Formative Assessment in Mathematics for Engineering Students

Eabhnat Ní Fhloinn[1] and Michael Carr[2]

[1]*School of Mathematical Sciences, Dublin City University, Dublin, Ireland*

[2]*School of Multidisciplinary Technologies, Dublin Institute of Technology, Dublin, Ireland*

Corresponding Author: Dr. Eabhnat Ní Fhloinn, Room X138A, School of Mathematical Sciences, Dublin City University, Glasnevin, Dublin 9, Ireland. eabhnat.nifhloinn@dcu.ie

Other Author: Dr. Michael Carr, Room 403, School of Multidisciplinary Technologies, Dublin Institute of Technology, Bolton St, Dublin 1, Ireland. michael.carr@dit.ie

Formative Assessment in Mathematics for Engineering Students

In this paper, we present a range of formative assessment types for engineering mathematics, including in-class exercises, homework, mock examination questions, table quizzes, presentations, critical analyses of statistical papers, peer-to-peer teaching, online assessments and electronic voting systems. We provide practical tips for the implementation of such assessments, with a particular focus on time or resource constraints and large class sizes, as well as effective methods of feedback. In addition, we consider the benefits of such formative assessments for students and staff.

Keywords: formative assessments; engineering mathematics; low-stakes assessment; assessment for learning

1. Introduction

First published almost forty years ago, Rowntree's seminal text on assessment opens with the words *"If we wish to discover the truth about an educational system, we must look into its assessment practices"* (Rowntree 1977). However, academic staff are under increasing amounts of time pressure while trying to juggle an ever-growing number of competing demands (Spurling 2015; Menzies and Newson 2007; Fitzgerald, Gunter, and White 2012). As a result, one of the most critical issues for educators in higher education is that the assessment process should not be excessively time-consuming in any given academic year (Vos 2000). In addition, class sizes are growing and so is the number of students for whom lecturers are expected to provide meaningful feedback (Ecclestone and Swann 1999; Hazelkorn 2015). Students entering into engineering programmes in higher education are used to a school system in which they submit work and receive feedback on a weekly basis (or more often than that) (Jones 2008, 341). Finally, providing high-quality feedback to students, something that Black (1998, 104) argues is essential to effective teaching and learning, comes with its own particular challenges in mathematics, given that *"the difference between levels of performance is* *not a matter of 'more of something', but a matter of 'something different'*" (Gadanidis 2003). As a result, the introduction of formative assessment into engineering mathematics classes can seem like a daunting task for many lecturers.

2. Formative assessment

Much has been written about formative assessment over the past fifty years or more, with seminal texts such as Bloom (1969), Sadler (1989), Black and William (1998a; 1998b; 2003) adding particular depth to the area. There is some discussion in the literature as to the precise definition of formative assessment (Wiliam 2007), what it is that makes an assessment summative or formative (Ramsden 1992; Bennett 2011), or indeed whether the terms "formative assessment" and "assessment for learning" can be used interchangeably (Black et al. 2003; Wiliam 2007), with arguments made on both sides. However, for the purposes of this paper we will rely on the recent definition given by Schoenfeld (2015) (heavily influenced by the work of Black and Wiliam (1998a; 1998b)), who states that formative assessments are *"examinations or performance opportunities the primary purpose of which is to provide students and teachers with feedback about the student's current state, while there are still opportunities for student improvement"*. As a result, a number of the assessment types described below have a small weighting of marks attached to them in order to increase student engagement, and some may be best implemented towards the end of a learning period, as revision aids; however, we would argue that these factors on their own do not make them summative assessments. As Sadler (1989, 120) observed, *"the primary distinction between formative and summative assessment relates to purpose and effect, not to timing"*. In addition, it should be noted that in higher education, a greater responsibility for independent learning falls upon the student than in earlier years of education (Stephenson and Yorke 2013), meaning that assessment can still be formative even towards the latter half of a teaching semester.

Cauley and McMillan (2010) note that there are four main reasons why students tend to learn more through formative assessment. Firstly, it allows students to focus on progress, while allowing the lecturer to focus on tweaking the method of instruction; secondly, because the assessment is immediate, the feedback is generally more meaningful; thirdly, students have a better idea how they can improve as the assessments are specific rather than global; and finally, it is consistent with constructivist theories of teaching, learning and motivation. Although sometimes seen as time-consuming, McIntosh (1997) argues that formative assessment should not be thought of as an add-on to an already full curriculum but rather as a part of *"good teaching"*.

Feedback plays a vital role in effective formative assessment. Stull et al (2011) identified two main functions of feedback for students and two others for lecturers. For students, feedback helps to pinpoint problem areas while also reinforcing successful learning. For lecturers, feedback shows the success of their instruction to date and identifies what areas need further modification. Numerous researchers have explored the challenges of providing high quality feedback with which students engage, particularly when student numbers are large, providing suggestions such as implementing peer-feedback (Nicol 2010), improving students' abilities to assess their own work (Sadler 2010) and the utilisation of technology to enhance feedback (JISC 2010).

3. Assessment in engineering mathematics

A report of the Mathematics Working Group (MWG) of the European Society for Engineering Education (SEFI) in 2013 highlighted the importance of assessment within engineering mathematics for achieving progress in the range of mathematical competencies they identified as core to engineering (Alpers et al. 2013, 12). They state that for many of these competencies, the traditional end-of-year assessment is not sufficient on its own to ensure mastery, although for others it is possible to design appropriate questions. For example, ‘handling mathematical symbols and formalism’ is assessed by a written examination, but ‘communicating in, with and about mathematics’ might be better assessed by students solving problems either individually or in small groups and presenting their solutions orally or in a short report.

A number of researchers have argued that mathematics is a special case (London Mathematical Society 2011), and as such may be better suited to the more traditional system of lectures (Pritchard 2010) and assessments than other disciplines, although there has been some disagreement in relation to this, with Barton (2011) notably providing a counter-argument. Regardless, a study by Iannone and Simpson (2011) showed that closed-book assessments are the most common and most highly weighted form of assessment in mathematics in higher education in the U.K., and went on to show that student preference was for a continuance of this (Iannone and Simpson 2015a). However, students in their study did welcome the idea of more variety in assessment and for a tailoring of assessments to suit the topic in question, in particular in relation to statistics. Lawson (2004) found a similar result in a SEFI-MWG survey of engineering mathematics lecturers about assessment practices, which showed many different approaches in use across Europe, with written closed-book examinations most commonly used but oral assessments, open-book examinations, take-home assignments and computer-based assessments also in use.

4. Formative assessment ideas for mathematics for engineers

Although Schoenfeld (2015, 193) observed that formative assessment can assist educators to build *“rich mathematical classroom environments”*, Hassan (2011, 335) posed the question as to how effective formative assessment processes can be undertaken with classes of several hundred students, as is often the case in engineering mathematics. One suggestion he made in this regard was to assign a team of lecturers to a single group to assist with this, but he acknowledged that many universities are not in a position to do this. He concluded that each lecturer must instead design *"a personalised formative assessment to fit the situation in question"*.

We have attempted below to provide some suggestions in this regard. We will consider in broad detail nine approaches to formative assessment that could be implemented in an engineering mathematics class, highlighting some practical tips for implementation in each case to aid practitioners, particularly those working with large groups of students. We will begin with assessment formats that do not require extensive use of technology, before moving on to consider assessment opportunities involving technology. This list does not claim to be exhaustive, but rather provides an outline of tried and tested methods in a practical setting. We would advise using a variety of formative assessments in the course of a semester, as advocated by Hassan (2011, 334). As well as allowing students to display their learning in a mixture of ways, the novelty factor of different formative assessment types may help to increase student engagement, as it has long been known that varying the stimulus within lectures helps to increase the students' attention (Bligh 1972, 46).

4.1 Individual/Paired in-class exercises

This is the most familiar form of formative assessment for higher education students, as it reflects common practice in mathematics education in schools internationally, often known as seatwork (Serrano 2012) or perhaps more accurately as Kikan-Shido (between desks instruction) from the Japanese (Clarke 2004), which encapsulates more of the actions of the educator during the process.

A problem or technique is demonstrated on the board and then students are allowed to attempt a similar problem. They may be encouraged to do this individually or in pairs or small groups, depending on the situation. The lecturer circulates around the room while the exercise is undertaken, observing students' work and offering either individual or whole-class feedback. The lecturer subsequently adjusts their teaching as a result of observations made during this process.

4.1.1 Practical notes for implementation

This method can be highly effective in terms of engaging students while also acting as a formative assessment mechanism to provide both the student and lecturer with immediate feedback as to whether or not they can successfully complete the given exercise. However, Clarke (2004) observed that once class sizes were greater than 30 students, this technique became more challenging to implement, turning "extremely problematic" for groups larger than 60. Indeed, while Banky (2007) found compelling evidence of its use in an electronics class in engineering, there were only 21 students involved in the study. Often the physical set-up of a lecture in higher education (tiered rooms, continuous desks) makes circulation difficult - although asking students to leave every third row empty can rectify this issue, where practical to do so, but this is only an option where no more than two-thirds of the seats will be filled.

There is considerable scope for variation within this approach to formative assessment: the exercises given can be similar in difficulty to those demonstrated on the board, or slightly easier/more difficult, depending on the approach of the lecturer. Some decide to give a range of exercises which are scaffolded in terms of difficulty, in order to provide more challenging problems for the most able students in the class.

If the exercises are to be completed in pairs or small groups, this can present both benefits and challenges, with Sheryn and Ell (2014) reporting that students struggled with the varying abilities within groups and the speed at which the other students worked. However, in that same study, students also mentioned the benefits of being able to address misconceptions with their group, as well as sharing thinking strategies and approaches. Yoon et al (2011) reported similar findings, with students also including affective benefits to group-work of this sort, such as increased confidence and an allaying of anxieties when they realised other students might also not know immediately how to tackle a problem.

4.2 Homework

The concept of "homework" is again a familiar one for incoming university students, and one which may well have been employed as a formative assessment during school years (Black and Wiliam 2009; Boston 2002), provided the focus was on deepening their learning rather than receiving a high grade (Vatterott 2014; Zmuda 2008). At post-primary level, homework has been shown to be a motivating factor in student learning (Bempechat 2004) and to improve academic achievement (Cooper, Robinson, and Patall 2006), although some studies have shown little impact (Kohn 2006).

The students are given a homework assignment sheet, usually with a number of questions on it, and a relatively short timeframe in which to complete these questions (one to two weeks), as it is likely that engineering undergraduates may have mathematics classes only two to three times per week. Students must then submit their full worked solutions to the lecturer for grading and feedback. Online approaches to this are also possible, but these will be discussed in a later section.

4.2.1 Practical notes for implementation

In order to increase student engagement with homework, these usually need to be treated as "low-stakes" assessments (Seaton 2013, 966). Often this results in each homework assessment being worth as little as 1-2% of the final module grade, which appears to be sufficient motivation for the majority of students to complete these.

The main reservation of many lecturing staff with assigning homework to students relates to the time investment required for marking and feedback to students on a regular basis throughout the semester. This is a particular issue for staff with large class sizes and no teaching assistant support. One workaround to this issue is to only mark part of each assignment (Seaton 2013), without students being notified in advance as to which question will be marked. In order to increase the formative nature of the assessment, Seaton (2013) recommends reserving some marks for “completeness” and “presentation”, and providing full model answers to students after the submission date, as well as a comment box which highlights elements where their attempt could be improved. To help with the perception of fairness in relation to which question is marked, one suggestion is to use a random number generator in front of the class after the submission date to select the question that will be marked.

An alternative approach to homework marking was suggested by Alpay et al. (2010), who devised a tutorial system which provided small-group tutorial support in mathematics to first-year computer science students, through the use of peer-tutors from third and fourth year in conjunction with their regular tutors. These peer tutors were responsible for marking and feedback on assignments, which were zero-weighted. However, engagement with this process remained high on the part of the students, largely due to the interactive and small-group nature of the tutorials. Student responses to the scheme were favourable, in particular in relation to the support given to them by the peer tutors.

4.3 “Mock” examination question

This approach involves the use of a question from one of the module’s past terminal examination papers, in an adaptation of what Black et al. (2003, 53) referred to as “the formative use of summative tests”. Students work in small groups on this question for a specified amount of class-time and hand up a single solution per group. Each group is then given another group’s solution, along with the correct solution, and must provide written feedback on the errors that were made. No grade is given on the question, but errors or inaccuracies are highlighted.

4.3.1 Practical notes for implementation

This activity can be quickly prepared, as both the question and full solution are already in existence since the previous terminal examination. The knowledge that this question was taken from a summative examination appears to increase the engagement and motivation of students when attempting the solution (Hassan 2011, 335). Asking the students to correct another group’s solution has the dual benefit of allowing for more immediate and detailed feedback to be provided to students when class numbers are large, while also focusing the students on the importance of accuracy and clarity as they mark another group’s work. However, as a formative assessment practice, it is generally useful only two to three times per semester, as it appears to lose its effectiveness if overused.

4.4 Table quiz

As a revision exercise several weeks in the semester, a mathematical table quiz can prove to be an engaging formative assessment. For example, the class could be divided into teams of three to four students, and one question be presented every five minutes for the duration of a normal lecture. A variation of this is to provide each team with a sheet of questions to work through and some group space to tackle the problems in a given time (separate rooms if practical, corners of rooms more likely) as described by

Berry and Nyman (2002).

4.4.1 Practical notes for implementation

This is most effective if it is possible to design questions that are too long for one student to do on their own in the time allotted for the question, for example if 10 similar calculations are required and the results must then be collated. A strategy like this encourages the team to subdivide the work and involve all members to get the answer, increasing the engagement of all students involved with the task.

In terms of creating effective teams so that students derive as much benefit as possible from the assessment, there are a number of good suggestions given by Felder et al (2000), such as forming the groups yourself, ensuring groups are of heterogeneous ability and explaining to students what you are doing and why.

Numerous variations on the marking scheme are possible, from a straightforward allocation of points per correct answer to a more complex “Who wants to be a millionaire” format, for example involving mini chocolate bars as prizes as described by Thomas (2003). Feedback can be given to all groups simultaneously by providing a detailed correct solution for each problem at the end of the quiz and allowing time for students to review these solutions in their groups before the end of the lecture. Student feedback about this approach was very positive (Thomas 2003), with attendance levels almost 100% and student interactions with each other high throughout the class.

4.5 Presentations of earlier material

Students work either individually or in pairs to produce a short (five – ten minute) presentation on a mathematics topic that was covered earlier in the semester (Carr and Ní Fhloinn 2009). Alternatively, students may present their solution to a homework problem or prior examination question to the rest of the class (Berry and Nyman 2002; Kågesten and Engelbrecht 2007). Including such low-stakes oral mathematics assessments regularly in a student's degree programme may help to prepare them to undertake higher-stakes presentations at a later date (Iannone and Simpson 2015b, 984).

4.5.1 Practical notes for implementation

This can also be a useful way of encouraging students to revise core mathematical skills, as it can be challenging to motivate students to spend time on such topics. Carr and Ní Fhloinn (2009) report on an initiative in which the engineering mathematics lecturer liaised with the communications skills lecturer to undertake such an assessment. They found that this helped students to see the links between their different modules, increasing engagement with the process, and also allowed both lecturers to gain a better insight into student understanding in their area. Students also reported that the presentations helped them to clarify basic mathematics rules, as well as practise public speaking. A small amount of credit was allowed for both modules from the same presentation.

Kågesten and Engelbrecht (2007) found that students rated listening to other students presenting homework problems even more highly than presenting themselves, in terms of being the most productive as a learning experience. Thomas (2003) introduced a "Who wants to be a millionaire"-style quiz after presentations, with the questions devised by the student presenters and involving all those students in attendance working in groups.

In terms of software restrictions, Härterich et al. (2012, 263) observed that some students struggled with the depiction of formulae on Powerpoint, although this could be resolved with the use of a Latex-based package such as Beamer if it proved to be a serious concern.

4.6 Critical analyses of statistical methods

Many engineering students will study statistics at some point during their degree, and it is important that they learn to critically analyse a given situation in order to be able to implement the correct statistical test under a particular set of conditions (Hogg 1985; Snee 1993). One approach to formative assessment for engineering students studying statistics was described by Carr (2011). Students were given a research paper or newspaper article that contained a significant element of statistical analysis. They were asked to critically analyse this paper, noting why the statistical methods therein were used and, in particular, if these methods were appropriate. They then prepared a short presentation on the topic which was delivered to the entire class-group. This may be done individually or in pairs. They received feedback from the lecturer, both on the quality of their presentation and on their critical analysis of the statistics involved. When surveyed about this assessment approach, students were generally positive about the experience and felt they had learned a lot about statistical methods through participating in the process, although some were less positive about what they learned from the other presentations.

4.6.1 Practical notes for implementation

To help the students to focus primarily on the statistical analysis within the paper, rather than the study design, Carr (2011) advised choosing a paper from a different area of engineering (or even science) to their own native discipline. In relation to the implementation of the presentation itself, many of the comments made in the previous section would also apply in this case.

In larger classes, this activity could also be designed as a paired or group assessment, which students work on over the course of several weeks. In such a case, it is important to clarify the assessment guidelines by including some forms of peer- and self-assessment within the marking scheme (Vos 2000, 232), even for low-stakes assessments such as this, to avoid disengagement due to any perceived unfairness in the marking of the group versus individual effort.

4.7 Peer-to-peer teaching

This assessment involves a form of co-operative learning known as "jigsaw" (King 1993, 34), where the sum of what students teach each other adds up to knowledge of a particular topic. Each student in a pair is given a handout on a different topic. The students have, say, 20 minutes to prepare and they must then teach their topic to their partner. These topics may be complementary or involve different sections of an approach to a problem. By circulating throughout the class while this activity is undertaken, much like the Kikan-Shido approach, the lecturer can provide feedback to students about areas in which they are still unsure or have misconceptions.

The students must then either complete a short written exercise on the topic they were "taught" by their partner, or else give a short explanation of it to the class, depending on what is most appropriate to the topic in question.

4.7.1 Practical notes for implementation

There is a wealth of clear, concise resources available for use in such an approach, so this activity need not require extensive preparation on the part of the lecturer. Two extremely useful resource banks for engineering mathematics are mathcentre (www.mathcentre.ac.uk) and the Helping Engineers Learn Maths project (www.lboro.ac.uk/research/helm), both of which are available free-of-charge.

This form of collaborative learning can be highly effective, with high levels of engagement and positive student attitudes reported (Van Tran 2012). One potential disadvantage is that misunderstandings can arise when a topic is being taught by a student who has not yet mastered this topic themselves (Hassan 2011, 334; Sheryn and Ell 2014, 872), although this can be overcome by the lecturer engaging with the pair throughout the process in order to give help as needed (Hassan 2011). This is more easily done when class numbers are not overly large, or when a tutor is available to assist the lecturer so that there are two or more people circulating among the student pairs.

4.8 Online assessment

The range of online assessment tools available in mathematics is ever-growing, with a wide variety of free tools available to lecturers, ranging from the open-source WeBWorK (http://webwork.maa.org/) to closed standalone systems such as Khan Academy (https://www.khanacademy.org/), as well as a huge number of packages which can be purchased, either by a department within a university or in the form of licences purchased by individual students. A full discussion of the available tools and approaches is beyond the scope of this paper but some recent overviews can be found in Abdulwahed, Jaworski and Crawford (2012), Juan et al. (2011), Joubert (2013) and Greenhow (2015).

Online formative assessment can be used to ensure students know basic mathematical concepts covered in previous years or to revise material from the current module. Systems can be set up to allow students multiple attempts at any given assessment, or just a single attempt where appropriate, perhaps with some practice tests included (Carr, Bowe, and Ní Fhloinn 2013; Marjoram et al. 2008). The instant feedback provided by many online systems is another attractive element of such assessment, but it should be remembered that, if an answer is simply marked as incorrect in mathematics, there is no real feedback given to students about where in the series of intermediate steps they may have made an error (Hubbard 1995, 45). This should be taken into account in the design of the assessment in order to ensure high quality feedback for both student and lecturer, allowing it to be truly formative.

4.8.1 Practical notes for implementation

To increase student engagement with this type of formative assessment, some lecturers give a small number of marks for having taken such tests, without their actual score on the test being taken into account (Currell and Dowman 2003). Others allow multiple attempts but only award a mark once a perfect or almost perfect score has been achieved (Carr, Bowe, and Ní Fhloinn 2013).

Although there is a wide choice of options available within online assessment engines, their success has been variable, due in part to local issues such as poor agreement with the needs of staff or students, difficult interfaces, or limited technical support (Masouros and Alpay 2010), emphasising the need for careful selection of the correct tool for each individual situation. There is also a significant time investment needed for both the set-up and question development in any such online system (Burrow et al. 2005), although once developed, it can result in a reduction in contact time with students.

In terms of improving student learning, Hannah, James and Williams (2014) found that, in their weekly formative online assessments which allowed multiple attempts, some students spent too long perfecting these exercises at the expense of studying higher-level examination-type questions, which Lingard et al. (2009, 604) observed are difficult to set in this format. This danger was flagged by Ramsden (1992, 189), when he stated that the *"separate assessment of basic skills and knowledge, unless clearly flagged as a relatively unimportant part of the whole assessment process, leads to a focus by students on these activities rather than on more complex ones that are* *related to understanding."* Furthermore, without a specific time limit on questions, Shorter and Young (2011) suggested that some students became accustomed to having this amount of time on a question and subsequently struggled to complete questions when time was more limited, such as during a summative terminal assessment.

Jones (2008) recommended allowing students several attempts at practice questions initially in any online system, in order to avoid any problems with input issues, where students have correctly solved an exercise but, due to some perceived discrepancy in the format of their inputted answer, are marked as incorrect. He observed that, unless such inaccurate feedback is immediately addressed, it can lead to frustration among students and eventual disengagement with the assessment process.

In terms of the student experience in relation to online assessment, reports are generally very positive, with students stating that they enjoy doing such assessments (Burrow et al. 2005), that they find them to be useful (Currell and Dowman 2003) and that they help them to achieve a better result overall (Brito et al. 2009).

4.9 Electronic voting systems (EVS)

Electronic voting systems (also known as audience response systems or clickers) can be used to provide lecturers with immediate feedback from a large group of students, by posing questions related to course material and having students enter their selected answer on their keypad (King and Robinson 2009). Questions are generally posed in a multiple-choice format with the responses received displayed on a bar-chart beside the question (Kay and LeSage 2009). Student responses to EVS are always anonymous to their classmates (although the lecturer may later be able to link a response to a student, depending on the system), which can help to overcome the common issue reported by Yoon et al (2011) of students being reluctant to answer questions posed by mathematics lecturers during lectures, due to fear of being incorrect. MacArthur and Jones (2008) reviewed 56 publications regarding the use of EVS in undergraduate science lectures and found students to have been overwhelmingly positive in relation to their usage, although measurable increases in student learning had not always been shown.

King and Robinson (2009) introduced the use of electronic voting systems EVS in engineering mathematics in Loughborough and subsequently provided a bank of questions online for downloading.

4.9.1 Practical notes for implementation

Clearly, there is a cost involved in the purchase of an electronic voting system if one is not already in use in a university setting. There are three main approaches to the acquisition of handsets: students purchase their own for the duration of their programme; students borrow handsets (e.g. from the library) for the duration of a module; and students collects handsets at the beginning of a lecture in which they will be used and return them at the end of the lecture (King and Robinson 2009). If the purchase of an EVS is prohibitive, no-cost solutions that have long been in use include the use of coloured cards (with each student having 4 different coloured cards, each of which represents a different multiple-choice option, with students holding one card aloft to show their opinion); free apps such as Socrative (www.socrative.com); web-based EVS such as Polleverywhere (www. polleverywhere.com) in which students use their mobile phones to vote; or setting up a Twitter hashtag related to the module that allows students to ask or answer questions through their Twitter account (Junco, Elavsky, and Heiberger 2013).

Issues identified in various studies (MacArthur and Jones 2008) included set-up time, development of suitable questions and technological issues specific to certain brands. However, formative assessment was seen to be the most suitable use of this technology. This was also the finding of King et al (2008) when they conducted a smallscale review of the experiences of lecturing staff using EVS for engineering mathematics. They also reported that engagement levels appeared to wane slightly once students were familiar with the technology, particularly in classes of mixed ability, where some students became distracted and began chatting once they had selected their answer, suggesting that some care must be taken in the design of the questions to minimise the chances of this occurring.

4.10 Combinations of approaches

Many of the above approaches deal with only a single type of formative assessment, but a number of studies have been conducted in which a range of such assessments in mathematics are carried out in a single module. The results from these studies have been positive overall. For example, Stephens and Konvalina (2001) studied the impact of short weekly quizzes, computer algebra software projects and a “mock” final examination on student learning in a university algebra course and concluded that all three factors significantly influenced student learning. Berry and Nyman (2002) used a combination of oral presentations, poster presentations and team test taking in a mathematical modelling course and found that student motivation was increased. Shorter and Young (2011) introduced daily in-class quizzes, online homework, and project-based learning into an undergraduate mathematics module, reporting that a combination of the in-class quizzes with the students’ project marks were the best predictor of students’ final scores. As a result, they advised using a combination of these two methods to allow lecturers to judge how students will perform in terminal examinations.

5. Concluding remarks

Despite the inherent challenges, the importance of formative assessment for both lecturer and students cannot be understated, and the above examples show a range of possible approaches to its successful integration into engineering mathematics modules. The exact methods suitable for any individual module will be dependent on both the needs of the students and those of the lecturer in question, and are likely to vary between different stages of students' engineering undergraduate careers. In this paper, we have detailed a range of formative assessment approaches for engineering mathematics, as well as some practical points for consideration, in order to allow each lecturer to design the most appropriate assessment programme for their own situation, as advocated by Hassan (2011). By regularly including carefully planned and designed formative assessment within engineering mathematics modules, we are sending a message to students about our own perception of its importance and the benefits that can be accrued by both staff and students as a result. As Ramsden (1992, 187) memorably observed *"From our students' point of view, assessment always defines the actual curriculum"*.

References

Abdulwahed, Mahmoud, Barbara Jaworski, and Adam Crawford. 2012. Innovative approaches to teaching mathematics in higher education: A review and critique.

Alpay, E., P. S. Cutler, S. Eisenbach, and A. J. Field. 2010. Changing the marks-based culture of learning through peer-assisted tutorials. *European Journal of Engineering Education* 35 (1) (03/01): 17-32, http://dx.doi.org/10.1080/03043790903202983.

Alpers, Buckhard, Maria Demlova, Carl-Henrik Fant, Tommy Gustafsson, Duncan Lawson, Leslie Mustoe, Brita Olsson-Lehtonen, Carol Robinson, and Daniela Velichova. 2013. *A framework for mathematics curricula in engineering education.* Brussels: European Society for Engineering Education (SEFI), , http://www.sefi.be/wp-content/uploads/Competency%20based%20curriculum%20incl%20ads.pdf (accessed 24th March 2016).

Banky, George P. 2007. Looking for kikan-shido: Are elements of it detectable in tertiary engineering pedagogy? Paper presented at Australasian Association for Engineering Education 2007 Conference, Melbourne, Australia.

Barton, Bill. 2011. Growing understanding of undergraduate mathematics: A good frame produces better tomatoes. *International Journal of Mathematical Education in Science and Technology* 42 (7): 963-73.

Bempechat, Janine. 2004. The motivational benefits of homework: A social-cognitive perspective. *Theory into Practice* 43 (3) (08/01): 189-96, http://dx.doi.org/10.1207/s15430421tip4303_4.

Bennett, Randy Elliot. 2011. Formative assessment: A critical review. *Assessment in Education: Principles, Policy & Practice* 18 (1) (02/01): 5-25, http://dx.doi.org/10.1080/0969594X.2010.513678.

Berry, John, and Melvin A. Nyman. 2002. Small-group assessment methods in mathematics. *International Journal of Mathematical Education in Science and Technology* 33 (5) (09/01): 641-9, http://dx.doi.org/10.1080/00207390210144034.

Black, Paul. 1998. *Testing: Friend or foe? theory and practice of assessment and testing*. Master classes in education. Abingdon: RoutledgeFalmer.

Black, Paul, Christine Harrison, Clare Lee, Bethan Marshall, and Dylan Wiliam. 2003. *Assessment for learning: Putting it into practice*. 1st ed. Berkshire: Open University Press.

Black, Paul, and Dylan Wiliam. 1998a. Assessment and classroom learning. *Assessment in Education* 5 (1): 7-74.

Black, Paul, and Dylan Wiliam. 1998b. *Inside the black box: Raising standards through classroom assessment*London: King's College School of Education.

Black, Paul, and Dylan Wiliam. 2003. ‘In praise of educational research’: Formative assessment. *British Educational Research Journal* 29 (5): 623-37.

Black, Paul, and Dylan Wiliam. 2009. Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability (Formerly: Journal of Personnel Evaluation in Education)* 21 (1): 5-31.

Bligh, Donald A. 1972. *What's the use of lectures?*. San Francisco: Jossey-Bass.

Bloom, Benjamin S. 1969. Some theoretical issues relating to educational evaluation. *Educational Evaluation: New Roles, New Means: The 63rd Yearbook of the National Society for the Study of Education*(part II): 26-50.

Boston, Carol. 2002. The concept of formative assessment. ERIC digest.

Brito, Irene, Jorge Figueiredo, Maria Flores, Ana Jesus, Gaspar Machado, Teresa Malheiro, Paulo Pereira, Rui MS Pereira, and Estelita Vaz. 2009. Using e-learning to self regulate the learning process of mathematics for engineering students. In A. Bulucea, V. Mdladenov, E. Pop, M. Leba & M. Mastorakis (Eds.), Recent advances in applied mathematics. Proceedings of the 14th international conference on applied mathematics (MATH -09), Puerto de la Cruz, Spain (pp.165–169). WSEAS Press.

Burrow, Michael, Harry Evdorides, Barbara Hallam, and Richard Freer-hewish. 2005. Developing formative assessments for postgraduate students in engineering. *European Journal of Engineering Education* 30 (2) (05/01): 255-63, http://dx.doi.org/10.1080/03043790500087563.

Carr, Michael. 2011. Critical analysis of statistical methods used in research papers. *MSOR Connections* 11 (2): 7-9.

Carr, Michael, and Eabhnat Ní Fhloinn. 2009. Alternative forms of continuous assessment in mathematics. *Trends in Science and Mathematics Education* 1 : 51-63.

Carr, Michael, Brian Bowe, and Eabhnat Ní Fhloinn. 2013. Core skills assessment to improve mathematical competency. *European Journal of Engineering Education* 38 (6) (12/01): 608-19, http://dx.doi.org/10.1080/03043797.2012.755500.

Cauley, Kathleen M., and James H. McMillan. 2010. Formative assessment techniques to support student motivation and achievement. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas* 83 (1) (01/01): 1-6, http://dx.doi.org/10.1080/00098650903267784.

Clarke, David. 2004. Kikan-shido - between desks instruction. Paper presented at “Lesson Events as the Basis for International Comparisons of Classroom Practice” at the Annual Meeting of the American Educational Research Association, San Diego.

Cooper, Harris, Jorgianne Civey Robinson, and Erika A. Patall. 2006. Does homework improve academic achievement? A synthesis of research, 1987–2003. *Review of Educational Research* 76 (1) (March 01): 1-62.

Currell, Graham, and Tony Dowman. 2003. A learning framework for basic mathematics and statistics in science. *LTSN MathsTEAM Project: Maths for Engineering and Science.*

Ecclestone, Kathryn, and Joanna Swann. 1999. Litigation and learning: Tensions in improving university lecturers' assessment practice. *Assessment in Education: Principles, Policy & Practice* 6 (3) (11/01): 377-89, http://dx.doi.org/10.1080/09695949992801.

Felder, Richard M., Donald R. Woods, James E. Stice, and Armando Rugarcia. 2000. The future of engineering education II. teaching methods that work. *Chemical Engineering Education* 34 (1): 26-39.

Fitzgerald, Tanya, Helen M. Gunter, and Julie White. 2012. *Hard labour? academic work and the changing landscape of higher education: Academic work and the changing landscape of higher education*. International perspectives on higher education research. Vol. 7. Bingley: Emerald Group Publishing.

Gadanidis, G. 2003. Tests as performance assessments and marking schemes as rubrics. *Reflections: Journal of the Mathematical Association of New South Wales* 28 (2): 35-40.

Greenhow, Martin. 2015. Effective computer-aided assessment of mathematics; principles, practice and results. *Teaching Mathematics and its Applications* 34 (3) (September 01): 117-37.

Hannah, John, Alex James, and Phillipa Williams. 2014. Does computer-aided formative assessment improve learning outcomes? *International Journal of Mathematical Education in Science and Technology* 45 (2) (02/17): 269-81, http://dx.doi.org/10.1080/0020739X.2013.822583.

Härterich, Jörg, Christine Kiss, Aeneas Rooch, Martin Mönnigmann, Moritz Schulze Darup, and Roland Span. 2012. MathePraxis–connecting first-year mathematics with engineering applications. *European Journal of Engineering Education* 37 (3): 255-66.

Hassan, O. A. B. 2011. Learning theories and assessment methodologies – an engineering educational perspective. *European Journal of Engineering Education* 36 (4) (08/01): 327-39, http://dx.doi.org/10.1080/03043797.2011.591486.

Hazelkorn, Ellen. 2015. *Rankings and the reshaping of higher education: The battle for world-class excellence*Palgrave Macmillan.

Hogg, Robert V. 1985. Statistical education for engineers: An initial task force report. *The American Statistician* 39 (3) (08/01): 168-75, http://www.tandfonline.com/doi/abs/10.1080/00031305.1985.10479423.

Hubbard, Ruth. 1995. *53 ways to ask questions in mathematics and statistics (interesting ways to teach).* Bristol: Technical & Educational Services Ltd.

Iannone, Paola, and Adrian Simpson. 2011. The summative assessment diet: How we assess in mathematics degrees. *Teaching Mathematics and its Applications* 30 (4) (December 01): 186-96.

Iannone, P., and A. Simpson. 2015a. Students' preferences in undergraduate mathematics assessment. *Studies in Higher Education* 40 (6) (07/03): 1046-67, http://dx.doi.org/10.1080/03075079.2013.858683.

Iannone, Paola, and Adrian Simpson. 2015b. Students' views of oral performance assessment in mathematics: Straddling the 'assessment of' and 'assessment for' learning divide. *Assessment & Evaluation in Higher Education* 40 (7) (10/03): 971-87, http://dx.doi.org/10.1080/02602938.2014.961124.

JISC. 2010. *Effective assessment in a digital age: A guide to technology-enhanced assessment and feedback.* HEFCE, .

Jones, I. S. 2008. Computer-aided assessment questions in engineering mathematics using MapleTA ®. *International Journal of Mathematical Education in Science and Technology* 39 (3) (04/15): 341-56, http://dx.doi.org/10.1080/00207390701734523.

Joubert, Marie. 2013. Using digital technologies in mathematics teaching: Developing an understanding of the landscape using three "grand challenge" themes. *Educational Studies in Mathematics* 82 (3): 341-59.

Juan, Angel A., Cristina Steegmann, Antonia Huertas, M. Jesus Martinez, and J. Simosa. 2011. Teaching mathematics online in the european area of higher education: An instructor's point of view. *International Journal of Mathematical Education in Science and Technology* 42 (2) (03/15): 141-53, http://dx.doi.org/10.1080/0020739X.2010.526254.

Junco, Reynol, C. Michael Elavsky, and Greg Heiberger. 2013. Putting twitter to the test: Assessing outcomes for student collaboration, engagement and success. *British Journal of Educational Technology* 44 (2): 273-87.

Kågesten, Owe, and Johann Engelbrecht. 2007. Student group presentations: A learning instrument in undergraduate mathematics for engineering students. *European*

Journal of Engineering Education 32 (3) (06/01): 303-14, http://dx.doi.org/10.1080/03043790701276833.

Kay, Robin H., and Ann LeSage. 2009. Examining the benefits and challenges of using audience response systems: A review of the literature. *Computers & Education* 53 (3): 819-27.

King, Samuel O., L. Davis, Carol L. Robinson, and J. P. Ward. 2008. Use of voting systems in lectures at loughborough university - a review of staff experiences. Paper presented at Mathematical Education of Engineers (MEE 2008): 14th Joint Conference of Mathematics Working Group (MWG) of the European Society for Engineering Education (SEFI) and the Institute of Mathematics (IMA), Loughborough.

King, Alison. 1993. From sage on the stage to guide on the side. *College Teaching* 41 (1): 30-5, http://www.jstor.org/stable/27558571.

King, Samuel O., and Carol L. Robinson. 2009. 'Pretty lights' and maths! increasing student engagement and enhancing learning through the use of electronic voting systems. *Computers & Education* 53 (1) (8): 189-99.

Kohn, Alfie. 2006. *The homework myth: Why our kids get too much of a bad thing*. Philadelphia, PA: Da Capo Books.

Lawson, Duncan. 2004. Assessment in engineering mathematics. Paper presented at 12th SEFI Maths Working Group Seminar, Vienna.

Lingard, Jennifer, Laura Minasian-Batmanian, Gilbert Vella, Ian Cathers, and Carlos Gonzalez. 2009. Do students with well-aligned perceptions of question difficulty perform better? *Assessment & Evaluation in Higher Education* 34 (6): 603-19.

London Mathematical Society. 2011. *Comments on review of the UK professional framework for higher education.*

MacArthur, James R., and Loretta L. Jones. 2008. A review of literature reports of clickers applicable to college chemistry classrooms. *Chemistry Education Research and Practice* 9 (3): 187-95.

Marjoram, Martin, Denise Moore, Ciaran O'Sullivan, and Paul Robinson. 2008. Implementing a key skills in mathematics initiative. Paper presented at Proceedings of Mathematical Education of Engineers (MEE), Loughborough, United Kingdom.

Masouros, Spyridon D., and Esat Alpay. 2010. Mathematics and online learning experiences: A gateway site for engineering students. *European Journal of*

Engineering Education 35 (1) (03/01): 59-78, http://dx.doi.org/10.1080/03043790903428729.

McIntosh, Margaret E. 1997. Formative assessment in mathematics. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas* 71 (2) (11/01): 92-6, http://dx.doi.org/10.1080/00098659709599333.

Menzies, Heather, and Janice Newson. 2007. No time to think: Academics' life in the globally wired university. *Time & Society* 16 (1) (March 01): 83-98.

Nicol, David. 2010. From monologue to dialogue: Improving written feedback processes in mass higher education. *Assessment & Evaluation in Higher Education* 35 (5) (08/01): 501-17, http://dx.doi.org/10.1080/02602931003786559.

Pritchard, David. 2010. Where learning starts? A framework for thinking about lectures in university mathematics. *International Journal of Mathematical Education in Science and Technology* 41 (5) (07/15): 609-23, http://dx.doi.org/10.1080/00207391003605254.

Ramsden, Paul. 1992. *Learning to teach in higher education*. London; New York: Routledge (accessed 2016-03-01T14:28:38+0000).

Rowntree, Derek. 1977. *Assessing students: How shall we know them?*. 1st ed. London: Harper and Row Ltd.

Sadler, D. Royce. 2010. Beyond feedback: Developing student capability in complex appraisal. *Assessment & Evaluation in Higher Education* 35 (5): 535-50.

Sadler, D. Royce. 1989. Formative assessment and the design of instructional systems. *Instructional Science* 18 (2): 119-44.

Schoenfeld, Alan H. 2015. Summative and formative assessments in mathematics supporting the goals of the common core standards. *Theory into Practice* 54 (3) (07/03): 183-94, http://dx.doi.org/10.1080/00405841.2015.1044346.

Seaton, Katherine A. 2013. Efficacy and efficiency in formative assessment: An informed reflection on the value of partial marking. *International Journal of Mathematical Education in Science and Technology* 44 (7) (10/01): 963-71, http://dx.doi.org/10.1080/0020739X.2013.831490.

Serrano, Ana M. 2012. A cross-cultural investigation into how tasks influence seatwork activities in mathematics lessons. *Teaching and Teacher Education* 28 (6) (8): 806-17.

Sheryn, Louise, and Fiona Ell. 2014. Teaching undergraduate mathematics in interactive groups: How does it fit with students' learning? *International Journal of Mathematical Education in Science and Technology* 45 (6) (08/18): 863-78, http://dx.doi.org/10.1080/0020739X.2014.884647.

Shorter, Nichole A., and Cynthia Y. Young. 2011. Comparing assessment methods as predictors of student learning in an undergraduate mathematics course. *International Journal of Mathematical Education in Science and Technology* 42 (8) (12/15): 1061-7, http://dx.doi.org/10.1080/0020739X.2010.550946.

Snee, Ronald D. 1993. What's missing in statistical education? *The American Statistician* 47 (2) (05/01): 149-54, http://amstat.tandfonline.com/doi/abs/10.1080/00031305.1993.10475964.

Spurling, Nicola. 2015. Differential experiences of time in academic work: How qualities of time are made in practice. *Time & Society* 24 (3) (November 01): 367-89.

Stephens, Larry J., and John Konvalina. 2001. Factors influencing success in intermediate algebra. *Computers in the Schools* 17 (1-2): 77-84.

Stephenson, John, and Mantz Yorke. 2013. *Capability and quality in higher education*Routledge.

Stull, Judith C., Susan Jansen Varnum, Joseph Ducette, John Schiller, and Matthew Bernacki. 2011. The many faces of formative assessment. *International Journal of Teaching & Learning in Higher Education* 23 (1) (03): 30-9, http://search.ebscohost.com/login.aspx?direct=true&db=ehh&AN=67214871&site=ehost-live.

Thomas, Colin. 2003. A game show format for first year problem classes in mathematical modelling. *LTSN MathsTEAM Project: Maths for Engineering and Science.*

Van Tran, Dat. 2012. The effects of jigsaw learning on students' attitudes in a vietnamese higher education classroom. *International Journal of Higher Education* 1 (2): p9.

Vatterott, Cathy. 2014. Student-owned homework. *Educational Leadership* 71 (6) (03): 39-42, http://search.ebscohost.com/login.aspx?direct=true&db=ehh&AN=94925704&site=ehost-live.

Vos, Henk. 2000. How to assess for improvement of learning. *European Journal of Engineering Education* 25 (3) (09/01): 227-33, http://dx.doi.org/10.1080/030437900438658.

Wiliam, Dylan. 2007. Kepping learning on track: Classroom assessment and the regulation of learning. In *Second handbook of research on mathematics teaching and learning.*, ed. Frank K. Lester Jr. Vol. 2, 1053-1098. Charlotte, NC: Information Age Publishing Inc.

Yoon, Caroline, Barbara Kensington-Miller, Jamie Sneddon, and Hannah Bartholomew. 2011. It's not the done thing: Social norms governing students' passive behaviour in undergraduate mathematics lectures. *International Journal of Mathematical Education in Science and Technology* 42 (8) (12/15): 1107-22, http://dx.doi.org/10.1080/0020739X.2011.573877.

Zmuda, Alison. 2008. Springing into active learning. *Educational Leadership* (November): 38-42.