This model is intended for evolutionary methods in technology R & D, and is proposed for compilation, manipulation, and computation of genetics-equivalent information in technology. The model is based on axioms of controlled physical matter, material parts, and scientific relations being the performance of those parts. A technological system is defined as a collection of two or more material parts with functions in composition, and as a chronologically-finite class of mathematical sets in a habitat and a technocoenosis. The mechanism of technology speciation is proposed to be the evolutionary functional set that is computable across all technological systems.

*Keywords*: set theory, sets, matter, parts, function, relation, system, artifact, evolution, evolutionary, computation, structural, functional, parametric, technological, technology.
1. Introduction

Although the role of combinatorial use of technologies in the evolutionary approach to development of more advanced technologies has been recognized [Arthur (2009)], implementation of such ideas into usable methods in R & D of emerging technologies has not yet been successful in the conducted literature search.

From biology, the DNA molecule, genotype, and species genomic sequence, are very hard to ignore for their efficiency in describing speciation in a vast biodiversity. If DNA-like evolutionary coding were written for computational usage by the contemporary computing machines for the task of manipulating with available technologies in order to invent and design new products, it could (i) push technology innovation, (ii) enable a methodology of systematic invention, and (iii) produce technology roadmaps that are essentially not brainstormed by human creativity but that are rather the result of mathematical computations.

There seems to be a major theoretical obstacle in translating what has been so far a subject of unstructured verbal discourse into a precise mathematical form that would enable a universal typology of technologies with the tools to transcribe the genome equivalents of technological systems, trace lineages, calculate possible future combinations of technologies, and make comparisons among technologically diverse yet directly competing products.

There are emerging at least two schools of evolutionary thought in the R & D context. The first is the Delphi method, which due to its focus on expert opinions of groups of individuals, has grown into a methodology [Karvonen (2009)] that can be used for analyzing technology evolution without the need to operate with a genetics-equivalent code for technologies, particularly when it concerns technology forecasting and “strategic planning for development of emerging technologies” [Gerdsri (2007)]. In such school of thought, analytical models revolve around methods of corporate decision-making, such as proposed for establishing “R&D priorities of emerging technologies in the technology foresight context” [Lee and Lee (2008)].

The second emerging school of thought relies on a different theoretical approach – and this is also pursued by the author in this paper – to seek out information that is intrinsic to technologies and products incorporating them, rather than extrinsic in the form of expert panels. This is exemplified by Grienitz and Blume [2010] in terms of Product Structure Segments and Product-Structure-Segment-Attributes.

Once it is clear that information that is directly attributable to products is pursued, that information has to be structured into a common mathematical model that can be applied to any technology and to any past or future product under consideration. For such universal applicability, the model in question must be composed of basic concepts that are common to all technologies and technological artifacts. The goal here is that such model would become for technology research what genome is for genetics.

This paper presents such model with the view that it is conducive to mathematical proofs, quantifiable empirical data, and reproducible experiments. In order to address these theoretical needs, this model resorts to the deductive method [Tarski (1946)] and the mathematical set theory [Lipschutz (1964)] for computations employing mathematical formulas, equations, operations, term manipulations, and Venn diagrams.

The advantage of the proposed model is reliance on mathematically formulated scientific relations that (i) directly relate to the material parts used in technological systems (as opposed to consensual convergence of experts’ ideas in the Delphi method), and that (ii) can be translated into a computing language for performing fast-speed computations by a...
machine, so that an exponentially improved processing rate and high precision of evolutionary forecasting of technologies can be achieved as compared to the case-by-case interpersonal evaluation by an expert panel.

2. Branching Off from the Historical Method

Basalla, in his praiseworthy “The Evolution of Technology” [1988], applied historical method to technology evolution and treated the material artifact as a relic, as “the fundamental unit for the study of technology” [1988, p. pvii] that permits to trace technology’s continuity and lineages among artifacts over a historical period.

In contrast to Basalla’s investigation into relics and the historical criticism of the sources that assist in finding possible explanations for evolution of artifacts, this model applies mathematical analysis to (i) the isolated material artifact (e.g. a manufactured product) as a physical object in natural sciences and ontology [Hirsch (2005)] and not in history, and (ii) the scientific relations that are associated with the artifact in disciplines that are relevant to that artifact. Furthermore, in contrast to Basalla’s “centrality of the artifact” [1988, p. 30], in the presented model the artifact is a defined term and is only one of several fundamental terms.

Whereas a historian’s analysis considers science as one of the sources of novelty in technology [Basalla (1988)], it is posited here that science in general is an analytical reference frame that is consistent and complete for the purpose of this model, despite the incompleteness of transient human knowledge about the scientific relations that govern the phenomena in the universe. A model, such as this presented here, which treats the subject of technology evolution in mathematical terms and enables computations, can only be based on scientific relations and explanations involving various scientific disciplines that are relevant to a particular technological system under investigation.

Such distinction between the role of science as a source of novelty and its role as an analytical reference frame can be demonstrated on a simple artifact – a teaspoon. Inventing the teaspoon does not require the scientific knowledge of a planet’s gravity, free fall acceleration, and normal force, yet in order to investigate and analyze the workings of the teaspoon as an artifact, a scientific explanation has no substitute. Furthermore, the burden of proof of a scientific relation in technology in the context of this model remains on the body of science that is used to explain that relation, such as a proof from physics is required to prove friction in a disk brake.

To clarify the author’s position: this paper is not about science and it does not present a new scientific theory. This paper is about a proposed method for practice in research & development of new technologies, so any mention of science in this paper is only methodological.

Considering science as a reference frame, some axioms need to be developed, on which the audience can agree. For simplicity, there are certain chronological factors that will be omitted for now, despite being equally axiomatic, and that will be introduced further in this paper in the treatment of technological systems.

3. Axiom of Controlled Physical Matter in Technology

At first, consider that there exists a universal set \( U \) of all detectable physical matter in the universe regardless of methods and variables of quantification. Given this, the first axiom can define the substance of the building blocks of technology:
Technology is composed of the set \( \{ \text{controlled matter} \} \) that is a proper subset of \( U \).

\[
\{ \text{controlled matter} \} \subset U
\]  

(1)

It can be commonly agreed that control of physical matter is in primitive terms defined as making the matter do what is sought by the controlling life form, using the abstract term ‘life form’ to interchangeably denote one or more biological organisms of one or more biological species, such as the case of stone tools that were developed and used by some species in the genus of great apes that can also be used by homo sapiens. Such control is both governed and limited by the scientific relations that apply to that matter in the universe, and this in turn results in the diversity of control methods and their effectiveness.

Example 1.

(i) solid: machine tools shaping solid objects, electronic components controlling electron flow, road surface with visual markings, solid rocket fuel, nanotechnology, technical textiles and fashion textiles;

(ii) liquid: water for cleaning, aviation fuels, coolant in radiators, lubricating oils, hydraulic working fluids;

(iii) gas: natural gas as combustion fuel, breathable gas mixture for pressurized spacecraft, compressed air in pneumatic systems, inert gases in Geiger-Muller tubes for measuring radiation;

(iv) plasma: ions in ion thrusters on spacecraft, ion sputtering, plasma arc welding;

(v) nuclear technology;

(vi) atomic clock;

(vii) industrial chemical processes;

(viii) materials engineering;

(ix) biological life: biological macromolecules (DNA, RNA), cells, tissues (transplants, ivory, leather, fur), organs (bodily functions), entire living organisms including among those also homo sapiens (all occupations, product users, operators, inventors, engineers, assembly-line workers) and domesticated animals (horse, mule, bull, cow, pig, goat, sheep, dog, etc.), silkworm larva, cotton and flax plants, medicinal plants, salmon aquaculture, agriculture (of cereals, vegetables, fruits), natural-forest or farm-grown tree timber, causing this model to treat all biological life and its processes as equally technological systems in order to maintain this model’s consistency, which is particularly important due to bio-technological processes gaining value for their lower environmental impact and greater sustainability.

Control of matter in technology raises philosophical and ethical questions that are beyond the scope of this model. What entity, as an element of what set, exercises this control? Is it entitled to control certain or all matter in some or all ways that the controlling entity has learnt of? Which reasons lead it to seek and exercise this control and are those reasons objectively legitimate?
4. Axiom of Parts from Matter

The next axiom is necessary to identify those elements of the universal set \( U \) of all detectable matter in the universe that can be theoretically used in technology as building blocks:

There exists a set of material parts \( \{ \text{parts} \} \) such that the set’s member parts are composed of a subset of the universal set \( U \) of all detectable matter in the universe.

\[
\exists \{ \text{parts} \} \text{ such that } \{ \text{parts} \} \subseteq U
\]  

(2)

where each and every material part is represented by a pair of a numeric value \( m \) of a metrological variable that describes the material part, and of a physical object \( o \) to which this numeric value in this metrological variable applies:

\[
(m, o)
\]  

(3)

Considering that more than one metrological variable is necessary to ontologically identify one material part in the universe, there exists a set of such indexed variables \( \{ m_p \} \), where each variable takes in relation to every part a numeric value belonging to the set of real numbers \( m \subseteq \{ \mathbb{R} \} \). The set \( \{ m_p \} \) can be called the structural image of one part that describes and identifies the part through such variables as physical dimensions, (sub)atomic and molecular structure, mass, structurally relevant material properties, etc.

More than one physical object can be found or made to satisfy the numeric value \( m \) in a given metrological variable. All the physical objects, which satisfy the full set \( \{ m_p \} \) of the numeric values of such metrological variables of the part, belong to the set of indexed physical objects \( \{ o_u \} \).

It needs to be noted that not every material part that belongs to the universal set \( U \) of all detectable matter in the universe also belongs to the subset \( \{ \text{controlled matter} \} \) of \( U \), as for example not every oxygen molecule found in the atmosphere is used by a jet aircraft burning fuel inside its jet engine. To further explain this example: a particular oxygen molecule out there in the Earth’s atmosphere is not controlled by any life form yet until it is delivered inside a specific jet engine of a specific aircraft that a life form built and launched to fly through the atmosphere at that spatial coordinate at that moment.

Example 2.

The 789\textsuperscript{th} fabricated automotive disk brake rotor has the diameter of 12 inches, as per the specifications for this rotor model:

\[
(12 \text{ in.}, \text{observed physical object with number 789 jotted on it by marker})
\]

A material part is thus defined as a physical object that matches the given structural image set, which in turn makes the part reproducible, replaceable, and enables a cardinality of such identical (or nearly identical) physical objects to exist as a set. Mapping of a new structural image set onto a physical object involves such processes as: fabrication (casting, machining, 3D printing), assembling an artifact from made parts, copy reproduction, software
installation; natural processes, such as geological deposits and biological growth; handpicking from a heap of gravel a stone within the ranges of appropriate size, hardness, and mass, for a slingshot.

A material part is materialized at a coordinate in time and space when a structural image is mapped onto a physical object. The materialization is a scientific fact and as such is validated through observation and measurements by instrumentation. An artifact that is a collection of material parts is materialized when each of its individual parts is materialized and all the parts are assembled into one collection that is capable of operation.

Conversely, as an illustration, if a solid object that previously was established to be a material part is broken into two pieces, or has a piece of material broken off, it no longer matches the structural image set and therefore no longer satisfies the pair \((m_p, o_u)\), despite remaining a physical object. Since a structural image is mapped onto matter (during materialization of a part), it may also be unmapped through the same or other fabricating or natural processes at various unmapping rates, resulting in a finite duration of existence of a material part.

Although from Basalla’s historical study of technology evolution [1988], a general reader would imagine a part as a detail in a relic from an earlier century, in this model the definition of a material part also extends to the most advanced and complex technologies of the present and of the future, also covering as material parts the gases, plasmas, liquids, nano structures, with all of their constituents also being treated as parts, and all particles, including electrons, atoms, ions, photons (special-case particle objects), and even a boson in the Large Hadron Collider. This includes matter in any of its phases (states) and at any scale, including the fundamental constituents of physical matter, as per a scientific explanation of physical matter.

If, \(A, B, C\), are subsets of one universal set \(\{\text{parts}\}\), and if \(m\) is a metrological variable that describes one material part \(a \in A\), for every part \(a\) there is a dissimilar part \(b \in B\) such that \(m_a \neq m_b\), and a similar part \(c \in C\) such that \(m_a = m_c\), so that \(A = C\), within the framework of scientific metrology. Thus, for every variable \(m\) that is mapped onto physical objects, the universal set \(\{\text{parts}\}\) is divided into two partition sets \(A\) and \(B\). Every material part is either an element of \(A\) or is not an element of \(A\).

From here onward, metrological variables that describe a material part or an artifact will be called structural variables in order to avoid confusion in terminology with metrology involved in measuring a part’s performance that will be introduced in the next axiom.

5. Axiom of Scientific Relations as Parts’ Purposes

The third and last axiom will define how the substance of technology, physical matter, is controlled, and how the building blocks of technology, material parts, interact with one another and with the world around them:

There exists a body of science as a universal set of discovered scientific relations that apply (are related: \(R\)) to the universal set \(U\) of all matter:

\[
\exists \{\text{scientific relations}\} \text{ such that } \{\text{scientific relations}\}RU
\]  
(4)

The universal set of scientific relations also applies to the subset \(\{\text{parts}\}\) of \(U\):
\[ \exists \{\text{scientific relations}\} \text{ such that } \{\text{scientific relations}\} R \{\text{parts}\} \quad (5) \]

where each and every scientific relation is a pair of a type of a scientific function \( f \) and of a numeric value \( \pi \) that this function takes, where the numeric value (decimal or binary) belongs to the target set (range) of this function:

\[ (f, \pi) \quad (6) \]

Considering that more than one scientific relation applies to every material part, there is a set \( \{f_r\} \) of the indexed scientific functions of one part that in total are the part’s potential functionality.

Each scientific function \( f \) in the set \( \{f_r\} \) forms a pair with every numeric value \( \pi \) in this function’s target set. Every numeric value \( \pi \) is such that it belongs to the set of real numbers \( \pi \subset \{R\} \). The respective numeric values from the target sets of all functions belonging to the set \( \{f_r\} \) form the set of indexed parameters \( \{\pi_r\} \) as a collection of real numbers. (In other words, a numeric value \( \pi \) can be said to belong to the solutions set for the equation of a function \( f \).)

Example 3.

Angular velocity \( \omega \) takes on the numeric value of 12 radians per second:

\[ (\omega, 12 \text{ rad/sec}) \]

This axiom implies that each and every purpose of each and every part is a scientific relation. A scientific relation, a function, is necessary in order to map from the controlled physical matter of the part to a measurable result, the parametric numeric value, of the part. In turn, the controlled physical matter of a physical object with a structural image mapped onto it is only needed in order to generate the function that produces such parameter and that can be experimentally and statistically validated to be related to the physical object’s matter.

Functions, as scientific relations, can be found in all disciplines of relevance to technological systems and artifacts, including mathematics, informatics, chemistry, biology, economics, anthropology, aesthetics, and all branches of engineering. Among scientific functions is also energy, including such kinetic and potential forms as mechanical, elastic, gravitational, chemical, nuclear, electrical, magnetic, thermal, radiant, sound. Among scientific functions can be, if it is scientifically proven by color psychology, a psychological effect on human brain of visually perceived red color of the coating on an artifact. Scientific functions also include material properties in parts, in other words the physical or chemical properties of materials used in the parts.

Furthermore, expression (5) in this axiom can be restated as follows to emphasize that for every material part, there exists some purpose:

(i) For each pair ‘part’ and ‘scientific relation’, the binary relation is true

\[ \text{(part, scientific relation)} \quad (7) \]

(ii) \( (f_r, \pi_r) \) is true for every \( o \) onto which \( m \) is mapped:
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∀(mp, o5)∃(f2, π2) (8)

Expression (8) characterizes the control of matter in technology over the course of technology’s evolution. The term \( m \) implies the capability to map a structural image onto a physical object, and using some mapping method to tame matter into an artifact. The term \( o \) implies control of physical objects in the universe. The term \( f \) implies a body of science as a set of scientific relations (that can be used to create new results by the parts), and this explains why aggregation of science contributes to technological progress. The term \( π \) implies the kind and magnitude of a result that technology can produce.

The numeric values in the terms \( m \) and \( π \) in this expression can be obtained theoretically from engineering calculations and experimentally by taking measurements in metrology.

Example 4.

This example is theoretical and is intended to clarify the relations among the introduced four sets.

Consider the arrow diagram in Fig. 1 that shows a set of two nearly identical physical objects \( o_5 \) and \( o_6 \) that have been materialized with one structural image set \( \{m_p\} \) mapped onto each of them. This mapped structural image set consists of one numeric value in each of the two collected variables \( m_1 \) and \( m_3 \). Each of the two physical objects generates the same set of two scientific functions \( f_2 \) and \( f_4 \) and by doing so produces numeric values in two parameters \( π_2 \) and \( π_4 \) which in turn belong to the respective target sets of the given functions.

Fig. 1. Arrow diagram for physical objects \( o_5 \) and \( o_6 \).

6. Technological System as a Class of Sets

It is probably impossible to find a technology or a product that is not a system of material parts. Even if it is granted that a teaspoon is one physical object, it still cannot be used without such complex system as a human organism. Even if physical structures that appear as one whole to an unaided eye, such as a rod of a nuclear isotope or a blub of a liquid chemical substance, at a different scale they are clearly collections of interacting component parts. So it is safe to posit that, until proven to the contrary, every technology and product or process needs to be approached with the framework of a technological system.

When each of its terms is considered, \( ∀(m_p, o_5)∃(f_r, π_r) \) in expression (8) by same logic must be valid for a complex material part that is a collection, a set, of component material parts. Such complex material part’s four sets \( \{m_p\}, \{o_5\}, \{f_r\}, \{π_r\} \) contain elements
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belonging to the material parts in the collection, and in such case there exists a multiple-part artifact, as a physical object that is a collection of physical objects.

A union of two or more material parts, or more precisely unions of respective sets \( \{m_p\}, \{o_u\}, \{f_r\}, \{\pi_r\} \) of two or more material parts, is a technological system if at least one function of each one part is in a composition, including identity, with at least one function of some other part:

\[
\text{(part a)} = \{^a \{m_p\}, \{^a \{o_u\}, \{^a \{f_r\}, \{^a \{\pi_r\}\}\} \}
\]

\[
\text{(part b)} = \{^b \{m_p\}, \{^b \{o_u\}, \{^b \{f_r\}, \{^b \{\pi_r\}\}\}\}
\]

(System: \(a \cup b\) = \{\{m_p\} \cup ^b \{m_p\}, \{^a \{o_u\} \cup ^b \{o_u\}, \{^a \{f_r\} \cup ^b \{f_r\}, \{^a \{\pi_r\} \cup ^b \{\pi_r\}\}\}\}

if \(g \circ f\) (or \(g=f\)) is true for at least one \(g\) in \(^b \{f_r\}\) and at least one \(f\) in \(^a \{f_r\}\).

(9)

Here \(g \circ f\) means that the target set (range) of at least one function of a part in the system is the domain of at least one function of some other part in the system. And \(g=f\) means that one function is common to two or more parts according to the relation (7).

Example 5.

Two automotive parts are considered:

(part 1: brake pad) with a set of functions \( \{^1 \{f_r\}\} = \{^1 F_A, ^1 F_r, \ldots\} \), where force \(^1 F_A\) is applied against part 2, and \(^1 F_r\) is the force of friction on part 1 when it is pushed against part 2.

(part 2: disk rotor) with a set of functions \( \{^2 \{f_r\}\} = \{^2 F_N, ^2 F_r, \ldots\} \), where normal force \(^2 F_N\) = \(^1 F_A\), and \(^2 F_r\) is the force of friction such that \(^2 F_r = \mu \cdot ^2 F_N\).

The function force of friction \(^2 F_r\) is generated by both parts during their interaction, so it is included in the set of functions of each part as \(^1 F_r\) and \(^2 F_r\) respectively.

The function \(^2 F_N\) of part 2 is related to the function \(^1 F_F\) of part 1 by equation \(^1 F_F = \mu \cdot ^2 F_N\), where \(\mu\) is the coefficient of friction.

In the context of the terms appearing in this model, a definition of a technological system is in order:

Considering the above and assuming that no collection of component parts has an infinite duration, the technological system can be defined as an existing for a finite interval of time class \( S \) as a collection of the following four related fundamental subclasses (that for simplicity will be further referred to as the four sets): the subclass/set \( \Xi \) as a collection of component parts’ structural variables \( m_1, \ldots, m_\Xi \) (structural set) mapped onto every physical object \( o \) that is an element of the subclass/set \( A \) of multiple-part artifacts \( o_1, \ldots, o_{|A|} \) (artifacts set) to generate the subclass/set \( \Theta \) of functions \( f_1, \ldots, f_{|\Theta|} \) (system functional set) in order to produce the subclass/set \( \Pi \) of parameters \( \pi_1, \ldots, \pi_{|\Pi|} \) (parametric set) at an instant in time \( t \) (or during a time interval \( \Delta t \)):

\[
S_{\Xi} = \{\Xi, A, \Theta, \Pi\}
\]

where \( \Xi = \{m_1, \ldots, m_\Xi\}\), \( A = \{o_1, \ldots, o_{|A|}\}\), \( \Theta = \{f_1, \ldots, f_{|\Theta|}\}\), \( \Pi = \{\pi_1, \ldots, \pi_{|\Pi|}\}\)

It can be experimentally validated under controlled laboratory conditions by an independent examiner that each and every member function of a system’s functional set \( \Theta \) is
generated by the physical objects that are parts of a multiple-part artifact, and the parametric set $\Pi$ can be demonstrated by taking sensor readings with scientific instrumentation.

Conversely, in absence of a scientific inquiry into the specific functional set and specific parametric set, multiple-part artifacts belonging to different technological systems would be compared by some physical resemblance and by some distinct physical features in their respective constituent parts (including the number of respective parts), making the task of defining an archetype that is common to a number of technological systems both objectively impossible, due to heterogeneous physical features among all examined systems, and self-defeating, because the totality of physical features can be posited to be unique to every technological system by definition.

Finally, a discussion of inputs and outputs of a technological system as a class composed of four subclasses was avoided here, because the terminology of inputs and outputs has been briefly tested on the proposed model and found (i) to not coincide with any of the four subclasses of a technological system, and (ii) to be too abstract for the manipulations sought under this model. Generally, the terms inputs and outputs can be equated with the domains and ranges of functions, yet it is more convenient to continue to think in terms of mathematical domain and target (range) sets, rather than use redundant here terms inputs and outputs. Yet one observation should be kept in mind regarding domains of functions: the numeric values in at least some structural variables may serve as the domains of some functions performed by the respective parts.

7. System’s Chronology

In this model, a technological system is viewed and understood not only in physical space but also in time.

A physical object that is an artifact does the operation, which is the process of generating the system’s functional set by the structural set and performing the parametric set. An artifact can exist without subsequent operation, yet operation cannot occur prior to the materialization of the artifact.

An artifact can exist and yet simultaneously not generate its functions at an instant in time; on the other hand, it cannot generate its functions while not existing as an artifact at the time instant of function generation, in other words it cannot generate its functions when the artifacts set is an empty set.

The non-infinite duration of the relation between the finite numeric values of structural variables and the respective physical objects (onto which these numeric values are mapped) results in a finite lifespan of an artifact belonging to a finite-cardinality artifacts set. The generation of functions by such structurally and chronologically finite physical objects as parts and artifacts results in a finite time interval of function generation with a corresponding finite time interval of parameter performance.

The system’s functions are only generated when the structural set permits and when the parametric set requires, leading to the assumption that the lifespan of an artifact is a longer time interval than the total sum of time intervals during which a function is generated by the artifact.

When the function generation duration equals to zero, no corresponding to it performance parameter can be produced, and the performance duration also equals to zero. In other words, at an instant in time, when no function is generated, no corresponding to it performance parameter is produced. A system cannot produce performance parameters without simultaneously generating the functions that produce those parameters, which is not
to be confused with the possibility of generating a cardinality of functions and simultaneously producing a cardinality of parameters in a chronological order. The latter possibility also supports the assumption that the lifespan of an artifact is a longer time interval than the total time interval during which a function is generated by the artifact.

Hence the following fundamental categories of concurrent time intervals can be identified for a technological system (with a corresponding instantaneous relation between them): artifact lifespan \( \Delta t_o \) and function generation duration \( \Delta t_f \), where the former is always longer than the latter, and where function generation duration \( \Delta t_f \) is always equal to performance duration \( \Delta t_\pi \):\n
\[
\Delta t_o > \Delta t_f \quad \text{where} \quad \Delta t_f = \Delta t_\pi
\]

Because the artifact lifespan is finite, and because the function generation duration is also finite, a performance parameter also has a finite duration, which for one artifact cannot exceed the artifact lifespan and cannot exceed the total function generation duration of the related function. Conversely, if the artifact lifespan equals to zero, function generation duration equals to zero, consequently performance duration equals to zero, and no performance parameter can be produced.

It should be noted, however, that if a technological system is designed to generate a cardinality of functions and produce a cardinality of respective parameters simultaneously (in parallel) and or consecutively (in series) in shorter time intervals, such system design represents a chronological multiplication of performance.

8. System’s Performance

The practical problems to be solved by the technological system determine the required parametric set, which measures the totality of the combined productive and counterproductive performance that is produced by the technological system.

Because the functions of material parts in a technological system are scientific relations, some scientific relations of the parts will be productive to the system, while other scientific relations of the same parts will be counterproductive to the system. For example, two moving mechanical parts that are linked through an intended functional composition also generate undesirable friction, which despite being counterproductive to the system still remains a scientific relation between these two parts, and as a consequence results in a measurable performance parameter of these parts.

This totality of performance can be viewed as a complex solution to the practical problems to be solved by technology. The elements of the parametric set are performance parameters, including all aspects of performance, such as efficiency, costs, aesthetics, psychological and sociological factors, and so forth. The parametric set is a collection of scientific facts that is validated by observation and measurements by instrumentation of the parameters produced and performed.

Because a parametric set is a set of performance parameters where each parameter has a numeric value, performance of a technological system can be measured laterally, as a cardinality \( |\Pi| \) of the parameters in the parametric set, and vertically, as the numeric values in each parameter \( \pi \). Similarly, an increase in performance can be defined as expansion of an existing parametric set that is achieved by addition of new performed parameters to the set (lateral changes involving set membership and \( |\Pi| \)), and or by improvement of the numeric values of the existing parameters in the set (vertical changes in one or more \( \pi \)), where the
numeric value in each and every parameter in the set can be improved as tending to infinity or negative infinity or an optimal numeric value, whether decimal or binary.

A function that is generated in each of two or more similar technological systems will naturally have different parametric numeric values in each of those technological systems. One possible way to explain this is to say that types of functions, as scientific relations, are transferrable among systems, yet parameters are dependent on the system structural set of engineered material parts that generate those functions, and on the presence or absence in the system’s functional set of other functions (and their respective parts).

A function that is generated by one artifact of one technological system may, regardless of the causes, have different parametric values at different coordinates in spacetime.

A function that is generated in each of two or more artifacts of one technological system may have different parametric values due to habitat and technocoenosis factors when these artifacts are operating simultaneously.

Moreover, a change in the functional set membership may result not only in a change in the parametric cardinality of the system, but also in a change in the numeric values of the retained parameters, because a parameter (and the corresponding function) can be produced by diverse compositions of functions (functional sets).

Furthermore, the cardinality of the artifacts set of a technological system is a performance multiplier of the technological system in the spacetime continuum, with such consequences as:

(i) Cardinality of artifacts that multiplies performance at spatial proximity: increase in performance at one spatial coordinate by several artifacts that are grouped to operate together at that spatial coordinate. For example, 4 outboard motors on one high-speed motorboat, 4 AA batteries connected in series, 3 astronauts performing diverse tasks inside one spacecraft module.

(ii) Cardinality of artifacts that multiplies performance at spatial separation: performance achieved at various distant coordinates, whereby several spatially spread out artifacts of one technological system operate and deliver performance locally at their respective coordinates, rather than as one technological system that covers an entire spatial area. For example, radio repeaters, street lights, airliners used for connecting flights.

(iii) Cardinality of artifacts that multiplies performance at simultaneity: increase in instantaneous performance due to a number of artifacts performing simultaneously. For example, 10,000 mobile phones of one model being used by 10,000 individual users at one instant, or one million software copies running on one million personal computers.

(iv) Cardinality of artifacts that multiplies performance at time separation or over consecutive time intervals: one artifact replacing another artifact of one technological system in order to sustain the performance of the technological system over an interval of time (thus extending the performance of the technological system past one artifact’s life expectancy or past one artifact’s presence at a required coordinate). For example, replacement of retired navigation satellites in a GPS constellation, magazine of ammunition in a handgun, replacement of a squadron of downed jet fighter aircraft either by another squadron of the same aircraft model on stand-by at an air force base or by newly manufactured aircraft of the same aircraft model.
The above four aspects also co-occur: simultaneous performance at spatial proximity or at spatial separation, and performance at time separation at spatial proximity or at spatial separation.

According to this model, the ‘primary dimensions’ of technological competition that had been empirically researched as part of the investigation by Sood and Tellis [2005, p. 156] into the S-curve have been identified as singled out parameters shared by technological systems. The findings by Sood and Tellis [2005, p. 161] of ‘a sequence of random, unpredictable dimensions of competition’ further confirm such identity. Because the S-curve acknowledges only one subjectively selected shared parameter and ignores the existence of the entire parametric set, of which at least a subset is considered and calculated by engineering practitioners during the engineering cycle of a technological system, the S-curve is both inadequate and counterproductive for explaining the technology evolution on the systems it represents.

9. System’s Physical Habitat

The physical objects that are material parts, and by extension the artifacts as sets of such physical objects, exist in physical environments that include natural and artificial conditions, such as the ambient temperature, humidity, partial pressure of a gas in a gas mix (as in atmosphere), friction (motion over gas, liquid, solid), meteorological conditions, outer space environments, electric conductivity of a medium (water spill on a printed circuit board), sizes and tolerances of adjacent parts and artifacts, gravity, inertia, vibrations (caused by other technologies), radiation, magnetic field, and so on.

Each and every artifact of a technological system exists, and the system’s functional set is generated by it, in some simultaneous environment, effective habitat $S^H$, which is a collection of matter that (i) through scientific relations (functions) and their compositions interacts with the artifact, and that (ii) is a subset of the universal set $U$ of all matter in the universe. A technological system is designed for an artifact’s existence and generation of functions in a specific habitat, and the longer an artifact of the system exists, the greater is the probability that the artifact will enter a different habitat.

Recalling from expression (4) that there exists a universal set of scientific relations that apply to the universal set $U$ of all detectable matter in the universe, and recalling expression (8) that $\forall (m_p, o_o) \exists (f_r, \pi_r)$, each of the four sets $\{m_p\}$, $\{o_o\}$, $\{f_r\}$, $\{\pi_r\}$ of every part has a complement set provided by the surrounding environment. In a system of parts, any part may represent a surrounding environment to some other part, and yet all surrounding environment that does not include any of the system’s parts will be called the effective habitat $S^H$ as a complement of the system class $S$ at an instant in time $t$ (or during a time interval $\Delta t$).

For every artifact $o$ in the artifacts set $A=\{o_1, \ldots, o_{|A|}\}$ of a technological system, there exists a complement habitat $S^H_o$, hence a set $\{(S^H_o)_o\}$ of habitats with set cardinality that equals the cardinality $|A|$ of the system’s artifacts set $A$ at an instant in time $t$ (or during a time interval $\Delta t$):

\[
\forall S_t=\{\Xi, \{o_1, \ldots, o_{|A|}\}, \Theta, \Pi\} \exists \{(S^H_o)_o\} \text{ such that } S^H_t(o)=\{\Xi', A', \Theta', \Pi'\} \quad (12)
\]

There is a habitat only if there is an artifact, and there is no habitat if there is no artifact. A habitat surrounding an artifact may undergo changes during the three previously identified time intervals - artifact lifespan $\Delta t_o$, function generation duration $\Delta t_f$, and
performance duration $\Delta t$. Each habitat $S^H$ that is encountered by a technological system interacts with all four subclasses of the system.

Prescriptively, a technological system must be given a structural set (choice of materials for the parts; the architecture of parts) and a functional set (functions generated by parts) in order to nominally operate in a required habitat, such as a satellite will feature a thermal control subsystem and parts from space-rated materials with material properties that must be treated as scientific functions generated by the respective parts.

As a collection of material parts, each and every artifact is a subset of the universal set $U$ of all matter in the universe, and the complement of said subset is all the other matter in the universe. In the spatial reference frame of the artifact, such complement is all the surrounding environmental matter, which also includes both naturally spatially-positioned matter and all other existing artifacts of all technological systems at that instant of time. Of particular interest is a subset of this complement – such environmental matter that has scientific relations that affect the artifact from a spatial position relative to the artifact’s structural set.

10. System’s Technological Coenosis

Because all possibly conceivable structural images of parts, all possibly conceivable functions, and all possibly conceivable parameters, cannot be collected in one artifact of one technological system, for each and every technological system there exists the first-order technological coenosis $S^C$ (technocoenosis), which is a complement to the system class $S$, as a collection of other technological systems $S_1, ..., S_{|C|}$ with their artifacts materialized in order to realize their respective system functional sets, which in turn are necessary for the realization of the system functional set of the system in question at an instant in time $t$ (or during a time interval $\Delta t$), and $S_t^C$ is such that its member systems are members of the population set $P_t$ of technological systems at the time:

$$\forall S_t={\Xi, A, \Theta, \Pi} \exists S_t^C={S_1, ..., S_{|C|}}=\{S_j\} \text{ such that } S_j \subset P_t$$

Example 6.a.

$S_t^C: \text{airliner A-380} = \{\ldots, S_{800:road_trailer TII_NICOLAS}, S_{900:trailing_edge_wing_assembly_jig TRITEC}, S_{1100:passenger_boarding_bridge A380_Jetway:AppronDrive}, S_{1101:tow_bar_tractor AM500}, S_{1102:aircraft_refueler_truck F-TSA80}, S_{1103:baggage_conveyor_belt_loader TK-XC80}, \ldots \}$

Example 6.b.

$S_t^C: \text{teaspoon X} = \{\ldots, S: \text{metallurgical_mold Y}, S: \text{hand/ homo_sapiens}, \ldots \}$

The first-order technocoenosis provides further required functions to the system by functional composition of the functions provided by the members of the technocoenosis with the system’s functions.

The membership of a technological system’s four sets ($\Xi, A, \Theta, \Pi$) determines the system’s technocoenosis, which can be prescriptively viewed as a set of complement
extensions of the four sets of the technological system, and as outsourcing of elements for those sets to other technological systems in the population. Each technological system that belongs to this technocoenosis is in turn dependent on its own first-order technocoenosis, forming the second-order technocoenosis for the system under consideration (and so forth order-wise), as a traceable chain of dependency on other technological systems, where dependency will at the very least include materialization of the system’s parts and possibly even include generation of the system’s functions. The system’s artifacts are dependent on the artifacts of the members of the system’s first-order technocoenosis, therefore the system artifacts’ lifespan, function generation duration, and performance duration, are limited by the duration of existence of the members of the system’s technocoenosis.

A technological system is a class with four finite subclasses. The finiteness of the subclasses leads to the impossibility of a human-designed stand-alone technological system. Observations of finiteness include: (a) the numeric values taken by the structural variables in every known human-designed technological system are finite \( |m| \neq \infty \); (b) no artifact yet exists that experimentally demonstrates the generation of an infinite cardinality of functions \( |\Theta| \neq \infty \); (c) there is a paradox in attempting to validate by observation an infinite cardinality \( |A|=\infty \) of the system’s artifacts set and in attempting to measure the performance of an infinite cardinality of parameters \( |\Pi|=\infty \) with infinite values \( |\pi|=\infty \).

11. Evolutionary Functional Sets

One widely held misconception that has been obscuring the introduction of mathematical analysis into the subject of technology evolution has been the prevailing concept of a broadly defined technology, such as a fixed-wing aircraft, jet aircraft, combat aircraft, etc., that is elevated on the abstraction ladder over all the corresponding physical objects (artifacts) and production models (technological systems). This model is contrary to that prevalent concept – a so-defined technology is only a subset of every technological system, not an umbrella concept covering many technological systems. This will be explained as follows.

All technologies evolve in evolutionary functional sets, where each such evolutionary functional set \( E \) is defined as a mathematical set of functions that is copied as a proper subset by a number of technological systems. This evolutionary functional set is the core evolutionary mechanism in this model that enables copying of functions from one technological system to another. In this model’s terms, the evolutionary functional set is the underlying cause of the omnipresent trend of artifact resemblance – evolutionary continuity among artifacts, as discussed by Basalla [1988]. Whereas a technological system’s functional set contains all the functions that are necessary for and specific to that particular system, an evolutionary functional set \( E \) is a narrow set of functions, which can be theoretically extracted from, copied, and inserted into, a technological system’s functional set.

If \( E=\{\text{evolutionary set’s functions}\} \) and \( \Theta=\{\text{system set’s functions}\} \), then the evolutionary functional set is a proper subset of a system’s functional set:

\[
E=\{f_1, \ldots, f_{|E|}\} \\
E \subset \Theta
\]  

and conversely, a system’s functional set contains evolutionary functional sets among the system’s functions.
Scientific relations are independent of technological systems. As a consequence, technological systems in the population have shared scientific functions, shared in the sense that one function is copied and then used in more than one system, and a function that belongs to only one system is only a temporary empirical exception that is still reproducible in another system that can be designed in the future. Thus any one function can be considered an evolutionary functional set $E_f = \{f\}$ with set cardinality $|E_f| = 1$, and a system’s functional set can be defined as a set of evolutionary functional sets:

$$\Theta = \{E_1, \ldots, E_{|\Theta|}\} = \{E_i\}$$

An evolutionary functional set is realized as a proper subset of the system functional sets of a number of technological systems $S_1, \ldots, S_n$:

$$E_i \subset S_j \quad (17)$$

The evolutionary functional set is united with other evolutionary functional sets in the population $P_t$ of technological systems, including evolutionary set unions that produce new evolutionary sets:

$$S_j = \{\Xi, A, \{\ldots, E_i \cup E_k, \ldots\}, \Pi\} \quad (18)$$

Mathematical operations on functions as elements of sets and on resultant sets of functions can trace and form new evolutionary sets, modify an already invented evolutionary set’s membership, and unite sets.

Example 7.

Consider that $E$ denotes an evolutionary functional set, which is further labeled with a working name, description, or numerical designation. For example, an evolutionary functional set such that collects the functions commonly agreed on to be pertaining to the fixed wing of aircraft: $E:\text{fixed}_\text{wing}$. On a side note, properly sorted numerical designation of sets facilitates high-volume data processing by computer software of evolutionary functional sets, systems, functions, material parts, parameters, and artifacts, so that $E:\text{fixed}_\text{wing}$ can be equally written as, picking a random catalogue number, $E:32701$.

The feasibility of such identification of common functions is demonstrated by recalling the following functions:

- $L$ - aerodynamic lift
- $V$ – true airspeed
- $E:\text{fixed}_\text{wing}=\{\ldots, L, V, \ldots\}$

It can be proven using the above example that the evolutionary functional set $E:\text{fixed}_\text{wing}$ is a proper subset of the system functional set $\Theta$ of a technological system – airliner model A-380. For convenience during initial analysis, functional sets can be drawn around a material part or subsystem that generates them, as shown below:
If \((E:\text{fixed\_wing}) = \{\ldots, L, V, \ldots\}\) and \((\Theta:A-380) = \{\text{functions of aircraft sys } A-380\}\),
then \((E:\text{fixed\_wing}) \subset (\Theta:A-380)\), or alternatively written:
\(\{\ldots, L, V, \ldots\} \subset \{\text{functions of aircraft sys } A-380\}\),
and \((E:\text{fixed\_wing}) \neq (\Theta:A-380)\),
and \((\Theta:A-380) \not\subset (E:\text{fixed\_wing})\),
because \((E:\text{landing\_gear}) \subset (\Theta:A-380)\) and \((E:\text{turbofan\_engine}) \subset (\Theta:A-380)\),
so that \((\Theta:A-380) = \{E:\text{fixed\_wing}, E:\text{turbofan\_engine}, E:\text{landing\_gear}, \ldots\}\),
and \((E:\text{fixed\_wing}) \neq \{E:\text{fixed\_wing}, E:\text{turbofan\_engine}, E:\text{landing\_gear}, \ldots\}\),
and \(\{E:\text{fixed\_wing}, E:\text{turbofan\_engine}, E:\text{landing\_gear}, \ldots\} \not\subset (E:\text{fixed\_wing})\).

Technology evolution is an advancing sequence of invention of new evolutionary functional sets, for which in turn new technological systems are designed with the system functional set containing the invented evolutionary functional set.

A new evolutionary functional set can only be invented by a union if the evolutionary functional sets to be united have already been invented by the time \(t\) of invention. Conversely, if any of the evolutionary functional sets to be united does not yet exist at the time \(t\), said new evolutionary functional set cannot be invented and cannot be realized in a technological system, and a system containing the required union of evolutionary sets cannot be designed, and no artifact of such system can be materialized:

\[E_i \cup E_k = E_q \text{ only if } \exists E_i \text{ and } \exists E_k\]  \hspace{1cm} (21)

Because evolutionary functional sets are realized through technological systems, the population \(P_{\Delta t}\) of technological systems can be surveyed for necessary evolutionary functional sets \(E_i\) and \(E_k\) to establish if they have already been invented.

Once invented, a new evolutionary functional set can be united with as many existing system functional sets as logically possible in order to improve the performance (parametric set as defined above) of other and further technological systems, even transcending industries and past applications. Such new evolutionary functional set brings about functional changes into a subset of the population \(P_t\) of technological systems.

Where possible, the artifacts that already existed at the time of invention of the new evolutionary functional set will be retrofitted, or existing systems will be modified in order to materialize further artifacts with the new evolutionary functional set. Where this is not possible, the existing technological systems will be discontinued in the sense that no more of their artifacts will be materialized, and new systems that offer an increase in performance (cardinality and or numeric values) over the existing systems, will be designed, leading to materialization of their respective artifacts.

An invented evolutionary functional set is realized through design of technological systems that contain this functional set. Realization of an evolutionary functional set continues in successively designed technological systems for as long as there is no other evolutionary functional set invented that offers new technological systems being designed improved performance in the numeric values and or cardinality of prioritized parameters in comparison to the existing evolutionary functional set. It is possible that no increase in
performance is achievable (or even sought) during that interval of time, and the parametric sets of successively designed technological systems are inherited with minimal changes and are limited by existing systems’ functional sets. Appearance of such other evolutionary functional set in successively designed technological systems can be exemplified on the historical cases of diesel locomotives replacing steam locomotives and transistors replacing vacuum tubes.

The duration of realization of an evolutionary functional set is evidenced and measured using the existing artifacts of the technological systems that contain and realize this evolutionary set in the population.

If realization of an evolutionary functional set ceases, new technological systems will be designed with some other (new) evolutionary functional set, and will eventually replace the artifacts of existing technological systems that have been designed with the ceased evolutionary functional set.

Example 8.

This example demonstrates that evolutionary functional sets and system functional sets can be computed using the mathematical set theory for the purposes of empirical analysis and invention. (In addition to the demonstrated use of Venn diagrams for these purposes, truth tables are also recommended but will not be considered here.)

Recall the evolutionary set \( E: \text{fixed\_wing} \) that was compiled in the previous example:

\[ E: \text{fixed\_wing} = \{ \ldots, L, V, \ldots \} \]

As demonstrated above, the common functions of aircraft wings of fixed-wing aircraft are identifiable by a practicing engineer, and are described as one evolutionary set \( E: \text{fixed\_wing} \).

The common functions of helicopter rotors are posited to be similarly identifiable by a practicing engineer, and are described as one evolutionary set \( E: \text{rotary\_wing} \) (for example by introducing the torque and angular velocity of the main rotor in helicopters).

The common functions of atmospheric reentry vehicles are posited to be likewise identifiable by a practicing engineer, and are described as one evolutionary set \( E: \text{atm\_reentry\_vehicle} \).

Drawing a Venn diagram in Fig. 2 of a population of technological systems in the year 2012, the following regions are denoted:

Region \( P \) is the total population of technological systems with artifacts existing in the year of AD 2012.

Region \( A \) represents the set of all systems that feature the evolutionary functional set \( E: \text{fixed\_wing} \) as a subset of the system functional set:

\[ A = \{ S: (E: \text{fixed\_wing}) \subseteq \Theta \} \], so that

\[ A = \{ A320, PBY-5, F-35C, A-90, DG-800, EA300, X-48B, WK2, LongEZ, AGM-86, X-47B, \ldots \} \]

Region \( B \) represents the set of all systems that feature the evolutionary functional set \( E: \text{rotary\_wing} \) as a subset of the system functional set:

\[ B = \{ S: (E: \text{rotary\_wing}) \subseteq \Theta \} \], so that

\[ B = \{ H-47, Mi-10, H-5, R-22, Ka-52, ELA07, \ldots \} \]
Region C represents the set of all systems that feature the evolutionary functional set $E:\text{atm.\,reentry\_vehicle}$ as a subset of the system functional set:

$$C = \{ S : (E:\text{atm.\,reentry\_vehicle}) \subset \Theta \}, \text{ so that } C = \{ \text{ApolloCM, SoyuzSA, DC-X*, DragonC1, ...} \}$$

*only parts from a damaged artifact remain in existence*

Consider that each of the three sets A, B, C, may intersect with either of the other two sets. The regions of intersect are marked i, ii, iii.

If each set A, B, C, is a collection of technological systems with one common evolutionary set, then such regions of intersect as i, ii, iii, indicate that there may be systems that contain unions of both evolutionary sets, and those systems are elements of both sets, A and B, or A and C, or B and C, respectively. If such systems exist, then the evolutionary sets that are respectively unions of evolutionary sets $A \cup B$, $A \cup C$, and $B \cup C$, have already been invented and realized. If no such systems have been found in said regions of intersect, then these evolutionary set unions have not yet been realized.

Empirical search reveals existing systems in the three regions i, ii, and iii:

Systems in region i, where sets A and B intersect, include such member as tiltrotor V-22, because:

$$\Theta : V-22 = \{ ..., E:\text{fixed\_wing}, E:\text{rotary\_wing}, ... \}, \text{ where } E:\text{tiltrotor} = (E:\text{fixed\_wing}) \cup (E:\text{rotary\_wing})$$

Systems in region ii, where sets A and C intersect, include such member as STS Orbiter, because:

$$\Theta : \text{STS\_Orbiter} = \{ ..., E:\text{fixed\_wing}, E:\text{atm.\,reentry\_vehicle}, ... \}, \text{ where } E:\text{spaceplane} = (E:\text{fixed\_wing}) \cup (E:\text{atm.\,reentry\_vehicle})$$

**Fig. 2. Venn diagram of a population of technological systems in Example 8.**
Systems in region iii, where sets B and C intersect, include such member as Roton, because:

\[
\Theta: \text{Roton}=\{\ldots, \text{E:rotary_wing, E:atm.reentry_vehicle}, \ldots\}, \text{where E:rotary_reentry_vehicle=}(\text{E:rotary_wing}\cup\text{E:atm.reentry_vehicle})
\]

Although this example was handpicked for reasons of illustration, this method is applicable not only to aerospace technologies, but to all technologies as diverse as industrial process technologies, nano- and bio-technologies, and information technologies.

12. Discussion and Conclusions

The presented model decontextualizes technological artifacts from historical epochs of their origins and extracts from technologies their genome-like information that is grouped into systems of interrelated mathematical sets. The model was developed by starting with three axioms, which were used to define the classes of elements of a technological system, with a subsequent identification within such technological system of the evolutionary functional set that can be manipulated to produce progressing speciation of technological systems with traceable lineages. The next challenge is to translate all of these terms into a computer language to enable computer-performed computations to draw future technology roadmaps that can provide guidance to present-day R & D efforts.

When a technological system is defined in terms of matter governed by scientific relations (functions), one fundamental question arises: if functions and their composition link the parts within a technological system, if functions and their composition link the parts of a technological system and the matter in an artifact’s habitat, and if functions and their composition link the parts of a technological system and the matter in the system’s technocoenosis – how can the boundaries of a technological system be unambiguously defined in terms of matter and scientific relations to objectively separate the system from its habitat and from its technocoenosis? One solution to this theoretical dilemma is to define a technological artifact as a reproducible local spacetime redistribution of physical matter for the purpose of using the redistributed physical matter’s scientific relations to produce the results that are sought by the controlling life form.

On the basis of the proposed model, and to distinguish technology from the mechanistic existence of inanimate matter in the universe, technology can be abstractly defined as a life form’s control of physical matter, which in turn is objectively bound by scientific relations. Two or more quantities of physical matter are combined to interact as parts of a system in order to produce a result that is only sought by the controlling life form. Even more abstractly, it can be said that technology is an aspect of existence of life. This holistic view of technology also opens possibilities not only for commercial but also for academic applications of this model, such as computing theoretical possibilities in extraterrestrial technologies in conjunction with the ongoing astronomical search for exoplanets, as would be necessary to predict detectable signatures of activity of extraterrestrial civilizations [Davis (2012)].
References