A project-based-learning approach to teaching first-order and second-order differential equations to engineers

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ABSTRACT

In an attempt to increase engagement in a third-year mechanical-engineering mathematics module, a project was introduced in 2015/16. Students were asked to use the solution of a second-order differential equation (previously introduced in lectures) to solve a problem in which they must design a simple spring-damper system for one of a lorry, digger, truck, tractor, car, motorbike or pogo stick. In 2016/2017 this approach has been extended to a second project to use a first order differential equation to design a parachute. These projects were worth a total of 20% of the module. A detailed description of the implementation of this assessment is given along with an analysis of how students perform on questions on similar topics in the terminal examination for this module. Secondly we outline feedback received from a survey of the students. Finally we discuss the possibility of teaching the entire maths module via a project based learning approach.

INTRODUCTION

Many engineering students struggle to see the relevance of mathematics (Hidi and Harackiewicz 2000). This is turn may lead to low levels of engagement. A recent study of the First Year Experience (FYE) in the eight third level institutions in the Dublin Regional Higher Education Area (DRHEA) found that one of the key problem areas identified by academics across all eight institutions was lack of "student engagement" (Roper et al 2013, Cusack et al 2013). This lack of engagement often results in poor performance and ultimately impacts upon retention. Since September 2012, incoming first-year students to higher education in Ireland have studied a revised mathematics curriculum (Project Maths) in second-level (Jeffes et al 2013, Prendergast et al, 2017). This new approach to the teaching and learning of mathematics in Ireland aims to situate mathematics in everyday contexts where possible, so that students will be better able to understand the uses and relevance of mathematics. However, these students entered a higher education system that was accustomed to a mathematics curriculum that had not changed substantially in almost fifty years. Much of the material taught in the early years of mathematics is not explicitly mapped at that point to modules or applications in later years, making it difficult for students to understand the importance of what they are learning at this early stage in their careers. It is commonly the case that, in later years, lecturers will refer back to material covered in first year and show students how they will now use this mathematics in more advanced applications. However, from the point-of-view of student engagement and retention, this seems to be done at too late a stage, and needs to be dealt with in early years instead. This is particularly important in engineering as it relies heavily on mathematics throughout the degree programme. Successful service-teaching of mathematics relies heavily on a "sufficient supply of discipline related problems" (Yates 2003). This changing mathematical landscape in Ireland provided the motivation for the development of the project-based-learning approach described in this paper.

Engineering mathematics is generally either taught by engineering lecturers (who are usually not experts in mathematics) or by lecturers from mathematics departments who are not embedded within the students' home departments, and naturally may not be experts in the overall discipline being studied by the students. As a result, the balance between theory and practical applications is often skewed (Sazhin 1998, p.145). Lecturers may forget the significance of students "getting a feeling for the importance of the subject" (Rota). The level of interaction between mathematics lecturers and staff members in the students' home departments can vary widely from institution to institution.

In Dublin Institute of Technology, students are offered two main routes to obtain a Level 8 engineering qualification: via direct entry onto a four-year Honours degree programme (Level 8) or alternatively through a three-year Ordinary degree programme (Level 7) followed by a transfer into third year of the Honours degree (Llorens et al. 2014, Carr et al. 2013). This project is thus an attempt to evolve the teaching of engineering mathematics at Level 7 to both improve the engagement of students in engineering mathematics classes and to provide a deeper understanding of the material, which may ultimately help these students to progress onto a Level 8 degree programme if they so desire.

BACKGROUND

There exist many examples in the literature on the need to make the mathematics we teach to engineers more applied. The development of such examples and projects can be challenging for those teaching engineering mathematics as they may not be engineers or familiar with all aspects of engineering. However, there is a full spectrum of initiatives available in the literature, from improved examples in the classroom (e.g. Helm, Young et al 2012, Robinson 2008) to teaching the material via problem-based learning (e.g. Rooch et al. 2012). We provide a brief overview of some of these approaches.

Helping Engineers Learn Mathematics (HELM) is a major curriculum development project undertaken by a consortium of five English universities - Loughborough, Hull, Reading, Sunderland and Manchester - led by Loughborough. This project provided a huge list of engineering mathematics resources including a set of good examples of engineering applications of mathematics (http://helm.lboro.ac.uk/). Although this work is of a high standard, many of the examples contained therein are more relevant to later years of a Level 8 engineering programme, and the examples suitable for Level 7 students are limited in number.

Young et al (2012) at the University of Central Florida developed a bolt-on single-credit module called "applications of calculus", taught in parallel with their calculus modules. This has been shown to be effective in terms of retention of students within STEM subjects, although there are no projects introduced, simply a range of problems completed in class that are relevant to applications.

Robinson (University of Loughborough) used sports-based group projects for undergraduate students in sports science (Robinson 2012). These projects consisted of teamwork, use of software and application of mathematics to realistic problems. This not only improved engagement, it also introduced a range of important skills for engineers, such as technological and communication skills as well as collaborative and analytical techniques.

Rooch et al. (2012) developed a series of projects (ribbed cooler and a Segway) for teaching mathematics to first year engineering students in Germany. However, the mathematics required is quite involved in each case, meaning that they must supply the students with a number of different formulae in order to allow them to complete the projects. This is a common difficulty when designing "real-life" mathematics projects for students to attempt, due to the scaffolded nature of mathematical knowledge.

Within Dublin Institute of Technology itself, some example of project-based learning already exist. For example, design projects were introduced into first-year physics lab sessions for engineering students. These projects relied upon material covered in mathematics, physics and mechanics modules, bringing them all together in a single design project (Sheridan et al. 2010). A range of other variations exist between teaching applications and full problem-based-learning from Verner et al. (2008) and Mills and Treagust (2004), through to Abramovich and Grinshpan (2008).

AIMS AND SCOPE

In a move towards a more student-centred-learning approach for the teaching of engineering mathematics across all three years at Level 7 we have piloted a project based learning approach in the third year of the programme (Carr & Ni Fhloinn, 2016). We ultimately will work backwards to first year, once initiatives have proven to be successful. Much work has been done on using project-based/application-based learning as a method for teaching mathematics in higher education, but in the main, these modules have many mathematical pre-requisites, so they are in essence only suitable for later years of a programme and/or essentially being "bolted on" (Young et. al 2011) to pre-existing modules. Similar work has been done in the third year of the Level 8 degree programme to teach mathematical modelling with good success (Keane, Carr and Carroll, 2008), so it was of interest to introduce at a similar stage in a Level 7 programme and monitor its impact. Given that the standard of first year in a Level 7 programme is not high enough for many of these existing resources to be used, by trialling this approach in third-year, we can learn valuable lessons before considering earlier years.

The aim of this work is to use a hybrid approach to "project-based-learning" where a significant amount of the pre-requisites is taught over several weeks in a more standard approach and then a realistic project is introduced that consolidates the material that has been covered in class and provides an opportunity to learn applications of the material.

In 2015 one such project was introduced. Given the success of such a project we have introduced a second project this year.

The objectives of the projects are to improve engagement and ultimately retention of students; to give students a deeper understanding of the material; to introduce problem-solving, teamwork and communication skills; to move towards a more student-centred environment within the existing structure of lectures and tutorials; and to create a series of resources that could be used by lecturers teaching at Level 7.

PROJECT OVERVIEW

A series of two-hour group-work sessions were introduced, focused on the topic of second-order differential equations. Following a number of standard lectures, students were assigned to groups of four to work on a short project together during the group-work sessions, with additional work to be completed outside of class time.

The outline of the project given are listed on the website below. We hope in time to populate this website with more examples

PROJECT 1

As a group derive the first order differential equation for a falling body. Your derivation should include a diagram of the forces acting on this body. You will then solve this equation and use the result to design a parachute for one of the following an aid cargo, a large man on a charity jump, a small woman on a charity jump, a tandem jump, a paratrooper, a sky diver or a BASE jumper.

After a break of several weeks the groups were assigned a second project.

In order to design the parachute, needed to consider the mass of the jumper, what sort of velocity would be appropriate on landing etc.

PROJECT 2

In the second project we asked students to use the solution of a second order differential equation (previously introduced in lectures) to design a simple spring-damper system for a vehicle from the following list: lorry, digger, truck (large), truck (small), motorbike, motorbike (scrambler), bus (large), bus (small), moped, quad bike, tractor, tractor (seat), car (large), car (small), pogo stick, racing bike or standard bicycle. No two groups were assigned the same vehicle, and the different masses, number of wheels involved and type of damping required meant that the projects were sufficiently different that each group had to work independently on their solution.

The full details of the projects given may be found here

(https://sites.google.com/site/ditprojectbasedlearning)

We also intend to publish sample examples of students work on this website once we have received the permission of the students.

The projects were worth 10% of the students' final grades for that module, with the marks awarded per group. At the end of three weeks, each group presented their solution to the class during a ten-minute presentation slot, as well as handing up a short (four-ten page) report. The variation in report-length was chosen to allow students to include detailed diagrams and additional information where needed. The mixture of assessment methods included within the project gave students the opportunity to display their skills in a range of areas, while providing them with useful practice of presenting technical data in a clear and coherent manner. Students were obliged to attend all the presentations given, which also provided a valuable opportunity for peer-learning, as they heard how different groups had approached a similar problem, and allowed for some class discussion about optimum approaches afterwards.

In order to design a simple spring-damper system, students needed to first consider the mass of the vehicle, calculate an appropriate spring constant, and decide on what type of damping would be ideal for the vehicle in question. For example, the damping needed by a scrambler motorbike is different from that of a family car, where a smoother ride would be required. This was a multi-layered problem, which allowed students to investigate a number of areas in greater depth, considering aspects relevant to the generation of the second-order differential equation. Once they had solved the differential equation, they were then required to sketch the analytic solution by hand, to investigate if the resultant sketch resembled the type of damping they hoped to produce. If so, they then needed to plot a graph of the analytic solution and relate these back to the original problem, giving an interpretation of their results.

OVERVIEW OF PREVIOUS WORK

In our previous paper on this topic (Carr & Ni Fhloinn, 2016) and analysis was carried out examining the relationship between project mark and exam result.

When each student's project mark was plotted against their performance on this exam question, the result was statistically significant, with a Pearson correlation of 0.453 found with p=0.006. Similarly, when each student's overall performance was plotted against their performance on this exam question, another statistically significant correlation was found, with a Pearson correlation of 0.71 with p=0.000. While this is not surprising, showing that the most capable students performed well on all components of the assessment, it does contrast with previous years, when even strong students avoided or did poorly on the differential equations question in the examination.

To complement this work this year we have carried out short survey of the students to get their opinions of this approach.

OVERVIEW OF THE SURVEY QUESTIONS

The participants were approached during class and were given a paper survey with 4 questions addressing the project objectives of increasing student engagement, identifying the value and relevance of mathematics in the engineering discipline and improving basic communication and teamwork skills. They were given an explanation at the start of the survey capturing its purpose and stating that the information would remain anonymous, participation was voluntary and that it would not affect their module grade. Each thematic question consisted of three or four subsections. Three questions were structured using a likert 5-point scale with a comment section provided for respondents to explain or justify their selection. A final open-ended question required students to sum up their overall experience in completing the mathematical projects; what they particularly enjoyed and what they found challenging.

The development of the survey questions targeting engagement, value, relevance and teamwork was influenced by the work of Clem *et al*, (2014) who developed and validated an experiential learning survey, who used Carver's (1996) four pedagogical principles that help outline experiential learning[1] as their guiding framework. Predominantly this influenced the questions around value and relevance. This research also highlighted the importance of the project tutor in the effectiveness of student engagement. Lin & Huang's (2017) work focused on course engagement in general whilst Burch *et al* (2014) concentrated specifically on engagement and experiential learning. Both these influenced the identification of questions around student engagement. From a teamwork perspective, Lurie *et al's* (2014) 'reliable five –question' survey was partially applied along with Hughes & Jones (2014) work, who clearly define teamwork as a set of critical skills that can be developed and the need to focus on the process and provide 'meaningful feedback' during administered assessments.

In total there were 19 respondents, giving a 47.5% response rate. Below is a synopsis of the responses under each theme.

The full list of questions asked can be viewed here (https://sites.google.com/site/ditprojectbasedlearning/student-survey)

Theme 1: Increased level of Student Engagement

Students agreed (3.6 average) that they took the time to look over the class notes in order to ensure they understood the material before attending the class where teams were allocated time to work on the mathematical project. The comments captured such as *"it was necessary to complete the project"* and "*the notes contained the material I needed*" also gives somewhat of an indication of the importance and effectiveness of the class material. Nevertheless, some students stated they didn't bother at all with the notes with one identifying their Youtube preference! The project also encouraged attendance with a 4.5 average of students agreeing they always attended class during project time offering two main reasons. One centred on the point that it was the best way to get the work done and the other reason identified was that they didn't want to let their teammates down.

Students were in strong agreement (4.05 average) that the project gave them the opportunity for greater interaction with the material as it facilitated greater ease to ask questions to help understand the material than during normal class sessions. Students also agreed that they were fully absorbed in the activities and discussion enabled by these sessions.

Theme 2: The Learning Value and Relevance of Mathematics in the field of Engineering

Having practical examples was a consistent reason given as to why students strongly agreed (4.5 score) that learning mathematics in this format assisted in

[1] – authenticity, active learning, drawing on student experience and connecting that experience to future opportunity.

greater understanding of its relevance in their engineering discipline "*it gives a stepping stone into what you will be doing in engineering*". Students also agreed (4.2) that this approach helped to stimulate their thinking due to the hands on approach and the clear overlap with other engineering subjects. From a terminal examination viewpoint students were neutral (2.57) as to how this activity helped them prepare for their exams. Notwithstanding this however, some students who had agreed that it was difficult to see how the project based approach helped in exam preparation made comments that did not align to the Likert score for this question. It may in fact highlight a limitation with the use of reverse questioning.

Theme 3: Improving Communication and Teamwork skills

Students responded with a broad range of answers when asked to describe the role they played on their project team. From designing the poster presentation to solving the equations and plotting the graphs to project manager. It indicates somewhat the range of work tasks involved in a project of this nature and the breath of distribution amongst teams.

Students agreed (4.15) that they were encouraged to share their ideas and commented on the "*friendly"* environment in general indicating a healthy working team, although some students "*didn't understand what was going on"* and therefore got carried by the team. In relation to problem solving, students strongly agreed (4.4) that they made a consorted effort to overcome problems they encountered with comments such as "*the team pulled together*" emphasizing positive teamwork in action. Students disagreed (2.15) with the question that it was challenging to get their point across with comments such as "*we all listened to each other*", confirming positive teamwork.

Overall Student Experience

The main theme emerging when asked about the positive aspects of their experience was that the students really appreciated the opportunity to work on a real world problem. All of those who commented referred to this "*hands-on approach"*. Only one comment emerged around the appreciation towards getting the opportunity to practice presentation skills.

On the flip side, students found many aspects of the project difficult. There was a multitude of challenges captured with no dominant theme emerging. The issues captured were around those not attending class sessions, uneven effort of team members, the fact that a lot of the information provided wasn't specifically required to solve the problem and the variation across sources on the letters used to symbolise the same things caused confusion. Students found it difficult to wade through all the information and sift out the relevant parts. Another difficulty they captured was beginning the derivations required as they needed to apply information in the notes rather than just directly copying the derivations. The information captured in this section of the survey highlighted some additional skills that are not necessarily required or acquired in the traditional classroom.

RESULTS

As part of analysis we plotted the marks the students received for project 1 (first order differential equations) against the exam (figure 1) and project 2 (second order differential equations) against the exam (figure 2).

In figure 1 we see very little correlation between the project mark and the exam mark (R^2= 0.006). By the time of the second project we are seeing a much stronger correlation between the exam mark and the project mark (R^2=0.24) . One explanation of this may be that the project work requires a different skill set to standard exams but already by the second project, the more conscientious students are starting to develop this skill set, or at least or working hard at developing this skill set.

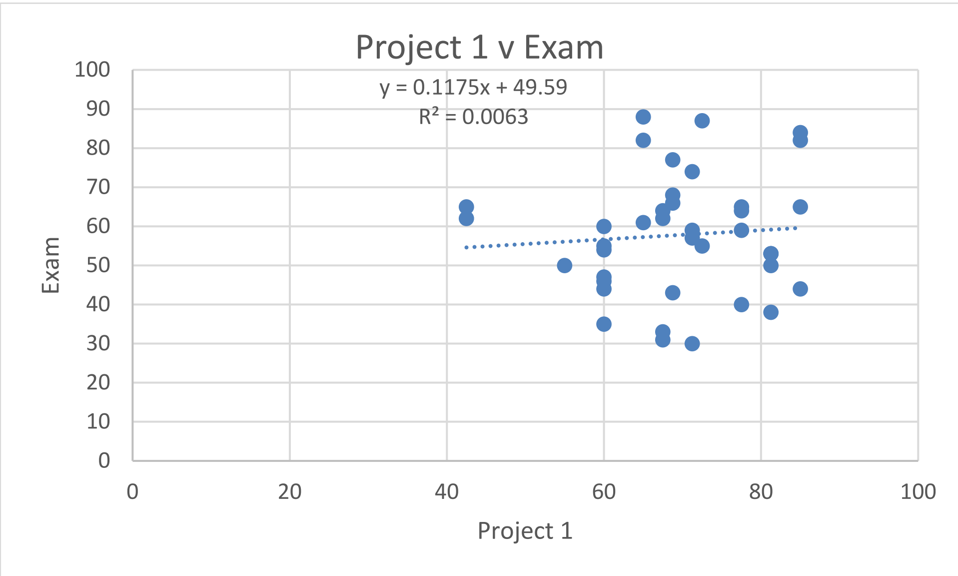

Figure 1: Project 1 (first order differential equations) against the exam

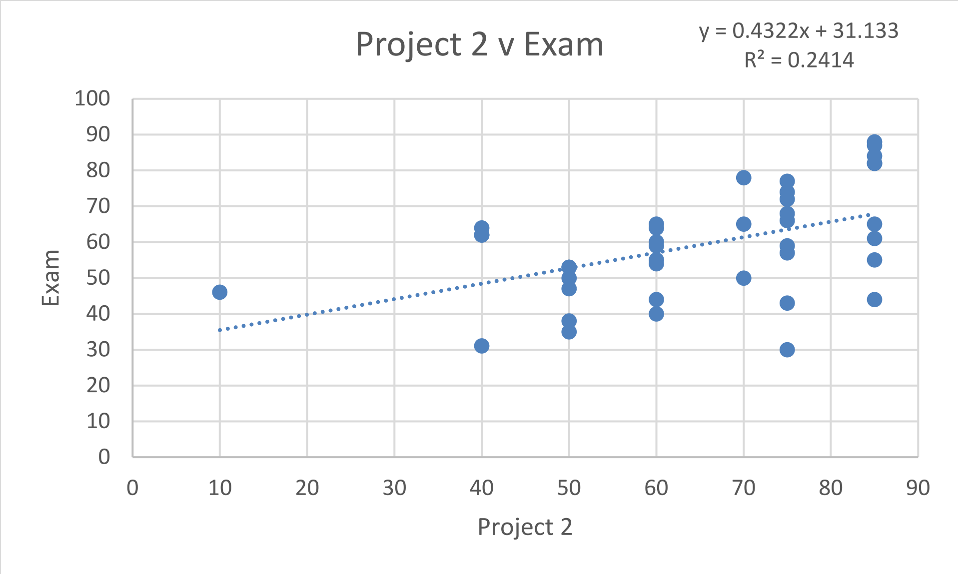

Figure 2: Project 2 (second order differential equations) against the exam

CONCLUSIONS AND FUTURE WORK

In our previous paper we looked at the introduction of a short project-based-learning element into a mathematics module for third-year Level 7 mechanical engineering students. This was well-received by students and resulted in greater engagement with examination questions on the same topic in comparison with previous years when this material was taught in a standard lecture environment. Student performance on these examination questions was also improved which is consistent with the findings of Reyes, M. R., Brackett, M. A., Rivers, S. E., White, M., & Salovey, P. (2012). However, the strongest students overall performed the best on these questions. Through the inclusion of a presentation element within the project assessment, the "*active verbal involvement*" of students advocated by Kwon (2000) was addressed, allowing students the opportunity to explain and justify their thinking, which is particularly important for engineering students while studying mathematics. This year we introduced a second project. We surveyed the students on four themes namely engagement, the learning value and relevance of mathematics in engineering, teamwork skills and the overall experience.

Students felt the project increased their engagement by giving them the opportunity for greater interaction with the material and agreed that they were fully absorbed in the activities and discussion enabled by these sessions.

The students agreed that having practical examples of engineering mathematics assisted in greater understanding of its relevance in their engineering discipline and that this approach helped to stimulate their thinking due to the hands on approach and the clear overlap with other material from other engineering subjects.

Students were generally positive about the value of working on real world problems, this was commented on by a large number of responders. There was a wide variety of difficulties in transitioning away from a structured environment to more open ended questioning

In terms of teamwork they were happy with the environment in general indicating a healthy working team, although some students admitted to being carried by the team. In relation to problem solving, students strongly agreed that concerted effort was made to overcome problems they encountered and with their supporting comments emphasized positive teamwork in action. This highlights the value of this approach and some of the limitations of the pure classroom approach.

Although it is challenging to develop projects of this type, the aim was to create a scenario that would be "experientially real to students and…take into account students' current mathematical ways of knowing" (Rasmusen & Kwon 2007). We will now introduce a further project into the module in third year and then proceed to introduce this approach to earlier years of the programme. In addition further quantitative analysis of the performance of the students is required and we hope to complement this with focus groups and in depth interviews.

Kaur (2010) concludes that in order to work towards excellence in teaching mathematics, dual responsibility between the teacher and learner needs to be encouraged. This classroom project, which encourages the students to take ownership for finding and presenting a practical solution to an engineering problem using mathematical theory, is a step in the right direction.

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