DEVELOPMENT OF A COMPUTER-BASED TEACHING TOOL IN PROBABILISTIC MODELLING

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ABSTRACT
A computer-based teaching tool is being developed to teach Markov Chains in a practical and intuitive manner via the example of the board game, Snakes and Ladders. In addition, an analogous example of an industrial application in the form of a laboratory scale, rotary processor will be analysed. The electronic format of this teaching tool allows it to be used as a non conventional teaching tool via for example, CD Rom format or via the internet.

INTRODUCTION
Development of graduate education is major priority, both nationally and for UCC, as reflected in the University (College Cork) Strategic Framework Plan, 2006-2011, [1]. This states that developments in this area will '*involve the provision of additional generic and specialist skills training through graduate programmes/graduate schools.*' The aim is to build a suite of tailored modules across the University which addresses the education and training needs of postgraduate students in all disciplines and fields.

Key points and characteristics of such modules is that they should benefit as large a group of postgraduate students as possible; very narrow disciplinary courses are likely to be less favoured than interdisciplinary courses. This means the courses must be accessible to students from disparate backgrounds. Also the courses should be transferable as it is likely that, in the future, some postgraduate training courses will be offered across institutions. Hence modules must be developed in a manner that they can be accessed readily by students outside the traditional classroom arrangement that has heretofore been the default model of delivery at UCC and elsewhere. Furthermore, for modules with a mathematical basis, course content must very much stress applications of tools in the research domain and be concerned with getting the students started rather than rigorous proving of the underlying theorems.

Random process theory is applicable to many diverse areas that harbour significant research nodes in UCC; such areas include Applied Biology, Renewable Energy, Software Design, Financial Mathematics, Marine Studies, Signal Analysis, Epidemiology and Process Engineering. Many of the researchers active in these areas supervise research students and the need has been identified to provide background education in random process theory and probabilistic modeling for these students. This paper focuses on the development of a teaching tool for one such technique, Markov Chains. The objective is to make Markov Chain simulation understandable to people with a limited mathematical background. Thus the technique will be introduced and explained through the simulation of the well known, children's board game, Snakes & Ladders. A more realistic engineering demonstration of its capabilities will then be carried out by using it to determine the Residence Time Distribution of particles in a rotary processing unit, [2].

TEACHING TOOL DEVELOPMENT

The tool is currently being developed and will feature a number of distinct components:

- Outline of the Markov Chain simulation method.
- Appreciation of the Board Game of Snakes & Ladders as a Markov Process
- Understanding of the Unpredictability in the Operation of a Rotary Processor
- Computer implementation of the Teaching Tool
- User Learning with the Tool

1] MARKOV CHAINS

Markov Chains (or Processes) were first defined and studied by the Russian mathematician A. A. Markov (1856 – 1922). They can be used to analyse stochastic processes i.e. systems where some variable changes in a random fashion with time. The key Markov Property that defines such systems is that the future state of the variable only depends on its current state and is not dependent on the history of states that have been passed through. The variable of interest is presupposed to exist within finite discrete intervals, known as the states of the system. Time is also considered to be discrete so the variable switches from one possible value to the next at each time step. The construction of a Markov Model can be considered in two stages; determination of an initial probability vector ('the initial condition') and formulation of a stochastic transition matrix. The square transition matrix, **P**, is the key to analysing the probabilistic progress of the process. Each element, p_{ij}, of this matrix is the conditional probability that the variable in state i, will move to state j, in a single step (time interval).

2] SNAKES & LADDERS AS A MARKOV PROCESS

The game of Snakes & Ladders has long been known to constitute an interesting example of a Markov Chain [3, 4]. The board typically consists of 100 squares numbered sequentially as seen in figure 1.

Figure 1: Snakes and Ladders Board Layout

In its simplest form, a counter is placed on square 1 (system entry) and the outcome of a dice throw determines its next position. The game continues until the counter reaches the last square (system exit). The amount the counter moves is random and depends on the number that shows on the dice. Every throw of the dice will move the counter along a different amount. If the counter lands on a ladder it moves along or up the board a very large amount; if it lands on a snake it moves back a large amount. What is of interest is how long the game takes i.e. how many throws of the dice are needed to move the counter from state 1 to state 100. Each time the game is played, the number of dice throws will be different; i.e. the residence time of the counter moving along the board will vary from game to game.

From a Markovian perspective, the variable of interest in this case is the position or displacement of the counter and with a board of 100 squares it clearly can take 100 different possible values. Each dice throw can be considered as a new time step. The initial vector will have a 1 in the first state and the other 99 entries will be zero. The entries in the transition matrix can be found by considering the probability outcomes for the standard dice. It is clear there is a 1/6 chance of moving from the current state (square on which the counter lies) to the next six sequential states. Thus each row of the transition matrix will have 1/6 on each successive entry to the right of the main diagonal. Ladders can be accommodated by placing a 1 in the state at the bottom of a ladder and zero at the top state. Snakes by the reverse arrangement.

3] ROTARY PROCESSOR

In its simplest form, a rotary processor consists of a long, cylindrical shell with longitudinal flights running internally along its length. The cylinder is usually inclined at a small angle to the horizontal. The solids to be processed are introduced at one end (the higher end) of the cylinder and are carried up the sides of the unit by the flights as a result of the rotational motion. They then slide off the flights at a certain incline, fall through the drum interior to the base of the unit, (making intimate contact with the gas during their descent). Crucially, they can also roll along the floor some considerable distance after each fall. They are then picked up by the flights and the motion is repeated until the particles exit the drum at the other, lower end.

Rotary processors are industrially used to carry out drying, cooling, incineration and chemical reactions. From considerations of process safety, quality and efficiency, every particle should remain long enough in the unit so that it is fully processed. However one of the disadvantages of such units is the non-uniform residence time of the particles; some may travel quite rapidly through the system and some more slowly. Rapidly moving particles have a short residence time and thus may be dangerously under-processed when they exit the system. The distribution in the Residence Time of the system arises from the unpredictable nature of the motion of the particles.

For this project, a laboratory scale model of a rotary processor was built. Material of construction was Perspex to permit video of the motion of the particle within the unit to be taken. The system is illustrated in figure 2.

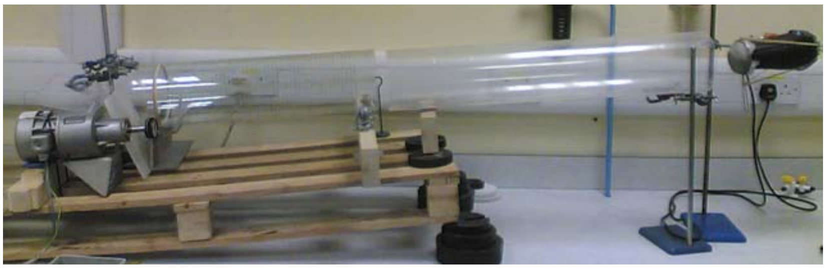


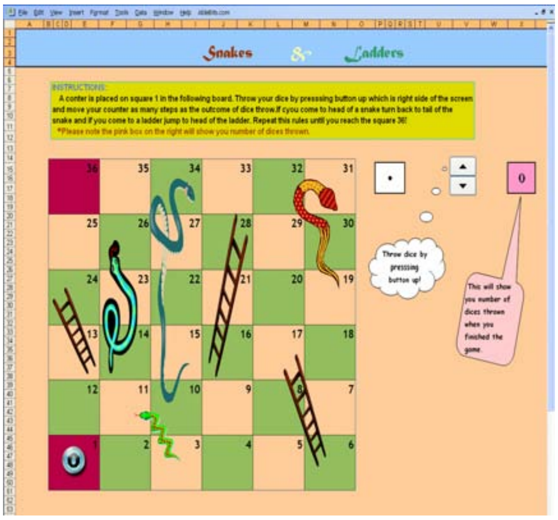
Figure 2: Experimental Rotary Processor

Video footage was taken of the motion of the particle from entry to exit in the rotary unit, digitized and stored on a computer as an accessible video file. The number of half revolutions (corresponding to time steps) of the unit that occur is displayed. Viewing the operation of the rotary unit enables the system user to obtain a strongly intuitive feeling of the inherent uncertainty in system dynamics.

To apply the Markov Chain to this system, the rotary unit can be divided into bands or zones so that the position or displacement of the particle is discretized. As with the counter moving along the Snakes and Ladders board, the motion of a particle in the rotary unit is random i.e. varies from trial to trial. Hence we obtain a distribution in the Residence Time in both systems. Each half revolution of the unit, the particle falls vertically down and then tumbles or rolls a certain distance that changes every time. Thus each half revolution of the rotary unit, corresponds to a throw of the dice. The distance that it tumbles or rolls along the floor of the unit corresponds to the movement of the counter along the board. Occasionally, the particle may tumbles a long way forward (a ladder) and other times it rolls a long way back (a snake).

4] COMPUTER IMPLEMENTATION OF THE TEACHING TOOL

A computer animation of the board game was built with the software, Excel and MATLAB. A 100 square board game with 100 states would have a corresponding 100 by 100 entry transition matrix and this was regarded as too large to appreciate by a new user. So a 6 by 6 board game was developed. The board and dice appear on the screen. The user clicks on the dice icon and a random number generator returns a number to the screen. The counter is simultaneously moved along the same amount on the board. The process continues until the game finishes with the number of dice throws recorded. The game can be repeated as many times as desired and the distribution in the number of required dice throws (the residence time distribution) shown in frequency histogram form. Sample screen output is shown in figure 3.



TRANSITION MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 3: Screen Output of Snakes & Ladders Simulation with Corresponding Transition Matrix

The initial state vector and probabilistic transition matrix of the corresponding Markov chain can be viewed in a separate window. Each click on the time step icon, updates the state vector (by post multiplication by the transition matrix) to give the probabilities that the counter lies in each of the possible states. This information is also displayed in frequency histogram form. On each successive frequency histogram, the mean displacement is marked by a red line and the 'average' time for the game to finish can be ascertained.

In the board game, the transition matrix that governs the stochastic motion of the counter or particle comes from analysis of the possible dice outcomes. For the rotary processor, the transition probabilities must come from experimental analysis of the motion of the particle. A transition matrix was built from recorded probabilities and can be displayed to the system user. As with the Markov model of the board game, by clicking on an icon the user can advance the simulation by one time step with the state vector being updated and displayed in frequency histogram form. Figure 4 shows a snapshot of the dynamic evolution of the displacement state vector.

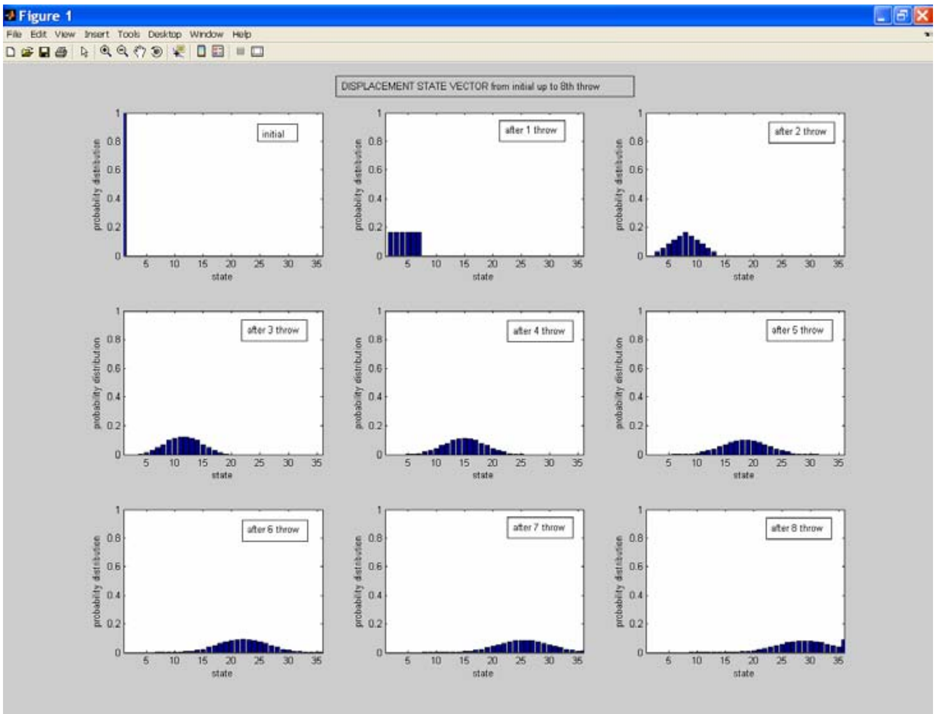


Figure 4: Screen Output of Markov Simulation

5] USER-LEARNING WITH THE TOOL

The tool will be configured so that the user can explore various aspects of Markov Chains. As with all analogies, the behaviour of a particle in a rotary unit is not identical to that of a counter in Snakes and Ladders. However by investigating modifications of the board game, the agreement between both systems can be improved. This study also enriches the learning experience. The behaviour of a counter on a board without the Snakes and Ladders or with the Ladders but without the Snakes can be easily simulated with a slightly modified transition matrix. Also the effect of changing the dice characteristics can be studied. With the standard dice, irrespective of the outcome of the throw, the counter must always move forward. The dice could be changed so instead of the six possible outcomes being (1, 2, 3, 4, 5, 6) they are (-3, -2, -1, 1, 2, 3). Now the counter can move back as well as forward and forward and reverse motion are equally likely so we have a pure *diffusion* system. Alternatively, the six outcomes could be arranged to be (-1, 0, 1, 2, 3, 4) where forward motion is more probable than backward motion so we have *convection* superimposed on *diffusion*. All these dice arrangements give six Uniform or Equi-distributed outcomes. A Non-Uniform dice would favour some outcomes more than others; for instance (0, 1, 1, 2, 2, 2) where the probability of moving forward two squares is now 0.5 rather than 1/6.

In the board game, either a single person can play with a single counter or multiple persons can play, each with their own counter. Each player's counter moves independently of the other persons. In a real rotary processor, thousands of particles will be present simultaneously and they will interact significantly. To maintain the analogy with the board game only the motion of a single particle will be simulated.

Various aspects of Markov theory can be illustrated by facets of the game. In some versions, any dice throw that gives a number greater than that needed to move the counter to state 100 will finish the game. In Markovian terminology, the final state is treated as an *absorbing* barrier. In other versions, if a number greater than that needed is obtained, the counter bounces back from the final state by an amount equal to the excess; here the final state has the characteristics of a *reflecting* barrier. Very significantly, the rule that if a 6 is thrown then the dice can be throw again, cannot be included in the model. Such a rule is a non Markovian feature as it introduces a history effect (i.e. the fact that the player previously threw a 6) into the progress of the counter. It acts as an illustration of an effect that violates the basic Markov assumption.

DISCUSSION

The development of the tool is near the completion stage and it is anticipated it will be ready for the forthcoming academic year. It has yet to be decided whether it will reside on the web or on the University's Blackboard system. For the pilot trial, there will be an initial contact hour with the prospective students to explain the tool. An on-line self-test exam will be added to the system to permit the students to test their own knowledge. A questionnaire will be distributed to the all participants to assess its use and to suggest changes.

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