St Patrick's College,

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Spoon-Feeding to Tongue-Biting and Beyond: Implementing a Reform Approach to Mathematics Teaching in an Irish Primary School

Thesis

by

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ABSTRACT

This research aimed to examine teachers' experiences whilst implementing a reform approach to mathematics teaching in an Irish primary school. In particular, it aimed to examine the constraints and affordances pertaining to changing mathematics practice in this case study school. The school engaged in professional development that focused on using an instructional framework (Hiebert et al., 1997) in the school-identified strand units of length and weight. Four classes were tracked throughout the study and each class acted as a sub-case within the larger study. Data collected through lesson observations, teacher interviews, pupil work samples, and focus group interviews with pupils were used to compare teachers' experiences and to identify what supported and enabled them to change their practice, in addition to highlighting challenges that may have prevented such change. Findings were that shifts in practice were evident in the strand units of length and weight to different degrees in the four classes during the study. Teachers reported that tailored professional development, peer support, and the use and refinement of an instructional framework contributed to enabling them in changing their mathematics teaching. Factors that may have restricted the potential for embedding and enhancing this changing practice included a textbook dependent culture, teacher discomfort with facilitation, and possible limitations regarding the professional development. Possible implications of this study include the need for schools to have a number of factors in place for the successful implementation of a reform approach to mathematics teaching including peer support, tailored professional development, and a focus on a specific area of the mathematics curriculum. Furthermore, this study highlights the need for all stakeholders to consider textbook dependency, teacher facilitations skills, and the timing and content of professional development when implementing a reform approach to mathematics teaching.

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No set of goals and objectives includes everything.

This is what we are about:

We plant seeds that one day will grow.

We water seeds already planted,

knowing that they hold future promise.

We lay foundations that will need further development.

We provide yeast that produces effects beyond our capabilities.

We cannot do everything and there is a sense of liberation in realizing that.

This enables us to do something,

And to do it very well.

It may be incomplete,

but it is a beginning,

a step along the way...

(Archbishop Oscar Romero)

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CHAPTER 1: INTRODUCTION

In this chapter, the research topic is outlined including a brief summary of the historical context from both an international and an Irish perspective. Additionally, the research problem is outlined and relevant terms for the study are defined. Finally, the organising framework for the thesis is described by providing the purpose and structure of each chapter.

RESEARCH TOPIC

International Context

Mathematics has played a prominent and consistent role on international educational and social policy agendas for a considerable time. Many reasons probably contribute to this prominence—two possible reasons are discussed here. First, as an integral part of the STEM (Science, Technology, Engineering, and Mathematics) subjects, mathematics is viewed as central to commerce, industry, the knowledge economy, global competitiveness, and ultimately future national and individual prosperity. In the following assertion, the National Council for Curriculum and Assessment (NCCA) echoes this emphasis on mathematics as an economic imperative coupled with the commercial importance of the STEM subjects: "The development of mathematical skills impinges on the individual's opportunities for development, with consequent economic implications in a society increasingly reliant on and influenced by advances in science and technology, which have a high dependency on mathematical principles" (2005a, p.2). Second and as a partial by-product of the former, mathematics teaching and learning is viewed as a way of a) improving mathematical outcomes and b) facilitating and boosting productivity and capacity in the STEM areas. This study focuses on the latter—teaching and learning in mathematics. Despite an almost global-wide emphasis on teaching and learning in mathematics, little if any change is evident on international performance assessments regarding mathematics for the majority of countries. The exception being for countries from the Asia-Pacific region such as Singapore, Hong Kong, Korea, China, and Japan who continue to outperform other countries in mathematics international assessments both at primary and post-primary level.[1] The number of Asia-Pacific countries in the top ten performing countries in mathematics has increased steadily since the inception of the Programme for International Student Achievement (PISA) in 2000. In 2000, only three of the top ten countries were from the Asia-Pacific region; however, this increased to four out of ten in 2003, six out of ten in 2006 and 2009, and seven out of ten in 2012. In stark contrast to this improved performance, most other countries including Ireland have remained stagnant in relation to international performance on mathematics assessments. Therefore, with the exception of the countries from the Asia-Pacific region, little appears to have changed internationally with regard to mathematics performance. Some commentators blame this stagnation on the "distorted image" (e.g. Boaler, 2009; Pratt, 2002; Schackow & O'Connell, 2008) of mathematics that many pupils experience in schools. However, changing such practice has proved challenging.

Agreement exists in much of the literature that traditional mathematics lessons are out-dated and may no longer be fit for purpose. Additionally, there is general agreement in much of the literature that mathematics lessons should focus on advancing mathematical understanding rather than replicating taught rules and procedures (e.g. Askew, 2012; Boaler, 2009; Pratt, 2002; Wood, 1999), which is often the core component of traditional lessons. Mathematics that focuses on mathematical understanding, the overt expression of

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[1] These countries were the top performers in both the TIMSS (2011) and PISA (2012) with regard to mathematics.

mathematical thought, the co-construction of knowledge, and problematic tasks[2] are accepted generally as being true to a reform agenda (see chapter two for a detailed exploration of reform-based mathematics lessons). The literature refers to this approach to mathematics in many ways including progressive mathematics teaching, a reform approach to mathematics teaching, and reform-oriented mathematics teaching. Extensive literature exists, particularly in the US, relating to reform-oriented initiatives in mathematics teaching. The literature strongly suggests that mathematics practice has proven to be particularly difficult to change (e.g. Fraivillig, Murphy & Fuson, 1999; Frykholm, 2004; Hamm & Perry, 2002; Pratt, 2002; Stigler & Hiebert, 1999; Warfield, Wood & Lehman, 2005). Furthermore, the literature suggests that the actualisation of traditional and reform-oriented classrooms can take very different forms; and indeed both classroom cultures can exhibit tenets that are synonymous with the other (e.g. Boaler, 1999; Ma (1999); Wood, Williams & McNeal, 2006). To similar effect, the literature outlines that such hybrid lessons are commonplace and sometimes reflect teachers that are in transition from a more traditional culture to a more progressive one (e.g. Hamm & Perry, 2002; Warfield et al. 2005). Essentially, it is not always *either or*, the reality of mathematics classrooms can be *both and*. The literature also suggests that the teacher has a pivotal role to play in changing such classroom practice. Specifically, many studies identify the pivotal role played by teachers in cultivating classroom cultures where classroom dialogue is truly democratic and where pupils play an increased participatory role (e.g.

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[2] Problematic tasks refers to tasks that require pupils to think and come up with a solution to the problem, not necessarily by using an algorithm but by using any method that pupils deem useful. "Problematic tasks challenge students' thinking and involve them in testing, proving, explaining, reflecting, and interpreting" (accessed from http://nzmaths.co.nz/problems-are-problematic).

Fraivillig et al., 1999; Walshaw & Anthony, 2008; Warfield et al., 2005; Wood et al., 2006). However, affording such opportunities to pupils means that teachers need to share mathematical authority with pupils. There is evidence in the literature that teachers often find this particular role change difficult (e.g. Frykholm, 2004; Hamm & Perry, 2002; Nathan & Knuth, 2003; Walshaw & Anthony, 2008). Another challenge for teachers that is highlighted in the literature is the role of skilled facilitator (e.g. Lampert, 1999; Rittenhouse, 1998; Walshaw & Anthony, 2008). In this role, teachers need to share responsibility and authority with pupils yet still steer mathematical discussions in productive directions. This skilful negotiation between facilitator and educator often requires teachers to step in and out of mathematical discussions at opportune moments. Knowing when to intervene and when to abstain can be challenging for teachers. Studies have also identified other challenges to the successful implementation of reform approaches to mathematics. In particular, the literature highlights the importance of teacher beliefs in the effective implementation or otherwise of reform-oriented mathematics classrooms (e.g. Warfield et al., 2005). Similarly, teacher discomfort is identified in the literature as a potential obstacle when teachers are required to change from the traditional role of mathematics teacher (e.g. Frykholm, 2004).

However, despite much of the literature strongly advocating a move away from traditional approaches to mathematics teaching, vigorous arguments against such moves are also evident in the literature. Considering the emphasis for almost two decades on reform-oriented mathematics teaching in the US, much of the opposition to reform mathematics teaching originates in the US. More specifically, the nature of mathematics and the nature of mathematics teaching and learning are hotly contested issues in the literature. Consequently, theorists, researchers, and educators disagree about the relevance and potential of reform-based mathematics teaching on several issues. The US math wars during the 1990s reflected heated controversies about the teaching of mathematics, specifically, between those advocating a reform curriculum who believe that it reflects a deeper, richer view of mathematics, and traditionalists who believe that such a curriculum is superficial and undermines classical mathematical views (Schoenfeld, 2004). However, Schoenfeld (2004, p.253) contends that although such controversies came forcefully to the fore in the 1990s as a result of the reform stimulated by the National Council of Teachers of Mathematics, the underlying issues being contested are more than a century old—these issues include:

- Is mathematics for the elite or for the masses?
- Are there tensions between "excellence" and "equity"?
- Should mathematics be seen as a democratizing force or as a vehicle for maintaining the status quo?

Another issue relates to the absolutist view of mathematics that exists in the Western world and the tacit understanding that such knowledge is infallible (Lerman, 1998). This is challenged by the belief in reform-based mathematics that knowledge is socially constructed and so uncertain and fallible. Likewise, Ernest (1998) is sceptical about the practicalities in realising specific aspects of progressive or reform-based mathematics in classrooms. Specifically, he highlights the reform notion that mathematics lessons should more closely resemble the work of mathematicians; something he concludes is difficult considering the very differing contexts between knowledge genesis and knowledge justification in mathematics research. He surmises that "...for educational reasons we may prefer to let the ethos of the mathematics classroom match that of the context of the genesis of mathematical knowledge rather than that of justification, in the culture of research mathematics" (p.258). Moreover, Kirshner (2010) highlights two flaws that he associates with the pedagogy underpinning reform. First, he criticises the contradictory ways that metacognition is incorporated into reform pedagogies—on the one hand, metacognition serves as an enculturation goal of instruction achievable through discussion and argumentation; and on the other hand, metacognition is a prerequisite for students' construction of conceptual content whilst engaged in collaborative activities. Consequently, he claims that reform pedagogy is a "...fundamentally incoherent agenda for student learning, like a cat trying to catch its tail, always just out of reach" (p.5). Secondly, he claims that the reform discourse does not support teachers in the development of the expertise necessary for coordinating independently coherent agendas of enculturation (dispositions) and construction (concepts). Another prominent critic—Hirsh (1996)—is more scathing of reform pedagogies on the basis that he believes content is more important than process and so disagrees with the emphasis on abstract, generalised thinking skills in reform efforts. Moreover, even advocates of reform efforts such as Stigler and Hiebert (1999) contend that the approach to reform of mathematics teaching in the US has been fundamentally flawed:

> The approach in the US has been to write and distribute reform documents and ask teachers to implement the recommendations contained in such documents. Those who have worked on this problem understand that this approach simply does not work. The teaching profession does not have enough knowledge about what constitutes effective teaching, and teachers don't have a means of successfully sharing such knowledge with one another. To really improve teaching we must invest far more than we do in generating and sharing knowledge about teaching. This is another sort of teaching gap (p.12).

Stigler and Hiebert conclude that misinterpretation by teachers and subsequent superficial change is a fundamental problem in the US approach to reform: "Teachers can misinterpret reform and change surface features—for example, they include more group work, use more manipulatives, calculators, and real-world problem scenarios; or include writing in the lesson—but fail to alter their basic approach to teaching mathematics" (p.106/107). They conclude that on the one hand considering the energy invested in reform in the US, it is shocking to realise how little penetration there has been into the classroom. On the other hand, they conclude that given the cultural, systemic nature of teaching it would be surprising if reform efforts were successful. Consequently, they contend that disseminating models of effective teaching through static documents will not work for a cultural activity such as teaching.

Despite all of these criticisms of reform-based mathematics teaching, Schoenfeld (2004) makes a case for the middle ground. Specifically, he argues that the two perspectives, taken to extremes, are nonsensical and untenable; however, he contends that each camp—the extreme reform camp and the extreme traditionalist camp—are not uniform and are instead a confederation of strange bedfellows:

> Some of the reformers are committed to a pure vision of discovery learning, consistent with the traditionalists' caricatures. Some are committed to the democratic, equity vision enunciated in the *Standards*. Some were provoked by the traditionalists and took a strong defensive stance. Similarly, some of the traditionalists feel that the mathematical values they cherish are being challenged. Some feel that equity and excellence are in tension, and that the reform curricula, whatever their social goals, must perforce weaken mathematics (and U.S. national security and the economy) (p.281).

Schoenfeld contends that despite the extremist proposals on both sides, a rational middle ground exists. This is important because he claims that the casualties from the math wars are children—children who do not receive the kind of robust mathematics education they should.

Irish Context

"Significant concerns" regarding the development of literacy and numeracy in Ireland have been identified by the Department of Education and Skills (2011, p.7). These substantial concerns resulted in the implementation of a nine-year national strategy that aims to improve literacy and numeracy standards amongst Irish young people – Literacy and Numeracy for Learning and Life 2011-2020. These concerns are discussed from the perspective of pupil performance in mathematics and mathematics pedagogy.

Pupil Performance in Mathematics

A genuine concern regarding mathematics achievement has existed in Ireland for a number of years, for example, on international assessments Irish 15-year old pupils perform significantly better at reading than at mathematics and have done for more than a decade (e.g. PISA, 2000, 2003, 2006, 2009, 2012). In essence, this discrepancy in mathematics performance is evident in all rounds of PISA. Correspondingly, the latest round of assessments in 2012 indicates that once more Irish pupils perform better on reading than mathematics. Specifically, out of 34 OECD countries Irish pupils' performance ranked 4th on print reading and 5th on digital reading but only 13th and 15th respectively on print mathematics and digital mathematics. Furthermore, one-fifth of Irish students who participated in PISA 2009 did not have sufficient mathematical skills to cope with every-day life; and at the other end of the spectrum Ireland had "...significantly fewer high performing students than other countries" (DES, 2011, p. 13). This underperformance of Irish high-achieving pupils was again evident in PISA 2012 when comparing these pupils with their international counterparts.

At primary level, Ireland participated on Trends in International Mathematics and Science Studies (TIMSS) in 1995 and again in 2011. In 1995, Fourth Class children performed significantly above the international average[3] in mathematics and ranked 6th of 17 countries. Third Class children attained a mean score which was similar to the international average[4] and were ranked 7th of 16 countries. In 2011, only Fourth Class pupils took part in Ireland and their performance was ranked 17th of 50 participating countries (Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1997). Similar to 1995, Irish Fourth Class pupils

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3 The average international mean score was 529 and Ireland's mean score was 550.

4 The average international mean score was 520 and Ireland's mean score was 527.

performed significantly above the international average[5] (Eivers & Clerkin, 2012). Although Irish primary school pupils performed above average on mathematics, a performance gap exists for reading and mathematics. On the Progress in International Reading Literacy Study (PIRLS) Irish Fourth Class pupils were amongst the top performers and ranked 10th of 45 participating countries (Eivers & Clerkin, 2012). Furthermore, only five countries achieved mean scores that were significantly higher than Ireland's on the reading assessment (PIRLS) whilst thirteen countries achieved mean scores that were significantly higher than Ireland's on the mathematics assessment (TIMSS). Regarding TIMSS, Irish pupils performed best on the subscales of Knowing, Applying, and Number and least well on the subscales of Reasoning, Measures, Data Display, and Geometric Shapes. Ireland's strong performance on Number and relatively poor performance on Geometric Shapes and Measures aligns generally with the 1995 TIMSS findings (Eivers & Clerkin, 2012). The relatively low score on Reasoning is noteworthy because it is deemed the most cognitively challenging of the subscales. Therefore, although Irish pupils have performed relatively well on two cycles of TIMSS, consistent weaknesses are apparent. National assessments of primary school mathematics have also identified weaknesses in these areas of Geometric Shapes and Measures. National Assessments of Mathematics Achievement (NAMA) have been conducted in Irish primary schools since 1977, however, the target class levels have varied between second, fourth and sixth classes[6]. In all three of the latest rounds of mathematics assessment, pupils performed best on *Data* and least well on the *Measures* strand (similar to

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5 Ireland's mean score was 527 and the TIMSS mathematics centrepoint was 500.

6 1977: Second and Fourth Classes; 1979: Sixth Class; 1984: Sixth Class; 1999: Fourth Class; 2004: Fourth Class; 2009: Second and Sixth Classes.

the 1995 TIMSS findings), in addition to performing poorly on higher-level mathematics skills such as *Problem-Solving*[7].

Mathematics Pedagogy

This concern regarding mathematics teaching and learning in Ireland progressed into a policy decision that resulted in major changes in mathematics at second level. The Project Maths initiative was introduced in twenty-four schools in 2008 and extended to all post-primary schools in 2010 and is described as a "...major national reform of post-primary mathematics curriculum and assessment in the Republic of Ireland for both junior and senior cycles" (Jeffes, Jones, Wilson, Lamont, Straw, Wheater & Dawson, 2013). However, similar to the US, such reform efforts in mathematics received criticism in Ireland. An example of such being the criticisms from the School of Mathematical Sciences in University College Cork who raised concerns about the extent to which students will be prepared for the breadth and depth of third level mathematics (Grannell, Barry, Cronin, Holland & Hurley, 2011). Furthermore, they criticise the claims being made that Project Maths can contribute to a deeper understanding of mathematics and the acquisition of skills necessary for the development of a smart economy. They also criticise the emphasis on real-life contexts and lament the exclusion of some mathematical content. However, despite such contestations, Jeffes et al. (2013, p.7) highlight the major alteration in teaching and learning that this initiative requires at post-primary level and describe it as a "...philosophical shift in Irish post-primary education towards an investigative, problem-focused approach to learning mathematics, emphasising its application in real-life settings". However, just because an initiative is progressed, this is no guarantee of effective implementation. Jeffes et al. (2013)

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[7] The skill of *Communicating & Expressing* was not included as it was judged that it could not be tested using a pen and paper test.

were commissioned to review the impact of Project Maths on student achievement, learning, and motivation. They found that although students reported frequently undertaking activities aligned with the revised syllabus, approaches that are more traditional continue to be widespread. In particular, they describe these traditional approaches as being those associated with using textbooks and copying from the board. Furthermore, they found that there did not appear to be a substantial shift in what teachers are asking students to do. Accordingly, they found evidence of students' mastery of procedures and problem solving but very little evidence of students involved in the higher-order skills such as reasoning and proof, or making connections between mathematics topics. The authors emphasise the need to engage students in high quality tasks that require them to problem solve; draw out connections between mathematics topics; communicate more effectively in written form; and justify and provide evidence for their answers. Although a reform-oriented agenda has been progressed in the post-primary mathematics arena in Ireland, little evidence exists of substantial change to traditional mathematics practice. These findings underscore the difficulty associated with changing mathematics pedagogy. Interestingly, there appears to be a general assumption that the teaching and learning espoused in this revised post-primary syllabus is similar to that experienced by pupils in mathematics lessons in Irish primary schools, in that Project Maths builds upon the "…foundations of mathematical knowledge acquired at primary school" (Jeffes et al., 2013, p.7). Furthermore, reflecting on the discussion paper that preceded the introduction of Project Maths illuminates this assumption further; in particular, it unearths the supposition that problem solving is an inherent component of primary school mathematics lessons, or more specifically, the assumption that teachers are implementing the primary school mathematics curriculum as it was intended:

> The revised Primary School Curriculum is more in line with the RoyceME philosophy and, in particular, with the problem-solving approaches to mathematics education. In time, it may eventually permeate second level education 'from the bottom up' according as students transferring to post-primary schools have had longer experience of such approaches at primary school...(NCCA, 2005a, p.6).

However, assuming that such mathematics practice is commonplace in Irish primary schools is greatly misaligned with the reality as identified by national studies and reports.

The available evidence does not support the assumption that Project Maths is building upon progressive mathematical experiences in primary schools. In fact, the findings of many reports and studies focussed on Irish primary school mathematics classrooms suggest that, like post-primary schools, a traditional approach to teaching mathematics continues to prevail. This is despite the fact that a revised mathematics curriculum was introduced in all Irish primary schools in 1999 and ostensibly implemented officially in classrooms over a decade ago (2002). This Primary School Mathematics Curriculum (PSMC) (Government of Ireland, 1999) advocates an approach to teaching and learning mathematics where the child is an active participant in the learning process, which includes meaning making and the construction of knowledge. Accordingly, it advocates a constructivist approach to mathematics learning involving the child as an active participant in the process where "information acquired is interpreted by the learners themselves, who construct meaning by making links between new and existing knowledge" (PSMC, 1999, p.5). This approach accentuates conceptual understanding and cautions against pushing children to "...premature mastery of computational facts and procedures" (p.5). It suggests that before finding expression in written form, concepts should be adequately developed through extended periods of oral reporting and discussion. The curriculum continuously emphasises the importance of productive mathematical discussion during meaning making and the co-construction of knowledge. Specifically, it underscores the role that this type of interaction plays in facilitating the testing of mathematical ideas, the subsequent modification of ideas, and the interrogation of reasoning and justifications. Furthermore, it promotes guided discovery, experimentation, and the use of mathematical discourse to advance mathematical thinking: "Experimentation, together with discussion amongst peers and between the teacher and the child, may lead to general agreement or to the re-evaluation of ideas and mathematical relationships. New ideas or concepts may then be constructed" (p.5). To similar effect, it states that the importance of structured opportunities for engaging in exploratory mathematical activity cannot be overemphasised. To this end, it endorses the constructive use of mathematical equipment: "The experience of manipulating and using objects and equipment constructively is an essential component in the development of both mathematical concepts and constructive thought through the strands of the mathematics programme" (p.6). Finally, the curriculum highlights the crucial role the teacher plays in the development of children's construction of meaning, problem solving strategies, and self-motivation. However, the actualisation of this role, or specifically what type of practice it requires of the teacher, is not necessarily clear. The curriculum emphasises the need for exploratory mathematical activities, the use of mathematical equipment, and the importance of discussion in the development of mathematical thought. However, with the exception of a reference to the teacher's role in supplying the child with appropriate mathematical language where necessary and the children's need for guidance when formulating theories about discoveries, the teacher's role within any of these activities remains ambiguous. A possible contradiction to this role of the teacher as guide, supporter, and helper is evident in the advocacy of scaffolding as a form of instruction. In this approach, the curriculum advocates the teacher modifying the support according to the needs of the child including breaking the task down into manageable parts for the child. Furthermore, it supports the "very important" role played by direct instruction in mathematics. These very different approaches may cause confusion for teachers: on the one hand the importance of children co-constructing knowledge and making meaning for themselves through exploration and discussion; and on the other hand the need for teachers to use direct instruction and to scaffold tasks that are deemed too challenging or unmanageable for children. The curriculum is unclear with regard to balancing such instruction, in particular, *when* and *how* such instruction can be effective is ambiguous.

Considering the approaches advocated in the PSMC, it is unsurprising that an assumption exists that Project Maths is building upon mathematical experiences of primary school. However, numerous reports and studies suggest that the reality of Irish primary schools mathematic classrooms differs greatly from the experiences espoused in the curriculum. Fullan and Pomfret (1977) differentiate between the intended and implemented curriculum: "Implementation refers to the actual use of an innovation or what an innovation consists of in practice. This differs from intended or planned use... (p.336). The previous section focuses on the *Intended* mathematics curriculum for Irish primary schools, whilst the following section highlights the reported pedagogical shortfalls in the *Implemented* mathematics curriculum in Irish primary schools. Since the implementation of the 1999 Primary School Curriculum, the Inspectorate has published several reports regarding teaching and learning in mathematics.[8] These reports highlight similar concerns in mathematics lessons:

- an over-reliance on whole-class teaching in a majority of classrooms including teacher-dominated discussion (DES, 2005a, 2005b, 2010);

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8 These reports include an Evaluation of Curriculum Implementation: English, Mathematics and Visual Arts (2005); Literacy and Numeracy in Disadvantaged Schools: Challenges for Teachers (2005); Incidental Inspection Findings: A Report on the Teaching and Learning of English and Mathematics in Primary Schools (2010); and the Chief Inspector's Report 2010 - 2012.

- classroom environments where pupils are not provided with opportunities to work collaboratively (DES, 2005a, 2005b, 2010, 2013);
- an over-reliance on textbooks as the chief teaching aid (DES, 2005a, 2005b, 2010, 2013);
- insufficient provision and use of resources, in particular, concrete materials (DES, 2005b, 2010, 2013); and
- insufficient differentiation to meet the needs of children with varying learning abilities and needs (DES, 2005a, 2005b).

Furthermore, the *Review of English, Mathematics and Visual Arts (2005b)* reported that in more than two-thirds of mathematics classrooms, teacher talk dominated where pupils worked individually and silently for excessive periods. The *Chief Inspector's Report 2010-2012* reported insufficient opportunities for pupils to a) handle learning materials and manipulatives (with "serious deficiencies" reported in one in five mathematics lessons) and b) engage in active learning in mathematics. It also highlights unsatisfactory use of talk and discussion in one in five mathematics lessons. Finally, it reports that the use of assessment and the quality of planning are not satisfactory with one in five mathematics lessons not being adequately prepared. Unsurprisingly, the recommendations from these reports include that:

- the over-reliance on textbooks as the primary teaching aid should be discontinued;
- teachers should plan adequately for mathematics lessons and the outcome of assessments should be used in planning programmes of work;
- pupils should be encouraged to use a range of reasoning and problem-solving strategies;
- teachers' awareness of the potential of collaborative learning should be heightened;

- talk and discussion should feature more prominently in mathematics lessons; and
- pupils should '...have access to the objects, equipment and materials necessary for them to discover, learn and consolidate their learning' (DES, 2010, p.17).

The findings of national and international mathematics assessments mirror these findings regarding the over-reliance on textbooks in Irish primary classrooms (e.g. NAMA, 1999, 2004, 2009; NAMIS, 2010; TIMSS, 1995). Similarly, the Inspectors' findings regarding the insufficient use of concrete materials are also corroborated in national assessments (e.g. NAMA, 2004, 2009). Finally, the findings regarding the insufficient opportunities for collaborative learning are mirrored by the NCCA (2005b) findings that whole-class teaching was the organisational setting most used, followed closely by individual work; whilst there was only limited use of pair or group work.

Several of these findings are mirrored in Murphy's (2004) study that aimed to investigate the fidelity of curriculum implementation in Irish Senior Infant classrooms[9], specifically, to explore the extent to which the curriculum guidelines were informing classroom practice. He undertook a Target Child Observation Study of pupil activity across 15 Senior Infant classrooms that comprised 50 hours of observation of the classroom activity of 150 pupils. He found that despite the curriculum emphasis on child-centred learning, most of the activities and patterns of interaction that he observed were generally teacher-focused. Moreover, instead of more progressive, child-centred approaches as advocated in the 1999 curriculum, he found teacher-focused traditional practice. Several of his findings mirror those of the Inspectorate as discussed earlier including a) a prevalence of whole-class teacher-focused instruction with little evidence of pupil interaction; b) an over-reliance on textbooks and worksheets; and c) a limited supply and use of concrete materials. He found that

[9] Senior Infants is the second year of formal schooling in Ireland.

mathematics practice in Senior Infant classrooms had not yet fully embraced the "...various dimensions of the interactive model of numeracy development outlined in the 1999 curriculum" (p.252). In particular, he found an over-reliance on numerical arithmetical number operations, a distinct emphasis on procedural rather than conceptual understanding, and a classroom approach and methodology that "...appears to remain teacher-directed and focussed, with pupil activity consisting of the widespread use of worksheets and textbooks...rather than on guided discovery, activity and practical application of numeracy concepts as envisaged" (p.252). He concludes that professional development programmes that help teachers re-construct their own understandings of child-centred instructional practices, in addition to improved resourcing of classrooms would contribute to advancing a more faithful implementation of the PSMC in Irish infant classrooms.

Therefore, general agreement exists that the pedagogical approaches employed in Irish primary mathematics lessons are misaligned with the constructivist principles which are advocated in the PSMC and so need to be reformed and enhanced (e.g. DES, 2005a, 2005b, 2010; Murphy, 2004; NAMA, 1999, 2004, 2009; NAMIS, 2010). Achieving this in individual classrooms is challenging; attempting to do this at a whole-school level is even more ambitious. Thus, teachers require guidance and support in attempting to address these perennially reported pedagogical shortcomings. Essentially, these findings suggest that Irish primary school teachers need support in implementing the *Intended* mathematics curriculum.

RESEARCH PROBLEM

The concern with mathematics education in Ireland highlights a complex, multi-faceted issue. Concern exists regarding a number of issues including disappointing pupil performance on national and international assessments, in addition to reported mathematics teaching in Irish classrooms. Although an overview was provided earlier with regard to mathematics performance, the focus of this study is on mathematics classroom practice. Although the mathematics curricula at both primary and post-primary level align generally with the principles that underpin reform-oriented or progressive mathematics, it appears that the reality of mathematics teaching and learning in Irish classrooms differs greatly from these principles. The research problem of this study encompasses the teaching and learning of mathematics in Irish primary school classrooms. Despite the approach advocated in the Irish primary school mathematics curriculum, evidence suggests that Irish primary school mathematics lessons remain traditional in nature. In particular, the problem seems to relate to mathematics classrooms in which teachers dominate discussions, mathematical authority is not shared, textbooks determine the mathematical content and tasks, pupils work individually and silently for long periods, and pupils regurgitate procedures without necessarily understanding the mathematical concepts that underpin them. Some mathematics educators regard this approach to mathematics as not suiting the needs of today's pupils (e.g. Askew, 2012, Boaler, 2009). Similar assertions prevail in Irish literature. After studying mathematics pedagogy in Irish post-primary schools Lyons, Lynch, Close, Sheerin, and Boland (2003) argue that the traditional approach to teaching mathematics is viewed increasingly as dysfunctional in educational terms and that there is growing recognition that mathematics teaching and learning takes place in a social context through the interactional experiences in the classroom. Consequently, it appears that pedagogical practice in mathematics is a problem in Ireland. Tangible efforts have been made at post-primary level to tackle this problem through the national rollout of a revised syllabus called Project Maths that is accompanied by long-term professional development for teachers[10]. Paradoxically, no such

[10] Professional development for Project Maths is provided by a dedicated national support service that focuses solely on supporting post-primary mathematics teachers. This

concerted national reform initiatives for mathematics are evident yet in Irish primary schools[11]. Accordingly, this study is timely in exploring the reality of implementing a reform approach to mathematics teaching in an Irish primary school. As a result, policy makers, teacher educators, schools, and teachers can learn from this study with regard to the constraining and enabling factors that may exist whilst attempting to implement change in Irish primary school mathematics practice.

ORGANISING FRAMEWORK

In chapter two, the pertinent literature is reviewed with a specific focus on relevant empirical studies. Three distinct sections are reviewed in this chapter beginning with an exploration of various types of mathematics lessons. This includes details of lessons that are based on a reform approach to mathematics teaching and a rationale is provided for using such lessons. Similarly, traditional mathematics lessons are outlined in addition to the existence of hybrid mathematics lessons that comprise features of both traditional and reform-based mathematics lessons. The second section focuses on implementing a reform approach to mathematics teaching and is explored through pertinent studies. Specifically, challenges that teachers face when attempting to implement reform approaches are investigated. Finally, effective features of professional development are identified through a review of the literature.

In chapter three, the research problem is outlined in addition to the four research questions. Furthermore, the research design that was used in the study is detailed. A rationale is provided for the use of qualitative research as I attempt to situate myself within the study

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professional development has taken place incrementally over a number of years.

[11] Although a new mathematics curriculum for the junior classes in primary schools is due for completion in 2017 (personal communication with the NCCA).

including my views on teaching mathematics, knowledge, and research. In this chapter, a conceptual framework for the study is provided and an overview of the research cycle and the participants is included. Furthermore, a description of data collection and data analysis is provided, whilst quality assurance and ethical considerations are also detailed.

In chapter four, the findings of the study are reported in three distinct sections. The first section focuses on the findings with regard to each case study teacher, in particular, shifts in practice in the observed mathematics lessons and each case study teacher's experience of implementing the reform approach to mathematics teaching. In the second section, factors that contributed to supporting shifts in mathematics practice are explored. The final section focuses on exploring factors that may have constrained the potential for embedding and enhancing changing mathematics practice. In all sections, triangulation of data is used where appropriate in an attempt to provide a holistic view of the study, the ultimate aim being to answer the research questions.

In chapter five, the main findings of the study are outlined and conclusions are drawn from these findings. The implications of this study for policy makers, the Inspectorate, teacher educators, and schools are also discussed in this chapter. The limitations of this study are also outlined. Finally, the need for further research in relation to a number of key areas is highlighted at the end of this chapter.

CHAPTER 2: LITERATURE REVIEW

In the previous chapter, the problem with regard to mathematics teaching and learning in Irish primary schools is detailed. Specifically, the prevalence of traditional mathematics is highlighted, classrooms in which procedural fluency is strongly emphasised[12]. The need for Irish primary school teachers to adopt practices that are more aligned with reform-based mathematics teaching is evident. This literature review aims to explore the implementation of a reform approach to mathematics teaching following professional development through pertinent studies. There are three sections in this literature review: types of mathematics lessons; implementing a reform approach to mathematics teaching; and effective professional development. The section based on types of mathematics lessons outlines the conceptualisation of traditional mathematics lessons and from the literature identifies common features of such lessons. It explores reform-based mathematics lessons and outlines the rationale for such approaches. Furthermore, it highlights the existence of hybrid mathematics lessons and explores categories of both traditional and reform-based classroom cultures. The section on the implementation of reform approaches of mathematics teaching outlines studies that identify challenges faced by teachers when attempting to implement reform approaches. Finally, this section explores the possible constraints that may exist when implementing a reform approach to mathematics teaching including the challenge of teacher discomfort. In the section on effective professional development, the findings of a

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12 The National Council of Teachers of Mathematics define procedural fluency as the "ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another".

comprehensive study that synthesised international research on professional development studies are outlined. Specifically, features associated with professional development that was successful in affecting student outcomes are highlighted. To similar effect, features of effective professional development in mathematics are identified.

TYPES OF MATHEAMTICS LESSONS

Several types of mathematics lessons are identified in the literature. These lessons fall into three main categories: traditional mathematics lessons, reform-based mathematics lessons, and hybrid lessons that comprise elements of both traditional and reform-based mathematics lessons. This section explores all three types.

Traditional Mathematics Lessons

Traditional mathematics lessons sometimes are referred to as conventional mathematics lessons in the literature. Traditional mathematics lessons are very different to those espoused in the literature as being useful in advancing pupils' mathematical understanding. Boaler (2009) contends that a huge gap exists between what we know works for children when learning mathematics and what actually happens in most classrooms. There is general agreement in the literature that traditional mathematics lessons usually involve:

- the teacher demonstrating and explaining lessons which pupils reproduce accurately over and over again, often reproducing these lessons in silence (Askew, 2012; Boaler, 2009);
- talk is often limited to answering closed questions (Askew, 2012);
- uninspiring 'drill and kill' methods which encourage passive learning (Boaler, 2009);
- pupils following specific procedural instructions (Stigler & Hiebert, 1999);
- an over-emphasis on isolated procedural skills (Stigler & Hiebert, 1999);
- prevalence of pupils working silently and individually (Askew, 2012; Boaler, 2009);

- problem solving as the endpoint of mathematical application (Askew, 2012);
- little mathematical discussion (Askew, 2012);
- pupils being taught to follow the rules thus negating the necessity for thinking (Boaler, 2009);
- the teacher being viewed as the sole source of mathematical knowledge and the analytic centre for mathematics (e.g. Hamm & Perry, 2002; Nathan & Knuth, 2003; Wood, Williams & McNeal, 2006).

Boaler (2009) suggests that the uninspiring 'drill and kill' methods that often feature in traditional mathematics lessons sometimes encourage passive learning where students quickly learn that thought is not required. She further highlights the irony that "mathematics, a subject that should be all about inquiring, thinking and reasoning is one that students have come to believe requires *no thought*" (p.37). She also expresses a concern about disempowerment because "many students learn to suppress their thoughts, ideas and problem solving abilities in mathematics classes" (p.40) which she claims is one of the most serious indictments of our education system. This disempowerment results from situations where pupils work in silence, without offering their ideas and perspectives. She suggests that there are many problems with this 'silent approach' to mathematics; in particular, she argues that mathematicians highlight the essential role of talking over listening, however, "listening is the signature of the passive mathematics approaches that are the norm for students" (p.41). Askew (2012, p.xii) echoes these sentiments and also highlights concerns about the lack of talk in mathematics lesson: "Children sat in groups but did not really work together, despite what I'd read about the power of group work. Talk was mostly limited to answering closed questions, not the lively discussion of mathematical ideas that research advocated". Furthermore, traditional mathematics lessons assume a close link between high levels of teacher control and minimal pupil discourse (Walshaw & Anthony, 2008) which is indicative of an expositional model of instruction that limits democratic participation by pupils.

Although Anghileri (2007, p.1) argues that mathematics teaching has changed over the decades and that "children in the mathematics classroom today will be involved in many more thinking activities and much less drill and practice"; many of her contemporaries disagree (e.g. Askew, 2012; Boaler, 2009; Ma, 1999; Stigler & Hiebert, 1999; Wood, 1999). Wood (1999) claims that in order for children to develop their conceptual understanding of mathematics, changes in existing classroom practices are required. However, changing mathematics practices has proven to be difficult and "many children are still subjected to an out-dated and narrow form of teaching" (Boaler, 2009, p.3). Furthermore, Stigler & Hiebert (1999) claim that teaching is the next frontier in the continuing struggle to improve schools; particularly because teaching is the key lever for improving learning. However, they acknowledge that there is very little evidence that change is occurring and if change does occur, teacher's practice often only changes in superficial ways. Regarding the slow rate of change, Wood (2001, p.116) offers a plausible explanation and suggests that teaching mathematics differently is counterintuitive to time-honoured pedagogical practices:

> ...as adults we are aware when children do not know what we know. When we see children struggling with a new skill or knowledge, we typically take steps to alleviate the situation by "teaching" – telling and showing children what it is they need to know.

Nonetheless, there is general agreement that the mathematics that many pupils encounter in schools is an impoverished version of the subject, which little resembles the creative work of mathematicians (Askew, 2012; Boaler, 2009; Pratt, 2002; Schackow & O'Connell, 2008). Boaler (2009) refers to school mathematics as being a distorted image and a mutated version of the subject - a fake version that misrepresents the subject. In contrast, the processes used by mathematicians illuminate the exploratory and creative nature of mathematics. These processes include the ability to problem solve; communicate their ideas; reason through mathematical situations; share solutions and strategies; talk about patterns; jointly create conjectures; refine with counter examples; prove their conjectures; make connections between and among mathematical concepts, and represent their mathematical thinking (Askew, 2012; Boaler, 2009; Schackow & O'Connell, 2008). In particular, the collective endeavour of mathematicians is noteworthy:

> ...mathematicians jointly create conjectures through talking about patterns, sharing methods and discussing strategies. Indeed it is the collective activity that drives much of this. There is little point in describing or conjecturing for its own sake. It is through being immersed in such group activities that individuals come to take on board such habits of mind (Askew, 2012, p.44).

The mathematics that people need for life and work is not the type of mathematics learned in most classrooms – "people do not need to regurgitate hundreds of standard methods, they need to reason and problem solve, flexibly applying methods in new situations" (Boaler, 2009, p.9). Askew (2012) further validates the notion that current mathematics teaching is not fit for purpose – preparing pupils for life. He draws parallels between the drill of endless calculations with 'mindless-ness' and argues that such chores are out-dated and obsolete. The misalignment between what many pupils experience in mathematics classrooms and the type of mathematics that they require to be flexible and autonomous thinkers is a common theme in the literature. Fundamentally, there is agreement in much of the literature that traditional mathematics lessons are not 'fit for purpose'.

This mismatch between traditional mathematics lessons and the type of mathematics teaching needed for 21st century living is unsurprising when the learning theory underpinning such traditional approaches is considered. The traditional practice encompassing "drill and practice" and "reinforcement" aligns most favourably with the behaviourist theory of learning. Behaviourist theories of learning are influenced by the belief that learning constitutes the acquisition of skills (Greeno, 1998) and that all learning can be explained as a response to external stimuli. This theory extends to the view that 'knowledge' can be transferred from the teacher to pupils and that pupils can acquire this knowledge through 'drill and practice' (Greeno, 1998; Lerman, 2000) and positive and negative reinforcement (Lerman, 2000). Furthermore, this theory is based on the educational premise that complex skills are learned by acquiring simpler components followed by combinations of these into more complex behavioural abilities (Greeno, 1998). In relation to behaviourism, Lerman (2000, p.22) claims that this theory of learning supplied the "...psychological rationale both for the building blocks metaphor for the acquisition of mathematical knowledge and the pedagogical strategies of drill and practice, and positive and negative reinforcement" (p.22). Accordingly, the links between behaviourist theories of learning and traditional mathematics lessons are apparent. However, Anghileri (1995) outlines the shortcomings of the behaviourist theory of learning for children and contends that this theory is inadequate in explaining the way in which children learn to use language, to have ideas, to be creative, and to solve all kinds of problems in mathematics.

Reform-Based Mathematics Lessons

Reform-based mathematics lessons are referred to also in the literature as progressive mathematics lessons, reform-oriented mathematics lessons, and reform approaches to mathematics teaching. The goal of reform-based mathematics teaching is to enable pupils to "...think creatively and flexibly about mathematical concepts and solve mathematical problems with understanding"; however, for this to come to fruition, teachers need to create situations in which students are involved in exploring mathematical ideas, making conjectures about those ideas, and justifying their mathematical reasoning (Warfield, Wood & Lehman, 2005, p.439). It is suggested that when mathematics is taught in this way – "the whole subject that involves problem solving, creating ideas and representations, exploring puzzles, discussing methods and many different ways of working", then more people are successful (Boaler, 2009, p.2; see also Stigler & Hiebert, 1999). Many contestations exist in the literature regarding the type of mathematics with which contemporary pupils should engage. Boaler (2009, p.13) contends that children need "to solve ill-structured problems, to ask many forms of questions, to draw and visualize mathematics and to use, adapt and apply methods". Askew (2012) argues that children should be engaged in problem solving and in meaningful dialogue around big ideas in mathematics – mindful mathematics lessons where someone (not just the teacher) is explaining and everyone (including the teacher) is following the explanation. He also argues for engaging children in mathematical activity rather than assuming that "knowledge is something that they lack, we have and simply need to pass on to them. We have to help children to become knowing, through collective activity, rather than see them as passive recipients of knowledge" (p.100). Similarly, Schoenfeld (2002) suggests that children should develop deep understandings about important mathematical ideas in which skills and understandings deeply intertwine. Ma (1999) echoes this aspiration and draws a clear distinction between "practical arithmetic" and "arithmetic with a reasoning system"; claiming the latter as being the core mathematical content for high-achieving countries in mathematics comparison studies. Finally, Wood (2001) contends that learning mathematics with understanding best occurs in situations where children problem solve, reason and communicate their ideas and thinking to others. Accordingly, the literature repeatedly advocates collective endeavours such as a) problem solving, b) reasoning, and c) communicating and refining thinking about big mathematical ideas. In summation, reform-based mathematics lessons focus on understanding of mathematical concepts in a democratic environment that demands pupil participation. The successful implementation of such an approach appears to require teachers to adopt a very different role to that which is synonymous with traditional mathematics classrooms.

Boaler (2009) conducted an eleven-year longitudinal study with students from two UK secondary schools focused on different ways of learning mathematics. One school used a project-based approach (PBA) whilst the other school used a more traditional approach (TA). The PBA involved students working on open-ended projects that needed mathematical methods instead of students practising procedures. The TA comprised students practising methods that were demonstrated by teachers and working through short exercises in their books. On the General Certificate of Secondary Education (GCSE) mathematics examination, the students who experienced the PBA attained significantly higher grades and attained higher grades than the national average despite having started at significantly lower levels than the national average. The PBA students out-performed the TA students on all of the different assessments designed to assess students' use of mathematics in 'real world' situations. All 40 of the students from the TA said they would never make use of their school learned methods outside of school. Contrastingly, the students from the PBA were confident that they would use the methods they learned and furthermore, could give examples of using mathematics in their jobs and lives. Eight years later these gains remain: young adults who had experienced the PBA were working in more highly skilled or professional jobs than those that had experienced the TA. This is despite the fact that the school attainment range of the survey respondents was equal across both schools. Moreover, when comparing the jobs of children to their parents, 57% of the PBA adults were working in jobs that were more professional compared to only 23% of the TA adults. Similarly, 52% of the TA adults were in jobs that were less professional than their parents compared with only 15% of the PBA adults. Finally, the PBA adults communicated a positive approach to work and life, describing ways that they used their school-based methods to solve problems and make sense of mathematical situations in their lives; whilst the TA adults expressed concerns that their school-based mathematics had not prepared them for the demands of the workplace. Boaler concludes that the contrasting teaching approaches resulted in students developing different ways of knowing mathematics, and profound differences in the ways that young adults interacted with knowledge.

The theoretical and philosophical underpinnings of the reform curriculum are more complex and often more contested in the literature than those underpinning traditional mathematics approaches. Therefore, drawing a direct parallel between reform approaches and one distinct theory of learning is not an easy task. It may be more helpful to consider a number of related learning theories that are likely to have influenced reform approaches—the constructivist theory of learning, the sociocultural theory of learning, and the situated perspective. Not only do multiple variations regarding the epistemological underpinnings of the constructivist theory of learning exist in the literature, contestation is also evident regarding the potential of merging these theories of learning. The constructivist theory of learning emphasises the mind and cognition as opposed to behaviours. This theory is most influenced by Piaget who was the pioneer of this approach to cognition in this century (von Glasersfeld, 1994a). Piaget explains that, in his view, knowledge arises from the active subject's activity, either physical or mental, and that it is goal-directed activity that gives knowledge its organisation (von Glasersfeld, 1994a). The emphasis on cognition in Piaget's constructivist perspective is illuminated by von Glasersfeld[13] (1994a, p.74): "Piaget's position

13 Von Glasersfeld (1994a) acknowledges the difficulty in abstracting a coherent theory of cognitive development from Piaget's enormous body of work that spanned a period of over 70 years and in which he published 88 books and hundreds of articles. Hence, the danger in attempting to summarise his work based on two or three of his books is highlighted by Von

can be summarily characterized by the statement: 'The mind organises the world by organising itself' (1937, p.311). The cognitive organism shapes and coordinates its experience and, in doing so, transforms it into a structured world". In contrast to behaviourist principles, constructivist theorists view learning as more than just transmitting knowledge from one person to another; instead, the learner is viewed as an active participant in constructing their own knowledge from personal experience (Cobb, 1994; Ernest, 1994). This construction of knowledge requires the learner to engage in meaning making (Ernest, 1994; Greeno, 1998) and is underpinned by the psychological paradigm (Cobb, 2000; Lerman, 2000). The focus on the psychological perspective is often equated to an emphasis on the individual in constructivism (Cobb 1994; Ernest, 1994a; Lerman, 1994). It is this emphasis on the individual to the detriment of the social aspect of learning in constructivism, which appears to have incited most of its critics. For example, Lerman (1994) claims that the mathematics education research community adopted a trivial form of constructivism—focusing on the active child constructing his/her knowledge and viewing the teacher's role as setting up a 'constructivist' classroom and making models of children's understanding. Furthermore, he explains that radical constructivists (e.g. Bauersfeld; Voigt; Confrey; Ernest; Cobb) recognise the diminished role played by language in constructivism and so have attempted to include social interactions in constructivism. He claims that they work on the assumption that the cognizing individual is central as the meaning-maker and that the environment, including other people, contributes to the construction of that personal meaning. Consequently, the

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Glasersfeld who claims that such efforts can lead to limited perspectives that at best provide incomplete views of Piaget's theory and at worst can perpetuate the distortion of key concepts. Nonetheless, Von Glasersfeld outlines his interpretation, subjectivity included, of Piaget's work, which he refers to as a theory of the knowing mind.

most fundamental criticism of constructivism in the literature relates to the emphasis on the individual to the detriment of a sociocultural perspective. Such criticism acted as a catalyst for, and contributed to, the development of another theory of learning—social constructivism. This perspective emphasises the socially and culturally situated nature of mathematics (Cobb, 1994) but includes a focus on the individual constructing his/her own meaning in response to experiences in social contexts (Ernest, 1994b). It is apparent in the literature that social constructivism is used as an umbrella term that covers vastly divergent views. For example, Ernest (1994b) proposes that social constructivism can be viewed with a Piagetian theory of mind or with a Vygotskian theory of mind. He further contends that social constructivism with a Piagetian theory of mind can be delineated into a) a radical constructivist position that prioritises the individual aspects of knowledge construction but acknowledges the important, but secondary role, of social interaction; and b) a complementarist position that accommodates both an individual and a social construction of knowledge. Finally, he argues that social constructivism with a Vygotskian theory of mind views individual subjects and the realm of the social as inextricably interconnected, with human subjects formed through their interactions with each other (in addition to their internal processes) in social contexts.

However, Waschescio (1998) persuasively argues against the distinctions that both constructivist and sociocultural theorists often make, based on the premise that their underlying assumptions are flawed. Specifically, Waschescio queries the tenet that constructivism considers cognitive psychological and sociological aspects whereas sociological (Vygotskian) approaches focus solely on the latter. For Waschesico, the main concern here is that this distinction appears to be artificial, in that, it suggests a separation between cognitive, psychological, and individual processes on the one hand and between sociological, cultural, and interactional on the other. Furthermore, such a distinction engenders a view that "...cognitive psychological processes are internal whereas external processes are sociological; that is, they are noncognitive, nonpsychological" (p.239). Waschesico explains that in fact the distinction Vygotsky makes between internal and external processes is that between inner-psychological and inter-psychological ones; consequently, Vygotsky locates psychological processes not just within individuals but also between individuals. Consequently, Waschesico concludes that it is not

> ...justified to create a contrast in which individual and cognitive aspects of development are labeled psychological whereas interindividual and cultural aspects are labeled sociological, and then ascribe the first label to constructivism and the second label to Vygotskian approaches. Indeed, the central claim of Vygotskian theory is that these levels – cultural, interpsychological, and inner-psychological – cannot be separated (p.240).

Echoes of the reform approach to mathematics teaching are evident in Shulman's (2000) extolling of social processes in developing and refining understanding. Specifically, he promotes verbal interaction such as dialogue, exchange, conversation, discussion, and alternating argument as contributing to the prevention of illusory understanding[14]. He contends that it is through this verbal interaction that deep understanding becomes apparent because you do not know whether people really understand something until you push them in discussion, conversation, and dialogue. For him, the essence of pedagogy is putting the inside out (getting students to say what they know with greater precision and rigour), working on it together while it is out (social interaction), then putting the outside back in (the new or refined understanding) so that the intellectual and social cycles of learning can begin all over again.

Similar to Waschescio, Greeno (1998) also criticises the forced separation of learning theories. Specifically, he criticises the habitual portrayal in research literature and the popular press of the behaviourist skill-oriented and cognitive understanding-oriented perspectives as

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14 Shulman (2000) defines illusory understanding as (appearing to know something but not really knowing it).

diametrically opposed. He disagrees that learning according to one perspective precludes learning according to the other. Instead, he argues that important strengths and values of behaviourist and cognitive practices can be included in the practices drawn from the situative perspective, which values students' learning to participate in inquiry and sense-making. He contends that the situative perspective on learning and cognition can provide a synthesis that subsumes the cognitive and behaviourist theoretical perspectives. The aspiration presented by Greeno undermines the argument that behaviorist and cognitive perspectives are in conflict. Specifically, he suggests that students can learn to engage in activities in which technical skills support individual contributions and in which conceptual understandings are both used and constructed. This theory of learning called the situative perspective proposed by Greeno (1998) is endorsed by Cobb (2000) and Putman and Borko (2000). In this theory, attention is focused on "…systems in which people interact with each other and with material, informational, and conceptual resources in their environments" (p.23). Cobb (2000) contends that one of the strengths of the situative perspective is that it highlights systems as well as individuals. Putman and Borko (2000) outline three conceptual themes that are central to the situative perspective; these themes include that cognition is a) situated in particular physical and social contexts, b) social in nature, and c) distributed across the individual, other persons, and physical and symbolic tools. This notion of cognition as situated and shared differs from traditional cognitive perspectives that focus on the individual as a unit of analysis. It also highlights the importance of 'others' in the learning process, in particular, the importance of discourse communities that provide "…the cognitive tools—ideas, theories, and concepts—that individuals appropriate as their own through their personal efforts to make sense of experiences" (Putman & Borko, 2000, p.5).

In relation to traditional didactic instruction, the display of skill is often treated as the most important form of social participation, and skills are often divorced from their connections to activities in communities outside the classroom (Greeno, 1998). However, Greeno contends that although technical skills have value, technical skills are not the only basis for making valuable contributions. In particular, he argues that in the situative perspective, skills are understood as aspects of a person's participation in social practices. In relation to reform efforts in mathematics, Greeno (1998) relates these to the participation-oriented educational practices advocated in the situative perspective:

> ...these practices encourage students to participate in processes that include conceptual inquiry and use of skills in problem solving that emerge in meaningful projects. Their intent is to extend students' learning activities to include formulation and evaluation of conjectures, examples, applications, hypotheses, evidence, conclusions, and arguments, and to have conceptual growth and skill acquisition occur in relation to these participatory activities (p.15).

Similarly, in conducting design experiments in mathematics classrooms, Cobb (2000) reports that he and his colleagues have come to reject purely individualistic approaches, and instead find it more useful to view students' mathematical reasoning as "...acts of participation in communal practices that they and the teacher establish in the course of their ongoing interactions" (p.76). Boaler (2000b) who also extols the potential of using situative perspectives for mathematics teaching, learning, and research echoes this broader conceptualisation of what it means to learn mathematics:

> A focus on the broader systems in which students operate will involve consideration not only of the concepts and procedures that students learn but also of the practices in which they engage as they are learning and the mediation of cognitive forms by the environments in which they are produced (p.118).

A reform approach to mathematics teaching is the focus of this study and forms the basis of the approach advocated with teachers during the professional development programme.

Hybrid Lessons

Although seemingly obvious, it is worth noting that similar to theories of learning, mathematics lessons do not always fall neatly into one category or another – traditional or reform-based. Aspects of mathematics lessons can fall into either category and indeed lessons could be categorised as being anywhere on a continuum between traditional and reform-oriented. Ma (1999) validates the notion of hybrid mathematics lesson in that she suggests that this change may be an evolutionary process where elements of both traditions can prevail:

> ...the change of a classroom mathematics tradition may not be a "revolution" that simply throws out the old and adopts the new. Rather, it may be a process in which some new features develop out of the old tradition...the two traditions may not be absolutely antagonistic to each other (p.153).

A study by Warfield, Wood & Lehman (2005) suggests that hybrid lessons can be indicative of a transition from traditional mathematics lessons to more reform-based ones. Consequently, hybrid lessons may be considered a manifestation of change. Over a two-year period, they investigated the learning of seven beginning elementary teachers who participated in a professional development project designed to help them to learn to teach mathematics according to reform recommendations. Analysis of the mathematics lessons revealed distinct differences in some lessons between years one and two. In year one, teachers were in transition from conventional mathematics teaching; however, their ways of working still maintained some features of conventional mathematics lessons. So although these lessons contained features of interaction and discourse that are consistent with traditional mathematics lessons, they were within a "reform class discussion structure" (p.445). In particular, the focus in these classes was on pupils' reporting of different strategies to the rest of the class. Resulting from the teachers' transition role, the teachers often took over by a) repeating or echoing what the pupil said to make sure that everyone heard the strategy; b) adding detail to the pupils' strategy; and c) providing a rationale for the pupils' strategy. Although teachers would often then be true to the reform agenda by asking if anyone had a different strategy, teachers in year one were still dominating the discourse in these mathematics lessons. In year two, distinct differences emerged between two groups of teachers in relation to their practice. One group developed a form of practice that was consistent with the reform agenda whilst another group maintained the same form of teaching as before - "...a strategy-reporting environment with characteristics from conventional instruction" (p.448). In the latter group, teachers curtailed their children's thinking and introduced problems in such a way as to indicate how they should be solved. In one teacher's class, the pupils' explanations were "...neither complete nor clear, but the teacher did not question them in ways that enabled them to further explain" (p.448). In this group, teachers viewed their role as managing the interactions in the class and were primarily concerned with classroom management issues and the mechanics of implementing the reform approach, for example, how to encourage pair work and collaborative problem solving. They also considered discussions as opportunities for pupils to learn and adopt new strategies with no focus on involving pupils in questioning and challenging one another. In stark contrast, teachers in the other group developed practice that also comprised strategy-reporting characteristics but was enhanced by opportunities for justification. Listeners asked questions for the purposes of clarification and understanding, and the explainer provided reasons for their thinking. Teachers in this group did not focus on classroom management issues; instead, they focussed on children's understanding of mathematics. Regarding classroom discussions, these teachers saw their role more broadly than just establishing routines. Their expectations of their role included allowing pupils to think for themselves and explain their thinking, encouraging pupils to work through problems and clarify unclear issues, and standing back so that pupils could take control. These teachers learned that "children are capable of solving problems and of explaining and justifying their own thinking, and they learned that their actions played a role in allowing that to happen" (p.449). Warfield et al. (2005) conclude that although all seven teachers adopted practices advocated in the reform approach, some of these teachers did not learn to teach in ways that encouraged children to become autonomous learners. They propose divergent conclusions for the remaining teachers who they claim began to learn to teach mathematics in ways that enabled their children to become autonomous learners. This included allowing children to solve problems in their own ways, expecting children to explain and justify their thinking, and to listen to and question the reasoning of others. They claim that these findings offer explanations as to a) how teachers' beliefs, learning, and practice might be linked and b) why learning differs for teachers in the same professional development project. They conclude that teacher beliefs about the autonomy or authority of individuals can support or constrain their learning. This study illuminates the relationship between teacher beliefs and practice when implementing a reform-based approach to mathematics teaching. In particular, the findings suggest that when some teachers were in transition from conventional mathematics teaching to more progressive approaches; their ways of working still maintained some features of conventional mathematics lessons. Specifically, the researchers found that teachers adopted aspects of the reform approach, such as, reporting strategies but this was embedded within structures that are more traditional. Furthermore, this study is useful in identifying ways in which teachers respond differently to the same professional development programme. In this way, this study is pertinent when considering the current study.

Consistent with this notion of hybrid mathematics lessons and a continuum of classroom cultures, Wood, Williams and McNeal (2006) further categorise traditional or conventional mathematics lessons into a) conventional textbook and b) conventional problem solving; whilst sub-dividing progressive or reform-oriented mathematics lessons into a) strategy reporting and b) inquiry/argument. These four broad classification categories for classroom culture are based on different patterns of social interaction. Their findings suggest that children's expressed mathematical thinking is related closely to the types of interaction patterns that differentiated class discussions among the four classroom cultures. In the conventional textbook class discussion, children's participation was limited to responding to teachers' questions by giving known answers or predetermined information whilst in the conventional problem-solving class there were more opportunities for children to participate in the discourse, in particular through the exploration of methods. However, they found that the challenge was removed for pupils in the latter classes because teachers often 'hinted' at the solution method. In the strategy reporting class, children's participation again increased substantially, which is unsurprising considering the focus on sharing strategies. Interestingly, they found that the "...interaction patterns found in conventional classroom environments (IRE, Funnel, Give Expected Information) rarely exist in the strategy reporting classroom culture" (p.235). Instead, pupils reported and explained their solutions. However, teachers still dominated part of the discussion by elaborating on and extending children's explanations to convey important ideas to other children. Finally, the inquiry/argument class demonstrated not only the highest participation by children but also the biggest shift in the nature of this participation. The interaction patterns in these classes suggest a shift from an emphasis on children reporting different strategies to "...children as listeners taking over the role of the teacher in questioning, clarifying, and validating mathematical ideas" (p.235). Children's participation rates in the different classroom cultures is important because Wood et al. found considerable differences in incidents of expressed mathematical thinking in the various classes. They found that more mathematics was expressed in the conventional problem-solving lessons than in the conventional textbook lessons, and this mathematics was at a higher level. They found that the amount of mathematics expressed in conventional problem-solving classes and strategy reporting classes was similar, however the mathematics expressed in the strategy reporting classes was at a higher level involving the application of mathematical ideas to new situations, breaking mathematical tasks into component parts, and analysing the relationship between the parts. Furthermore, they found that the highest amount of mathematics was expressed in inquiry/argument lessons and the mathematics expressed was substantially higher than that expressed in the strategy reporting classes. The incidences of mathematical thinking increased across the four classroom cultures and so did the quality of thought (Wood et al., 2006). The researchers emphasise the differences they found between reform-oriented classroom cultures and conclude that only the inquiry/argument lessons provided opportunities for all children to be involved in meaning-making and to develop common ground on which to build shared understanding. This research is relevant to the current study because the findings suggest the important role that teachers play in creating classroom environments that demand pupil participation in discussions, which in turn increases opportunities for the expression of sophisticated mathematical thought.

IMPLEMENTING REFORM BASED MATHEAMTICS LESSONS

Peppered throughout the literature are studies that detail the complexity of implementing a reform approach to mathematics teaching. In this section, Irish studies that provide empirical evidence regarding the implementation of reform-based mathematics teaching approaches in Irish primary schools are explored. Furthermore, studies that identify the challenges faced by teachers when attempting to implement reform approaches in their mathematics teaching are detailed.

Irish Empirical Evidence

A number of Irish studies have investigated aspects of reform-based mathematics lessons in Irish primary classrooms. In this section, relevant findings from studies by Dooley (2010), NicMhuirí (2012), and O'Shea & Leavy (2013) are highlighted. One of the most common themes marbled throughout the empirical literature on reform approaches to mathematics teaching is the prevalence of traditional instruction despite teachers' engagement in reform initiatives. An Irish study with primary teachers effectively highlights this dilemma. A study conducted by O'Shea and Leavy (2013) investigated primary teachers' understanding of organising learning from a constructivist perspective following their engagement with constructivism and mathematical problem-solving in a professional development initiative. Although a small-scale study some interesting findings emerged regarding reform-based teaching, in particular, the use of hybrid approaches (a blend of traditional and reform approaches); the prevalence of traditional approaches throughout the study; teachers' apparent unfamiliarity with constructivist teaching approaches; and teachers' difficulty in moving from a didactic to a more facilitative role. More specifically, the study found that four of the five teachers reported finding the professional development that focused on organising learning from a constructivist perspective 'refreshing', whilst all five participating teachers found discussing and examining constructivism and its implications for their teaching exciting. The authors contend that this finding is surprising considering the central role played by constructivism in the Irish Primary School Mathematics Curriculum (1999); consequently, constructivism should not have been revolutionary to these teachers. The findings in this study suggest that although constructivism is a primary principle of the mathematics curriculum, it has yet to make the transition from the curriculum to the classroom. The study also found that blending of traditional practices and reform practices were evident across all five cases of Irish primary teachers. Moreover, this study suggests a number of significant factors that affected the successful transition to using constructivism as a basis for teaching in classrooms. One of the main factors hinged on translating the theory of constructivism—a theory of learning—into a theory of teaching. Other factors include the large number of students in each class; the breadth of the curriculum; the wide range of abilities in each class; and managing student learning in group situations. Other interesting findings emerged from different cases. For example, one teacher—Emily—explained that she teaches the number strand thoroughly to students using traditional approaches, and would only consider engaging students in learning from a constructivist perspective when she feels confident her students have significant background knowledge. Another teacher—Susan—viewed constructivism as primarily an enrichment activity for capable students of mathematics. Finally, despite being excited about constructivism and its implications for practice, the study found that teachers retained a deep respect for traditional mathematics instruction including the need for significant focus on computation and the importance of students recalling basic mathematical facts acquired through rote learning. The authors conclude that although teachers were inspired by learning from a constructivist perspective, it was evident that methodologies that reflect constructivist principles would not replace deeply rooted traditional methodologies used in their classrooms. Traditional approaches to teaching mathematics were evident amongst participating teachers, and in particular, "teachers found it difficult to redefine their relationships with students during instruction and from a didactic to a more facilitative role" (p.313). This resulted in reduced opportunities for students to fully experience a constructivist approach to learning because the teachers involved maintained a traditional didactic relationship with the students. The authors conclude that teachers faced many dilemmas in facilitating learning from a constructivist perspective including finding a balance between individual and group learning, in addition to defining appropriate constructivist learning experiences. The authors concede that these dilemmas are difficult to resolve considering the nature of the Irish primary classroom and societal expectations. They suggest that professional development focused on developing teachers' understanding of constructivism is not sufficient in helping them overcome the challenges of adopting it in their teaching of problem solving. Furthermore, they suggest that in order to make meaningful changes, professional development needs to deal with teachers in more holistic ways in order to address the multiple factors that influence their practice.

NicMhuirí's (2012) study included implementing a reform-oriented mathematical practice—the discourse community approach. This study comprised two stages. Stage 1 involved the analysis (in relation to discourse) of audio recordings of six mathematics lessons carried out by four teachers in two different schools. Stage 2 comprised a teaching experiment conducted by the author after which five lessons were analysed. The findings from Stage 1 suggest lessons in which little if any mathematical authority was devolved to students; student thinking was rarely an object of discussion; all teachers appeared to position themselves as mathematical authorities and played a strong role in evaluating student thinking; and students appeared to be positioned and to position themselves as received knowers. Furthermore, in Stage 1 of this study the mathematical discourse focussed mainly on numerical answers or computation strategies. Findings from Stage 2 suggest mathematics lessons in which mathematical power was devolved to students to some extent; students' mathematical thinking was valued; students were positioned as a source of mathematical ideas; and students were encouraged in their responsibility for their own learning and for the learning of their peers. In such a classroom, the study found that children build on each other's mathematical ideas in lessons and across lessons, and that a communal sense of responsibility for learning is developed. However, similar to other studies outlined in this review, implementing reform-based mathematics lessons sometimes proved challenging in this study. In particular, NicMhuirí identified a number of tensions involved in facilitating a mathematical discourse community. These tensions include arranging a balance between cognitively demanding and routine tasks; maintaining cognitive demand whilst ensuring the discourse was accessible to all students; knowing when to be explicit and use 'direct telling'; and refraining from 'direct telling'. The author maintains that the latter tension resulted in some issues not being addressed—issues that she believes possibly should have been addressed more directly.

In another study, Dooley (2010) investigated the construction of mathematical insight by pupils in whole-class conversation. She found that "…extended whole-class conversation can be a vehicle for the construction of mathematical insight. The insight that is thus constructed is frequently embedded in and instigative of constructions by other students" (p.229). Furthermore, this study indicates that construction of insight by pupils in whole-class settings is a complex interaction of task, classroom discourse style, and pupil engagement. Other interesting findings also emerged from this study. For example, the findings of this study indicate that mathematical insight is usually distributed amongst a few participants and that student agency is the overarching and critical variable in fostering mathematical insight. Furthermore, in this study, the effect of group-work increased pupils' contribution in whole-class discussion. Interestingly, this study suggests that frequently, pupils' reasoning of a task was more sophisticated at a second session, leading the author to conclude that time is an important consideration in the construction of insight. In this study, students assumed various roles, for example, some assumed the role of "task doers", others were "observers" who monitored the situation and regulated comments, whilst others were "tacit participants" who made few contributions in whole-class discussions yet engaged with and built with the ideas generated in follow-up work. Furthermore, this study indicates that students' roles varied from lesson to lesson and that different students assumed the lead role in different lessons. However, despite the construction of mathematical insight, similar to other studies underpinned by reform approaches, challenges were evident. In particular, one of these challenges relates to pupils who resisted engagement—a problem that the author claims happened to some extent in each of the lessons. Therefore, "if mathematics for insight is to be embraced, ways of dealing with such challenges need to be addressed" (p.236).

These studies involving reform-based mathematics teaching in Irish primary schools suggest that whilst challenges may present themselves, positive outcomes are also associated with these types of mathematics lessons. However, in both NicMhuirí's and O'Shea and Leavy's studies the prevalence of deep-rooted traditional mathematics teaching in Irish primary classrooms is highlighted.

Challenges When Implementing Reform Based Mathematics Lessons

Many studies in the literature identify the challenges faced by teachers when attempting to implement reform approaches in mathematics lessons. These challenges range from specific challenges such as sharing mathematics authority with pupils and facilitating mathematical discussion to more generic challenges that teachers face such as teacher discomfort in relation to reform based lessons. These challenges are explored in this section through pertinent empirical studies.

Relinquishing culturally embedded mathematical authority in lessons can prove challenging for teachers, as evidenced in a qualitative study by Hamm and Perry (2002). They investigated classroom discourse processes and participatory structures that grant sources of mathematical authority in six first-grade classes in five urban elementary schools in the US. After detailed coding of the lessons, the researchers found that only one of the six teachers granted authority to her pupils and created a classroom environment in which pupils participated in mathematical analysis and discourse. This teacher created multiple opportunities for her pupils to adopt a sense of mathematical authority. However, it is noteworthy that even though this one teacher invited pupils to assume responsibility; these practices were not consistently prevalent and most of her behaviour reinforced herself rather than the classroom community as the source of mathematical authority. Despite these attempts at sharing mathematical authority with the class she ultimately retained the mathematical authority. Therefore, "despite engaging in multiple practices that granted students a sense of mathematical authority, Teacher C rarely stepped away from the central role as classroom leader to allow true public discourse about mathematical ideas" (Hamm & Perry, 2002, p.136). Nonetheless, the practice of the other teachers in the study contrasted sharply with this teacher's practice in that the "...observed lessons were bereft of behaviors supporting these principles" (p.135). The teachers' practices firmly established and maintained the teacher as the source and authority of mathematical knowledge. Hamm and Perry derive a number of conclusions from this study. Firstly, they conclude that teachers may find it challenging to relinquish authority to the classroom community. Secondly, they conclude that the implementation of a reform approach to mathematics teaching is more than just the number or percentage of reform-minded behaviours. They propose that a small number may be adequate to make a difference if they are salient and not undermined by counter-productive behaviours; equally, a large percentage of these behaviours may be of little use if they are accompanied by behaviours that firmly communicate that the teacher is the sole mathematical authority in the classroom. In referring to this issue as "veneer versus substance", they highlight the need for a consistent and complementary approach by teachers when implementing a reform-based approach to mathematics teaching. Indeed, they refer to this as the "depth of teachers' commitments to reform-based practice in mathematics" (p.135). Their third conclusion is based on the procedures-based instructional practice that they observed with five out of the six teachers. They found that "students were engaged almost exclusively in approaching learning with the expectation that everything they were to learn was already known and the students had few opportunities to move beyond learning simple procedures" (p.136). Consequently, they conclude that although it is likely that these students will learn the mathematical procedures they have been taught, it is significantly less likely that they will learn that they can apply these procedures flexibly across mathematical topics. In essence, they conclude that multiple sources conspire to send mixed messages to students about their authority to develop and to verify mathematical knowledge.

A study of an experienced middle school mathematics teacher's efforts to change her classroom practice highlights that the quality of student talk as orchestrated by the teacher is paramount in enhancing mathematical understanding. Nathan and Knuth (2003) compared two years of a teacher's practice following an intervening professional development programme. The teacher taught sixth-grade mathematics for seventeen years in a US public school. In particular, this study focused on whole-class interactions with the teacher. In year one, they found that the teacher adopted a very familiar role – a strong centralised role where she served as a) the mathematical authority and b) the hub of the mathematical conversations in that nearly all exchanges flowed through her. They concluded that the teacher dominated both the analytical and the social realms of the classroom. Consequently, students a) rarely participated in exchanges unless called upon; b) relied predominantly on the teacher for information rather than other students; and c) rarely used discourse as a means to construct their own conceptions, test out hypotheses, or question other students' ideas. Following the professional development programme in the intervening summer, they found that several marked differences emerged as the teacher's role and actions changed in year two. They found substantial changes in the patterns of whole-class interactions. This shift in practice included a) a removal of the teacher as the central role in mathematical discussions; and b) social facilitation of student-driven discussions. This resulted in "...tremendous growth in student-to-student mathematical talk" (p.195) with a suspected decrease in teacher-led discourse. They found that the teacher "...removed herself from the analytic aspects of the classroom discourse and gave her attention primarily to the social aspects" (p.198) resulting in no clear authority for students to turn to when presented with uncertainty. With a mathematical authority vacuum, they found that these students did not always have the resources to construct or verify correct mathematical ideas and conventions, nor did they have a way of resolving differences or addressing their confusion. Therefore, they found that although students were interacting more frequently with each other there was a dearth in mathematical rigour:

> ...there was often a lack of rigorous argumentation and evidence in the discussions, and a lack of convergence toward acceptable mathematical ideas and conventions. With no clear mathematical authority participating, student ideas were offered publicly for others to pick up, refute, or ignore, with no basis for evaluation other than opinion (p.198).

This uncertainty was illustrated when one student suggested a vote as a means of gaining consensus in relation to the definitions of mathematical terms. Nathan and Knuth suggest that although students are able to fill the conversational void, they may not be able to serve as the analytical authority necessary to promote correct understanding about all of the content matters. They conclude that despite the goal of sharing mathematical authority, there is still a need to support students' learning during classroom interactions and that teachers need to monitor continually where the discourse is going. They contend, "...teachers need to learn the stepping in and out that is paramount to promoting productive discourse" (p.204). Furthermore, they suggest that teacher educators should address the following questions when providing tools for teachers to facilitate productive mathematical discourse: 1) when does the teacher step in and out; and 2) how does the teacher participate in classroom discourse?

Similarly, Walshaw and Anthony (2008) highlight the important role that effective mathematical discourse plays in developing a mathematical community and shaping mathematical identities and suggest that:

> Effective use of classroom discourse makes students' mathematical reasoning visible and open for reflection. In an environment where ideas are shared, students' own ideas become resources for their own learning. But more than that: Their explanations stimulate, challenge, and extend other students' thinking (p.539).

However, they suggest that the quality not frequency of this mathematical discourse is the most important feature and furthermore, they identify what the teacher does as being an integral contributing factor to the quality of this dialogue: "...more talk in classrooms does not necessarily enhance student understanding. Better understanding is dependent on particular pedagogical approaches, purposefully focused on developing a discourse culture that elicits clarification and produces consensus within the classroom community" (p.522). Furthermore, they claim that teachers must not just hear, but also listen attentively to the mathematics in students' talk. Essentially, they emphasise that despite mathematical authority being shared (or an attempt at such), teachers and pupils play very different roles in mathematical dialogue; hence the unequal relationship that they propose.

The challenges that teachers face in extending children's thinking is highlighted in a US study of first-grade teachers' implementation of a new curriculum (*Everyday Mathematics*) that was conducted by Fraivillig, Murphy and Fuson (1999). Findings were that teachers often elicited children's mathematical thinking but less often supported or extended children's thinking. They found the six teachers exhibited potentially productive methods for supporting children's mathematical thinking; however, many teachers "...smoothly conducted their lessons without ever explicitly focusing on children's mathematical thinking for extended periods of time" (p.168). The researchers suggest that many teachers in the study supported children's thinking to a significant degree; however, very few elicited student explanations of solution methods or extended children's mathematical thinking to higher levels. Conversely, one particular teacher emerged as the sole figure that consistently used practice that accessed children's understanding; and assisted and challenged children in their thinking. The researchers conducted a case study on this teacher and categorised her practice into three overlapping components:

- eliciting children's solution methods;
- supporting children's conceptual understanding; and
- extending children's mathematical thinking.

The researchers conclude that the successful implementation of such an approach as exhibited by this teacher requires specific pedagogical skills; and teachers who are ready to relinquish mathematical authority and to empower their students to conceptualise mathematics in meaningful ways will require guidance in proceeding with these steps. They propose two supports to aid this teacher guidance. They suggest that teacher-education programmes should address the difficult skills of elicitation and extension. Additionally, they propose providing robust images of effective mathematics practice for teacher reflection through descriptions of the strategies outlined in Advancing Children's Thinking (ACT) pedagogical framework that emerged during the study.

The studies so far have highlighted the challenges of sharing mathematical authority and orchestrating productive mathematical discussions when implementing reform approaches to mathematics teaching. Another specific challenge identified in the successful implementation of reform-based approaches to mathematics teaching relates to the role of teacher uncertainty and discomfort. In a US study, Frykholm investigated teacher discomfort when implementing a reform approach to mathematics teaching. Frykholm (2004, p.126) suggests that curricular innovations that challenge the traditional role of the teacher can be discomforting for teachers: "This kind of teaching necessarily fosters dissonance in the classroom that is both challenging and often unnerving for teachers who seldom before have been asked to relinquish control of the substantive content". In relation to his findings, Frykholm (2004) categorised teacher discomfort into four chief domains:

- Cognitive discomfort
- Beliefs-driven discomfort
- Pedagogical discomfort
- Emotional discomfort

Teachers' own mathematical thinking and understanding emerged as a cognitive discomfort in the study. In particular, due to the nature of the curriculum, teachers were forced to grapple with the differences between the procedural mathematics with which they were familiar and a broader conceptual understanding of mathematics that endorsed understanding over algorithmic proficiency. Teachers' beliefs about teaching mathematics, and indeed the nature of mathematics was also a source of teacher discomfort. In particular, some teachers struggled with implementing the curriculum because its basic philosophy regarding students constructing their own understanding of mathematics was at odds with their belief that mathematics constitutes a collection of basic skills and is a "...static body of truths to be conveyed by the teacher and retained by the learner" (p.135). The study found pedagogical issues in relation to classroom environments were also a recurring source of teachers' discomfort whist they implemented this reform-based curriculum. This discomfort included basic issues such as the need for classroom environments that look very different to traditional classrooms and subsequently teachers' ability to manage these environments. Specifically, teachers struggled with the changing roles of teachers and pupils in this new classroom dynamic.

> As they diminished their subject-matter authority in the classroom, their students simultaneously became more autonomous as they explored curricular activities and participated in classroom discussions. Often four or five different solutions would surface simultaneously. How to manage these moments of uncertainty became a source of discomfort for some teachers (p.136).

Frykholm admits that the category (emotional discomfort) is difficult to disentangle from the previous category about pedagogical discomforts; however, he notes that emotional discomfort was evident in the work and words of many of the teachers throughout the study, even if this emotional discomfort was evidenced subtly. In particular, the changing role of the teacher and the students was a source of emotional discomfort for teachers. This included the loss of ritual in classrooms, anxiety around guiding mathematical discussion, and the vulnerability experienced by teachers when engaged in mathematical exploration with students. Finally, another source of emotional discomfort for teachers related to conflicting expectations held by the educational community – the call for innovation yet the pressure and accountability associated with high-stakes testing. Fryholm (2004) claims that although some of the teachers in the case study appeared to be quite similar in terms of their conceptions and teaching experiences, they responded quite differently in classrooms when faced with moments of potential discomfort or uncertainty. These case studies reveal the complex nature of teacher discomfort when teaching and learning mathematics. It is clear that even when teachers appear to have similar traits, they can act very differently when confronted by possible uncertainty in mathematics classes. For example, two teachers with strong mathematical content knowledge (one of the common sources of teacher discomfort) acted very differently in mathematics classes that required them to implement the reform-curriculum – one teacher possessed minimalist views of her students and the curriculum resulting in convergent teaching, whereas the other teacher created "...open-ended learning opportunities where students generated and directed much of the discourse" (p.145). To similar effect, both teachers that admitted a) having poor mathematical content knowledge and b) experiencing resulting moments of uncertainty were very different in the way they engaged students. Despite the reform-curriculum one teacher continued to transmit learning in a traditional, undemocratic style (supplemented by textbook use) whilst the other teacher used her lack of mathematical understanding as an opportunity to create a safe environment where teacher and students could explore the mathematics together. Therefore, making links between teacher discomfort and learning opportunities for pupils is not easy (Frykholm, 2004). In light of these findings, Frykholm proposes the notions of "debilitating" and "educative" discomfort. In particular, he stresses the impact that beliefs and knowledge have upon teacher uncertainty. Frykhom outlines three scenarios where debilitating discomfort might emerge for teachers:

1. lack of sufficient mathematical content knowledge which can be exacerbated by a teacher's conflicting philosophical beliefs to that of the curriculum;
2. irrespective of content, a belief that the scope, design, and philosophy of the curriculum are inappropriate for student learning; and
3. emotional and pedagogical influences.

In line with his assertion that discomfort in mathematics classes does not have to be inherently negative but in fact, can be a welcome scenario, Frykhom highlights the advantages of educative discomfort:

> What is most notable about teachers who have an ability to tolerate discomfort – if not use it as a pedagogical tool – is the alignment of their belief structures with the philosophy of the curriculum...regardless of a teacher's knowledge base, discomfort can become educative as long as teachers believe the uncertainty is a necessary and natural component of learning (p.146).

Similar to Frykholm (2004), Nathan and Knuth (2003, p.204) acknowledge the difficulties and challenges associated with the changing role of teachers in reform-based mathematics. In particular, they highlight the ambiguity that exists both in theory and practice in relation to the teacher's role. However, they also illuminate the opportunity that exists for researchers within this challenging struggle: "...as teachers wrestle with how to reconceptualise their pedagogy in light of reform recommendations, their practices provide investigators of teaching a valuable window into the various forces that shape reform-based instructional practices". However, they further contend that much of the valuable research to date has focused on the teaching of 'experts' who are already part of the reform agenda; and consequently, they identify a palpable need for investigations with ordinary teachers:

> As the research base extends from these exemplars to include ordinary teachers operating in more typical settings, we can expect to learn a great deal more about the relation between teachers' beliefs and their instructional practices that will lay the groundwork for further advancements in reform-based teacher education and professional development (p.204).

Similarly, in relation to Ireland, a growing body of research exists regarding the implementation of reform-based mathematics teaching (e.g. Dooley, 2010; NicMhuirí, 2012; O'Shea & Leavy, 2013); however, much of this research relates to mathematics 'experts'. For example, NicMhuirí and Dooley both conducted recent research (as outlined earlier); however, they are both currently lecturers in mathematics education and they both taught the lessons that comprised elements of reform-based mathematics. Consequently, implementing a reform approach to mathematics teaching with 'ordinary' teachers is an area for further development in Ireland. In the following section of the literature review, international evidence in relation to effective professional development is outlined.

EFFECTIVE PROFESSIONAL DEVELOPMENT

This section of the literature review explores empirical findings in relation to features of effective professional development. General findings in relation to effective professional development are explored whilst specific findings in relation to effective professional development in mathematics are also outlined from a number of studies.

Features of Effective Professional Development

Several studies have aimed to investigate features of effective professional development. These include recent research (e.g. Porter, Garet, Desimone, Yoon & Birman, 2000;

Timperley, Wilson, Barrar & Fung, 2008) in addition to more established empirical work (e.g. Joyce, Showers & Bennett, 1987). Despite being almost 30 years old, Joyce, Showers & Bennett's research on staff development is acknowledged as some of the seminal work in the area of teacher professional development. Following a meta-analysis of nearly 200 research studies, in addition to a review of the literature on staff development, they suggest that:

- What the teacher *thinks* about teaching determines what the teacher *does* when teaching;
- Almost all teachers can take useful information back to their classrooms when training includes four parts: 1) presentation of theory 2) demonstration of the new strategy, 3) initial practice in the workshop, and 4) prompt feedback about their efforts;
- Teachers are likely to keep and use new strategies and concepts if they receive coaching while they are trying out the new ideas in their classrooms;
- Competent teachers with high self-esteem usually benefit more from training[15] than their less competent, less confident colleagues;
- Flexibility in thinking helps teachers learn new skills and incorporate them into their repertoires;
- Individual teaching styles and value orientations do not often affect teachers' ability to learn from staff development;
- A basic level of knowledge or skill in a new approach is necessary *before* teachers can 'buy-in' to it;

15 The word 'training' is used consistently instead of professional development in the work of Joyce, Showers & Bennett and is likely to be reflective of its time—the 1980s.

- Initial enthusiasm for training is reassuring for organisers but has relatively little influence on learning;
- Where or when training is held seems to matter little, whereas training design does matter;
- Effects of training do not depend on whether teachers organise and direct the programme, although social cohesion and shared understandings do facilitate teachers' willingness to try out new ideas (1987a).

Furthermore, they claim that the important components of teaching practice are cognitive in nature, and so:

> The purpose of providing training on any practice is not simply to generate the external visible teaching 'moves' that bring that practice to bear in the instructional setting but to generate the cognitions that enable the practice to be selected and used appropriately and interactively (p.85).

Consequently, they claim that combinations of the four components—theory, demonstration, practice, and feedback—appear necessary in staff development in order to develop the levels of cognitive and interactive skills that facilitate such practice in classrooms. Finally, their findings suggest that commitment follows competence and so competence may be a precondition for commitment. They conclude that although pre-competence 'commitment' may result in more pleasant initial training sessions, it may not be a substitute for appropriate training components. This highlights the importance of the design and content of effective professional development.

In a more recent research study, Timperley et al. (2008) synthesised the literature based on 97 studies of professional development and learning, primarily studies carried out in economically advanced countries. They analysed 97 individual studies and groups of studies that met a set of methodological criteria and had substantive student outcomes associated with teacher professional learning and development. The range of student outcomes included personal, social, and academic attributes. Their findings are categorised into three chief aspects:

- the professional learning context;
- the content of the professional development/learning opportunities; and
- the activities constructed to promote the learning.[16]

These categories are used as subheadings for the exploration of their findings.

Professional Learning Context

Timperley et al. (2008) identified seven elements in the professional learning context that were important for promoting professional learning in ways that impacted positively and substantively on a range of student outcomes. First, they found that the learning opportunities documented in the core studies occurred over an extended period of time (commonly between six months and two years although some extended to five years). In relation to extended time, they conclude that extended timeframes and frequent contact were probably necessary because, in most core studies, the process of changing teaching practice involved substantive new learning that, at times, challenged existing beliefs, values, and/or the understandings that underpinned that practice. Second, they found that engagement of external expertise, often researchers, was a feature of nearly all core studies but that the presence of external experts did not guarantee success. Consequently, they contend that experts need more than knowledge of required changes to teaching practice; they also need to know how to make the

[16] The researchers also draw conclusions about the learning process and teachers' responses; however, they caution that due to limited availability of evidence, the findings are based on a mix of theory and evidence and should be considered conjecture, not definitive. Considering this section of the literature review focuses on empirical findings, these conjectures are not included.

content meaningful to teachers and manageable within the context of teaching practice. Third, their findings suggest that the learning content and the activities to support it, together with the rationale for participating, had a greater influence on student outcomes than the circumstances of initial engagement (whether teachers volunteered or not). They found that these factors determined whether teachers engaged sufficiently during the learning process to deepen their knowledge and extend their skills in ways that improved student outcomes. Fourth, they found that two types of prevailing teacher discourses were challenged in the core studies: a) teachers' social construction of students and b) how to teach particular curricula effectively. In relation to the latter, they found that in mathematics and science there were large discrepancies between prevailing discourses about what counted in learning and teaching and the approaches promoted in the professional development. The required shift involved moving the focus from facts, procedures, and memorisation to processes of inquiry and the development of students' conceptual understandings. They found that as teachers became more skilled in implementing the approaches, and student learning deepened, teachers' perspective regarding what constitutes effective teaching in mathematics and science changed. Fifth, they found that a common feature of the core studies was the opportunity for participation in some form of professional community of practice; however, they found that in no case did participation alone result in improved student outcomes. In fact, they found that in some cases participation had the effect of reinforcing the ineffective status quo. They identified two characteristics of effective professional communities:

- participants were supported in processing new understandings and their implications for teaching, which sometimes included challenging problematic beliefs and testing the efficacy of competing ideas; and

- the focus was on analysing the impact of teaching on student learning including grounding discussions on artefacts representing student learning, and teachers holding high but realistic expectations for students.

Sixth, they found that the pedagogical approaches promoted in mathematics and science professional development did not occur in isolation of a wider research/policy environment and were consistent with policy emphases and recommendations by national subject associations and/or were based on generally accepted research findings. Their seventh finding in relation to context is that effective leaders actively supported the professional learning of their staff and, at times, participated themselves.

Content of the Professional Development

In relation to the content of the professional development, Timperley et al.'s (2008) synthesis of the evidence suggests that four key features are evident in effective professional development content. They found that aspects of content were integrated in all of the core studies. More specifically, they found that integration occurred with respect to theory and its translation into practice, and in relation to teacher understanding of pedagogical content knowledge, how students learn particular curricula, and how that learning should be assessed in order to focus teaching. Additionally, they found that a focus on the relationship between teaching and learning was one of the key features of the core studies. They found that part of the shift in teacher beliefs came about when teachers took greater responsibility for the learning of all students. In one such study in New Zealand, it was the students' stories about their experiences in the classroom that provided the catalyst for teacher engagement in the professional development. They conclude that identifying a problem only provides a catalyst for finding a solution if there is a vision of a realisable alternative. Their findings suggest that such a catalyst for engagement needs to be coupled with a vision for better student outcomes, and support for teachers to pursue such a vision. Furthermore, they found that approximately 50% of the core studies made specific reference to teachers developing their understanding of and use of assessment, with part of the professional development focused on the skills of interpreting and using data. Finally, in relation to sustainability only seven of the core studies satisfied the criterion so Timperley et al. (2008) advise caution about these findings. Nonetheless, they found that features of professional development that were associated with sustained student outcomes included a strong theoretical base that provided the foundation for principled decisions about practice, and the skills to collect relevant evidence and use it to inquire into the impact of teaching on student learning, particularly in relation to understanding students' problematic thinking or achievement.

Activities constructed to Promote Professional Learning

One of Timperley et al.'s (2008) chief findings in relation to activities that promote professional learning is that teachers were able to engage in multiple and aligned opportunities that supported them to learn and apply new understandings and skills. They highlight six key features of activities that were identified as most effective in promoting professional learning. The first key feature relates to a clear alignment between the intended learning goals and the activities was evident in the core studies and that individual activities often served multiple purposes. In addition, they found that teachers were provided with a variety of ways to understand the content. These activities included:

- listening to others with greater expertise;
- discussing practice with colleagues and with someone with specific expertise;
- having opportunities to see real or simulated practice;
- examining student understandings and outcomes;
- being observed and receiving feedback;

- discussing teachers' own theories of practice and their implications for teaching and learning;
- receiving student activities and materials;
- participating in activities positioned as students; and
- engaging with professional readings.

They conclude that teachers need multiple opportunities to learn through a range of activities. Furthermore, they found that content is more important than any particular activity because every type of activity that was part of the core studies with positive outcomes was also associated with studies with low or no impact. They conclude that the understandings promoted through engagement in these activities are more important than the activities themselves. Additionally, they found that for most of the core studies professional instruction was sequenced in a typical format. This format included beginning with some kind of rationale or catalyst to engage in the professional learning, followed by front-loading of new learning in relatively formal ways, followed by more individualised opportunities to learn where activities were provided for teachers to translate new knowledge into practice. This included a range of activities to refine new practice in classrooms, in addition to repeated opportunities to revisit and refine new knowledge. In relation to catalysts for mathematics professional development, they found that providers frequently challenged teachers' definitions of what mattered in mathematics, for example, teachers traditionally focused on computation, with providers advocating in-depth understanding. Finally, they found that effective professional development pedagogies provided teachers with opportunities to discuss and negotiate the meaning of the new learning and its implications for practice.

Whilst Timperley et al.'s (2008) synthesis focused on studies that met a set of methodological criteria and had substantive student outcomes associated with teacher professional learning and development, an American longitudinal (1996-1999) study examined the effects of professional development on improving classroom teaching practice. This study by Porter, Garet, Desimone, Yoon and Birman (2000) comprised approximately 300 teachers and aimed to evaluate the Eisenhower Professional Development Program, which is a federal government investment that focuses solely on developing classroom teachers' knowledge and skills. They found that professional development focused on specific, higher order teaching strategies (for example, the use of problems with no obvious solutions) increased teachers' uses of these strategies in the classroom. Furthermore, they found that this effect was even stronger when the professional development includes the following six dimensions:

1. the activity is reform type (for example, as part of study groups or teacher networks), rather than traditional (workshop or seminar);
2. is sustained over time;
3. provides opportunities for active learning;
4. is coherent with teachers' goals and other reforms;
5. involves groups of teachers from the same school; and
6. is focused on specific content and teaching strategies.

They found little change in overall teaching practice from 1996 – 1999; however, despite little change over time in teaching practice, individual teachers varied in their classroom practices, and moderate variation occurred in the classroom practices of individual teachers from year to year. Interestingly, they found that most of the variation across teachers in their classroom teaching occurred between teachers in the same school, not between schools. Considering this finding, the authors conclude that both teaching and professional development are typically individual experiences.

Although the extent to which these studies identify features of effective professional development varies, all three studies identify similar features of effective professional development. These studies suggest that effective professional development focuses on teaching and learning; engages teachers in actively learning new knowledge and skills; and expects teachers to engage in some form of community of practice (even if this merely comprises social cohesion within a staff).

Features of Effective Professional Development in Mathematics

A number of studies that specifically focus on features of effective professional development in mathematics are outlined in this section. These studies include Timperley et al. (2008); Loucks-Horsley and Matsumoto (1999); Back, De Geest, Hirst, Joubert and Sutherland (2009); and Doerr, Goldsmith and Lewis (2010). Eleven of the core studies in Timperley et al.'s (2008) synthesis of the literature focused exclusively on mathematics professional development, the majority of which promoted ideas about the nature of mathematics and mathematics teaching that are fundamentally different to traditional forms of mathematics teaching. They found that in all eleven studies the shifts that teachers were asked to make were challenging and required significant external support. However, they found that the professional development in all eleven studies was successful in supporting teachers to change their practice in ways that had positive outcomes for students. They conclude that significant and positive shifts in mathematics teaching practice are possible when the right content is provided with the right kinds of supports in the right kinds of circumstances. They found that these 'right' conditions include:

- a focus that is specific to mathematics;
- clearly articulated goals to teachers that relate to student outcomes in mathematics;
- a range of mathematics-based content;

- tailoring of generic pedagogy to a mathematical context (often to pedagogical approaches that are specific and exclusive to mathematics);
- a supportive environment;
- the provision of sufficient time;
- approaches consistent with those of the wider mathematics community;
- assessment as a core component;
- content that serves to develop teacher understanding of the theoretical basis for the practices being promoted; and
- content that serves to develop teacher understanding of the complex relationship between the key elements of teacher subject knowledge, pedagogy, assessment, and how students learn.

They found that the professional development that was associated with changes to teachers' practice that led to improved student outcomes focused on conceptual understanding of mathematics.

In contrast to the Timperley et al. study (2008), which focused solely on professional development that was associated with substantive student outcomes, Loucks-Horsley and Matsumoto (1999) chose not to limit their review of research on professional development for teachers of mathematics and science to studies that link professional development to student learning. Their review of the literature discusses the content (what is to be learned), processes (how content is to be learned), strategies and structures (how content is organised for learning), and contexts (conditions under which content is learned) in which professional development is effective. In relation to content, they suggest that it should help teachers understand a) subject matter, b) learners and learning, and c) teaching methods. Regarding effective processes through which teachers learn, they propose four effective learning experiences for teachers. Effective learning experiences are learner-centred (acknowledging what teachers know and are able to do and building on this for new understandings); and they are knowledge-centred (providing opportunities for teachers to develop well-organised bodies of knowledge within their discipline). Furthermore, they are assessment-centred (providing opportunities for feedback, revision, and reflection); whilst finally, they are community-centred (building in time for teachers to work together and provide each other feedback). Specifically, Loucks-Horsley and Matsumoto claim teachers need these types of learning experiences in order to implement challenging reforms in science and mathematics. Based on Loucks-Horsley et al.'s (1998) 15 strategies that are used in combination for professional development for teachers of science and mathematics, they synthesise these into five categories. Immersion strategies involve teachers 'doing' science and mathematics, for example, mathematics teachers solve mathematical problems. Curriculum strategies involve teachers with the actual learning experiences and materials they will use with their students. The strategy of examining practice focuses on teachers examining their own practice through the use of 'artifacts of practice' such as students' work, students' responses to assessments, students' thinking, video or narrative cases of teaching dilemmas, or teacher questions on student learning. Collaborative strategies for professional learning include within-school and across-school professional networks, partnerships with scientists and mathematicians, and coaching and mentoring. Finally, vehicles and mechanisms are structures through which learning of various kinds occurs including workshops and institutes, and technology. In relation to contexts of professional development, they suggest that supportive contexts in which collaborative cultures prevail support teacher learning. Furthermore, research on successful schools with high student achievement highlights the need for strong leadership for professional development. Despite advocating the above characteristics as contributing to effective professional development, Loucks-Horsley and Matsumoto caution that these characteristics cannot simply be applied to every teacher learning situation in the same way—each situation calls for a unique design that combines elements of professional development in different ways. Although reviewing studies that were and were not tied to student outcomes, they suggest an emerging consensus with regard to effective professional development, in that; the findings of studies that examine student learning are similar to those that do not. For example, studies that attempt to link professional development directly to student learning outcomes identify the importance of curriculum-based professional development; focusing professional development on student thinking; goal-focused, collaborative, supportive school and district environments; and paying attention and aligning other elements in the system e.g. assessment, curriculum, administrative support.

Back, De Geest, Hirst, Joubert & Sutherland (2009) conducted a research report that aimed to provide advice, guidance, and recommendations for the UK-based National Centre for Excellence in the Teaching of Mathematics (NCETM) in order to inform future plans and to illuminate the types of evidence that could demonstrate that professional development is informing teachers' practices and students' learning. The study investigated 30 professional development initiatives for teachers of mathematics, in different locations in England. When considering teacher views, their findings suggest that factors that contribute to effective professional development of mathematics include leadership, in particular, leaders that have knowledge and understanding of current practice, in addition to a practical approach, involving advice that is directly applicable to the classroom including the identification of resources. Furthermore, teachers value professional development that is stimulating and challenging, although such challenge is sometimes uncomfortable for teachers. Another contributing factor identified by teachers is time—time out of the day-to-day routine to focus on their professional practice and to reflect. Opportunities for networking within or between schools is highly valued by teachers. In addition to these factors identified through teacher views, other factors also emerged from the data as contributing to effective professional development in mathematics. These include focusing on mathematics, specifically, on ways of teaching the subject and on students' mathematical conceptions. Another factor is engaging in cycles of planning teaching, and predicting student responses to particular mathematical activities, followed by teaching and reflecting on the actual student responses, seemed to provide teachers with ways in which to talk about student learning. Furthermore, encouraging teachers to become more reflective and engaging them with research and professional literature related to teaching and learning mathematics appears to be beneficial. They also found that expecting and supporting change was a key feature of many of the professional development initiatives. Finally, they found that supporting the embedding of change through staff discussions, sharing of approaches, and designing new ideas and resources contributed to effective professional development.

These findings about effective mathematics professional development are supported by research undertaken by Doerr, Goldsmith and Lewis (2010) who analysed the evidence regarding the nature and impact of professional development in mathematics. They found that substantial support exists for focusing mathematics professional development on the four broad goals of developing:

- teachers' mathematical knowledge and capacity to connect it to practice;
- teachers' capacity to notice, analyse, and respond to student thinking;
- the beliefs and dispositions that foster teachers' continued learning; and
- collegial relationships and learning structures that can support and sustain teachers' learning.

Additionally, they found that three features of professional development design appear to be important for supporting progress toward these goals. These features include time; systemic support for teachers' learning; and opportunities for teachers' active learning. Finally, lesson study is a form of professional development in mathematics that first gained prominence in Japan. Promising work has taken place with regard to lesson study in Ireland where studies found that lesson study enabled pre-service teachers to deepen their mathematical subject knowledge (e.g. Corcoran, 2007; Leavy, 2010; Leavy, McMahon & Hourigan, 2011). Furthermore, some studies have been undertaken with practising teachers, which also indicate evidence of teacher learning (Corcoran, 2011)[17] and additionally suggest that engaging in lesson study can enhance teacher self-efficacy in relation to teaching mathematics (McLoone, 2011). From an Irish perspective, with the exception of lesson study, there is a dearth of studies in Ireland in relation to professional development in mathematics in a primary school context. Hence, the findings from the international professional development literature influence the design and content of the professional development in this study. The links between this literature and the professional development employed in this study are explored in chapter three.

CONCLUSION

Much of the literature suggests that the types of mathematics that is required for life in the 21st century is very different from the out-dated version that many children experience in school; however, changing mathematics practice has proven to be particularly challenging across most jurisdictions. However, the type of mathematics required for schooling is a contested area in the literature. Examples of this contestation include the 'Math Wars' in the US, in addition to criticisms of Project Maths—a reform based second-level syllabus—in

[17] Corcoran's study comprised practising teachers engaged in five cycles of lesson study over a three-year period.

Ireland. Nonetheless, the literature advocating reform approaches highlights the critical importance of thinking in mathematics. This thinking requires pupils to communicate, reason, argue, and conjecture – skills which are most effective when executed in collaboration with others. The studies outlined in this review highlight the integral role that teachers play in cultivating classroom cultures in which this type of pupil-centred mathematics learning thrives. This teacher role is very different to that with which most teachers are familiar. The teacher's role needs to progress from that of transmitter of knowledge to that of facilitator of understanding. In particular, it requires teachers to adopt a more facilitative, intuitive role that requires them to scaffold learning by stepping in and out of mathematical discourse at crucial moments. Facilitating the social aspects of mathematical discourse is not enough; teachers must play a pivotal role in ultimately ensuring that mathematical understanding is advanced. This requires sensitive balancing on the part of teachers—too much scaffolding and pupils' agency and autonomy can be stifled; too little scaffolding and the mathematical tasks or discourse can be unproductive. Furthermore, teachers need to provide challenging tasks that require pupils to grapple with mathematics and ultimately to think; rather than merely reproducing and applying standard algorithms. Mathematical expectations for pupils need to increase to a level where there is 'cognitive conflict' and children are mindfully engaged in tasks that require them to really think – to problem solve, to reflect, to reason, to refine ideas and to make connections. Furthermore, the reform-oriented mathematics literature accentuates the need for teachers to move away from teacher-dominated lessons to lessons that are more democratic where pupils and teachers share responsibility and authority for mathematical correctness. The literature suggests that sharing mathematical authority needs to be tangible and authentic; otherwise, pupils are confronted by confusing practice that on the surface appears to be more democratic but in reality retains the teacher as the ultimate analytical centre. In essence, this is an issue of "veneer versus substance". Finally, the challenges and difficulties in successfully implementing reform approaches to mathematics teaching are prevalent in the literature. In particular, studies highlight the importance of teacher beliefs when attempting to change mathematics practice and also the role that teacher discomfort (cognitive, beliefs-driven, emotional, and pedagogical discomfort) plays in the successful implementation or otherwise of reform approaches to mathematics teaching. However, in conclusion, notwithstanding the Irish studies outlined (e.g. Dooley, 2010; NicMhuirí, 2012; O'Shea & Leavy, 2013) a dearth exists in the literature regarding studies that examine the implementation of reform-based mathematics teaching in an Irish context. Moreover, much of the growing Irish research (e.g. Dooley, 2010; NicMhuirí, 2012) comprise mathematics 'experts' implementing elements of reform-based mathematics. This illuminates the need to conduct studies focusing on reform-based approaches to mathematics teaching with 'ordinary' teachers in Irish primary school classrooms. The following chapter details the findings from such a study. Finally, in relation to professional development, a large number of features associated with effective professional development are identified in the literature including a focus on teaching and learning; active engagement of teachers in their learning; and some form of cohesive group or community of practice that supports teachers. Specifically in relation to effective professional development in mathematics, common features identified in the literature include a focus on the nature of mathematics and mathematics teaching and learning; a focus on the relationship between teacher subject knowledge, pedagogy, assessment, and student learning; active learning for teachers; and supportive, collegial structures that promote learning. Furthermore, other studies have highlighted additional features of effective professional development in mathematics including tailored pedagogy specific to mathematics; a focus on conceptual understanding; and exploration of the theoretical understandings associated with the promoted practices. As outlined in the following chapter, these features as identified in the literature on effective professional development informed the planning and design of the professional development aspect of this study.

CHAPTER 3: METHODOLOGY

In the previous chapter, the literature regarding reform approaches to teaching mathematics was reviewed. Specifically, the benefits of implementing reform-based mathematics classrooms are detailed. Furthermore, the changes in practice that are necessary for reform-based mathematics lessons to flourish are highlighted, in addition to exploring some of the challenges associated with such change. This chapter details the methods used to research the implementation of a reform approach to mathematics teaching in an Irish primary school. The research problem is framed in the Irish context and the design of the research is outlined including an exploration of my views of knowledge and research in addition to an exploration of the key concepts associated with this study. The research cycle is outlined and details are provided of the participants involved in the study. The data collection procedures are detailed including the use of interviews, observations, document review, and audio-visual methods whilst the issue of quality assurance in this study is also documented. Finally, the ethical considerations are explored and the steps taken during data analysis are outlined.

RESEARCH PROBLEM

The research problem is explored in detail in the chapter one; however, a summary of the research problem is outlined here. Performance on national and international attainment tests, national attainment tests, national reports, and empirical studies all indicate a problem with mathematics teaching and learning in Ireland. In particular, international assessments consistently highlight an underperformance by Irish pupils in mathematics when compared to their performance in both English reading and science. Furthermore, the continuation of a traditional approach to teaching mathematics—despite evidence that it ill prepares pupils for the demands of the 21^{st} century—is highlighted repeatedly in studies and reports as a cause for concern. Although the mathematics curricula at both primary and post-primary level align generally with the principles that underpin reform-oriented or progressive mathematics, it appears that the reality of mathematics teaching and learning in Irish classrooms differs greatly from these principles. The research problem of this study encompasses the teaching and learning of mathematics in Irish primary school classrooms. Specifically, the problem relates to mathematics classrooms in which teachers appear to dominate discussions (e.g. DES, 2005a, 2005b, 2010; Murphy, 2004); textbooks are used as the chief teaching aid (DES, 2005a, 2005b, 2010, 2013; Murphy, 2004; NAMA, 1999, 2004, 2009; NAMIS, 2010; TIMSS, 1995; pupils work individually and silently for long periods (DES, 2005b); there is insufficient provision and use of resources (DES, 2005b, 2010, 2013; Murphy, 2004; NAMA, 2004, 2009); and pupils do not engage actively with mathematics (DES, 2013). However, the literature identifies the challenges faced by teachers when attempting to change their mathematics teaching including teacher discomfort (e.g. Frykholm, 2004) and difficuly in sharing mathematical authority with pupils (e.g. Hamm & Perry, 2002; Nathan & Knuth, 2003). This study aims to explore the experiences of an Irish primary school during the implementation of a reform approach to mathematics teaching. This includes the experiences of pupils, teachers, and the principal. The main question of the study investigates the experience of implementing a reform approach to mathematics teaching in an Irish primary school. More specifically:

- In what ways does teachers' mathematics teaching change (if at all)?
- In what ways do teachers' experiences of implementing a reform approach to mathematics teaching differ?
- What factors contribute to enabling teachers to change their mathematics teaching?

- What factors contribute to restricting changes to mathematics teaching?

RESEARCH DESIGN

Qualitative research is employed in this study in an attempt to unearth the meaning ascribed by teachers and pupils to the implementation of a reform approach to mathematics teaching. Specifically, qualitative research is used in an attempt to establish the meaning of the phenomenon from the perspectives of the participants in their own setting. Agreement exists in the literature that the researcher holds a central role in qualitative research and therefore the researcher is often considered to be the primary data collection instrument (Borman, Clarke, Cotner & Lee, 2006; Creswell, 2009; Lincoln & Guba, 1995; Robson, 2002;). Considering the centrality of the researcher in qualitative data, this section outlines my background and my views of research and knowledge. Specifically, my experience and beliefs as a teacher of mathematics, a professional development provider, and a research participant observer are outlined.

I worked as a primary school teacher for nine years, both as a class teacher and as a learning support teacher. During my time as a class teacher, I taught both junior and senior classes and taught in both a socially disadvantaged urban school and a rural school with multi-class settings. As a teacher of mathematics I tried to make mathematics meaningful for pupils, in that, I aimed for conceptual understanding instead of relying solely on algorithms and procedural fluency. I attended Froebel College of Education as an undergraduate and so I believe that the Froebelian philosophy of child-centred education that values active learning and the use of concrete materials has greatly influenced my work as a teacher, including that as teacher of mathematics.

In terms of my current employment, I am on secondment to a professional development service for schools that is operated by the Department of Education and Skills. During the study, I was in my seventh year of secondment. During my secondment, I have worked closely with teachers and schools in attempting to refine and change their teaching, specifically in relation to English and mathematics so that the best possible student outcomes can be achieved. I believe that improving student outcomes comprises much more than increasing test scores and consider attitudinal, social, and conceptual changes as equally important. In terms of teacher's practice, I believe that many factors contribute to effective practice and that contexts differ from classroom to classroom. I believe that teaching is a multi-dimensional social construct that is difficult to delineate into cause and effect. Accordingly, I agree with Lincoln and Guba (1985) that only time- and context-bound working hypotheses are possible and that all entities are in a state of mutual simultaneous shaping. In relation to the reform approach to mathematics that is advocated in this study, this is similar to approaches that I advocate as a teacher educator. However, the professional development in this study differs from that possible in my current teacher educator setting. Although sustained, on-site, whole-staff professional development is valued in my daily work, the realities of financial and human resources mean that this type of professional development is rarely possible. Instead, much state-funded professional development in Ireland is currently either a) for one member of staff (the cascade model) who is expected to disseminate the learning at school level or b) elective after-school workshops which are held in education centres and rarely attended by whole staffs. The emphasis currently appears to be on episodic professional development for individual teachers at an off-site venue. Contrastingly, this study provided me with an opportunity to work with a whole-staff over the course of two terms in their school setting.

I believe that my work with schools as a teacher educator has contributed to informing my beliefs about research and the nature of knowledge. In particular, my beliefs align well with the naturalist paradigm as outlined by Lincoln and Guba (1985). In relation to knowledge, I believe that there are not one, but multiple realities. Furthermore, I believe that the knower and known are interactive and inseparable (Lincoln & Guba, 1985) meaning that both the knower and the known influence each other. From a research perspective, I believe that all research is value-bound and that decisions made in research, for example, the research questions, data collection tools, and data analysis are influenced by the researcher's own values and beliefs. Consequently, this should be borne in mind when considering research findings. As a research participant observer in this study, I encountered a number of challenges including adopting a neutral stance during lesson observations and teacher interviews. In particular, I struggled with striking a balance between supporting and challenging teachers in changing their mathematics practice (whereby I sometimes took a lead role during the professional development) and researching the teachers' experiences of implementing a reform approach to mathematics teaching (whereby I took on a "pure observer" role rather than a "participant observer" role).[18] This role-duality was one of the most difficult challenges that I encountered during this study.

In an attempt to aid consistency regarding understanding and implementation of a reform approach to mathematics teaching across the school, I chose Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier and Human's (1997) research-based framework for teaching and learning mathematics with understanding. Another framework was considered for use in this study—Fraivillig, Murphy and Fuson's (1999) Advancing Children's Thinking (ACT) pedagogical framework—however, it was deemed less suitable because the framework emerged from the pedagogical actions of just one teacher who was teaching first-grade at the

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18 Robson (2002) distinguishes between a "pure observer" role where coding schemes are sometimes used to structure the observation and a "participant observer" role where the observer seeks to become some kind of member of the observed group.

time of the study . Hiebert et al.'s (1997) framework highlights core features of classrooms that promote understanding in mathematics and is based on the findings of four research and development projects—Cognitively Guided Instruction, Conceptually Based Instruction, Problem Centred Learning, and Supporting Ten-Structured Thinking. As well as aiding consistency, I chose this framework because it aligns well with the social constructivist philosophy of the Irish Primary School Mathematics Curriculum. In particular, the curriculum advocates that children must first construct their own internal structures to learn mathematics and then through social interaction test the ideas they have constructed and modify them. Furthermore, the curriculum advocates that children should respect one another's solutions, not discredit partners' reasoning, and discuss the train of thought used in the process. The mathematics curriculum is underpinned by a sociocultural theory that views "...cognitive development as a product of social interaction between partners who solve problems together" (p.4). The philosophy underpinning Hiebert et al.'s framework is similar to this in that the processes of reflection and communication are deemed of high importance in making connections in mathematics. The core features of this framework are outlined in Table 3.1. Hiebert et al. (1997) contend that these features are essential in classrooms that promote and support students' understanding

Table 3.1 Hiebert et al.'s Framework of Dimensions and Core Features of Classrooms that Promote Understanding

Dimensions	Core Features
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave something behind of mathematical value
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes

This study employs the case study strategy. The unit of study or "case" is a school as represented by its teaching staff (including principal) and its pupils[19]. This study employs an embedded single-case study design (Yin, 2006), where the "case" is a primary school, and the classrooms are embedded "subcases". In this study, the four subcases are compared and contrasted in order to "allow for greater opportunity to generalize across several representations of the phenomenon" (Borman et al., 2006, p.123). For the purpose of the current study, the most relevant setting was considered to be an Irish primary school that a) expressed a desire to improve mathematics teaching and learning; and b) had a minimum number of teachers to allow for sicknesses, leave, and natural attrition during the study.

[19] The focus on the teaching staff and pupils rather than others within the school community (e.g. Board of management, parents, etc.) relates to the fact that the research project is linked to classroom practice.

Therefore, a school with a minimum of ten teachers was selected which served the dual purpose of also ensuring that each class teacher was teaching a single class.

PARTICIPANTS

This study uses purposive sampling to identify the case study school. The selection of the school for the case study was based on the researcher's judgement regarding a) suitability and b) typicality. The school were judged to be suitable because a) they expressed a desire to improve teaching and learning in mathematics and b) teachers in the school considered they were 'traditional' mathematics teachers and so were thought to be suitable candidates for trialling a reform approach to mathematics. At the beginning of the study, the school was in existence for three years resulting from an amalgamation of an all-girls and an all-boys school. There were 205 pupils enrolled in the school during the project: 103 girls and 102 boys. Teachers, pupils, and the principal all participated in the study. There were thirteen teachers on staff during the study: an administrative principal, eight class teachers and four support teachers. Two of these support teachers were shared with other schools and one teacher taught a special class for pupils with severe emotional behavioural difficulties. Four of the permanent teachers were on maternity leave during the majority of the study, including the deputy principal. All of the class teachers and the principal[20] were female whilst two male teachers worked as support teachers. All class teachers volunteered for their classes to be 'tracked' throughout the study. Being 'tracked' meant that the teacher was interviewed, two

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[20] The principal had over forty years' teaching experience when the study began: twenty-nine years as a class teacher, eight years as a teaching principal and three years as an administrative principal. The principal had taught in both schools prior to amalgamation. She also held leadership roles in both schools prior to amalgamation: one as deputy principal and one as principal.

mathematics lessons were observed in her class, pupil focus group interviews were conducted with pupils from her class, and documents associated with the observed lessons were reviewed. Accordingly, data from these teachers and their classrooms form most of the data in this study. For the purposes of this study, four teachers were selected for tracking purposes. The following selection criteria were used in an attempt to capture a breadth of experiences of implementing a reform approach to mathematics teaching:

- a spread of classes including junior, middle and senior classes;
- a spread of teaching experience amongst the teachers;
- a spread of 'position' in the school including permanent and substitute teachers; and
- a spread of school culture with regard to the school's recent amalgamation.

It was not possible to use gender as criteria for selection because all class teachers were female. Initially it was decided to track Senior Infants, Second, Fourth, and Sixth Classes to ensure a spread of classes across the school; however, a student teacher was on placement in Fourth Class during the study so Fifth Class was chosen instead. Table 3.2 provides an overview of the tracker teachers.

Table 3.2 Overview of Tracker Teachers

	Class	Position	Teaching Experience	Prior to Amalgamation
Ann	Senior Infants	Permanent	10+ years	All-girls school
Bernie	Second Class	Permanent	>5 years	Neither school
Catherine*	Fifth Class	Substitute	>3 years	Neither school
Deirdre	Sixth Class	Permanent	5-10 years	All-boys school

* Catherine was also undertaking the Teaching Diploma during the study.

Although all pupils in the school may have experienced a reform approach to mathematics teaching, data were collected from the pupils in the tracker classes (see Table 3.4). Follow-up focus group interviews took place after lesson observations with a representative sample from each of tracker class. The focus groups comprised a range of attainment levels in each class grouping based on their latest performance on school-administered standardised mathematics tests, for example:

- 2 pupils who performed in the 10th to 20th percentile rank;
- 2 pupils who performed in the 40th to 60th percentile rank; and
- 2 pupils who performed in the 80th to 90th percentile rank.

This composition ensured a spread of scores across the normal distribution range. Where possible, gender was considered as selection criteria. Using the same focus groups throughout the study (initial focus group, Phase 1 focus group, and Phase 2 focus group) allowed for consistency and facilitated comparisons throughout the process. The chief rationale for including pupil voice in this study relates to a reported dearth of pupil voice in the literature based on professional development in mathematics. In particular, Joubert & Sutherland (2008) highlight that although the universally agreed ultimate aim of professional development for teachers of mathematics is to improve learning opportunities for students; in general, research does not attend to the voice of the students. Consequently, they contend that the voice of students could add a valuable perspective on changes in student learning when considering professional development in mathematics. Based on this premise, pupils were included in this study to ascertain their views and experiences of mathematics lessons before and during the study. This allowed for triangulation of data between teacher reports of their experiences, lesson observation of mathematics lessons, and pupils' reported experiences. Furthermore, including pupils in the data collection facilitated the tracking of their experiences of mathematics lessons from more traditional oriented lessons to lessons that comprised some reform elements.

RESEARCH CYCLE

The research took place over a year and comprised two chief phases. Phase 1 took place from June to December 2012 and Phase 2 took place from January to June 2013.

Phase 1 of the research comprised a number of aspects, an overview of which can be seen in Table 3.3.

Table 3.3 Overview of Research Phase 1

PHASE 1

June 2012 – December 2012

June

- Meeting with the principal and teachers of the proposed case study school in order to outline the study.
- Administration and analysis of Sigma-T standardised test in First to Sixth Classes by case study school in order to identify the area most in need of improvement. The strand of Measures was identified as the strand that pupils performed least well on. The teachers then chose the strand units of length and weight as the focus for attempting to change mathematics practice.

July – August

- Investigation by the researcher to identify:
  1. an evidence-based framework for teaching and learning mathematics with understanding (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human, 1997) – this instructional framework is detailed further in chapter four;
  2. recommended approaches in the school-selected strand (Measures); and
  3. pupil conceptions/misconceptions in the school-selected strands (length and weight).

September

- A more detailed outline of the proposed study with the school staff including a question and answer session.
- Identification of 'tracker' teachers.
- Distribution and collection of Informed Consent Forms and Plain English Statements to the chairperson of the Board of Management, Principal, teachers, parents, and pupils.
- Exploration of beliefs underpinning mathematics teaching and learning in the school; exploration of mathematics practice in the school.
- Dialogue with the school regarding the sequence of professional development, for example, the strand unit that the teachers wished to begin with was length.

October

- Initial pupil focus group interviews regarding current maths lessons.
- Teacher reflection on mathematics teaching and learning at first professional development session, for example, in discussion groups and using a reflection sheet drawn from Hiebert et al.'s (1997) instructional framework
- Overview of the instructional framework: professional development session based on using approaches to teaching and learning mathematics with understanding
- Overview of the Measures strand: professional development sessions based on mathematical content knowledge and pedagogical content knowledge:
  - general subject knowledge about Measures;
  - student conceptions/misconceptions in relation to Measures; and
  - possible approaches for teaching Measures.
- Professional development session on mathematical content knowledge and pedagogical content knowledge for the strand unit of length including:
  - general subject knowledge about length;
  - student conceptions/misconceptions in relation to strand unit of length; and

- possible approaches for teaching the strand unit of length.

- Professional development session based on teaching the strand unit length including:
    - sharing of ideas and expertise;
    - pupil voice from initial focus groups;
    - collaborative reflection on pupil voice; and
    - collaborative generation of lesson plans.

November

- Continuation of professional development sessions based on the strand unit length including:
    - sharing of ideas and expertise;
    - pupil voice from initial focus groups; and
    - collaborative generation of lesson plans.
- Observation of mathematics lessons in the four 'tracker' classes.
- Pupil focus group interviews from 'tracker' classes immediately after observation of mathematics lessons.
- Analysis of work samples, learning logs, and lesson plans for each of the observed lessons.

December

- Face-to-face individual interviews with the four 'tracker' teachers.

Phase 2 mirrored that of Phase 1 with a few exceptions (see Table 3.4 for an overview of Phase 2 of the research). The mathematics focus was on the strand unit of weight during Phase 2 instead of that of length. The principal was interviewed at the end of Phase 2, as were the class teachers who were not observed during the research. This data collection took place in order to explore the experiences of teachers, other than the tracker teachers, whilst implementing a reform approach to mathematics teaching. Furthermore, I took a more facilitative role during the professional development in Phase 2. During Phase 1, a lot of the professional development comprised my exploration of the reform approach with the teachers. During Phase 2, I still provided input on the mathematical content (weight); however, unlike Phase 1, I encouraged teachers to engage in selected readings in their own time and I asked them to devise lesson plans collaboratively outside of the professional development sessions. The content of each professional development session is detailed in Appendix A (including the selected readings with which the teachers engaged) whilst the collaborative planning is outlined further in chapter four.

Table 3.4 Overview of Research Phase 2

PHASE 2

January 2013 – June 2013

January

- Professional development session on revised instructional framework (particularly revised to include teacher talk based on feedback from teachers and observations of mathematics lessons – in an effort to prompt and enhance facilitative language and questioning):
  1. pupil voice – feedback from Phase 1 pupil focus group interviews;
  2. reflection and comparisons between pupil voice from initial focus groups and Phase 1 focus groups;
  3. professional development on revised instructional framework including video footage, the concept of revoicing, and prompt language/questions; and
  4. personal reading time – journal articles on whole-class discussion and mathematical thinking.
- Professional development session on mathematical content knowledge and pedagogical content knowledge for the strand unit of weight including:
  - feedback/discussion from journal articles;
  - general subject knowledge about weight;
  - student conceptions/misconceptions in relation to strand unit of weight; and
  - possible approaches/activities for teaching strand of weight.
- Professional development based on the strand unit of weight including:
  - sharing of ideas and expertise;
  - pair/group research into activities/problems for weight, for example, in textbooks, websites, etc.;
  - collaborative generation of lesson plans; and
  - collaborative reflection on lessons.

February

- Lesson implementation by all class teachers.

March

- Observation of mathematics lessons in the four 'tracker' classes.
- Pupil focus group interviews from tracker classes immediately after observation of mathematics lessons.
- Analysis of work samples, learning logs, and lesson plans for each of the observed lessons.
- 'Tracker' teacher face-to-face individual interviews.

April – May

- Initial data analysis.

June

- Principal teacher face-to-face individual interview.
- Teacher focus group interview (with all class teachers excluding the tracker teachers).

Professional Development

As outlined in the overviews of each phase, professional development featured in both phases of the study. A detailed overview of the content and processes involved in the professional development programme are outlined in Appendix A. I created this professional development programme, which was chiefly influenced by the evidence of effective professional development outlined in the literature. In particular, it comprised participation in a form of professional community of practice which the literature suggests is an effective element of professional development (e.g. Back et al., 2009; Koellner, Jacobs & Borko, 2011; Joyce & Showers, 2002; Loucks-Horsley & Matsumoto, 1999; Loucks-Horsley et al., 2010; Porter et al., 2000; Timperley et al., 2008). Similarly, the professional development involved teachers discussing practice with colleagues and with someone with specific expertise, which is also outlined in the literature as being effective (e.g. Loucks-Horsley & Matsumoto, 1999; Loucks-Horsley et al., 2010; Timperley et al., 2008). Furthermore, content was integrated in the professional development—theory and practice; subject content knowledge, pupil learning, and instructional practices—which is highlighted in the literature as being effective an effective element of professional development (e.g. ACME, 2002; Borko, 2004; Koellner, Jacobs & Borko, 2011; Loucks-Horsley & Matsumoto, 1999; Loucks-Horsley et al., 2010; Timperley et al., 2008). As suggested by a number of studies (e.g. Back et al., 2009; Borko, 2004; Joyce & Showers, 2002; Porter et al., 2009; Timperley et al., 2008) the professional development focused on the relationship between teaching and learning. Another feature of the professional development required teachers to participate in activities positioned as students in order to stimulate and challenge their thinking; a feature espoused in the literature (e.g. Back et al., 2000; Borko, 2004; Loucks-Horsley & Matsumoto, 1999; Loucks-Horsley et al., 2010; Timperley et al., 2008). Although attempting to satisfy aspects of effective features of professional development from the literature, this study also faced challenges and restrictions in relation to professional development. One example pertains to the duration of the professional development—in the literature (e.g. Back et al. (2009); Guskey (2002b); Porter et al. (2000); Timperley et al. (2008) an extended period of time is advocated. However, although the professional development in this study lasted seven months, this is at the lower end of the duration that research suggests is effective (e.g. Timperley et al. (2008) suggest between 6 months and 2 years as being effective). Examples of where the above elements relating to effective professional development occur in this study, in addition to further elements of effective professional development employed in this study and how they relate to the literature are explored in more detail in Appendix B.

DATA COLLECTION

In an attempt to provide "thick descriptions" (Geertz, 1973) and to explore all of the research questions, multiple sources of data were collected in this study. Considering the multiple sources of data, the data collection process was complex and required balancing the perspectives of teachers, pupils, principal, professional development provider, and researcher. This process was complicated further by multiple data collection methods including interviews, document analysis, observation, and audio-visual methods. However, using Hiebert et al.'s (1997) instructional framework as a lens for this data collection helped to alleviate this complexity because it influenced the pupil focus group interviews, teacher interviews, teacher reflection sheets, and lesson observation (these methods are detailed later in this chapter). The use of multiple sources of evidence and data collection methods in this study allowed for triangulation of evidence. The following section outlines these data collection methods in more detail. Table 3.5 outlines the data collected, the participants involved, in addition to when the data were collected.

Table 3.5 Data Collected during the Study

Data Method[21]	Activity	Participants	Before Study	Phase 1	Phase 2	Study End
Interviews	Focus Group Interview	Pupils (24)	√	√	√	
		Teachers (4)				√
	Face-to-face Interview	Tracker Teachers (4)		√	√	
		Principal				√
Document Review	Teacher Reflection Sheets	Class Teachers (7)	√			
	Learning Logs	Pupils (80)		√	√	
	Work Samples	Pupils (24)		√	√	
	Collaborative Lesson Plans	Teachers (13)		√	√	
	Field Notes	Researcher	√	√	√	√
	Reflective Journal	Researcher	√	√	√	√
Observation	Mathematics Lessons	Teachers (4)		√	√	
		Pupils (104)		√	√	
	Engagement in Professional Development	Teacher & Principal (13)		√	√	
Audio-visual Methods	Digital Photographs of Mathematical Tasks during Lesson Observation	n/a		√	√	

Interviews

The advantages of conducting interviews are well documented in the literature (e.g. Creswell, 2009; Robson, 2002). In this study, nine interviews were conducted. Two semi-structured interviews were conducted with each of the four tracker teachers – one during Phase 1 and another during Phase 2. The purpose of these interviews was to explore the experiences of teachers during the implementation of the reform approach to mathematics teaching. Furthermore, one semi-structured interview was conducted with the principal at the

[21] Creswell (2009) categorises the types of qualitative data collection methods into four broad headings: observations, interviews, documents, and audio-visual materials.

end of the study. The purpose of this interview was to ascertain the principal's experience of the reform initiative including any observed changes in teachers' practice and attitude to teaching mathematics. The interview schedule used followed a broad structure and can be viewed in Appendix C. I conducted and transcribed all of the interviews. This facilitated immediate initial analysis of the data. With the permission of the interviewees, I took notes at each interview and interviews were also recorded on an audio device. The purpose of note taking was twofold: a) to ensure that responses were recorded in the event that the recording device failed and b) to provide additional thinking or response time to interviewees so that the interview did not become a succession of rapid questions. I transcribed each interview, normally within 48 hours of the interview. This quick transcription allowed for ongoing analysis of the data, in addition to providing immediate feedback with regard to a) interview technique; b) questions that should be included or omitted at the next round of interviews; and c) emerging themes from the data. The tracking codes assigned to the teachers throughout the study are outlined in Table 3.6.[22]

Table 3.6 Tracking Code Assigned to Teachers

Interview/Focus Group Interview	Phase 1 Interview	Phase 2 Interview	Focus Group Interview at Study End
Ann (Tracker Teacher)	Int.1 A	Int.2 A	
Bernie (Tracker Teacher)	Int.1 B	Int.2 B	
Catherine (Tracker Teacher)	Int.1 C	Int.2 C	
Deirdre (Tracker Teacher)	Int.1 D	Int.2 D	
Class Teachers			FG.CT

22 Tracking codes are not included when discussing each teacher in detail because each teacher is given a pseudonym in these sections.

Observations

Observation was used in this study in two settings: a) the engagement of the teachers at the professional development session was observed and recorded in field notes (these field notes are referenced throughout chapter four) and b) eight mathematics lessons were observed in total - two mathematics lessons in each of the four tracker classes (one in each of the classes during Phase 1 and one during Phase 2). The chief purpose of this observation was to confirm any shifts in mathematics practice. I took on the role of 'pure' observer (Robson, 2002) or 'complete' observer (Creswell, 2009) during these mathematics lessons. Accordingly, I did not intervene or interfere with the classroom interactions or practice. The following protocols (based on Creswell's (2009) recommendations) were used when recording lesson observations in this study:

- Demographic information as outlined in Table 3.7

Table 3.7 Demographic Information for Lesson Observation

Date	
Class	
Number of Pupils	
Location	e.g. hall, classroom, corridor, etc.
Mathematical Focus	e.g. content such as Weight, Length; processes such as estimation, measuring, etc.; skills such as implementing, problem solving, reasoning, etc.
Organisation Settings	e.g. whole class, group work, pair work, individual work, etc.

- Descriptive notes: these notes were made during, and immediately after each lesson observation. They included descriptions of:
  - tasks;
  - perceived difficulty of tasks by pupils; and
  - interesting language used by pupils (this included language with regard to the mathematical concept, for example, length or weight, in addition to language

associated with mathematical thinking such as discussing solution methods, reasoning, etc.)

- Reflective notes: these notes were extensive and were made a few hours after each lesson observation. These included my thoughts, ideas, feelings, questions, concerns, etc.

Hiebert et al. (1997) contend that their instructional framework can be useful in analysing classroom practice regardless of the instructional approach; hence, their instructional framework was adapted and used as a coding scheme in this study. Considering the coding scheme was based on the instructional framework, it evolved during the study in conjunction with the evolution of the instructional framework. Inter-observer reliability can be an issue with coding schemes (Robson, 2002); however, this is not an issue in this study because the researcher observed all eight lessons. The coding scheme used for lesson observation in Phase 1 was based on the instructional framework (Hiebert et al., 1997) and is outlined in Table 3.8.

Table 3.8 Coding Scheme used for Lesson Observation during Phase 1

Dimensions	Core Features	Present : △ Partially Present: ⋀ Not Present: — Not Applicable: n/a
Nature of Classroom Tasks	Make mathematics problematic	
	Connect with where students are	
	Leave something behind of mathematical value	
Role of the Teacher	Select tasks with goals in mind	
	Share essential information	
	Establish classroom culture	
Social Culture of the Classroom	Ideas and methods are valued	
	Students choose and share their methods	
	Mistakes are learning sites for everyone	
	Correctness resides in mathematical argument	
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user	
	Used with purpose – to solve problems	
	Used for recording, communicating and thinking	
Equity and Accessibility	Tasks are accessible to all students	
	Every student is heard	
	Every student contributes	

The coding scheme used for lesson observation during Phase 2 was based on the evolved instructional framework – the *4Ts Instructional Framework for Maths* (which is adapted from Hiebert et al.'s (1997) instructional framework (see Table 3.9). The evolution of the instructional framework, including the rationale for such revisions, is detailed in chapter four.

Table 3.9 Coding Scheme used for Lesson Observation during Phase 2

	A Pre-requisite	**Present : △ Partially Present: ∧ Not Present: ___ Not Applicable: n/a**
Classroom Environment	ideas and methods are valued	
	students choose and share their methods	
	mistakes are learning sites for everyone	
	correctness resides in mathematical argument, not the popularity of the speaker	
	tasks are accessible to all students	
	every student is heard	
	every student contributes	
	students often work in small groups but they can also choose to work independently	
	norms for working effectively in small groups are established	
	whole-class dialogue encourages reflection, communication and reasoning	
	students share their mathematical thinking with the whole class, not just the teacher	
4Ts	**Core Features**	**Present : △ Partially Present: ∧ Not Present: ___ Not Applicable: n/a**
Tasks	Make mathematics problematic	
	Connect with where students are	
	Select tasks with goals in mind	
	Leave something behind of mathematical value	
Teacher	Take on an active, skilled facilitator role	
	Share essential information	
	Establish classroom culture	
	Encourage revision of conjectures	
Tools	Meaning for tools must be constructed by each user	
	Used with purpose – to solve problems	
	Used for recording, communicating and thinking	
Talk	Teacher talk encourages reflection and communication	
	Students use language to refine, revise, clarify and communicate mathematical thinking	
	Talk is used to encourage and communicate reasoning	

Document Review

A number of documents were reviewed in order to inform the study. The reflection sheets that teachers used to record their mathematics practice before the study began were reviewed. Pupils' learning logs and work samples including copies, task sheets, and photographs of activities were reviewed. Additionally, teachers' collaborative lesson plans were reviewed (these lesson plans are detailed in chapter four) whilst my field notes and my reflection journal were analysed during and after the study. As opportunities arose during the course of the data collection visits, which were spread over three school terms, I made brief, in situ, notes regarding incidents, activities, comments, conversations, physical details, and visual information that I considered to be relevant to the research. Clarification and expansion of these notes was undertaken as soon as possible, almost exclusively each evening after a data collection visit. Additionally, issues, ideas, and practical research difficulties and considerations were recorded in my journal notes. The observational notes and journal entries were used as a focus for reflection during the duration of the field visits. The purpose of the document review was threefold: a) to inform the design and use of other data collection instruments and procedures; b) to contribute to the identification of themes for further exploration; and c) to inform the content of the ongoing professional development.

Focus Group Interviews

Fourteen focus group interviews in total were conducted during the study. Thirteen were conducted with pupils from the four tracker classes at three different stages in the study – one before the implementation of the reform approach, one during Phase 1, and one during Phase 2. Furthermore, a focus group interview was conducted with all of the class teachers (when excluding the tracker teachers this amounted to four teachers) at the end of the study to ascertain their experiences of implementing the reform approach to mathematics teaching. Table 3.10 outlines the number of focus groups and participants involved.

Table 3.10 Focus Group Interviews Conducted during the Study

Focus Group Interviews	No. of Participants	No. of Focus Group Interviews	Before Study	Phase 1	Phase 2	Study End
Senior Infant Pupils	6	4[23]	√√	√	√	
Second Class Pupils	6	3	√	√	√	
Fifth Class Pupils	6	3	√	√	√	
Sixth Class Pupils	6	3	√	√	√	
Class Teachers	4	1				√
TOTAL	**28**	**14**	**5**	**4**	**4**	**1**

Considering the recommended number of participants for focus groups in the literature (e.g. Kvale & Brinkman, 2009; Robson, 2002), six pupils were selected from each of the four tracker classes. Furthermore, Kvale and Brinkman (2009) suggest that specific considerations are important for interviewing children and so during the pupil focus group interviews attempts were made to:

a) avoid the use of long and complex questions;

b) pose only one question at a time;

c) use age-appropriate questions;

d) avoid being associated with the classroom teacher; and

e) refrain from conveying that there is only one correct answer to the question.

The purpose of the initial focus group interviews was to ascertain the pupils' experiences of mathematics lessons before the research study began. This facilitated a) the setting of the pedagogical context including a reported glimpse of current mathematics practice and b) making comparisons between pupils' experiences as the study progressed and the possible

23 Two focus group interviews were conducted with Senior Infants in the initial stages of the study because it emerged after the first focus group interview that pupils were unclear with regard to what constitutes 'maths'.

change in mathematics practice. The purpose of the focus group interviews during the implementation of the study was to illuminate pupils' experiences of the reform approach to mathematics teaching, in particular, to amplify insights emerging from the observed mathematics lessons and the pupil work samples. Accordingly, these focus group interviews were conducted immediately after mathematics lessons were observed. The same implementation, recording and transcribing procedures that were used for the interviews were also used for the focus group interviews. The focus group interview schedules for pupils and for teachers are outlined in Appendix D.

QUALITY ASSURANCE

In qualitative research, Borman et al. (2006) claim that although researchers are concerned with maintaining rigor, the emphasis is on trustworthiness. Furthermore, they claim that such trustworthiness can be achieved through a) careful work of constructing the research design and approach, b) conducting the research ethically and honestly, c) analysing findings carefully, and d) providing a presentation of results informed by rich descriptions which leads to appropriate extrapolations from the data. Based on Lincoln and Guba's (1985) four criteria for generating trustworthiness in naturalistic inquiry or qualitative research, Table 3.11 provides an overview of the techniques used in this study for establishing trustworthiness.

Table 3.11 Techniques used in this Study for Establishing Trustworthiness

Criterion Area	Technique	Current Study
Credibility	Prolonged engagement in the field	I engaged with the school over a year, two terms of which required regular school visits including nine contact hours with the whole staff. Furthermore, I conducted fourteen focus group interviews, nine interviews, and observed eight lessons.
	Persistent observation	I observed mathematics lessons and observed the engagement of teachers with the professional development programme.
	Triangulation of sources and methods	I attempted to triangulate all data sources and methods during and after the study to identify themes and to highlight discrepancies.
	Peer debriefing	I debriefed with members of the numeracy team at the Professional Development Service for Teachers where I work
	Negative case analysis	I was aware of the possibility of contradictory data and so searched for this when triangulating evidence. An example of contradictory data in the study was when one of the case study teacher's reports of her mathematics teaching were contradicted by the pupil reports from her class. Furthermore, I gave attention to outliers, and findings that were at odds with the general themes that were emerging and then reported on these.
	Member checks (in process and terminal)	Throughout each phase of the study and again at the end of the study, I checked the accuracy of the findings by checking themes and interpretations with the participants.
Transferability	Thick description	I provided detailed descriptions of the setting, participants, lessons (including tasks), professional development, etc. Furthermore, I used digital photography in an attempt to provide further detail of the mathematics lessons.
Dependability	Audit trail	I documented a full record of the activities whilst carrying out the study, for example, records of raw data (focus group and interview transcripts, detailed records of the content and processes of professional development, field notes, work plans, pupil work samples, learning logs); research cycle, research journal; and details of coding.
Confirmability	Audit trail	
All of the above	Reflexive journal	Throughout the study, I kept a reflection journal in which I documented thoughts, concerns, and tentative plans after each site visit. Furthermore, I documented retrospective reflections a few weeks after the completion of each phase.

ETHICAL CONSIDERATIONS

Ethical issues were considered at all stages of this research (Creswell, 2009). I was conscious of the necessity to ensure that the conduct of the research, at a minimum, would avoid causing harm to participants during the case study. Consequently, prior to commencement, ethical approval was sought and approved from the Research Ethics Committee of St. Patrick's College, Drumcondra. Ethical issues were considered during the data collection stage in the form of informed consent, in the design of instruments, in addition to risk management procedures. During the reporting of findings real names were replaced by pseudonyms, for example, a pseudonym for each teacher and numerical numbering for pupils (for example, Pupil A) and every effort was made to maintain the anonymity of the school. More details of the ethical considerations for this study are outlined in Appendix E. Regarding ethical value, the execution of this study has relevance and importance for the Irish education system as a whole. In addition, it is understood that teachers in the case study school benefited directly from their participation through individualised and sustained professional development, in addition to reflection on their own practice. Furthermore, pupils benefited from the study in a number of ways, for example, pupils had the opportunity to report their experiences of mathematics lessons before and during the study. Consequently, teachers accepted much of the pupils' feedback on mathematics lessons, and subsequently some of their ideas and suggestions were incorporated into new mathematics lessons. Furthermore, pupils had the opportunity to experience a reform approach to mathematics teaching – an approach that affords greater agency to pupils. At all times, I attempted to "...respect the rights, needs, values and desires of the informants" (Creswell, 2009, p. 198).

In the context of this study, I was acutely aware of role duality or role conflict. This dual role required reflection on the perceptions *of* and biases *towards* the teachers: I was an 'outsider' as far as I was not a member of staff but an 'insider' in terms of professional biography. My shared background as a primary teacher is likely to have enabled me to see things from the teacher's perspective and to appreciate the enabling and constraining factors that operate in classrooms. Similarly, my non-teaching knowledge of the educational system assisted in adopting the role of non-participant but sympathetic observer. Robson (2002, p.169) asserts that relevant professional or practitioner knowledge is beneficial in qualitative research because the researcher needs to be able to grasp the issues so that the information can be interpreted, not just recorded: "Without a firm grasp of the issues (theoretical, policy etc.) you may miss clues, not see contradictions, requirements for further evidence, etc.". Similarly, Cochran-Smith and Donnell (2006, p.510) highlight the benefits of practitioner knowledge in research and assert that practitioner inquiry has boundless potential:

> ...boundaries between teaching and inquiry must blur so that practitioners have opportunities to construct their own questions, interrogate their own assumptions and biographies, gather data of many sorts, develop courses of action that are valid in local contexts and communities, and continuously reevaluate whether a particular solution or interpretation is working and find another if it is not.

Nonetheless, whilst acknowledging the advantages that background experiences bring to the role, considering my involvement in education, I cannot claim to have been an entirely objective observer. Therefore, I incorporated reflexivity into the work throughout the study. The use of detailed field notes, in conjunction with constant reflection after each site visit including writing a research journal, allowed me to unearth personal biases, perceptions, and observations. Acknowledgement of these subjective views allowed me to face each site visit with a renewed freshness, openness, and willingness to gather all data.

DATA ANALYSIS

Data analysis was an ongoing process throughout the study and involved multiple levels of analysis. The initial stages of analysis took place continuously for example, during data collection in the field; whilst transcribing focus group and interview transcripts; whilst writing field notes; and during the reflective process of writing a researcher journal. The qualitative data were analysed by organising it categorically and chronologically, reviewing it repeatedly, and continually coding (Creswell, 2009). Consequently, the data analysis required "multiple levels of abstraction" (Robson, 2002). The data analysis framework used for the in this study comes from Creswell (2009) and is outlined in Table 3.12. Data analysis in this study was supplemented by procedures advocated by Eisenhart (2006), Nespor (2006), and Robson (2002)—as referenced throughout this section.

Table 3.12 Data Analysis Framework

Creswell's (2009) Framework for Data Analysis	
Organising and Preparing Data for Analysis	Transcribing interviews, optically scanning material, typing field notes, sorting and arranging the data into different types
Reading through All Data	Getting a general sense of the information and then reflecting on the overall meaning, may include writing notes in margins and recording general thoughts
Coding the Data	Organising data into categories and labelling these categories with a term. A preliminary codebook that contains predetermined codes can be used but should be allowed to change and develop based on the information learned during the data analysis.
Generating Descriptions	Using the coding process to generate a description of the setting or people in addition to categories or themes for analysis
Generating Themes	This coding and descriptions can then be used to generate a number of themes—perhaps five to seven—that display multiple perspectives from individuals and are supported by diverse quotations and specific evidence
Interrelating Themes	Themes are analysed for each individual case and across different cases, for example, into a storyline or narrative in order to build additional layers of complex analysis
Representing Themes	Using a narrative passage to convey the findings of the analysis, for example, a discussion that focuses on a chronology of events or a discussion of several themes—including subthemes, specific illustrations, multiple perspectives from individuals, and quotations
Interpreting the Meaning of Themes	This is the final step in data analysis and is based on what was learned from the data—these lessons can be the researcher's personal interpretation or meaning derived from a comparison of findings with information gleaned from the literature or theories

The general steps of data analysis in this study included:

Organising and Preparing Data for Analysis

1. Interview data and focus group interview data were transcribed from audio tapes whilst field notes, notes from lesson observations, details of the professional development sessions, and the researcher reflective journal were typed up. Data were

then stored electronically in folders according to a) the data source e.g. teachers/pupils/researcher/principal and b) the phase of the study e.g. at the beginning of the study/during Phase 1/during Phase 2/at the end of the study.

Reading through all Data

2. The data were read and re-read and reflections were recorded in the researcher reflective journal.

Coding the Data

3. Classification or broad coding: the data were organised into broad categories and then further categorised according to chronology. Considering the dimensions of Hiebert et al.'s (1997) instructional framework influenced much of the data collection tools (pupil focus group interviews, teacher interviews, and lesson observation), these dimensions were used for the purposes of broad coding. Accordingly, the broad categories included:

    - nature of classroom tasks;
    - role of the teacher;
    - social culture of the classroom;
    - mathematical tools as learning supports; and
    - equity and accessibility.

These categories of pre-determined codes resemble Creswell's "preliminary codebook".

Generating Descriptions and Themes

4. Identifying themes: using the research question as a lens, themes or patterns were identified from the data and then the broad categories from step 3 were categorised under relevant themes. Some of these themes mirrored those of the broad coding, for example, nature of classroom tasks, role of the teacher, social culture of the

classroom, and mathematical tools as learning supports. Equity and accessibility did not emerge as a theme from the data. Other themes emerged and were added to this list including:

- pupil attitudes to mathematics;
- the use of talk in mathematics lessons; and
- the role of textbooks in mathematics lessons.

This exemplifies the evolutionary nature of the preliminary codebook.

5. Describing themes: the names outlined in step 4 were assigned to each of these themes and a short descriptive statement was generated for each theme. For example, the descriptive statement for the *use of talk in mathematics lessons* theme was "Pupils in all four focus group interviews report not getting many opportunities to talk about mathematics in lessons, most of the talk appears to be teacher-led and teacher-dominated with the exception of 6th class where some group work appears to take place when doing puzzles".
6. Coding by perspective: each of the themes was divided based on participant perspectives (in some instances these themes were grouped based on a number of participants, for example, a group of pupils that took part in a particular focus group). Accordingly, the themes were divided based on the following perspectives:
    - senior infant pupils;
    - second class pupils;
    - fifth class pupils;
    - sixth class pupils;
    - senior infant teacher;
    - second class teacher;

- fifth class teacher;
- sixth class teacher;
- other teachers;
- principal; and
- research observer (particularly in relation to lesson observation and field notes on professional development sessions).

7. Summary statements: summary statements were generated chronologically for each participant perspective within a theme – these were statements that attempted to reflect participants' experiences and attitudes at a particular time in the study. This often required three statements for each participant that was pertinent to a theme – one associated with before the study, one associated with Phase 1, and one associated with Phase 2. An example of this for senior infant pupils in relation to the nature of mathematical tasks would be a summary statement on the nature of mathematical tasks a) before the study began, b) during Phase 1, and c) during Phase 2.
8. Cross-case analysis: summary statements for each theme were analysed across participant perspectives in an attempt to synthesise themes through comparing and contrasting perspectives within each theme. For example, summary statements in relation to the nature of mathematical tasks from the senior infant pupil perspective and the senior infant teacher perspective were compared and contrasted. A further example was the comparing and contrasting of the summary statements relating to the nature of mathematical tasks from the senior infant teacher, second class teacher, fifth class teacher, and sixth class teacher.

Interrelating Themes

9. Narrative synthesis: summary statements were used as a basis for developing a cohesive narrative for each theme whereby supporting data such as evidence from lesson observations and relevant quotations were used to strengthen the overall judgements. For example, regarding the nature of mathematical tasks in senior infants all of the following were used to support the summary statement:
    - quotations from the senior infant pupils;
    - quotations from the senior infant teacher;
    - photographs of the tasks during observed lessons;
    - relevant abstracts from the lesson plans; and
    - evidence from lesson observation
10. Testing the narrative: the narrative for each theme was tested by re-interrogating the data to ensure the judgement was robust and reliably reflective of the data; accordingly, some narrative and summary statements had to be altered or discarded.

This level of analysis was successful in identifying themes and creating a chronological narrative; however, in an attempt to build additional layers of analysis the following steps were also included:

Representing Themes

11. Explaining themes: in an attempt to understand and explain the themes that emerged from the data, themes were compared to the literature so that a deeper understanding of the data might be possible. Creswell (2009) refers to this as moving deeper and deeper into understanding the data. For example, the role of the teacher theme was compared to the literature on teachers' experiences of attempting to change their mathematics practice. The challenges highlighted in the literature with regard to

teacher discomfort and hybrid lessons allowed for a fresh analysis of the data and facilitated a deeper understanding of this theme. Eisenhart (2006) describes representation as a "...descriptive summary with interpretation, constructed by the researcher to reveal what has been learned and filtered through the researcher's choices of what is important for readers to know – [it] is the heart and bulk of almost every...qualitative case study" (p.569-570).

12. Synthesising themes: for purposes of clarity, themes were synthesised in order to make a coherent, logical argument. This sometimes required a) blending themes that were similar and where crossover was evident; b) discarding themes that were not within the scope of the research question; and c) extracting strong patterns within themes to create new themes. For example, the theme of textbooks was merged with the theme of the nature of mathematical tasks due to their similarity and crossover, thus blending a theme. Similarly, discarding the pupil attitude to mathematics theme is an example of discarding themes. This theme was discarded because although it emerged strongly as a theme before the study and was used as a catalyst for change within the professional development, it emerged less strongly as a theme in Phases 1 and 2. Pupils reported enjoying the lessons during these phases; however, no richness emerged from the data. Finally, emergence of peer support and tailored professional development as strong patterns in the data is an example of creating new themes. Consequently, these were included as new themes.
13. Re-interrogating the data: in an attempt to ensure relevance and robustness of these themes, all of the data were re-read with the refined themes in mind. Accordingly, new perspectives arose from some of the themes, which subsequently were incorporated into the findings, whilst other themes were discarded. Referring to field notes, Nespor (2006, p.300) acknowledges this analytical oscillating and claims that

analytical patterns are "intrinsically tentative and open to reconstruction" so that things that seemed marginal at first can become important, old data based on now discarded assumptions can become irrelevant, and rediscoveries across successive readings of notes. For example, teacher reflection sheets that were used for data collection on the first session of professional development were rediscovered during this re-interrogation stage. Another example of a rediscovery relates to the power of the pupils' reports of mathematics lessons to act as a catalyst for teacher change. Re-interrogating the data allowed for this connection to be made based on evidence from the interview with the principal, field notes associated with the professional development, and my researcher reflective journal.

Interpreting the Meaning of Themes

14. Finally, using the research questions as a lens, these themes were described in a narrative in chapter four. This narrative outlines shifts in teachers' practice throughout the study (for each of the four tracker teachers), factors that contributed to changes in teachers' practice and factors that restricted such changes. Furthermore, conclusions relating to these themes are outlined in chapter five—these conclusions represent the learning derived from the data analysis.

CONCLUSION

Qualitative research is used in this study in an attempt to unearth the reality behind the research questions. Furthermore, it allows multiple perspectives to be explored in the participants' natural setting. The research questions focus on discovering and analysing the experiences of teachers whilst implementing a reform approach to mathematics teaching for the strand units of length and weight. The study comprises two phases and includes an on-site professional development programme that aims to support teachers in implementing the reform approach to mathematics teaching. The use of qualitative data over the course of an academic year, coupled with the use of multiple data collection tools enables a detailed exploration of this phenomenon. These data collection tools include lesson observations in four classes, focus group interviews with pupils from each of these classes, teacher interviews, an interview with the principal, a focus group interview with class teachers, and document review of lesson plans and samples of pupils' work related to the observed lessons. Furthermore, my research journal and my field notes are reviewed. With the exception of the interview with the principal and the focus group interview with class teachers, these data collection tools are used in both phases of the study. The breadth of data collection tools in addition to their use over two phases contributes to the complexity of the data collection and analysis in this study. In the following chapter, the findings of the study are detailed; in particular, Irish primary school teachers' experiences of implementing a reform approach to mathematics teaching are outlined. Specifically, shifts in mathematics practice are identified and analysed. Finally, factors that may have contributed to enabling or restricting such shifts in practice are considered.

CHAPTER 4: FINDINGS

In this chapter, the findings of the study are reported. Teacher interviews, lesson observations, and pupil focus group interviews were carried out during Phases 1 and 2 of the study. Furthermore, pupil focus group interviews were undertaken before the implementation of the reform approach to mathematics teaching in an attempt to capture pupils' experiences of mathematics lessons before the study. Similarly, teacher reflection about mathematics teaching was conducted at the first professional development session. Moreover, an interview with the principal, in addition to a focus group interview with teachers took place at the end of the study. Three distinct sections outline the findings. In the section that dominates this chapter, each case study teacher's experience of implementing the reform approach to mathematics teaching is outlined including shifts in practice during Phases 1 and 2. Furthermore, the factors that contributed to supporting and enabling teachers to change their mathematics practice are outlined. Finally, the factors that may have restricted the potential for embedding and enhancing changes to mathematics practice are outlined. In all sections, triangulation of data is used where appropriate in an attempt to provide a holistic view of the study, the ultimate aim being to answer the research questions: What is the experience of implementing a reform approach to mathematics teaching in an Irish primary school? More specifically:

- In what ways does teachers' mathematics teaching change (if at all)?
- In what ways do teachers' experiences of implementing a reform approach to mathematics teaching differ?
- What factors contribute to enabling teachers to change their mathematics teaching?
- What factors contribute to restricting changes to mathematics teaching?

CASE STUDY TEACHERS

Following engagement in professional development as outlined in chapter three, the tracker teachers in the study appeared to have varying experiences whilst implementing the reform approach to mathematics teaching and at times seemed to demonstrate conflicting views about this approach to mathematics teaching. Furthermore, shifts in practice in the observed mathematics lessons varied during the study. In an attempt to acknowledge this heterogeneity, an overview of each tracker teacher's journey throughout the study is provided in this section. Accordingly, it is useful to consider these teachers as sub-cases within the school case study. Before an exploration of each teacher's journey, an overview of the type of mathematics lessons that appeared to be typical in the school before the study began is outlined.

Mathematics Lessons Prior to the Study

Data from the initial focus group interviews with pupils, teacher reports during the professional development, teacher reflections, and the interview with the principal suggest that pupils in this school experienced a traditional form of mathematics teaching before the study began. Due to the sensitive nature of some of the pupil comments regarding their experiences of mathematics lessons (some of the reports are less than favourable and could be deemed to be critical of the teacher), pupil reports of their mathematics lessons before the study began are anonymised in this section of chapter four. This is an attempt to protect the anonymity of teachers considering it is a single stream school and so there is a possibility that individual teachers could be identified. In particular, the data suggest a number of features synonymous with traditional mathematics lessons. For example, the teacher appeared to dominate the mathematics lessons with a subsequent restricted role for pupils. This teacher dominance is prevalent in the following pupil comments that exemplify the restricted role that pupils appeared to play in mathematics lessons in addition to the teacher's role as the sole validator of mathematical knowledge:

Interviewer:	Do you ever get a chance of sharing the way that you solved a problem with other children in your class?
Pupil E:	That would be at the end.
Several Pupils:	Ya.
Pupil E:	At the end like when it's over. The majority of people do it the way the teacher does it - which is the easiest way.
Pupil F:	Ya coz like it's the way the teacher taught it to us...(Initial pupil focus group).

This teacher dominance is also reflected in the principal's views of the type of mathematics lessons that existed in the school before the study began:

> I would think they were very much teacher-driven...much like the way teachers would have experienced maths lessons themselves where the teacher would do the teaching and the children were listening...because even from some of the layout of the classrooms, it would indicate that (Principal interview).

Aligned with traditional mathematics lessons, teachers appeared to demonstrate and show the 'correct' way of doing things whilst pupils appeared to watch on silently before transcribing into copies. The principal raised the concern that pupils "...have the idea that the answer is right or wrong – you are good at maths if you can get the right answer or have the perception that they are bad at maths if they don't get the right answer" (Principal interview). In general, the teacher's role appeared to be that of corrector and explainer – correcting sums when they are completed and explaining what to do in the textbook or on the worksheet. The following pupil comments from different classes exemplify this:

Pupil A:	Like sometimes when she is telling us she kind of shows what we have to do ... then she shows what we have to do then she shows us up in the front of the room.
Pupil E:	On the board.
Pupil A:	Or sometimes like she gives out the sheets or like if it's a book we just do what we saw like that we had to do.

Pupil E: Or sometimes if it's like really, really hard she would put down about two sums so that we can copy and we can do the rest on our own.

Pupil F: And sometimes she just tells us to copy and she writes it on the board (Initial pupil focus group).

Furthermore, the data suggest a heavy reliance on textbooks and workbooks where pupils often appeared to work silently and individually on textbook tasks. This emerged as a very strong theme throughout the initial focus groups with pupils. Similarly, this reliance on textbooks and workbooks suggests a focus on procedural fluency. The following pupil comments reflect the focus on procedural fluency and the reliance on textbooks and worksheets:

Pupil C: There are some parts of the book that there is loads of sums and take-away sums and plus sums.

Interviewer: What type of other work?

Pupil C: We have our Mathem… some other maths to do and some more pages of maths to do as well.

Pupil E: Sometimes we get pages (*worksheets*) and sometimes we do work in our books.

Pupil A: But most of the time it is on pages (Initial pupil focus group).

The principal echoed this reliance on textbooks and expressed concerns that the textbooks were dictating the mathematics programme before the study began: "…the overreliance on the textbook…the textbook dictating the programme where some people (*teachers*) would start at the beginning of the textbook and work through it" (Principal interview).

Another feature of traditional mathematics lessons appeared to be the limitation of problem solving to problems outlined in the textbook resulting in problems that tended to be simple, one-step, procedural-type problems based on number operations. Teachers reported this realisation during the professional development when presented with challenging problems to solve (M. Treacy, field notes, 28/1/13). Furthermore, a teacher interview comment underlines the prevalence of simple, procedural problems before the study began when the teacher highlighted the need "...to move away from the book problems because I only realise now that really they are all just operation problems really in a way. They are not really related to the different topics" (Int.2 B). Synonymous with traditional mathematics lessons, there appeared to be little or no collaboration between pupils and similarly, little or no opportunities for communicating about mathematics. The data from the pupil focus group interviews suggest that opportunities for talk were limited to correcting homework and checking answers. When asked, "Do you ever get a chance to talk about your maths ideas?" typical pupil responses included:

Pupil E:	We never get to.
Pupil C:	Never.
Pupil B:	Not really.
Pupil E:	It's just the teacher does it (*mathematics*) up on the board and you have to do it. That's the end of it (Initial pupil focus group).

Furthermore, one of the tracker teacher's comments about her old mathematics practice resonates with those of the pupils when she concluded that "...before it was all about the teacher-talk and you never thought about letting them (*the pupils*) explain how they did anything. It was more give me the answer, not tell me how you actually came up with that" (Int.2 B).

Another feature of traditional mathematics lessons relates to the limited focus on practical tasks. There appeared to be little opportunities for pupils to use concrete materials or engage with practical mathematical tasks. The following pupil comments act as an apt summary and highlight two chief aspects of this traditional mathematics classroom: teacher dominance and little opportunities for pupils to use concrete materials:

Interviewer:	How often do you use any sort of practical things in your maths class?
Pupil A:	Never.

Pupil B:	Not that often.
Pupil F:	The only time we really used them is last year.
Pupil E:	...was weight. We only use them for the weighing.
Pupil C:	Ya. We used the trundle wheel.
Pupil A:	Ya well the teacher really used it – we didn't.
Pupil B:	Ya.
Pupil E:	We don't really get to use it. If we were using it, it would be the teacher using it...if we were using it, it would be the teacher handling it and showing it to us (Initial pupil focus group).

Furthermore, this restricted use of mathematical materials is mirrored in this retrospective teacher report, which emphasises that one of the benefits of the reform approach to mathematics teaching is pupils' autonomous use of materials:

> ...before I would have told them how to use them and what you wanted them to do with them whereas now it was a case of giving them the materials, giving them the problem and observing to see how they approach the problem and use the materials themselves...(Int.2 D).

Again synonymous with traditional mathematics lessons, getting mathematics 'right' or 'wrong' emerged as a strong theme in the initial focus group interview data from pupils and further fortifies the suggestion of traditional mathematics teaching. The following comment exemplifies the emphasis on mathematics being 'right' or 'wrong':

> If you get something wrong like the teacher might come over and do it right but then you might not understand (Initial pupil focus group).

Finally, teachers completed an anonymous reflection regarding their mathematics teaching at the first professional development session. This reflection was based on Hiebert et al.'s (1997) instructional framework and provides teacher reports about mathematics lessons before the study began. The reflections of seven out of the eight class teachers are included for data analysis purposes.[24] Table 4.1 provides a composite overview of these reflections.

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24 One of the class teachers did not offer the reflection for data analysis purposes.

Table 4.1 Composite Overview of Teacher Reflection on Mathematics Teaching prior to the Study

Dimensions	Features	Always	Sometimes	Never
Nature of Classroom Tasks	Tasks are problematic	1	6	
	Tasks encourage reflection and communication		5	2
	Tasks connect with where students are	1	6	
	Something of mathematical value is left behind (residue)[25]		7	
Role of the Teacher	Tasks are selected with goals in mind - sequences of tasks, not just individual tasks	5	1	1
	Essential information is shared with the pupils	5	2	
	Explanations and demonstrations by the students become more important than those by the teacher		1	6
	Teacher has removed him/herself from a position of authority[26] in order to promote the student intellectual autonomy		2	5
Social Culture of the Classroom	Ideas and methods are valued	2	5	
	Students work together to solve problems and interact intensively about solution methods		5	2
	Students choose and share their methods		3	4
	Mistakes are used as learning sites for everyone	1	4	2
	Correctness resides in mathematical argument, not in the popularity of the speaker	2	2	3
	Students have learned to live with a certain amount of uncertainty	1	2	4
Mathematical Tools as Learning Supports	Meaning for tools is constructed by each user		4	3
	Students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems		7	
	Tools are used for recording, communicating and thinking		5	2
Equity and Accessibility	Tasks are accessible to all students	4	3	
	Every student is heard	2	5	
	Every student contributes		7	
	Other students' thinking is respected and valued	3	4	

25 There are two types of residue: 1) insights into the structure of mathematics (mathematical relationships and 2) strategies or methods for solving problems.

26 A position of mathematical authority is where the teacher decides whether answers are correct.

In alignment with the data from pupil focus group interviews, teacher interviews, and professional development sessions, the data from the teacher reflection checklists support the suggestion that traditional mathematics lessons prevailed in this school before the study began. A number of noteworthy features emerge from this data. The dominant role of the teacher is evident in that six of the seven teachers reported that explanations and demonstrations by the student were never more important than those by the teacher with only one teacher reporting that they were sometimes more important. Furthermore, five of the seven teachers reported that they never removed themselves from a position of mathematical authority (deciding whether answers are correct) in order to promote the intellectual autonomy of students whilst only two reported sometimes doing this. Another noteworthy feature relates to communicating mathematical ideas. Four out of the seven teachers reporting that students never chose and shared their methods and three teachers reporting that students only sometimes did this suggests a dearth of student autonomy in addition to little or no opportunities for communicating mathematical ideas. Additionally, three out of the seven teachers reporting that the user never constructed meaning for mathematical tools and five teachers reporting that the user only sometimes constructed it suggests the restricted use of mathematical materials. Moreover, two of the seven teachers reported that mathematical tools were never used for recording, communicating, and thinking whilst five teachers reported that they were sometimes used for these purposes. Another notable feature relates to the cognitive challenge associated with mathematics lessons. Four out of the seven teachers reporting that students never had to live with a certain amount of uncertainty suggests mathematics lessons that were predictable and unchallenging. Two teachers reported that students sometimes lived with a certain amount of uncertainty with only one teacher reporting that students always did. An additional feature regarding the poor emphasis on mathematical argument is reflected in the reports of three of the seven teachers that correctness never resided in mathematical argument as opposed to the popularity of the speaker; with two teachers reporting that it sometimes did and only two teachers reporting that it always did. Another notable feature is the dearth of tasks that encourage reflection and communication. Two of the seven teachers reporting that tasks never encouraged reflection and communication and five teachers reporting that they only sometimes did, suggests mathematics lessons that were procedural-based and focused on individual work. Finally, the dearth of collaborative work during problem solving is reflected in the reports of two of the seven teachers that students never worked together to solve problems and interact intensively about solution methods and five teachers reporting that students only sometimes did this.

The individual experiences of implementing the reform approach to mathematics teaching are outlined in the following section for each of the four tracker teachers. The lessons that each of these teachers implemented were devised collaboratively, that is, three teachers (two class teachers and one support teacher) devised these lessons. Ann devised the lesson with the teacher of Junior Infants and a support teacher, Bernie devised the lesson with the teacher of First class and a support teacher, whilst Catherine and Deirdre devised the lessons together with a support teacher. These lessons were devised using three main sources:

- Hiebert et al.'s (1997) instructional framework (this framework is explained in detail later in this chapter) which contributed to the pedagogy adopted in the lessons;
- the Primary School Mathematics Curriculum (objectives for length and weight) which contributed to the focus and the content of the lessons; and
- the booklets on length and weight (the content of which is described later in this chapter) which contributed to the content of the lessons.

During Phase 1, this collaborative lesson planning began during the professional development sessions and was completed by teachers outside of the professional development sessions;

whilst during Phase 2, this collaborative planning took place outside of the professional development sessions.

As outlined in the previous chapter, before the implementation of the reform approach to mathematics teaching, the professional development focused on pedagogical content knowledge (Hiebert et al.'s instructional framework) and subject content knowledge (facts, pupil misconceptions, and tasks in relation to the topic of Measures, in particular, length and weight). Furthermore, during the professional development teachers analysed the pupil reports regarding mathematics teaching before the study began. This activity appeared to be a catalyst for teacher engagement and buy-in (this is discussed later in the chapter). Before the implementation of lessons during Phase 2, teachers contributed to refining and reshaping the instructional framework to suit their context (this revised instructional framework was used in Phase 2). Similar to Phase 1, there was an emphasis on the instructional framework and the content of Measures (albeit weight) in Phase 2. Likewise, teachers analysed pupil reports; however, during the Phase 2 professional development these reports related to pupil reports regarding the collaboratively devised lessons that were implemented in Phase 1. One of the chief differences during the Phase 2 professional development was the exploration of international scholars with regard to mathematics teaching and learning. This included the use of video footage, podcasts, and selected readings as outlined in chapter three.

Ann

Ann taught a junior class at the time of the study and had been teaching at this class level since the school amalgamation three years earlier. She was a permanent teacher in the school and had over 10 years teaching experience.

Phase 1

The observed lesson during Phase 1 focused on length, in particular, using the story of Santa and toys to provide pupils with a reason for measuring length. The main task required pupils to help Santa by measuring pictures of toys so that he could fit them all into his toy sack. In groups, pupils had to first estimate and then measure the length of the toy in their picture using cubes as the unit of measurement. Interesting discussions ensued about length and width because some groups measured both length and width whilst other groups only measured length (as illustrated in Image 4-1 where one group only measured the length of the toy hammer). However, whether width was an option to measure depended on the toy picture.

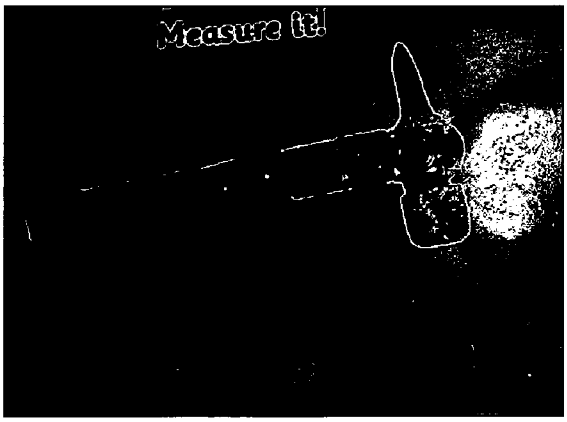

Image 4-1 Senior Infant measuring task during the Phase 1 observed lesson

Pupils then had to report their findings to the whole class. Next, groups of pupils had to order the pictures of toys including the corresponding number of cubes from the smallest to the tallest. Finally, pupils recorded their findings on paper. The sophistication of these recordings ranged greatly between:

- simple drawings of the toys without any reference to the cubes (see Image 4-2 of the toy caterpillar);

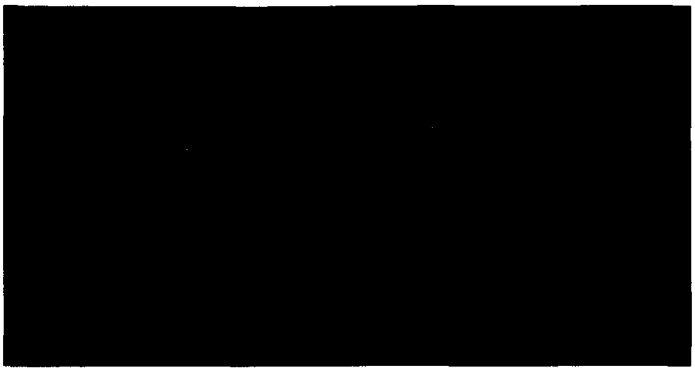

Image 4-2 Senior Infant representation of a toy caterpillar

- to representations of both the toy and the cubes, with the pictorial representation of the cubes superimposed on the toy and again above the drawing of the car (see Image 4-3 of the toy car);

Image 4-3 Senior Infant representation of a toy car with cubes on and above the car

- to representations of the toy with a separate but related representation of the cubes (see image 1.5 of the toy butterfly);

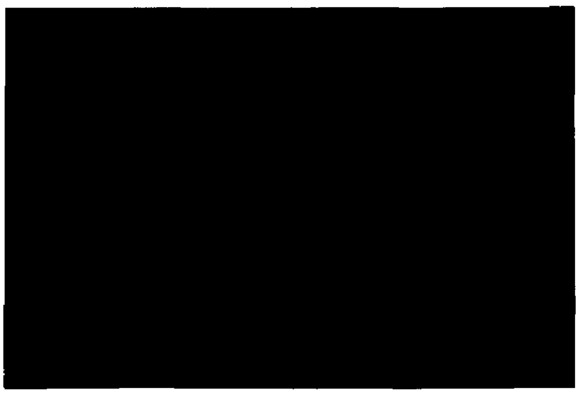

Image 4-4 Senior Infant representation of a toy butterfly with cubes represented separately

- to representations of the cubes in the shape of the toy (see Image 4-5 of the toy hammer);

Image 4-5 Senior Infant representation of a toy hammer using only representations of cubes

- to representations of the toy with only the numerical representation of the cubes (see Image 4-6).

Image 4-6 Senior Infant representation of a toy caterpillar with cubes represented only in numerical form

During the observation of this length lesson in Phase 1, mathematics teaching appeared to have changed from the didactic, individual, textbook focus that was reported before the study. Ann had sourced a problem based on length and so textbooks did not feature in the lesson. The pupils worked collaboratively in groups of two or three, rather than individually whilst using cubes to measure items was a central component of solving the problem so pupils were involved in informal, practical measuring tasks. However, despite such progress a number of anomalies prevailed in these lessons. Instead of sharing mathematical authority with pupils, Ann appeared to have relinquished it. An example of this was evident when a group of pupils was struggling; one of the pupils in the group was persuasively leading the others in the wrong direction and although the teacher observed this, she did not intervene at the point of need. Afterwards, Ann articulated that she felt that she "wasn't teaching" and that she thought that she could not or should not intervene (M. Treacy, field notes, 16/11/12). On the one hand, this demonstrates teacher uncertainty, which is understandable considering the teacher was implementing a new and unfamiliar approach to mathematics. On the other hand, it suggests a possible diminishing of the teacher's autonomy. This is a worrying finding because it is not aligned with the intentions of the professional development, which is to contribute to enabling teachers to implement a reform approach to mathematics teaching. It is unclear from the available data what exactly constrained the actions of this teacher. The second anomaly in this lesson relates to the fact that although pupils worked in groups on a practical task, there was little evidence of whole-class discussion or development of mathematical thinking. Completing the practical task appeared to have been the chief purpose of the lesson (although pupils also reported how many cubes long their toy was and recorded their findings). In relation to the teachers' experience of implementing this approach to mathematics teaching, a number of challenges are evident in the data from the Phase 1 teacher interview. Ann was the only tracker teacher to question the 'newness' of the reform approach to mathematics teaching. She suggested that teaching mathematics this way, and specifically using the instructional framework, was little different to the way she already teaches mathematics. She reported struggling to identify the differences between her original practice and the new approach considering that she already used concrete materials and group work. Specifically, she queried what was different about the new methodology. A number of observations may assist in understanding this teacher's struggle better. Infant Classes in the school have used Ready, Set, Go Maths (Pitt, 2001) for a number of years; this programme promotes the focused use of concrete materials to develop number concepts, encourages higher-order teacher questioning whilst scaffolding tasks, and emphasises the importance of pupils communicating and justifying their thinking. It is possible that the teacher was drawing parallels between the approaches advocated in Ready, Set, Go Maths, and those espoused in the instructional framework. However, it may also be possible that the teacher had not yet fully grasped the depth of the reform approach to mathematics teaching. She referred to the new approach as using concrete materials and group work, which is true; however, the approach is much more than that. It further advocates a focus on developing conceptual understanding of mathematics and on the advancement of mathematical thinking by communicating mathematical ideas including conjecturing, argument, refining thinking, etc. Another possibility is that the teacher was in fact already using these approaches and so, as she asserted, the methodology was no different to her existing practice. Considering data from lesson observations are not available from before the study, it is difficult to determine the validity of this argument; however, it appears to be unlikely considering data from the pupil focus group interviews suggest that a traditional, didactic approach prevailed in this classroom. The second challenge evident in the interview data was the constraint on teacher autonomy that first became evident during the lesson observation. In the interview, Ann again referred to her uncertainty with regard to "not being allowed to tell". She twice referred to not knowing if she were "allowed" to intervene.

> I just found it hard to know what was my role...I was kind of like I'm not allowed tell them, they have to figure it out for – I can say to them what do you think but I won't be able to intervene but then actually I was intervening.

This reduction in autonomy was a regrettable and an unintended consequence of the professional development programme, which aimed to empower teachers. For Ann, it appears that teacher uncertainty may have heightened her fear and constricted her role in the mathematics lesson. Accordingly, for Ann the degree of her uncertainty appears to have resulted in a form of pedagogical paralysis where she desired explicit direction with regard to what she could or could not do. Such paralysis is inherently disempowering; and is indicative of a dependency culture where teacher autonomy is almost non-existent. Therefore, this teacher's experience of discomfort is similar to debilitating discomfort as described by Frykholm (2004). Furthermore, a third constraining factor was evident in the interview data, that of time pressure with the teacher specifically referring to the amount of time that the reform approach takes and the corresponding pressure it puts the teacher under to get mathematics 'done'. This suggestion may somewhat contradict Ann's earlier assertion that she was already teaching in this way before the study began. Equally, it may demonstrate a time or curriculum overload pressure that she experiences continuously, regardless of which approach is employed. Finally, Ann identified collaborative planning as an enabling factor during the study. Specifically during the interview, she commended the peer support that she derived from this process and expressed a desire to continue planning in this way despite the fact that it was the first time she had experienced this type of pedagogical peer support whilst planning for specific lessons.

Phase 2

The observed lesson during Phase 2 focused on weight, in particular, groups of three pupils investigating the heaviest and lightest objects from a set of objects. Each group had access to a bucket balance, a container of 'weights' (cubes, links, or bears), a bag of three objects, and two cards with the letters L and H. Each child had to estimate which object was heaviest or lightest by holding an object in each hand whilst his/her arms were outstretched. Subsequently, the group had to discuss their estimates and come to a consensus before ordering the objects from lightest to heaviest using the cards to show which objects were heaviest and lightest (see Image 4-7).

Image 4-7 Senior Infant ordering activity during Phase 2 observed lesson

This was followed by whole-class feedback where the teacher facilitated the discussion by a) asking pupils to explain what they did, and what they found out; b) using revoicing; and c) asking groups how they could check their answers. Finally, the teacher highlighted a group that was having difficulty and asked other groups to offer solution methods, for example, "Does anyone have any idea what this group could do?"

The observed lesson during Phase 2 had some similarities with that during Phase 1, for example, a) Ann sourced a problematic task and textbooks were not used; b) pupils worked in groups of three; and c) pupils were required to use concrete materials to solve the problem. However, differences were also evident between lessons during Phases 1 and 2. Ann appeared to be considerably more confident using the reform approach than during Phase 1. This was particularly evident in the way in which she facilitated the discussion and intervened where necessary without undermining pupils' autonomy (M. Treacy, field notes, 1/3/13). This facilitation was in stark contrast to the complete stepping away from the lesson that was evident during Phase 1. In particular, Ann's use of effective questioning and prompts that facilitated discussion was noteworthy during this lesson. Another difference relates to whole-class discussion and sharing of ideas and methods, which was evident but had not been as strongly evident during the Phase 1 lesson where pupils had just reported the number of cubes that they needed to measure the length of a toy picture. Again, Ann's use of effective questioning contributed to the success of this discussion, for example, open-ended prompts and questions such as "Explain what you did."; "What did you find out?"; "How could you have checked?"; "How do you know it did not work?"; "What could we do?"; etc. In contrast, teacher questioning and probing was not evident during the Phase 1 lesson because Ann reported being unsure of whether she "could" intervene. Ann's role in the process appeared to be more defined and proactive and there was evidence of more whole-class sharing of mathematical thinking during the observed lesson in Phase 2. This increase in teacher confidence was again evident in the teacher interview during Phase 2 where Ann reported being more confident with regard to her role in the reform approach:

> I thought initially when I was doing the length that I had to let them off on their own completely and I wasn't sure when I could intervene or when I could guide them whereas with the weight I think I had a better idea of how much help I could give or how much guidance I could give.

Ann attributed this increased confidence to a number of factors including the focus on facilitation and teacher language in the revised instructional framework and in the Phase 2 professional development in addition to her familiarity with the approach considering it was no longer "new". However, despite this increase in teacher confidence, during the interview Ann expressed doubt about the relevance and practicality of using this approach to teach mathematical concepts. Specifically, she equated the reform approach to "discovery learning" and suggested that it could not teach certain concepts to children that she herself could teach them and that there is a need for explicit instruction by the teacher before using this approach.

Consequently, she suggested using this approach as an addition to her traditional teaching instead of as a replacement. She expressed the opinion that it would work well as an assessment tool, specifically at the end of a topic to check whether she had "taught" the topic well. This finding resonates with a similar finding by O'Shea and Leavy (2013) who found that one of the teachers in their study would only consider engaging pupils in learning by constructivist approaches after she had employed traditional approaches and on the condition that she was confident that the pupils had significant background knowledge. Although the data suggest that Ann's confidence in using the approach increased during Phase 2, she expressed her intent to return to teaching the way she had always taught, with the possibility of using the reform approach as an assessment tool at the end of a topic.

In summation, Ann's mathematics lessons appear to have been traditional in nature before the study began with a heavy reliance on textbooks and a propensity for teacher demonstration and telling, with minimal opportunities for pupils to engage in mathematical discussion. The data suggest mathematics lessons changed in Ann's class during Phase 1. Specifically, it appears that textbooks were less dominant and pupils worked collaboratively on practical tasks. Furthermore, it appears that teacher uncertainty with regard to a) her role and b) what she could and could not do, resulted in an unintended consequence. This manifested itself in Ann almost fully removing herself from the mathematics lesson and therefore, her autonomy to make pedagogical and epistemic decisions may have been undermined. Although more shifts in practice were evident during Phase 2 including whole-class sharing of mathematical ideas and solution methods, the data suggest that a degree of teacher uncertainty or doubt remained. Despite the apparent success of moving from a traditional mathematics class to a class demonstrating aspects of a reform approach, Ann indicated her intent of returning to a more traditional approach (albeit supplemented by some aspects of the reform approach). This finding supports that of Warfield et al. (2005) who found that hybrid lessons can play a part in changing mathematics practice. Thus, there may be discord between Ann's beliefs about mathematics teaching and learning and those underpinning the reform approach to mathematics teaching. Nonetheless when asked during Phase 2 whether her mathematics lessons had changed, Ann concluded that:

> Ya. I suppose my questioning of the children would have changed...I am giving more open questions and letting them think of their answers without just guiding them first. I suppose also after spending time with a new topic I would now try to have a lesson where they are using what they learned...you know as more of a discovery lesson I suppose you know for me to see what they know but for them also to problem solve...to see the skills that they have learned and maybe the language that they have picked up, to use that (Phase 2 teacher interview).

Ann's conclusions regarding the changes to her mathematics lessons highlight a number of notable shifts in practice. Furthermore, her conclusions suggest that these shifts in practice align with the reform approach to mathematics teaching as outlined in the instructional framework. The change to more open-ended questioning is a significant one because it requires the teacher to take on an active, skilled facilitator role during mathematical discussions whilst it also encourages pupils to reflect, communicate their solutions and reasoning, and revise their solutions and conjectures. Furthermore, providing lessons where pupils have the opportunity to use mathematics that they have learned means connecting with where students are. Additionally, providing opportunities for pupils to problem solve suggests tasks that encourage reflection, communication of mathematical thinking, and the purposeful use of tools – to solve problems. Finally, encouraging pupils to use the skills and language they have learned is significant because it can provide opportunities for pupils to use language to refine, revise, clarify, and communicate their mathematical thinking.

Bernie

Bernie taught a junior class at the time of the study. This class comprised 31 pupils; a considerable number of whom had diverse needs. She was a permanent member of the teaching staff and had more than five years teaching experience.

Phase 1

The observed lesson during Phase 1 focused on length, in particular, using concrete materials to measure length followed by a problem task card. The main task required pupils to individually measure cut-outs of their hands and feet using a variety of concrete materials including cubes, lollipop sticks, pipe cleaners, cars, and matchsticks (see Image 4-8).

Image 4-8 Second Class measuring activity during Phase 1 observed lesson

Pupils then recorded their findings in writing; that is, comparing the different findings for their feet and hands based on which measuring tool they used. Once this activity was completed, pupils worked either in pairs or in groups of three to complete the following problem[27]:

My mum is taller than my dad. I'm smaller than my brother but bigger than my little sister. Draw my family. Try this: Our dog is half the size of me.

[27] The teacher sourced this problem from a Hawker Brownlow Education mathematics task book (2006).

Pupils discussed this problem in pairs or groups; however, during the observed lesson one pupil generally took the lead in drawing the family (see Image 4-9 for samples of these drawings). In some groups, the other pupils appeared to be engaged, for example, they continued to make comments, suggestions, and reminders such as "The mum is taller than the dad" or "The dog is supposed to be only half the size of you". Contrastingly, in other groups, some pupils who were not drawing appeared to disengage, for example, some were talking to each other and some were staring into space. There was no evidence in the observed lesson of pupils sharing the drawing task between them.

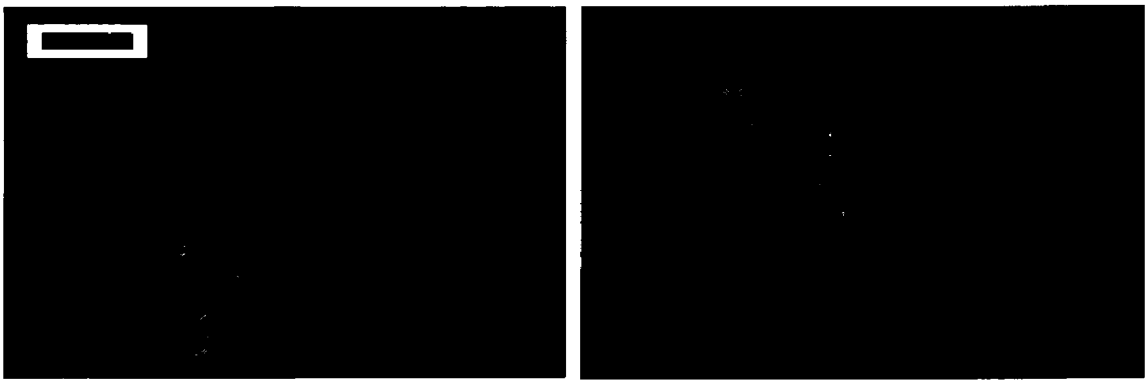

Image 4-9 Second Class family drawings during Phase 1 observed lesson[28]

During observation of the length lesson during Phase 1 mathematics teaching appears to have changed in this classroom from the textbook dependent, procedures focused mathematics classes that were reported before the study. Bernie created the tasks collaboratively with other teachers and so the use of textbooks or workbooks was not evident. For the drawing task pupils worked in groups of two or three as opposed to individually as was the case before the study began. In addition, pupils were engaged in practical tasks that

28 The textbox on the first image is to preserve the anonymity of the pupils whose names appear on the work sample.

required them to use concrete materials. To similar effect, pupils were engaged in tasks that required some level of problem solving. The greatest change in emphasis in this lesson appeared to be a focus on conceptual understanding of length as opposed to a focus on procedural fluency. These lessons still appeared to lack certain important features espoused in the reform approach to mathematics teaching. In particular, although focusing on conceptual understanding and comprising some level of problem solving, the tasks could have been more challenging with more opportunities for rich problem solving. Consequently, there appeared to be minimal sharing of ideas within groups and no sharing of ideas across groups, which is unsurprising considering the nature of the tasks did not necessarily encourage or require sharing. The measuring task was completed individually and although the drawing task was completed in pairs or groups of three, there was little evidence of discussion in many groups. Additionally, the teacher's role appeared to be that of organiser and classroom manager rather than mathematics teacher. Specifically, her role appeared to include supplying concrete materials to pairs or groups, keeping some pupils on task, and organising follow-up activities for early finishers. The data suggest that pupils worked on their own individually or in groups with little mathematical interjection or enhancement by Bernie. However, a possible reason for Bernie's lack of engagement with the mathematics became evident during the teacher interview. Similar to Ann, Bernie reported not knowing whether she was "supposed to" guide the pupils or by guiding them would she be inadvertently controlling the mathematics class. This teacher uncertainty relates to guidance, questioning, and ultimately the role of the teacher in the process and is exemplified in Bernie's comment:

> Should you ask any question or by asking a certain question are you giving them (*the pupils*) too much…are you like helping them to discover? Do you want to ask the bare essentials and let them completely come up with everything or do you want to steer the question?...so you want questions that are going to get them completely thinking kind of openly or do you want a kind of steering question? It is important to know what type of question to ask because you weren't sure like, should I even have said that, or is that still me being in control…

However, unlike Ann, Bernie's uncertainty with regard to her role did not appear to undermine her autonomy in the mathematics lesson. The following comment highlights how Bernie appeared to embrace and enjoy the change to mathematics practice:

> So the teacher trying to talk less and giving control over to the children was great, it took that little bit of biting your tongue but once they got into it like they were great…and rather than you telling them, they were telling you. It was great.

Furthermore, she asserted that she wanted support and guidance with regard to "the language of questioning and affirming comments" so that she could affirm pupils and give them confidence without being the mathematical judge and saying whether it is correct or incorrect and without telling them how to do it. In this instance, teacher uncertainty or dissonance appears to have been a catalyst for teacher learning because it appeared to prompt teacher reflection and subsequently the identification of personal learning needs for the teacher. Therefore, in Bernie's case, her uncertainty appears to be similar to "educative discomfort" as described by Frykholm (2004). Finally, in relation to supports, the teacher reported finding the instructional framework very useful (although there appeared to be little evidence of the core features in the observed lesson) and she highlighted teacher reflection and peer support from the collaborative planning process as important learning and development opportunities.

Phase 2

The observed lesson in Phase 2 was based on weight and the main task required pupils to weigh various objects using a bucket balance and then to record their findings individually in writing. Pupils recorded items in their copy that they believed to be lighter than their shoe and items that they believed to be heavier than their pencil. Next, the teacher distributed a set of five or six cards to each group, for example, the cards for one group included 6 cars, 2 pencils, 14 lollipop sticks, 2 copies, and 1 pencil case whilst another group's cards included 8 cars, 5 pencils, 20 matchsticks, 4 copies, 3 rubbers, and 9 cubes. Pupils then compared the weight of the objects on the cards using a bucket balance (see Image 4-10).

Image 4-10 Second Class weighing activity during Phase 2 observed lesson

Subsequently, the pupils recorded their findings individually in their copy books, for example, 3 scissors were heavier than 2 pencils; 6 scissors are heavier than 10 cars; 18 lollipop sticks are heavier than 3 rulers; etc. The data from the lesson observation during Phase 2 suggest *further* development in mathematics teaching. In line with the Phase 1 lessons, these lessons continued to include the use of practical tasks that were sourced by Bernie rather than coming from the textbook; however, differing from the Phase 1 lessons, these tasks appeared to be more problematic in nature and also required pupils to work collaboratively (even though pupils recorded these findings individually). Sharing solution methods across groups was not necessarily appropriate for the type of tasks considering each group received *different* task cards. However, groups of pupils engaged in discussion about the weights of objects within their group during the activity, for example, predicting which would be heavier/lighter before weighing, refining their predictions during weighing and then clarifying their findings before recording these. In some groups, there was evidence of pupils justifying their thinking based on previous findings, for example, justifications such as "I think they (*2 pencil cases*) will be heavier than the (*14*) lollipop sticks because they (*2 pencil cases*) were heavier than the (*9*) cubes and the cubes were heavier than the lollipop sticks". However, although the pupils worked in mixed ability groups, this type of justification was not evident in all groups. In some groups, the pupils gave predictions but did not give reasons for their thinking. Therefore, it appears that opportunities for this type of mathematical discussion were incidental rather than structured in the observed lesson during Phase 2. However, incidental or not, such opportunities for mathematical discussion did not exist in the Phase 1 observed lesson because pupils worked on the measuring task individually. Although not appearing to adopt an active role in this mathematics lesson, Bernie reported being more comfortable with the reform approach, and her role within it during Phase 2. Specifically, she reported feeling more confident in Phase 2 than Phase 1 with regard to "guidance and not giving them (*the pupils*) too much". Particularly interesting is the teacher's reference to being reassured and not feeling guilty, which is reflected in the following comment:

> When we were using it the last time...you felt almost as if you were neglecting or stepping back too much whereas when we were doing it in Phase 2 I felt more reassured that the teacher was the facilitator but like, you were still the teacher but you were giving them that freedom and you weren't feeling guilty that you weren't answering their questions.

The teacher attributed much of this increased confidence to a) teacher reflection, b) peer support, and c) teacher learning with regard to facilitation, teacher talk, guidance, and questioning. Furthermore, she reported finding the instructional framework very useful for her lessons, although in this observed lesson and similar to the observed lesson in Phase 1, only a small number of the core features were evident. Interestingly, despite the data suggesting a heavy reliance on textbooks and worksheets in this class before the study began, Bernie's experience of implementing the reform approach appears to have fortified her intention to use textbooks more judiciously. Bernie appeared to find her decision liberating. However, despite this intention and her advice that other teachers should leave the textbook aside, a number of factors appear to undermine the reality of doing this. Bernie indicates that teachers will have to "...learn to leave the book aside" intimating the reliance and possible reluctance that such a move might entail for some teachers. Similarly, the struggle between the teacher's autonomy and the power wielded by the textbook is evident when she says "you know you are the teacher, the book isn't the teacher...". Moreover, she refers to the financial cost of the textbooks on parents, which may force teachers to continue using it. The "systemic silence" (Sugrue & Gleeson, 2004) is broken inadvertently when the teacher states, "I know we have to use the textbook". This in fact is not officially true; however, subliminal messages may result in this being the perceived reality. The latter point is discussed in more detail later.

In summation, the mathematics that pupils experienced in Bernie's classroom before the study began appears to have been very traditional, in that it focussed on reproducing procedures, in particular, reproducing number operations repeatedly. Pupils' reference to textbooks, worksheets, maths sheets, and "sums" in the initial focus group was striking. There appears to have been a slight shift in practice in mathematics lessons as the study progressed. The Phase 1 lessons focused on conceptual understanding rather than procedural fluency and involved hands-on activities and some collaborative work whilst the Phase 2 lessons appeared to involve tasks that were slightly more problematic, more collaborative work, with some opportunities for mathematical discussion. However, scope exists in this classroom for a) mathematical tasks to be more problematic and challenging; b) more structured opportunities for mathematical discussion; and c) more in-depth involvement of the teacher in productive mathematical discussion. Finally, similar to the experience of Ann, teacher uncertainty during Phase 1 appears to have been an issue for Bernie. However, unlike Ann, who appeared paralysed by this discomfort, Bernie appeared to use the discomfort productively by considering specific pedagogical questions. Rather than wanting direct answers to these questions, she appeared to use these questions for self-reflection. Consequently, Bernie reported being confident, assured, and comfortable with regard to her role in the reform approach during Phase 2. Her uncertainty appeared to have diminished. Considering the authority wielded by the textbook before the study began Bernie appeared to possess more autonomy at the end of the study. Finally, when asked during Phase 2 whether her mathematics lessons had changed, Bernie stated that:

> Well, definitely since doing the length lessons I have stepped back and given more control to the children; like my maths lessons were always teacher-talk, teacher asked the questions and teacher said if it was right or wrong. Whereas, now it is more open-ended and there is a lot more children talking and explaining how they did it rather than me just saying ya well that's right or that's wrong. There is a lot more children taking over and the teacher stepping back, more so...Well I would do a lot more pair work and group work. Before it would have been a lot more individual tasks... and more concrete materials now (Phase 2 teacher interview).

Bernie's conclusions regarding changes to her mathematics lessons also confirm a slight shift in practice (although not all of these features were evident in the two observed lessons but may have been present in other mathematics lessons). Furthermore, her conclusions suggest that these shifts in practice are aligned with the reform approach to mathematics teaching as outlined in the instructional framework. Stepping back and giving more control to the pupils can elevate the importance of pupil explanations and demonstrations whilst moving away from the teacher as the position of mathematical authority can provide opportunities for promoting the autonomy of pupils' mathematical thinking. Furthermore, using open-ended questions can encourage pupils to explain and communicate their thinking and reasoning whilst moving from individual tasks to more pair and group tasks can provide more opportunities for pupils to refine, revise, clarify, and communicate mathematical thinking.

Finally, Hiebert et al. (1997) contend that using tools such as concrete materials can free thinking for more creative activities.

Catherine

Catherine was a newly qualified teacher teaching a senior class at the time of the study and was working as a substitute teacher covering a maternity leave. Furthermore, she was undertaking her Teaching Diploma[29] during the study. Mathematics lessons appear to have changed in this class as the study progressed.

Phase 1

The observed lesson during Phase 1 focused on problematic tasks associated with the concept of length. Pupils were divided into groups of three or four and each group was assigned a different task. For each task of the tasks outlined below, pupils were encouraged to discuss and choose which measuring tools or equipment would be most suited to their task.

Task A

This task was divided into two parts. Pupils were required to estimate, record, measure, and re-record the length and width of a table and to calculate the difference between their estimate and the actual measurement. Subsequently, pupils were required to estimate, record, measure, and re-record the perimeter of the top of the table. Once more, pupils had to calculate the difference between their estimate and the actual measurement.

Task B

In this task, pupils were required to complete two parts. They had to estimate, record, measure, and then re-record the length and width of the classroom whilst similar

[29] Newly qualified teachers have to undertake a Teaching Diploma, which comprises external evaluation from the Inspectorate of the Department of Education and Skills.

to Task A they then had to calculate the differences between their estimates and the actual measurement. Next, they had to estimate, record, measure, and re-record the perimeter of the classroom and then calculate the differences between their estimates and actual measurements. The pupils choose to use the trundle wheel for this measuring activity.

Task C

In this task, pupils were required to complete two parts. Pupils had to estimate, record, measure, and then re-record each other's heights whilst similar to Tasks A and B they then had to calculate the differences between their estimates and the actual measurement. Furthermore, they had to calculate the differences between the heights of the pupils in the group. Then, they had to draw an outline of their hands and feet and then estimate, record, measure, and re-record the perimeter of these outlines. Once again, they had to compare their estimates with the actual heights and similar to the first part of the task, they had to compare the perimeters of hands and feet within the group.

Task D

Two groups completed this task, which required pupils to choose a shape to create on the floor using masking tape. Pupils were required to create the shape, estimate the perimeter, measure the perimeter, and calculate the differences between the estimates and the actual measurements. Both groups completed the task; however, the sophistication level varied between the two groups. One group chose to create a straight line on the floor using masking tape and so completed the task on a simple level. Meanwhile the other group completed the same task but they engaged with it in a slightly more sophisticated way. They not only created a square instead of a straight line but they also discussed their plans in detail before they began and drew out a plan of their proposed shape (see Image 4-11).

Image 4-11 Fifth Class creating a square during Phase 1 observed lesson

In summation, the data from the Phase 1 observed lesson on length suggest that for all of the tasks pupils a) worked collaboratively in groups of three or four, b) used concrete materials to solve practical tasks, and c) recorded solutions to problematic tasks. Talk appeared to be limited to within individual groups; however, when compared with previous evidence for this class, this suggests a considerable increase in the opportunities for pupils to talk about mathematics. The following pupil report illuminates the type of talk that took place in one particular group: "We were talking about our estimates and stuff. And you were asking like 'What do you think?' like 'Why do you think that?' and 'What is your estimate?' and everything" (Pupil focus group interview during Phase 1). Similar to the observed lessons in other classes during Phase 1, the teacher appeared to have adopted the role of organiser and classroom manager rather than that of mathematics educator. The teacher distributed the task cards and clarified any queries regarding completion of these tasks but there was no evidence of the teacher engaging in any mathematical discussion with these groups. Furthermore, like the other Phase 1 lessons, there appeared to be no sharing of solution methods or ideas between groups and no evidence of communicating or refining mathematical thinking. Similar to Second Class, completing the practical task appeared to have been the endpoint in itself. However, the tasks in Fifth Class were more problematic in nature and required some calculations using the findings.

Although not directly involved in the mathematics discussions within the groups, the teacher did not appear to be uncomfortable with this role. However, the teacher interview data from Phase 1 suggest that Catherine may have struggled with the collaborative planning, reporting that she found it "...difficult enough at times" because she had different ideas to the other teacher and subsequently, it took extra time to find a balance and to make decisions. The data also suggest that Catherine did not appear to be bound by the instructional framework, for example, unlike Ann and Bernie she did not refer to being unsure of whether she could or could not intervene or what she should or should not say. A possible reason for this might be the fact that she reported using the instructional framework judiciously to reflect on her mathematics practice before and after the lesson, and she commended its use on reminding her of certain practices that she often forgets in her "everyday teaching". In line with this, she referred to "reminders" several times during the interview. To similar effect, she referred to the instructional framework as a useful assessment tool for her teaching. However, similar to Ann and Bernie, one of the challenges she identified in implementing the reform approach was the change to the teacher's role. Furthermore, she reported not knowing how this approach would work for other topics, for example, with regard to teaching the concept of fractions. She suggested that lots more ideas needed to be given to teachers for other areas of the mathematics curriculum because she "...wouldn't have a clue how to teach them practically". Interestingly, the data suggest that Catherine appears to equate a considerable proportion of this reform approach to the use of practical tasks, which in fact is only one aspect of a multi-dimensional approach.

Phase 2

The observed lesson during Phase 2 focused on a conceptual understanding of weight. The lesson was divided into a number of parts. Initially, pupils worked in groups of five on gaining an understanding of the 'feeling' for certain weights (50g, 100g, 200g, 250g, 500g). Pupils took turns to fill up a bag with rice so that it would approximately weigh 50g. Pupils estimated the weight against a benchmark of a container that weighed 50g and compared the 'feeling' of this bag of rice by passing both the container and the bag of rice around the group. A lot of discussion was evident in the groups during this task, in particular with regard to the quantity of rice, for example, the pupils used language such as more, less, enough, not enough, too heavy, too light, and just right.

The second part of the lesson involved the teacher weighing the bag of rice from each group and recording on a table on the whiteboard. After each recording, the teacher posed questions such as:

- Is this bag heavier or lighter than 50g?;
- How much heavier/lighter is it in grammes?;
- How many grammes are the group out by?; and
- Which group is closest to 50g?

This activity was repeated for 100g, 200g, 250g, and 500g. There was a competitive element to this activity because the teacher awarded a point to each group who got closest to the benchmark weight.

The third part of the lesson involved a rich problem solving task which pupils had to try to solve in their groups. The teacher sourced the problem called 'What's my weight?" from www.nrich.maths.org (see Image 4-12 for the task that was presented to the pupils).

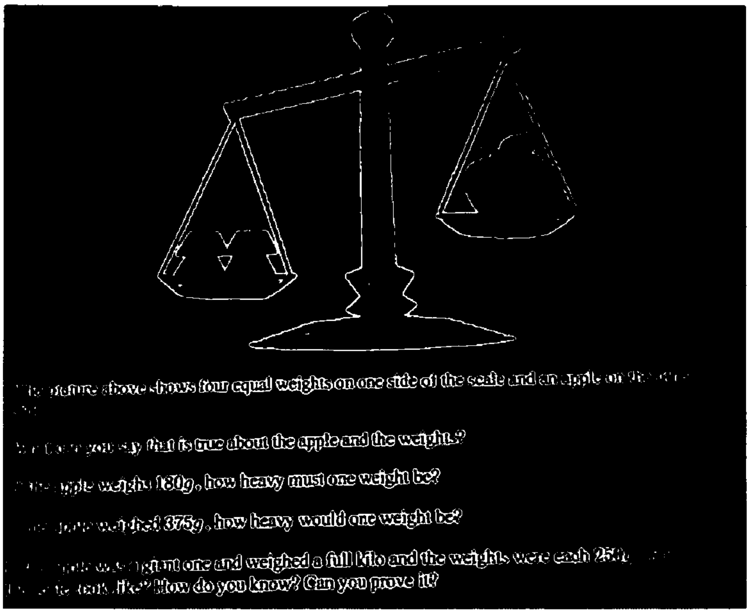

Image 4-12 Fifth Class problem solving task during Phase 2 observed lesson

Some groups used concrete materials to represent the weights and the apple. Lots of discussion was evident in each of the four groups as they attempted to solve this problem. Interestingly, the discussion suggested that pupils assumed that the answer had to be a specific number and this proved to be an obstacle for many of them. In fact, the problem had missing data and so the answer could simply be "each weight must weigh *more* than 45 grammes". The pupils seemed determined to get 'the answer', which they seemed to believe should be an exact number. This suggests that pupils believed that answers in mathematics have to be exact. It also suggests an over-reliance on procedures for solving a problem where thinking outside the box is not considered. The following comment from one of the pupils during the group discussion exemplifies this over-reliance on procedures for solving problems: "We don't know whether to multiply or divide" (Lesson Observation, Fifth Class, 1/3/13).

The final part of the lesson involved a whole-class plenary based on the problem-solving task. Through questioning, Catherine provided opportunities for pupils to share their strategies for solving the problem. The following questions are typical of those she used:

- Will someone give us ideas of how they went about solving this problem?;
- Did any group do it differently?;
- Did any group come up with anything different?; and
- What did your group do?;

Through teacher prompting and questioning, there was evidence of pupils explaining and clarifying their solution strategies and in some cases their thinking. Furthermore, there was evidence of some pupils communicating their reasoning in this whole-class discussion. The following pupil comment communicates his reasoning as to why his group could not solve the problem:

> Well the weights are equal but are heavier than the apple so we can't find the weight of each weight but if the apple and weights balanced then we could find out how much each weight weighs (Lesson Observation, Fifth Class, 1/3/13).

However, other groups gave suggested weights that would satisfy the problem; for example, one group suggested that if all weights were equal to 50g then their combined weight would still be heavier than the apple (see Image 4-13 for this group's pictorial representation of this solution).

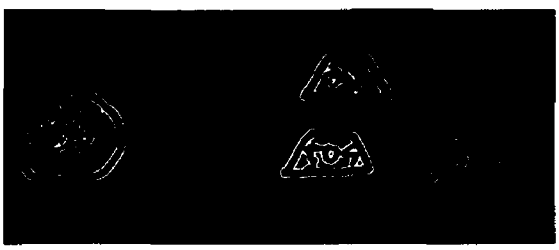

Image 4-13 Fifth Class problem solving solution during Phase 2 observed lesson

In summation, the data from the observed weight lesson during Phase 2 suggest that like the other classes, there appeared to be shifts in practice in mathematics lessons between

Phases 1 and 2. Similarities between the lessons in Phases 1 and 2 include a) exploration of the concept through practical tasks and b) pupils working collaboratively in groups. However, the difference between the lessons is considerable. For example, the sophistication level of the lesson appeared to increase between Phases 1 and 2, for example, there were a number of sections to the weight lesson including:

- mixed-ability group work based on practical tasks and the use of concrete materials;
- a whole-class plenary with regard to findings;
- group work based on a challenging problem that encouraged mathematical discussion; and
- whole-class discussion about the problem and various solution strategies.

The observed lesson during Phase 1 only comprised the first section – groups working on a practical task. Another difference relates to the inclusion of a challenging non-routine problem because this was not evident during Phase 1. The third difference between the lessons relates to whole-class discussion. This was not evident in the Phase 1 lesson; however, it was evident in two distinct sections of the Phase 2 lesson. Furthermore, the discussion with regard to the problem appeared to be rich, with evidence of pupils explaining their thinking, highlighting the challenges they faced, reasoning, clarifying their thinking, and refining their thinking. The following comments give a flavour of the type of discussion with which pupils were engaged, in particular, it highlights the communication of mathematical thinking and the reasoning that took place both within the group and then during whole-class discussion:

Pupil B: We were kind of talking about will we use…if the apple is 180 grammes how much will one weight be…and we were thinking will we divide it or multiply it…what number will we multiply or divide it by. We were discussing how much each weight would be…how big it would have been. All that sort of talk.

Interviewer:	In the whole group feedback, what kind of things were you talking about?
Pupil A:	Well we were like saying the apple was 180 grammes so then the weights in the picture weren't that far underneath it so then they couldn't have been that much each...they probably like we were saying were only a small bit heavier than the apples so.
Pupil B:	We were explaining how we solved it.

This contrasts sharply with the evidence suggesting that before the implementation of the reform approach, pupils in this class mainly talked to give answers to procedural operations. Therefore, this lesson demonstrates considerable progress in pupils' level of communication and in the quality of expressed mathematical thinking. Finally, during the observed weight lesson, the teacher appeared to be comfortable with her role and although she did not contribute to the mathematical discussion as a participant; she skilfully facilitated the discussion ensuring the effective communication of pupils' mathematical thinking. Pupil reports further strengthen the suggestion that the teacher was more comfortable in her role during the Phase 2 weight lesson. In particular, pupils considered the teacher's role to be that of helper, prompter, and advisor rather than that of teller. The following comment exemplifies this revised perception:

> When she (*the teacher*) was coming around giving us advice kind of...she would give us maybe step one but then we would have to figure out step two or something and then she'd maybe give us steps on how to do step three but we'd have to do step four.

The data from the Phase 2 teacher interview support the finding that there appeared to be a considerable shift in practice in mathematics lessons in this class. In particular, Catherine reported a) including more problem solving in her lessons; b) moving away from textbook problems to richer, more challenging problems; c) using more talk and discussion; and d) using practical activities to explore the mathematical concepts of weight and length. She attributed some of this change to the learning that took place during the professional development in realising that a variety of tasks exist to explore different topics and so she reported being inspired to "…go out and look for more ideas for different topics". Aligned with Phase 1, Catherine identified the instructional framework as also contributing to the change in her mathematics practice. However similar to Phase 1 and unlike Ann, she did not appear to feel pressurised by the instructional framework in that she reported merely glancing over it now and again. Importantly, despite experiencing such success, the continuation and progression of this approach to mathematics appears to be under threat because Catherine appeared to feel pressurised by a number of factors including time, textbook use, and presumptions from other teachers. She reported feeling under pressure to get everything associated with weight covered when a full week was spent exploring the concept through practical activities and solving rich problems. Her concern related to the amount of time that this takes, time that she reported normally would be spent doing activities in the textbook. A similar concern relates to her understanding that despite exploring the concept of weight in detail she still had to complete the weight chapter in the textbook: "I feel like I have to do both". Catherine's concerns are evidenced further when she suggests that either there should be less to cover in mathematics or alternatively teachers should not have to use textbooks. However, Catherine poses the latter as a question rather than a suggestion, which highlights her perceived lack of autonomy with regard to textbook use: "maybe we should have less to cover so that we can get all the practical activities in as well or else is it a case we don't have to use the books?" Catherine asked this question tentatively and appeared to be waiting for an answer. It appears to be more akin to a genuine question than a suggested solution. This again underpins the lack of autonomy that many teachers appear to experience in relation to the ingrained culture of prolific textbook use. Another of her concerns appears to be the presumptions of other teachers, although this obstacle cannot be divorced completely from the previous one with regard to textbook use, for example, Catherine reported that "when they go on to Sixth Class next year it will be presumed they have done everything that is in the Fifth Class book".

In summation, there appeared to be a considerable shift in practice in Catherine's mathematics lessons during the study. Before the study began, this class appeared to be traditional in orientation with a strong emphasis on procedures, writing number operations into copies, working individually and silently, and getting mathematics "right". The data suggest that some shifts in practice were evident during the Phase 1 observed lesson, in that lessons appeared to comprise more practical tasks and collaborative group work. However, these lessons appeared to lack a focus on problem solving, communicating mathematical thinking, and discussion. Catherine appeared comfortable in this role and reported finding the instructional framework helpful in changing her practice. Phase 2 data suggest further shifts in practice to include more mathematical communication and discussion, challenging problematic tasks, and a focus on developing understanding of a concept rather than a procedure. Once again, Catherine appeared confident in her role as facilitator albeit that similar to the other tracker teachers there was little evidence of her contributing as a participant to the mathematical discussion. Catherine's newfound autonomy with regard to pedagogy and the nature of mathematical tasks appears to be under threat from the possible return to a situation where the textbook holds the pedagogical and epistemic power in mathematics classes. This is disappointing considering when asked during Phase 2 whether her mathematics lessons had changed, she reported that:

> Well firstly probably since doing the length lesson my lessons have become more practical and there'd be a lot more concrete activities and then probably since we started working on weight I've kind of brought more problem solving into the lessons...you know aside from the book problems that we would have been doing up to then...richer problems (Phase 2 teacher interview).

Catherine's conclusions regarding changes to her mathematics lessons support the finding that considerable shifts in practice were evident in her lessons throughout the study.

Furthermore, her conclusions suggest that these shifts in practice are aligned with the reform approach to mathematics teaching as outlined in the instructional framework. Catherine's report that mathematics lessons are more practical suggests a focus on conceptual understanding rather than procedural fluency whilst the use of concrete materials may allow pupils to free their thinking for more creative tasks. To similar effect, the move towards rich problem solving in lessons can provide opportunities for reflection, communication, and clarifying and refining mathematical thinking.

Deirdre

Deirdre taught Sixth Class at the time of the study and had worked previously as a learning support teacher shared between two schools. She was a permanent member of the teaching staff and had between five and ten years teaching experience.

Phase 1

The observed lesson on length during Phase 1 took place between the classroom and the school hall. The first part of the lesson, which took place in the classroom, comprised whole-class revision of language associated with length from previous lessons, for example, decametre, hectometre, etc. Pupils then worked in groups of five where first they had to order themselves based on their height (either from tallest to smallest or from smallest to tallest). Subsequently, they had to attempt to make all five pupils in each group appear to be the same height (for example, some pupils stood with their knees bent whilst others stood on the top of their toes, etc.). The second part of the lesson took place in the school hall, with pupils working in six ability groups of four or five pupils on practical, problematic tasks. These tasks, which are outlined below, varied with regard to the type of task, the type of equipment needed, and the difficulty level of the task. Each group received a differentiated task card and pupils were assigned roles in each of these groups. Pupils were encouraged to discuss their task, devise a plan, and choose which measuring instruments would be most suited to their task.

Task A

This task required pupils to find the perimeter of a tile in the hall ceiling. Pupils were given further prompts such as: a) estimate the perimeter of the tile first; b) discuss how you would go about this; c) decide which instruments to use; and d) calculate the difference between your estimate and the actual measurement. The pupils discussed in detail various ways they could complete this task. After considerable debate, they agreed on the method that is outlined in the group's work record (see Image 4-14).

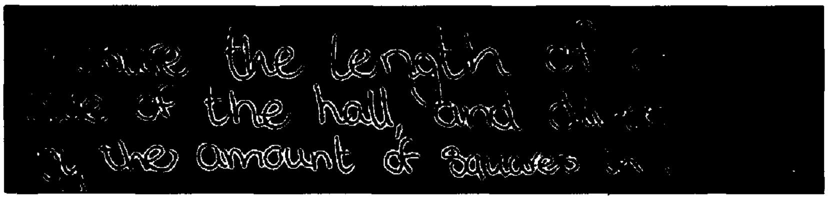

Image 4-14 Sixth Class work record for length task A during Phase 1 observed lesson

After attempting to measure the length of one side of the hall with a measuring tape, the group re-considered and decided to measure the length of the floor in the hall using a trundle wheel. They then counted the amount of tiles in one row on the ceiling (directly under where they measured the floor) and divided the number of tiles into the measurement they got for the length of the hall. Subsequently, they multiplied this figure by 4 to determine the perimeter of the tile (they had decided earlier that the tiles were square and so each side was of equal length). Image 4-15 represents a learning log from one of the pupils in this group.

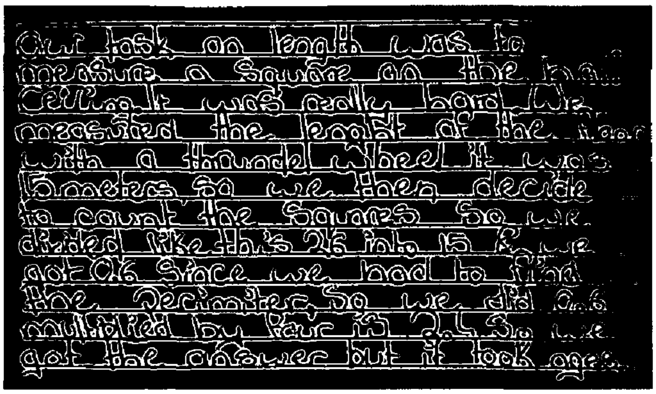

Image 4-15 Sixth Class learning log for length task A during Phase 1 observed lesson

As evidenced in the learning log (Image 1.23) it appears that this pupil initially found this task challenging, "It was really hard…So we got the answer but it took ages". The learning logs of two other pupils in this group mirror this sense with regard to the difficulty level of the task, for example, "At first we didn't know what to do…" and "We thought it looked very hard to do…".

Task B

This task required pupils to find the optimum height that each ball would bounce, for example, a basketball, volleyball, football, plastic ball. Pupils were given further prompts such as: a) estimate first; b) discuss how you would go about this; c) decide which instruments to use; and d) calculate the difference between your estimates and the actual measurements. The optimum heights ranged from 1.54cm for the basketball (see Image 4-16) to 34cm for the plastic ball.

Image 4-16 Sixth Class length task B during Phase 1 observed lesson

Task C

This task required pupils to find the perimeter/circumference of four different sized hoola hoops. Pupils were given further prompts such as: a) estimate the perimeters/circumferences of each hoop first; b) discuss how you would go about this; c) decide which instruments to use; and d) calculate the difference between your estimates and the actual measurements. Interesting discussion ensued in this group with regard to the most appropriate measuring instrument to use, for example, one pupil suggested using a trundle wheel whilst another pupil disagreed and instead suggested using a measuring tape "...because it is flexible" (M. Treacy, field notes, 16/11/12). Image 4-17 shows pupils measuring one of these hoops with a measuring tape.

Image 4-17 Sixth Class length task C during Phase 1 observed lesson

Pupils measured the hoops using both a trundle wheel and a measuring tape; however, they got conflicting answers. Consequently, another pupil asked, "Could we now use non-standard measuring tools?" (M. Treacy, field notes, 16/11/12). The pupils then decided to measure the hoops using wool and then measured the wool using a measuring tape. Although this task only required pupils to measure three hoola hoops, the pupils also measured the circumference of sticky tape.

Task D

This task required pupils to find the perimeter of the four group members positioned together leaving no gaps. Pupils were given further prompts such as: a) estimate the perimeter first; b) discuss how you would go about this; c) decide which instruments to use; and d) calculate the difference between your estimate and the actual measurement. In particular, this task generated a lot of discussion amongst the group because initially they appeared to grapple considerably with how this task could be completed. After about ten minutes of discussion, the group decided to lie on the ground beside each other and use masking tape to make an outline of the perimeter of this combined body shape or as one pupil described it: "like crime scene investigation marking the position of the body". Image 4-18 illustrates their solution method for this task. This presented further difficulties because the group then had to decide how to use the masking tape whilst they were all lying down. After discussing this considerably, they decided that the two pupils at either end of the group shape could do the marking at the opposite end of the shape. In their discussion, the pupils concluded that this would not disrupt the outline. Next, they placed wool on the masking tape outline, then straightened out the wool, and measured the wool with a trundle wheel.

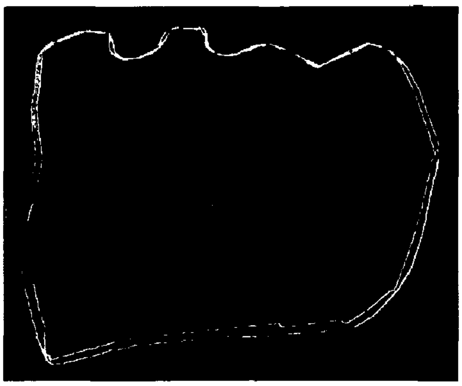

Image 4-18 Sixth Class solution method to length task D during Phase 1 observed lesson

Task E

This task was divided into two parts. Pupils had to draw two regular shapes that were three times longer than their width and then calculate the perimeter. Prompts on the task card included a) discuss how you would go about this and b) decided which instruments to use. During the discussion on how they would complete this task, one pupil concluded that the shape "needs to be extra-long" (M. Treacy, field notes, 16/11/12). Next, pupils were instructed to draw two irregular polygons free hand. Prompts on the task card included a) estimate the perimeter; b) discuss how you would go about this; c) decide which instruments to use; c) measure the perimeter; and d) calculate the difference between your estimates and the actual measurements. One pair of pupils drew the polygons free hand as instructed on the task card whilst another pair used concrete materials to create the regular polygons and used sticky tape to keep the pieces in place (see Image 4-19). During this task, I overheard one pupil explaining to another pupil "I am doing a hexagon and he is doing an octagon" (M. Treacy, field notes, 16/11/12).

Image 4-19 Sixth Class polygon activity for length task E during Phase 1 observed lesson

Task F

This was an open-ended task divided into two parts. Pupils were instructed to fold a one-metre measuring tape into equal parts. The associated questions on the task card were a) how many parts can you fold it into and b) how long is each part? Once completed, pupils had to identify nine items and then categorise them into three categories (easy to measure, ok to measure, and difficult to measure). Finally, pupils were asked to prove why each item was in a particular category.

In summation, mathematics lessons appear to have changed in this class during Phase 1 compared to the type of lessons that teachers and pupils reported were in existence before the study began. In this lesson, there appeared to be a particular focus on enhancing the concept of length and its associated language using problematic tasks. Pupils appeared to have been fully engaged in creative, teacher-designed problematic tasks based on the school environment. Consequently, textbooks did not feature in this lesson. The need for genuine communication about mathematics in this lesson is evident in the following pupil comment: "(*We were talking*) all the time because...the way we were measuring stuff we all thought of different instruments to do it and we all had different ways" (Phase 1 Sixth Class pupil focus group). Similarly, the rich nature of the tasks required pupils to work collaboratively to devise solutions. The following comment illuminates this joint effort: "Our group all had different ideas and then we put them all together and we tried each one and then we got the best one in the end" (Phase 1 Sixth Class pupil focus group). The tasks were extremely well organised and Deirdre displayed adept classroom management skills. However, although appearing relatively comfortable with this role as organiser, like the other teachers, there was no evidence of Deirdre contributing to the participation of the mathematical discussion in any of the groups. Nonetheless, this is a considerable change from the traditional role of teller. Pupil reports strengthen this finding and their conceptualisations of the teacher's role during Phase 1 include that of helper and supervisor – a role where the teacher might "give you a hint" rather than that of teller. Pupils reported liking this challenge when the teacher is not telling: "It was good because ...we had to do it and it became more of a challenge rather than her (*the teacher*) just telling us what to do" (Phase 1 Sixth Class pupil focus group). Although this lesson comprised many features espoused in the instructional framework, evidence did not emerge of a) pupils sharing strategies and ideas across groups or b) whole-class discussion. The Phase 1 teacher interview supports the finding that Deirdre did not contribute to the mathematical discussion. In fact it appears that similar to the other tracker teachers, Deirdre believed that she was supposed to refrain from getting involved, for example, when asked what she found useful she referred to "...the parts where we withdrew from the language aspect of it and let the children come around to it themselves". She also highlighted the different role that the teacher plays in such an approach, that is "...a more facilitatory role rather than an instructional role". Deirdre seemed to enjoy this different role and gave examples of encouraging phrases that she used repeatedly to help her with this facilitation when pupils asked for assistance such as "See where that takes you"; "Go with that"; and "That's very interesting". She suggested that having a prompt sheet of similar phrases and questions would be very useful for teachers. Similar to Bernie, it appears that Deirdre's uncertainty prompted her to reflect on productive ways of improving her practice. Accordingly, her uncertainty with the new approach aligns with Frykholm's (2004) educative discomfort rather than debilitating discomfort. She also acknowledged the support of a) having the instructional framework as a guide and b) having peer support through the collaborative lesson planning process. Nonetheless, despite her apparent success and similar to Catherine, Deirdre queried the relevance of this approach to other areas of the mathematics curriculum, for example, when teaching fractions and decimals.

Phase 2

Similar to the observed lesson on length during Phase 1, the observed lesson on weight during Phase 2 included whole-class discussion and group tasks. However, unlike the observed lesson during Phase 1 it also comprised other sections that will be detailed later (there were six distinct sections to this observed lesson). Initially, pupils discussed the concept of weight as a whole class, for example, what it is, how it can be measured, and possible misconceptions about weight. As a whole class, pupils then began to build a fact file about weight. This comprised pupils making statements about weight and other pupils giving a thumbs up or thumbs down depending on whether they agreed with the statement, for example, "A standard bag of sugar is 1 kilogramme"; "My pencil case is light, my pen is lighter but my topper is the lightest"; etc.

Subsequently, mixed ability groups comprising four pupils engaged in varied tasks based on weight. Two tables of equipment were set up from which the groups could choose materials. This equipment included:

- a variety of kitchen scales;
- a variety of bathroom scales;
- a variety of spring balances;
- a variety of bucket balances;
- timers;
- tubs of materials such as cubes, marbles, counters, batteries, etc.; and
- items such as bags of sugar, jelly, packets of wafers, a coconut, etc.

Six groups completed one task each so there were six tasks in total; however, two of these tasks were the same whilst one task was only slightly different from another. Hence, four tasks are detailed here.

Task A

Two different groups completed this task even though the weights given to each group varied slightly. Each group was given four cards (0.2kg, 60% of 1kg, 3/8kg, 1/10kg for one group and 0.5kg, 5/8kg, 150g, 25% of 1kg for another group) and four containers. Each group was instructed to order the cards from lightest to heaviest (left to right) without using a benchmark weight. Next, each group had to estimate the weight of the containers using their hands only, record these estimates, and then place the container in front of the corresponding weight (effectively ordering these containers from lightest to heaviest). Pupils then had to weigh the containers using a kitchen scales, record these weights, compare these weights with the estimated weights, and communicate the findings to the other group.

Task B

Two different groups completed this task; the only difference being that one group was instructed to use marbles whilst another used batteries. Each group was instructed to use a benchmark weight, for example, a bag of sugar and then estimate and record how many marbles/batteries would be needed to balance the benchmark weight. Each group then had to check using a kitchen scales, compare the actual number required to their estimated number, and communicate these findings to the other group. Furthermore, the task card suggested that pupils could estimate individually and then have a competition within the group to ascertain who had the closest estimate.

Task C

This task required pupils to find something in the class that might weigh the same as a) the class coconut, b) the hour glass, and c) the ice-cream wafers. Pupils then had to record these choices and check whether each of these pairs balanced using a bucket balance.

Task D

In this task, pupils were instructed to choose five containers of different contents and to compare the weights using only their hands. Pupils had to record these estimates and then based on these estimates, order these containers from lightest to heaviest. Next, pupils had to compare the weight of these containers using only a bucket balance and then compare these findings with their estimated ordering.

The third section of the observed lesson on weight was an optional task for groups that had finished the weighing tasks. These groups were directed to discuss how you might make a spring balance in the classroom. The fourth section of the observed lesson involved each group reporting to the whole class regarding their weighing task. Each group explained their task, reported what they did, and reported their findings, for example, their findings such as estimates, actual weights, differences, etc. Deirdre was actively involved in facilitating this whole class reporting session, for example, she asked questions such as "What helped you to complete this task?"; "What else did you do?"; "How did you record your findings?"; etc.

The fifth section of the lesson comprised a word problem that was adapted from a problem on www.nrich.maths.org.

> The pupils in 6th class fill 9 containers of sand. They are all the same weight when full. While the pupils are out of the room, the teacher puts a marble in one of the containers. How will they find which container has the marble?
> **They can only use the bucket scales twice.**

Having distributed the problem to each group, Deirdre expressed her concern to the researcher that the problem would be too difficult for the mixed ability groups to solve, particularly, because the teachers had grappled considerably with this problem at the preceding professional development session[30] (M. Treacy, field notes, 1/3/13). However, this concern proved to be unfounded because four out of the six groups reported partial findings before the end of the lesson and one group had solved the problem within a few minutes.

30 Only one group of teachers out of four groups managed to solve the problem.

Only one group was unable to report any progress.[31] The groups tackled this problem in different ways, for example:

- one group discussed various possibilities orally;
- two groups discussed various possibilities and then decided on one solution which they recorded in writing (see Image 4-20);

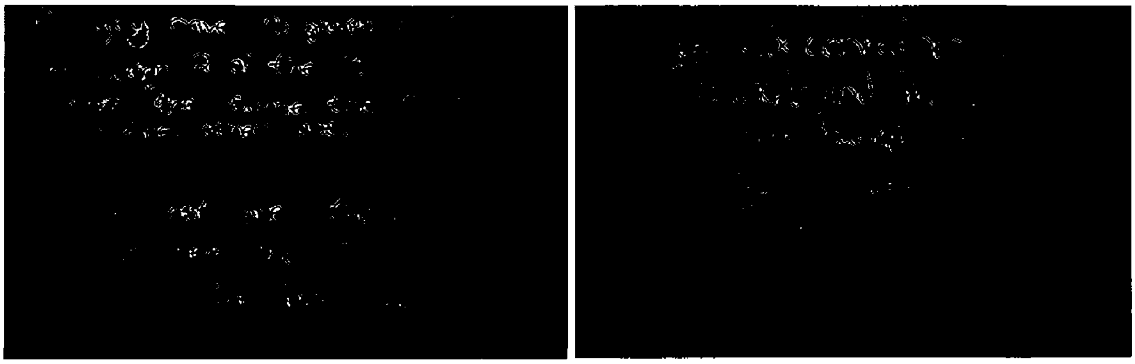

Image 4-20 Sixth Class samples of writing for problem solving activity during Phase 2 observed lesson

- one group used manipulatives (counters, miniature cars and houses) to represent each of the nine containers and then discussed various possibilities using the manipulatives;
- one group drew a pictorial representation of the nine containers (see Image 4-21); and

[31] The teacher indicated that if the lesson were not being observed she would have given the other groups more time to find a solution; however, lunchtime was approaching and the teacher wanted to complete the final section of the lesson.

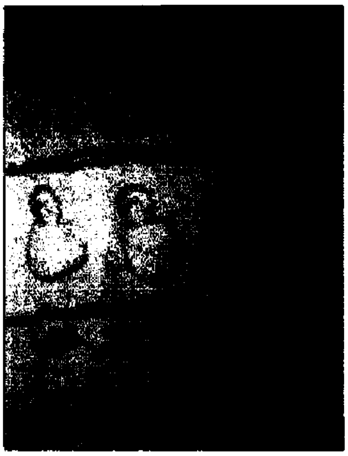

Image 4-21 Sixth Class drawing for problem solving activity during Phase 2 observed lesson

- one group drew a pictorial representation of the nine containers and also used manipulatives (counters) to represent the nine containers (M. Treacy, field notes, 1/3/13).

In the learning logs that pupils completed after the lesson, a number of pupils referred to using manipulatives as helping them in solving this problem:

> It also helped when we had the little cars and houses to figure out the puzzle.
>
> The little minupulitives (*sic*) are useful because you can see it instead of just seen (*sic*) a page full of numbers.

Similarly, the benefits of group work when solving problems emerged as a strong theme from the learning logs. Nine out of the twenty-three learning logs extoled the benefits of working in groups or as part of a team. Some of the most apt quotations include:

> I learned that working out problems in a group is easier than working it out alone.
>
> I learned that working in a group is better and more effective than working individually.

The final section of this observed lesson involved whole-class feedback on each group's progress. The teacher played an active role in the facilitation of this discussion, for example, she asked for clarification, sought alternative solution methods, and asked pupils to revoice ideas from other groups.

The data from the Phase 2 lesson on weight suggest that like the other classes, there appeared to be a shift in practice between the observed mathematics lessons in Phases 1 and 2. Similar to the Phase 1 observed mathematics lesson, this lesson also comprised a) a whole-class discussion on the mathematical concept with a specific focus on language and b) pupils working collaboratively on one practical task. However, it differed from the Phase 1 lesson in that it comprised many other aspects including:

- mixed-ability rather than ability group work (the Phase 1 lesson involved ability groups);
- an open-ended task: "How might you go about making a spring balance in your classroom?";
- groups reporting back to the whole class including which strategy they used, what helped them, what their findings were, and how they recorded their findings;
- a multi-step complex problem that required pupils to grapple, work together, and use trial and error;
- whole-class feedback on the strategies used to solve the problem; and
- the learning logs focused on learning (see Image 4-22. The Phase 1 learning logs were generally descriptive in nature and outlined the task that was completed rather than focusing on learning).

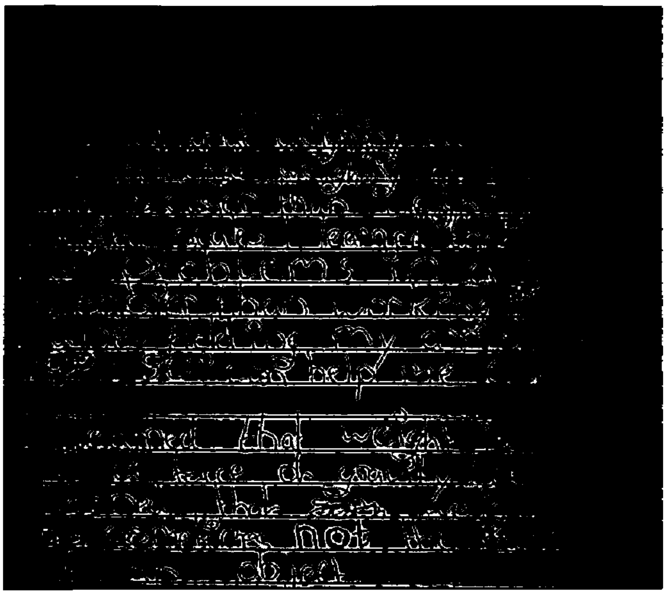

Image 4-22 Sixth Class learning log after Phase 2 observed lesson

Furthermore, the following pupil report confirms the continued facilitation role adopted by Deirdre: "every once in a while she would come around and just see what we were doing and say what if ye tried something else" (Phase 1 Sixth class pupil focus group). Although this lesson is a progression from Phase 1 and generally aligns with a strategy-reporting classroom (as described by Wood et al., 2006), it lacked the components of an argument/inquiry classroom in that although pupils reported their strategies, they had not taken on the role of the teacher in questioning, clarifying, and validating mathematical ideas. Nonetheless, the data from the Phase 2 teacher interview suggest that Deirdre attributes two chief reasons for the change in mathematics lessons. She indicated that teacher reflection contributed substantially to the change in mathematics lessons. Specifically, she reported a) being more willing to accept that things can be done differently; b) realising that thought and effort has to be put into sourcing rich problems; and c) focusing more on mathematical discussions. Deirdre also attributed using the instructional framework as another catalyst for her change in mathematics practice. In particular, she emphasised how her use of it changed during the study, in that she reported initially relying on it quite heavily when planning lessons but later used it only as a mental checklist. She explained that she used it more frequently until she became used to it and internalised it. Therefore, aligned with the intention of the instructional framework, it appears to have acted as a temporary scaffold for Deirdre until she became more comfortable with teaching mathematics this way. She suggested that other teachers should use the instructional framework and reported that it "definitely works for me…it's brilliant". This confidence in teaching reform-based mathematics was evident in all of the Phase 2 data associated with Deirdre.

In summation, the data suggest a considerable shift in practice in the mathematics lessons in this class throughout the study. Like the other classes, it appears to have progressed from a traditional lesson that focused on procedures and was dominated by teacher talk and demonstrations to lessons where pupils actively collaborated on challenging mathematical tasks and reported their strategies and ideas in structured whole-class discussion. Unsurprisingly, the teacher's role also appeared to change considerably from a traditional role of telling, showing, and being the validator of mathematical knowledge to a more facilitative, proactive role that held high expectations for pupils. However, Deirdre differed from the other tracker teachers, in that she appeared to embrace her new role fully in both phases of the study. None of the data indicates that she over-relied on either the instructional framework or the professional development in her implementation of the reform-based approach to mathematics teaching; nor does it indicate a debilitating fear or a quest for external validation or guidance. Essentially, implementing the reform approach to mathematics teaching appears to have enabled Deirdre to exercise her autonomy fully from the beginning of the study. When asked during Phase 2 whether her mathematics lessons had changed she reported that:

> Most definitely, they have changed. A lot more focus on language…for them to focus on the revoicing of language and…being conscious of the whole idea of when disagreeing with something that they disagree with what they are saying

> rather than the person. Although I did a lot of problem solving more focus on groups and more of a willingness to accept that things can be done differently. You know, that groups can solve a problem differently and to make the problem, I suppose a rich one. Although, I don't use that word but I think that I had to pace that a little bit maybe and not start off with the really heavy ones because I thought I'd lose them and they'd lose confidence so I'm nearly up there now with the richer problems. I thought that was very good and I probably wouldn't have done that before this project. I wouldn't have put as much thought into the problems and staging them, you know (Phase 2 teacher interview).

Deirdre's conclusions regarding changes to her mathematics lessons support the finding that shifts in mathematics practice were evident in the observed mathematics lessons in this class. Furthermore, her conclusions suggest that these shifts in practice are aligned with the reform approach to mathematics teaching as outlined in the instructional framework. For example, focusing more on language and discussion can provide more opportunities for pupils to communicate, revise, and refine mathematical thinking whilst creating a culture around respectful argument can ensure that correctness resides in the mathematical argument, rather than the popularity of the speaker. Similarly, this can elevate the importance of pupils' explanations and demonstrations. Furthermore, greater use of group tasks can increase opportunities for sharing and communicating mathematical thinking whilst a willingness to accept that things can be done differently means that ideas and methods are valued and so multiple solution methods are encouraged. Finally, the focus on rich problem solving can increase the opportunities for refection, refining, revising, clarifying, and communicating mathematical thinking.

In conclusion, mathematics lessons appeared to change in the four tracker classes during the study. Furthermore, the data suggest that mathematics lessons further progressed between Phases 1 and 2, which supports the findings of Ma (1999) that changing mathematics practice is an evolutionary process. However, the extent of the shifts in practice varied as did teachers' experiences of implementing the reform approach to mathematics teaching. In particular, some teachers appeared to welcome the opportunity to implement the reform approach whilst other teachers, although implementing the approach during the study, were circumspect about its suitability for teaching mathematics. Similarly, some teachers gave examples of how they had already used the approach in other areas of the mathematics curriculum whilst some teachers reported their intentions of returning to mathematics lessons in which textbooks feature strongly.

ENABLING FACTORS

This section reports on the factors that contributed to supporting and enabling teachers to change their mathematics teaching in this study. The data suggest that several factors contributed to changing mathematics teaching including peer support, tailored professional development, and the contextualised use of an instructional framework. This data mainly draws from teacher reports and is best summarised by two teacher reports. For example, when asked at the end of the study, what helped teachers in changing mathematics lessons, one class teacher who was not tracked during the study concluded that "...everything helped...the talks with you (*professional development*) and the group work together (*lesson planning with other teachers*)" (Teacher focus group interview). Another teacher lent further legitimisation to this by stating that "the instructional framework...also the collaborative planning that we did" were the most beneficial aspects of the study (Phase 1 teacher interview). The findings in relation to these factors are discussed in more detail in this section.

Peer Support

Peer support emerged as a strong theme in the data from teachers. After Phase 1, teachers were asked at a professional development session to identify what worked well whilst implementing the reform approach to mathematics teaching. Teachers almost unanimously agreed that the collaborative lesson planning was worthwhile, useful, and a positive experience (M. Treacy, field notes, 21/1/13). This is an interesting finding because teachers reported never having engaged in this sort of planning before. Some teachers reported having experience of planning whole-school topics for History and Geography at the beginning of the year, for example, "I don't think we did much (*collaborative*) planning. At the start of the year maybe for SESE[32] topics only so that we wouldn't cross over" (Phase 1 teacher interview). Another teacher reported having experience in a previous school of planning on a weekly basis; however, she reported that this planning was confined to sharing topics they intended to cover rather than collaboratively planning lessons (Phase 1 teacher interview). Consequently, all teachers reported that collaboratively planning lessons was a new experience for them. Furthermore, three of the four tracker teachers reported finding collaborative planning very beneficial (Catherine reported finding it difficult at times). Specifically, teachers highlighted opportunities within collaborative planning for bouncing ideas off one another. Examples included where one teacher reported, "you could throw out an idea and then talk through it - like would that work or why would it" (Int.1 A) whilst other teachers suggested, "you can trash out ideas…" (FG.CT) and "…see what works" (FG.CT). Another teacher reported that the collaborative planning was interesting and she highlighted the potential for sharing expertise: "It was great to sit down and do that (*planning lessons*) with others. It is just to get the expertise and the sharing" (Int.1 D). This supportive nature of collaboratively planning lessons was also evident in the following teacher comment that intimates the potentially isolating nature of teaching:

> So it was great to have someone else rather than you sitting on your own with the curriculum books. It was great to have the teacher who has the stage before you, there you know just to bounce ideas off them…I would love to do more of it. (Int.1 B).

32 SESE refers to Social, Environmental, and Scientific Education and includes a number of subjects in the Irish Primary School Curriculum: History, Geography, and Science.

The principal's comments about collaborative planning resonate with those of teachers and further emphasises the supportive nature of working together in addition to the potential for learning within such activities: "...collaborative planning...I think people really appreciated that and I think that working together some of the teachers who may not have fully grasped what was involved learned from the others in the group..." (Principal interview). This finding regarding the benefits of peer support when attempting to change practice is supported by similar findings in the literature. In a synthesis of the research on professional learning and development that has been demonstrated to have a positive impact on student outcomes, Timperley et al. (2008) emphasise the importance of teachers having opportunities to process new learning with others. Specifically, they suggest that collegial interaction that is focused on student outcomes can help teachers integrate new knowledge into practice. The collaborative planning in this study focused on student outcomes because the teachers devised lesson plans based on the objectives in the mathematics curriculum. More specifically to mathematics practice, Borko, Frykholm, Pittman, Eiteljorg, Nelson, Jacobs, Koellner-Clark & Schneider (2005) found that peer support from others on a similar trajectory of development with regard to mathematics teaching helped teachers to reach new plateaus in their understanding of mathematics.

Tailored Professional Development

Similar to peer support, the benefits of professional development also emerged as a strong theme from the data (mainly based on teacher reports). In particular, all of the four tracker teachers identified the professional development as contributing to changes in their mathematics teaching whilst professional development also emerged as a strong theme in data from the teacher focus group interview. Teachers identified a number of aspects of the professional development that they found useful including the fact that all teachers in the school engaged in the professional development. One teacher emphasised that this provided opportunities for a whole-school approach to mathematics (FG.CT) whilst the principal also highlighted the need for whole-staff input when attempting to change mathematics practice (Principal interview). This engagement of the whole school staff in the professional development programme cannot be divorced from the previous finding with regard to the benefits of peer support and collaboration.

Additionally, teachers commended the fact that the professional development was tailored to the needs of the school. Typical comments included the fact that the professional development was devised based on the school-identified areas for improvement in mathematics which allowed the school to "...pick one area and work together on it" (Teacher focus group) considering how difficult it is "...to target every area and so instead target the weak areas and work together on them" (FG.CT). These areas were identified by the school based on poor student outcomes for the measures strand in mathematics assessments. Likewise, the principal identified the use of the pupil reports from the school as an effective stimulus in the professional development:

> I thought some of the input they (*teachers*) got shocked them to be honest...out of their complacency. As in realising how the children felt about the maths – the feedback from the children was a real eye-opener to the teachers and I think it really made them reflect on their practices...(Principal interview).

In support of this, a change in engagement levels by the teachers became evident after the professional development session that analysed the pupil reports of mathematics lessons. In the two preceding sessions, some of the teachers appeared unresponsive during the professional development but in subsequent sessions, their engagement levels in discussions and activities appeared to increase (M. Treacy, field notes, 15/10/12). These findings regarding the centrality of student learning outcomes in professional development endeavours are echoed in the literature. Mujis, Kyriakides, Van der Werf, Creemers, Timperley & Earl

(2014, p.249) contend that student learning and well-being cannot be seen as "...by-products of effective teaching and professional learning, but rather as the reason to engage, the basis for understanding what needs to change, and the criteria for understanding whether those changes had been effective". To similar effect, based on the synthesis of research described earlier Timperley et al. (2008) claim that the extent to which student outcomes form the rationale for, and ongoing focus of, teacher engagement influences whether the professional learning activities have a positive impact on student outcomes. Furthermore, Guskey (2002a) contends that demonstrable results with regard to student learning outcomes are central to the endurance of any change in instructional practice. Consequently, the powerful contribution of pupil voice in this study may be attributed to what Timperley et al. (2008) refer to as a professional learning community that is focused on becoming responsive to students.

Another aspect of the professional development that teachers reported finding useful was their active contribution to its progress. A number of teachers reported that using the instructional framework in Phase 2 was easier because they had contributed to the changes based on their experiences of using it (M. Treacy, field notes, 14/1/13). Teachers had identified areas in which they needed further support and clarification, for example, the use of language and questioning in addition to more guidance in the skill of facilitation (this is outlined in more detail later in the chapter). The need for teachers to be active in the professional learning process is emphasised continuously in Timperley et al.'s (2008) findings. Specifically, they found that successful professional learning programmes allow teachers to identify further learning that they themselves need in order to assist their students' learning. Furthermore, they claim that successful professional development programmes involve teachers in discussing and developing understandings that are meaningful in their particular practice contexts. Finally, they propose that professional development providers need to support teachers as they develop "...the theoretical understandings and tools that will enable them to take a self-regulated, inquiry approach to their everyday practice" (p.21). Similarly, Higgins and Parsons (2011) found that taking a teacher-centred approach that was responsive to the needs of teachers was an important underpinning of a successful system-wide mathematics professional development in New Zealand.

Another beneficial factor of the professional development was the timeframe involved. The principal acknowledged the role that longer-term professional development played in the successful implementation of changes to mathematics teaching: "...a one-off input can really open people's eyes to things but I think there really is need for follow-up support" (Principal interview). The professional development aspects of this study spanned seven months. Guskey (2002a) acknowledges this need for continued follow-up support and claims that sustaining change is the most neglected aspect of professional development. Furthermore, Timperley et al. (2008) found that to make significant changes to their practice, teachers need multiple opportunities to learn new information and understand its implications for practice. They claim that because learning is cyclical rather than linear "...teachers need to be able to revisit partially understood ideas as they try them out in everyday contexts" (p.15). Furthermore, they found that for substantive learning (including that involved in improving their students' mathematical problem solving) teachers need extended time in which to learn and change, typically one to two years. Specifically in relation to mathematics professional development, Askew, Browne, Rhodes, Wiliams & Johnson (1997) found that short professional development courses have little impact on teachers' beliefs and practices. This finding is supported by Joubert and Sutherland (2008) who, having synthesised the literature regarding effective mathematics professional development, concluded that sustained, longer term professional development that unfolds over time is more effective than one-time events.

Finally, teachers' reports on the professional development suggest teacher learning had taken place and was reflected in their changing practice. The data suggest that this learning was in relation to a) knowledge and b) linked to teacher reflection. In relation to increased knowledge, typical comments included:

> Like when we actually went through everything that was involved in length. Like you wouldn't think that there was so much – like normally you just go straight to measuring…(Int.1 B).
>
> I got better at developing questions to kind of challenge their thinking than when we were doing length. When we were doing weight I felt I got more out of it even kind of using the language…I learned from the first phase even ways of getting them to use the language that they needed rather than in the first phase I think I was giving it to them (FG.CT).

This suggestion that teachers' learning during the professional development had an impact on their practice is echoed in other findings in the literature. Timperley et al. (2008) found that in effective professional development, theories of curriculum, effective teaching, and assessment are developed alongside their applications to practice. In this study, teachers engaged in professional development over a seven-month period whilst simultaneously applying associated changes to their classroom practice. Furthermore, the principal highlighted the importance of teacher reflection during the professional development: "I think it is very important that teachers are asked to reflect. I think that was a very effective part of the whole project" (Principal interview). Mirroring these sentiments, another teacher welcomed the opportunity for teachers to take stock and reflect on their teaching:

> I suppose just to stop and focus on the way you teach something…not just opening the book…so ok, you get out your metre sticks and you get out your things but this (*professional development*) made us focus a little bit on what we are doing, and how to make it a bit more realistic to them, and how to let them take it on more and do it. So that focus was good (FG.CT).

Although initiated during the professional development sessions, the data suggest that some teachers then extended this reflection to their own classrooms where they reflected on their teaching. Typical comments included:

> And even just getting us to reflect was great – coz you never as a teacher kind of think oh I'll think about the lesson, you know you just say 'well that's done now,

> that was great' and you never kind of...you never really fully think about the lesson (Int.1 B).
>
> And I suppose reflection from lesson to lesson...saying oh well that went well or I wouldn't do that or you might do the same task again and maybe change things again and then reflect on that and see if you were happy with the outcome based on what you had planned on doing (Int.2 C).

Findings in the literature support the finding that in effective professional development teachers monitor and reflect on the effectiveness of changes they make to their practice. Timperley et al.'s (2008) findings suggest that in order to make a difference to students, teachers need to engage in professional inquiry that identifies the pedagogical content knowledge and skills that they need to assist their students to achieve the valued learning outcomes.

Using and Refining an Instructional Framework

The data suggest that another contributing factor to enabling teachers to make changes to their mathematics teaching was the use of the instructional framework. Additionally, teachers reported that using and refining the instructional framework to suit their own needs and context contributed to supporting them and enabling them to implement the reform approach to mathematics teaching. During Phase 1 Hiebert et al.'s (1997) instructional framework based on the book *Making Sense: Teaching and Learning Mathematics with Understanding* was used as a stimulus for changing mathematics teaching (see Table 4.2 which outlines the instructional framework used during Phase 1).[33]

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[33] The additional information column was extrapolated from various chapters of the book and was included as a more detailed reference guide for teachers.

Table 4.2 Dimensions, Core Features, and Additional Information of Classrooms that Promote Understanding (Hiebert et al., 1997)

Dimensions	Core Features	Additional Information
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave something behind of mathematical value	Tasks should encourage reflection and communication Tasks should allow students to use tools Tasks should leave behind important residue - there are 2 types of residue: a) insights into the structure of mathematics (mathematical relationships) and b) strategies or methods for solving problems
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture	Explanations and demonstrations by the students become more important than those by the teacher Teachers need to select sequences of tasks not just individual tasks Teachers should remove themselves from a position of authority (deciding whether answers are correct) in order to promote the autonomy of students' intellectual activity
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument	Students work together to solve problems and interact intensively about solution methods Collaboration depends on communication and social interaction (individual work can be followed by small group or whole class discussion of methods and ideas) Students must learn to live with a certain amount of uncertainty
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking	Tools include oral language, physical materials and symbols Students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems Using tools can free our thinking for more creative activities
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes	All ideas and methods are potential learning sites A variety of ideas are essential for fuelling rich discussions Each person learns to respect and value each other's thinking

Following several weeks of implementation, the tracker teachers were very complimentary about the instructional framework, in particular, its use as a teaching aid. The reported benefits of using the instructional framework included a) using it as a reflection tool for teaching mathematics; b) ensuring a focus on problem solving; and c) increasing pupil participation in mathematics classes. In relation to its use as a reflection tool for mathematics teaching, the following comment is typical:

> It kind of gave you an area like that you knew if you weren't doing it well then to focus on it the next time. You know this is what I need to focus on...this is where I need to put the attention in specifically (FG.CT).

A number of the tracker teachers reported that using the instructional framework contributed to ensuring a focus on problem solving in their mathematics lessons as indicated in the following comment:

> It (*the instructional framework*) was so focused on problem-solving...and the children just get used to problem-solving being the norm I suppose for maths... so I suppose the nature of the classroom tasks then you are always thinking problematic (Int.1 D).

Teachers also highlighted the benefits of increased pupil participation when using the instructional framework for mathematics lessons. This increased pupil participation ranged from teachers reporting that children had control over their own learning to children enjoying working collaboratively to children contributing more to mathematics lessons (Phase 1 teacher interviews). Furthermore, the following teacher reports highlight the active role that pupils played in mathematics lessons that were planned using the instructional framework:

> So definitely, I've been using it *(the instructional framework)* and I think the children enjoy it a lot more when they get a chance to give their views and explanations (Int.1 C).

> (*The instructional framework allows for*) more autonomy, the children will be able to choose instruments or to choose tools in general. Giving it (*the tools*) to them – not saying 'we will use this to do this' – let them figure out what they need to use to perform whatever task or to solve whatever problem that you have in front of them (Int.1 D).

Nonetheless, teachers also reported challenges in using the instructional framework during Phase 1. The reported challenges mainly related to the role of the teacher, in particular, changing from the traditional role of teller, demonstrator, and corrector in mathematics lessons to a role that emphasised more pupil participation and required more facilitation skills. The following comments exemplifies this challenge:

> Well the biggest challenge would be the teacher stepping back and trying to give that control…rather than the teacher voice it is the pupil voice that needs to be heard. That was the biggest challenge (Int.1 B).

Teachers referred to this change in many ways during the Phase 1 teacher interviews from 'stepping back' to 'handing over' to 'letting them off' to 'biting your tongue' to 'observation' to 'facilitation'. The data from teachers suggest that the skill of facilitation and the language associated with facilitation proved challenging for teachers during Phase 1. The following teacher reports highlight the challenging nature of knowing how much guidance to give pupils: "I found it hard to know, can I say anything at all, do I let them completely find it out on their own or can I still guide them anyway" (Int.1 A) and 'it is just to know have you said too much or too little' (Int.1 B). Teachers suggested the need for additional guidance by giving "…pointers on how much you can guide them or how much has to be left to them" (Int.1 A) and by including "…the language of questioning and affirming comments" (Int.1 D) in any revision of the instructional framework. The uncertainty regarding questioning and guidance was a very real one for teachers and is highlighted by one of the tracker teachers:

> Should you ask any question or by asking a certain question are you giving them (*the pupils*) too much…are you like helping them to discover? Do you want to ask the bare essentials and let them completely come up with everything or do you want to steer the question?...so you want questions that are going to get them completely thinking kind of openly or do you want a kind of steering question? It is important to know what type of question to ask because you weren't sure like, should I even have said that, or is that still me being in control…(Int.1 B).

In addition to these teacher reports, data from the lesson observations suggest that although ample collaborative opportunities were evident, there was very little cross-fertilisation between the groups, in that learning seemed to remain within each small group and was not shared with other groups or with the whole class (M. Treacy, field notes, 16/11/12). Arising from this, sharing and building on mathematical thinking was not obvious at a whole-class level. Moreover, the lesson observation data suggest that the types of problems were not particularly rich or challenging (M. Treacy, field notes, 16/11/12). In conclusion, based on teacher reports and suggestions and augmented by data from lesson observations, it was agreed to refine the instructional framework for Phase 2. The refining and contextualising of the instructional framework emerged out of a necessity to further progress mathematics lessons after Phase 1 of implementation. The chief additions deemed necessary to refine and contextualise the instructional framework included:

- an emphasis on the teacher's role as an active, skilled facilitator;
- inclusion of teacher talk that encourages reflection and communication, in particular, the language needed for facilitation and the questioning needed for discovery learning;
- an emphasis on whole-class discussion in developing mathematical thinking including reasoning, and revision of conjectures and solutions;
- an emphasis on students using language to refine, revise, clarify, build on and communicate mathematical thinking;
- inclusion of revoicing to a) deepen mathematical understanding and to enrich mathematical thinking and b) to encourage sharing of mathematical ideas; and
- an emphasis on students choosing to record verbally, concretely, pictorially/graphically, symbolically or in written form (see Appendix F for a further exploration of these additions).

Finally, the data from lesson observations and teacher interviews suggest that two dimensions of Hiebert et al.'s (1997) instructional framework appeared to be afforded less importance by teachers for mathematics teaching in this school context – Social Culture of the Classroom, and Equity and Accessibility. In particular, teachers reported using these as guiding principles for classroom culture generally instead of specific dimensions for mathematics lessons. For example, one teacher reported that: " the social culture as regards the ideas and methods are valued, you'd always try your best to do that anyway" (Int.1 D). In essence, the dimensions appeared to be pre-requisites for a certain type of classroom. Therefore, in the revised instructional framework, these dimensions were adapted and combined to form a classroom environment foreword that focused on pre-requisites (see Table 4.3).

Table 4.3 Foreword of the 4Ts Instructional Framework for Maths

Classroom Environment: A Pre-Requisite

A conducive, inclusive, and supportive classroom environment is a pre-requisite for using the 4Ts instructional framework for maths. In this type of classroom:

- ideas and methods are valued
- students choose and share their methods
- mistakes are learning sites for everyone
- correctness resides in mathematical argument, not the popularity of the speaker
- tasks are accessible to all students
- every student is heard
- every student contributes
- students often work in small groups but they can also choose to work independently
- norms for working effectively in small groups are established:
  1. You are responsible for your own behaviour.
  2. You must be willing to help anyone in your group who asks.
  3. You may not ask the teacher for help unless everyone in your group has the same question.[34]
- whole-class dialogue encourages reflection, communication and reasoning
- students share their mathematical thinking with the whole class, not just the teacher

[34] Lampert (2001, p.81) suggests these norms be established for small group work in mathematics.

Resulting from this merging, three dimensions remained, so combined with the new dimension regarding teacher talk, the revised instructional framework had four dimensions all beginning with T – Tasks, Teachers, Tools, Talk; hence, the revised instructional framework evolved into the 4Ts Instructional Framework for Maths (see Table 4.4).

Table 4.4 The 4Ts Instructional Framework for Maths

4Ts	Core Features	Additional Information
Tasks	Make mathematics problematic Connect with where students are Select tasks with goals in mind Leave something behind of mathematical value	Tasks should encourage reflection and communication Tasks should allow students to use tools Teachers need to select sequences of tasks not just individual tasks Tasks should leave behind important residue - there are 2 types of residue: a) insights into the structure of mathematics (mathematical relationships) and b) strategies or methods for solving problems
Teacher	Take on an active, skilled facilitator role Share essential information Establish classroom culture Encourage revision of conjectures	Teachers facilitate discussion and value silence (Pratt, 2002) in the course of mathematical discussions Explanations and demonstrations by the students become more important than those by the teacher Teachers remove themselves from a position of authority (deciding whether answers are correct) in order to promote the autonomy of students' mathematical thinking Revising conjectures (Lampert, 2001) and solutions is encouraged Reasoning is used to judge whether a conjecture/solution is mathematically sound
Tools	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking	Tools include oral language, physical materials, pictures/diagrams (Askew, 2012) and symbols Students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems Using tools can free our thinking for more creative activities Students choose to record verbally, concretely, pictorially/graphically, symbolically or in written form
Talk	Teacher talk encourages reflection and communication Students use language to refine, revise, clarify and communicate mathematical thinking Talk is used to encourage and communicate reasoning	Teacher questioning is open-ended and probing Teacher responses are neutral (Pratt, 2002) and encourage further discussion Students share, clarify and refine their mathematical ideas through facilitated discussion Revoicing is used to deepen mathematical understanding and to share mathematical thinking Students build on the mathematical ideas of others Talk is used to encourage and communicate reasoning – both in verbal and in written form (either in a journal or a notebook)

This revised instructional framework (which was referred to as the 4Ts Instructional Framework for Maths during the study) was used to teach the strand unit weight during Phase 2. All of the tracker teachers reported being more confident in using the instructional framework during Phase 2 (Phase 2 teacher interviews). The teacher feedback regarding the usability of the instructional framework was even more positive than during Phase 1. The four teachers referred to it as being "brilliant (Int.2 D)", "fantastic" (Int.2 B), "very good (Int.2 C)", "very clear…very good (Int.2 A) and "very much a positive experience" (Int.2 D). They also reflected on the refinements to the framework since Phase 1, in particular, several teachers highlighted the talk dimension as being a noteworthy improvement. Typical comments included:

> I think the talk part might have brought the framework together (Int.2 D).

> There wasn't the talk element in it (*during Phase 1*) so I wasn't sure how much I could intervene or what questions to ask…but I thought this time it worked really well. I wouldn't change anything (Int.2 A).

As detailed in the section outlining the experiences of the tracker teachers, many of the teachers also reported being more confident regarding their role as a facilitator during Phase 2. All of this feedback from teachers was mirrored in the observed lessons during Phase 2 where teachers appeared to take on a more active, facilitative role regarding mathematical discussion than was evident during Phase 1 where many teachers adopted the role of classroom organiser and did not appear to engage in mathematical discussions with pupils. Teachers' experiences of using the 4Ts Instructional Framework for Maths were extremely positive in this case study school. Similar to Phase 1, teachers highlighted the benefits of using the instructional framework as a reflection tool. Typical comments included:

> The instructional framework would have helped…just even to kind of glance over it now and again to make sure that I am bringing in every element of it into the maths lesson (Int.2 C).

> I suppose the instructional framework made me focus my maths lessons a bit more so I was conscious all the time of the 4Ts – the tasks, the tools, the talk, the teacher. I was conscious of those all the time and conscious of tools you know…just in the front of my mind all the time was the 4Ts (Int.2 D).

Furthermore, teachers highlighted their use of the instructional framework during planning for mathematics lessons. The following comment underlines this and highlights how one teacher reported using the instructional framework as an aid for planning her mathematics lessons:

> Well I like it when it's broken down into the 4Ts. I get out a page for every lesson and then divide the page up into four parts and then I plan the tasks like the materials, the tools that we will need and like I would be envisaging how it would work but like you still have to give them the freedom so you have to plan the talk that you are going to say…and the questions that you are going to ask so that you can lead them into the tasks without telling them what to do (Int.2 B).

Despite teachers' reporting that the revised instructional framework contributed to enabling them to change their mathematics teaching, the importance of professional discretion cannot be over emphasised. Successful mathematics classrooms do not mean conforming to a highly prescribed method of teaching (Stigler & Hiebert, 1999; Hiebert et al., 1997). Instead, it means, "…taking ownership of a system of instruction, and then fleshing out its core features in a way that makes sense for a particular teacher in a particular setting" (Hiebert et al., p. 14). In this way, the 4Ts Instructional Framework for Maths can act as a temporary bridge or scaffold between recommended pedagogical features and pupil learning where planning, implementation, and reflection feature in an iterative feedback loop. Equally, it has the potential (in the words of one of the teachers) to guide teachers in progressing from "spoon-feeding" to "biting your tongue" (Int.2 B) and beyond.

In summation, the data suggest that peer support, tailored professional development, and using and contextualising an instructional framework were contributing factors in enabling teachers to implement the reform approach to mathematics teaching and in some cases to embed and enhance these changes.

RESTRICTIVE FACTORS

This section reports on factors that may have restricted the potential for embedding and enhancing changing mathematics practice during the study. These restrictive factors include a textbook dependent culture, uncertainty regarding teacher facilitation, and possible limitations of the professional development.

Textbook Dependent Culture

In relation to the reliance on textbooks, the data suggest widespread use of textbooks in the school before the study began resulting in mathematical tasks that were often simple and procedural-based. The data from multiple sources such as the initial pupil focus groups, teacher reports during the initial stages of professional development, teacher reflections, tracker teacher interviews, and interview with the principal suggest that before the implementation of the reform approach, the textbook appeared to be the chief, if not the only, source of mathematical tasks. Unsurprisingly, the principal's main concern before the study began related to the over-reliance on textbooks. This data from the principal corroborate the pupils' and teachers' reports regarding the prolific use of textbooks:

> ...the reliance on the textbook...the over-reliance on the textbook...and the textbook dictating the programme where some people (*teachers*) would start at the beginning of the textbook and work through it.

These findings regarding the over-reliance on textbooks and worksheets are echoed in many similar findings regarding mathematics lessons in Irish primary schools (e.g. DES, 2005a, 2005b, 2010, 2013; Murphy, 2004; NAMA, 1999, 2004, 2009; NAMIS, 2010; TIMSS, 1995). This finding is also supported by Dunphy's (2009) finding that 96% (n=266) of Junior Infant teachers (from a national sample) indicated they were using textbooks. Furthermore, she found a textbook-centred rather than a child-centred pedagogy; and consequently, she recommends that teachers need to be strongly encouraged to move away from a textbook dependent culture whilst also acknowledging the potential difficulties associated with such a change. Specifically, she raises concerns about teachers' confidence when teaching mathematics considering the extent to which she found that textbooks appeared to determine the enacted curriculum in Junior Infants. Worryingly, she found that in most instances teachers appeared to consider the textbook and the curriculum as being the same thing. This finding that textbooks appeared to determine the enacted mathematics curriculum in some Irish primary classrooms is concerning because a recent Irish study undertaken by Eivers, Delaney and Close (2014) found that the content of a Canadian mathematics programme[35] aligned more closely with the strand emphases of the Irish primary school mathematics curriculum than the three Irish textbooks analysed. Furthermore, they found that the textbook most widely used in the schools in the study was the least similar to the strand emphases in the curriculum. For example, 24% of mathematics curriculum objectives address Shape and Space compared to 8% of pages in the textbook. They also found that this textbook was the one most heavily targeted at Number and Algebra (65% of pages), and at isolated computation in particular (29% of pages).

Not surprisingly then the systemic, ingrained nature of textbook use for mathematics in this case study school (and indeed in Irish primary schools generally) meant that some teachers did not feel the need to question the prolific use of textbooks because it had "become" the mathematics lesson. Equally, some teachers questioned its blanket use but appeared to believe that they had little or no say in this matter because they "had to use them" (Phase 2 teacher interview). This finding mirrors that of Dunphy (2009) who found that many Irish teachers of Junior Infants felt constrained by the textbook but felt compelled to use it

[35] The Junior Undiscovered Math Prodigies (JUMP) programme, which originated in Canada, was piloted in Third Class in a sample of Irish primary schools.

anyway. This is an important finding and may be explained by Sugrue and Gleeson's (2004, p.293) contestation that textbook culture is one of several "systemic silences" in the Irish education system. Specifically, they highlight the silences or virtual silences with regard to the

> ...degree of influence exerted by textbook publishers on a relatively small market and the manner in which the virtual monopoly of a few reduces risk-taking in the production of texts and materials; thus paradoxically, experimentation is further limited by market forces!

In a retrospective reflection, one of the teachers captures the domineering force that the textbook exerted in her class before the study began:

> The book – you know we didn't go near the book once. We were doing Length for about three and a half weeks – the pre-measuring vocabulary, then the non-standard units and onto the standard – and they said 'oh miss we haven't used our book with ages'. Like, they are in the same frame of mind that we are – that maths is just the book. Like they have got out of that habit now and we are only using the bare essentials in the book ... But like for Length I would never, ever dream of touching the book. Like every other year, you were doing it for the sake of just covering the pages and making sure that the parents saw that – 'oh ya we bought this book now she has used it you know'. It doesn't come to the end of the year and they are like why did we buy that – she didn't use it. So, we haven't touched any of the Length in the book (Int.1 B).

The elevated and often uncontested central role of textbooks in these classes suggests that the textbook rather than the teacher often held both the pedagogical and epistemic authority. Consequently, teachers may have had little or no control over the mathematical content or methods used in classes. However, the data from Phases 1 and 2 suggest that textbooks were no longer used as the main source for mathematical tasks as the study progressed. Furthermore, it appears that this trend continued after the study because the Whole School

Evaluation[36] report also commends teachers on a) moving away from textbook use and b) generating lessons themselves rather than using textbooks. Despite this finding, some teachers even after implementation of the reform approach to mathematics teaching, expressed a concern that they still "had to do" the textbook. Such a perception, real or otherwise, is likely to have restricted the potential for embedding and enhancing changing mathematics practice. More worryingly, this is a concern considering the findings by Eivers et al. (2014) that textbooks in Third Class are not closely aligned with the strand emphases of the Irish Primary School Mathematics Curriculum.

Teacher Discomfort with Facilitation

The data suggest that adopting a facilitative role in mathematics lessons proved challenging for teachers. In particular, the uncertainty of teachers in their newly adopted role emerged very strongly as a theme in the data from Phase 1. For example during lesson observations, some teachers appeared to be uncomfortable with their role and many appeared to be uncertain of what to do or say when pupils (following established classroom norms) asked teachers for guidance. Teachers appeared to be hesitant to 'tell' and so many teachers opted to keep quiet completely, thus shutting down the possibility of providing necessary guidance and advancing mathematical understanding. For the majority of the observed lessons, teachers had completely 'stepped back' and were providing little or no guidance to pupils. Teachers appeared to have taken on an observational role rather than a proactive, facilitative role. This is despite the fact that some teachers had reported in interviews taking on a facilitative role. This ambiguity may also have been confusing for pupils considering the widespread practice of 'telling' that prevailed in these classrooms before the implementation

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[36] The Inspectorate from the Department of Education and Skills undertook a Whole School Evaluation in the year directly after the completion of the study.

of the reform approach to mathematics teaching. Supporting the data from lesson observations, teachers reported confusion regarding their role, in particular, whether to intervene and if so, when to intervene. Examples of teacher reports in this regard are outlined in the previous section on the refinement of the instructional framework. This finding supports that of Frykholm (2004) who found that pedagogical practices that challenge the traditional role of the teacher might cause teacher discomfort. The data suggest that not just were some teachers uncertain about how much direction to provide, but also when to step in and out of certain mathematical conversations. Walshaw and Anthony (2008) highlight this dilemma and claim that teachers have to nudge conversations in mathematically enriching ways whilst sensitively stepping in and out of classroom discourse. This confusion regarding 'telling' or 'not telling' mirrors teachers' uncertainty regarding the difference between relinquishing mathematical authority and sharing mathematical authority. These uncertainties fall under Frykholm's (2004) Pedagogical Discomfort domain. Unsurprisingly, during Phase 1 the teachers in this study suggested that the role of the teacher needed further clarification in any revised instructional framework (as discussed earlier). This finding regarding the difficulty and uncertainty that many teachers experienced when attempting to share mathematical authority supports similar findings by Hamm and Perry (2002). Furthermore, the different findings from Phases 1 and 2 are supported by comparable findings from the literature. The Phase 1 findings support those of Nathan and Knuth (2003) who found that when teachers remove themselves completely from the analytic aspects of classroom discourse, no clear authority is present for students to turn to in times of uncertainty. Similarly, Walshaw and Anthony (2008) conclude that despite attempting to share mathematical authority, teachers must play a different role than pupils in mathematical dialogue. The important facilitation role that is required by teachers for beginning the development of effective mathematical discourse was evident in the findings from Phase 2.

Nonetheless, the challenges that teachers reported in relation to adopting a more facilitative role in mathematics lessons are likely to have restricted changes to mathematics practice.

Limitations of Professional Development

Finally, possible limitations of the professional development may also have been a restrictive factor in embedding and enhancing changing mathematics practice. A number of possible limitations with regard to the professional development are detailed in this section. Although some subject specific content focused on pupils' thinking, in particular, pupils' conceptual misunderstandings in relation to length and weight, more emphasis on pupils' thinking and how students learn may have enhanced the professional development. For example, Timperley et al. (2008) found that effective professional development in mathematics focused on developing teachers' understanding of the complex relationships between the key elements of teacher subject knowledge, pedagogy, assessment, and how students learn. Another limitation relates to the limited focus on assessment in the professional development, which may also have restricted its impact on classroom practice. Although assessment was used as the rationale for engaging in the professional development[37] and some attitudinal outcomes such as pupils' enjoyment of mathematics were evident during the focus group interviews, assessment of student outcomes did not directly feature during the rest of the study. Considering Timperley et al. (2008) found that effective professional development in mathematics included assessment as a core component; a minimal focus on assessment in this study may have restricted potential changes to classroom practice. A further limitation relates to the timing of the professional development, which may have restricted its impact on classroom practice. Each professional development session

[37] On analysis of standardised test results, the school identified the strand unit of Measures as an area for improvement in mathematics.

took place after school hours (generally beginning at 3.15pm) and some evidence suggests that teachers were tired having taught since 9.30am. One example of this was where teachers seemed distracted, and appeared reluctant to engage in group work or whole group discussion. Another being where a teacher asked what time the session would be finished before she had even taken a seat in the room. To similar effect, the timing of the professional development did not provide much scope for prolonged sessions because often teachers had childcare and personal commitments. Consequently, the duration of the professional development sessions ranged from between 1 and 1½ hours, which did not lend itself to deep engagement with the content. Finally, varying levels of teacher engagement during the professional development sessions may also have restricted the potential for changing practice. As suggested in the last point, some teachers appeared disinterested at times and reluctant to engage in activities. This was more evident at the initial stages of the professional development and as discussed earlier a turning point was evident when the teachers analysed the pupil reports of mathematics lessons. The week after the analysis of pupil reports there appeared to be a greater willingness to engage from the teachers, a willingness that continued and increased throughout the professional development. This finding mirrors the findings of a New Zealand study highlighted by Timperley et al. (2008) where the students' stories about their experiences in the classroom provided the catalyst for teacher engagement in the professional development. Nonetheless, almost four of the nine hours of professional development had passed at this stage, much of which focused on subject content knowledge and pedagogical knowledge, in particular the exploration of Hiebert et al.'s (1997) instructional framework. Considering the instructional framework was the basis of the reform approach to mathematics teaching as outlined earlier in the chapter, it seems fair to assume that poor engagement during these professional development sessions may have restricted the embedding and enhancing of changes to mathematics practice.

CONCLUSION

A number of distinct findings emerge from this study. The data suggest that shifts in practice were evident in the mathematics lessons of all four tracker classes over the course of the study. In particular, shifts in mathematics practice appear to have been evident in the tracker classes for the strand units of length and weight. Moreover, these shifts in practice appeared to continue throughout the study, in that, shifts in practice appeared to be evident between Phases 1 and 2. Although the extent to which shifts in practice varied across the four tracker teachers, these shifts in practice were aligned with the reform approach to mathematics teaching as outlined in Hiebert et al.'s (1997) instructional framework. The mathematics lessons in tracker classes appeared to change from traditional, textbook-dependent ones towards lessons comprising elements of reform-type strategy reporting lessons. The data also suggest that teachers' experiences of implementing the reform approach to mathematics teaching varied. Some teachers appeared to welcome the opportunity to implement the reform approach to mathematics teaching whilst other teachers, although implementing the approach appeared circumspect about the approach. Furthermore, the data suggest that a number of factors contributed to these shifts in practice in the tracker classes. These enabling factors included peer support, tailored professional development, and the use and refinement of an instructional framework for mathematics. Contrastingly, the data suggest that a number of factors may have restricted the potential for embedding and enhancing this changing practice. The findings suggest that a textbook dependency culture, uncertainty regarding teacher facilitation, and possible limitations of the professional development may have been restrictive factors in successfully implementing a reform approach to mathematics teaching in this school. In relation to the conceptual framework of this study, the findings suggest that the professional development influenced mathematics teaching and learning but equally mathematics teaching and learning in this school influenced the professional development. For example, initially the prevalence of traditional mathematics practice in the school coupled with a desire to improve student outcomes for Measures influenced the professional development. Subsequently, pupil reports of their experiences of mathematics lessons influenced the professional development. Similarly, after Phase 1, teachers' experiences of implementing a reform approach to mathematics teaching, in particular, the need for more guidance on facilitation and more emphasis on talk in mathematics lessons influenced the professional development. Likewise, professional development during Phase 2 appears to have further influenced mathematics lessons. Accordingly, professional development and mathematics teaching and learning in this study influenced each other in an iterative loop. A similar influential relationship was evident for the instructional framework, which initially influenced both the professional development and mathematics lessons but then was refined based on experiences of implementing a reform approach to mathematics teaching. In the next chapter, the findings are outlined in relation to the four key questions of the study. Furthermore, conclusions are drawn based on these findings and implications for policy makers, the Inspectorate, teacher educators, and schools are highlighted. Finally, the limitations of the study are outlined and the need for further research in a number of key areas is highlighted.

CHAPTER 5: CONCLUSION

In this chapter, the main findings of the study are outlined and conclusions from these findings are drawn. The findings relate to four questions, the first of which is: in what ways does teachers' mathematics teaching change (if at all)? The second question explores ways in which teachers' experiences of implementing a reform approach to mathematics teaching might differ. The third question relates to factors that contribute to enabling teachers to change their mathematics teaching; whilst the fourth question relates to factors that contribute to restricting such change. Additionally in this chapter, the implications of this study for policy makers, the Inspectorate, teacher educators, and schools are discussed in light of the conclusions. Finally, the limitations of this study are outlined and the need for further research in relation to a number of key areas is highlighted.

SUMMARY OF FINDINGS

One of the main findings of the study indicates that the extent to which mathematics teaching displayed reform features varied considerably between the four tracker teachers. Specifically, in all classes during the Phase 1 observed lessons, the findings suggest that teachers moved away from using textbooks for the topic of length. Also, there appeared to be more use of concrete materials in mathematics lessons and tasks appeared to be focused on developing the concept of length through practical tasks as opposed to focusing solely on calculations. Furthermore, teachers appeared to remove themselves from a position of mathematical authority during the Phase 1 observed lessons. Some classes also demonstrated group work and involved some challenging problem solving tasks. Teachers' mathematics teaching continued to demonstrate these features during the Phase 2 observed lessons; however, mathematics teaching appeared to evolve further. For all classes, collaborative group work was evident in the Phase 2 lessons. Furthermore, problem-based tasks were evident during these lessons in all classes, although the extent to which these tasks challenged pupils appeared to vary considerably across the four classes. For three of the classes, more whole-class discussion was evident in these lessons where teachers played a more active role in encouraging pupils to share and clarify their solution methods and mathematical thinking. Similarly, in three of the four classes, although removing themselves from a position of mathematical authority, teachers played a key role in facilitating mathematical discussion. This had a direct impact on pupil participation in these three classes. This latter finding supports that of Wood, Williams and McNeal (2006) who found differences in pupils' expressed mathematical thinking across the four classroom cultures—conventional textbook, conventional problem solving, strategy reporting, and inquiry/argument. The findings in this study suggest that features of the reform approach were evident, albeit evident to different degrees, in these lessons, in addition to some traditional features—hybrid mathematics lessons. This finding is supported by similar findings by O'Shea and Leavy (2013) who found that the blending of traditional practices and reform practices were evident across all five participants in their study. Moreover, this finding supports Ma's (1999) claim that traditional and reform mathematics traditions may not be antagonistic to each other when changing mathematics practice. However, the findings in this study raise queries regarding Ma's (1999) claim that changing mathematics practice may be an evolutionary process in addition to Warfield, Wood and Lehman (2005)'s suggestion that hybrid lessons can be indicative of a transition from traditional mathematics lessons to more reform-based ones. In particular, evolution implies progression or advancement (or a transition to reform-based lessons) whereas in this study regression to a mathematics lesson that was more traditional in nature and depended considerably on the textbook was an outcome of changing mathematics practice for one teacher. Consequently, it may be more accurate to suggest that changing mathematics practice can act as a springboard for future pedagogical decisions—ones that can be either progressive or regressive. The findings in this study also suggest that despite teachers participating in the same professional development programme, the extent to which mathematics teaching displayed reform features varied considerably between the four tracker teachers. This finding is consistent with that of Warfield, Wood, and Lehman (2005) who found that mathematics teaching varied considerably over a two-year period for teachers who participated in a professional development project designed to help them to learn to teach mathematics according to reform recommendations. To similar effect, this finding supports that of Porter et al. (2000) that after participating in a professional development programme individual teachers varied in their classroom practices.

Another finding relates to teachers' experiences of implementing the reform approach to mathematics teaching. Specifically, teachers' experiences appeared to vary between the four tracker teachers. Some teachers appeared to welcome the opportunity to implement the reform approach to mathematics teaching whilst other teachers, although implementing the approach, appeared circumspect. For example, during the study one tracker teacher gave examples of how she had already begun using the reform approach for other topics in the mathematics curriculum whilst contrastingly another tracker teacher reported her intention of returning to her traditional teaching methods after the study (albeit supplemented with some aspects of the reform approach). Similarly, the tracker teachers' tolerance for discomfort appeared to vary, for example, all teachers reported struggling with facilitation during Phase 1, which supports the finding by O'Shea and Leavy (2013) that Irish teachers experience difficulty in moving from a didactic to a more facilitative role; however, teachers' reactions to this discomfort appeared to vary in this study. Some teachers acknowledged the struggle and reported devising their own strategies to overcome the obstacles whilst other teachers appeared to be fazed by this struggle and sought answers from the researcher instead. This finding regarding the variance in teachers' experiences of implementing the reform-based mathematics lessons is supported by similar findings by Frykholm (2004).

A number of factors were identified in this study as contributing to enabling teachers to change their mathematics teaching. These include peer support, tailored professional development, and the contextualised use of an instructional framework. In relation to peer support, teacher reports indicate that the opportunity to collaborate on lesson planning with their peers contributed to enabling them to change their mathematics teaching. This finding reflects similar findings in the literature where Loucks-Horsley and Matsumoto (1999) found that collaborative practices contributed to professional development that resulted in changes in mathematics and science lessons, and where Timperley et al. (2008) found that effective professional development involves discussing practice with colleagues. Furthermore, many studies highlight the central role that engaging in some form of community practice plays in effective professional development (Back et al., 2009; Koellner, Jacobs & Borko, 2011; and Porter et al., 2000).

Additionally, teachers indicated that professional development that was contextualised for their school, and tailored to their specific needs, contributed to changes in their mathematics teaching. This finding supports that of Koellner, Jacobs and Borko (2011) who found that adapting professional development to support local goals and interests is one of three features identified in the literature as being critical for effective professional development in mathematics. In this study, the professional development was tailored to the needs of the school in that the school identified the mathematics focus, and the context of mathematics lessons in the school was identified and used as a starting point in the professional development. Furthermore, pupil reports of their views and experiences of mathematics lessons in this school were included as a fundamental component of the professional development. The inclusion of pupil voice in the professional development is important because this study highlights that pupil voice can act as a catalyst for teacher change in mathematics. This finding contributes to the literature in identifying the importance of pupil voice as a catalyst for teacher change (pupil voice is discussed in more detail in the section on conclusions). Finally, another factor that contributed to enabling teachers to change their mathematics teaching relates to the instructional framework. In particular, teachers reported that using and then refining an instructional framework to address their specific needs contributed to enabling them to make changes to their mathematics teaching. This finding aligns with Fraivillig, Murphy and Fuson's (1999) conclusion that the successful implementation of a reform approach in mathematics lessons requires specific pedagogical skills, skills in which teachers require guidance. One of the supports they propose involves providing robust images of effective mathematics practice for teacher reflection through descriptions of the strategies outlined in their instructional framework that emerged during their study. Similarly, Hiebert et al. (1997) suggest that their framework can be used for facilitating changes to mathematics teaching by providing a lens for examining the development of understanding and for guiding teachers in providing classrooms that aim to develop understanding—teachers can use the framework to reflect on their mathematics teaching and to consider how this might change.

Whilst factors that supported teachers in changing their mathematics teaching were identified in the study, factors that may have restricted such change were also identified. The findings suggest that a number of factors may have restricted potential changes to mathematics practice. These factors include the prevalence of a textbook dependent culture, teacher discomfort with facilitation, and possible limitations of the professional development. The dominant role of the mathematics textbook in this school continually emerged as a theme throughout the study. Some teachers reported feeling pressurised to use the textbook for length and weight in addition to implementing the reform approach, thus restricting the time that they could explore the topics using the reform approach to mathematics teaching. This finding regarding a textbook dependent culture supports that of Dunphy (2009) who found a textbook-centred rather than a child-centred pedagogy in Irish mathematics lessons for pupils of Junior Infants. The findings in this study go one step further and suggest that not only is pedagogy textbook-centred rather than child-centred, textbook-centred pedagogy can undermine the teacher's role in mathematics lessons, in particular, teacher autonomy. Considering the restrictive role that a textbook dependent culture appeared to play, this study highlights teacher autonomy as a critical factor in changing mathematics teaching (this finding is discussed further in the section on conclusions). Another finding relates to teacher discomfort with facilitation in mathematics lessons. This discomfort may have restricted changes to mathematics teaching (links to Frykholm's (2004) findings are detailed earlier in this section). Finally, limitations of the professional development may have restricted potential changes to mathematics practice. Specifically, the timing of the professional development after school hours could mean that teachers were less engaged having taught all day and also meant that extended sessions were not possible. Furthermore, the focus of the professional development was on pedagogy and the mathematical concepts of length and weight, and did not focus considerably on pupils' thinking in mathematics. This lack of attention on pupils' mathematical thinking, and how pupils learn may have also restricted changes to mathematics practice considering the importance of focusing on pupils' thinking that is evident in the literature on professional development in mathematics (e.g. ACME, 2002; Borko, 2004; Loucks-Horsley & Matsumoto, 1999). Similarly, assessment was not a core component of the professional development and considering findings suggest that assessment is a core component of effective professional development in mathematics (e.g. Timperley et al., 2008), this may have restricted potential changes to mathematics practice.

All of these restrictive factors reinforce the need for extended, tailored professional development when attempting to implement reform approaches in mathematics lessons.

Finally, similar to identifying teacher autonomy as a factor in changing mathematics teaching, this study also identifies power displacement as a factor in changing mathematics teaching. This power displacement was an unintended consequence of the study whereby in the initial stages of implementation some teachers appeared to replace the power of the textbook in their mathematics lessons with the power or authority (perceived or otherwise) of the instructional framework.

CONCLUSIONS

In this section, pertinent findings from this study are utilised in determining assumptions and opinions with regard to the main themes that emerged during the study. In particular, inferences are drawn with regard to the findings of the study.

Impact of Professional Development

Despite professional development that spanned seven months and was designed specifically to meet the needs of the case study school, the extent to which mathematics teaching displayed reform features varied considerably between the four tracker teachers. Although shifts in practice were evident in all of the four tracker classes, changes in some of these classes were minor and may have been transient in nature. It seems fair to conclude that despite professional development that spanned seven months, the impact on some mathematics lessons was small. A further conclusion can be drawn that professional development over a shorter period of time, which is not tailored to the schools' needs, would have even less impact. This conclusion has consequences for most stakeholders including policy makers such as the Department of Education and Skills and the Teaching Council, in addition to teacher educators and schools. The value of providing episodic, generic professional development to schools over a short timeframe needs to be robustly interrogated. As suggested in the literature, effective professional development in mathematics takes place over an extended timeframe (e.g. Back et al., 2009; Timperley et al., 2008) and is learner-centred—based on what teachers know and are able to do and building on this for new understandings (Loucks-Horsley & Matsumoto, 1999).

Pupil Voice as a Catalyst

In this study, teachers' engagement levels with the professional development programme increased following the session in which teachers actively analysed and drew conclusions based on pupils' reported experiences of their mathematics lessons. Consequently, a conclusion can be drawn that pupil voice can act as a catalyst for teacher engagement during professional development. It is unclear why pupil voice is so influential in this regard; however, it may be possible that teachers do not believe that practice needs to change in their mathematics lessons until they hear pupils' views and experiences of such lessons. Such a realisation may encourage teachers in attempting to enhance their mathematics lessons and so teachers engage more fully in the professional development in order to achieve the aim of changing practice. To similar effect, it is possible that teachers believe that their mathematics lessons display certain reform features already, and the views of pupils dispel such beliefs, again prompting teachers to engage more deeply with the professional development. In an Irish context where the Teaching Council is currently devising a national professional development policy, this conclusion is important, in that, it can highlight the unique revolutionary and stimulating role that pupil voice can play in professional development. The importance of including pupil voice in professional development should be included both in professional development policy and practice, as a means of increasing buy-in from teachers and deepening their engagement with professional development; the ultimate aim being to then improve teaching practice with a corresponding improvement in learner outcomes. Consequently, this conclusion has implications for the Teaching Council and professional development providers in Ireland.

Varying Outcomes from the Same Professional Development

This study indicates that the extent to which mathematics teaching displayed reform features varied considerably between the four tracker teachers, even though these teachers taught in the same school and participated in the same professional development programme. A conclusion can be drawn that even when professional development is implemented at whole school level, the degree to which teachers make associated changes to their classroom practice can vary. A number of reasons may account for this variance including the possibility of varying starting points for individual teachers. For example, teachers' mathematics practice may have already varied considerably before engagement with the professional development; consequently, the degree to which mathematics lessons displayed reform features may not be solely attributable to the professional development but could be impacted by teachers' past practice. Similarly, teachers' mathematical knowledge for teaching may have varied between the tracker teachers resulting in varying degrees of implementation of the reform approach to mathematics teaching. This argument is supported by Delaney's (2010) and Corocran's (2005) Irish studies that found considerable variance in Irish teachers' (practising and pre-service) mathematical knowledge for teaching. In relation to practising teachers, Delaney (2010) found that mathematical knowledge for teaching varies widely amongst Irish primary teachers with the highest scoring teachers responding correctly to over 60% more of the mathematical knowledge for teaching measures than the lowest scoring teachers. This is an important finding considering Delaney also found that in general, teachers who scored higher on this measure exhibited instruction of a higher mathematical quality than teachers with lower scores on the measure. Although assessing the mathematical subject knowledge of Irish student teachers (those in initial teacher education), Corcoran's (2005) findings support those of Delaney. Corcoran found considerable variation between the performances of Irish student teachers on the SKIMA[38] self-audit. Furthermore, she found marked variations between scores by the same students on different elements of the audit. In relation to performance, she found that Irish student teachers generally performed best on number and operation on number—although she makes distinctions between operative mathematical understanding and descriptive understanding. Contrastingly, her study suggests that Irish student teachers in the areas of Measures, and Shape and Space whilst mathematical reasoning also emerged as an area of concern. Considering Delaney's (2010) and Corcoran's (2005) findings, the possibility that teachers' mathematical knowledge may have varied amongst the teachers in this study is an assumption that has merits. Consequently, implications for policy makers and teacher educators are apposite. As advocated by Delaney (2010) and Corocran (2005), pre-service teachers should be required to study mathematical knowledge for teaching as part of their initial teacher education programme. Similarly, this conclusion highlights the need for similar interventions for practising teachers. In particular, mathematics professional development and support materials should include an emphasis on mathematical knowledge for teaching.

It is also conceivable that teacher discomfort (as described by Frykholm, 2004) and teacher beliefs (including beliefs about mathematics, beliefs about mathematics teaching, and pedagogical beliefs) may have varied between the tracker teachers resulting in varying degrees of implementation of the reform approach in mathematics lessons. Equally, teachers'

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[38] The SKIMA (Subject Knowledge in Mathematics) is a tool developed in the UK for the purposes of auditing trainee teachers' mathematical subject knowledge.

engagement levels with the professional development may also have varied with a corresponding impact on implementation of reform practices in their mathematics lessons. Professional development providers should consider and explore all of these possibilities in order to identify the reasons for such variance in practice. Identifying such reasons would pave the way for differentiated professional development, which has the potential to address the specific needs of individual teachers or groups of teachers within a school. This conclusion has immediate implications for the Irish context in which curricular changes at primary level are imminent—curricular changes that may require whole-staff professional development. The Primary Language Curriculum (Junior Infants to Second Class) is scheduled for completion in 2015 and the Primary Language Curriculum (Third to Sixth Class) is scheduled for completion in 2017. Moreover, the Primary Mathematics Curriculum (Junior Infants to Second Class) is scheduled for completion in 2017 whilst the Primary Mathematics Curriculum (Third to Sixth Class) is scheduled for completion in 2019. These curricular changes will require substantial professional development, which may include whole-school staff professional development.[39]

Instructional Framework as a Teacher Aid

Teachers in this study reported that using and then refining an instructional framework to address their specific needs contributed to enabling them to make changes to their mathematics teaching. It seems fair to conclude that an instructional framework that illuminates espoused mathematics practice can aid teachers in changing their mathematics teaching. The reasons why an instructional framework can aid teachers in such a way may include the fact that in this instance the instructional framework explicitly highlighted a) the

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[39] Personal e-mail communication with the Deputy CEO of the National Council for Curriculum and Assessment (May 2015).

advocated culture of mathematics lessons; b) specific pedagogy suitable for mathematics; c) dimensions that are important in mathematics lessons; and d) teachers' and pupils' roles within mathematics lessons. In so doing, the instructional framework may have provided teachers with an alternative visualisation of mathematics lessons—one that differs greatly from textbook-dominated lessons. This visualisation may have been a stimulus for teachers to realise that an alternative approach to mathematics is possible, and may have provided them with a structure and a context in which to experiment with such lessons. In such a way, an instructional framework can become a vehicle through which teachers implement aspects of a reform approach to mathematics teaching. The instructional framework can provide a vision and a structure to mathematics lessons that are not dependent upon textbooks—this supporting scaffold is vitally important for teachers who may never have experienced such mathematics lessons, either as teachers or as pupils. In this study, the instructional framework was 'flat', in that it was outlined in a document (albeit supplemented by professional development that included selected readings, podcasts, video footage, and teacher resources). It seems fair to conclude that such an instructional framework could be enhanced by illuminating such classroom culture and pedagogy using videos of mathematics lessons. Professional development providers, in addition to curriculum developers such as the National Council for Curriculum and Assessment, should consider devising such 'three-dimensional' models of instructional frameworks in an attempt to further support teachers in changing their mathematics teaching. This would contribute to making the implicit explicit and provide teachers with a robust visualisation of espoused mathematics lessons. Like the teachers in this study, teachers could use the instructional framework for planning and reflecting on mathematics lessons. Furthermore, using such an instructional framework as a lens through which Inspectors evaluate mathematics lessons in Irish primary schools would contribute to a focus on classroom culture, conceptual understanding, and higher order mathematics skills in school evaluations. Evaluating mathematics lessons in such a way would emphasise the importance of these dimensions and therefore contribute to a system that values these types of mathematics lessons more. Finally, regardless of professional development, curriculum development, or external evaluation, individual teachers and schools should consider using instructional frameworks as an aid to changing mathematics teaching. An instructional framework such as the one used in this study could be used by schools and teachers in the school self-evaluation process for gathering evidence regarding current mathematics teaching, setting targets for improvement, changing mathematics teaching, and then re-evaluating mathematics teaching after a period of time. In this way, the instructional framework can form the basis for evaluating, planning, implementing, and reflecting on mathematics lessons, in addition to providing a basis for professional discourse around mathematics lessons in schools.

Textbook Dependency and Teacher Autonomy

This study identifies teacher autonomy as a critical factor in changing mathematics teaching. The dominant role of the mathematics textbook in this case study school continually emerged as a theme throughout the study. Some teachers reported feeling pressurised to use the textbook for length and weight in addition to using the reform approach to mathematics teaching, thus restricting the time that they could explore the topics using the latter. A conclusion can be drawn that a textbook dependent culture can undermine teacher autonomy in mathematics lessons. This may mean the undermining of teachers' autonomy not just regarding the content of mathematics lessons, but also in relation to when this content is introduced in mathematics lessons and how much of this content is emphasised during mathematics lessons. Similarly, in a culture in which textbooks dominate, mathematics lessons may focus less on conceptual understanding and more on procedural fluency and the mechanics of 'doing' mathematics. Furthermore, a textbook dependent culture may undermine teachers' pedagogical decisions including choices around differentiation, classroom organisation, classroom culture, and assessment practices. In such a way, textbooks hold the mathematical and pedagogical power in mathematics lessons.

The role of textbooks in undermining teacher autonomy has implications for most stakeholders in the Irish educational system including members of the Inspectorate, schools, and parents. In particular, Inspectors should continue to highlight the textbook dependent culture that exists for mathematics in many schools but additionally they should provide tangible alternative solutions for schools in their recommendations. Additionally, Inspectors should incorporate return visits to schools in order to assess any progress in relation to textbook dependency in mathematics. Likewise, schools need to consider their complicity in the prolific use of textbooks and workbooks in all areas of the primary school curriculum but in particular in mathematics. Schools need to reflect on the pedagogical value of the tasks outlined in textbooks and workbooks, in addition to comparing the requirements of the mathematics curriculum with the textbook-type tasks. Armed with this knowledge, schools should then plan appropriate mathematics programmes for various class levels. In so doing, teachers can avail of opportunities to be actively involved in mathematics lessons through decision making thus increasing a currently diminished autonomy. Principals have a unique role to play in this cultural shift considering their overall responsibility for teaching and learning in schools as outlined in the Education Act (Government of Ireland, 1998). Such a concerted, whole-school effort to improve mathematics programmes by being less textbook-dependent requires dynamic leadership. Finally, considering teachers in this study reported feeling pressurised to use textbooks; parents have a role to play in enabling teachers to regain autonomy from textbooks in mathematics lessons. In the first instance, this would require schools communicating with parents about the mathematics curriculum, the content of textbooks, and in particular, any changes to the traditional use of textbooks in schools. Such discussion might then lead to a school policy, understood by all of the school community, regarding the use of textbooks in mathematics lessons.

Teachers' Facilitation Skills

This study identifies teacher discomfort with facilitation as a critical factor in changing mathematics teaching. Specifically, teacher discomfort with regard to facilitation emerged as a restrictive factor when attempting to implement a reform approach to mathematics teaching. It seems fair to conclude that considering the specific role of the teacher in reform-based mathematics lessons, teachers need to be skilled at facilitation if they are to effectively implement the approach. This discomfort may stem from a variety of causes including the teacher's conceptualisation of what constitutes a teacher's role in mathematics lessons—if a teacher is more aligned with the traditional conceptualisation where teachers tell and show, then a discomfort with a role in which teachers facilitate and prompt learning is unsurprising. Likewise, teacher beliefs about mathematics, mathematics teaching, and general pedagogy may undermine their willingness to embrace facilitation skills in mathematics lessons. For example, a teacher whose worldview aligns with a social constructivist theory of learning may consider facilitation in mathematics lessons very differently from a teacher whose worldview aligns more with a behaviourist theory of learning. Moreover, discomfort with facilitation in mathematics lessons may stem from fear of exposure, particularly with regard to teachers who have poor mathematical knowledge for teaching or limited competence in mathematics. Finally, lack of familiarity with facilitation in mathematics lessons might be the genesis for teacher discomfort—teachers may display reluctance or discomfort because they do not fully understand what is meant by facilitation in mathematics lessons or indeed struggle to reconcile its place because they have never experienced in such a setting.

Facilitation skills in mathematics lessons need to be considered as a critical factor by teacher educators including professional development providers and pre-service provides, in addition to curriculum developers. In the first instance, pre-service providers should ensure the facilitation skills required for mathematics teaching are explored meaningfully through theory and practice in their programmes. Additionally, in designing professional development, providers should identify teachers' needs regarding facilitation and incorporate elements into the professional development programme that will support teachers in developing these skills. Regarding curriculum developers, such as the National Council for Curriculum and Assessment in Ireland, the skills that teachers require to implement the new mathematics curriculum should not only be included in the curriculum specification but should also be emphasised as fundamental skills on which effective implementation depends. Similar to the recommendation for a three-dimensional or 'live' instructional framework, the advice and requirements for facilitation should be supplemented with video footage of classroom practice in addition to witness statements from teachers and pupils.

LIMITATIONS OF THE STUDY

A number of limitations exist in relation to this study. These limitations include mathematics practice, teacher implementation, pupil attitudes, the role of the researcher, and the size and duration of the study.

Mathematics Practice

Although a number of data sources such as the initial pupil focus group interviews, teacher reports at the professional development sessions and interviews, and the interview with the principal suggest that traditional mathematics teaching prevailed in the school before the study began, no lesson observations took place before the implementation of the reform approach to mathematics teaching. This is a limitation of the study for two reasons. The extent to which traditional mathematics lessons prevailed in this school is not available from the current data. Moreover, any variations in mathematics practice that may have existed before the study began are not available from the data and so direct comparisons with regard to shifts in practice before and during the study are not possible. This limitation could be eliminated if lesson observations had taken place prior to the study. Such observations could also have provided baseline evidence for an in-depth analysis of lesson progression throughout the study.

Implementation

In this study, there is no way of knowing from the available data whether this approach to mathematics was used in other areas of the mathematics curriculum or just in the targeted strand units of length and weight. Furthermore, it is unclear from the available data to what degree this approach was implemented in the classes that were not being tracked. Therefore, it is not possible to draw conclusions with regard to implementation at whole-school level or in other areas of the mathematics curriculum. To similar effect, the relatively small number of lessons observed in each of the tracker classes (one during Phase 1 and one during Phase 2) means that the observed lessons may not have been reflective of all of the length or weight lessons in that class. Accordingly, it may be possible that some teachers merely complied with the approach for the two observed lessons only. Observing more lessons, both in the tracker classes and in other classes, throughout the study would eliminate this limitation.

Pupil Outcomes

Although shifts in practice were evident in the four tracker classes, no quantitative data are available that measured pupil outcomes such as attitudes or attainment. This is a limitation of the study. Considering the shifts in practice that were evident, the availability of quantitative data on pupils' attitudes to mathematics and pupil attainment in mathematics would have enhanced the findings. Such data would allow correlations to be made between mathematics practice, pupil attitudes to mathematics, and pupil attainment in mathematics.

Size and Duration of the Study

Possibly the most striking limitation of all is the duration of the study. The study lasted for one year; however, this comprised just two terms of professional development. This is a relatively short period in the life of a school and in the implementation of any reform initiative. It is also a short timeframe from which to expect significant changes to well-established mathematics practice. A more prolonged study with a case study school that spanned a number of years could yield interesting findings. Similarly, the study was limited to one school and so is reflective of the experiences of this school only. The study could be expanded to include a number of different schools in an attempt to gain a broader understanding of schools' experiences of this approach to mathematics teaching. Ideally, a variety of schools could be chosen to represent schools of varying size, socio-economic status, and geographical location.

Role Duality

Another limitation of the study (although it could also be cogently proposed as an advantage) was my role-duality as researcher and professional development provider. As outlined in chapter three, I sometimes struggled with this role-duality, in particular, adopting a neutral stance during lesson observations and teacher interviews. Furthermore, this role-duality could be considered a limitation because it could be argued that considering I provided the professional development, I expected to find evidence of shifts in practice. Finding a school that is already implementing a reform approach to mathematics teaching and so do not require professional development would alleviate this limitation; however, at the time of the study I was unaware of any such primary school. Another aspect of my role-duality relates to my being a teacher, so I was an insider in terms of being a teacher but an outsider with regard to: a) not being a member of the school staff, b) being out of the classroom for a number of years, and c) being in a research role. The confusion experienced by some researchers in similar situations is acknowledged by Brannick and Coghlan (2007, p.70) who contend that researchers "...are likely to encounter role conflict and find themselves caught between loyalty tugs, behavioural claims and identification dilemmas". This outside role may have contributed to teachers and pupils being more honest and forthcoming with their views and opinions; however, equally, it may have prevented my accessing some nuances within the school. To similar effect, the fact that I knew some of the staff[40] may be considered a limitation to the study from a perspective of neutrality; however, familiarity with parents and teachers may also have contributed to such high consent rates for the study. Despite all of these dilemmas, Merton (1972, p.44) diminishes such dichotomisation and instead focuses on the potential for understanding that such research makes possible: "Insiders and outsiders in the domain of knowledge, unite. You have nothing to lose but your claims. You have a world of understanding to win". Bearing this in mind, it is possible that such an in-depth understanding of the Irish primary school system enhanced rather than limited the study.

RECOMMENDATIONS FOR FURTHER STUDIES

Arising from this study, a number of questions have arisen and some areas of ambiguity have been identified. Further research may be warranted in a number of these areas.

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[40] I knew some of the teachers in the case study school because some of the teachers had taught me when I attended primary school, I had worked with some of the teachers, and I knew some of the teachers from professional development settings where they had attended courses that I provided. Equally, I had supported both schools prior to amalgamation.

- A longitudinal study is required to explore the relationship between reform-based mathematics practice in Irish primary schools and pupil outcomes. Such outcomes could include pupil attitudes, performance on attainment tests, and pupil motivation. Ideally, this longitudinal study should comprise a number of schools rather than just one case study school. The inclusion of a number of schools would also allow for further comparisons of learning experiences and performance on tests.
- It was unclear from the available data in this study whether teacher beliefs affected the extent to which teachers implemented the reform approach to mathematics teaching. Ascertaining such beliefs through a specific measurement tool could contribute to understanding the degree to which beliefs affect the implementation of a reform approach to mathematics teaching. Furthermore, exploring possible changes to teacher beliefs as teachers become more proficient and confident in implementing such an approach would be apposite.
- There was some evidence in this study from pupil focus groups that attitudes to mathematics improved because of the reform approach to mathematics teaching. Further research on the impact of this approach on pupil attitudes would be pertinent. Accordingly, a study is needed to ascertain pupils' attitudes to the type of mathematical learning environment espoused in reform-based mathematics. In particular, this study could consider whether this type of learning environment affects pupil motivation in addition to pupil attitudes. Furthermore, it could explore which particular aspects of reform-based mathematics teaching affect pupil attitudes and motivation.
- A by-product of this study is the 4Ts Instructional Framework for Mathematics. Teachers identified this tool as contributing to shifts in their mathematics practice.

Further research into the practical use and possible modification of this framework could assist teachers in their continual quest to improve mathematical learning environments and increase conceptual understanding.

- There was evidence of a textbook-dependent culture in this study that appeared to restrict teachers' influence over the mathematical content and pedagogy. A study is required to explore the extent to which a textbook dependent culture may reduce teacher autonomy in mathematics lessons. In particular, the study could identify differences in teacher autonomy, perceived or otherwise, across a number of schools. Including schools from a broad spectrum where different cultures exist with regard to textbook use would contribute to the richness of the findings.
- Some evidence existed in this study that the quality of mathematical tasks differed greatly between the four tracker classes. Further exploring the nature of mathematical tasks and any associated potential for developing mathematical understanding and expressed thinking would be an important study. In particular, this study could attempt to identify the types of tasks that are most beneficial at different stages in primary school.
- This study provides evidence that despite participating in the same professional development programme, the extent to which mathematics teaching displayed reform features varied considerably between the four tracker teachers. A study is warranted to explore the reasons for such variance in practice. This study could explore factors such as teacher beliefs and mathematical knowledge for teaching as an initial point of comparison amongst teachers.

EPILOGUE

Conducting this study was both a privilege and a rich learning experience for me. The willingness of the teachers in this case study school to engage in a reform approach to mathematics teaching was uplifting. Similarly, I was encouraged by the shifts in mathematics practice, and in particular, the change in pupils' engagement in mathematics over the course of the study. Nonetheless, this journey was not without its emotional challenges for me. In particular, I sometimes struggled with the dual-role of researcher and professional development provider. Furthermore, I was dismayed to learn that because of the study one teacher reported not knowing what she "was allowed" to do in mathematics lessons. Such teacher disempowerment was in direct contrast to what I had hoped to achieve with the professional development programme. Accordingly, this realisation disoriented me to a considerable degree. However, despite my learnings (both pleasant and uncomfortable) throughout the study, I hope that I have in some small way contributed to empowering these teachers with regard to their mathematics practice. If nothing else, I hope I have planted a seed of reflection in their minds with regard to the potential opportunities for developing mathematical understanding.

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Appendix A: Professional Development Programme

The professional development programme is detailed in this Appendix. Each professional development session is outlined in a separate table.

Table A.1 Professional Development Day 1

	Professional Development Day 1
Date	24.9.12
Time	3.15 – 4.45
Stage in Study	Phase 1 – before implementation of reform approach to mathematics teaching
Attendance	12 (all teachers including the principal)
Purpose	1. To present a rationale for changing mathematics teaching 2. To provide an overview of Hiebert et al.'s (1997) instructional framework
Resources	• Slideshow (rationale for changing mathematics teaching)
Hand-outs	• Summary chapter of the instructional framework from the book • Overview of instructional framework – dimensions and core features (taken directly from book) • Overview of instructional framework – dimensions and core features with additional information (I summarised individual chapters) • Teacher reflection checklist – based on the instructional framework • Overview of learning journals – including a possible structure
Teacher Activities	1. Group discussion regarding the current challenges in mathematics teaching 2. Teach to Learn (jigsaw) activity – in pairs/groups of three, the teachers read about one dimension in the summary chapter in the book (each group had a different dimension). They noted a) the main points and b) considered any possible challenges/concerns regarding the suggestions. Each group then 'taught' the other groups about their dimension based on their reading and notes. 3. Individual teacher reflection – teachers anonymously reflected on their own practice using the teacher reflection checklist, which is based on the instructional framework.

Table A.2 Professional Development Day 2

Professional Development Day 2	
Date	1.10.12
Time	3.15 – 4.45
Stage in Study	Phase 1 – before implementation of reform approach to mathematics teaching
Attendance	12 (all staff including principal)
Purpose	1. To present an overview of the concept of Measures 2. To generate reflection and discussion regarding the teaching and learning of Measures 3. To provide an overview of the topic length
Resources	• Slideshow on Measures • Slideshow on length
Hand-outs	• An activity sheet for Definitions (Measures) • A booklet[41] on Measures • Measures and length slideshows
Teacher Activities	Measures: 1. Individual activity – devise definitions for 5 out of 8 words on the hand-out 2. Group discussion regarding the understanding of terms and the importance of mathematical language 3. Group discussion regarding what it means to 'measure' Length: 1. Group brainstorming of language needed for length 2. Group callout for possible definitions for some of this language, for example, height, distance, perimeter, radius, diameter, circumference, etc. 3. Group callout regarding suitable resources for measuring standard units and non-standard units of length

[41] I created this booklet to support teachers in their understanding of Measures.

Table A.3 Professional Development Day 3

Professional Development Day 3	
Date	15.10.12
Time	3.15 – 4.15
Stage in Study	Phase 1 – before implementation of reform approach to mathematics teaching
Attendance	12 (all staff including principal)
Purpose	1. To include pupil reports in the professional development 2. To encourage teachers to compare pupil reports of mathematics lessons with the approaches recommended in the instructional framework 3. To collaboratively devise lesson plans for length
Resources	• Laminated double-sided instructional framework poster for each teacher • Booklet on length[42] • Excerpts from pupil reports
Hand-outs	• Collaborative planning template • Curriculum objectives for each class level for length
Teacher Activities	1. In class level groups of 3 (infant teachers and one support teacher; 1st and 2nd class teacher and one support teacher; 3rd and 4th class teacher and one support teacher; 5th and 6th class teacher and one support teacher), teachers read and analysed the excerpts from the pupil reports. 2. Whole group feedback was taken regarding pupil reports. 3. Each group compared the feedback/findings regarding pupil reports with the instructional framework and highlighted areas for improvement in their mathematics teaching (based on the reports from the pupils). 4. The areas which the teachers highlighted for improvement were: ▪ The role of the teacher ▪ Sharing mathematical ideas – refection and communication ▪ Use of tools, in particular, the use of concrete materials 5. Teachers began to collaboratively devise a lesson plan for length using: ▪ Curriculum objectives ▪ Length booklet ▪ Instructional framework 6. The collaborative planning was scheduled to continue the following week during a Croke Park hour[43]

[42] I created this booklet to support teachers in their mathematics lessons. It comprised a) pupil conceptions and misconceptions about length, b) mathematical knowledge for teachers, and c) suggested activities and length-based tasks.

[43] Croke Park refers to an industrial relations agreement that requires teachers to work one extra hour per week – this hour is outside of normal school hours.

Table A.4 Professional Development Day 4

Professional Development Day 4	
Date	12/11/12
Time	3.15 – 4.45
Stage in Study	Phase 1 – before implementation of reform approach to mathematics teaching
Attendance	12 (all staff including principal)
Purpose	1. To continue collaborative lesson planning
Resources	• Length booklet • Boxes of resources for length • Open-ended task cards for length
Hand-outs	• Curriculum objectives • Ideas for projects on length
Teacher Activities	1. Teachers continued collaborative lesson planning in groups of 3 2. Teachers discussed and used the resources for length in order to prompt ideas for planning (one box of resources was distributed to each group) 3. Teachers completed a reflection journal on collaborative planning at the end of the session.

Table A.5 Professional Development Day 5

Professional Development Day 5	
Date	14.1.13
Time	3.15 – 4.45
Stage in Study	Phase 2 – after implementation of length lessons
Attendance	11 (including principal but one teacher absent due to sick leave)
Purpose	1. To review and analyse the new pupil reports from Phase 2 2. To introduce a revised IF – the *4Ts instructional framework for teaching maths*[44] 3. To provide opportunities for teachers to engage in professional reading 4. To critically analyse video footage
Resources	• Excerpts of pupil reports • Video footage (Deborah Ball)[45]
Hand-outs	• Instructional framework • 4Ts • Readings (Nick Pratt's (2002) article *Mathematics as Teaching* and Therese Dooley's (2011) article *Telling Matters in Mathematics Teaching and Learning*
Teacher Activities	Pupil Reports 1. In groups of three, teachers re-analysed the initial pupil report excerpts (as used on day 3 of professional development before the implementation of the reform approach). 2. Oral feedback was taken to establish themes 3. In groups of 3, teachers analysed the pupil report excerpts from Phase 1 (after the implementation of the lessons on length) 4. Oral feedback was taken to establish themes 5. Teachers compared and contrasted the pupil report excerpts from before and after implementation of reform approach to mathematics teaching 6. Teachers gave feedback on their observations 7. Teachers also viewed some sample learning logs 8. Teachers gave feedback on these and commented that the learning logs appeared to be mainly descriptive /narrative in nature (describing the tasks they engaged in rather than on learning). 9. Discussion with teachers regarding a) possible need for a template/structure for learning logs and b) what might be included in a learning log. Suggestions included: ▪ Reference to mathematical understanding

[44] Specifically revised to include a) teacher talk (including prompt questions and suggested language); and b) a more explicitly facilitative role for the teacher. This revision was based on the experiences in Phase 1.

[45] Video footage of classroom practice (3rd grade discussing odd and even numbers) that Deborah Ball used at a conference in Marino, Dublin. Video footage was accessed at http://www.ncca.ie/en/conference

- What was learned
- Any misconceptions/partial understandings
- What do I still not understand, etc.?

Revised instructional framework (4Ts)

1. Teachers reviewed the original IF individually
2. Teachers read Pre-requisites section in 4Ts and made links with original IF
3. Teach to Learn (jigsaw) Activity – each group a) read one 'T' section; b) took notes for feedback; c) made comparisons with original IF (except Talk group as they have no comparison to make in original); and d) 'taught' this 'T' section to the other groups.
4. Discussion about the revised instructional framework (4Ts), in particular:
   - Role of the teachers as an active skilled facilitator (not just taking a back seat)
   - Inclusion of 'talk' element
5. Discussion about the need to progress teaching and learning from Phase 1, for example:
   - Phase 1 was focused on collaborative group work; hands-on tasks; developing concept of length; discovery learning, etc.
   - Phase 2 will need to also include a) whole class discussion; b) reasoning; c) problem solving; and d) a more active role by the teacher
6. Analysis of video footage to explore the role of the teacher in whole class discussion (with a focus on reasoning and mathematical discussion)
7. Discussion regarding a) was there teaching in this lesson?' and b) if so, was this teaching skilled? The points that teachers raised included:
   - There was teaching evident
   - It was very skilled teaching
   - This type of teaching requires a lot of patience, for example, this type of discussion may not have been in the teacher's plans; the discussion takes a lot of time; fear that the discussion may not be 'going' anywhere
   - It was not evident/clear that all pupils were engaged/interested in the discussion
   - The behaviours and language of the pupils (for example, *I disagree; thank you for pointing that out; I disagree with myself;* etc.) suggest that pupils are used to working in this way – this type of discussion is not new to them
   - This type of facilitative teaching could expose a teacher in that it would require a deeper understanding of mathematics
   - The culture of the classroom is all important for this type of teaching to be successful
   - The language the teacher uses, in addition to the language pupils use, will significantly impact the quality of the discussions

Professional Reading

1. Teachers read summary section of Nick Pratt's (2002) article *Mathematics as Teaching*
2. Teachers were asked to read the rest of the article in their own time
3. Teachers were asked to read Therese Dooley's (2011) article *Telling Matters in Mathematics Teaching and Learning* in their own time

Table A.6 Professional Development Day 6

Professional Development Day 6	
Date	21.1.13
Time	3.15 – 4.15
Stage in Study	Phase 2 – after implementation of length lessons
Attendance	10 (including principal but two teachers absent due to sick leave)
Purpose	1. To further explore revised instructional framework (4Ts) through readings, feedback and discussion 2. To reflect on length lessons in Phase 2 3. To provide further opportunities for to engage in professional reading 4. To critically analyse a podcast 5. To explore teacher and pupil 'talk'
Resources	• Podcast of Seán Delaney interviewing Jo Boaler
Hand-outs	• Revised instructional framework (4Ts) teacher reflection checklist • Readings (Therese Dooley's (2011) article *Telling Matters in Mathematics Teaching and Learning;* Lampert's (2001) *Teaching Problems and the Problems of Teaching* (p.70 -72 and p.80-83) • Sample of teacher talk/questions
Teacher Activities	1. Feedback from Dooley (2011) article, for example, interesting points, anything that resonated, any challenges/concerns, etc. 2. Individual reflection on Length lessons in Phase 2: ▪ What worked well (WWW) ▪ Even better is (EBI) 3. Teachers shared reflections in pairs in this activity and then whole group feedback 4. Teachers read a section of Lampert's (2001) book - section on disagreement (p.70-72) 5. Whole group discussion 6. Teachers listened to podcast of Seán Delaney interview with Jo Boaler[46] 7. Discussion in pairs 8. Teachers read a section of Lampert's (2001) book - section on group work (p.80-83) 9. Teachers took away hand-out on sample teacher talk/questions to read in their own time

46 This podcast was accessed at http://insideeducation.podbean.com/2012/11/08/programme-150-jo-boaler-on-teaching-mathematics-4-11-12/

Table A.7 Professional Development Day 7

Professional Development Day 7	
Date	28.1.13
Time	3.15 – 4.15
Stage in Study	Phase 2 – before implementation of weight lessons
Attendance	12 teachers (all staff including principal)
Purpose	1. To explore possibilities for weight 2. To highlight rich, mathematical problems 3. To analyse problems in textbooks 4. To illuminate 4Ts (revised instructional framework) through weight tasks and problems
Resources	• Booklet on weight[47] • Maths problems based on weight from www.nrich.maths.org ▪ Cherry Buns ▪ Grandma's Pies ▪ Weights Combination
Hand-outs	• Copies of maths problems
Teacher Activities	1. Sample activity from Stage 1 of the weight booklet: Direct handling (page 7) 2. Sample activity from Stage 2 of the weight booklet: Ordering objects using various non-standard units (page 11) 3. Sample activity from Stage 3 of the weight booklet: Kilogramme benchmarks (page 16) 4. Sample problem from weight booklet: 9 sand-filled drinks, cartons all of equal weight, one has a washer inside, find which one but balance can only be used twice.[48] 5. Maths problems based on weight from www.nrich.maths.org ▪ Cherry Buns ▪ Grandma's Pies ▪ Weights Combinations 6. Teachers analysed problems in textbooks from a 'thinking' perspective – are the pupils thinking or are these problems suited to procedural practice? 7. Teachers compared these textbook problems with the problems from the nrich website which they just experienced 8. Discussion regarding possible structure of a weight maths lesson: ▪ Some tasks that explore the concept of weight ▪ Some problems where these concepts are used – also developing problem solving, reasoning and justification skills 9. Teachers agreed to collaboratively plan weight lessons (in same groups as they had done for length) during the next Croke Park hour

[47] Similar to the length booklet, I devised this and included mathematical knowledge for teachers, pupil conception's about weight, possible misconceptions, and possible classroom tasks and activities.

[48] The 4Ts was modelled during all of these activities.

Appendix B: Elements of Effective Professional Development

Table B.1 Elements of Effective Professional Development derived from the Literature

Elements	Literature Sources	Examples from Professional Development (PD) in this Study
Context		
Extended period of time	• Back et al. (2009) • Guskey (2002b) • Porter et al. (2000) • Timperley et al. (2008)	• The professional development spanned seven months
Engagement of external expertise	• Timperley et al. (2008)	• As an external teacher educator, I designed and facilitated the professional development
Challenging prevailing teacher discourse	• Timperley et al. (2008)	• One of the purposes of PD day 1 was to present a rationale for changing mathematics teaching • During PD day 3 teachers compared pupil reports of mathematics lessons with the instructional framework and highlighted areas for improvement in their mathematics teaching • During PD day 5, teachers compared and contrasted the pupil report excerpts from before and after implementation of the reform approach to mathematics teaching
Participation in some form of professional community of practice	• Back et al. (2009) • Koellner, Jacobs & Borko (2011) • Joyce & Showers (2002) • Loucks-Horsley & Matsumoto (1999) • Loucks-Horsley et al. (2010) • Porter et al. (2000) • Timperley et al. (2008)	• All of the seven PD sessions involved some form of professional community of practice • Teachers collaboratively designed lesson plans for length and again for weight
Consistency with the wider research/policy agenda	• Loucks-Horsley & Matsumoto (1999) • Porter et al. (2000) • Timperley et al. (2008)	• As evidenced in the literature review, despite some contestation, reform approaches to mathematics teaching are promoted consistently in many jurisdictions
Leaders actively supporting the professional learning	• Back et al. (2009) • Timperley et al. (2008)	• The principal took part in all of the professional development and provided opportunities for collaborative planning and financial support for mathematical materials
Content		
Integration of content (theory and practice; subject content knowledge, pupil learning, and instructional practices)	• ACME (2002) • Borko (2004) • Koellner, Jacobs & Borko (2011) • Loucks-Horsley & Matsumoto (1999) • Loucks-Horsley et al. (2010) • Timperley et al. (2008)	• The pedagogy (instructional framework) and the subject content knowledge (measures, length, weight) were integrated throughout the PD sessions
Focus on the relationship between	• Back et al. (2009)	• During PD days 3 and 5 the pupil reports acted as a catalyst for discussions about teachers'

teaching and learning	• Borko (2004) • Joyce & Showers (2002) • Porter et al. (2009) • Timperley et al. (2008)	practices and pupils' learning experiences in mathematics lessons
Focus on assessment	• Timperley et al. (2008)	• Teachers identified the Measures strand as an area for improvement following analysis of standardised test results across the school
Records of classroom practice	• Borko (2004) • Loucks-Horsley & Matsumoto (1999)	• Reports from pupil focus groups were used during PD days 3 and 5 in order to bring teachers classrooms into the professional development • Teachers also critically analysed some textbook-based tasks that they used in their classrooms during PD day 7
Strong theoretical base	• Back et al. (2009) • Joyce, Showers & Bennett (1987) • Loucks-Horsley et al. (2010) • Timperley et al. (2008)	• Hiebert et al.'s (1997) instructional framework based on a theory for developing conceptual understanding strongly influenced the PD
Activities		
Opportunities for active learning	• Porter et al. (2000) • Joyce, Showers & Bennett (1987)	• All seven PD sessions comprised opportunities for teachers' engagement in active learning
Learner-centred (building on what teachers know and are able to do)	• Back et al. (2009) • Koellner, Jacobs & Borko (2011) • Loucks-Horsley & Matsumoto (1999)	• In the first PD session teachers were encouraged to identify their knowledge and needs in relation to Measures • The instructional framework was refined in conjunction with teachers partly to address needs identified by them
Multiple opportunities to revisit learning	• Timperley et al. (2008)	• The principles underpinning reform approaches to mathematics teaching were revisited continuously through a variety of activities, for example, analysis of the instructional framework, analysis of video footage, engagement with a podcast, opportunities for professional reading, and engagement in mathematics activities and tasks during the PD
Listening to others with greater expertise	• Timperley et al. (2008)	• During PD day 6 listening to a podcast of Jo Boaler and Seán Delaney
Discussing practice with colleagues and with someone with specific expertise	• Loucks-Horsley & Matsumoto (1999) • Loucks-Horsley et al. (2010) • Timperley et al. (2008)	• During all seven PD sessions, teachers discussed practice with me and other colleagues • During the collaborative lesson plans teachers discussed their practice in relation to length and weight
Having opportunities to see real or simulated practice	• Timperley et al. (2008)	• During PD day 5 teachers critically analysed video footage of Deborah Ball teaching a mathematics lesson
Being observed and/or receiving feedback	• Joyce, Showers & Bennett (1987) • Loucks-Horsley & Matsumoto (1999) • Timperley et al. (2008)	• The four tracker teachers were observed and received indirect feedback during group discussion about the Phase 1 lessons during PD day 5.
Receiving student	• Timperley et al. (2008)	• All teachers received a booklet on length and a

activities and materials		booklet on weight in which student activities were detailed
Participating in activities positioned as students (to stimulate and challenge)	• Back et al. (2009) • Borko (2004) • Loucks-Horsley & Matsumoto (1999) • Loucks-Horsley et al. (2010) • Timperley et al. (2008)	• During PD day 2 teachers engaged in two definitions activities positioned as students: one based on definitions for Measures and one based on definitions for length • During PD day 7 teachers engaged in many activities positioned as students including o sample activity from Stage 1 of the weight booklet: Direct handling (page 7) o sample activity from Stage 2 of the weight booklet: Ordering objects using various non-standard units (page 11) o sample activity from Stage 3 of the weight booklet: Kilogramme benchmarks (page 16) o sample problem from weight booklet: 9 sand-filled drinks, cartons all of equal weight, one has a washer inside, find which one but balance can only be used twice o maths problems based on weight from www.nrich.maths.org (Cherry Buns; Grandma's Pies; and Weights Combinations)
Engaging with professional readings	• Back et al. (2009) • Timperley et al. (2008)	• Teachers read chapters from Hiebert et al.'s (1997) book during PD day 1 • Teachers read Nick Pratt's (2002) article Mathematics as Thinking during PD day 5 • Teachers were encouraged to read Therese Dooley's (2011) article Telling Matters in Mathematics Teaching and Learning between PD days 5 and 6 • Teachers read sections of Lampert's (2001) book Teaching Problems and the Problems of Teaching - section on disagreement (p.70-72) and section on group work (p.80-83) during PD day 6
Sequencing of professional instruction	• Timperley et al. (2008)	• Findings in relation to mathematics in Irish primary classrooms were used as a catalyst for changing practice in PD day 1 whilst pupil reports of mathematics lessons were used as a catalyst in PD day 2 • The PD was front-loaded with the new knowledge, for example, the instructional framework during PD days 1 and 2 • During PD days 3 to 7 teachers got opportunities to apply this new knowledge to their practice
Discuss and negotiate new learning	• Timperley et al. (2008)	• Throughout the PD, teachers got opportunities to discuss the new learning including raising concerns and highlighting opportunities • Between the implementation of the length lessons and the weight lessons teachers contributed to the refinement of the instructional framework in an attempt to make it more meaningful for their context

Appendix C: Interview Schedule

Robson (2002) suggests that a semi-structured interview schedule should have introductory comments, a list of topic headings or key questions, a set of associated prompts, and closing comments. The key questions in the interviews during Phase 1 were based on a) the professional development and b) the instructional framework; whilst in Phase 2 they were based on a) mathematics lessons and b) the instructional framework. The interview with the principal mainly focused on a) mathematics lessons; b) the impact of the reform approach; and c) future plans.

Tracker Teacher Interview Schedule: Phase 1

Professional Development

1. Which aspects of the professional development did you find most useful regarding the teaching of maths?
2. Tell me about your experience of the collaborative planning?
3. Did your teaching change as a result of the professional development, and if so, in what ways did it change?

Instructional Framework

4. What was your experience of using the instructional framework?
5. What were the benefits of using the instructional framework?
6. What were the challenges of using the instructional framework?
7. In what ways could the instructional framework be improved?

Closing Question

8. Is there anything else that you would like to add?

Tracker Teacher Interview Schedule: Phase 2

Maths Lessons

1. Have your maths lessons changed since you started this project and if so, in what ways have they changed?
2. Have there been changes in your maths lesson between phases 1 and 2?
3. What has helped you to change these lessons?

Instructional Framework

4. What has been your experience of using the instructional framework?
5. Have you any suggestions on how the instructional framework could be improved?

Closing Question

6. What advice would you now give other teachers regarding the teaching of maths?

Principal Interview Schedule: End of Study

Opening Question

1. As a principal, what was your experience of the project?

Maths Lessons

2. What do you think maths lessons were like before the project began?
3. What do you think maths lessons are like now after the project?
4. If so, what has helped to change these lessons?

Project

5. Which aspects of the project do you think worked well?
6. Which aspects of the project could be improved?

Future

7. What next steps do you intend taking in relation to the teaching and learning of maths?

Appendix D: Focus Group Interview Schedule

The focus group interviews opened with a general or "grand tour question" and gradually delved deeper into the area the researcher wished to explore (Brenner, 2006). Although the schedule evolved over the course of the study, the key questions in the pupil focus group interviews were based generally on the five dimensions in the *Framework of Dimensions and Core Features of Classrooms that Promote Understanding* (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human, 1997). The key questions in the teacher focus group interview were based on teachers' experiences of mathematics lessons as the study progressed in addition to questions about using the instructional framework.

Initial Pupil Focus Group Schedule

Opening Question

1. What do you like most/least about mathematics?

Nature of Classroom Tasks

2. How often do you talk about your maths ideas in class?
3. If you talk in maths class, who do you talk to?
4. What kind of maths do you talk about?

Role of the Teacher

5. How are ways of doing maths explained or demonstrated?
6. Who shows you ways of doing maths?
7. How does this person show you?

Social Culture of the Classroom

8. How do you solve problems in maths class?
9. Do you solve problems on your own or do you solve them with other children?
10. How do you know that a certain way of solving a problem works?
11. How do you share your way of doing the problem?

Mathematical Tools as Learning Supports

12. What kinds of things help you to do your maths?
13. How often do you use practical things or concrete materials?
14. How often do you use drawing in maths?

Equity and Accessibility

15. Who talks the most in maths class?
16. What do they talk about?

Concluding Question

17. What could your teacher do to make it easier for you to understand maths?

Post-Lesson Pupil Focus Group Schedule: Phase 1

Opening Question

1. What did you like most about today's maths lesson?
2. Is there anything you didn't like about today's maths lesson?

Talk

3. In today's maths lesson how often did you talk about your maths ideas or talk about maths?

Tasks

4. How are ways of doing maths demonstrated and explained in your maths class? Did anyone explain it or demonstrate it to you?
5. In today's maths lesson how did you solve the problem?

Tools

6. What kind of things helped you? And the type of things I'm talking about here is tools or objects.

Role of Teacher

7. Who talked the most in the maths lesson toady?

Closing Question

8. How did today's maths lesson make you feel about maths?

Post-Lesson Pupil Focus Group Schedule: Phase 2

Opening Question

1. What did you learn in today's maths lesson?

Tasks

2. How did the activities/tasks help you to learn maths today?

Tools

3. How did tools (materials) help you to learn maths today?

Talk

4. What did you talk about in today's maths lesson?

Teacher

5. What role did the teacher have in today's maths lesson?

Closing Question

6. What advice would you now give to teachers about maths lessons?

Teacher Focus Group Schedule

Maths Lessons

1. Have your maths lessons changed since you started this project and if so, in what ways have they changed?
2. Have there been changes in your maths lesson between phases one and two – between Phase 1 being *Length* and Phase 2 being after Christmas when we did *Weight*?
3. If so, what helped you to change your lessons?

Instructional Framework

4. What has been your experience of using the instructional framework?
5. Have you any suggestions on how the instructional framework could be improved?

Closing Question

6. What advice would you now give other teachers regarding the teaching of maths?

Appendix E: Ethical Considerations

Ethical issues were considered at all stages in the study including during the data collection stage and the report writing stage. These issues included consent, design of instruments, risk management procedures, and reporting of findings.

1. Consent

Prior to the data collection stage, the Board of Management and teachers were provided with written documentation outlining the research purpose and procedures. An oral presentation, including a question and answer session, was made to the school staff. It was emphasised that the intention was not to pass judgement on the school but to attempt to understand key issues from teachers' and pupils' experiences. Furthermore, reassurances were given in relation to confidentiality. Written consent was received from the Board of Management, the principal and all teachers. The letter sent to the Chairperson of the Board of Management/Teachers/Principal can be viewed in Table D.1 whilst a copy of all consent forms can be viewed in Table D.2.

Table E.1 Letter to Chairperson of Board of Management/Principal/Teachers

Dear [Name of principal/chairperson/teacher]

I am a primary school teacher and am on secondment with the Professional Development Service for Teachers (PDST). I am currently doing doctoral research in St. Patrick's College, Drumcondra. The aim of my research project is to investigate the perspectives and experiences of teachers and pupils in primary classrooms during mathematics lessons.

I am interested in working with the whole school teaching staff during the current academic year. In order to up-skill and support the teachers, I will work collaboratively with them over the course of two school terms. My role will be that of supporter and facilitator. In order to gain the perspectives and experiences of the pupils and teachers, I would like to visit three or four classes for one or two visits during this timeframe. My role will be that of observer of mathematics lessons. These classes would be self-selected by the teachers. After these lessons, there will be some follow-up interviews with the teachers and some focus groups with samples of pupils. These pupils will be chosen using various criteria including gender and attainment. Data will be collected by means of interviews with class teachers and groups of children, digital photographs of children's mathematical work, samples of written work, teacher reflective journals and pupil journals. You are welcome to receive feedback on the project on its completion. Data collected will be analysed for the doctoral project and also for further publications. Reports on the project will be given at conferences related to mathematics education. However, the name of the school will not be revealed and, in order to safeguard children's anonymity, individuals' names will not be used.

All participants have the right to withdraw from the research at any time and the researcher has the right to discontinue working if the need arises. Participants will be given as much 'say' as possible in achieving the aims of the study. It is hoped that this research will be viewed as a joint enterprise in which the researcher and the teachers learn more about mathematics teaching and learning.

I would be most grateful to you for confirmation that you are happy [for the school] to participate in this project during this year. I will contact you next week to make an appointment to meet you but, if you wish, you may email me at miatreacy@pdst.ie or contact me by telephone on 087-2724363.

Thank you for your time

Mia Treacy

Table E.2 Written Consent Forms

Teacher Consent Form

Consent Form (Teachers)

I have read about the research project on *Teaching and Learning Mathematics* and I understand what is involved.

I agree to take part in the project
Yes ☐ No ☐

Signed ____________________________________

Parent Consent Form

Consent Form (Parents)

I have read about the research project on *Teaching and Learning Mathematics* and I understand what is involved.

I agree to let my child take part in the project
Yes ☐ No ☐

Signed ____________________________________

Pupil Consent Form

Consent Form (Senior Pupils)

Dear Pupil,

I am a primary school teacher and I also work with teachers. I am trying to find out what is the best way to teach mathematics to children in primary schools.

Your teacher has agreed to help me with my work. You will be very helpful to me by showing how you work on lots of different mathematics activities and telling me what you think about them. I would also like to tell other teachers about the work you do but I will not use your real names when I am doing this. You can drop out of the project whenever you wish. When the project is finished, I will come back to the school to tell you what I have found out about the teaching of mathematics.

I would be grateful if you would complete the form to show that you agree to take part in the project.

Thank you

Mia Treacy

....

Please tick the Yes or No box:

- I understand what the project is about

Yes ☐ No ☐

- I know that some of my mathematics work and journals might be used

Yes ☐ No ☐

- I know that I will be taped if the researcher wants to talk to me

Yes ☐ No ☐

- I know that photographs might be taken of me working on mathematics but my face will not be used

Yes ☐ No ☐

- I know that my real name will not be used

Yes ☐ No ☐

- I know that I can drop out if I want to

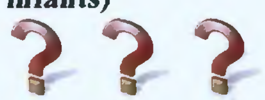
Yes ☐ No ☐

- I want to take part

Yes ☐ No ☐

Signed ____

Consent Form (Junior Pupils) (*to be read with senior infants*)[49]

I want to ask you lots of questions about maths. 

I will ask you these questions in groups with other children.

[49] The design of the consent form for junior classes, in particular, the use of pictures to enhance the meaning of the form, is generally based on those used by Edel Collins in her Master's thesis.

I will use a tape recorder so that I can remember your answers. 

I might take photographs of your maths work.

I will keep your name secret when I tell other people about this. 

Thank you
Mia Treacy

My Name is ______________________

Please circle the face that shows how you feel about talking to Mia about maths.

Mia will ask 

Mia will use a 

My name will be a 

Furthermore, informed written consent was received from the parents of pupils who were directly involved in the study, that is, the pupils in tracker classes.[50] In all of the tracker classes, I also read out Plain Language Statements (see Table D.3). Consent was then also received from pupils on age-appropriate forms. Only one pupil from the four tracker classes denied consent. This pupil did not take part in the data collection; however, the pupil did complete the classroom tasks because they were considered to be part of the mathematics programme for the year. The pupil's written work or classroom interaction was not used. Furthermore, two photographs of this pupil's work were deleted at the end of one lesson observation.[51]

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50 With the exception of one pupil, all parents gave consent.

51 The pupil was working in a group of four and the group asked for a photograph to be taken of their work.

Table E.3 Plain Language Statements

Plain Language Statement (Senior Pupils)

To be read to senior pupils of primary age:

I am interested in doing a project on mathematics.
I am interested in what you think of the mathematics which you are learning, for example, what you like or dislike; what you find easy or difficult; what helps you to learn mathematics.
I will be working with all the teachers in the school and will be helping them to plan mathematics activities.
Sometimes, I will come to your classroom to watch these mathematics activities.
I will also ask you to write about these mathematics activities in journals.
Sometimes I might need to interview you in groups about these mathematical activities but I will only do this with your permission. I will need to audio-tape these interviews.
I might also take digital photographs of your work. If I take a photograph I will show it to you and your teacher and I will delete it if you do not want me to use it. Your real name will not be used with the photograph and I will not show your face in the photograph.
I may need to ask your teacher how you have been getting on in mathematics over the last year.
You do not have to take part in the project. If you decide not to take part you will still have to do the tasks because they are part of your mathematics programme for the year but I will not be using your written work.
At the end of the project I will tell you about some of my findings and I would be delighted if you shared with me some of your thinking about what helps you to learn and enjoy mathematics.
I will be telling other people who are interested in mathematics education about this project but I will not use your real names or the name of your school when I do this.

Plain Language Statement (Junior Pupils)

To be read to junior pupils of primary age (senior infants):

I want to ask you lots of questions about maths. I will ask you these questions when you are in groups with other children.
I will use a tape recorder so that I can remember your answers.
I might take photographs of your maths work.
You do not have to take part.
I will keep your name secret when I tell other people about this.

Plain Language Statement (Parents/Guardians)

Dear Parent/Guardian

I am a primary school teacher and am on secondment with the Professional Development Service for Teachers (PDST). I am currently doing doctoral research in St. Patrick's College, Drumcondra. Your school has kindly agreed to take part in my research project. The aim of my research project is to investigate the perspectives and experiences of teachers and pupils during mathematics lessons.

In order to gain the perspectives and experiences of the pupils, I will be observing mathematics lessons in your child's class. After these lessons, there will be some follow-up interviews with some pupils. These interviews will be with groups of pupils who are chosen for a number of reasons, for example, gender and attainment. Results achieved by pupils in their class mathematics tests may be used to select pupils for the interviews. The interviews will be audio-taped. Data will also be collected by means of observation notes, digital photographs of children's mathematical work, samples of written work, and pupil journals. You are welcome to receive feedback on the project on its completion. Data collected will be analysed for the doctoral project and also for further publications.
If you wish, you may contact me to receive information or feedback on the project. Data collected will be used for my doctoral thesis and for related publications. In any reports on the project, individual children's names will not be used in order to safeguard anonymity.

I would be grateful if you would complete the form below indicating that you give permission for your child to participate in this project. If you do not give permission, your child will continue to be involved in the lessons as they are part of the mathematics programme for the year but I will not be using any of his/her written work.

Thank you in anticipation

Mia Treacy

2. Design of Instruments

The possibility of bias and subjectivity was counteracted when designing the data collection instruments by using semi-structured interviews in which the main questions were based on findings from the literature. Both the interview and focus group questions were based chiefly on the five dimensions from Hiebert et al.'s (1997) instructional framework.

3. Risk Management Procedures

Ground rules for interviews and focus groups clarified that personal or sensitive professional matters would not be discussed. However, social risks still exist in research. In order to address any potential social risks, the following risk management procedures were deemed necessary:

a) In order to minimise teachers' potential apprehension surrounding changes to classroom practice, a relatively small area of the mathematics curriculum was identified for exploration. This facilitated teachers in experimenting with a new approach to mathematics whilst still engaging with his/her familiar practice for the other areas of the mathematics curriculum.
b) In an attempt to decrease the risk of teacher vulnerability whilst being observed during mathematics lessons, lesson plans were designed collaboratively. This shifted sole responsibility for the success or otherwise of the mathematics lesson from the individual teacher being observed towards a more collective responsibility.
c) The focus group questions focused on the actual lesson that took place, that is, the mathematical approaches and content, rather than the teaching in order to minimise the risk of a teacher being exposed as ineffective. Despite this, such criticisms emerged on one occasion. I used discretion to discard the content because it may have been psychologically damaging for the teacher.
d) Pupils were assured that all opinions are valued and indeed necessary for the success of the focus groups and accordingly, I assured them that there were no correct or incorrect answers.
e) Pupils were given permission to turn off the tape recorder during the focus group interviews should they feel uncomfortable or should they prefer that certain

aspects of the discussion are not audiotaped. This did not happen in any of the focus group interviews.

f) Both teachers and pupils were assured that they could decline from participating in the research at any time, and that I would view any such actions sympathetically and sensitively. This did not happen at any stage in the research.

g) No videotaping of lessons was used because teachers expressed concern and discomfort at this type of data collection.

h) In an attempt to ensure that the principal had an active role in the process, the principal, like the other teachers, was encouraged to engage with the professional development and take part in an interview.

4. Reporting of Findings

During the reporting stage, real names were replaced by pseudonyms, for example, Pupil A or pseudonyms for teachers. Despite this, and chiefly due to the small sample size, it may not be possible to guarantee anonymity regarding participant identity; however, every effort was made to ensure that the identity of the participants was protected. The case study school was not identified by name in the report. Copies of transcripts were checked with interview participants. Draft sections of the report were provided, for comment, to relevant participants. All data will be retained for five years and then destroyed (Creswell, 2009). Schools and teachers are not identifiable in the written report, which is written in language that is respectful to all. Copies of the completed draft report will be provided for comment to individual key participants in the school. Finally, in a further attempt to maintain anonymity direct quotations were not used from the Whole School Evaluation that was carried out in the school by the Inspectorate of the Department of Education and Skills.

Appendix F: Additions to the Instructional Framework after Phase 1

The chief additions deemed necessary to refine and contextualise the instructional framework are explored in this Appendix.

The Teacher's Role as an Active, Skilled Facilitator

This addition was deemed necessary for a number of reasons. First, all four of the tracker teachers reported finding the facilitation role challenging during Phase 1. Teachers appeared to struggle with the meaning of facilitation and in particular, what facilitation might look like in a mathematics lesson. Furthermore, teachers reported struggling with how much guidance, if any, to provide pupils. Second, although all four teachers reported finding facilitation a challenge, some teachers appeared to be frozen by their uncertainty whilst other teachers actively sought their own solutions, such as, devising a menu of non-committal statements that could be used by the teacher. Consequently, in consultation with the teachers it was decided to a) include facilitation more explicitly in the instructional framework and b) explore the skill of facilitation more explicitly during the Phase 2 professional development. Finding facilitation a challenge when attempting to change mathematics practice is not surprising considering it occurs regularly in the literature as a challenge for teachers (e.g. Fraivillig et al., 1999; Rittenhouse, 1998; Walshaw & Anthony, 2008).

Teacher Talk that Encourages Reflection and Communication

Similar to the skill of facilitation, and possibly related to it, teachers reported not only being unsure of when to intervene but also being unsure of what to say during whole-class and small-group mathematical conversations. In particular, in consultation with the teachers, it was agreed to include the language needed for facilitation and the questioning needed for discovery learning in the revised instructional framework. This addition was deemed necessary because during the Phase 1 observed lessons; although pupils often worked collaboratively, there was little evidence of pupils sharing mathematical ideas. This is important considering Wood, Williams and McNeal's (2006) findings in relation to increased incidences of expressed mathematical thinking in strategy reporting or inquiry/argument classrooms. Similarly, they found that inquiry/argument classrooms provided opportunities for the highest participation by children across all classrooms and children acted as listeners, taking over the role of the teacher in questioning, clarifying, and validating mathematical ideas. Furthermore, they found that inquiry/argument classrooms provided opportunities for all children to be involved in meaning-making and to develop common ground on which to build shared understanding. Consequently, teacher talk was included in the revised instructional framework to remind teachers of the importance of encouraging pupils to reflect on and communicate their mathematical ideas. Furthermore, this was also included in the Phase 2 professional development sessions (in the main, whilst also exploring facilitation).

Whole-class Discussion in Developing Mathematical Thinking

During the Phase 1 observed lessons, there was no evidence of pupils engaged in whole-class mathematical discussions. Similarly, although many pupils worked collaboratively during these lessons, there was little evidence of discussion that developed or advanced mathematical thinking. In addition to the observed lessons, a number of other reasons contributed to the decision to include an emphasis on developing mathematical thinking through whole-class discussion that would incorporate mathematical skills such as reasoning, and revising conjectures and solutions. One reason relates to the fact that reasoning is deemed one of the most difficult subscales on TIMSS and is the one that Irish pupils performed least well on in TIMMS, 2011 (Eivers & Clerkin, 2012). Similarly, in their study on the implementation of Project Maths Jeffes et al. (2013) found little evidence of Irish students using reasoning in their mathematics lessons. Another reason relates to the emphasis on mathematical discussion and the development of reasoning in the Primary School

Mathematics Curriculum. The curriculum continuously emphasises the importance of productive mathematical discussion during meaning making and the co-construction of knowledge. Specifically, it underscores the role that this type of interaction plays in facilitating the testing of mathematical ideas, the subsequent modification of ideas, and the interrogation of reasoning and justifications. Another reason relates to the *Chief Inspector's Report 2010-2012,* which makes a number of recommendations including that talk and discussion should feature more prominently in mathematics lessons, and that pupils should be encouraged to use a range of reasoning and problem-solving strategies. Finally, the importance of reasoning in productive mathematics lessons is highlighted in much of the literature (e.g. Boaler, 2009; Lampert, 2001; Mercer, 2008; Warfield et al., 2005; Wood, 1999).

Students using Language to Refine, Revise, Clarify, Build on, and Communicate Mathematical Thinking

It was evident during the Phase 1 observed lessons that little opportunities existed for pupils to communicate deeply about mathematics. Consequently, opportunities for pupils to refine, revise, clarify, build on, or communicate their mathematical thinking were not prevalent. This addition was included in the instructional framework as a reminder that these opportunities are important for pupils (as evidenced in the previous two sections) and can occur at whole-class, small group, or individual level. Furthermore, the importance of language in supporting meaning making in mathematics emanates strongly from much of the literature. For example, Pratt (2002) attributes the indeterminacy of language as the driving force in the meaning making process of mathematics and recommends that teachers need to allow children "...more space and time both to find adequate words in giving explanations and to make sense of the words of others in the context of their own knowledge" (p. 37).

Other mathematical educators such as Askew (2012), Boaler (1999), and Wood (1999) illuminate the important role that language plays in conceptual understanding in mathematics. Finally, Anghileri (2007, p.14) claims "opportunities to share their thinking with others will encourage children to reflect on the methods and language they themselves use and become aware of alternative interpretations and strategies". Similarly, these opportunities allow for revision in mathematics or more specifically revising thinking about mathematics (Lampert, 2001).

Revoicing

Revoicing is 'the reporting, repeating, expanding or reformulating a student's contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students' (Forman & Larreamandy-Jones, 1998, p. 106). Little revoicing was evident during the Phase 1 observed lessons. Furthermore, teachers reported not being familiar with the concept of revoicing. Revoicing was chosen to be included in the revised instructional framework so that it might contribute to a classroom culture in which communicating and clarifying mathematical thinking prevailed. Specifically, it might contribute to deepening mathematical understanding, enriching mathematical thinking, and encouraging sharing of mathematical ideas.

Students Choosing to Record Verbally, Concretely, Pictorially/Graphically, Symbolically or in Written Form

During the Phase 1 observed lessons, some evidence existed of pupils choosing how to record their findings; however, in most cases, pupils followed the teacher's lead on which way to record their findings. For example, in Second Class although recording the findings of the task concretely, pupils were merely following directions from the teacher. Similarly, the pupils then followed the teacher's instructions by recording their findings in their copies using numbers. In this instance, pupils did not choose to record their findings either concretely or in symbolic form—they merely followed the teacher's instructions. Similar examples were evident in the other tracker classes. The importance of pupils recording their mathematical ideas in a variety of ways is a common theme in much of the literature (e.g. Askew, 2012; Boaler, 2009). Furthermore, considering the Primary School Mathematics Curriculum (1999) advocates the importance of verbal communication in mathematics and also states that children should be enabled to record the results of mathematical activities concretely, using diagrams, using pictures, and using symbols (p.18/38) it was decided in consultation with the teachers to include this in the revised instructional framework.

Appendix G: Sample Photographs from Lesson Observations

In this appendix, sample photographs from the observed lessons during Phases 1 and 2 are provided.

Image E.1 Senior Infant ordering task during the Phase 1 observed lesson

Image E.2 Senior Infant estimation activity during Phase 2 observed lesson

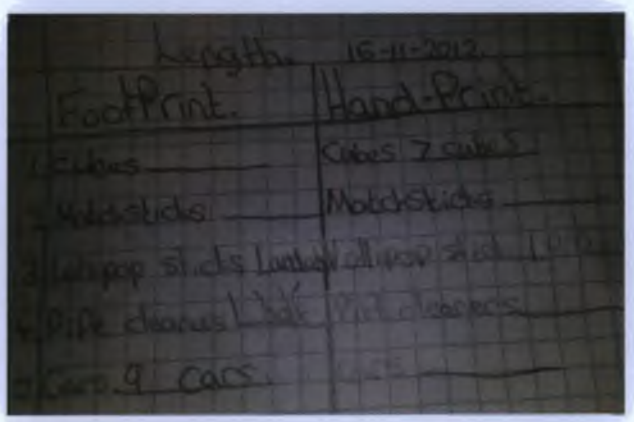

Image E.3 Second Class pupil's record of findings during Phase 1 observed lesson

Image E.4 Second Class pupil's record of estimates and findings during Phase 2 observed lesson

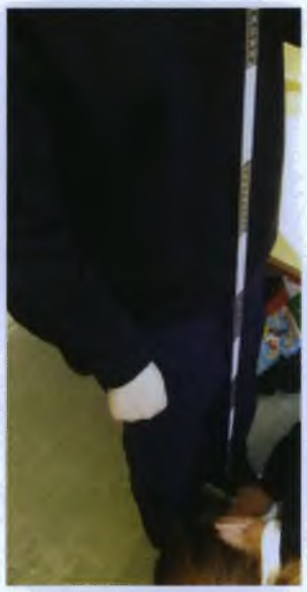

Image E.5 Fifth Class height activity during Phase 1 observed lesson

Image E.6 Fifth Class creating a line during Phase 1 observed lesson

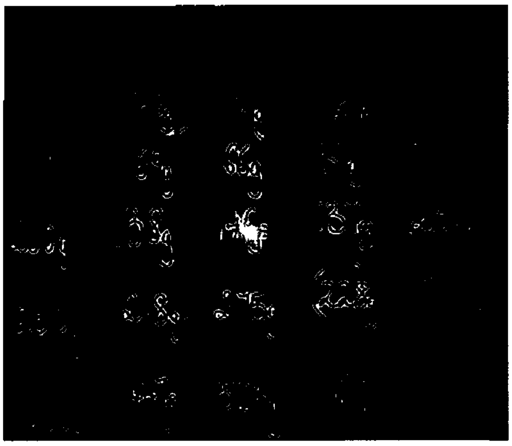

Image E.7 Fifth Class teacher recording during Phase 2 observed lesson

Image E.8 Sixth Class hall layout during Phase 1 observed lesson

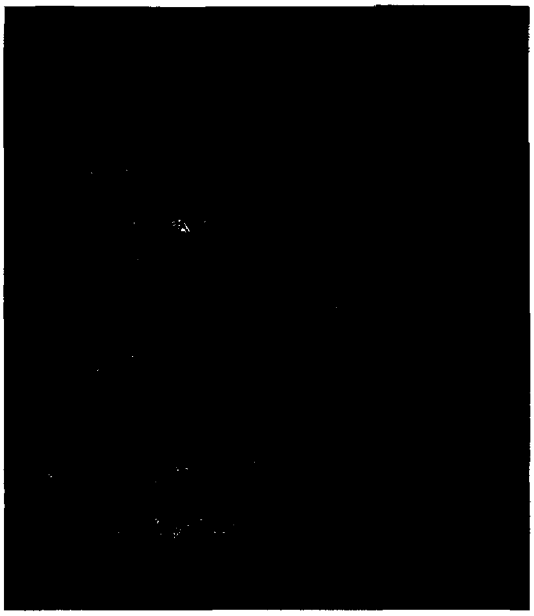

Image E.9 Sixth Class record of work for length task C during Phase 1 observed lesson

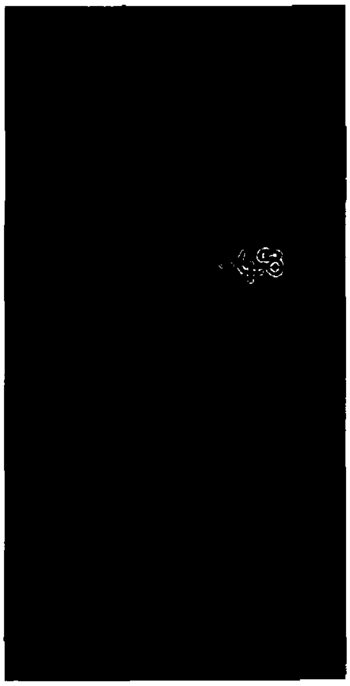

Image E.10 Sixth Class drawing for length task E during Phase 1 observed lesson

Image E.11 Sixth Class weighing activity for task A during Phase 2 observed lesson

Image E.12 Sixth Class weighing activity for task B during Phase 2 observed lesson

Image E.13 Sixth Class weighing activity for task D during Phase 2 observed lesson