Young Children's Identifications of the Most and Least Likely Outcomes of Experiments

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The aim of this study was to investigate the probabilistic thinking of young children, focusing in particular on the judgements that influence their identifications of the most and least likely outcomes of experiments. Research studies present conflicting results pertaining to young children's potential to engage in probabilistic thinking and a wide variance exists across international mathematics curricula regarding the age at which children receive formal probabilistic instruction. At present, young children in Ireland are not formally introduced to probability until Third class when they are approximately 8 or 9 years old. In this study, the probabilistic thinking of 16 children aged 5-6 years was examined using task-based group interviews. The results suggest that young children are capable of engaging in sophisticated probabilistic thinking and highlights that the current practice of formally introducing children to probability in Third class warrants further investigation.

Introduction

Assessing the probability of an event is an everyday occurrence and both adults and children encounter regular opportunities to construct probabilistic understandings and to develop probabilistic thinking skills in their daily lives. However, research into young children's probabilistic thinking has produced inconclusive and conflicting results regarding the potential of children to understand probabilistic situations and, consequently, further research is required to identify the strengths and limitations of young children's probabilistic thinking (Bryant & Nunes, 2012).

The study described here sought to examine the probabilistic thinking of young children through investigating their responses to probabilistic tasks. The children were asked to identify the most and least likely outcome of a variety of experiments and to justify their thinking. In this paper we examine the literature pertaining to young children's probabilistic thinking. Drawing upon this literature, we present findings from our examination of children's probabilistic thinking, as evidenced through their engagement in four tasks.

Literature Review

Probability is not a new mathematical concept. As such, this literature review includes recent research studies along with literature that gives a historical perspective. We draw largely on the work of international researchers due to the absence of research that has been undertaken into probabilistic thinking in Ireland, particularly relating to young children. Throughout this paper, the term *young children* refers to children aged between 3 and 8 years.

Piaget and Inhelder are widely recognised as the first researchers to study the development of probabilistic thinking in children and their research paved the way for further research into this area (Ben-Zvi et al., 2018). Through conducting clinical interviews with children aged between 4 and 12 years, they concluded that children's ability to engage in probabilistic thinking is linked to their cognitive development and that the systematic understanding of probability commences between the ages of 9 and 12 years (Piaget & Inhelder, 1975). However, Piaget and Inhelder's findings have also been contested by various researchers. For example, Bryant and Nunes (2012) argued that it involved the use of an unfamiliar context while others have contended that the questions were based on children's verbal abilities and, as a result, may not reflect the children's probabilistic thinking because the verbal abilities of children develop later (e.g. Fischbein & Gazit, 1984).

In contrast to Piaget and Inhelder, several researchers have suggested that children possess basic notions of probability from a young age. Fischbein (1975) systematically studied the literature relating to children's probabilistic thinking and was among the first researchers to contend that even preschool children can possess an intuitive understanding of probability. For example, a study by Nikiforidou et al. (2013) found that children aged between 4 and 6 years express stable understandings of probability and can identify the most likely outcome of events.

Theory about early probability learning remains relatively new and further research is required into how young children's probabilistic thinking develops over time (Supply, 2020). However, a framework designed by Jones et al. (1997) almost twenty-five years ago may be useful for describing and predicting young children's responses to probabilistic scenarios. The role of this framework in guiding the task design and data analysis processes in this study are discussed in the methodology section of this paper.

Methodology

Participants

The participants were selected from a Senior Infant class in the school where one of the authors of this paper was teaching. Within this convenience sample, a smaller sample of 16 children were chosen to participate through the use of stratified sampling. This allowed for an equal number of boys and girls to be chosen at random and led to the creation of groups comprising of two boys and two girls. This gender balance was sought as unequal numbers of boys and girls have been found to disadvantage certain group members (Swann, 1992).

Data Collection

Task Design. The task design process was guided by the probabilistic thinking framework designed by Jones et al. (1997). The tasks used related to a single construct to allow for a fine-grained analysis of the children's thinking. This study focused on the probability of an event construct because a number of researchers have investigated young children's probabilistic thinking in relation to this construct and their findings differ regarding the types of reasoning demonstrated by young children when identifying the most/least likely outcome (e.g. Nikiforidou & Pange, 2010; Piaget & Inhelder, 1975). Jones et al. (1997) presented learning descriptors at each level of the probability of an event construct which acted as a guide when designing the tasks. In order for the children's responses to be mapped onto the framework, it was necessary to provide the children with opportunities to identify the most/ least likely outcome of an experiment and to examine if the children's justifications involved subjective, quantitative, or numerical judgements, or a combination of these judgements. The tasks that were completed during the original study in which this probabilistic thinking framework was formulated typically involved the children making a prediction, carrying out an experiment, and comparing the results to their predictions (Jones et al., 1997). The tasks in the current study were modelled on a similar format.

The Interview. Task-based group interviews were utilised as the primary method of data collection in this study. Interviewing children about their mathematical thinking enables researchers to look beneath the surface and can reveal insights into a child's learning that otherwise may go undetected (Ginsburg, 1997). This research tool involves the interviewer and participants interacting in relation to tasks which are introduced in a pre-planned manner (Goldin, 2000). The interviews were conducted in small groups. Group interviews have been shown to generate richer responses than individual interviews, providing opportunities for children to share ideas, hear opposing views, and challenge each other's thinking (Littleton & Mercer, 2013). The limitations of group interviews were also recognised throughout the study. In a group interview it can be difficult to ascertain if children are sharing their own thoughts or if they are agreeing with the views of others, repeating these ideas with little understanding. Thus, the children's comments were not analysed in isolation. Their thinking was tracked throughout each task and the potential impact of the group on their thinking was examined.

Data Sources. Video-recording was the primary method of data collection utilised in this study as it captured the children's behaviour in audio and visual form. The children's utterances, gestures, pauses, intonations, and expressions provided insights into their thinking and assisted the researchers in elucidating the meaning of their spoken words. Photographs were taken of the children's use of resources and copies of their drawings were collected. These artefacts supported inferences made from the children's spoken ideas and enabled a more rigorous analysis than could be afforded by examining the transcripts in isolation.

Data Analysis

Data from the interviews were drawn upon to generate the most accurate interpretation of the children's thinking. A deductive approach to data analysis was chosen to allow the children's probabilistic thinking to be examined in relation to the aforementioned framework designed by Jones et al. (1997). The chosen codes of 'subjective', 'transitional', 'informal quantitative', and 'numerical' related to the four levels of probabilistic thinking identified by Jones and his colleagues.

Findings and Analysis

The children were presented with four tasks and each task was broken into three smaller tasks. For example, Task 1 was broken into Task 1.1, 1.2, and 1.3. The tasks varied in complexity and a range of resources were utilised to explore the children's thinking. For example, spinners were introduced in Task 2.2. The children were asked to identify the most and least likely outcome from a spinner that had two possible outcomes. As this was the children's first use of a spinner during the interview, a spinner that displayed three cats and one dog was used, thereby creating a discrete scenario in which the children could count the number of animals to express the probability quantitatively or numerically. However, in Task 4.2, the children were presented with a spinner for the final time and the segments were not of equal size to explore how the children would respond to a continuous situation in which the events were not equally likely. The children's responses were analysed using the probabilistic thinking framework designed by Jones et al. (1997). The focus was on gathering examples of the children's thinking under each level rather than on identifying a dominant level of thinking for each child because children's thinking is fluid and these levels represent an approximation of their thinking at a particular moment, in response to a particular set of tasks.

Evidence of Subjective Thinking (Level 1)

The children's comments were classified as representing subjective thinking when their probabilistic judgements were based on personal beliefs and preferences (Jones et al., 1997). The children expressed subjective thinking 50 times during the interviews which represented 9% of the children's probabilistic judgements. The children expressed a variety of subjective beliefs when justifying their choice of the most/least likely outcome. The most common form of subjective reasoning used related to the position of a particular object within a bag or its location on a dice or spinner:

Sarah: Yellow because, probably all the yellows will probably be in the corners and all the blues will be in the inside *(bag of bears).*

Jane: I think four because, emm, I actually think two because two is normally at the bottom *(dice)*.

Mark: Purple because it mostly starts at purple at the top and then goes back around and goes at the top again *(spinner)*.

This form of subjective reasoning was most prevalent during the tasks that involved identifying which colour bear was most/least likely to be chosen from a bag. However, in Task 3.3 the children were asked to identify the colour counter that was most/least likely to be chosen from a bag of counters and none of the children solely justified their thinking by referring to the position of the counters in the bag. Consequently, the increased use of subjective thinking during the tasks involving drawing a bear from a bag cannot be directly attributed to the context of drawing items from a bag.

The children's use of subjective thinking was also evidenced when their choice of the most/least likely outcome was influenced by their favourite colour. For example, when asked which colour on a spinner was the most likely outcome, Alex justified his choice of colour by stating that it was his favourite colour on the spinner (Task 4.2). During a task involving drawing a bear from a bag, Jane claimed that she didn't know which colour was most likely because blue and yellow were her favourite colours (Task 1.2).

The subjective judgements used by the children in this study were not restricted to the position or colour of a particular outcome. The children's thinking was also influenced by other factors such as the power they used when rolling a dice, the potential impact of previous outcomes, the size of the numerals on the dice, and external factors such as the impact the wind could have on the spinner. For example, three of the children indicated that they held a belief that previous outcomes could have an impact on future events:

Conor: Because it won last time.

Ben: I think I'm, well, going to lose because I landed on a six last time and that means I might land on a six again.

Mark: Emm, lose because I already won.

From the above extracts, it appears that Ben and Conor exhibited a *positive recency bias* because they held the belief that a previous outcome is more likely to occur again. In contrast, it appears that Leah exhibited a *negative recency tendency* as she believed that because she won previously, she was less likely to win again. It is surprising that only three children in the study based their probabilistic reasoning on previous outcomes because research has found that children are often influenced by previous experiences (e.g. Kazak & Leavy, 2018).

Piaget and Inhelder (1975) claimed that young children have subjective tendencies because they lack a grouped organisation of thought which does not develop until later. However, the fact that only 9% of the children's probabilistic judgements reflected subjective thinking appears to indicate that the children in this study have developed deeper levels of thinking than Piaget and Inhelder perceived as possible for their age.

Evidence of Transitional Thinking (Level 2)

The children's comments were classified as representing transitional thinking when they exhibited a readiness to recognise the significance of quantitative measures while also reverting to subjective reasoning (Jones et al., 1997). Comments that referred to uncertainty without quantification were also considered to represent transitional thinking, as recommended by Polaki et al. (2005). The children expressed transitional thinking 58 times during the interviews which represented 10% of the children's probabilistic judgements.

Six of the children referred to informal quantitative reasoning while also expressing subjective reasoning, as demonstrated by the following comments during Task 1.1 in which the children were asked which colour bear was most likely to be drawn from a bag:

Daniel: There's more green and there's only one red and it might be buried.

Emma: The red because there's only one and it might be at the bottom.

Mark: 'Cause there's more green and the red could be at the bottom.

Jane: And there's more red and probably everyone likes greens.

These comments reflect that the children were in a period of transition, beginning to recognise that the quantity of each colour bear influences its chance of being chosen, while continuing to be bound by subjective reasoning.

The majority of the children's comments that were classified as transitional thinking referred to uncertainty without quantification. On 45 occasions, when asked to identify the most/least likely outcome, the children acknowledged that the outcome was uncertain, as evidenced in the following extracts:

Conor: Because there's any number that you could get.

Grace: Because you don't know what you're going to land on.

Hannah: We don't know, because like you could get any colour.

The children most commonly referred to uncertainty without quantification during tasks involving equally likely outcomes. For example, during Task 3.2, the children were asked for the most/least likely outcome when a traditional six-sided dice is rolled. Most of the children recognised that that a most/least likely outcome did not exist. While some children stated that there was just one of each number on the dice or that each number had the same amount, in many cases the children did not refer to quantities, instead discussing the unpredictability of the outcome. For example, Hannah referred to the uncertainty associated with rolling a dice, stating that "no one knows what they're going to get". This aligns with previous research findings that children often equate equal likelihood with uncertainty (Watson, 2005).

Evidence of Informal Quantitative Thinking (Level 3)

The children's comments were classified as representing informal quantitative thinking when they used quantitative reasoning to justify their choice of the most/least likely outcome (Jones et al., 1997). The children expressed informal quantitative thinking 463 times during the interviews which represented 80% of the children's probabilistic judgements. In tasks involving discrete situations, the children made regular references to part-part relationships in justifying their choice of the most/least likely outcome. For example, in Task 2.2, Emma identified the cat as the least likely outcome, stating that "there's only two cats and there's four dogs on the dice". On several occasions, the children also made explicit comparisons between quantities, using words such as more, most, less, and least:

Sarah: Because there's *more* dogs than cats.

Ben: Because there's four reds, so reds are the *most* so I think that.

Tom: There's *less* cats than dogs.

Mark: You've *least* purple so everyone knows you're least likely to get purple.

The above extracts appear to indicate that the children recognised that the quantity of each part impacts its chance of occurrence in discrete situations.

Task 4.2 involved a continuous situation as the children were presented with a spinner that was shaded one-half orange, one-third blue, and one-sixth green. The children's informal quantitative justifications pertaining to this task differed to those shared during discrete situations in which the children referred to specific quantities to justify their thinking. Contrastingly, the children's informal quantitative judgements in response to the continuous situation presented in Task 4.2 involved general references to the comparable sizes of the segments, as can be seen from the following extracts in which Tom and Sarah, during two separate interviews, were justifying their choice of green as the least likely outcome:

Tom: Because green is smaller than blue and orange.

Sarah: Green because green is tiny and the orange and blue are much bigger.

While the children appeared to recognise that the size of each segment should be considered when identifying the most/least likely outcome, their limited knowledge of fractions appeared to prohibit them from making explicit references to the quantity represented by each segment.

Evidence of Numerical Thinking (Level 4)

The children's comments were classified as representing numerical thinking when they assigned valid numerical measures to describe the probability of an event occurring (Jones et al., 1997). The children exhibited numerical thinking on five occasions which represented just one percent of the children's probabilistic judgements. This type of thinking was only used by the children in response to Task 4.2 and 4.3 in which the children were discussing spinners that were shaded one-half orange. For example, during Task 4.2, Daniel identified orange as the most likely outcome "because orange is half.", while during Task 4.3, Shane stated that "half is orange and half is blue so they have an equal chance". These children recognised that the orange segment represented half of the spinner and, through doing so, identified the part-whole relationship between the orange segment and the entire spinner. These comments represented the only times when the children referred to part-whole relationships. Throughout the interviews, most of the children's judgements were based on comparisons between each part rather than comparing the quantity of one part to the overall quantity. This echoes findings of several studies that children understand proportions as part-part relations before they understand part-whole relations or fractions (e.g. Nunes & Bryant, 1996). Consequently, it appears that the children's use of numerical thinking may have been limited by their previous mathematical experiences, in particular in relation to fractions.

Conclusion

The children's engagement in probabilistic tasks revealed detailed information pertaining to their probabilistic judgements. Many of the children demonstrated robust probabilistic thinking despite their limited experiences of probability. The children referred to uncertainty throughout the interviews, suggesting an awareness of the unpredictability associated with random phenomena. The children's dominant level of thinking was identified as informal quantitative reasoning. However, some of the children appeared to be in a period of transition between subjective and informal quantitative reasoning because although they predominantly attempted to quantify probabilities, at times they exhibited unpredictable tendencies to regress to subjective judgements. Numerical judgements were used infrequently by the children due to their limited knowledge of fractions which restricted their use of part-whole reasoning.

This study demonstrated the potential of Senior Infant children to exhibit robust reasoning in response to probabilistic tasks. The children appeared motivated by, and interested in, the probabilistic tasks. This raises questions regarding the age at which children are introduced to formal probabilistic instruction. However, further research is required into how children's probabilistic thinking develops in order to design instruction that is appropriately challenging and that will have positive implications for the children's everyday lives and their probabilistic understandings.

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