Development of a Novel Rheometric Device for the Determination of Pressure Dependent Viscosity of non-Newtonian Fluids

by

Salma Akhter, B.Sc. Eng.
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This thesis is submitted to Dublin City University as the fulfilment of the requirement for the award of the degree of

Master of Engineering

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School of Mechanical and Manufacturing Engineering
Dublin City University

February, 2000
DECLARATION

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of *Master of Engineering*, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed  
Salma Akhter

ID No 96971509

Date February 2000
This thesis is dedicated to my parents who took all the troubles in the world with smile for the advancement of their children's knowledge and to my husband and daughter
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The author is grateful for the many sacrifices made by her husband and daughter so that this work might be completed.

Finally, all praise to God almighty for enabling me to complete this work.

Salma Akhter
**Notation**

- $p_1$: Pressure gradient in the first section of the unit
- $p_2$: Pressure gradient in the second part of the unit
- $\tau_1$: Shear stress in the fluid in the first part of unit
- $\tau_2$: Shear stress in the fluid in the second part of unit
- $K$: Non-Newtonian factor
- $\tau_{c1}$: Shear stress on the shaft in the first section of the unit
- $\tau_{c2}$: Shear stress on the shaft in the second section of the unit
- $\tau_{ca}$: Critical shear stress
- $\mu_0$: Initial viscosity of polymer melt
- $a$: Viscosity constant
- $b$: Pressure coefficient of viscosity
- $V$: Velocity of the shaft
- $Q_1$: Flow of fluid in the first section of the unit
- $Q_2$: Flow of fluid in the second section of the unit
- $U_1$: Velocity of polymer in the first section of the unit
- $U_1$: Velocity of polymer in the second section of the unit
- $P_m$: Maximum pressure in the step
- $h_1$: Radial gap in the first section of the unit
- $h_2$: Radial gap in the second section of the unit
- $L_1$: Length of the first section of the unit
- $L_1$: Length of the second section of the unit
Development of a Novel Rheometric Device for the Determination of Pressure Dependent Viscosity of non-Newtonian Fluids

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ABSTRACT

A new type of Rheometer has been designed based on hydrodynamic principles. Hydrodynamic pressure technique is a relatively new and innovative technique for rheological studies of viscous non-Newtonian fluids. These principles have been extensively used for the last ten years for drawing and coating of strips and wires. The Rheometric Device consists of a rotating inner cylinder (shaft) in a fixed hollow outer cylinder. The complex geometry gap between the two cylinders is filled with a viscous non-Newtonian fluid. When the surface of the shaft is rotating inside the hollow cylinder filled with a viscous fluid, shearing takes place and hydrodynamic pressure develops the magnitude of which is dependent on the shape of the surfaces, the viscosity of the fluid as well as the shear rate; i.e., the speed with which the inner solid cylinder is rotated. The Rheometer has been developed to determine the Rheological properties of viscous fluid at pressures of up to 100 bar and a shear rate range of 500 to 4000 sec⁻¹.

Experimental procedure and methods have been outlined and a number of experiments have been carried out to determine the effects of pressure and shear rate on viscosity. Included in the work are measurements of the pressure with variation of the shearing speed and inserts. Three different non-Newtonian fluids, glycerine, and silicon with two different viscosities were used as the pressure medium. The experimental works were undertaken with glycerine keeping temperature at 18 ± 1°C and silicon at 25 ± 1°C for shearing speeds of between 0.25 m/sec-2 0 m/sec. In the present study, theoretical models have been developed based on the non-Newtonian characteristics and a shear rate viscosity relationship was determined using the rheometer at different pressures by comparing the calculated theoretical pressure distribution with the experimental results.
Additional experimental work has been carried out using a Brookfield viscometer to determine the effect of shear rate on viscosity at lower shear rates. The Newtonian or non-Newtonian characteristics of glycerine, silicon was investigated by using this viscometer. Using the theoretical model, the viscosity of the fluid was determined at lower shear rates at atmospheric pressure. Results from the new Rheometric Device and the Brookfield viscometer have been presented graphically and new type of equations have been developed based on the experimental results using BDV and new Rheometer at the shear rates between 0-4000 Sec⁻¹ and only at atmospheric pressure.
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Chapter One

INTRODUCTION AND LITERATURE REVIEW

1. Introduction

1.1 Hydrodynamic phenomena

Hydrodynamic means dynamics of fluids under certain flow conditions. When a viscous fluid is present between the gap of the two solid surfaces and one of the surfaces moves relative to the other, hydrodynamic pressure develops. A common situation is when the fluid is flowing through a converging gap (like a journal bearing). The magnitudes of this hydrodynamic pressure are dependent on the shape of the surfaces, the viscosity of the fluid as well as the relative speed of movement. The mechanics of fluid pressure generation can be explained by the fact that the moving surface drags the fluid into the gap formed between it and the fixed surface and the relative motion between the moving surface and the fluid gives rise to the pressure. If the fluid is oil type then the generated pressure is not so high. But if it is of polymer solution type then the generated pressure is many times higher. However, for plastic processing equipment and food and mineral processing industries, the pressure generation due to hydrodynamic phenomenon could be a significant design factor for the processing equipment. This principle should be advantageous for designing a new rheometer for determining the rheological properties of viscous fluids such as glycerine and silicone, etc. at higher than
atmospheric pressures. The flow characteristics of non-Newtonian fluids are influenced by many factors, which are described in relation to the present work for rheological studies of these fluids.

1.2 Rheology of non-Newtonian fluids

"Rheology" is a general term for the study of deformation and flow of materials, it originates from the Greek word "rhein", which means, "to flow". Rheology is concerned with the flow and deformation of materials experiencing an applied force. The term 'Rheology' was invented by Professor Bingham of Lafayette college, Easton, PA, USA on the advice of a colleague, a Professor of Classics. It means the study of the deformation and flow of materials as mentioned earlier. This definition was accepted when the American Society of Rheology was founded in 1929. That first meeting heard papers on the properties and behaviour of such widely differing materials as asphalt, lubricants, paints, plastics and rubber, which gives some idea of the scope of the subject and also the numerous scientific disciplines which are likely to be involved. Nowadays, the scope is even wider. Significant advances have been made in biorheology, in polymer rheology and in suspension rheology. There has also been a significant appreciation of the importance of rheology in chemical processing industries. Opportunities no doubt exist for more extensive applications of rheology in the biotechnological industries. Rheology is becoming more and more important in plastic industry, both in areas of development and processing [1]. Other applications of rheology are as follows (i) Pumping slurries - materials transport (ii) Thickening and de-watering of mineral slurries.
(iii) Filtration (iv) Forming materials e.g. brick and ceramic products (v) Paint manufacture e.g. non-drip paints (vi) Reactions involving mineral slurries e.g. gold extraction (vii) Food chemistry and manufacture—texture of ice cream, pasta, desserts, processed meats, cosmetics chemistry (viii) Drilling mud for petroleum industry (ix) Polymer chemistry—solution and melts (x) Plastohydrodynamic wire drawing and wire coating i.e. rheological properties required for good coating performance.

In recent studies of thermoplastic injection moulding, sophisticated rheological techniques have been employed to characterise the viscous behaviour. Since the findings of rheology are of fundamental importance for the development, manufacture and processing of innumerable products, without rheology, nothing in materials and process engineering can function today. Rheometry is the technology, which involves the rheological measurement of fluids. The flow characteristics of non-Newtonian fluids are influenced by many factors, which are described in relation to the present work for developing a new type of rheometer for rheological studies of glycerine, silicone, liquid honey, polymer melt etc.

1.3 Variables which affect viscosity

1.3.1 The effect of pressure on viscosity

Several theories suggest that the free volume [2-6] determines the viscosity of a fluid. The free volume of a fluid is defined in various ways, but a common definition is the difference between the actual volume and a volume in which such close packaging of the molecules occurs that no motion can take place. The greater
the free volume the easier it is for flow to take place. The viscosity of liquids increases when the pressure is increased because the distance between the molecules, and hence their mobility decreases. At very high pressures no free volume is left in the liquid which becomes solid [7]. Free volume increases with temperature because of thermal expansion. However, the most direct influence on free volume should be of the pressure. An increase in hydrostatic pressure decreases the free volume and increases the viscosity of a fluid. Viscosity, by definition, is the internal resistance to shearing stress due to intermolecular forces of attraction. It is thought that if those forces of attraction are encouraged, the apparent viscosity of the polymer, which is one of the most important properties of these materials, may be increased.

Westover [8] described a double piston rheometer for measuring viscosity as a function of pressure at 172 MN/m². He found that the viscosity of polystyrene increases by over a hundred times and the viscosity of polyethylene increases by a factor of five at a constant rate of shear as the pressure was increased. Maxwell and Jung [9], and Choi [10] demonstrated that the effect of hydrostatic pressure on the apparent viscosity of branched polyethylene and polystyrene at constant shear and temperature are appreciable and should not be neglected. Cogswell [11] suggested that the effect of an increase in pressure may be linked to that due to a drop in temperature. He observed that for low-density polyethylene, an increase in pressure of 100 MN/m² had the same effect on viscosity as that due to a drop in temperature of 53°C within the melt range. It had been noted that at very high pressure (above 140 MN/m²) the melt tended to recrystallise and in consequence the melt acted melt acted like a solid plug [12]. For this reason, pressure viscosity measurements are often conducted at relatively high temperature. Since the work carried out by Westover appears to be the most comprehensive, his results are used to determine the
pressure coefficient of viscosity. Figure 1.1 shows the effect of pressure on shear stress-shear rate curves and figures 1.2 shows how shear rate effects the influence of pressure on viscosity.

13.2 Effect of Shear rate

Table 1.1 shows the approximate magnitude of shear rates encountered in a number of industrial and everyday situations in which viscosity is important and therefore needs to be measured. The approximate shear rate involved in any operation can be estimated by dividing the average velocity of the following liquid by a characteristic dimension of the geometry in which it is flowing (e.g., the radius of a tube or the thickness of a sheared layer).

<table>
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<th>Situation</th>
<th>Typical range of shear rates ($s^{-1}$)</th>
<th>Application</th>
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<tr>
<td>Extruders</td>
<td>$10^0 - 10^2$</td>
<td>Polymers</td>
</tr>
<tr>
<td>Chewing and swallowing</td>
<td>$10^1 - 10^2$</td>
<td>Foods</td>
</tr>
<tr>
<td>Dip coating</td>
<td>$10^1 - 10^2$</td>
<td>Paints, confectionary</td>
</tr>
<tr>
<td>Mixing and stirring</td>
<td>$10^1 - 10^3$</td>
<td>Manufacturing liquids</td>
</tr>
<tr>
<td>Pipe flow</td>
<td>$10^0 - 10^3$</td>
<td>Pumping, Blood flow</td>
</tr>
<tr>
<td>Spraying and brushing</td>
<td>$10^3 - 10^4$</td>
<td>Spray-drying, painting Fuel atomisation</td>
</tr>
<tr>
<td>Lubrication</td>
<td>$10^3 - 10^7$</td>
<td>Gasoline engines</td>
</tr>
</tbody>
</table>
Since the non-Newtonian viscosity behaviour will be dealt with in this project, it is important to first define what the Newtonian behaviour is, in the context of the shear viscosity. A Newtonian fluid has the characteristic that the shear viscosity does not vary with shear rate [1]. For non-Newtonian behaviour, the apparent viscosity decreases as the rate of shear increases. Figures 13, 14, 15 and 16 show the effect of shear rate on shear stress, where the influence of temperature may be noticed. These curves were produced by extruding polymer melts (Alkathene WVG23, polypropylene KM61, Rigidex (HDP), and Polystyrene) through an extrusion rheometer at different temperatures. A non-linear relationship is seen to exist between shear stress and shear rate. The viscosity of the polymer may be calculated at any known shear rate by measuring the slope of the curve. A Newtonian fluid under shear stress condition exhibits linear relationship with shear rate, where the slope of the line represents the viscosity of the fluids. Figures 17, 18, 19 and 110 show another way of representing the effect of shear rate on viscosity, where the viscosity may be read off directly from known shear rate values. (For a Newtonian fluid, this curve would be a horizontal straight line.)

1.3.3 Effect of Temperature on Viscosity

An increase in temperature generally decreases the viscosity of the fluids. This effect vastly differs for different types of polymer. Figure 111 shows typical changes in viscosity against temperature at zero shear rates. The slope of each line measures the activation energies. This energy has more deterioration effect on viscosity compared to those of lower activation energies. This energy is a function of polarity in
polymers and the most non-polar polymers such as polyethylene with very small inter-molecular forces have low activation energy.

Flow occurs when polymer molecules slide past each other and the ease of flow depends on the mobility of the molecular chains and the forces of holding the molecules together. That an increase in temperature reduces viscosity is widely known and in general is a property easily understood. Viscosity and temperature for a Newtonian fluid may be related by an Arrhenius equation of the form

\[ \mu = Ae^{E/RT} \]

This equation together with a knowledge of A, a constant, and E, the activation energy, enables to calculate the coefficient of viscosity \( \mu \) for an absolute temperature \( T \), \( R \) is the universal gas constant. A second (empirical) equation is often used for melts. This is \( \mu = a^{-bT} \) where both \( a \) and \( b \) are constants. To obtain a fundamental explanation of the difference in temperature dependence of viscosity between different polymers a number of attempts have been made. The application of "free volume theory" is most successful. This theory suggests that at some temperature \( T_0 \) there is no "free volume" between the molecules. This free volume, \( f \), is postulated to increase linearly with temperature so that at \( T_g \) the fractional free volume has a value \( f_g \).

The expansion coefficient \( \alpha_f \) is defined by the equation,

\[ F = f_g + \alpha_f (T - T_g) \]
It has been proposed that $f_g$ has the universal value of $0.0025$ and $\alpha_t$ is a universal value of $4.8 \times 10^{-4}$. Williams, et al. [13] have argued from this that the viscosity $\eta$ of a polymer at temperature $T$ may be related to its viscosity $\eta_a$ at an arbitrary reference temperature $T_a$ by the equation,

$$\log \frac{\eta}{\eta_a} = - \frac{C_1^\eta (T - T_a)}{C_2^\eta + T - T_a}$$

If the arbitrary temperature is taken as $T_g$ then the above equation becomes

$$\log \frac{\eta}{\eta_a} = - \frac{C_1^\eta (T - T_g)}{C_2^\eta + T - T_g}$$

In this case $C_1^\eta = 1/2303$, $f_g = 17.44$, and $C_2^\eta = 1/2303$, $C_1^\eta = 51.6$ have been proposed as universal constants. The equations are known as the WLF equation [14].

DIENES [15] believed that as the temperature increases the molecular arrangements within the polymer change more towards random configuration, therefore, it becomes easier for the polymer to flow at higher temperatures. Figures 1.11 and 1.12 show the variation of viscosity versus temperature for these polymers at zero shear rates. These graphs do not represent the complete behavior since viscosity measurements are affected by pressure, shear rate, temperature, etc. and it is necessary to include these effects on viscosity of polymer melts.
13.4 Effect of Flow Characteristics

In the plasto-hydrodynamic pressure unit the polymer melt is subjected to very high shear rates and pressures, much greater than those capable of being developed in any extrusion rheometer. The presence of critical shear stress at low shear rate decreases the coating thickness, as concluded by Crampton[16]. However, it is also believed that, the poor performance of the unit at higher drawing speeds is related to a combination of factors such as shear rate, melt flow instability, partial crystallisation, compressibility etc, and not just by the critical shear stress. The high-pressure generation is believed to have the effect of increasing the melt viscosity in the unit. Crampton maintained the temperature at a steady value when the tests were conducted, thus minimising the effects inherent with changing temperature. However, to understand these effects fully, further investigation is needed.

13.5 Critical Shear Stress

The critical shear stress is the stress at which the uniformity of the non-Newtonian fluids such as polymer melt flow ceases to exist. Several workers [17-19] investigated this phenomenon and they showed that certain flow defects are associated with the polymer Ripple, Bamboo, Spiral, ZigZag or Helix for different types of polymers. The terms “Melt fracture”, “Elastic turbulence” and “Flow distortion” have been used to describe this effect.
However, there is a general agreement on the following points

1. The critical Shear Stress (CSS) has values in the region of 0.01 to 1.0 MN/m² for most polymers.
2. The CSS is independent of the die length and diameter.
3. The CSS does not vary widely with temperature.
4. The flow defects always take place when non-Newtonian fluids are involved.
5. A discontinuity in the viscosity shear stress curves occurred.
6. The CSS was shown by the cinematography method to take place in the die.

1.4 Literature Review

1.4.1 Historical background of hydrodynamic lubrication

As the demand for higher quality wire increased, better lubrication proved necessary to promote efficiency, surface finish, quality, heat dissipation and a reduction in production time. The latter may be summarised as (i) Reduced drawing time (ii) Reduction of number of inter-pass heat treatment (iii) Elimination of redrawing time (iv) Reducing down time due to changing dies because of excessive wear. Christopherson and Naylor [20] pioneered the development of hydrodynamic lubrication in wire drawing. They employed a long tube, with very close tolerances, attached to the front end of a conventional die as shown in Figure 1.13. Oil was used for lubrication purpose and as the wire was pulled through the dies, it pressurised the lubricant by viscous action and fed into the die inlet. This pressurised lubricant then completely separated the wire from the die, preventing metal to metal contact.
Experimental results provided by them showed evidence of deformation of the wire in the tube before the die entrance. Although it was shown that hydrodynamic lubrication was achieved under the designed conditions, the pressure nozzle had to be placed vertically and was of such a length that the wire industry found it too inconvenient to put it into practice.

Wisterich [21] carried out experimental work on the forced lubrication based on a pressure tube system. Soap powder was used as the lubricant in a short nozzle, which was attached to the entry side of the die. Experimental results showed that the speed, temperature and the tube gap had a direct effect on the property of the film thickness produced. He also noted that oil produced a thicker film than soap. A schematic diagram of the unit used by Wisterich is shown in Figure 1.14.

Tattersal [22] published a detailed analysis of plasto-hydrodynamic lubrication action in wire drawing taking some rheological and metallurgical properties of the process into account. Experimental results and theory showed that these were in reasonable agreement. More recently Chu [23] using the work of Tattersal presented a chart for the inlet tube he designed. Subsequently Bedi [24] introduced an analysis for wire drawing assuming complete hydrodynamic lubrication. Studies of the lubricant film thickness which would be developed under practical drawing conditions have been presented in references [25-26]. Middlemiss [27] improved the previously used drawing unit by using an externally pressurised lubrication system.

Kalmogov et al [28] also conducted experimental work on tube sinking under conditions of hydrodynamic action. In contrast to similar devices for wire drawing, there was no seal between the nozzle and the die, because the lubrication process in tube sinking was much lower. Soap powder was used as the lubricant in that work.
Bloor et al [29] produced a theoretical analysis for elasto-plasto hydrodynamic lubrication action for strip drawing through wedge shaped dies. They took account of elastic component in the strip at entry and exit to the die and the pressure and viscosity characteristics of the lubricant. It has shown that by comparing the magnitude of the predicted lubricant film thickness that hydrodynamic lubrication could be accomplished during the process.

Double die arrangements were developed by Orlov et al [30]. Their version of the pressure nozzle/die is shown in Figure 1.5. The lubricant is transported into the chamber formed by the exit cone of the pressure die and the entry part of the drawing die, where the pressurised lubricant provides the hydrodynamic lubrication during drawing. Although claim was made for reduced power consumption and reduced die wear, there was however a lack of substantial evidence to support this claim.

1.4.2 Background Literature of Polymer Melt as a Lubricant in Drawing Process

Polymers were first used as lubricants in deep drawing and hydrostatic extrusion. They were chosen because of the radically differing characteristics from either soap or oil. There are many important differences between the rheology of molten polymer and conventional lubricants such as oil. The most obvious of these is the very high viscosity of polymer melts at higher temperatures, which would preclude the use of oil as a lubricant. Non-Newtonian lubricants have been previously investigated for journal bearing [30] and these have been found to be advantageous for bearings subjected to oscillatory load which induce fatigue loading. The use of a polymer melt as the coating material in the drawing process was
suggested by Symmons and Thompson [31] They investigated the adherence of a polymer coat onto the drawn wire There was some hydrodynamic lubrication achieved but not as much as expected Both Stevens [32] and Crampton [33] conducted experiments which showed that polymer coating of wire was possible, depending on the temperature, viscosity of the polymer and drawing speed of the wire The experiments they carried out reduced the cross-sectional area of the wire They also noted that the polymer coat thickness on the wire decreased as the drawing speed increased The apparatus used was a modification of Cristopherson tube A schematic diagram of this unit is shown in figure 1 16

Other experimental work carried out by Crampton et al [34] has shown that effective reduction of the wire diameter should be possible using a polymer melt in conjunction with a stepped bore tubular unit only Panwher et al [35-37] reported work carried out on the dieless tube sinking process and presented an analytic solution based upon Newtonian characteristics Subsequently Panwher [38] analysed the system taking account of the Non-Newtonian characteristics of the polymer Symmons et al [39-40] presented analysis of a die-less wire drawing process using a viscosity-pressure and viscosity - temperature relationship of experimental form For the deformation of strip using a stepped parallel bore unit, Memon et al [40-44] also published theoretical and experimental works

Parvinmehr et al [45-49] carried out similar experiments to Crampton’s but used two different types of pressure tubes in which the smallest diameter of the tube was greater than the initial diameter of the wire being drawn The units had no reduction die since Crampton and other researchers previously noted that deformation started to occur before the entrance to the reduction die The two pressure tubes he used in his experiment were of stepped and tapered bored nature
His experimental results showed that polymer-coating thickness varied with the wire drawing speed, the polymer type and the polymer temperature. Hashmi and Symmons [50] developed a numerical solution for a conical tubular orifice through which a continuum is drawn. Very good agreement between the theoretical and experimental results was observed at low drawing speed, whereas for higher drawing speeds the agreement between the predicted and the actual percentage reduction in area was very poor.

Investigation was carried out by Al-Natour and Hashmi [51-52] using a combined parallel and tapered bore pressure unit in conjunction with a polymer melt. They developed a theoretical analysis assuming Newtonian pressure medium. Also, hydrodynamic lubrication was achieved. Nwir and Hashmi [53-56] then published the pressure model considering the polymer as a non-Newtonian fluid. Akhter and Hashmi [57] developed new mathematical models for different types of pressure unit using the polymer as non-Newtonian fluid.

14.3 Background literature of rheometers and rheological research

Many methods have been devised to study the flow properties of liquids. Traditionally, rheological measurements are performed with Capillary rheometers (viscometers) and Rotational rheometers (viscometers). The earliest attempts to measure the flow properties of fluids dealt mainly with the principles of Capillary viscometry. Probably the first scientific experiment in which a capillary or tube was used to measure viscosity was made by Hagen in 1839, followed shortly thereafter by Poiseuille’s work. Poiseuille studied capillary flow problems in order to understand better the circulation of blood through capillary vessels in the human body.
body. He discovered the relation slip (known as the Hagen Poiseuille law) between flow rate and pressure drop. This discovery was the foundation of capillary viscometry. With developments in hydrodynamics and advances in the technology of non-Newtonian fluids, capillary viscometry became a reliable method of measuring the flow properties of a fluid [58]. The capillary viscometer provides an improvement in shear capability. Novak and Winer [59] used a capillary viscometer to measure the viscosity of polymer/mineral oil mixtures at pressures to 0.3 GPa and shear stresses to $10^5$ Pa. They reported Newtonian behaviour in two ranges of shear stress at low shear stress a “first Newtonian” associated with the viscosity of the blend and above about $10^4$ Pa shear stress, a transition to a “second Newtonian” for which viscosity was again rate independent but with a value reduced to nearly that of the mineral oil solvent. With the capillary technique the hydrostatic pressure varies across the length of the capillary requiring foreknowledge of the effect of pressure on material parameters.

Couette devised the first practical rotational viscometer in 1890 [60]. Couette’s Concentric cylinder viscometer consisted of a rotating cup and an inner cylinder which was supported by a torsion wire and rested in a point bearing in the bottom of the cup (fig 117). 23 years later Hatcheck described a modified version of the Couette viscometer, which improved on the design of the guard rings and minimised effects with very low viscosity fluids. Many aspects of the new theories (non-Newtonian or complex fluids) pioneered by Reiner, Bingham, et al., required innovations in design. Barber et al. [61] operated a rotational viscometer at atmospheric pressure with gaps approaching one micron. Shear rates as high as $10^6$ sec$^{-1}$ were obtained. However, the shear stress was limited to $10^4$ Pa. Galvin [62] developed a high pressure, high shear rate capillary viscometer. It was used in a
lubrication research laboratory that covers a wide range of rate of shear at ambient pressures substantially above atmospheric pressure and at various temperatures.

A new slit rheometer with a continuously adjustable gap was developed to characterise the shear viscosity material functions of structured fluids (suspensions, dispersions, gels) as a function of deformation rate and temperature [63]. The rheology and hydrodynamic analysis of grease flows in a circular pipe was investigated by Young [64]. Rheological properties and pumping characteristics of two greases were investigated in the temperature range of 5°C to 35°C. The high pressure rheology of a soap-thickened grease and the base oil has been characterised using a falling-body viscometer, high pressure couette viscometer, and a high pressure penetrometer by Scott [65]. He demonstrated the modified Bingham model to describe the experimental observation. A modified Bingham type rheometer, designed for operation at temperatures up to 500°F and pressures of up to 200 psi is described by Nanan [66].

The relationship of wall shear rate for closed couette flow has been determined for power law fluids with a helical barrel rheometer [67]. The principle of this rheometer is based upon measuring the pressure differential across one flight of special extruder wherein the helix is cut in the barrel rather than in the screw. A correlation for the pressure and temperature dependence of viscosity is demonstrated by Jagadish [68]. Mamdouh et al. [69] studied the rheological properties of silicone fluids (polydimethylsiloxanes). Their studies include the depiction of the rheological fingerprints of two PDMS solutions with the zero-shear viscosities 1000 and 30000 mPa's. For the sake of brevity they call these solutions PDMS1 and PDMS30, respectively. Also included in the study are the transient shear stress responses, the measurements of yield stresses, the thixotropy, and the creep-recovery tests. Their
experiment shows the influence of temperature on the rheological fingerprints of PDMS1 and PDMS30. PDMS1 solutions exhibit Newtonian behaviour and their viscosities decrease by almost 45% when the temperature is raised from 10 to 50°C. However, PDMS30 shows two distinct ranges of Newtonian and non-Newtonian flow behaviour. Over the initial range of Newtonian flow, the viscosity of PDMS30 decreases with temperature from 10 to 40°C and slightly increases at 50°C.

Compamk et al [70] studied the effect of chain length and ion size on the viscosity and ion conductivity of poly (dimethylsiloxane) systems. They applied free volume, Arrhenius type viscosity, and ion conductivity models. Graebhing et al [71] studied the viscoelastic properties of poly (dimethylsiloxane)-poly(oxyethylene) blends. They characterised the polymer blends by high values of the storage modullus at low frequencies and by long relaxation times. They related the rheological behaviour to a few physically significant parameters such as the zero-shear viscosity of phases, the dispersed particle size, and the interfacial tension between phases. Allcock et al [72] described the importance of PDMS applications in medicine, which is due to its resistance to blood fluids.

1.5 Present Project and its aim

This thesis outlines the author’s effort in the area of non-Newtonian fluid rheology. A novel rheometric device, using a stepped bore unit, is developed based on hydrodynamic principles. This new technique would be useful in determining the viscosity of non-Newtonian fluids at different shear rates and high pressures. The experimental work was carried out and the theoretical model has been developed in conjunction with the experimental work. Included in this work are,
measurement of the variation of pressures with speed and changing the fluid, the
insert and determination of viscosity and coefficients of the viscosity equation in conjunction with the experimental results

The scope of the present design and research is to

a) Design and development of an experimental device permitting the determination of pressure-dependent viscosity of non-Newtonian fluids

b) Measure the developed pressure in the Rheometric unit for different types of fluids at different with the shear rates, and for different gap geometries

c) To develop a mathematical model for the prediction of various parameters involved in the process such as pressure, viscosity, shear rate, viscosity, coefficients of the viscosity equation in conjunction with the experimental results

d) To determine the viscosity of the different types of fluids at different pressures developed at different shear rates

e) To study the effects of pressures on the viscosity of the fluids which is developed in the unit

f) To measure the viscosity of different fluid with a Brookfield viscometer with different shear rate and to determine the effect of shear rate on viscosity at the lower shear rate range

g) To develop a new type of equation at the shear rate range 0-4000 Sec\(^{-1}\) and only at atmospheric pressure

h) To discuss the viability of the process and its limitations in relation to practical applications
16 Overview of the Project

The details and background of the investigation are presented in this thesis as following.

In chapter one, a background literature of the rheology of non-Newtonian fluid and the effect of pressure, shear rate and the temperature on viscosity of fluids as well as the literature review of the hydrodynamic phenomena and its presence in wire drawing process have been presented. The aim of the research is also included in the first chapter. The theoretical analysis for the rheometer is given in chapter two. A discussion on the equipment used during the course of tests is given in chapter three, also the design of the pressure unit is discussed in details in chapter three. The materials and the equipment used during the course of the tests and the experimental procedure is given in chapter four. The experimental results using the new rheometric device and the Brookfield Viscometer are given in chapter five. The discussions are also given in this chapter. Finally, chapter six states relevant conclusion and gives suggestions for both theoretical and experimental future work.
FIG 11 Flow Curves for 0.92 polyethylene at 130°C [49]
Figure 1.2 Viscosity vs Pressure for different shear rates (0.92 Polyethylene 130°C)[49]
Shear Stress (N/m²) x 10⁵

- 130°C
+ 150°C
* 180°C

Fig 1 3
Flow Curve for ALKATHENE WVG23

Shear rate ($10^4$/sec)
Fig 14 Flow curve for POLYPROPYLENE KM61
Flow curves for RIGIDEX
Fig 1

Shear Stress (N/m²) x 10^5

0  0.2

0  0.5  1  1.5  2
MELT FRACTURE FOR 230°C  →  MELT FRACTURE FOR 240°C

\[ \mu_0 = 300 \text{ N sec/m}^2 \\
K_1 = 2410 \]

\[ \mu_0 = 280 \text{ N sec/m}^2 \\
K_1 = 3810 \]

\[ \mu_0 = 255 \text{ N sec/m}^2 \\
K_1 = 510 \]

Flow curves for POLYSTYRENE
Fig 17 Effect of shear rate on viscosity of ALKATHENE WVG23
Fig 18  Effect of shear rate on viscosity of POLYPROPYLENE KM61
Fig. 19 Effect of shear rate on viscosity of RIGIDEX
Fig 110  Effect of shear rate on viscosity of POLYSTYRENE
Fig. 111 Effect of temperature on viscosity of polymer
Fig. 1.12 Effect of temperature on viscosity of WYG23, KM61, RIGIDEX and POLYSTYRENE.
Fig 1.13 Typical pressure tube and die

Fig 1.14 Nozzle-die unit (BISTRA)
Fig 1.15  Pressurised chamber
Fig 1.17  Couette's viscometer, A is the inside guard rings F and F'
Chapter Two

HYDRODYNAMIC ANALYSIS

2.1 Introduction

When a solid surface is rotated within another surface (orifice) filled with a viscous fluid, shearing takes place and in addition to the shear force a hydrodynamic pressure is generated. The magnitude of the shearing and the pressure depends upon the speed with which the solid surface is rotated, the viscosity of the fluid and the geometrical configuration of the unit. In order to verify the mechanics of the process of hydrodynamic pressure development within the unit it is important to develop a suitable mathematical model assuming non-Newtonian characteristics. This chapter outlines details of a model developed by the author for the proposed high-pressure rheometer.

Rheologies of fluids were discussed and several mathematical models developed, for wire drawing were reviewed in chapter one. Theoretical and experimental works have also been carried out to observe the effectiveness of different lubricants and to calculate the film thickness during deformation process in the presence of hydrodynamic lubrication. Different types of Rheometers have been discussed in chapter one. Theoretical solutions for a novel plasto-hydrodynamic die-less drawing process were presented in references [46-50] for circular cross-section wires and rectangular strips.

In the present study the geometrical configuration is different from the previous researchers. A non-Newtonian plasto-hydrodynamic analysis of the process is presented for
a stepped parallel gap in which glycerine and silicone is used as the pressure medium. The effects of shear rate and pressure on the viscosity of glycerine and silicone, together with the shear stress are included in the analysis.

2.2 Theoretical analysis

The following analysis is based on the geometrical configuration shown in figure 1.

To formulate the analysis the following assumptions were made:

1. Flow of non-Newtonian fluid is isothermal.
2. Flow of non-Newtonian fluid is axial.
3. Flow of polymer is laminar.
4. Thickness of the fluid layer is small compared to the dimension of the stepped gap rheometric unit.

Analysis of the parallel gap rheometric unit is presented in two parts:

1. Determination of the pressure in the unit.

2. Determination of the viscosity of polymer within the unit and the coefficients of the viscosity equation.
2.2.1 Determination of the pressure and coefficients of viscosity equation of non-Newtonian fluids

Two different equations are generally used to express the shear stress and shear rate relation for a polymer solution. The first is a power law equation given by

\[ \tau = \mu \left( \frac{dU}{dy} \right)^n \]  \hspace{1cm} (1)

This equation is applicable for any type of fluid. Here \( n \) is the power law index which equals to 1 for non-Newtonian fluid, greater than 1 for a dilettante fluid and less than 1 for pseudoplastic fluid. In this equation \( \tau \) is the shear stress, \( \mu \) is the viscosity coefficient for power law fluid and \( \frac{dU}{dy} \) is the shear rate.

The second is an empirical equation relating shear stress and shear rate of a viscous fluid suggested by Rabinowitch [12] in the form,

\[ \tau + K \tau^3 = \mu \left( \frac{dU}{dy} \right) \]  \hspace{1cm} (2)

This equation was later used by Hung and Muster (74) to determine the non-Newtonian flow in a step bearing and also by Swamy et al (75) to calculate the load capacity of a finite width journal bearing. Other workers have also used this equation to analyse the behaviour of non-Newtonian fluids [33,49]. In equation (2) \( K \) is the non-Newtonian factor, \( \mu \) is the viscosity of the fluid, \( \tau \) and \( \frac{dU}{dy} \) are the shear stress.
and shear rate respectively. In the analysis, which follows, is based on the second empirical equation.

Flow of non-Newtonian fluid in the parallel stepped gap unit is divided into two sections. The first section being the entry part of the unit before the step and the second section being the tube after the step.

\[ \frac{dp}{dx} = \frac{d\tau}{dy} \]

Integration with respect to \( y \) gives,

\[ \tau_1 = P_1' y + \tau c_i \]

where \( \tau c_i \) is the shear stress at \( y = 0 \) and \( P_1' = (dp/dx) \) is constant across the gap.
Substituting equation (3) into equation (1) becomes,

\[ P_1 \dot{y} + \tau \beta + K(P_1 \dot{y} + \tau \beta) = \mu \left( \frac{dU}{dy} \right) \]

or

\[ \mu \left( \frac{dU}{dy} \right) = p_1 \dot{y} + \beta + K(p_1 \dot{y} + \beta)^3 + 3p_1 \dot{y}^2 \beta + 3 \beta p_1 \dot{y} \]

After integration equation (4) becomes

\[ U_1 = \frac{p_1 \dot{y}^2}{2\mu} + \frac{\beta \dot{y}}{\mu} + \frac{K(p_1 \dot{y}^4)}{4} + \frac{\beta \dot{y}^3}{\mu} + 3p_1 \dot{y}^2 \beta + 3 \beta p_1 \dot{y} + C_1 \]

where \( C_1 \) is the constant of integration.

The boundary conditions are,

(a) at \( y = 0 \), \( U_1 = V \) and (b) at \( y = h_1 \), \( U_1 = 0 \)

Since at \( y = 0 \), \( U_1 = V \) hence \( C_1 = V \) hence equation (5) becomes

\[ U_1 = \frac{p_1 \dot{y}^2}{2\mu} + \frac{\beta \dot{y}}{\mu} + \frac{K(p_1 \dot{y}^4)}{4} + \frac{\beta \dot{y}^3}{\mu} + 3p_1 \dot{y}^2 \beta + 3 \beta p_1 \dot{y} + V \]

where

\[ \frac{\tau}{2} (p_1 h_1) \beta^2 + \left( \frac{1}{K} + p_1 h_1 \right) \beta + \left( \frac{\mu \nu}{Kh_1} + \frac{p_1 h_1}{2K} + \frac{p_1 h_1^3}{4} \right) = 0 \]
Now, let

\[ J_i = \frac{3}{2} P_i h_i, \quad M_i = \frac{1}{K} P_i^2 h_i^2, \]

And

\[ N_i = \frac{\mu v}{k h_i} + \frac{P_i h_i}{2k} + \frac{1}{4} P_i^3 h_i^3, \]

So that,

\[ \tau c_i^3 + J_i \tau c_i^2 + M_i \tau c_i + N_i = 0 \quad (8) \]

Also, let

\[ \tau c_i = \phi_i - \frac{J_i}{3} \quad (9) \]

Substituting for \( \tau c_i \) from equation (8) into (9) we get,

\[ (\phi_i - \frac{J_i}{3})^3 + J_i (\phi_i - \frac{J_i}{3})^2 + M_i (\phi_i - \frac{J_i}{3})^2 + M_i (\phi_i - \frac{J_i}{3})^2 + N_i = 0 \]

Which, after substitution for \( J_i, M_i, and N_i \) from above and simplification becomes,

\[ \phi_i^3 + \phi_i \left( \frac{1}{k} + \frac{1}{4} P_i^2 h_i^2 \right) + \frac{\mu v}{k h_i} = 0 \]
which may be written as

\[ \phi_i^3 + A_i \phi_i + B_i = 0 \]  \hspace{1cm} (10)

\[ A_i = \frac{1}{k} + \frac{1}{4} P_i^2 h_i^2 \]

and

\[ B_i = \frac{\mu^2}{kh_i} \]

Equation (10) is a cubic equation, which can be solved by applying Cardan's formula as outlined in Appendix B

\[ \phi_i = \left[ -q + \left( q^2 + P_i^3 \right)^{\frac{1}{3}} \right]^\frac{1}{3} + \left[ -q - \left( q^2 + P_i^3 \right)^{\frac{1}{3}} \right]^\frac{1}{3} \]

Substituting the values of \( \phi_i, A_i, B_i \) and \( J_i \) in above equation becomes,

\[
\begin{align*}
\pi_i &= \left( -\frac{\mu v}{2Kh_i} + \frac{\mu^2 v^2}{4K^2 h_i^2} + \frac{1}{27} \left( \frac{1}{K} + \frac{1}{4} P_i \right)^3 \right)^{\frac{1}{3}} + \\
&\left( -\frac{\mu v}{2Kh_i} - \frac{\mu^2 v^2}{4K^2 h_i^2} + \frac{1}{27} \left( \frac{1}{K} + \frac{1}{4} P_i \right)^3 \right)^{\frac{1}{3}} - \frac{1}{2} P_i h_i
\end{align*}
\]

(11)

The flow of polymer gives

\[ Q_1 = \int_0^{h_i} U_1 \, dy \]  \hspace{1cm} (12)

Substituting for \( U_1 \) from equation (6) into the above equation and integrating gives,
Also the steady state flow gives, \( \frac{d}{dx}(Q) = 0 \) Differentiating equation (13) and setting it equal to zero we note that the right hand side of equation (13) is constant. As such \( \tau_{c1} \) and hence from equation (11)

\[
p_{1}' = \left( \frac{dp}{dx} \right)_{1} = \text{const} = \frac{p_{m}}{l_{1}}
\]

where \( p_{m} \) is the pressure at the step and \( l_{1} \) is the length of the first section of the unit.

Continuity of flow gives, \( Q_{1} = Q_{2} \) and therefore it is necessary to establish the flow equations for the second part of the unit before \( p_{m} \) can be predicted. The steady state flow in the second part of the unit gives,

\[
\left( \frac{dp}{dx} \right)_{2} = -\left( \frac{d\tau}{dy} \right)_{2}
\]

The boundary conditions are as for part one, so that shear stress in the second unit becomes
\[
\tau c_1 = \left( -\frac{\mu v}{2K} + \left( \frac{\mu^2 v^2}{4K^2 h_1^2} + \frac{1}{27} \left( \frac{1}{K} + \frac{1}{4} p_1 h_1^2 \right) \right)^{\frac{1}{3}} \right) + \left( -\frac{\mu v}{2K} - \left( \frac{\mu^2 v^2}{4K^2 h_1^2} + \frac{1}{27} \left( \frac{1}{K} + \frac{1}{4} p_1 h_1^2 \right) \right)^{\frac{1}{3}} \right)\frac{1}{2} p_1 h_1 \quad (14)
\]

And the flow becomes,

\[
Q_2 = -\frac{p_2 h_2^3}{6\mu} + \frac{\tau c_1 h_2^2}{2\mu} + K \left( -\frac{p_2 h_2^5}{20} + \frac{\tau c_1 h_2^2}{2} + \frac{p_2 h_1^4}{4} \tau c_1 - \frac{\tau c_1 p_2 h_2^3}{2} \right) + Vh, \quad (15)
\]

Again from steady state flow, \( \frac{d}{dx}(Q_2) = 0 \), Thus differentiating equation (15) and noting that \( \tau c_2 = \) constant we have

\[
p_2' = \left( \frac{dp}{dx} \right)_2 = \text{const} = \frac{p_m}{l_2}
\]

and \( l_2 \) is the length of the second section of the unit. It is shown that the pressure profile in the first and the second section of the unit is linear. It is known that pressure increases the viscosity of polymers and this effect should be included in the analysis for a satisfactory solution. With reference to Figure 1.2, a generalised equation relating viscosity and pressure may be shown to take the form [49]

\[
\mu = \mu + \frac{a + bp^2}{\gamma}
\]

where, \( a \) and \( b \) are constants and \( \mu \) is the initial viscosity of non-Newtonian at ambient pressure, \( \gamma \) is the apparent shear rate and is given by \( \gamma = v/h \)
Hence, substituting \( \gamma \) into above equation gives,

\[
\mu_1 = \mu + \frac{h_1}{V} (a + b p^2)
\]

for the first section and

\[
\mu_2 = \mu + \frac{h_2}{V} (a + b p^2)
\]

for the second section.

The apparent shear rate in the first section is \( v/h_1 \) and the apparent shear rate in the second section is \( v/h_2 \). Since \( v/h_2 \) is much higher than \( v/h_1 \). Also the length of the first segment is 4 times greater than the length of the second segment at the lower shear rate the effect of pressure on viscosity is more influential than at the higher shear rate. So, considering \( \gamma = v/h_1 \),

\[
\mu = \mu + \frac{h_1}{V} (a + b p^2)
\]

Equation (16) enables to calculate the viscosity of the polymer at known pressure and shear rate. This equation together with equations (12), (13), (14) and (15) may be solved simultaneously. Numerical values of \( p = p_m \) and hence \( p_1 \) and \( p_2 \) may be substituted into the above equations by assuming the value of 'b' in the same manner as in reference [49] and then, progressively changing the value of 'a' until the theoretical and experimental results show close agreement and the equation \( Q_1 = Q_2 \) is satisfied 'a' is obtained by using an iteration technique. Therefore the pressure in the unit, viscosity of the polymer and shear stresses (\( \tau_{c_1} \) and \( \tau_{c_2} \)) can be determined. The pressure values for different velocities when compared with those obtained experimentally from the rheometer enable determination of the constants, \( a \) and \( b \). The value of constants, pressure and viscosity can of course be determined for any other form of the viscosity – shear rate – pressure relationship, eg,
\[ \eta = K_2 \gamma \]  

[17]

This is the well known "power-law" model and \( n \) is called the power-law index. \( K_2 \) is called the consistency (with the strange unit of \( \text{Pa}s^n \)) [1]

**Newtonian model**

If the polymer is considered as a Newtonian fluid, the relation between the shear stress, viscosity and shear rate is given by \( \tau = \mu \left( \frac{du}{dy} \right) \) and the pressure at the step is [49]

\[
P_{n} = \frac{6 \mu V}{h_1} \left( \frac{h_1 - h_2}{L_1} \right) + \frac{h_2}{L_1} \left( \frac{h_2}{L_2} \right)
\]  

(18)
Chapter Three

DESIGNING AND COMMISSIONING OF THE RHEOMETRIC DEVICE AND TEST PROCEDURE

3.1 General Description

Experiments were carried out using a general purpose test bench, which has been newly constructed. The general view of the test bench is shown in plate 3.1 and Fig 3.1 (Schematic diagram). The test bench is approximately 1.5 m long and 1 m wide. The Rheometric Device is driven by means of an electric motor, squirrel cage 3kW, 380 volts, 3 phase power supply. The build-up of speed is determined by the accelerator time set on the frequency inverter, which varies the motor speed settings. The actual motor speed (rev/min) is obtained using a remote hand held digital tachometer (Shimpo type DT-205) which measures the speed of a mark rotating on the output shaft of the motor. The above arrangement facilitated shearing speeds between 0.25 to 2 m/sec. The accelerator time and remote hand held digital tachometer are shown in plate 3.2. Two pressure transducers were mounted in two different locations in the pressure unit, which enabled the pressure distributions along the unit to be measured. The output from the transducers was taken from the RDP data recording unit. One thermocouple was mounted to monitor the temperature within the unit. The test bench supported all electrical and mechanical equipment and ensured that the drive system was rigid. It contains a flat surface area on which experiments can be done safely.
3.2 Instrumentation

In order to determine the feasibility of the system and the correlation between the theoretical and experimental results, a number of piece of equipment and various devices were used to monitor, control, display and record various parameters during the experimental tests. These included pressure transducers, digital magnetic pickup, thermocouple, electric motor, digital tachometer, RDP data unit. Details of each are given below.

3.2.1 Pressure Transducers

Two pressure transducers (model S/1542-09G) were used to monitor the fluid pressure in the rheometric pressure unit. The pressure transducers feature small physical size with an excellent frequency response and are eminently suitable for the measurement of dynamic pressures and pressure transients. The S/1542-09G pressure unit is threaded M8X 1, the front face of the unit providing a flat surface for sealing. The bonded strain gauge system results in a robust transducer of high accuracy that is suitable for both static and dynamic pressure measurements. The maximum working pressure limit of 1000 PSIG, calibrated at 1000PSIG and with a maximum excitation voltage of 5 volts, calibration factor 2 3614MV/V, shunt resistor 59KΩ. Plate 3.5 shows the pressure transducers.
3.2.2 Digital Magnetic Pickup

The model RS304-172 pickup provides a digital pulse output whenever there is an abrupt change from non-magnetic to magnetic material moving past the piece. The rise–fall times and amplitude of the output pulse are independent of the characteristics and speed of the magnetic discontinuity. The maximum rise time is 1000 nanoseconds, the maximum fall time is 50 nanoseconds. The magnetic pickup was connected between the motor and the shaft. The shaft of the machine was driven by means of the motor via the digital magnetic pickup. Plate 33 shows the magnetic pick up in place.

3.2.3 Thermo-couple

To monitor the temperature continuously, a K type thermocouple “fibre glass insulated with 2 mm insulation diameter” was used to carry out the experiments. The working temperature range of the thermocouple is 0 to +45°C. This is shown in plate 3.

3.2.4 The Variable Speed Motor

A variable speed motor, standard squirrel cage 3 kW, 380 volt, 3 phase power supply was used. This was estimated to give the motor an operating speed range of between 150-2800 RPM, but in reality gave a speed range of 100-2600 RPM. The motor’s direction can be altered easily. The electric motor can be over speeded. The sizing of the motor was carried out allowing 50% factor of safety. The motor is shown in plate 33.
3.2.5 Digital Tachometer

A digital tachometer was used to either monitor or to control the speed of the shaft. It had a frequency regulator to control the speed. The frequency was directly read from the tachometer. It was manually controlled to increase or to decrease the speed of the shaft. The frequency inverter was normally operated from the remote control unit. From the remote control unit, one could run the motor forward or reverse, start or stop the motor, adjust the speed of the motor and read the frequency at which the motor was being run at.

3.2.6 RDP Unit

The E308 is a comprehensive transducer indicator instrument providing signal conditioning, excitation and ±19999 digit indication for strain gauge type transducers such as load cells, etc. and for transducers with built-in electronics producing high output (e.g., 5 volts) signals. The two pressure transducers were connected to the RDP unit. Transducer 1 was connected to RDP1 and Transducer 2 was connected to RDP2. RDP1 was calibrated at 631 and RDP2 was calibrated at 275 according to their shunt resistance. The RDP unit is shown in Plate 3.2.

3.3 Design of the Rheometric Device

The principal objective of the experimental programme was to develop a novel Rheometric Device and to investigate the pressure dependent rheological
properties of non-Newtonian fluids. The Rheometric Device developed is actually a stepped gap pressure hydrodynamic unit made from mild steel parts. The total length of the pressure unit is 15 cm which consists of the pressure cylinder, two sets of inserts, the central shaft, the pressure end plate, and the shaft end plate. The original layout of the pressure unit can be seen in plate 3.2 and plate 3.4. The detailed design of the pressure unit is shown in Figs. 3.2 and 3.3. It was designed so that one could gain easy access to the pressure unit without having to dismantle the whole unit.

3.3.1 Insert 1

The length of each of the three Insert 1 is 40 mm and the widths are of insert 1 is 24.2, 24.5 and 24.7 mm. The Insert 1 is attached to the outer casing by two M6 socket head cap screws (SHCS). The screws were drilled in 15 mm deep and copper sealing was used to prevent leakage through the threads of the screws. There is a hole in the middle of Insert 1 to mount the pressure transducer. When the shaft rotates, pressure develops within the units and the pressure reading was obtained from the RDP unit. The details of the insert 1 are given in Fig. 3.4. And a photograph of the insert 1 is shown in plate 3.7.

3.3.2 Insert 2

The length of the insert 2 is 10 mm and width is 24.9 mm. The Insert 2 is attached to the outer casing of the pressure unit by SHCS in two places. One pressure
transducer was mounted in the middle of the insert 2. The screws were drilled in 15mm deep and again copper sealing was used to prevent leakage through the threads of the screws. The pressure reading was obtained from the RDP unit. The Insert 2 was attached adjacent to insert 1 within the pressure cylinder. The detailed drawing is shown in Fig 3.5. The photograph of the insert 2 is shown in plate 3.7

3.3.3 shaft

The Pressure chamber consists of two inserts, shaft and pressure end plate. The shaft is rotated inside the pressure chamber. The shaft is a solid cylinder whose diameter is 50mm and the length is 90mm and is made of stainless steel. The shaft was supported by two FAG6012 type bearings. The shaft was attached to the shaft end plate and there is a V groove for fixing a ‘O’ ring. The dimension of the O ring is 3.0 wide x 2.0 deep. The shaft is attached to the motor via the digital magnetic pick-up. A frequency inverter to regulate the frequency adjuster can regulate the shaft speed. The output shaft speed was monitored using the tachometer. The detailed design of the shaft is given in Fig. 3.6. Plate 3.3 shows a photograph of the shaft.

3.3.4 Pressure End Plate

The pressure end plate was made of mild steel. The diameter of the pressure end plate is 150mm. The thickness is 6mm. The end plate is attached to the pressure chamber by four M8 bolts. The bolts were drilled through 6.5mm dia on a 125mm
The diameter of the shaft end plate is 150mm The thickness is 15mm The end plate is attached to the pressure cylinder by four M8 bolts The bolts were drilled through 8.5mm diameter on a 125 PCD in 4 places The drawing is shown in Fig 3.8 and plate 3.8 shows the shaft end plate.

3.4 Design of the Electrical Installation

The electrical system for the bench were designed so as to allow multi-user facilities with associated interlocks so that equipment cannot be turned on without its control sensor to monitor the process.

The electrical installation basically is divided into two parts -

1. The electrics for the motor
2. The electrics for the RDP unit

The bench was to be connected by a 3 phase plug to the main supply and all electrical on the bench were isolated from the outside by means of a 3 phase isolated switch and a fuse box. From the fuse box, electricity was fed to the motor controller and the RDP unit. From the fuse box, the 3 phases were brought to the KEB.
combivert 56-frequency inverter. The 3KW motor was then connected from this inverter by means of a shielded cable. Photo 3.6 shows the shielded electrical insulation.

3.5 Test procedure with the Novel Rheometric Device

Experimental work was carried out using the purpose built test bench instrumented to measure the drawing speed, the temperature and the hydrodynamic pressure distribution within the stepped gap pressure unit. The schematic diagram of the test bench is shown in Figure 3.1. The motor was attached to the shaft of the pressure unit via a coupling (magnetic pick up) for running the machine.

The test bench is instrumented to facilitate continuous monitoring and recording of the pressure with the pressure transducers, which were mounted on the pressure unit at two different points to measure the pressure. Thermocouples were used to monitor the temperature. The shearing speed of the shaft was measured using a digital tachometer.

In carrying out the experiments, the following procedure was followed:

First the hydrodynamic pressure chamber was cleaned out mechanically and assembled together. To prevent leakage during the test period, all the parts of the unit were sealed properly. Then the pressure chamber was filled with enough fluid for a complete test programme. The pressure transducers were connected to the RDP unit. The RDP1 and RDP2 were calibrated at a desired value. After switching on, it was ensured that the readings from the RDP were not stable. The readings from the RDP unit were made at the calibrated value and after one hour the readings were at the
desired calibrated value. Then the RDP units were switched on from the calibration level and the readings were made to reach zero level. About an hour was needed for the RDP reading to reach zero level. The speed of the motor was increased slowly to a certain speed by the accelerator set on the frequency inverter. The pressure within the unit was recorded from the RDP unit keeping the temperature constant at 25±1°C for the selected speed. The temperature was monitored from the thermocouple. The speed of the motor was monitored from the digital tachometer. For each speed and every fluid, at least 6 data were recorded. At the end of each test, the speed of the motor was increased and the pressure reading and the temperature were recorded. Tests were carried out for at least 4 speeds for the same fluid and the same insert combinations. After completing the tests, the insert 1 was changed and a similar number of data were recorded for the same fluid. Then both the fluid and the insert 1 were changed and tests were carried out following the same procedure. At the end of each test, the motor and the display units were switched back to their original positions, the test number was recorded on the data sheet, and all the parameters were recorded for subsequent collection and analysis.
Fig 3.1: Schematic diagram of the drawing bench
Fig. 3.2: Pressure Unit.
Fig 3.3 Pressure Cylinder
Fig.3.4: Insert 1.
Fig. 3.5: Insert 2.

60
Fig 36: Shaft
Fig. 3.7: Pressure End Plate.
Fig 38 Shaft End Plate
Fig 3.8: Shaft End Plate

- Third Angle Projection

- Tolerances unless specified

- Fractional decimals ± 0.1

- Angle tolerances ± 1

- Surface finish

- Drill thru 5 dia on a 1250 PCD

- 4 Places

- D Ring Groove

- On a 1085 PCD

- 30 Wide x 20 Deep

DCU MECHANICAL ENG DEPT

END PLATE

DRAWN

DATE 18/6/97

CHECKED

APPROVED

ASS'T ENG

ENG

PROJECT 0.3/0.4 5/12 - 3/9
Plate 3.1: Overall view of the rig.

Plate 3.2: RDP Unit, and frequency Inverter.
Plate 3.3: Electric motor and magnetic pickup.

Plate 3.4: Pressure Chamber.
Plate 3.5: Pressure transducer and thermocouple.

Plate 3.6: Electric Controller.
Plate 3.7: Insert 1.

Plate 3.8: End Plate.
Chapter Four

EXPERIMENTAL WORK AND MATERIAL

4.1 The high Pressure Rheometric Device and the Test Procedure

The details of the new rheometric device and the experimental procedure were described in chapter three. However, a brief description of these is repeated here. The experimental set-up consists of the test bench, the electrical installation, the drive system, the motor, the pressure unit, thermocouple, pressure transducer. Schematic diagram of the process is shown in Fig. 3.1

The pressure chamber was filled with the selected fluid for a complete test programme. Two-pressure transducers and one thermocouple were connected to the pressure unit.

The pressure transducers were connected to RDP unit and calibrated at a desired value. After switching on, the reading of the RDP unit was to set the desired values. It takes about one hour to establish the RDP setting. Then, switching on the frequency inverter in reverse direction for running the motor it was set at a desired speed, keeping the temperature at 25 ± 1°C. The pressure readings from the two transducers were taken. For each set of inserts at least 6 readings were taken at the same speed for the same fluid. For different fluids and different speeds a number of experiments were carried out.
4.2 Working Principle of the Device

The new rheometric device (Fig 4.1) consists of two cylinders, one is the hollow fixed cylinder and the other is a rotating solid shaft. The gap between the two surfaces was filled with viscous non-Newtonian fluid. The shaft is attached to the motor via a coupling. Two pressure transducers were attached to the pressure unit to monitor the pressure developed in the unit. The shaft can be rotated at different speeds to generate different shear rate and due to drag force, hydrodynamic pressure develops. At different shear rates, different pressure readings were obtained. A thermocouple has been inserted into the pressure unit at the position A shown in Fig 4.1 to monitor the fluid temperature continuously.

4.3 The Pressure Fluid

Polymer melts have been used in die-less drawing process by previous workers because of their inherent high viscosity, which permitted reduction of length of the required pressure unit, offered the flexibility and the provision of a coating of the product for several industrial applications. Several types of polymer melts have previously been tested to meet various industrial requirements in which a melt chamber was heated to a temperature suitable for the particular polymer, which has been used at the entry end of the pressure unit.

The present experimental investigation concentrates on the performance of the high-pressure rheometric unit using glycerine and two silicones with vastly different viscosities, 5 and 12.5 N Sec/m². Glycerine, a very important polyhydroxy alcohol, has been traditionally produced as a by-product in soap and fatty acid.
industry. It was produced for the first time on a large scale using the sulphite-steered yeast process during World War I when demanded for glycerol as explosives for wartime use exceeded the supply from the soap industry [73].

The term glycerol applies only to the pure chemical compound 1,2,3-propanetriol \( \text{CH}_2\text{OH} \ \text{CHOH} \ \text{CH}_2\text{OH} \). The term glycerine applies to the purified commercial products normally containing >95% of glycerol. Several grades of glycerine are available commercially, which differ somewhat in their glycerol content and in other characteristics such as colour, odour and trace impurities. Glycerine is a clear, water-white, viscous, hygroscopic liquid with a sweet taste at ordinary room temperature above its melting point. Glycerine is completely soluble in water and alcohol, lightly soluble in diethyl ether, ethyl acetate and dioxane and insoluble in hydrocarbons. Glycerine has a high boiling point of 290 °C at atmospheric pressure [74].

The study of the chemistry of silicon and its compounds began with the discovery of the element in 1824. Silicon polymers are made from organosilicon intermediates prepared in various ways from elemental silicon, which is produced by reducing quartz in an electric furnace. The silicon fluids are low polymers produced by the hydrolysis reaction [75].

The silicon fluids (polydimethyl siloxane) are widely used in many industrial applications due to their numerous advantages. In particular, they are transparent liquids that have remarkable mechanical, chemical, and thermal stability. Among other features, they have low surface tension coefficients, around 20 dyn/cm, and high zero shear viscosity values of up to 30,000 mPa s.

Silicons are polymers melted at ambient temperature. Consequently, they can be used in cold state. They have been commonly used in the production of fibre-optic
cables. They are common place in many industrial-coating mixtures in paper coating processes. In general, silicon fluid has been used in coating, seals, gaskets, adhesives, and medicine [76]. These compounds are used for paper and textile treatment. In the present study, the rheological properties of two silicon fluids with viscosities of 5 and 12.5 N Sec/m² have been discussed. For the sake of brevity, these fluids will be called silicon 5 and silicon 12.5 respectively.

4.4 Brookfield Digital Viscometer (Model DV-1+ Version 2.0)

A number of tests were carried out using a BDV to determine the viscosity of the fluids used in this study. Three tests provided viscosity values at shear rates of up to 2-29 sec⁻¹ and at atmospheric pressure only.

The Brookfield DV-1+ Viscometer measures fluid viscosity at lower shear rates. The principle of operation of the DV-1+ is to drive a spindle (which is immersed in the test fluid) through a calibrated spring. The viscous drag of the fluid against the spindle is measured by the spring deflection. Spring deflection is measured with a rotary transducer. The measuring range of a DV-1+ is determined by the rotational speed of the spindle, the size and the shape of the spindle, the container and the full scale torque of the calibrated spring. The viscometer can measure the viscosity in the range 15 m Pas to 2,000,000 m Pas and with the spindle no. 16 for the shear rate range of 0 to 29.0 per second. For this shear rate range, the viscosity will be between 120 to 400,000 m Pas [77]. Plate 4.1 shows the Brookfield viscometer.
45 Test Procedure with the Brookfield Viscometer

The power cord to the socket on the back panel of the instrument is connected and plugged into the appropriate AC line.

451 Autozero

Before taking any reading, the viscometer must be Autozeroed. Once power switch is turned to the on position, the DV-1+ is in autozero sequence. After removing the spindle and the pressing key, the viscometer becomes autozeroed.

452 Spindle Selection

Spindle selecting screen displays a screen indicating as shown below:

<table>
<thead>
<tr>
<th>Cp 0 0</th>
<th>sp 01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 RPM</td>
<td>%0 0</td>
</tr>
</tbody>
</table>

where cp = viscosity, sp = select spindle, RPM = speed, % = % of torque. The spindle was chosen LV4 and it was attached to the lower shaft. The shaft was lifted slightly, held firmly with one hand while screwing the spindle on with the other hand. The spindle was inserted and centered in the test material until the fluid level is at the immersion groove in the spindle's shaft. The code was 64 for the spindle LV4. Pressing the "SELECT SPINDLE" key and using the up and down arrow, the code of the spindle was made 64.
4 5.3 Speed Selection

There are 18 speeds programmed into the standard DV-1+. The viscometer speed was chosen by pressing the "UP" and "DOWN" arrow keys. The speed and the viscosity readings were selected or noted when the % torque was greater than 10%. For different sets of speeds, the viscosity readings have been recorded. The same procedure was carried out for different types of fluids.
Figure 4.1  Schematic diagram showing the pressure unit.
Plate 4.1: BROOKFIELD Viscometer.
51 Introduction

Experimental results obtained by using the novel Rheometric Device (the stepped gap pressure unit) and the Brookfield Viscometer are presented in graphical form in this section. A number of experiments were carried out to investigate the performance of the unit by using glycerine, silicon 5 and silicon 12.5 as the pressure medium. In obtaining experimental results, the inlet gap \( h_1 \) was changed whilst the outlet gap \( h_2 \) was kept constant at 0.1 mm.

To achieve change in gap ratios, the parameter \( h_1 \) was changed to 0.8 mm, 0.5 mm and 0.3 mm. The temperature was kept constant at 18 ± 1°C for glycerine and 25 ± 1°C for the silicon 5 and silicon 12.5. For each gap ratio, tests were performed at linear speeds of between about 0.25 m/sec and about 2.0 m/sec for both types of fluids. Using the Brookfield viscometer, the viscosity values were measured at atmospheric pressure and at shear rates between 2 to 29 sec\(^{-1}\).

52 Results of Pressure

The generated pressures in the stepped parallel gap pressure unit of the Rheometric Device were measured by means of pressure transducers, which were mounted on the unit.
The results of the pressures are divided into two sections

(i) The pressure profiles along the length (pressure distribution)

(ii) Maximum pressure Vs speed

5.2.1 Results of the Pressure Distribution

Pressure distribution within the stepped gap pressure unit, based on experimentally measured pressure at two different locations, one in the middle of the first insert of the unit and the other in the middle of the second insert of the unit, have been obtained and plotted for the gap ratio $h_1/h_2 = 8$, $5$ and $3$. These results exhibited similar trends with slight differences as the ratios and speeds were altered. Pressures were found to increase from the entry point of the stepped parallel gap in the unit towards the location of the first pressure transducer and the step and decreases after the step. A straight line is drawn through the zero pressure at the entry part of the unit and the pressure reading at the location of the transducer 1 and is extended up to the step. A second line is drawn from the step to the pressure reading at the transducer 2 and is then extended to the exit end of the unit where the pressure is again zero. The pressure distribution line from the location of transducer 1 to step and from the step to the location of transducer 2 is shown as a dotted line in the figures. The pressure readings at the entry and the exit points of the unit are zero. The pressure at the step could not be measured since it was found difficult to mount a pressure transducer there without damaging the step configuration.

Fig. 5.1 shows the pressure profiles for glycerine for different velocities at the gap ratio ($h_1/h_2=3$). It is evident from fig 5.1 that for the range of shearing speeds considered, the maximum pressure occurs at the step and that this maximum pressure
increases with the shearing speed. For example, at the shearing speed of 0.5 m/sec, the maximum pressure is about 4 bar whereas at 1.5 m/sec shearing speed, a maximum pressure of 13 bar is obtained. Fig 5.2 shows the pressure distribution of glycerine for different velocities at the gap ratio of 5. The maximum pressure is about 3 bar when the velocity is 0.5 m/sec and the maximum pressure is 9 bar at the velocity of 1.5 m/sec which is higher than the maximum pressure at the speed of 0.5 m/sec. Fig 5.3 shows the pressure distribution for glycerine at the gap ratio of 8 for different velocities. Fig 5.3 shows the similar trends as those in Figs 5.1 and 5.2, i.e., as the shearing speed increases, the maximum pressure increases. Fig 5.4 shows the pressure profiles for glycerine at the speed of 0.5 m/sec at three different gap ratios. For a higher gap ratio (gap ratio of 8) the maximum pressure 2.3 bar was obtained at the shearing speed of 0.5 m/sec which is low compared to the maximum pressure at the lower gap ratio. From the figure 5.4 it is evident that the higher the gap ratio lower is the maximum pressure. Figs 5.5 and 5.6 show the pressure profiles for glycerine at the speed of 1.0 m/sec and 1.5 m/sec at three different gap ratios. These two figures (Figs 5.5 and 5.6) show the similar trend as in Fig 5.4.

Figs 5.7 to 5.12 show the pressure distribution curves for silicone fluid whose viscosity is 5 Ns/m². Fig 5.7 shows the pressure distribution curve for silicone 5 at the gap ratio of 3, the shearing speed is 0.25 m/sec and 0.5 m/sec. Fig 5.8 shows the pressure profile at the same gap ratio and the shearing speed of 1.0 to 2.0 m/sec. These figures show that the pressure increases with the increase of speed. Figs 5.9 and 5.10 show the pressure profile for silicone 5 at the gap ratio of 5. From the Fig 5.9 results of maximum pressure is 16.1 bar at the speed of 0.25 m/sec and the maximum pressure is 18.6 bar at the shearing speed of 0.5 m/sec. Speed appears to be the dominant factor for the pressure distribution. Figs 5.11 and 5.12 also show the
pressure profile for silicon 5 for different shearing speeds at the gap ratio of 8. These two figures show the similar trend as in Fig 5.1. Fig. 5.13 shows the pressure distribution of silicon 5 at the speed of 0.25 m/sec at different gap ratios. It is found that if the gap ratio is 5 then the maximum pressure is 16.1 bar, at the gap ratio of 3 the maximum pressure is 15.1 bar and at the gap ratio of 8 the maximum pressure is 12 bar. At the same speed the maximum pressure is higher at the gap ratio of 5. Fig 5.14 shows the pressure profile of silicon 5 at the speed of 2 m/sec for different gap ratios. The pressure values are 35.83 bar, 33.07 bar and 26.18 bar for the gap ratio of 5, 3, and 8. The gap ratio 5 is the optimum gap ratio. As the inlet gap decreases, the fluid flow rate also decreases in the unit, which in turn is thought to be responsible for lower pressure. An increase in the inlet gap may cause back flow of the polymer, which would also reduce pressure.

Figs 5.15, 5.16 and 5.17 show the pressure distribution for silicon 12.5 at the speeds of between 0.5 m/sec and 2.0 m/sec at different gap ratios. For the gap ratio of 3, the pressure distribution curves for different shearing speeds such as 0.5 m/sec, 1.0 m/sec, 2.0 m/sec are presented in Fig 5.15 for silicon 12.5. For the gap ratio of 5, the pressure distribution curves for different shearing speeds are presented in Fig 5.16. Fig 5.17 shows the pressure profile for the three different velocities at the gap ratio of 8. The pressure profile for silicon 12.5 at the speed of 0.5 m/sec for different gap ratios are presented in Fig 5.18. The maximum pressure is 34.5 bar, 38.5 bar and 30.3 bar for the gap ratio of 3, 5, and 8 respectively. The pressure profiles for this fluid at the velocity of 2.0 m/sec have been presented in Fig 5.19 for different gap ratios. Fig 5.20 shows the pressure profiles for glycerine, silicon 5, silicon 12.5 at the speed of 1 m/sec and at the gap ratio of 5. The maximum pressure for silicon 12.5 is higher than the other two fluids. This is due to the higher viscosity of silicon.
12.5 Almost similar trends of pressure distribution could be noticed from these figures even for these gap ratios, the difference is that the overall gap ratio affects the magnitude of the pressure generated.

5.2.2 Results of Maximum Pressure Vs Speed

Experimental results were obtained in terms of the maximum pressure within the pressure unit of the Rheometer. Experiments were carried out by varying \( h_1 \) in a systematic manner keeping \( h_2 \) constant. For different shearing speed of up to 2m/sec, the maximum pressures were monitored for three different viscous fluids. These results are presented in the following figures. Fig 5.21 shows the effect of speed on the maximum pressure generated within the unit when glycerine is used as the pressure fluid at different gap ratios. This figure shows that at gap ratio of 3, 5 and 8 the pressure Vs velocity curves are almost linear. Fig 5.22 shows the effect of speed on pressure for the silicone 5 fluid. The pressure vs speed curves are almost linear for all three gap ratios, and the maximum pressures occur at the gap ratio of 5. It is evident that the speed increases, the maximum pressure increases. Fig 5.23 shows the effect of speed on pressure for silicone 12.5. At all the gap ratios, the maximum pressure readings are increasing almost linearly with the increase of speed and maximum pressures are obtained for the gap ratio of 5. For the non-Newtonian fluid the developed pressure increases because viscosity changes with the shear rate and affects the pressure. It is evident that as the speed increases, the maximum pressure also increases. Fig 5.24 shows the comparison of the effect of speed on the maximum pressure for glycerine, silicone 5 and silicone 12.5. This figure shows that the maximum pressure is highest for silicone 12.5. The generated pressure depends upon
the viscosity of the fluid, the geometry of the unit and the relative speed of movement

5.3 Determination of the Rheology (Values of the Coefficient a and b)

The values of the coefficients of ‘a’ and ‘b’ for three different viscous fluids have been calculated in conjunction with the experimental and the theoretical results. In reference [49] it was suggested that the value of ‘b’ in equation (16) for a viscous fluid may be taken as equal to $4 \times 10^{11} \text{m}^2/\text{N}$ for calculating the pressure distribution for theoretical results. The initial viscosity, $\mu_0=1 \text{N sec/m}^2$ and the non-Newtonian factor $K=5 \times 10^{-11} \text{m}^4/\text{N}^2$ were taken from reference [49]. Fig 5.25 shows the comparison of experimental pressure distribution of glycerine with the theoretical result at the velocity of 1.5 m/sec for different values of ‘a’ and ‘b’. It is clear that for $b = 4 \times 10^{11} \text{m}^2/\text{N}$, the value of ‘a’ is $300 \text{N/m}^2$, which satisfies the continuity equation $Q_1 = Q_2$ and also gives the theoretical results very close to the experimental results. Therefore, this value of b has been chosen for further iteration of results. For ‘b’ = $4 \times 10^{11} \text{m}^2/\text{N}$, the value of ‘a’ has been changed until the values of theoretical and experimental maximum pressure were very close. Fig 5.26 shows the theoretical pressure distribution for different values of ‘a’ (for $b = 4 \times 10^{11} \text{m}^2/\text{N}$) and the experimental pressure distribution at the speed of 1.5 m/sec. When the value of a = 500 N/m$^2$, the experimental and theoretical results are almost same for the shearing speed of 1.5 m/sec for the gap ratio of 3. The experimental and theoretical pressure values are very close for all the speeds between 0.5-1.5 m/sec, when the values of $a=300 \text{N/m}^2$. As such $b = 4 \times 10^{11} \text{m}^2/\text{N}$ and $a = 300 \text{N/m}^2$ have been accepted as the value for glycerine. Fig 5.27 shows the comparison of experimental pressure
distribution with the theoretical results at the velocity of 1.0 m/sec of silicon 5 for different values of 'a' and 'b'. For the value of \( a = 7000 \text{ N/m}^2 \) and \( b = 5 \times 10^{-11} \text{ m}^2/\text{N} \), the experimental and the theoretical results are very close. Fig 5.28 shows the theoretical pressure distribution for the different values 'a' (For \( b = 5 \times 10^{-11} \text{ m}^2/\text{N} \)) and the experimental pressure distribution at the speed of 1.0 m/sec. When the value of \( a = 7340 \text{ N/m}^2 \), the theoretical results are almost same for the gap ratio of 5 at the shearing speed of 1.0 m/sec. The experimental and the theoretical pressure results are very close for all the speeds between 0.25-2.0 m/sec, when the value of \( a = 7100 \text{ N/m}^2 \) and \( b = 5 \times 10^{-11} \text{ m}^2/\text{N} \). So that \( a = 7100 \text{ N/m}^2 \) and \( b = 5 \times 10^{-11} \text{ m}^2/\text{N} \) have been accepted as the value of silicon 5.

Fig 5.29 shows the comparison of experimental pressure distribution with the theoretical results at the velocity of 1.0 m/sec of silicon 12.5 for different values of 'a' and 'b'. Fig 5.30, the theoretical pressure distribution for different values of 'a' (For \( b = 5.6 \times 10^{-11} \text{ m}^2 \)) and the experimental pressure distribution at the speed of 1.0 m/sec. When the value of \( a = 17100 \text{ N/m}^2 \) and \( b = 5.6 \times 10^{-11} \text{ m}^2/\text{N} \), the experimental and the theoretical values are almost same. For \( a = 17,000 \text{ N/m}^2 \) and \( b = 5.6 \times 10^{-11} \text{ m}^2/\text{N} \), the experimental and the theoretical results are very close for the speeds between 0.5-2.0 m/sec. So that \( a = 17000 \text{ N/m}^2 \) and \( b = 5.6 \times 10^{-11} \text{ m}^2/\text{N} \) have been accepted as the value of Silicon 12.5. For the different values of 'a' and 'b', the comparison of experimental and theoretical pressure values for the three different types of fluids at the different speeds and the error analysis are given in Appendix C.
5.4 Effect of Pressure on Viscosity

Viscosity, by definition, is the internal resistance to the shearing stress due to intermolecular forces of attraction. It was thought that when the molecular attractions are encouraged, the apparent viscosity of the fluids, which is one of the most important properties of these materials, might be increased.

The effect of pressure on viscosity at different shear rates for glycerine is presented in Fig 5.31. The developed pressure for glycerine in the rheometer was in the range 2 to 13 bar. The viscosity of glycerine has been calculated using the coefficient values of the viscosity equation within the range of 0 to 100 bar and from the figure it is evident that with the increase of pressure, the viscosity values are increasing for a given shear rate. The effect of pressure on viscosity for silicon 5 for different shear rates are presented in Fig 5.32. The experimental pressure values were between 10 to 35 bar. Using the coefficient values the viscosity values were calculated and plotted. This figure shows similar trend like those in Fig 5.31. Fig 5.33 shows the effect of pressure on viscosity for silicon 12.5 at different shear rates. The developed experimental pressures were between 30-60 bar. Within these pressure values the viscosities were calculated and are presented in graphical form and from the figure it is shown that with the increase of pressure the viscosity increases at the same shear rate. Fig 5.34 shows the comparison of viscosity values of glycerine, silicone 5 and silicone 12.5 at the shear rate of 2000 sec⁻¹.

83
5.5 Effect of Shear Rate on Viscosity

Newtonian fluids under shear stress condition exhibit linear relationship with shear rate where the slope of the line represents the viscosity of the fluids. For the non-Newtonian behaviour, the apparent viscosity decreases as the rate of shear increases. From the experimental results, using a Brookfield viscometer, viscosity values can be determined for low shear rates. Fig 5.35 shows shear stress vs shear rate curves obtained using the Brookfield viscometer for glycerine, silicon 5, and silicon 12.5. From the figure, it is evident that the curves are almost linear for all three fluids. Using the viscometer, the viscosity has been measured for the three different types of fluids at the lower shear rates. The effect of shear rate on viscosity is presented in Fig 5.36. From the figure, the viscosity increases with the decrease of shear rate.

Experimental work carried out using the Brookfield viscometer was at the shear rate range of up to 30 per second whereas tests carried out using the new Rheometer was in the shear rate range of 500-4000 per second. The values of the coefficients in equation (16) have been determined according to the pressure values obtained experimentally at shear rate range of between 500-4000 sec⁻¹.

Fig 5.37 shows the effect of shear rate on viscosity of glycerine at atmospheric pressure. Using the viscosity equation \( \mu = \mu_0 + \left( \frac{a + bp^2}{\gamma} \right) \) and the viscosity values are much higher than the values obtained at lower shear rate range obtained by using the Brookfield viscometer. Figs 5.38 and 5.39 show the effect of shear rate on viscosity of silicon 5 and silicon 12.5 also at the atmospheric pressure. Once again the viscosity results from the Brookfield viscometer are much lower than
those predicted according to equation (16) is for shear rate range between 500 and 4000 per second as the novel Rheometer can not be operated at lower than about 500 per second Furthermore, the new Rheometer is more appropriate for viscosity measurements at higher than atmospheric pressure As such the comparison of low pressure and low shear rate viscosity values may not be appropriate The experimental data from the novel Rheometer were used to determine the viscosity-shear rate equation given in reference [1] as,

$$\eta = K_2 \gamma^n$$

(17)

The results according to this equation are also shown in figs 5.37, 5.38 and 5.39 and are similar to those given by equation (16) The constants of the equation for three fluids are given:

As K_2 = 7.36, n = 75 for glycerine
K_2 = 1634, n = 234 for silicon 5
K_2 = 7855, n = 12 for silicon 12.5

A curve fitting exercise using the experimental points from the novel Rheometer and the Brookfield viscometer produces the following form of equations

($$\mu = A\gamma + B$$)

$$\mu = -7\times10^{-5}\gamma + 1.3237$$ For glycerine
$$\mu = -0.0012\gamma + 7.8883$$ For silicon 5
$$\mu = -0.0016\gamma + 12.265$$ For silicon 12.5

These equations may be taken as valid over the entire shear rate range of up to 4000 per second but only at atmospheric pressure.
Fig 5.1: Pressure distribution of glycerine for different velocities at the gap ratio of 3.
Fig 5.2: Pressure distribution of glycerine for different velocities at the gap ratio of 5.
Fig 5.3: Pressure distribution of glycerine for different speeds at the gap ratio of 8.
Fig 5.4: Pressure distribution of glycerine at the velocity of 0.5m/sec for different gap ratios.
Fig 5.5: Pressure distribution of glycerine at the velocity of 1.0m/sec for different gap ratios
Fig 5.6: Pressure distribution of glycerine at the speed of 1.5 m/sec for different gap ratios
Fig 5.7: Pressure distribution of silicon 5 for different velocities at the gap ratio of 3.
Fig 5.8: Pressure distribution of silicon 5 for different shearing speeds at the gap ratio of 3.
Fig 5.9: Pressure distribution of silicon 5 for the different shearing speeds at the gap ratio of 5.
Fig 5.10: Pressure distribution of silicon 5 for the different shearing speeds at the gap ratio of 5.
Fig 5.11: Pressure distribution of silicon 5 for different shearing speeds at the gap ratio of 8.
Fig 5.12: Pressure distribution of silicon 5 for different velocities at the gap ratio of 8.
Fig 5.13: Pressure distribution of silicon 5 at the velocity of 0.25m/sec for different gap ratios.
Fig 5.14: Pressure distribution of silicon 5 at the velocity of 2m/sec for different gap ratios.
Fig 5.15: Pressure distribution of silicon 12.5 for different velocities at the gap ratio of 3.
Fig 5.16: Pressure distribution of silicon 12.5 for different velocities at the gap ratio of 5.
Fig 5.17: Pressure distribution of silicon 12.5 for the different velocities at the gap ratio of 8
Fig 5.18: Pressure distribution of silicon 12.5 at the velocity of 0.5m/sec for different gap ratios
Fig 5.19: Pressure distribution of silicon 12.5 at the velocity of 2.0m/sec for the different gap ratios.
Fig 5.20: Comparison of Pressure distribution of glycerine, silicone 5, silicone 12.5 at the gap ratio of 5 and at the velocity of 1.0m/sec
Fig 5.21: Maximum pressure vs speed of glycerine for different gap ratios.
Fig 5.22: Maximum pressure vs speed of silicon 5 for different gap ratios.
Fig 5.23: Maximum pressure vs speed of silicon 12.5 for different gap ratios.
Fig. 5.24: Maximum pressure vs speed of glycerine, silicon 5 and silicon 12.5 at the gap ratio of 5.
Fig 5.25: Experimental and theoretical pressure profile of glycerine for the different values of 'a' and 'b' at the velocity of 15m/sec.
Fig 5.26: Experimental and theoretical pressure profiles of glycerine for the different values of 'a' at the velocity of 1.5m/sec.
Fig 5.27: Experimental and theoretical pressure profile of silicon 5 with different values of 'a' and 'b'.

- $a=4000\text{N/m}^2$, $b=4\times10^{-11}\text{m}^2/\text{N}$
- $a=7000\text{N/m}^2$, $b=5\times10^{-11}\text{m}^2/\text{N}$
- $a=8000\text{N/m}^2$, $b=7\times10^{-11}\text{m}^2/\text{N}$
- Experimental
Fig 5.28: Experimental and theoretical pressure profile of silicon 5 for the different values of 'a' at the velocity of 10m/sec.
Fig 5.29: Experimental and theoretical pressure profile of silicon 12.5 for different values of 'a' and 'b' at the velocity of 1.0 m/sec.
Fig 5.30: Experimental and theoretical pressure profile of silicon 12.5 with different values of 'a'.
Fig 5.31: Effect of pressure on viscosity for glycerine.
Fig 5.32: Effect of pressure on viscosity for silicon 5
Fig 5.33: Effect of pressure on viscosity for silicon 125
Fig 5.34: Effect of pressure on viscosity for silicon 12.5, silicon 5, glycerine at the shear rate of 2000 per second
Fig 5.35: Shear rate vs Shear stress of glycerine, silicon 5, silicon 12.5 using Brookfield viscometer.
Fig 5.36: Effect of shear rate on viscosity of glycerine, silicon 5, silicon 12.5 using the Brookfield viscometer.
Fig 5.37: Effect of shear rate on viscosity of glycerine at atmospheric pressure.

\[ \mu = -7 \text{E}^{-05} \gamma + 13237 \]
Fig 5.38: Effect of shear rate on viscosity of silicon 5 at atmospheric pressure.
Fig 5.39: Effect of shear rate on viscosity of silicon 12.5 at atmospheric pressure.
5.6 Discussion

5.6.1 Introduction

Hydrodynamic principle has been introduced in a novel rheometric device to study the rheological behaviour of non-Newtonian fluids using glycerine and silicon as examples. To investigate the performance of the device, an extensive experimental and theoretical programme was conducted during which a considerable amount of data were obtained. Limited experiments were also done by using a Brookfield viscometer at atmospheric pressure only. This section is aimed to highlight the important results obtained experimentally and theoretically to determine the viscosity-shear rate relationship for glycerine, silicon 5 and silicon 12.5 fluids.

The discussion chapter is divided into three sections: (i) test procedure and experimental work, (ii) analysis and theoretical results, and (iii) error analysis.

5.6.2 Discussion on Test Procedure and Experimental Work

A number of interesting results have been observed while carrying out the experimental tests, using the new rheometric device and glycerine and silicon as the test fluids. During the course of the experimental programme, parameters such as gap ratios, shearing speed, fluid type were varied in order to investigate their effect on the performance of the pressure unit. The length and the length ratio, the outlet gap $h_2$ were constant throughout the tests. Experimental work was first done with the two pressure transducers, their rating was 2000 bar using glycerine as the test fluid. The readings were not changing with the change of the speed. The pressure transducers
were over rated and hence new lower capacity 70 bar ones were selected. These new transducers were found to be sensitive to the variation of pressure with changing shear velocity.

The results of the pressure versus shear rate using silicon 5, silicon 12.5 and glycerine as test fluids are shown in Fig. 5.1 to 5.20. For the gap ratio of 5 the generated pressure is found to be higher. As the inlet gap decreases, the fluid flow rate also decreases into the gap, which in turn is thought to be responsible for lower pressure. An increase in the inlet gap without increasing the outlet gap may cause backflow of the polymer which would also reduce the pressure. From Fig 5.35 it is seen that glycerine and silicon 5 show Newtonian behaviour as the shear stress and shear rate curves are almost linear for shear rate values of less than about 30 per second. Silicon 12.5 shows non-Newtonian behaviour. This is also confirmed from the results in Figs 36. Figs 5.37 to 5.39 show the viscosity vs shear rate curves. At lower shear rate the predicted viscosity values are much higher than those measured using the Brookfield viscometer. The difference in results is due to the fact that experimental work was carried using the new rheometer at higher shear rates and the coefficients were calculated according to theoretical and experimental results at higher shear rate. The thermocouple was connected where the pressure reading is zero, so that the actual temperature cannot be measured. With an increase of one degree centigrade at that location the temperature may increase by several degrees which may decrease the viscosity and hence the pressure readings.
A model has been developed based on the assumption that the pressure medium demonstrates non-Newtonian characteristics. The following assumptions were made in order to simplify the analysis:

(I) The flow of the fluid medium in the present pressure unit is laminar. This seems to be a reasonable assumption since the shearing speeds of the shaft are relatively low and the gaps are small.

(II) The flow of the fluid pressure medium is axial. Once flow through the pressure unit has commenced, little or no back flow is expected. This assumption allowed one-dimensional flow to be considered.

(III) The thickness of the fluid layer is small compared to the dimensions of the Rheometric pressure unit. This assumption enabled the analysis to be conducted in Cartesian rather than cylindrical co-ordinates.

(IV) The pressure in the fluid medium is uniform in the thickness direction. This assumption simplified calculation of the pressure in the unit and allowed the viscosity constant to be evaluated.

(V) The flow of the fluid through the unit is isothermal. This may introduce some error in the results since viscosity of the fluids is sensitive to the temperature change.

The empirical non-Newtonian equation commonly used to represent the behaviour of the fluid, as proposed by Rabinowitch [12], takes the form:

\[ \tau + K \tau^2 = \mu \frac{du}{dy} \]

This equation was used in the analysis to predict the pressure. The non-Newtonian factor 'K' and the initial viscosity \( \mu \) were both determined experimentally by curve...
fitting the above equation over the results obtained from the flow characteristics of polymer melt

The above equation represents the effect of shear stress on the viscosity only. It is known that the pressure increases the viscosity of the fluid. To incorporate this effect in the analysis, the data provided by Westover for the Alkathene polymer was used to produce Fig 1 2. Which shows that the influence of pressure on the viscosity is dependent on the shear rate. The accuracy of Westover's results is not known and unfortunately the polymer used for his experiments was not WVG 23. Therefore, this may have introduced some discrepancy in the theoretical results in Fig 1 2. A generalised equation was derived to predict the viscosity of the polymer melt at known pressure and the shear rates which is shown to take the form

\[
\mu = \mu_0 + \frac{a + bp^2}{\gamma}
\]

where \( \gamma \) is the apparent shear rate in the first part of the unit and assumed to be \( \frac{V}{h_1} \).

The theoretical value of \( \mu_0 \) and non-Newtonian factor 'K' have been taken from ref [49] in which similar fluids were used. However, there were some differences in the properties, which may produce some error in the theoretical results.

5 6 4 Error Analysis

Errors always occur in experimental work owing to inaccuracies of other variables, however small they may be, which the final results depend upon. Investigation of the probable error is therefore necessary before results can be considered with some degree of confidence. Theoretical results were predicted using
numerical techniques incorporating iteration methods. In conjunction to the experimental and theoretical results, the values of the pressure coefficients of viscosity have been determined using an iteration technique. For different values of 'a' and 'b' different theoretical pressure values have been obtained at the same shearing speed. Error has been calculated within the difference between the theoretical and the experimental pressure values. Appendix C shows the error analysis for different types of fluid. For glycerine (a = 300 N/m², b = 4 x 10¹¹ m²/N) at the gap ratio of 8, 5 and 3 at the speed of 0.5 m/sec the percentage of errors are 12, 14 and 11.8 respectively. On the other hand for the same fluid for the same value of 'a' and 'b' at the gap ratio of 8, 5 and 3 at the speed of 1.5 m/sec, the percentage of errors are 7, 5.5 and 2.4. A possible arithmetic error of about ±15% were occurred in the theoretical and experimental pressure values while calculating the values of the pressure coefficients of viscosity. A considerable difference has been observed between some of the experimental and the theoretical results, based on mean values of 'a' and 'b' of equation (16). This suggests that the analysis carried out for the rheometric device needs more refinement. It is believed that the discrepancies may have arisen due to the value of non-Newtonian factor 'K', the absence of the parameter in the analysis which affect the flow properties of fluids.
Chapter Six

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

6.1 Introduction

This chapter presents a summery of the current thesis. It is divided into two sections. The first section represents the investigation under consideration, the techniques to carry out this task and a brief summery of the conclusion, and the second identifies further studies that can be pursue in future works.

6.2 Conclusion

This thesis outlines the author's research effort in the area of high-pressure non-Newtonian fluid rheology.

- This work involved measurement of the variation of pressure with shear rate and gap ratio for glycerine and silicon using a novel rheometer designed on the basis of hydrodynamic principles. The range of shear rate is 500 to 4000 per second.

- A theoretical analysis based on the non-Newtonian characteristics for a new rheometric device has been presented where the effect of the hydrodynamic pressure influences the viscosity of the fluids.

- The influence of the geometry of the unit and the speed on the maximum pressure generation, which affect the viscosity, has also been discussed. Experimental results from the pressure distribution within the stepped
parallel gap Rheometric device have been used in conjunction with the theoretical pressure results to determine the coefficients for a viscosity shear rate

- Using these coefficients, the viscosity at different pressures was calculated and the effect of shear rate and pressure on viscosity was demonstrated

- Using the coefficients the viscosity of these fluids was predicted at lower shear rates at atmospheric pressure. Using the Brookfield viscometer, viscosity values of the three different types of fluids were measured at different lower speeds and the shear rate vs viscosity curve has been plotted. These values were much higher than those measured using the Brookfield viscometer

- New type of equations have been developed at the shear range 0 to 4000Sec\(^{-1}\) based on the experimental results using the BDV and New Rheometer

- The new Rheometric device is an innovative facility in high-pressure fluid rheology

Although considerable amount of work on the process was done so far, there are more avenues of work on this process in the future

6.3 Suggestions For Future Work

- In the present work the analysis was developed assuming isothermal condition. It is suggested that a great deal of knowledge could be obtained about the rheology of different non-Newtonian fluids at elevated temperatures
• The viscosity of a fluid is known to be sensitive to the temperature change, hence a relationship predicting the viscosity change due to the temperature variation may improve the results.

• The rheology of the fluid should be better understood with respect to the effect of pressure and temperature on viscosity. So, the theoretical model should be based on both the temperature and pressure.

• In the present study the temperature of the fluid was monitored outside the pressured gap where the hydrodynamic pressure is zero. The temperature should be measured within the gap. But it is difficult to insert a thermocouple in the step without damaging the step configuration. So, finite element modelling can be used for predicting the theoretical temperature in the step.

• The geometry of the pressure unit could be changed to tapered, combined or parabolic types.

• The pressure transducers could be placed at the nearest position of the step.

• The experimental work could be done at the lower shearing speeds modifying the thickness of the entry gap of the unit and at the lower shear rate, the coefficient values can be obtained.
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Publications Arising From the Present Work


APPENDIX A

Listing of Computer Programme for the analysis based on
non-Newtonian Fluid Characteristics
Listing of Computer Programme for the analysis based on non-Newtonian Fluid Characteristics

10 DIM P(500),H(50),V(50),A(100),B(100)
20 PRINT "THIS PROGRAMME CALCULATES THEORETICALLY THE MAXIMUM PRESSURE, VISCOSITY, COEFFICIENTS OF VISCOSITY"
30 PRINT
40 AL1= 04 AL2= 01
50 H1= 0008 H2= 0001
60 AVIS=1 PC=5.6E-11
70 V=4
80 V=V+1
90 VEL=V/10
100 PRINT "THE VALUE OF B"
110 INPUT B
120 PRINT "THE VALUE OF A"
130 INPUT A
140 SRT=VEL/H1
150 PSTEP=1000000
160 REM1=1E-9
170 PEX=479613
180 P1=100
190 P1=P1+PSTEP
200 VIS=AVIS+(((P1^2)*(B))+A)/SRT)
210 DP1=P1/AL1
220 POIS1=(4+(PC*(DP1^2)*(H1^2)))/(12*PC)
230 QUET1=(VIS*VEL)/(2*PC*H1)
240 T11=QUET1+(SQR((POIS1^3)+(QUET1^2)))
250 T12=QUET1+(SQR((POIS1^3)+(QUET1^2)))
260 PHY1=(T11^3333)-(T12^3333)
270 TC1=PHY1-(DP1*H1* 5)
280Q1=(((DP1^3)*(H1^3))/(6*VIS))+((TC1*(H1^2))/(2*VIS))+((VEL*H1)+(PC/VIS))*

(((DP1^3)*(H1^5)/20)+((TC1^3)*(H1^2)/2)+((DP1^2)*(H1^4)*TC1^2 25)+( 5*(TC1^2)*

DP1*(H1^3)))
290 DP2=P1/AL2
300 POIS2=(4+(PC*(DP2^2)))/(12*PC)
310 QUET2=(VIS*VEL)/(2*PC*H2)
320 T21=-QUET2+(SQR((POIS2^3)+(QUET2^2)))
330 T22=QUET2+(SQR((POIS2^3)+(QUET2^2)))
340 PHY2=(T21^3333)-(T22^3333)
350 TC2=PHY2+(DP2*H2^5)
360 Q2=-(DP2*(H2^3))/(6*VIS)+(TC2*(H2^2)/(2*VIS))+(VEL*H2)+(PC/VIS)*
    (-(DP2^3)*(H2^5)/20)+((TC2^3)*(H2^2)*5)+((DP2^2)*(H2^4)*TC2^25)*
    (5*(TC2^2)*DP2*(H2^3)))
370 RES1=Q1-Q2
380 IF (ABS(RES1)<=REM1) THEN GO TO 450
390 IF RES1<0 THEN GO TO 410
400 IF RES1>0 THEN GOTO 190
410 P1=P1-PSTEP
420 PSTEP=PSTEP/10
430 GO TO 190
440 ER=P1-PEX
450 PRINT "VEL=", VEL, "PM=", P1, "TC1=", TC1,"TC2=", TC2, "Q1=",Q1,"Q2=", Q2,
    "Q1-Q2=", Q1-Q2
500 END
APPENDIX B

SOLUTION OF THE CUBIC EQUATION
In chapter 2, Equation (10) is a cubic equation which can be solved by applying Cardens formula. This equation however, is shown to have two imaginary roots and one real root.

The cubic equation is of the form,
\[ \phi_1^3 + 3p\phi_1 + 2q = 0 \]

Where \(3p = A_1\) so that \(p = A_1 / 3\),

And

\[ 2q = B_1 \text{ such that } q = B_1 / 2, \]

The discriminant of the equation \((A_1 - 1)\) in the number
\[ J_1 = -p^3 - q^3 \]

Substituting the value of \(p\) and \(q\) in the above equation gives,
\[ J_1 = -(A_1^{3/27}) - (b_1^{2/4}) \]

According to Carden's formula the real root of equation \((A_2 - 1)\) is
\[ \phi_1 = U_1 + V_1 \]

Where \(U_1 = [-q + (q^3 + p^3)^{1/2}]^{1/3}\) and \(V_1 = [-q - q^2 + p^3]^{1/2}\)^{1/3}
Appendix C

Error Analysis
Comparison of experimental pressure with the theoretical pressure for different values of 'a' and 'b' (Error analysis)

Table C 1: For Glycerine

<table>
<thead>
<tr>
<th>Speed m/sec</th>
<th>Values of Pressure Coefficients</th>
<th>Gap ratio H₁/H₂</th>
<th>Experimental Pressure (Pex) bar</th>
<th>Theoretical Pressure (Pth) bar</th>
<th>Percentage of error = (Pex-Pth)/Pex</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>( a = 300 \text{ m}^2/\text{N} ) ( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>2.54</td>
<td>2.07178</td>
<td>15</td>
</tr>
<tr>
<td>0.5</td>
<td>( a = 150 \text{ m}^2/\text{N} ) ( b = 2 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>2.54</td>
<td>1.73412</td>
<td>31.7</td>
</tr>
<tr>
<td>0.5</td>
<td>( a = 600 \text{ m}^2/\text{N} ) ( b = 7 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>2.54</td>
<td>2.7498</td>
<td>10</td>
</tr>
<tr>
<td>1.0</td>
<td>( a = 300 \text{ m}^2/\text{N} ) ( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>3.58</td>
<td>3.47</td>
<td>3.07</td>
</tr>
<tr>
<td>1.0</td>
<td>( a = 150 \text{ m}^2/\text{N} ) ( b = 2 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>3.58</td>
<td>3.133</td>
<td>12.4</td>
</tr>
<tr>
<td>1.0</td>
<td>( a = 600 \text{ m}^2/\text{N} ) ( b = 7 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>3.58</td>
<td>4.1593</td>
<td>16.1</td>
</tr>
<tr>
<td>1.5</td>
<td>( a = 300 \text{ m}^2/\text{N} ) ( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>5.09</td>
<td>4.9</td>
<td>3.7</td>
</tr>
<tr>
<td>1.5</td>
<td>( a = 150 \text{ m}^2/\text{N} ) ( b = 2 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>5.09</td>
<td>4.531</td>
<td>10.9</td>
</tr>
<tr>
<td>1.5</td>
<td>( a = 600 \text{ m}^2/\text{N} ) ( b = 7 \times 10^{11} \text{ N/m}^2 )</td>
<td>8</td>
<td>5.09</td>
<td>5.57</td>
<td>9.4</td>
</tr>
<tr>
<td>0.5</td>
<td>( a = 300 \text{ m}^2/\text{N} ) ( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>3.2</td>
<td>3.64</td>
<td>13.75</td>
</tr>
<tr>
<td>0.5</td>
<td>( a = 150 \text{ m}^2/\text{N} ) ( b = 2 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>3.2</td>
<td>3.2206</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>( a = 300 \text{ m}^2/\text{N} )</td>
<td>( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>6.27</td>
<td>6.46</td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------</td>
<td>-----------------------------------</td>
<td>----</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>10</td>
<td>( a = 600 \text{ m}^2/\text{N} )</td>
<td>( b = 7 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>6.27</td>
<td>7.36</td>
</tr>
<tr>
<td>15</td>
<td>( a = 300 \text{ m}^2/\text{N} )</td>
<td>( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>9.5</td>
<td>9.3</td>
</tr>
<tr>
<td>15</td>
<td>( a = 300 \text{ m}^2/\text{N} )</td>
<td>( b = 4 \times 10^{11} \text{ N/m}^2 )</td>
<td>5</td>
<td>9.5</td>
<td>10.2</td>
</tr>
<tr>
<td>05</td>
<td>( a = 300 \text{ m}^2/\text{N} )</td>
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**Note:** The table represents data with some examples of forces and areas in different units, with calculations and results provided in numerical form.
### Results For Glycerine

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<th>Speed m/sec</th>
<th>Values of Pressure Coefficients</th>
<th>Gap ratio $H_1/H_2$</th>
<th>Experimental Pressure (P+h) bar</th>
<th>Theoretical Pressure (p+h) bar</th>
<th>Percentage of error =</th>
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## DATA FOR SILICON 5

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