Identifying problematic mathematical topics and concepts for first year students

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Abstract

There is ample evidence to support the fact that students transitioning from post-primary to mathematics in higher education struggle with the requirement for an increased level of independent thinking and a move away from rote learning. As a result we are undertaking a large-scale project focussed on the development of interactive formative assessment techniques that aim to improve the teaching and learning experience of first year undergraduate mathematics modules.

As a first step, in order to identify the mathematical topics and concepts that are problematic for first year undergraduate students two surveys were developed, distributed and analysed: one aimed at students and another at lecturers.

Overall, students reported little difficulty with many of the mathematical topics and concepts presented in the survey. Further analysis revealed that students who completed Ordinary Level Mathematics in their final year at post-primary perceived that they had greater difficulty than those of their peers who took the Higher Level course. There was a marked contrast between the concepts identified by the lecturers and the students. Lecturers reported that many students struggle with basic arithmetic and algebra whereas students identify calculus as the main problem.

Background

It has been recognised that the transition from post-primary to higher education mathematics is problematic for many students. Evidence has shown that students grapple with first year mathematics, in particular the transition from rote learning to independent thinking. Gill and O'Donoghue (2007) investigated this well-known 'Mathematics problem' in an Irish context and it has been well documented in the UK and elsewhere (Lawson et al., 2012; OECD, 2004).

The National Forum for the Enhancement of Teaching and Learning in Higher Education in Ireland selected 'Teaching for Transitions' as their first enhancement theme. They identified the 'need to integrate approaches to building digital capacity across the sector' as one of the principles of their roadmap for enhancement in a digital world. (National Forum, 2015). In line with these policies they funded this project as one of their Teaching and Learning Enhancement Projects and it concerns the development of digital formative assessment techniques to improve the teaching and learning experience of first year undergraduate mathematics modules.

Black and William (1998) defined formative assessment as 'encompassing all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged' (p7-8).

This formative assessment steers students to learn, enables teachers to make informed teaching strategies and motivates students. 'This is assessment for learning.' (Bloxham, 2008 p. 14). Schoenfield (2015) defines formative assessments as 'examinations or performance opportunities the primary purpose of which is to provide student and teachers feedback about the student's current state, while there are still opportunities for student improvement'. In this project we aim to identify mathematical topics and concepts that are problematic for first year undergraduate students in Higher Education Institutes (HEIs) in Ireland, and to use this information in the development of formative assessment techniques and resources consisting of online activities and interactive tasks to improve student understanding of the topics identified. We will evaluate the effectiveness of these resources before developing a shared digital platform for all the HEIs in Ireland.

This paper reports on the process of identifying the problematic mathematical topics and concepts identified by students and lecturers.

Methodology

There are five HEIs in Ireland involved in this project; Athlone Institute of Technology (AIT), Dublin City University (DCU), Dundalk Institute of Technology (DKIT); Maynooth University (MU) and St. Patrick's College Drumcondra (SPD).

In order to identify the problematic concepts, two surveys were developed: one aimed at students and another at lecturers. A total of 460 students, attending mathematics modules in four of the HEIs involved in the project, completed the student survey in the spring of 2015. Most of the students were just finishing their first year in higher education. A small number (circa 20) were at the end of their second year.

Data was gathered using 23 Likert item questions followed by 7 open-ended questions (n= 460). The Likert items concerned mathematical topics from the typical first year undergraduate curriculum such as using the laws of logarithms to simplify expressions (Question 5) and finding and graphing the tangent to a curve (Question 15). An example sheet with worked examples of each type of question was given to the students to remind them of the ideas and skills involved in each question. Students were asked to rate their ability to (a) understand the ideas involved and (b) do the questions, using a 5-point Likert scale; strongly agree, agree, neutral, disagree, and strongly disagree. The open questions asked which topics caused the students most difficulty, what types of resources they currently use and what new resources they would like to be made available. There were 5 questions at the beginning of the survey asking students about their background; institute, course, gender, mature student and Leaving Certificate mathematics level taken. Appendix A contains a copy of the Likert item questions contained in the survey handed out to students.

All of the Likert survey data was entered into Excel and analysis of this data was performed using charts and chi-squared tests. The responses to the open-ended questions were imported to Nvivo where coding was completed.

The second survey was carried out to ascertain the main concepts, procedures and tasks that mathematics lecturers identify as being problematic, the types of resources that are already in use in relation to these topics and how they are disseminated. Data was gathered using a Google form that was shared amongst the HEIs in Ireland. 33 responses were received from mathematics lecturers in 15, out of a total of 21, HEIs in Ireland (n=33). All data was input into Nvivo for coding. Appendix B contains a copy of the questions asked.

These questions were selected and discussed by members of the project group who teach first year undergraduate mathematics.

Student Survey Results

Table 1 shows the breakdown of the students' backgrounds captured as part of the surveys.

Category	Gender	Mature	Leaving Cert Mathematics
Males	293 (65%)		
Females	160 (35%)		
Non Mature		368 (84%)	
Mature		69 (16%)	
Higher			282 (64%)
Ordinary			142 (32%)
Foundation			5 (1%)
Did not take			15 (3%)
Total number of responses	**453**	**437**	**444**

Table 1: Student background data.

The category of most interest is the mathematics level completed by students in their Leaving Certificate. This is the terminal post-primary state examination in Ireland. Students can opt for three different levels in this examination; Higher, Ordinary and Foundation Level. Entry requirements in mathematics for higher education depend both on the institute and the student's course of choice. In this paper we will examine any difference between responses from students who had completed Higher Level mathematics and those who completed Ordinary Level mathematics.

When we considered the responses to the Likert scale questions we noticed that the majority of students' answers were in the strongly agree (44%) and agree (33%) categories, with 15% of responses being neutral, and only 7% in the disagree category and 1% in the strongly disagree category. It seems that the vast majority of students do not encounter problems with this material or do not perceive that they have problems with the mathematics question types presented in the survey or perhaps are unwilling to rate their own ability poorly.

Figures 1 and 2 show the percentage of student responses, per Likert scale, for each of the questions. Figure 1 shows responses from students who took Ordinary Level mathematics, while Figure 2 shows Higher Level.

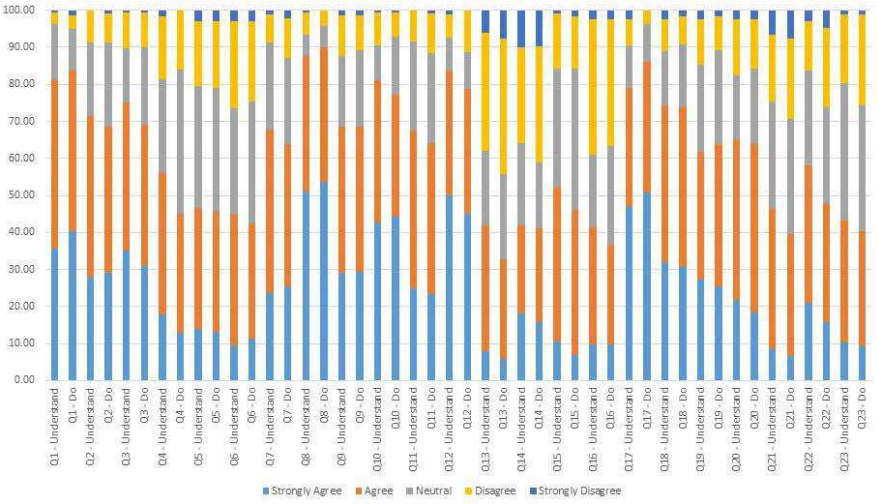

Figure 1: Survey Responses for students who took Ordinary Level Mathematics at Leaving Certificate. The % of student responses, per Likert scale, is shown for each of the 23 question types. The % for Understand and Do, for each question, are displayed in separate columns.

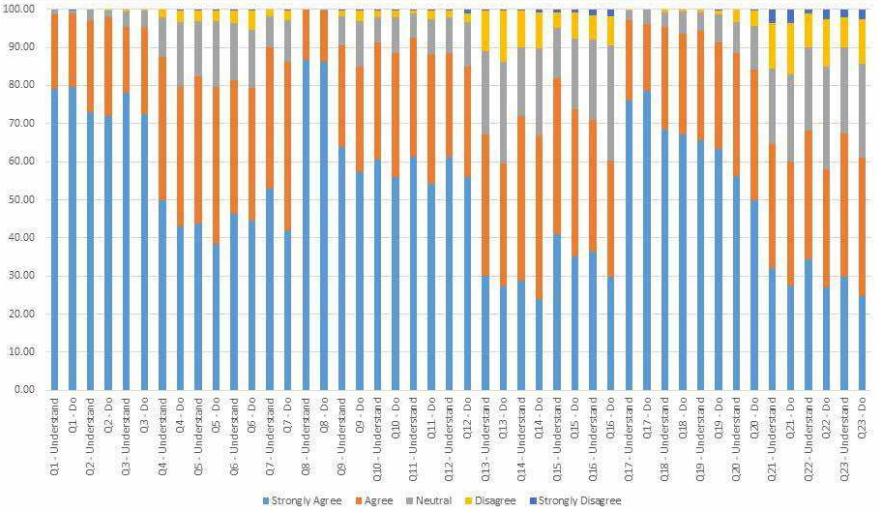

Figure 2: Survey Responses for students who took Higher Level Mathematics at Leaving Certificate The % of student responses, per Likert scale, is shown for each of the 23 question types. The % for Understand and Do, for each question, are displayed in separate columns.

Looking at the data in Figure 1, the question types that Ordinary Level students may perceive they have difficulties with are:

- Q4: Powers – using laws of indices
- Q5: Logs – using the laws of logarithms to simplify expressions
- Q6: Using the connections between logs and exponents
- Q14: Finding limits of functions using rules of limits
- Q16: Deciding whether a function is continuous or not
- Q21: Finding stationary points
- Q22: Optimisation (max/min) problem
- Q23: Graph sketching using derivatives

For each of the above question almost 20% or more of Ordinary Level students who responded to the question either disagreed or strongly disagreed that they could (a) understand and (b) do the question types. There were no question types where more than 20% of students who had taken Higher Level Mathematics responded in this way.

A chi-squared test, comparing Higher Level and Ordinary Level overall response rates per scale, showed a statistically significant difference between the responses from students who took Higher Level and those who took Ordinary Level ($p=0.004$). Note that the 5-point scale was reduced to 3 (agreement, neutral, disagreement) to ensure no item had less than 5 responses for this test. Thus students who had taken Ordinary Level mathematics at post-primary are more likely to perceive that they have difficulty with powers, logs, limits, functions and calculus than those who took Higher Level.

There were 414 student responses to the open-ended question asking students about the topics that caused most difficulty. Clearly students recognise that they have some difficulties, but as indicated above may not be willing to admit this. The responses to the open questions were coded according to the most reported topics and are shown in Figure 3 below.

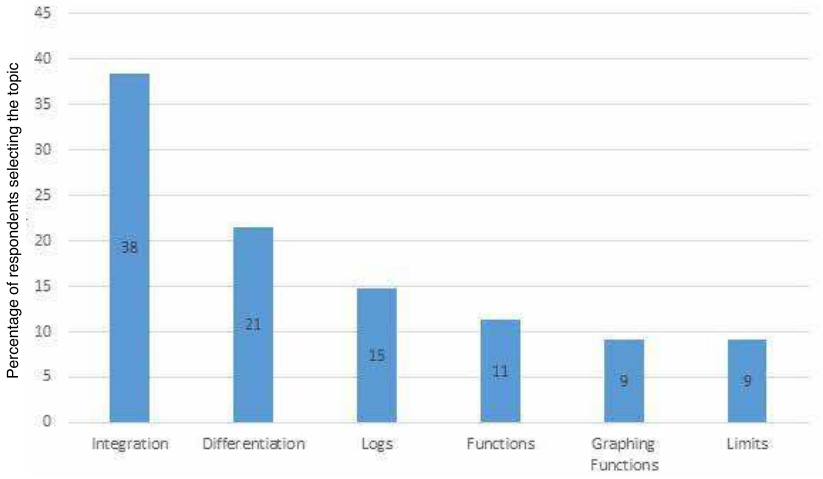

Figure 3: Percentage of responding students identifying these topics (students may have mentioned more than one topic). The % of responding students who identified the problem topics of integration, differentiation, logs, functions, graphing functions and limits in response to the open question on topics causing difficulty.

These topics are displayed in decreasing order of frequency reported; 159 students reported problems with integration, 89 with differentiation, 61 with logs, 47 with functions, 38 with graphing functions, 37 with limits. These are shown as a percentage of the overall number of students responding to the particular question (414). Some students mentioned more than one topic in their response to this question.

In the comments section many of the students referred to problems with topics they had not covered prior to attending higher level education:

'Optimization and graphing functions. It was one of the topics that was least covered in secondary school, therefore coming into university, studying maths at a higher level was a bit difficult to grasp. The formulas and when to use the right ones was the difficulty.' (Student 31)

The idea of selecting the right formula/method came up a few times:

'Graphing functions and interpreting data from graphs; unsure of what methods to use. Any topics not covered in leaving cert maths' (Student 135)

Lecturer survey results

Figure 4 shows the results of the coding.

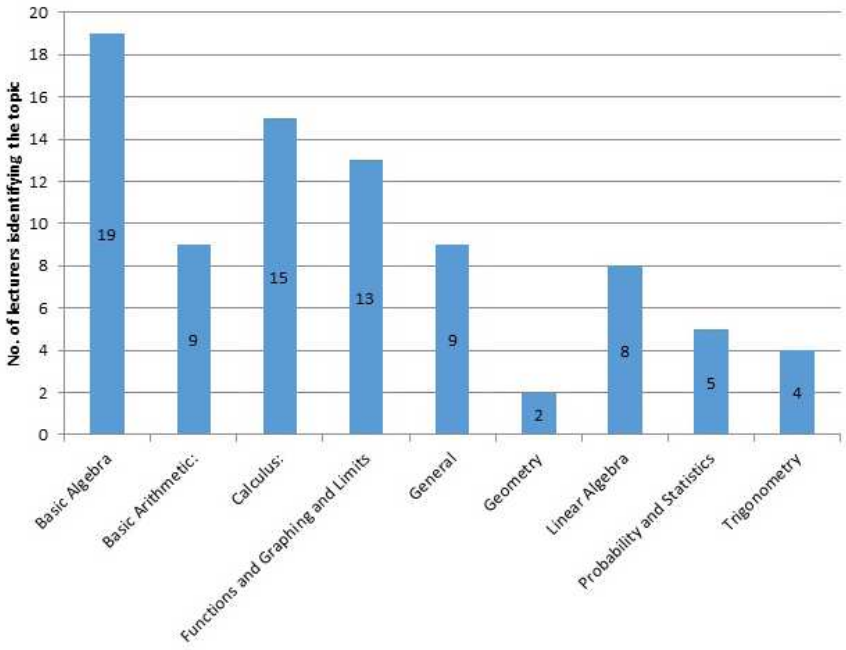

Figure 4: The number of lecturers who identified the displayed categories as problematic. Lecturers mentioned more than one topic.

19 lecturers reported that students have problems in basic algebra; this is the biggest concern for lecturers. This is in contrast to the reporting from students where Calculus is the most reported problem (see Figure 1 and 3).

In order to compare the lecturer and student open-ended data, the student data was recoded in Nvivo using the lecturers' codes. For example, of the 19 lecturers who reported problems in basic algebra, 13 (42%) specifically mentioned logs, indices, exponents or powers in their response. In stark contrast, only 74 from the 414 (18%) of students responding to the corresponding open question contained referred to either logs, indices, exponents or powers.

In relation to logs, one of the lecturers reported:

'Logarithms - general understanding and how to use rules to solve equations; application to experimental laws' (lecturer 8)

And a student who recognised the importance of understanding indices:

'Mostly ideas of topic confuse me most. Most difficult was algebraic manipulation of rules of indices. Without knowing these it was hard to do a lot of the maths' (Student 30)

Conclusions and way forward

In our survey, most students highly rated their ability to understand and do a variety of Firstt year undergraduate mathematics question types. There was a significant difference between the ratings provided by students who took Ordinary Level mathematics at Leaving Certificate and those who completed the Higher Level. 20% of students who completed Ordinary Level rate themselves poorly in powers, logs, functions and some aspects of calculus. This is in line with Breen et al. (2009), who noted that students who take Ordinary Level mathematics display lower levels of confidence than those who take Higher Level. In addition, Faulkner et al. (2014) have shown that students entering higher education with Ordinary Level mathematics perform less well than Higher Level students in the mathematics diagnostic tests. In Australia, Rylands et al. (2009) found that a student's mathematical level in secondary school is a good indicator of preparedness for third level mathematics.

We were surprised that the students did not report having problems with many of the topics listed in the Likert questions, however the lecturer data shows that lecturers across the country are seeing students struggle with basic mathematical skills as well as higher order concepts.

It may be that students having problems with differentiation etc. are actually having problems with the basic concepts, such as rearranging a formula, rather than differentiation itself.

Students and lecturers were also asked about resources they find useful and how they would like to see them delivered. The results of this data are not reported here but have been used to help shape the proposed delivery of the resources.

Now that we have identified the main concepts and topics and the preferred delivery mechanism we plan to develop online activities and tasks to promote understanding of these concepts, and evaluate the effectiveness of these resources.

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Appendix A: Student survey questions

List of the 23 question types students were asked to rate

- Q1: Systems of Linear Equations 2 variables
- Q2: Systems of Linear Equations 3 variables
- Q3: Rearranging Formula
- Q4: Powers – using laws of indices
- Q5: Logs – using the laws of logarithms to simplify expressions
- Q6: Using the connections between logs and exponents
- Q7: Solving inequalities
- Q8: Solving quadratic equations
- Q9: Graphing basic functions
- Q10: Interpreting graphs of basic functions
- Q11: Algebraic fractions

- Q12: Dealing with percentage change
- Q13: Finding limits of functions using graphs
- Q14: Finding limits of functions using rules of limits
- Q15: Finding and graphing the tangent to the curve
- Q16: Deciding whether a function is continuous or not
- Q17: Basic Differentiation
- Q18: Differentiation: the product rule
- Q19: Differentiation: the quotient rule
- Q20: Differentiation: the chain rule
- Q21: Finding stationary points
- Q22: Optimisation (max/min) problems
- Q23: Graph sketching using derivatives

Question 14 format and an example of this question type

14) Finding limits of functions using rules of limits

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
I understand the ideas in questions like this.					
I am able to do questions like this.					

14. Finding limits of functions using rules of limits

Example: Use the rules of limits to evaluate

$$\lim_{x\to 2} \frac{3x^2 - 4x + 9}{x + 5}.$$

Figure 5: Question example on Finding Limits and Rules of Limits

Example questions were given to the students as part of the survey to enable them identify the questions types.

Open Questions

1. What topics in first year mathematics caused you most difficulty? (Feel free to include topics that are not on the list above.) Please indicate whether it was the methods or the ideas involved that made the topic difficult for you
2. What topics in first year mathematics did you find most easy?
3. Please list any resources that you have found helpful for dealing with first year mathematics topics.

(Please give as much detail (e.g. web address) as possible.)

Books:_____

Handouts: _____

Videos: _____

Websites: _____

Other (please specify): _____

If possible, please indicate why the resources listed above were useful.

4. Are there any gaps in the resources available? Please explain.
5. Have you any advice on the resources you would like us to develop?
6. How should these resources be made available?

Print-based:

Videos:

Websites:

Other (please specify):

7. Any other comments

Appendix B: Lecturer survey questions

1. Institution Name
2. Department
3. Please list the names of first-year service Mathematics modules that you teach or have taught recently and the student groups/programmes involved.
4. What concepts in the first-year curriculum do your students find most difficult to understand?
5. What procedures and tasks in the first-year curriculum cause most difficulty for your students?
6. Please list any resources that you have found helpful to aid students with the difficulties outlined above.
7. If possible, please indicate why the resources listed above were useful.
8. Are there any gaps in the resources available? Please explain.
9. Have you any advice on the resources you would like us to develop?
10. How should these resources be made available?
11. Do you agree to participate in this study?