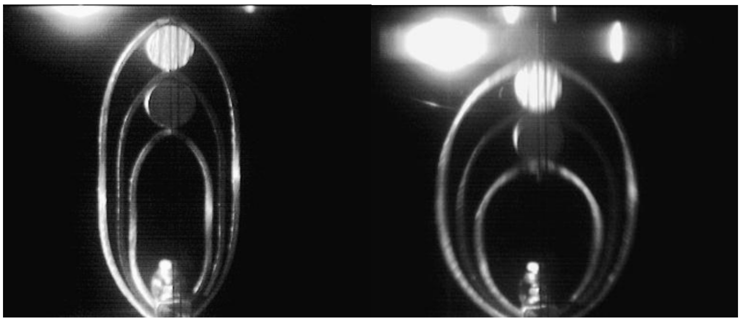


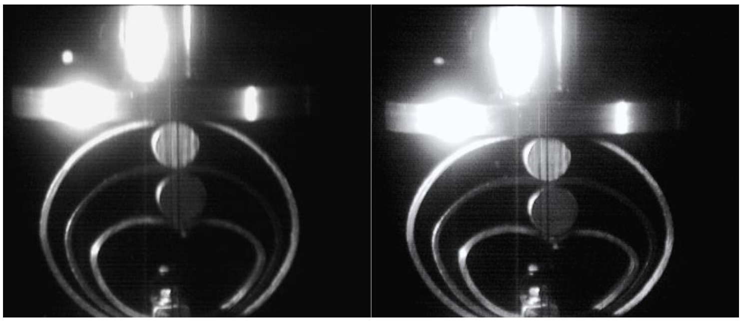
Fig. 4. A schematic of both the standard and optimised design. [28]



(a)



Frame 1: 0ms. Frame 5: 6ms.



(b) Frame 11: 18ms. Frame 15: 20ms.

Fig. 5. **(a) Force time curve for an OIPDS.**

(b) Experimental displacement evolution of sample 9. [28]

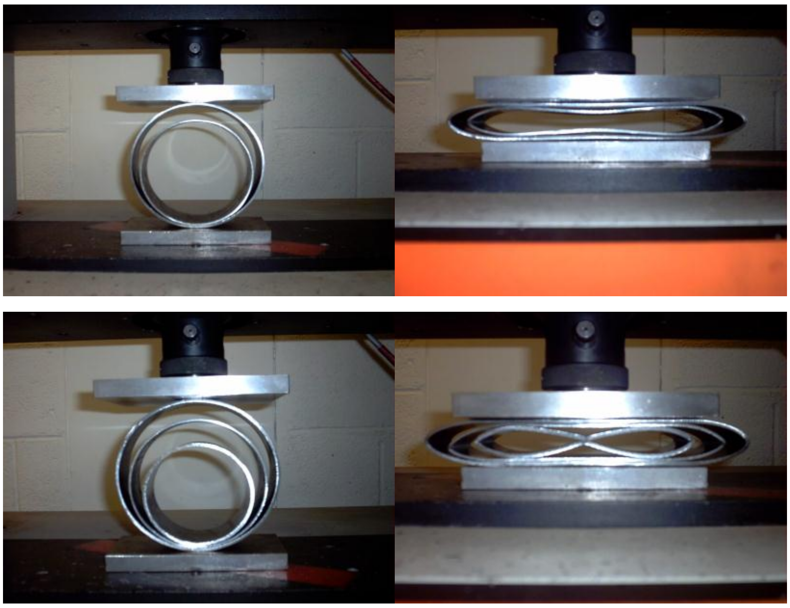


Fig. 6. Initial and final stages of compression for a two and three-tube system [29].

Fig. 7. Initial and final stage of compression for a 3-tube system with external constraints [30].

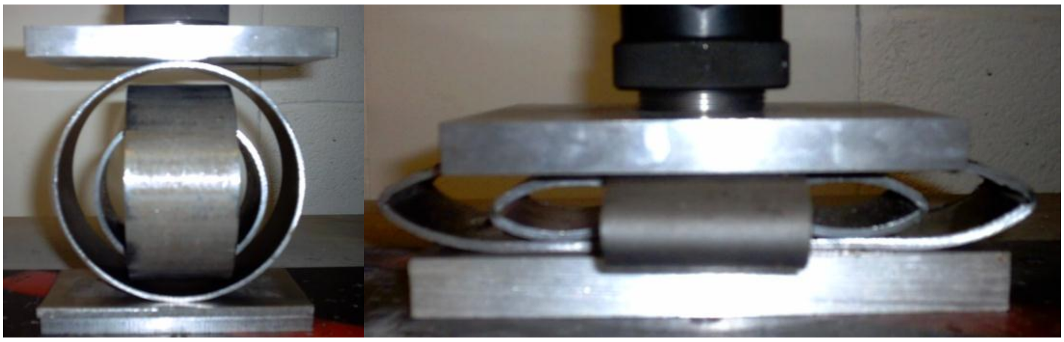


Fig. 8. Initial and final stage of compression of an Out of plane system [31].

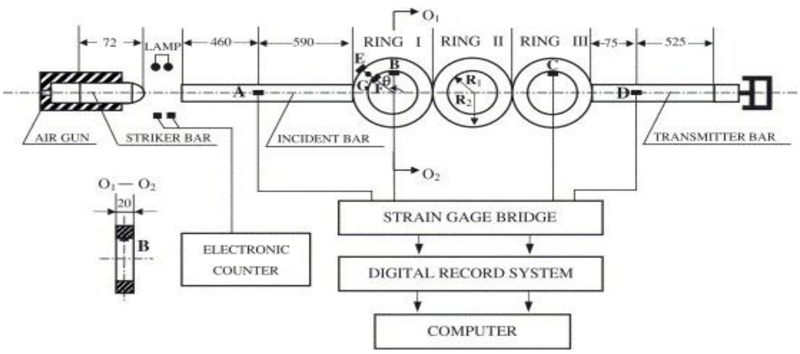


Fig. 9. Modified split Hopkinson pressure bar test apparatus and recording system [42].

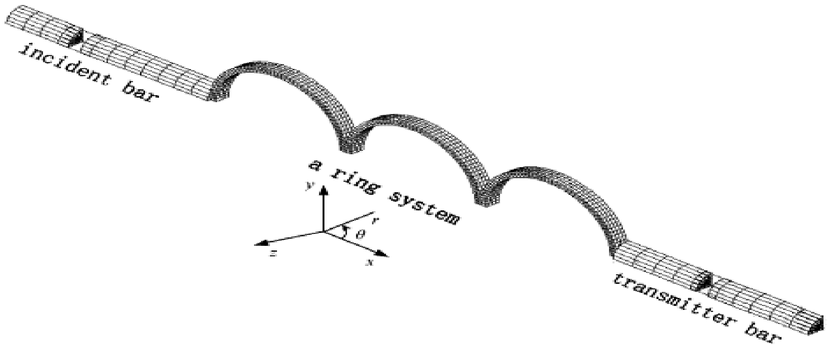


Fig. 10. Sketch of the Finite element model [42].

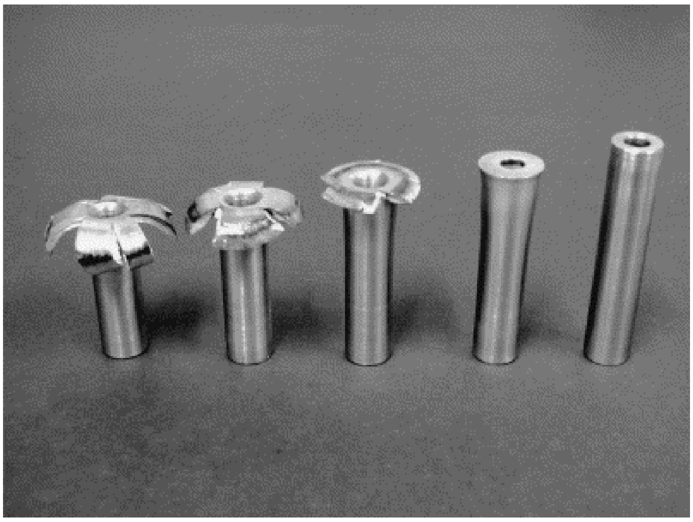


Fig. 11. Mild steel samples: Deformation states at velocities of 385 m/s, 277 m/s, 227 m/s, 173 m/s and 0 /ms respectively [50].

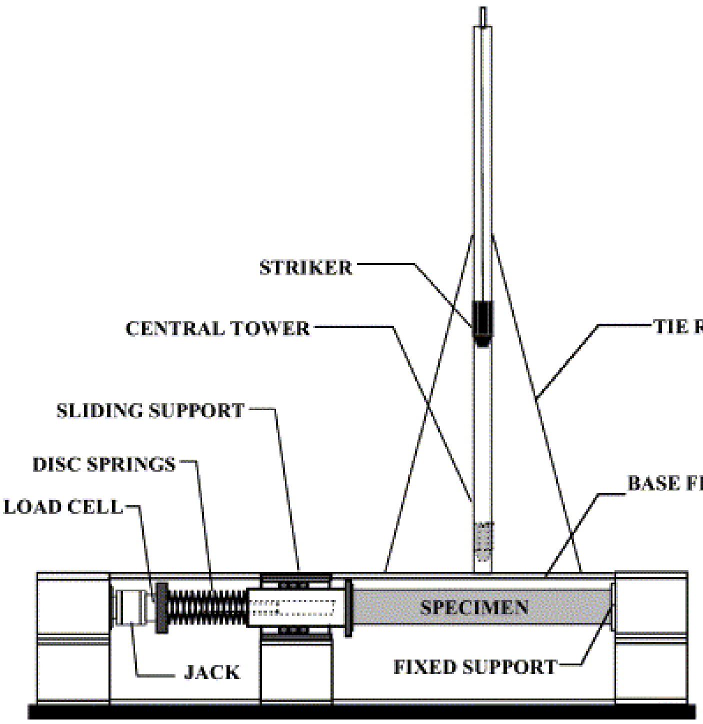


Fig. 12. Schematic view of the impact rig for testing of axially pre-loaded tubes subject to lateral impact [52].

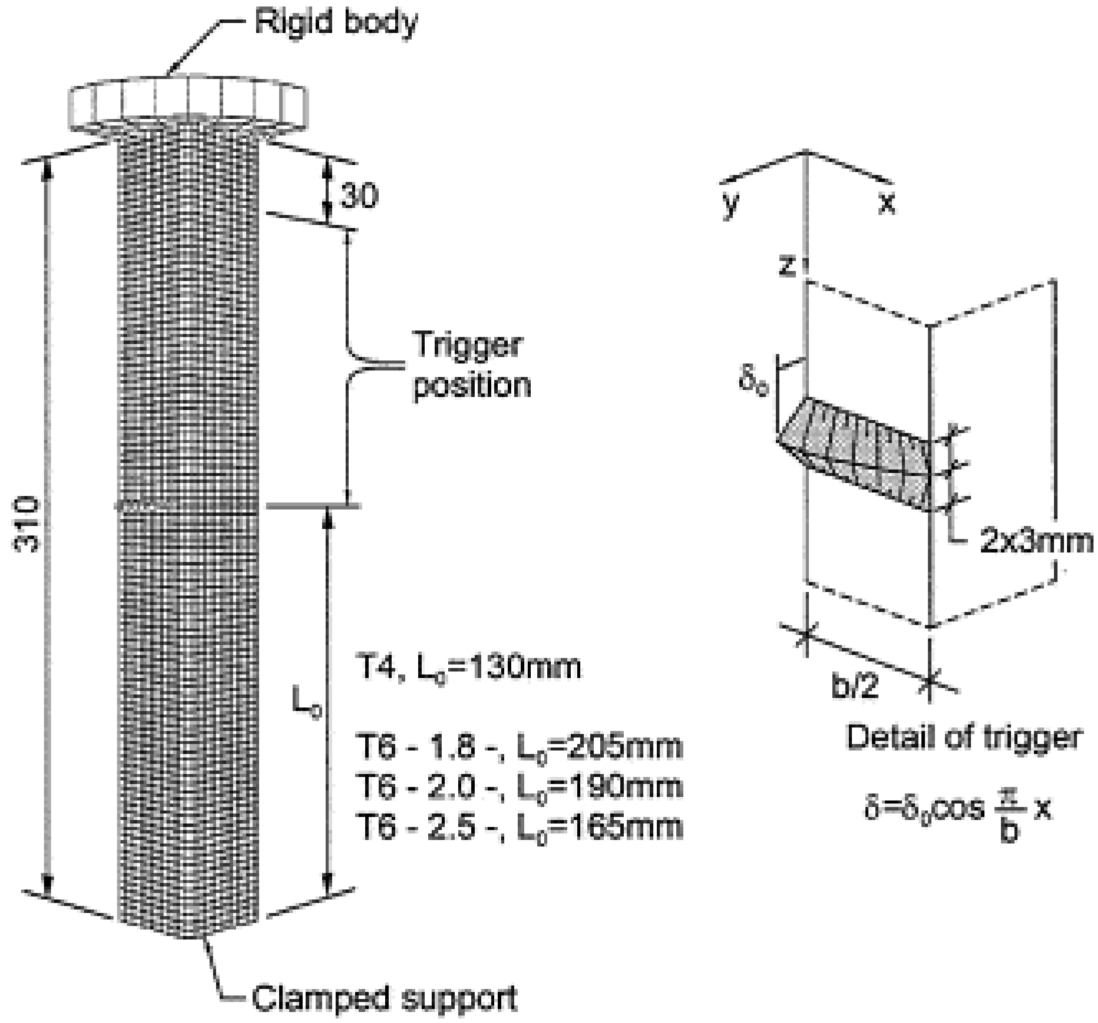


Fig. 13. One quarter finite element model including trigger position [53].

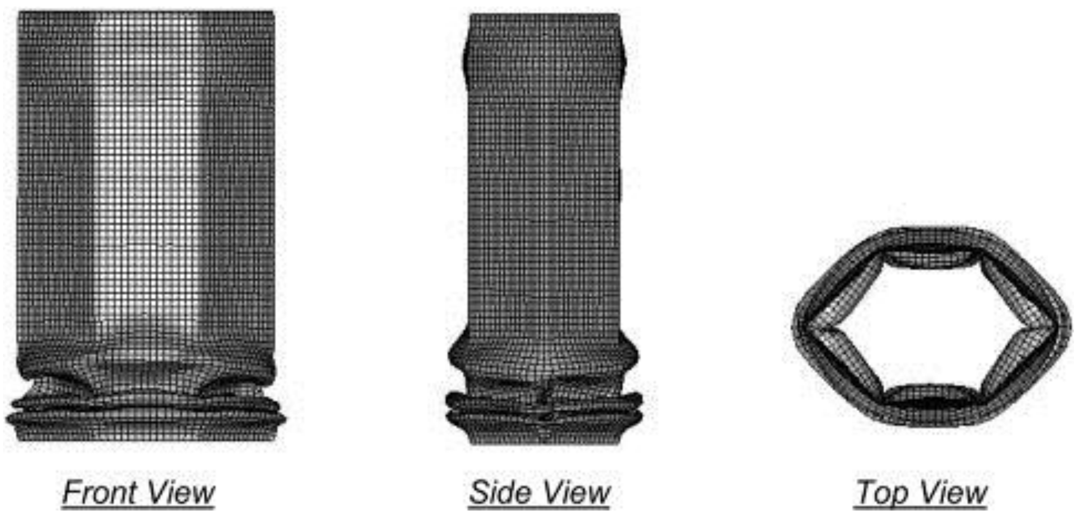


Fig. 14. Final post-buckling deformation of an LS-DYNA hexagonal section model [54].

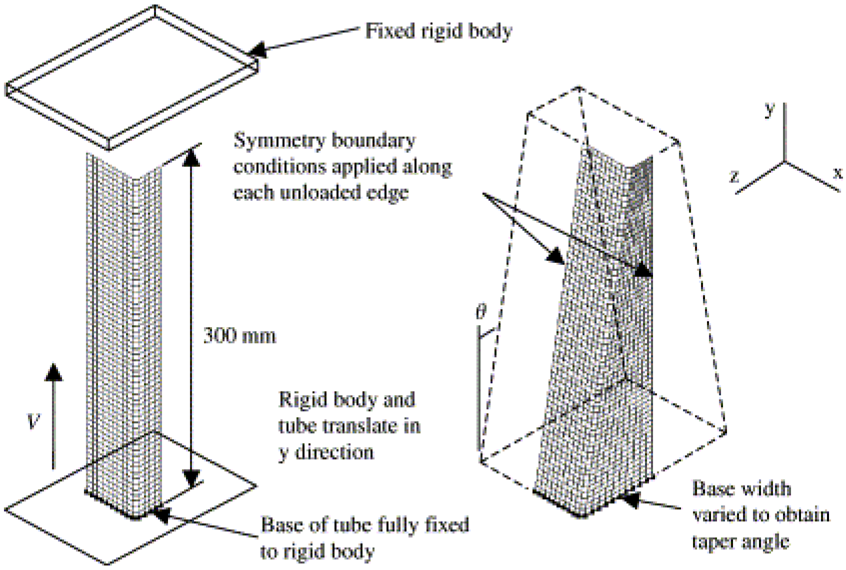


Fig. 15. Tube mesh, geometry and loading arrangement used in the FE model [55].

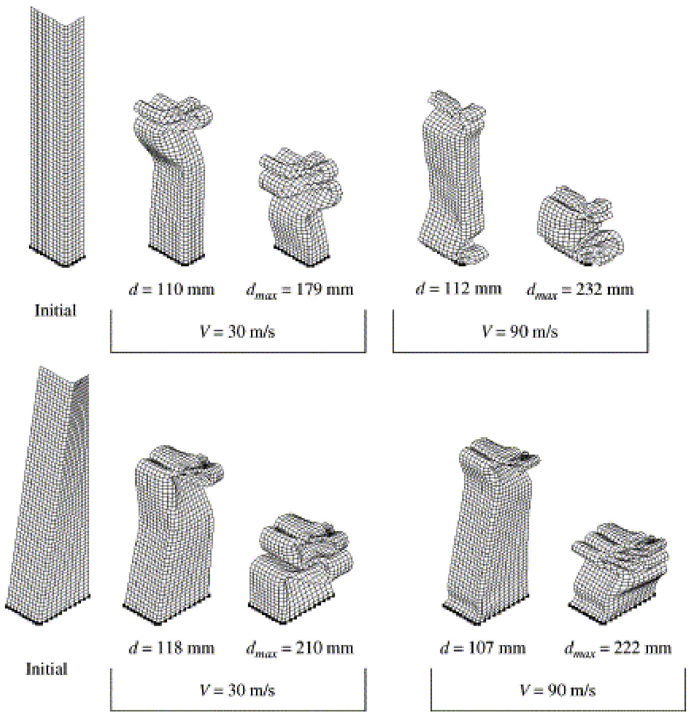


Fig. 16. Deformation profiles for the straight and tapered tubes at high and low impact velocities [55].

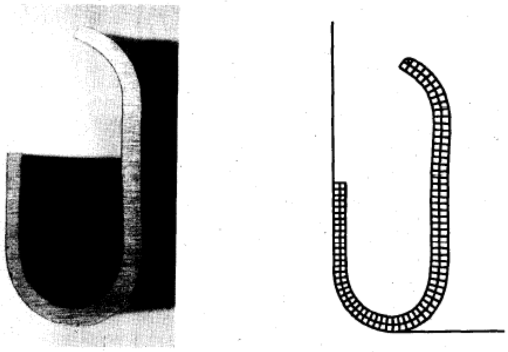


Fig. 17. Comparison between ABAQUS and experimental deformed shape [63].

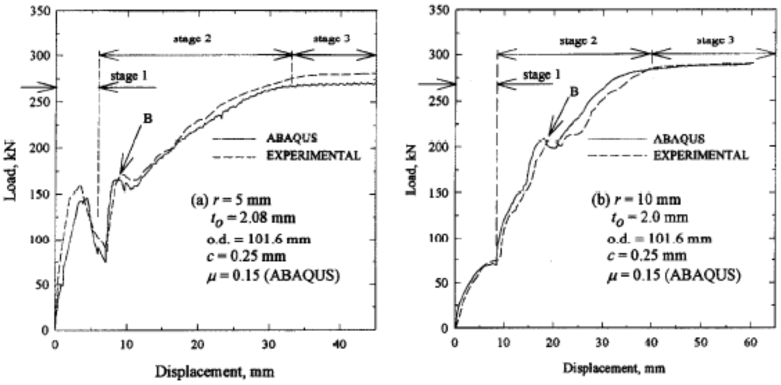


Fig. 18. Experimental results and ABAQUS solutions for quasi-static internal inversion of mild-steel tubes [63].

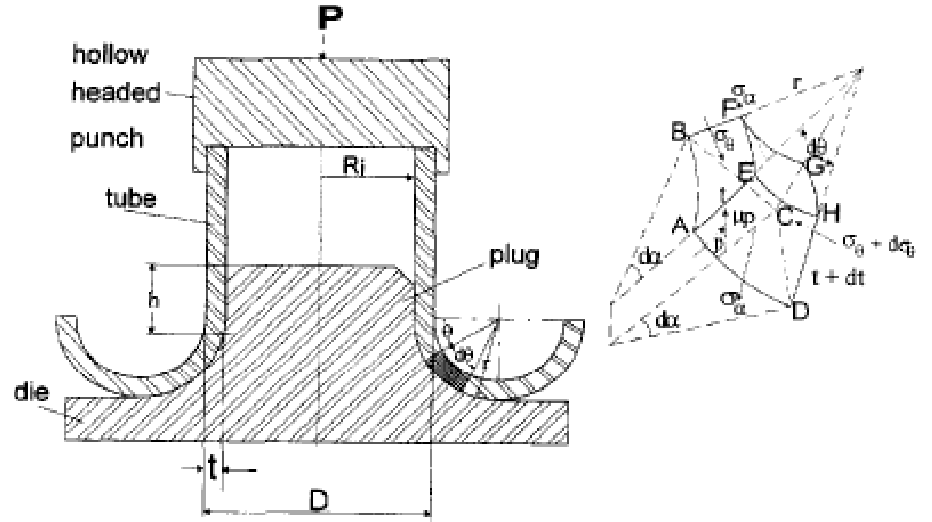


Fig. 19. Schematic cross-section of test arrangement for tube inversion process and stress on an infinitesimal element [65].

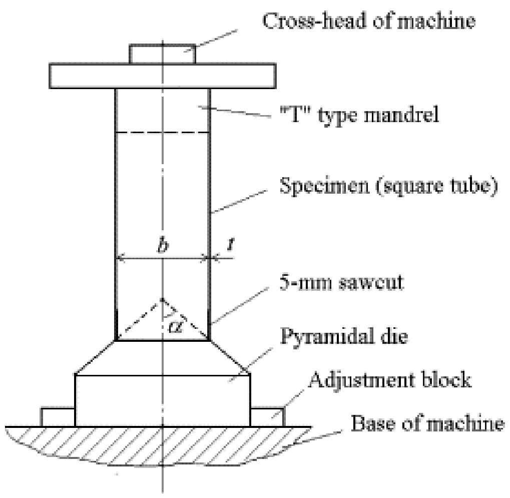


Fig. 20. Sketch of the experimental set-up [70].

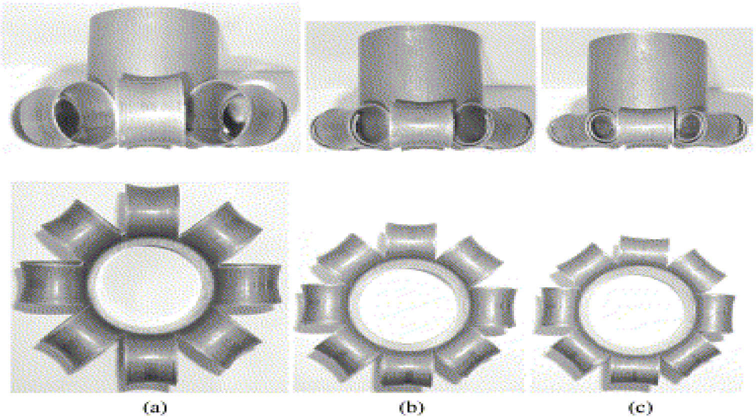


Fig. 21. Typical mild steel specimens in their final deformed stages [71].

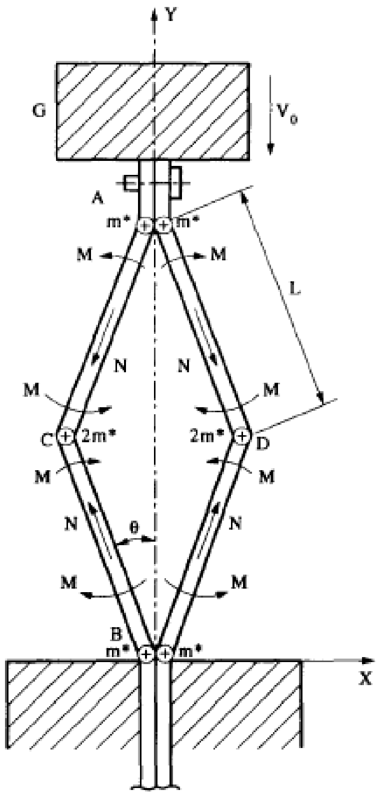


Fig. 22. A structural model for analysing a typical type II structure [76].

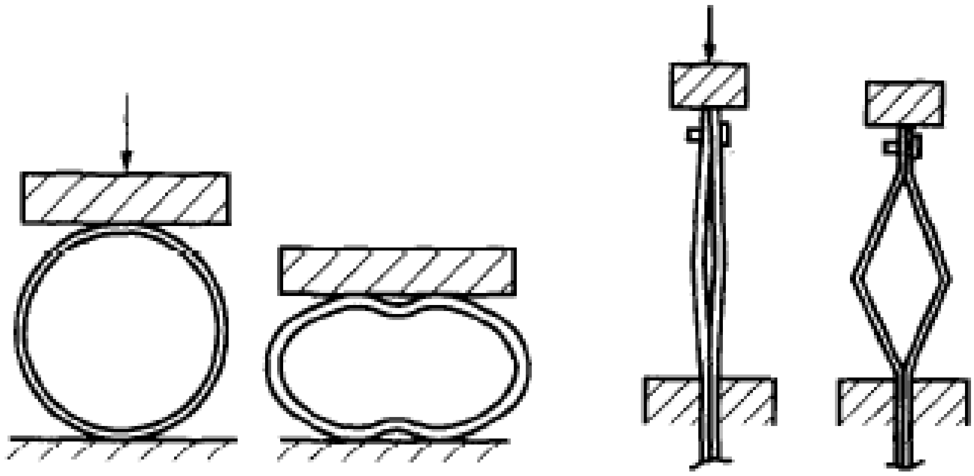


Fig. 23. Type I and Type II structures: Initial and final stages respectively [76].

Table 1: Description of Finite Element Code for High Velocity Impact.[5]

The Characteristics of the Computer Code - High Velocity.

• Mesh Description	Eulerian and Lagrangian.
• Spatial Discretization	Finite Difference / Finite Element.
• Temporal Integration	Explicit.
• Artificial Viscosity	Explicit Formulation.
• Material Model	Incremental elastic-plastic.
• Failure Criteria	Principle stress, strain, plastic work, damage mechanics
• Methods of Material Characterisation	
•	Wave propagation methods.
•	Split Hopkinson Bar.
•	Plate Impact.
•	Bar-Bar impact.
•	Exploding cylinder.
• Boundary Conditions	Reflective and transmittable.
• Initial Conditions	Velocity.

Phenomena associated with High Velocity Impact.

• Extent of Deformation	Local.
• Modal response	High frequency.
• Loading/response time	Sub milliseconds.
• Strains	>60%.
• Strain rates	$>10^5$/s.
• Failure	Physical separation of material.

Table 2: Description of Finite Element Code for Low Velocity Impact.[5]

The Characteristics of the Computer Code - Low Velocity.

• Mesh Description	Predominantly Lagrangian.
• Spatial Discretization	Predominantly Finite Element; high order elements
• Temporal Integration	Predominantly implicit.
• Artificial Viscosity	None, or implicit through discretization scheme.
• Material Model	No definitive model.
• Failure Criteria	Plastic flow.
• Methods of Material Characterisation	Conventional hydraulic testing machine.
• Boundary Conditions	Wide selection of internal and boundary constraints.
• Initial Conditions	Force, displacement, velocity.

Phenomena associated with Low Velocity Impact.

• Extent of Deformation	Global.
• Modal response	Low frequency.
• Loading/response time	Milliseconds-seconds.
• Strains	0.5 – 10%.
• Strain rates	$10^{-2} – 10^{1}$ / s.
• Failure	Large plastic flow.

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