DEVELOPMENT OF A COMPUTER-BASED TEACHING TOOL IN PROBABILISTIC MODELLING

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This work was part founded by Science Foundation Ireland under the STAR Programme

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$$P = \begin{pmatrix} p_{11} & p_{12} & . & . & . \\ p_{21} & p_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & p_{nn} \end{pmatrix}_{n\mathbf{x}n}$$

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$$a_k = a_0 P^k \quad , \qquad k = 1, 2, 3, ...$$

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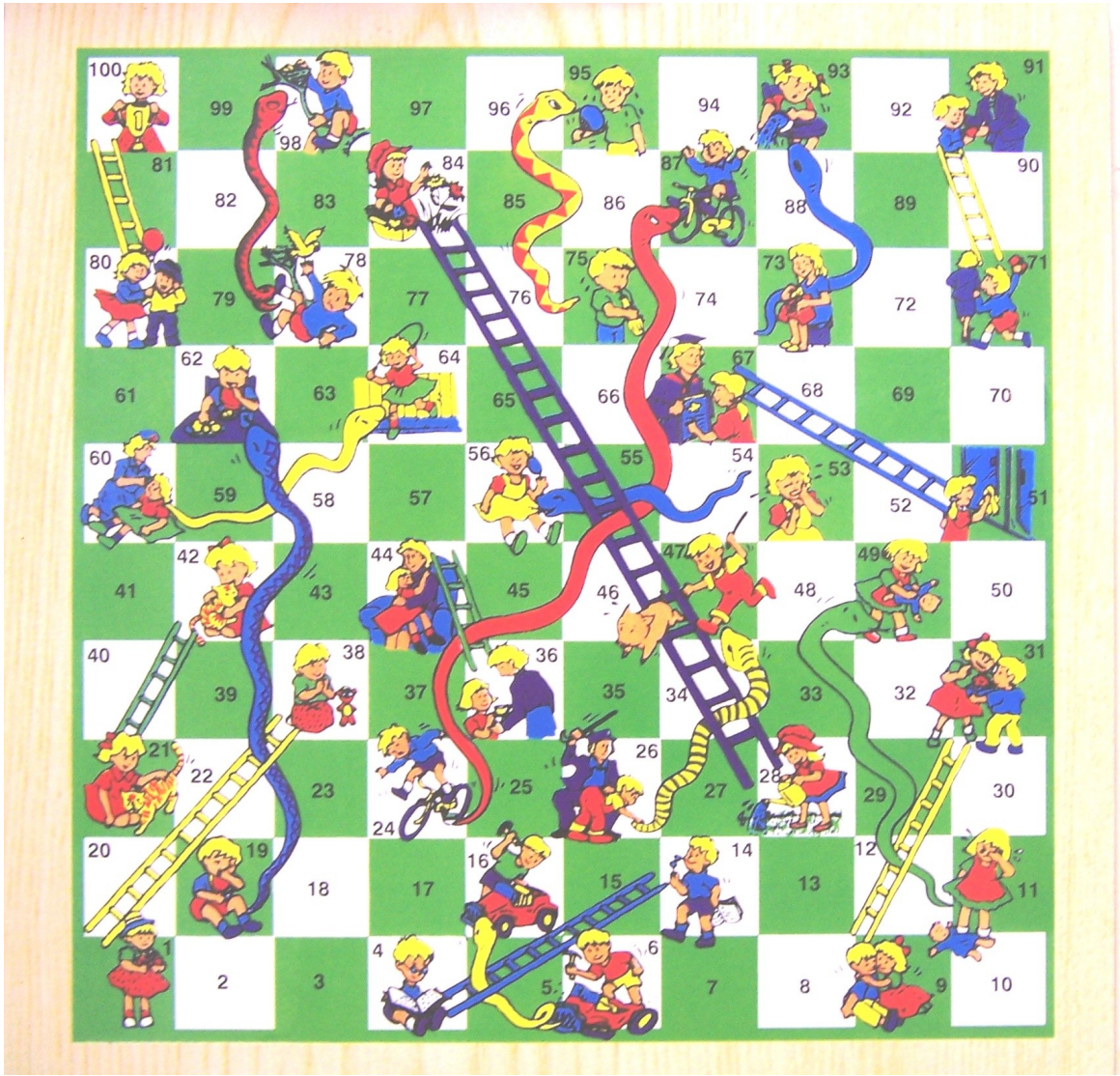
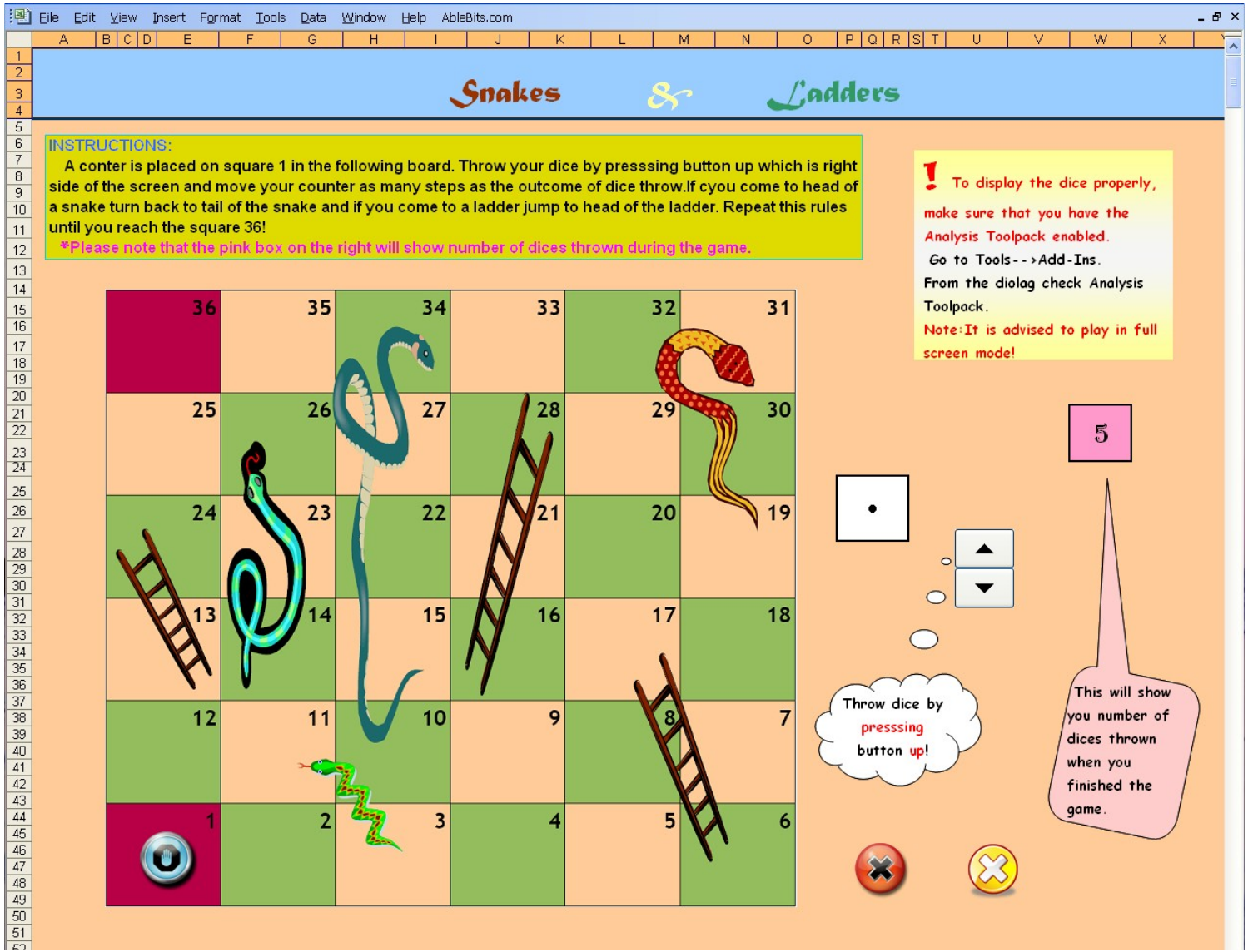
- to make Markov Chain understandable to people with limited mathematical background
  - well known children's board game, *Snakes & Ladders*
- to determine Residence Time Distribution of particles in a *rotary processing unit*

Game Board


100 99 98 97 96 95 94 93 92 91
81 82 83 84 85 86 87 88 89 90
80 79 78 77 76 75 74 73 72 71
61 62 63 64 65 66 67 68 69 70
60 59 58 57 56 55 54 53 52 51
41 42 43 44 45 46 47 48 49 50
40 39 38 37 36 35 34 33 32 31
21 22 23 24 25 26 27 28 29 30
20 19 18 17 16 15 14 13 12 11
1 2 3 4 5 6 7 8 9 10

Snakes & Ladders Game



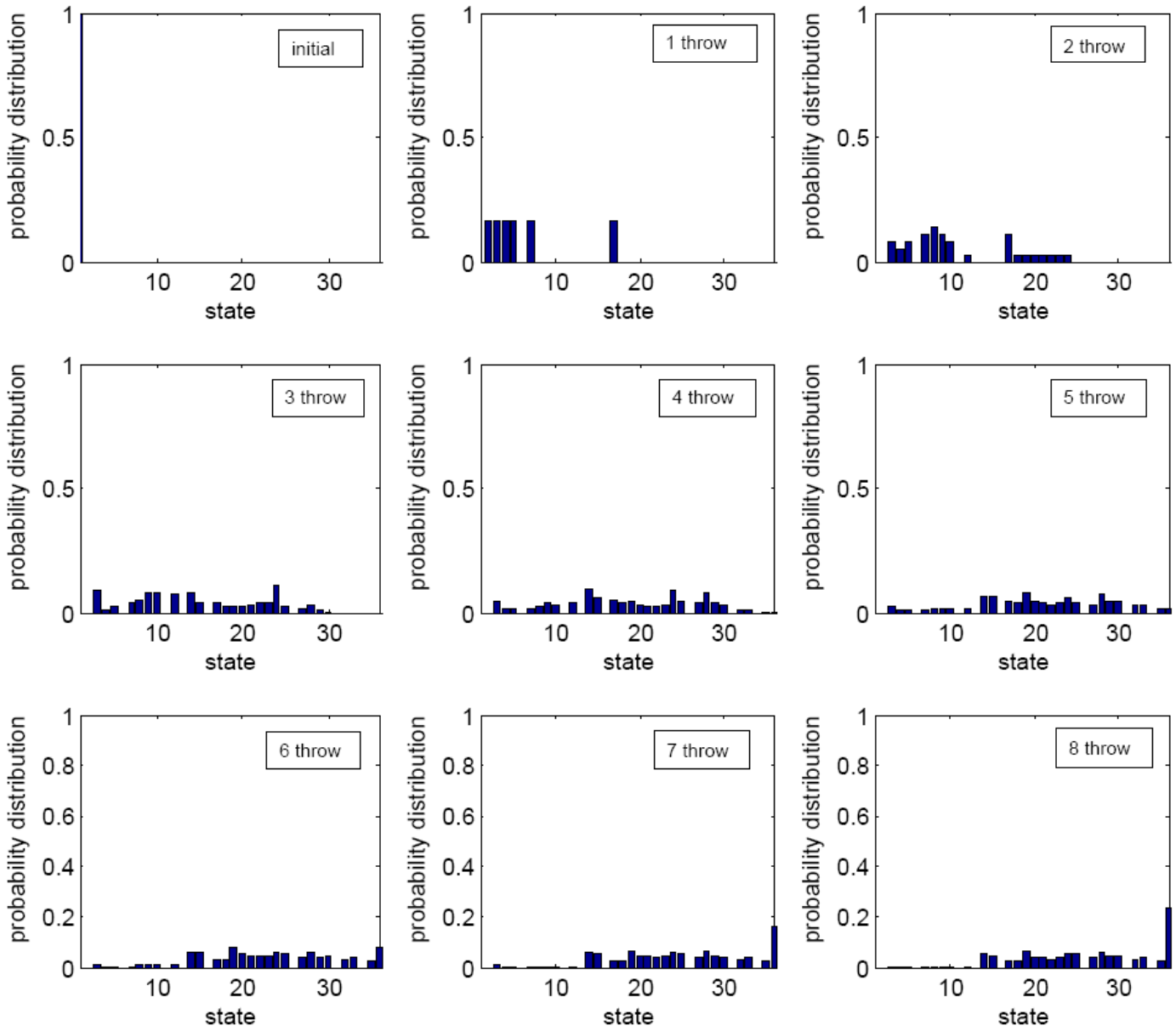
Transition Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	1/6	0	0	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	1/6	0	0	0	0	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	1/6	0	0	0	0	0	1/6	1/6	0	1/6	0	1/6	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	1/6	0	0	0	0	0	0	1/6	0	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	1/6	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	1/6	0	0	0	1/6	0	0	0	0	0	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	0	0	0	0	0	1/6	0	0	0	1/6	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	1/6	0	0	0	0	0	0	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	1/6	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	0	1/6	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/2
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	2/3
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

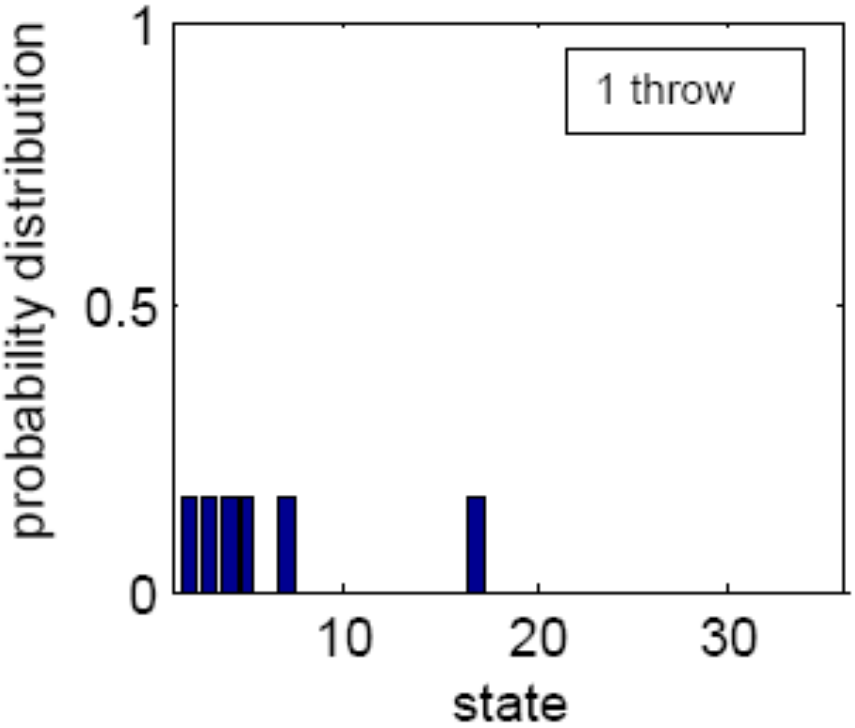
Transition Matrix

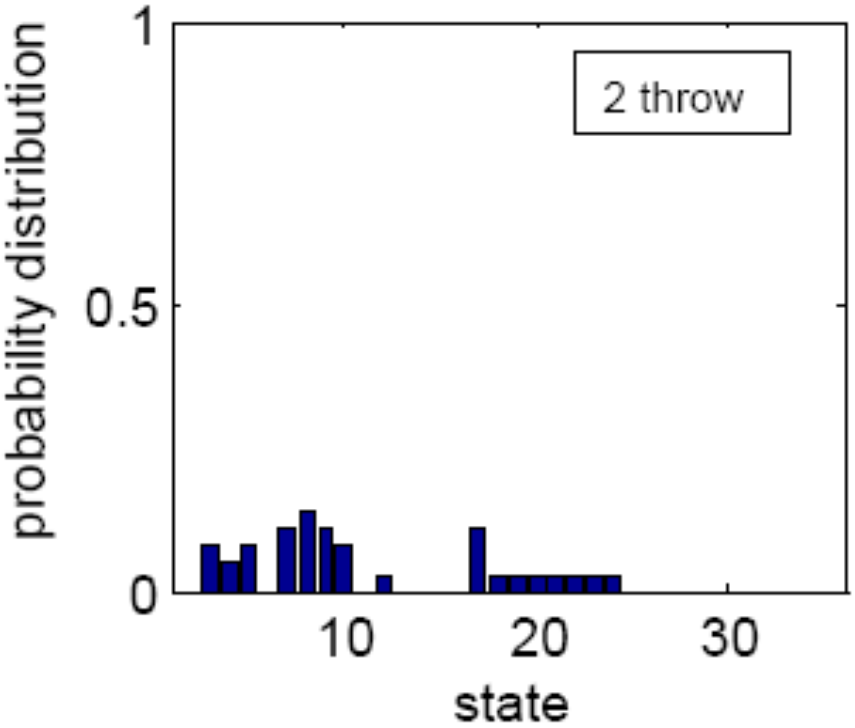
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1
1	0	1/6	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	0	0	0	1/6	0	
2	0	0	1/6	1/6	1/6	0	1/6	1/6	0	0	0	0	0	0	0	0	1/6	0	
3	0	0	0	1/6	1/6	0	1/6	1/6	1/6	0	0	0	0	0	0	0	1/6	0	
4	0	0	0	0	1/6	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	1/6	0	
5	0	0	1/6	0	0	0	1/6	1/6	1/6	1/6	0	0	0	0	0	0	1/6	0	
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	
7	0	0	1/6	0	0	0	0	1/6	1/6	1/6	0	1/6	0	0	0	0	0	0	
8	0	0	1/6	0	0	0	0	0	1/6	1/6	0	1/6	0	1/6	0	0	0	0	
9	0	0	1/6	0	0	0	0	0	0	1/6	0	1/6	0	1/6	1/6	0	0	0	
10	0	0	1/6	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	0	0	0	
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
12	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	0	1/6	1/6	
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	0	1/6	1/6	1/
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/6	1/
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/6	1/
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/

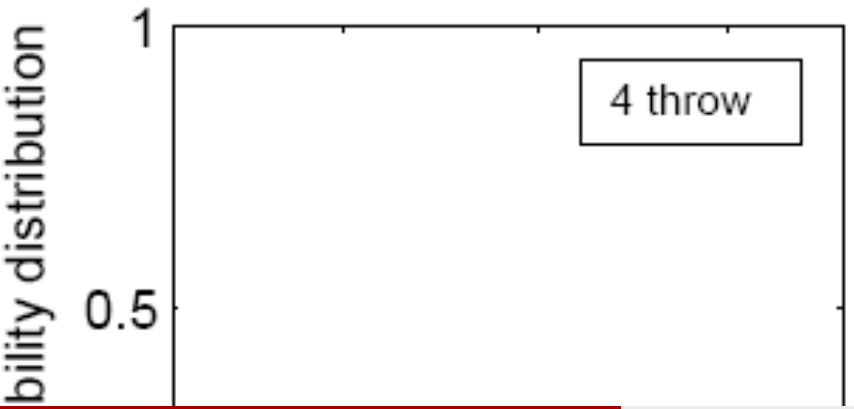
Markov Simulation

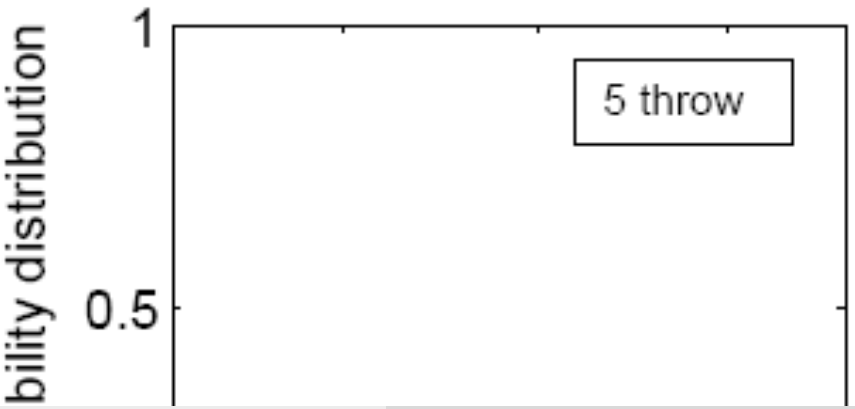


Markov Simulation

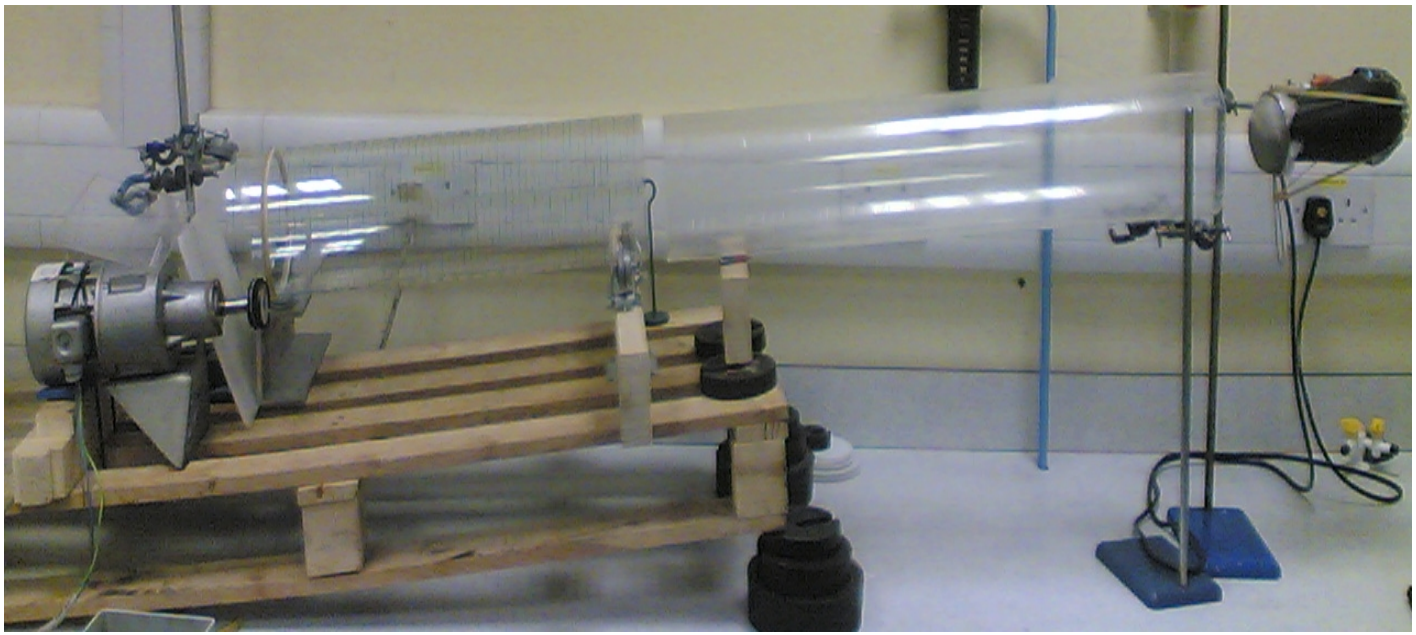








Rotary Processor



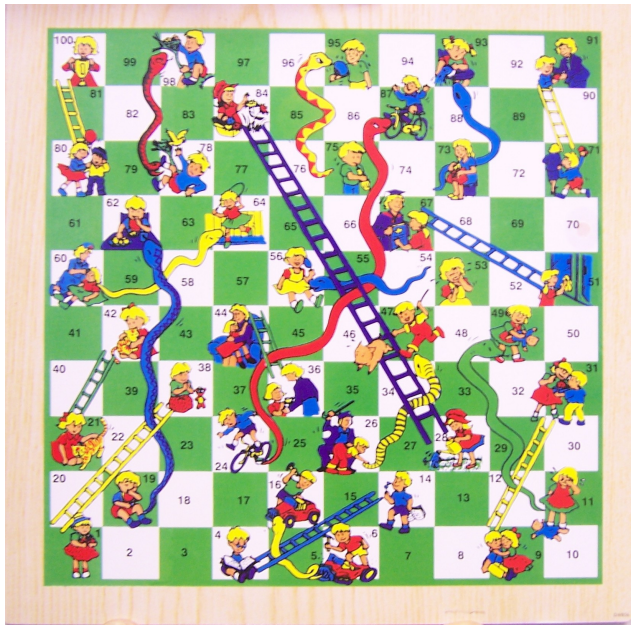
Transition Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	26/75	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	13/50	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	15/79	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	13/95	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	2/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1/23	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1/60	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1/75	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/75	3/857
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	1/99	1/60
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303	2/85
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	2/75
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/23
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/20
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	5/68
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	10/91
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	4/25
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	12/59
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	24/91
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	32/95
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	25/63
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	11/24
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	25/49
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	53/93
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

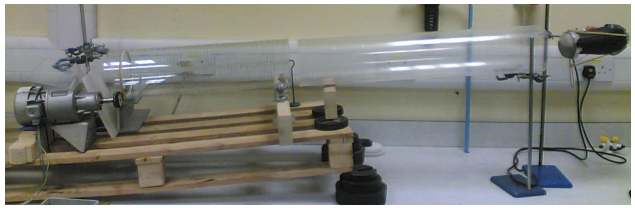
Transition Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	26/75	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60	1/303
3	13/50	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99	1/60
4	15/79	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43	1/99
5	13/95	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82	1/43
6	2/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20	3/82
7	1/23	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23	1/20
8	1/60	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50	1/23
9	1/75	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41	3/50
10	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50	3/41
11	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50	3/50
12	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75	3/50
13	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50	4/75
14	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12	3/50
15	0	0	0	0	0	0	1/99	1/303	2/75	1/23	1/20	4/75	4/57	2/23	1/12

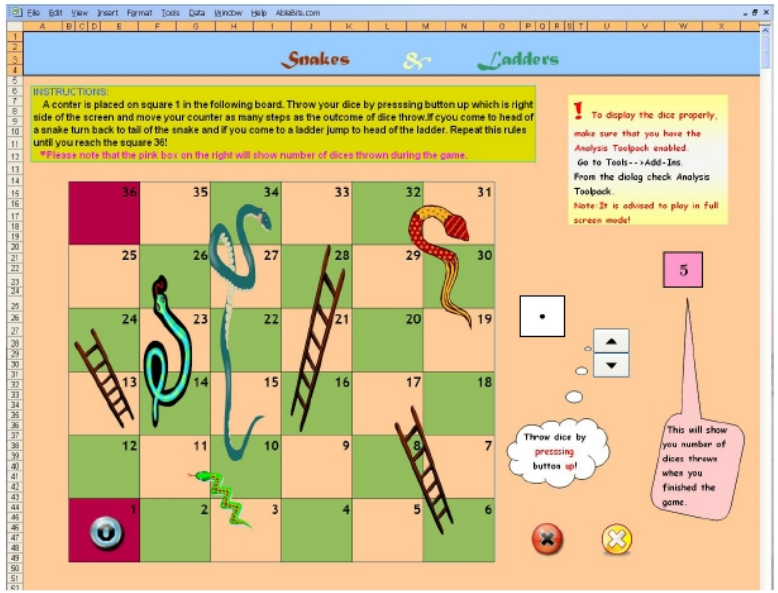
- In the board game;
  - a single person or multiple persons
  - uniform dice or non-uniform dice
  - without Snakes & Ladders or with the Ladders but without the Snakes



- In the board game;
  - a single person or multiple persons
  - uniform dice or non-uniform dice
  - without Snakes & Ladders or with the Ladders but without the Snakes
- In rotary process;
  - thousands of particles
  - a single particle



- In the board game;
  - a single person or multiple persons
  - uniform dice or non-uniform dice
  - without Snakes & Ladders or with the Ladders but without the Snakes
- In rotary process;
  - thousands of particles
  - a single particle
- In Markov theory;
  - Absorbing barrier
  - Reflecting barrier



Thank You!

Questions??