Improving the Problem-solving Potential (PsP) of Highly-able Transition Year Students through Participation in a Mathematics Intervention

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Declaration

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Abstract

Title: Improving the Problem-solving Performance (PsP) of Highly-able Transition Year Students through Participation in a Mathematics Intervention.

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There is widespread agreement in education about the existence of a cohort of students with ability or potential above that of their peers. In this thesis, they will be referred to as “highly-able” students, and the focus will be on those students whose high ability is in mathematics. Contrary to popular belief, highly-able students have additional educational needs to be catered for. In Ireland, classroom differentiation remains the sole in-school measure available, despite post-primary mathematics education undergoing large-scale changes over the past decade. In the mid-2000s, research highlighted the performance of Ireland’s top students in international mathematics assessments as an area of concern, yet these have failed to improve and, in some cases, have actually disimproved.

When the additional needs of highly-able students are not met, they are at risk of negative traits of perfectionism, under-achievement, behavioural problems in class, and so on. This research focussed on addressing the additional educational needs of highly-able students in Ireland by targeting an improvement in their mathematical Problem-solving Potential (PsP), a newly-designed triad construct with three influencing factors: problem-solving skills, mindset, and mathematical resilience. To facilitate the development of potential, external resources and supports are needed to act upon and encourage the abilities and traits of an individual. In order to achieve that, a mathematics intervention was designed.

The intervention utilised collaborative problem-solving as the pedagogical approach, following the Collaborative Problem-solving (CoPs) model specifically designed for this research, to outline the problem-solving process in a group dynamic. Although it is sometimes presumed that highly-able students prefer to work alone, research has found that they are willing collaborators with like-minded peers. The intervention was implemented across six cohorts of students, accessed through the Centre for Talented Youth Ireland, over a three-year period, and resulted in an increased PsP for 97% of participants.

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1. Introduction

1.1. Background to this research

In education, there is a general consensus that there are students in regular classrooms with the potential to display high ability (Riedl Cross et al., 2014; Tomlinson, 2005; Van Tassel-Baska & Stambaugh, 2005). However, there is little consensus as to what to call these students, with labels such as "*gifted*", "*talented*", and "*gifted and talented*" being fraught by often contradictory definitions or negative connotations (Eyre, 2017; Smedsrud, 2020). The multitude of labels is also accompanied by a melee of qualification criteria, which range from parental/teacher nomination to the stringent adherence to cut-off points from standardised test scores (O'Reilly, 2014; Peters et al., 2020). Regardless of the label or qualification criteria, however, it is believed that this group of students benefits from additional support in education, with methods such as acceleration, enrichment and differentiation used throughout the world (Ledwith, 2013; van Tassel-Baska, 2000). Whilst early definitions focussed on a generalised ability (Terman, 1921; Terman et al., 1926), prominent theories now refer to domain-specific abilities or "*multiple intelligences*", with mathematics recognised within these (Gagné, 2004; Gardner & Hatch, 1989; van Tassel-Baska, 2005). In an Irish context, the National Council for Curriculum and Assessment (NCCA) utilises the term "*exceptionally-able*" to refer to those students that "*require opportunities for enrichment and extension that go beyond those provided for the general cohort of students*" (NCCA, 2007, p. 7). However, the term "highly-able" has gained traction in recent years, partly due to its less conflicting past in comparison to other aforementioned terms with their conflicting definitions (Mellroth, 2020; Montacute, 2018; Sutherland, 2011), and as a result, this was the term chosen for this project, as it best encapsulates the potential of the students in this study.

Mathematics at second-level in Ireland is divided into two cycles: three years of study in the Junior Cycle, which culminates with the Junior Certificate examinations; and the Leaving Cycle, with two years of study leading to the Leaving Certificate examinations, but which may be preceded by the optional fourth year known as Transition year. While Irish students have regularly performed above average in mathematics in the OECD international assessments, the percentage of Irish students recognised as "*top performers*" has been consistently below average (OECD, 2006; OECD, 2010; OECD, 2014; OECD, 2016; OECD, 2019). The top Irish students have also underperformed on the TIMSS assessments, relative to the performance of Irish students overall and also relative to similar countries in this percentile (Mullis et al., 2020; Perkins & Clerkin, 2020). The primary strategy employed in Ireland to cater for highly-able students in school is differentiation by the classroom teacher (CIDREE, 2010). However, while a large percentage of teachers reported differentiating their lessons on a regular basis, they perceived mixed levels of effectiveness in this approach (Riedl Cross et al., 2014). Furthermore, a report found that teachers believed highly-able students were unchallenged throughout the Junior Cycle (Shiel & Kelleher, 2017). While out-of-school programmes do exist, for example as operated by the Centre for Talented Youth Ireland (CTYI) and the Maths Enrichment Programmes (https://www.irishmathstrust.com/mathematics-enrichment), these have certain limitations attached, such as travel distances, cost (for CTYI only), student selection (certain programmes require high scores on standardised testing), etc.

Mathematics is best learned through the construction of knowledge in the learner's own context (Cobb, 1994), and problem-solving is an important educational tool in allowing this to occur (J. W. Wilson et al., 1993). While 'problem-solving' has been used through the years to mean a variety of things, including a misrepresentation as drill-based exercises, it is perhaps best described as "*…a process…by which an individual uses previously learned knowledge, skills and understanding to satisfy the demands of an unfamiliar situation*" (Krulik & Rudnick, 1989, p. 5). Well-chosen problems are fundamental to the success of problem-solving. Krulik & Rudnik (1989) outlined that, for a problem to be successful, the following criteria should be found: the student must want to solve the problem; the student should experience failure during their attempts; and this failure should lead the student to attempt new methods. Polya (1945) was instrumental in the development of mathematical problem-solving as an educational tool, and created a set of heuristics for the problem-solving process that remain relevant and influential in modern mathematics. Mason et al (1985, 2010) built upon the work of Polya to introduce another structured approach to problem-solving, drawing attention to the importance of being "*stuck*". However, both of these approaches focus on problem-solving as a more solitary pursuit. Collaborative problem-solving, where students work in groups on the same problem, requires students to develop their individual problem-solving skills, while simultaneously developing communication skills in order to function in a group with common learning objectives and goals (Häkkinen et al., 2017). While it is often believed that highly-able students prefer to work alone (Ruf, 2003), there is evidence to suggest that they are willing participants in collaborative activities when they feel their work is appreciated by their peers (Davis & Rimm, 1989; French et al., 2011).

The early stages of this project were shaped by the research outlined in this section, which subsequently developed the scope of the project, discussed in the next section.

1.2. Scope and context of this research

The failure to address the additional needs of highly-able students and subsequent failure to encourage the development of their ability may be of significant loss to the student and the society around them. In the opening to their 'draft guidelines for teachers', the NCCA addressed the myth that highly-able students will succeed even without additional support, and suggested that, in fact, they may become "*bored and disruptive*" if their additional needs are not catered for (NCCA, 2007, p. 7). They elaborated that the further highly-able students are able to progress in their education without encountering challenging work, the more they may struggle to cope when they finally are challenged. Within the classification of highly-able students there exists a sub-group known as underachievers and, while the reasons for this underachievement are vast and numerous (Çakır, 2014; Schultz, 2002; White et al., 2018), their existence suggests that more can be done to cater for their needs (NCCA, 2007; Rubenstein et al., 2012; Wellisch & Brown, 2012).

The prevalence of 'perfectionism' amongst highly-able students is often detrimental to their development, whereby they avoid challenging work for fear of failure or to preserve the façade of 'looking smart' (Council of Curriculum Examinations and Assessment (CCEA), 2006; Mofield & Parker Peters, 2018; O'Reilly, 2014). O'Reilly continues that these students may benefit further by studying challenging content in an environment that has no bearing on their grades, thus alleviating certain concerns. Research into mindsets has been spearheaded through the years by Carol Dweck (Dweck, 2006; Dweck et al., 1995a; Dweck & Leggett, 1988). Originally developed as implicit theories of intelligence, mindsets are categorised predominantly along a scale between fixed or growth. A person with a growth mindset believes that intelligence is malleable and that they learn through effort and determination; they value learning, including from mistakes; and they respond positively to challenge. Conversely, a person with a fixed mindset believes that intelligence is a fixed construct that cannot be changed through greater effort; they prefer to 'look smart'; and they avoid challenging work that may tarnish how smart they appear. Blackwell et al (2007) found that the implementation of a mindset intervention improved motivation amongst students in a mathematics classroom. Johnston-Wilder et al (2010b) defined mathematical resilience through three domains: *growth*, alluding to the aforementioned concept of a growth mindset; *value*, which stipulates that those who value the study of mathematics to their lives are more mathematically resilient; and *struggle*, which explores the ways through which mathematically resilient people respond when they are challenged. The encouragement of attributes related to mathematical resilience and growth mindsets, such as the acceptance of struggle and mistakes as necessary for learning within mathematics, are also believed to be of benefit to highly-able students and may attempt to address the difficulties brought about by perfectionism (Yeager & Dweck, 2012).

Providing highly-able students with the opportunity to study previously-learned material or skills in greater depth is often categorised as "enrichment" in gifted education (Siegle et al., 2013; Smithers & Robinson, 2012), and, in mathematics, problem-solving holds the potential for such an opportunity. The NCCA wrote that "*problem solving tasks activate creative thinking*" and that "*reasoning mathematically about tasks empowers learners to make connections within mathematics and develop deep conceptual understanding*" (NCCA, 2013, p. 11). A student engaging in problem-solving should encounter a challenge during the process, which therefore creates an environment in which those with characteristics of a growth mindset and high mathematical resilience will thrive. Collaborative problem-solving has a positive impact on cognitive, metacognitive and affective domains in students, with benefits to students in their critical thinking, communication, and reasoning (Diezmann & Watters, 2001; Gokhale, 1995). While this section has presented some of the potential benefits of a growth mindset, mathematical resilience and problem-solving to highly-able students, it has also highlighted some risks involved where these concepts are not developed. These risks, amongst others, formed the development of the research problem, discussed in the next section.

1.2.1. Research Problem

The risks of failing to address the additional needs of highly-able students were outlined in the previous section, with perfectionism, under-achievement and educational stagnation being just some of the concerns that may have long-lasting and damaging effects. In the Irish context, highly-able students currently rely on classroom differentiation and out-of-school programmes, despite the former being fraught by inconsistency, and the latter affected by costs and travel distances for students. Irish students' results on international assessments, such as PISA and TIMSS, have been consistently above average over the past 30 years, and the performance of Ireland's weakest students in this period have improved dramatically. However, Ireland's top students have consistently performed below average in the PISA rankings, and their results on the TIMSS assessments have disimproved between 1995 and 2019. Furthermore, Irish students competing in the International Maths Olympiad, a competition for the top mathematics students around the world, have underperformed (McGrath, 2017). Teachers (Riedl Cross et al., 2014), parents (O'Reilly, 2010) and students (McGrath, 2017; O'Reilly, 2010) have overwhelmingly stated that the additional education needs of highly-able students in Ireland have not been catered for. These concerns emerged at an early stage of this research study, which shaped the research towards addressing the additional needs of highly-able mathematics students.

The characteristics of a growth mindset are beneficial to highly-able students: the acknowledgement that mistakes are valuable to learning may aid in the struggles of perfectionism; and a positive response to adversity is important when they eventually encounter challenging work. This final attribute is further influenced by the concept of mathematical resilience, which relies upon a student's value of mathematics, their mindset, and the extent to which they believe struggle is important to the learning of mathematics. While there is a plethora of research on mindsets, there is no current research on the mindsets of highly-able students in Ireland; nor is there research on the mathematical resilience of this student cohort, although mathematical resilience is a research area in its early stages.

1.2.2. Research Aim

After a rigorous review of literature in the aforementioned areas, this research took on the aim of investigating and improving highly-able students' problem-solving skills, mathematical resilience, and mindsets – a newly-developed triad construct of 'Problem-solving Potential' (PsP) created during this study. In order to facilitate this aim, an intervention was developed for highly-able mathematics students in Transition Year in post-primary school in Ireland (aged approximately 16 years). The intervention was rooted in constructivist and socio-constructivist principles, with a primary teaching style of collaborative problem-solving. Characteristics related to a growth mindset and mathematical resilience were encouraged throughout the intervention, particularly at points in which students experience struggle, seeking to address the fear of failure or avoidance of challenge for some highly-able students. The employment of collaborative problem-solving as the foundation for the intervention aims to develop students' abilities to think critically and mathematically, while also highlighting the importance of peer-to-peer collaboration and communication. The need to interact as part of a group throughout the problem-solving process further challenged these students to embrace a growth mindset and engage with the process, without fearing the judgement of others if they failed to immediately supply the correct answer. The intervention was designed for students in Transition Year partly due to the flexibility of content for these students, but the selection of this year further addresses the previously-mentioned concerns of O'Reilly (2014), as it is removed from the pressure of grades related to both the Junior and Leaving Certificates. Further, while a single school may not have sufficient numbers of highly-able students to operate an intervention programme, the flexibility of Transition Year may allow schools to cluster together and offer a combined programme. Junior Cycle mathematics in Ireland has increased the emphasis on problem-solving in the first three years of students' post-primary education (DES, 2017), and thus the intervention, as an option for Transition Year students, is ideally placed to continue the development of students as effective problem-solvers in mathematics. Through the investigation of PsP, this project also aimed to present the first furore into the necessary research of highly-able students' mindsets and mathematical resilience in Ireland, which was outlined in the previous section as an area currently devoid of any research.

1.2.3. Research Objectives

Following on from the primary research aim identified in the previous section, the following objectives were identified:

- To conduct a thorough literature review that would inform the intervention design and research practices;
- To create a theoretical framework upon which the intervention and its principles would rely;
- To design a mathematics intervention for highly-able students that focussed on investigating and improving their problem-solving skills, mathematical resilience, and mindsets;
- To implement and facilitate the mathematics intervention with the desired cohort of students;
- To gather data that may be used to investigate the effectiveness of the intervention.

1.2.4. Research Questions

There were three main 'phases' in this research project, as will be discussed in further detail in the next subsection. The first three research objectives relied heavily upon the thorough literature review that was conducted in Phase 1 of this research, as this informed the development of the theoretical framework that underpinned the entire research study, which in turn informed the design of the intervention for this research. The final two research objectives were satisfied in Phases 2 and 3, as the intervention was implemented across six cohorts of students, and data was collected via the various research instruments outlined in Chapter 3. Over the course of the study, three major research questions emerged, with each one fragmenting into further sub-questions. The following are the research questions that this study seeks to address:

1. Has the student sample shown an improvement in their Problem-solving Potential (PsP) over the duration of the intervention?

    A Has the student sample shown a measurable difference in their mindset over the duration of the intervention?

    - Has the student sample shown a measurable difference in their mindset towards general intelligence over the duration of the intervention?

- Has the student sample shown a measurable difference in their mindset towards mathematical talent over the duration of the intervention?
- Do the students perceive a change in their mindset after studying the intervention? Are these perceptions consistent with the changes in students' mindsets identified by the research instruments?

B Has the student sample shown a measurable difference in their mathematical resilience over the duration of the intervention?

- Do the students perceive a change in their mathematical resilience after studying the intervention? Are these perceptions consistent with the changes in students' mathematical resilience identified by the research instrument?

C What categories of the problem-solving grading rubric, if any, has the student sample improved in?

- Do they perceive a change in their problem-solving skills after studying the intervention? Are these perceptions consistent with the changes in students' problem-solving skills identified by the research instrument?

D What proportion of the student sample has shown a measurable increase in their Problem-solving Potential over the duration of the intervention?

2. What opinions do the student sample have about the intervention after completing it?

A Do they perceive any long-term benefits to undertaking the intervention?

B Is there evidence to suggest that the CoPs model was beneficial to the student sample's experiences on the intervention?

3. Are there statistically significant differences in the results of the student samples between different variations of the intervention?

A To what extent, if any, are results different between the student-sample to have taken part in the fourteen-week intervention compared to those who studied the three-week intervention?

B Does the use of an independent facilitator have any measurable difference on the student sample results in comparison with those who studied under the author?

1.3.Research Methods

This section displays the phases of this research study, and the aspects of the study specific to each phase (Figure 1). The research phases are further detailed in Section 3.6. Each of the phases of this research study were informed by relevant literature, with the most prominent research studies underlying each phase outlined in Table 1. A fuller description of the corresponding literature can be found in the coming chapters, with an in-depth outline of the methods for this research study, and the related theory, presented in Chapter 3.

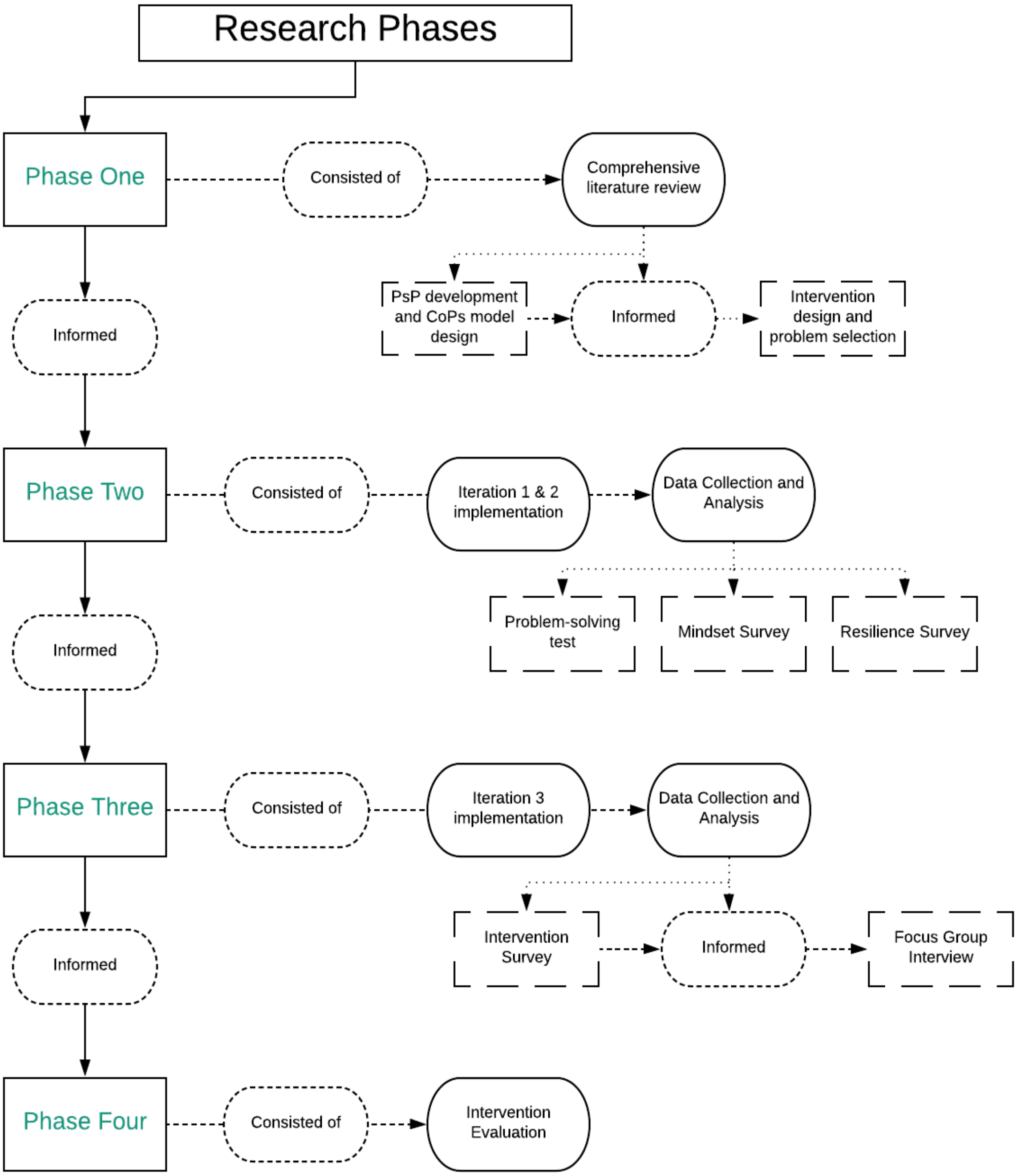

Figure 1 Research Phases

Phase	Aspect of the Research	Main Theory	Relevance to the research
1	Comprehensive literature review	(Dai, 2020; Dweck, 2006; Dweck & Leggett, 1988; Johnston-Wilder & Lee, 2010b; Mason et al., 2010; Polya, 1945)	Informed the development of theoretical framework and the CoPs model
		(Blackwell et al., 2007; Dolmans et al., 2005; Krulik & Rudnick, 1989; Mason et al., 2010; Polya, 1945; Posamentier & Krulik, 2015; Schoenfeld, 1992)	Informed the design of the intervention
2	Quantitative data collection and analysis	(Cohen et al., 2007; Docktor & Heller, 2009; Dweck, 2006; Kooken et al., 2016)	Informed the selection and analysis of research instruments
3	Qualitative data collection and analysis	(Cohen et al., 2007; Kitzinger, 1995; Powell & Single, 1996; Secor, 2010; Zohrabi, 2013)	Informed the development of the intervention survey and the focus group interview and their analysis
4	Intervention evaluation	(O'Meara, 2010; Shapiro, 1987; Walsh, 2015)	Evaluated the intervention

Table 1 Research Phases and Relevant Theory

1.4. Research Contributions

The contributions of this research study are in the field of research into mathematics education, and more specifically, in the educational research area of highly-able mathematics students in Ireland, although the results are likely to be applicable more generally also. The primary contribution of this research was the development of the Problem-solving Potential (PsP) triad, a construct that highlights the importance of problem-solving skills, a growth mindset, and high mathematical resilience to a student's overall ability to problem-solve in mathematics. While each of these aspects has been researched individually and their benefits to students explored, this research is the first to present them as a singular construct with mutually beneficial attributes that impact upon one another. The development of the PsP was informed by the review of literature into gifted education, and was deemed the most appropriate approach to address the additional needs of highly-able mathematics students. This research study presents the PsP and its relevance to highly-able students; however, the construct and its applications are relevant to all learners.

Collaborative problem-solving has been highlighted by the OECD as an important skill for students in the 21st century. Our Collaborative Problem-solving (CoPs) model takes its foundation from prominent theories in the field, but adapts them to a collaborative problem-solving dynamic, and provides a detailed path for a group of students engaged in problem-solving. This model is not only a contribution to the field of mathematics education, but also to any field that benefits from collaborative problem-solving. Further, it may be of benefit internationally where the development of collaboratively problem-solving is being emphasised following OECD recommendations.

While research into the field of mindsets is vast, this doctoral study marks the first investigation into the mindsets of highly-able students in Ireland, and adds to the body of research internationally, where prior research has called for growth (Esparza et al., 2014). The research instrument developed by Dweck (Dweck, 2000, 2006; Dweck & Leggett, 1988), and utilised across numerous projects (Campbell et al., 2021; Esparza et al., 2014; Karwowski, 2014; Lee et al., 2012; Mofield & Parker Peters, 2018), was used to investigate the mindsets of the student cohort towards general intelligence and towards mathematics. The results were also triangulated through qualitative research methods. The results calculated provide insights into the mindsets of this group of highly-able students in Ireland.

Mathematical resilience is a relatively new research area, with much of the literature only in existence since 2010. In light of this, the data collected and analysed for this doctoral study adds significantly to the area. It not only utilises the research instrument designed to investigate mathematical resilience amongst students, but also triangulates the data with qualitative research methods, which was a recommendation of Kooken et al (2016). Furthermore, this research study represents the first application of the area of mathematical resilience amongst highly-able post-primary students and raises some important questions about the applicability of such instruments for students at the upper-end of such scales.

The intervention developed for this project was grounded in prior theory (Cobb et al., 1998; Jafari Amineh & Davatgari Asl, 2015), and each design feature guided by research (P. Heller et al., 1992; Hmelo-Silver & Barrows, 2006; Tang, 2000). This intervention has the potential for significant impact to the study of mathematics for highly-able students in Transition year in Ireland. It also provides the platform for the development of PsP amongst this student cohort, and the introduction of the CoPs model into the study of mathematics in schools.

1.5. Chapter Synopsis

This research is composed of seven chapters, including this current chapter. The organisation of the remaining chapters, and a brief outline of their content, is as follows.

Chapter 2 – Literature Review - is a comprehensive review of the literature that was relevant to this research. It is subdivided into six areas of discussion: gifted education; mathematics in second-level education in Ireland; constructivism; mathematical problem-solving; mindsets; and resilience. Most of these subsections are also further subdivided.

Chapter 3 – Methodology – provides an outline of the research methods utilised by the author. It outlines the theoretical framework designed – the PsP – and the CoPs model, a purpose-built model for collaborative problem-solving. It also provides a detailed overview of the research instruments and the analysis procedure for each one. Finally, it identifies the research issues.

Chapter 4 – Intervention – is a detailed breakdown of the intervention designed for this research. It explores and justifies each decision taken in the design process.

Chapter 5 – Results – reports the results collected through the instruments outlined in Chapter 3.

Chapter 6 – Discussion – discusses the results from Chapter 5 in the context of the research questions.

Chapter 7 – Conclusion – concludes this work by discussing the intervention evaluation, the impact and scope of this research, and future work to be completed.

2. Literature Review

In this chapter, the relevant literature that has been reviewed for this study will be presented through five major headings, each of which is further expanded upon through subheadings. First, a detailed outline of the field of gifted education, and, further, its relevance within Ireland, as this is an important field of focus for this research project. Second, an introduction to the study of mathematics at second-level in Ireland to provide a detailed breakdown of the system in its current from. Each of the remaining sections provide a review of the literature relevant to each factor of the PsP: first, an overview of constructivism, with particular attention to problem-solving; followed by a discussion of mindsets and how they relate to highly-able students; and finally, the concept of mathematical resilience.

2.1. Gifted Education

"In the field of gifted education conceptions of giftedness have fragmented rather than converged over time and definitions are now numerous and often contradicting" (Eyre, 2017, p. ix).

Upon a review of the literature, there were multiple common terms utilised to describe students with the potential to display high ability, including "*gifted*", "*talented*", "*gifted and talented*", "*highly able*", "*exceptionally able*", and "*very able*" (Gagné, 2004; NCCA, 2007; Sutherland et al., 2009; van Tassel-Baska, 2000; Worrell et al., 2019). However, while labels for those represented may be numerous and varying, the field of study itself is universally referred to as 'gifted education' (Gagné, 2004; Monks & Katzko, 2005; Sternberg & Davidson, 2005; Subotnik et al., 2011).

The earliest definitions of "*gifted*" (Terman, 1916, 1921; Terman et al., 1926) are now considered to represent the conservative end of the scale in gifted education. Terman (1921) categorised "*gifted students*" as those who achieved in the top 1% on his Standford-Binet standardised test, an adaptation of the Binet-Simon test. Stern & Kluver (1925) built upon the use of standardised tests to identify gifted students, and introduced Intelligence Quotient (IQ) scoring as a means of easy categorisation. Brown et al. (2005) outlined the continued use of this identification process for much of the 20th century, with Subotnik et al. (2011) highlighting that the link between IQ and giftedness remains *"entrenched in the minds of the public"* (p.5). Indeed, standardised testing remains an integral part of the "*Talent Search*" model (Assouline & Lupkowski-Shoplik, 2012; Lee et al., 2008) utilised by many out-of-school programmes in selecting students (Alamprese et al., 1989; Ledwith, 2013; O'Reilly, 2010).

These early definitions of gifted represented it as a "*unidimensional*" construct (Subotnik et al., 2011), where students possessed a superior, natural general ability (Plomin & Price, 2003). Renzulli's "*Three-Ring conception of giftedness"* represented a move away from this, as it outlined an individual's potential to achieve in general performance areas such as mathematics, art or languages; or more specific fields such as poetry or sculpture (Renzulli, 1978). More importantly, the theory relied upon the idea that high ability came about via an intersection of a person's above-average ability, creativity and task commitment (Brown et al., 2005; Renzulli, 1978, 2012). Renzulli classified giftedness as either "*schoolhouse*" or "*creative-productive*". Students who displayed high ability through testing and IQ achievement were represented within the former classification; while those who solved problems and were original thinkers were classified as the latter (O'Reilly, 2010). The Three-Ring Model emphasised the influence psychological characteristics (such as task persistence, creativity or motivation) had on the prevalence of "*gifted behaviour"* in students, and the requirement for school programmes to nurture these factors (Subotnik et al., 2011).

Gardner's "*theory of multiple intelligences*" (Gardner, 1999; Gardner & Hatch, 1989) also deviated from Terman's definitions (Terman, 1921), moving again towards a more liberal conception of giftedness as a multidimensional and complex model (Brown et al., 2005). The theory of multiple intelligences redefined how human intelligence was perceived (Gardner & Hatch, 1989), and explored the concept of domain specific high-abilities that manifested in one, or more, of the following areas: logical/mathematical; linguistic; musical; spatial; body-kinaesthetic; interpersonal; and intrapersonal. Gardner (1999) later added naturalist, spiritual, and existential intelligences to cater for an evolved society about to enter the 21st Century. In 2003, Sternberg warned that the field risked missing out on large numbers of individuals with high ability by reducing intelligence to "*a single number*" (Sternberg, 2003). He based this warning upon decades of research into conceptions of giftedness, in particular his "*theory of successful intelligence*", which emphasised four elements: ability to succeed in life based on a person's own standards and context; ability to succeed by capitalising on strengths and compensating weaknesses; ability to select, shape and adapt to one's environment; and ability to succeed by achieving a balance of analytical, creative and practical abilities (Sternberg, 1999, 2003). It is this final element which forms Sternberg's Theory of Triarchic Intelligence, in which a person may display high ability academically, creatively, practically, or through some combination of these (Sternberg, 1984, 2003).

In modern literature, a consensus has emerged that a student may have the "*potential"* for high ability (Simonton, 1998, 2005), yet fail to achieve that which becomes expected of them (Feldhusen, 2005; Subotnik et al., 2011). Tannenbaum (1983) wrote that "*gifts"* may exist within children as potential to grow into *talents* possessed when they are adults. Similarly, Gagne (1995) differentiated between "*gifted*" and "*talented*" as labels for students in his 'differentiated model for gifted and talented' (DMGT). Gagne regarded the "*gifted"* as those who possessed the potential to have high-ability in singular or multiple areas. The fruition of such "*gifts*" into recognised high-ability constituted the development of "*talents*" in the individual (Gagné, 1995, 2004). Unlike Tannenbaum, Gagne did not specify that talents could only exist in adulthood (Subotnik et al., 2011). These models represent a departure from other literature in which gifted and talented were often used as synonyms (Rotigel & Fello, 2004), or as a combined phrase of "*gifted and talented*" (Heller, 2004; Subotnik et al., 2011).

One of the early definitions for potential was that by Scheffler (1985), who defined potential as capacity, propensity and capability. Dai & Coleman introduced their definition as "*what people can accomplish given relevant experiences, resources and support and how this potential develops*" (Dai & Coleman, 2005, p. 255), while Dai later updated this to "*any latent qualities that can be realised or developed through experiences, leading to some objectively defined success or achievement*" (Dai, 2020, p. 20). Furthermore potential may be affected on four levels: aptitudes and dispositions; characteristic adaptations; self and future; and sociocultural mediations (Dai, 2020). Aptitudes refer to the cognitive traits, while dispositions refer to the affective-conative aspects of a person, similar to the intrapersonal catalysts within Gagne's DGMT (2004). Characteristic adaptations are the "*self-organisation of abilities, interests, self-concepts, preferences and personality characteristics in response to environmental opportunities and challenges*", and represent the development of "*niche potentials*" in a person (Dai, 2020, p. 25). Self and future encapsulates the degree of a person's commitment towards a particular area. Finally, the influence of tools, resources and supports to allow for the development and manifestation of the other aspects of potential, is known as the sociocultural mediation.

The term highly-able has been used repeatedly over recent decades (Hockett, 2009; Smithers & Robinson, 2012), often as an umbrella term that may include many of the contrasting definitions discussed thus far (Alamprese et al., 1989; Tomlinson, 2005). Hockett (2009) utilised the term highly-able due to the presumption throughout literature that students within gifted education are more *"cognitively advanced"* than their peers. Tomlinson (2005) alluded to the "*high potential*" or "*high performance*" of these students as qualifiers for the interchangeable use of *highly-able* over *gifted*. The complex and contrasting nature behind definitions of gifted, talented, and gifted and talented, led Smithers & Robinson (2012) to use *highly-able* in a bid to remove any advances or reforms in gifted education in England from its own negative past, and Montacute (2018) implored reforms to cast a wide net in the identification of students. For similar reasons, the author has chosen to use the term *highly-able* in reference to these students throughout this work. Furthermore, the author agrees with the literature that these students are those who display high potential or high performance (Montacute, 2018; NCCA, 2007; Tomlinson, 2005), and particularly in mathematics for the context of this project. The NCCA (2007, p. 12) draft guidelines suggest that identification may be achieved through a variety of means, backed by research in the field, and the author conforms to this belief to give the best opportunity to classify highly-able students who may be under-achieving.

Despite the contrasting, and often contradicting, definitions presented throughout the field of gifted education, there exists a constant amongst most modern theories: certain students can possess potential for high-ability in domain-specific areas of academics, and these students require additional resources to aid in the development of such abilities (Tomlinson, 2005; van Tassel-Baska, 2008). In both old and newer theories, mathematics has remained a domain through which high-ability is recognised, with standardised testing still used in some modern identification methods, albeit often not as the sole measure of high-ability. Furthermore, students recognised as being highly-able in mathematics require additional support in order to develop their abilities. We will now consider the teaching of highly-able students, before focusing in on the particular situation of mathematics in Ireland, where the intervention described in this thesis is situated.

2.1.1. Teaching Highly-able Students

"*Only when individual differences are acknowledged, embraced and acted on in the classroom, will gifted students be adequately served*" (Van Tassel-Baska & Stambaugh, 2005, p. 216). Although students thought to be highly-able may often be grouped to allow for easier differentiation within lessons, the differences between these students may be as complex as those of students with specific learning difficulties (O'Reilly, 2014). Different students may have different needs or ability levels and thus catering for these within the regular classroom is an increasingly difficult task, and certainly not one capable of being accomplished without the backing of peers and school leaders (Riedl Cross et al., 2014). Teacher subject knowledge is just one of a number of factors that must be considered in addressing this situation (Van Tassel-Baska & Stambaugh, 2005), while also taking into consideration that teachers have other students to cater for, and must deal with other aspects of a regular classroom, such as discipline (Hertberg-Davis, 2009).

Beyond regular classroom differentiation, pull-out programmes and out-of-school programmes are popular in teaching highly-able students (O'Reilly, 2010). Pull-out programmes withdraw highly-able students from the regular classroom for certain periods, grouping them with other highly-able students to be taught an altered curriculum (Council of Curriculum Examinations and Assessment (CCEA), 2006; Ledwith, 2013). Out-of-school programmes operate similarly in grouping highly-able students together, but take place outside of regular school time (O'Reilly, 2010). Examples of both types of programmes can be seen in various countries around the world: AP Potential in the United States (Van Tassel-Baska, 2006); pull-out programmes in The Netherlands (CIDREE, 2010); CTYI Saturday classes in Ireland (O'Reilly, 2014); and pull-out programmes in Singapore (Ponnusamy & Tan, 2017).

When programmes for highly-able students are discussed, the first question is usually analysing whether they are enrichment or acceleration programmes (Schiever & Maker, 2003). By and large, the goal of enrichment programmes is to provide students with a learning experience that is "*greater in depth or breadth than that which is generally provided*" (Schiever & Maker, 2003, p. 164). For this reason, inquiry- or problem-based teaching approaches are commonly associated with enrichment (Renzulli, 1987). Acceleration is a broad term that denotes many different approaches that alter curricula, teaching practices, or the timeline of school-life for a student (Ledwith, 2013). Types of acceleration include grade-skipping; early-entrance to school or university; and curriculum compacting, amongst other approaches (Ledwith, 2013; Schiever & Maker, 2003). While enrichment and acceleration are often discussed as mutually exclusive to one another, many researchers advocate that successful programmes for highly-able students draw on characteristics of both (Renzulli, 1987; van Tassel-Baska, 2008).

Renzulli (2012) addressed the requirement that domain-specific practices or programmes, evolved from theory, must meet the needs of the students the theory recognised as highly-able – hence programmes of acceleration or enrichment in mathematics must be aimed at students recognised as highly-able in mathematics. In summarising best practice for educating highly-able students, Van Tassel-Baska & Brown (2007) emphasised the need to group highly-able students by subject area, and deliver to them a curriculum entrenched in higher-order thinking, tailored to that subject specific domain. Bosse & Rotigel (2006) advocated for programmes for highly-able mathematics students that utilised problem-solving to "*study mathematics in greater depth*". Heinze (2005) found that highly-able mathematics students possessed greater problem-solving abilities than their peers, and that careful task selection may further enhance these abilities. Sriraman (2003, p. 163), through an investigation of attributes of problem-solving amongst a group of nine freshman students from a rural Mid-Western high school in the United States, found that highly-able students flourish when their attention is captured in questioning, and thus called on teachers to "*create learning opportunities that allow for mathematically gifted students to develop and apply their talents*". Mathematical problem-solving is further discussed in Section 2.3.4 below.

2.1.2. Gifted Education in Ireland

In 1993, the Irish government published its first definition of "*exceptionally able or talented*" students (DES, 1993):

"Pupils… who have demonstrated their capacity to achieve high performance in one or more of the following areas: General intellectual ability; Specific academic ability; Creative or productive thinking; Leadership ability; Visual and performing arts; Mechanical aptitude; Psychomotor ability, e.g. in athletics, gymnastics."

In 2007, the NCCA expanded on the definition for "*exceptionally-able*" students with the publication of draft guidelines for teachers, a report outlining identification, profiles, and strategies for highly-able students (NCCA, 2007). The aims of the report were to "*raise awareness*", "*support management and teachers*", and to "*provide models of good practice*" (NCCA, 2007, p. 7). Subsequently, in 2014, a team working with CTYI surveyed more than 800 educators from around Ireland - ranging from school leaders, to teachers at both primary and secondary level, to those working with children with special educational needs (Riedl Cross et al., 2014). While most respondents were in favour of support for highly-able students, they cited reasons such as overloaded curricula, large class sizes, and an emphasis on catering for the weaker students as some of the reasons why it was not always possible. Almost 85% of teachers reported carrying out differentiated lessons that sought to challenge highly-able students; however, the report questions the "*adequacy*" of the differentiation utilised. It also found that teachers at second-level were less likely to differentiate their lessons, with just 73% (n=95) willing to differentiate their lessons as compared to 90% at primary level (n=257) (Riedl Cross et al., 2014, p. 69).

In 2010, a report from the Consortium of Institutions for Development and Research in Education in Europe (CIDREE) studied the education systems of Ireland, the Netherlands and Switzerland; and, in particular, how each caters for their highly-able students (CIDREE, 2010). Whilst the Netherlands and Switzerland had a wide range of strategies and programmes available to highly-able students, Ireland's students relied upon differentiated learning plans within the regular classroom, as suggested by the guidelines for the exceptionally-able (NCCA, 2007). In 2015, a joint report from the Education and Training Inspectorate and the Department of Education and Skills (DES) outlined pedagogy, cross-curricular links, and the transition from primary to second-level, as important areas of focus in the teaching of mathematics (DES, 2015a). Although the report dealt comprehensively with the pedagogy required for effective mathematics teaching and learning, it failed to address the needs of the highly-able student. Students with learning difficulties in Ireland may be able to avail of additional teaching within a resource classroom setting, but highly-able students have no such additional support (NCSE, 2013). This is due to highly-able students' exclusion from the National Council for Special Education (NCSE) definition of students with special educational needs.

As highlighted by Riedl-Cross et al (2014), the greatest obstacle to gifted education in Ireland is the lack of cohesion and clarity between the government, school leaders and teachers. Whilst the NCCA draft guidelines outline differentiation of lessons as the chosen method to cater for highly-able students, the reality is that teachers find this strategy only partially effective and are constrained by many other factors. Students are therefore reliant on out-of-school programmes (McGrath, 2017), such as those operated by CTYI, to bridge gaps in their education. The mathematics-specific programmes available will be discussed as part of the next section.

2.2. Mathematics in Second-level Education in Ireland

In Irish secondary schools, mathematics is divided into three cycles: the Junior Cycle, a three-year period of study culminating in the Junior Certificate examinations; a stand-alone, voluntary programme called Transition Year (TY); and a two-year period of study leading to the Leaving Certificate examinations (O'Reilly, Dooley, Oldham, & Shiel, 2017). Junior Certificate mathematics is further subdivided into two levels of difficulty: ordinary and higher; while Leaving Certificate mathematics may be studied at foundation, ordinary or higher levels. Second-level mathematics education in Ireland has undergone a period of change over the past decade with, first, the phased implementation of a new curriculum commonly referred to as Project Maths (PM) (DES, 2010); and, even more recently, the introduction of Junior Cycle Mathematics (JCM) (DES, 2015b, DES, 2017).

A report from Conway & Sloane in 2005 highlighted shortcomings in mathematics education in Ireland at the time, while also discussing trends in mathematics education around the world (Conway & Sloane, 2005). The NCCA, who oversee changes to curricula in second-level education, also released a discussion paper in 2005 that suggested a review of some important areas, such as: the role mathematics education holds; issues within schools regarding mathematics education; assessment; the syllabi; and the "*teaching and learning culture*" in schools (NCCA, 2005, p. 25). Following on from the recommendations in these reports, and through consultations with relevant parties, the Project Maths curriculum reforms were developed (DES, 2010).

The main features of the reforms, as reported by the Project Maths Implementation Support Group (DES, 2010, p. 13), were designed to:

- Support students' transition from primary to secondary education;
- Emphasise the relational understanding of mathematics and its wider applications;
- Promote logical reasoning and problem-solving;
- Increase the numbers of students studying higher level mathematics at Leaving Certificate;
- Prepare students for mathematics in future careers.

The implementation of Project Maths began in 2008 on a pilot scheme with 24 schools, before a nationwide roll-out in 2010. The reforms have been the subject of debate amongst researchers in Ireland (Cunningham et al., 2016; Kirwan, 2015; Shiel & Kelleher, 2017), even though accurate depictions of the impact of educational upheaval require time, according to research (Treacy et al., 2016). The mathematics syllabus at both Junior Cycle and Leaving Certificate was divided into 5 contextual strands of study: statistics and probability; geometry and trigonometry; number; algebra; and functions. The Junior Cycle syllabus was further reformed in 2017, combining algebra and functions into one strand (DES, 2017). This latest reform also introduced a "*unifying strand*" that ran concurrently through each other strand, composed of six elements: building blocks, representation, connections, problem-solving, generalisation and proof, and communication (DES, 2017, p. 10). Grannell at al. (2011) discussed the strain that Project Maths posed on mathematics teachers due to its apparent transformation of teaching styles from those of teacher-orientated classrooms and rote learning, to student-centred and problem-solving; although Oldham & Close (2009) highlighted that problem-solving has been intended to be entrenched in mathematics at second-level for decades.

Problem-solving has been at the forefront of most publications around Project Maths (NCCA, 2012, 2013, 2014). Indeed the NCCA confirmed the curriculum had been reduced in size to cater for the "*development and assessment of conceptual understanding and of the higher-order thinking and problem-solving skills…*" (NCCA, 2012, p. 13). The further Junior Cycle reforms also placed problem-solving firmly at the core of mathematics education as one of the elements of the "unifying strand" (DES, 2017). The Junior Cycle specification for mathematics also introduced eight key skills to be developed within students through their Junior Cycle studies (DES, 2017, p. 8): being creative; being literate; being numerate; communicating; managing information and thinking; managing myself; staying well; and working with others. Many of these skills are essential within mathematical problem-solving, and are discussed further in Section 2.3.4, and in collaborative problem-solving. Mathematics teachers at second-level outlined the time-commitment for authentic problem-solving as an obstacle for its regular use in their classes (Shiel & Kelleher, 2017). Whilst in another study, teachers were found to be desirable of more time for mathematics, particularly double periods, to allow for more problem-solving (O'Meara & Prendergast, 2017), although the allocation is below what may be required (Prendergast & O'Meara, 2016; Shiel & Kelleher, 2017). This raises a further concern that authentic problem-solving is not as readily utilised in the regular classroom as it should be.

Transition Year (TY) is a voluntary year offered in most schools (92%) in Ireland (Jeffers, 2019). The TY guidelines offer suggestions as to what should be taught to students, but also firmly state that curricula and teaching strategies are to be decided by each individual school (DES, 1994). The guidelines do, however, also emphasise that TY should not be equipped as a third year of study towards the Leaving Certificate (DES, 1994, p. 2). It is known that both the Junior Cycle and the Leaving Certificate are time-constraining for teachers (NCCA, 2013). TY serves as the ideal stage of second-level education for the existence of a programme for highly-able mathematics students. The flexibility within the TY guidelines would also allow schools to adopt such a programme to fit their needs, while not placing pressure on schools by introducing one as mandatory. It is for this reason the author chose TY as the optimum time to introduce a mathematics invention for highly-able students.

2.2.1. Mathematics for Highly-able students in Ireland

Whilst reports into post-primary mathematics education in Ireland indicated a clear need for change (Conway & Sloane, 2005; NCCA, 2005), some parties have expressed fears that, although this change may benefit students as a whole, it may not prove sufficiently challenging for highly-able students (Lubienski, 2011; NCCA, 2012). A report from the Project Maths Implementation Support Group (a team collaborating from education and industry tasked with informing the Minister for Education how Project Maths can reach its goals) also highlighted the apparent lack of attention paid to the needs of highly-able students (DES, 2010). This raised concerns that neither the old nor new mathematics systems were equipped to help Ireland's highly-able mathematics students (DES, 2010, p. 28). In response to criticism during the implementation period, the Project Maths team released a report outlining the reasoning behind the change and what would be different in the new syllabus (NCCA, 2012). There lacked, within this publication, any reference as to how this new approach to the teaching and learning of mathematics would enhance the learning of the highly-able student. A report from the NCCA in 2017 on the impact of Project Maths on students' performance at Junior Cycle level found that, not only were highly-able students underperforming, but that teachers felt these students were unchallenged in the three years of Junior Cycle mathematics (Shiel & Kelleher, 2017).

The Programme for International Student Assessment (PISA) is a set of assessments carried out by the Organisation for Economic Co-operation and Development (OECD) once every three years to measure international standards of 15-year-olds abilities in reading, mathematics, and science (https://www.oecd.org/pisa/). Although Ireland has performed slightly above average in the PISA mathematics rankings since 2006 (OECD, 2006; OECD, 2010; OECD, 2014; OECD, 2016; OECD, 2019), an area of concern exists regarding highly-able students' performance in these assessments. In each set of rankings, Ireland finished below the OECD average in terms of the percentage of students rated as top performers (those who score in the top two levels of competency, out of the total six levels) as evident in Figure 2:

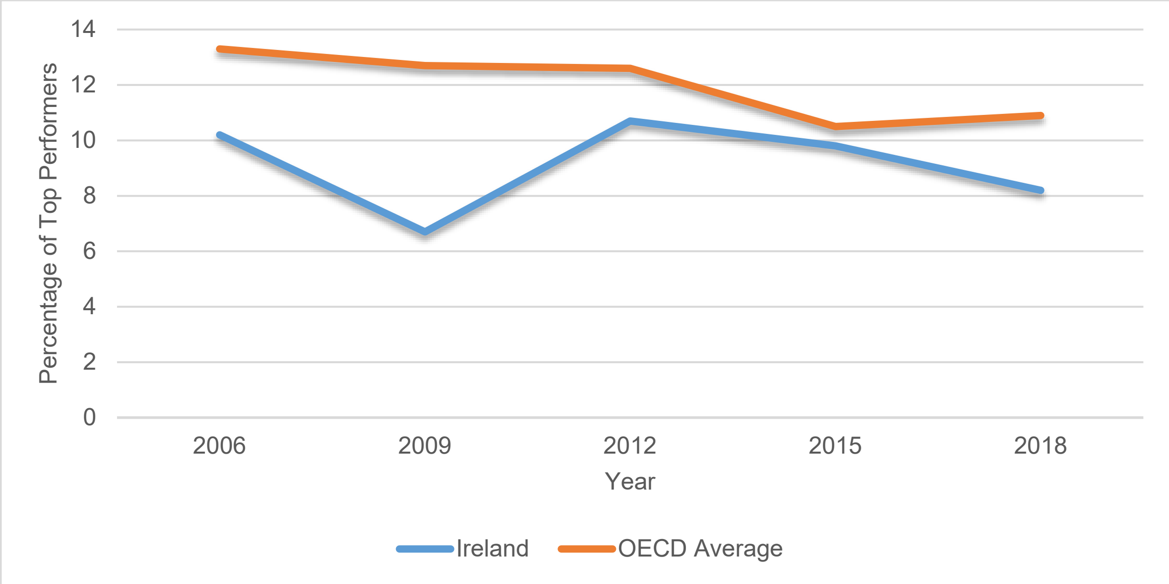

Figure 2 Percentage of Top Performing Students, PISA 2006-2018

The Trends in International Mathematics and Science Study (TIMSS) is another set of international student assessments, conducted every 4 years for students in 4th class in primary school (ages 9/10) and 2nd year in secondary school (ages 13/14). Irish students have completed these assessments on four occasions (1995, 2011 (only 4th class), 2015 and 2019). Perkins & Clerkin (2020, pp. 66–67) outlined the results of Irish students on the TIMSS 2019 mathematics assessment, and analysed them in comparison to prior assessments, and to countries with statistically similar overall scores, and found that:

- Our 4th class students performed well at the lower end of the distribution (5th percentile), relative to similar countries; however, they performed less well at the upper end (95th percentile)
- The performance of low-achieving 4th class Irish students improved substantially between 2011 and 2019, whilst high-achieving Irish students registered no change over this period
- Low-achieving Irish students in 2nd year performed considerably better than their counterparts in similar countries, and displayed an improvement between each set of assessments from 1995 to 2019

- In contrast, high-achieving Irish students in 2nd year - showed no change between 2015 and 2019; disimproved between 1995 and 2019; and scored lower in comparison to similar countries

Further to this, at both age groups, Irish students displayed competency in basic skills and understanding, but were found to be considerably weaker in reasoning and generalisation (Mullis et al., 2020).

The International Mathematical Olympiad (IMO) is a competition for the top-performing mathematics students around the world, with 112 countries participating in 2019. Whilst Ireland has competed in the IMO since 1988, the highest position obtained was 51st in 2005, out of 91 countries (https://www.imo-official.org/results.aspx). Furthermore, McGrath (2017) found that Irish students' results were consistently below those of countries with similar PISA rankings. One of the measures aimed at preparing Irish students for the IMO are the Senior Mathematical Enrichment classes operated by the Irish Mathematical Olympiad (IrMO), which take place in five universities around Ireland and are taught by lecturers at each university. Most students who participate in the Senior Mathematical Enrichment classes are invited due to their high performance in their Junior Certificate or in IrMO competitions, although the classes are available to any student (per https://www.irishmathstrust.com/mathematics-enrichment). In recent years, Junior Mathematical Enrichment classes were launched in the five universities for students with an interest in mathematics. The Enrichment classes, operating as out-of-school programmes that take place predominantly on Saturday mornings, are one of the few options available for highly-able mathematics students in Ireland, but are somewhat limited due to their occurrence in just five universities, which may lead to long commutes for students.

In terms of further acceleration/enrichment opportunities open to highly-able students in Ireland, CTYI, based in Dublin City University, offers out-of-school programmes for highly-able students at both primary and post-primary levels (www.dcu.ie/ctyi). Students may qualify for CTYI programmes through a talent search, completion of entry assessment, or recommendation from an educational psychologist. The Preliminary SAT (PSAT) is the standardised test used by CTYI in the talent search model and is widely used for other programmes in gifted education identification processes (Ledwith, 2013). Ledwith (2013) designed the Early University Entrance (EUE) programme for high-achieving TY secondary-school students. Students who qualify through the aforementioned criteria can enrol in modules based on university degree courses, such as psychology, engineering, law, etc. EUE was based on the acceleration model of dual enrolment of students often seen in the United States (Ibata-arens, 2012); however, such programmes in the United States offer students the opportunity to progress in school beyond this level, whereas in Ireland the programme is not linked to schools, and therefore cannot be continued as study over the final two years of secondary school (Ledwith, 2013).

Apart from these, there are a couple of other initiatives in operation. The Irish Mensa operates occasional workshops for students who register an IQ in the top 2% of the population, measured through internationally recognised standardised tests (McGrath, 2017). However, McGrath noted that these were infrequent. Maths Circles is an initiative launched in Cork to promote lunchtime mathematics clubs within schools, although it exists predominantly within schools in Cork (http://mathscircles.ie/#!/) and has not become widespread throughout the country.

As previously discussed, differentiation of lessons is the primary measure available within schools to cater for highly-able students, and there exists an open debate as to its effectiveness (Riedl Cross et al., 2014). Out-of-school programmes, such as those operated by the IrMO and CTYI are advantageous for highly-able students, but are not without their limitations, such as:

- Travel distances for students, as both organisations run their programmes through universities or other higher education institutions
- Costs - travel costs incurred, or the fees required for most CTYI programmes
- Personnel required - CTYI programmes are taught by part-time lecturers or post-graduate students (O'Reilly, 2010); Senior Maths Enrichment classes are taught by university lecturers (Kreussler, 2019); and Junior Maths Enrichment classes are taught by students on concurrent teacher undergraduate degrees who have been trained for the programme (http://www.nuigalway.ie/cki/servicelearning/casestudies/juniormathsenrichment/#).

The remaining portion of this chapter outlines literature relevant to the design of the intervention and the overarching aim of this research study, as presented in Section 1.2.3 – to improve highly-able students' problem-solving skills, mindsets and mathematical resilience.

2.3. Constructivism

In education, two prominent learning theories have dominated research over the past century: firstly, behaviourism, which focussed on observable behaviour rather than mental activity or understanding; and constructivism, which defined learning as a process of constructing new knowledge (Pritchard, 2008). Behaviourists believed that curricula should be divided into a set of skills to be mastered, from simplest to most complex, which may be learned through listening to the teacher and completion of sufficient practice examples (Fosnot & Perry, 2005). While behaviourism dominated much of the 20th century, educational research began to move away from these ideas in the 1970s and '80s (Jafari Amineh & Davatgari Asl, 2015). In contrast to behaviourism, constructivists required students to take a more active participation in the learning process (Poplin, 1988), which has spawned multiple teaching approaches to facilitate this, such as: problem-based learning, project-based learning, active learning and guided-inquiry, amongst others (Hyland, 2018). Pritchard (2008, p. 18) categorised learning in school as knowledge, concepts, skills (both mental and physical) or attitudes. Furthermore, he defined constructivism as the learning process when "*new information is built into and added onto an individual's current structure*" of these areas. Social constructivism was first heralded by Vygotsky (1978), and considered the constructivist approach with an emphasis on the interaction between the learner and those around them (Pritchard, 2008). Vygotsky's theory centred on the "*zone of proximal development*", in essence outlining one's ability to "master concepts and skills" through interaction and collaboration with a facilitator or a group of peers (Schreiber & Valle, 2013).

Poplin (1988) credits Jean Piaget with the founding theory behind constructivism in the mid-20th century, and the work of Gallagher & Reid (1983) in disseminating these ideas amongst educationalists. Constructivism has been widely recognised as important to mathematics education due to its assertion that new knowledge is best understood when constructed by the learner in their own context (Cobb, 1994; Hyland, 2018; Wilson, Fernandez, & Hadaway, 1993). The NCCA review of mathematics education in Ireland called on curricular changes that heralded a constructivist approach to learning (NCCA, 2005). Following on from this, in 2013 the NCCA recognised constructivism as one of the lead cultures in mathematics education today, and emphasised its importance in developing a context for problem-solving within the classroom (NCCA, 2013). Wilson et al (1993) heralded problem-solving as the perfect mathematical approach to encompass constructivist principles. While much research exists to discuss differences between constructivism and social constructivism, Cobb et al (1998, p. 72) highlighted the agreement amongst theorists in that "*the construction and validation of mathematical concepts are collective as well as individual activities and that they occur via a process of argumentation within a community*"

2.3.1. Collaborative learning

With the growth in popularity of social constructivism, teaching styles have developed that hinge on the social interaction and collaboration of a group of students (Bayat & Tarmizi, 2012; Schreiber & Valle, 2013). Laal & Ghodsi (2012) defined collaborative learning as *"an educational approach to teaching and learning that involves groups of learners working together to solve a problem*". Dillenbourg (1999) went into great detail to outline the complexity within such a definition, whereby multiple words may be interpreted on large scales – such as the size of groups, or the manner of learning. He continued that most empirical research in the area has supported the formation of small groups (3 to 5 students), and that learning may come as a side effect of problem-solving whereby students improve their problem-solving skills, or learn to work well with others (1999, p. 4). Furthermore, collaborative learning has been found to benefit students academically (develop critical thinking and problem-solving techniques), socially (develop support system and learning community), and psychologically (reduced anxiety and improved self-esteem) (Laal & Ghodsi, 2012). The 2015 PISA report discussed collaborative problem-solving in-depth, and outlined its importance in the field of mathematics education (OECD, 2017). Before discussing this further in Section 2.3.5, we must first outline what is meant by problem-solving in the field of mathematics.

2.3.2. Collaborative Problem-solving

Care & Esther (2014) defined collaborative problem-solving as the "*search for relevant information from another person, joint use of different resources, and agreement on strategies and solutions*" . Indeed, the PISA 2015 Framework offered a similar definition, albeit with increased focus on the individual's capacity to operate in this setting (OECD, 2017, p. 6):

> *"Collaborative problem solving… is the capacity of an individual to effectively engage in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and resources to reach that solution"*

Further to this, Rosen & Foltz (2014) believed that for collaborative problem-solving to succeed, students must develop a shared understanding of a problem, and must persist to maintain this shared understanding throughout the problem-solving process. Collaborative problem-solving retains the cognitive skills of strategy-selection and self-regulation utilised in individual problem-solving, and demands the development of social skills to compliment them (Häkkinen et al., 2017). Communication within a group is fundamental to the success of collaborative problem-solving (Häkkinen et al., 2017; Hesse et al., 2015).

Reflection within collaborative problem-solving may occur independently and personally, as with individual problem-solving; or by the process of reflective verbalisation, whereby students must reflect via communication with their group or the teacher (Clarke, 2011). Wetzstein & Hacker (2004) outlined the requirement for students to explain and justify their solution in order for successful reflective verbalisation to occur. Where students merely vocalised their workings, without due care as to "*why*" each step was taken, there was no perceived effect on performance (Wetzstein & Hacker, 2004, p. 146). Care & Griffin (2014) outlined collaborative problem-solving as a vital 21st century skill, transferrable from the classroom to modern employment. In recent years, the OECD has prioritised assessment of students' collaborative problem-solving, and also outlined communication as a key skill for students in the 21st century (OECD, 2017).

2.3.3. Facilitator

Krulik & Rudnick (1989) asked the important question "*what can the teacher do…?*" when it comes to problem-solving as a teaching strategy. Kalpana (2014) described the role of the teacher in a constructivist classroom as that of a "*facilitator or guide*" to students, responsible for creating the environment in which students may work collaboratively. The role of the facilitator was well-detailed by Hmelo-Silver & Barrows (2006), with prominence placed on the use of questioning to promote deeper learning. A facilitator should provide guidance to students when they are stuck, without solving the problem directly for the students (Schreiber & Valle, 2013); and encourage alternative paths to a solution (Krulik & Rudnick, 1989). Krulik & Rudnick (1989) also advised teachers within this role to encourage student creativity and make them create their own problems to be solved, both of which have been discussed in Section 2.3.4 as problem-solving extensions.

The role of the facilitator should organically lessen over a period of time (Hmelo-Silver & Barrows, 2006). For example, Isoda & Katagiri (2012) believed that facilitator questioning is of utmost importance to student learning, but students should begin to anticipate the incoming question after repeated exposure, and thus seek to find an answer before the question has been asked. The facilitator scaffolds student learning to encourage their development as autonomous learners, and thus the role diminishes as this progresses (Hmelo-Silver & Barrows, 2006).

2.3.4. Mathematical Problem-Solving

In recent decades, problem-solving has been at the forefront of mathematics education (Carillo & Cruz, 2016; Heller, Keith, & Anderson, 1992; Krulik & Rudnick, 1989; Posamentier & Krulik, 2015; Schoenfeld, 1992), as researchers sought to establish its importance to the study of mathematics. Krulik and Rudnick (1989) defined a problem as a situation for which no "*apparent path to a solution*" appears obvious to the individual. They elaborated that a problem must pass three criteria for a student to engage in problem-solving:

- Acceptance – the student accepts the challenge to find a solution
- Blockage – initial attempts to solve the problem have failed, thus moving the problem beyond routine, skill-driven exercises

- Exploration – the desire to find a solution, spurred on by the acceptance of the challenge, drives the student to seek new methods of attack for the problem

Well-chosen problems should provide an opportunity for students to inspect the information given, select what is important, determine relevant mathematical principles, and allow for reasonable assumptions that may lead to a meaningful solution (Heller & Hollabaugh, 1992). Well-chosen problems require a level of higher-order thinking that aims to develop the conceptual understanding of the learner (Isoda & Katagiri, 2012). Mason et al (2010) extolled the virtues of how certain problems lend themselves to the discovery of new problem-solving techniques and strategies; for example, the problem 'palindromes' (Appendix G) was utilised by Mason et al to help introduce both specialising and generalising.

George Polya's steps to solving a problem were first published in 'How to Solve It' in 1945, but their influence on modern problem-solving is still widely acknowledged (Mason, Burton, & Stacey, 2010; Posamentier & Krulik, 2015; Schoenfeld, 1992). Polya (1945, 1957) outlined the process as: understanding the problem; devising a plan; carrying out the plan; and looking back. To understand the problem, a student should not only be able to interpret its meaning but should also desire to find its solution (Polya, 1957); and thus the importance of well-posed and well-chosen problems are paramount to effective problem-solving (Cai et al., 2016).

Polya's (1945) problem-solving steps, and accompanying discussion into the heuristics of problem-solving, formed the basis of the concept for most researchers in recent decades (Krulik & Rudnick, 1989; Mason, Burton, & Stacey, 1985; Schoenfeld, 1987, 1992). It established mathematical problem-solving as a cognitive process, whereby students utilised their knowledge of strategies and content to develop a deeper understanding of a topic; rather than merely the repetition of exercises with their emphasis on procedural acquisition (Schoenfeld, 1992). Mason et al. (2010) developed a more structured approach to problem-solving with Rubric Writing (Figure 3), and highlighted the importance mathematical language may play in the process. Further building upon the work of Polya, the OECD (2014a, p. 31) created a set of "*competencies*" that students must meet to develop as problem-solvers, and grouped these competencies into four categories: exploring and understanding; representing and formulating; planning and executing; monitoring and reflecting.

According to Polya, traditional views on mathematics questions, where even the best students complete a question and look at it no further, hold a missed learning opportunity (Polya, 1957). '*Looking back*' on the solution was emphasised as an important educational tool by Polya, and one that helped to further develop a student's understanding of the problem. Further to this, Mason et al (2010) stressed the requirement to review the solution and reflect on '*what was done'* and '*why it was done*'. The '*review phase*' within Rubric Writing built upon Polya's '*looking back*' by introducing mathematical extensions, whereby the original problem was altered to suit the needs of the problem-solver – either creating an easier problem, or one more difficult to solve (Mason et al., 2010, p. 39). Furthermore, the process of developing new insights into a problem was argued to encompass mathematical creativity (Nadjafikhah et al., 2012); a concept that, until recent decades, was reserved solely for new discoveries in mathematics (Sriraman, 2004).

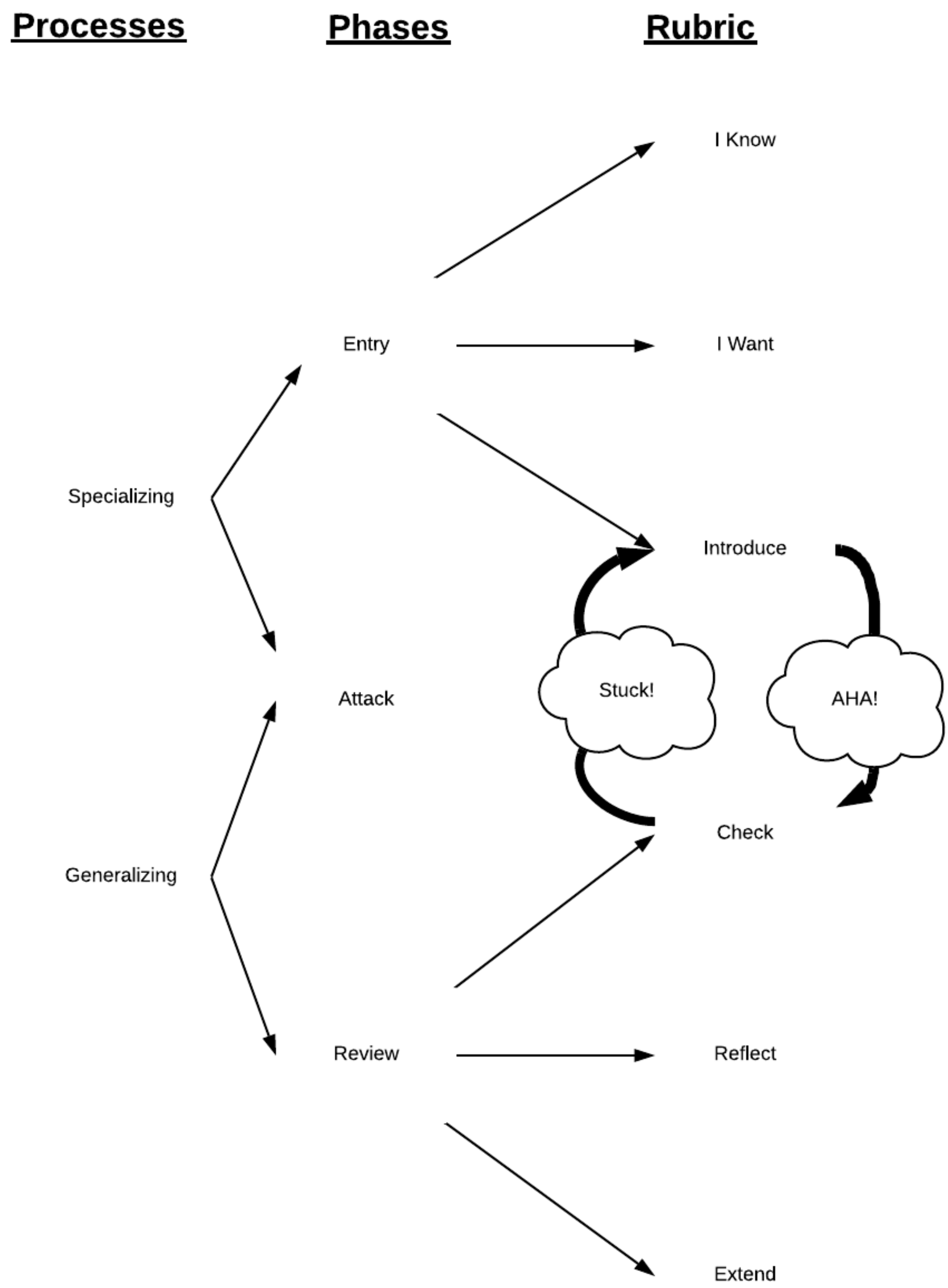

Figure 3 Rubric Writing (Mason et al., 2010)

The importance of strategy-selection within mathematical problem-solving has been well-researched and discussed (Krulik & Rudnick, 1989; Mason et al., 2010; Shen, 2012; Sternberg, 1999). Krulik & Posamentier (2015) chose 10 such strategies and utilised carefully-selected problems that championed each strategy, while also acknowledging that most problems may be solved through multiple strategies. In fact, the path towards a solution often requires the employment of multiple strategies (Krulik & Rudnick, 1989). Mason et al (2010) also introduced strategies through problem-selection, and discussed their role in the grand scheme of mathematical thinking – often discussed as some combination of deeper thinking, reasoning, conceptual understanding and questioning (Breen & O'Shea, 2010; Schoenfeld, 1992). Mathematical problem-solving strategies were identified by the author as an important component to effective problem-solving teaching, and thus they are discussed in detail in Chapter 4.

2.3.5. Collaborative Problem-solving in Mathematics

The importance of collaborative problem-solving to the field of mathematics is highlighted by the emergence of online communities of mathematicians who collaborate to solve new "*micro-problems*" (Tausczik et al., 2014), and its importance within mathematics education is repeated throughout research (Bjuland, 2004; Care et al., 2016; Harding et al., 2017). However, Graesser et al (2018) suggested that collaborative problem-solving is under-researched in educational settings. The application of collaborative problem-solving within mathematics education often utilises the heuristics developed for individual problem-solving (Bjuland, 2004; Graesser et al., 2018; OECD, 2017), but a clearly defined set of heuristics specific to collaborative problem-solving in mathematics has not yet been adapted. The PISA Framework for Collaborative Problem-solving (OECD, 2017), which included mathematics as one of a number of disciplines, created a matrix of collaborative problem-solving proficiencies that were the intersection of their previously defined problem-solving proficiencies with collaborative proficiencies (2017, p. 11); however, these are intended as competencies to be assessed rather than heuristics. As with individual problem-solving, well-chosen tasks have been highlighted as imperative to the success of collaborative problem-solving (Care et al., 2015; Harding et al., 2017).

One aspect of the primary aim of this research study was to improve highly-able students' problem-solving skills in mathematics, and these recent sections have provided some theoretical background as to how this may be achieved through the application of collaborative problem-solving. The theory of the other two aspects of the primary aim - mindsets and mathematical resilience - will now be detailed in Sections 2.4 and 2.5 respectively.

2.4. Mindsets

In recent decades, Carol Dweck has been leading a growing body of research into intelligence malleability, and its perceived effect on achievement (De Castella & Byrne, 2015; Dweck, 1986; Dweck et al., 1995a; Hochanadel & Finamore, 2015; Yeager & Dweck, 2012). Dweck & Leggett (1988) investigated two major patterns of belief in individuals that affected goals and motivation in learning: helplessness, or those who avoided challenge and faced deteriorated performance when challenged; and mastery-orientated, or those who sought challenge and thrived when facing obstacles. Further to this, they outlined the effect that these attributes may have on goal orientation of students. "Helpless" students were found to strive towards performance-based goals, where testing measures were perceived as the measure of one's competence or incompetence (Dweck, 1986; Dweck & Leggett, 1988). In contrast, "*mastery-orientated*" students tended towards goals of learning, with less dependence on grades. These contrasting goal-orientations were summarised as a comparison of those trying to "*prove*" their ability (helpless) versus those trying to "*improve*" their ability (mastery-orientated) (Dweck & Leggett, 1988, p. 259). The pursuit of performance goals was outlined to have negative effects on cognitive and affective processes within students when faced with difficulty (as seen in Table 2 in (Dweck & Leggett, 1988, p. 262)). Senko (2019) highlighted that performance goals may serve a purpose when a student must perform well in a high-stake single exam.

Performance Goals	Learning Goals
Loss in belief in efficacy of effort, given low ability attribution	Continued belief in efficacy of effort: Effort self-instruction instead of low-ability attribution; positive rule emphasises utility of effort
Defensive withdrawal of effort: Effort confirms low ability judgement; inverse rule creates conflict between task requirements and goal	No defence required: Effort is consonant with task requirements and goal
Attention divided between goal (worry about outcome) and task (strategy formulation and execution)	Undivided, intensified attention to task that directly serves goal
Negative effect can interfere with concentration or can prompt withdrawal	Affect channelled into task
Few intrinsic rewards from effort (or high effort progress) to sustain process	Continuous intrinsic rewards for meeting challenge with effort

Table 2 Cognitive and affective effects when faced with challenge (Dweck & Leggett, 1988)

Through the research into motivation and goal-orientation when challenged, theories of intelligence were generated to represent the set of beliefs held by students: those who believe intelligence is "*a malleable, increasable, controllable quality*" (incremental theory); and those who believe intelligence is a "*fixed or uncontrollable trait*" (entity theory) (Dweck et al., 1995a; Dweck & Leggett, 1988). These theories are not intended as rigid predictions of student behaviour, but rather a core assumption, or framework to further build judgements (Dweck et al., 1995a). Yeager & Dweck (2012) summarised the patterns through four core headings of: goal-orientation; value of effort, help and strategy; response to challenge; and achievement during adversity, as seen in Table 3:

	Entity Theory	**Incremental Theory**
Goals	Look smart	Learn
Value of effort, help and strategy	Higher	Lower
Response to challenge	Tendency to give up	Work harder and smarter
Achievement when faced with adversity	Decrease or remain low	Increase

Table 3 Theories of Intelligence Beliefs (Yeager & Dweck, 2012)

Dweck (2006) built upon the implicit theories of intelligence, and concluded that people may hold the belief that their intelligence and personality can develop and persevere through adversity. This new model was branded as "*mindsets*"; with a "*growth mindset*" being *"based on the belief that your basic qualities are things that you can cultivate through your efforts, your strategies, and help from others"* (Dweck, 2017). In contrast to this, a "*fixed mindset*" represented the belief that a person's qualities were unable to change. However, research (Dweck, 2017; Yeager & Dweck, 2012) indicates that we can alter our mindset through active thought in our decisions and actions, and thus transition from emphasis on performance goals to those of learning goals, as seen in Table 3.

Research into mindsets has proven particularly useful in establishing the impact of growth or fixed mindsets on achievement motivation (Blackwell et al., 2007; De Castella & Byrne, 2015; Dupeyrat & Mariné, 2005; Dweck, 2017). Blackwell et al. (2007) tracked the mathematics grades of 373 7th grade students in the United States over a two-year period, and found that those with a growth mindset (incremental theory) followed an upward trajectory, while those with a fixed mindset (entity theory) followed a flat trajectory. Following on from these findings, Blackwell et al. (2007) implemented an eight-week intervention to teach students about intelligence malleability. They found that students who were taught about incremental theory showed improvements in motivation and achievement. Schmidt, Shumow & Kacker-Cam (2017) utilised a mindset intervention to better understand the effect it may have on student behaviour and interest during regular classroom activities. The six-week intervention was delivered to 370 7th grade students and 356 9th grade students in the United States, and found a positive impact on students exposed to the intervention. In the intervention, positive aspects of a growth mindset were encouraged through activities and tasks in one 50-minute lesson per week.

Over the past three decades, research into mindsets has yielded a strong link between our intrinsic belief about intelligence and how we set goals and overcome challenges (Dweck, 1986, Dweck, 2017; Dweck, 1995a; Dweck & Leggett, 1988). Yeager & Dweck (2012) highlighted the role mindsets play in academic resilience, and thus how they shape students' reactions to failure. They outlined how students with a growth mindset place greater value on the effort required to learn and, in doing so, view setbacks as an indication they must work harder. When faced with setbacks, students with a fixed mindset have been found to be not as academically resilient as those with a growth mindset, and view failure as a sign of being "*dumb*" (Blackwell et al., 2007; Yeager & Dweck, 2012).

2.4.1. Mindset and Highly-Able Students

While research into mindsets has grown rapidly in recent decades, research into the mindsets of highly-able students has been lacking (Esparza et al., 2014). One of the greatest risks to the mindsets of highly-able students is that of praise for ability rather than effort, according to Dweck (2007) and Boaler (2013). Praising ability and test scores may encourage students to value test performance rather than learning, and thus develop goals based on performance that may have a detrimental effect should they fail (Dweck, 2007). Further to this, highly-able students are at risk due to a pursuit of "*perfectionism*" that is driven by performance (Esparza et al., 2014). Perfectionism is usually defined as the pursuit of "*excessively high standards*", and may carry the implication that students shy away from tasks that provide challenge to maintain their "*smart identity*" (Mofield & Parker Peters, 2018, pp. 329, 331). Students seeking perfection have also been found to link their self-worth to their performance in tests, which may have a detrimental effect on their well-being when faced with failure (Sowa et al., 1994).

Within gifted education, there exists a subset of students who are recognised as highly-able, yet underachieve in school. Clinkenbeard (2012) investigated highly-able students' motivation in school, and outlined how underachievers may feel disillusioned within poorly-differentiated classrooms, leading to their mental withdrawal from tasks. McCoach & Siegle (2003) collated common attributes associated with underachievers from research, and investigated each attribute through a sample of 178 highly-able students (56 classified as underachievers). They found that motivation and goal-orientation were the two factors that showed noticeable difference between underachievers and their peers, and further outlined that interventions for highly-able students should address these factors. Further to this, there was a strong correlation between these two factors, which the researchers suggested was due to goal orientation directly impacting upon students' motivation to achieve said goals. The role mindsets play in both of these factors has been well-discussed above. Much of the literature suggests that understanding how mindsets affect highly-able students is an area that requires substantially more research (Esparza et al., 2014; Schultz, 2002).

2.5. Mathematical Resilience

The development of "*mathematical resilience*" (Johnston-Wilder & Lee, 2010b) as a construct bloomed from decades of research into resilience, in its primary field of psychology, as to how an individual responds to setbacks, disappointments or trauma (Goldstein & Brooks, 2005; Toland & Carrigan, 2011). In a report investigating numeracy in Ireland, and how it may be improved in schools, the Department of Education and Skills highlighted students' lack of resilience in mathematics as an area of concern (DES, 2015a, p. 11). Johnston-Wilder & Lee (2010b) outlined the requirement for mathematics-specific resilience due to "*the type of teaching often used, the nature of mathematics itself, and the pervasive beliefs about mathematical ability being 'fixed'*"; and, consequently, used "*mathematical resilience*" to define student attitudes and responses towards learning mathematics when they were challenged in the learning process. They outlined behavioural attributes found within students who displayed high mathematical resilience - smart strategy selection, craving discussion, questioning mathematical ideas, developing growth mindsets – and, further, methods through which mathematical resilience may be developed (Johnston-Wilder & Lee, 2010b). Johnston-Wilder et al (2013) proposed that, not only could students' mathematical resilience be increased, but also that parents, who are themselves not mathematically resilient, may be coached in methods of developing student resilience. Hutauruk & Priatna (2017) discussed both struggle and a positive response to mistakes as important to resilience within mathematics. Yeager & Dweck (2012) outlined that a growth mindset may be hugely beneficial to the development of resilience within students. Those with a fixed mindset are more likely to give up when challenged academically (Table 3), and therefore show less mathematical resilience. Indeed, research into mindsets is intrinsically linked to the study of resilience, where responses to challenges or set-backs are linked to the intrinsic theory of intelligence held by students (Dweck et al., 1995a; Hochanadel & Finamore, 2015; Yeager & Dweck, 2012).

While research into the importance of mathematical resilience began to grow, there lacked any subject-specific measurement that may be used to study students' attitudes and responses while studying mathematics (Kooken et al., 2013). Further to this, an increased desire to improve student performance in STEM (Science, Technology, Engineering and Mathematics) education emerged over the past two decades due to its importance to many 21st century jobs (Kooken et al., 2016). The Mathematical Resilience Scale (MRS) was developed to measure mathematical resilience through three factors (Kooken et al., 2016, 2013):

- Value – the degree to which a student believes studying mathematics is important to their life
- Struggle – student understanding of struggle as a natural process within the learning of mathematics, and acknowledgement that struggle does not indicate failure
- Growth – the importance a growth mindset may have on the study and learning of mathematics

The validation study outlined the usefulness of the MRS in: identifying students lacking in mathematical resilience; exploring demographics of those who continue to engage with STEM; and, as of interest in this project, identifying students who are highly-able in mathematics but "*whose affective traits make it less likely for them to persevere*" in the study of mathematics (Kooken et al., 2016). Johnston-Wilder et al (2013) developed a model (Figure 4), adapted from Vygotsky's 'Zone of Proximal Development', that depicted the optimal conditions for students to learn. The 'comfort zone' represented the skill level at which a student is independently competent, and therefore does not require peer or teacher support and is unchallenged by the work (Lee & Johnston-Wilder, 2018). Conversely, the 'anxiety zone' was outlined as the level unattainable for a student even with support, and was believed to result in task avoidance and stress (Thomas, 2020). The 'growth zone', similar to the ZPD, was discussed as the space in which learning is best-achieved, with regular support and scaffolding for the learner. Lee et al (2018) highlighted the importance of getting "*stuck*" within the 'growth zone' and of strategies to relieve this feeling before anxiety or stress set in.

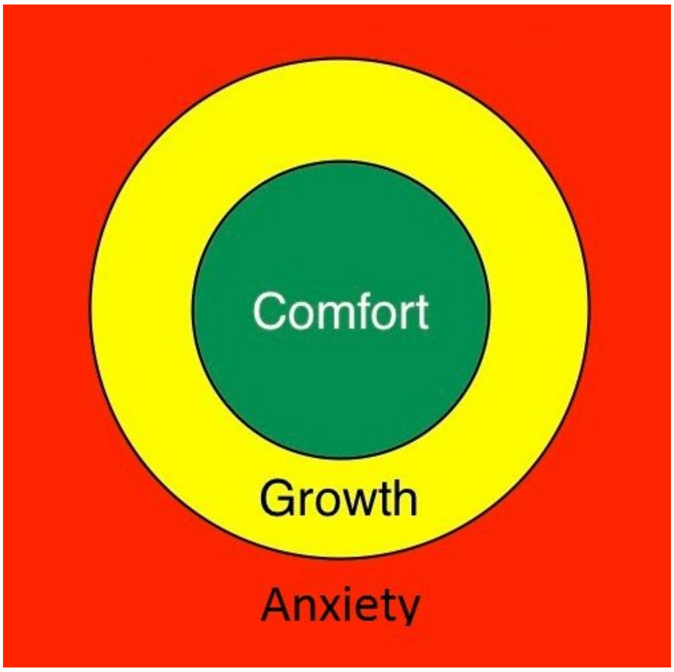

Figure 4 Growth Zone Model (Johnston-Wilder et al., 2013, p. 4)

2.5.1. Mathematical Resilience and Highly-able Students

Kooken et al (2013) identified boredom within the regular classroom in mathematics as one of many forms of adversity that students may have to overcome. Boredom within the regular classroom is a well-researched contributory factor to the under-achievement of highly-able students across all subjects (CIDREE, 2010; Ledwith, 2013; Phillips & Lindsay, 2006; Riedl Cross et al., 2014). Much of the prior research into resilience of highly-able students has focussed on emotional or social concerns and their impact on underachievement (Bland et al., 1994; Ford, 1994; Sowa et al., 1994). While certain studies have utilised the MRS and analysed results (Gürefe & Akçakın, 2018; Hutauruk & Priatna, 2017; Johnston-Wilder, Brindley & Dent, 2014), no such study has, at time of writing, focussed on highly-able students.

The 'growth zone model' may be acutely important for highly-able students, who may regularly inhabit the 'comfort zone' if they do not encounter a challenge in mathematics. In a small case study, Thomas (2020) believed an intervention for highly-able students that introduced them to the 'growth zone model' yielded positive responses from the students. Baker (2019) found that some highly-able students reported their mathematical anxiety, and hypothesised that an intervention would be of benefit to this cohort to offer them further support.

2.6. Conclusion

The review of literature as presented in this chapter informed the decisions for the rest of this research study. Section 0 sought to navigate the numerous definitions and labels associated with 'gifted education'; provide the context behind the author's use of 'highly-able' within this project; and highlight the difficulties for this student cohort within Irish second-level education. It was identified during the literature review that highly-able students may benefit from an improvement in their problem-solving. Whilst problem-solving skills were of obvious importance to students' attempts at problem-solving, the prevalence of attributes related to 'perfectionism' and the existence of 'underachievers' within the student cohort led the author to the concepts of mindsets and mathematical resilience, and, further, how they may influence highly-able students' attempts at problem-solving. In order to explore this further, the decision was made to design a research-based mathematics intervention that sought to improve the problem-solving skills, mindsets and resilience as a single, unifying construct within highly-able students. The decisions taken in the design process will be outlined in detail in Chapter 4, but first, we must examine the research methodology undertaken within this project.

3. Methodology

The purpose of this chapter is to outline in detail the research assumptions made by the author; the theoretical background upon which the research is designed; the research problem and subsequent questions; and the research methods employed by the author to answer these questions. It then concludes by addressing the limitations to the research.

3.1.Research Assumptions

Paradigms are used within research as guides or models as to how to conduct research (Mackenzie & Knipe, 2006). Scotland (2012) defined a paradigm as consisting of four components: ontology, epistemology, methodology and methods; where the ontological and epistemological assumptions made within research guide the selection of methodology and methods. It is therefore appropriate to discuss the assumptions made within this work, before then discussing the methodology and methods employed.

Ontological assumptions are based around the nature or essence of the subject of the research, or more basically defined by Scotland as "*perceptions of how things really are and how things really work*" (Scotland, 2012, p. 9). Mathematics education research studies the factors affecting the teaching and learning of mathematics and the development of methods to improve instruction (Godino et al., 2007). An objectivist view of ontology believes that social phenomena exist independent of the perceptions of social "*actors*" (Saunders et al., 2009) in an objective reality (Scotland, 2012). Conversely, a subjectivist impression interprets reality as a construction of social interactions, individual perceptions and consequent actions (Packer & Goicoechea, 2000; Saunders et al., 2009). How we observe and learn mathematics is largely defined by our interactions within the classroom with our peers and teachers (Walsh, 2015). The emergence of constructivism and social constructivism in mathematics education emphasises the growing movement towards a more subjectivist viewpoint (Packer & Goicoechea, 2000); this research project upholds this viewpoint, with the intervention entrenched in constructivist approaches to teaching, and thus it is underpinned by subjectivist assumptions. It is expected that student interactions with peers and the facilitator within the intervention will help to shape their learning.

Epistemological assumptions are interpreted as those on the nature of knowledge, what constitutes acceptable knowledge in a discipline, and also how to acquire this knowledge (Saunders et al., 2009; Scotland, 2012; Walsh, 2015). The positivist viewpoint is often referred to as scientific, due to the reliance on scientific method to accrue knowledge that is then viewed in an objectivist reality (Saunders et al., 2009). Such knowledge relies heavily on quantitative analysis and testing of hypotheses (Bahari, 2010). In contrast, interpretivists believe that "*knowledge is constructed personally and subjectively by an individual*" (Walsh, 2015, p. 101), and thus human experience plays a large role in this viewpoint. This research follows the belief that mathematics is a personal construction by individuals, following interactions with their environment, and thus follows an interpretivist viewpoint. Furthermore, instruction should follow constructivist principles such as those outlined by Lebow (1993):

- Instructional design should not be restrictive to students' ability to self-regulate the learning process;
- Design the learning environment to enhance the interest and engagement of the learner, and promote self-autonomy;
- Utilise task design to challenge learners and embed the goals of learning in the learning activity;
- Support student ownership of the learning process by promoting the skills and behavioural responses that allow students to take responsibility for their own development;
- To develop students' reflection on the learning process, with particular attention on response to mistakes.

These principles, and the instructional design of the intervention, will be further discussed in relation to the intervention design in Chapter 4.

3.2. Research Problem

In Chapter 1 we presented the research problem, and in Chapter 2 we further elaborated upon the literature that led to this problem emerging in this study. While changes occurred in mathematics education at second-level in Ireland in the past decade, there have been no changes to the education of highly-able students, with classroom differentiation remaining as the sole means of catering for their additional educational needs in school. Ireland's top performers have underachieved in recent decades in international assessments and competitions: scores in PISA have been consistently below the mean; scores in TIMSS are comparatively worse to other similar nations, despite our average and low-performing students achieving comparatively better than these same nations; and our performances in the International Mathematics Olympiad have been much lower than similar nations. Highly-able students are at risk of developing negative attributes of a fixed mindset if they receive praise within the classroom for test scores rather than effort or learning, and these attributes may result in the avoidance of challenging work at a later point. The development of growth mindset related attributes, however, may encourage highly-able students to persist with challenging work, and may help these students to overcome negative traits related to perfectionism. The nurturing of strategies to help cope with challenging work, and improvement in students' mathematical resilience, is also required to prepare this student cohort for when they may be challenged.

A survey of primary and secondary teachers found that, while many wanted to provide adequate support for highly-able students, the emphasis on weaker students did not mean this was always possible (Riedl Cross et al., 2014). A small-scale survey of highly-able students attending Mensa workshops found that the students believed their needs were not being catered for in school (McGrath, 2017); while parents and students interviewed by O'Reilly felt that the regular classroom was not providing a challenge for the students. It was for these reasons that this research focussed on addressing the additional educational needs of highly-able students in second-level education in Ireland.

3.3. Research Aim

The aim for this research was to address the additional educational needs of highly-able students in second-level education in Ireland by targeting an improvement in any combination of their mathematical problem-solving, their mindsets, or their mathematical resilience. This triad construct, known as Problem-solving Potential (PsP), was defined for this research, and will be outlined further in the next section. To achieve this improvement in PsP, a mathematics intervention was designed based on research. The design of the intervention is discussed at length in Chapter 4.

3.4. Theoretical Framework

In the early stages of this research, the author began to question the role of students' mindsets in a problem-solving setting, and in particular with students with recognised high-ability. Past research has warned of the dangers that exist by labelling students as highly-able or by praising ability over effort, and therefore increasing the risk of promoting fixed mindset attributes in these students (Section 2.4.1) (Dweck, 2000; Mofield & Parker Peters, 2018; Niiya et al., 2004). Further research yielded the importance of a growth mindset to being mathematically resilient (Section 2.5) (Johnston-Wilder & Lee, 2010b; Yeager & Dweck, 2012); where, conversely, students with a fixed mindset may avoid challenge or give up more quickly. Problem-solving within mathematics requires determination to reach a solution 2.3.4. In this thesis, it is conjectured that a student's potential in mathematical problem-solving is greatly affected by a combination of the problem-solving techniques or skills they possess, their mindset, and their level of mathematical resilience.

In Section 0, the four levels of complexity for potential were outlined as: aptitudes and dispositions; characteristic adaptation; self and future; and sociocultural mediation (Dai, 2020). According to Dai, the levels at which potential may be acted upon escalate in complexity, and represent the "*main endogenous and exogenous forces propelling talent development*". At the first level, cognitive, affective and conative traits of each individual are nourished, and thus it is at this level that the attributes of each influencing factor of PsP exist and may be nurtured. The manner in which an individual intrinsically prioritises and organises their abilities, traits, self-concepts and motivations is referred to as the characteristic adaptations (Dai, 2020). It is at this level that the positive influence of a growth mindset or high mathematical resilience in the *struggle* domain may take hold, as research has shown that these attributes shape a person's response to challenge (Boaler, 2013; Johnston-Wilder & Lee, 2010b; Yeager & Dweck, 2012). Development at this second level of complexity may also help to relieve negative traits that hinder cognitive progression (Ackerman, 2010; Lubinski & Persson Benbow, 2001). The *value* domain of mathematical resilience determines the degree to which a student values mathematics in their life and further study (Johnston-Wilder & Lee, 2010b), and thus the development of this domain will impact upon the third level of complexity (Dai, 2017).

This triad of influencing concepts is discussed as Problem-solving Potential (PsP) throughout this document, and is represented in Figure 5. While there are other external factors that may impact upon a students' potential to problem-solve - from something as simple as how much sleep they get at night to something more complex as their mental health (Tayraukham et al., 2009) - the three influencing factors of the PsP are the most prominent and impactful as they influence other factors often listed by research, such as motivation and self-regulation (Guven & Cabakcor, 2013). The final level of complexity is the result of external components, such as resources, tools and supports, acting upon the individual to encourage the development of their potential across the first three levels (Dai, 2020). Each of these factors of PsP may be improved through participation on an intervention, as found in prior research (Blackwell et al., 2007; Goodall & Johnston-Wilder, 2015; Schoenfeld, 2013). In order to improve upon their PsP, a student must improve their problem-solving skills, mindset or mathematical resilience, or any combination of these concepts, and to achieve this in this research, a mathematics intervention was designed.

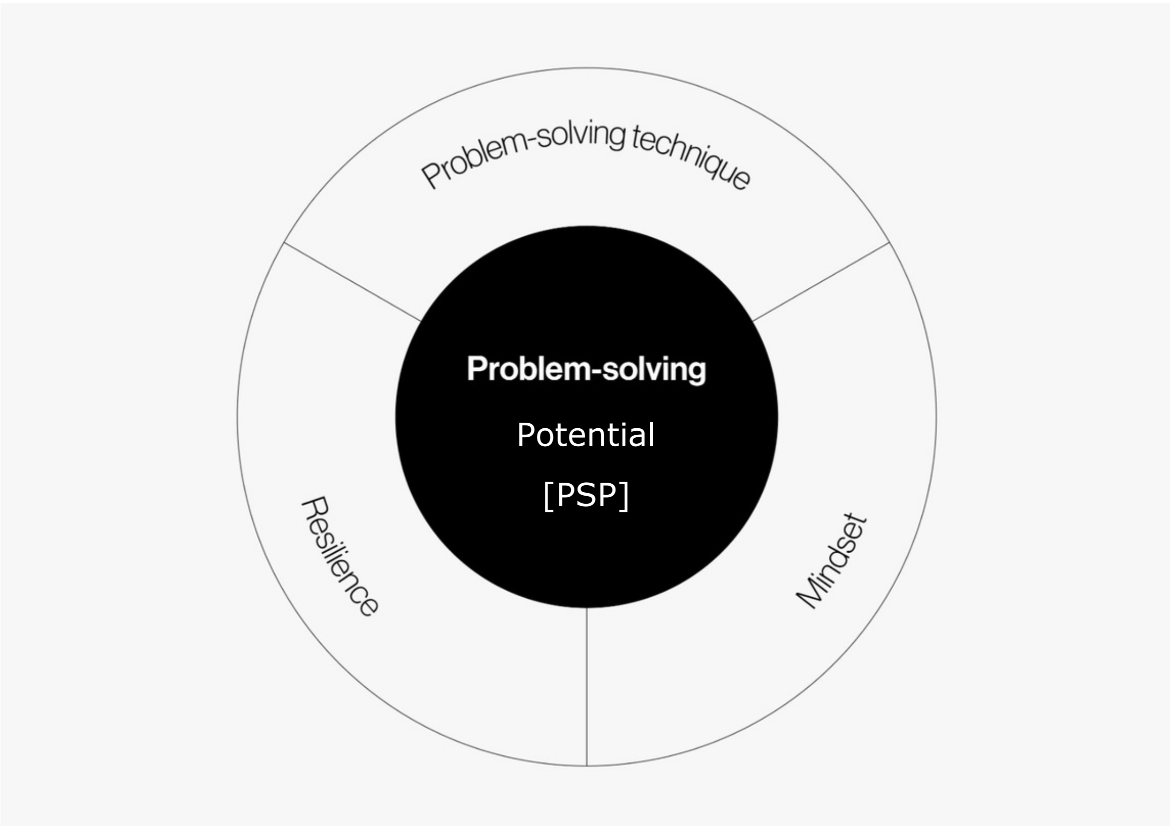

Figure 5 Problem-solving Potential (PsP)

Research into these concepts helped to shape the nature of the intervention design, and also the data collection instruments utilised.

3.5. Research Questions

To adequately address the research problem outlined in Section 3.2, it was necessary to develop related research questions, and to carry out the research with the aim of answering these questions over its course. The following are the research questions to be answered in this document:

1. Has the student sample shown an improvement in their Problem-solving Potential (PsP) over the duration of the intervention?

    A Has the student sample shown a measurable difference in their mindset over the duration of the intervention?

    - Has the student sample shown a measurable difference in their mindset towards general intelligence over the duration of the intervention?
    - Has the student sample shown a measurable difference in their mindset towards mathematical talent over the duration of the intervention?
    - Do the students perceive a change in their mindset after studying the intervention? Are these perceptions consistent with the changes in students' mindsets identified by the research instruments?

    B Has the student sample shown a measurable difference in their mathematical resilience over the duration of the intervention?

    - Do the students perceive a change in their mathematical resilience after studying the intervention? Are these perceptions consistent with the changes in students' mathematical resilience identified by the research instrument?

    C What categories of the problem-solving grading rubric, if any, has the student sample improved in?

- Do they perceive a change in their problem-solving skills after studying the intervention? Are these perceptions consistent with the changes in students' problem-solving skills identified by the research instrument?

D What proportion of the student sample has shown a measurable increase in their Problem-solving Potential over the duration of the intervention?

2. What opinions do the student sample have about the intervention after completing it?

A Do they perceive of any long-term benefits to undertaking the intervention?

B Is there evidence to suggest that the CoPs model was beneficial to the student sample's experiences on the intervention?

3. Are there statistically significant differences in the results of the student samples between different variations of the intervention?

A To what extent, if any, are results different between the student-sample to have taken part in the fourteen-week intervention compared to those who studied the three-week intervention?

B Does the use of an independent facilitator have any measurable difference on the student sample results in comparison with those who studied under the author?

For the duration of this document, the author will address each of these questions and discuss them in relation to the research conducted.

3.6. Research Phases

There are four main 'phases' within this project, as outlined in Figure 6, with each phase informing the following one. The intervention, designed in Phase 1, was adjusted twice during the project, from its original form. The three versions of the intervention are referred to as Iterations 1, 2 and 3 for ease of differentiation in discussion. Iterations 1 and 2 took place during Phase 2, with Iteration 3 in Phase 3, and these are discussed in Chapter 4. The modifications and extensions in data collection between Phases 2 and 3 are outlined in more detail in Section 3.8.2.

Phase 1 began with a comprehensive literature review, centred on gifted education, the teaching of highly-able students, and designing an intervention in mathematics for these students. When viewed through the lens of the Irish context, the decision to focus on problem-solving was made. A review of the prominent literature in this area led to the design of the PsP, the theoretical framework that underpins this research; and also of the CoPs model, outlined in the next section. Decisions regarding the design of the intervention were also made in this phase – such as the role of the teacher, the classroom dynamic, and the selection of adequate problems – with these decisions discussed at length in Chapter 4 of this work.

The creation of the PsP in Phase 1 informed the selection of research instruments for Phase 2, with each branch of the framework requiring a suitable instrument to test student progress. Phase 2 began the implementation of the intervention, with the first four student cohorts participating during this time. Data was collected for each of these cohorts using the test instruments outlined from Section 3.8.3 to Section 3.8.5, and analysed to investigate potential changes in students' PsP. Following Cohort 2, changes were made to the intervention based on observations made by the author, moving the intervention from Iteration 1 to Iteration 2. These changes involved the moving or removing of problems, or the addition of new problems.

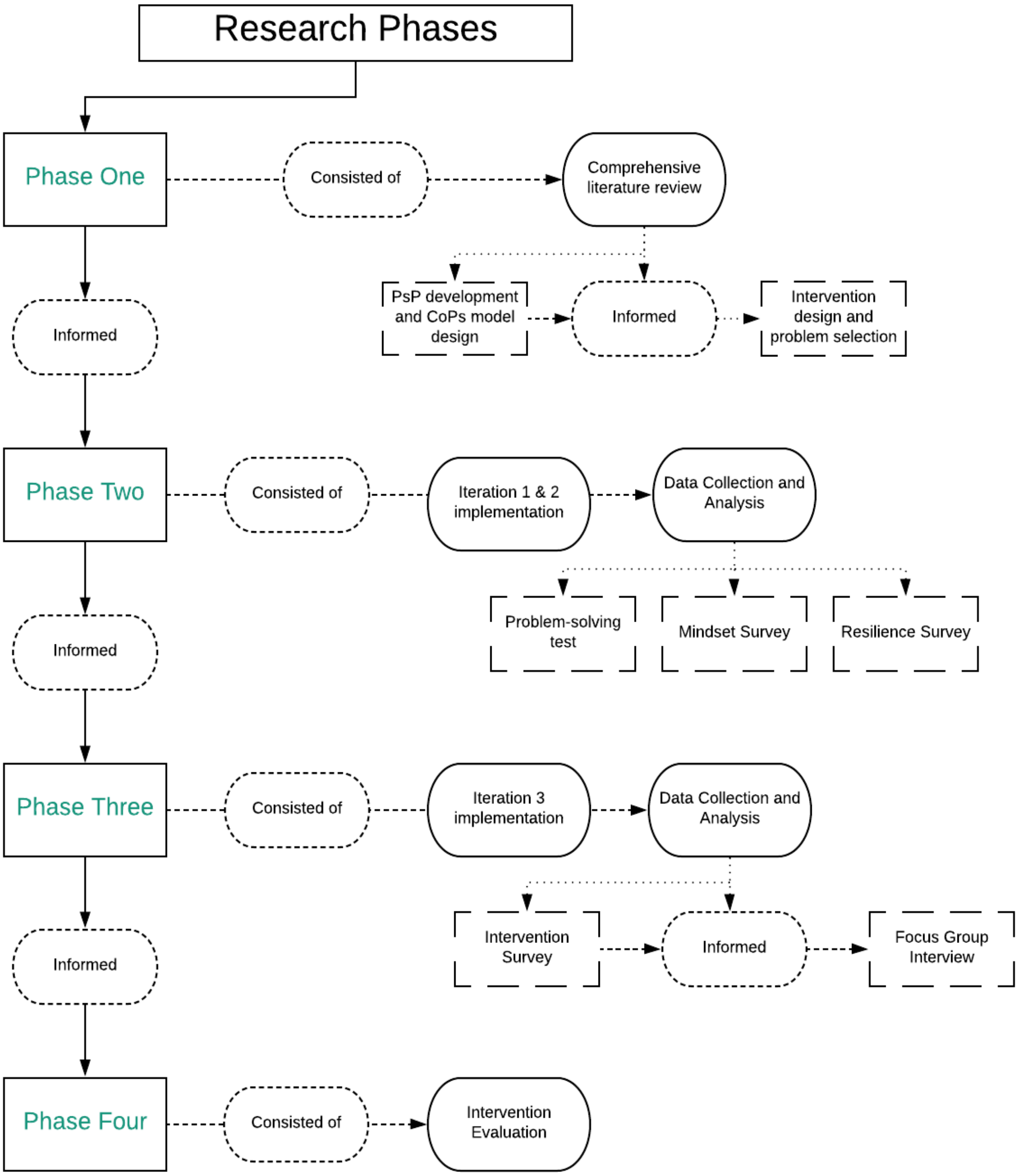

Figure 6 Research Phases

The data collected during Phase 2 of the intervention was largely quantitative in nature, and investigated potential changes to students' PsP over the duration of the intervention. During Phase 2, it was decided that it would be of merit in the next phase to investigate whether or not the students perceived any change in their PsP to have occurred. An intervention survey and an interview were the research methods chosen to address this question, with both being implemented during Phase 3 of the research. The intervention survey was first piloted, and then administered to students from Cohorts 1 to 5 via email, with the responses used to develop four areas of questioning for the focus group interview. Phase 3 also contained the final two cohorts of students participating in the intervention.

Finally, Phase 4 of the research was the evaluation of the intervention for this research, which was accomplished through the use of an evaluation framework for educational interventions, adapted from the work of Shapiro (1987). The evaluation of the intervention is presented in detail in Section 7.1.

3.7. Collaborative Problem-solving (CoPs) model

The main influences in the development of the CoPs model for this research - the problem-solving phases of Mason et al (2010) and the steps of Polya (1945) - have been discussed at length in Chapter 2. These models outline the problem-solving process for an individual, and offer a guide as to how to progress through a problem. The intervention developed for this research draws on the communal aspects of socio-constructivism and collaborative problem-solving, encouraging communication within a group during the problem-solving process, and thus it was deemed necessary to develop a model for problem-solving in a group setting (Figure 7). Each stage of the CoPs model will now be discussed, accompanied by the typical actions of students during each stage in a text box.

There is general acceptance that a problem must be understood before it can be solved (Schoenfeld, 1982), and this is evident within the 'entry' phase of Mason et al (2010) and Polya's first step (1945). In a group setting, an opening discussion encourages students to vocalise their understanding of a problem, and thus creates an opportunity for misinterpretations to be addressed. Collaborative problem-solving relies on the members of a group working together, sharing knowledge and pooling skills and resources, to reach a common goal of solving a problem (Care & Griffin, 2014; Goos & Galbrath, 1996; Rosen & Foltz, 2014).

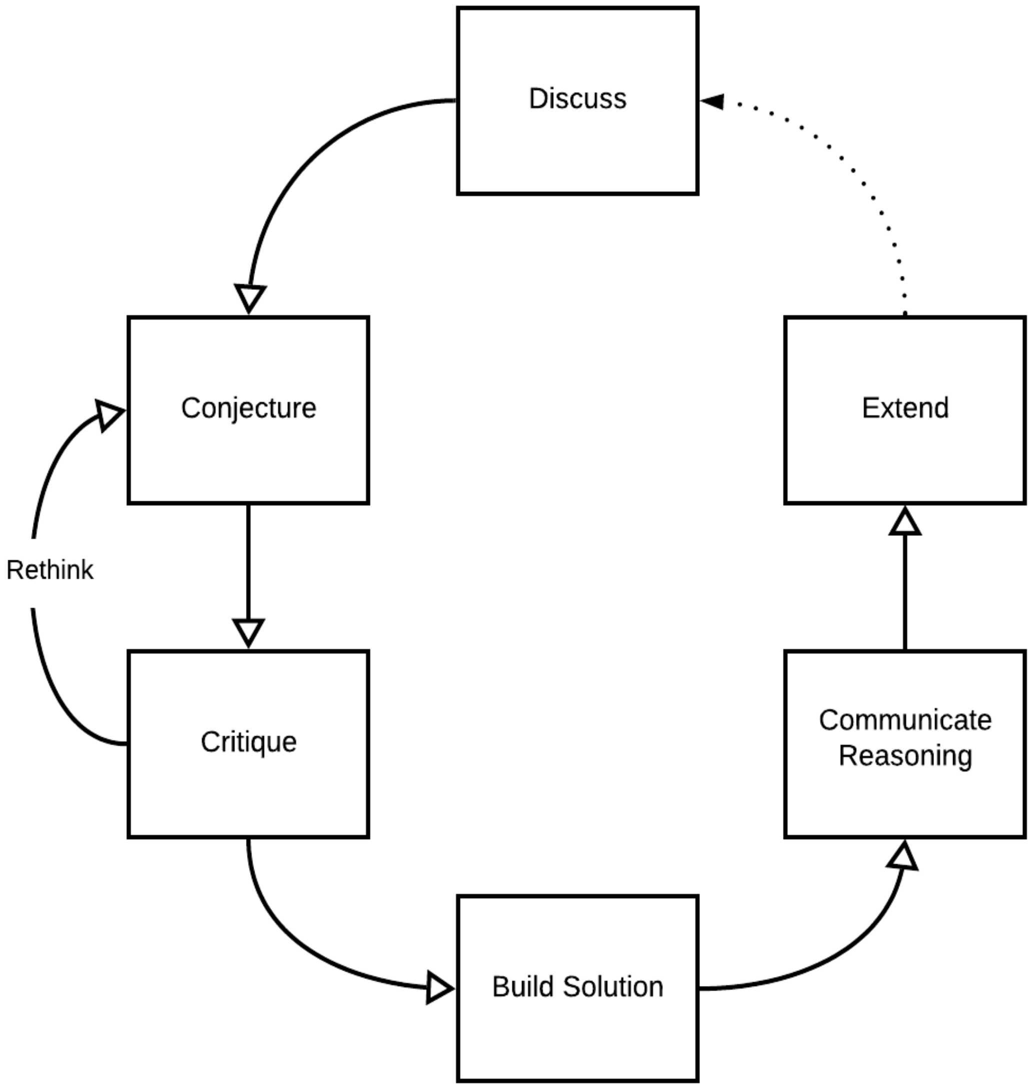

Figure 7 CoPs model

In the "*entry*" phase (Mason et al., 2010), problem-solvers are encouraged to "*introduce*" strategies that may help to solve the problem. In our CoPs model, *discuss* has the same expectation of students, where they introduce their thoughts on how to approach a problem to their group, and, later in the intervention, discuss potential assumptions within a problem.

Discuss: *Read the problem individually; discuss interpretations of the problem; correct any misconceptions; discuss possible approaches to the problem; discuss possible assumptions*

Once a clear understanding of the problem has been developed, students must create a plan to attempt to solve it. Mason et al (2010, p. 58) defined a conjecture as "*a statement which appears reasonable, but whose truth has not been established*". In the second stage of the CoPs model, students develop conjectures for a problem and devise their method(s) of proving these statements. Polya (1957, p. 9) outlined the creation of a conjecture in a problem as the "*main achievement"* in the path towards a solution. Various strategies may be utilised by different students within the same group, or the whole group may work on the same strategy with different parts of the problem. This stage may be revisited on numerous occasions in the problem-solving process of a single problem.

Conjecture: *Strategy, or multiple strategies, chosen; group members allocate tasks to be completed; individual work carried out*

Mason et al (2010) outlined the importance of developing an "*internal monitor*" so that a problem-solver may become more self-aware throughout the process. Self-regulation has been a well-researched area in education. The development of metacognitive strategies such as self-questioning, self-monitoring and self-evaluation, amongst others, can improve performance in academic and non-academic domains (Montague, 2008). In the opening weeks of the intervention, the facilitator encourages students to develop a relational understanding of a problem. Problem-solving solutions or partial solutions are met with the question 'why', prompting students to examine their reasoning and seek a deeper understanding of their approach. This reinforcement to self-regulate encourages students to develop their "*internal monitor*", but also to question and 'critique' the work of those in their group. The *critique* stage highlights the role the student has in monitoring their own reasoning, and the reasoning of their group, as they attempt to solve a problem. Their progress within this stage may result in them being "*stuck*" and returning to previous stages to clarify their work or attempt a new strategy; or proving their conjecture and progressing to the next stage.

Critique: *Self-regulation of own work; critique of group members' work; may become stuck and need to rethink strategy, or utilise another strategy previously dismissed*

Students within the same group may utilise different strategies towards a problem, or members of a group may work on various aspects of the problem with the same strategy (e.g.: working on different patterns for even or odd numbers). *Building a solution* refers to students pooling together all the relevant information for a problem into their complete solution, and checking to ensure it is mathematically accurate.

Build a solution: *Collect all relevant information for the solution; check the solution*

Mason et al (2010) and Polya (1945) emphasised the importance of reflection to the problem-solving process and to improving problem-solving. Polya characterised this step as "*surveying and scrutinizing*" a solution in order to better one's understanding and to create an even better solution (1957, p. 36). Mason et al proposed a structured approach to reflection, whereby the problem-solver systematically analyses the key moments of their solution to better understand them (2010, p. 38). Students participating on the intervention were required to reflect on the problem-solving process, but also to explain it. Collaborative problem-solving requires a shared understanding to exist throughout the process (Rosen & Foltz, 2014), and the *communicate reasoning* stage of the CoPs model exists as the culmination of that understanding. Students are expected to develop their ability to communicate reasoning in several ways: to communicate their own reasoning to the rest of their group to ensure the group understands their work; to communicate the reasoning behind the group solution from the previous phase to the facilitator; and to communicate the group solution in a clear and concise written diary entry (see Section 4.4.5).

Communicate reasoning: *Reflect on the problem-solving process; explain student's own workings; explain the group solution; write diary entry to explain the group solution*

Extend may be seen as the last stage in the CoPs model, or the stage that restarts the process for a new problem. Mathematical extensions to problems are further questions that take us beyond the regular scope required of the original problem, and may be used to create entirely new, yet related, problems (Mason et al., 2010). The creation of such extensions can invoke mathematical creativity amongst students. The concept can also be utilised where a problem was too difficult, and an extension may be applied to simplify the problem. Well-chosen problems may prove exhaustless (Polya, 1957), and thus provide further challenge for students who find a solution quickly.

Extend: *verbally, and occasionally written; reflect on important points in solution; assess mistakes made; extend the problem*

These six stages are combined as shown in Figure 7 above to create the CoPs model first used in this research study.

3.8. Research Methods

Weaknesses in the use of mono-method data collection have been widely published over recent decades, with validity concerns the prominent issue. A mixed methods research approach utilises both quantitative and qualitative techniques. Onwuegbuzie & Johnson (2006) argued this addresses non-intersecting weaknesses of each method, while combining their strengths; while Matthews & Ross (2010) highlighted the possibility for data triangulation to improve validity of findings. This study used a mixed-methods approach to utilise data triangulation, but also due to the varying nature of the research questions. We will now discuss the data collection process, and the instruments used throughout this project.

3.8.1. Participants

In order to test the effectiveness of the intervention, the author accessed participants through CTYI, which operates programmes for highly-able students throughout primary and secondary levels of Irish education and is based predominantly on the Dublin City University (DCU) campus in North Dublin city. Students who wish to participate in their programmes must qualify through one, or more, of the following, which reflects the literature (Ledwith, 2013):

- Educational Psychologist report
- Teacher recommendation
- Previous test scores on the PSAT

Over the course of this doctoral programme, students had the opportunity to study our intervention either with the Early University Entrance (EUE) programme – one day per week for 14 weeks; or as a summer programme – 14 days over a three-week period. Students who opted to study mathematics on either programme were the participants for this project. Of the six cohorts to complete the intervention, the author facilitated on all six, while an independent facilitator was also used for two cohorts. In total, 89 students participated on the intervention:

- Of these, 56 were male and 33 were female;
- 63 students completed the 14-week programmes, while 26 completed the 3-week programme;
- 70 students participated under the author, while 19 participated with the independent facilitator.

3.8.2. Data Collection

All data was collected with permission from CTYI. Each student received a student assent form; a parental consent form; a plain language statement for students; and a plain language statement for parents. The results published in this work are based solely on those participants who returned both the student assent form and parental consent form. In line with best practice, results obtained from students who failed to return either form are omitted from this report. This applies to data collected from the test measures outlined in Sections 3.8.3 through 3.8.5. Results are also omitted if students failed to attend the intervention during any one of the three days of data collection. Data for Phase 2 of this project was collected during three rounds of testing. Table 4 outlines when each round occurred in the two variations of the intervention:

Round of Testing	14-week intervention	3-week intervention
Round 1	Week 1	Day 1
Round 2	Week 8	Day 8
Round 3	Week 13	Day 13

Table 4 Testing Rounds

The instruments used during this phase were the mindset survey (Section 3.8.3), the mathematical resilience scale (MRS) (Section 3.8.4), and a problem-solving test question (PTQ) (Section 3.8.5). Test conditions remained consistent through each cohort and round of testing, with the time breakdown allotted in Figure 8 below:

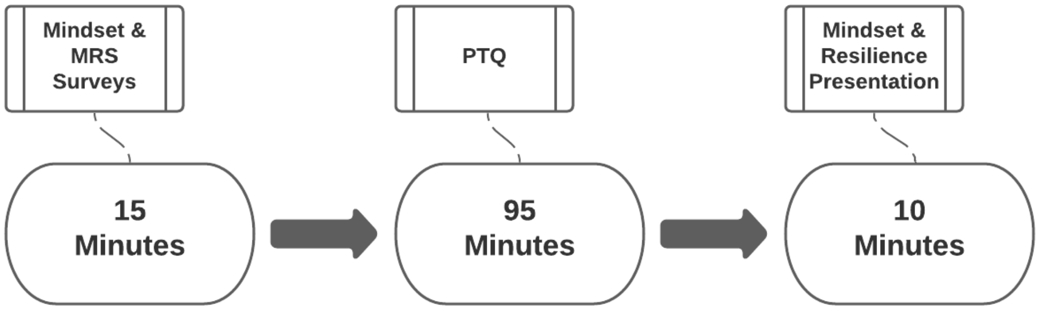

Figure 8 Instrument Distribution Timescale

The mindset and resilience presentation was presented to the students at the end of the data collection in Round 1. This presentation contained the main attributes and benefits of a growth mindset and mathematical resilience, as outlined by the author throughout this work thus far. Any questions from students regarding these concepts were also answered at this time.

In Phase 3, these test measures and conditions were maintained; however, additional data was also collected in response to the observations of the author. The pilot intervention survey was distributed on the final day of Cohort 5. This survey was then used to inform the questions for the focus group interview, and for alterations to the finalised intervention survey. The intervention survey was distributed to parents via email by CTYI, and accessed as a Google Form. This form contained a checklist of consent and assent for parents and students to complete before the survey was taken. Within the consent form was an option for a parent to permit their child to participate in the focus group interview, through which times and dates of availability were declared. Student participants in the focus group interview were selected based on the day and time with highest availability. Diary entries were completed by students at the end of the intervention each week, and collected by the author (Section 3.8.6). The following sections outline in greater depth the instruments used for data collection, and how they were analysed.

3.8.3. Mindset Survey

Test instruments in the study of mindsets have predominantly come from the research of Dweck (1986, 2006). In the process of reviewing relevant literature, numerous versions of mindset survey were found, varying in the number of questions from 3 to 20. For this research, a 16-item mindset survey (Appendix B) was administered to students during sessions one, eight and 13. This survey was a combination of various instruments used by Dweck and other researchers (De Castella & Byrne, 2015; Dweck, 2000, 2006; Dweck & Leggett, 1988; Yeager & Dweck, 2012) through research into malleable intelligence and mindsets:

- Questions 1-8: The first eight questions of the survey are widely used in research investigating students mindsets in relation to general intelligence (De Castella & Byrne, 2015; Dweck & Leggett, 1988). It consists of four fixed-mindset questions, and four growth-mindset questions.
- Questions 9-16: Dweck (2006) outlined the adaptability of the intelligence scale to specific domains; applied by simply replacing the word *intelligence* in each item with the desired domain. For the purposes of this project, *intelligence* was replaced by *mathematical talent* in each of the questions 1-8.

3.8.3.1. Analysis

Each question was answered on a 6-point Likert scale, from strongly disagree to strongly agree. Fixed-mindset questions were assigned a score from 1-6, while growth-mindset questions were reverse scored (6-1) (Blackwell et al., 2007; De Castella & Byrne, 2015; Lee et al., 2012). A mean score was then calculated for each student, with a low score (1) attributed to a more fixed mindset, and a high score (6) attributed to a more growth mindset (Blackwell et al., 2007). Students were then divided into three categories based on their scores (Lee et al., 2012): scores of 1-3 representing a fixed mindset; scores of 4-6 representative of a growth mindset; and scores of 3-4 exclusive displaying attributes of both growth and fixed mindsets.

Each scale of the mindset survey was first analysed independently of the other, and then the correlation between the pair of scales was calculated. The number of students registering in each category was calculated for each scale. To investigate if significant change in students' scores had been obtained, two statistical tests were used. Firstly, a paired t-test was calculated for each scale comparing the data of each pair of testing rounds: Rounds one and two; Rounds two and three; and Rounds one and three. The paired t-test is a parametric statistical test, and assumes that a data set is normally distributed. Each test was calculated for a 95% level of significance, and the null hypothesis assumed that there was no statistically significant difference between the means. Secondly, Cohen's d was calculated for each pairing to investigate the size of the effect of the treatment (the intervention) on the students' scores. Cohen's d works by standardising the difference between two means, resulting in a set of scores that are easily comparable between instruments of various scaling (Goulet-Pelletier & Cousineau, 2018). The resulting score was then categorised based on the guidelines of Cohen et al (2007).

Results obtained were also analysed comparatively between the two time-variations of the intervention, and between the different facilitator groups. Normality could not be assumed for these comparisons, as certain groups were not sufficiently large, i.e. $N < 30$ (Ghasemi & Zahediasl, 2012). For this reason, a Kruskal-Wallis test was used to compare the data sets of each group for each round of testing. The Kruskal-Wallis is a non-parametric test that determines if a statistically significant difference exists between the medians of the data sets (Cohen et al., 2007).

3.8.4. Mathematical Resilience Scale

In order to measure any changes in mathematical resilience, it was necessary to find an instrument that focussed specifically on resilience within the subject of mathematics, rather than more general instruments, such as the Grit Scale (Duckworth & Quinn, 2009). The Mathematics Resilience Scale (MRS) was developed and validated in 2016 (Kooken et al., 2016), and thus at time of writing there exists very few publications yielding MRS results. The MRS consists of 24-items, which are further divided into three domains: value (8-items), which investigates how valued mathematics is to the responder, and thus how motivated they may be in its study; struggle (9-items), or the understanding that mathematics as a subject requires struggle or challenge, and a positive reaction to it; and growth (7 items), referring to the role a growth mindset plays in the overall concept of mathematical resilience. The responses were recorded on a 7-point Likert Scale with a higher score indicating a higher mathematical resilience (Kooken et al., 2016). Kooken et al (2016) highlighted the potential for the MRS to be used to identify highly-able students whose progression in mathematics is at risk due to the negative affective traits they possess. Due to this, its specificity towards the study of mathematics, and its acknowledgement of the links between mindsets and resilience, the MRS was chosen as a test instrument for this research.

3.8.4.1. Analysis

Student responses were recorded on a 7-point Likert scale, with responses coded as: 1 = strongly disagree, 2= disagree, 3 = somewhat disagree, 4 = neutral, 5 = somewhat agree, 6 = agree, 7 = strongly agree. Six items on the scale were reverse scored (Gürefe & Akçakın, 2018; Kooken et al., 2016). A sum total was then calculated for each student (Johnston-Wilder et al., 2014), with a minimum of 24 and a maximum score of 168. With regards to statistical testing, the MRS and each of its individual domains were analysed with the same tests and conditions as the mindset scales outlined in Section 3.8.3.1, i.e. paired t-tests were used to investigate the potential significant differences between overall scores, but also each individual scale, between the rounds of testing; Cohen's d was calculated for each round of testing to determine the size of the treatment effect; and the non-parametric Kruskal-Wallis test was utilised in the comparative analysis.

3.8.5. Problem-Solving Test

The students completed the problem-solving test question (PTQ) directly after the surveys during each round of testing. While investigating possible test questions, it was decided that the question needed to fulfil three main criteria:

1. It needed to be attainable for all students (they needed to possess the pre-requisite mathematical knowledge to attempt the question);
2. It had to provide an opportunity for simplification and extension, to allow students to attempt an easier or harder version of the question depending on their progress;
3. There needed to be more than one path to the solution (to allow students to approach the question in different ways as needed).

As a result, the following question (Gardner, 1961) was chosen to be used for the PTQ:

> *Prof. Henry walked very slowly down the down-moving escalator and reached the bottom after taking 50 steps. As an experiment, he then ran up the same escalator, one step at a time, reaching the top after taking 125 steps. Assuming the professor went up five times as fast as he went down (that is, took five steps to every one step before), and that the escalator moved at a constant speed in both trips, how many steps would be visible if the escalator stopped running?*

This question was chosen because:

1. It was attainable – one solution to the question requires knowledge of the relationship between distance, speed and time; and of simultaneous equations. Each of these are prescribed on the Junior Certificate mathematics syllabus (DES, 2012), and thus it is reasonable to assume that the students involved possessed this knowledge.
2. It allowed for simplification and extension – for students who were having difficulty with the problem, it may be made easier by inserting values for certain variables; and it may be made harder by altering some of the given information, such as the uniform speeds of the escalator.

3. There existed numerous paths towards a solution – the author found two separate solutions prior to the implementation of the intervention involving the use of algebra, but various students studying the intervention have found solutions utilising different areas of mathematics than that of the author.

It was decided to administer the same PTQ for all three rounds of testing for all cohorts, guided by other research in the study of problem-solving improvement (Ali et al., 2010; Hendriana et al., 2018). Whilst students receive the same question three times, and therefore may be aware of the answer to the question by the second or third attempt, the analysis of their solution should focus mainly on aspects other than the actual answer. Furthermore, any student's effort to ask questions related to the PTQ, or to open a discussion during class time regarding it, was dismissed by the facilitator. The students were not made aware of the manner in which the tests were marked at any point in the intervention. One of the main issues with assessing problem-solving in mathematics is the "*failure of students to communicate clearly what they have done or what they are thinking*" (Szetela & Nicol, 1992). Szetela & Nicol continue that an answer alone to a problem is near meaningless, but rather must be accompanied by clear and concise argument from the student. Our CoPs model addressed this shortcoming by requiring students to communicate their reasoning, with the written diary entries encouraging this trait in written form. This aimed to prepare the students to improve their communicated reasoning throughout the intervention. We will now discuss how exactly the analysis of the PQT was undertaken.

3.8.5.1. Analysis

The PTQ was analysed using a pre-existing rubric (Education, 2011) designed to assess problem-solving (Appendix D). The rubric offers a score of 1-6 in each of five areas:

1. **Making sense of the task**: Interpret the concepts of the task and translate them into mathematics
2. **Representing and solving the task**: Use models, pictures, diagrams and/or symbols to represent and solve the task situation and select an effective strategy to solve the task

3. **Communicate reasoning**: Coherently communicate mathematical reasoning and clearly use mathematical language
4. **Accuracy**: Support the solution/outcome
5. **Reflecting and evaluating**: State the solution/outcome in the context of the task. Defend the process, evaluate and interpret the reasonableness of the solution/outcome.

The rubric outlines in detail how students may achieve each mark within each category. The categories selected within this rubric are consistent with those in other problem-solving rubrics – such as the rubric used by PISA (OECD, 2014a), or that developed by Heller et al (1992) to evaluate "*expert problem-solving*" in physics. Hull et al (2013) discussed the need for detailed rubrics that evaluated not only student solutions, but also the conceptual understanding they displayed within these solutions. The ease-of-use of detailed rubrics makes them more accessible to teachers and researchers alike (Docktor & Heller, 2009). The chosen rubric rewards students for communicating and providing clear mathematical evidence, rather than for providing a correct answer, and thus seeks to address the main concern raised in the previous section.

In a study of 713 first year university students, Cooper et al. (2008) found that working in collaborative problem-solving groups improved students' individual problem-solving abilities. For this research, the categories of the chosen rubric may be closely linked to the stages of the CoPs model discussed in Section 3.7. Students are tasked with transferring the skills they developed through group problem-solving into written form. The *discuss* stage of the model encouraged students to think in-depth about their interpretation of a problem, and is scored through the rubric as 'making sense of the task'. In 'representing and solving the task', students are moving through the stages of the model as they develop conjectures and critique their own work as they test them; followed by building their solution. Students are scored on how effectively they 'communicate reasoning' within their solution, just as they communicated reasoning to their group or the facilitator during the intervention. Top marks may only be achieved in categories 1 and 4 through the use of extensions to the problem. The final category rewards students for reflecting on their process, as they had been required to do throughout the intervention. A random sample of test questions were independently graded by the author's supervisor as a measure of research validity.

The test scores, and the scores in each individual category, were analysed for statistical differences between using the same statistical tests as the mindset scales and the MRS.

3.8.6. Diary Entries

In Section 3.7, the *communicate reasoning* stage of the CoPs model outlined why it is important for students to reflect on their attempts and processes towards solving a problem by learning to communicate their understanding of a solution with their group or the facilitator. However, it is also important to prepare students for reflecting or "*looking back*" during individual problem-solving (Mason et al., 1985; Polya, 1945). To facilitate this, students were asked to write a reflective diary entry each week. Hensberry & Jacobbe (2012) found that diary writing during problem-solving aided in student understanding, but also helped students to structure their problem-solving processes.

The 'diary problem' was attempted each week by students in the third hour of the problem-solving intervention, with the final 10-15 minutes reserved for students to write a reflective diary entry on their processes towards solving that problem. Students were given the following prompts on day one as to what to write about: 'what worked, and why'; 'what didn't work, and why'; and 'how did you feel about working as a group?' The diary entries were collected by the facilitator at the end of the hour, and were safely stored in a locked cabinet until the analysis process. The student diaries are further outlined in the context of the intervention in Section 4.4.5.

3.8.6.1. Analysis

A thematic analysis was chosen as the optimum method to analyse the student diaries due to its reliance on themes emerging from the data, which later may be valuable in the triangulation process (Braun & Clarke, 2012). The diary entries were read by the author, then coded and re-coded by hand by the author in order to conduct a thematic analysis (Cohen et al., 2007; Matthews & Ross, 2010), and the emergent themes were then recorded with their frequency of occurrence. While the themes were being defined, certain codes were combined together to further develop their narrative under their relevant theme. A random sample of the diary entries was also coded by the research supervisor as a measure of validity. The diaries were then re-read by the author to extract relevant quotes under each theme, which were collated into a document. The results of the thematic analysis, and a subsequent discussion under each theme, will be outlined in Section 5.5.

3.8.7. Intervention Survey

A survey was developed to gather data on students' experiences of studying the intervention using a mix of closed and open questions (Zohrabi, 2013). The questions were chosen by the author in consultation with their research supervisor, and sought student opinions on any perceived changes in resilience and mindset, amongst other items. Baker (1994) described "*pilot-testing*" as a means of "*trying out*" a research instrument before its eventual use in a study. A pilot version of the intervention survey was developed and administered to Cohort 5 at the end of the last day of the intervention. The intended purpose of the pilot survey was to investigate the clarity of the questions being asked. Following the analysis of the survey, several changes |(alterations to scale descriptors and question-wording) and additions (further detail in mindset question, and addition of "stuck" questions) were made to produce the intervention survey (Appendix A).

The finalised intervention survey was first administered to all past students of Cohorts 1 to 5. The survey was conducted using a Google Form, and emailed to students' parents by CTYI. The Google Form required the parent to complete a consent form, and each participating student to complete an assent form, before access to the survey was granted. The intervention survey was also administered to Cohort 6 at the end of their participation on the intervention, with one additional question to gauge their opinion of mindset-effectiveness in various subject matters. This final question was added following the focus-group interview, and in response to conflicting statements made by students during the interview in relation to mindsets.

3.8.7.1. Analysis

Responses to the pilot survey were collated into a document by the author, and discussed with the research supervisor. Changes were made to several questions, where potential issues were discovered in the language used, and questions were added to extract further detail. In particular, several new questions were added to gain insight into students' beliefs about their mindset.

Whilst closed questions are more easily analysed, open questions often grant better opportunities for participants to state their opinion, and thus a mix of both types of questioning is preferable (Zohrabi, 2013). The online intervention survey gathered quantitative data, where responses were given on a scale for each closed-question, and qualitative data in the form of accompanying statements of opinion from students. The open-responses were coded by the author using grounded theory in order to conduct a thematic analysis for each question (Cohen et al., 2007). Results obtained will be discussed in Chapter 5.

The responses to the online intervention survey were utilised in the preparation for the focus group interview, with four main areas chosen for interview questioning due to the intervention survey data. These will be outlined further in the following sections.

3.8.8. Focus Group Interview

A focus group is a type of group interview that relies on the interaction and discussion between participants and the interviewer in order to gather data relating to a particular subject matter (Kitzinger, 1995). Focus groups allow for participants to discuss common ideas, but also conflicting opinions, in a controlled setting (Secor, 2010). Secor (2010, p. 199) suggested that individual interviews are best-served where the interviewees are particular experts, where a focus group may be difficult to bring together, or where the backstory of an individual is relevant data; however, focus groups allow for "*experiences [to be] talked about and debated*". In the latter stages of the research project, it was decided to invite students who had participated in the intervention to take part in a focus group interview. Students from Cohorts 1 to 5 were invited (as Cohort 6 was only beginning the intervention), with eventual participants emerging from Cohorts 3, 4 and 5. The aim of the focus group was to gather information from students regarding their experiences studying the intervention, and any perceived benefits to it. The role of the moderator was to facilitate open dialogue in a semi-structured interview, consisting of pre-prepared questions and spontaneous sub-questions that arose due to conversation (Powell & Single, 1996). Open-ended questions were used to encourage conversation amongst participants.

Student participation in the focus group was determined via the Google Form outlined in Section 3.8.6. Parents were requested to issue consent for their son/daughter, and then prompted to suggest dates and times of availability. The researcher compiled this information into a spreadsheet to determine the optimum selection. The focus group interview was then organised for 6pm on Tuesday, 20th November 2018. Parents were contacted directly by the researcher (email addresses were submitted together with availability) to inform them of the scheduled interview. A list of interview questions was prepared by the researcher (Appendix E), consisting of four main question areas – new skills, mindsets, challenge, and problem-solving in school - and optional follow-up questions. The moderator for the interview, a staff member in the School of Mathematical Sciences, Dublin City University, was not familiar with the students and had no prior interaction with them. The use of an external moderator allowed participants to express their opinions more freely (Onwuegbuzie & Leech, 2007) as they had no direct involvement with this research.

3.8.8.1. Analysis

The focus group was recorded using three recording devices, with permission from all participants. The recording was transcribed by the author, and a form of inductive analysis was then used to extract meaning from the empirical data (Powell & Single, 1996). The data collected was coded and re-coded several times by the author to conduct a thorough "*thematic analysis*" (Kitzinger, 1995; Powell & Single, 1996; Secor, 2010). Thematic analysis involves the flagging of themes, words, phrases or interpretations through the transcript, which are then gathered into categories and sub-categories if patterns emerge (Secor, 2010). The patterns found from the interview data for this research are outlined in Chapter 5, and utilised in discussion to triangulate data collected through the other instruments utilised in this research.

3.9. Intervention Evaluation

"*Educational evaluation is the systematic appraisal of teaching learning*", and new educational interventions must be evaluated to assess their relevance and appropriateness in the field (Wilkes & Bligh, 1999, p. 1269). A form of intervention evaluation for medical treatments used by Shapiro (1987) has since been adapted to the field of education (O'Meara, 2010; Walsh, 2015), and evaluates an intervention through under four headings: intervention effectiveness; intervention integrity;

intervention acceptability; and social validity. The evaluation of the intervention for this research will be discussed in depth in Section 7.1 under each of the four headings, each of which are further subdivided. Data collected through the research instruments outlined in this chapter will be utilised where applicable in the evaluation process.

3.10. Research Issues

In this section, any issues regarding the reliability or validity of data collection and analysis are outlined, with justification given as to why decisions were made in relation to the development of the intervention and the chosen data collection tools. Limitations to the study, its intended role in mathematics education in Ireland, and the ethical implications of the intervention are also discussed below.

3.10.1. Data Collection and Analysis

Data collection tools for mindset and resilience were consistent with those already used in research, and their analysis followed a strict method used with no issues of interpretation, as outlined in Section 3.8.3. The Problem-solving Test Question (PTQ) was used consistently through each cohort of students, with test conditions maintained each time. The PTQ was analysed using a pre-existing problem-solving rubric discussed in Section 3.8.5.1, and found in Appendix D. The focus group interview was conducted by an independent moderator, and followed the research recognised dynamic of a semi-structured interview (Powell & Single, 1996), with both pre-planned questions and questions arising from participant responses. The intervention survey was first piloted to investigate potential language or question issues (Baker, 1994). The online version of the survey was distributed by CTYI due to data protection rights of the participants. The diary entries were completed by students during their participation on the intervention, and are further discussed in Section 4.4.5.

3.10.1.1. Researcher Distance

While the author was heavily involved in the implementation of the intervention and the analysis of data, certain measures were in place to maintain objectivity, with those pertaining to the analysis outlined in the previous section. In relation to the PTQ, for Rounds 2 and 3 the author analysed each students' work without checking their score from previous rounds of testing. The author's qualification as a mathematics teacher for second-level education in Ireland meant they were adequately qualified to facilitate the intervention. The intervention was also taught by an independent facilitator, and the results remained largely consistent with those obtained under the facilitation of the author, as will be discussed in Chapter 5. The focus group interview was conducted by an independent member of staff from the School of Mathematical Sciences in DCU, and the author transcribed and analysed the audio using approved research methods.

3.10.2. Ethics

Ethical approval for the research was required in order to teach the intervention to the desired cohort of students (Transition Year; aged approximately 15/16). An application for an expedited-review was submitted to Dublin City University in accordance with the DCU Ethics Guidelines in January 2017. Approval was granted in early February 2017. The application concerned the quantitative data collection tools and the diary entries.

An amendment to the ethics for the research was submitted in September 2018 to facilitate the qualitative data collection involved in the intervention survey and focus group interview. The ethics amendment was approved in September 2018.

Due to the participation of minors in the data collection, a number of ethical criteria needed to be adhered to. The most important of these were:

- Any adults involved in the data collection or teaching of students were Garda vetted, in accordance with DCU research policy (https://www.dcu.ie/researchsupport/rec_guidance.shtml). The author and alternate facilitator were Garda vetted through CTYI; while the interviewer was Garda vetted through DCU.
- Students who wished to participate in the research were issued with assent forms to be signed with a witness, and a plain language statement to ensure accurate communication of what the research entailed.
- Parents were issued with a consent form to allow their son/daughter to participate in the research, and a plain language statement to explain the research.
- Students had the option to opt out of the research at any period.

- Data collected was stored in a locked cabinet, solely accessed by the author or their supervisor; and data analysis was stored on the author's password-protected computer.
- Data published in this work is anonymous, and the identity of the participating students is protected in accordance with ethical guidelines.

Consent and assent forms are available in Appendix I, and plain language statements are available in Appendix J.

3.10.3. Reliability

In research, reliability is defined as "*the degree to which measures are free from error and therefore yield consistent results*" (Thanasegaran, 2009, p. 35). Each cohort participated on the intervention under the same conditions, relevant to the time variation chosen. The rounds of testing for each cohort were also conducted under the same test conditions throughout the duration of this research project. The maintenance of conditions within the intervention and over the rounds of testing were important features of reliability for the results collected. Each of the data-collection instruments, and their relevant analysis methods, have been detailed in this chapter thus far, with past literature drawn upon to strengthen their reliability. The reliability of the intervention was further tested through the use of an independent facilitator, where similar results were obtained.

3.10.4. Validity

According to Zohrobi, "*validity is concerned with whether or not our research is believable and true and whether it is evaluating what it is supposed… to evaluate*" (2013, p. 258). While various types of validity exist within current research, those most important within this research are descriptive, generalizability, internal, and statistical conclusion (Cohen et al., 2007). It is now important to outline each of these validity types concerned with this project, and how they have been addressed:

- Descriptive validity – the "*factual accuracy*" of the research and the results reported. The validity of the instruments used during this project, and the objectivity of the author throughout the analysis process, was discussed in Section 3.10.1.
- Generalizability – the degree to which the theory created by the research may be utilised outside of the research. The CoPs model was designed to build upon current prominent

problem-solving theory, and to introduce a model that may be applied within any group problem-solving setting. The PsP is a triad construct that outlines three possible areas for students to improve if they wish to become better problem-solvers, and thus is widely applicable within this context.

- Internal validity – the demonstration that reported results are corroborated by the raw data. Chapter 5 outlines in detail the results of this research, with raw data supplied in support or as an accompanying appendix where applicable.
- Statistical conclusion validity – how statistically strong reported results or relationships may be. The statistical tests chosen for this research are outlined earlier in this chapter, together with the reasoning behind each test selection. Where there was uncertainty with the statistical strength of a test, a follow-up test was used to confirm the result.

3.10.5. Triangulation

Triangulation is defined by Jick (1979) as the combination of different methods to "*improve accuracy of… judgements… on the same phenomenon*". The four most commonly discussed forms of triangulation are: data triangulation, investigator triangulation, methodological triangulation, and theory triangulation (Flick, 2004). This research employed each of data, methodological and investigator triangulation.

Data triangulation is associated with the use of multiple sources in a study to gather data (Flick, 2004; Jick, 1979). This research utilised this method of triangulation through data collection in the form of surveys, a focus group and diary entries. Additionally, Turner & Turner (2009) outlined the "*variance in events, settings, times*" as a method of data triangulation, which this study utilises through the two variations of the intervention outlined in Section 4.3.

Bekhat & Zauszniewski (2012) found methodological triangulation to be "*beneficial in providing confirmation of findings, more comprehensive data, increased validity, and enhanced understanding of the studied phenomenon*". This research employed 'between-methods' methodological triangulation, with use of both quantitative and qualitative methods to corroborate findings.

Investigator triangulation is defined as the use of multiple researchers or investigators in a study (Flick, 2004; Turner & Turner, 2009). Within qualitative research, investigator triangulation often takes the form of multiple coders of data. As previously mentioned, a random sample of any qualitative data for this research was coded independently by the research supervisor to ensure reliability and to corroborate the findings of the author. Further to this, an independent facilitator was used during Cohorts 3 and 6. A comparative analysis of the student results between classes operated by this facilitator versus those of the researcher is presented later in this thesis.

3.10.6. Limitations

The research contained a number of limitations, although the author has sought to address each limitation where it has presented itself in the writing, and discussed each decision made with knowledge of the limitations they created:

- Student cohorts: The students in the programme entered via the Centre for Talented Youth in Ireland (CTYI), and as a result the author did not have a say in the selection criteria for students to participate. The criteria used are consistent with Talent Search models used in educational research (O'Reilly, 2010), and also with Irish standards for selecting students with high ability (NCCA, 2007). The student cohorts chose to study an extra mathematics module out of school, either over their summer holidays or in-place of school one day per week. Due to this, there is a possibility that these students were more inclined towards growth mindsets and high mathematical resilience than students from the wider population of highly-able students.
- Time constraints: CTYI programmes granted 5 hours of teaching per day for the intervention. The problem-solving intervention discussed in this research encompassed 3 hours of this time. During the three data collection weeks, students completed the test measures during the remaining 2 hours so as not to interfere with the study of the intervention. Students were constrained to complete all test measures in this two-hour period. The length of the intervention was also constrained to 14 days, either over the 3-week or 14-week period discussed in Section 3.8.1, as this was the length of time for CTYI programmes.

- Cost: The CTYI programmes from which participants were accessed are fee-paying programmes, and therefore may not be accessible to all students. For those living outside Dublin, there is also a cost involved in daily travel to DCU for the programme.
- While students of the 3-week intervention participated during the summer, those of the 14-week intervention attended for one day per week during the school term. Therefore, it cannot be stated with certainty that they were not studying mathematics in school during the rest of the school-week.
- Response rates:
    - Students were not obligated to participate in the research. The data collected for problem-solving, mindset and resilience presented in this work is solely representative of the students who completed all 3 test weeks, and also returned signed assent and consent forms.
    - The Intervention Survey completion and subsequent participation in the focus group interview were achieved via an external email sent to parents of former students through CTYI. Participation in the online survey was voluntary, with a response rate of 28%. Suggested practice for dissemination of online surveys, including reminders to boost response rates, were used (Baruch & Holtom, 2008). Non-responses may have been the result of: lack of parental consent; lack of student assent; time constraints of students in the Leaving cycle of secondary school; parent email account out-of-use; or email sent to junk mail. Of those who completed the survey, 18 agreed to participate in the focus group interview; however, date and time availability reduced this to 8 students.
    - As a direct result of this, discussion in Chapter 6 between qualitative and quantitative findings are between different sample sizes.

3.11. Conclusion

The purpose of this chapter was to outline the methods undertaken by this research to investigate the research problem. The theoretical framework underpinning this research - a triad construct defined for this research known as PsP - and the CoPs model – a problem-solving solving process specific to group-work – were presented. The majority of this chapter, however, outlined the chosen research instruments, their theoretical background, and the analysis process used for this research; and the results of these analyses are provided in full in Chapter 5. As has been discussed thus far in this work, the aim of this research was to address the additional educational needs of highly-able students, by targeting an improvement in their PsP, and to achieve this a mathematics intervention was designed, We will now discuss the design of the intervention.

4. Intervention

This chapter gives an overview of the design of the intervention, from its beginning phases, and how it was adjusted to take its final form at time of writing. The intervention was designed under a mix of constructivist and socio-constructivist paradigms, implementing problem-solving as the teaching approach to foster the development of highly-able students' Problem-solving Potential (PsP) in mathematics. The intervention emphasised: problem-solving strategies; higher-order thinking skills, such as reflection and extension; collaborative learning skills; mathematical resilience; and characteristics related to a growth mindset. Problems were explored by students in groups in a student-centred learning environment, with a teacher acting as facilitator, following the CoPs model created during this research study.

Over the duration of this project, students had the opportunity to study the intervention in one of two formats:

- A 14-week programme: one day per week; three hours each day
- A 3-week programme: five days in two weeks, and four days in the final week; three hours per day

This chapter aims to express the reasoning for each decision made in designing the intervention.

4.1. Background

In Chapter 1, the research problem was first outlined, and drew attention to the risks for highly-able students if their additional educational needs are not catered for. This was followed by an in-depth literature review in Chapter 2, through which the state of gifted education in Ireland was discussed, and the oversights with regards to highly-able students were reviewed. It was found that the main strategy employed within schools at present to cater for these students is classroom differentiation by the teacher, and thus they rely heavily upon out-of-school programmes for additional support, although these are not always suitable for students for a variety of previously discussed reasons. It was therefore deemed necessary to design a mathematics intervention for this research that sought to address important areas of development for highly-able students in Ireland.

During Phase 1 of the research, an in-depth literature review was undertaken, which informed the design of the intervention and the selection of problems. This study aligns with the popular belief that mathematics as a subject is best learned through the construction of knowledge, as discussed in Section 2.3, and thus this intervention was designed from a constructivist viewpoint. The development of the intervention was also shaped by this project's theoretical framework, discussed in Section 3.4, whereby students' PsP is affected by: the problem-solving skills they possess; their mindset; and their level of mathematical resilience. Vygotsky's "*zone of proximal development*", discussed in Section 2.3, suggests that students learn best through their interactions with a facilitator or other students (Vygotsky, 1978). Collaborative problem-solving was identified as the optimum teaching-style for the intervention. It encourages students to develop the same problem-solving skills as individual problem-solving, whilst also enhancing communication and cooperation skills (Häkkinen et al., 2017). Aspects of the instructional design, such as group-work or the facilitator, are discussed later in this chapter.

4.2. Target Demographic

In Section 2.2, Transition Year (TY) was identified as an optimal time to introduce a mathematics intervention for highly-able students for two reasons. Firstly, there is greater flexibility within the guidelines of study for TY (DES, 1994). While there exists no predetermined syllabus for the year, the guidelines are clear in the assertion that TY should not be considered a third year of study towards the Leaving Certificate. Secondly, highly-able mathematics students undertaking TY have a common level of mathematical content knowledge as they have all completed Junior Cycle mathematics. Problems were chosen for the intervention under the assumption that all students had completed the higher-level curriculum due to their recognised high ability in mathematics.

The NCCA (2007) listed eight methods of identification for highly-able students: observation, parent referral, peer referral, self-referral, referred by other individual or organisation, psychologist referral, teacher referral, and a school-wide process. These methods of identification are in-line with much of the literature in the field (O'Reilly, 2010; Renzulli, 2012; Swiatek, 2007). Many highly-able students may perform to a high standard in testing scenarios – such as standardised testing or exams – whilst others may underperform, and thus rely on observations by parents, teachers, or others, who identify the potential for high-ability within a student. Finally, a student self-referring may possess the motivation to develop high-ability, which Sternberg (1984) believed was an important characteristic.

As discussed in Section 3.8.2, the student cohorts were accessed through CTYI, whose guidelines for qualification for their programmes fit into the NCCA identification parameters. The majority of students had also completed third year at the time of participation. A further breakdown of the student cohorts is discussed in Chapter 5.

4.3. Time Layout

The intervention was designed as a three-hour per day programme, to run over fourteen days; and was tested as both a 3-week and 14-week programme (as discussed in Section 3.8.1). The exact length of fourteen days was determined by the restrictions of both the summer CTYI programme and the Early University Entrance (EUE) programme. During the first session of each cohort, the author delivered a presentation to students on the concepts of mindsets and mathematical resilience. In the presentation, the author provided details of the benefits of both a growth mindset and high mathematical resilience specific to highly-able students, such as those discussed previously in Sections 2.4 and 2.5. After this presentation, the intervention was designed such that each day did not require a formal introduction from the facilitator, but rather began with the first problem of the day.

In total, there were six different cohorts of students who completed the intervention, which occurred during Phases 2 and 3 of the research. Although each cohort of students spent an equal amount of time studying the intervention, the layout varied somewhat between the 14-week and 3-week programmes. In the 14-week format, the mathematics intervention was held one day per week over a course of 14 weeks from September to December, or January to April. One week in each of Cohorts 1 and 4 was postponed due to circumstances outside the control of the author, with an extra week added to the end of the intervention in replacement. Students studied the intervention from 10am to 2pm, with a 1 hour break at 12 noon. The hour after the break was referred to as a 'tutorial'. This predominantly operated identically to the regular class, with one additional feature: the reflective diary entry (discussed in Section 4.4.5).

Cohort 2 was the first to complete the intervention as part of the CTYI Summer programme over a 3-week period, with two weeks of five days and a third of four days, with weekends taken as a break from class. Students studied the intervention from 9am to 1pm, with a one-hour break at 11 am. Cohort 5 studied the intervention as part of a free CTYI Summer programme from 10am to 2pm, with a one-hour break at 12 noon. Similar to the 14-week format, those studying the intervention during the summer programme had a two-hour class in the morning, followed by a one-hour tutorial directly after the break. We will now outline the instructional aspects of the intervention.

4.4. Instructional Design

The coming section outlines the instructional tools used in the intervention, and the reasoning behind each decision. It begins by discussing the structural breakdown of the intervention into themes, each of which is elaborated upon later in this chapter. It then lays out the functions of both group-work and the facilitator within the intervention.

4.4.1. Themes

In order to create a formal structure for the intervention, the decision was made early in the development stage to create 'themes' to underpin each session, with each serving a distinct purpose. The themes referred to either strategies, topics, or a combination of both. The use of strategies for effective problem-solving is a well-researched area within mathematics, with many suggesting similar or identical strategies (Krulik & Rudnick, 1989; Posamentier & Krulik, 2015; Schoenfeld, 1982). Seven of the most prominent problem-solving strategies identified from the literature were each assigned to one of the first seven days of the intervention (Table 5). Each strategy is further discussed in relation to intervention content in Section 4.5.

Day	Theme
1	Visual
2	Patterns
3	Generalising & Specialising
4	Conjectures
5	Assumptions & Questioning
6	Structure
7	Working Backwards

Table 5 Daily Strategy Themes

Problems were chosen each time based on their suitability to the theme, while bearing in mind that carefully-chosen problems should be designed such that the path to a solution (of which many may exist) is not immediately obvious. Indeed, this is to be encouraged as the means to an effective problem-solving experience (OECD, 2004; 2014a; 2017). Such is the nature of problem-solving however, that given that problems within each session lend themselves to multiple different solutions, sometimes they did not specifically require the use of the specified strategy at all times. Alterations were thus made to the intervention at two stages (identified as Iterations 1, 2 or 3; as discussed in Section 3.6), with these changes occurring as a result of observations by the author regarding the suitability of problems to particular themes. On other occasions, encouraging students to seek multiple solutions to certain problems often yielded the discovery of a solution within the parameters of the chosen theme.

Once students had been introduced to the chosen strategies, themes became more generalised. Days eight to 11 altered the categorisation of problems to meet those within the Junior Cycle Strands: problems in Day 8 focused on Number Theory; Day 9 encouraged the development of algebraic generalisations; Day 10 explored problems involving geometry and shape; and Day 11 focused on Probability and Statistics. Through each of these sessions and the remainder of the intervention, students were tasked with choosing effective strategies from those discussed in the opening seven days in order to solve the problems. The remaining three days of the intervention served to unify the students' learning to-date, with problems coming from any topic, solvable with multiple strategies previously covered.

4.4.2. Problems & problem-solving

The use of the term "*problems*" within mathematics has a confusing and troubled history (Schoenfeld, 1992). To those mathematically inclined, problems are what we create when the outcome or solution is unclear, and the process to finding this outcome or solution is the art of problem-solving. The misuse of these terms throughout the decades in reference to procedural mathematical tasks (Mason, 2016) however, has created the necessity to outline what is meant in the context of this intervention by problems, and problem-solving. The author offers this definition of what constitutes a problem from Posamentier & Krulik's (2015) work in outlining problem-solving and its benefits for teachers:

> *"A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent path to the solution."*
>
> *(Posamentier & Krulik, 2015, p. xv)*

They made the distinction between those problems, which require "*analysis and synthesis of previously learned knowledge to resolve*"; and "*questions*" (situations involving recall) or "*exercises*" (practice to reinforce knowledge).

The CoPs model designed for this research formed a unifying thread throughout the intervention due to its application within every problem, and through every theme. While it was not explicitly introduced to students, they were encouraged to move through problems in the manner expressed by the model. This was done to develop a habitual problem-solving process amongst the students, rather than relying on a 'rule' they must follow.

4.4.3. Group-work

The benefits of constructivist and socio-constructivist approaches to mathematics have been discussed in Chapter 2, with particular attention drawn to collaborative problem-solving as an important 21st Century skill for students. Collaborative problem-solving leads to the development of problem-solving skills, just as in individual problem-solving, but also stimulates communication skills (Häkkinen et al., 2017; OECD, 2017). Vygotsky's theory on the "*zone of proximal development*" emphasised the interaction between a learner and either a facilitator or their peers, as a fundamental component in the mastery of skills or concepts (John-Steiner & Mahn, 1996; Vygotsky, 1978). To facilitate collaborative problem-solving as the primary teaching and learning approach for this intervention, group-work was employed throughout the students' participation. Dolmans et al (2005) outlined group-work as a *"stimulus for interactions"*, with students required to work collaboratively and develop their skills of communication in order to communicate reasoning. Edwards and Jones (1999, p. 284/5) examined students' experiences of group-work in small collaborative groups through semi-structured interviews, with emergent themes from the data – such as "*benefits of group-work*", "*confidence building*", "*sharing knowledge*" – displaying overwhelming positivity for the process. Group-work has been found to increase students' understanding of concepts within mathematics, aided by the requirement for discussion, rather than the rote-learning of material (Koçak et al., 2009; Sofroniou & Poutos, 2016).

While it has been widely mentioned in media that highly-able students prefer to work alone (French et al., 2011), it has been suggested in research that they are willing participants in group-work if: they are surrounded by like-minded peers (Davis & Rimm, 1989); their work is appreciated by their peers or the teacher (French et al., 2011); or they have had positive prior experiences of group-work in that environment (Neber et al., 2010). Further, their preference for group-work may strengthen when dealing with open-ended questions, and in a situation where their learning goals are equal to those of their group members (French et al., 2011). It was therefore deemed beneficial to utilise group-work during the intervention to provide the students with the opportunity for collaboration amongst like-minded, equally motivated peers, which they may not have encountered in a school setting.

As highlighted in Section 2.3, student interactions and their environment play an important role in their mathematical development. Students were assigned to groups on the first day of the intervention, and remained in their group for the duration; with exceptions occurring where absentees required a group's remaining members to temporarily join another group. During Cohort 1, students were divided into groups of four or five students; however during subsequent cohorts this was altered to three to four students per group. This alteration was made following observations by the author, such as increased distraction and greater difficulty in subdividing workload within a problem for the larger group size of five.

The CoPs model, as set out in this research (Section 3.7), was specifically designed to cater for a group dynamic in solving mathematical open-ended problems. For this reason, communication within a group was encouraged throughout each cohort. On the first day of the intervention, a set of 'house rules' was also introduced to the students with regard to the classroom dynamic. While most of these rules were classroom management in nature (toilet policy, etc), there were also two rules in relation to the CoPs model: an opening discussion for each problem was required; and each student must have a functioning understanding of the group's solution to each problem. These two rules aided in the encouragement of communication from the beginning, and it was further encouraged through the role of the facilitator, which will now be discussed.

4.4.4. Facilitator

The main role of the facilitator within the classroom was that of providing scaffolding for student learning as they solved problems. Dolman (2005, p. 734) defined the facilitator as necessary *"to keep the learning process going, to probe the students' knowledge deeply, to ensure that all students are involved in the process, to monitor educational progress of each student in the group and to modulate the challenge of the problem"*. With this description, it is clear that the facilitator takes a secondary role in the student-centred classroom, but rather must be available to encourage and question students to aid in their progression.

Schoenfeld (1992, p. 63) emphasised the questioning nature of the facilitator – asking students to explain their reasoning throughout the problem-solving process, despite how uncomfortable this made them in the early stages – and how this encouraged students to seek the conceptual understanding for themselves over time. Mason et al (2010) believed in a similar process, whereby utilising an external "*enemy*" to question your work would lead to the development of an "*internal monitor*". The CoPs model requires students to "communicate their reasoning" to the facilitator, their group-mates or themselves through the diary entry. This step was designed to emphasise the development of the inner-questioner within students. Similar to Schoenfeld (1992), a level of discomfort with this communication was observed in the early stages with each cohort, but once this subsided, the practice became habitual for students.

Blackwell et al (2007) found that a mindset intervention may improve student motivation within the classroom. In our project, a further role of the facilitator was to introduce students to the concepts of mindsets and mathematical resilience at the beginning of the intervention, and to encourage behaviour consistent with a growth mindset and mathematical resilience throughout. Students were encouraged to seek to understand and accept their mistakes; to accept struggle as important for learning and devise plans to overcome this; and to be resilient in the face of failure within a problem.

The principal facilitator for each cohort of students on the intervention was the author for this project. The author is a fully-qualified mathematics teacher for secondary-level education, with a high content knowledge of mathematics. The early stages of research into the pedagogical approaches taken universally in the mathematics classroom, and also specifically with highly-able students, meant the author was justifiably qualified to facilitate the intervention. During Cohorts 3 and 6, a secondary facilitator was also used due to the increase in the number of participants who opted to study mathematics. The secondary facilitator had a background in mathematics, and a PhD qualification in engineering, combined with several years of teaching highly-able students with CTYI. Prior to commencing teaching Cohort 3, the author had a detailed meeting with the secondary facilitator to discuss the aims and learning objectives of the intervention. Two separate classes operated concurrently and identically throughout these two cohorts, with the author responsible for Cohorts 3A and 6A, and the secondary facilitator responsible for 3B and 6B.

4.4.5. Diary Entry

Hensberry & Jacobbe (2012) found that the introduction of a reflective diary may improve students' problem-solving skills. Further to this, Mason et al (2010) suggested a problem-solver should write a reflection "*for someone else to read*". The reflective diary was introduced to students of this project during the 'tutorial' hour. Each day, one question was designated as a 'diary problem', with students providing a written reflection on the chosen-problem. Mason et al also suggested developing a structure in the reflection, and thus, on day one, students were given prompts for the diary entry. The students completed the problem in their groups and then wrote a reflection on the problem-solving processes utilised to reach their solution.

The aim of the diary was to further encourage students to communicate their reasoning, and to ensure each student invested in the problem-solving process. The diary entries were collected by the facilitator, and analysed by the author to investigate any patterns that may occur. In Cohorts 5 and 6, prompts were intermittently placed within each day to encourage students to reflect on their work. The aim of these prompts was to promote deeper thinking during regular problems, similar to what was required in the diary problem. The following prompts were used at various points: what worked in your solution and why; what didn't work in your solution and why; how did you work today; how did your group work; did you get stuck and how did you get through this.

4.5. Intervention Content

Due to the vast array of resources available under the ever-growing topic of mathematical problem-solving, the author did not create new problems for the intervention, but rather carefully selected problems (Appendix G) for each day and theme from sources such as 'Thinking Mathematically' (Mason et al., 2010) and 'Mind Your Decisions' (https://mindyourdecisions.com/blog/). Each iteration of the intervention was run with two consecutive cohorts of students. Over the course of this project, the author observed the students' reactions to and performance in each problem. Alterations to the problems used were made after Iteration 1 (Cohorts 1 and 2) and Iteration 2 (Cohorts 3 and 4) based on the observations of the author. These alterations were made due to reasons such as, but not limited to: a problem being better suited to an alternative day or theme than originally assigned;

students' lack of understanding of a problem; a problem proving to be overly simple; a problem being replaced by a more suitable one.

A primary concern of the author during the problem-selection phase of this research was the level of mathematical content within each problem. The target demographic for the intervention was Transition Year students, and thus care was taken in problem selection such that each problem was solvable with the content knowledge gained through the Junior Cycle of mathematics education in Ireland (DES, 2012). As discussed earlier, the aim of the intervention was not the introduction of new content, but rather the development of relational understanding and improvement of problem-solving abilities utilising prior knowledge. These considerations were also accounted for in the selection process.

There are three iterations of the intervention, which are discussed in the coming sections:

Iteration	Cohorts	Timeframe	Changes made
1	1 & 2	Jan 2017 – Aug 2017	N/a
2	3 & 4	Sept 2017 – June 2018	Questions & facilitator
3	5 & 6	July 2018 – present	Questions

Table 6 Intervention Iterations

The first iteration of the intervention contained large volumes of questions each day. This was a precautionary measure by the author to ensure sufficient challenge and content for students. Alterations to the intervention reduced the number of questions in each day, following observation. The following sections outline each of the first seven themes of the intervention, with carefully chosen problems specific to each theme highlighted in textboxes.

4.5.1. Visual

The visualisation of a problem is one of the most widely discussed strategies in mathematical problem-solving (Docktor & Heller, 2009; Hull et al., 2013; Mason et al., 2010, p. 33). In 'How to Solve It', Polya (1957, p. 33) outlined visualisation as the starting point within the problem-solving process. For this reason, it is apt that the first theme students encountered on this intervention was that of visualisation. Van Garderen & Montague (2003, p. 246) defined visualisation as the *"construction and formation of internal images and/or external images and then using those images effectively for mathematical discovery and understanding"*. Visualisation within mathematical problem-solving often refers solely to use of diagrams or drawings as external imagery (Hegarty & Kozhevnikov, 1999) – in other words, visual aids accomplishable through the use of paper and pencil – however, the use of physical models has also been discussed within the research of mathematical problem-solving (Krulik & Rudnick, 1989; Stylianou & Silver, 2004), and the author has grouped this under the theme of 'visual'.

Visual diagrams were not supplied for the majority of problems, and thus students' correct interpretations of a problem were paramount to the determination of a solution. The role of the facilitator in the beginning stages of a problem may be to guide students towards the correct interpretation.

Solids & Frames:

I can make any size cube from smaller 1x1x1 cubes. What relationship exists between solid cubes and cubes made of just a frame, each with equal dimensions?

In 'Solids and Frames' (Mason et al., 2010), the definition of a "*cube made with just a frame*" caused uncertainty amongst students, which was alleviated through facilitator intervention. The uncertainty surrounded the "*frame*" being made up of individual blocks, or simply an outline.

As previously mentioned, Day 1 in Iterations 1 and 2 of the intervention contained a greater number of problems than required. Thus, many of these were moved in Iteration 3 to other days, or removed completely, as detailed in Appendix F. A total of six problems from the 'visual' session of Iterations 1 and 2 were moved in Iteration 3. There was also a further addition to Day 1 within Iteration 3 – 'Tetris' – which was one of the few problems over the course of the intervention to include a visual diagram.

4.5.2. Patterns

Schoenfeld (1992) denoted definitions and beliefs as to the nature of mathematics as the "*science of patterns*". The study of patterns has been at the forefront of mathematics research and problems for as long as research has been documented, and thus the ability to recognise or search for patterns is a widely-established technique in the problem-solving process (Mason et al., 2010; Polya, 1957; Posamentier & Krulik, 2015; Schoenfeld, 1992). Mason et al (2010, p. 8-9) outlined the importance of patterns to the generalisation of a problem, but also in providing a conjecture within a problem that then must be attacked.

The problems selected under this theme contained multiple kinds of patterns for students to explore, with two examples discussed in Table 7:

Problem	**Rationale for inclusion**
<u>Leapfrogs:</u> *Ten pegs of two colours are laid out in a line of 11 holes, with the colours kept together and separated by the spare hole. I want to interchange the black and white pegs, but I am only allowed to move pegs into an adjacent empty hole or jump over one peg into an empty hole. Can I make the interchange?*	Pattern exists for generalisation for the fewest number of moves required (based on number of pegs). Patterns exist for the different kinds of move allowed. Further patterns within extensions to the problem.
<u>Circle and Spots</u> *Place N spots around a circle and join each pair of spots by straight lines. What is the greatest number of regions into which the circle can be divided by this means?*	False pattern presents itself within the early cases of the problem.

Table 7 Pattern Problems

Numerous classifications of revisions were made to Day 2 of the intervention (Appendix F). Due to the high volume of problems included in the early days of Iterations 1 and 2, many problems were not attempted during class despite their inclusion in the list. Certain problems therefore were utilised during latter days of the intervention.

4.5.3. Generalising and Specialising

Isoda & Katagiri (2012, p. 78) defined the specialisation of a problem as considering *"a smaller subset included in that set, or a single phenomenon in that set (a special case)"*. Further to this, they outlined the use of "*extreme cases*" as one example of specialisation, utilised in assessing the validity of a conjecture. Mason et al (2010) offered specialisation as a strategy to utilise when "*stuck*". Through the use of specific examples, or altering the conditions of a problem, it may be utilised to aid progression (Mason et al., 2010, p. 99). The effectiveness of specialisation throughout the problem-solving process is summed up in 'Thinking Mathematically' (Mason et al., 2010) as it can be used *"randomly, to get a feel for the question"*; *"systematically, to prepare the ground for generalizing"*; or *"artfully, to test a generalization"*. Generalising was often the consequence of accurate specialisation, whereby patterns may have developed allowing for a statement to be made about a broad range (Mason et al., 2010). The generalisations often developed in our project took the form of formulae that could then be tested further. 'Consecutive Sums' (Mason et al., 2010) featured during Day 3 in all three iterations of the intervention, and displays multiple facets of specialisation and generalisation.

Consecutive Sums:

Some numbers can be expressed as a sum of consecutive positive numbers.

Exactly which numbers have this property?

This problem began each time with students attempting numbers at random, before they usually began to group numbers in smaller sets to test in a more systematic manner, and finally leading to generalisations or formulae created.

4.5.4. Conjectures

Barton (2020) considered conjecturing as vital within the mathematics classroom, and highlighted task-selection as an important element in its development. Indeed, a good task should *"promote conjecture and argumentation"* (Hmelo-Silver, 2004). The process of conjecturing involves making some logical guesses to create an idea that may lead to a solution, testing this idea to be true, distrusting this idea, and trying to then prove it to be true (Mason et al., 2010). Developing a conjecture within a problem may provide a pathway to a solution, but it may also yield a dead-end and disprove an idea. Mason et al (2010) believed that experiences of failed conjectures are necessary to become better problem-solvers.

'Furniture' (Mason et al., 2010) provided a very useful illustration of conjecturing.

Furniture:

A very heavy armchair needs to be moved, but the only possible movement is to rotate it through 90° about any of its corners. Can it be moved so that it is exactly beside its starting position and facing the same way?

Most students believed the move was possible, and thus went about attempting to attack the problem with numerous different methods to test this out – drawings, physical chairs, coordinate diagrams, etc. The varying styles and strategies used, only to then realise their belief to be false, offered a perfect experience of conjecturing in problem-solving.

4.5.5. Assumptions and Questioning

Our reading and understanding of a problem is fundamental to the problem-solving process, as emphasised in the first step of Polya's heuristic (1945). Sometimes, however, we place hidden constraints or assumptions on a problem that are not only unnecessary, but also hinder our progress towards a solution (Mason et al., 2010). In 'River Crossing' (Mason et al., 2010), every group began the problem with all three men on the same side of the river, when this information is not stated.

River Crossing

Three men desperate to cross a river encounter two small boys on a homemade raft. The raft will carry only one man or both boys. Can the men cross the river?

While this yields a solution, it is not the optimal solution, and thus assumptions may be a hindrance, but they may also provide important information towards numerous solutions. Solutions may be thought of as a consequence of assumptions made in the problem-solving process (OECD, 2017). Becoming aware that we place assumptions on a problem may be helped by developing how we question our work (Mason et al., 2010). The role of questioning is not only to understand assumptions, but also to develop self-regulation skills in students, whereby they habitually question their methods (Pate & Miller, 1995). The decision was made to combine questioning with assumptions to highlight the importance it holds in identifying routes towards multiple solutions based on the assumptions we choose to place on problems. The facilitator was tasked with encouraging students to find multiple solutions during this session, to allow students the freedom to organically notice assumptions they had placed on each problem.

4.5.6. Structure

Posamentier & Krulik (2015) stressed structure in problem-solving as "*one of the most important strategies*" available. They believed that unstructured problem-solving may lead to confusion within a problem, and thus learning to structure problem-solving in a meaningful way is pivotal. Understanding the mathematical structure within a problem also falls under this theme, as outlined by Mason et al (2010). Within collaborative problem-solving, a group also must choose how they structure their efforts – collectively, assigning roles, etc. – and this too may become a valuable tool to their problem-solving (OECD, 2017)

With a problem as vague as ‘Jacobean Village’ (Mason et al., 2010), it is very easy to lose sight of a solution if there is no structure to your working, as was usually the case with the students in this study.

Jacobean Village:

A certain village in Jacobean times had all the valuables locked in a chest in the church. The chest had a number of locks on it, each with its own individual and distinct key. The aim of the village was to ensure that any three people in the village would amongst them have enough keys to open the chest, but no two people would be able to. How many different locks are required and how many keys?

Early attempts mostly involved randomly choosing numbers for different variables, without a clear or organised manner. Only by structuring the process and attacking the problem systematically did students find a solution to this problem.

4.5.7. Working Backwards

Working backwards in a mathematics problem may be defined as finding the start-point of a problem, when the end goal is already known (Katz et al., 2016). Posamentier & Krulik (2015) believed this strategy to be important within problem-solving, yet confusing as a concept to many students studying mathematics. They found this to be due to the routine-style questions often utilised in mathematics, whereby the path is straightforward from start to finish. Katz et al (2016) found that students’ self-regulation and self-efficacy improved when emphasis was placed on utilising working-backwards.

‘Milk Carton’ (Mason et al., 2010) was chosen for each iteration as the opening problem for this theme.

Milk Carton:

How much cardboard do you need to make a milk carton to hold 1 litre of milk?

It satisfied the theme, but also reinforced 'questioning and assumptions' when students began to ponder the shape of the end product. In Iteration 3, 'your move' (https://mindyourdecisions.com/blog/) was added as a new diary problem. Similar to previously encountered problems, it allowed students to play the game involved in the problem in order to develop their understanding.

Your Move:

Alice and Bob are playing a game using a chess board. Alice starts by placing a knight on the board. Then they take turns moving the knight to a new square (one it has not been on before). Standard chess rules apply: the knight can only move in an "L" shape, 2 squares in one direction and one square to the side. The first player who cannot move the knight to a new square loses the game. Who wins if both players play optimally, and what is the winning strategy?

4.6. Contextual Themes

As was mentioned in Section 4.4.1, days 8 to 11 on the intervention were themed contextually based on the four strands of the Junior Cycle mathematics curriculum in Ireland (DES, 2017): number; algebra and functions; geometry; and probability and statistics. The problems for each day were selected as they required students to utilise their prior knowledge in each of these topics of mathematics together with the problem-solving strategies they had learned thus far. While the problems often utilised aspects from multiple topics of mathematics, they were chosen for a particular theme due to their suitability to that topic, in the opinion of the author. Two examples will now be outlined, the first of which is displayed in the text box below (Mason et al., 2010):

Cycling Digits:

I have a number in mind which, when you remove the units digit and place it at the front, gives the same result as multiplying the original number by 2. Am I telling the truth?

This problem appeared on Day 8 (Number), and students required a prior knowledge of place value and other aspects of number systems to proceed with the problem. While this was a challenging problem, it allowed for a simplification extension by seeking answers that were simply larger than the original number.

A further example (www.mindyourdecisions.org) utilised on Day 11 is shown below, where students required a knowledge of probability to find a solution. This problem also organically introduced students to Hamiltonian Paths. As these do not occur on the Leaving Certificate curriculum, each facilitator gave a very brief outline of them to each group once the students had discovered the paths.

Ants:

Eight ants start at different corners of a cube. Suddenly, each ant moves to an adjacent corner at random. That is, each ant walks along one of its three adjacent edges with equal chance. What is the probability that none of the ants collide?

4.7. Classroom Outline

This section will outline further details from the classroom that have not been previously discussed. The classroom was prepared each day before the arrival of the students. Four individual tables were combined in a square pattern for each group. The problems were introduced through a presentation broadcast by a projector. Groups completed their workings on A1 size sheets of paper that were distributed by the facilitator, with extra sheets provided when necessary. While the application of student roles in groupwork may be utilised in research to ensure all members of a group are motivated and participating (Huss, 2006), the 'house rules' and dynamic of the CoPs model require this of the students on this intervention, and thus the introduction of student roles was not deemed necessary. After observation of six cohorts of students participating on the intervention, the author believes this was the correct decision.

4.8. Conclusion

The purpose of this chapter was to outline and discuss each aspect of the design of the intervention for this research, and to provide a theoretical overview of each aspect. The intervention was designed with the aim of addressing the additional needs of highly-able mathematics students by improving their PsP, and thus each decision in the design process was taken through this lens. The intervention was implemented through six cohorts of students, who participated with either the author or the independent facilitator, and as either a 3-week or 14-week programme. Data was collected during and after their participation, through instruments discussed previously, and the results gathered will be outlined in the next chapter.

5. Results

In this chapter, the data collected over the duration of the study is presented. We begin by reviewing the participants in the sample, and then move on to reporting the quantitative results from the mindset survey, the mathematical resilience scale, and the problem-solving test question. We then outline the results of the student diaries, the intervention survey undertaken with the participants, and those of the in-depth focus group with a small number of participants.

5.1. Participants

In this chapter, the results presented are those of the 89 students who: studied the intervention; returned the signed assent and consent forms; and participated in the three rounds of testing. The following are statistics pertaining to the breakdown of students into particular categories that may be utilised throughout this chapter:

- The group was comprised of 56 male students and 33 female students;
- 12 students were in Cohort 1; 12 in Cohort 2; 18 in Cohort 3; 14 in Cohort 4; 14 in Cohort 5; and 19 in Cohort 6;
- 63 students participated in the 14-week intervention, while 26 participated in the 3-week intervention;
- 70 students studied the intervention under the author's facilitation, with the remaining 19 studying under the independent facilitator;

The rest of this chapter will present comparisons collected through the chosen research instruments, beginning with the mindset survey.

5.2.Mindset

Students completed a 16-item mindset survey, based on the research of Dweck (Dweck, 2006; Dweck et al., 1995b; Hong et al., 1999). As outlined in Section 3.8.3, this survey consisted of an early version of the 8-item "*theories of intelligence*" scale (De Castella & Byrne, 2015; Hong et al., 1999), and an 8-item adaptation of the theories of intelligence scale, focussed specifically on mathematical talent (Dweck, 2000; Ingebrigtsen, 2018). Each of these versions will now be discussed independently.

Theories of Intelligence Scale

The Theories of Intelligence (TI) scale is colour-coded blue in Appendix A. A 6-point Likert scale was used, as per previous research (Blackwell et al., 2007; De Castella & Byrne, 2015; Spinath et al., 2003), and responses were weighted 1-6 for growth mindset questions, and reverse scored 6-1 for fixed mindset questions. Thus, the higher the score received, the stronger the growth mindset of the student. An average score for each student was then calculated, and categorised as growth if inside the range 4-6; fixed if between 1 and 3; and neutral if falling between 3 and 4, exclusive (Lee et al., 2012). The categorisation of students for rounds 1, 2 and 3 of testing is outlined in Figure 9**.**

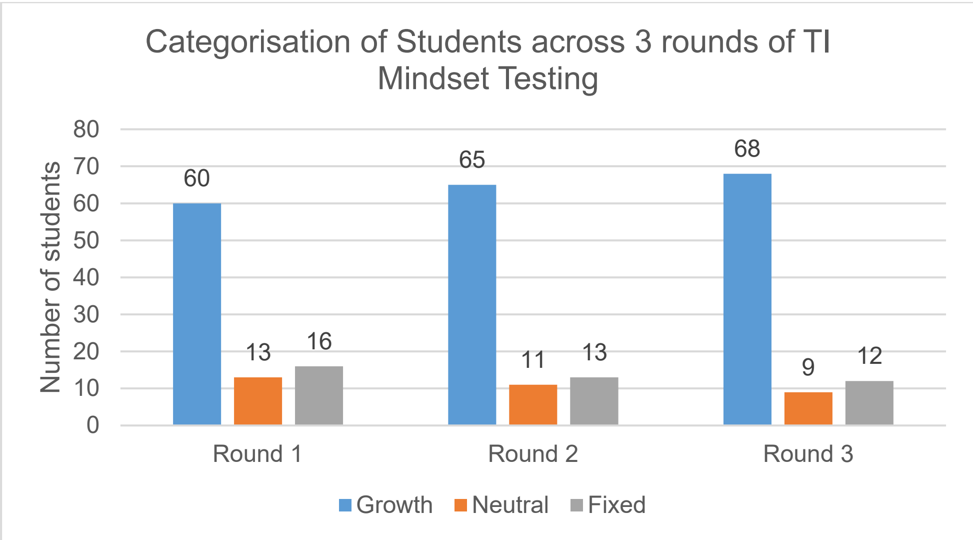

Figure 9 TI Mindset Categories for students across the three rounds of testing undertaken during the intervention

Prior to studying the intervention, a majority of students' mindsets were already registered as growth, as seen in Figure 9. Paired t-tests were then used to determine the statistical significance of any differences observed in mean scores between the rounds of testing. The null hypothesis stated that there was no significant difference between the means, and was tested under a 95% significance level, meaning that a p-value below 0.5 is rejected. Results of the paired t-tests are outlined in Table 8:

Round	Initial		Final		P-value	Null Hypothesis
	Mean	**SD**	**Mean**	**SD**		
1 vs 2	4.2	0.974	4.387	0.981	0.0114	Rejected
2 vs 3	4.387	0.981	4.478	1.022	0.891	Failed to reject
1 vs 3	4.2	0.974	4.478	1.022	0.0018	Rejected

Table 8 Theories of Intelligence Paired T-test Results

These results suggest a significant increase in students' scores on the TI scale over the course of the intervention. However, the degree of improvement between Rounds 2 and 3 of testing was not statistically significant, and therefore any change in mindset may have occurred during the first half of the intervention.

While the paired t-tests reported whether a statistically significant change had indeed occurred between the rounds of testing, Cohen's d was calculated between each round of testing to measure the effectiveness of the intervention. A guideline for interpretation of the effect sizes calculated was outlined by Cohen et al (2007), although they warned against rigid adherence to these ranges (2007, p. 522):

- 0 – 0.20 Weak effect
- 0.21 – 0.50 Modest effect
- 0.51 – 1.00 Moderate effect
- >1.00 Strong effect

The effect size for the TI scale is presented in Table 9. It may be interpreted from these results that the intervention had a modest effect on students' mindsets towards general intelligence between Rounds 1 and 3 of testing. In the context of this research, it was previously reported that the majority of students presented with a growth mindset at the beginning of the intervention, which strengthened over the intervention's duration; and the modest effect size calculated would appear to confirm this.

Scale	Round	Cohen's d	Effect Size
TI	**1-3**	0.341	Modest
	1-2	0.274	Modest
	2-3	0.116	Weak

Table 9 TI Scale Effect Sizes

Mathematical Talent Scale

The mathematical talent (MT) scale contains the same layout and scoring system as the TI scale. The MT scale has been tailored to determine if students respond differently to the questions from the TI scale by replacing the word *intelligence* with *mathematical talent* for each question. This practice was introduced by Dweck (2000), and researchers have since created multiple versions of the scale tailored to a wide range of domains (De Castella & Byrne, 2015; Ingebrigtsen, 2018; Lee et al., 2012). The MT scale is colour-coded yellow in Appendix B.

Students' scores on the MT scale are recorded in Figure 10. In comparison with scores on the TI scale, a larger number of students began the intervention with neutral mindsets (13 on TI scale) regarding malleability of mathematical talent. However, over 80% of students registered a growth mindset in this area by the end of the intervention.

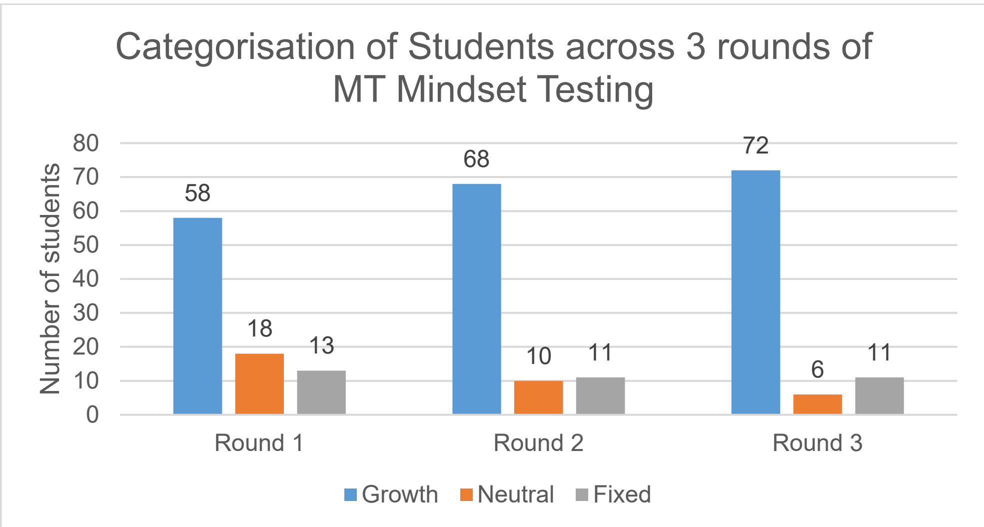

Figure 10 MT Mindset Categories for students across the three rounds of testing undertaken during the intervention

The paired t-tests were calculated under the same conditions as those for the TI scale.

Round	**Initial**		**Final**		**P-value**	**Null Hypothesis**
	Mean	**SD**	**Mean**	**SD**		
1 vs 2	4.291	1.032	4.458	0.98	0.048	Rejected
2 vs 3	4.458	0.98	4.54	0.958	0.932	Failed to reject
1 vs 3	4.291	1.032	4.54	0.958	0.005	Rejected

Table 10 Mathematical Talent Paired T-test Results

The results displayed in Table 10 suggest students' mindsets towards mathematical talent have improved over the course of the intervention. However, similar to scores on the TI, improvements in mindset may have been achieved in the first half of the intervention.

To determine the effect size of the intervention on the students' mindsets towards mathematical talent, Cohen's d was calculated under the same conditions as the TI scale. The results, presented in Table 11, reflect those of the TI scale such that students' scores registered a modest effect size from Rounds 1 to 3 of testing.

Scale	**Round**	**Cohen's d**	**Effect Size**
MT	**1-3**	0.304	Modest
	1-2	0.213	Modest
	2-3	0.114	Weak

Table 11 MT Scale Effect Sizes

Scale Comparison

The scales within the mindset survey each focussed on students' mindsets in relation to either general intelligence (TI) or mathematical talent (MT). To determine the consistency of students' mindsets between the aforementioned areas, the Pearson correlation coefficient was calculated between the pair of scales for each round of testing (Table 12). Correlations of this kind are adjudged on a scale between negative and positive 1. The closer the score resides to 0, the weaker the connection between the two factors. Positive scores indicate factors with a positive impact on one another, while negative scores carry a negative impact.

	TI / MT
Round 1	0.7396
Round 2	0.8187
Round 3	0.7912

Table 12 Mindset Scales Correlations

A strong correlation existed in Round 1 between mindsets on general intelligence and mathematical talent, and, although there was a slight decrease between Rounds 2 and 3, it also strengthened over the course of the intervention. It is clear from these scores that the students' mindsets are quite consistent between general intelligence and mathematical talent.

As previously discussed in Section 5.1, the intervention may be further examined through its different variations: those who studied the 3-week intervention as opposed to the 14-week intervention; and those who studied under the independent facilitator compared to those who studied under the author. The following two subsections will present these results.

5.2.1. Time Layout Comparison

This section sets out to compare the mindset results of students who participated on the 14-week (n=63) intervention to those who participated on the 3-week intervention (n=*26*), in order to ascertain if there is any discernible difference based on the time layout of the intervention. Table 13 displays the results for each scale:

Scale	Round	14-Week		3-Week	
		Mean	SD	Mean	SD
TI	1	4.161	0.973	4.295	0.952
	2	4.433	0.988	4.274	0.934
	3	4.496	1.016	4.433	1.015
MT	1	4.234	1.062	4.428	0.918
	2	4.5	1.033	4.356	0.803
	3	4.502	1.014	4.63	0.775

Table 13 Time Comparison Scales

For students of the 14-week programme, results on the TI and MT scales showed a progressive increase from Rounds 1 to 2, and Rounds 2 to 3. The results for both scales for students of the 3-week programme showed a slight decrease from Rounds 1 to 2, but then a greater increase from Rounds 2 to 3, resulting in an overall increase.

Further differences between the two formats are evident in students' score changes over the course of the intervention. Figure 11 displays the percentage of students' scores that increased (moved towards growth), decreased (moved towards fixed) or remained unchanged on the TI and MT scales from rounds 1 to 3 for both the 14-week and 3-week programmes. For the MT scale, the percentage changes are largely the same for both intervention formats. However, the percentage changes for the TI scale showed greater increases overall for students on the 14-week intervention.

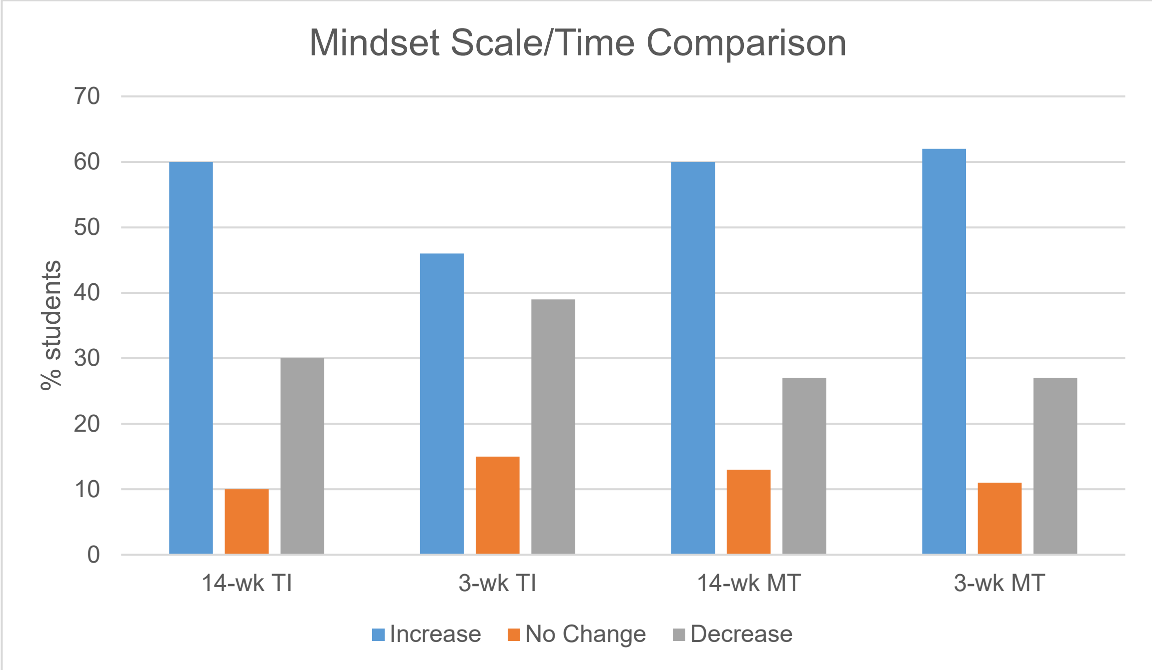

Figure 11 Mindset Scale/Time Comparison

Due to the size of the 3-week sub-group (*n*=26), it could not be assumed that the data followed a normal distribution, and further statistical testing of skewness and kurtosis also suggested this. For this reason, a non-parametric test was required to investigate potential differences between the data for the 14-week intervention and the 3-week intervention. The Kruskal-Wallis test – a non-parametric equivalent to an ANOVA test (Cohen et al., 2007) – compares the medians of two or more independent samples, and determines if statistically significant differences exist. For this section, the Kruskal-Wallis test was used to determine whether there was a statistical difference between the 14-week and 3-week intervention variations on students' results on each of the mindset scales. The null hypothesis stated that there was no statistically significant difference between the medians of each data set, and was tested under a 95% level of significance. The results of each test are reported in Table 14, and, for reference, a null hypothesis is rejected if the p-value is less than 0.05.

Scale	Round	Kruskal-Wallis Test Statistics			
		H	**df**	**P-value**	**H_0**
TI	**1**	0.435	1	0.510	Failed to reject
	2	0.366	1	0.545	Failed to reject
	3	H<0.001	1	0.996	Failed to reject
MT	**1**	0.745	1	0.388	Failed to reject
	2	0.585	1	0.444	Failed to reject
	3	0.284	1	0.594	Failed to reject

Table 14 Kruskal-Wallis Mindset Results

Each test failed to reject the null hypothesis, and thus it may be interpreted that no significant difference existed between the 14-week and 3-week variations of the intervention, with respect to the changes in mindset of the student populations.

5.2.2. Facilitator Comparison

The independent facilitator was important in establishing validity and reliability in the results obtained. Of the 89 students whose results are presented thus far in this research, 19 participated on the intervention with the independent facilitator, comprised of Cohort 3B and Cohort 6B. These 19 students completed the test measures under the same test conditions as those studying with the author (n=70). The means and standard deviations for students' scores on each mindset scale obtained under each facilitator are outlined in Table 15.

Scale	Round	Author		Independent Facilitator	
		Mean	SD	Mean	SD
TI	1	4.235	0.934	4.081	1.07
	2	4.412	0.923	4.3	1.132
	3	4.527	0.897	4.306	1.335
MT	1	4.321	0.993	4.188	1.126
	2	4.422	0.94	4.581	1.075
	3	4.591	0.881	4.363	1.147

Table 15 Facilitator Comparison Scales

Figure 12 displays the percentage of students whose scores increased, decreased or remained unchanged for each group. Due to the small size of the group studying under the independent facilitator, the results of any one student have a much more dramatic effect on given percentages. A greater number of students showed a decrease in their score on both the TI scale and the MT scale. Whilst 9 students showed a decrease on the TI scale, 5 of these maintained a growth mindset towards general intelligence; 1 moved from a growth to neutral mindset; 1 remained a neutral mindset; and the remaining 2 moved from neutral to fixed mindsets.

As the parameters for normality could not be guaranteed, a Kruskal-Wallis test was once again used to investigate any potential differences between students' scores under each facilitator for each round of testing. The null hypothesis stated that there was no statistically significant difference between the medians of each data set, and was tested under a 95% level of significance, whereby a p-value of less than 0.05 would reject the null hypothesis. The results of these tests are displayed in Table 16.

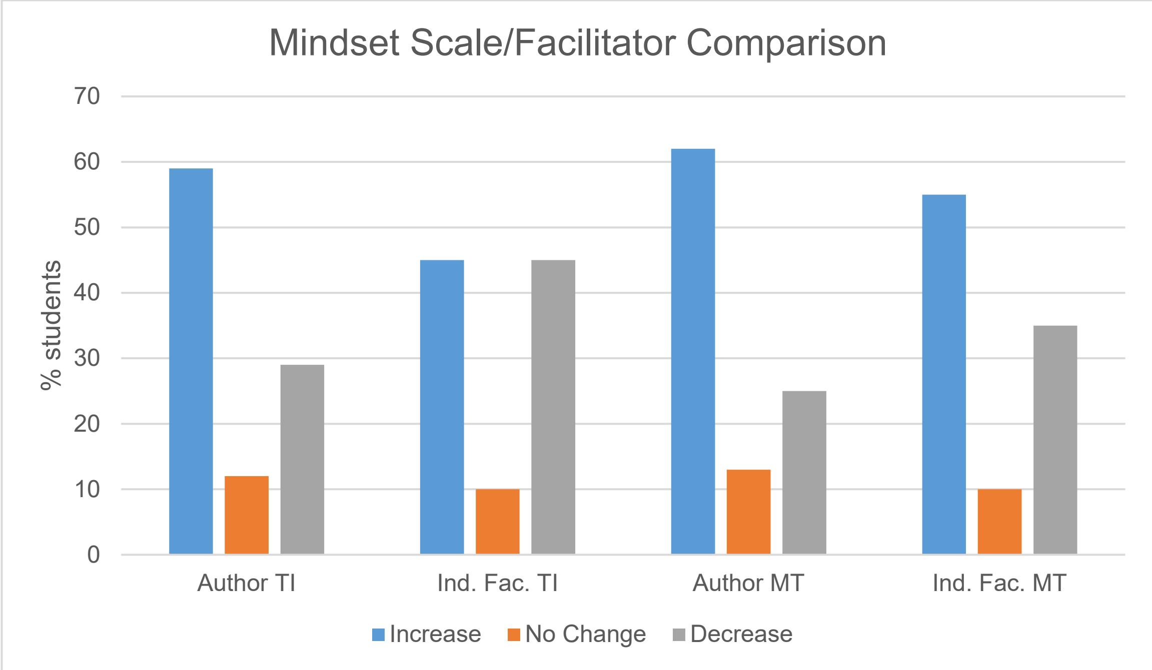

Figure 12 Mindset Scale/Facilitator Comparison

Scale	Round	Kruskal-Wallis Test Statistics			
		H	df	P-value	H_0
TI	1	0.096	1	0.757	Failed to reject
	2	0.272	1	0.602	Failed to reject
	3	0.076	1	0.783	Failed to reject
MT	1	0.109	1	0.741	Failed to reject
	2	0.300	1	0.584	Failed to reject
	3	0.261	1	0.609	Failed to reject

Table 16 Kruskal-Wallis results for different facilitators

All Kruskal-Wallis tests failed to reject the null hypothesis. It may be therefore surmised that no statistically significant difference existed between the facilitator results for either scale. This is significant in terms of the transferability of the intervention, whereby the independent facilitator's students were found to increase to a statistically significant degree to the results of students who participated with the author.

5.3. Resilience

Students' mathematical resilience was measured using the Mathematical Resilience Scale (MRS), as discussed in Section 3.8.4. Students' scores have been analysed as a total mark, and also further divided into the three domains of the scale: value (V), growth (G) and struggle (S). A 7-point Likert scale was used, with 18 questions scored 1-7, and the remaining 6 reverse-scored 7-1 (as per Kooken et al., 2016). Struggle scores range from 9 to 63 (9 questions); value ranges from 8 to 56 (8 questions); and growth ranges from 7 to 49 (7 questions). Thus the overall scores range from 24 to 168, with a higher score indicating a stronger mathematical resilience.

From Rounds 1 to 3 of testing, of the 89 students to complete the MRS: 49 (approx. 55%) students' overall scores increased; 32 (approx. 36%) scores decreased; and 8 (approx. 9%) remained unchanged. All 89 students scored in the upper-half of the scale range in all three rounds of testing. Paired t-tests were used to determine the statistical significance of differences in mean scores between the rounds of testing. The null hypothesis stated that there was no significant difference between the means, and was tested under a 95% significance level, meaning that a p-value below 0.05 would be rejected. Results of the paired t-tests are outlined in Table 17:

Round	**Initial**		**Final**		**P-value**	**Null Hypothesis**
	Mean	**SD**	**Mean**	**SD**		
1 vs 2	142.539	12.019	142.472	11.691	0.8975	Failed to reject
2 vs 3	142.472	11.691	144.292	12.813	0.0051	Rejected
1 vs 3	142.539	12.019	144.292	12.813	0.0795	Failed to reject

Table 17 MRS Paired T-test Results

The test failed to reject the null hypothesis calculated from Rounds 1 to 2 – the mean score decreased marginally over this period – and Rounds 1 to 3. The result of the paired t-test between Rounds 1 and 3 failed to reject the null hypothesis. As a result of the slight decrease from Rounds 1 to 2, the subsequent increase from Rounds 2 to 3 was enough to be statistically significant and reject the null hypothesis.

Although much of the current, albeit limited, research considered the three domains of the MRS as a single construct in their analysis, the validation study (Kooken et al., 2016) discussed each domain individually in relevance to the student sample and their importance to mathematical resilience. Thus, the author also analysed the results separately to gain an insight into how students responded in each of the domains. The separation also allows for comparison between the growth domain and the results previously discussed in Section 5.2. Under the same conditions as previously mentioned, paired t-tests were used to determine the statistical significance of differences in mean scores between the rounds of testing. The results are illustrated in Table 18. All three paired t-tests calculated for the *value* domain failed to reject the null hypothesis. In direct contrast, the results observed in the *struggle* domain rejected the null hypothesis for all three paired t-tests. This indicates a significant increase in the students' scores in this domain over the course of the intervention. Finally, the results of the paired t-tests in the *growth* domain, found one statistically significant change in students' scores. However, this result was a slight decrease in students' scores from Rounds 1 to 2. The results during Round 3 produced a marginal increase that led to the paired t-test between Rounds 1 and 3 failing to reject the null hypothesis.

Domain	**Round**	**Initial**		**Final**		**P-value**	**Null Hypothesis**
		Mean	**SD**	**Mean**	**SD**		
Value	**1 vs 2**	46.764	6.274	46.494	6.064	0.5139	Failed to reject
	2 vs 3	46.494	6.064	46.787	6.446	0.432	Failed to reject
	1 vs 3	46.764	6.274	46.787	6.446	0.966	Failed to reject
Struggle	**1 vs 2**	54.663	5.246	55.528	4.467	0.0006	Rejected
	2 vs 3	55.528	4.467	56.843	4.815	0.0003	Rejected
	1 vs 3	54.663	5.246	56.843	4.815	0.0001	Rejected
Growth	**1 vs 2**	41.112	5.704	40.449	5.847	0.0244	Rejected
	2 vs 3	40.449	5.847	40.596	5.929	0.6845	Failed to reject
	1 vs 3	41.112	5.704	40.596	5.929	0.3024	Failed to reject

Table 18 Paired t-test results for each domain

The *growth* domain of the MRS was intended to test students for a growth mindset (Johnston-Wilder & Lee, 2010b; Kooken et al., 2016). Pearson correlation calculations were carried out by the author to investigate potential relationships between the three mindset scales analysed in Section 5.2 and the *growth* domain of the MRS. The *growth* domain was positively correlated with each scale during all rounds of testing, with each correlation strengthening in Rounds 2 and 3, as displayed in Table 19:

	Growth/ TI	**Growth/ MT**
Round 1	0.4676	0.5974
Round 2	0.5120	0.6359
Round 3	0.5931	0.6700

Table 19 Growth domain correlations with mindset scales

While strong, positive correlations exist between the *growth* domain results and each mindset scale, each mindset scale registered a statistically significant increase between Rounds 1 and 3, while the *growth* domain failed to do so. The improvement in correlation between Rounds 1 and 3, however, appears to indicate that students' opinions on mindset towards each of general intelligence and mathematical talent became more consistent with their views on the *growth* domain. This strong correlation will be of significance in the discussion of Research Question 1B in Section 6.1.2.

Students' mean scores on the MRS seem to suggest a strong pre-existing mathematical resilience. In each round of testing, every student was recorded as scoring in the upper half of the scale, with approximately 90% of students falling in the 4th quartile in all three rounds of testing. Cohen's d was calculated for each domain, and the overall MRS, for each round of testing to determine the effect size of the intervention on each score, with the results and interpretation outlined in Table 20.

For each comparison where the paired t-tests rejected the null hypothesis, a modest effect size was also found. The strongest effect sizes were all found within the *struggle* domain.

Scale	Round	Cohen's d	Effect Size
Value	**1-3**	0.005	Weak
	1-2	-0.107	Weak
	2-3	0.084	Weak
Struggle	**1-3**	0.489	Modest
	1-2	0.379	Modest
	2-3	0.404	Modest
Growth	**1-3**	-0.110	Weak
	1-2	-0.243	Modest
	2-3	0.043	Weak
MRS	**1-3**	0.188	Weak
	1-2	-0.014	Weak
	2-3	0.304	Modest

Table 20 MRS Effect Sizes

5.3.1. Time Layout Comparison

We will now compare the MRS results for the two time-variations of the intervention – 14-week (*n=63*) and 3-week (*n=26*) – as previously discussed for the mindset scales in Section 5.2.1. Within this section, scores will be broken down into each domain of the MRS, and also discussed as the combined scale.

Scale	Round	14-week		3-week	
		Mean	SD	Mean	SD
Value	1	47.286	5.915	45.500	7.033
	2	47.254	5.459	44.654	7.110
	3	47.143	6.208	45.923	7.037
Struggle	1	54.984	4.699	53.885	6.421
	2	56.000	3.935	54.385	5.470
	3	57.079	4.523	56.269	5.510
Growth	1	41.444	6.010	40.308	4.897
	2	41.079	5.686	38.923	6.059
	3	40.825	5.827	40.038	6.251
MRS	1	143.714	16.624	139.692	12.348
	2	144.333	15.081	137.962	12.012
	3	145.048	16.559	142.231	13.489

Table 21 MRS Time Variation Means & SD

In Section 5.2.1, the results of each mindset scale were discussed for students of the 3-week intervention, with the mean score for each scale decreasing in Round 2, followed by a more substantial increase in Round 3, leading to an overall increase in each scale. As displayed in Table 21, this is also evident for the MRS scale, and for the *value* domain. While the *growth* domain also decreased in Round 2, the subsequent increase in Round 3 was not great enough for an overall increase. For those on the 14-week intervention, the *growth* domain decreased in Round 2 and 3 of testing. In contrast to this, scores in the *struggle* domain for both 3-week and 14-week students increased between each round of testing.

As the parameters for normality could not be guaranteed, a Kruskal-Wallis test was once again used to investigate any potential differences between students scores' for each time layout for each round of testing. The null hypothesis stated that there was no statistically significant difference between the medians of each data set, and was tested under a 95% level of significance, whereby a p-value of less than 0.05 would reject the null hypothesis. The results of these tests are displayed in Table 22:

Scale	Round	Kruskal-Wallis Test Statistics			
		H	df	P-value	H_0
Value	**1**	0.885	1	0.347	Failed to reject
	2	2.768	1	0.096	Failed to reject
	3	0.605	1	0.437	Failed to reject
Struggle	**1**	0.158	1	0.691	Failed to reject
	2	0.732	1	0.392	Failed to reject
	3	0.193	1	0.660	Failed to reject
Growth	**1**	1.676	1	0.195	Failed to reject
	2	2.683	1	0.101	Failed to reject
	3	0.296	1	0.587	Failed to reject
MRS	**1**	1.041	1	0.308	Failed to reject
	2	4.321	1	0.038	Rejected
	3	0.890	1	0.345	Failed to reject

Table 22 Kruskal-Wallis MRS results - Time Comparison

As previously mentioned in this section, scores from the 3-week intervention decreased in Round 2 for the MRS, and the v*alue* and *growth* domains. This is reflected in the Kruskal-Wallis results, with Round 2 of each of these registering higher than usual H-statistics; resulting in the rejection of the null hypothesis for Round 2 of the MRS score. The failure to reject the null hypothesis in Round 1 and Round 3, however, suggests that students' scores, and any increases, over the entire duration of the intervention were not statistically different. This failure to reject the null hypothesis between Rounds 1 and 3 is significant as, while the scores in the 3-week programme decreased in Round 2, they further increased in Round 3 to a degree that offset the earlier decrease.

5.3.2. Facilitator comparison

In this section, students' scores on the MRS are divided into two groups – those who studied under the author (*n=70*) and those who studied under the independent facilitator (*n=19*) – and compared to ascertain whether or not evidence exists that there was a difference between the two facilitators. The means and standard deviations for each group are displayed in Table 23, which shows a total score on the MRS but also their scores for each of its domains:

Scale	Round	Author		Independent Facilitator	
		Mean	SD	Mean	SD
Value	1	46.5	6.431	47.738	5.714
	2	46.014	6.385	48.263	4.395
	3	46.2	6.839	48.947	4.183
Struggle	1	54.7	5.554	54.526	4.033
	2	55.5	4.775	55.631	3.183
	3	56.786	5.042	57.053	3.979
Growth	1	41.114	5.576	41.105	6.315
	2	40.3	5.767	41	6.263
	3	40.557	5.737	40.737	6.756
MRS	1	142.314	12.466	143.368	10.468
	2	141.814	12.314	144.895	8.894
	3	143.543	13.586	146.737	8.837

Table 23 Facilitator Breakdown Means & SD

There are several minor differences in the increases and decreases of means between the two groups in different categories. Both groups increased their *struggle* scores between each round of testing and both decreased their *growth* scores between Rounds 1 and 3.

As previously outlined in Section 5.2.2, conditions for normality within the independent facilitator group could not be assumed, and thus a Kruskal-Wallis test was used to compare the medians of both groups to investigate any potential differences between them for each round of testing. Each test was conducted under a 95% level of significance, with a null hypothesis stating that no statistically significant difference existed between the medians. A p-value of less than 0.05 would reject the null hypothesis. The results of the Kruskal-Wallis tests are outlined in Table 24:

Scale	Round	Kruskal-Wallis Test Statistics			
		H	df	P-value	H_0
Value	1	0.413	1	0.521	Failed to reject
	2	1.611	1	0.204	Failed to reject
	3	2.692	1	0.101	Failed to reject
Struggle	1	0.113	1	0.737	Failed to reject
	2	0.045	1	0.833	Failed to reject
	3	0.004	1	0.952	Failed to reject
Growth	1	0.082	1	0.775	Failed to reject
	2	0.552	1	0.457	Failed to reject
	3	0.120	1	0.729	Failed to reject
MRS	1	0.024	1	0.877	Failed to reject
	2	1.510	1	0.219	Failed to reject
	3	0.813	1	0.367	Failed to reject

Table 24 Kruskal-Wallis results for facilitator MRS

The Kruskal-Wallis tests failed to reject the null hypothesis for all three domains, and for the overall MRS scores, for all rounds of testing. This indicates that there was no statistically significant difference between the two groups' test scores on the MRS, regardless of the facilitator present.

5.4. Problem-solving test

The problem-solving test question (PTQ) was attempted by students immediately after completion of the mindset survey and MRS, with the test conditions outlined previously in Section 3.8.5. The same question was used for each of the rounds of testing, and a score was calculated using a problem-solving rubric (Appendix D) with a maximum score of 30. A score of 0 was possible where a student did not attempt the problem at all, although this did not occur during any round of testing. The rubric was divided, with equal score-weighting, into each of the following categories: making sense of the task; solving the task; communicating reasoning; accuracy; and reflecting and evaluating. Later in this section, the results of the PTQ are presented as a breakdown of each of these categories, but, first, as an overall test result for problem-solving.

Of the 89 students to complete the PTQ, 70 students registered an improved score between Rounds 1 and 3 of testing, with a mean increase of 5.296; 11 students registered an overall decrease by the end of the intervention, with a mean decrease of 2.182; and 8 students' scores remained the same. Of the 11 students to show a score decrease, 9 studied the intervention in the first three cohorts, 1 in Cohort 5 and 1 in Cohort 6. Paired t-tests were used to determine the statistical significance of any differences observed in mean scores between the rounds of testing. The null hypothesis stated that there was no significant difference between the means, and was tested under a 95% significance level, meaning that a p-value below 0.05 resulted in the null hypothesis being rejected. Results of the paired t-tests are outlined in Table 25:

Round	**Initial**		**Final**		**P-value**	**Null Hypothesis**
	Mean	**SD**	**Mean**	**SD**		
1 vs 2	10.843	3.532	12.427	3.985	0.0001	Rejected
2 vs 3	12.427	3.985	14.742	4.491	0.0001	Rejected
1 vs 3	10.843	3.532	14.742	4.491	0.0001	Rejected

Table 25 PTQ Paired T-test Results

The results of the paired t-tests suggest a statistically significant increase overall in students' test scores on the PTQ between each round of testing. We will now report the results of each category separately to investigate if students scored a uniform increase in each category, or if particular categories were the leading cause in the overall student increase. Table 26 shows the breakdown of marks across each of the five different rubric categories. Each of the categories is assigned a colour code (and abbreviation).

	Rubric	Round	Mean	SD
M.S.	Making sense of the task	1	2.697	0.76
		2	2.91	0.748
		3	3.337	0.988
S.T.	Solving the task	1	2.427	0.81
		2	2.753	0.816
		3	3.169	0.956
C.R.	Communicating reasoning	1	2.135	0.846
		2	2.584	0.915
		3	3.056	0.934
Acc.	Accuracy	1	2.101	0.923
		2	2.494	1.023
		3	3.169	1.208
R&E	Reflecting and evaluating	1	1.393	0.717
		2	1.697	0.982
		3	2.023	1.128
	Total	1	10.843	3.532
		2	12.427	3.985
		3	14.742	4.491

Table 26 Rubric category results

Paired t-tests were also calculated for the results in each category to investigate potential differences between the mean scores in each round of testing. The paired t-tests were calculated at a 95% significance level, under the null hypothesis that there was no statistically significant difference with the means. A p-value below 0.05 would suggest such a difference did exist, and thus reject the null hypothesis.

Colour Code	Round	Initial		Final		P-value	Null Hypothesis
		Mean	SD	Mean	SD		
M.S.	1 vs 2	2.697	0.76	2.91	0.748	0.001	Rejected
	2 vs 3	2.91	0.748	3.337	0.988	$p < 0.001$	Rejected
	1 vs 3	2.697	0.76	3.337	0.988	$p < 0.001$	Rejected
S.T.	1 vs 2	2.427	0.81	2.753	0.816	$p < 0.001$	Rejected
	2 vs 3	2.753	0.816	3.169	0.956	$p < 0.001$	Rejected
	1 vs 3	2.427	0.81	3.169	0.956	$p < 0.001$	Rejected
C.R.	1 vs 2	2.135	0.846	2.584	0.915	$p < 0.001$	Rejected
	2 vs 3	2.584	0.915	3.056	0.934	$p < 0.001$	Rejected
	1 vs 3	2.135	0.846	3.056	0.934	$p < 0.001$	Rejected
Acc.	1 vs 2	2.101	0.923	2.494	1.023	$p < 0.001$	Rejected
	2 vs 3	2.494	1.023	3.169	1.208	$p < 0.001$	Rejected
	1 vs 3	2.101	0.923	3.169	1.208	$p < 0.001$	Rejected
R&E	1 vs 2	1.393	0.717	1.697	0.982	$p < 0.001$	Rejected
	2 vs 3	1.697	0.982	2.023	1.128	0.001	Rejected
	1 vs 3	1.393	0.717	2.023	1.128	$p < 0.001$	Rejected

Table 27 Paired T-test Results for PTQ Categories

For all but two tests, the resulting p-value was significantly small and thus was reported as ‘$p < 0.001$’. The paired t-test results suggest that there was a statistically significant increase in students’ scores between each round of testing.

Cohen’s d was calculated on the differences between each round of testing to quantify the effect size of the intervention on the scores, with the results presented in Table 28. Utilising the previously mentioned guide for interpreting the Cohen’s d results, we can see that the intervention has had a strong effect on the scores in each category of the grading rubric.

Scale	**Round**	**Cohen’s d**	**Effect Size**
M.S	**1-3**	0.944	Strong
	1-2	0.574	Strong
	2-3	0.796	Strong
S.T.	**1-3**	1.017	Strong
	1-2	0.765	Strong
	2-3	0.781	Strong
C.R.	**1-3**	0.916	Strong
	1-2	0.76	Strong
	2-3	0.692	Strong
Acc.	**1-3**	1.175	Strong
	1-2	0.912	Strong
	2-3	0.876	Strong
R&E	**1-3**	1.016	Strong
	1-2	0.697	Strong
	2-3	0.902	Strong

Table 28 Cohen's d for PTQ

Three samples of student work are included as Appendix : the first represents the standard required for a score of 4 or more in each category; the second represents the standard that merited 2 out of 6 in each category; and the final sample represents how students can build upon their solution through generalisations and extensions.

It is important to consider what the score improvements mean in the context of each category, and these will now be briefly outlined.

Making Sense of the Task

Students are awarded scores in this category for their interpretation of the task, and their ability to translate it into mathematics. Figure 13 displays the number of students who achieved each score for each round of testing. There is no column for a score of 0 or 6 as no student received either score in any round of testing. A score of 6 in this section would require a student to outline connections between this problem and either a real world construct or another problem they have encountered before.

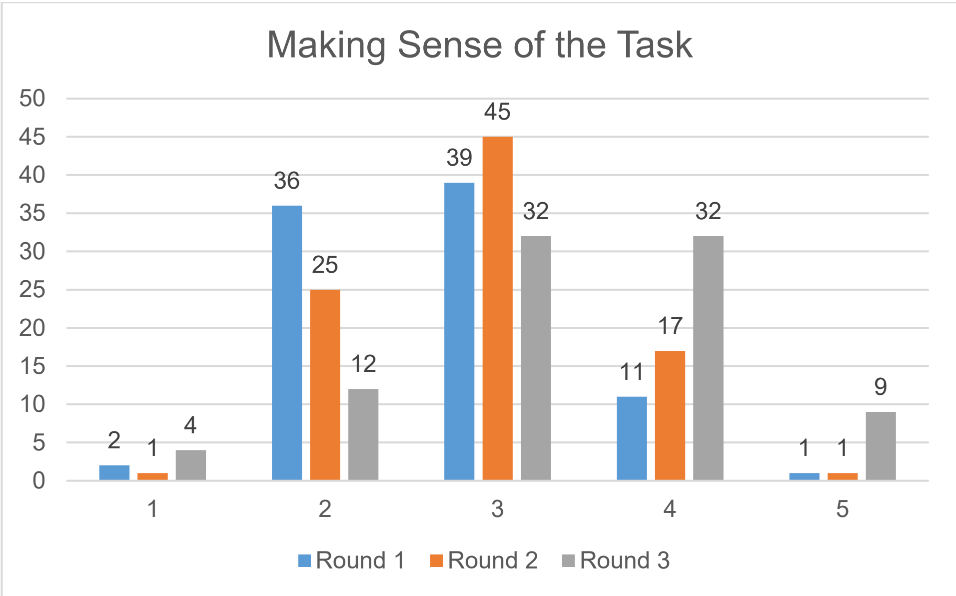

Figure 13 Making Sense of the Task: Frequency of each score per round of testing

The Round 1 mean score of 2.697 marks suggests the majority of students' work was "*underdeveloped*" or "*partially developed*", and this is clearly evident by the number of students scoring 2 or 3 on the rubric. In Round 3, however, the majority of students scored 3 or 4 on the rubric, with the mean score of 3.337, which indicates that students' work progressed to "*partially*" or "*adequately*" developed. There were also a further 8 students whose work was deemed "*thoroughly developed*".

Solving the Task

This category rewards the selection of strategies to solve the problem, particularly when these strategies prove to be effective or complete. The number of students who scored 1 to 5 are displayed in Figure 14; once again, columns were not required for a score of 0 or 6 as no student received this mark.

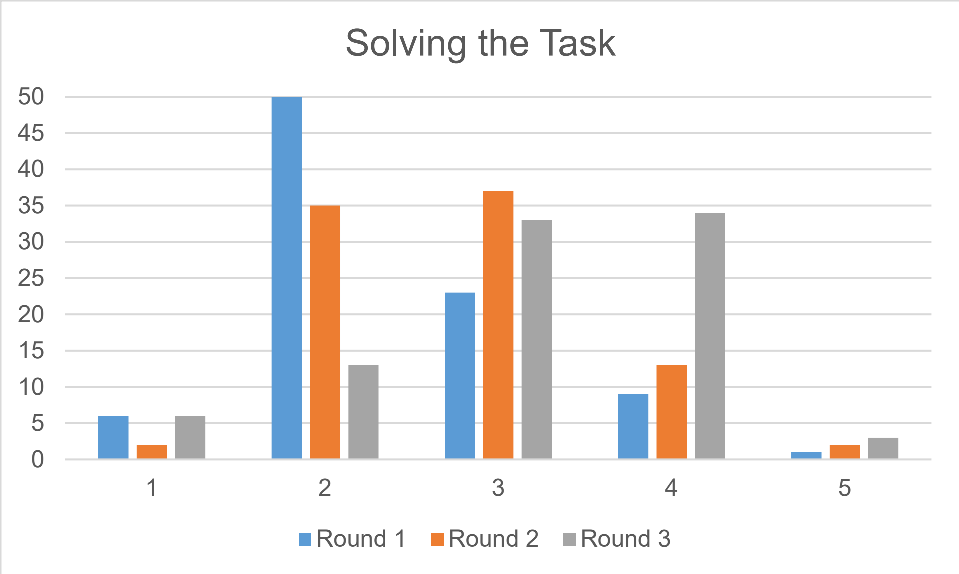

Figure 14 Solving the Task: Frequency of each score per round of testing

Over half of the students received a score of 2 in this category in Round 1, indicating "*underdeveloped*" or "*sketchy*" strategy selection in the process of solving the problem. This was also evident in the mean score of 2.427 presented earlier in this section. By Round 3, 34 students utilised a strategy that proved "*complete*", whilst 33 students' work was "*partially effective*" in solving the problem. A score of 5 was achieved in this category through the use of "*elegant*" or "*complex*" strategies in solving the problem. In Round 1, one student achieved this score, although in subsequent rounds of testing they selected a simplified strategy and thus scored 4 in each of these rounds.

Communicating Reasoning

In this category, the coherent use of mathematical language and effective communication of students' reasoning for their work was graded. On the intervention, students were encouraged to explain their attempts throughout the problem-solving process, and seek to understand *why* a particular solution may work for a problem. This was also evident in the CoPs model discussed in Section 3.7.

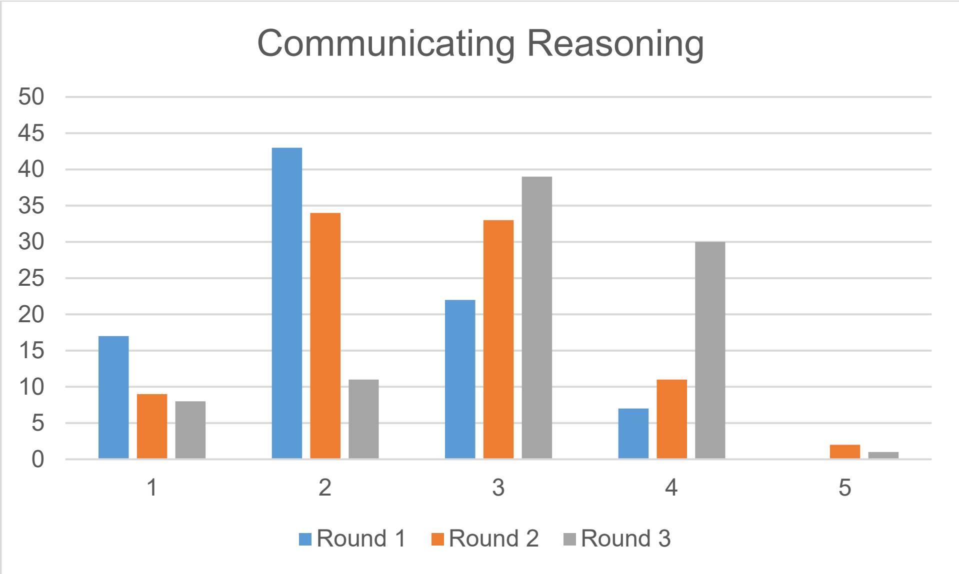

Figure 15 Communicating Reasoning: Frequency of each score per round of testing

A majority of students (n=60) presented work that was "*underdeveloped*" (score = 1) or "*minimal*" (score = 2) in Round 1; whereas this reduced to 19 students by Round 3, with 30 students' work following a "*clear and coherent path*" (score = 4) in their communicated reasoning. Similar to the previous section, a score of 5 required an "*elegant*" communication of reasoning. One of the students who achieved this score in Round 2 maintained a clear and coherent communication in Round 3, but did not present it in an elegant manner, and thus decreased their score to 4.

Accuracy

This is the primary category in which students are rewarded for a correct solution, with a score of 4 received if one is presented and mathematically justified. A score of 5 or 6 is achieved by seeking to work beyond the scope of the given problem, by generalising or extending it to create new problems. This is the sole category in which a score of 6 was achieved, and thus Figure 16 displays the number of students who scored each mark from 1 to 6:

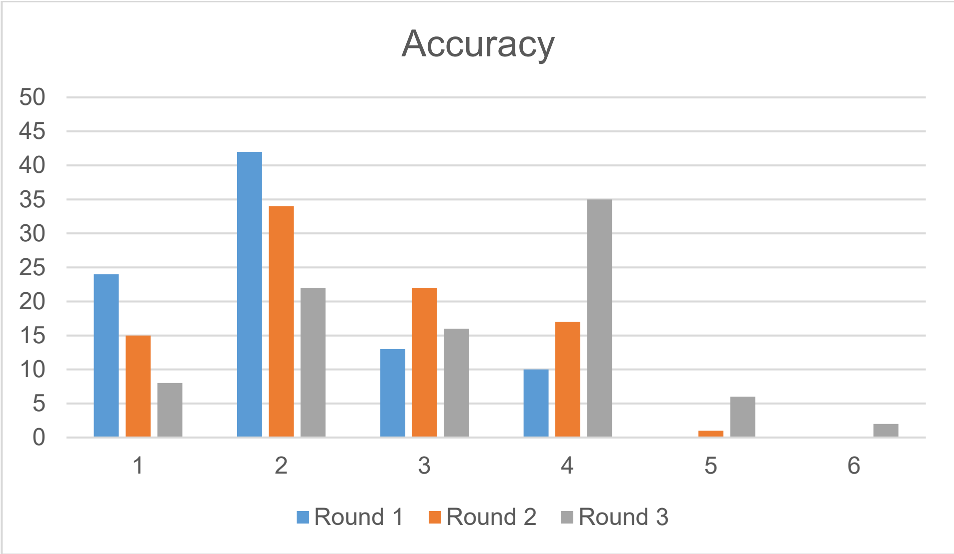

Figure 16 Accuracy: Frequency of each score per round of testing

In Round 1, 10 students presented a correct solution that was justified mathematically by their workings, with this increasing to 35 students in Round 3. If students presented an answer without sufficient and justified work they received a score of 2 in this section. As the same problem was used for all three rounds of testing, it is possible that students may have been aware of the answer after Round 1 (although the answer was never given in class or discussed during class time). However, students were still required to display an understanding of the problem in their solution to achieve a higher mark.

Reflecting and Evaluating

In this category, students must state the solution in the context of the task and justify their solution process, with a score of 5 requiring a second solution method for the problem or in-depth evaluation of the effectiveness of chosen strategies. As the *communicating reasoning* category awarded marks for coherent communication throughout the solution, this category was focussed solely on reflection, comment or justification at the end of a solution. Figure 17 displays the number of students who received each score 1 to 6:

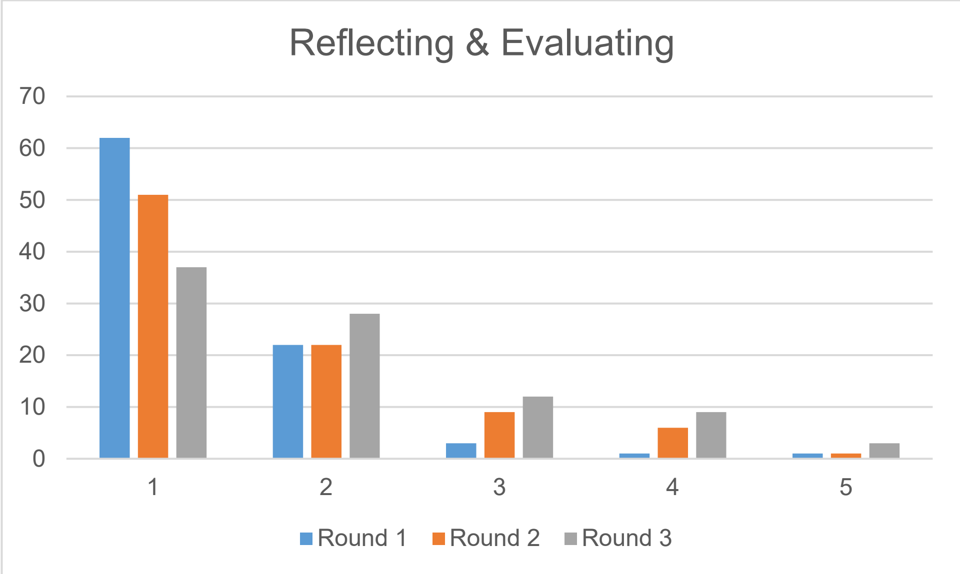

Figure 17 Reflecting & Evaluating: Frequency of each score per round of testing

With a mean score of 1.393 in Round 1, this was by a large margin the lowest scoring category on the rubric. In Round 3, 37 students scored 1 mark, indicating "*minimal*" effort to justify their solution at the end of the problem. There is a slight increase in the number of students who scored 3, 4 or 5 between the rounds of testing, but the increase is much lower than each of the other categories. The student with a score of 5 in Rounds 1 and 2 presented a simplified and less reflective solution in Round 3, and thus decreased their score to 4 in this round. This student was also mentioned in *communicating reasoning* for a similar decrease. Three other students, however, did rework the task with multiple methods in Round 3, and thus scored 5 marks in this category.

5.4.1. Time Layout Comparison

This section will examine and portray the results of the PTQ for each time variation of the intervention, as previously completed for the mindset survey and MRS. 63 students studied the 14-week variation of the intervention, with the remaining 26 studying the 3-week variation. The means and standard deviations for the PTQ and each of the individual rubric categories are outlined in Table 29 for the 14-week and 3-week variations of the intervention:

Scale	Round	14-week		3-week	
		Mean	SD	Mean	SD
M.S.	1	2.714	0.792	2.654	0.689
	2	2.968	0.803	2.769	0.587
	3	3.413	0.994	3.154	0.967
S.T.	1	2.381	0.762	2.538	0.859
	2	2.73	0.827	2.808	0.801
	3	3.079	1.005	3.385	0.804
C.R.	1	2.111	0.845	2.461	0.811
	2	2.571	0.979	2.615	0.752
	3	3	0.984	3.192	0.801
Acc.	1	2.016	0.924	2.308	0.928
	2	2.444	1.044	2.615	0.983
	3	2.984	1.225	3.615	1.061
R&E	1	1.429	0.797	1.308	0.471
	2	1.81	1.045	1.423	0.758
	3	2.19	1.148	1.615	0.983
PTQ	1	10.651	3.612	11.269	3.207
	2	12.524	4.173	12.231	3.386
	3	14.667	4.813	14.961	3.649

Table 29 PTQ Time Variation Means & SD

As the parameters for normality could not be guaranteed, a Kruskal-Wallis test was once again used to investigate any potential differences between students scores' under each facilitator for each round of testing. The null hypothesis stated that there was no statistically significant difference between the medians of each data set, and was tested under a 95% level of significance, whereby a p-value of less than 0.05 would reject the null hypothesis. The results of these tests are displayed in Table 30:

Scale	Round	Kruskal-Wallis Test Statistics			
		H	**df**	**P-value**	**H_0**
M.S.	**1**	0.003	1	0.953	Failed to reject
	2	1.257	1	0.262	Failed to reject
	3	1.181	1	0.277	Failed to reject
S.T.	**1**	0.914	1	0.339	Failed to reject
	2	0.442	1	0.506	Failed to reject
	3	2.24	1	0.134	Failed to reject
C.R.	**1**	3.975	1	0.046	Rejected
	2	0.097	1	0.756	Failed to reject
	3	0.588	1	0.443	Failed to reject
Acc.	**1**	2.02	1	0.155	Failed to reject
	2	0.604	1	0.437	Failed to reject
	3	6.091	1	0.014	Rejected
R&E	**1**	0.028	1	0.866	Failed to reject
	2	2.735	1	0.098	Failed to reject
	3	6.134	1	0.013	Rejected
PTQ	**1**	1.372	1	0.241	Failed to reject
	2	0.002	1	0.967	Failed to reject
	3	0.084	1	0.772	Failed to reject

Table 30 PTQ Kruskal-Wallis results - Time variation

The Kruskal-Wallis test rejected the null hypothesis on three instances, and these will now be further examined. Firstly, the results for Round 1 of *communicating reasoning* were found to differ to a statistically significant degree, with this likely due to the students of the 3-week variation starting with a higher mean score in this category. As tests for Rounds 2 and 3 failed to reject the null hypothesis, however, it may be surmised that those on the 14-week variation had caught up by Round 2, and any subsequent increases were not different to a statistically significant degree. Secondly, the test for Round 3 of the *accuracy* category rejected the null hypothesis. Whilst students' scores from the 3-week variation were higher in Round 1 to begin with, they also increased by a greater amount over the course of the intervention. A precise reason for this is not immediately clear. Finally, the test for Round 3 of the *reflection* category rejected the null hypothesis, with students' scores from the 14-week variation increasing to a greater extent than their 3-week comparison. Once again, there does not seem to be an obvious reason for this.

5.4.2. Facilitator Comparison

In this section, students' PTQ results will be compared based on the facilitator they studied with – the author (*n=70*) or the independent facilitator (*n=19*). The mean scores and their respective standard deviations for the PTQ and each individual rubric category are displayed in Table 31:

Scale	Round	Author		Independent Facilitator	
		Mean	SD	Mean	SD
M.S.	1	2.686	0.733	2.737	0.872
	2	2.914	0.756	2.895	0.737
	3	3.386	0.906	3.158	1.259
S.T.	1	2.471	0.793	2.263	0.872
	2	2.786	0.832	2.632	0.761
	3	3.229	0.920	2.947	1.079
C.R.	1	2.329	0.812	1.789	0.855
	2	2.629	0.887	2.421	1.017
	3	3.086	0.864	2.947	1.177
Acc.	1	2.214	0.915	1.684	0.885
	2	2.529	0.989	2.368	1.165
	3	3.229	1.206	2.947	1.224
R&E	1	1.371	0.641	1.474	0.964
	2	1.686	0.941	1.737	1.147
	3	2.029	1.179	2	0.943
PTQ	1	11.071	3.303	10	4.155
	2	12.5	3.893	12.158	4.295
	3	14.943	4.266	14	5.161

Table 31 PTQ Facilitator comparison: means and SD

As the parameters for normality could not be guaranteed, a Kruskal-Wallis test was once again used to investigate any potential differences between students scores' under each facilitator for each round of testing. The null hypothesis stated that there was no statistically significant difference between the medians of each data set, and was tested under a 95% level of significance, whereby a p-value of less than 0.05 would reject the null hypothesis. The results of these tests are displayed in Table 32:

Scale	Round	Kruskal-Wallis Test Statistics			
		H	df	P-value	H_0
M.S.	1	0.011	1	0.918	Failed to reject
	2	0.012	1	0.913	Failed to reject
	3	0.513	1	0.474	Failed to reject
S.T.	1	1.481	1	0.224	Failed to reject
	2	0.67	1	0.413	Failed to reject
	3	.721	1	0.396	Failed to reject
C.R.	1	6.789	1	0.009	Rejected
	2	0.552	1	0.458	Failed to reject
	3	H<0.001	1	0.991	Failed to reject
Acc.	1	5.906	1	0.015	Rejected
	2	0.513	1	0.474	Failed to reject
	3	0.691	1	0.406	Failed to reject
R&E	1	0.019	1	0.891	Failed to reject
	2	0.002	1	0.964	Failed to reject
	3	0.098	1	0.755	Failed to reject
PTQ	1	3.145	1	0.076	Failed to reject
	2	0.219	1	0.64	Failed to reject
	3	0.146	1	0.703	Failed to reject

Table 32 PTQ Kruskal-Wallis results: Facilitator comparison

The null hypothesis was rejected for Round 1 of two categories: communicating reasoning, and accuracy; indicating that the median scores at the start of the intervention are statistically different. However, as the tests in Rounds 2 and 3 fail to reject the null hypothesis, it is reasonable to assume that those who studied with the independent facilitator had caught up by Round 2, and both groups experienced a similar increase by Round 3.

5.5. Student Diaries

The student diaries – first outlined in Section 3.8.6, and further specifically discussed in the context of the intervention in Section 4.4.5 – were coded by the author to uncover prevalent themes or subthemes in the data. As a result of the analysis, 12 major themes emerged, with one theme composed entirely of two subthemes – positive and negative responses to group-work. The final themes, and the frequency of their occurrence, are displayed in Figure 18. We will now discuss each of these themes in turn.

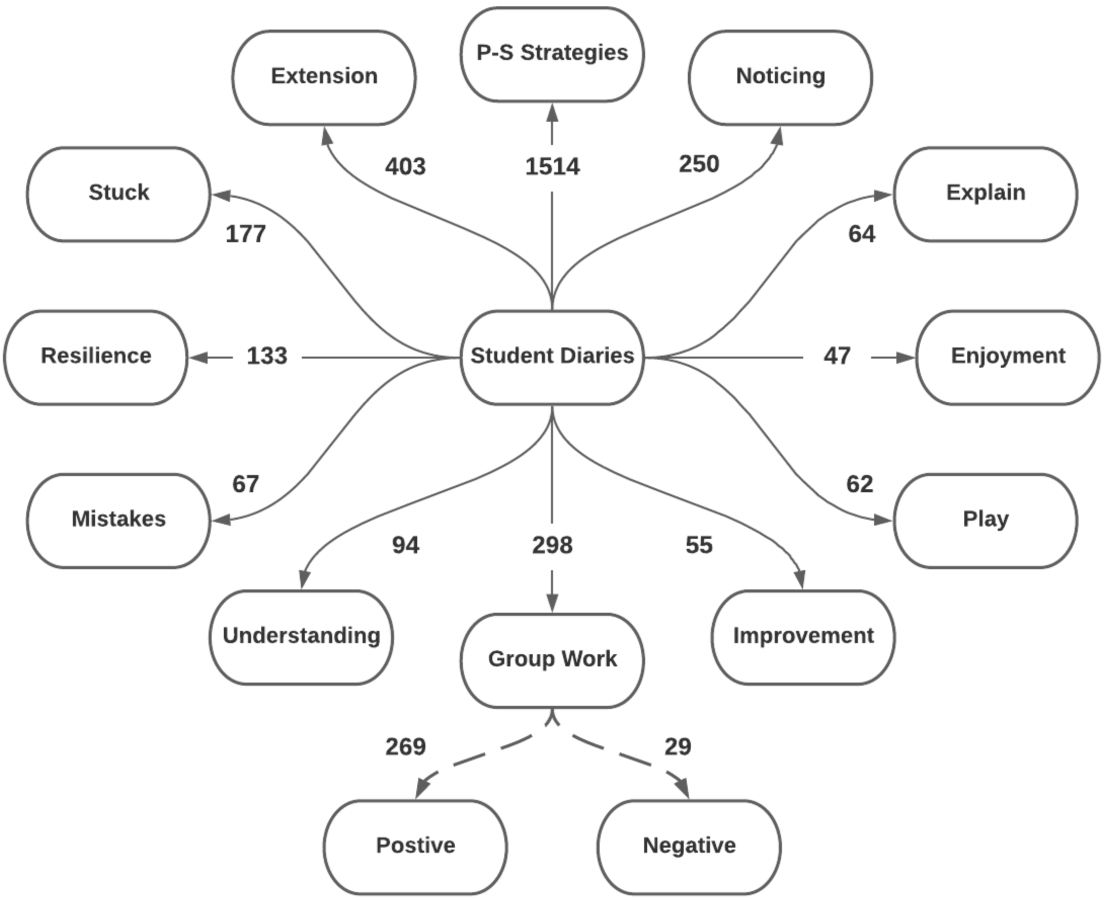

Figure 18 Diary Themes

Problem-solving Strategies (*n*=1514)

Students were instructed to outline their processes towards a solution for each 'diary problem' in their diary entry each day. For this reason, it would have been expected that the chosen problem-solving strategy, or other strategies attempted, would feature heavily in students' writing, and this is reflected in the high frequency of occurrence (*n*=1514). In the first round of the thematic analysis, each problem-solving strategy was given its own code, but the decision was made to combine these into one larger theme. Each formal problem-solving strategy introduced in the intervention was identified at some point in the diary entries, although some were more prevalent than others, as presented in Table 33.

Problem-solving Strategy	Frequency
Visual	437
Patterns	341
Generalising & Specialising	597
Conjectures	12
Assumptions & Questioning	71
Structure	31
Working Backwards	25

Table 33 Diaries: Problem-solving Strategies Frequency

Many of the codes under this theme simply involved mentioning which strategy was utilised in the process towards finding a solution, such as: "*we began by drawing out…"; "we decided to start with small numbers"*; "*we has assumed that…*". However, there were also occurrences that highlighted the importance of the strategies to students in the problem-solving process. The following are just three such instances:

- The importance of visuals, recorded in the penultimate session

> *We were quite well prepped for this challenge as over the course of this course thus far we realised the importance of diagrams. This was crucial for solving this problem*

- Relying on a proper structure to problem-solving

> *It took us a very long time to figure this problem out as we kept forgetting what we had done… We found this question to be very tedious and long-winded but it was just a matter of keeping everything neat and easy to read*

- The role assumptions play

> *I think by making the assumption we made, we made the problem harder for ourselves. I think had we more time it would have been a good idea to attempt the problem using a different assumption. I feel that any in-school maths I have done up to this point have not been as vague as this problem and haven't needed you to make an assumption*

While these quotes are but a small reflection on the overall frequency of problem-solving strategies mentioned in the diaries, they highlight student acknowledgement of the importance of the strategies in the process towards a solution.

Extension (*n*=403)

Similar to *problem-solving strategies*, the theme of *extension* was likely to occur frequently due to the nature of the diary reflections. While there were many instances where the student merely listed their extension at the end of the reflection to add further depth to the original problem, there were also a large variety of diary entries that offered in-depth analysis and exploration of, or a full solution to, the chosen extension. For some students, the extension allowed them to create their own challenging problems, e.g. "*I liked this question as it was simple, yet we could go much further in our extensions*"; whilst also letting students simplify an overly-challenging problem, e.g. "*as an* *extension we would alter the question so that it would state that there were four people living in the village. This would make the problem easier*".

While common themes did not occur in the style of extensions created by the different student cohorts, some common extensions occurred to particular problems. Table 34 displays some of these common extensions that reoccurred, and the problem they extended (Appendix G):

Problem	Extension
How many squares are there on a regular chessboard?	How many rectangles are there on a regular chessboard?
My friend claims all 4-digit palindromes are divisible by 11. Are they?	Are 5-digit palindromes divisible by 11? 6-digit? N-digit?
Two piles of matches are on a table. A player can remove a match from either pile or a match from both piles. The player who takes the last match loses. If there are two players, how should you play?	What about if there are three players? What about if there are three piles?

Table 34 Extension Examples

Group-work (*n*=298)

Group-work was a fundamental aspect in the development of the intervention, reflected in the development of the CoPs model for this research discussed in Section 3.7, and thus its prevalence as a major theme was somewhat to be anticipated. It is also the only theme to be divided into two subthemes, to reflect the overwhelmingly positive response to group-work from students, but also acknowledge the small amount of negative responses. Amongst the positive responses, there were many instances (*n*=44) where students found a benefit to the differing perspectives of their groupmates, such as: "*Personally, it's good to learn other people's way of approaching a problem*"; with the following student attributing his growth to these differing perspectives:

"At the start of the day I was unable to change my thought process in order to solve the problem, but [when] I began to hear how others solved the problem I began to be able think in new ways and use new techniques to solve new problems"

There were many more student reflections on their improvements of different aspects of group-work (*n*=24), but none more so than the following, which ties together many of the desired effects of this intervention:

"I think I am getting better at working as a group in this course. It's a new experience but I feel the group is getting better at dividing up the jobs. I'm also improving in trusting other people's work without having to check over everything they've done. I'm also getting better at explaining my work as before I just needed to get a right answer, but now I have a group that need to understand it."

In the development of the CoPs model (Section 3.7), it was speculated by the author that the stages of the model may yield certain benefits to students, and examples of these benefits were found in the student diaries. One intention of the *discussion* stage of the model is to alleviate misinterpretations of a problem, which is evident in the following student quote:

"After reading the problem at first I was confused by the question but my group helped me to understand better what was being looked for in the problem".

The diaries showed that, during the *conjecture* stage, students attempted different strategies in the same group (e.g. "*This question was made easier by group-work because we all attacked the question with different methods")*, or simply worked together on one strategy by subdividing the workload amongst the group members:

"While answering this question, we each experimented the number of interactions for different amounts of old men to save time and energy… It would have taken me a lot longer to answer this question if I had not answered it with my group"

Indeed a huge emphasis is placed on communication within a group throughout the problem-solving process on the intervention, culminating in the *communicate reasoning* stage where students must be able to explain their processes and understanding of a problem, e.g.:

> *"As a group we worked well through communication and explanation. We were each able to explain our thoughts and opinions to one another. This allowed for a faster understanding of the given question".*

Of the negative responses to group-work, many of these were attributed to a "*lack of focus*" (*n*=12) on one particular day. There were also several instances (*n*=7) whereby a negative response early in the intervention, e.g.: "*I'm not working that good with my group and write things out instead of discussing them with my peers*" (Day 1), was followed by an acknowledgement of improvement in a subsequent session, e.g.: "*I felt my team work has gotten better as we all need to work together to solve most problems*" (Day 2). The remaining ten instances were miscellaneous negative responses.

Resilience (*n*=133)

Mathematical resilience was discussed in detail in Section 2.5. One important aforementioned characteristic of mathematical resilience is the development of strategies to help when faced with challenging work (Johnston-Wilder & Lee, 2010a). Some of these strategies were evident in students' diaries, and a selection are outlined through the following:

- Group discussion (*n*= 21)

> *When we looked at the question first it was quite confusing. We didn't understand it so we talked about it in our group for a fairly long time which was very helpful*

- Ask for help (*n*=11)

> *We were stuck immediately trying to figure out… We had to ask for help in which we discovered…*

- Start again (*n*=6)

> *We struggled with this question, but once we took a step back and looked at it a different way we were able to solve it.*

In many instances, *resilience* was cross-coded with *noticing*, whereby a student acknowledged their feeling of struggle within a problem, individually or collectively, leading to a member of the group having a 'eureka' moment to further progress the problem, e.g. "*We struggled getting the reason why for a while but then we realised…*" and "*We got stuck at this point for a long time, until I realised that…*". In the former quote, the student registered their struggle with developing a conceptual understanding of a solution, and this will be further discussed under the theme of *understanding*.

In Section 2.5, the significance of a growth mindset to mathematical resilience was discussed. One particular aspect of this is how a student responds to failure, whereby those with a fixed mindset may be more prone to giving up, and those with a growth mindset may learn from their mistake and continue in their work. This was epitomised by two students in the following reflections: "*I'm coping better with not getting the answer the first time around*" and "*We failed a few times but we kept attempting this question until we could come up with a solution and also until we had understood the pattern and were able to explain the pattern…*". Further to this, there were numerous examples in diary entries of students showing mathematical resilience through their determination to find a solution despite repeated setbacks in their problem-solving attempts, as represented by the following quote. While portions of the quote that are specific to the problem are removed, the sentiment of the student's writing remains, such that their group made continuous attempts at the problem to reach their solution.

> *"Today's challenge was the most difficult thus far. At first we thought it to be Fibonacci. And that did work until we paused and realised [a contradiction]…. So we tried a few low numbers… [but this] caused a huge struggle for us…. We then tried to separate odds from evens… but even still caused us great struggles…. We tried more and more numbers…Finally, we figured out that…"*

Stuck (*n*=177)

We have already seen quotes under the theme of *resilience* that contained the word ‘stuck’, and indeed many of the quotes from student diaries that contained *stuck* as a theme also contained resilience. However, there were enough instances where they were not cross-coded to warrant separate themes. The most common occurrence (*n*=22) in which an admission of being “*stuck*” was not followed up by continued attempts at solving a problem, was the student running out of time, e.g. “*At this point we were stuck… Unfortunately we ran out of time before being able to produce a formula…”*. Due to the limitations around the timing for the ‘tutorial’, this could not be avoided and was therefore somewhat expected in the diary reflections. There were also two occurrences where students reported that they became “*distracted*” following getting stuck, and did not get back on track with the problem before the time was up. Both of these instances occurred in the first two weeks of the intervention.

Noticing (*n*=250)

Mason et al (2010, p. 128) highlighted noticing as a fundamental tool within problem-solving, whereby the problem-solver takes note of important points in the process towards a solution, or *notices* something that may help them. This process was emphasised by the facilitator on the intervention as they encouraged students to understand *why* a solution worked, or to reflect on how they overcame the feeling of being stuck. In doing so, or in simply discussing their solutions, the students utilised the words “*noticed*” or “*realised*” with such regularity that they emerged as the theme *noticing*. Many of the instances are documented under various other themes, such as *resilience* and *mistakes*.

Mistakes (*n*=67)

The theme of *mistakes* was created for students outlining and reflecting on “*mistakes*” or doing something “*wrong*” in their work, rather than simply mentions of a failed strategy attempt. The acknowledgement that a mistake had been made, addressed, and corrected, was important due to the reported prevalence of perfectionism amongst highly-able students (as discussed in Section 2.4.1). Whilst some instances (*n*=11) of mistakes were simply mathematical errors, the most common occurrence (*n*=31) was due to a misinterpretation of the problem, particularly in the opening stages of a group's attempts.

> *"At first we found no connection… We then tried to identify where we had gone wrong. We reread the problem, we realised the wording had confused us the first time and had led to the problem not working. After we had discovered this we went back and tried again."*

Of these 31 instances, 20 occurred in the first four days of the intervention, which may indicate that the students became more competent at addressing misinterpretations during the opening discussion of a problem.

Further examples (*n*=8) of students reflecting on mistakes were linked to the problem-solving strategy themes (see Section 4.4.1). On the first day of the intervention, when the students would not have been introduced to any of the chosen strategies, the following quote highlights how 'specialising' and 'structure' would have helped to alleviate the mistake:

> *"The first attempt we made was to just try and individually count each square, but we quickly realised it would be too easy to make a mistake and ditched the idea".*

Similarly, on the day where students encounter problems rich with 'assumptions', but before it had been discussed as theme, the following was identified by a student:

> *"We came up with an answer straight away and agreed that it was possible. However, we had made [an] assumption… Once we realised we had made an assumption… we discovered that the problem was not possible"*

Explain (*n*=64)

In what was a consequence of the group-work dynamic, there were instances in the student diaries where one student had to explain part of the problem, e.g. "*I was really stumped by this problem and had real trouble with it. Luckily some members of my group understood it far better than me and were able to explain it to me*"; or part of their solution, e.g. "*I didn't understand someone's reasoning* *to figure out the problem, but my teammates simply explained it to me again and I understood*". Throughout the diaries, it became evident that the students reacted overwhelmingly positively to working in groups, as was previously discussed under the theme of *group-work*. This was further evident through many of the codes for the theme of *explain*, where students reflected on their improved ability to explain their ideas to the group (as shown in quotes under *groupwork*), or the group's achievement in explaining their ideas amongst each other on the path to a solution, e.g.:

> *"As a group we worked well through communication and explanation. We were each able to explain our thoughts and opinions to one another. This allowed for a faster understanding of the given question"*

Understanding (*n*=94)

In the CoPs model, the '*communicate reasoning*' stage requires students to have an understanding of their solution such that they can communicate "*why*" it works to their group, the facilitator, or in their diary. The theme of *understanding* emerged in the diaries due to students drawing attention to their search for "*why*" a particular solution, or part of a solution, worked, e.g. "*I then looked for a pattern and why this was true*". There were several occurrences of students outlining their "*struggle*" as they sought to develop a conceptual understanding of a problem, as previously outlined in a quote under the theme of *resilience*. The following quote succinctly describes the attitude of most students at the beginning of the intervention when they were asked to seek a conceptual understanding to a problem, rather than merely being told if their answer was correct:

> *"In math, I usually just give the answer and get it right or wrong… for this question we had to find out why it worked and that was weird but I think we did ok"*

Improvement (*n*=55)

One further theme to emerge in the data was that of *improvement*, whereby a student registered in their writing a particular aspect that they felt they had improved upon over the course of the intervention, or even over a particular day of problem-solving. Whilst there were some instances (*n*=16) of general statements of improvement, e.g. "*I think we have improved in our attempts at* *problem-solving*"; there were also students that reflected on details of the intervention that they had noticed improvements in:

- Discussion: "*I found that we actually spent more time reading and talking about the problem more than we had in previous weeks*"
- Group work: *"I feel that we have improved this week as a group, and are able to bring our own individual strengths to the group to help each other"*
- Explaining: *"I'm also getting better at explaining my work"*

Play (*n*=62)

In Phase 3 of the research, a new diary problem was added to Day 12 of the intervention that focussed around a game that could be played. Due to this, the theme of *play* emerged in the diary entries of Cohorts 5 and 6, with most students reporting how they began the problem by "*drawing out the board and playing the described game*".

Enjoyment (*n*=47)

The final theme from Figure 18 is that of *enjoyment*. This recognises students' expressions of enjoyment or pleasure at a particular aspect of the intervention, such as: a problem – "*[the problem] was enjoyable, because although it was difficult it was doable*"; group-work – "*I enjoyed this question because we were able to work together so well*"; or being challenged and showing resilience – "*This was a very difficult question and I didn't like it at first, but once we started doing [a strategy] it was exciting making progress*".

Summary

Through reading and analysing the student diaries, it became obvious that the student sample had embraced many of the ideals that the intervention was designed to encourage. The prevalence of the *problem-solving strategies* and *extensions* are major features of the skills emphasised by the author, while their willingness to seek the conceptual *understanding* was also evident. Whilst mindsets were not produced as a theme, the acknowledgement of *mistakes*, and their desire to *improve* mathematically and as part of a group, displayed positive attributes of a growth mindset.

Furthermore, the emergence of *resilience* and *stuck* highlighted the students' desire to persevere towards a solution.

The themes that emerged from the diaries offered an insight into students' experiences of the intervention during their participation. We will now present the data collected post-intervention through the intervention survey and the focus group interview.

5.6. Intervention survey

As mentioned in Chapter 3, a survey was administered to students to ascertain their opinions of the intervention itself. Recall that this was emailed as an online survey to most participants, while students from Cohort 6 undertook it immediately upon completion of the intervention. The results outlined in this section are those obtained through the online survey (28 students), and the survey completed by Cohort 6 (29 students) at the end of their participation. Of the 28 students to complete the online survey, 18 studied the 14-week variation of the intervention and 10 studied the 3-week variation. Cohort 6 studied the 14-week variation. Where relevant, the results are also presented as a comparison between the data sets, to observe any differences in opinions collected at the immediate end of participation versus those who completed the intervention three or more months prior to the survey. The comments provided by students for each question were coded to produce prevalent themes in the data. The most common themes, with a selection of student quotes, for each question will also be discussed. The questions are discussed under three subheadings: students' opinions of the intervention (Section 5.6.1); students' opinions or beliefs prior to participation (Section 5.6.2); and students' opinions or beliefs affected by the intervention (Section 5.6.3).

5.6.1. Opinion of Intervention

To ascertain students' opinions of the intervention itself, they were asked a series of questions regarding their enjoyment of the intervention; their perception of the workload involved; any changes they would like to see made to the intervention; and which variant (3-week or 14-week) they would recommend to others to study.

Enjoyment

Students were asked to rank their enjoyment of studying the intervention on a scale from 1 (did not enjoy at all) to 5 (enjoyed a lot). One student (from Cohort 6) selected a 3, but all other students chose either 4 or 5, with 37% opting for 4, and 61% choosing 5. From this, it is clear that student enjoyment of the intervention does not appear to have been affected by studying different variations of the intervention, or by time elapsed before completing the survey. 50 respondents (21 from the online survey and 29 from Cohort 6) added further comments, with 25 of these specifically citing "*group-work*" as a contributing factor to their enjoyment of the intervention, making this the most common theme for this question. Students stated that they found the group-work *"very helpful to learning as we all approached problems in different ways"* and that it meant *"different approaches at solving a problem could be discussed and tackled simultaneously within the group. This helped me to broaden the way I view problems*." They also contrasted this with their usual experience, saying *"The fact that it was group-work and encouraged conversation provided stark contrast with a classroom environment"*. One student also expressed their enjoyment for working in groups with "*people with like-minded attitudes*". The two next most common themes mentioned were "*problem-solving techniques*" (*n*=12) and "*challenges*" (*n*=8). For the former, students noted the development of skills and techniques for problem-solving on the intervention, such as reflection, extension and problem-solving strategies, as aiding in their enjoyment of the intervention; whilst for the latter, students were found to enjoy the "*challenging level of maths on the module",* with one student adding that they enjoyed learning "*how to overcome a problem I'm stuck on"*.

Workload

Figure 19 outlines students' responses to the weekly workload required on the intervention, on a scale of 1 (light) to 5 (heavy). The spread of responses is quite wide to this particular question.

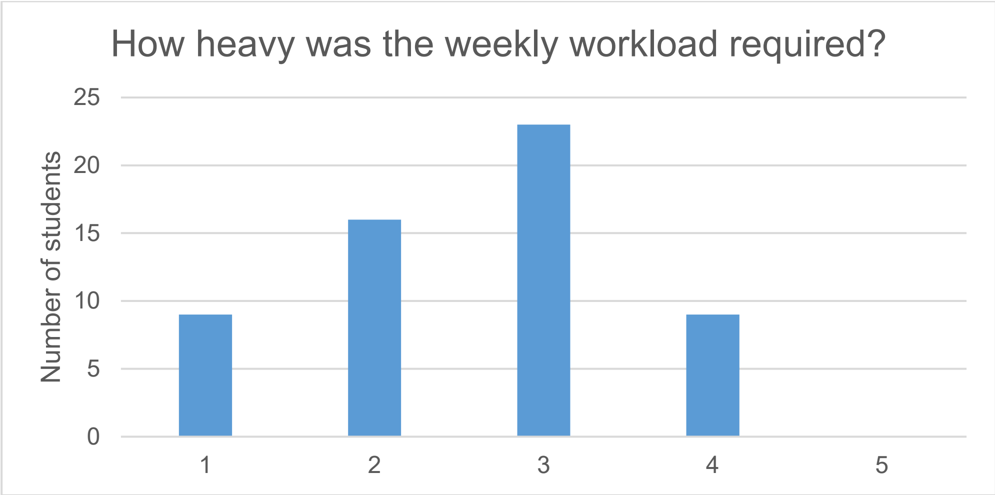

Figure 19 Student perceptions of the workload

In the open responses, of those registering a light workload, seven attributed this to the lack of homework or external work required e.g. *"Because it didn't require a lot of homework which was good. I could expand more on something I was stuck on if I wanted, but I didn't feel pressured or anxiety from a s**t ton of overwork. I could also reflect on how to improve if I wanted."* Those towards the middle or upper-end of the scale were more inclined to balance the lack of homework with the challenge required within the classroom e.g. *"Although the module did not require much outside preparation, the problems given were often quite challenging."* The wide variety of responses to this question, combined with the clear lack of cohesion amongst students as to what constituted the intervention's workload, led to the development of the second major question on the focus group, and its accompanying sub-questions (Appendix E).

Changes

When asked about what changes they would make to the intervention, 75% of students believed no changes were necessary. Of the eight open responses to the online survey, two believed short breaks should be applied to increase concentration (short breaks were introduced from Cohort 4 onwards); five requested changes beyond the scope of the intervention; and one wanted shorter amounts of time spent on each problem. Four of the six student responses from Cohort 6 believed that groups should be changed each day.

5.6.2. Prior to Intervention

In the next section of the survey, students were asked about circumstances prior to the intervention: their perceived mindset, resilience, and whether they had ever become "*stuck*". A short explanation of growth and fixed mindsets, identical to that encountered by students at the start of the intervention, was outlined at the beginning of the mindset questions as a reminder to students.

Mindset

Students' responses to their own perception of their mindset prior to the intervention are shown in Figure 20, with those who answered the online survey and those who were in Cohort 6 shown separately.

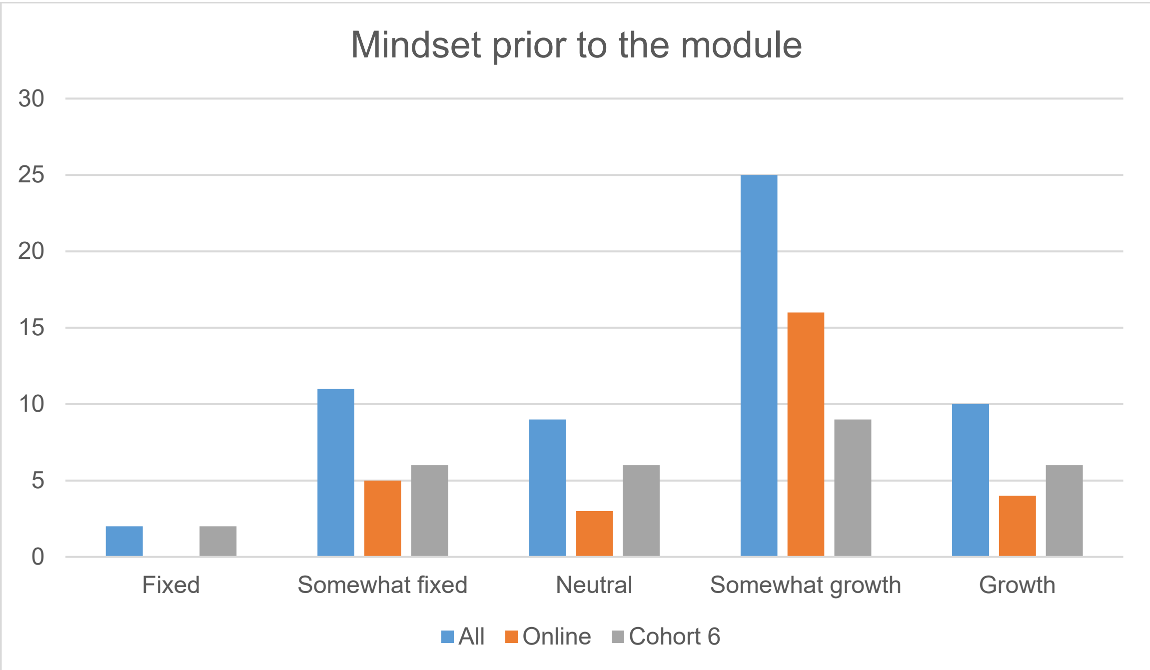

Figure 20 Student perception: Mindset prior to intervention

Here, it can be seen that there are some differences between the 'online' responses (where students had completed the intervention at least 3 months earlier, and in certain cases, up to a year or more before) and the 'Cohort 6' responses, who had just completed the intervention. The latter group were more likely to identify their mindset prior to the intervention as having been towards the fixed end of the scale. One possibility here is that this was an accurate reflection for the students who answered the survey; another is that students who had just completed the intervention remembered their original mindset more accurately than those who had done it a long time ago. This mindset data formed one part of a question, and was in conjunction with students' beliefs about their mindsets post-intervention. Therefore, the open-ended comments for this question are further discussed in Section 5.6.3, alongside other questions exploring perceived effects of the intervention.

Stuck

Students were asked if they became "*stuck*" prior to the intervention and, if so, asked to provide further detail on a scenario in which they became stuck and how they overcame it. 71% (n=41) of respondents acknowledged being "*stuck*" prior to the intervention. To overcome their scenario, ten students made a reference to studying or practicing the area of maths to help overcome being stuck; whilst eight students asked a teacher, a family member or a friend for help. Three students reported that they gave up once they became stuck.

5.6.3. Perceived Intervention Effect

In the final section of the survey, students were again asked about their perceived mindset, resilience and becoming "*stuck*" – but this time, it was in relation to how they perceived these attributes *after* the intervention. In addition, they were also asked if they felt they had developed any new skills as a result of the intervention, or if they believed the intervention had a long-term benefit to their study of mathematics.

Mindset

Firstly, all students were asked to rank their mindset immediately after studying the intervention. Figure 21 displays a comparison between the mindsets self-identified by students prior to and after studying the intervention, in terms of percentages, with a clear movement collectively away from a fixed mindset evident in the graph.

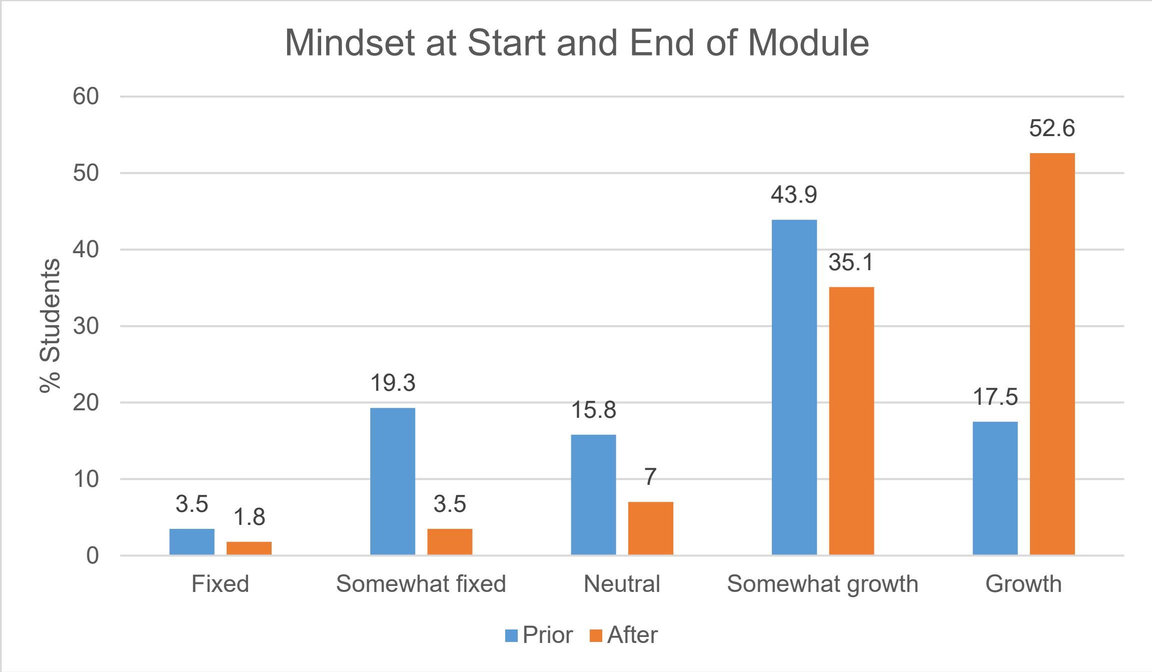

Figure 21 Student perception: Mindset change from start to end of intervention

Seven students on the online survey, and one student from cohort 6, did not leave an open-ended comment for this question. Of the other 49 other responses, only one theme emerged, as four students expressed a belief that natural ability is integral to a person's mathematical ability. Of those, two students with fixed-leaning mindsets believed skills were learned merely to "*compliment"* natural ability, rather than increasing it. Conversely, a student with a growth-leaning mindset believed that natural ability simply affected how quickly a person's mathematical ability may improve, and that some require greater effort than others. Whilst expressing the opinion that the intervention helped them to see "*how much more [they] could do to improve [their] mathematical ability*", one student also addressed the difficulty around this for students who are weak at mathematics, where "*the stigma around being bad at maths definitely doesn't help those struggling*". This first statement encapsulates the purpose of the intervention, given that it is targeted at highly-able students who have already displayed their ability in mathematics, but designed to provide a challenge to them and encourage their Problem-solving Potential.

As a minimum of three months had elapsed since the respondents of the online survey had completed the intervention, they were also asked to rank their current mindset. Here, there was a further movement towards a growth mindset between finishing the intervention and completing the survey (Figure 22), with no student from this group identified as leaning towards a fixed mindset at either stage. The open-responses from students for this question alluded to the necessity for "*hard work*" and "*effort*" to improve upon one's mathematical ability.

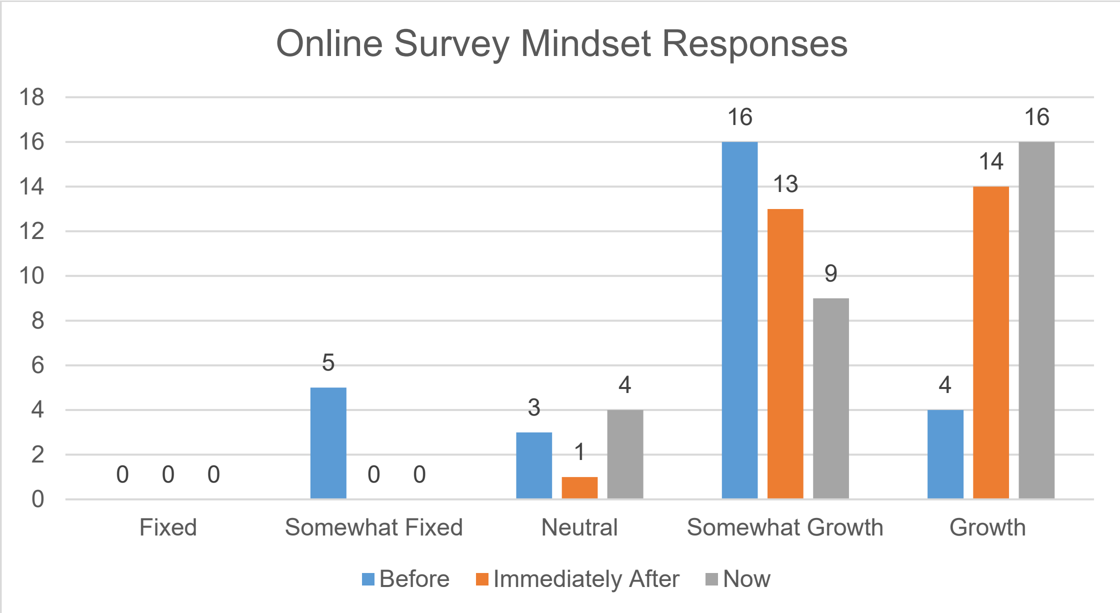

Figure 22 Online Intervention Survey: Mindset change

Resilience

Students were questioned on any perceived change in their resilience when faced with challenging work; 47 of the students stated the intervention did affect their resilience, nine believed it did not, and one did not answer. Of those to respond 'no', four believed they were already resilient, two stated that they always enjoyed challenging work, and three did not provide an open response. Four of the nine aforementioned students provided an open-response, with all four suggesting they were already mathematically resilient and thus the intervention could not improve upon this. The positive responses were analysed for major themes, yielding the following:

- Fifteen students believed they were more resilient as they had learned new strategies on the intervention to help them deal with challenging work (e.g.: "*Before studying this module, I'd give up on questions very easily, but now I find myself taking a break and coming back to it*").

- Thirteen said they learned to "*not give up*" with one student believing they are now "*better coped to deal with failure and try again*"
- Six students believed experiencing being stuck on the intervention helped them to become more resilient.
- Six students expressed their willingness to spend more time on challenging work due to the time-intensive problems they had encountered on the course
- Five students felt the challenging problems on the intervention helped to improve their resilience.

Stuck

In a separate question, students were asked if they became stuck while studying the intervention, which 54 students acknowledged had occurred. When asked to outline how they overcame this, the most common solutions were group discussion (*n*=25), and to try to approach the problem with a different strategy (*n*=21). Six students also asked the facilitator for help when they became stuck.

Benefit

The students were asked if they felt the intervention would have a benefit to their study of mathematics in school, with a 100% positive response rate.

- Thirteen students felt the problem-solving skills, and particularly the new strategies, learned will benefit them:

> *I think the module will benefit my school maths because it really helped me to understand problem-solving and the steps/stages of solving problems and also it will help me with all the new skills that I learned to put them into practice.*

- Thirteen students believed the intervention taught them to 'think' differently about mathematics:

> *"It has taught me to think mathematically rather than just memorising things which is applicable to any problem in maths"*

- Seven students felt they will benefit from the increase in their resilience:

> *"I have gained a better resilience when it comes to completing challenging maths problems as well as a more positive and curious outlook"*

- Seven students believed the intervention has helped to make mathematics in school seem easier to do

> *The lateral thinking approach employed in the module has been very useful in solving the trickier problems in higher level.*

New skills

Finally, students were asked if they had learned new skills while studying the intervention, with a 100% positive response. The open responses were categorised under: problem-solving strategies, problem-solving skills, group-work, resilience or mathematical skills. Figure 23 displays the categories, and specific skills mentioned by students.

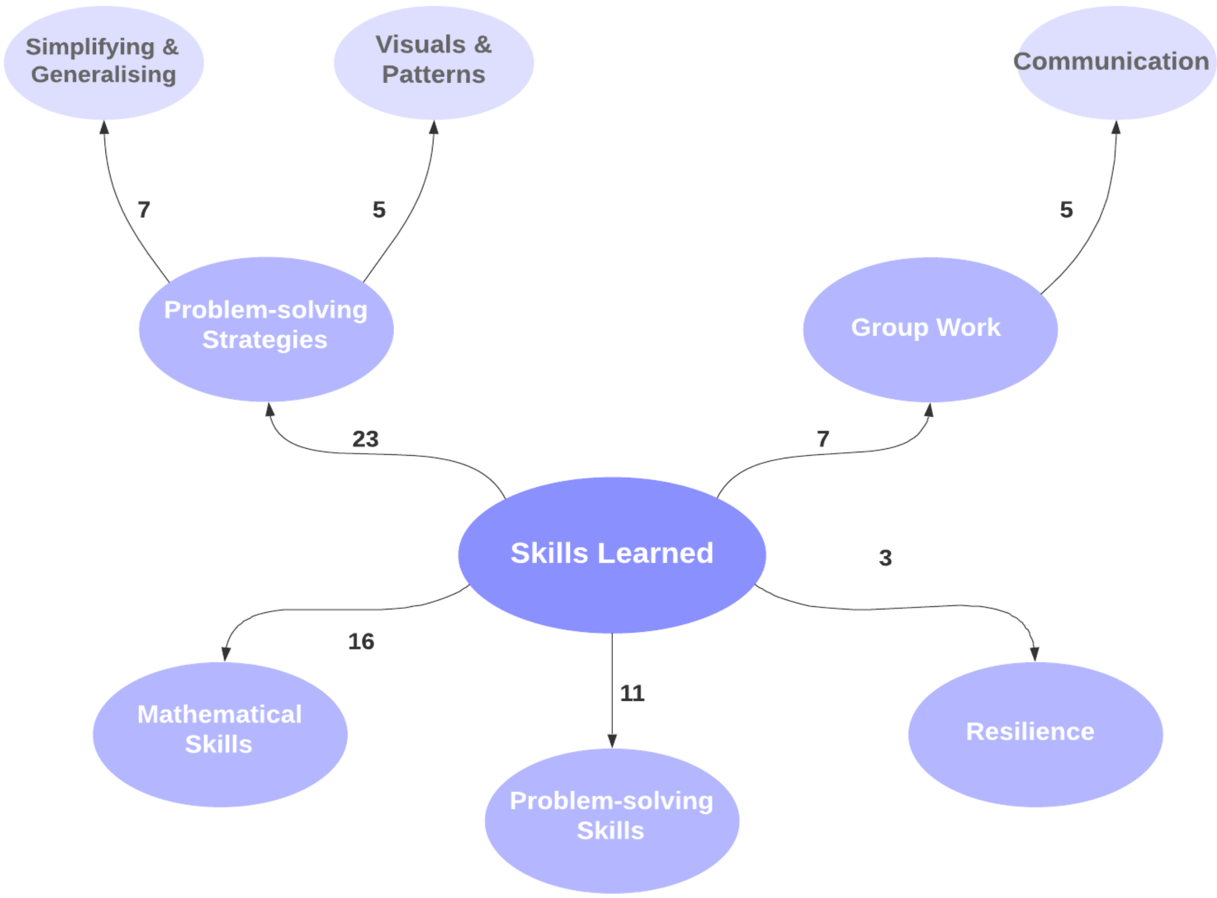

Figure 23 Student Perception: Skills learned

While 23 students made reference to learning new "*problem-solving strategies*", 12 of these specifically mentioned strategies they had learned – simplifying & generalising (n=7), and visuals & patterns (n=5). Five students believed they had learned new "*communication skills*" on the intervention, with one student stating that "*[they] can communicate [their] process to others better because of working in* groups". While many of the instances under the theme *problem-solving skills* were utilising this as a general phrase, there were also mentions for "*reflecting*" (n=2), "*critical thinking*" (n=3), and "*extending*" problems (n=2). The 16 mentions of *new mathematical skills* were all either unique mentions (e.g. "*algebra skills*", "*perfect squares*", etc) or the general phrase "*maths skills*". Of the three mentions of resilience, two students felt they had learned how to approach a problem when stuck; and one student believed they were more resilient when they set their mind to it.

The intervention survey was utilised to recruit participants for the focus group interview, and the students' responses on the survey helped to guide the questions for the interview. Therefore, it is appropriate to now present the data collected through the focus group interview.

5.7. Focus group interview

Through the online survey, students of Cohorts 1 to 5 were invited to participate in a focus group interview. The interview was conducted in Dublin City University in mid-October 2018, by a member of staff of the School of Mathematical Sciences who had no prior interaction with the students. Eight students (one female and seven males) participated in the interview – five from the 14-week variation of the intervention, and three from the 3-week variation (Table 35). The students will henceforth be referred to as S1 to S8, for anonymity purposes.

The interview was audio-recorded on several devices, and subsequently transcribed by the author. A general inductive analysis was completed on the raw data, whereby themes and sub-themes were identified through the reading of the text and the coding and re-coding of the data by the author (Cohen et al., 2007). As a measure of validity to the findings of this analysis, the data was also analysed by the author's supervisor and adjustments made accordingly. The themes and sub-themes are collectively discussed under the overarching general theme "*the impact of the module on the students*".

Code	Gender	Intervention Variation
S1	Male	14-week
S2	Male	3-week
S3	Male	14-week
S4	Male	14-week
S5	Female	3-week
S6	Male	14-week
S7	Male	14-week
S8	Male	3-week

Table 35 Student sample who took part in the focus group

Figure 24 provides a detailed outline of the themes (major and minor) and sub-themes identified through the analysis process. The major themes emerged due to their higher frequency of occurrence in coding, or were generated by the amalgamation of sub-themes under a relevant theme. Each theme is also labelled with the frequency it appeared in the coding process. The impact the intervention had on the students' experiences of, or feelings towards, 'resilience', 'mindsets', 'group-work' and 'problem-solving', was prevalent throughout the data, and thus these categories formed the major themes. The sub-theme 'CoPs Model' is cross-categorised between the themes of 'group-work 'and 'problem-solving,' and occurred where students made direct reference to features of the CoPs model used in the problem-solving process.

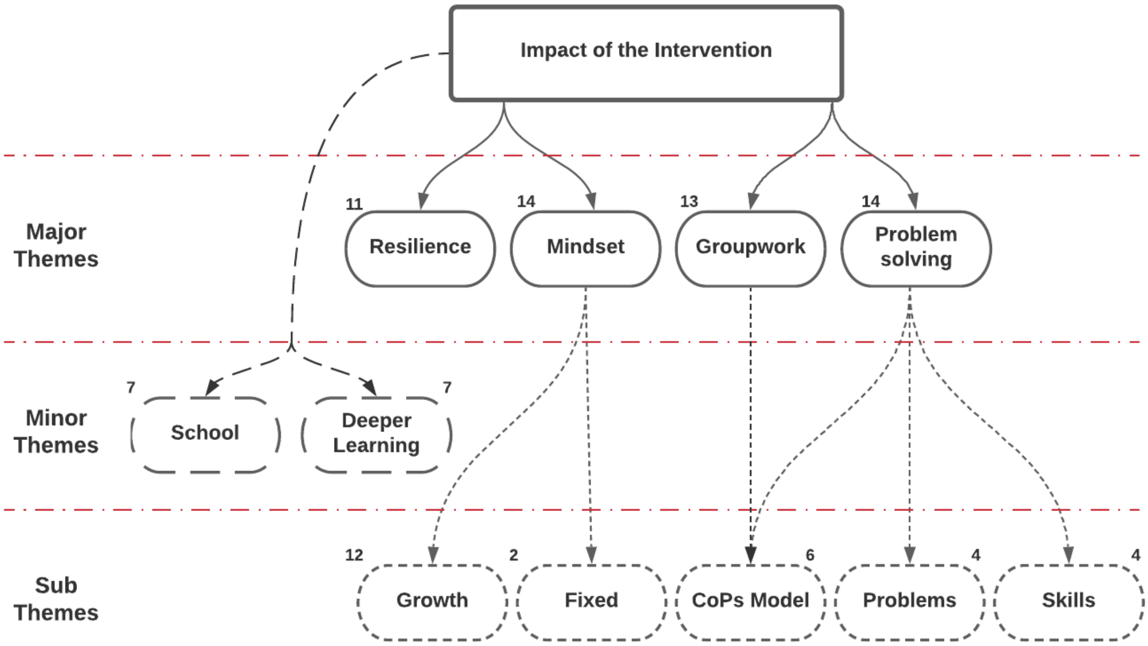

Figure 24 Impact of the Intervention

From the earliest stages of coding, it became evident that 'group-work' was emerging as a major theme, due to its prominence in discussion at various stages of the interview. In the context of group-work in the intervention, students spoke with resounding positivity. There were four references to group-work as an intrinsic part of successful problem-solving, as it offered the students a forum to communicate their opinions and thoughts on each problem, and work together to reach a conclusion:

> S2: *"…the groups were really, really good. Because I don't think that I could have solved any one of those problems on my own, maybe perhaps one; but since I had other people to vent my opinions and thoughts and have them vent their thoughts onto me, and then solve the problem because of group-work, means I felt that was one of the best aspects of the course."*

It was also further expressed by three students (S2, S4, and S5) that the group dynamic gave them the opportunity to see different approaches to problems that they themselves would not normally have discovered (e.g. S4: *"…I found that like other people's outlook on a problem helped me much more than me just sitting there doing it myself, because they'd think of things that I wouldn't"*). S3 and S8 believed that the discussion within the group also had a positive impact on their problem-solving attempts, whilst S4 felt the group discussion helped to clarify misinterpretations of the problems (an aforementioned expectation of Stage 1 of our CoPs model).

In Section 3.7, discussion and communication within a group were outlined as paramount to the implementation of our CoPs model, and thus student references towards aspects of the CoPs model were cross-categorised under the themes of 'group-work' and 'problem-solving'. One such example, given below, showed S3 reflecting on the importance of communication to the improvement of the group:

> *"…the group setting definitely like helped a lot at it because it felt like all of your skills were kind of combined and you were constantly explaining ideas that maybe you had heard about or elements of maths that maybe one person was better at than another, and there's kind of a constant like dialogue back and forth, and you're sort of improving the group as an entity overall."*

Whilst the positive group-work environment created during the intervention was well-received overall it did, however, lead one student (S1) to reflect on their past experience of group-work at school:

> *"If people think you can solve the problem a lot of the time they just won't be bothered. Which then puts more pressure on you to solve the problem and if you can't solve the problem you're screwed. Because then it makes you stupid"*

This also marks one of the two occurrences of a fixed mindset attribute being displayed by a student. There were 12 references to a growth mindset or growth-mindset-related attributes (Table 3) in the interview, and one further reference to a fixed mindset attribute. Despite the two references to fixed mindset attributes, the group unanimously felt the intervention had a positive impact on their mindset. Both S2 and S6 believed the effort required to solve the problems on the intervention helped to instil a growth mindset (e.g. S2: "*…had you had a fixed mindset before, you are basically forced into a growth mindset with the problems 'cause you do have to stick at it…*"). There were also three references to an improved mindset proving beneficial beyond the intervention: S4 paid greater attention to mistakes made during schoolwork to develop their understanding; S8 focussed on developing areas of maths they found difficult, rather than trying to avoid them as they previously had done; and S2 stated:

> *"… after the maths programme I did, I decided that I'd take back up piano because I dropped it when I was like 7. And then I decided that I'd take it back up, because I was like, I might as well go for it… And in the sense of piano, it's just like you have to just choose to believe that if you put the effort into it, you'll get, you'll get to where you need to be. I feel that the maths kind of portrayed that eloquently, because I feel that if you believed that you could solve the problem, most of the time you could just plough through – just use as many different methods as you could and get that final answer. And so that does apply outside of maths – outside of school even."*

The concept of domain-specific high-ability was discussed at length in Chapter 2. S2 outlined their belief that, whilst domain-specific high-ability may exist and thus give the impression that some people are better than others at a topic, the presence of a growth mindset will still allow everyone to improve at any topic with enough effort. In contrast to this, S8 felt that ability within certain domains was capped, and that no amount of effort would improve upon this. They did, however, outline mathematics as one such domain that could be continuously improved with the presence of a growth mindset, and, later in the interview, expressed the importance of teacher feedback in developing growth mindsets in students from a young age. This was corroborated by S6, and further linked to effects on student motivation:

> *"if you're not as fully motivated to do something and everyone around you is telling you you'll never be able to do it, why would you even try? So it's this idea that the people around you shape your mindset, and their opinions influence your motivation…"*

As outlined in Section 2.5, mathematical resilience may be subdivided into struggle, growth and value. For the coding process, references to *struggle* and *value* were categorised under the theme of 'resilience', whereas references to *growth* were already within the theme of 'mindset'. S1, S2, S3, S4, S6 and S7 became involved in a discussion of being stuck during the intervention, and, importantly, how the intervention facilitated their methods for working beyond this. S2 outlined the group dynamic as pivotal in working through struggle, in that different viewpoints may be combined to produce a solution as opposed to the work of a single individual. For S3 and S4, the experience of attacking challenging problems on the intervention gave them the confidence to tackle problems individually, utilising the techniques and strategies learned to approach a problem from different angles in the search of a solution. S1 cherished being stuck on a problem, describing it as "*no fun*" if struggle did not occur. Further to this, they believed the extension of a problem allowed them to be creative and make a problem much more difficult to produce struggle for themselves:

> *"I mean, like the extension stuff- considering we could make it as painful for us to solve as possible, just because we were told like make an extension, we were like, ok – how hard can we make this? …So in that aspect, like the fact that you were able to do an extension means that even if you are finding it easy, you make it as hard as you need it to be."*

The final major theme of 'problem-solving' emerged through the amalgamation of sub-themes in the data: 'skills', referring to the problem-solving skills and techniques the students encountered on the course; the 'problems' themselves; and references to the CoPs model utilised on the intervention. S1, S3, S5 and S6 believed the problems chosen were fundamental to the success of the intervention, because they did not specify how to approach each problem, and thus the group or individual had to create a plan to attack each problem, e.g.

> *S5: "when you realised there was a different way of thinking of the questions, that you weren't like 'oh yeah I know how to do that, I just do the same as I've done every other time', you realise 'oh I have to try to think of another way myself to figure out how to do it'. You weren't just told 'yeah that's the way to do it'. You didn't just know instantly."*

This was also of particular importance to S1 who expressed that "*it's not problem-solving if you're told how to solve the problem*" – something they believed to be an issue with problem-solving in school, where either the teacher or the textbook provide too much information to allow for authentic problem-solving.

While communication was previously discussed under the theme of 'group-work', it is a significant skill for collaborative problem-solving, which was noted in the interview by S2:

> *"…being able to communicate your thoughts to other people who are stubborn and who also think that they can solve the problem better than you can. So it's kinda like, you almost have to side develop the skills of communicating your thing as best you can and proving it as best you can in a limited amount of time"*

S4 reflected on the necessity in the intervention to develop one's conceptual understanding and communication of reasoning (Stage 5 of the CoPs Model), where

> *"there was more of an emphasis on the why, rather than getting an answer…We'd get the answer in our group and we'd give it to [the facilitator] and [they'd] just ask us why… and we'd have to just go away and figure out the why"*

Similarly, both S2 and S6 felt the diary entry forced them to look at problems in different ways, which expanded their understanding of the problems. The skill of extending the problems, previously mentioned under the theme of 'resilience', was highlighted by S1 and S6 as a valuable skill learned on the intervention that allowed them to continuously explore a problem in-depth (S6: "*There was, either it was generalising it, or you add an extra step to the problem, how does that change it. There was always something else that was there to… that could be played with the problem*").

There were two minor themes to also emerge from the interview transcript – 'school' and 'deeper learning'. Some of the instances where 'school' arose have already been discussed, where a student has outlined how the intervention differs to their school experience - in 'group-work' and 'problem-solving', as previously quoted by S1 under each respective theme; and in student motivation, quoted by S6 under the theme of 'mindset'. There were also four instances where a student mentioned how they apply the skills learned on the intervention to their school work. Of the seven occurrences of ‘deeper learning’:

- Three referenced a desire to continually strive for a deeper understanding of a problem
- Two called upon critical thinking in problem-solving
- Two were mentions of future self-directed learning influenced by attributes learned on the intervention.

5.8. Conclusion

This chapter presented the results collected over the duration of this doctoral study through the research methods that were outlined in Chapter 3. First, the quantitative data collected during the intervention via the mindset survey, MRS, and PTQ were displayed and analysed using appropriate statistical testing. Second, the results of the diary entries, which were the sole qualitative data collection method employed during the intervention, were analysed thematically. Lastly, the data collected post-intervention, through the intervention survey and focus group interview, was presented and analysed for themes. The research methods were selected such that the results collected may contribute to the Research Questions for this study. The following chapter will discuss each of these Research Questions, and draw upon the results where appropriate.

6. Discussion

The aim of this chapter is to discuss the results obtained in this research and how they combine to answer the research questions that arose over this project, previously outlined in Section 3.5. There are three major research questions to be addressed, with each further divided into sub-questions.

6.1. Research Question 1

Has the student sample showed an improvement in their Problem-solving Potential (PsP) over the duration of the intervention?

In order to effectively quantify changes in students' PsP, it is necessary to investigate changes in the individual aspects of the PsP – problem-solving skills, resilience, and mindset. Questions 1A to 1C address each individual aspect of the PsP, by first discussing the quantitative results obtained and any measured differences within them, and then discussing these in relation to students' perceptions as measured by the qualitative methods. Question 1D then discusses these aspects in connection to one another.

6.1.1. Research Question 1A

As previously discussed in Section 3.8.3, students' mindsets towards general intelligence and mathematical talent were tested, utilising a scale designed and extensively used by Carol Dweck (2000, 2006; 1988), as well as many other researchers (Blackwell et al., 2007; Esparza et al., 2014; Lee et al., 2012). In the context of this research, the students were introduced to the concept of a growth mindset in relation to general intelligence immediately after the pre-test survey, and characteristics of a growth mindset were encouraged throughout intervention participation. Recall that mindset scores were categorised as growth if inside the range 4-6; fixed if between 1 and 3; and neutral if falling between 3 and 4, exclusive (Lee et al., 2012). This section will discuss the analysis of students' results on each scale to determine potential measurable differences in their mindsets.

Has the student sample shown a measurable difference in their mindset towards general intelligence over the duration of the intervention?

As presented in Section 5.2, students' test scores on the Theories of Intelligence (TI) scale were found to increase significantly in an 8-session period (from an initial Round 1 mean of 4.2, with standard deviation 0.974, to a Round 2 mean of 4.387, with standard deviation 0.981). This echoes the findings of a similar-size study conducted by Blackwell et al (2007), who studied the changes in mindset in seventh-grade students (*n*=91) over an 8-week period, with students pre- and post-tested using the same TI scale utilised for this research (Appendix B). Their experimental group was exposed to growth-mindset-related material and lessons during the 8-week period and their mean score was found to increase significantly; whereas those of the control group, not introduced to the concept of growth mindsets, did not see a significant increase.

While our mean score increased further over the next 5-session period of this research (with a final Round 3 mean of 4.478, with standard deviation 1.021), the increase from Round 2 to Round 3 was not statistically significant. It should be noted that the period of time between Rounds 2 and 3 of testing was shorter than Rounds 1 and 2 (5 sessions instead of 8 sessions), a possible contributing factor. It may also simply be the case that most of the shift in students' mindsets towards general intelligence that was going to happen as a result of a singular, continuous intervention had happened by the time of Round 2 testing. In major studies into the effectiveness of mindset interventions (e.g. Aronson et al., 2001; Blackwell et al., 2007; Paunesku et al., 2015), there has been no research into the upper limit of sessions an intervention should be for maximum effect; conversely, Orosz, et al (2017) warned that interventions too short in nature (5 weeks) may produce short-term effects to mindsets that dissipate over time. We did not find this to be an issue with the students who studied the intervention over a 3-week period (as evidenced through opinions reported on the intervention survey and focus group interview), but it should be remembered that it was a highly intensive version of the intervention they studied, which would not be typical in many initiatives of this kind .

We previously reported, in Chapter 5, the number of students displaying growth, neutral or fixed mindsets after each round of testing. As illustrated in Figure 9, there was an increase in the number of students with a growth mindset round-on-round, and decreases in the student numbers of both neutral and fixed mindsets over the same periods. Delving further into these numbers, Figure 25 displays the number of individual students who moved between each mindset category between Rounds 1 and 3 of testing. The green directed (or outer) arrows signify a positive change towards a more growth mindset, while the red directed (or inner) arrows represent a movement towards a more fixed mindset; the black, looped arrows represent no change; and the labels accompanying each arrow show the number of students who moved between these categories.

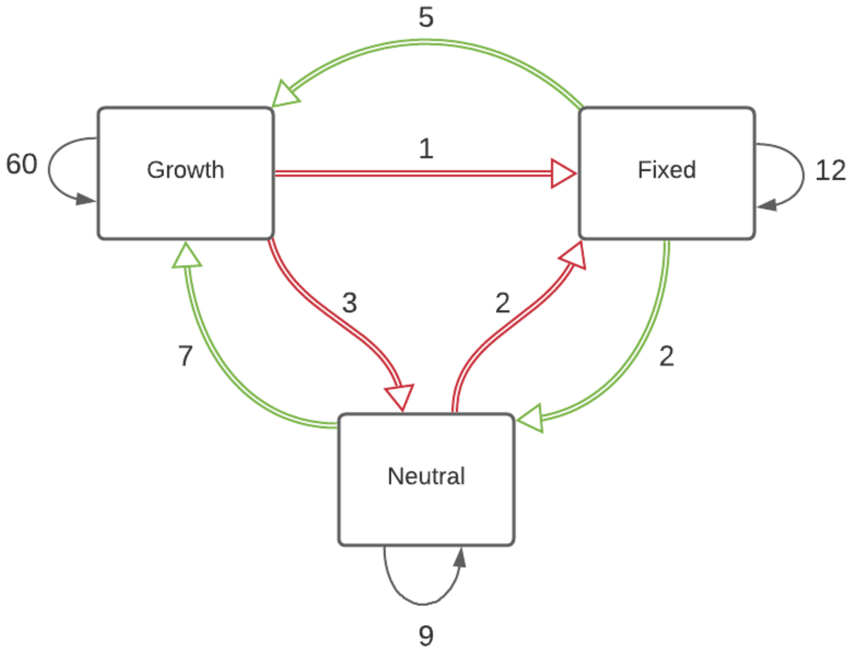

Figure 25 Student movement in Mindset (TI) categories

Of the 14 students who moved in a positive direction (fixed to neutral/growth; or neutral to growth) their mean score in Round 1 was 3.23, representing a neutral mindset; whilst their mean score in Round 3 was 4.572, representing a definitive growth mindset, as shown in Figure 26. In contrast to this, the six students who moved in a negative direction (growth to neutral/fixed; or neutral to fixed) had a mean score of 4.043 in Round 1, and 2.98 in Round 3. While this represents a movement from a growth mindset to a fixed mindset, the scores are only marginally inside each category band, and thus it may be possible that this group of students were overall tending towards a neutral mindset.

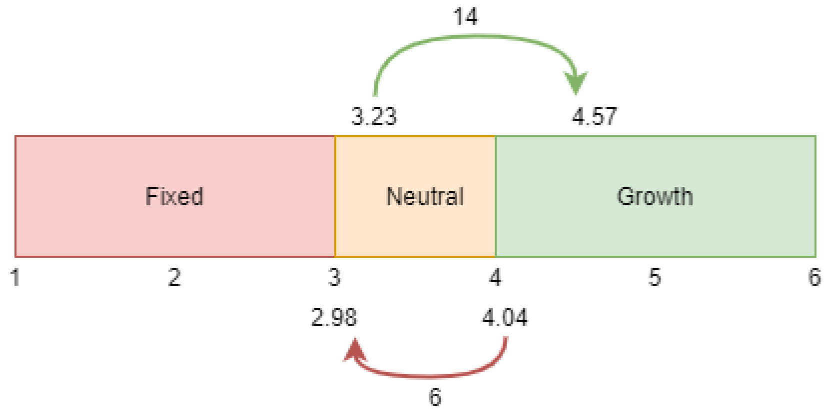

Figure 26 TI Scale: Positive & negative student movements

In summary, with an initial mindset mean score of 4.2 (standard deviation 0.974) in Round 1 testing and a final mindset mean score of 4.478 (standard deviation 1.021) in Round 3 testing, which is statistically significant at 95% confidence, it would seem that the student sample has shown a measurable difference in their mindset towards general intelligence over the duration of the intervention.

As has been highlighted throughout this work, the development of a growth mindset within the students on this research may have far-reaching benefits for them. While the intervention was focussed on development in a mathematics context, the increases discussed in this section suggest the impact of the intervention in a more general context also, and this may benefit their study of other subjects, or their lives outside school.

Has the student sample shown a measurable difference in their mindset towards mathematical skill over the duration of the intervention?

Students' mindsets towards mathematical talent (MT) were largely consistent with those of general intelligence (TI), as evidenced by the strong correlations displayed in Table 12. The mean score on the MT scale in Round 1 (4.291, with standard deviation 1.032) was slightly above that of the TI scale discussed in the previous section. Figure 27 outlines the overall student numbers that moved from one mindset category to another between rounds 1 and 3 of testing on the MT scale, with a large proportion of those with neutral mindsets in Round 1 moving to a growth mindset by the end of the intervention.

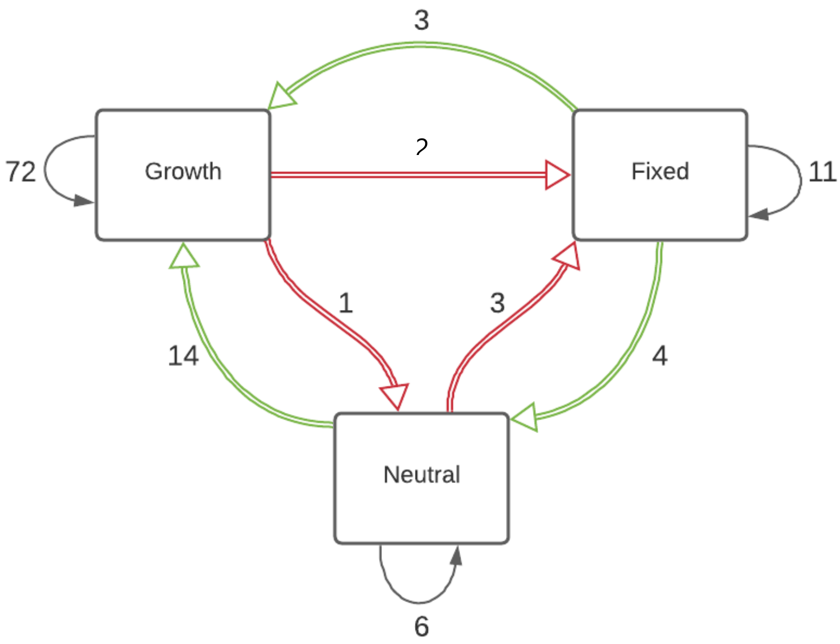

Figure 27 Student movement in Mindset (MT) categories

Of the 21 students who moved in a positive direction, they had a mean score of 3.179 in Round 1, and a mean score of 4.351 in Round 3, representing a movement from a neutral mindset to a definitive growth mindset, as shown in Figure 28. The five students to move in a negative direction decreased from a mean score of 3.876 in Round 1, to a mean score of 2.626 in Round 3. In contrast to the decreased scored on the TI scale, those on the MT scale scored more definitely as fixed mindset in Round 3.

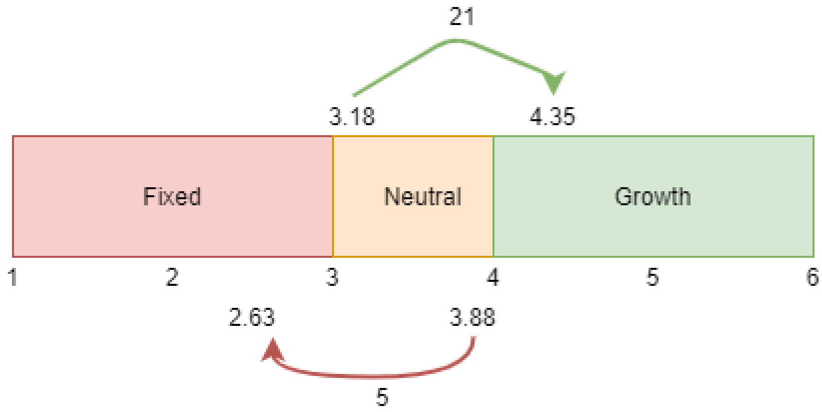

Figure 28 MT Scale: Positive & negative student movements

The results of the MT scale analysis were discussed in Section 5.2, outlining a statistically significant increase in the overall mean score from Round 1 (mean of 4.291, with standard deviation 1.032) to Round 3. Our findings echo those of Esparza et al (2014), who introduced gifted science students (n=80) to a mindset intervention for six weeks and found their overall scores on a science-specific mindset survey increased to a greater extent than those of a control group, although ours focused on mathematics rather than science.

In summation, the results of the paired t-tests, in combination with the analysis of students movement through mindset categories, indicates a measurable difference in students' mindsets towards mathematical talent. Willingham et al (2021) suggested that it was important to study mindsets towards mathematical talent, and to nurture attributes specifically to this domain, rather than just generically for general intelligence. While highly-able students may not be overly challenged by the Project Maths curriculum (McGrath, 2017; Shiel & Kelleher, 2017), it is necessary for us to prepare them to cope with future challenges they may encounter in mathematics, such as the study of highly specific and complex mathematics degrees at university. The development of a growth mindset towards mathematics is one way to do so, such that they value their mistakes when they make them; they understand that effort may be required in order to learn; and they develop goals related to learning rather than performance (Boaler, 2013). While the quantitative results suggest the intervention resulted in an improvement in the students' mindsets towards mathematical talent, we will now consider their opinions towards this, as gathered through the qualitative research methods.

Do the students perceive a change in their mindset after studying the intervention? Are these perceptions consistent with changes in students' mindsets identified by the research instruments?

As previously outlined in Section 5.2, of the 57 students to complete the intervention survey, 13 students believed their mindsets to be fixed prior to studying the intervention. However, only three students believed their mindsets to be fixed (or somewhat fixed) after studying the intervention, with 10 students recording their mindsets moving from fixed to growth (or somewhat growth). The number of students believing their mindset to be neutral also fell, from nine prior to studying the intervention to three afterwards. No student believed their mindset was negatively affected by studying the intervention, i.e. moving from growth to neutral or fixed, or moving from neutral to fixed. 38 (66.67%) students believed their mindsets improved; 15 (26.32%) of those who registered no improvement already believed their mindset to be growth. Of the remaining four (7.02%) students, two remained with fixed mindsets, and two remained with neutral mindsets. The largest proportion of students believed their mindsets were growth prior to studying the intervention (n=31), with this further increasing after studying the intervention (n=51).

The percentage of perceived change presented in the above paragraph (66.67%) is slightly above the percentage improvement on each of the two scales: 60.67% on the MT scale; and 56.18% on the TI scale. Table 36 displays the percentage of students from the intervention survey that identified within each mindset category, cross-tabulated with the percentage of students who were actually in each category based on their scores in each respective mindset scale in Rounds 1 and 3 of testing.

	Mindset	Survey %	TI %	MT %
Before	*Growth*	61.4	67.42	65.17
	Neutral	15.8	14.61	20.22
	Fixed	22.8	17.98	14.61
After	*Growth*	87.7	76.4	80.9
	Neutral	7	10.1	6.7
	Fixed	5.3	13.5	12.4

Table 36 Student percentages in each mindset category: intervention survey, TI, MT

Although this illustrates that the students somewhat over-estimated the level of change in their mindsets, due to the lower numbers that took part in the intervention survey, this difference is equivalent to approximately 4 students (7.02%) misinterpreting their mindsets.

During the focus group interview, no respondent believed their mindset to be fixed after studying the intervention, with four students expressing their belief that the intervention helped to improve their mindset. Two students also expanded upon this, and suggested that the experience of being stuck on the intervention, and being required to persist and learn ways to overcome this, helped to improve their mindset.

As the intervention survey was anonymous, we cannot link the responses back to students' mindset scores; however, this can be done with the focus group participants, and so we will now look at the isolated results of the students who participated in the focus group interview, in order to develop a comparison between their registered mindset scores and their opinions observed through the focus group. Of these eight students, seven registered as growth mindset towards general intelligence in Round 1 of testing while the remaining student was categorised as having a fixed mindset, albeit on the boundary between fixed and neutral, with a score of 3. By Round 3, all eight students were categorised as having a growth mindset towards general intelligence. The student moving from a fixed mindset to a growth mindset had a score increase of 1.875 (to a new score of 4.875), while the other seven students showed a minimal overall increase (Round 1 mean 4.536 to Round 3 mean 4.571). In Round 1, five of the eight students were categorised as having a growth mindset towards mathematical talent, with two students registering as a fixed mindset, and the remaining student as neutral. Whereas by Round 3, only one student was not categorised as growth mindset, as they moved from fixed to neutral.

These selected results coincide with the beliefs about mindsets expressed by the students during the focus group, whereby no student identified as having a fixed mindset by the end of the intervention, and none of these eight students were categorised as fixed mindset on any of the three scales in Round 3. Further to this, there were only two instances relating to fixed mindsets in the focus group transcript. One related to a fixed mindset towards English, which is beyond the scope of this research as it only addressed mindsets towards general intelligence and mathematical talent. The second reference outlined motivations within a group as a paramount to the success of group-work, whereby the over-reliance of the group on one particular member may lead to them feeling "*stupid*" if they failed to succeed in their work. This echoes the findings of prior research presented in Section 4.4.3, that highly-able students are more-willing participants within a group dynamic where they are working with like-minded and equally-motivated peers (Davis & Rimm, 1989; French et al., 2011). The effectiveness of group-work is maximised when the individuals of a group strive towards common goals or share equal motivation (Järvelä et al., 2010). The positivity towards the group-work on the intervention was also further enforced through the student diaries, where it emerged as a major theme. Conversely to this, and reflected in the student quote, where students in a mixed-level group hold varying levels of motivation to their goal orientation, there may be a negative impact on the more highly motivated or higher-cognitive student (Costley & Lange, 2018). Johnson & Johnson (2009) highlighted the role of the teacher or facilitator to help guide groups in their collective goal orientation, and thus subvert conflict within a mixed-motivation group. Within the context of this research, the role of the facilitator was outlined in-depth in Section 4.4.4, and included the task to "*ensure all students are involved*" in the problem-solving process. Furthermore, the 'communicate reasoning' stage of the CoPs model required all students to be able to explain the reasoning behind the groups solution of a problem.

Prior research on mindsets has indicated that the acceptance of mistakes as valuable for learning is an important characteristic of a growth mindset (Boaler, 2013; Yeager & Dweck, 2012). In the student diaries (Section 5.5), *mistakes* emerged as a major theme due to students reflecting on their mistakes within their diary entries. This is perhaps of greater note given that students wrote the reflections after completing the problem, and therefore could have omitted any mistakes. Their acknowledgement of the mistakes they made, some examples of which have been outlined previously, show characteristics of a growth mindset.

In summary, the students were quite accurate in their opinions of their mindsets and any perceived changes to them were quite consistent with the data collected through the mindset survey, and triangulated by the interview data.

Summary

It is clear from the overall mean scores on each scale that the student sample for this research presented with a collective growth mindset at the beginning of the intervention. However, both statistical analysis and an examination of student movement between the mindset categories for each scale indicate a general movement towards a stronger growth mindset amongst the students throughout the intervention. Of those presenting with fixed or neutral mindsets, there is a clear shift for the majority of students towards a growth mindset; and for those already presenting with a growth mindset, their scores suggest that it strengthened over the course of the intervention. While the paired t-tests for each scale appear to suggest that any overall change in mean score had already taken place by Round 2, the continued encouragement of growth mindset attributes for the duration of the intervention may help to achieve long-term benefits, and avoid the possible relapse associated with a shorter intervention (Orosz et al., 2017). The calculation of Cohen's d for all three scales suggested that the intervention yielded a 'modest' effect on students' scores. Furthermore, the majority of students perceived an improvement in their mindset also, as evidenced in the intervention survey and the focus group interview. As both of these measures were administered to students a minimum of three months after their completion of the intervention, and with over half of the respondents on the intervention survey being six months or more removed from their participation, it is evident that the improvements in mindsets achieved during the intervention were long-lasting. The TI scale was included for this research due to its prevalence in research over recent decades, with the related mathematics-based scale (MT) used less frequently. However, due to the high level of correlation and similarity between the data results collected, the MT scale will be discussed solely for the remainder of this work.

6.1.2. Research Question 1B

Has the student sample shown a measurable difference in their mathematical resilience over the duration of the intervention?

In order to assess students' mathematical resilience in this study, the MRS was used. In their original validation study, the scales of the MRS were discussed individually to assess whether each domain could be used as a predictor of a student's resilience in mathematics (Kooken et al., 2016). In that study, students who had chosen to study mathematics to a high degree at third level were found to score highly on the MRS, and in each individual domain. The *value* domain investigates "*the extent to which students find studying mathematics important*". It therefore would make sense for students who have chosen to study mathematics at third level to score highly in this domain. In the context of this research, a similar hypothesis exists. The participants for this intervention chose to focus on mathematics from a wide range of subjects, with some students on the 14-week programme travelling for over three hours for each class, and those on the 3-week programme devoting three weeks of their summer holidays to the study of mathematics. In this context, it is fair to assume that these students already placed a high value on the importance of studying mathematics. This is reflected in the mean score of 46.764 (with a standard deviation of 6.274) in Round 1, or 83.5% out of the possible score of 56. The Round 3 mean score of 46.787 (with a standard deviation of 6.445) indicates that the student sample maintained their high opinion in the study of mathematics over the duration of the intervention.

The *growth* domain of the MRS was designed to assess whether a student possessed a growth mindset towards mathematics (similar to that of the MT mindset scale presented in Section 5.2). The *growth* domain and the MT scale were positively correlated in all three rounds of testing with the student sample (*n*=89), with the correlation increasing from 0.597 in Round 1 to 0.67 in Round 3. Both instruments aim to measure the same characteristic within students; and as such, many of the comments from the mindset discussion in the previous section are also applicable here. Therefore, students' scores in the *struggle* domain will solely be considered for the remainder of this discussion.

Kooken et al (2016, p. 237) believed that the *struggle* domain represented an "*important dimension in identifying attributes*" of mathematical resilience, as it assessed how students responded when they were faced with challenges in mathematics. Of the 89 students presented in this research: 55 (61.8%) improved their score in the *struggle* domain; nine (10.1%) remained unchanged; and the remaining 25 decreased in score from Rounds 1 to 3. However, of these 25 students, four had achieved a maximum score of 63 in Round 1; a further four had scored 61 or 62; and these 25 students still had a mean score of 53.84 in Round 3, with a standard deviation of 5.5. This mean score suggests that, despite their decrease, this group of students still maintained a high level of mathematical resilience. In the total student sample, scores in this domain were found to increase to a statistically significant degree over the course of the intervention, from a Round 1 mean score of 54.663 (with a standard deviation of 5.246), to a Round 3 mean score of 56.843 (with a standard deviation of 4.815). Furthermore, the calculation of Cohen's d revealed a modest effect size between the rounds of testing (Table 20). While the validation study for the MRS did not outline specific scores, Kooken et al (2016) found that students studying STEM subjects in university scored highly in the *struggle* domain. In contrast to this, apprentices in STEM programmes in England were found to have scores on the *struggle* domain of 30.5, with standard deviation of 3.62, although anxiety towards mathematics was found to be prevalent within this group of apprentices (Johnston-Wilder et al., 2014). As questions in the *struggle* domain pertained to how students respond or feel about struggle, challenge and mistakes within mathematics, this was of significant relevance to this research. Over the course of the intervention, students were encouraged by their facilitator to continue working through challenging problems, and to develop strategies to implement when they became stuck.

The results obtained in the *struggle* domain presented the student sample as very mathematically resilient at the start of the intervention, yet the analysis still showed statistically significant increases over the course of the intervention, and a modest effect size on their scores also. STEM undergraduates were found to score highly on the *struggle* domain on the validation study, such that Kooken et al (2016) suggested that future research should collect qualitative data to further understand this domain, and the qualitative data obtained through the student sample for this research confirms this need. This research has collected said data, and will now discuss it, first as students' opinions of their change in resilience, and then in its relevance to the quantitative data collected.

Do the students perceive a change in their mathematical resilience after studying the intervention? Are these perceptions consistent with changes in students' mathematical resilience identified by the research instrument?

As presented in Section 5.7, *resilience* emerged as a major theme following a thematic analysis of the focus group data. During the focus group, six students became involved in an in-depth discussion about being "*stuck*" while attempting to solve problems on the intervention. While two students suggested that the problems selected were the reason for becoming stuck, two other students credited their potential for extension as the major source of struggle. They outlined how the introduction of an extension gave them the opportunity to "*make [each problem] as hard as you need it to be*". The 'extend' stage of the CoPs model (Section 3.7) was created for this precise purpose, to allow students to explore their creativity within mathematics, and to encourage them to challenge themselves further. The careful and deliberate selection of problems, as discussed in-depth in Chapter 4, was also important to the extension phase, as they needed to have the potential to be expanded upon by students (Mason et al., 2010)

It was also imperative that the intervention introduced students to strategies to employ when they did become stuck while problem-solving, and the 'stuck' discussion during the focus group revealed some of the strategies they had learned: they relied upon their group to explain the reasoning to them when 'stuck' in their individual understanding of a problem; they tried to "*just keep writing*" and attempted every approach that they could think of; they asked someone else who was working on the problem; or they started the problem "*afresh*". Johnston-Wilder et al (2010b) highlighted the development of strategies as an important characteristic of mathematical resilience.

Students who completed the intervention survey were also asked if they became stuck at any time on the intervention, with 54 of the 57 respondents admitting they had. When asked how they overcame this, 25 students mentioned group discussion; 21 believed a change of strategy helped them to became unstuck; and a further six students asked the facilitator for help. During individual problem-solving, Mason et al (2010) suggested several different ways to try to become unstuck, such as: searching for hidden assumptions; taking a break from the problem; further specialising; and more. On the intervention, two of these methods were introduced through themes, providing the students with the tools to help tackle later problems. The third, taking a break, was reflected upon by students on the intervention survey, and listed as a method of overcoming being stuck on a problem. The main principle for students during the moment of being stuck, however, is to acknowledge that they became stuck so that they may internalise it and seek a solution to the feeling (Johnston-Wilder & Lee, 2010b; Mason et al., 2010, p. 45). There was clear evidence of this acknowledgement from students, particularly through the intervention survey and the diaries. One of the three students who said they did not become stuck listed group-work as the reason for this, while the other two did not further elaborate. In comparison, when asked if they became stuck prior to the intervention, only 40 of the 57 students agreed, while the most common solution mentioned was to practice more exercises in the topic (n=10 students), and just eight students mentioned that they asked for help from someone.

In a separate question, students were also asked if they believed their resilience had increased after studying the intervention, and 47 students believed it had. As previously detailed in Section 5.6.3, fifteen of these students believed their resilience had improved as they had learned new strategies on the intervention to help them cope with challenging work. The problems on the intervention were also heralded as a reason for the increase in resilience, due to how time-intensive they often were and because they exposed students to the feeling of being stuck. Similarly, from the focus group discussion, when asked whether they believed being stuck on the intervention had helped them to cope with similar feelings afterwards, the six students involved in the discussion agreed with this statement. One student elaborated that he was now more likely to persist in the face of challenge, whereas before he would "*just drift away from the problem*"; and a second believed the intervention gave them "*more confidence that like you actually could solve a problem no matter how much time it would take*". This discussion was a clear indication that the students believed their resilience had improved over the course of the intervention, and indeed due to the intervention and the features it incorporated.

It is clear from the discussion of the results on the *struggle* domain that the student sample for this research presented, and maintained, a high level of mathematical resilience when measured by the MRS. While the paired t-tests found a significant increase in the students' collective resilience, and Cohen's d showed a modest effect size on resilience, the actual percentage of students who increased their score (65.47%) was much lower than the percentage of those who perceived to have increased their resilience (82.46%). However, as the scores on the *struggle* domain began at such a high point in Round 1, the discrepancy between perceived versus actual increase in resilience may be outside the scope of the instrument – something that was alluded to by Kooken et al (2016) when they suggested future research incorporated qualitative data collection. One particular aspect for consideration that is beyond the current scope of the instrument is the students' knowledge of strategies for coping with challenging work, which we will return to shortly.

'Across method' triangulation is the use of both quantitative and qualitative research methods to confirm findings. The analysis of the *struggle* domain results yielded statistically significant increases in students' mathematical resilience. However, we will now examine the qualitative data gathered through the student diaries to corroborate these findings.

As previously outlined in Section 5.5, both *resilience* and *stuck* emerged as major themes from the student diaries. Students' quotes were often coded under both themes where they highlighted how they became stuck on a problem but then persisted and found a resolution by "*realising*" or "*noticing*" something within the problem, e.g. "*We got stuck at this point for a long time, until I realised that…*". Mason et al (2010) believed that learning began when a student becomes stuck, and Johnston-Wilder et al (2010b) continued this sentiment and expressed that the mathematically resilient student recognises when they are stuck but continues to work through this by implementing different strategies they have learned. Thus far in this research we have seen the student sample not only recognise their feelings of being stuck through both the focus group and intervention survey, but also elaborate on the strategies they learned through the intervention to overcome this feeling. Further to this, the student diaries had been found to present strategies that students utilised as a means of overcoming struggle during their problem-solving: group discussion; starting again using a different approach; and asking for help. These are consistent with the strategies mentioned by students in the post-intervention qualitative methods.

Another characteristic of mathematically resilient students is the ability to "*discuss and question their mathematical ideas*", leading to an enhanced ability to "work collaboratively" (Johnston-Wilder & Lee, 2010b). Despite this, there is no question on the MRS that assesses a student's ability to discuss mathematics or their ability to work as part of a group. As has been discussed at length in this research, through the CoPs model, students on the intervention were required to discuss their workings and communicate their reasoning for each problem they encountered. Evidently, this was a successful feature of the intervention as the positive response to group-work was a major theme to emerge from the student diaries. Further to this, group-work was identified by students in the intervention survey as a new skill learned, but also as the leading cause of enjoyment on the intervention. *Group-work* was also one of the major themes of the focus group interview, in which the students believed it was fundamental to their problem-solving on the intervention. In Section 4.5.5, the theme of 'assumptions and questioning' was discussed, and the importance of 'questioning' to problem-solving was highlighted. The development of students' questioning of their solution, and their desire to seek an understanding of that solution or alternatives, was evident in the student diaries (as the theme of *understanding*), the intervention survey (as they found long-term benefit to a new style of thinking), and in the focus group interview (in the minor theme of *deeper learning*).

In summary, the analysis of the results in the *struggle* domain suggest that a statistically significant increase occurred in students' mathematical resilience over the course of the intervention. Although this increase is significantly below the perceived increase by students, as registered in the intervention survey, the reason for this appears to be due to the increase in students' mathematical resilience occurring in areas beyond the scope of the research instrument. In particular, students improved their knowledge of strategies for coping with challenging work, and their ability to discuss and question mathematics. While these were specified as important characteristics of mathematically resilient students, they were not assessed by the MRS.

6.1.3. Research Question 1C

What categories of the problem-solving grading rubric, if any, has the student sample improved in?

The problem-solving grading rubric (Appendix D) is comprised of 5 categories – making sense of the task; solving the task; communication of reasoning; accuracy; and reflecting and evaluating – with a maximum score of 6 in each category. In Section 5.4, students' results on the Problem-solving Test Question (PTQ), calculated using the rubric, were presented and analysed as an overall score and also as individual scores for each of the five categories. Paired t-tests were used to determine whether statistically significant differences had occurred in students' scores between the rounds of testing, and the null hypothesis (H_0 = no difference between the means) was rejected in all cases, indicating that a statistically significant increase in students' scores in all categories had occurred over the course of the intervention. Cohen's d was also calculated to determine the effect size of the intervention on students' scores, with each test returning a 'strong' effect size (Cohen et al., 2007) for improved problem-solving abilities. Past research has suggested numerous benefits to an improved ability to problem-solving (Lester Jr, 1994; NCCA, 2013; Oldham & Close, 2009; Wearne et al., 1996), such as: thinking critically; relational understanding of mathematics; encourages creativity; increased reflective practice; better self-regulation and self-monitoring; greater estimation abilities; and many more.

Of the 89 students to complete the PTQ for all rounds of testing: 70 students increased their score, with a mean increase of 5.314; 11 decreased their score, with a mean decrease of 2.091; and eight remained unchanged. 'Making sense of the task' was the highest scoring category in Round 1, and the only category to achieve a Round 1 modal score of 3, which indicated that the largest proportion of students were able to make a good attempt at interpreting the task and translating it into mathematics. In contrast, 'reflecting & evaluating' was the lowest scoring category in Round 1, with a modal score of 1. The other three categories had modal scores of 2, meaning a large proportion of students had shown underdeveloped workings under the criteria of each respective category. However, these categories registered the three highest mean increases in scores by Round 3.

Students were encouraged to communicate their reasoning throughout the intervention, either in discussion with their group or the facilitator, or through their diary entries. In Round 1, a majority of students (n=60) showed underdeveloped or minimal communication within their PTQ attempt; whereas this dropped to 19 students by Round 3, with more students (n=57) increasing their score in this category than in any other.

Although students may have had some awareness of the answer to the problem in Round 2 or 3, due to the same problem being used for all rounds of testing, they were required to provide mathematically accurate and justified work to receive credit for a correct answer. In the 'accuracy' category, a correct answer without supporting work gained a score of 1. This category had the greatest mean increase (1.067) over the course of the intervention.

'Reflecting and evaluating' was the lowest scoring category through all three rounds of testing in a variety of different ways: it maintained the lowest mean (Round 1 = 1.393, Round 3 = 2.022) and mode (Round 1 = 1, Round 3 = 2) of any category; it registered the lowest mean increase of any category (0.629); and it had the fewest number of students to increase their score (n=40) between Rounds 1 and 3 of testing. To score above a 2 in this category, students had to provide a reflection at the end of their solution and meet certain criteria within this reflection. This was only produced by 24 students in Round 3, increased from five students in Round 1. While the paired t-test and Cohen's d show statistically that the student sample improved in this category, it appears that this improvement is minimal, but also that it is only evident for a small number of students.

Do they perceive a change in their problem-solving skills after studying the intervention? Are these perceptions consistent with changes in students' problem-solving skills identified by the research instrument?

Thus far in this research, we have identified and discussed many different skills of mathematical problem-solving, such as strategy-selection, extension, reflection, communication, and so on. At no point have we attempted to create an exhaustive list of these skills due to the abundance and variety that exist in research. In this section, we will discuss any change the student sample has perceived in the skills previously identified, and in any others identified by the students through the intervention survey or focus group interview.

As previously displayed in Figure 23, when asked via the intervention survey if they had learned new skills on the intervention, the most common open-response (n=23) from students was that they had learned new "*problem-solving strategies*", with twelve of these further detailing exact strategies that were introduced through the sessional themes on the intervention (Section 4.4.1). The selection of strategies to employ in order to attempt to solve a problem is fundamental to problem-solving (Mason et al., 2010; Polya, 1945), as are well-chosen problems that do not make it immediately obvious to the problem-solver as to what strategy should be used (Krulik & Rudnick, 1989). Therefore, for authentic problem-solving, the problem-solver must develop their strategy knowledge to utilise so they may attack a given problem in a variety of ways. This was emphasised by two students on the focus group interview, with one student declaring that "*it's not problem-solving if you're told how to solve the problem*".

Further references to 'problem-solving skills' or 'techniques' were widespread throughout the intervention survey. Twelve students believed the development of their problem-solving skills was a principle reason for their enjoyment of the intervention, while 11 students outlined "*problem-solving skills*" as newly-learned skills on the intervention. Although many of these were mentions of the generic phrase, there were also specific mentions of extending problems, reflection and critical thinking as newly learned problem-solving skills. Similarly, participants of the focus group highlighted each of these as newly-learned skills from the intervention, with particular emphasis placed on the use of extensions to provide challenge to themselves within mathematics.

During the focus group, a participant expressed their belief that the intervention required them to "*side develop*" their communication skills due to the emphasis placed on group-work. Another student emphasised how group-work led to them "*constantly explaining ideas*" and a "*constant… dialogue back and forth*" within the group, which emphasises their desire within the group to communicate their reasoning to one another. Whilst five respondents on the intervention survey listed *communication* as a new skill they had learned, this was put in context by one student who expressed that it was the communication of their "*processes*" that they had improved upon.

All 57 students to complete the intervention survey believed the intervention would benefit their study of mathematics in school. Thirteen respondents further expressed the belief that their participation on the intervention would be of benefit due to the problem-solving skills they learned and, in particular, the strategies they learned. Certain individual responses also referenced an increased ability in "*critical thinking*" and to "*think mathematically*" as beneficial effects of the intervention.

In Section 6.1.2, quantitative and qualitative research methods were utilised by 'across triangulation' to further validate research findings. Similarly, we will now discuss whether students' perceived changes in problem-solving skills are consistent with changes measured through the PTQ, but we will also utilise the student diaries for triangulation of the data.

It was evident in the data collected post-intervention that the student sample believed they had improved in their knowledge of problem-solving strategies. Prior research has indicated that problem-solving strategies may be uncovered and learned through the act of problem-solving (Mason et al., 2010; Polya, 1945; Posamentier & Krulik, 2015; Schoenfeld, 1992), and thus it is reasonable to assume that the student sample have indeed increased their knowledge of problem-solving strategies. The category 'solving the task' on the PTQ awarded marks for the selection of an effective strategy in solving the problem. In Round 1 a majority of students (n=50) utilised "*underdeveloped*" strategies in their attempts to solve the PTQ. This category witnessed significant improvement in Round 3, with the largest proportions of students implementing "*complete*" (n=34) or "*partially complete*" (n=33) strategies. Statistical analysis also further evidenced an improvement in this category, with the paired t-test rejecting the null hypothesis and Cohen's d showing a 'strong' effect size. *Problem-solving strategies* was also, by a large margin, the most prevalent theme in the student diaries, where students reflected upon the individual or combined strategies they utilised in solving problems. Problem-solving strategies were given prominence in the intervention through their role as sessional themes, and it is evident that this resulted in an improvement of students' knowledge of these strategies.

The CoPs model designed for this research placed an importance on students' ability to problem-solve collaboratively. The importance of this was reflected in the post-intervention data collection, through which the positive response to group-work emerged as a major theme in the focus group and in two separate questions of the intervention survey (enjoyment and learned skills). One fundamental aspect of the implementation of the CoPs model was the encouragement for students to understand their processes and communicate them accurately with their group, the facilitator or as a written diary entry. Previously in this section it was reported that the 'communicating reasoning' category had the highest number of students with increased scores. An increase in the ability of the student sample to communicate their reasoning is also evident through the results of the paired t-test and Cohen's d for this category. The PTQ allowed the students to show a measurable improvement in their communication of reasoning; however, it is through the qualitative methods that this improvement is further evident. Throughout the student diaries, there is evidence of the communication of reasoning, or the explanation of a process, by a student (either by the diary entrant or a group member) that has led to an increased understanding of a problem or towards a solution for a problem, e.g. "*As a group we worked well through communication and explanation. We were each able to explain our thoughts and opinions to one another. This allowed for a faster understanding of the given question".* Both *understanding* and *explain* also emerged as themes in the student diary analysis due to the students' reflecting on the many instances of communicated reasoning within their group.

In both the intervention survey and focus group, students identified the extension of problems as a valuable skill learned on the intervention. However, this was not evident in the PTQ. Only eight students created an extension to the problem, which resulted in a score of 5 or 6 in the 'accuracy' category. No student had achieved either mark in this category in Round 1. This does not appear entirely reflective of students' ability to extend problems, however, as the *extension* of problems was the second most prevalent theme in the student diaries. There was also a degree of consistency between the diaries and the opinions in the post-intervention data, where students utilised their extension to provide further challenge for themselves, as seen in the following quotes from a student diary and the focus group, respectively: "*I liked this question as it was simple, yet we could go much* *further in our extensions*"; "*the fact that you were able to do an extension means that, even if you are finding it easy, you make it as hard as you need it to be*". Due to the prevalence of problem extensions in the diaries, and the student opinions on extensions in the intervention survey and focus group, it would appear that many of the students did not attempt to extend the PTQ due to the test-scenario in which it was used.

The development of "*critical thinking*" or "*thinking mathematically*" skills is somewhat more nuanced than those skills previously discussed. These improved skills are self-reported by students and, due partly to the numerous definitions in research for both, cannot be further clarified at this point.

6.1.4. Research Question 1D

What proportion of the student sample have shown a measurable increase in their Problem-solving Potential over the duration of the intervention?

A student's (PsP) has been defined for this research as a construct affected by a combination of their knowledge of problem-solving skills, their mindset, and their mathematical resilience. Prior research has hypothesised that the act of labelling a student as 'highly-able' may cultivate a fixed mindset within them, such that they value the feeling of looking smart rather than learning (Dweck, 2007). Furthermore, those with a fixed mindset may possess characteristics that lead them to give up when faced with challenging work, rather than persisting when they struggle (Blackwell et al., 2007). However, a majority of the student participants for this research presented at the start of the intervention with growth mindsets (n=58) and high mathematical resilience (mean score of 54.663, from a possible 63), as measured by the relevant research methods, and thus they had less scope for change within these factors.

We have thus far analysed and discussed the students' results for each of the three factors independently, and will now consider whether, for each individual student, their scores increased, decreased or remained unchanged in the PTQ, the MT scale, and the *struggle* domain of the MRS, to gain an insight into the effectiveness of the intervention for increasing PsP. As outlined in Section 3.4, it is not necessary to improve in all three aspects in order to improve one's PsP. In fact, an improvement in any of the three factors results in a student's increased PsP. In some instances, students decreased their scores in one or two factors of PsP, but improved in the third, resulting in an improvement in their PsP. In this section, these instances will be discussed, and possible reasons for score decreases offered. Table 37 displays the number of students to increase in the three aspects of PsP:

MT	Struggle	PTQ	Total Students
		✓	12
	✓		7
✓			3
✓	✓		6
✓		✓	16
	✓	✓	13
✓	✓	✓	29
55	55	70	86

Table 37 PsP Factor Increases. An increase in a factor is represented by a tick.

As seen above, of the 89 students presented in this research, all but three increased their score in at least one of the three factors, meaning that 97% of students in the sample increased their PsP by the end of the intervention. A third of students increased their scores on all three test measures over the duration of the intervention. In Round 1 of testing, these 29 students displayed scores consistent with those of the overall student sample, including a collective growth mindset and high mathematical resilience. We will now consider each of the other combinations of improvement within the PsP.

We have already discussed, in Section 6.1.2, the possibility that changes may have occurred in mathematical resilience that were beyond the scope of the research instrument, but which became apparent through triangulation with the qualitative data, as was suggested by the validation study of the MRS (Kooken et al., 2016). Sixteen students increased their scores on the MT scale and the PTQ, but failed to do so in the *struggle* domain, with a Round 1 mean score for these students on the *struggle* domain of 56.125 (standard deviation 4.768), which was slightly above that of the student sample. A score of 56 or greater equates to selecting 'agree' for every question on the *struggle* domain. A similar situation can be seen in that of the 12 students who increased in just the PTQ, where the mean score of these students on the *struggle* domain in Round 1 was 58.25, well above the mean score of the student sample. Ten of these 12 students began and finished the intervention with a growth mindset, with only two finishing with a fixed mindset. These scores indicate students with growth mindsets and high mathematical resilience prior to their participation on the intervention increased their problem-solving skills during the intervention, representing an increase in their PsP.

We now consider those who did not increase their MT score. The mean score on the MT scale was found to increase over the course of the intervention, reflecting findings from other research (Esparza et al., 2014; Mofield & Parker Peters, 2018); however, a collective increase does not indicate that every student increased their score. 13 students increased their scores on the PTQ and on the *struggle* domain during the intervention, but not on the MT scale. These 13 students began the intervention with high mathematical resilience, as evidenced by a Round 1 mean score of 55, but still registered an increase by all 13 students to a Round 3 mean of 59.154. 11 of these students began the intervention with a growth mindset, with 9 maintaining their growth mindset throughout, and two moving to a fixed mindset, albeit both with a score on the upper limit of a fixed mindset (score = 3). The remaining two students began with and maintained a fixed mindset throughout their participation. The high variability amongst students' scores has been recognised in other prior research (Mofield & Parker Peters, 2018), and remains consistent with our findings. In addition, seven students increased their scores only in the *struggle* domain. Two of these moved from neutral to fixed mindsets, and also finished the intervention with scores of 6 and 7 on the PTQ, which equates to "*minimal*" workings in most of the rubric categories. Meanwhile, four other students maintained their growth mindsets throughout the intervention, despite minor decreases, with no student scoring less than 10 on the PTQ in Round 3.

In terms of those who did not register an increase on the PTQ, only six students combined an increase in both the *struggle* domain and the MT scale, yet failed to increase on the PTQ, although these six students registered a mean score of 14.667 in Round 1, above the student sample mean of 10.843. Three of these students moved from neutral mindsets to growth, while two more strengthened their growth mindset. The final student maintained a fixed mindset, despite it increasing slightly, but showed a greater increase on the *struggle* domain than any other student in the sample, improving from 37, the lowest score in Round 1, to 54 in Round 3. This student may be an example of those mentioned by Kooken et al (2016), who possess high ability in mathematics, but whose progress is hindered by negative affective traits. Many of the other lowest-scoring students from Round 1 also registered large improvements, showing that, while the majority of the student sample may have started with and maintained high mathematical resilience, there was also a group whose progress in mathematics may have been at risk due to their own beliefs. A further three students increased their scores on the MT scale only, with two of these students' scores on the PTQ decreasing by 1 mark, to 12 and 10, and one student's remaining unchanged at 9. Each of these three students decreased their scores on the *struggle* domain, but finished the intervention with growth mindsets, with one student moving from neutral to growth.

Finally, only three students failed to increase their score in any of the three factors. Therefore, when measured strictly by increases in each of the quantitative methods, these are the only three students to not improve their PsP. Therefore, it has been shown that 86 out of the 89 students on this research increased their scores in one or more of the research methods, and thus improved their PsP.

Research Question 1 Summary

The intervention for this research was designed as the conduit to impart upon the students the benefits of being mathematically resilient, possessing a growth mindset and developing a strong level of knowledge of skills for problem-solving, with the overall aim of improving their PsP. While the students presented with high scores in mindsets (similar to those presented by Esparza et al (2014)) and mathematical resilience (compared to scores of STEM apprentices (Johnston-Wilder et al., 2014)), the quantitative data collected suggests that their scores still showed an increase over the course of the intervention through the varying combinations of the factors of PsP, and further evidence beyond the scope of the research instruments, collected via the qualitative methods, corroborates this. The improvement in problem-solving skills was also evident through the PTQ, but more substantially so in the qualitative methods, through which some students also attributed their increased resilience to their increased knowledge of problem-solving skills. The diaries showed that, in the face of challenging work on the intervention – often due to their own developed extension – the students utilised the problem-solving strategies they had learned; communication skills to discuss the problem with their group; and other strategies they had devised due to the experience of being stuck already on the intervention. The author does not suggest that these are the sole contributors to a person's potential to problem solve, but that they are pivotal.

As discussed earlier in this work, potential is developed through four levels of complexity (Dai, 2017). The first three levels are intrinsic within the individual, through which traits and abilities related to each factor of the PsP may act. The final level, however, represents the influence of external factors upon these first three levels, where supports, resources and tools are utilised to develop traits and abilities at each of the other levels (Dai, 2020). For this research, the intervention was designed to act as this fourth level of complexity, and to influence any development of problem-solving skills, mindsets or mathematical resilience in the student sample in order to lead to an overall improvement in their potential.

Building upon the prior research of others outlined throughout this work, and as further evidenced and discussed in Chapters 5 and 6, the author suggests that an improvement in a person's PsP, and subsequently their ability to engage productively with problems, is the result of an improvement in any combination of: developing knowledge of problem-solving skills, cultivating a growth mindset, and displaying resilience when faced with challenging mathematics. Furthermore, based on the discussion throughout this Research Question, it is obvious that the intervention for this research was successful in improving the PsP amongst the student sample.

6.2. Research Question 2

What opinions do the student sample have about the intervention after completing it?

Post-intervention qualitative data was collected via the intervention survey and the focus group interview. Through the analysis of this data, certain opinions of the students were evident, as presented previously in Sections 5.6 and 5.7. The students' opinions with regards to changes in their mindset, resilience and problem-solving have already been discussed in this chapter. The following two subsections address research questions that emerged following the analysis of the post-intervention data.

6.2.1. Research Question 2A

Do they perceive of any long-term benefits to undertaking the intervention?

In both the intervention survey and the focus group, the students were asked if they believed the intervention would have long-term benefits to their study of mathematics, or their lives beyond it. There were two themes of equal frequency (n=13) as the most common open-responses on the intervention survey: problem-solving strategies; and thinking differently about mathematics. With regard to the former, students in particular highlighted the strategies they had been introduced to on the intervention. The importance of problem-solving strategies has been repeated throughout (Mason et al., 2010; Polya, 1945; Posamentier & Krulik, 2015); and also has been at the forefront of the students' opinions through the different research instruments. It is therefore no surprise that they believe it to be of long-term benefit to their study. Those on the focus group expanded upon the benefit to a strong knowledge of strategies, by addressing their role in resilience towards challenging work, e.g.:

> *"I think the programme definitely did like give you more confidence that like you actually could solve a problem like no matter how much time it would take. And it would sorta give you the motivation to apply more and more different angles of thinking about it and more and more techniques because you sorta were like really, really driven to get the answer to the problem and understand it"*

An increased resilience was also further echoed on the intervention survey, with seven students identifying this as a benefit from their participation on the intervention.

Thirteen students from the intervention survey believed the different manner of thinking they had encountered on the intervention would be of long-term benefit to them. Some further elaborations on this were mentions of "*critical thinking*" and "*thinking mathematically*". Similarly, those on the focus group discussed the "new" aspects of mathematics they encountered on the intervention, such as extension, reflection and being asked 'why' a solution worked. Webster & Willett (2019) lauded the benefits of reflective diary writing in the teaching of critical thinking skills, and thus the diary entries for this research may have contributed towards students' perceived improvement in this area. Furthermore, the communication of reasoning embedded in the CoPs model encouraged students to reflect on their work, thus encouraging critical thinking.

As the students participating in focus group and the online version of the intervention survey were a minimum of three months removed from their study of the intervention, they had returned to school, with most attending fifth or sixth year at the time of completion of either of these methods. Due to this, there are occurrences where students outlined where they had already noticed a benefit to their participation on the intervention. The following is a non-exhaustive list of some of these benefits (previously presented in Sections 5.6 and 5.7):

- Growth mindset in mathematics extends beyond its domain:

> *"After the maths programme… I decided to take back up piano… because I was like I might as well go for it... In the sense of piano it's just like you have to just choose to believe that if you put the effort into it you'll get it… I feel like the maths portrayed that eloquently because I feel that if you believed that you could solve the problem most of the time you could… just use as many different methods as you could… And so that does apply outside of maths, outside of school even"*

- Learn to seek an understanding

> *"The module also helped me to see maths as less of a do this, do that subject, but to develop a better understanding of maths in general"*

- Different ways of thinking

> *"The lateral thinking approach employed in the module has been very useful in solving the trickier problems in higher level."*

6.2.2. Research Question 2B

Is there evidence to suggest that the CoPs model was beneficial to the student samples experiences on the intervention?

The rationale for the development of the CoPs model was outlined at length in Section 3.7. During the analysis of the post-intervention data, it became clear that the process involved in the CoPs model had an effect on the students, and on their opinions of the intervention. The model was designed to adapt other problem-solving processes, such as those of Polya (1945) and Mason et al (2010), to a group-work environment. The positive response to group-work emerged as a major theme in the focus group, and two separate questions of the intervention survey; whilst also featuring extensively in the student diaries. The students were appreciative that group-work allowed them to discuss their ideas, and to learn from the other members of the group. The notion that highly-able students may prefer to work alone was previously addressed in Section 4.4.3, with prior research outlining a number of conditions that may strengthen their willingness to engage in group-work. Many of these conditions were met through the student cohorts participation on the intervention for this research, such as: working with like-minded peers (Davis & Rimm, 1989); open-ended problems (French et al., 2011); and their work being appreciated by their peers or the teacher (Neber et al., 2010). Each of these were evident in the qualitative data presented in Chapter 5, with students on the focus group drawing particular attention to the influence of working with like-minded and equally motivated peers to the overall positive result of group-work. Positive responses to the problems on the intervention were found throughout the diary entries, and also in the focus group. The final condition may be summed up by the final quote in the *group-work* theme in the diaries (Section 5.5), where the student acknowledged that the improvement in their communication of reasoning was brought about by the group's need for their workings.

When the author introduced each stage of the model earlier in this research, they outlined a list of processes they believed might occur at each stage, and many of these were confirmed through the qualitative data collected. For example, it was speculated that the 'discuss' stage may help to alleviate students misconceptions of a problem, and this proved to be true, e.g. "*After reading the problem at first I was confused by the question but my group helped me to understand better what was being looked for in the problem*".

A student on the focus group reported that being asked to explain "*why*" their solution worked for a problem was a new and somewhat confusing experience, but further elaborated how "*it made it easier to understand*". The 'critique' stage of the model focusses on the development of students' self-regulation, and this development is encouraged by the facilitator's questioning. Mason et al (2010) defined this as convincing "*an enemy*" and an important step in developing internal questioning. The recurrence of "*why*" in the student diaries also led to *understanding* emerging as a theme during their analysis. The diaries also gave an insight into how students moved between the 'conjecture' and 'critique' stages to attempt and evaluate different strategies.

The 'communicating reasoning' stage required students to be able to explain their own work but also the overall group solution. Kostos & Shin (2010) highlighted mathematical communication, both orally and in writing, as having ever-growing importance in mathematics education. There were many instances already discussed in this research where students outlined their improvement at communicating their own work, but there were also others, elaborated upon under the theme *explain* in the diaries, where students required a group member to explain their part of the solution in order to develop a full group understanding. This is known as peer-teaching, in which students must develop their understanding to the extent that they are comfortable relaying their work to others, and has been found to increase students' motivation in mathematics (Lim, 2014).

It has previously been discussed that the students of the focus group found the creation of their own extensions beneficial as it allowed them to alter a problem to challenge themselves. Sheffield (2009) outlined the use of problem-solving and problem-posing in mathematics to develop students' creativity, and Leiken & Lev (2013) found a strong relationship between high ability and creativity in mathematics. Thus, when presented with the opportunity and encouragement to extend mathematical problems through the 'extend' stage, it may not be surprising that the students on this intervention flourished, and found the practice beneficial. Furthermore, highly-able students have educational needs beyond those of regular students due to the speed at which they may progress through their work. Students often found complete solutions in vastly different timeframes on the intervention, but the opportunity to create extensions meant those who finished quickly were able to create a new problem and thus restart the problem-solving process.

Research Question 2 Summary

This research question was created towards the introduction of the post-intervention research methods, as it became possible to gather data to explore students' opinions of the intervention. Throughout the qualitative data the students' opinions of the intervention and its features were universally positive, as presented in Chapter 5 and discussed in the sub-questions 2A and 2B. The students believed that their participation on the intervention resulted in long-term benefits to their study of mathematics, with some of these benefits already transpiring as they progress through the Senior Cycle in school. Further, the data collected through the intervention survey and focus group contained student quotes that alluded to aspects of the CoPs model introduced through this research that aided in their problem-solving attempts, and thus the model was beneficial to their study.

6.3. Research Question 3

Are there statistically significant differences in the results of the student samples between different variations of the intervention?

As was outlined in detail in Section 3.8.1, there were two variables in the intervention format over the duration of this research. The first was the time layout, discussed in detail in Section 4.3, whereby students had the option to study the intervention as either a 3-week or 14-week programme. Cohorts

2 and 5 studied the 3-week variation. Students on both programmes studied the same material as any other cohort in the same iteration of the intervention – e.g. both Cohort 1 and 2 were in Iteration 1, and therefore studied the same material despite being two different time variations. For the purposes of this research, the data collected through the research instruments for both time layouts will be discussed comparatively in order to ascertain if one layout is more effective than the other.

The second variable in the intervention was the facilitator with whom students studied – the author or the independent facilitator. The results obtained under each facilitator will be compared and discussed in Section 6.3.2 as a measure of external validity (Calder et al., 1982) for the effectiveness of the intervention.

6.3.1. Research Question 3A

To what extent, if any, are results different between the student-sample to have taken part in the 14-week intervention compared to those who studied the 3-week intervention?

Students of Cohorts 1, 3, 4 and 6 participated on the 14-week intervention (n=63), which operated one day per week for 14 weeks. Students of Cohorts 2 and 5 attended the 3-week intervention (n=26), which operated for 3 weeks – 5 days per week for two weeks, and 4 days for the third week. As previously mentioned, the data collection process was consistent for each time variation, and the same test instruments were utilised for both variations.

The results collected through the mindset survey, the MRS and the PTQ were each presented as a comparison between the time variations in Sections 5.2.1, 5.3.1 and 5.4.1 respectively. For the purposes of statistical comparison, a Kruskal-Wallis test was calculated for each test instrument to investigate if there was a statistically significant difference between the medians in any round of testing. With regards to each of the mindset scales, the statistical test failed to reject the null hypothesis. This indicates that there was no statistically significant difference between the data sets in any of the rounds of testing, and therefore it may be assumed that the data sets had a similar starting point and increased at the same rate over the course of the intervention. The Kruskal-Wallis tests for the *struggle* domain in the MRS also failed to reject the null hypothesis in all rounds of testing.

In the PTQ, a statistically significant difference was found to exist between the Round 1 data sets for *communicating reasoning*. However, as noted in Section 5.4.1, as the tests for Rounds 2 and 3 failed to reject the null hypothesis, it is reasonable to assume that the difference had been reduced by Round 2, and subsequent increases in both data sets were similar in Round 3. Such an explanation, however, could not be offered for the rejected null hypotheses for the Round 3 results in *accuracy* and *reflection & evaluation*. The mean score for students of the 14-week variation in the *accuracy* category was lower in Round 1 than that of the 3-week variation – indicating that those on the 3-week variation used more accurate mathematics on their first attempt at the PTQ – and also appeared to have a lower mean increase. The same situation occurred in *reflecting & evaluating*, albeit this time with the students of the 3-week variation producing a lower mean score, and lower mean increase. In this scenario, the author suggests a hypothesis that the skill of written reflection may require a longer period of implementation to produce sustained and obvious improvement. However, this remains a hypothesis as the data collected for this research could not support or refute the claim. Upon the review of literature (Hensberry & Jacobbe, 2012; Jurdak & Zein, 1998), the author found no study that investigated diary writing in mathematics that had duration of less than 6 weeks. There is no obvious reason for the difference in the *accuracy* category. Despite these results in individual categories, the tests calculated for the PTQ overall failed to reject the null hypothesis for all three rounds of testing.

As no statistically significant differences were found in the mindset scales, the PTQ, and the *struggle* domain, it is reasonable to assume there was no significant difference in students' results regardless of the time variant through which they studied the intervention.

6.3.2. Research Question 3B

Does the use of an independent facilitator have any measurable difference on the student sample results in comparison with those who studied under the author?

External validity is concerned with measures of generalisability of research, whereby the variation of "*measures, persons, settings and times*" are used to ensure research findings are accurate (Calder et al., 1982). While this research had already achieved a certain degree of external validity through the implementation of the intervention across six different cohorts of students over a time-span of three years, and with the application of the intervention over two different time variants (further discussed in Section 6.3.2), the introduction of the independent facilitator brought with it one further aspect of external validity. As the author of this research was also the principle designer of the intervention, they were familiar with the materials and methods of the intervention. The degree to which the intervention was transferable to a more generalised setting, in which the instructor and the participants had no prior knowledge of the research, could be assessed with the use of the independent facilitator. Thus, this section will examine any potential differences between the quantitative results of students who studied with the author (n=*70*) versus those who studied with the independent facilitator (n=*19*).

The independent facilitator had no prior connection to the author, the research supervisor or the students participating on the intervention. The results collected through the mindset survey, MRS and PTQ for each round of testing were presented in Chapter 5. Students' results were separated based on their facilitator, and a Kruskal-Wallis test was calculated for each round of testing to evaluate any statistically significant differences between the data sets.

The facilitator comparison for each of the mindset scales was presented in Section 5.2.2. The Kruskal-Wallis test failed to reject the null hypothesis in each round of testing for the scales for general intelligence (TI) and mathematical talent (MT). It may therefore be inferred that the students' improvements in mindset were consistent regardless of which facilitator they studied with. Similarly, the Kruskal-Wallis tests for the MRS and each of its domains failed to reject the null hypothesis for each round of testing.

In Section 5.4.2, the PTQ results were broken down into the individual categories of the grading rubric. Similar to the TI scale, two Round 1 data sets were found to contain statistically significant differences – *communicating reasoning* and *accuracy*. However, once again the Rounds 2 and 3 test results failed to reject the null hypothesis, indicating that any differences in students' scores had been overcome by Round 2, and subsequent increase in Round 3 were mirrored in both groups. In summary, based on the evidence presented, it is reasonable to assume that there is no significant differences in the results regardless of the facilitator.

Research Question 3 Summary

By and large, there are no statistically significant differences between the different variations of the intervention as measured by the results of the student sample on the mindset survey, the MRS and the PTQ. The introduction of an independent facilitator was an important step in the validation of the results of the invention, and the results obtained showed that the student sample performed and improved to a degree under each facilitator that was not statistically different. Any differences in the results of the student sample in the two time variants of the intervention were found not to be statistically significant in both the mindset survey and the MRS, and, while there were minor differences in the *accuracy* and *reflecting & evaluating* categories of the PTQ, there was no obvious reason for this difference.

6.4. Further Discussion

This research set out to address the additional educational needs of highly-able students, as it was apparent within the literature that this cohort's needs were largely not being catered for, with parents (O'Reilly, 2010), educators (Riedl Cross et al., 2014) and the students themselves (McGrath, 2017) offering opinions to confirm as such. During the analysis of the qualitative data for this research, the student sample expressed their opinions at various points regarding the study of mathematics at school, without being prompted to do so. In Section 5.7, 'school' emerged as a minor theme in the focus group data, where students expressed firm opinions on the state of problem-solving and group-work from their experiences in school. There was a general belief amongst the students that problem-solving in school was largely formulaic and can be summed up by the quote: "*it's not problem-solving if you're told how to solve the problem*". This has been a longstanding issue with problem-solving in a school setting, where routine tasks are encountered but labelled as problem-solving (Krulik & Rudnick, 1989; Posamentier & Krulik, 2015; Schoenfeld, 1992). It had been hoped, in Ireland, that problem-solving would garner renewed emphasis through the implementation of 'Project Maths' and the new Junior Cycle (DES, 2010, DES, 2017). However, intensive problem-solving activities are more time-consuming to implement than regular teacher-led instruction, and despite a desire amongst teachers for double periods in mathematics to allow for this, teachers reported low levels of implementation of double periods across both Junior and Senior Cycles

(O'Meara & Prendergast, 2017). In addition, there are concerns about the variability between schools, and even within the same school, as to the number of mathematics classes each student receives (Prendergast et al., 2018; Shiel & Kelleher, 2017). In fact, Prendergast et al (2018) found that students in exam years (3rd and 6th) were less likely than first years to hold belief in their ability to solve time-consuming tasks and students in first year believed word problems were solvable through routine steps. Each of these concerns may be addressed by regular problem-solving activities, as evidenced by the student opinions for this research, where both of these issues were raised in the intervention survey and the focus group interview. Indeed, open-responses on the intervention survey showed positivity towards the challenge involved in the problems during the intervention, particularly when compared to work encountered in school.

The students on the focus group also reported negative experiences of group-work, where mixed-ability groups led to added pressure on them to solve the problem individually, which in-turn led to one of the expressions of a fixed mindset trait, where the student felt "*stupid*" if they could not solve the problem. Conversely, one student on the intervention survey expressed their enjoyment at getting to work with "*people with like-minded attitudes*". In Section 4.4.3, it was noted that highly-able students are willing to engage in group-work if they have had a positive experience in that setting in the past, and if they are working with like-minded peers (French et al., 2011; Walker et al., 2011). Based on these conditions and the experiences reported by the students on the focus group, these students may fail to engage in group-work in a classroom scenario in the future unless their needs are catered for. The students on the focus group also identified teacher feedback as important in tackling fixed mindsets amongst the less-able students, affirming findings from that research that constant, negative feedback from a young age leads to a lack of motivation (Blackwell et al., 2007; Smith et al., 2018; Truax, 2018).

The opinions outlined in this section appear to mirror those of previous research: that the needs of highly-able students of mathematics are largely not being catered for within schools.

7. Conclusion, Thesis Contributions & Future Work

In this chapter, the research conducted for this doctoral study will be summarised, and the findings emphasised, with particular attention to their relevance to wider research. Firstly, however, the intervention designed for this research will be evaluated.

7.1. Intervention Evaluation

A form of intervention evaluation, first implemented by Shapiro (1987) in the field of medicine, has since been adapted for the purpose of evaluating educational interventions (Faulkner, 2012; O'Meara, 2010; Prendergast, 2011; Walsh, 2015). The evaluation process consists of four criteria:

- Intervention effectiveness
- Intervention integrity
- Intervention acceptability
- Social validity

Each of these criteria will now be assessed in the following subsections.

7.1.1. Intervention Effectiveness

The effectiveness of the intervention refers to the change or improvement made as a result of the intervention undertaken (O'Meara, 2010). The 'change' in relation to this research is that of students' PsP, and the improvement experienced in this construct was evaluated and discussed in Chapter 6, through Research Question 1 (A-D). In light of this discussion, it is evident that the intervention was effective, with 97% of the student sample improving their PsP over the course of the intervention. Further to this, the student sample overwhelmingly attested to improvements in problem-solving skills, mindsets, and mathematical resilience through their responses on the intervention survey and focus group interview, as presented in Chapter 5. It is therefore evident that the intervention was effective and achieved its aim.

7.1.2. Intervention Integrity

The integrity of the intervention is determined through its appropriateness, its importance, and the significance of the goals it accomplished (O'Meara, 2010). The design of the mathematics intervention for this research was the culmination of a thorough literature review, and every decision in the design process was rooted in prior research, as outlined at length in Chapter 4. This intricate design process ensured the appropriateness of the intervention for the student demographic. As the author is a qualified mathematics teacher for second-level education in Ireland, he was able to ensure the appropriateness of the content in the intervention for the student demographic. The appropriateness of the intervention may be further evidenced in the student responses to questions on the intervention survey, such as:

- 100% positive response for their enjoyment of the intervention
- 100% positive response for new skills learned on the intervention
- 100% positive response for long-term benefits to their study of mathematics due to participation on the intervention

To discuss the importance of the intervention, we return to the research problem and subsequent aim for this study. The aim of this research was to cater for the additional educational needs of highly-able students by targeting an improvement in their PsP, and the intervention was the conduit through which this was achieved. The risks of failing to address the additional education needs of highly-able students have been outlined throughout this work, and thus it was abundantly clear that this intervention was important for the student demographic. The significance of the goals of the intervention is intertwined with the importance, as the opportunity to address the needs of highly-able students should be of greater importance in education in Ireland than it sometimes appears to be. Also, as summarised by Walsh: "*It is significant as the problem exists*" (Walsh, 2015, p. 282).

7.1.3. Intervention Acceptability

In order to evaluate the acceptability of an intervention, the following must be taken into consideration:

- Time and cost of the intervention
- Method of delivery
- Effectiveness and integrity
- Possible side effects
- Understanding of the intervention
- Is it replicable and transportable?

(as seen in, O'Meara, 2010, p. 270)

The first year of this research study was devoted to the review of literature and the design of the intervention. It was therefore time intensive for the author. As outlined in Section 4.2, the participants on the intervention studied the intervention for three hours per day, either over a 3-week period, for five days in two weeks, and four days of the other week; or as a 14-week programme, one day per week. Those on the 3-week programme attended during the school holidays in the summer, whilst those on the 14-week programme attended the intervention on a weekday in place of attending school for that day. Thus, regardless of the time variant studied, the intervention represented a significant time commitment from the participants. The cost of the invention was outside the control of the author or the research supervisor, due to the reliance on the CTYI for access to the participants.

The method of delivery of the intervention was the subject of Chapter 4. As previously mentioned, each decision in the design process was informed by research, and thus the method of delivery was in-keeping with these standards. Some of the features of delivery, such as the initial presentation of the problems and the questioning nature of the facilitator, were discussed in a positive light by students on the focus group interview. Collaborative problem-solving was the main pedagogical approach on the intervention, and the student sample responded to this with resounding positivity, as presented in the diaries, the intervention survey, and on the focus group interview.

The effectiveness and integrity of the intervention were discussed in Sections 7.1.1 and 7.1.2.

No negative side effects were apparent to the author during students' participation on the intervention, and no side effects emerged from the evaluation of the post-intervention qualitative data.

Due to the author's qualification as a second-level mathematics teacher, they could ensure that each of the problems selected for the intervention were solvable with the level of mathematical knowledge acquired in Junior Cycle mathematics. As the participants on the intervention were predominantly Transition Year students, this common level of mathematical knowledge was assumed. While some of the other features of the intervention may not have been familiar to the participants, such as reflective diaries, extensions, relational understanding, and so on, the active role of the facilitator near the beginning of the intervention was utilised to address any issues that arose. As the participants became more familiar with the required processes, the role of the facilitator became more subtle. This increased understanding from students over time was evident through numerous quotes identified in Chapter 5, but perhaps best embodied by the following from a student diary:

> *"I think I am getting better at working as a group in this course. It's a new experience but I feel the group is getting better at dividing up the jobs. I'm also improving in trusting other people's work without having to check over everything they've done. I'm also getting better at explaining my work as before I just needed to get a right answer, but now I have a group that need to understand it"*

The transportability of the intervention is very strong, as it requires little more than a classroom and a projector. In fact, it could also be easily delivered without the use of a projector, with the problems being printed or distributed through an educational app. The replicability of the intervention was assessed during this research study with the independent facilitator. As discussed in Section 6.3.2, there was no statistically significant difference found in the results achieved by students studying with either facilitator, indicating a high level of replicability for the intervention.

Upon review of the intervention through each of the headings presented at the beginning of this subsection, it is obvious that the intervention was acceptable.

7.1.4. Social Validity

The social validity of the intervention is determined through: the immediacy and degree of change; the effort of implementation; the theoretical orientation; and the intervention facilitator (O'Meara, 2010).

The immediacy of the change was discussed under each of the each of the individual aspects of the PsP in Sections 6.1.1 through 6.1.3. The degree of change in all three aspects was evident by Round 2, with further change also apparent in the PTQ in Round 3. The data collected through the post-intervention methods also indicated that the students' perceived this change to have occurred due to the intervention and, furthermore, that it was still in effect three months or more after participation.

As previously mentioned, participation on the intervention was a huge commitment for the students, with those on the 3-week variation attending during their summer holidays, and some students on the 14-week variation travelling over three hours per day to attend. Each class was also three hours of problem-solving, which is a high level of task-commitment per day. The development of the intervention required a high level of effort from the author. Each day of the intervention required three hours of facilitation; however, the author's background as a mathematics teacher at second-level meant they were comfortable with the duties of facilitation, and thus the effort required was primarily due to it being time-intensive.

Chapter 4 was devoted in its entirety to the explanation of each instructional decision taken in the design of the intervention for this research. Every decision was grounded in relevant literature and theory. The PsP and the CoPs model were created following the literature review in Phase 1 of this research, and were based solely on theory. The methods and data-collection tools chosen for this research were also informed by theory, as detailed in Chapter 3.

The primary facilitator for this intervention was the author, as has been discussed at length throughout this work. The author is a fully qualified second level mathematics teacher, and thus is suitable for the role of facilitator for this intervention. This qualification also meant the author was aware of the level of content knowledge of the participants. The author was also familiar with the requirements of the facilitator as set out in Chapter 4 as they were designed the intervention in collaboration with the research supervisor. The independent facilitator held a PhD in engineering, and had several years of experience teaching with CTYI. They were informed of the requirements of the facilitator in a meeting with the author prior to the intervention. The results obtained through the quantitative data collection methods were compared for each facilitator, and the findings were discussed under Research Question 3B. There was no statistically significant difference found between the data sets for the facilitators.

Summary

The adapted Shapiro model allots the opportunity to systematically discuss the evaluation of educational interventions, which should prove to be valuable all future interventions. With regards to this research project, the intervention may be regarded as a success based on this evaluation. The effectiveness was well discussed throughout Chapter 6, but the acceptability and social validity are strong features of this research. The ease-of-replicability, and the confirmation of transferability through the use of an independent facilitator, are particular strengths of this intervention.

7.2. Summary of this Work

The main aim of this research was to address the additional educational needs of highly-able students in Ireland by targeting an improvement in their PsP, and to achieve this a mathematics intervention was designed.

It is well-understood in research that highly-able students have additional educational needs that go beyond the scope of regular classroom instruction (Van Tassel-Baska & Stambaugh, 2005), with educational programmes around the world employing methods such as acceleration and enrichment to cater for their needs (Ledwith, 2013). In Ireland, classroom differentiation is the sole measure available for highly-able students in school (CIDREE, 2010), although a large-scale study suggested that teachers believe this is only sometimes effective (Riedl Cross et al., 2014). While second-level mathematics education in Ireland has undergone changes in the past decade, there exists concerns as to how the new syllabus challenges highly-able students (Lubienski, 2011). There are numerous risks involved when highly-able students are unchallenged, such as: they may underachieve; they may develop negative behaviours, such as being disruptive due to boredom; they may develop traits of a fixed mindset; they may develop superficial ideals of perfectionism (Chan, 2012; NCCA, 2007), and so on.

Through international assessments, such as PISA and TIMSS, it has been found that Ireland's average and low-performing students achieve well comparatively with other similar nations, with our low-performing students improving over the past decade (Mullis et al., 2020; OECD, 2019). However, our high-performing students perform comparatively worse than similar nations, and have disimproved on one assessment tool since 1993 (Perkins & Clerkin, 2020). From the review of literature, it is clear that the issue of catering for the additional educational needs of highly-able students in Ireland must be addressed. Whether adjudged by results in international assessments or competitions (Mullis et al., 2020; OECD, 2016, OECD, 2019), or by the students' opinions of their education (McGrath, 2017), or by the opinions of their educators (Riedl Cross et al., 2014), or by the opinions of their parents (O'Reilly, 2010), it is obvious that highly-able students in Ireland are in need of further supports. While out-of-school programmes may be of benefit to many students, it should not be necessary for students to attend these programmes in order to have their educational needs catered for. In order to address these needs, this research focussed on the development of: their problem-solving skills, which can lead to an improvement in critical thinking and higher-order thinking skills (Cooper et al., 2008; Snyder & Snyder, 2008); a growth mindset, to help deal with failure and negative attributes of perfectionism (Mofield & Parker Peters, 2018; Yeager & Dweck, 2012); and their mathematical resilience, to encourage them to persist when challenged and to value struggle for learning (Johnston-Wilder & Lee, 2010b; Lee & Johnston-Wilder, 2018). This triad construct has been defined for this research as the PsP that a student possesses.

To facilitate the development of students' PsP, a mathematics intervention was designed for this research, based upon constructivist and socio-constructivist principles, utilising collaborative problem-solving as the pedagogical method within the classroom. The CoPs model was designed for this research to define the problem-solving process within a group environment. The intervention introduced students to problem-solving strategies through weekly themes, and encouraged positive attributes of a growth mindset and mathematical resilience through facilitator intervention. In line with Dai's final level of complexity for potential (Dai, 2020), this intervention provided resources, in the form of mathematical problems; tools, through the introduction of problem-solving strategies, skills and the CoPs model; and supports, as the encouragement from the facilitator, to nurture the students' PsP. The intervention was implemented with six cohorts of highly-able mathematics students over a three year period through cooperation with CTYI.

Over the course of this work, three major research questions emerged that we have sought to address through the collection and analysis of quantitative and qualitative data. The findings of each major research question will now be summarised.

Has the student sample showed an improvement in their Problem-solving Potential (PsP) over the duration of the intervention?

The traits and abilities associated with each factor of the PsP are developed intrinsically within the individual, as outlined in the first three levels of complexity in the development of any potential (Dai, 2017; Dai & Coleman, 2005). However, to stimulate the development of these traits, external factors act upon the individual (Dai, 2020; Subotnik et al., 2011). These factors then encourage the improvement of said traits and abilities to shape their response to challenge and opportunity, and inevitably how they value progression in a subject or topic (Dai, 2020; Johnston-Wilder & Lee, 2010a; Yeager & Dweck, 2012). For this research, the intervention provided the external factors to encourage the development of the PsP amongst the student sample. The quantitative results collected through the test-instruments during the intervention suggested that 97% of students improved their PsP through participation on the intervention, with an improvement achieved through an increase in one or more of the influencing factors of the PsP. For mindset, the results of both the TI and MT scales were discussed; however, as there was a high degree of similarity between the results on each scale, the MT results were utilised in the overall discussion of the students' mindsets due to its specificity to mathematics. The student sample began the intervention with a collective growth mindset, mirroring similar results from Mofield & Parker Peters (2018) and Esparza et al (2014), but an improvement was still measurable to a statistically significant degree upon completion of their participation. This was further reflected in students' opinions of their mindsets, with students on the intervention survey and focus group interview attesting to an improvement in mindset over the course of the intervention. The student sample also began the intervention with a high level of mathematical resilience. Resilience results were discussed primarily through the *struggle* domain of the MRS as attitudes measured by the *growth* domain were represented by the results of the MT scale (Johnston-Wilder & Lee, 2010a), and there was a belief that the student sample for this research had demonstrated a high value placed on mathematics in their lives given the commitment required for participation on the intervention, similar to findings on the *value* domain for mathematics undergraduates (Kooken et al., 2016). Despite the high starting point, the student sample showed a collective improvement in the *struggle* domain. A majority of students also expressed the belief that their resilience improved through their participation on the intervention, citing improved problem-solving strategies and resilience strategies, amongst other reasons for this improvement (Section 5.6.3). Finally, a large scale improvement was also found in problem-solving skills through the grading rubric for the PTQ, with the greatest improvement in the categories for *accuracy* and *communicating reasoning*. Improvements in various aspects of problem-solving skills were widely evident in the student diaries, the intervention survey, and the focus group interview.

What opinions do the student sample have about the intervention after completing it?

This research question explored the opinions of the students that had not already been addressed in the previous question, particularly their opinion of long-term benefits to studying the intervention, and perceived benefits of the CoPs model evident within the data. In both the intervention survey and focus group interview, the student sample believed the intervention would be of long-term benefit to their study of mathematics, with some recounting how it was already helping in their study for the Leaving Certificate. In developing the CoPs model, it had been theorised that certain actions may be taken by students at each stage, and post-intervention qualitative data was used to confirm many of these. While it has been reported in the media that highly-able students prefer to work alone (as rebuffed by (French et al., 2011; Walker et al., 2011)), it has been found that this is not true when they are working collaboratively with like-minded and equally-motivated individuals (Kanevsky, 2011), which was also the case on the intervention for this research. In the student diaries, the intervention survey, and on the focus group interview, the students' overwhelmingly positive reaction to working in groups on the intervention was particularly evident.

Are there statistically significant differences in the results of the student samples between different variations of the intervention?

Largely, no differences were found in the different variations of the intervention. In terms of the time variants for the intervention, this suggests that the intervention was successful as an intensive 3-week programmes, and also as a 14-week programme. The facilitator comparison was important as a feature of validity and reliability for the findings of this research, as it removed the author as a factor in the improvement achieved by the students.

7.3. Research Contributions

This doctoral study has had an impact on a number of facets of research in mathematics education, and particularly mathematics education in Ireland. Each of these has already been addressed at length throughout this work, but will now be summarised under their relevant heading.

Problem-solving Potential

The development of the PsP built upon prior research in mindsets (Dweck, 2006, 2007; Yeager & Dweck, 2012), mathematical resilience (Johnston-Wilder & Lee, 2010b; Kooken et al., 2016) and mathematical problem-solving (Mason et al., 2010; Polya, 1945) to define a singular construct with these three major influencing factors. In order to improve upon one's PsP, it is necessary to improve in any one, or combination, of the factors, and for this research this was achieved through the students' participation on a mathematics intervention. The development of a growth mindset by participation on an intervention is a well-researched area (Blackwell et al., 2007; Esparza et al., 2014); early interventions to improve mathematical resilience have been quite successful, with positive attitudes towards problem-solving also being nurtured (Johnston-Wilder & Lee, 2017); and the development of problem-solving skills is at the heart of all mathematics education (Krulik & Rudnick, 1989; Mason et al., 2010; Schoenfeld, 1992). While it has been recognised in this research that these are not the only factors that affect a student's potential to problem-solve, it has also been highlighted that these are important factors, and worthy of consideration as a singular construct for development.

With regards to highly-able students, the three aspects of PsP are designed to address these students' additional educational needs, in part by providing them with skills to solve problems in mathematics, and by encouraging the attributes of a growth mindset and mathematical resilience such that they may react positively when they struggle and value the effort involved in learning. While this research has focussed on the needs of highly-able students, the development of PsP amongst all students may help to combat mathematics anxiety, a prevalent issue within the study of mathematics that has been defined as an "*inconceivable dread of mathematics*" (Buckley & Ribordy, 1982, p. 1). Students who suffer from this anxiety have been found to actively avoid mathematics, which results in difficulties of competence in school, but also in everyday life (Ashcraft, 2002; Buckley & Ribordy, 1982; Furner & Duffy, 2002). Studies in the United States discovered that just 7% of people had a positive experience of mathematics at school and two-thirds "*loathe*" the subject (Furner & Duffy, 2002), whilst a high level of mathematics anxiety was also found amongst a study of 3880 Irish students (Lee, 2009). On the intervention survey (Section 5.6.3) and focus group interview (Section 5.7) for this research, students commented on the negative relationship that many other students have with mathematics, despite their own positive attitude towards it. Mathematics anxiety has been shown to negatively impact a students' problem-solving ability due to its influence on working memory, while it also inhibited the use of advanced problem-solving strategies amongst some students with high-cognitive function. However, growth mindsets (Boaler, 2013; Mitchell, 2018; Peterman & Ewing, 2019) and high mathematical resilience (Goodall & Johnston-Wilder, 2015; Johnston-Wilder et al., 2014; Thomas, 2020) are believed to have a positive impact upon highly-anxious students, and thus an improvement in these either of these areas would also impact upon a student's problem-solving. It is for these reasons that the PsP would be beneficial for use amongst all student cohorts.

CoPs Model

As has been noted throughout this research, collaborative problem-solving is repeatedly referred to as an important 21st Century skill for students. Yet, prior to this research, no well-defined set of heuristics existed for the process of problem-solving within a group environment. The CoPs model was developed based on prior research in mathematical problem-solving (Mason et al., 2010; Polya, 1945) and adapted to highlight the role of collaboration in the problem-solving process. While the model has been discussed at length throughout this research, the following are some important attributes encouraged within the model:

- Critical thinking - the ‘critique’ stage encourages students to think critically about their own processes, but also those of their group-members
- Conceptual understanding - in order to ‘communicate [their] reasoning’, it is necessary to develop a conceptual understanding of the mathematical processes being used
- Communication - the development of communication skills are fundamental to the success of collaborative problem-solving
- Mathematical creativity - the ‘extension’ stage creates an opportunity for mathematical creativity for students, through which they may also expand a problem to suit their own ability level

The CoPs model is a guide through the problem-solving process that may be implemented in any mathematics setting that utilises collaborative problem-solving, and potentially to other subjects also.

In the Irish context, problem-solving has been at the forefront of the recent large-scale changes to the second-level mathematics curriculum (DES, 2017; NCCA, 2013), with teachers eager to implement group-work in the regular classroom (Shiel & Kelleher, 2017). The CoPs model offers teacher a set of heuristics to apply within the classroom for collaborative problem-solving. As has been extensively discussed throughout this work, the CoPs model was designed to aid in the development of skills of problem-solving, such as communication of reasoning, extending problems, reflecting, strategy-selection, and more. Student groups can be encouraged to proceed through the stages, just as they were on the intervention for this research project. Beyond the Irish context, the CoPs model would be beneficial in mathematics classrooms around the work for collaborative problem-solving.

Mindsets of Highly-able Students in Ireland

In the process of the thorough literature review that was conducted in Phase 1 of this research, the author found no research study that examined the mindsets of highly-able students in Ireland at any age-level. Thus, this research has presented the first set of results collected with this particular cohort of students. It had been theorised, and discussed in education research, that students who have been labelled as 'highly-able' may tend towards a fixed mindset as they wish to maintain a façade of "*looking smart*" (Dweck, 2007; Mofield & Parker Peters, 2018). However, the results collected for this research found that the student sample displayed a collective growth mindset before their participation on the intervention, and achieved a statistically significant increase over the intervention. This echoed the findings of some previous studies that found that highly-able students may embrace the principles of a growth mindset through an intervention, albeit with high variability in the students' scores (Esparza et al., 2014; Mofield & Parker Peters, 2018). This research also corroborated the results of the mindset scales with the qualitative data collected. The qualitative data also represents the first study in Ireland to present highly-able students' opinions of their mindsets.

The intervention for this research was designed as a conduit to encourage, amongst other things, the positive attributes of a growth mindset with the student cohort and, based on the discussion in Section 6.1.1, it is fair to say that the intervention achieved this goal amongst a large portion of the student sample. Similar ideals may be further applied to problem-solving scenarios within a school setting, particularly given the prominence of problem-solving within the new Junior Cycle mathematics (Byrne et al., 2021; Faulkner et al., 2020; Oldham & Close, 2009), with the teacher encouraging the positive traits of a growth mindset actively during each problem-solving session. Further research would be needed to investigate if this practice would have a positive effect on the student population.

Research in the field of mindsets towards mathematics in Ireland is under-developed for all cohorts of students. As previously stated, this research presented the first study into the mindsets of highly-able students in Ireland. However, the cohort of students who participated on the intervention for this research chose to study an extra, time-intensive mathematics programme outside of school. There is therefore the potential that this cohort of students are particularly prone to growth mindsets, as they have maintained their ability and desire to learn so late into their school life. It is therefore important that this research marks just the first step into an investigation of the mindsets of highly-able students in Ireland, with a large scale study required to provide an in-depth understanding of the area.

Mathematical Resilience of Highly-able Students in Ireland

Similar to the study of mindsets, this research presents the first set of results that examine the mathematical resilience of highly-able students in Ireland. Much of the research that utilises the mathematical resilience scale (MRS) has focussed on those who experience mathematical anxiety (Johnston-Wilder et al., 2014, 2015); however, the risks of low mathematical resilience to highly-able students has been outlined in prior research (Yeager & Dweck, 2012). This doctoral study yields the first results of highly-able students through measurement by the MRS.

The validation study for the MRS (Kooken et al., 2016) noted that further analysis of the *struggle* domain would benefit from qualitative research methods, and this research was able to achieve this through the diary entries, the intervention survey and the focus group interview. This qualitative data also confirmed that some aspects of mathematical resilience may be outside the scope of quantitative data collection.

Further research is required into the mathematical resilience of highly-able students in Ireland. Students with low mathematical resilience may struggle to deal with challenge in mathematics (Johnston-Wilder & Lee, 2010b), and this is particularly important within highly-able students as they may not meet a challenge within mathematics for long periods of time.

Intervention

The mathematics intervention designed for this doctoral study is a purpose-built programme for highly-able mathematics students in Transition Year in Ireland, which, as has been discussed throughout this project, is of great importance to these students. The intervention has possible implications for the encouragement of mathematical talent and potential amongst highly-able students in schools. As was presented in detail in Chapter 4, each decision in the design process was based on relevant research. Participation on the intervention can provide students with stronger problem-solving skills that may benefit their study of mathematics in the Leaving Certificate, as was reported by students on the intervention survey (Section 5.6.3). Faulkner et al (2021) recently found that a sample of undergraduate students in Ireland did not display a level of improvement in problem-solving that was expected following the overhaul of the second-level mathematics curriculum. Further to this, they outlined how students may be experiences "*helplessness*" when they encounter unfamiliar problems. Therefore, the development of PsP, through participation on our mathematics intervention, may be of benefit to students in their future study at second-level, but also in tertiary education.

7.4. Recommendations

Following the completion of this doctoral study, the author has a number of recommendations for based on the findings of this research:

- The author recommends the implementation of his mathematics intervention in schools to develop highly-able students' PsP as a means of catering for their additional educational needs. This implementation should take place during TY and may involve individual schools or an amalgamation across multiple schools if possible. Students should be identified per the NCCA guidelines. Cohorts should not exceed 20 students, unless multiple facilitators are present. Group sizes should not exceed four students, as discussed in Section 4.4.3.
- As previously discussed, the student sample for this research may have been prone to growth mindsets as they had selected to study an extra mathematics course outside of school. The author recommends that further study is conducted into the mindsets of highly-able students in Ireland in a more general setting to investigate if the mindsets presented in this research are representative of the wider student cohort. If the mindsets of wider population of this student cohort differ to a significant extent, then it may be necessary to address negative mindset traits earlier in their education.
- This study is the first step into research of the mathematical resilience of students in Ireland, but merely examined the mathematical resilience of one student cohort at one age level. This field has a huge scope for research that needs to focus on the mathematical resilience of varying student cohorts, and at varying age levels. The author recommends that a study is conducted amongst the wider population of highly-able students at second-level using the mathematics resilience scale.

- The CoPs model represents a blueprint for collaborative group-based problem-solving. The model has thus far been used solely for the research intervention. The author recommends the use of the model in the regular mathematics classroom during problem-solving, but also its implementation at third level. Further to this, the author recommends group sizes of 3-4 students, and the teacher acting as a facilitator within the classroom.

7.5. Future Work

There are numerous avenues for future research stemming from this doctoral study, for the author and the wider mathematics education community:

- A large-scale investigation into the mindsets of students in primary, secondary and tertiary education in Ireland is required.
- A large scale investigation into the mathematical resilience of students at both primary, secondary and tertiary education in Ireland is required. This should also be accompanied by a study of the levels of mathematical anxiety amongst students in Ireland.
- While this research tracked the PsP of students over the course of the intervention, and several months after their participation, there is scope for a longitudinal study that examines the long-term benefits of an improvement in PsP.
- Problem-solving skills, mindsets and resilience are not exclusive to the study of mathematics. Future work is required to determine the effectiveness of the construct of PsP within other relevant subject areas.
- Further study of the intervention is required within a school environment. This may result in further adaptations being made to the intervention, as occurred through each iteration,

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Appendices: Appendix A

Intervention Survey

This survey refers to the two-hour Problem Solving Intervention, together with the tutorial hour. Please answer based on this 3 hours of study only. Please place a tick in just one box per question. If you make a mistake, cross out the tick and place one in the correct box.

Please indicate using a tick, whether you studied the problem-solving mathematics module as 14 days over a 3 week or 14 week programme:			
3 week programme		14 week programme	

Did you enjoy studying the module?				
Not at all	Slightly	Moderately	Very	Extremely
Further Comment:				

How did you feel about the weekly workload required while studying the module?				
Too intense	Slightly intense	OK	Slightly easy	Too easy
Further Comment:				

Would you make any changes to the module?			
Yes		No	
Further Comment:			

Based on your thoughts about the module, would you advise someone to study it as a three week or fourteen week programme?			
3 week		14 week	
Further Comment:			

Did you learn new skills while studying the module?			
Yes		No	
Further Comment:			

Do you feel studying the module will benefit your study of mathematics in school?			
Yes		No	
Further Comment:			

Do feel your mindset has changed over the course of studying this module?			
Yes		No	
Further Comment:			

Do you feel your resilience towards challenging work has changed over the course of studying this module?			
Yes		No	
Further Comment:			

Prior to studying this module, was there a time when you became stuck while studying maths?			
Yes		No	
If yes, please provide details of when or where this happened			

While studying this module, did you become stuck at any time?			
Yes		No	
If yes, what did you do to overcome this?			

In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work. In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. Please answer the following based on this:				
Before studying the module, my mindset was:				
Fixed	Somewhat fixed	Neutral	Somewhat growth	Growth
After finishing the module, my mindset was:				
Fixed	Somewhat fixed	Neutral	Somewhat growth	Growth
Now, my mindset is:				
Fixed	Somewhat fixed	Neutral	Somewhat growth	Growth
Further Comment:				

Note: The two rows that have been shaded grey appeared on the online survey only

Appendix B

Mindset Survey

Name: ________________________________

Please place a tick in just one box. If you make a mistake, cross out the tick and place one in the correct box.

		Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
You have a certain amount of intelligence, and you can't really do much to change it							
Your intelligence is something about you that you can't change very much							
No matter who you are, you can significantly change your intelligence level							
To be honest, you can't really change how intelligent you are							
You can always substantially change how intelligent you are							
You can learn new things, but you can't really change your basic intelligence							
No matter how much intelligence you have, you can always change it quite a bit							
You can change even your basic intelligence level considerably							

		Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
You have a certain amount of mathematical talent, and you can't really do much to change it							
Your mathematical talent is something about you that you can't change very much							
No matter who you are, you can significantly change your level of mathematical talent							
To be honest, you can't really change how talented you are at maths							
You can always substantially change how talented you are at maths							
You can learn new things, but you can't really change your basic level of mathematical talent							
No matter how much mathematical talent you have, you can always change it quite a bit							
You can change even your basic level of mathematical talent considerably							

Appendix C

Mathematics Resilience Scale (MRS)

Name: ___________________________

Please place a tick in just one box. If you make a mistake, cross out the tick and place one in the correct box.

		Completely disagree	Disagree	Somewhat disagree	Don't know	Somewhat agree	Agree	Completely agree
Maths is essential for my future	V							
Maths will be useful to me in my life's work	V							
Maths courses are very helpful no matter what I decide to study	V							
Knowing maths contributes greatly to achieving my goals	V							
Having a solid knowledge of maths helps me understand more complex topics in my field of study	V							
Thinking mathematically can help me with things that matter to me.	V							
It would be difficult to succeed in life without maths	V							
Maths develops good thinking skills that are necessary to succeed in any career	V							
Everyone struggles with math at some point	S							
Good mathematicians experience difficulties when solving problems.	S							
		Completely disagree	Disagree	Somewhat disagree	Don't know	Somewhat agree	Agree	Completely agree

People who work in maths related fields sometimes find maths challenging.	S							
Everyone makes mistakes at times when doing maths	S							
Struggle is a normal part of working on maths	S							
People in my peer group struggle sometimes with maths	S							
Maths teachers are sometimes stumped by a maths problem	S							
When someone struggles in maths, it doesn't mean they have done something wrong.	S							
Making mistakes is necessary to get good at maths	S							
Maths can be learned by anyone	G							
If someone is not a maths person, they won’t be able to learn much maths	G							
If someone is not good at maths, there is nothing that can be done to change that.	G							
People are either good at maths or they aren’t	G							
Everyone's maths ability is determined at birth	G							
Some people cannot learn maths.	G							
Only smart people can do maths	G							

Appendix D

PTQ Rubric

2011-2012 **Mathematics Problem Solving Official Scoring Guide** 2011-2012

Apply mathematics in a variety of settings. Build new mathematical knowledge through problem solving. Solve problems that arise in mathematics and in other contexts. Apply and adapt a variety of appropriate strategies to solve problems. Monitor and reflect on the process of mathematical problem solving.

Process Dimensions	**6/ 5	4	3	*2 / 1
Making Sense of the Task *Interpret the concepts of the task and translate them into mathematics.*	The interpretation and/or translation of the task are • thoroughly developed and/or • enhanced through connections and/or extensions to other mathematical ideas or other contexts.	The interpretation and translation of the task are • adequately developed and • adequately displayed.	The interpretation and/or translation of the task are • partially developed, and/or • partially displayed.	The interpretation and/or translation of the task are • underdeveloped, • sketchy, • using inappropriate concepts, • minimal, and/or • not evident.
Representing and Solving the Task *Use models, pictures, diagrams, and/or symbols to represent and solve the task situation and select an effective strategy to solve the task.*	The strategy and representations used are • elegant (insightful), • complex, • enhanced through comparisons to other representations and/or generalizations.	The strategy that has been selected and applied and the representations used are • effective and • complete.	The strategy that has been selected and applied and the representations used are • partially effective and/or • partially complete.	The strategy selected and representations used are • underdeveloped, • sketchy, • not useful, • minimal, • not evident, and/or • in conflict with the solution/outcome.
Communicating Reasoning *Coherently communicate mathematical reasoning and clearly use mathematical language.*	The use of mathematical language and communication of the reasoning are • elegant (insightful) and/or • enhanced with graphics or examples to allow the reader to move easily from one thought to another.	The use of mathematical language and communication of the reasoning • follow a clear and coherent path throughout the entire work sample and • lead to a clearly identified solution/outcome.	The use of mathematical language and communication of the reasoning • are partially displayed with significant gaps and/or • do not clearly lead to a solution/outcome.	The use of mathematical language and communication of the reasoning are • underdeveloped, • sketchy, • inappropriate, • minimal, and/or • not evident.

Accuracy
Support the solution/outcome.
	The solution/outcome is correct and enhanced by
	• extensions,
	• connections,
	• generalizations, and/or
	• asking new questions leading to new problems.
		The solution/outcome given is
		• correct,
		• mathematically justified, and
		• supported by the work.
			The solution/outcome given is
			• incorrect due to minor error(s), or
			• a correct answer but work contains minor error(s)
			• partially complete, and/or
			• partially correct
				The solution/outcome given is
				• incorrect and/or
				• incomplete, or
				• correct, but
				  ○ conflicts with the work, or
				  ○ not supported by the work.

Reflecting and Evaluating *State the solution/outcome in the context of the task.* *Defend the process, evaluate and interpret the reasonableness of the solution/outcome.*	Justifying the solution/outcome completely, the student reflection also includes • reworking the task using a different method, • evaluating the relative effectiveness and/or efficiency of different approaches taken, and/or • providing evidence of considering other possible solution/outcomes and/or interpretations.	The solution/outcome is stated within the context of the task, and the reflection justifies the solution/outcome completely by reviewing • the interpretation of the task • concepts, • strategies, • calculations, and • reasonableness.	The solution/outcome is not stated clearly within the context of the task, and/or the reflection only partially justifies the solution/outcome by reviewing • the task situation, • concepts, • strategies, • calculations, and/or • reasonableness.	The solution/outcome is not clearly identified and/or the justification is • underdeveloped, • sketchy, • ineffective, • minimal, • not evident, and/or • inappropriate.

****6 for a given dimension would have most attributes in the list; 5 would have some of those attributes.**

***2 for a given dimension would be underdeveloped or sketchy, while a 1 would be minimal or nonexistent.**

For use beginning with 2011-2012 Assessments Office of Assessment and Evaluation

Oregon Department of Education Adopted May 19, 2011

Appendix E

Questions for Focus Group Interview

Thank you for agreeing to participate in this focus group interview. Before we begin I would like to welcome you all, and go through some guidelines. This interview is being audio recorded using this Dictaphone, but your names will not be used in any report or publication. I will be asking questions based on your experiences of studying problem solving in mathematics with CTYI, either with the Early University Entrance Programme, or the summer programme. The problem solving module, and its tutorial, comprised 3 hours of your study each day – the questions are not concerning your experiences with the processes and skills aspect of study. The questions I ask will be open, and your opinion matters to us. There are no right or wrong answers. You may also add to the opinion expressed by other members of the discussion. The interview should take approximately one hour, we ask you to switch off your mobile phones during this time. Please give everyone the chance to express their opinion during the conversation.

To begin, please state your name and whether you studied the module over 3 weeks, or 14 weeks

Did you learn any new skills while studying the module?

- What did you learn?
- Have you been able to use these skills while studying in school?
- Has studying the module been of benefit to your study of mathematics in school?
- How did you find being asked to extend problems?
    - Was this a new concept to you?
    - Is this skill transferable to mathematics in school?
- How did you find being asked to reflect on questions?
    - Was the concept of the diary new to you?
    - Is this skill transferable to mathematics in school?
    - Have you tried to reflect on any other aspects of school?

Did you find the module to be challenging?

- What aspects?
  - Problems/ length of work/ extension/ reflection/ group work
- Was this a new kind of challenge?
  - Did you find Junior Cert maths challenging in any way?
  - How did you try to overcome it?
- Did you become 'stuck' while working through the problems?
  - How did you overcome this?
  - Is being stuck a valuable process? Why/ why not?
  - Have you become stuck on anything since? In school or otherwise?
    - Has being stuck made you more comfortable with asking for help from others, or a teacher, in a group work setting? What about in a regular classroom? **Note: ask this if asking for help does not come up organically. If it does, just ask are they more comfortable asking for help in classroom.**
- Do you feel like your resilience with challenging work has changed due to studying the module?
  - Has a change in resilience impacted on your study at school? If so, how?

Did you feel your mindset changed over the course of the module?

- Would you have considered your mindset closer to growth or fixed before the module?
- And after?
- Has your mindset changed between completing the module and now?
- What effects has a change in mindset had on your study in school, if any?
- Can you give me an example of a time where you showed growth mindset in relation to something? It might have been in school, or at a hobby, or work etc.

Have you studied problem-solving in school?

- Was problem-solving studied as part of the Junior Cert?
- Have you studied it in school since finishing the module?
- Would you consider the concepts and skills aspects of school mathematics to be problem-solving?
- Do you feel studying the module has made you a better problem-solver in mathematics?
  - What about as general problem-solvers?
- Did working in groups aid in your problem-solving attempts?
- Did you learn or improve any particular skills through working in groups that you could see yourself using in the future?

Appendix F

Iterations 1, 2 & 3 Problems

KEY: Change Category						
A = Too easy; B = Better fit to another theme; C = Unsuitable to theme						
	Title	Source	Iteration 1	Iteration 2	Iteration 3	Change category
			Week each problem appeared (x = unused)			
1	Hexagons	Text and Tests	1	1	1	
2	Lockers	Thinking Mathematically	1	1	1	
3	Radiation	Text and Tests	1	1	1	
4	Solids and Frames	Thinking Mathematically	1	1, 2, 3	2	B
5	More Cubes	Nrich	1	1	1	
6	Crates of Bottles	Thinking Mathematically	1	1, 12	12	B
7	Paper Cups	Thinking Mathematically	1, 3	1, 3, 4	3	B
8	Cricket Balls	Thinking Mathematically	1	1	x	A
9	Ladies who Lunch	Thinking Mathematically	1	1	1	
10	Cubes again… I'm sorry	University module	1	1	1	
11	Creature Collection	Thinking Mathematically	1	1	x	A
12	Matchstick Squares	Thinking Mathematically	1, 2, 3	1, 2, 3	3	B
13	Tetris	Mind your Decisions	x	x	1	
14	Chessboard	Thinking Mathematically	1	1	1	
15	Census Worker	PDMT Workshop	2	2	5	B
16	Next line?	Nrich	2	2	2	
17	Leapfrogs	Thinking Mathematically	2	2	2	
18	Rectangles	Thinking Mathematically	2	2	2	
19	Circle and Sports	Thinking Mathematically	2	2	2	
20	Paper Strips	Thinking Mathematically	2, 3	2, 3	2	B
21	Polygons	Nrich	2	2	5	B
22	To wake you up	Nrich	3	x	x	A
23	Bees	Thinking Mathematically	3	3	3	
24	Consecutive Sums	Thinking Mathematically	3	3	3	
25	More consecutive sums	Thinking Mathematically	3	3	3	
26	Step Up	Mind your Decisions	x	x	3	
27	Squares	Thinking Mathematically	4	4	4	
28	Furniture	Thinking Mathematically	4	4	4	
29	Fifteen	Thinking Mathematically	4	4	4	
30	Fare is fair	Thinking Mathematically	4	X	x	A
31	Palindromes	Thinking Mathematically	4	4	4	

	Title	Source	Iteration 1	Iteration 2	Iteration 3	Change category
32	More furniture	Thinking Mathematically	4	4	4	
33	Race to 32,768	Mind your Decisions	x	x	4	
34	Nine Dots	Thinking Mathematically	5	5	5	
35	River Crossing	Thinking Mathematically	5	5	5	
36	Triangles	Thinking Mathematically	5	5	5	
37	Cut-away	Thinking Mathematically	5	5	5	
38	Arithmegons	Thinking Mathematically	5	5	x	C
39	Hopping Flies	Thinking Mathematically	5	5	5	
40	Cartesian Chase	Thinking Mathematically	6	6	6	
41	Coin Slide	Thinking Mathematically	6	6	6	
42	Desert Crossing	Thinking Mathematically	6	6	6	
43	Jacobean Village	Thinking Mathematically	6, 11	6, 11	6	B
44	Diagonals of a Rectangle	Thinking Mathematically	6	6	x	A
45	Gossips	Thinking Mathematically	6, 7	6, 7	13	B
46	Basketball	Mind your Decisions	x	x	6	
47	Room Painting	Mind your Decisions	x	x	6	
48	Milk Carton	Thinking Mathematically	7	7	7	
49	Taking Matches	Thinking Mathematically	7	7	7	
50	Sequence	Thinking Mathematically	7	7	7	
51	Divisibility	Thinking Mathematically	7, 8	7, 8	8	B
52	One Sum	Thinking Mathematically	7	7	7	
53	Your move	Mind your Decisions	x	x	7	
54	Finger Multiplication	Thinking Mathematically	8	8	8	
55	Reversals	Thinking Mathematically	8	8	8	
56	Cycling Digits	Thinking Mathematically	8	8	8	
57	Chinese Remainders	Thinking Mathematically	8	8	x	C
58	Divisors	Thinking Mathematically	8	8	8	
59	Perfect Squares	Mind your Decisions	x	x	8	
60	Find A, B, C	Mind your Decisions	x	x	8	
61	Moving Mean	Mind your Decisions	9	9, 11	11	B
62	Weighing fish	Thinking Mathematically	9	9	x	A
63	Mean Maths	Mind your Decisions	x		9	
64	Polygonal Numbers	Thinking Mathematically	9	9	9	
65	Sum of Squares	Thinking Mathematically	9	9	9	
66	Cubed numbers	Thinking Mathematically	9	9	9	
67	Horse Racing	Mind your Decisions	x	x	9	
68	Knotted	Thinking Mathematically	10	x	x	C

	Title	Source	Iteration 1	Iteration 2	Iteration 3	Change category
69	Folding Polygons	Thinking Mathematically	10	x	x	A
70	Ins and Outs	Thinking Mathematically	10	10	10	
71	Nullarbor Plane	Thinking Mathematically	10	10	x	A
72	Sticky Angles	Thinking Mathematically	10	x	x	C
73	Tethered Goat	Thinking Mathematically	10	10	10	
74	Areas	Mind your Decisions	x	x	10	
75	More areas	Mind your Decisions	x	x	10	
76	Water Tank	Mind your Decisions	x	x	10	
77	Shape Partitioning	Nrich	10	10	x	C
78	How many triangles?	Thinking Mathematically	x	10	10	
79	Painting Cubes	Youcubed	x	10	3	B
80	Blood Type	Nrich	11	x	x	A
81	Students	Text and Tests	11	x	x	A
82	Oceans	Nrich	11	x	x	C
83	Club Lotto	Nrich	11	11	11	
84	Match Boxes	Thinking Mathematically	11	11	x	A
85	Ants	Mind your Decisions	x	11	11	
86	Lock Codes	Mind your Decisions	x	11	11	
87	Taxi	Youcubed	x	11	11	
88	Geysers	Mind your Decisions	x	x	11	
89	Productive Exchange	Thinking Mathematically	12	12	12	
90	Remainders of the day	Thinking Mathematically	12	12	12	
91	Rational Divisors	Thinking Mathematically	12	12	x	A
92	Warehouse	Thinking Mathematically	12	12	x	A
93	Hamburger	Thinking Mathematically	12	12	x	A
94	Fred and Frank	Thinking Mathematically	12	12	12	
95	Find the values	Mind your Decisions	x	x	12	
96	Cat in a box	Mind your Decisions	x	x	12	
97	Clocks	Thinking Mathematically	13	13	x	A
98	Patchwork	Thinking Mathematically	13	13	13	
99	Painted Tyres	Thinking Mathematically	13	13	13	
100	Average Speed	Mind your Decisions	13	13	x	A
101	Rule of thumb	Thinking Mathematically	13	13	x	A
102	Handshakes	Can you solve my problems?	x	x	13	
103	Who is who?	Mind your Decisions	x	x	13	
104	Motorcycles	Mind your Decisions	x	x	13	

	Title	Source	Iteration 1	Iteration 2	Iteration 3	Change category
105	Square Takeaway	Thinking Mathematically	14	14	14	
106	Triangular Count	Thinking Mathematically	14	14	x	A
107	Repaint	Thinking Mathematically	14	14	x	A
108	Square bashing	Thinking Mathematically	14	14	14	
109	Wool winding	Thinking Mathematically	14	14	x	C
110	Travelling through time	Thinking Mathematically	14	14	14	
111	Paper Clip Chains	University module	x	x	14	
112	Batteries	University module	x	x	14	
113	How many children?	Mind your Decisions	x	x	14	

Appendix G

List of Problems

	Title	Problem
1	Hexagons	I have a hexagon of perimeter 24cm. I place 3 such hexagons together. Each one must be touching the other two, and where they touch must be a full side touching (not an edge or partial side). What is the perimeter of the shape?
2	Lockers	You are in a hallway with N lockers, all closed. A student walks along and opens them all. The next student closes every second locker. The third student goes to every third locker- if it is open, they close it; if it was closed, they open it. The fourth student does the same to every fourth locker. This continues until student N. What lockers are open, and why?
3	Radiation	The international radiation symbol first appeared in 1946. It is coloured black. The symbol consists of a central circle of radius r, an internal radius of 1.5r and an external radius of 5r for the blades, which are separated from each other by 60˚. Calculate, in terms of r and π, the area of the black region.
4	Solids and Frames	I can make any size cube from smaller 1x1x1 cubes. What relationship exists between solid cubes and cubes made of just a frame, each with equal dimensions?
5	More Cubes	We had interlocking cubes in 10 different colours, up to 1000 of each colour. We started with one yellow cube. This was covered all over with a single layer of red cubes. This was then covered by a single layer of blue. Followed by green, black, brown, white, orange, pink and purple. The many layered cube was then broken up and each colour made into cube These were just of one colour, and the largest possible cubes were made. Eg: red layer made three 2x2x2 cubes and a 1x1x1. What colour was the largest cube made? What colour had no 1x1x1 cubes? What colour was made into the most cubes?
6	Crates of Bottles	I have a crate that can hold 36 bottles of water. Is it possible to arrange 14 bottles in the crate such that each row and column has an even number of bottles?
7	Paper Cups	In preparing for a party, I opened a package of paper cups and put them on a table. Some were the right way up, others upside down. By flipping two cups at a time, can I get them all upright?
8	Cricket Balls	Amongst 9 apparently identical cricket balls, one is lighter than the others. How quickly can you uncover which is the light one using a makeshift balance?
9	Ladies who Lunch	Five women have lunch together seated around a circular table. Ms O'Shea is sitting between Ms Lehane and Ms Mahoney. Eimear is sitting between Carina and Ms Nugent. Ms Lehane is between Eimear and Aine. Carina and Deirdre are sisters. Bea is seated with Ms Price on her left and Ms Mahoney on her right. Match the first names with the surnames.
10	Cubes again… I'm sorry	I have eight cubes, all the same size. Two of them are coloured red, two are green, two are white and two are yellow, but otherwise, they are indistinguishable. I want to build one big cube out of these, in such a way that each colour appears at least once on each face. How many different ways can I do this?
11	Creature Collection	Ross has a collection of lizards, worms and beetles. He has more worms than beetles and lizards combined. In total his collection has 26 legs and 12 heads. How many lizards does he have?
12	Matchstick Squares	How many matchsticks are required to make 14 squares in a row? How many are required to make an NxN square?
13	Tetris	In a game of tetris, there are 7 pieces, each made up of 4 square units. Can you use one of each piece to make a 4x7 rectangle?
14	Chessboard	How many squares are there on a regular chessboard?
15	Census Worker	A census worker walks up to a woman's house CW: "How many children do you have, and what are their ages?" Woman: "I have 3 children, and the product of their ages is 36" CW: "I still don't know their ages" Woman: "The sum of their ages is equal to the number of the house next door" CW: "I still don't know their ages" Woman: "I have to go, my eldest is asleep" CW: "Thank you, I now know their ages"

16	Next line?	Graphic 1 below this table
17	Leapfrogs	Ten pegs of two colours are laid out in a line of 11 holes, with the colours kept together and separated by the spare hole. I want to interchange the black and white pegs, but I am only allowed to move pegs into an adjacent empty hole or jump over one peg into an empty hole. Can I make the interchange?
18	Rectangles	How many rectangles are there on a regular chessboard?
19	Circle and Sports	Place N spots around a circle and join each pair of spots by straight lines. What is the greatest number of regions into which the circle can be divided by this means?
20	Paper Strips	Imagine a long thin strip of paper stretched out in front of you, left to right. Fold it in half from right to left. Repeat this two more times. How many creases are there? How many creases would there be if we repeated this 10 times?
21	Polygons	Given the number of sides of a polygon, what is the maximum number of right angles it can have?
22	To wake you up	If I cut a piece off a 2-D shape, does it decrease the shapes area or perimeter?
23	Bees	Male bees hatch from unfertilised eggs, and so have a mother but no father. Female bees hatch from fertilised eggs. How many ancestors does a male bee have in the twelfth generation back? How many are males?
24	Consecutive Sums	Some numbers can be expressed as a sum of consecutive positive numbers. Exactly which numbers have this property?
25	More consecutive sums	In how many different ways can a number be represented?
26	Step Up	Liam's house has a staircase with 12 steps. He can go down one at a time or two at a time. In how many different ways can Liam go down the 12 steps, taking one or two steps at a time?
27	Squares	Which numbers can be expressed as the difference of two perfect squares?
28	Furniture	A very heavy armchair needs to be moved, but the only possible movement is to rotate it through 90˚ about any of its corners. Can it be moved so that it is exactly beside its starting position and facing the same way?
29	Fifteen	Nine counters marked with the digits 1 to 9 are placed on a table. Two players alternatively take on counter from the table. The winner is the first player to obtain, amongst his or her counters, three with the sum of exactly 15. What is the best move to make?
30	Fare is fair	I wish to share 30 identical individual sausages equally amongst 18 people. What is the minimum number of cuts I need to make? What is the minimum number of pieces I need to create?
31	Palindromes	My friend claims that all 4 digit palindromes are divisible by 11. Are they?
32	More furniture	Try our original question with a sofa of dimensions 2 unit x 1 unit
33	Race to 32,768	On a table are nine face-up cards. Each card displays a different number from the list: 2, 4, 8, 16, 32, 64, 128, 256, 512. Alice and Bob alternatively take turns. Each picks a single card on a turn, and Alice goes first. Once a card is chosen it cannot be selected again. The game ends when someone wins or all the cards have been selected. The winner is the first person to collect a set of 3 cards with a product of 32,768. If neither player does this, the game ends in a draw. If both play optimally, does either player have a winning strategy? Or does the game always end in a draw? How should you play this game?
34	Nine Dots	Nine dots in a 3 by 3 square array are to be joined by four consecutive straight lines, without removing the pencil from the paper
35	River Crossing	Three men desperate to cross a river encounter two small boys on a homemade raft. The raft will carry only one man or both boys. Can the men cross the river?
36	Triangles	With six matches, make four equilateral triangles
37	Cut-away	Two diagonally opposite corners of a chessboard are removed. Can you cover the remaining board with dominoes each of which covers two squares?
38	Arithmegons	A secret number is assigned to each vertex of a triangle. On each side of the triangle is written the sum of the two vertices at its ends. Find a simple rule for revealing the secret numbers

39	Hopping Flies	There are 25 coins in a 5x5 array. A fly randomly lands on a coin and wants to hop to every other coin exactly once, at each stage moving only to an adjacent coin in the same row or column. Is it possible?
40	Cartesian Chase	This is a game for two players on a rectangular grid with a fixed number of rows and columns. Play begins in the bottom-left square where the first player places a mark. On their turn, a player may put his mark directly above, directly to the right of, or diagonally above and to the right of the last mark made by their opponent. Play continues and the winner is the person who gets their mark in the upper-right hand corner first. Find the best way of winning.
41	Coin Slide	Select three large and three small coins, and place them in a row so that consecutive coins are touching, and so that large and small coins alternate. A move consists of sliding a pair of adjacent coins to a new position in the row, without interchanging them. Can you, by a sequence of moves, put all the large coins at one end, and the small at the other. Consecutive coins in the final row must be touching.
42	Desert Crossing	It takes 9 days to cross a desert. A man must deliver a message to the other side, where no food is available, and then return. One man can carry enough food to last for 12 days. Food may be buried and collected on the way back. There are two men ready to set out. How quickly can the message be delivered with neither man going short of food.
43	Jacobean Village	A certain village in Jacobean times had all the valuables locked in a chest in the church. The chest had a number of locks on it, each with its own individual and distinct key. The aim of the village was to ensure that any three people in the village would amongst them have enough keys to open the chest, but no two people would be able to. How many different locks are required and how many keys?
44	Diagonals of a Rectangle	Draw a rectangle 3 squares by 5 squares, and draw in a diagonal. How many squares are touched by the diagonal.
45	Gossips	Each evening in a certain village, the old men gather in pairs to exchange gossip about village activities. At each exchange, each one passes on all that he has learned about the day's events. What is the fewest number of exchanges needed so that everyone is up to date on all possible news?
46	Basketball	A, B and C play a series of 1-on-1 basketball games. In a given game, two of them play against each other while the third person rests. The loser of the game then goes to rest while the winner continues to play, facing off against the person who just rested. After they were done, here is how many games each person played: A played in 8 games B played in 11 games C played in 15 games The puzzle is: who lost the 4th game?
47	Room Painting	Team A has 4 girls and 7 boys. Team A takes 5 days to paint 4 rooms. Team B has 7 girls and 10 boys. Team B takes 4 days to paint 5 rooms. Team C has 8 girls and 5 boys. How long will it take team C to paint 6 rooms? Assume every girl works at a constant rate, every boy works at a constant rate, and every room is identical.
48	Milk Carton	How much cardboard do you need to make a milk carton to hold 1 litre of milk?
49	Taking Matches	Two piles of matches are on a table. A player can remove a match from either pile or a match from both piles. The player who takes the last match loses. If there are two players, how should you play?
50	Sequence	Write down a sequence of 0s and 1s. Underneath each consecutive pair, write a 0 if they are the same and a 1 if not. Repeat this process until you are left with a single digit. Can you predict what the final digit will be?
51	Divisibility	To check whether a number is divisible by 11, sum the digits in the odd positions counting from the left (the first, third,…) and then sum the remaining digits. If the difference between the two sums is divisible by 11, then so is the original number. Otherwise, it is not. Why does this work?
52	One Sum	Take any two numbers that sum to one. Square the larger and add the smaller. Square the smaller and add the larger. Which do you expect to be bigger?
53	Your move	Alice and Bob are playing a game using a chess board. Alice starts by placing a knight on the board. Then they take turns moving the knight to a new square (one it has not been on before). Standard chess rules apply: the knight can only move in an "L" shape, 2 squares in one direction and one square to the side.

		The first player who cannot move the knight to a new square loses the game. Who wins if both players play optimally, and what is the winning strategy?
54	Finger Multiplication	The following technique was widely used in medieval Europe. Knowing how to multiply two numbers less than 6, you can multiply two numbers between 5 and 10 as follows. Open both palms towards you. To calculate 7 x 9, say, put 7 – 5 = 2 fingers down on the left hand, and put 9 – 5 fingers down on the right. Count the number of down fingers (4 + 2 = 6) and multiply together the number of up fingers (3 x 1 = 3). Put the two answers together (63). Does this work, and why?
55	Reversals	Take a three-digit number, reverse its digits and subtract the smaller from the larger. Reverse the digits of the result and add the two. What happens? Why?
56	Cycling Digits	I have in mind a number which, when you remove the units digit and place it at the front, gives the same result as multiplying the original number by 2. Am I telling the truth?
57	Chinese Remainders	The set of numbers { 3 x 5 x 3 x 2 + 3 x 11 x 2 x 2 + 5 x 11 x 1 x 2 + 3 x 5 x 11n: n is an integer} are exactly the integers that leave a remainder of 2 on dividing by 3, 5 and 11. Why? Generalise
58	Divisors	Take any number and find all of its positive divisors. Find the number of divisors of each of those divisors. Add the resulting numbers and square the answer. Compare it with the sum of the cubes of the numbers of divisors of the original divisors.
59	Perfect Squares	Find all positive integers n for which n2 +45 is equal to a perfect square
60	Find A, B, C	A x B x C = ABC/5. Find A, B, and C (digits 0-9)
61	Moving Mean	In a list of positive integers, the arithmetic mean is 5, and the number 16 is known to appear. If the 16 is removed, the mean drops to 4. What is the largest possible number that could occur in the original list, and how many numbers were in that list?
62	Weighing fish	A fisherman caught 3 fish. The fish were not weighed separately but instead in pairs. The big and medium fish together weighed 16kg. The big and small fish weighed 14kg. The middle and small fish weighed 12kg. How much did each fish weigh?
63	Mean Maths	In a class of p students, the average (mean) of the test scores is 70. In another class of n students, the average of the scores for the same test is 92. When the scores of the two classes are combined, the average of the test scores is 86. What is the value of p/n?
64	Polygonal Numbers	A number which can be represented as the number of dots in a triangular array is called triangular. A number which can be represented as the number of dots in a pentagonal array is called a pentagonal number. Which number are triangular, which are pentagonal, and more generally, which are P-polygonal?
65	Sum of Squares	Notice that 22 + 32 + 62 = 72 32 + 42 + 122 = 132 42 + 52 + 202 = 212 Is this part of a general pattern? Notice also that 32 + 42 = 52 102 + 112 + 122 = 132 + 142 212 + 222 + 232 + 242 = 252 +262 +272 Is this part of a general pattern?
66	Cubed numbers	Someone noticed that 2 x 3 x 4 + 4 x 10 = 43 And that 5 x 6 x 7 + 7 x 19 = 73 Are these examples of some general pattern, or are they simply anomalies?
67	Horse Racing	There are 25 horses in a stable. You want to find the fastest 3 horses. What is the miniumum number of races needed to figure this out? You can not race more than 5 horses at a time, and you do not have a watch.
68	Knotted	By how much is a rope shortened when a simple overhand knot is tied in it?
69	Folding Polygons	Which polygons can be folded (by one straight fold) along a line of symmetry so that the resulting pieces are both similar to the original? What if the fold does not have to be along a line of symmetry?
70	Ins and Outs	Take a strip of paper and fold it in half several times. Unfold it and observe that some of the creases are IN and some are OUT. For example, three folds produce the sequence In in out in in out out

		What sequence would arise from 10 folds? (if that many were possible)
71	Nullarbor Plane	A man lost on the Nullarbor Plain in Australia hears a train whistle due west of him. He cannot see the train but he knows that it runs on a very long, very straight track. His only chance to avoid perishing from thirst is to reach the track before the train has passed. Assuming that he and the train both travel at constant speeds, in which direction should he walk?
72	Sticky Angles	Given a supply of sticks, all the same length, and a supply of angles, all the same length, can you join the sticks together end to end at the given angle to make a closed ring?
73	Tethered Goat	A goat is tethered to the edge of a circular silo in a grassy field by a rope which reaches just halfway round the silo. How much grass can the goat reach?
74	Areas	Graphic 3 below this table
75	More areas	Graphic 4 below this table
76	Water Tank	Graphic 5 below this table
77	Shape Partitioning	What shapes have the property that they can be cut along a single straight line to form two congruent shapes similar to the original? Two similar shapes similar to the original? Two similar shapes?
78	How many triangles?	Graphic 2 below this table
79	Painting Cubes	If you paint a larger cube that is made up of smaller 1x1x1 cubes. How many of the smaller cubes have no painted faces, or 1 painted face, or 2…?
80	Blood Type	Blood comes in four types: O, A, B, and AB. The percentages of people in the United States with each blood type are shown below: O : 46 A : 40 B : 10 AB : 4 What is the probability that two people getting married 1) both have blood type O, 2) both have the same blood type?
81	Students	In a class of 30 students, there are 17 girls and 13 boys. Five are A students, and three of these students are girls. If a student is chosen at random, what is the probability of choosing a girl or an A student?
82	Oceans	Show that there are more possible seating arrangements in a classroom with 30 desks than there are drops of water in all the Earth's oceans.
83	Club Lotto	A local sports club is planning to run a weekly lotto. To win the Jackpot of €1000, contestants must match one letter chosen from the 26 letters in the alphabet and two numbers chosen, in the correct order, from the numbers 0 to 9. In this lotto, repetition of numbers is allowed (e.g. M, 3, 3 is an outcome). If a contestant matches the letter only, or the letter and one number (but not both numbers), they will win €50. Find how much the club should expect to make or lose on each play, correct to the nearest cent, if they charge €2 per play
84	Match Boxes	Match boxes customarily have their length, width and depth of different lengths. Three such boxes can be assembled into a rectangular box with all 3 boxes parallel, in three distinct ways. In how many ways can 36 boxes be similarly assembled?
85	Ants	Eight ants start at different corners of a cube. Suddenly each ant moves to an adjacent corner at random. That is, each ant walks along one of its three adjacent edges with equal chance. What is the probability that none of the ants collide?
86	Lock Codes	A safe has a code lock that unlocks if you unput the correct 4 digits, in any order. The lock has a keypad with the digits 0-9. How many different unlock codes are there?
87	Taxi	In your city there are two ride-share companies: Uber and Lyft. Your father uses one of those to get back from the airport, but leaves his phone in the car. You are given the following data: 85% of the taxis in the city are uber, and 15% Lyft. Your dad thinks he left his phone in a Lyft car, but he's not sure. In your experience your dad is correct about 80% of the time, and incorrect 20% of the time. He wants to get his phone back. Which company should he call first? That is, is it more likely that he left his phone in an uber car or a Lyft car?
88	Geysers	At the beautiful Three Geysers National Park, a placard explains that its three geysers erupt at precise intervals of time. Geyser A erupts exactly every 2 hours, geyser B erupts exactly every 4 hours, and geyser C erupts exactly every 6 hours.

		However, you have no idea how the three eruptions are staggered. Assuming each geyser started erupting independently at a random point in history, what are the probabilities that each (A, B, and C) will be the first to erupt after your arrival?
89	Productive Exchange	27 x 18 – 28 x 17 = 10 37 x 18 – 38 x 17 = 20
90	Remainders of the day	Find: A number that leaves a remainder of 1 on dividing by 2. And for which that quotient (the whole number result of dividing by 2) leaves a remainder of 1 on dividing by 3 And for which that quotient leaves a remainder of 1 on dividing by 4. Why must such a number be divisible by 3?
91	Rational Divisors	Given that 14/15 divides into 28/3 a whole number of times, we might say that it is a rational divisor of 28/3. Find all rational divisors of 28/3, ½, and of both. Does it make sense to talk about the greatest common rational divisor of two fractions, and the lowest common rational multiple of two fractions?
92	Warehouse	In a warehouse you receive 20% discount, but you must pay 15% sales tax. Which is better to be calculated first?
93	Hamburger	A hamburger is made of bread, tomato, lettuce and meat. If each of the four ingredients increases in price by 5%, by how much does the total cost of the ingredients rise?
94	Fred and Frank	Fred and Frank are two fitness fanatics on a run from A to B. Fred runs half the way and walks the other half. Frank runs for half the time and walks the other half. They both run and walk at the same speeds. Who finishes first?
95	Find the values	AA + BB + CC = ABC. Find the unique values
96	Cat in a box	A cat is hiding in one of five boxes that are lined up in a row. The boxes are numbered 1 to 5. Each night the cat hides in an adjacent box, exactly one number away. Each morning you can open a single box to try to find the cat. Can you win this game of hide and seek? What is your strategy for finding the cat?
97	Clocks	When my son was born my wife and I agreed that if he woke up before 5am, she would go and feed him. However if he woke after 5am, I was to go and bring him into our bed. One night when he woke up my wife looked at the clock and said it was my turn. I found this odd as it was quite dark but did anyway. It turns out my wife had mistaken 12:30 for 6:00 as she looked at the clock upside down. When will the hands of an upside down clock show a proper time?
98	Patchwork	Take a square and draw a straight line right across it. Draw several more lines in any arrangement so that the lines all cross the square, and the square is divided into several regions. The task is to colour the regions in such a way that adjacent regions are never coloured the same. (regions having one point in common are not considered adjacent). How few different colours are needed to colour any such arrangement?
99	Painted Tyres	Once while riding on my bicycle along a path I crossed a strip of wet paint about 6 inches wide. After riding a short time in a straight line I looked back at the marks on the pavement left by the wet paint picked up on my tyres. What did I see?
100	Average Speed	Driving through roadworks on a motorway, the sign said 'speed limit 50; average speed calculated'. I noticed that for a certain number of minutes I was going at 60mph. For how long do I have to go at 30 to be legal? At 35? Generalise. I noticed that for a certain distance I was going 60mph. How far do I have to go at 30? At 35? Generalise.
101	Rule of thumb	It is rumoured in some countries that the police will not stop you for speeding unless you are going at least 10% over the limit. One such country recently changed from miles to kilometres on all road signs. What is the new rule of thumb?
102	Handshakes	Mr. Lars and his wife Mrs. Lars went to a party and met 4 other married couples. Some people shook hands with each other, but no person shook hands with his or her spouse, and no person shook his or her own hand. Mr. Lars then asked each person, including his wife, "How many distinct people did you shake hands with?" Each person answered honestly, and surprisingly

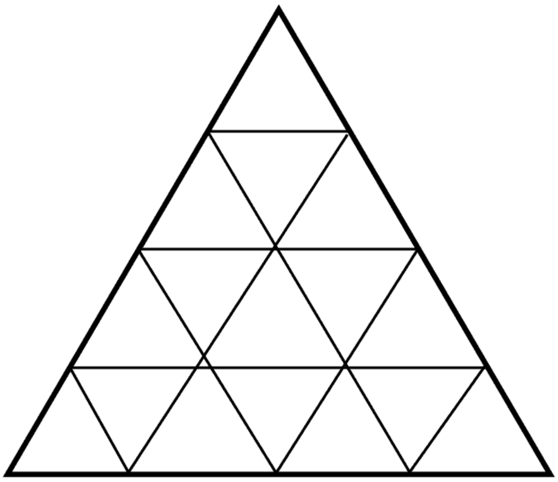
		each person gave a different answer! How many distinct people did Mrs. Lars shake hands with? And how many distinct people did Mr. Lars shake hands with?
103	Who is who?	Mr. White, Mr. Blue, Mr. Brown, and Mr. Pink are at a restaurant. Mr. Pink said, “I can’t believe it. The boss gave us names that matched our shirt colours, but no one got the same name as their own shirt colour. My name is terrible.” “Who cares what anyone’s name is?” said the person in the blue shirt. “Yeah, that’s easy for you to say. You have a cool sounding name. Maybe if the cleaners hadn’t messed up my dark coloured shirts I could have worn a different shirt and gotten a better name,” Mr. Pink replied. “Yeah, I don’t like my name either,” said Mr. Brown. What colour shirt was each person wearing?
104	Motorcycles	A cyclist ascends a long climb at a constant speed of 18 kilometers per hour (km/h). A motorcycle rally starts 1 minute after the cyclist. Every minute, for the entire day, a motorcycle starts the same long climb and goes until it reaches the end of the climb, which is the finishing line for the motorcycles. When the cyclist reaches the top of the long climb, he arrives at the same time as a motorcycle that is reaching the finish line. The cyclist then turns around and descends to the starting point of the climb with double of the speed of his ascent. The cyclist keeps track of when motorcycles pass him. By funny coincidence, the number of motorcycles that overtook him on his ascent exactly equals the number of motorcycles that pass by him on his descent. All motorcycles have the same constant average speed. The question is: what is the speed of a motorcycle?
105	Square Takeaway	Take a rectangular piece of paper and remove from it the largest possible square. Repeat the process with the left over rectangle. What different things can happen? Can you predict when they will happen?
106	Triangular Count	How many triangles are there on an 8-fold triangular grid?
107	Repaint	The squares of a chessboard are repainted randomly in black and white. Must there be a rectangle all of whose corner squares are the same colour?
108	Square bashing	Take any numbers satisfying a pattern of the form 42 + 52 + 62 = 22 + 32 + 82 Pair up the left and right numbers in any way at all, for example 42, 53, 68. Notice that 422 + 532 + 682 = 242 + 352 + 862 Why?
109	Wool winding	Wool bought in bulk for a knitting machine comes on cones. The machine uses several cones at one time but, once set up, it is tedious to have to replace an empty cone, and unpleasant to have to rewind wool on to an empty cone. If I have c cones with varying weights of wool on them (all the same colour), and if my pattern calls for k cones at once on the machine, what calculation will predict whether I can use all the wool on the cones without rewinding.
110	Travelling through time	In the 19th century, before the invention of motor vehicles, people would sometimes share a horse when making a long journey. One person would ride while the other walked; the rider would tie the horse at a convenient point and walk on, while the other would pick up the horse and ride on. The horse could rest whilst waiting for the second rider. This action might be repeated several times. How must the ride and tie options be adjusted so that the two travellers arrive at their destination at the same time?
111	Paper Clip Chains	A stationery shop receives a delivery of paper clips attached to each other in chains. During a “move”, the stationer may select a clip and untie it at both ends, which leaves him with this clip now alone plus two smaller chains, which were once on the right and on the left of the selected clip. 1. From a chain of 63 paper clips, how many successive “moves” does he need to perform, at least, so as to be able to supply, without any further manipulation, any required number of paper clips between 1 and 63? 2. What is the longest initial chain that allows one, in 8 manipulations, to supply any number of paper clips between 1 and the length of the initial chain?
112	Batteries	A drawer contains an even number of seemingly identical batteries. We know however that there are as many good as bad ones. To test them, Orlaith has a flashlight, which works only when the two batteries inserted are good. She tells John that she will need a maximum of 6 attempts to make the lamp work. 1. How many batteries does the drawer contain?

		A few weeks later, John finds himself in the same situation, while there are 20 stacks in the drawer (10 good and 10 bad). 2. How many tests will it take before John can make the lamp work?
113	How many children?	The average age of a family (two parents and their children) is 20. If we exclude one parent, who is 40, the average age drops to 15. How many children are in the family?

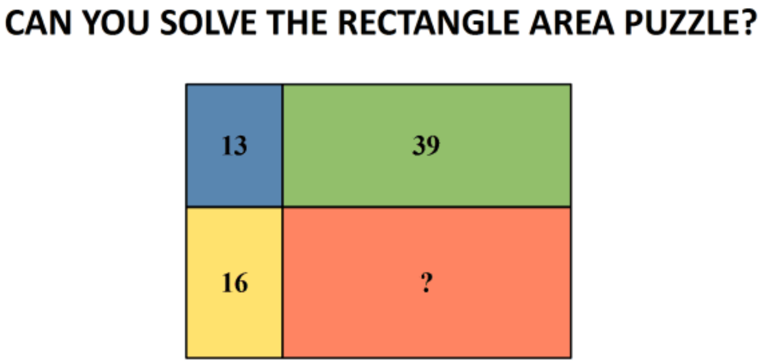
Graphic 1

1
11
21
1211
111221
312211
13112221
1113213211
31131211131221
……………………………………?

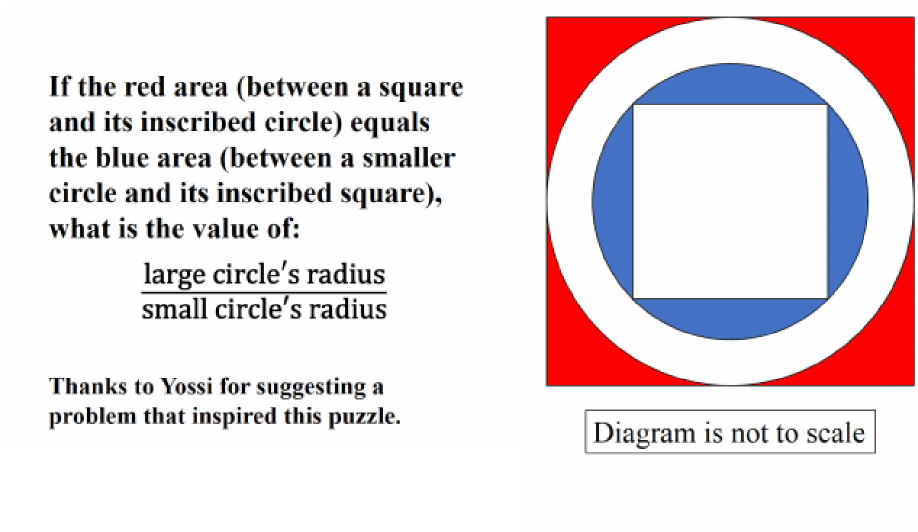
Graphic 2



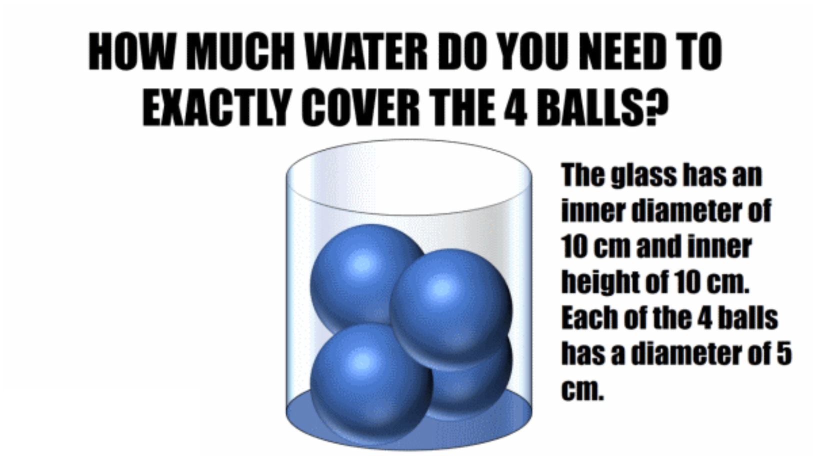
Graphic 3



Graphic 4



Graphic 5



Appendix H

Interview Transcript

Int: Yeah so, thanks very much for coming. Welcome along, and so my name is [Interviewer (Int)], and I'm asking the questions, or I suppose more accurately leading the focus group on behalf of Aidan. So I have these list of questions here and I promise not to keep you longer than an hour. There's a limit on how long (laughter)… if I have to stop halfway through a sentence, I'll do it for you. As Aidan said, we have 3 devices here, all just recording audio- no visual element at all. Any of the data harvested from this will be used anonymously (low giggling) – that's some industry jargon for ya, "harvested" (laughter). Yeah so there's anonymity attached to this as well so feel free to say whatever you want to say.

S1: My name is S1 and I'm… (Inaudible; laughter)

Int: I'll get to that. (Laughter)

S2 hiiiii S1 (laughter).

Int: Yeah so, I'll be asking you questions based on your experiences of studying problem solving maths with CTYI with either the Early University Entrance or the summer programme. The questions asked are specifically in relation to the 3 hours of problem solving studied – so the two hour class and one hour tutorial. The other two hours studied are not part of the research – so I think Aidan just said that far better than I just did, but just to reiterate. Yeah and so how do these work. So first there are no right or wrong answers to these things.

S1: Well… (laughter).

Int: Contributions…

S1 There's always a wrong answer (laughter).

Int: I feel we'll bridge that quite soon (laughter). It's about expressing opinions; it's about building on other people's opinions and it's about speaking honestly. And respecting of everyone. What we want here is that everyone contributes somewhat. So what I might do just to begin if you wouldn't mind going from left to right – if you could slowly state your name, and maybe tell me if you were on the 3 week or the 14 week module. Just so I get to know everyone.

S3 I'm S3, I was on the 14 week module

Int: Thanks, S3

S4 I'm S4, I was on the 14 week module as well.

Int: Thanks, S4

S5 I'm S5 and I was on the 3 week module

Int: Cool

S6 I'm S6 and I was on the 14 week module

Int: Thanks … (inaudible)

S1 I'm S1. I was on the 14

Int: S1, thanks very much

S2 I'm S2 and I was on the 3 week

Int: S2. 3. Sound.

S7 And I'm S7 and I was on the 14 week

Int: S7 on the 14. Cool. Thanks very much guys. So I suppose we'll hop straight in. And S5, I suppose I'm looking straight at you so I might start with yourself (laughter). And I would like to know, whether or not you learned any new skills while you were studying the module.

S5 I did learn new things, and I find that from what we did when we were doing the programme – like in school now I find I realise things and I recognise "oh, I did that before" and… yeah.

Int: Brilliant. Guys, a lot of nodding heads, and so, S7, would you agree?

S7 Yeah, I actually would, yeah

Int: S1, you agree too?

S1 Yep

Int: Ok, so we're talking about new skills. Has anyone any specific skills they'd like to mention?

S2 Yeah, I think that like a lot of the sorta like number theory stuff that came… sort of proving that something could be divisible by something for example like. I choose to not do TY, so now I'm in 5th now and I'm seeing a lot of that is coming up back up in fifth year and I'm noticing that that's... like I've seen this before, I know how to do this, and it was taught very well by Aidan, so very much it's a…

Int: Very good

S1 Take his words, and pretend I said them (laughter).

Int: Very good, very good. Yeah and so, I suppose one of the aspects would be I think ye did a lot about extending problems throughout this course, and so S3 you're nodding at me now, so would you maybe elaborate on the extension of tasks and maybe whether or not you consider that a skill or what skills are involved in that?

S3 Yeah, so I definitely think that was a very useful element to… I think it allowed you to separate out the important and the not important elements of a particular problem. And like understand what… like what was sort of underlying your solution to it, and make it much more general than like "how would you solve this"; for example if the numeric case that you were asked about was different to the specific question you were asked. Like what are the principles; how do they differ and how are they the same, from that?

Int: I think… generalised ability. So I can look at a problem and it's quite specific- but from that I can extrapolate outwards and say well this is the structure maybe.

S3 (noise of agreement)

Int: And so I think when we talk about generalised ability, we might talk about transferability. And so, S6, would you be able to say that those skills you learned here (5:00), have you seen yourself or have you been aware of yourself using them maybe outside of the maths module where you learned them?

S6 I suppose so, yes. A lot of the logic used would very clearly be used elsewhere, and the whole idea of thinking of the problem critically- so it's not just taking the numeric case, it is generalising it. It's realising that it is… there is more going on maybe when as more elements get added in… how that changes. And… yeah.

Int: I think I agree, and I understand what you're saying. So that's really cool. So S4, I managed to skip by you there and you were delighted with yourself. I'd like to ask you – how did you find being asked to reflect on questions, because I think that was an element of your course.

S4 It was weird at first because I had never been asked to do it before, but like I suppose it kind of helped me because if I (6:00) didn't maybe realise I didn't understand it fully then someone else might be able to like tell me why it was and then I could help him, and it kinda just… it was a different way of doing problems. Instead of just looking at it and doing it on your own, you were cooperating with people.

Int: Yeah, I think… I think "weird at first" to me may imply that it was new.

S4 Yeah

Int: It was probably a new skill, and yeah I think we've probably (inaudible CT)… more or less. Cool. And I suppose we talked about transferability of extension of problems – with reflection, just off the top of your head, has anyone an instance where they might have been able to take that ability to reflect and apply that maybe in other aspects of their school life?

S2 I mean it's come up a lot like… even like during the course when we'd be doing a problem, and we'd be asked to do the journal entry, and then as soon as… like, cos you'd have to spell out what the problem was, then during the spelling out of this problem – exact step by step what this problem is, you kinda can see it in a new different light because you are telling yourself basically every single thing that exists in this problem. And then upon doing that you can look at a new angle, which you can then use to solve the problem, which has happened a few times with the tutorial question. That like, after you do the journal entry, then and only then will you realise how to actually solve the problem. And then it comes up an awful lot in school as well, because like if you write a really long essay in Irish or French or in English, once you read over it you realise that you've hit all the key points and you can see how you can improve on that.

Int: Wow, that's really cool. So it's kinda like a meta-awareness. (laughter). Powerful, powerful stuff. (laughter). S2 you're gonna have some huge soundbites today. (laughter). Cool, that's brilliant, guys. What I'd like to move onto now, and I suppose this might not occur to ye very often in school, but I want to ask ye about challenges, and I suppose, S6, I might just go to you again – did you find the module to be challenging, and, if so, what aspects of it?

S6 Well, yes, because of the whole extension aspect of it, every problem… there was always a little bit more that could be asked. There was, either it was generalising it (8:00), or if you add an extra step to the problem how does that change it. There was always something else that was there to… that could be played with with the problem.

Int: Yeah, extension is definitely one of them. S5, with respect to its being challenging, is there any aspects that jump out to you?

S5 (pause). As in like…

Int: As in like the problems themselves, the length of the work, reflection, group work, any of these things…

S5 Well I suppose with the questions however challenging it was varied on the type of question, and so some would be more challenging than others, and some would be easier, and…

Int: I think that's where S6 was going with the extension stuff

S5 Yeah

Int: It's that, regardless of how the problem is initially posed, that I can make it somewhat challenging, or somewhat less challenging (S5 noise of agreement) if the need arise. S1 would you have any parts of it that you found challenging at all?

S1 I mean like… (pause) not particularly (laughter). But that's just because I don't find it hard. I mean like, the extension stuff- considering we did everything we could to make it as painful for us to solve as possible, just because we were like told like make an extension so we were like ok – how hard can we make this? And then we'd just go… like you get the marker, you throw it as far as you can across the room and then try to solve it from there. So in that aspect, like the fact that you were able to do an extension means that even if you are finding it easy you just make it as hard as you need it to be. So… yeah… we made it hard for ourselves.

Int: Yeah, but that… and that's exactly it. It's… I suppose one of the skills is the ability for you to take something and challenge yourself with it, because it's not a typical problem that may challenge you. I think that's perfectly valid. I think when you talk about things that may be challenging and they're often even just isolated things within a class which you may not have (10:00) found overly challenging, this notion of being 'stuck' either during, or at the beginning or at the end of a problem, can often be used. And so the language of 'being stuck' – did anyone ever, I suppose, experience that while doing a problem, even if it was a problem… yeah so…?

S7 (interjects) Yeah

Int: Sorry S7 do you want to elaborate slightly?

S7 Yeah like I have found that like in maths and even in this course the problems took like an hour so like – the problems that he gave us – so like half an hour into that you'd encounter difficulties where you'd be stuck and wouldn't know where to go and you'd just have to try everything and just, throw the kitchen sink at it.

Int: yeah and so, that was the very next question I was going to ask was – how do you overcome this? So you're saying, some sort of resilience or productive struggle…?

S7 Yeah just keep going, and just try to do everything until you get to a solution.

Int: Ok. Rough show of hands for who'd agree with that, being stuck or that sort of thing?

S1 (interjects) I mean, you just keep writing. If you're stuck just keep writing every tiny thought that pops (D: yeah) into your head, and eventually like half an hour later you'll realise… oh wait it's the… (sound of obvious realisation). And then you'd…

Int: (interjects) It's the first thing I've written down. (Inaudible CT).

S2 Yeah that happened to us aswell. 'Cause since like if like my group of 4 got stuck there's 4 different people with 4 very different ideas on how to solve a problem. So like, surely at some combination of what has been written down can solve, and often times that was the case – that it wasn't just one person's entire thought process that was cleaned straight through and got the solution it was like everybody sorta like pitching in. like 'cause, people understand and people can sorta like do maths differently to one another, so it's kinda… so everybody helped one another because of people being different and because different people were there to help out.

Int: Cool. So from your answers, something I'd like to say and you can say whether it's true or false – this notion of getting stuck can often be productive, it can often be helpful. (12:00)

S1 If you don't get stuck it's no fun.

Int: well there we go, 'cause otherwise you're just bored.

S1 it's not fun to just write numbers. It's fun to like… writing the numbers isn't the fun part. That's the bit that's like ah c'mon, there ya go now I did this, now this is the fun part – how am I gunna use this. That's the fun bit. (Int: ok) The fun bit is getting stuck so you can get over the fact that you're getting stuck.

Int: Yeah and so, in-built in that is this notion - it's a valuable thing to experience because it's an experience in itself. It allows you to kind of excel or move on at least, and move forward. That's really cool. Before I move off this notion of being stuck – for fear of irony – (laughter) I would ask, I suppose, has anyone, again coming back to this notion of in school or otherwise in life, has anyone since doing this module and having experienced that notion of being stuck and moving through it. Has anyone an instance where that may have happened where they became stuck, but now they have that tool that could now get them through it or allow them to persevere?

S1 I mean you use the same methods – which is keep writing. You just… it's the same technique though you learnt while we were there, you just use it with... yeah, yeah, it does help.

S2 It just shows up all the time like even if you're not stuck, even if you just need to attack a problem from afresh, then even just knowing what helps you get stuck, so if you kinda catch yourself before you do get stuck you can apply those methods and that kinda either eliminate you getting stuck or it'll just help you plough through the problem further than you would have before.

Int: and so, what I'm getting, particularly from your answer S1, but kinda there's this agreement in the room that this notion of being stuck, you persevere through it. And what I'd like to know is whether it's fair to say "do you know what, my experiences on the module, where I was deliberately placed in a situation where I may be stuck, whether that has helped me and maybe I've become more comfortable with that aspect and maybe I can now feel… and so S4 you're nodding there, and S5 aswell. You think that yeah the module probably has helped me with when I get stuck now

S5 Yeah

S4 Yeah I suppose, (14:00) I mean like before if I got stuck I might just kinda like… just drift away from the problem but now I like, I kinda just keep going at it and just try to solve it. Just try come at it from different angles and stuff, 'cause that's what the module taught me, like if my approach isn't working then maybe someone else's might. I suppose.

Int: S3 would you agree?

S3 Yeah… yeah I think the programme definitely like did give you more confidence that like you actually could solve a problem like no matter how much time it would take. And it would sorta give you the motivation to apply more and more different angles of thinking about it and more and more techniques because you sorta were like really, really driven to get the answer to the problem and to understand it.

Int: Sure. And so S6 you're… do you agree with that?

S6 Yeah I would. That… (long pause)

Int: Cool. (S8 inaudible) The word that's kinda on the edge of my tongue – you're kinda describing to me a… building a resilience. (murmured agreement). And kind of a resilience through this kind of struggle and this being stuck. And learning to cope and learning to deal with problems and probably grapple with large problems. That's just really cool, and I wanted to encapsulate it for myself there. So I want to move, I want to start to discuss mindset with you. But before that I want to just read out a very short definition that, I suppose, explains what a growth mindset is, at least in Aidan's words. So he has written – "a growth mindset indicates that you believe you can get smarter, or better at something, through effort and over time. A fixed mindset is the belief that you only have a certain degree of intelligence or ability, and no amount of effort can change that". So we have this, I suppose, dichotomy of fixed vs growth mindset. And so S5 I'd like to ask you first, if that's ok, do you feel that your mindset changed over the course of this module?

S5 Yeah, 'cause before I did it I'd find (16:00) if you were doing maths in school I'd be like if I couldn't do something I wouldn't… I'd just say "oh like I can't do it" or something, but now if I can't do something I think aw but I can like, and I try and find a way to… like I don't say no I can't do it, I just try… (Int: sound of agreement). (Inaudible)

Int: Brilliant, and that's cool. So you're actually saying the module has helped and yano what I probably was fixed before whereas now I'm closer to growth mindset. (S5 noise of agreement). And is there anyone here who's mindset would still be fixed? (long pause). So I'm kind of getting no's. S7 you're…?

S7 Yeah I'd say growth mindset.

Int: You have a growth mindset now? And would you say the module helped you with that?

S7 Yeah because I believe you can always get smarter, like you can always just increase like what you're doing and your intelligence and this course would have helped. So…

Int: (Noise of agreement). And S1 you may not have been overly challenged for certain aspects of it. Would you still feel that it may have helped you?

S1 I had a growth mindset before, (Int: Brilliant) and this didn't make it worse. So… (laughter).

Int: But it may have instilled in a belief that yeah that idea I had about this growth mindset it's probably true.

S1 Yeah

Int: Great so it didn't actively hurt. I'll take that.

S1 There ya go (laughter)

S2 It's a very conscience… had you not… had you had a fixed mindset before, you are basically forced into a growth mindset with the problems 'cause you do have to stick at it, and then if you do end up solving it, that is almost a proof in and of itself that there is a growth mindset there. If that makes sense.

Int: yeah yeah, I think you're actually acting out the proof of that growth mindset.

S1 We are the proof

Int: There ya go. There ya go. Well there maybe… S6 would you believe it's fair to at least hypothesise about some connection between these moments of getting stuck, maybe if I'm able to get through those, I'm probably have a growth mindset, or develop a growth mindset. Is there some link (18:00) between those stuck moments and your outlook on mindset?

S6 Yes, because, if when you get stuck and you just abandon it you're just accepting 'I'll never get over that (Int: yeah), I'm never going to grow'. Whereas if you keep at it and you do overcome that challenge, then you have grown and you've proved to yourself that you can grow further than what you may have thought before.

Int: Yeah, brilliant. And now, and S3I might start with you, but I'll open this to the room eventually – we're all agreeing that growth mindset is how we choose to view things, and that the module has helped us with our growth mindset, but growth mindset should probably permeate maths and go into other aspects of our lives. So firstly, would you agree with that, and then secondly do you have an example of where you may have done that in your life?

S3 Yeah well, I mean it definitely is something that is very beneficial and I'd say like… just in other things, sort of, and also things vaguely related to maths, like in other sort of scientific and physics problems I've been working on. Particularly with like, I did the Young Scientist last year, and like at times it seemed like we were never gunna get the project done or it was just gunna turn out terribly or whatever, but it was sorta like the persistence that we had like more confidence and we knew sort of how we could latterly think about the problems that sort of, that sort of motivated us to keep going. I suppose.

Int: Brilliant. That's really cool. S4, would you agree that the module has helped our growth mindset approach and then can you think of anywhere where you may have subsequently used your growth mindset to your own benefit?

S4 Yeah. I suppose, just in normal school work maybe. Just 'cause I know that if I keep working at it and keep working at it, I'll get an answer. I might not get the right one, but

I'll get somewhere near the right one. (20:00) And I'll understand where I went wrong because if I just leave it I won't understand it… I won't understand how to get the right answer and I won't understand why I got the wrong answer.

Int: That's really… to just channel S1's "it didn't hurt" type of thinking (laughter). (door opening). Hey!

S8 Sorry I'm late.

Int: That's alright we might have been early. Do you mind if I ask you your name?

S8 S8

Int: S8, how are you S8?

S8 I'm good thanks

Int: S8 we're talking about growth mindsets (S8ok), and that means that 'yes I can get better' and 'yes I can improve' vs… (S1: inaudible). (laughter) I have the feelings stick you're not allowed talk right now. So yeah we're just talking about this notion that we have this growth mindset where I can improve; I can get better; I can increase my intelligence; I can do all of these positive things, vs a fixed mindset where you're as smart as you are; you're as good as you are type of thing. And so, yeah while I have you caught off guard – I'd ask you what your outlook would be; would you be… would you consider your outlook to be, or your mindset to be a growth one or a fixed mindset?

S8 Depends what you're looking at. Like for example with maths the more you practise the better you get at it, but stuff like English I think there's a degree where you can't get better. Like you can improve to a certain point your vocabulary or your grammar. (Int: sure) Yeah so that's sorta my opinion on it.

Int: Ok, so allow me to paraphrase and correct me – yes growth mindset but it varies within a certain domain.

S8 Yes

Int: Fantastic. I like that. I like that a lot S8. And so, can I ask you with respect to the module that you did with Aidan – would that have had an impact positively or negatively on your mindset?

S8 It would have had a slight positive one. Yeah I definitely noticed that I was getting better as the weeks went on, and I was able to do things I wasn't able to do, but then you'd notice that when you were studying aswell, but it definitely reinforces it. And probably would definitely help people who maybe wouldn't put much time into studying or spend studying where that doesn't help them as much because it's organised.

Int: And now I'll let you take a breath. And S1 I want to pick up on something S8 said and I want to ask you your opinion on it – which was that 'yes growth mindset, but maybe domain specific', or the factor at which you can it's very it's severity may be domain specific.

S1 Well I mean, the rate at which you get better would probably vary, but like even if… even if you feel like you're not improving at all, if you keep doing it over and over again you'll like figure little bits out. You'll slowly improve. It's like there's always… Having a growth mindset will always benefit you. You always can improve, it's just that maybe you don't… some people improve faster than others - which is why some people seem better than others. it's 'cause they just happen to improve faster in that area.

Int: Ok. And so my… the severity of my growth mindset… so I've a really aggressive… I have really a growth mindset – does that… and then if I kinda plot that vs effort put it, will that correlate to improvement, yano what I mean? So do those two… are they the two key factors that decide I suppose (S1 well) that decide how aggressively you improve

S1 well, how much you think you can improve and then just like your also just your natural ability with that area. (Int: ok) Some people are just better at maths and so like they're gunna improve faster at maths than people who aren't as strong in maths so… yeah… so based on your growth mindset and your natural ability in that area probably pretty well correlates to how fast you improve in that area.

Int: Good. S5 can I ask you the question I started to ask the guys, which was whether or not you think you have a growth mindset; whether or not you think the module helped you; and then, if yes to both, have you an instance you may have benefited from your growth mindset since you began, or finished, the module?

S5	Yeah I do think that after doing the programme that I have more of a growth mindset and just in like day life I remember doing I don't just like stop I just keep trying to improve on things.

Int:	Brilliant. So S2 and S7, I haven't necessarily asked ye questions on this type of stuff. Have ye an anecdote or anything that has occurred to you since you've finished the module that kind of encapsulates that 'yeah I have a growth mindset' (25:00).

S7	Well just say with like tests in school, like the first few tests you might not have done as well and then you're getting better and better as the year progresses.

Int:	ok

S2	I feel for me that's kinda like, sort of, like after CTYI, after the maths programme I did, I decided that I'd take back up piano because I dropped it when I was like 7. And then I decided that I'd take it back up because I was like I might as well go for it and I think that combined with… I think that there's just some things you have to choose to believe, like there's no proof you just have to choose to believe in. And in the sense of piano it's just like you have to just choose to believe that if you put the effort into it you'll get, you'll get to where you need to be. I feel that the maths kind of portrayed that eloquently, because I feel that if you believed that you could solve the problem most of the time you could just plough through – just use as many different methods are you could and get that final answer. And so that does apply outside of maths – outside of school even.

Int:	That's profound. I like that. It's kind of like proof by exhaustion only its piano by exhaustion. Ya still get there anyone, that's really cool. Brilliant so I think I've annoyed ye enough about mindset and resilience and all of these type of things. And maybe it's no harm to talk about the primary object of this, which I think was problem-solving. But first I would like to know, and maybe S6 you're the man to answer this for me – what approaches are taken in secondary schools in your experience to problem-solving, did ye do it any junior cert level or…

S6	Yes it is a thing at junior cert level but with the way that with problem solving… you're taught how to solve problems of that type first and then you're given numbers to apply it. Whereas with the problems we were given you were just given the problem straight – you had to find the ways to solve it yourself and also it was about more than just being able to solve it with those numbers that were given, which took the part of the extension, the whole idea of generalising the problem. So that was something that was quite different.

Int:	S4 would you have experienced problem-solving or would you have been taught p-s at junior cert level?

S4	I suppose yeah, but like I would have been taught kind of… it would have been an individual p-s instead of doing it in groups like I did in the module so it was just my outlook a problem instead of other people's outlook on a problem. I found that like other people's outlook on a problem helped much more than me just sitting there doing it myself (Int: sound of acknowledgement), because they'd think of things that I wouldn't and I'd think of different things they wouldn't.

Int:	Yeah absolutely, so S3would you pick up on that and maybe speak to the effect of group settings on problem-solving – so how would that change the way you approach a problem, and how did you feel about it?

S3	Well the… the group setting definitely like helped a lot at it because it felt like all of your skills were kind of combined and you were constantly explaining ideas that maybe you had heard about or elements of maths that maybe one person was better at than another, and there's kind of a constant like dialogue back and forth, and you're sort of improving the group as an entity overall.

Int:	Ok, and so if I harp back to my first question – S5 would you feel comfortable saying that, well I suppose assuming do you agree that you feel the group-work had that… a similar effect with you, or I was in this kind of collaborative mind-frame. And so if I was to ask you about the skills you learned on the module, and I told you suppose communication skills or verbal skills were a thing, would you agree that the module probably helped you there?

S5	Yeah, because you were never on your own doing the problems (29:00), so you were always talking to at least 3 or 4 different people and you were always discussing the problem and getting other people's opinions. So you were constantly talking and discussing.

Int: Ok, I'd like to put forward maybe an anecdote from my past and maybe it's from your past as well. My memory of being put into groups, sometimes it has the exact same effect that it had on ye, sometimes you're sitting looking at two people who yano you're kind of… you're kind of like yano I'm going to solve this problem anyway, and sometimes there can be this strange group dynamic where it's not a tripod of people collaborating together, sometimes it's something else. So, what are your… (laughter) (S6 yes) S6 you have whiplash you're nodding that hard.

S6 See, the reason I think that the group-work worked in this setting is because everyone was there wanted to be there. We were already equally motivated. Everyone wanted to learn, whereas that's not something you get at school all the time (30:00). Groups are just chosen willy-nilly and it's just, a lot of the time it's just most of the people sitting back while one person does the work.

Int: Yeah, I do think intrinsic motivation is a huge thing. S1 you're kind of giggling as well. You've been in groups I'd say that have this dynamic?

S1 Like, so many times you're in school like we're doing debates in TY and they're like "ah sure S1 will do it" and they just go off into the corner and have a chat and I'm sitting here with my copy being like "what am I supposed to do?". Because the whole thing of like… there's some people… if people think you can solve the problem a lot of the time they just won't be bothered. Which then puts more pressure on you to solve the problem and if you can't solve the problem you're screwed. Because then it makes you stupid. But like… if everyone in the group is like… if everyone in the group is motivated towards solving a problem it makes everyone in the group feel like they're better at that thing, just because everyone is contributing something. Like if you're sitting there on your own and everyone else is doing something all the pressure is on you to solve it, and also there's just one person excelling ahead above everyone else then no one else really has a chance to show that they can do it. But then if everyone is working together that means that everyone gets to contribute… everyone gets to actually improve. And yeah it's beneficial for them.

Int: Absolutely. S8 can I ask you a question that's kind of parallel to this but not necessarily linked in any other way is… there's this thing in secondary school syllabus isn't there where there's 'concepts and skills' - I think it appears on one of the papers as such - would you consider that to be p-s?

S8 Well what I think happens is a lot of these courses is they teach you how to do something and then everything is very clear once you know how to do it. So for example, if you get compound interest in the question and it's very clear that it is compound interest, so you're not really solving a problem you're just turning words into numbers. Rather than having to… very often you know how to put the words into numbers rather than having to figure out for yourself

Int: Sure. It can be somewhat superficial.

S8 Yes

Int: S7 would you agree?

S7 Yeah… (Inaudible). The concept and skills I think you said, they're just like theorems and proofs and everything and they don't actually teach you… they just teach you the layout of what to do and get you to learn that off instead of actually teaching you the skills to like be able to do it yourself.

Int: S2 do you agree aswell?

S2 Yeah I agree with that, because it is sorta like a lot of it is sorta just learn this off, don't question why just learn this off. This is how you do it. But in secondary schools you're not really taught who came up with it or like why do you do it, or maybe like why is this the most optimal way, or is it the most optimal way we don't know that. But we're just taught learn it off as such and don't (33:00) learn the whys or the reasons why, just kind of learn it as is and that's about it.

Int: I'm going to reward you candour with the most open-ended question I think I've ever given in any of these instances, and I would ask you to tell me what authentic p-s is to you, or means to you or what would that look like?

S2 As in… I mean… I feel that that's where the branch of applied maths comes into play, 'cause a lot of applied maths is taking these real world problems, a lot of them word based with a couple of numbers in between; then you have to draw the diagrams, you have to do all the maths to work it out and then and only then do you get your final answer. I think in terms of just raw what is p-s, it is solving problems, it is dealing with real world issues or like superficial issues, such as like I dunno somebody's car broke down but it can also be about like how to solve global warming. That is p-s, and I feel that if you learned to problem solve on a basic… on a superficial level you can apply… because everything can be broken down into fundamental steps. So if you sort of learn the most fundamental part of everything you can apply that to a much larger and larger and larger problem. I feel.

Int: Ok. S5 can I ask you, and picking up on the themes of S2's answer, were that p-s can't be this precooked, well-written question that has all the ingredients in it. So it can't be precooked, and it has to have some tangible real life meaning to the person who's answering the question. So typically you don't really care about like payments on a mortgage or something like that. So would you believe that for it to be a real problem it can't be precooked and it probably has to be some kind of context based issue?

S5 Yeah I think, but… yeah just like sometimes it could be a little bit if its… if there's no way of you getting a certain answer without some information there might be some information already in the question but it's good to not have the information that you think about it and you think how can I do this and work it out yourself. And then you learn by doing that.

Int: Ok, that's cool. So S3 can I pick up on things that you may have implied and tell me if I'm right or wrong or anything else, but I think that you do a decent amount of work in physics, you have an interest in physics and you've done some BT stuff; and so to harp a really good scientific question is a kind of problem solving mentality. So, I'd like to ask you is there anything that you took from this module, with respect to p-s, that has helped you I suppose in your studies of physics.

S3 I think so yea. It's all just like the general idea of looking for the subtle ways in which concepts or ideas are manifested in a given problem or a given thing that you're trying to gain insight into. It's like, that obviously not a technique that's not just specific to a mathematical problem, and I don't think it's even just specific to a scientific problem. It's like looking at a certain issue from as many angles as you can to understand what ideas underlie it, and, I mean fundamentally from that, how it works… I suppose.

Int: Yeah that's very good. I moved that around a good bit before I got a chance to get back to everyone. So I suppose if I bring this back to within a general sense, p-s either in secondary schools or as a result of this module, has anyone any comments they'd like to add in given how far that moved?

S1 I don't think there is much p-s in secondary schools. In the junior cert you just get given… I mean the entire book is just 'these are the problems that they will ask you, this is how you solve those problems'. (37:00) It's not problem solving if you're told how to solve the problem. All you're doing is taking numbers and plugging them into this thing that you have. Even in applied maths, like the is a problem solving aspect to applied maths, but the only way to do that is to just read the question and just ignore so much it telling you how to do it. Like our maths teacher gives us a booklet and it'll have all the different problems of this type and it'll have this is how you solve this problem, this is how you go through it; but the problem is to actually have a p-s aspect, you have to stop listening to the teacher and just look at the page and think 'ok try and solve this myself'. Because like there's a bunch of different types of problems you can get in applied maths, like ok you're crossing a river, what's the shortest distance you can… like… they can ask you, given these measurements what's the optimal route… what's the… what direction do they go at to get to the other side as quickly as possible.

Int: Launch angle isn't it?

S1 Yeah, so like if you listen to the teacher they'll go ok so this is how you draw it, this is the diagram you draw; put the numbers here and then you go through this method and a number comes out. Whereas to actually have a problem solving aspect you have to figure out where to put the numbers; you have to figure out oh this is how I should draw it; this is how I should turn this concept into a mathematical thing and then you have to figure out the steps from the question to the solution. If you're given the steps it's not really problem-solving.

Int: Yeah, I think that last line really summed up your argument quite well.

S2 I think sorta on that note is sorta that it varies greatly from school to school. Like I go to effectively a DEIS school in Blanchardstown right, so it's kinda… they're incentivized to keep as many people at higher level as they can… higher level in whatever subject so the school can get grants or anything like that. So, and because of that, my year of like 230, there's like 30 people who do higher level maths, and that number is going to drop. The teacher has said that there will be more people who will drop. And because of that, and because they try to keep and sustain as many people as they can, they don't really have time to teach you the whys and hows; they only have time to teach you like how to do a problem and not why that problem exists, why that problem could be useful, and like how that formula was derived or anything like that. They just have to go straight through. It varies the same from school to school, because in another one of my friends schools they have a … in TY they have a computational thinking module, like a really well done (laughter)… posh school, very posh school. But it just, it just, I feel it shows the discrepancy between schools and that it just… it also kinda shows that the government doesn't really care much for this. 'Cause like my school is sorta just like bare bones, barely functions as it's own school, and it really can't quite teach… (laughter and voice: you know this is being recorded).

Int: Anonymity!! (laughter)

S2 I mean it's being taught as bare bones so that's sort of why they don't have the time to teach you the whys and hows. So if you were to ask like where was this formula derived from the teacher has told me a few times 'go look it up online', 'cause he doesn't have the time to like show me or to show anyone in the class (40:00).

Int: So I think, for a variety of reasons that is beyond even the capabilities of this group, what happens in school can be some, depending on the strength of language, at the very least watered down up as far as neutered p-s – where you're provided with a streamlined solution technique.

S6 Even with applied maths, the whole idea of question one you're immediately thinking these 4 formulae will come up

S1 Yeah!

S6 These are the only ones I'll ever need for this question (laughter). So that's not problem-solving.

S1 I mean to a degree…

S6 To a degree it is, but it's not fully p-s.

Int: nah I think…

S1 Like they could take those four equations and built a problem out of them, that you're not specifically told like how to solve the problem beforehand, but like…

Int: You'll still know the foundations are those 4 equations

S6 Yeah I do because question 1 is written on the top.

Int: You were going to say something

S8 I know what happens in my school it's actually almost the opposite of S2's – tell everyone to go down to (laughter) lower level if you can't handle higher level. It's like they tell everyone after a month you should go down, and they keep saying it to them if they don't improve. (Int: ok) But what happens in our tests… so in class it's very much similar. Then in the test what they do is they give tests where they won't give you what you're normally given, but then it's like another stage upon that; so it's sort of like you have to figure out a small bit and then you put it into the actual p-s. So then… it's sorta…

Int: So they're using this technique whereby we scare people… kinda like the Scooby-doo approach – where we scare them away, but the people who stay around really benefit.

S8 Well it's not… it's not really that because the approach does work – it gets people the best marks they're gunna get based on… well like I went to *school name* so I know that they tell people to go down and if they go down it's because they can't do higher level, and they can't… they will do better in ordinary level. But then the tests that you do get, they're… a lot of the questions will be harder than the usual questions of that type on the test we're given. So it sort of prepares you and you're given a small bit of p-s, but not too much.

Int: Ok, ok.

S2 And I think another thing to add just onto that, is that like I see a lot of people in my classes that they do try their absolute hardest to sustain higher level and I fully salute them for that, but the issue becomes is that if they ever do get a test back and they don't do so well on it, instead of saying 'ok where did I go wrong' they're like 'oh man I'm so stupid I should just drop'. But the know, they say to themselves they can't drop for whatever reason it may be, but it almost kinda like feeds back into the growth mindset sorta thing, that because they don't sort of like think that they can improve on in maths or in English or in irish or whatever, they just kind of like stay where they are almost.

Int: Yeah, no I like that you linked it back to the growth mindset thing, so I think we were tipping around that for a few minutes where…

S1 I was just thinking there, he's like they just tell everyone if you can't… if you're not doing good enough just drop, you're never gunna do good (laughter).

S8 No but it's because the ordinary level maths course, and the higher level maths course, they don't overlap everywhere, so (S3 yeah my school is kinda similar) the longer you're in higher level, the more likely you are to just have no idea what's going on because you have no idea what's going on for 6 months, then suddenly you drop to OL and you still have no idea what's going on because you haven't been able to figure… because you have missed out on what they've done and you haven't gotten anything gotten any benefit out of the higher level anyway.

Int: And S1 reminds me there, because we spoke at length I think just before you came in about those notions of growth mindset and fixed mindset. And what we all, I know now, that we all in the room have in common, an appreciation that we all have a growth mindset, that the module helped us with it and then we went on to talk about p-s and we kind of looked at the sunny side of it. And so maybe you can empathise with people who may not have that growth mindset, and so then p-s to them, even (44:00) this watered down version, might actually be a significant thing.

S1 But the problem is they don't know how to problem solve, like as much as problem solving is like oh yeah you're meant to figure out the solution on your own, yano you can be taught how to problem solve – you can be taught methods you can use to like understand a problem, because the main thing about, with problem solving is you need to understand what the problem actually is. And I feel like that's the problem with a lot of people that struggle with maths is that they learn the methods like 'I do this and I do this' but you go to them the second you say 'why are you doing that' and they haven't a clue because they just learnt it off. The better you understand the concepts you're dealing with, the easier you're gunna find maths.

Int: Can I tweak the language slightly and point it at you S5. 'They don't know how to problem solve' – would it be fair to say, because this is what happened to ye, they may not have been exposed to p-s, because I think expose is a nice way of (45:00) saying what actually goes on. You're thrown in at the deep end and told find your own armbands; it's not that the armbands are here. So it's an exposure to p-s, and that brings me back to the group thing. When you were first exposed to p-s in this module, what was that like for you?

S5 it was… when you realised there was a different way of thinking of the questions, that you weren't just like 'oh yeah I know how to do that I just do the same as I've done every other time', you realise oh I have to try to think of another way myself to figure out how to do it. You weren't just told yeah that's the way you do it. You didn't just know instantly.

Int: Do you agree S4?

S4 Yeah I suppose, and also with the kinda people not being exposed to problem-solving. It also depends on what kind of teacher you get 'cause if your teacher just draws things up on the board and says' right lads take that down, this'll do ya for this question, this question and this question' or getting a teacher who will explain why you do that question; and it's only pot luck on which question you get. So your level of maybe attitude to p-s could be maybe shaped by the kind of teacher you get. And it's kind of random in most schools.

Int: And so, taking that a half step further S6, it would be reasonable maybe to say that your mindset, be it fixed or growth, might be a reflection of people you are exposed to, probably most commonly in that regard. Like I can see yano… so a teacher who doesn't I suppose expose you to p-s or allow you to have this productive struggle or get stuck for extended periods of time, that might not necessarily foster that growth mindset that ye have all spoke about developing as a result of these things.

S6 Yes, because the whole idea of if you're not… if you're not as fully motivated to do something and everyone around you is telling you you'll never be (47:00) able to do it, why would you even try? So it's this idea that the people around you shape your mindset, and their opinions influence your motivation… as such.

Int: And I think, if your mindset becomes sufficiently growth orientated or sufficiently strong in your resolution and your resilience, you'll eventually become resistant to those types of thoughts and persevere regardless is kind of the endpoint. So you're nodding there S8, would you agree with that?

S8 Yeah like if a teacher just told you you're terrible you're not gunna and that you're not able to do it, especially like for example as a 10 year old - you're very impressionable and you're gunna think 'oh they'll be able to do that'. But if a teacher maybe sort of like gives you a hint even and maybe or show you how to do it, and especially with a growth mindset I think you could definitely improve in p-s, but very often a growth mindset to do with how we're taught maths – sorta like once you're taught something once you always know how to do it – so if you figure out how to do it once then you can repeatedly do it you've permanently improved at maths.

Int: And one question I'd like to ask you before we bring the group back together at the very end was, we talked a good bit about challenges and what we found challenging in the course, and I'd like to know what you found most challenging or did you find something challenging about the module you did with Aidan and if so what was it?

S8 Yeah well I hate algebra. So like any… so like what I found almost everything we did used algebra. So normally I'd find every way I can to go around without using it, and it sorta forced me to try to do that. Also a lot of the questions you had to write some things in an algebraic fashion so that kind of forcing me to do it definitely helped me to improve at using it. Like there would be an awful lot of questions in school where I would go 'I think I can do this another way' so I'd do it like that.

Int: Yeah, and so since the module do you find you're doing that less and less – avoiding algebra?

S8 Well it depends on what I think is easier. Sometimes I use it because I've realised it is actually the easier way of doing it and then there are still some questions where I think a method that I, well maybe I saw somewhere when I was younger or I just thought oh I might be able to do it like that, worked. (D: ok) But yeah, I do that if I think it's faster normally is the most important thing.

Int: Great. What I'm trying to, I suppose, say here is that beforehand we had this distaste of and avoidance for a certain branch of mathematics…

S8 Yeah

Int: I do this module and I've switched away from, if not avoidance at least towards objectively looking at the potential solutions I have and choosing the quickest as opposed to the one… or avoiding one over the other

S8 Yeah

Int: I think that might be an example of where a growth mindset was somewhat developed during this module and has been applied further down the road. And so, I'm feverishly trying to impress S2 and not run over time. And so what I would like to just say, and I'll talk for a minute and then I'll allow everyone to have a last closing word as some sort of summary. We began with questions asking you about your… did you learn any skills. And so there I think we spoke about p-s and I think S5 then spoke afterwards about communication; and those were the main things there. If there are any others keep them in the back of your mind and say them in a second. We then spoke about parts of the module that were challenging. Within challenging we talk about being stuck and then what we did when we were stuck. From there we moved on to this idea of mindsets – growth mindset and closed mindset. We spoke a good bit about that. And then we ended up going into the weeds a bit of p-s in secondary schools, but I thought that was quite fun aswell. So S3 (51:00) with those four overarching themes, is there anything that still sticks out to your or is profound, maybe that we skipped over?

S3 Well I think the question of difficulty was sort of an interesting and nuanced one in that the module was done in such a way where you didn't by any means need exceptional ability at maths to like follow everything that was going on; and there was nothing conceptionally particularly complicated about it; but it was that a lot of the problems had a lot of depth to them. To the point where a lot of the time like you'd finish up on a problem and you're still like if you had spent longer on it there is still even more depths of the problem you could have gone into. Like I remember this particular problem where we were looking at it, and just kept simplifying it and simplifying it and simplifying it and trying to understand the mechanisms behind it and then we realised that there was even more depth than we thought there was to the initial problem (52:00) – I think it required like integrated curves or something that we didn't know how to do. But that like… but at the same time if we didn't wanna like… like there's also… we could have gotten an answer that was somewhat correct, it was sufficient, very quickly without too much effort but then there's so much depth to it that you wouldn't run out of things to do yano.

Int: I think in task design, it's very easy to build a pseudo-problem that's very closed and obvious and has one solution technique. It's very subtle however and… creativity I think lives in those grey spaces between ambiguity and ill-defined problems and extendable problems. I think that's what you're… you're describing how a group of people can approach quite an ill-defined or an open-ended problem and investigate it in a variety of ways. I do think that's one of the key factors in the enjoyment I think for these types of questions. That's really… I like that point a lot. S4 I suppose the same question to you – the four major topics we talked about; is there anything you want to leave out on the table or forever hold your piece about?

S4 I suppose it's more like the fact that the module, there was more of an emphasis on the why, rather than getting the answer; which I hadn't experienced before. We'd get the answer in our group and we'd give it to Aidan and he'd just ask us why and we'd just sit there like 'ehhh what'. (Int: 'cause!!!) And then we'd have to go away and figure out the why expand it again and it just kind of… it made it easier to understand maths in kind of like a general way – in like how the basic things work and how it's built up to the question that we're answering I suppose.

Int: Absolutely, I think that… I think that that was underneath a lot of the cynical comments that we had earlier on about reductionist teaching where I can teach a monkey four different tricks and he'll get some procedural competence, but the why and the understanding they're big words, and they're far more powerful words. And I think that's really cool. If you're getting that from this module, that's where the interest is, because to be honest with you if my interest isn't peaked by something why would I care? I think ye have experienced a module where yer people whose interest is maybe exceptionally difficult to peak at certain things and yet yer grappling with these questions. 5…

S5 Yeah I agree with everything that was said about the why and the teamwork and just everything… everything that we did we didn't just… once you finish the problem you didn't just forget about it, it stuck with you.

Int: S6… (55:00)

S6 Yeah so, one of the things was – I generally wouldn't think of a problem graphically, but that was something that you had to do a lot of the time to help. And even with the reflections it was the whole idea… because you had to explain it step by step it was easier maybe to describe the graphic method. So then I got used to thinking of the problems in a different way. And that wasn't something I would have been exposed to before, was being forced to do that.

Int: So like, algebraic solution techniques are one way, graphical solution techniques are another way. If I have, some people say nine different approaches to solving a problem, the more of those I have at my disposal the more robust I'll be and the more open-ended the problem I could probably attack. So, might you have ever approached a problem from one direction and maybe hit a barrier and said well what will this other direction glean for me and try this multi-faceted approach?

S6 Since yes, but during it I would more so try it and then go and talk to the group to see what they came up with at solving it, and then try working at that method.

Int: Whoever drew the prettiest picture would win (laughter)

S6 Whoever drew the prettiest picture yep… not S1. (Laughter)

Int: Just as S1 is about to speak.

S1 Now my self-esteem is just gone so I can't talk any more. (Laughter)

Int: I don't believe that. Have you any parting thoughts?

S1 Not really, I think I hit on everything.

Int: Cool, yeah. Great. S2?

S2 I mean, I think that one of the better aspects of the course I feel, was like you've heard my description of my school – I'm not all too fond of it. But I'm sorta… now I don't have PTSD any time group work is assigned, that it's kinda like shaking realising that I might have to do everything here as is usually the case, but it was very much not the case which I thought was really good. It shows good communication skills, being able to communicate your thoughts to other people who are stubborn and who also think that they can solve the problem better than you can. So it's kinda like, you almost have to side develop the skills of communicating your thing as best you can and proving it as best you can in a limited amount of time. And I think that the group-work, again to hit on that, because I did thoroughly enjoy that; that the groups were really, really good. Because I don't think that I could have solved any one of those problems on my own, maybe perhaps one; but since I had other people to vent my opinions and thoughts and have them vent their thoughts onto me, and then solve the problem because of group-work, means I felt that was one of the best aspects of the course.

Int: Well said.

S8 Thanks S2.

Int: Let the record show S8 took all the credit for that. (Laughter) S7?

S7 I was just thinking about an example of how say my problem-solving skills improved – well say we got the same question 3 times, it was the escalator question (loud laughter and responses of "oh yeah").

Int: Took you to new heights did it?

S7 We got it in like week 1, week 8 and week 13 or something. The first week it was the first thing we did – I had no idea what to do. I could just not think of it. After an hour and a half I was still staring at my page being like 'what the hell am I supposed to do here?'. Then after 8 weeks where we tackled it again, and it still took some time, but gradually after a while I began to realise what I was supposed to do, after 8 weeks of experience with these questions. So it just gave me the tools to do questions I wasn't able to do before.

Int: Great and I'd just like to thank you all – as soon as you said escalator everyone got out of their seats with memories (laughter), but when I was asking were you ever stuck you're all like 'oh I dunno'. (Laughter) Sound!

S8 Yeah well I was gunna say the fact that we did the escalator question three times definitely reinforces a growth mindset…

Int: Unless you just take the stairs for the rest of your life… (laughter)

S8 Yeah, and I definitely think the group-work… like one thing that it does is when you're forced to like sort of work with people that want to do it, instead of what everyone here has probably already done and just died inside if they got someone that they knew just wasn't going to do anything – I think that when you're given people that you want to work with it definitely helps you improve because you see something and then you can sort of… like people mimic other people's… the people around them. So you sorta amalgamate their mind, or the way their mind works into how you work and then you can see a problem from five different angles, instead of two for example if you had 3 people in a group and did share all the angles that they saw. But that multiplies over all the different problems and you figure out ways to do a problem and to solve problems in general by working with people.

Int: Cool. That's brilliant guys. I just want to thank you again, both from Aidan but also from me.

Appendix I

Consent & Assent Forms

Informed Assent Form- Interview

Dear Student

I would like to invite you to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving second level students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted in collaboration with CTYI. This study is open to you as you have previously studied the module during the Early University Entrance or summer mathematics programmes with CTYI. You are invited to attend a focus group interview taking place in DCU. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)	
I have read the Plain Language Statement (or had it read to me)	*Yes/No*
I understand the information provided	*Yes/No*
I have had an opportunity to ask questions and discuss this study	*Yes/No*
I have received satisfactory answers to all my questions	*Yes/No*
I am aware that data collected can be requested by the court or may be legally required if my safety is believed to be at risk	*Yes/No*
I assent to participate in a focus group interview	*Yes/ No*

Your identity will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password collected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. You may withdraw from the project at any time.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I assent to take part in this research project

Student's Signature: ______________________________

Name in Block Capitals: ______________________________

Witness: ______________________________

Date: __________________

Informed Assent Form- EUE

Dear Student

I would like to invite you to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving Transition year students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted as part of the CTYI Early University Entrance Programme, for which you have signed up. Within this programme, you will participate in 5 hours of mathematics each week for 14 weeks. 3 hours of this time will be directly related to the research being undertaken, while the other 2 hours will be based upon prior modules developed for high achieving students. Classes will take place every Friday in Dublin City University for the duration of the programme.

You will be asked to complete two surveys and a short test on your mathematical problem solving abilities at the beginning and end of the programme. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)

I have read the Plain Language Statement (or had it read to me)	*Yes/No*
I understand the information provided	*Yes/No*
I have had an opportunity to ask questions and discuss this study	*Yes/No*
I have received satisfactory answers to all my questions	*Yes/No*
I am aware that data collected can be requested by the court or may be legally required if my safety is believed to be at risk	*Yes/No*
I am aware I may be asked to complete surveys and a test that will focus on their problem solving mathematical abilities	*Yes/No*

Your identity will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password collected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. You may withdraw from the project at any time, and this will not influence their participation in the Early University Entrance programme.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I consent to take part in this research project

Student's Signature: ______________________________

Name in Block Capitals: ______________________________

Witness: ______________________________

Date: ______________________________

Informed Assent Form- Summer

Dear Student

I would like to invite you to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving second level students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted in collaboration with CTYI, for which you have signed up. Within this programme, you will participate in 5 hours of mathematics for 14 days over a three week period. 3 hours of this time will be directly related to the research being undertaken, while the other 2 hours will be based upon prior modules developed for high achieving students. Classes will take place in Dublin City University for the duration of the programme.

You will be asked to complete two surveys and a short test on your mathematical problem solving abilities at the beginning and end of the programme. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)	
I have read the Plain Language Statement (or had it read to me)	*Yes/No*
I understand the information provided	*Yes/No*
I have had an opportunity to ask questions and discuss this study	*Yes/No*
I have received satisfactory answers to all my questions	*Yes/No*
I am aware that data collected can be requested by the court or may be legally required if my safety is believed to be at risk	*Yes/No*
I am aware I may be asked to complete surveys and a test that will focus on their problem solving mathematical abilities	*Yes/No*

Your identity will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password collected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. You may withdraw from the project at any time, and this will not influence their participation in the programme.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I consent to take part in this research project

Student's Signature: ______________________________

Name in Block Capitals: ______________________________

Witness: ______________________________

Date: ______________________________

Informed Consent Form- Summer

Dear Parent
I would like to invite your child to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving second level students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted in collaboration with CTYI. Within this programme, students will participate in 5 hours of mathematics for 14 days over a three week period. 3 hours of this time will be directly related to the research being undertaken, while the other 2 hours will be based upon prior modules developed for high achieving students. Classes will take place every day in Dublin City University for the duration of the programme.

Students will be asked to complete two surveys and a short test on their mathematical problem solving abilities at the beginning and end of the programme. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)

I have read the Plain Language Statement (or had it read to me)	*Yes/ No*
I understand the information provided	*Yes/ No*
I have had an opportunity to ask questions and discuss this study	*Yes/ No*
I have received satisfactory answers to all my questions	*Yes/ No*
I am aware of the legal limitations to the protection of data as outlined in the Plain Language Statement	*Yes/ No*
I am aware my child may be asked to complete surveys and a test that will focus on their problem solving mathematical abilities	*Yes/ No*

Students' identities will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password protected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. Students may withdraw from the project at any time, and this will not affect their participation in the programme.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I consent to take part in this research project

Parent's Signature: ________________________________

Name in Block Capitals: ________________________________

Date: ________________________________

Informed Consent Form- Interview and online survey

Dear Parent

I would like to invite your child to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving second level students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted in collaboration with CTYI. This study is open to students who have previously studied the module during the Early University Entrance or summer programmes with CTYI. Students are invited to complete the attached online survey, but also to attend a focus group interview taking place in DCU. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)

I have read the Plain Language Statement (or had it read to me)	*Yes/ No*
I understand the information provided	*Yes/ No*
I have had an opportunity to ask questions and discuss this study	*Yes/ No*
I have received satisfactory answers to all my questions	*Yes/ No*
I am aware of the legal limitations to the protection of data as outlined in the Plain Language Statement	*Yes/ No*
I consent for my child to complete an online survey investigating their experiences while studying the module	*Yes/ No*
I consent for my child to participate in a focus group interview	*Yes/ No*

Students' identities will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password protected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. Students may withdraw from the project at any time, and this will not affect their participation in the programme.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I consent to take part in this research project

Parent's Signature: ________________________________

Name in Block Capitals: ________________________________

Date: ________________________________

Informed Consent Form- EUE

Dear Parent

I would like to invite your child to participate in a research study that is being carried out to develop an intervention module to be taught to high achieving Transition year students. This research is being carried out by Aidan Fitzsimons of the School of Mathematical Sciences in DCU, as part of a doctoral programme under the supervision of Dr Eabhnat Ni Fhloinn.

The research will be conducted as part of the CTYI Early University Entrance Programme. Within this programme, students will participate in 5 hours of mathematics each week for 14 weeks. 3 hours of this time will be directly related to the research being undertaken, while the other 2 hours will be based upon prior modules developed for high achieving students. Classes will take place every Friday in Dublin City University for the duration of the programme.

Students will be asked to complete two surveys and a short test on their mathematical problem solving abilities at the beginning and end of the programme. Details with regards to these can be found on the Plain Language Statement. Please take your time to read and answer the questions below. If you have any questions or concerns regarding the research please do not hesitate to contact Aidan Fitzsimons by email on aidan.fitzsimons4@mail.dcu.ie or by phone on 01-7005021.

Participant – please complete the following (Circle Yes or No for each question)

I have read the Plain Language Statement (or had it read to me)	*Yes/ No*
I understand the information provided	*Yes/ No*
I have had an opportunity to ask questions and discuss this study	*Yes/ No*
I have received satisfactory answers to all my questions	*Yes/ No*
I am aware of the legal limitations to the protection of data as outlined in the Plain Language Statement	*Yes/ No*
I am aware my child may be asked to complete surveys and a test that will focus on their problem solving mathematical abilities	*Yes/ No*

Students' identities will be kept confidential through the use of pseudonyms in any publications or reports relating to the research. All information collected during this research will be stored in a locked cabinet and room in the offices of the School of Mathematical Sciences. Any Electronic data will be password protected. Data will only be available to the lead researcher Aidan Fitzsimons and his supervisor, Dr Eabhnat Ni Fhloinn. Students may withdraw from the project at any time, and this will not affect their participation in the Early University Entrance programme.

I have read and understood the information in this form. My questions and concerns have been answered by the researchers, and I have a copy of this consent form. Therefore, I consent to take part in this research project

Parent's Signature: ________________________________

Name in Block Capitals: ________________________________

Date: ________________________________

Appendix J

Plain Language Statements

Plain Language Statement- Parents – EUE

This project is being undertaken as part of a doctoral programme being studied by Aidan Fitzsimons. Aidan is a member of staff in the DCU School of Mathematical Sciences, and also with CTYI, where he has taught the Maths Magic programme for primary school students. It is hoped that this project will help to create an effective programme to challenge high achieving students and develop their problem solving abilities in mathematics. This project is called "Designing an intervention module to challenge high achieving transition year students in mathematics". Aidan can be contacted by email at aidan.fitzsimons4@mail.dcu.ie or by phone at 01-7005021.

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Participation in this module is entirely voluntary and students can ask to opt out at any time. This will not affect their participation in the Early University Entrance programme. **If students, or their parents, do not wish to participate in the research they will remain a part of the module. They will engage in data collection, but their results will be used as a class evaluation and not as part of the research project.**

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Plain Language Statement- Parents - Summer

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Plain Language Statement- Parents – Module Survey & Focus Group

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Students who have previously studied the problem-solving module as part of the CTYI Early University Entrance (EUE) or summer programmes are the desired participants for this study. Those students are now invited to complete either, or both, a survey and a focus group interview. These new data-collection measures aim to investigate students' experiences of the module, while also seeking their responses to the mathematics studied.

A mix of both tick-the-box and comment answers are used on the module survey. Students may answer each question by selecting the option most applicable to them, and adding further comment if they so wish. The survey is completed via google forms, and responses will be anonymous. The focus group interview will be conducted by Dr. Diarmaid Hyland, a member of the school of mathematics in DCU. Dr. Hyland is Garda vetted through DCU, and is an independent party due to having no contact or communication with the students at any time prior to the interview. Similar to the survey, the questions during the focus group will explore the students' experiences of the module studied, but gives an opportunity for students to further elaborate on their answers and those of the other participants.

The Google form requires parental consent to the survey to begin. You will then be asked for consent for your son/daughter to participate in the focus group interview. If you consent, you will be asked to select dates and times of availability. If you do not consent, you will be taken to the survey for students to complete. Student names are not required at any point, to ensure anonymity. Students will be asked to sign an assent form for the interview if they choose to attend. These will be kept by the supervisor of this project rather than the principal investigator, to further ensure anonymity of students.

All student names will remain unpublished in reports or publications related to this project. Pseudonyms will be used in reports in reference to students where necessary, to protect their identities. All information collected will be kept in a locked cabinet and room, and only available to Aidan and his supervisor, Eabhnat Ni Fhloinn. Any data contained in electronic form will be password protected. **All raw data collected will be destroyed by a confidential shredding company, and any electronic data deleted by the principle researcher under the supervision of his supervisor. Legal limitations to confidentiality are as follows: information collected can be subject to subpoena and statutory reporting under child protection legislation.**

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Plain Language Statement- Students – EUE

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The benefits to the programme for students are the development of their problem solving abilities in mathematics, and the participation in an educationally stimulating environment that seeks to aid in their academic development. No risks are associated with this study.

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As you have previously studied the problem-solving module as part of the Mathematics course for CTYI Early University Entrance (EUE) or summer programmes, you have qualified as a participant for this study. You are now invited to complete either, or both, a survey and a focus group interview. These new data-collection measures aim to investigate your experiences of the module, while also seeking your responses to the mathematics studied.

A mix of both tick-the-box and comment answers are used on the module survey. You may answer each question by selecting the option most applicable to you, and adding further comment if you so wish. The survey is completed via google forms, and responses are anonymous. The focus group interview will be conducted by Dr. Diarmaid Hyland, a member of the school of mathematical sciences, DCU. If you choose to participate, you will be interviewed by Dr. Hyland in a group with other students. Similar to the survey, the questions during the focus group will explore your experiences of the module studied, but gives an opportunity for you to further elaborate on your answers and the thoughts of the other participants.

The Google form requires parental consent to the survey to begin. The survey begins at section 5, and should be completed by you from this point on. Your name is not required at any point, to ensure anonymity. You will be asked to sign an assent form for the interview if your parent/guardian has consented for your participation. These will be kept by the supervisor of this project rather than the principal investigator, to further ensure anonymity of students.

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Appendix K

PTQ Student Sample A

1) Professor on an escalator (3) [redacted]

Prof Henry walked very slowly ↓ 50 steps.

Prof Henry ran up quickly ↑ 125 steps

Prof Henry ran up 5 times as quickly as he walked down.

Plans

Okay so my plans involve putting them in terms of eachother by multiplying the walking distance by 5 or dividing the running distance by 5.

Consider 

triangle. D = Distance, S = Speed, T = Time.

Try using a simultaneous equation to find the number of steps that would be visible if the elevator were to stop.

I know the answer has to be between 50 and 125 steps, the question tells us this.

I know the answer to the question is 100 steps, however this has just been confusing me as I don't remember how I got to this answer before. I would prefer to be working with no prior knowledge of the question.

Solving

Okay a good starting point is to keep in mind

$50 \leq x \leq 125$ $(x \in \mathbb{R})$ where x = no. of steps

The thing we are looking for is the no. of steps i.e distance

I firstly want to allocate hypothetical speeds to the problem.

If Prof. Henry walks at a speed of 50 steps/min then:

He will have walked down in 1 min.

To find his running speed multiply by 5 to get 250 steps/min

Now to find how long he spent on each journey.

Walking Down

$\frac{50s}{50s/m} = 1$ minute

$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$

Running Up

$\frac{125s}{250s/m} = .5$ minute

$= 30$ seconds.

MS = 5
S = 4
C = 4 21
A = 4
R = 4

[redacted]

② Now we have the distance, the speed and the time for both journeys

	Distance	Speed	Time
Walking Down	50s	50s/min	1min
Running Up	125s	250s/min	30sec

I want to try figure out if Prof. Henry's walking speed would be quick enough for him to go up the escalator.

I might take sample numbers and using trial and error try to figure out where these two figures conject to give a number of steps.

e.g at 75 steps does this problem make sense?

for starters 75 is between 50 and 125

Nevermind, I've just realised that to balance the Walking Down and Running Up speeds the escalator must run at a constant rate.

Divide 125s by 50s and you get 2.5. I think this figure is important.

2.5 = 5/2

~~If you want the time in both equations to be equal multiply the running~~

You have to divide the distance and speed of the running by 2.5 to get that of the walking.

To get the time's equal multiply the Running Up by 2.

Now the distance will be 250s and speed will be 500s/min

I say divide both of these by 2.5 to find ~~the distance~~ how many steps would be visible and the speed the escalator moves at

250s ÷ 2.5 = 100 steps visible ✓

500s/min ÷ 2.5 = 200s/min

Reflection

To begin with, I laid out my prior knowledge of the question and how to attack it.

I stated that my final answer must be between 50 and 125, my final answer of 100 fits this perfectly.

I didn't quite know how to find my answer so I started doing some basic things that might help me down the line e.g explanations of what might be needed.

I started to focus on Distance, Speed and Time

[redacted]

③ I subbed in hypothetical speeds. Now I realise that at the end I should have given the speed in terms of x i.e the escalator moves at 5x steps /minute.

- I drew out a table of Distance, speed and Time.

Distance and speed were proportional but time was not. So then, I let both their times be equal to 1min.

I could then divide my new running up figure (~~500~~ 250) by the proportional ratio of 7.5 that I found earlier, leaving me with a final answer of 100 steps

- Making Sense of Task: thoroughly developed through the "plan" (5)
- Solving the task: effective and complete, not 'elegant' (4)
- Communicate Reasoning: clear and coherant communication (4)
- Accuracy: Correct solution, but not enhanced by extension (4)
- Reflection: Solution is reflected upon, but not reworked (4)

PTQ Student Sample B

Round 2

Test Question 2 [redacted]

x = speed $5x+50$ (down) (up 5xspeed) $125-x$

$5x+50=125-x$

$6x=75$

$x=12.5$

$5x+50=125-x$

$\Rightarrow 62.5+50=125-12.5$

$\Rightarrow 112.5=112.5$

answer = 112.5 ?

'Making sense' only gets 2, as student appears to add speed with distance (steps)

ms = 2
S = 2
C = 2
a = 2
r = 2/10

All sections underdeveloped or sketchy.

$12.5=\frac{125}{10}$

$\Rightarrow$ 1 stairs down for every 10 steps up (upwards)

$\Rightarrow \frac{62.5}{50}=\frac{5}{4}=1.25$

1.25 stairs down for every step down

112.5 stairs showing ?

~~10-1.25 = 8.75~~

PTQ Student Sample C

Round 2

[redacted] Some Logic Stuff [redacted]

If the escalator stopped moving the professor would have to take, in one case, the 50 steps we are told he took + whatever y steps the escalator brought him down when it was on. If he (without skipping any steps) takes 50+y steps to walk down an escalator, one step at a time, there must be 50+y steps on the escalator.

When moving down the escalator the professor is taking z steps while there are y extra steps on the escalator that the escalator is bringing him down also. In total he has travelled z+y escalator steps and thus there are z+y steps visible on the escalator if it stopped running.

When moving upwards he is travelling w steps but in that time being brought down u steps as the escalator is moving in the opposite direction thus in total he is travelling w−u steps.

In Round one, no further work was done. In Round two, these generalised thoughts were provided, to get a score of 5 in 'accuracy'.

In Round 3, the following extension was explored, gaining a score of 6 in 'accuracy'.

Round 3

[redacted] (2) Test Question [redacted]

Extension: Lets try creating a similar question of ~~on~~ a professor going up/down an escalator except this time we have a new rule.

~~When the professor steps out~~

The escalator only begins to move after the professor steps on and then begins to ~~considerably~~ linearly accelerate.

So for example when the professor has taken 0 steps ~~acceleration would be 0~~ a the escalators rate of movement would be 0 steps/s however after 50 steps it may be 2 steps/s having linearly accelerated in the time he took those steps.

To create a formula to find the total steps moved using a given number of steps (s) and an acceleration rate (a) we need to find a formula for the total number of steps the escalator has moved along. To do this we can use the ~~acceleration~~ distance formula:

$$\text{distance} = (\text{start velocity})\text{time} + \frac{(\text{acceleration})(\text{time})^2}{2}$$

Our distance is S (number of steps) (this s)
start velocity is 0 (escalator isn't moving at the start)
acceleration is a

Time can be written using the number of steps taken by the ~~proffes~~ professor as he takes them at a consistent rate so time is the number of steps taken by the professor
= s

So substituting these in we get:

$$S = 0s + \frac{as^2}{2} \quad \text{or} \quad S = \frac{as^2}{2}$$

S (big s) is the total number of steps the escalator moves along with a given time taken and acceleration. The total number of steps ~~taken~~ moved along the escalator in total then is $S+s$ or ~~as~~ $s+\frac{as^2}{2}$

For example if the escalator is accelerating at 2 steps per steps taken and ~~[illegible]~~ 10 steps are taken in total then

Total steps moved $= 10+\frac{2(100)}{2} = 110$ steps total

So when given a case like this the total number of steps moved is $s+\frac{as^2}{2}$ where s is the steps taken by the professor and a is the acceleration rate with regards to the steps taken by the professor. To solve a question where the escalator is accelerating from a constant rate starting from zero a person would have to ~~[illegible]~~ find this equation and use it to find the total steps ~~on the~~ visible on the escalator if it were to stop moving.

Ex. A professor takes 10 steps to reach the bottom of a down moving escalator accelerating ~~[illegible]~~ by 2 steps/step for every step the professor takes. How many steps would be visible ~~on~~ if the escalator stopped moving.

Ans: Use the distance formula $s=ut+\frac{1}{2}at^2$ to ~~[illegible]~~ determine $s+\frac{as^2}{2}$ = total steps visible and plug in the numbers:
$10+\frac{2(100)}{2} = 110$ steps

[redacted] ③ Test Question [redacted]

This question can be made more difficult however by not directly giving the acceleration and rather asking the person solving the question to use simultaneous equations to find it. They now must find the total steps and acceleration.

For example: First case professor takes 10 steps down to reach the bottom.
Speed resets then second case professor takes 20 steps to reach the top.

Firstly you find the equation $s + \frac{as^2}{2}$ = Total steps.

You then ~~sub~~ substitute the values remembering to subtract the second part in the second case as you are moving against the escalator:

$10 + \frac{100a}{2}$ = total steps $= 10 + 50a$

$20 - \frac{400a}{2}$ = total steps $= 20 - 200a$

Using simultaneous equations:

$10 + 50a$ = Total steps (T)

$40 + 200a = 4T$

$+20 - 200a = T$

$60 = 5T$ $\quad 12 = T$

$10 + 50a = T = 12$ And to verify: $20 - 200a = 12$

$50a = 2$ $\quad -200a = -8$

$a = \frac{1}{25}$ $\quad 200a = 8$

$a = \frac{8}{200} = \frac{4}{100} = \frac{1}{25}$

So we find that the total number of steps visible if the escalator stopped moving is 12 and the acceleration rate of the escalator is $\frac{1}{25}$ steps for every step the professor takes.

Another thing we can do with this question ~~(steps~~ (similar to the original) have the professors movement speed be different in both cases.

Ex. First case: Professor takes 10 steps to reach bottom
Second case: Professor takes 20 ~~steps~~ steps to reach top moving 5 times as fast.

~~A~~ Escalator is moving ~~down~~ downwards, Find Acceleration and Total steps.

The ~~question~~ answer method remains mostly the same bar a change to the second equation:

~~Su~~ Total steps (T): $s + \frac{as^2}{2}$

First equation: $10 + \frac{100a}{2} = 10 + 50a$

In the second equation ~~e~~ the acceleration rate will be five times slower per step the professor takes as it takes five times as many steps to accelerate the same amount

5a

Second equation: $20 - \frac{\frac{a}{5}(400)}{2} = 20 - \frac{a(400)}{10} = 20 - 40a$

Using simultaneous equations

$$10 + 50a = T \rightarrow 40 + 200a = 4T$$
$$20 - 40a = T \quad + \; 100 - 200a = 5T$$
$$140 = 9T$$
$$T = \frac{140}{9}$$

$$10 + 50a = T$$
$$10 + 50a = \frac{140}{9}$$
$$90 + 450a = 140$$
$$450a = 50$$
$$a = \frac{1}{9} \text{ steps / step taken}$$

Verify: $20 - 40a = \frac{140}{9}$
$180 - 360a = 140$
$360a = 40$
$a = \frac{1}{9}$ steps / step taken

Total steps = $\frac{140}{9}$
Acceleration rate = $\frac{1}{9}$ steps / step taken