Object Polygonization in Traffic Scenes using Small Eigenvalue Analysis

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Abstract

Shape polygonization is an effective and convenient method to compress the storage requirements of a shape curve. Polygonal approximation offers an invariant representation of local properties even after digitization of a shape curve. In this paper, we propose to use universal threshold for polygonal approximation of any two-dimensional object boundary by exploiting the strength of small eigenvalues. We also propose to adapt the Jaccard Index as a metric to measure the effectiveness of shape polygonization. In the context of this paper, we have conducted extensive experiments on the semantically segmented images from Cityscapes dataset to polygonize the objects in the traffic scenes. Further, to corroborate the efficacy of the proposed method, experiments on the MPEG-7 shape database are conducted. Results obtained by the proposed technique are encouraging and can enable greater compression of annotation documents. This is particularly critical in the domain of instrumented vehicles where large volumes of high quality video must be exhaustively annotated without loss of accuracy and least man-hours.

Keywords: Dominant Point, Shape Representation, Shape polygonization, Small Eigenvalue

1 Introduction

Polygonal approximation is a convenient way to compress the digital representation of a closed shape curve. The polygonal approximation is achieved through detecting dominant points on the two-dimensional shape curve. Dominant points are some times termed significant points on the shape curve that are capable of representing the curvatures in the shape curve or contours. The local properties of a closed shape curve are preserved even after compression due to polygonal approximation. An ideal polygonal approximation should not be affected by linear transformations such as rotation, scaling and translation.

Polygon approximation methods can be applied to shape analysis, pattern classification, image understanding, 3D reconstruction, cartography and computer simulation applications. One example for computer simulation is emulation of traffic utilising the polygonal approximation of objects in the scene. Furthermore, for training machine learning systems in autonomous driving scenarios, labelled polygonized objects in traffic scenes play a major role in detection and recognition activities. An instrumented vehicle can collect up to 50TB of data per 8 hours of operation using 4-8 cameras. This video data then needs to be semantically annotated to be used. The number of objects per video frame will vary considerably but using a sample from a standard dataset [Cordts et al., 2016]. We observe an average of 7 and 11.8 instances per image of humans and vehicles that may require huge man-hours of annotation with fully described polygon boundaries. High quality fully annotated video with 30 frames per second will therefore generate significant quantities of annotation data.

Effective polygonal approximation that doesn't reduce the accuracy of modelling will greatly improve the efficiency of these processes. With this motivation, we address the polygonal approximation of objects in traffic scenes.

The rest of this paper is as follows: Section 2 gives an overview of prior work on polygonal approximate of shape curves. Section 3 describes the polygonal approximation technique and the need for post-processing to reduce duplication of vertices. Section 4 presents detailed experiments based on the proposed technique. Section 5 gives a conclusion on the proposed work.

2 Related Work

Polygonal approximation methods can be classified as sequential, merge based (heuristic) and split and merge methods [Morgera, 2012]. Alternatively, these algorithms can be classified [Madrid-Cuevas et al., 2016] on two criteria polygon approximation [Perez and Vidal, 1994] having number of dominant points to be fixed or specifying maximum allowable distortion after polygonal approximation. A brief review of current methods according to this classification is presented in this section.

One of the first heuristic polygonal approximation techniques [Ramer, 1972, Douglas and Peucker, 1973] proposed using an iterative method to divide a closed shape curve into a polygon with minimal number of vertices or dominant points. [Masood and Haq, 2007] proposed an heuristic method by elimination of break points (vertices of polygonised boundary) iteratively for polygonization of digital shape curve. Similarly, by eliminating one break point based on associated error value is proposed in [Masood, 2008]. [Carmona-Poyato et al., 2011] used an iterative method that optimises an objective function known as reference approximation for polygonal approximation of digital shape curve using the method proposed in [Perez and Vidal, 1994]. [Marji and Siy, 2003] proposed a method based on region-of-support of every boundary point capable of preserving the symmetry of the shape after polygonal approximation. Similarly, [Wu, 2003] proposed a method of detecting dominant points using region of support at every point of the boundary instead of considering a fixed length of support region and [Bhowmick and Bhattacharya, 2007] proposed geometric constraints for straightness properties of digital curve to detect the dominant points. Polygon approximation by specifying the maximum allowable distortion is derived based on sum of squares deviation criterion [Salotti, 2002] to detect the dominant point in a shape curve.

There are some methods that do not use any user defined parameters as criterion for finding optimal dominant points. The methods [Ramer, 1972, Douglas and Peucker, 1973, Carmona-Poyato et al., 2011] are modified using a non-parametric approach [Prasad et al., 2012] that utilises the theoretical bound of the deviation of the pixels obtained by the digitisation of a line segment. Another method [Madrid-Cuevas et al., 2016], which is also a non-parametric approach, that exploits the split/merge technique and a quality measure known as Figure-of-Merit. In spite of many algorithms for polygonal approximation, still there is scope for improvement in addressing near real-time polygonization of shape curves or boundaries of objects. The methods in [Bhowmick and Bhattacharya, 2007, Dinesh et al., 2005] exploit the straightness of the line segment to detect dominant points in the shape curve. In [Dinesh et al., 2005] employed a split based method that uses small eigenvalue for polygonization of a shape curve. It recursively splits a closed boundary curve considering a threshold for small eigenvalue. The value of the threshold and its computation is not specified. In split based techniques, duplication of vertices arises due to recursive splitting of boundary based on a decision criteria. This disadvantage shall be overcome using a merge based technique. Furthermore, suppression of collinear points after polygonal approximation assist in avoiding duplicated dominant points in a straight line segment.

3 Proposed Method

The proposed method utilizes a merging technique to detect dominant points of an object boundary in two dimensional space. In this technique, a decision is made at every boundary point to choose to merge the point in the set of dominant points to form a vertex of the approximating polygon. Theoretically, the eigenvalue associated with a straight line is equal to zero [Guru et al., 2004, Dinesh et al., 2005]. However, the eigenvalue associated with a digital line will be slightly more than zero. The proposed method uses small eigenvalue [Tsai et al., 1999, Guru et al., 2004] of the co-variance matrix of boundary points of an object as a decision criterion.

3.1 Small Eigenvalue Computation

The small eigenvalue is computed by adopting the methodology proposed in [Tsai et al., 1999]. The choice of parameters is carried out similarly as specified in [Guru et al., 2004]. Let $B = \{b_k(x_i, y_i) | i = 1, 2, ..., N_k\}$ be N number of boundary pixels belonging to k_{th} boundary of an object, the small eigenvalue λ of the covariance matrix of B is computed as following:

$$\lambda = \frac{1}{2}\left[C_{11} + C_{22} - \sqrt{(C_{11} + C_{22})^2 - 4C_{12}^2}\right] \quad (1)$$

where

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

is a covariance matrix of B and the coefficients in the matrix are computed as follows:

$$C_{11} = \frac{1}{N}\sum_{i=1}^{N}(x_i^2 - c_x^2) \quad (2)$$

$$C_{12} = C_{21} = \frac{1}{N}\sum_{i=1}^{N}(x_i.y_i - c_x.c_y) \quad (3)$$

$$C_{22} = \frac{1}{N}\sum_{i=1}^{N}(y_i^2 - c_y^2) \quad (4)$$

c_x and c_y are the mean values of the x and y co-ordinates respectively.

3.2 Polygon Approximation Algorithm

The input for the proposed technique is a set of boundary/contour pixels b_k of an object in the traffic scene and the output is a set of dominant points P resulting in polygonal approximation of the object. The algorithm for the proposed polygonal approximation of a given boundary is as follows:

Algorithm 1 Polygonal Approximation

Require: $B = \{b_k(x_i, y_i) | i = 1, 2, ..., N_k\}$
Ensure: $P = P_{k1} \ldots P_{kM}$
$StartPoint \leftarrow 0$
$Index \leftarrow 0$
for $i \leftarrow 1$ to N_k **do**
    $Points \leftarrow b_k[StartPoint : i + 1, :]$
    Compute λ for $Points$ using Eqn.(1)
    if $\lambda > Threshold$
        $StartPoint \leftarrow i$
        $P[Index] \leftarrow b_k(x_i, y_i)$
        $Index{+}{+}$
    end if
end for

3.3 Post-processing of vertices

The vertices obtained after applying the polygonal approximation on the boundary pixels will have closer vicinity at curvature extreme portions of the boundary pixels as shown in Figure. 1(a). The points shown in the highlighted portion of Figure. 1(c) are nearly collinear. It is necessary to reduce such points by retaining certain points that are not collinear. This can be achieved by re-applying the same polygonal approximation method mentioned in 3.1 with the smallest threshold as a parameter for small eigenvalue as mentioned in [Guru et al., 2004]. This results in effective suppression of collinear points in the curvature extreme region as shown in Figure. 1(d).

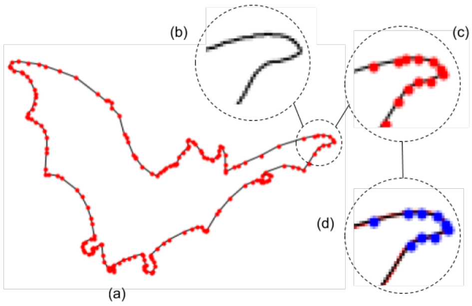


Figure 1: Illustration of collinear points suppression after polygonal approximation

4 Experiments

To validate the efficacy of the proposed polygon approximation technique, we have conducted experiments on the standard MPEG-7 shape database and on semantic segmentation images from the validation set of the CityScapes dataset [Cordts et al., 2016].

According to our literature study, only certain contours like hammer, plane, chromosome, leaf, semicircles and infinity are used. In our experiments, we have also considered more complex shapes and the result on a subset of data is shown in Figure. 2. The polygon approximation technique is evaluated in terms of Compression Ratio (CR) and Integral Square Error (ISE). Compression Ratio (Eqn. 5) is defined as the ratio between the number of points in the contour (N) and the number of points(P) obtained after polygonal approximation.

$$CR = \frac{N}{P} \tag{5}$$

The Integral Square Error ISE is the sum of square error and is defined as follows:

$$ISE = \sum_{i=1}^{N} e_i^2 \tag{6}$$

where e_i is the distance between the original contour point and the approximated line segment.

In assessment of semantic segmentation, the Jaccard index, also known as Intersection over Union (IoU), is popularly used. This is a statistical measure used to define the proximity of two sample sets that helps in understanding the approximation or prediction quantitatively. According to our literature review, Jaccard index is not found as a metric to evaluate the proximity between the original shape curve and its respective polygonal approximation. However, in semantic segmentation [Badrinarayanan et al., 2015], the Jaccard index effectively explains the proximity of actual regions and their respective predicted regions by the segmentation techniques. Therefore, in our experiments, we use Jaccard index to find the ratio of the symmetric difference between the original shape curve and its respective polygonal approximation to their union. Hence, Jaccard index explains the empirical proximity between a shape curve and its polygonal approximation. A shape curve, S, is represented by a set of boundary points and its respective polygonal approximation, P, yields a set of dominant points that effectively preserves the curvature information. The Jaccard index, $J(SR, PR)$, is given by:

$$J(SR, PR) = \frac{SR \cap PR}{SR \cup PR} \tag{7}$$

where SR, is the region of original shape contour (S) and PR is the region of polygonal shape approximation (P) using the proposed method.

4.1 Experiments on MPEG-7 Shape Database

Several methods [Prasad et al., 2012, Carmona-Poyato et al., 2011] used the Compression Ratio (CR) and Integral Square Error (ISE) as the quality metrics to present the efficacy of their methods on a small set of contours rather than the whole dataset. In our experimentation, we have considered all the objects mentioned in Table. 2 in the dataset for evaluation of the performance of the proposed technique. Figure 2 shows the dominant point extraction on some of the shapes – spring, bat, elephant and guitar. The above mentioned shapes have curvature extremes in their shape. Hence, these shapes are considered for visualisation. It can be observed from figure 2 that the polygonal approximation of all the shapes is effective in preserving the curvature extreme of original shape curve. Table. 1 shows the number of contour points (CountourPts), approximated dominant points (M), compression ratio (CR) and integral square error (ISE) of the polygonal approximation and its post-processing for the shapes shown in figure 2. Table. 2 shows overall mean of Jaccard Index (JD), Compression Ratio (CR) and Integrated square error (ISE) of polygonal approximation and its post-processing for every class of shape in the MPEG-7 dataset. It can be observed that the performance of the proposed technique is efficient in terms of CR and JD on most of the object classes in the MPEG-7 dataset. However, for the object class $Glass$, performance of the proposed technique decreases in terms of the Jaccard Index after post-processing. Figure. 4(b) shows the drawback of the proposed post-processing step where one of the dominant points is removed resulting in deformation of the shape curve after polygonal approximation.

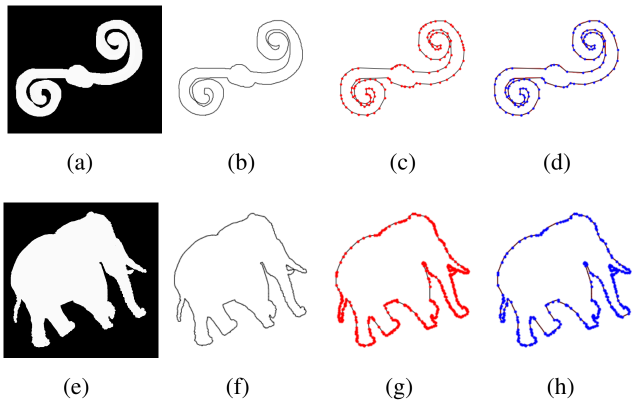


Figure 2: Dominant point extraction on some of the shapes in MPEG-7 dataset possessing curvature extreme. 1^{st} column: Input images, 2^{nd} column: contour extraction output, 3^{rd} column: dominant points and 4^{th} column: dominant points after post processing.

Input	ContourPts	M	JD	CR	ISE	M^P	JD^P	CR^P	ISE^P
Spring-1	1948	122	0.956	15.967	1.926	61	0.864	31.934	5.493
Elephant-1	3317	386	0.99	8.593	1.81	182	0.982	18.225	6.093

Table 1: Metric on shapes considered in figure 2 from MPEG-7 dataset. $*^P$ represents post-processed values

4.2 Polygonization of Semantically Segmented Objects

In order to show the applicability of the proposed technique in real-time visual analysis and automatic annotation tasks, we have conducted experiments on semantic segmentation results on the CityScapes Dataset. We present the results of our polygonal approximation on object shapes yielded by semantic segmentation [Badrinarayanan et al., 2015]. Table. 3 shows the Jaccard Index and compression ratio of polygonal approximation and after its post-processing. Figure. 3 illustrates polygonization of objects in a traffic scene. The performance

Input	JD	CR	ISE	JD^P	CR^P	ISE^P
Apple	0.98	16.44	5.51	0.98	17.75	5.64
Bat	0.98	10.31	2.17	0.98	15.23	3.08
Beetle	0.95	7.83	2.34	0.94	11.47	3.04
Bell	0.98	12.21	1.48	0.97	15.88	1.86
Bird	0.98	12.72	3.49	0.98	15.92	4.79
Bone	0.95	56.42	226.98	0.92	60.18	252.98
Bottle	0.96	20.8	0.65	0.95	26.24	0.82
Brick	0.97	16.28	1.88	0.97	19.72	2.57
Butterfly	0.98	8.99	2.05	0.98	13.22	2.64
Camel	0.97	11.48	1.36	0.97	13.95	1.77
Car	0.97	12.28	1.21	0.97	13.91	1.21
Carriage	0.95	11.18	1.27	0.94	12.31	1.33
Cellular Phone	0.98	31.55	0.88	0.97	39	0.86
Chicken	0.97	8.76	2.08	0.97	11.36	2.57
Children	0.96	10.31	0.57	0.96	12.96	0.65
Chopper	0.96	12.85	1.78	0.96	14.56	2.03
Classic	0.99	15.56	1.72	0.98	18.8	1.92
Comma	0.98	49.99	15.25	0.98	55.16	17.71
Crown	0.94	8.2	2.35	0.94	9.7	2.69
Cup	0.98	20.53	1.84	0.98	23.72	2.31
Deer	0.93	5.57	2.28	0.92	8.33	3.53
Device	0.98	23.17	29.95	0.96	40.04	28.72
Dog	0.97	10.25	1.47	0.97	13.34	1.85
Elephant	0.96	9.76	2.34	0.96	13.84	3.74
Face	0.99	15.19	3.44	0.99	18.31	4.04
Fish	0.96	11.42	2.01	0.96	14.24	2.41
Flatfish	0.99	15.27	5.65	0.99	20.81	10.67
Fly	0.94	9.83	3.44	0.93	12.85	4.78
Fork	0.94	26.61	2.89	0.93	32.43	4.14
Fountain	0.97	14.98	1.73	0.97	17	1.98

Input	JD	CR	ISE	JD^P	CR^P	ISE^P
Frog	0.98	12.57	3.19	0.98	14.93	3.98
Glass	0.96	83.62	30.33	0.85	88.95	31.8
Guitar	0.97	15.9	5.73	0.97	18.95	7.81
Hammer	0.91	30.51	4.51	0.87	34.46	4.95
Hat	0.97	18.65	1.37	0.97	21.68	1.48
Hcircle	0.98	38.48	11.29	0.98	40.86	12.06
Heart	0.99	29.11	39.3	0.99	30.55	36.39
Horse	0.98	11.19	1.91	0.98	16.17	2.7
Horseshoe	0.93	13.87	1.45	0.92	16.71	2.21
Jar	0.97	13.04	3.18	0.97	14.95	3.81
Key	0.98	19.55	2.21	0.98	22.68	2.46
Lizzard	0.97	9.4	2.35	0.97	13.76	3.43
Lmfish	0.97	10.3	2.15	0.97	14.27	3.22
Misk	0.99	20.1	6.41	0.99	22.2	8.85
Octopus	0.96	13.25	2	0.95	14.91	2.19
Pencil	0.95	39.63	12.77	0.95	48.3	16.54
Personal Car	0.99	18.14	3.87	0.98	20.72	4.49
Rat	0.95	12.74	1.66	0.95	14.37	1.83
Ray	0.99	11.36	5.39	0.98	16.29	8.73
Sea Snake	0.95	13.53	4.3	0.95	18.43	7.4
Shoe	0.99	19.79	1.93	0.99	23.02	2.37
Spoon	0.96	17.35	9.96	0.95	20.98	23.81
Spring	0.92	13.13	1.52	0.91	15.75	1.99
Stef	0.9	10.44	1.73	0.9	12.15	2.19
Teddy	0.98	11.86	1.88	0.98	13.55	2.26
Tree	0.97	14.38	3.38	0.96	17.89	4.84
Truck	0.95	9.27	1.31	0.95	11.25	1.33
Turtle	0.98	8.51	1.66	0.98	12.42	2.3
Watch	0.97	20.09	1.04	0.97	27.41	1.44
–	–	–	–	–	–	–

Table 2: Metric extracted on MPEG-7 dataset

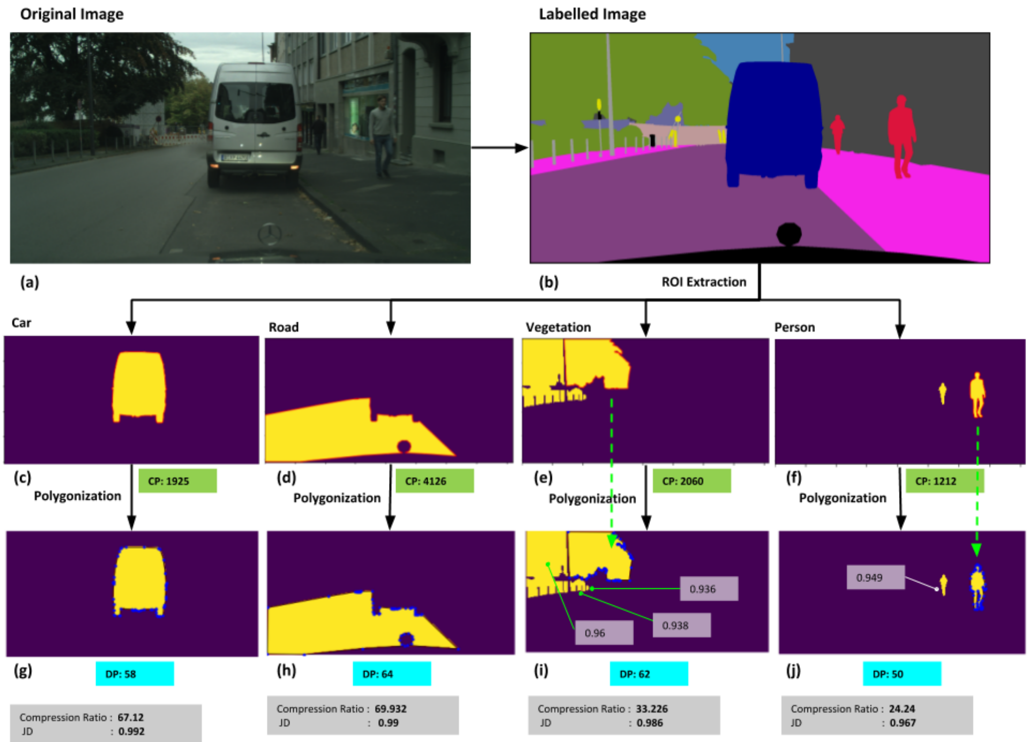
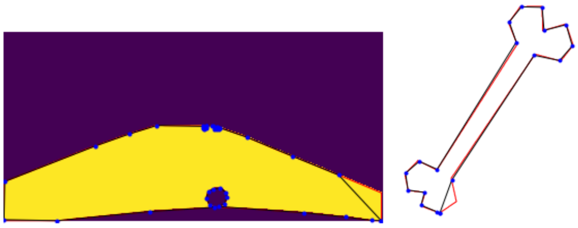


Figure 3: Polygonal Approximation of objects in a traffic scene

of the proposed algorithm for most of the object classes in the CityScapes dataset is considerably more effective in compressing and retaining the shape of the digital curve. From Table. 3, it can be observed that the proposed technique is effective on most of the traffic object classes however on the object class $Pole$ and $Sidewalk$ are slightly low. This is due to post-processing step which results in extra compression of near rectangular shapes and shapes of small objects which possess minimal boundary points in the boundary curvature. The application of a post-processing step on certain shapes, which are near regular shape, leads to filtering out (removing) dominant points that are important to preserving the curvature extreme of the shape curve. Figure. 4 shows two examples that highlight the drawback of this post-processing method. It can be clearly observed that one of the dominant points is filtered out affecting the shape of the curve which results in clamping of the shape structure. Figure. 4(a) shows the clamping effect on the object class $Road$. One of the dominant points which preserves the curvature extreme has been filtered out and resulted in deformation of the shape after polygonal approximation.



(a) (b)

Figure 4: Illustration of a drawback of the proposed post-processing method

5 Conclusion

A simple yet effective technique that exploits the merge based method using small eigenvalue of a straight line for polygonal approximation has been proposed. This technique helps in retaining the curvature extreme of the shape contour with minimum points. We have shown that Jaccard Index is an effective metric to assess polygonal approximation. The proposed technique has been applied on semantic segmentation results on CityScapes dataset for representation of object shape in the scene which enables effective shape recovery of the objects during visual analysis and annotation tasks. Hence, the proposed technique plays a key role in automatic annotation tasks involved in ADAS (Advanced driver-assistance systems) and autonomy applications.

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Object Class	JD	CR	JD^P	CR^P
Bicycle	0.923	13.215	0.91	14.775
Building	0.921	23.853	0.894	25.704
Bus	0.943	23.495	0.933	25.518
Car	0.949	19.083	0.942	21.028
Fence	0.917	32.323	0.884	34.157
Motorcycle	0.922	12.662	0.914	14.016
Person	0.924	13.008	0.913	14.863
Pole	0.779	51.683	0.771	52.317
Rider	0.92	12.113	0.909	13.947
Road	0.947	46.548	0.933	49.814

Object Class	JD	CR	JD^P	CR^P
Sidewalk	0.915	32.875	0.872	35.567
Sky	0.931	22.503	0.907	24.62
Terrain	0.909	30.098	0.875	32.461
Traffic Light	0.918	16.512	0.885	17.686
Traffic Sign	0.92	26.681	0.91	27.385
Train	0.93	26.54	0.924	28.243
Truck	0.936	22.195	0.932	24.339
Vegetation	0.925	21.259	0.902	23.111
Wall	0.916	35.176	0.886	36.722
–	–	–	–	–

Table 3: Metric extracted on Object Classes in CityScapes Dataset

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