# Computational Analysis of Gene Expression Data 

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## Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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## Abstract

Gene expression is central to the function of living cells. While advances in sequencing and expression measurement technology over the past decade has greatly facilitated the further understanding of the genome and its functions, the characterisation of functional groups of genes remains one of the most important problems in modern biology. Technological advancements have resulted in massive information output, with the priority objective shifting to development of data analysis methods. As such, a large number of clustering approaches have been proposed for the analysis of gene expression data obtained from microarray experiments, and consequently, confusion regarding the best approach to take. Common techniques applied are not necessarily the most applicable for the analysis of patterns in microarray data. This confusion is clarified through provision of a framework for the analysis of clustering technique and investigation of how well they apply to gene expression data. To this end, the properties of microarray data itself are examined, followed by an examination of the properties of clustering techniques and how well they apply to gene expression.

Clearly, each technique will find patterns even if the structures are not meaningful in a biological context and these structures are not usually the same for different algorithms. Also, these algorithms are inherently biased as properties of clusters reflect built in clustering criteria. From these considerations, it is clear that cluster validation is critical for algorithm development and verification of results, usually based on a manual, lengthy and subjective exploration process. Consequently, it is key to the interpretation of the gene expression data. We carry out a critical analysis of current methods used to evaluate clustering results. Clusters obtained from real and synthetic datasets are compared between algorithms.

To understand the properties of complex gene expression datasets, graphical representations can be used. Intuitively, the data can be represented in terms of a bipartite graph, with weighted edges between gene-sample node couples corresponding to significant expression measurements of interest. In this research, this method of representation is extensively studied and methods are used, in combination with probabilistic models, to develop new clustering techniques for analysis of gene expression data in this mode of representation. Performance of these techniques can be influenced both by the search algorithm, and, by the graph weighting scheme and both merit vigorous investigation. A novel edge-weighting scheme, based on empirical evidence, is presented. The scheme is tested using several benchmark datasets at various levels of granularity, and comparisons are provided with current a popular data analysis method used in the Bioinformatics community. The analysis shows that the new empirical based scheme developed out-performs current edge-weighting methods by accounting for the subtleties in the data through a data-dependent threshold analysis, and selecting 'interesting' gene-sample couples based on relative values.

The graphical theme of gene expression analysis is further developed by construction of a one-mode gene expression network which specifically focuses on local interactions among genes. Classical network theory is used to identify and examine organisational properties in the resulting graphs. A new algorithm, GraphCreate, is presented which finds functional modules in the one-mode graph, i.e. sets of genes which are coherently expressed over subsets of samples, and a scoring scheme developed (using bi-partite graph properties as a basis) to weight these modules. Use of this representation is used to extensively study published gene expression datasets and to identify functional modules of genes with GraphCreate. This work is important as it advances research in the area of transcriptome analy-
sis, beyond simply finding groups of coherently expressed genes, by developing a general framework to understand how and when gene sets are interacting.

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## CHAPTER 1

## InTRODUCTION

Approximately a century and a half ago an Augustinian monk, Gregor Mendel, hypothesized, through the study of Pisum Sativum (Pea plants), that there were units of heredity that controlled how traits were passed from one generation to another. Unknowingly he was preparing the core of genetic theory. In the intervening time, this discipline has changed substantially, although the analysis of genes in the nucleus of the cell has remained a fundamental concern in the study of biological organisms. Understanding how, why and when genes are expressed ${ }^{1}$ is critical to the understanding of the functioning of the cell, and hence of biological organisms. Experimental work suggests that biological networks are modular, (Barabasi and Oltvai, 2004; Petti and Church, 2005) with modules defined over groups of genes and proteins, as well as other molecules that are involved with a common subcellular process. The underlying idea in clustering genes is that genes that are coregulated will be grouped together, and if co-regulation indicates shared functionality, then clusters defined at gene level represent biological modules. Understanding how, why and when these groups operate is one of the most important questions in modern Biology.

[^0]
### 1.1 Motivation for High Level Computational Analysis of Gene Expression Data

What is High Level Computational Analysis of Gene Expression Data?
Analysis of the transcriptome of an organism or group of cells to infer how gene expression affects it's function can take many forms. These include laboratory experiments, which might include gene "knock outs" to analyse the effect on phenotype, or time consuming, gene-by-gene investigations experiments. Computational analysis, involving simultaneous analysis of multiple gene expression can also be carried out. Computational algorithms and techniques include (i) extraction and estimation of expression levels - normally referred to as "low level" analysis, or (ii) identification and linkage of patterns of expression in the data - referred to as "high level" analysis. This categorisation, though crude, is useful and is expanded upon throughout the thesis.

Why is high level analysis of gene expression data important?
Single gene experiments can reveal only a limited amount of information. Genes do not work in isolation, but rather in modules in which the products of a number of genes come together. Furthermore, genes may be expressed coherently under one condition, but diverge under another. Identifying functional groups of genes can shed new light on the prognosis of a disease, (identifying for example targets for treatments), can elucidate functions of unknown genes, determine sets of genes involved in regulation of a particular process and so on. For example, in a study of the Saccharomyces Cerrvisae genome, hypoxic genes, which are transcriptionally repressed during aerobic growth (through recruitment of the Ssn6-Tup1 repression complex by the DNA binding protein Roxl), where investigated, (Klinkenberg et al., 2005). In an oxygen deprived environment, cells are unable to main-
tain oxygen-dependent heme (a complex containing iron, among other elements, to which oxygen binds) biosynthesis and heme accumulates in the cell, which serves as an effector for the transcriptional activator Hapl, Fig. 1.1. A heme-Hapl complex activates transcription of the ROX1 gene that encodes the repressor of one set of hypoxic genes. Under hypoxic conditions, heme levels fall, and a heme-deficient Hapl complex represses ROX1 expression. As a consequence, the hypoxic genes are derepressed (Klinkenberg et al., 2005). Put quite simply, if we examine the transcriptome of S. Cerrvisae during hypoxic and aerobic conditions, and we have prior information that the ROX1 gene encodes a repressor for hypoxic genes, we have the potential to evaluate which set of genes are affected by ROX1.


Figure 1.1: Through the repression or activation of ROX1 gene, hypoxic genes can be regulated. Identification of potential functional and/or regulatory groups is one of the aims of cluster analysis. Blue indicates repression, while green indicates activation.

What 'tools' are used for high level analysis of gene expression data?

High level analysis of gene expression data combines methods from Biology, Statistics and Computer Science to derive a picture of what genes are expressed in the nucleus of a living cell. Biological Science poses questions and hypotheses about active processes of the cell and gives us experimental tools to test these hypotheses. Statistical quantification of evidence is standard, not least because new automated tools mean that experimental techniques are rapidly becoming high-throughput in nature, resulting in a vast amount of data. Finally, Computer Science offers tools to organise, analyse and visualise the information generated in addition to the potential to simulate abstractions if biological theory and the theoretical implications. Statistical robustness is crucial, and results of analysis can provide a further basis for biological experiments. This cyclical process, with each disciplinary combination both directly and indirectly reinforcing investigations as a whole is a powerful combination. The focus here is computational although the context of the work is core to understanding the choices made.

### 1.2 Scope and Contribution

With the explosion of data following the typing of the human genome in 2000, much effort has focused on data generation, rather less on appropriate methods of analysis, with the initial assumption being that standard methods would apply. Systematic assessment of analytical techniques for high throughput technologies, such as microarray data has attracted limited interest, (Kerr and Churchill, 2001; van Bakel and Holstege, 2004; Zakharkin et al., 2005; Pham et al., 2006; Datta and Datta, 2006; Giancarlo et al., 2008). The view that clustering methods are universally applicable is a common mis-conception and recently, has provoked considerable controversy among practitioners, not least in the biological context, (Levsky
and Singer, 2003; Shendure, 2008). The main objectives of this thesis are thus: (i) An assessment of common unsupervised clustering methods and their applicability to gene expression data, (ii) an understanding of the purpose of such analysis, i.e. an extensive examination of the properties of the gene expression dataset, (iii) the development of a robust solution to computational analysis of these data, (iv) testing the solution proposed for diverse data.

### 1.3 Layout of Thesis

Chapter 2 introduces the main concepts and defines the terminology used throughout the thesis. Many of the definitions introduced in this Chapter can be found in the glossary for ease of reference.

Chapter 3 examines the theoretical state of the art in gene expression clustering. In particular, commonly-applied clustering techniques are examined for their appropriateness for gene expression data, and alternative, notably biclustering and graphical methods, are considered.

Evaluation of clustering results is non-trivial for gene expression data as very little may be known about the data before hand. In Chapter 4 we thus propose a practical approach for the evaluation of clustering techniques. We assess results obtained with selected clustering algorithms (identified in Chapter 3) for real benchmark and synthetic datasets. We demonstrate that recognition of valid clusters is problematic, and results frequently misleading in the context of gene expression data.

In Chapter 5, we adopt a framework for graphical modelling to carry out robust and extensive analyses of gene expression data behaviour. This allows us to link the theory of the gene expression data, identified in chapters 2 and 3 , with the "realistic"
organisational properties and patterns found in large gene expression datasets.
In Chapter 6 we develop algorithms which draw on properties, identified in Chapter 5 , to extract meaningful groups of genes and samples from the datasets. The ultimate goal of this analysis is to group subset of genes and samples, for which the genes show correlated behaviour, i.e. to extract bi-clusters or functional modules from the graph.

Chapter 7 discusses the challenges in analysing gene expression data and what can be achieved through appropriate computational analysis. Overall results are evaluated, and areas for future work highlight on the basis of conclusions drawn.

Due to the fact that this research deals with large datasets and each analysis is quite detailed and to maintain continuity of text, a lot of information has been enclosed into various appendices. Appendix A contains details of mathematical formulae. Details of the benchmark and synthetic datasets are given in Appendix B. Additional graphs from various analysis can be found in Appendix C.

## CHAPTER 2

## BACKGROUND

In this Chapter, we briefly introduce some key concepts concerning gene expression, its role in a biological cell and tools used for its measurement. For a more extensive overview, additional references are cited.

### 2.1 Introduction

Early observers using microscopes noted that living cells contained a light grey sap encapsulating a darker, denser globule of floating matter. In 1831, the botanist, Robert Brown, used the word nucleus to describe this dark, central globule, while the sap is the cytoplasm. Adding stains or dyes to thinly sliced tissue caused chromatin material in the nucleus to stand out. To early observers, chromatin appeared to be tiny granules or delicately intertwined threads scattered about inside the nucleus. These long entangled threads are what we now know as chromosomes.

Today we know that the chromosome structures, found in the nucleus of a cell, consist of linear deoxyribonucleic acid (DNA) polymers, in which the monomeric subunits are four chemically distinct nucleotides that can be linked together, in any order, into chains up to millions of units in length. Each nucleotide in a DNA polymer is made up of three components: a phosphate, a deoxyribose sugar and one of
four nitrogenous bases: adenine, thymine, cytosine or guanine, usually abbreviated to A, T, C and G respectively, (Brown, 2002d).


Figure 2.1: Left - Molecular structure of the four nitrogenous bases, which differentiate the DNA polymers and can be categorised as Purine's (double ring) or Pyrimidine's (single ring). Right - "The Double Helix"

DNA polymers assemble together in pairs within the nucleus of a cell to form a double stranded structure known as the Double Helix, (Figure 2.1). The double helix has structural flexibility due to base-pairing and base-stacking. Base-pairing between the two DNA polymer strands involves the formation of hydrogen bonds between an adenine on one strand and a thymine on the other strand, or between a cytosine and a guanine. Only the A-T and C-G pairs are permissible, partly because of the geometries of the nucleotide bases and the relative positions of the groups that are able to participate in hydrogen bonds, and partly pairing must be between a purine and a pyrimidine, (resulting in the distinctive helix structure). Base-stacking then involves hydrophobic interactions between adjacent base-pairs, adding stability to the double helix, (Walker and Rapley, 1997).

The limitation that only base pairs A-T and C-G are permissible has significant biological implications. It results in perfect copies of a parent molecule during DNA replication through the simple expedient of using the sequences of the pre-existing
strands to dictate the sequences of the new strands. This is template-dependent DNA synthesis and is the system used by all cellular DNA polymerases; (an enzyme fundamental to DNA replication). Further details of DNA polymerase function can be found in Brown (2002c) and Hartl and Jones (2002).

Genes are a DNA sub-segment of the genome which contain important biological information. The vast majority of genes code for proteins and a few for noncoding Ribonucleic acid (RNA) ${ }^{1}$. The so-called "expression" of protein-specifying genes involves intermediate messenger RNA (mRNA), or coding RNA. This is transported from the nucleus to the cytoplasm of the cell where it directs synthesis of the protein coded by the gene. The structure of RNA is similar to that of DNA in that it is also a polynucleotide. However, the sugar in the RNA nucleotide is a ribose sugar and, further, RNA contains the monomeric subunit uracil (U) instead of the base thymine found in DNA. Additionally, RNA is found in single strands in the cell, while DNA is usually double-stranded. RNA is transcribed from DNA by RNA polymerase, facilitating the biological encoding of DNA to be realised, (Brown (2002g,e) for more information)

Template-dependent RNA synthesis is used by RNA polymerases to make RNA copies of genes: these copies preserve the biological information contained in the sequence of the genomic DNA molecule, meaning that the sequence of nucleotides in a DNA template dictates the sequence of nucleotides in the RNA that is created. During transcription, ribonucleotides are added one after another to the RNA transcript, the identity of each nucleotide being specified by the base pairing rules: A-U, G-C ${ }^{2}$. RNA polymerase is the central component of the transcription initiation com-

[^1]plex. Every time a gene is transcribed, a new complex is assembled immediately upstream of the gene. The initiation complexes are constructed at specific positions on the genome, marked by specific nucleotide sequences called promoters, which are only found upstream of the gene, (Brown, 2002b).

Genes are said to be expressed when the product they code for are expressed (see below for details). A large percentage of protein-coding genes are involved in expression, replication and maintenance of the genome. A smaller percentage specifies components of the signal transduction pathway that regulates genome expression and other cellular activities in response to signals received from outside the cell. Other genes code for enzymes, responsible for the general biochemical functions of the cell, while the remainder of the genes are involved in activities such as transport of compounds into and out of cells, the folding of proteins into their correct three dimensional structures, the immune response and synthesis of structural proteins, such as those found in the cytoskeleton and in muscles, (Petsko and Ringe, 2004b).

### 2.2 The Central Dogma of Protein Synthesis

The genome is a repository of biological information, where utilization of this requires the coordinated activity of enzymes and other proteins. These participate in a complex series of biochemical reactions collectively referred to as genome expression.

The initial product of genome expression is the transcriptome, a collection of RNA molecules derived from those protein-coding genes, having biological information required by the cell at a particular time. These RNA molecules direct the synthesis of the final product of genome expression, the proteome, (the cell's reper-


Figure 2.2: Central Dogma of protein synthesis
toire of proteins), which specifies the nature of the biochemical reactions that the cell is able to carry out. Transcription, (where individual genes are copied into RNA molecules, see above) constructs the transcriptome. Construction of the proteome involves translation of these RNA molecules into protein (Figure 2.2). Transcription does not result in synthesis of a new transcriptome but rather (i) its maintenance, (replacing mRNA that have been degraded), and (ii) changes to its composition, (by switching on and off different sets of genes).

It is an inadequate over-simplification to describe synthesis and maintenance of the transcriptome and proteome as the two-step process "DNA makes RNA makes protein", as the series of events involved is much more complex (Figure 2.3). Nevertheless, the Central Dogma has considerable acceptance for its simplicity. In reality, genome expression comprises the following steps, discussed by Brown (2002d,g,a,b,e,f,c).

1. Accessing the genome - Involves processes influencing chromatin structure in the parts of the genome that contain active genes, ensuring that these genes are accessible and are not buried deep within highly packaged parts of the chromosomes.
2. Assembly of the transcription initiation complex - this comprises a set of proteins that work together to copy genes into RNA. Assembly of initiation complexes is a highly targeted process because these complexes must be con-
structed at precise positions in the genome, adjacent to active genes.
3. Synthesis of RNA, during which the gene is transcribed into an RNA copy.
4. Processing of RNA molecule - Involves a series of alterations made to its sequence and chemical structure, and which must occur before the RNA molecule can be translated into protein or, in the case of non-coding RNA, before it can carry out other functions in the cell.
5. RNA degradation - The controlled turnover of RNA molecules, (plays an active role in determining the makeup of the transcriptome), (Lorkowski and Cullen, 2006).
6. Assembly of the translation initiation complex - Occurs at the termini of coding RNA molecules, and is a prerequisite for translation of these molecules.
7. Protein synthesis - The synthesis of a protein by translation of an RNA molecule, (Brown, 2002f).
8. Protein folding and protein processing. Folding results in the protein taking up it's correct three-dimensional structure. Processing involves modification of the protein by addition of chemical groups and, for some proteins, removal of one or more segments of the protein, (Branden and Tooze, 1999).
9. Protein degradation has an important influence on the composition of the proteome and, like RNA degradation, is an integral component of genome expression, (Petsko and Ringe, 2004a).

Control mechanisms exist for regulation of each step, (Figure 2.3), allowing the cell to 'adjust expression' in response to changes in its environment and to signals


Figure 2.3: Detailed Central Dogma
received from other cells. These regulatory events determine not only function of individual cells but also the processes of differentiation and development.

Processing of mRNA has an important influence on the composition of the transcriptome. RNA editing, for example, can result in a single pre-mRNA being converted into two different mRNAs, coding for distinct proteins. Splicing, in which one pre-mRNA gives rise to two or more mRNAs by assembly of different combinations of exons ${ }^{3}$, resulting in protein isoforms ${ }^{4}$, is fairly widespread, (Modrek and Lee, 2002; Lee and Wang, 2005; Birzele et al., 2007). The mRNA resulting from both editing and alternative splicing often displays tissue specificity. These processing events increase the coding capabilities of the genome without the requirement for increased gene number. To some degree, this explains the "surprise" discovery of the Human Genome Project that an estimated 20, $000 \sim 25,000$ protein-coding

[^2]genes only are responsible for the synthesis of many thousands of functional proteins, (Pennisi, 2000; Lander et al., 2001; Consortium, 2004).

Many genes are active at any one time in a given cell. Transcriptomes are therefore complex, containing copies of hundred, if not thousands, of different mRNAs. Usually, each mRNA contributes a small fraction only of mRNA abundance, with the most common type rarely contributing more than $1 \%$ of the total. Overall, mRNA itself typically accounts for $\sim 4 \%$ of total abundance of RNA in a cell, while non-coding RNA makes up the remainder, (Brown, 2002g)

### 2.3 Analysing Gene Expression

Identification of which genes are active, and under what circumstances, is a constant goal of the scientific community. The development of high-throughput technologies for the measurement of the entire transcriptome has evolved from gene-by-gene experimental methods, such as reverse transcriptome polymerase chain reactions (RTPCR) (Erlich, 1989), reverse northern (Alwine et al., 1977) and Southern hybridisation (Southern, 1975). These relatively new high-throughput tools have broadened the size and scope of biological questions scientists can pose.

### 2.3.1 Microarray Technologies

One-at-a-time study of gene expression is time consuming and limited as multiple gene expression is typical for an organism. Microarray and DNA chip technology have facilitated determination of transcriptome composition, enabling comparisons between them.

Various manufacturers provide a large assortment of different microarray plat-
forms (Aligent and Affymatrix for instance). Fundamentally, all platforms take advantage of the specificity and affinity of complementary base-pairing of nucleic acid. Thousands of known discrete DNA sequences (known as probes) are attached, (via print, jet, photolithography), at known positions to a solid surface. To measure the quantity of transcripts of specific genes in a sample of interest, genetic material is extracted from the sample and labelled with a fluorescent dye. The labelled genetic material is referred to as the target. The target is then allowed to hybridise ${ }^{5}$ to the probes on the slide. After hybridisation, a specialised scanner is used to measure the amount of fluorescence (i.e. amount of target) at each probe, which is reported as intensity. The "raw" or "probe-level" data are the intensities read for each of these components. As the address and sequence of each probe is known, probe intensities determine the abundance of the sequence of each specific gene in the target.

## Microarray Platforms

Different platforms can be divided into two main classes that are differentiated by the type of data they produce.
(A) The high density oligonucleotide array platform contains probes of length 25 base-pairs $(\mathrm{bp})^{6}$ that are synthesised directly onto the array surface using photolithography. Rather than discrete spot association with a transcript of interest, a combinatorial probe is used for each gene, i.e. each gene on the array is represented by a series of smaller oligonucleotides that span different parts of the gene. Most probes are designed to represent the most common

[^3]transcripts expressed from a gene. Probe design is thus constrained by needing to meeting the most consistent hybridisation parameters across the entire set - as the size and concentrations of each probe is predetermined, the most critical factors are the Guanine-Cytosine content ${ }^{7}$, predicted secondary structure within the probe itself, as well as the uniqueness of the probe sequence itself, (Pham et al., 2006). There are typically mismatch probes and control probes incorporated in the design to measure the amount of non-specific binding and for use in normalisation procedures. Genetic material from one sample of interest is hybridised to the array resulting in one set of probe-level data per microarray. This platform is typically used for well-sequenced and annotated genomes. The data is, within reason, robust and reproducible between laboratories, with the design of each oligonucleotide being critical to the robustness of the array as a whole.
(B) For two-colour spotted (cDNA array) platforms, the probes may be as long as the gene product, $(\sim 2 k b)$. Probe sets are typically captured from gene products expressed in an organism of interest, so represent a set of likely gene expression patterns. The genetic material from two target samples are labelled with separate dyes (Cy3 (green) and Cy5 (red)), mixed together and hybridised to the same array, thus producing two sets of probe-level data per microarray (the red and green channels), (Gentleman et al., 2005a). This platform is typically used for incompletely sequenced genomes, but is often limited by poor probe annotation, redundancy in the array of gene products, cross-hybridisation of sequences common to different probes and sub-optimal

[^4]variation in hybridisation efficiencies ${ }^{8}$ across the probe set. The platform is useful for organisms not well represented in the public sequencing databases, (Pham et al., 2006).

The choice of platform (A or B) determines the type of experimental design (two colour comparison, or single colour indirect comparison), as well as choice of normalisation and filtering strategies employed. Sources of variation need to be accounted for, and data must be heavily manipulated before the genomic level measurements used for analysis can be obtained.

### 2.4 Design of Gene Expression Analysis Experiments

The size and scope of the questions the data can answer is dependent on the experimental design. Important features are: (i) choice and collection of samples (tissue biopsies or cell lines exposed to different treatments); (ii) choice of probes and array platform to use; (iii) choice of controls (to measure non-specific binding, noise, and for normalisation procedures etc.); (iv) RNA extraction, amplification, labelling, and hybridisation procedures; (v) allocation of replicates; and (vi) scheduling. Unsurprisingly, the quality of experimental design to a large extent determines the utility of the data, and avoidance of confounding between biological factors and/or measurement artefacts is important. Examples of biological factors include tissue heterogeneity, genetic polymorphism, and changes in mRNA levels within cells and among individuals due to sex, age, race, genotype-environment interactions and so on. The Biological variation between experimental units (i.e. individual mice, rats, tissue samples etc.) is of intrinsic interest to investigators. However, technical variation inherent in preparation of samples, labelling, hybridisation and other steps

[^5]of microarray experimentation, can significantly impact data quality, (Zakharkin et al., 2005) To minimise this quality control of RNA samples is required, (Gentleman et al., 2005a). For good reviews of microarray experiment design principles see Yang and Speed (2002), Churchill (2002) and Pham et al. (2006).


Figure 2.4: Microarrays have a large number of applications and expression data measurements can have an impact on a large spectrum of research areas.

Replicates: Biological replicates are samples extracted from independent ${ }^{9}$ biological units (cell line, organism etc.). Technical replicates are genetic material extracted from the same biological unit and hybridised to two different arrays. Independent biological replication is very important in experimental design, to achieve adequate power and validity in statistical inference and testing. For technical replicates, conclusions are descriptive and limited to the samples used (i.e. descriptive as opposed to inferential), (Churchill, 2002).

Comparison: Two-colour cDNA arrays are inherently directly comparable between samples. With oligonucleotide arrays, however, only one sample can hybridise to an array, thus only indirect comparisons are possible. The main design issue with cDNA microarrays is to determine which RNA samples should be hybridised together on the same slide to achieve the desired precision. Popular de-

[^6]signs and other practical considerations are discussed by Churchill (2002); Yang and Speed (2002) and references therein.

Reproduction and Analysis: The experimental design influences both data collected and the analysis performed. Comprehensive and meticulous details of the experimental procedures are vital for subsequent analysis of the data.

### 2.5 Microarray Data

### 2.5.1 Abundance Measures

High density oligonucleotide array and cDNA array platforms measure overall and relative abundance of a probe sequence in one and two target samples respectively, i.e. the former give absolute (log) intensities, while the latter give ratio (log) intensities. In many cases one of the samples in a cDNA array hybridization is a common reference used across multiple slides, with the sole purpose of providing a baseline for direct comparison of expression between arrays, (Gentleman et al., 2005b).

For oligonucleotide arrays, the direct comparison of expression measures within arrays is problematic, because fluorescent intensities are not the same across genes. The measured fluorescence intensities are roughly proportional to mRNA abundance but the proportionality factor, $(p)$, is different for each gene. When using short oligonucleotide arrays, $p$ is a function of the probes used and, in particular, of the frequencies of the different nucleotides in each. Specifically, the betweensample, within-gene comparisons are valid and sensible, but the within-sample, between-gene comparisons are not.

As an illustration of the difficulty, suppose that genes $a$ and $b$, have estimated expression measures 100 and 200 respectively, in sample $i$. These observed data tell
us nothing about the real relative abundance of mRNA for these two genes. There could in fact be more copies of the mRNA for gene $a$. On the other hand, if in a second sample, $j$, gene $a$ has an expression measure of 200 , we could conclude that the abundance of mRNA for $a$ in sample $j$ is likely to be higher than that observed in sample $i$, (Gentleman et al., 2005b).

For cDNA arrays the measure of interest is a ratio of abundance, typically calculated relative to a standard reference. Consider a sample, $i$, with estimated relative abundance of 1 for gene $a$ and 2 for gene $b$. It can be inferred that gene $a$ is expressed at approximately the same level in sample $i$ as in the reference sample, while gene $b$ has approximately twice the abundance of mRNA as the reference sample. Note: relative values if $a, b \mathrm{mRNA}$ abundance are unknown as the value in the reference sample is not specified here. Certain designs are also less readily interpretable e.g. dye swaps (Churchill, 2002) (a gene ratio may be recorded a 2 for one slide and 0.5 for another, corresponding to the same abundance of target). These simple examples serve to show that data from transcriptome analysis experiments need to be carefully interpreted in the context of the experimental design.

### 2.5.2 Gene Expression Data Characteristics

Once raw gene expression data are collected and processed, these are typically presented as a real-valued matrix, with rows corresponding to gene expression measurements over a number of experiments, and columns corresponding to the pattern of expression of all genes for a given microarray experiment, Figure 2.5. Each entry, $x_{i j}$, is the measured expression of gene $i$ in experiment $j$. Dimensionality of a gene or sample refers to the number of its expression or sample values recorded (number of matrix columns or rows respectively). A gene/gene profile, $\overrightarrow{g_{i}}$, is a single
data item (row) consisting of $p$ measurements, $\overrightarrow{g_{i}}=\left(x_{i 1}, x_{i 2}, \ldots, x_{i p}\right)$. An experiment/sample $\overrightarrow{s_{j}}$ is a single microarray experiment corresponding to a single column in the gene expression matrix, $\overrightarrow{s_{j}}=\left(x_{1 j}, x_{2 j}, \ldots, x_{n j}\right)^{T}$, where $n$ is the number of genes in the dataset. The notation adopted throughout this thesis is presented in Table 2.1.


Figure 2.5: Gene Expression Matrix

| X | Gene expression matrix |
| :--- | :--- |
| n | Number of genes |
| p | Number of samples |
| $x_{i j}$ | A cell in the gene expression matrix |
| $\overrightarrow{g_{i}}$ | Gene vector $i$ |
| $\overrightarrow{s_{j}}$ | Sample vector $j$ |

Table 2.1: Notation used throughout thesis

From the above discussion, we can summarise the properties of microarray data. Accuracy: The accuracy of gene expression data strongly depends on experimental design and minimisation of technical variation, whether due to instruments, observer or pre-processing ${ }^{10}$, (Zakharkin et al., 2005). It also depends on the number

[^7]of alterations of an mRNA molecule before it is measured by the array.
Incompleteness: Image corruption and/or slide impurities may lead to unusable or undetectable fluorescent intensities resulting in incomplete data, or missing values (Troyanskaya et al., 2001).

Noise: Due to the many uncontrollable factors in the experiments (biological variation, binding efficiencies, cross hybridisation etc.), gene expression data is intrinsically noisy, resulting in outliers, typically managed by: (i) robust statistical estimation/testing, (when extreme values are not of primary interest), (ii) identification, (when outlier information is of intrinsic importance), (iii) manual screening for defective slides, (Liu et al., 2002).

High Dimensional Data: The resulting dataset is of high dimension, with a few experiments, reporting on a large number of variables.

### 2.6 Beyond Microarrays

In addition to microarrays, other experimental methods for gene expression monitoring are continually under development. Serial Analysis of Gene Expression (SAGE), for example, is a technique used to produce a snapshot of the mRNA population in a sample, (Velculescu et al., 1995). The output of SAGE is a list of short sequence tags and the number of times each is observed. Using sequence databases a researcher can usually determine the original mRNA, (and therefore which gene), the tag was extracted from. Statistical methods can be applied to tag and count lists from different samples in order to determine which genes are more highly expressed between different samples. Several variants of SAGE have been developed, such as, LongSAGE (Saha et al., 2002), RL-SAGE (Gowda et al., 2004) and SuperSAGE (Matsumura et al., 2003). SuperSAGE advances its predecessors by using a
technique that expands the tag-size, (Matsumura et al., 2003). The longer tag-size allows more precise allocation of the tag to the corresponding transcript. By direct high-throughput sequencing techniques, thousands of tags can be analyzed in one run, producing precise gene expression profiles.

Many vendors provide an assortment of array platforms that are continually improved and upgraded. An alternative microarray, based on randomly arranged beads was developed by Illumina technologies. A specific 50bp oligonucleotide is assigned to each bead type, which is replicated approximately 30 times on an array, (Kuhn et al., 2004). The high degree of replication makes robust measurements for each bead type possible. Randomisation of the probe spots between arrays further avoids the potential systematic biases that could be introduced due to regular arrangements, (e.g. printing, scanning conditions etc.). Ideally, different arrays used in an experiment should have similar clones in different positions. Formerly, used for single nucleotide polymorphism detection amongst others. Illumina bead arrays can now be used to monitor the expression of genes in the entire genome.

Microarrays and their alterations offer a unique opportunity to analyse gene expression and regulation at a global cellular level. However, the generation of large datasets presents challenges in analysis and warehousing of the data, as well as its integration of that data with other high throughput platforms.

The "Central Dogma" of "DNA makes mRNA makes proteins" (that comprise the proteome) is overly simple. A single gene does not translate into one protein and protein abundance depends not only on transcription rates of genes but also on additional control mechanisms, such as mRNA stability, regulation of the translation of mRNA to proteins and protein degradation. Proteins can also be modified by post-translation activity (Brown, 2002(a)). Inevitably, the integration of transcriptome and proteome data will provide a more complete understanding of the
connection of gene expression to the physical chemistry of the cell. Integration and merger of proteomic ${ }^{11}$ and transcription data sources across platforms is needed, together with development of automated high-throughput comparison methods if detailed understanding of cell mechanisms is to be achieved. To this end, as well as to successfully integrate cluster information from different datasets, standardisation of gene and protein annotation methods across databases is overdue, (Waters, 2006). Finally, recent developments in new ontologies and databases facilitate storage of expression and meta information, which assists enormously in validation of exploratory analyses of gene expression datasets.

### 2.7 Summary

Gene expression analysis represents only one parameter by which cells or tissues may be characterised. Depending on the experiment, epidemiological or molecular pathological data, genomic changes or sensitivity to drugs may be additional parameters that will influence the interpretation of microarray data. The ability to combine RNA and protein expression data to comprehensively profile both transcriptional and post-transcriptional changes in cells and tissues is particularly appealing, although the number of proteins that can be profiled at this stage is substantially less than the number of genes. Although it is more difficult to identify proteins that are differentially expressed, techniques for rapid and reproducible two-dimensional gel protein separation and mass spectrometry-based protein identification make high throughput proteomics, not only desirable, but feasible in the short term as an adjunct to microarray transcriptome analysis, (Bowtell, 1999). Consequently, accurate algorithms and computing techniques are needed to measure and understand

[^8]transcript data before integration with other levels.
Measurement of gene expression is critical for the understanding of cellular biological processes, and although it is not a complete link between sequence and cell function, it is an important element in the chain of events. While high-throughput technology has "evolved" from its ancestor technologies, there are "side effects" which need to be dealt with, not least the challenge of analysing abundant data. Even as this work was completed, new and more precise methods of measurement have emerged, but the need for robust analysis techniques remains.

## CHAPTER 3

## A Review of Techniques

This Chapter examines the strengths and weaknesses of algorithms used in the analysis of gene expression data. The properties of these data were introduced in Chapter 2 and we investigate here what is required from pattern-finding algorithms to meet the challenges of their analysis. We focus on unsupervised pattern recognition algorithms, reviewing key concepts; (further details are also referenced appropriately).

### 3.1 Introduction

Array data are used to determine which genes are expressed under which conditions, and for comparisons between transcriptomes of different biological samples. Searching for meaningful information patterns and dependencies in gene expression data, to provide a basis for hypothesis testing is non-trivial. An initial step is to cluster or "group" genes, with similar changes in expression. Lack of a priori knowledge means that unsupervised clustering techniques, where data are unlabelled (un-annotated), are common in gene expression work.

Many excellent reviews of gene expression analysis, using clustering techniques, are available. Asyali et al. (2006) provide a synopsis of class prediction and dis-
covery; (respectively, supervised pattern recognition and clustering), while Pham et al. (2006) provide a comprehensive literature review of the various stages of data analysis during a microarray experiment. In a landmark paper, Jain et al. (1999) provided a thorough introduction to clustering, and gave a taxonomy of clustering algorithms, (used in this work). Reviewing the state of the art in gene expression analysis is complicated by the high level of interest in exploratory analysis and the consequent proliferation of techniques, Figure 3.1. We restrict our assessment to a selection of those methods, which illustrate the properties of each group in the taxonomy according to Jain et al. (1999), and also to those which address shortcomings of conventional approaches, by introducing modifications to account for properties specific to gene expression data.


Figure 3.1: Data from ISI web of science on number of papers published in the area with key words "Clustering (Pattern Recognition)" and "Gene Expression"

### 3.2 Pattern Recognition

Pattern recognition is an exploratory technique and assumes that there is an unknown mapping that assigns a group "label" to each gene, where the goal is to
estimate this mapping. However, common clustering approaches do not always translate well to gene expression data, and may fail significantly to account for the data profile.

To recapitulate from Chapter 2: properties (which any clustering algorithm in this domain must account for) include: (i) Accuracy: this is not absolute and depends on a number of factors, not least the experimental design and the platform used; (ii) Missing values: common in gene expression datasets, mainly because of the complex and specific nature of the experiments. Frequently, the measured intensity is not convincing and deemed "absent" by experimentalists. Dust, scratches or image quality may also render a large proportion of values unusable. Consequently, any clustering algorithm which can not account for missing values must be supplemented by missing value estimation procedures prior to any exploratory analysis of the data; (iii) Incorporated noise: this is typically multiplicative or additive, may be caused by measurement or experimental error, and affects accuracy. A good clustering algorithm should not be greatly affected by spurious measurements, and should treat outliers with caution.

As cluster analysis is usually exploratory, lack of a priori knowledge on gene groupings or the number of these, $K$, is common. Arbitrary selection of $K$ may undesirably bias the search, as pattern elements may be ill-defined unless signals are strong. Meta-data can guide choice of correct $K$, e.g. genes with common promoter sequence are likely to be expressed together and thus are likely to be placed in the same group. Methods for determining optimal number of groups, $K$, are discussed by Milligan and Cooper (1985) and Fridlyand and Dudoit (2001).

### 3.2.1 Cluster Types

In general, a cluster is defined as a group of objects, which exhibit similar properties according to some objective criterion. Clustering a gene expression matrix can be achieved in two ways:

1. genes can form a group which show similar expression across samples, (i.e. grouping rows of the gene expression matrix).
2. samples can form a group which show similar expression across all genes, (i.e. grouping columns of the gene expression matrix).

Both (i) and (ii) lead to global clusters, where a gene or sample is grouped across all dimensions. However, genes and samples can also be clustered simultaneously, with their inter-relationship represented by bi-clusters. These are defined over a subset of genes and a subset of samples thus focusing on a Section of the gene expression matrix and capturing local structure in the dataset. This is an important strength as cellular processes are understood to rely on subsets of genes, which are co-regulated and co-expressed under certain conditions and behave independently under others, (Ben-Dor et al., 2003).

Justifiably, this approach has been gaining much interest of late. For an excellent review on bi-clusters and bi-clustering techniques see Madeira and Oliveira (2004).

Additionally, clustering can be complete or partial, where the former assigns each gene to a cluster, and the latter does not. Partial clustering tends to be more suited to gene expression, as the dataset often contains irrelevant genes or samples. This allows: (i)"noisy genes" to be left out, with correspondingly less impact on the outcome and (ii) genes to belong to no cluster - omitting a large number of irrelevant contributions. This is important as microarrays measure expression for the entire
genome in one experiment, but genes may change expression independently of the experimental condition, (e.g. due to stage in the cell cycle). Forced inclusion, (as demanded by complete clustering), in well-defined but inappropriate groups may impact final structure found for the data. Partial clustering thus avoids the situation where an interesting sub-group in a cluster is obscured through forcing membership of unrelated genes.

Finally, clustering can be categorised as exclusive (hard), or overlapping. Exclusive clustering requires each gene to belong to a single cluster, whereas overlapping clusters permit simultaneous membership of numerous clusters. This membership may qualify additionally as crisp or fuzzy. Crisp membership is boolean either the gene does or does not belong to a group. In the case of fuzzy membership, each gene belongs to a cluster with a membership weight between 0 , (definitely excluded), and 1 , (definitely included). Clustering algorithms, which permit genes to belong to more than one cluster are typically more applicable to gene expression since: (i)impact of "noise" is reduced - the assumption is that "noisy" genes are unlikely to belong exclusively to any one cluster but are equally likely to be members of several, (ii) this supports the underlying principle that genes, with similar change in expression for a set of samples, are involved in a similar biological function. Typically, gene products are involved in several such biological functions and groups need not be co-active under all conditions. Thus gene groups are fluid, so that constraining a gene to a single group (hard cluster) is counter-intuitive.

### 3.2.2 Steps in Cluster Analysis

Cluster analysis includes several basic steps, (Jain et al., 1999). Initially, the data matrix is represented by number, type, dimension and scale of the gene expression profiles. Some features are set during the experiment, others are controllable, (e.g. scaling, imputation, normalisation etc.). An optional step of feature selection or feature extraction may also be carried out. The former refers to selecting, from the original features, a subset, which is most effective for clustering, while the latter refers to transformation of the input features to form a new set that may be more discriminatory in clustering, e.g. through Principal Component Analysis, (Yeung and Ruzzo, 2001).

Pattern proximity assessment is needed, usually provided by a "distance" measure between pairs of genes. (Alternatively, "conceptual" measures can be used to characterise similarity of gene profiles e.g. Mean Residue Score of Cheng and Church (2000), (see Section 3.3.1)). The next step is to apply a clustering algorithm to determine structure in the dataset. Methods can be broadly categorised according to taxonomy in Jain et al. (1999).

Structures are then described by data abstraction. For gene expression data, the context is usually direct interpretation by a human, so abstraction should ideally be straightforward, (for follow up analysis/experimentation). Required is usually a compact description of each cluster, through a prototype or representative selection of points, such as the centroid. Clusters are valid if they can not reasonably be achieved by chance or as an artefact of the clustering algorithm. Validation requires formal statistical testing, and can be categorised as Internal or External (see Chapter 4).

### 3.3 Clustering and Clustering Extensions

Analysis of large gene expression datasets is a relatively new task, although pattern recognition of complex data is well-established in a number of fields. Many common generic algorithms have, in consequence, been adopted for gene expression data, (e.g. Hierarchical (Eisen et al., 1998), SOM's (Kohonen, 1990), among others), but not all perform well. A good method must deal with noisy high dimensional data, be insensitive to the order of input, have moderate time and space complexity, (i.e. allow increased data load without breakdown or requirement of major changes), require few input parameters, incorporate meta-data knowledge (an extended range of attributes), and produce results, which are interpretable in the biological context.

### 3.3.1 Pattern Proximity Measures

The choice of proximity measure, needed to evaluate degree of expression coherence in a group of gene vectors, is as important as choice of clustering algorithm, and is based on data type and context of the clustering. Many clustering algorithms either employ a proximity matrix directly (e.g. hierarchical clustering), or use one to evaluate clusters during execution (e.g. K-Means). Proximity measures are calculated between pairs (e.g. Euclidean distance) or groups of genes (e.g. Mean Residue Error). For ease of reference, the mathematical formulae for each of the distances can be found in Appendix A.

Distances: Distance functions between two vectors include the so-called Minkowski measures, (Euclidean, Manhattan, Chebyshev, (Romesburg, 2004)), useful when searching for exact matches between two profiles in the dataset. These tend to find
globular structures and work well when these are compact and isolated. A drawback is that the largest feature dominates, so measures are sensitive to outliers, (Jain et al., 1999). However more sophisticated variants, such as Mahalanobis distance, also account for correlations in the dataset and are scale-invariant, (Romesburg, 2004). Different distance measures produce clusters of different shape, (e.g Euclidean are spherical, while Mahalanobis' are ellipsoidal). Alternatively, Kim et al. (2005) describe an adaptive distance norm (the Gaustafson-Kessel method). Here co-variances are estimated for the data in each cluster, (based on eigenvalue calculations), to obtain structure. Each cluster is then created using a unique distance measure.

Distances based on correlations reflect degree of similarity of changes in expression across samples, for two gene expression profiles, without regard to scale. For example, if, for a set of samples, gene $X$ is up-regulated, and gene $Y$ is downregulated, i.e. are negatively correlated, then $X$ and $Y$ would form a cluster. This would clearly not be the case if Minkowski distances were used, since the average absolute distance between the points would be large. Correlation coefficients commonly used include both parametric (standard Pearson, cosine), and nonparametric (Spearman's rank and Kendall's $\tau$ ), the latter used when outliers and noise are present, (Romesburg, 2004). In general, distance $=1-$ correlation $^{2}$, if sign is unimportant.

Conceptual Measures: As an alternative to measures of distance, "conceptual" measures of similarity can be used. Models are based on constant rows, columns and coherent values, (additive or multiplicative), (Madeira and Oliveira, 2004) (Fig. 3.2). A "good fit" indicates high correlation within a sub-matrix, (thus a possible cluster). These models are common to several clustering algorithms. For example, Cheng and Church (2000) and FLOC (Yang et al., 2003), use the additive model
(Fig. 3.2(C)), to evaluate biclusters obtained by determining the Mean Residue Score. Given a gene expression matrix, $X$, an element $a_{i j}$ in a sub-matrix, $A=$ $(I, J)$ is given by the constant additive model:

$$
\begin{equation*}
a_{i j}=\mu+\alpha_{i}+\beta_{j}+r_{i j} \tag{3.1}
\end{equation*}
$$

Note that that the mean value of $A$ corresponds to $a_{I J}$, the offset of row $i$ corresponds to $\alpha_{i}=a_{i J}-a_{I J}$, (the mean of row $i$ minus the overall mean of $A$ ), the offset for column $j$ corresponds to $\beta_{j}=a_{I j}-a_{I J}$, (the mean of column $j$ minus the overall mean of $A$ ) and $r_{i j}$ corresponds to unexplained error, which must be minimised. Simply rearranging equation 3.1 the residue of an element is calculated as:

$$
\begin{equation*}
r_{i j}=\left(a_{i j}-a_{i J}-a_{I j}+a_{I J}\right) \tag{3.2}
\end{equation*}
$$

The " $H$-score" of the sub-matrix is then the sum of the squared residues, given by:

$$
\begin{equation*}
H(I, J)=\frac{1}{|I||J|} \Sigma_{i \in I j \in J}\left(r_{i j}\right)^{2} \tag{3.3}
\end{equation*}
$$

A perfect bi-cluster gives a $H$-score equal to zero, (corresponding to "ideal" gene expression data, with constant or additive matrix rows and columns).

The Plaid Model bi-cluster variant, (Lazzeroni and Owen, 2000), builds the gene expression matrix as a sum of layers, where each layer corresponds to a bicluster. Each value $a_{i j}$ is modelled by $a_{i j}=\sum_{k=0}^{K} \theta_{i j k} \rho_{i k} \kappa_{j k}$ where $K$ is the layer (bicluster) number, and $\rho_{i k}$ and $\kappa_{j k}$ are binary variables representing membership of row $i$ and column $j$ in layer $k$. Here, the value of an element in the gene expression matrix is a linear function of the contributions of the different bi-clusters to which


Figure 3.2: (A) Bi-cluster with constant rows. Each row is obtained from a typical value $\mu$ and row offset $\alpha_{i}$, (B) Constant columns. Each value is obtained from a typical value $\mu$ and column offset $\beta_{j}$, (C) Additive model. Each value is predicted from $\mu$, and a row and column offset, $\alpha_{i}+\beta_{j}$. Similar model constructs apply for the multiplicative case with (A(i)) $\mu \times \alpha_{i}$, (B(i)) $\mu \times \beta_{j}$ and (C(i)) $\mu \times \alpha_{i} \times \beta_{j}$
the row $i$ and the column $j$ belong, (Fig. 3.3), (Lazzeroni and Owen, 2000). For layer $k$, expression level $\theta_{i j k}$ can be estimated using the general additive model, $\theta_{i j k}=\mu_{k}+\alpha_{i k}+\beta_{j k}$, in layer $k$, (Fig. 3.2 (C)).

$$
\begin{aligned}
& \mu_{2}=5
\end{aligned}
$$

Figure 3.3: Values at overlaps are seen as a linear function of different bi-clusters.

For the Coherent Evolutions model the exact values of $x_{i j}$ are not directly taken into account, but a cluster is evaluated to see if it shows coherent patterns of ex-
pression. In simplest model form, each gene expression value can have three states: up-regulation, down-regulation and no change. Thresholds between states are crucial and additional complexity results from extending model definitions to include further states such as "slightly" up-regulated, "strongly" up-regulated and so on, e.g. SAMBA, (Tanay et al., 2002).

Other Measures: Other measures used to evaluate coherency of a group of genes include conditional entropy: $H(C \mid X)=-\int \sum_{j=1}^{m} p\left(c_{j} \mid x\right) \log p\left(c_{j} \mid x\right) p(x) d x$, (the average uncertainty of the random variable $C$ (cluster category), when a random variable $X$ (gene expression profile) is known). The optimal partition of the gene expression dataset is obtained when this entropy is minimised, (Li et al., 2004) i.e. a partition is achieved where each gene is assigned with a high probability to only one cluster. This requires the estimation of the a posteriori probabilities $p\left(c_{j} \mid x\right)$, usually by non-parametric methods, as this avoids assumptions on the distribution of the underlying gene expression data).

Note: Pattern proximity measures described so far make no distinction between time-series data and those obtained from expressions of two or more phenotypes. Applying similarity measures to time series data is not straightforward. Gene expression time series have non-uniform intervals and are usually very short, (4-20 samples while classically even 50 observations is low for statistical inference). Furthermore, data are not independently, identically distributed. Similarity in time series should be viewed only in terms of similar patterns in the direction of change across time points (i.e. trends in the data), while robust measures must allow for non-uniformity, in addition to scaling, shifting and shape (internal structure of clusters), (Moller-Levet et al., 2003).

### 3.3.2 Conventional Methods

In this Section we examine popular clustering methods and their more recent developments. Each algorithm described below relies, by definition, on some choice of proximity measure and inherits the limitations of that choice.

## Agglomerative clustering

All agglomerative techniques naturally form a hierarchical cluster structure in which genes have crisp membership. Eisen et al. (1998) studied gene expression in the budding yeast, Saccharmyces Cerevisiae, using hierarchical methods, (which have been popularised due to ease of implementation, visualisation capability and availability). Methods vary with respect to choice of distance metric, decision on cluster merging, (linkage), as well as parameter selection affecting structure and relationship between clusters. Options include: single linkage (cluster separation as distance between two nearest objects), complete linkage (as previously, but between two furthest objects), average linkage (average distance between all pairs), centroid (distance between centroid's of each cluster) and Ward's method, (which minimises ANOVA Sum of Squared Errors between two clusters), (Sturn, 2001).

Distance and linkage determine level of sensitivity to noise: Ward's and the Complete method are particularly affected, (due to the ANOVA basis and outlier importance respectively, since clustering decisions depend on maximum distance between two genes). Single linkage forces cluster merger, based on minimum distance, regardless of other gene contributions to the cluster, so noisy or outlying values are among the last to be considered. Consequently, the "chaining phenomenon" may arise, (Romesburg, 2004). For commonly used Average and Centroid linkage
this problem is avoided, as no special consideration is given to outliers and clusters are based on highest density.

Results for agglomerative clustering may be intuitively presented by dendograms but there are $2^{n-1}$ different linear orderings consistent with tree structure, so care is needed in pruning. Dendrogram analysis, based on gene class information from specialised databases is presented by Toronen (2004), where optimal correlations are obtained between gene classes and used to form clusters from different branch lengths. Bar-Joseph et al. (2003) present an agglomerative technique for which each internal node has at most $N$ children, allowing up to $N$ genes (or clusters) to be directly connected, (extending traditional hierarchical concepts and reducing the effects of noise). Permutation is used to decide on the number of nodes $(\max N)$ to merge, based on a similarity threshold. Heuristically, algorithm complexity is comparable to traditional hierarchical clustering, although Bar-Joseph et al. (2003) also present a "divide and conquer" approach for optimal leaf ordering for small $N$, which has implications of increased time and space complexity.

Note: It should be stated that, such methods can not, in general, compensate for the greedy nature of the traditional algorithm, where mis-clustering at the beginning can not be corrected at a later stage and are magnified as the process continues. Further, Yeung et al. (2001) and Gibbons and Roth (2002) note that hierarchical clustering performance is close to random, despite its popularity and is poorer than other common techniques such as K-means and Self Organising maps (SOM).

## Partitive Techniques

Partitive clustering divides data by similarity measure, where typical methods measure distance from a gene vector to a prototype vector representing the cluster, and
intra-cluster/inter-cluster distance are respectively maximised/minimised. A major drawback is the need to specify the number of clusters in advance. Table 3.2 summarises algorithms discussed here.

- K-means produces crisp clusters with no structural relationship between these, (MacQueen, 1967). It deals poorly with noise, since outliers must belong to a cluster and this distorts the means. Equally, cluster inclusion is dependent on the cumulative values of genes already present, so order matters. Results are dependent on initial cluster prototype (which varies between clustering attempts); this leads to instability and, frequently, to a local minimum solution. Incremental approaches to refine local minima solutions converging to a global solution, include the Modified Global K-means (MGKM) algorithm (Bagirov and Mardaneh, 2006), which computes $k$-partitions of the data using $k-1$ clusters from previous iterations. A tolerance threshold must be set which determines the number of clusters indirectly, and, as with regular K-means, returns spherical clusters. For the six datasets reported the MGKM algorithm showed slight improvement over K-means, but at higher computational time cost, (Bagirov and Mardaneh, 2006).
- The prevalence of local minima for $K$-means is linked to initial prototype selection. Genetic algorithms (GAs), as an evolutionary approach, work well for small datasets, (less than 1000 gene vectors and of low dimension), but have prohibitive time constraints for anything larger, so are less desirable for gene expression analysis. Although GA's find the global optimum, they are sensitive to user-defined input parameters and must be fine tuned for each specific problem. Studies which have combined $K$-means and GA include Incremental Genetic $K$-Means Algorithm (IGKA), (Lu et al., 2004). This is a hybrid approach which converges to a global optimum faster than stand alone GA, and without the sensitivity to initialisation prototypes. The fitness function for the GA is based on Total Within Cluster Vari-
ance (TWCV), while the basis of the algorithm is to cluster centroids incrementally, using a standard similarity measure. The GA method requires the number of output clusters, $K$, to be specified, but is further complicated by inherent GA parameters (mutation probability rate, number of generations, size of the chromosome populations etc.), which influence time taken by the algorithm to converge to a global optimum.
- Fuzzy modifications of K-means include Fuzzy C-Means (FCM), (Dembele and Kastner, 2003), and Fuzzy clustering by Local Approximations of MEmberships (FLAME), (Fu and Medico, 2007). In both, genes are assigned a cluster membership degree indicating percentage association with that cluster, but the two algorithms differ in the weighting scheme used to determine gene contribution to the mean. For a given gene, FCM membership value of a set of clusters is proportional to its similarity to cluster mean. The contribution of each gene to the mean of a cluster is weighted, based on its membership grade. Membership values are adjusted iteratively until the variance of the system falls below a threshold. These calculations require the specification of a degree of fuzziness parameter which is problem specific, (Dembele and Kastner, 2003). As with K-Means, clusters are unstable, and considerably influenced by initial parameter values, while $K$, the number of clusters, must be specified a priori. In contrast FLAME requires membership of a cluster, $i$, to be determined by the weighted similarity of the gene to its $K$-nearest neighbours, and their membership of cluster $i$.

Note: This density-based approach (FLAME) further reduces noise impact, since genes with a density lower than a pre-defined threshold are categorised as outliers, and grouped with a dedicated 'outlier' cluster. FLAME produces stable clusters, but the size of the neighbourhood and the weighting scheme used affect $K$ (as above) and hence clustering achieved.

For both FCM and FLAME, genes may have multiple and varied degrees of membership, but interpretation differs. FCM and FLAME use averaging, where each gene contributes to the calculation of a cluster centroid, and its overall membership value set sums to 1 , (i.e. gene-cluster probability). Thus strong membership for a given gene does not indicate it to be more typical of the cluster, but rather the relative strength of its individual association, (Krishnapuram and Keller, 1993).

Table 3.1 illustrates three clusters of an FCM carried out on published yeast genomic expression data of Gasch and Eisen (2002), (available at http://rana. lbl.gov/FuzzyK/data.html). Membership values for genes $B$ and $D$ are very different for cluster 21, although both are approximately equidistant from the centroid of the cluster. Similarly genes $C$ and $D$ have comparable membership values for cluster 4, but gene $C$ is more typical (closer to the centroid) than gene $D$. With similar centroid distances, membership value for gene $B$ in cluster 21 is smaller than that for gene $A$ in cluster 46. These anomalies arise from the membership sum constraint, which decreases gene membership in one cluster to increase it in another. Listing genes in a cluster based on membership values is therefore counter-intuitive and does not reflect their compatibility with the cluster, but rather how they are shared between clusters. Similarly for FLAME, as the memberships are weighted relative to the $K$-nearest neighbours, so a low membership value indicates a high degree of cluster sharing among these and not a more typical value of a given cluster.

This interpretative flaw was recognised by Cano et al. (2007), who developed the possibilistic biclustering algorithm, which removes the sum rule restriction. The authors used spectral clustering principles, (Kluger et al., 2003), to create, from the original gene expression matrix, a partition matrix, $Z$, to which possibilistic clustering is applied. The resulting clusters were evaluated using the $H$-Score,

| GID | Cluster 4 |  | Cluster 21 |  | Cluster 46 |  |
| :--- | :--- | ---: | :--- | ---: | :--- | ---: |
|  | Centroid | Mem. | Centroid | Mem. | Centroid | Mem. |
|  | Dist. |  | Dist. |  | Dist. |  |
| A | 10.691 | 0.002575 | 8.476 | 0.002002 | 3.864 | 0.482479 |
| B | 6.723 | 0.009766 | 3.855 | 0.009341 | 6.33 | 0.007381 |
| C | 6.719 | 0.007653 | 5.29 | 0.00515 | 8.024 | 0.005724 |
| D | 7.725 | 0.007609 | 3.869 | 0.01782 | 6.279 | 0.010249 |

Table 3.1: Fuzzy Membership Interpretation. Membership of a gene and distance to cluster centroid, as calculated by Euclidean distance.
(Eq.3.3), and improved on traditional techniques. The algorithm requires, inter alia, two specific parameters, namely cutoff memberships for (i) gene inclusion and (ii) sample inclusion in a cluster. In this case, these cutoffs are intuitively reasonable as membership does indicate how typical a gene/sample is to a defined cluster, and not the degree to which it is shared between clusters.

## Neural Networks

Basic: Neural Networks (NN), loosely based on the biological parallel, can be modelled as a collection of nodes with weighted interconnections. Only numerical vectors are processed, so meta-information can not be included in the clustering procedure. Interconnection weights are adaptively learned i.e. features are selected by appropriate assignment of weights. In particular, Self Organising Maps (SOMs), a type of NN, have proved popular for gene expression, (Kohonen, 1990; Tamayo et al., 1999; Golub et al., 1999). A kernel function, that defines the region of influence, (neighbourhood), for an input gene, distinguishes SOM from K-means. Updating the kernel function causes the output node and its neighbours, to track towards the gene vector. The network is trained, (adjusting strengths of interconnections), from a random sample of the dataset. Once training is complete, all genes

|  | Cluster Mem. | Input | Proximity | Other |
| :---: | :---: | :---: | :---: | :---: |
| K-Means | Hard | Starting Prototypes, <br> Stopping Threshold, <br> K  | Pairwise Distance | Very Sensitive to input parameters and order of input. |
| MGKM | Hard | Tolerance Threshold | Pairwise Distance | Not as sensitive to starting prototypes. K specified through tolerance threshold. |
| IGKA | Hard | K, mutation prob., generation number, population size | TWCV | Time taken to converge to global influenced by parameters. |
| FCM | Fuzzy | Degree of fuzziness, Starting prototypes, Stop threshold, K | Pairwise Distance | Careful Interpretation of membership values. Sensitive to input parameters and order of input |
| FLAME | Fuzzy | $K_{n n}$ - number of neighbours | Pairwise Distance to $K_{n n}$ neighbours | careful interpretation of membership values. Output determined by $K_{n n}$. |
| Possibilisti biclustering | Fuzzy | Cut-off memberships, Max. residue, number of rows and number of columns | H-Score | Number of biclusters determined when quality function peaks by re-running for different numbers of eignevalues. |

Table 3.2: Summary of Partitive techniques. With the exception of FLAME and Possibilistic biclustering, all find complete global clusters.
in the dataset are then applied to the SOM. Cluster members, represented by output node $i$ are the set of genes causing $i$ to 'fire' (hard clustering).

SOMs are robust to noise and outliers, dependent on distance metric and neighbourhood function used. As for K-means, a SOM produces a sub-optimal solution if the initial weights for the interconnections are not chosen properly. Convergence is controlled by problem-specific parameters such as learning rate and neighbourhood function. A particular input pattern can fire different output nodes at different iterations; (while this can be overcome by gradually reducing the learning rate to zero during training, it can result in over-fitting, which leads to poor performance for new data). In specifying $K$, based on the number of output nodes, it should be noted that too few output nodes in the SOM gives large within-cluster distance, while too many results in meaningless diffusion across clusters.

Extended: The Self Organizing Tree Algorithm (SOTA), (Herrero et al., 2001), Dynamically Growing Self Organizing Tree (DGSOT) algorithm, (Luo et al., 2004) and, more recently, Growing Hierarchical Tree SOM (GHTSOM), (Forti and Foresti, 2006) were developed to combine strengths of NN, (i.e. speed, robustness to noise) and hierarchical clustering, (i.e. tree structure output, minimum a priori requirement for number of clusters specification and training) to deal with properties of gene expression data. Here the SOM network is a tree structure, trained by comparing only leaf nodes to input gene expression profiles (each graph node representing a cluster). SOTA and DGSOT result in a binary and n-tree structure respectively, while in GHTSOM, each node is a triangular SOM (3 neurons, fully connected), each having 3 daughter nodes (also triangular SOMs), Fig. 3.4. Tree growth strategy determines $K$.

At each iteration of SOTA the leaf node with the highest degree of heterogeneity is split into two daughter cells. In the DGSOT case, the correct number of daugh-
ters, ( $n_{d} \geq 2$ ), is determined dynamically by starting off with two and continually adding one until cluster validation criteria are satisfied. To determine $n_{d}$, a method was proposed, (Luo et al., 2004), based on geometric characteristics of the data (specifically, cluster separation in the minimum spanning tree of the cluster centroids). For this an empirical threshold, $\alpha$, value must be specified; (the authors propose 0.8 )). In SOTA and DGSOT, growth of the tree continues until overall heterogeneity crosses a threshold, $\beta$, or until all genes map onto a unique leaf node. The DGSOT method uses average leaf distortion to determine $\beta$ for growth termination, while, for SOTA, this threshold is determined by re-sampling, (with system variability defined to be the maximum distance among genes mapped to the same leaf node). By comparing distances between randomized data and those of the real dataset, a confidence interval and distance cut-off are obtained. In GHTSOT, growth occurs if a neuron is activated when a sufficient number of inputs map to it, (i.e. at least 3 or a user defined number, $\beta$ ), which determines the resolution of the system. Growth continues as long as there is one neuron in the system which can grow. The advantage of these methods over most partitive techniques is that $K$ is not pre-determined, but depends indirectly on the threshold, $\beta$, which is data dependant.

Key Features: SOTA, DGSOT and GHTSOM differ from typical hierarchical clustering algorithms in terms of adaptation. This occurs once a gene is mapped to a leaf node, but the neighbourhood of the adaptation is more restrictive than for SOM. DGSOT also overcomes the misclustering problem of traditional hierarchical algorithms, SOTA and GHTSOM, by specification of another input parameter, $L$ the immediate ancestor level in the tree of a given node which is growing. DGSOT then distributes all mapped values among the leaves of the subtree rooted at the $L^{t h}$ ancestor. In GHTSOM, new nodes (after growth) are trained using only those
inputs which caused the parent node to fire. Any neuron, which shows low activity, is deleted, and its parent is blocked from further growth. This has the advantage that inputs mapping to leaf neurons at the top of the hierarchy are usually noise, and clearly distinguishable from relevant biological patterns.


Figure 3.4: (A) SOTA. A binary tree structure. Neighbourhood of adaptation indicated for (i) node with sibling, (ii) node with no sibling, (B) DGSOT. N-ary tree structure. Neighbourhood of adaptation indicated when $L=2$, (C) GHTSOM. Each node represented by triangular SOM. Each layer indicated with line styles, (3 layers shown).

## Search Based

Solutions for a criterion function are found by searching the solution space either deterministically or stochastically, (Jain et al., 1999). The former exhaustive search is of little use for high dimensional gene expression analysis and, typically, heuristics are used. Simulated Annealing is well-known and has been applied by Lukashin and Fuchs (2001) and Bryan et al. (2006), using TWCV and H-Score (Eq. 3.3), respectively, as the fitness function, $E$, to be minimised. At each stage of the process, gene vectors are randomly chosen and moved to a new random cluster. $E$ is evaluated for each move and the new assignment is accepted if $E$ is improved or with a probability of $e^{-\frac{E^{n e w}-E^{o l d}}{T}}$ otherwise. The "temperature", $T$, controls readi-

|  | Structure | Proximity | Input | Other |
| :--- | :--- | :--- | :--- | :--- |
| SOM | None | Distance | Number of <br> output neu- <br> rons, Learning <br> rate | Careful considera- <br> tion of initialisation <br> weights |
| SOTA | Binary Tree | Distance | Threshold $\beta$ |  |
| DGSOT | N-ary Tree | Distance | Thresholds <br> and $L . \beta, \alpha$ | Corrects for mis- <br> clusterings |
| GHTSOM | Each node <br> triangu- <br> lar SOM, <br> arranged <br> in Tree <br> structure | Distance | Minimal re- <br> quirement - <br> learning rate |  |

Table 3.3: Summary of Neural Network techniques presented.
ness of the system to accept the poorer situation by chance, enabling the algorithm to avoid local minima. As the search continues, $T$, is gradually reduced according to an annealing schedule, and ultimately achieves the global minimum, where the annealing schedule parameters dictate performance and speed of the search. Choice of initial temperature $T_{i}$ governs convergence time and size of search space, (increased/decreased in the case of high/low $T$ respectively). Similarly for search termination, (final effective $T_{F}$ ). The user must specify the rate at which $T$ approaches $T_{F}$, which must be slow enough to guarantee a global minimum, as well as the number of swaps of gene vectors between clusters allowed in an iteration.

To determine $K$, a randomisation procedure is used, (Lukashin and Fuchs, 2001), to determine cut-off threshold for the distance, $D$, between two gene vectors in a single cluster. It is also necessary to determine $P$, the probability of accepting false positives, (e.g. $P=0.05$ ). Simulated annealing is then applied for different numbers of clusters, until the weighted average fraction of incorrect gene vector pairs reaches the $P$-value.

### 3.3.3 Biclustering Methods

Biclustering methods are important for gene expression data analysis (Section 3.2.1), so we deviate slightly from the known taxonomy of clustering algorithms, (Jain et al., 1999) to consider algorithms which adopt biclustering strategy 'in isolation'.

## 'Cheng and Church' Algorithm and FLOC

This algorithm, (Cheng and Church, 2000, : adapted from Hartigan (1972)) obtains H-scores, (Eq. 3.3, Fig. 3.2, (Madeira and Oliveira, 2004)) of the sub-matrices of the gene expression matrix. This method is initialised for the entire gene expression matrix and considers a sub-matrix to be a bi-cluster if $H(I, J)<\delta$ for some $\delta \geq$ 0 , (user defined). Each row and column of the original matrix is thus tested for deletion. Once a sub-matrix is determined to be a bi-cluster, its values are "masked" with random numbers in the initial gene expression matrix. Masking bi-clusters prevents the algorithm from repeatedly finding the same sub-matrices, but there is a substantial risk that this replacement will interfere with the discovery of future bi-clusters. To overcome this problem of random interference, Flexible Overlapped biClustering (FLOC) was developed - a generalised model of Cheng and Church incorporating null values, (Yang et al., 2003). FLOC constrains the clusters to both a low mean residue score and a minimum occupancy threshold of $\alpha, 0 \leq \alpha \leq 1$ (user defined).

Note: FLOC does not require pre-processing for imputation of missing values. Both these bi-clustering algorithms find coherent groups (Section 3.3.1) and permit overlapping.

## Coupled Two Way Clustering(CTWC)

Getz et al. (2000) adopted an iterative approach to find biclusters in the data. Firstly, all samples are clustered (using any clustering algorithm) using all genes, and vice versa to identify stable clusters of genes and of samples. Then, each gene cluster is used to cluster all stable sample clusters, and vice versa. Whenever a clustering operation generates a new stable subcluster, it is recorded and its members are used in the next iterative step. The process stops when no new stable clusters (that exceed a minimum size) are generated. This method is, of course, reliant on that algorithm used to cluster the data in the first place, and inherits the limitations of that method.

Inputs are determined by the clustering algorithm used. Unlike many others, the method adopted in Getz et al. (2000) (Superparamagnetic Clustering, (Blatt et al., 1996)) does not require the number of clusters to be specified before hand. However, if another clustering approach was used, for e.g. K-means, then the number of clusters would have to be specified. The results are also dependent on initial clusters found and the order used to cluster the samples etc. in the latter stages of the algorithm.

## The Plaid Model

The Plaid Model, (Lazzeroni and Owen, 2000), (Section 3.3.1), assumes that biclusters can be generated using a statistical model and aims to identify the parameter distribution that best fits the available data, by minimising the error sum of squares for the $k^{\text {th }}$ bi-cluster assuming that $k-1$ bi-clusters have already been identified.

Explicitly, it seeks to minimise for the whole matrix:

$$
\begin{equation*}
Q=\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m}\left(Z_{i j}-\theta_{i j k} \rho_{i k} \kappa_{j k}\right)^{2} \tag{3.4}
\end{equation*}
$$

where $Z_{i j}$ is the residual after deducting $k-1$ previous layers,

$$
\begin{equation*}
Z_{i j}=a_{i j}-\sum_{k=0}^{K-1} \theta_{i j k} \rho_{i k} \kappa_{j k} \tag{3.5}
\end{equation*}
$$

Parameters $\left(\theta_{i j k}, \rho_{i k}\right.$ and $\kappa_{j k}$, defined previously, Section 3.3.1) are estimated for each layer and for each value in the matrix, and are updated iteratively, providing refined estimates of $\mu_{k}, \alpha_{i k}$ and $\beta_{j k}$, (Fig: 3.2(C)) and $\rho_{i k}$ and $\kappa_{j k}$ to minimise $Q$, (Lazzeroni and Owen, 2000).

The importance of a layer is defined by:

$$
\begin{equation*}
\delta_{k}^{2}=\sum_{i=1}^{n} \sum_{j=1}^{p} \rho_{i k} \kappa_{j k} \theta_{i j k}^{2} \tag{3.6}
\end{equation*}
$$

To evaluate the significance of the residual matrix, $Z$ is randomly permuted and tested for importance. If $\delta_{k}^{2}$ is significantly better than $\delta_{\text {random }}^{2}, k$ is reported as a bi-cluster. The algorithm stops when the residual matrix $Z$ retains only noise, again with the advantage that the user does not need to specify the number of clusters beforehand. Another important property of this method is that statistical evaluation is intrinsic in the results.

### 3.3.4 Graph Theoretic Methods

Methods selected and detailed below also span agglomerative/partitive, global/local structures, crisp/fuzzy cluster membership etc. and, as with all others, have an

|  | Proximity | DeterministicClusters <br> Stochastic | Other |  |
| :--- | :--- | :--- | :--- | :--- |
| SA | Depends on <br> application | Stochastic | Depends on <br> application | Specification <br> of Annealing <br> Schedule |
| CC | Additive <br> Model | Deterministi | Overlapping, <br> partial bi- <br> clusters | $\delta$, random in- <br> terference |
| FLOC | Additive <br> Model | Deterministic | Overlapping, <br> partial | bi-clusters <br> Q and $\delta$ to <br> specify. Over- <br> comes random <br> interference, <br> allows missing <br> values. |
| CTWC | Depends on <br> application | Deterministi¢ | Crisp, partial <br> bi-clusters | Results de- <br> pendent on <br> underlying <br> algorithm <br> used and on <br> initial clusters <br> found. |
| Plaid | Additive <br> Model | Deterministic | Overlapping, <br> partial, <br> clusters bi- | Values seen as <br> sum of contri- <br> butions to bi- <br> clusters |

Table 3.4: Summary of biclustering techniques presented.
objective function to evaluate the clusters found. We isolate graph theoretic approaches for special consideration, due to their intuitive nature, interpretation and visualisation of gene expression matrices but also because the modelling paradigm is well established for analysis of other complex systems which exhibit similar properties to those found for gene expression.

## Data Organisation in Gene Expression Context

At a basic level, a graph, $G=(V, E)$, consists of two parts: a set of nodes (or nodes), $V$, and a set of edges (or links, connections), $\left(v_{i}, v_{j}\right) \in E$, which captures the concept of a 'relationship' between nodes $v_{i}$ and $v_{j}$, with $v_{i}, v_{j} \in V$.

A graph can be undirected (where edge ( $\mathrm{a}, \mathrm{b}$ ) is considered to be the same as edge ( $b, a$ ) and is only recorded once) or directed, (distinction is made between an edge $(a, b)$ and an edge ( $b, a$ ), i.e. both edges are recorded). A graph can be complete, with an edge between every node in a graph, or incomplete, with an edge between a subset of nodes in a graph - also referred to as strongly connected, connected or weakly connected.

Definition/Representation of graph edges and nodes in terms of gene expression data is a first consideration. Nodes may be viewed as either genes or samples, linked by edges. In the gene expression context an edge can have different meanings - depending on how the graph is designed. For example, it could indicate a similarity above some threshold of a similarity measure, or adherence to some cohesion model. We will see further examples of these in this Section, and later in Chapter 5 when we describe a new method for extraction of a graph from a gene expression matrix. A graph may be constructed from 'gene' nodes alone, with an edge which represents similarity of expression. Alternatively, the graph could be
constructed from both 'gene' and 'sample' nodes. Further, (Tanay et al., 2002) used additional 'condition' nodes and sample nodes, where condition nodes represented moderate and strong changes in gene expression, involvement in a protein complex etc.


(a) Gene Expression (b) Gene Expression Gr ap Graph containing only gene and sample nodes gene nodes

(c) Gene Expression Graph according to Taney et al. (2002) with both condition and sample nodes

Figure 3.5: Gene Expression Graphs. (a) is an example of an one-mode graph (one type of node). (b) and (c) are examples of bi-partite graphs (two types a node).

While benefits are associated with each representation, it is important to note that the final clusters/identification achieved follows from this choice. In graphical context, a cluster is defined to be connected components, i.e. a group of nodes that are connected to one another, but that have no connection to nodes outside the group. Examples include, contiguity-based clusters (connecting objects within a specified distance on one another), or a clique (a set of nodes in a graph that are completely connected to each other).

Given a dataset $X$, we construct an adjacency matrix, $A$, where $a_{i j} \in A$, $a_{i j}=f(i, j)$. The adjacency matrix can be a square $n \times n$ matrix (a One Mode Representation, Fig. 3.5a) or a $n \times p$ matrix (a Bipartite Representation, Fig. 3.5b and 3.5 c ), $n=$ cardinality of one set of nodes (e.g. number of genes), $p=$ cardinality of alternative set of nodes (e.g. number of samples). For some clustering schemes
each pair of nodes is connected by an edge with an assigned weight, $f(i, j)$, and thus the adjacency matrix records edge weights (and the structure is referred to as a weighted graph). In other instances, $f(i, j) \in\{0,1\}$ : edges are constrained to exist between objects $i$ and $j$ only if $f(i, j)=1$, and thus the adjacency matrix records whether an edge exists of not ${ }^{1}$. Additionally, each node may be assigned a weight ${ }^{2}$, $g\left(v_{i}\right)$ and this information can be used in any clustering process. The clustering problem is thus explicitly presented in terms of graph theoretical properties.

## One Mode Representation

A graph, $G=(V, E)$, is considered to be in one mode if an edge can exist between any two nodes, $v \in V$. A gene expression dataset can be modelled in this way where nodes in the graph represent genes and an edge exists between two gene nodes if they show common expression (e.g. measured as a distance function).

## Bipartite Representation

A graph, $G=(\top, \perp, E)$, is considered to be bipartite if there are two disjoint subsets of nodes, $\top, \perp$, and there is no edge between two nodes in the same subset. A gene expression dataset can be modelled in this way, where $T$ nodes represent genes and $\perp$ nodes represent samples. An edge $w_{i j} \in W$ is the weight matrix, were $w_{i j} \neq 0$ if there is an edge between $i \in \top$ and $j \in \perp$.

We consider applying these modelling ideas more specifically to gene expression data in Chapter 5.

[^9]
## Graph Theoretic Clustering Options

Graph theoretic approaches generally have gained ground recently in analysing large complex datasets and we consider in brief the principle options for the gene expression case.

- The Cluster Affinity Search Technique (CAST), (Ben-Dor et al., 1999), models data as an undirected graph, $G=(V, E)$, where $\{V, E\}$ is the set representing \{genes, similar expression\}. The model assumes that there is an ideal clique graph, (see 3.3.4), $H=(U, E)$, which represents the ideal input gene expression dataset, while data to be clustered is a "contamination" of the ideal graph $H$ by random errors. In a clique graph each clique represents a cluster. For a pair of genes in $G$, the model assumes that an edge/non-edge was assigned incorrectly, with a probability of $\alpha$. The true clustering of $G$ is assumed to be that which requires fewer edge changes to generate $H$.

CAST uses an affinity (similarity) measure, either binary or real valued, to assign a node to a cluster. This must be above a threshold, $t$ (user-defined, determining size and number of clusters). The affinity of a node $v$ to a cluster, is the sum of affinities over all objects currently in the cluster, so $v$ has high affinity with $i$ if $\operatorname{affinity}(x)>t|i|$, and low affinity otherwise. The CAST algorithm alternates between adding high affinity elements and removing low affinity elements, finding clusters one at a time. A disadvantage of this approach is that the result is dependent on the order of input, as once initial cluster structure is obtained, a node $v$ is moved to that cluster for which it has a higher affinity value.

- CLICK, (Sharan and Shamir, 2000), builds on the work of Hartuv et al. (2000), introducing a probabilistic model for edge weighting. Pairwise similarity measures between genes are assumed to be normally distributed: between 'mates'
$\left(N\left(\mu_{T}, \sigma_{T}^{2}\right)\right)$, and between 'non-mates' $\left(\left(\mu_{F}, \sigma_{F}^{2}\right)\right)$, where $\mu_{T}>\mu_{F}$. These parameters can be estimated via Expectation Maximisation methods, (Dempster et al., 1977). The weight of an edge is derived from the similarity measure between the two gene vectors, and reflects the probability that $i(\in V)$ and $j(\in V)$ are mates, specifically that:

$$
\begin{equation*}
w_{i j}=\log \frac{p_{\text {mates }} \sigma_{F}}{\left(1-p_{\text {mates }}\right) \sigma_{T}}+\frac{\left(S_{i j}-\mu_{F}\right)^{2}}{2 \sigma_{F}^{2}}-\frac{\left(S_{i j}-\mu_{T}\right)^{2}}{2 \sigma_{T}^{2}} \tag{3.7}
\end{equation*}
$$

Edges with weights below a user defined non-negative threshold are omitted from the graph. The graph is thus partitioned using a minimum weight cut algorithm, (Hartuv et al., 2000), and the weight of a partition is the average of these edge weights.

- The Statistical Algorithmic Method for Bi-cluster Analysis (SAMBA) method finds bi-clusters based on the coherent evolution model (Section 3.3.1), (Tanay et al., 2002). Firstly, the gene expression matrix is modelled as a bipartite graph, $G=$ ( $U, V, E$ ), where $U$ is the set of sample nodes, $U \cap V=\varnothing$ and an edge $(u, v)$ only exists between $v \in V$ and $u \in U$ iff there is a significant change in expression level of gene $v$, w.r.t. to its normal level, in sample $u$. Key to SAMBA is the scoring scheme for a bi-cluster, corresponding to its statistical significance, where a weight is assigned to a given edge, $(u, v)$, based on the log-likelihood of getting that weight by chance, (Tanay et al., 2002):

$$
\begin{equation*}
\log \frac{P_{c}}{P_{(u, v)}}>0 \text { for edges and, } \log \frac{\left(1-P_{c}\right)}{\left(1-P_{(u, v)}\right)}<0 \text { for non-edges. } \tag{3.8}
\end{equation*}
$$

The probability $P_{(u, v)}$ is the fraction of random bipartite graphs, with degree sequence identical to $G$, that contain edge $(u, v)$ (and can be estimated using Monte-

Carlo methods). $P_{c}$ is a constant probability $>\max _{(u, v) \in U x V} P_{(u, v)}$. Assigning these weights to the edges and non-edges in the graph, the statistical significance of a subgraph $H$ can be calculated:

$$
\begin{equation*}
\log L(H)=\sum_{(u, v) \in E} \log \frac{P_{c}}{P_{(u, v)}}+\sum_{(u, v) \in \bar{E}} \log \frac{1-P_{c}}{1-P_{(u, v)}} \tag{3.9}
\end{equation*}
$$

and, given that the expected degree for each node is $\hat{d}_{u}=\sum_{v \in V} \phi(u, v)$, where $\phi(u, v)$ is the probability that $u$ has an edge to $v$, the expected log-likelihood score of a subgraph is thus:

$$
\begin{equation*}
E(\log (L(H)))=\sum_{u, v}\left(\phi(u, v) \times \log \frac{P_{c}}{P_{u, v}}+(1-\phi(u, v)) \times \log \frac{1-P_{c}}{1-P_{u, v}}\right) \tag{3.10}
\end{equation*}
$$

The $K$ heaviest (largest weight) sub-graphs for each node in $G$ is found. Tanay et al. (2002) present two ways to calculate the weight of the resulting sub-graph.
(i) In the simpler model, bi-clusters, which reflect changes relative to normal expression level, without considering direction of change are sought.
(ii) The second model, focuses on consistent bi-cliques, targeting those samples which have the same or opposite effect on each of the genes.

|  | Mode | Proximity | Search | Other |
| :--- | :--- | :--- | :--- | :--- |
| CAST | One Mode | Similarity | Clique Graph | Parameters $\alpha$ and $t$. <br> Finds global, com- <br> plete, crisp clusters. |
| CLICK | One Mode | Distribution <br> based on <br> distance | Minimum <br> weight cut | Stat. Sig. of clus- <br> ters. EM to estimate <br> parameters. Finds <br> global, partial, crisp <br> clusters. |
| SAMBA | Bi-Partite | Probability | Heuristic <br> search of <br> neighbours | Stat. sig. of clusters. <br> Input $P_{c}$ difficult to <br> define. Finds par- <br> tial overlapping bi- <br> clusters. |

Table 3.5: Summary of Graph theoretic methods presented.

## Gene Expression Graphs and Random Graphs

Random graphs ${ }^{3}$ are proposed as the simplest and most straight forward realisation of a complex network with no apparent design principles. The theory of random graphs lies at the intersection between graph theory and probability theory and are used to explore the existence of properties of real world graphs. First studied by Erdös and Rényi (1959), the model they propose starts with $N$ nodes and connects every pair of nodes with probability $p$, creating a random graph with approximately $p N(N-1) / 2$ edges distributed randomly. Growing interest in complex systems has prompted revision of this as a suitable model. Questions have been raised as to whether the real networks behind diverse complex systems, such as gene coexpression in the cell, are fundamentally random? Intuitively, the random network model is insufficient for gene expression, since we expect these systems to display some organizing principles, which at some level are encoded in their topology. If

[^10]the topology does deviate from that of random graphs, we need to develop tools and measurements to capture, in quantitative terms, the underlying organisational principles. In Chapter 5 we investigate the potential of using random graph models to compare to real gene expression graphs in order to highlight interesting organisational properties in real gene expression graphs.

## Techniques for generating random graphs

There are a number of random graph models, here we list three.

- Erdös-Rényi Model There are two closely related variants of the Erdös-Rényi random model. In the first model, a graph, $G(n, M)$, is chosen uniformly at random from the collection of all graphs which have $n$ nodes and $M$ edges. In the second model, a graph $G(n, p)$ is constructed by connecting nodes randomly. Each edge is included in the graph with probability $p$, with the presence or absence of any two distinct edges in the graph being independent. An Erdös-Rènyi graph is not scale-free, (Erdös and Rényi, 1959), see below. Both of the two major assumptions of the $\mathrm{G}(\mathrm{n}, \mathrm{p})$ model (that edges are independent and that each edge is equally likely) are unrealistic in modeling gene expression data.
- Barabasi Albert Model This technique generates scale-free random graphs. In scale-free networks, some nodes act as "highly-connected hubs", although most nodes are of small degree (number of edges incoming and outgoing from a node). Scale free network structure and dynamics are independent of, $N$, the number of nodes. Their defining characteristic is that the degree distribution follows a power law relationship defined by $P(k) \sim k^{-\gamma}$, where
the probability $P(k)$ that a node in the network connects to $k$ other nodes is roughly proportional to $k^{-\gamma}$. The coefficient $\gamma$ varies from 2 to 3 for most real networks, or, in some cases, between 1 and 2, (Barabasi and Albert, 1999).
- Watts Strogatz Small World Model This model produces graphs with smallworld properties, i.e. where most nodes are not neighbours of one another, but most nodes can be reached from every other by a small number of hops or steps. Watts and Strogatz (1998) noted that graphs could be classified according to their clustering coefficient (the average proportion of edges shared by nodes neighbours in a graph) and mean shortest path length. Watts and Strogatz proposed a simple model of random graphs with (i) a small average shortest path and (ii) a large clustering coefficient .

Using graphical techniques to extract meaningful information from biological data is an intuitive and popular method. Bi-partite graphs representation, in particular, capture essential properties of the gene expression dataset, allowing for the extraction of biclusters, however, the alternative one-mode gene expression graph can also be been considered. This approach was used in Yip and Horvath (2007), where an edge existed between two genes nodes if they show similar expression across all samples, (measured e.g by a distance function). Yip and Horvath (2007) did not explicitly to investigate graphs with weighted edges. Carlson et al. (2006) and Zhang and Horvath (2005) studied weighted one-mode gene expression graphs (creating the graphs using a threshold on a similarity function) using classical network analysis techniques. Again, an edge existed between two genes nodes if they show similar expression across all samples. They found that, for the datasets tested, the gene networks generated by this technique were scale free, following a powerlaw distribution or an exponential. These results, however, are dependent the global
method used to extract a graph from the gene expression data. Analysis of complex weighted networks was also considered in Saramaki et al. (2007). All the aforementioned authors did not expand the investigation to the weighting scheme itself. It is important to recognise the essential role a weighting scheme itself plays on cluster determination from gene expression graphs, and thus it is important to investigate the intrinsic nature of these schemes in isolation.

### 3.4 Discussion

Despite shortcomings, application of clustering methods to gene expression data has proven to be of immense value, providing insight on cell regulation, as well as on disease characterisation. Nevertheless, not all clustering methods are equally valuable for high dimensional gene expression data. Recognition that well-known, simple clustering techniques, such as K-Means and Hierarchical clustering, do not capture complex local structure, has led to investigation of other options. In particular, bi-clustering has gained considerable recent popularity. Indications to date are that these methods provide increased sensitivity at local structure level in discovery of meaningful biological patterns.

An inherent problem with exploratory clustering is ab initio knowledge of $K$, the number of clusters. Consequently, those methods for gene expression analysis which do not need $K$ specified $a b$ initio have an advantage. Most algorithms seek empirically to determine this at run time, but derive complicated thresholds that may not make sense in the context of gene expression data. There is a risk that determination of these thresholds is not a one step process but requires testing and validation of clusters produced. A comprehensive survey of robust cluster validation and evaluation methods is given (Handl et al., 2005) but it seems clear that a
requirement for information-driven clustering is emerging, which integrates cluster and meta-information, (Choi et al., 2004; Liu et al., 2004; Kasturi and Acharya, 2005; Gamberoni et al., 2006; Kustra and Zagdanski, 2006). This provides a basis for validation, independent of the current problem, as well as interpretation of clustering results.

### 3.5 Summary

Cluster analysis, applied to gene expression data, aims to highlight meaningful patterns for gene co-regulation. The evidence suggests that, while commonly applied, agglomerative and partitive techniques are insufficiently powerful given the high dimensionality and nature of the data. While further testing on non-standard and diverse data sets is required, comparative assessment and numerical evidence, to date, supports the view that bi-clustering methods, although computationally expensive, offer better interpretation in terms of data features and local structure. While the limitations of commonly-used algorithms are well documented in the literature, adoption by the bioinformatics community of new (and hybrid) techniques, developed specifically for gene expression analysis has been slow, mainly due to the increased algorithmic complexity required. This would be catalysed by more transparent guidelines and increased availability in specialised software and public dataset repositories.

## CHAPTER 4

## Cluster Analysis: A Practical

## Evaluation

In the Assessment process, a clustering achieved is tested for specific properties. Assessment measures are rarely a fixed set but together form a diagnostic toolkit targeted at improving the clustering process. In general, clustering techniques optimise some form of this measure as a criterion function. Evaluation of clustering thus involves the synthesis of a number of assessment measures used to gauge final cluster quality in order to form an objective final judgement on the most suitable technique for the dataset involved. In this chapter we use these basic principals to investigate the applicability of clustering algorithms to gene expression data. The approach is to consider a series of measures which assess cluster quality on the basis of biological realism amongst other criteria. It also involves comparison of these measures between clustering algorithms and for different datasets. We evaluate clusterings obtained with selected algorithms ${ }^{1}$ identified in Chapter 3. Clusters obtained from real and synthetic datasets are compared between algorithms. We demonstrate the fact that, with so many classification criteria for clustering, no one

[^11]algorithm is good for all datasets, so that a preliminary review of the most appropriate methods is essential. Further, it is also strongly advocated that no single method or interpretation is sufficient and that recognition of valid clusters is frequently indefinite or misleading.

### 4.1 Introduction

Evaluation of clustering requires both internal/external assessments of clusters obtained, and comparison between algorithms. This is a complicated area for gene expression data due to its unique properties, due to the fact that little may be known about the data before hand. Many clustering algorithms are designed to be exploratory; so that clusters (dependent on given criteria) found will discover "a structure" which, while meaningful in the context of these, may yet fail to be optimal or even biologically realistic. Algorithms are inherently biased, as properties of clusters reflect built-in clustering criteria, while structures found are not usually the same for different algorithms. For example, with regard to the $K$-Means criterion the "best" structure is one that minimises the sum of squared errors (MacQueen, 1967), while for the Cheng and Church biclustering algorithm (Cheng and Church, 2000), it is that which minimises the Mean Residue Score (MRS, Eq. 3.3). The two assessments are generally not directly comparable, as the former highlights global patterns in the data and the latter local patterns, (Section 3.2). Also, large deviations from the mean may correspond to large residue scores, but this is not always the case. For example, Fig. 4.1(a), and the corresponding table, highlight a simple case of three genes in a cluster across four samples. According to the $K$-Means criterion, the cluster (Euclidean and centroid) distance is approximately 11.02, while MRS $=0$. In the second case (Figure 4.1(b)) the scale of profile 1 was reduced by
one third. In this case the cluster distance is decreased to 7.91 (indicating a better cluster), while the MRS is increased to 0.0168 (indicating an inferior cluster). Obviously, interpretation of cluster results relies at some level on subjective choice with regard to the assessment criterion to use. The greater the need, therefore, for independent validation by integration of findings with metadata. Subjective evaluation, (based on experience, background knowledge, expected results), even for low dimensional data, is non-trivial at best, but becomes increasingly difficult for high dimensional gene expression data. From these considerations, it is clear that cluster validation is critical for algorithm development and verification of results, with the latter usually based on a manual, lengthy and subjective exploration process.


| Profile 1 | 3.9 | 7.5 | 8.7 | 15.6 |
| :--- | ---: | ---: | ---: | ---: |
| Profile 2 | 3.1 | 4.1 | 5.2 | 6.5 |
| Profile 3 | 4.7 | 5.8 | 7.1 | 8.8 |

(A)


| 2.6 | 5 | 5.8 | 10.4 |
| ---: | ---: | ---: | ---: |
| 3.1 | 4.1 | 5.2 | 6.5 |
| 4.7 | 5.8 | 7.1 | 8.8 |

(B)

Figure 4.1: Cluster (B) has profile 1 scaled down by one third.

### 4.2 Assessment Methods

Once a non-random structure is distinguished in the data, a technique that finds the "best" structure is desirable - a vague expression - since 'best' may refer to novelty, stability, size, suitability, and is again dependent on the nature of the analysis and experimental purpose, and so on. Some non-trivial considerations might include: whether the method is exploratory or predictive (with results used as the
foundation for further investigations), whether all samples and genes should be grouped, whether a novel structure is to be assessed, and so on. Objective assessment measures of clustering quality fall into two categories - external and internal summarized in Table 4.1.

## Internal Measures:

These measures use only information from the clustering result and the dataset itself to assess cluster quality. Properties considered include:

- Compactness - intra-cluster homogeneity e.g. assessment of average or maximum pairwise intra-cluster distances, average or maximum centroid-based similarities.
- Separation - inter-cluster distance, e.g. average weighted distance where the distance between clusters can be computed as the distance between their centroids, or as the minimum distance between data items of each, e.g. the minimum distance between any two clusters.
- Connectedness - to what degree data items are grouped with their nearest neighbours in the data space, i.e. also known as connectivity (Handl et al., 2005).
- Combinations - as compactness usually improves with the number of clusters and separation usually deteriorates, linear/non-linear combination measures to assess both can be used, e.g. the SD-validity index (Halkidi et al., 2000), Dunn Indices (Dunn, 1974), Davies-Bouldin Index (Davies and Bouldin, 1979), Silhouette Index (Rousseeuw, 1987), C-Index (Hubert and Schultz, 1976).

An equivalent measure for fuzzy clusterings includes the Xie-Beni index (Xie and Beni, 1991).

- Fuzziness - applicable only to fuzzy partitions, these measures assesses sharing of membership between clusters, included are the partition coefficient and partition entropy (Bezdek, 1973, 1974).
- Stability/Predictive Power - based on repeatedly resampling the original data, this measures the consistency of the results, which in turn provides an estimate of the significance of the clusters obtained from the original dataset e.g. Ben-Hur et al. (2002) and Levine and Domany (2001). The jackknife approach, Yeung et al. (2001), forms clusters based on $p-1$ (with $p=$ number of samples) and uses the remaining sample to assess predictive power of the algorithm i.e. Figure Of Merit (FOM). Stability can also be assessed by perturbing data and comparing the different clusters found with the original partition, using external indices, (Bittner et al., 2000; Kerr and Churchill, 2001; Li and Wong, 2001).
- Preservation of distance information - the degree to which the distance information in the original data is preserved in a clustering and typically used for hierarchical clustering. Here, a cophenetic distance matrix is an $N \times N$ matrix where each entry $(i, j)$ records the level at which the data items $i$ and $j$ are grouped in the same cluster for the first time. The preservation is usually assessed using the cophenetic correlation coefficient i.e. the correlation between the entries in the cophenetic distance matrix and the original distance matrix, (Sokal and Rohlf, 1962).


## External Measures (Supervised):

These refer to assessments which reference external information (e.g. class labels or clusterings from alternative algorithms). The comparison of clusterings with external class labels is of critical importance as it provides a great deal of information to the user. For instance, genes that show similar pattern across clusters do not necessarily indicate the same pathway or similar function but could do. Examples of the properties key to external measurements include:

- Agreement with metadata - For biological function information included, in the gene list for each cluster, a more complete picture is inevitably provided of the dataset and the success of the technique. A number of functional annotation databases are available. The Gene Ontology database, (Ashburner et al., 2000), for example, provides a structured vocabulary that describes the role of genes and proteins in all organisms. The database is organised into three hierarchical ontologies: biological process, molecular function and cellular component. Several tools have been developed for batch retrieval of GO annotations for a list of genes ( e.g. tools DAVID, (Dennis et al., 2003), Babelomics, (Al-Shahrour et al., 2005) or Machaon CVE (Bolshakova et al., 2006)). Statistically relevant GO terms can be used to investigate the properties shared by a set of genes. These tools typically use comprehensive measures, like the F-measure (introduced by Rijsbergen (1975)), or hypergeometric tests, (Falcon and Gentleman, 2007), to test the significance of cluster purity, (the fraction of the cluster taken up by the predominant class label) and completeness, (fraction of items in a class grouped in the current cluster). This assessment can be adapted for partially annotated datasets, by only including that fraction of genes that are annotated in the calculation of
the measure. This facilitates the transition from data collection to biological meaning by providing a template of relevant biological patterns in gene lists.
- Agreement between clusterings (cluster runs) -In a simulation dataset, the true partition is known, and the performance of a technique can be assessed in terms of its clustering similarity to the true partition. There are several such indices to measure this in the literature, (Fridlyand and Dudoit, 2001). Most popular is the Rand Index (RI), (Rand, 1971) and a number of variations of this exist, including the adjusted RI, (Hubert and Arabie, 1985) and the weighted RI (Thalamuthu et al., 2006). In general, these determine the similarity between two partitions as a function of positive and negative agreement in pairwise cluster assignments. The Jaccard coefficient, (Jaccard, 1908), looks at similarity as a function of only the positive agreements in pairwise cluster assignments. Most of these can also only be used where a single class label is unequivocally assigned to a data item, thus are inappropriate for fuzzy clusterings or overlapping clusters, although a fuzzy extension has been proposed recently for the Rand Index, (Campello, 2007) Note: these measures can also be used where the gold standard is not known, to assess relative similarity of two clusterings obtained.

Table 4.1: Summary of Evaluation Measures, categorised into Internal and External. Assess' refers to which property the measure it assessing, Measure, refers to the popular name for the measure in literature, $G, L, C, C_{>1}, F$ indicates if the measure is suitable for assessing Global, Local structures, Crisp, Crisp in more than one cluster, and Fuzzy Membership respectively, Max/Min indicates whether the measure should be maximised or minimised, Bounds refers to the [maximum, minimum] possible value of the result.

| Assessment Measures |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Category | Assess' | Measure | G | L | C | $\mathbf{C}_{>1}$ | F | Max <br> /Min | Bounds |
| Internal | Connectedness | Conn | $\checkmark$ |  | $\checkmark$ |  |  | Min | $[0, \infty)$ |
|  | Compactness | Intra-cluster Distance | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  | Min | $[0, \infty)$ |
|  | Separation | Inter-cluster Distance | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | Max | $[0, \infty)$ |
|  |  | SD-validity Index | $\checkmark$ |  | $\checkmark$ |  |  | Min | $[0, \infty)$ |
|  |  | Dunn Indices | $\checkmark$ |  | $\checkmark$ |  |  | Max | $[0, \infty)$ |
|  | Combination | Davies-Bouldin Index | $\checkmark$ |  | $\checkmark$ |  |  | Min | $[0, \infty)$ |
|  |  | Silhouette Index | $\checkmark$ |  | $\checkmark$ |  |  | Max | $[-1,1]$ |
|  |  | C-Index | $\checkmark$ |  | $\checkmark$ |  |  | Min | [0, 1] |
|  |  | Xie-Bien Index | $\checkmark$ |  |  |  | $\checkmark$ | Min | $[0, \infty)$ |
|  | Fuzziness | Partition Coefficient | $\checkmark$ |  | $\checkmark$ |  | $\checkmark$ | Max | $[0, \infty)$ |
|  |  | Partition Entropy | $\checkmark$ |  | $\checkmark$ |  | $\checkmark$ | Min | $[0, \infty)$ |
|  | Continued on Next Page... |  |  |  |  |  |  |  |  |


| Category | Table 4.1 - Continued |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Assess' | Measure | G | L | C | $\mathbf{C}_{>1}$ | F | Max <br> /Min | Bounds |
|  | Stability | Cluster Overlaps <br> (Average Proportion Non-overlap, <br> Average Distance, <br> Average Distance <br> between Means) | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  | Min | $[0,1]$ |
|  |  | Figure Of Merit | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  | Min | $[0, \infty)$ |
|  | Distance <br> Preservation | Cophentic Correlation | $\checkmark$ |  | $\checkmark$ |  |  | Max | $[-1,1]$ |
| External | Purity | Biological Homogeneity Index | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  | Max | $[0,1]$ |
|  | Completeness | Biological Stability Index | $\checkmark$ |  | $\checkmark$ | $\checkmark$ |  | Max. | $[0,1]$ |
|  | Reconstruction of structure | Adjusted Rand Index Fuzzy Rand Index | $\checkmark$ $\checkmark$ | $\checkmark$ <br> $\checkmark$ | $\checkmark$ <br> $\checkmark$ | $\checkmark$ | $\checkmark$ | Max <br> Max | $\begin{aligned} & {[0,1]} \\ & {[0,1]} \end{aligned}$ |
|  |  | Jaccard Coefficient <br> Hubert $\Gamma$ Statistic |  | $\checkmark$ <br> $\checkmark$ |  | $\checkmark$ |  | Max <br> Max | $\begin{aligned} & {[0,1]} \\ & {[-1,1]} \end{aligned}$ |

These metrics are usually highly dependent on the number of clusters as an input parameter, (discussed in Chapter 3). The 'natural' number of clusters in the data depends on which clustering criterion are used in the algorithm and is not fixed between algorithms. For example, according to the $K$-Means criterion the optimal
number of clusters for a particular dataset may be 5 , while for CLICK it may be 10 for the same dataset. The 'optimal' number of clusters depends on the dataset and algorithm so that absolute choice is difficult. Assessment measures are also biased. The compactness index, e.g. is biased towards a large number of clusters, while the separation index is biased towards a small number of clusters. Formulae for each of the assessments can be found in Appendix A.

### 4.3 Tools and Packages

$\mathbf{R}$ is a powerful statistical computing language and environment, available for download from CRAN (http://CRAN.R-project.org/). This, and associated contributed packages, were used extensively for this analysis of clustering algorithms and evaluation methods. Primarily, esoteric packages, provided as part of the Bioconductor open source and open development project, (Gentleman et al., 2004), were employed. As a result of the Bioconductor project hundreds of packages for the analysis of gene expression data are publicaly available and, as the project is open development, updated regularly. Navigation and understanding of this abundance of contributed packages presents a serious challenge even for the experienced user.

For development of software for this analysis we used a number of contributed packages that perform cluster validation and which are available from CRAN or Bioconductor (http://www.bioconductor.org). These include packages $c l v$, (Nieweglowski., 2008), clValid, (Brock et al., 2008), e1071, (Dimitriadou et al., 2008), clusterSim, (Dudek, 2008) and biclust, (Kaiser et al., 2007). Not all assessment measures that we wanted to use were available in any package on CRAN or Bioconductor, and for those we implemented our own functions. These included
the C-Index, (Hubert and Schultz, 1976), and a form of the Xie-Beni index, (Xie and Beni, 1991).

### 4.4 A Framework for Evaluation

Each of the above measures assesses different properties of a clustering, but does not tell us which 'weigh' more heavily. The evaluation framework here provides a road-map to guide final judgement of quality and applicability. Steps in the evaluation framework, (Figure 4.2) are:

1. To understand the dataset properties (Chapter 2).
2. To understand the properties, (biases and cluster types found preferentially) of the chosen clustering algorithm (Chapter 3).
3. To make an educated guess for initial input parameters
4. To apply clustering algorithm.
5. For a range of input parameters to the clustering algorithm, to analyse internal assessment measures. Determine which type of internal measures are appropriate for the algorithm e.g. does it take into account fuzzy memberships, or overlapping clusters etc.?
6. To use optimal input parameters based on internal validation.
7. To apply external assessment measures.


Figure 4.2: Evaluation Work-flow

### 4.5 Datasets

Table 4.2 details the range of benchmark datasets used to evaluate clustering techniques. The range of datasets was chosen to reflect different platforms, experimental design, number and type of samples and genes. Details of datasets used can be found in Appendix B.

### 4.5.1 Creation of synthetic datasets

Synthetic datasets allow us to test assessment measures with a known partition. A two-step process was used to create two synthetic datasets, where the number, size and type of clusters could be controlled. The two-step process involved:

1. Data generated according to artificial patterns, such that the true class of each gene is known for 11 classes of various sizes, for which all genes in a class

| Author | Experiment | Type | Number of genes | Number of Samples |
| :--- | :--- | :--- | :--- | :--- |
| Alizadeth | Lymphoma | Spotted | $4026(3019)$ | 96 |
| Alon | Colon Cancer | Oligonucleotide | $2000(2000)$ | 62 |
| Cho | Yeast Cycle | Oligonucleotide | $6601(3000)$ | 17 |
| Gash | Yeast Stress | Spotted | $6152(3120)$ | 173 |
| Golub | Leukemia | Oligonucleotide | $7129(3571)$ | 72 |
| Hsiao | Human tissue | Oligonucleotide | $7070(2115)$ | 59 |
| Spellman | Yeast Cycle | Spotted | $6178(3049)$ | 82 |
| Stegmaier | Leukemia | Oligonucleotide | $22283(3145)$ | 22 |
| West | Breast Cancer | Oligonucleotide | $7129(3332)$ | 49 |
| Synthetic 1 | - | - | 3000 | 60 |
| Synthetic 2 | - | - | 3000 | 90 |

Table 4.2: Test datasets used for analysis. The value in brackets represents the number of genes after filtering
have identical patterns before error is added. Genes not contributing to the programmed pattern, so technically 'irrelevant' for cluster formation, were also included.
2. Error was added to the synthetic patterns, aimed to control the level of biological noise cluster each class (and hence, the signal-to-noise ratio), such that classes were less separable when affected by a higher error. Errors were added randomly, (uniform distribution), so that the signal-to-noise ratio was 20 at maximum.

Artificial patterns were created to reflect patterns of real gene expression datasets, based on the analysis in Chapter 2. The artificial patterns were encoded as follows, (refer to Figure 4.3 and for exact details on functions for artificial patterns see Appendix B):

- Synthetic dataset A was designed to represent data derived from two timecourse experiments, over 30 time points, creating a $60 \times 3000$ dataset, (see

Figure 4.3 (a)). The $11 \times 2$ groups were primarily modelled using $A \sin (2 \pi f+$ $\theta$ ), for various values of $A=$ amplitude, $f=$ frequency and $\theta=$ phase. Note that apart from pattern 'e', $A_{\text {time series 1 }} \neq A_{\text {time series 2 }}, f_{\text {time series } 1} \neq f_{\text {time series 2 }}$ and $\theta_{\text {time series } 1} \neq \theta_{\text {time series }}$, and that, in some groups (d and f, Fig. 4.3,(a)) time series 1 shows definite structure, where gene values are random in time series 2 and vice verse (group j Fig. 4.3(a)). For exact details on functions see Appendix B.

- Synthetic dataset B was designed to represent data derived from an experiment on phenotypic samples, with 3 'treatment groups'. In the first two groups samples relate to 6 individuals, with 5 tests carried out on each sample $(2 \times(6 \times 5)=60$ experiments). In the third group, samples relate to 4 individuals, again with 5 tests carried out on each sample ( $4 \times 5=20$ experiments). This created a $80 \times 3000$ dataset. Patterns were created to either (i) affect all samples between treatment groups similarly, (ii) affect a subset of treatment groups similarly, (iii) affect a subset of individuals in a treatment group, or (iv) have a different effect on individuals between treatment groups. Note also that intentional overlap between groups was added to the patterns, (see Figure 4.3 (b)).


### 4.5.2 Random Datasets

Random datasets were created by randomly sampling expression values from the original dataset without replacement, thus destroying any cluster structure in the data while retaining all other properties.


Figure 4.3: (a) Synthetic time series data, contained two time series over 30 time points. (b) Synthetic data from phenotypic samples. Contains data from 3 treatment groups, samples derived from 6 individuals in group 1 and 2, and 4 individuals in group 3, 5 tests carried out on each sample

### 4.5.3 Dataset pre-processing

An initial pre-processing step was applied to the 'real' datasets for testing, (Table 4.2) depending on whether the pre-processed data,due to the original authors, was available. Hence, the West, Golub, Hsiao, Stegmaier datasets were preprocessed as proposed by Speed (2000). This involves an initial step of applying a threshold on expression values of a floor of 100 and ceiling of 16000 . The data was then $l o g$-transformed and finally, standardized to have zero mean and unit variance. The Alon and Cho datasets were standardized to have zero mean and unit variance.

A filter based on variation in gene expression was applied to all datasets to focus computations on informative genes across the samples. Each dataset was also filtered based on the percentage of missing values (if $>25 \%$ missing values occur in the gene vector it was excluded analysis).

### 4.6 Evaluation

In the following sections, we assess a number of clustering applications using a variety of assessment measures. These are intended to give a 'flavour' of techniques and assessments, and the reader should be aware that there are a number of permutations of input parameters etc. that could affect the final clustering. The intention is to assess the metrics and the performance of the metrics when applied to specific clustering algorithms.

### 4.6.1 Hierarchical application

The agnes implementation of hierarchical clustering (available in the cluster package of Bioconductor (Maechler et al., 2005)) was used to carry out this analysis.

As stated previously, the cophenetic correlation is often used to decide between various linkage methods when applying hierarchical clustering. Table 4.3 provides the cophenetic correlation of this clustering on the test datasets. In all cases, the distance matrix used for the clustering correlates most with the cophentic distance derived when average linkage was used. However, tests on random datasets suggest that the cophenetic correlation measure is, in fact, biased towards this linkage method. There is a higher degree of correlation for the synthetic datasets compared to the real datasets, most likely because groups in these datasets are less complex than in real cases.

The dendogram produced by the hierarchical clustering can, of course, be 'cut' at various levels to produce clusterings with different cluster numbers, $K$. The trend of the internal assessments (SD-Validity, Dunn, Davies-Bouldin, Silhouette Index and C-Index) across various values of $K$ are given in Figures C.1-C.9, Appendix C. For a majority of these internal assessments, the values remain roughly constant for the entire range of $K$, (for all linkage methods), indicating that, according to these measures, hierarchical clustering techniques do not identify a compact and well separated structure in the data.

For a minority of datasets, however, the internal indices did indicate an optimal number of clusters. Table 4.4 summarises optimal $K$, suggested by internal measures for selected datasets. The SD-Validity index for single linkage, was minimised at 6 and 4 clusters for the Cho and Stegmaier datasets respectively. This value was corroborated by the Davies-Bouldin index for the Cho dataset for single linkage, Fig. 4.4. Again, the Dunn index indicates 4 clusters for the Cho dataset using average and ward linkage, Fig. 4.5.

For Synthetic B dataset, (were 11 groups is optimal, excluding outliers) the Dunn index suggested 15 and 10 clusters optimal for complete and ward linkage


Figure 4.4: Hierarchical Assessment (Single) of Cho and Stegmaier datasets using Internal measures. The x -axis indicates the cluster number while the y -axis indicates the score obtained.

|  | Single | Complete | Average | Ward's |
| :---: | :---: | :---: | :---: | :---: |
| Aliz | 0.363 | 0.211 | 0.485 | 0.178 |
| Alon | 0.245 | 0.304 | 0.544 | 0.428 |
| Cho | 0.296 | 0.332 | 0.558 | 0.335 |
| Gasch | 0.669 | 0.297 | 0.762 | 0.324 |
| Golub | 0.280 | 0.184 | 0.439 | 0.169 |
| Hsiao | 0.258 | 0.362 | 0.620 | 0.298 |
| Spell | 0.246 | 0.196 | 0.405 | 0.183 |
| Steg | 0.589 | 0.288 | 0.703 | 0.418 |
| West | 0.323 | 0.199 | 0.451 | 0.081 |
| Synth. A | 0.872 | 0.857 | 0.943 | 0.772 |
| Synth. B | 0.914 | 0.934 | 0.952 | 0.615 |
| Random Datasets |  |  |  |  |
| Aliz | 0.005 | 0.104 | 0.129 | 0.057 |
| Alon | 0.014 | 0.126 | 0.146 | 0.091 |
| Cho | 0.011 | 0.147 | 0.198 | 0.145 |
| Gasch | 0.006 | 0.1 | 0.12 | 0.057 |
| Golub | 0.008 | 0.1 | 0.116 | 0.066 |
| Hsiao | 0.014 | 0.129 | 0.150 | 0.092 |
| Spell | 0.004 | 0.106 | 0.142 | 0.048 |
| Steg | 0.006 | 0.121 | 0.170 | 0.104 |
| West | 0.006 | 0.092 | 0.11 | 0.063 |
| Synth. A | 0.009 | 0.106 | 0.122 | 0.07 |
| Synth. B | 0.009 | 0.110 | 0.128 | 0.076 |

Table 4.3: Cophenetic correlation coefficient. Agglomerative Clustering Techniques, internal assessment using the Cophenetic Correlation Coefficient measure.
respectively, Fig. 4.6. For the same dataset, the Silhouette index is maximised at 15 and 9 clusters for average and ward linkage respectively, Fig. 4.8. For Synthetic A dataset, the majority of indices do not change across clusters, with the exception of the Silhouette index, which indicates that the optimal number of clusters is 8 and 12 for complete and ward linkage methods respectively. Indeed, the negative Silhouette index for all the real datasets indicates that the clustering is very poor and that on average genes are placed in the wrong cluster, for all $K$, Fig. 4.8. With the exception of single linkage, the trend of the $S D$-Validity index is to increase

|  |  | Dunn | Davies-Bouldin | SD-Validity | Silhouette | C-Index |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cho | Single | - | 4 | 4 | - | 4 |
|  | Average | 4 | - | - | - | - |
|  | Complete | - | - | - | - | - |
|  | Ward | 4 | - | - | - | - |
| Steigmaier | Single | - | - | 4 | - | - |
|  | Average | - | - | - | - | - |
|  | Complete | - | - | - | - | - |
|  | Ward | - | - | - | - | - |
| Synthetic A | Single | - | - | - | - | $16^{\dagger}$ |
|  | Average | - | - | - | - | - |
|  | Complete | - | - | - | 8 | - |
|  | Ward | - | - | - | 12 | 7 |
| Synthetic B | Single | - | - | - | - | $16^{\dagger}$ |
|  | Average | - | - | - | - | - |
|  | Complete | 15 | - | - | 15 | 11 |
|  | Ward | 10 | - | - | 9 | 11 |

Table 4.4: K for selected datasets using HC Analysis. $\dagger$ - The value of C-index up to this point gradually declined.


Figure 4.5: Hierarchical Assessment (Average) of Cho dataset using Internal measures. The x -axis indicates the cluster number while the y -axis indicates the score obtained.
with the number of clusters in all linkage methods, i.e. the cluster variance to. distance ratio deteriorates, Fig. 4.7. The major trend of the C-Index is to decrease with the number of clusters, Fig. C.5, Appendix C. For ward and complete linkage methods, the $C$-Index indicates that the optimal number of clusters is 11 (optimal number) for Sythentic dataset B, Fig. 4.9.

Dunn Index


Figure 4.6: Hierarchical Assessment (Complete) of Synthetic dataset B using Dunn internal measures. The x -axis indicates the cluster number while the y -axis indicates the score obtained.

For a stability analysis ${ }^{2}$ of the hierarchical clustering methods, shows that the behaviour of $A D M$ measure across datasets is shown in Fig. 4.10. This measure generally increases with $K$ and it highlights larger effects of $K$ on the final clustering of Synthetic datasets A and B. For the single and average linkage methods this relationship is more gradual (range of values $0-1.2$ ) compared to complete and

[^12]
## SD-Validity Index



Figure 4.7: Hierarchical Assessment (Average) of datasets using SD-Validity Index. The x -axis indicates the cluster number while the y -axis indicates the score obtained.

Silhouette Index


Figure 4.8: Hierarchical Assessment (Ward) of datasets using Silhouette Index. The x -axis indicates the cluster number while the y -axis indicates the score obtained. For the majority of 'real' datasets the silhouette is negative, indicating a bad clustering. Similar results were obtained for all linkage methods.


Figure 4.9: Hierarchical Assessment (Ward) of datasets using $C$-Index. The x-axis indicates the cluster number while the $y$-axis indicates the score obtained. The trend of the index is to decrease as the number of clusters increases. Similar results were found for all linkage methods.
ward (range $0-5.3$ ), Fig. 4.10.
Similar behaviour of $A P N$ across linkage methods is observed, whereby the value of this index increases with $K$. Like the $A D M$ measure, this increase is more abrupt in the complete and ward linkage methods, Fig. 4.11. This indicates that as the number of clusters increases the consistency of results deteriorates. Exceptions include assessments of Synthetic datasets A and B, for complete and ward linkage.

Although there is a bias towards small $K$ with these measures, some information can be obtained. For Synthetic dataset $A$ the $A D M$ index indicates 13 clusters for ward linkage. For the same dataset, the $A D M$ and $A P N$ index indicate 9 clusters for complete linkage. For Synthetic $B$ dataset the optimal number of clusters of 8 as indicated by the $A D M$ and $A P N$ indices for complete linkage, while 17 clusters is indicated by the $A D$ and $F O M$ indices for the same linkage method (not shown). For ward linkage, all indices indicate 10 clusters for this dataset. For the Cho dataset, $A D M$ and $A P N$ indicate 3 clusters for average linkage, while the same measures indicate 5 clusters for the Alon dataset for ward linkage. For other datasets, these assessments give little information. Table 4.5 summarises results of the stability measurements.

External assessment is crucial in gene expression analysis, because it lets the experimenter link data to knowledge. External assessment results, using the Biological Homogeneity (BHI) and Biological Stability (BSI) measures, is given in Figures 4.12-4.13. It is evident that the biological stability of the hierarchical clusters is maximised when $K$ is small. The $B S I$ index inspects the cluster consistency of genes grouped by similar biological function, for $K=1$ stability $=1$ and gradually decreases as $K$ increases, i.e. the BSI index is biased towards a smaller number of clusters, Fig. 4.12. Single and Average linkage methods decrease linearly with $K$ while Complete and Ward deterioration is more pronounced. The


Figure 4.10: ADM measure across linkage methods. The x -axis indicates the cluster number while the y -axis indicates the score obtained. This measure highlights larger effects of $K$ on the clustering of Synthetic datasets A and B (smaller values preferred).


Figure 4.11: APM measure across linkage methods. The x -axis indicates the cluster number while the $y$-axis indicates the score obtained. For Ward and Complete linkage, similar to ADM behaviour, Synthetic datasets A and B show deviant behaviour compared to real datasets.

|  |  | ADM | AD | APN | FOM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Synthetic A | Single | 4 | - | - | - |
|  | Average | - | - | - | - |
|  | Complete | 9 | - | 9 | - |
|  | Ward | 13 | 12 | - | - |
| Synthetic B | Single | - | - | - | - |
|  | Average | - | - | - | - |
|  | Complete | 8 | 16 | 8 | 16 |
|  | Ward | 10 | 10 | 10 | 10 |
| Cho | Single | - | - | - | - |
|  | Average | 3 | - | 3 | - |
|  | Complete | - | - | - | - |
|  | Ward | - | - | - | - |
| Alon | Single | - | - | - | - |
|  | Average | 5 | - | 5 | 5 |
|  | Complete | - | - | - | - |
|  | Ward | - | - | - | - |

Table 4.5: $K$ selected by stability measures

BHI index measures whether genes placed in the same cluster belong to the same functional classes, through interrogation of GO ontologies. A value of unity indicates a biologically homogenous cluster. The maximum value for any technique is $\sim 0.35$, indicating that the clusters are not particularly homogenous. (This value was obtained for the Golub dataset, when $K=2$, Fig. 4.13.)

## Hierarchical Application Summary

Although there are some consistent predictions for $K$ within the stability analysis, these are not consistent with internal assessment results. For example, stability indices indicate 10 clusters is optimal for the Synthetic dataset B dataset, while internal indices indications range from $9-16$. Again, the external assessments are not consistent with either the internal or stability assessment results, and suggest that


Figure 4.12: Hierarchical Assessment using BSI. Top - bottom: Single, Average, Complete, Ward. The x -axis indicates the cluster number while the y -axis indicates the score obtained. It is clear there is a bias towards small values of $K$.


Figure 4.13: Hierarchical Assessment using BHI. Top - bottom: Single, Average. The $x$-axis indicates the cluster number while the $y$-axis indicates the score obtained.
clusters found by hierarchical clustering are not particularly homogenous. There are also obvious biases of indices (e.g. SD-Validity, C-Index, APN, BSI) towards a small number of clusters or linkage method (e.g. $A P N$ and $A D M$ towards single linkage).

### 4.6.2 Partitive Application

$K$-Means and SOTA were selected as representative techniques for the partitive analysis. Internal assessments (or classification criteria) have a more obvious role here as these are typically used to provide the required parameter, $K$, to the algorithm.

## K-Means Clustering

As discussed in Section 3.3.2, $K$-Means partitions the data by associating each gene vector with its nearest centroid and re-computing the cluster centroids. Thus, the $K$-Means technique is very sensitive to the random start positions and different executions will result in different clusterings. The $K$-Means algorithm available from the cluster package of Bioconductor (Maechler et al., 2005), was used for this analysis and starting positions were initalised using a technique which approximates the centres, based on the SPSS QuickCluster function, available in the clusterSim package (Dudek, 2008) of Bioconductor.

Figures C. 12 - C. 13 (Appendix C) summarises the internal assessments of clusterings obtained for various values of $K$. The Dunn index ranges from from $\sim 0.03$ to $\sim 0.1$ for $K=3$ to 16 , for all 'real' datasets, increasing slightly for Synthetic datasets A and B. This implies the datasets do not have compact and well separated
clusters when grouped according to the $K$-Means cost function. This assessment measure also indicates that the Synthetic B dataset (designed to have $K=11$ ) has optimal compactness to separation ratio at $K=4$. The Davies-Bouldin index indicates 6 for the same dataset, Fig. 4.14, (which measures the average error of each cluster group, rather than the maximum used by Dunn, thus incorrect grouping has less of an impact, see Appendix A for function details, (Davies and Bouldin, 1979)).


Figure 4.14: $K$-Means of selected datasets using Davies-Bouldin Index. The $x$-axis indicates the cluster number while the $y$-axis indicates the score obtained.

Overall, the range of values and trend for the SD-Validity index is similar to that obtained for the Hierarchical application - gradually increasing with $K$. For Synthetic A dataset, the $S D$-Validity index is minimised at $K=5$, before it increases (deteriorates) rapidly as $K$ increases. The $S D$-Validity index indicates $K=3$ for the Alizadeth and the Spellman datasets, before the value increasing rapidly, Fig. 4.15.

Unlike the hierarchical clustering methods, the Silhouette index is $>0$ for all


Figure 4.15: $K$-Means of selected datasets using SD-Validity Index. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.
datasets, which suggests a better clustering, Fig. 4.16. This index indicates that the best choice of $K$ is 14 for Synthetic B dataset. The Silhouette index indicates correctly that the optimal number of clusters for Synthetic A dataset is at $K=11$. For the Alon, Hsiao, Steigmaier and West datasets, the Silhouette index indicates $K=3$.

The C-Index is minimised at $K=9$ for Synthetic dataset B, while for Synthetic dataset A the $C$-Index indicates the optimal $K=5$. For the Alizadeth and Spellman datasets, the $C$-Index indicates $K=14$ and 15 respectively. In contrast to hierarchical methods, when this index is used with the $K$-Means algorithm, no bias towards small $K$ is indicated, Fig. C.13, Appendix C. Results of $K$-Means internal analysis are summarised in Table 4.6.

Unsurprisingly, the Connectivity measure increases with $K$, (when $K$ is low, each gene vector will more likely be grouped with its nearest neighbour). This

## Silhouette Index



Figure 4.16: $K$-Means assessment using Silhouette Index. The $x$-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.

## C-Index



Figure 4.17: $K$-Means assessment using $C$-Index for selected datasets. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.
indicated 4 clusters for Synthetic dataset B and 5 clusters for Synthetic dataset A. This measure is minimised for all other datasets at $K=3$.

Connectivity


Figure 4.18: $K$-Means assessment using connectivity index for selected datasets. The x -axis indicates the cluster number while the y -axis indicates the score achieved obtained.

Stability analysis of the $K$-Means algorithm show that the $A D$ measure does not determine $K$ for the $K$-Means algorithm, although the value is minimised for both Synthetic datasets A and B at $K=11$, Fig. 4.19. The $A D M$ measure selects $K=8$ for the Spellman dataset and $K=4$ for the Gasch dataset. Values of this measure are erratic across all the values of $K$, Fig. 4.20. The $A P N$ index is minimised at $K=6$ for Synthetic B dataset, $K=4$ for the Gash dataset and $K=5$ for the Alizadeth, Cho and Hsiao datasets, Fig. 4.21. The FOM index is not selective of $K$ across any dataset, see Table 4.7 for summary of stability indices.

As for hierarchical clustering, biological stability (BSI) and biological homogeneity ( BHI ) were assessed for the clustering produced by $K$-Means. As for hierarchical clustering, moreover, the biological measures identify no discernible struc-

|  | Dunn | Davies-Bouldin | SD-Validity | Silhouette | C-Index |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Synthetic A | - | - | 5 | 11 | 5 |
| Synthetic B | 4 | 6 | - | 14 | 9 |
| Alizadeth | - | - | - | 3 | 14 |
| Alon | - | - | 3 | - | - |
| Hsiao | - | - | - | 3 | - |
| Spellman | - | - | 3 | - | 15 |
| Steigmaier | - | - | - | 3 | - |
| West | - | - | - | 3 | - |

Table 4.6: Optimal $K$ identified by internal assessment measures for $K$-Means algorithm.


Figure 4.19: $K$-Means stability analysis using AD index. x -axis indicated cluster number and $y$-axis, score achieved.


Figure 4.20: $K$-Means stability analysis using ADM index. x-axis indicated cluster number and y -axis, score achieved.


Figure 4.21: $K$-Means stability analysis using APN index. x-axis indicated cluster number and $y$-axis, score achieved.

|  | AD | ADM | APN | FOM |
| :---: | :---: | :---: | :---: | :---: |
| Synthetic A | 11 | - | - | - |
| Synthetic B | 11 | - | 6 | - |
| Spellman | - | 8 | - | - |
| Gasch | - | 4 | 4 | - |
| Alizadeth | - | - | 5 | - |
| Cho | - | - | 5 | - |
| Hsiao | - | - | 5 | - |

Table 4.7: Stability summary of $K$-Means
ture in the datasets. $B S I$ values decrease with the number of clusters, $K$, consistent with the trend for hierarchical clustering, Fig. 4.22. The value of BHI increases slightly for the Golub dataset at $K=15$ (as opposed to $k=2$ found by hierarchical average linkage for this dataset).

## SOTA - Self Organising Tree Algorithm

Details of this technique were given in Section 3.3.2. A partitive technique, it uses self organising maps to discover hierarchical structure in the data. For this analysis, the Euclidean distance measure was used, with an ancestor height of two.

Figure C.15, Appendix C, summarises the internal assessment of the clusters obtained for various values of $K$. Again, the internal measurements tell us little about these data. The Silhouette index indicates the optimum choice of $K$ is 6 for both the Synthetic B and Stegmaier datasets. For all other datasets, the optimum choice is indicated at $K=3$. Again, this index is $>0$, for each datasets and across all $K$, similar to the results found for the $K$-Means algorithm, and again the maximum value found is for Synthetic B dataset. There is a large range in Silhouette values across $K$, for each dataset, however the trend is to decrease with increasing $K$ (exception is Synthetic B dataset which increases with $K$ and Synthetic A dataset


Figure 4.22: KMeans Assessment using Biological Homogeneity and Biological Stability Measures. The x -axis indicates the cluster number while the y -axis indicates the score achieved obtained.
which remains constant). The Dunn and Davies-Bouldin indices indicate $K=8$ for the Synthetic B dataset. The C-index is minimised for Synthetic dataset B at $K=$ 7. This value of this index increases with $K$ for Synthetic dataset A. Optimal $K$ indicated by the internal assessments for various datasets is given in Table 4.8. Unsurprisingly, the SD-Validity index tends to increase with $K$, Fig. 4.24.

|  | Dunn | Davies-Bouldin | SD-Validity | Silhouette | $C$-Index |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Synthetic B | 8 | 8 | - | 6 | 7 |
| Stegmaier | - | - | - | 6 |  |

Table 4.8: Optimal $K$ selected with internal assessments and SOTA algorithm

Table 4.9 shows the results for the stability analysis of selected datasets for the


Figure 4.23: Dunn index returned for various $K$ and SOTA algorithm.


Figure 4.24: SD-Validity returned for various $K$ using SOTA algorithm. This index is biased towards small values of $K$.


Figure 4.25: Silhouette index for various $K$ using SOTA algorithm. For all datasets this value is $>0$


Figure 4.26: $C$-index for selected datasets for various $K$, using SOTA algorithm

Connectivity


Figure 4.27: Connectivity measurements for selected datasets for various $K$, using SOTA algorithm

SOTA algorithm. The AD and FOM assessment indices were not informative of $K$ for this technique, hence not shown.

|  | AD | ADM | APN | FOM |
| :---: | :---: | :---: | :---: | :---: |
| Synthetic B | - | - | 5 | - |
| Alizadeth | - | 6 | - | - |
| Cho | - | - | 6 | - |
| Hsiao | - | 8 | - | - |
| Golub | - | - | 5 | - |
| Stegmaier | - | - | 7 | - |
| West | - | - | 6 | - |

Table 4.9: $K$ selected by stability measurements and SOTA algorithm

Biological Homgeneity (BHI) values, obtained for various values of $K$ with the SOTA algorithm, are of a similar range as those obtained for other clustering algorithms tested, Fig. 4.30. However, with previous techniques, this index did not change with $K$, whereas with SOTA there is an optimal. For example, this index is optimised at $K=16$ for the Golub dataset, corresponding to $B H I=0.25$. This


Figure 4.28: SOTA assessment using ADM index for values of $K$ for selected datasets.


Figure 4.29: SOTA assessment using APN index for values of $K$ for selected datasets.
value does not represent very homogenous clusters, or is it significantly different than the $B H I$ value found, for example, with $K$-Means.


Figure 4.30: SOTA assessment using Biological Homogeneity. The x -axis indicates the cluster number while the $y$-axis indicates the score obtained.

## Partitive Application Summary

A similar bias as observed for hierarchical algorithms occurs for the $K$-Means and SOTA algorithms whereby $S D$-Validity values increase with $K$. The Dunn index does not identify any compact and well separated clusters with the $K$-Means algorithm, again similar to hierarchical techniques. However, in contrast to hierarchical techniques, the silhouette values are $>0$ for all datasets across all $K . K$-Means and SOTA strive to find spherically linked compact clusters, and these results in agreement with Handl et al. (2005) in that the Silhouette index will perform better with these techniques.

### 4.6.3 Fuzzy Application

Examples of Fuzzy clustering algorithms include Fuzzy CMeans (FCM) and Fuzzy Local Approximation MEthod (FLAME). FCM is the most widely used fuzzy clustering method. We used the FCM implementation available in the el071 package of Bioconductor, (Dimitriadou et al., 2008), and similarly to the $K$-Means analysis, the initial cluster centres were estimated using the technique available in the clusterSim package. For these tests we used a 'fuzzification' parameter of 2, (see Section 3.3.2). The developers of Fuzzy Local Approximation MEthod (FLAME) committed code to open-source ${ }^{3}$, which we adapted for comparison in R. Recall that this method does not take $K$ as an input parameter, but determines the optimal number from the dataset. However, $K$ is dependent on a number of input parameters, primarily the number of nearest neighbours, $k n n$.

From an analysis of the effect of the $k n n$ parameter on FLAME output it was observed that $K$ decreases as $k n n$ increases, Table 4.10 (also noted in the original paper ( Fu and Medico, 2007)). This relationship arises for two reasons. In this algorithm, a cluster is determined from a 'Cluster Supporting Object' (CSO) and its relationship to $k n n$. Firstly, $k n n$ determines the smoothness of the cost function used by this algorithm, which in turn limits the maximum number of CSO's, and secondly, knn determines the range covered by one CSO - a larger value of $k n n$ results in a wider CSO range, therefore the fewer the CSO's, (Fu and Medico, 2007). As $K$ is not specified, the assessment indices are presented for various $k n n$. The reader can cross reference with Table 4.10 to check equivalent number of clusters. As this technique creates a dedicated outlier cluster, this was removed before assessment.

[^13]| knn $\rightarrow$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Alon | 35 | 29 | 22 | 22 | 23 | 20 | 19 | 17 | 17 | 15 | 15 | 14 | 14 | 14 | 14 | 14 | 14 | 13 | 13 | 14 | 12 | 11 | 11 | 11 | 10 | 9 |
| Aliz | 48 | 31 | 25 | 16 | 13 | 10 | 10 | 9 | 8 | 6 | 5 | 4 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Cho | 56 | 39 | 35 | 28 | 25 | 23 | 20 | 19 | 21 | 20 | 20 | 19 | 17 | 14 | 11 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| Gasch | 48 | 37 | 31 | 26 | 22 | 21 | 19 | 18 | 17 | 16 | 13 | 13 | 10 | 9 | 9 | 8 | 9 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| Golub | 25 | 22 | 19 | 13 | 15 | 12 | 12 | 11 | 11 | 9 | 9 | 9 | 8 | 7 | 7 | 8 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 5 |
| Hsiao | 36 | 31 | 28 | 28 | 26 | 25 | 25 | 24 | 21 | 22 | 21 | 20 | 19 | 16 | 16 | 15 | 15 | 15 | 15 | 14 | 13 | 13 | 12 | 12 | 12 | 11 |
| Iressa | 116 | 97 | 83 | 71 | 57 | 54 | 51 | 44 | 41 | 37 | 33 | 32 | 29 | 29 | 29 | 28 | 26 | 23 | 22 | 20 | 18 | 19 | 19 | 18 | 18 | 18 |
| Spell | 49 | 37 | 33 | 26 | 23 | 20 | 21 | 19 | 20 | 20 | 20 | 18 | 14 | 13 | 12 | 10 | 9 | 9 | 9 | 9 | 9 | 8 | 8 | 8 | 6 | 5 |
| West | 28 | 29 | 24 | 21 | 16 | 16 | 16 | 14 | 14 | 11 | 11 | 10 | 9 | 7 | 6 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| Synth A | 88 | 64 | 52 | 46 | 44 | 41 | 37 | 37 | 35 | 32 | 25 | 24 | 20 | 18 | 18 | 18 | 18 | 16 | 16 | 15 | 15 | 14 | 14 | 14 | 14 | 14 |
| Synth B | 67 | 59 | 52 | 50 | 36 | 38 | 34 | 30 | 27 | 28 | 28 | 26 | 24 | 25 | 24 | 24 | 21 | 23 | 23 | 21 | 22 | 21 | 21 | 20 | 21 | 20 |

Table 4.10: FLAME: Effect of $k n n$ on $K$. As the number of nearest neighbours increase, the number of clusters decrease. Due to the property that $k n n$ controls $K$, in further analysis of this method, we we use $k n n$ parameter in place of $K$.

The cluster separation and the fuzzy compactness ratio, (Xie-Beni index), estimate how spatially separated the clusters are. No well separated or compact clusters found for any $K>10$ by the FCM technique appearing as very large values of the Xie-Beni index, Fig. 4.31. For the Alon, Golub, Stegmaier and West datasets, this index indicates $K=3$ is optimal, while, for the FLAME method, it points to the partitioning again being more stable for small $K$ (i.e. for large values of $k n n$ in the graph). For large values of $K$, (i.e. small $k n n$ ) no compact or well separated clusters are found, Fig. 4.32.

The Partition Coefficient measures the amount of overlap, or sharing of gene membership, between clusters. As this value approaches unity the clustering approaches a crisp partitioning. For FCM this value monotonically decreases with $K$ (memberships of genes is spread roughly equally amongst the clusters), while for FLAME it remains roughly constant for each dataset for all values of $K$. This highlights the contrast FCM and FLAME membership requirements for a cluster. For the former, for a given gene, the membership value to a set of clusters is proportional to its similarity to cluster mean, and the latter requires membership of a cluster, $i$, to be determined by the weighted similarity of the gene to its $K$-nearest neighbours, and their membership of cluster $i$. The assignment of genes with few neighbours to a dedicated outlier cluster (not considered with this analysis) also contributes to stable results. This implies that for FCM that each gene does not particularly belong to any cluster, while for FLAME, memberships are similarly spread among a subset of clusters, regardless of the cluster number.

Similarly, the partition entropy measure monotonically increases with $K$ for FCM, where a small value indicates a good clustering, while for FLAME this value remains roughly constant for the majority of datasets. Again, highlighting the difference in the membership weighting schemes of the techniques. For example, in
the Alizadeth dataset, there is a minimisation of this value at $k n n=23$, which corresponds to $K=2$ (Table 4.10), while for the Spellman dataset this value is minimised at $k n n=30$, which corresponds to $K=5$.

### 4.6.4 Biclustering Application

Here the Plaid biclustering model is investigated ${ }^{4}$, significant because it uses a statistical model to capture the biclusters in the data.

## Plaid Model

Described in Sections 3.3.1 and 3.3.3, this method requires few input parameters. Code used for this method was made available by the original authors (Lazzeroni and Owen, 2000) and was used in this analysis. An advantage of this approach is the determination of $K$ directly from the dataset, however this algorithm uses a $K$-Means start to initialize clusters, thus it is sensitive to starting positions and different runs may produce different results. The number and size (\% volume of the dataset) of clusters returned for each dataset by the Plaid algorithm is given in Table 4.11

Notable of this technique is the large range in the size of the clusters returned. The average volume of clusters is quite high for each dataset. For example, the average volume of clusters found in the Alon dataset is $16.4 \%$ of the volume of the entire dataset and the largest bicluster found in the Stegmaier dataset is $36.6 \%$ of the total dataset size. This means that large proportions of the datasets are grouped into one cluster. Table 4.12 gives description of biclusters scores obtained with

[^14]

Figure 4.31: FCM Assessment using Internal measures. The x-axis indicates $K$ while the y -axis indicates the score achieved obtained.


Figure 4.32: FLAME Assessment using Internal measures. The x -axis indicates $k n n$ while the y -axis indicates the score achieved obtained.

| Dataset | K | Avg. Vol | Min. Vol | Max. Vol |
| :---: | :---: | :---: | :---: | :---: |
| Aliz | 3 | $27866(9.6 \%)$ | $12208(4.2 \%)$ | $41360(14.3 \%)$ |
| Alon | 37 | $20280.49(16.4 \%)$ | $11400(9.2 \%)$ | $32190(25.9 \%)$ |
| Cho | 9 | $5656.67(11.1 \%)$ | $1484(2.9 \%)$ | $9904(19.4 \%)$ |
| Gasch | 56 | $58626.75(10.8 \%)$ | $2087(0.4 \%)$ | $131442(24.4 \%)$ |
| Golub | 10 | $39528.9(15.4 \%)$ | $27144(10.5 \%)$ | $58240(22.6 \%)$ |
| Hsiao | 8 | $11359(9.1 \%)$ | $1788(1.4 \%)$ | $27081(21.7 \%)$ |
| Spell | 150 | $33423.83(13.3 \%)$ | $1256(0.5 \%)$ | $52836(21.1 \%)$ |
| Stegmaier | 9 | $10510.44(15.1 \%)$ | $3980(5.7 \%)$ | $25395(36.6 \%)$ |
| West | 63 | $23846.37(14.6 \%)$ | $2238(1.4 \%)$ | $38280(23.4 \%)$ |
| Synth. Data A | 8 | $21979.25(12.2 \%)$ | $5168(2.8 \%)$ | $34804(19.3 \%)$ |
| Synth. Data B | 7 | $20102.71(8.3 \%)$ | $12250(5.1 \%)$ | $30818(12.8 \%)$ |

Table 4.11: Plaid cluster statistics. \% indicates the percentage volume of the dataset.
this technique. Of note is the range of scores for biclusters found in the Spellman dataset. This dataset also presented the largest number of biclusters, 69 of which had a score $>150$. Biclusters with large scores were found in Synthetic B dataset, however these represented the smallest in terms of \% volume. However, $K=$ 3, found in Synthetic dataset B, is considerably less than the number designed. Biological homogeneity is again low for the biclusters returned by the Plaid Model, and is in fact lower than that obtained from more traditional methods (analysed above), Table 4.13.

### 4.6.5 Graphical Application

These evaluations were performed using software developed by the original authors and available with the Expander v4.3 suite of analysis tools ${ }^{5}$, (Sharan et al., 2003). Both algorithms examined here, CLICK and SAMBA, score clusters (biclusters in the case of SAMBA), using a probabilistic based scoring scheme (Section 3.3.4).

[^15]| Dataset | K | Avg. Score | Min. Score | Max. Score |
| :---: | :---: | :---: | :---: | :---: |
| Aliz | 3 | 12426.97 | 10893.62 | 13960.33 |
| Alon | 37 | 603.17 | 179.51 | 1727.81 |
| Cho | 9 | 3068.08 | 1444.87 | 4517.52 |
| Gasch | 56 | 6314.35 | 735.99 | 66365.16 |
| Golub | 10 | 2621.29 | 1433.33 | 4780.91 |
| Hsiao | 8 | 3136.63 | 1625.79 | 5768.42 |
| Spell | 150 | 306.44 | 38.13 | 2517.25 |
| Stegmaier | 9 | 1475.67 | 270.29 | 4155.55 |
| West | 63 | 774.28 | 238.38 | 3799.12 |
| Synth. A | 8 | 6653.08 | 3257.03 | 8576.35 |
| Synth. B | 7 | 38919.86 | 17629.02 | 68774.17 |

Table 4.12: Plaid cluster scores, a large score indicates a more significant cluster.

|  | BHI |
| :---: | :---: |
| Gasch | 0.110 |
| Golub | 0.197 |
| Stegmaier | 0.179 |
| Spellman | 0.103 |
| West | 0.176 |

Table 4.13: BHI values for selected datasets obtained with Plaid model algorithm

As these methods determine the optimal $K$ from the dataset, clusterings for multiple $K$ are not investigated.

## CLICK

| Dataset | $K$ | Avg. Score | Min. Score | Max. Score |
| :---: | :---: | :---: | :---: | :---: |
| Alon | 8 | 0.578 | 0.521 | 0.614 |
| Aliz | 13 | 0.535 | 0.423 | 0.57 |
| Cho | 24 | 0.725 | 0.615 | 0.788 |
| Gasch | 14 | 0.671 | 0.491 | 0.749 |
| Golub | 23 | 0.492 | 0.406 | 0.525 |
| Hsiao | 20 | 0.645 | 0.538 | 0.734 |
| Spell | 13 | 0.546 | 0.344 | 0.6 |
| Stegmaier | 18 | 0.702 | 0.583 | 0.78 |
| West | 14 | 0.501 | 0.447 | 0.553 |
| Synth. Data A | - | - | - | - |
| Synth. Data B | 7 | 0.844 | 0.685 | 0.94 |

Table 4.14: CLICK Cluster Statistics

This is a partial clustering method, in that not all the genes must be put into a cluster, however genes are grouped over all samples, i.e. global clustering. Notable from the results, and contrary to results obtained from previous methods, a structure was not identified in Synthetic dataset A (time series synthetic dataset), Table 4.14. Structure was, however, identified in 'real' time-series datasets, (Cho, Spellman, see Appendix B). Structure was found in Synthetic dataset B (7 clusters) with a higher significance compared to all real datasets. There is a small range in scores of clusters found in each of the datasets and the number of clusters found in each dataset is larger than those suggested by assessment indices for other global clustering techniques (hierarchical, partitive and fuzzy).

As this is a partial clustering method, and therefore clusters should in theory
only contain relevant genes, it is surprising that the Biological Homogeneity Index has a similar range to those obtained for traditional clustering techniques. Again, the largest BHI value is associated with the Golub dataset, Table 4.15 and these values are consistent with those obtained for other clustering algorithms tested.

|  | BHI |
| :---: | :---: |
| Gasch | 0.121 |
| Golub | 0.200 |
| Stegmaier | 0.185 |
| Spellman | 0.115 |
| West | 0.178 |

Table 4.15: BHI values obtained for selected datasets using the CLICK algorithm

## SAMBA

| Dataset | $K$ | Avg. Score | Min. Score | Max Score |
| :---: | :---: | :---: | :---: | :---: |
| Alon | 37 | 468.11 | 113.14 | 1217.62 |
| Aliz | 101 | 304.83 | 62.65 | 1114.9 |
| Cho | - | - | - | - |
| Gasch | 118 | 834.04 | 82.4 | 5780.29 |
| Golub | 49 | 490.78 | 32.91 | 1477.37 |
| Hsiao | 20 | 702.96 | 307.64 | 1452.83 |
| Spell | 88 | 348.2 | 83.43 | 984.93 |
| Stegmaier | 9 | 570.32 | 311.2 | 809.48 |
| West | 36 | 274.78 | 12.7 | 881.04 |
| Synth. Data A | 49 | 1823.27 | 39.55 | 5777.94 |
| Synth. Data B | 28 | 1027.81 | 55.78 | 4216.82 |

Table 4.16: SAMBA Cluster Scores

On average finds much smaller clusters compared to the Plaid technique, with most clusters $<1 \%$ of the total volume of the dataset. This is due to the negative effect of non-edges on bicluster scores, thus making larger clusters unfavourable, as
these would inevitably include more non-edges. Notable among the results of the SAMBA algorithm is the absence of structure found in the Cho dataset.

| Dataset | $K$ | Avg. Vol | Min. Vol | Max. Vol |
| :---: | :---: | :---: | :---: | :---: |
| Alon | 37 | $1432.3(1.1 \%)$ | $264(0.2 \%)$ | $3956(3.1 \%)$ |
| Aliz | 101 | $468.8(0.1 \%)$ | $60\left(2 e^{-3 \%}\right)$ | $2050(0.7 \%)$ |
| Cho | - | - | - | - |
| Gasch | 118 | $1170.34(0.2 \%)$ | $96\left(1 e^{-4 \%}\right)$ | $9454(1.7 \%)$ |
| Golub | 49 | $1699.08(0.6 \%)$ | $45\left(1 e^{-4 \%}\right)$ | $5190(2.0 \%)$ |
| Hsiao | 20 | $1636.3(1.3 \%)$ | $495\left(3 e^{-3 \%)}\right.$ | $5264(4.2 \%)$ |
| Spell | 88 | $482.2(0.2 \%)$ | $91\left(3 e^{-4 \%}\right)$ | $1648(0.6 \%)$ |
| Stegmaier | 9 | $1704.44(2.4 \%)$ | $931(1.3 \%)$ | $2592(3.7 \%)$ |
| West | 36 | $1067(0.6 \%)$ | $35\left(2 e^{-4 \%)}\right.$ | $3302(2.0 \%)$ |
| Synth. Data A | 49 | $1726.14(0.6 \%)$ | $55\left(2 e^{-4 \%)}\right.$ | $6000(2.2 \%)$ |
| Synth. Data B | 28 | $896.78(0.4 \%)$ | $104\left(3 e^{-4 \%)}\right.$ | $3400(1.2 \%)$ |

Table 4.17: SAMBA Cluster Statistics

Although, structures found with this biclustering technique are smaller compared to Plaid model, the $B H I$ values are marginally larger. As with all techniques examined, analysis of the Golub dataset for biological homogeneity returns the best result, albeit still small. Similar BHI results were found for this technique, when compared to more traditional methods, Table 4.18.

|  | BHI |
| :---: | :---: |
| Gasch | 0.178 |
| Golub | 0.209 |
| Stegmaier | 0.195 |
| Spellman | 0.157 |
| West | 0.180 |

Table 4.18: BHI values for selected datasets for biclusters obtained from SAMBA algorithm

## Graphical Methods Summary

The probabilistic scoring scheme employed by both models is a strength of these techniques, as it provides an estimate of significance of results. The final score of a cluster is a summation of significance of edges when using SAMBA, whereas for CLICK it is the average. Graph theoretic methods provide an intuitive method of gene expression analysis, owing to the modularity and inter-connectedness of gene expression in the cell. Moreover, the SAMBA technique is innovative as it finds local structures in a bipartite graph. In this analysis, this algorithm was applied to gene expression data only, however, it has been applied to compendium of biological data, (Tanay et al., 2005). The variation of clusterings found between the CLICK and the SAMBA algorithm, (although the former finds global structures, and the latter, local), highlight the need to investigate the techniques used to score gene interactions in the graphical domain. For example, the SAMBA algorithm identifies no biclusters in the Cho dataset, however, when applied to the same dataset the CLICK algorithm identifies clusters which has the highest overall homogeneity score of any real dataset tested. SAMBA edge-weighting is thoroughly investigated in Chapter 5. Using graphical techniques also presents the opportunity to explore the organisational properties of gene interaction using tools and ideas from classical graph theory.

### 4.7 Summary

It is important to understand that use of analytical validation techniques solely is not sufficient, but that an understanding of the working principles of clustering algorithms, validation measures and their intrinsic biases is critical to enable fair
and objective cluster validation. As shown in this investigation, many validation techniques are intrinsically biased, hence a careful analysis of the results obtained is required, and results should be corroborated using alternative validation techniques. Although research suggests (Chapters 2 and 3 and references therein) that clustering techniques which find a local structure in the data are more suitable for gene expression data, the topic of internal assessment measures for biclustering is not well developed and is of fundamental concern.

This analysis shows that the FCM algorithm does not partition the data well, where the membership of the genes is spread evenly across all clusters for all $K$, as indicated by the partition coefficient and the partition entropy measures. The Xie-Beni index has been found to be very eratic. However, the FLAME algorithm, which performs a partial clustering of the data by having a committed 'outlier' cluster, returns more stable results. FLAME was found to be more suitable for gene expression data clustering, which often contains a large noise component or irrelevant measurements. Most clustering techniques do not provide estimates of the significance of results returned. This is a strength of the exceptions CLICK, SOTA, Plaid and Samba algorithms, which do estimate significance in cluster scoring, as validation methods are inherently biased and misleading. A key strength of graphical techniques is that it transforms gene expression data into a network. These resulting networks can and should therefore be examined further using classical network analysis methods, to highlight the properties of gene interactions.There is no method investigated here which is optimal across all datasets, as indicated by the assessment indices. There is always a tradeoff between algorithm and assessment measure used, e.g. SD-Validity, BSI, Silhouette.

In this chapter assessment indices were evaluated for a range of cluster numbers $K$, for each dataset, to detect biases and trends. With many cluster validation
packages available, the returned result is simply the minimum or maximum for a particular range of values for $K$. However, we have shown that the trend and range of the values need to be accounted for. This analysis was carried out on comparably large sets of genes than typically reported in literature. It is expected that better results would have be obtained by limiting the number of genes being clustered to a small subset ( $100 \sim 600$ ) for computational and visualization purposes, however eliminating perhaps interesting genes in the filtering process.

Assessment indices measure the extent of a clustering algorithms's ability to find structures in a dataset. However, for clustering gene expression data, it is reasonable to consider external measures that use existing biological knowledge. Internal measures by themselves may not be suitable for gene expression data which are often subject to many sources of noise, (also argued by Handl et al. (2005)). The BHI index was used to quantify the association of gene expression profiles in a cluster with functional classes. The BHI index is, of course, greatly influenced by the annotation used. Here, all (Biological Process, Cellular Component and Molecular Function) categories from Gene Ontologies annotation database was used to define functional classes. If, for e.g. FunCat or EASE were used to determine functional classes the results may vary slightly.

Past studies have concluded that clustering of the gene expression profiles show that functionally similar genes are grouped together. This is often concluded by manually inspecting genes in a cluster. Validation of a clustering result in this manner is tricky, however, from our investigation, the biases and limitations of assessment indices suggest that validation through expert knowledge is the ideal, although time consuming and subjective. Manual interrogation will be aided as more advanced ontologies and detailed annotation databases become mainstream. External indices would be preferable over internal indices when there is a substan-
tial biological knowledge about the genome under investigated (i.e. proportion of annotated genes).

## CHAPTER 5

## In Depth: Constructing and

## Exploring Gene Expression

## Bi-Partite Graphs

Statistical methods can identify specific genes and groups of genes through coexpression analysis but are less reliable in terms of identifying pattern and dynamics of interaction. A natural representation of such inter-connectivity and an aid to its evaluation is a network. Practically, such a network can be considered as a (weighted) graph. Here, we introduce a new method for extracting graph structure from a gene expression dataset. We explore the organisational properties of graphs obtained, such as node degree distributions, edge distributions, clustering co-efficient information and amongst others and compare these for empirical data to those generated by a random graph model. We also describe, in detail, the motivation and implementation of a new edge-weighting scheme for the graph extracted from the data. Finally, we present results of an analysis of this weighting scheme and compare its performance with the well-known Tanay scheme, (Tanay et al., 2002, 2005).

### 5.1 Introduction

Traditionally, graph theory has been used to study complex networks ${ }^{1}$ and is proving a useful tool in the analysis of large complex biological datasets. For instance, protein-protein interactions can be modelled as an undirected graph (Pereira-Leal et al., 2004), where nodes represent proteins and an edge connects two nodes if the proteins physically combine. Transcription factor binding sites can also be identified through the use of undirected weighted graphs, where the weights of edges capture the similarity between aligned nucleotides in an input set of promoters, (Reddy et al., 2007). Additionally, metabolic networks can be represented as bi-partite graphs: In this case, an edge connects a reaction to a compound node, representing either substrate or product relationships, (Bourqui et al., 2007). As noted (Section 3.3), gene expression can also be modelled as a weighted bi-partite graph, where the two node types represent genes and samples. An edge is taken to exiss between a gene and a sample node, with the weight of the edge representing the effect of the experimental condition on the expression of the gene, (Tanay et al., 2002). Bipartite graph structures have also been been studied in a wide variety of contexts: for instance, in reference to company boards, (Robins and Alexander, 2004), to film actor social contacts, (Newman et al., 2002), to financial networks, (Caldarelli et al., 2004), in investigating word occurrences (i Cancho and Sole, 2001), in peer-to-peer networks, (Blond et al., 2005) and with respect to scientific co-authoring, (Newman, 2001b,a), amongst others.

The analysis of the gene expression graphs of interest to us is carried out on three levels, Fig. 5.1. Our analysis begins by extracting bi-partite graphs to investigate reactivity of genes at various response levels. Genes may be coherently

[^16]expressed at these different levels across samples: the next step is thus to investigate the bi-partite graph properties for combined levels of response. In Chapter 6, (mentioned here to give the complete roadmap), we also investigate the properties of, and extract coherent gene modules from, one-mode ${ }^{2}$ gene expression graphs.


Figure 5.1: Three level analysis of gene expression graphs on three levels.

### 5.2 Extracting a Graph from a Gene Expression Dataset

A gene expression graph can be organised as a bi-partite model, $G=(T, \perp, E)$ or a one-mode model, $G=(V, E)$, (Section 3.3.4). A large, and powerful, set of tools and ideas exist for one-mode graphs. A one-mode gene expression graph can be extracted from a dataset by e.g. applying a threshold to the distance matrix, and creating an edge between genes whose similarity exceeds this threshold, Section

[^17]3.3.4 and Zhang and Horvath (2005); Carlson et al. (2006), illustrated in Fig. 5.2a. However, this technique extracts a graph which encodes global structures from the dataset, so that it inherits drawbacks of the associated distance/similarity measure. Alternatively, two gene nodes can be linked in a one-mode projection of the bipartite graph, if they have a sample neighbour in common, Fig. 5.2a. However, some information encoded in the bi-partite graph may be lost by this projection, such as details on which or for how many samples gene expression is similar. In illustration, three gene nodes e.g. can form a clique even though not expressed under the same samples, (nodes 2,3 and 4 in Fig. 5.2a). This means that a number of bi-partite gene-sample graphs can give rise to the same one-mode projection, (e.g. Fig. 5.2b). Additionally, each sample node of degree $d$, (the number of edges incident to a node), can inflate to $\frac{d(d-1)}{2}$ edges in a one-mode projection, which can limit the number and type of the computations that are feasible. Bi-partite graphs capture local structures in the dataset, enabling identification and examination of meaningful local groups of gene-interactivity.

### 5.2.1 Node and Edge Definition

Critical to this analysis is the definition of a node and, in particular, the relationship between nodes defined through an edge. The edge definition and derivation will influence the final graph generated and hence the information retrieved. Both hard and soft threshold strategies can be used for definition of an edge in the network. A hard threshold is an all-or-nothing approach, where an edge is said to exist if the score of the gene in a particular sample exceeds a certain threshold. A soft threshold approach assigns an edge between each gene and sample node according to a function, $f(x) \rightarrow[0,1]$. This effectively ranks all nodes in a network. If a list

(a) Two methods of extracting a one-mode graph for gene expression. Top: extracting a graph from direct threshold analysis of the distance matrix, (dis = distance determined by some distance function, thres $=$ threshold, $T=$ TRUE). Bottom: Projecting bi-partite graph into one-mode by retaining links through second degree neighbours, e.g 2-D-4 produces link 2-4 in one-mode projection.


Figure 5.2: One-Mode Gene Expression Graphs
of neighbours of a particular node is required, a threshold must be determined for all edges so that these become the investigation focus.

There are two sets of nodes in our representation of a gene expression graph, $\top=$ the set of all genes and $\perp=$ the set of all samples. Fundamental to the method proposed here is that, for a given sample, genes having either high or low expression, (equivalent to induction or repression), are more likely to contribute to a function, or have a functional response, than for those for which expression values remain unaffected. Affected expression values can thus be extracted for further analysis. An edge $(i, j)$ is thus defined for the bi-partite graph, if gene $i \in \top$ is deemed to show significant change in expression relative to its normal level under sample $j \in \perp$. The edge set here is created using an empirical-based scheme. (The weight of the edge should then reflect how 'interesting' this change in expression is relative to other gene expression changes in that particular sample. Implementation of such a scheme is described, Section 5.3.) This approach differs from previous work, (Zhang and Horvath, 2005; Tanay et al., 2002, 2005), based, respectively, on (i) a soft thresholding approach applied to a distance matrix, based on an assumed power-law distribution, and (ii) defined nodes in a graph based on "properties" rather than samples.

In our method, a high/low expression value for gene $i$, under sample $j$, is determined relative to other expression values in gene vector $i$, (i.e. across rather than within samples). The motivation for this, (Chapter 2.5), is that, for microarray technology, direct comparison of expression measures within arrays is problematic, because fluorescent intensities are not the same across genes. While measured intensities are roughly proportional to mRNA abundance, the proportionality factor is different for each gene. Specifically, this means that between-sample, within-gene comparisons are appropriate, but within-sample, between-gene comparisons are
not straightforward, (Gentleman et al., 2005b).


Figure 5.3: Extracting expression values from a gene vector for further analysis

Hence, an edge $(i, j)$ exists when the $i^{\text {th }}$ gene shows "significant" induction or repression, relative to its mean level of expression, for sample $j$, Fig. 5.3. To estimate this significance, we make use of Chebyshev's inequality (Chebyshev, 1867), as no distributional form of expression values for each gene is assumed. Chebyshev's inequality, for any real number $\kappa>0$, can be written:

$$
\begin{equation*}
\operatorname{Pr}(|X-\mu| \geq \kappa \sigma) \leq \frac{1}{\kappa^{2}} \tag{5.1}
\end{equation*}
$$

with random variable $X, \mu$ the expected value of $X$ and $\sigma^{2}$ the variance.
Those expression values $X=x_{i j},(i=1 \ldots n, j=1 \ldots p)$, of interest, for a given sample $j$, are taken to be $\geq \kappa \sigma$ from the mean expression of gene vector i. From Eq. 5.1, for example, the associated probability of an expression value $\geq 4.47 \sigma$ from the mean of gene $i$ is less than 0.05 . Expression values $\geq 4.47 \sigma$ from the mean would indicate a strong response of gene $i$ to sample $j$.

Clearly, categories can be established to highlight those expression values which

Figure 5.4: Two-step process of empirical scheme. Step 1: A univariate analysis of each gene vector is carried out to determine strength of response. Step 2: A univariate analysis of each sample vector is performed, used to order gene response and assign weights (Section 5.3).
indicate a weak response, moderate response and strong response, where $\kappa$ indicates the threshold between categories, Fig. 5.4. For example, these categories could be defined by grouping expression values which are $\geq 2.58 \sigma, \geq 3.16 \sigma$ and $\geq 4.47 \sigma$ from $\mu$, into non-overlapping (mutually exclusive) categories of weak, moderate and strong respectively, corresponding to probabilities $=0.15,0.10$ and $0.05, \mathrm{Eq}$. 5.1. (The number of categories can clearly be extended for fine-grained response.) For the analysis described, the three categories weak, moderate and strong, as defined here were used as also in Tanay et al. (2002), to facilitate comparison. Threshold determination between categories is discussed further in the next subsection.

### 5.2.2 Threshold Estimation

To decide on thresholds between categories, i.e. the value of $\kappa$, graphs from real datasets, $G=(\top, \perp, E)$ are compared to graphs from random datasets ${ }^{3}$, $G_{\text {Rand }}=$

[^18]$\left(\top_{\text {Rand }}, \perp_{\text {Rand }}, E_{\text {Rand }}\right)$, for a range of possible thresholds, (Fig. 5.5). The null model assumes each edge in the graph was created with
$$
\text { probability }=\left(\left|E_{\text {Rand }}\right| / \text { Number of possible edges }\right)
$$
while the alternative model assumes an edge to be created with
$$
\text { probability }=(|E| / \text { Number of possible edges })
$$

The level at which the logarithm ratio of these two probabilities is maximised is taken to be 'optimal' in terms of any real effect observed.

Thresholds between categories are identified sequentially. Firstly, the strong response threshold, $\kappa_{s t r}$, is investigated and identified. One test criterion for maximum $\kappa_{s t r}$ for is that at least one edge must be identified in the real graph. Possible thresholds values are then tested in probability increments of 0.02 to determine percentage inclusion of expression values ${ }^{4}$. The threshold is then set at the level at which the $\log$ ratio is maximised. Once $\kappa_{s t r}$ is found, moderate and weak response thresholds ( $\kappa_{m o d}$ and $\kappa_{w k}$, respectively) are established, (in that order). The maximum value to test for $\kappa_{\text {mod }}$ is set to $\kappa_{s t r}$ and the maximum value to test for $\kappa_{w k}$ is set to $\kappa_{\text {mod }}$. In summary, the technique for identifying thresholds is:
for Induced and repressed categories do

- Identify threshold, $\kappa$, where at least one gene-sample couple identified.

Begin testing from this value.

- Test range of possible thresholds for $\kappa_{\text {str }}$, which correspond to probability

[^19]

Figure 5.5: Threshold analysis of selected input datasets Alizadeth, Alon, Hsiao and West. The x -axis is the probability threshold i.e. $\frac{1}{\kappa^{2}}$
increments of 0.02, Eq. 5.1

- Set $\kappa_{\text {str }}$ at the level which maximises the log ratio of probability of edge in real graph to random graph.
(Gene-sample couples corresponding to strong response have been identified and hence the strong response graph can be created. Remove those genesample couples from the dataset.)
- Starting from $\kappa_{s t r}$, test range of possible thresholds for $\kappa_{\text {mod }}$, which correspond to probability increments of 0.02 , Eq. 5.1.
- Set $\kappa_{\text {mod }}$ at the level which maximises the log ratio of probability of edge in real graph to random graph.
(The moderate response gene-sample couples have been identified and hence the moderate response graph can be created. Remove those edges, i.e. genesample couples, from the dataset).
- Starting from $\kappa_{\text {mod }}$, test thresholds for $\kappa_{w k}$, which correspond to probability increments of 0.02, Eq. 5.1.
- Set threshold for weak response at level where log ratio is maximised, $\kappa_{w k}$. (The weak response gene-sample couples have been identified and hence the weak response graph can be created. Remove those edges from the dataset). end for

Fig. 5.5 illustrates the results of a threshold analysis of four test datasets, where the probability of an edge for a range of values of $\kappa$ is plotted for each of the categories of response. Note that for each dataset, thresholds were identified sequentially, therefore the strong response threshold was identified and set before carrying out analysis for the moderate threshold, likewise for weak response. The x-axis in each of the plots indicats the corresponding probabilities for the $\kappa$ tested. Induced, repressed and random cases are shown in each plot. For the random case, there is
an expected gradual increase in the number of edges identified as $\kappa$ approaches 0 , mimicking the underlying normal distribution. The Alizadeth, West and, to some extent, Hsiao, are somewhat similar to the random profile, however differences exist on the vertical scale, indicating that there are more genes identified as repressed or induced across samples in the real datasets, compared to the random. In terms of comparative performance of real data versus random in the weak response category, the former, in general, led to identification of relatively more co-expressed genes, although the difference is less distinct: in some cases more edge were picked out by random selection.

The point at which the difference in the vertical scale is maximised is taken as the optimal threshold. In, for example, the Alizadeth dataset, Fig. 5.5a, this point occurs in the strong response category at probability $=0.06$ (for induced and repressed), corresponding to $\kappa_{s t r}=4.08$, Eq. 5.1. All gene expression values which are $\geq 4.08 \sigma$ from the mean are taken to be evidence of strong response and are then extracted as strong response from the dataset. All gene expression values which are $<4.08 \sigma$ from the mean are considered for inclusion in the moderate response category. Beginning at $\kappa=4.08$, possible values for $\kappa_{\text {mod }}$ are tested and the $\kappa_{\text {mod }}$ value at which the log ratio is maximised is identified - this occurs at probability $=0.10$ (for induced and repressed), corresponding to $\kappa_{\text {mod }}=3.162$. All gene expression values which, fall in the moderate category, i.e. are $\geq 3.162 \sigma$ (and $<4.08 \sigma$ ) from the mean are then extracted from the dataset. Beginning at $\kappa=3.162$ possible thresholds for weak response are tested, the threshold for the maximal $\log$ ratio is identified, and occurs at probability $=0.20$ (for induced and repressed), corresponding to $\kappa_{w k}=2.23$. All gene expression values in the weak response category: $\geq 2.23 \sigma$ (and $<3.162 \sigma$ ) s.d. from the mean are then extracted from the dataset.

For a number of datasets the thresholds are clear (e.g. Alizadeth and West datasets), however for others it is not so evident. For instance, the Alon dataset, (which represents data from a colon cancer study, see Appendix B), has anomalous behaviour, in that repressed genes are more evident, generating a large number of edges, while the induced genes closely follow random behaviour. Conversely, the Hsiao dataset, (which represents data from a compendium of normal tissue samples), evidently has more genes induced than expected while repressed genes are again close to random.

Table 5.1 shows $\kappa_{s t r}, \kappa_{\text {mod }}$ and $\kappa_{w k}$ representing the thresholds identified for each of the test datasets, i.e. for which the ratio of edges in real vs. random graphs is a maximum. For the majority of datasets thresholds are common, with $\mu \pm 3(4) \sigma$ a significant deviation the gene vector mean. The Cho and Stegmaier datasets are the smallest in terms of number of samples, (17 and 22 respectively, Appendix B), hence thresholds are less markedly difference from 0 . The anomalies of the Alon dataset are again reflected here in the lower threshold for the weak induced category, and, similarly, Hsiao dataset, with lower thresholds for strong response. These thresholds were applied for the respective datasets and six subgraphs were extracted for each test case, where an edge indicates a significant change in expression in that category.

### 5.2.3 Properties Of Gene Expression Graphs

Complex networks are usually analysed for a specific list of properties, whether bi-partite or one-mode, although basic analysis tools for bi-partite graphs have not previously been applied to bi-partite gene expression graphs.

In the following discussion, weak repressed, moderate repressed, strong re-

|  | Repressed |  |  | Induced |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Strong | Moderate | Weak | Strong | Moderate | Weak |
| Aliz | 4.08 | 3.16 | 2.23 | 4.08 | 3.16 | 2.23 |
| Alon | 4.08 | 3.16 | 2.23 | 4.08 | 3.16 | 1.82 |
| Cho | 3.16 | 2.88 | 2.23 | 3.16 | 2.88 | 2.23 |
| Gasch | 4.08 | 3.16 | 2.23 | 4.08 | 3.16 | 2.23 |
| Golub | 4.08 | 3.16 | 2.23 | 4.08 | 3.16 | 2.23 |
| Hsiao | 3.58 | 3.16 | 2.23 | 3.58 | 3.16 | 2.23 |
| Spell. | 4.08 | 3.16 | 2.23 | 4.08 | 3.16 | 2.23 |
| Steg. | 3.16 | 2.50 | 1.82 | 3.16 | 2.50 | 2.04 |
| West | 3.58 | 3.16 | 2.23 | 3.58 | 3.16 | 2.23 |

Table 5.1: $\kappa$, the number of s.d. units from the mean, that represent thresholds identified for each of the tested datasets.
pressed, weak induced, moderate induced and strong induced subgraphs, are the categorised subgraphs. A one-mode graph refers to the graph obtained by projection of the bi-partite graph. The term 'all-in-one' refers to a graph which is not split into categories, i.e. contains all nodes and edges from all subgraphs $G \in(T, \perp, E)$.

The main properties of the bi-partite gene expression graphs extracted from the test datasets are outlined in Table 5.2. Despite the size of the table, it is worthwhile considering it as a whole in order to identify common features across a wide range of data, as well as highlighting distinction.

Table 5.2: $n_{\top}=$ the active set of genes, $n_{\perp}=$ active set of samples, $m=$ number of edges, $d_{\perp}=$ average degree of sample nodes, $d_{\top}=$ average degree of gene nodes, $\delta$ $=$ bi-partite density i.e the fraction of existing links with respect to possible ones, cc $=$ clustering coefficient (Eq. 5.3), $c c_{\text {min }}=$ minimum clustering coefficient (Eq. 5.5)

| Graph Cat. | $n^{+}$ | $n_{\perp}$ | $m$ | $d_{\top}$ | $d_{\perp}$ | $\delta$ |  | $c c_{\text {min } T}$ | $c c_{\perp}$ | $c c_{\text {min } \perp}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Alizadeth |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 209 | 46 | 219 | 1.04 | 4.70 | 0.02 | 0.95 | 0.99 | 0.35 | 0.71 |
| Mod. Induced | 476 | 43 | 550 | 1.15 | 12.79 | 0.03 | 0.81 | 0.98 | 0.05 | 0.20 |
|  | Continued on Next Page... |  |  |  |  |  |  |  |  |  |


| Graph Cat. | $n_{\top}$ | $n_{\perp}$ | $m$ | $d_{\top}$ | $d_{\perp}$ | $\delta$ | ${ }_{c}{ }_{\text {T }}$ | $c c_{\text {min } T}$ | $c c_{\perp}$ | $c c_{\text {min }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wk. Induced | 2088 | 79 | 3713 | 1.77 | 47.00 | 0.02 | 0.44 | 0.81 | 0.02 | 0.09 |
| Str. Repressed | 63 | 16 | 64 | 1.01 | 4.00 | 0.06 | 0.97 | 1.00 | 0.06 | 0.33 |
| Mod. Repressed | 296 | 13 | 327 | 1.10 | 25.15 | 0.08 | 0.88 | 0.99 | 0.06 | 0.15 |
| Wk. Repressed | 1889 | 53 | 3227 | 1.70 | 60.89 | 0.03 | 0.48 | 0.85 | 0.02 | 0.12 |
| Alon |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 27 | 14 | 27 | 1 | 1.92 | 0.071 | 1 | 1 | 0 | 0 |
| Mod. Induced | 25 | 10 | 26 | 1.04 | 2.6 | 0.104 | 0.92 | 1 | 0.11 | 0.25 |
| Wk. Induced | 1167 | 28 | 1813 | 1.55 | 64.75 | 0.055 | 0.54 | 0.88 | 0.03 | 0.09 |
| Str. Repressed | 73 | 31 | 73 | 1 | 2.35 | 0.032 | 1 | 1 | 0 | 0 |
| Mod. Repressed | 221 | 29 | 226 | 1.02 | 7.79 | 0.035 | 0.96 | 0.99 | 0.04 | 0.11 |
| Wk. Repressed | 1338 | 56 | 2028 | 1.51 | 36.21 | 0.026 | 0.55 | 0.88 | 0.03 | 0.09 |
| Cho |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 351 | 9 | 351 | 1 | 39 | 0.111 | 1 | 1 | 0 | 0 |
| Mod. Induced | 227 | 9 | 227 | 1 | 25.22 | 0.111 | 1 | 1 | 0 | 0 |
| Wk. Induced | 805 | 12 | 2051 | 1.02 | 68.83 | 0.212 | 0.96 | 0.99 | 0.02 | 0.074 |
| Str. Repressed | 19 | 5 | 19 | 1 | 3.8 | 0.2 | 1 | 1 | 0 | 0 |
| Mod. Repressed | 21 | 2 | 21 | 1 | 10.5 | 0.5 | 1 | 1 | 0 | 0 |
| Wk. Repressed | 122 | 8 | 826 | 1.06 | 16.25 | 0.84 | 0.91 | 1 | 0.11 | 0.25 |
| Gasch |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 434 | 96 | 564 | 1.3 | 5.87 | 0.014 | 0.73 | 0.97 | 0.18 | 0.51 |
| Mod. Induced | 970 | 89 | 1644 | 1.69 | 18.47 | 0.019 | 0.55 | 0.89 | 0.08 | 0.30 |
| Wk. Induced | 2031 | 138 | 7017 | 3.45 | 50.84 | 0.025 | 0.25 | 0.59 | 0.04 | 0.19 |
| Str. Repressed | 645 | 93 | 848 | 1.31 | 9.12 | 0.014 | 0.74 | 0.97 | 0.22 | 0.51 |
| Mod. Repressed | 1258 | 85 | 2217 | 1.76 | 26.08 | 0.013 | 0.52 | 0.87 | 0.10 | 0.36 |
| Wk. Repressed | 2552 | 136 | 8448 | 3.31 | 62.11 | 0.024 | 0.27 | 0.61 | 0.03 | 0.16 |
| Golub |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 279 | 60 | 315 | 1.13 | 5.25 | 0.019 | 0.82 | 0.98 | 0.12 | 0.42 |
| Mod. Induced | 408 | 60 | 528 | 1.29 | 8.8 | 0.021 | 0.69 | 0.95 | 0.09 | 0.25 |
| Wk. Induced | 1641 | 70 | 2898 | 1.73 | 41.4 | 0.025 | 0.47 | 0.85 | 0.03 | 0.07 |
| Continued on Next Page... |  |  |  |  |  |  |  |  |  |  |


| Graph Cat. | $n_{\top}$ | $n_{\perp}$ | $m$ | $d_{\top}$ | $d_{\perp}$ | $\delta$ | ${ }_{c}{ }_{\text {T }}$ | $c c_{\text {min } T}$ | $c c_{\perp}$ | $c c_{\text {min } \perp}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Str. Repressed | 118 | 46 | 122 | 1.03 | 2.65 | 0.022 | 0.95 | 1 | 0.42 | 0.65 |
| Mod. Repressed | 312 | 45 | 348 | 1.12 | 7.73 | 0.025 | 0.84 | 0.99 | 0.09 | 0.30 |
| Wk. Repressed | 1411 | 68 | 2509 | 1.77 | 36.89 | 0.026 | 0.43 | 0.81 | 0.02 | 0.07 |
| Hsiao |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 576 | 57 | 758 | 1.32 | 13.3 | 0.023 | 0.72 | 0.96 | 0.13 | 0.35 |
| Mod. Induced | 399 | 57 | 555 | 1.39 | 9.7 | 0.024 | 0.68 | 0.95 | 0.15 | 0.36 |
| Wk. Induced | 1308 | 59 | 2826 | 2.16 | 47.89 | 0.036 | 0.41 | 0.81 | 0.06 | 0.13 |
| Str. Repressed | 13 | 9 | 13 | 1 | 1.44 | 0.111 | 1 | 1 | 0 | 0 |
| Mod. Repressed | 17 | 7 | 17 | 1 | 2.4 | 0.142 | 1 | 1 | 0 | 0 |
| Wk. Repressed | 22 | 17 | 296 | 1.34 | 17.41 | 0.079 | 0.65 | 0.94 | 0.05 | 0.16 |
| Spellman |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 207 | 36 | 218 | 1.05 | 6.05 | 0.029 | 0.93 | 0.99 | 0.07 | 0.45 |
| Mod. Induced | 597 | 35 | 657 | 1.1 | 18.77 | 0.031 | 0.86 | 0.99 | 0.03 | 0.19 |
| Wk. Induced | 2193 | 59 | 3661 | 1.67 | 62.05 | 0.028 | 0.49 | 0.84 | 0.02 | 0.11 |
| Str. Repressed | 316 | 49 | 323 | 1.02 | 6.59 | 0.02 | 0.96 | 0.99 | 0.06 | 0.17 |
| Mod. Repressed | 731 | 47 | 814 | 1.11 | 17.31 | 0.024 | 0.85 | 0.99 | 0.04 | 0.16 |
| Wk. Repressed | 2283 | 62 | 3934 | 1.72 | 63.45 | 0.028 | 0.45 | 0.82 | 0.02 | 0.09 |
| Stegmaier |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 9 | 4 | 9 | 1 | 2.25 | 0.25 | 1 | 1 | 0 | 0 |
| Mod. Induced | 60 | 4 | 60 | 1 | 15 | 0.25 | 1 | 1 | 0 | 0 |
| Wk. Induced | 1037 | 20 | 1249 | 1.2 | 62.45 | 0.06 | 0.88 | 0.99 | 0.04 | 0.12 |
| Str. Repressed | 19 | 9 | 19 | 1 | 2.21 | 0.111 | 1 | 1 | 0 | 0 |
| Mod. Repressed | 179 | 8 | 180 | 1 | 22.5 | 0.126 | 0.99 | 1 | 0.03 | 0.08 |
| Wk. Repressed | 705 | 19 | 832 | 1.18 | 43.79 | 0.062 | 0.69 | 0.96 | 0.03 | 0.08 |
| West |  |  |  |  |  |  |  |  |  |  |
| Str. Induced | 583 | 47 | 674 | 1.16 | 14.34 | 0.025 | 0.81 | 0.98 | 0.09 | 0.28 |
| Mod. Induced | 350 | 44 | 367 | 1.04 | 8.34 | 0.024 | 0.93 | 0.99 | 0.10 | 0.38 |
| Wk. Induced | 1489 | 48 | 2051 | 1.38 | 42.72 | 0.029 | 0.63 | 0.93 | 0.02 | 0.08 |
| Str. Repressed | 266 | 29 | 290 | 1.09 | 10 | 0.038 | 0.88 | 0.99 | 0.24 | 0.50 |
| Continued on Next Page... |  |  |  |  |  |  |  |  |  |  |


| Graph Cat. | $n_{\top}$ | $n_{\perp}$ | $m$ | $d_{\top}$ | $d_{\perp}$ | $\delta$ | $c c_{\top}$ | $c c_{m i n \top}$ | $c c_{\perp}$ | $c c_{\min \perp}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mod. Repressed | 195 | 24 | 239 | 1.22 | 9.95 | 0.051 | 0.74 | 0.97 | 0.09 | 0.28 |
| Wk. Repressed | 866 | 37 | 1426 | 1.64 | 38.54 | 0.045 | 0.49 | 0.85 | 0.03 | 0.12 |

## Descriptive Statistics:

Unsurprisingly, in all cases, the weak response categories have larger gene, $\left(n_{\top}\right)$, sample, $\left(n_{\perp}\right)$ and edge, $m$ sets, as thresholds are lower. Those datasets with a small number of samples tend to have a small $n_{\top}$ in the resulting graphs, e.g. Cho and Stegmaier datasets, which limits the type and amount of information that can be inferred from it. For a number of datasets there is a preference towards either induced or repressed categories, in terms of the number of edges identified, e.g. Alizadeth, Alon and Hsiao sub-graphs.

The average degree, $d_{\mathrm{T}}$, of the gene node set in the majority of datasets is $\sim 2$, increasing slightly for the weak response categories. This indicates that the majority of genes do not participate in more than two samples in each category (the Gasch dataset has the highest number of samples, with $d_{t o p}>2$ ). However, this still exceeds the expected degree (i.e. $n_{\text {top }} / m$ ). Again, for datasets with small number of samples, $d_{\top} \sim 1$. Unsurprisingly, the average degree of the sample node set, $d_{\perp}$ increases from Strong - Weak categories, meaning that there are more genes in a given sample responding in the weak category compared to the strong categories.

The density measure, $\delta$ evaluates overall how sparse the graph is, i.e. the number of edges compared to the number of possible edges, $m / n_{\top} n_{\perp}$. In most cases the gene expression graphs with $\delta<0.06$. This value increases in certain cases for $n_{\top}$ and $n_{\perp}$ small, indicating that most of the sample and gene nodes in the graph are connected, (e.g. Cho, Stegmaier and Alon repressed sub-categories). This measure
is more reflective of the number of gene and sample nodes in the graph and doesn't reveal much organisational information.

## Clustering:

The inherent tendency of real networks to form cliques (or clusters) is quantified by the clustering coefficient, cc. Traditionally, this is defined, for each node in a one-mode graph, $u \in N$, with at least two neighbours (i.e. degree, $d(u) \geq 2$ ), as the proportion of edges between its neighbours, (Guillaume and Latapy, 2004):

$$
\begin{equation*}
c(u)=\frac{|\{x, y\}, x, y \in N(u)|}{\binom{d(u)}{2}} \tag{5.2}
\end{equation*}
$$

where $N(u)$ is the set of neighbours of $u$. The clustering coefficient of a graph is the average over all nodes $u \in N$. Equivalently, in the case of a bi-partite graph, the clustering co-efficient, for two nodes $u$ and $v$, for either the gene or sample node set is:

$$
\begin{equation*}
c(u, v)=\frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|} \tag{5.3}
\end{equation*}
$$

This captures the probability that two nodes in the same node set (i.e. gene or sample) have a neighbour in common. The clustering for one node between all its neighbours is then:

$$
\begin{equation*}
c(u)=\frac{\sum_{v \cup N(N(u))} c(u, v)}{|N(N(u))|} \tag{5.4}
\end{equation*}
$$

where $|N(N(u))|$ is the number of second degree neighbours (i.e. neighbourhood of neighbour nodes). For the bi-partite case, a clustering coefficient can be obtained for the two sets of nodes separately, $(T, \perp)$, by taking the average of Eq. 5.4 over all nodes in each set, resulting in $c c_{\top}$ and $c c_{\perp}$. The definition pre-
sented in Eq. 5.3 for two nodes, has the disadvantage that the node with the largest neighbourhood will dominate the results, even if the smaller neighbourhood node is completely encapsulated in the larger. An alternative, introduced by (Latapy et al., 2006), is to scale by the minimum neighbourhood:

$$
\begin{equation*}
c(u, v)_{\min }=\frac{|N(u) \cap N(v)|}{\min (|N(u)|,|N(v)|)} \tag{5.5}
\end{equation*}
$$

This defines a more precise relationship between two nodes, and is more appropriate for gene expression data, given the fact that genes may participate in multiple samples and need not be co-active under all samples, with some samples having participation from more genes than others. Again, a summary value can be calculated for each node set separately by taking the average of Eq. 5.5 for each node, $c c_{T_{\text {min }}}$ and $c c_{\perp \text { min }}$.

In each of the subgraphs, Table 5.2, the clustering coefficient for gene nodes, $c c_{\top}$, is quite large, indicating that if two gene nodes participate in the same sample node, then a large proportion of their neighbourhood will be similar. The clustering coefficient for the sample nodes, $c c_{\perp}$, captures the overlap between gene subsets participating in the samples and is, unsurprisingly, smaller. Each node in this set has a much higher average degree (i.e. each sample node is typically connected to a large number of genes), whereas the average degree for gene nodes is $\sim 2$. Thus if two genes show similar expression in two or more samples, this reflects the 'entire neighbourhood’ of each gene node. In a random Erdös-Rényi Model graph, (Section 3.3.4), edges are distributed randomly between nodes with a probability $p$, so the clustering coefficient is $c c=p$, (probability that two nodes are connected). In real networks the clustering coefficient is typically much larger than it is for a corresponding random network, (i.e. one having the same number of nodes and
edges as the real network). Although the subgraphs are individually quite sparse (bi-partite density $\delta<0.1$ ), the relatively high $c c_{\top}$ and $c c_{\perp}$ indicate that there are denser local structures (more evident, as noted, for the gene node set), i.e. $c c>p$. The exceptions are the Cho and Stegmaier subgraphs and strong/moderate repression Hsiao subgraphs. These subgraphs also have the smallest $n_{\top}, n_{\perp}$ and $m$, and are quite dense, and $c c_{\perp}$ indicates that there is no overlap between gene subsets participating in the samples.

The minimum clustering coefficients, $c c_{\top_{\text {min }}}$ and $c c_{\perp \text { min }}$, capture the local interactions of two genes or two samples. This measure, for the sample nodes, $c c_{\perp \text { min }}$, is significantly larger than for $c c_{\perp}$ (at least double in most cases). This captures the fact that the $c c_{\perp}$ measure is dominated by sample nodes of high degree, while neighbourhoods of lower nodes of smaller degree do in fact overlap with other sample nodes.

## Degree Distribution:

To further understand the basis for gene expression networks, we examine the distribution of the node degrees. The spread in the node degrees is characterized by a probability distribution function, $P(d)$, which gives the probability that a randomly selected node, $u$, has a degree of exactly $d(u)$. For bi-partite graphs, there are two degree distributions, one for each node set, $\{T\},\{\perp\}$. Given there are substantially fewer samples than genes in the datasets, tail distribution statistics are unlikely for this case. For a large number of examples of real networks, the node degree distribution follows a power-law $P(d) \sim d^{-\gamma}$ and these networks are said to be scale-free. In an Erdös-Rényi model and Watts-Strogatz graph the edges are placed randomly, and degree distribution of the nodes is close to a Poisson distribution $p(d)=\lambda^{d} e^{-\lambda} / d$ ! (special case of Watts-Strogatz model), (Section 3.3.4)

Each subgraph may of course have alternative forms. Figures C. 18 - C. 21 (Ap-
pendix C) are the distribution plots found when the sub-graphs of strong - weak induction and strong - weak repression are extracted, for both sample and gene sets. It must be noted that clear identification of node degree distribution is difficult when the node set size in the graph is small. For subgraphs with larger node sets, (Aliz, Alon, Gasch, Golub, Hsiao, West), the node degrees are skewed to the right and are more accurately modelled by a Poisson distribution, most evidently in weak response categories, where $n_{\top}$ is largest. Overall, $n_{\perp}$ is small for a large number of the subgraphs: however, where large, (e.g. Figures C.20a, C.20b, C.21a,C.21c weak response categories) it again follows a Poisson distribution.

## All In One Graph

To examine whether there is a coherent response of genes across all categories, an all-in-one graph can be constructed for each dataset. For example, a gene with expression across samples in the moderate induced sub-graph, may be coherent with one expressed in the moderate repressed graph. This is captured in the all-inone graph which is constructed from the union of all sub-graphs, Table 5.3. The average degree of the gene nodes increases in the all-in-one graph indicating that genes identified are participating across categories. Likewise the average degree of the sample nodes increases, indicating that samples have overlapping gene sets across categories. The size of the edge set is simply the sum of the cardinality of each edge set from each sub-category.

## Clustering Coefficient:

Once again, we use the clustering coefficient information to examine the degree of sharing among node neighbourhoods. In the all-in-one graph the clustering coefficient of the sample nodes, $c c_{\perp}$, is $\sim 0.02$ for all dataset graphs (with the exception

| Dataset | $n_{\top}$ | $n_{\perp}$ | $m$ | $d_{\top}$ | $d_{\perp}$ | $\delta$ | $c c_{\top}$ | $c c_{\min T}$ | $c c_{\perp}$ | $c c_{m i n \perp}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Aliz. | 2870 | 84 | 8100 | 2.82 | 96.42 | 0.03 | 0.25 | 0.56 | 0.02 | 0.09 |
| Alon | 1857 | 57 | 4193 | 2.25 | 73.56 | 0.04 | 0.32 | 0.68 | 0.02 | 0.09 |
| Cho | 1537 | 12 | 3495 | 1.02 | 131.17 | 0.19 | 0.96 | 0.99 | 0.01 | 0.05 |
| Gasch | 3114 | 151 | 20738 | 6.66 | 137.33 | 0.05 | 0.18 | 0.35 | 0.02 | 0.13 |
| Golub | 2933 | 71 | 6720 | 2.29 | 94.65 | 0.03 | 0.31 | 0.67 | 0.02 | 0.05 |
| Hsiao | 1749 | 59 | 4465 | 2.55 | 75.68 | 0.04 | 0.39 | 0.78 | 0.06 | 0.13 |
| Spell. | 3024 | 71 | 9607 | 3.18 | 135.31 | 0.04 | 0.22 | 0.48 | 0.02 | 0.12 |
| Steg. | 1776 | 22 | 2349 | 1.32 | 106.77 | 0.06 | 0.67 | 0.94 | 0.02 | 0.07 |
| West | 2748 | 48 | 5047 | 1.84 | 105.15 | 0.03 | 0.43 | 0.79 | 0.02 | 0.07 |

Table 5.3: Statistics of All in One Graph. $n_{\top}=$ the active set of genes i.e. $d>0$, $n_{\perp}=$ active set of samples, $m=$ number of edges, $d_{\perp}=$ average degree of sample nodes, $d_{\top}=$ average degree of gene nodes, $\delta=$ bi-partite density i.e the fraction of existing links with respect to possible ones, $\mathrm{cc}=$ clustering coefficient (Eq. 5.3), $c c_{\text {min }}=$ minimum clustering coefficient (Eq. 5.5)
of the Hsiao graph), regardless of sample size - lower than the overall graph densities. This supports the idea that the clustering coefficient is independent of the size of the graph for most real world networks, (Guillaume and Latapy, 2004). However, the minimum clustering coefficients, $c c_{T_{\text {min }}}$ and $c c_{\perp \text { min }}$, are relatively high. $c c_{\perp \text { min }}$ trebles in most cases and is higher than the overall graph densities. This indicates again that the sample nodes of high degree are dominating the $c c_{\perp}$ measure, suggesting that there is a level of organization in the graph, such that if two nodes are linked, the neighbourhood of the smaller node will intersect the neighbourhood of the larger.

## Intersection Ratios:

Although the clustering coefficient information reveals the degree of sharing among node sets, information on the exact size of the intersection is lost. This level of organisation can be assessed through an examination of the size of the intersection neighbourhoods of the datasets. If two gene nodes have a sample neighbour in common, the size of this intersection neighbourhood will be significantly greater
than for a random graph ${ }^{5}$ of the same size and degree distribution, see Fig. 5.6.


Figure 5.6: Red = random, Black = real. Cumulative distribution of degree of intersection of neighbourhoods of gene nodes, i.e. if two gene nodes have a sample neighbourhood in common, there is a significantly greater chance that this neighbourhood will be larger than expected by chance. The plot shows, for each value $i$ on the $x$ axis, the ratio of all intersections greater or equal to $i$.

## Degree Distribution:

In terms of gene node degree distribution of the all-in-one gene expression graphs,
(Fig.C.22-C.23, Appendix C), these are in general right skewed: this is less evident in the Cho and Iressa datasets, which have the smallest number of sample nodes. Right-skewness is largely due to the higher cardinality of the gene sets in each dataset. For the gene node sets, the distributions approach the Normal - in those graphs where the average degree is high - and hence not particularly heterogeneous. The sample node distributions, in most cases, follow the Poisson. In Fig. 5.7 a plot of gene node degree distribution for four test datasets is given. In most cases the

[^20]gene node distribution is close to the Poisson in shape (solid line in plots), although tail effects are less extreme, indicating that approximation by the Normal is likely to be reasonable. The Gasch dataset, for example, is approximated well by a Normal, Table 5.3, and has the largest set of sample nodes: Note, Alizadeth $n_{\perp}=84$, Gasch $n_{\perp}=151$, Hsiao $n_{\perp}=59$, Spellman $n_{\perp}=71$. While this suggests that as the cardinality of the sample node set increases, the gene node degree distribution tends to Normal, the limited sample and gene set for most datasets means that we cannot conclude this is the case in general. Many other examples of real world one-mode networks approximate a power law. However, in support of our findings here it should be noted that a poor fit to a power-law distribution was also observed in other bi-partite models of real world complex networks, (Latapy et al., 2006; Guillaume and Latapy, 2004). Although, projection of the gene nodes into a one-mode graph is shown to follow a power-law in general (see Chapter 6), this conclusion cannot be drawn for the bi-partite case. However, we can conclude that the node degrees in a single graph can have alternative distributions.

Correlations between the bi-partite gene node degree and the degree in a onemode projection of the gene nodes, (Chapter 6), captures the notion of overlap, Fig. 5.8. The degree of a node in the one-mode projection is the sum of the degrees of the sample nodes to which it is connected in the bi-partite graph, minus the number of nodes in common in the neighbourhood of these nodes. That is to say, if a node, $u$ has a high degree in the bi-partite graph and a lower degree in the one-mode projection, there is an overlap between $u$ and its neighbour nodes, i.e. they are over/under expressed in the same samples, Fig. 5.8. Correlations between node degree in the bi-partite and one-mode graphs suggest that while some genes show little or no common sample activity, others are co-expressed across their entire sample neighbourhoods.


Figure 5.7: Distribution of gene node degrees for four test datasets. The solid line indicates a Poisson distribution. The distributions depart from the Poisson, despite shape similarity in part, with a higher \% distribution in the bulk and less extreme tail effect.

### 5.3 Edge Weights

The weighting scheme for a gene expression graph plays an important role in cluster determination, and thus merits independent investigation. To this end, we introduce the second step of our graph theoretic approach - a univariate analysis of expression values, within each sample vector, used to empirically weight the edges. We also discuss performance measures for edge-weighting schemes, and present a comparative evaluation based on application to several real datasets.


Figure 5.8: Correlation between node degrees in the bi-partite and one-mode projection for three test datasets. There is a large spread in the data indicating that although for some gene nodes there is overlap, for others there is none.

### 5.3.1 Definition of Assessment Properties

For evaluation of any clustering technique and for its reusability, it is important that weighting schemes are validated independently of the subsequent network analysis and good quality results reflect a well-designed weighting scheme, together with a reasonably robust and efficient search algorithm. Both aspects are usually susceptible to considerable refinement. Consequently, we propose an edge weighting assessment procedure, based upon four properties, as detailed below.

## Discrimination:

Ability of the method to "rate" highly those gene-sample couples which contribute to a cluster. The range and distribution of edge weights establish how well a given scheme distinguishes between relevant and irrelevant gene-sample couples.

## Reusability:

Independence of the proposed scheme and the subsequent clustering technique. This deals with how/if the weighting scheme must change to reflect additional layers of analysis.

## Robustness:

Ability of a given weighting scheme to deal with noise and missing values. This involves investigating the distortion of edge weights caused by different levels of noise and missing values.

Noise and missing values were added to the dataset to replicate measurement error of differing amounts. Noise was randomly "added/subtracted" to each value in the dataset as a percentage (up to $10 \%$ ) of the original value. To replace data with missing values, up to $10 \%$ of expression values from the original dataset were randomly selected and removed. Commonly, in cluster analysis, missing values in the gene expression matrix are replaced by zeroes or by an average expression level of the gene, ("row average"). More sophisticated options include methods of KNearest Neighbour (KNN) and Support Vector Decomposition type, (Troyanskaya et al., 2001). To test our weighting schemes the common practice of replacing missing values by the row mean was adopted. Replacing missing values with the gene vector mean, in this scheme, equate the expression as unresponsive.

For this analysis we define "Average Absolute Variation" as the average difference in edge weights compared to $0 \%$ noise/missing values, while "Stable weights" are defined to be those weights for which the variation is less than the \% level of noise/missing values added.

## Parameter Influence:

The weighting scheme ideally should require minimal specification of input parameters. This includes consideration of input parameter influence on discrimination and robustness, as well as on the distribution of weights themselves.

### 5.3.2 Edge Weights in Bipartite Subgraphs

For each sample vector, $j$, expression values $x_{i j}$ which indicate strong response of the $i^{\text {th }}$ gene under $j$, (as determined in step one, Section 5.2.1) are selected. Similarly, genes which show moderate and weak response under $j$ can be identified. For each of the three "strength of response" categories, a gene may be repressed or induced relative to the mean expression value; $(X-\mu<0$, or $X-\mu>0$ respectively), giving six sub-categories in total. For each sub-category, $C a t_{s}, s=$ $1 \ldots 6$ and for each sample variable, $j=1 \ldots p$, the empirical probability of $x_{i j} \in$ $C a t_{s}$ is calculated as:

$$
\begin{equation*}
\left|x_{i j} \geq x_{v j}\right| /\left|C a t_{s}\right|, x_{v j} \in C a t_{s}, i \neq v \tag{5.6}
\end{equation*}
$$

(probability $=1$ if $\left|C a t_{s}\right|=1$ ) and hence the edge weight in the bi-partite gene expression graph is obtained.

The weight is thus, directly related to obtaining a given expression level in a specific response category for a particular sample. So, if many genes react strongly in the sample, the weight is smaller, while if, more interestingly, only a few react strongly, the weight will be larger. Note that, with this weighting scheme, a given sample (experiment) may also have no reacting genes.

The graph is broken down into an independent subgraph for each sub-category.

Gene-sample edges in these different response groups may have similar weight "values" but are distinguished, in terms of absolute levels of expression ${ }^{6}$, by the category into which they fall, so that strength of response is important overall. Within a category however, the weight can be interpreted directly in terms of the relative probability of a gene-sample response. Thus the higher the weight, the more confidence that a relationship exists between gene and sample in that category.

### 5.3.3 Edge Weights in All In One graph

Obviously, transformation of the data into an all-in-one graph, requires scaling of edge weights to reflect the additional information in the graph.; Due to the design of the weighting scheme, an edge within the weak response category may have a larger weight (e.g. $e_{i j}=1$ ), than an edge within the moderate response category (e.g. $e_{i j}=0.2$ ), Fig. 5.9. These weights are rescaled in the all-in-one graph, to reflect the significance of an edge for a particular sample overall.

Each edge weight is rescaled to reflect the additional information in the graph. This is achieved simply by, (Fig. 5.10):

## for all Sample nodes, $j$ do

for all Induction or Repression Category, Cat $_{s}$ do
if $j$ is in $C a t_{s}$ then
Multiply each edge weight incident to $j$ by the degree of the sample node in the sub-graph

Add the degree of $j$ in each of the lower sub-graphs, e.g. for strong sub-graphs weights, add the degree of $j$ in the moderate and weak sub-

[^21]wk Repres

mod Repres

str Repres


Figure 5.9: Edge Weight Distribution of each Spellman subgraph for Induced genes graphs.

Likewise, for the moderate sub-graphs, add the degree of $j$ in the weak subgraph
end if
end for
Divide by the degree of $j$ in the all-in-one graph
end for
These edge weights still reflect the probability of getting that level of expression relative to the other genes being expressed in that sample.

### 5.4 Scheme evaluation

In this Section, we use the 4-point framework introduced above to analyse our novel weighting scheme, and to compare this with the scheme introduced in Tanay et al. (2002), (described in Section 3.3.4).


Figure 5.10: Rescaling edge weights from either Induced or Repressed Categories to weights in all-in-one graph, to reflect additional information.

### 5.4.1 Reusability

The empirical weighting scheme proposed here results in a partially-connected graph for each sub-category, since genes which do not show a significant change for a given sample do not generate an edge. Subsequent clustering techniques need to allow for this non-edge set, by optimisation of the objective function excluding nodes not connected by an edge. This scheme requires a dedicated algorithm for subsequent biclustering, which maintains the lists of gene-sample couples in each category, (i.e. strongly induced, moderately induced, etc.).

The Tanay scheme is independent of the subsequent clustering technique, as it results in positive edges for "interesting" gene-sample couples and negative edges for "non-interesting" gene-sample couples. Indeed, the scheme was specifically designed for an additive scoring system, where the sum of the edge weights in a subgraph corresponds to its statistical significance, ((Tanay, 2005) for more details). This scheme has also been applied to a compendium of information, and not just to gene expression data, (Tanay et al., 2005).

### 5.4.2 Parameter influence

The weighting scheme introduced here is controlled by a single parameter and is, therefore, easily configurable while offering some flexibility. This parameter is $\kappa$, (Eq. 5.1), which determines thresholds between categories. Table 5.4 illustrates the results of the threshold analysis for the Alizadeth dataset. The maximum threshold for which any gene-sample couple was identified in the real dataset was $\kappa=7.07$ (probability $\leq 0.02$, (Eq. 5.1)). Thresholds of $\kappa=5,4.08,3.58$ and 3.162, (i.e. probabilities $\left(\frac{1}{\kappa^{2}}\right) \leq 0.04,0.06,0.08$ and 0.10 respectively) were then tested. The

| Strong |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa$ | 7.07 | 5 | 4.08 | 3.58 | 3.16 |
| P | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e)}\right)$ | undef | undef | 1.69 | 1.02 | 0.62 |
| Moderate |  |  |  |  |  |
| $\kappa$ | 3.162 | 2.88 | 2.67 | 2.5 | 2.35 |
| P | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e)}\right)$ | 1.94 | 1.76 | 1.62 | 1.55 | 1.45 |
| Weak |  |  |  |  |  |
| $\kappa$ | 2.23 | 2.13 | 2.04 | 1.96 | 1.88 |
| P | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e)}\right)$ | 0.11 | 0.08 | 0.04 | 0.04 | 0 |

Table 5.4: Threshold Analysis, Alizadeth data. $\kappa=$ the number of standard deviations from mean (Eq. 5.1). $\mathrm{P}=\frac{1}{\kappa^{2}}$, is the probability that values are $\kappa \sigma$ from mean. The maximum log-ratio is taken as the threshold between categories.
log ratio of the probabilities is maximised at $\kappa=4.08$, and this was taken to be the strong response threshold. Thresholds for moderate and weak response were then similarly deduced to be $\kappa=3.162$ and $\kappa=2.23$, respectively. Table 5.1 provides results of the threshold analysis for three test datasets.

The main parameters for the Tanay weighting scheme are, similarly, thresholds between categories, and $P_{c}$, (the constant probability that an edge appears in a bicluster, Eq. 3.10). Thresholds between categories are arbitrarily chosen, based on normalized ranked values within each sample and are, therefore, not directly data dependent. This 'hard thresholding' has consequences for the deterioration of the scheme when noise and missing values are added to the data. As the threshold parameter is lowered, a higher percentage of edges will be identified, even if none exist. (For a more detailed discussion of parameter $P_{c}$ see Tanay et al. (2002)).

### 5.4.3 Robustness

The influence of noise and missing values are summarised respectively in Table 5.5 for the Alizadeth dataset. Results for other datasets are not displayed here but are broadly consistent with those presented.

The absolute variation in weights is extremely low for the empirical scheme since the technique examines extreme values, i.e. values which appear in the tail of the distributions of each gene variable. In addition, weights are not based directly on a given expression value, but on that expression value relative to other values in the category for a particular sample (Step 2 of scheme, see Fig. 5.4). The category is also defined relative to expected value of the gene variable, (Step 1 of scheme, see Fig. 5.4). As "missing" values are replaced by the row mean, this does not greatly affect extreme values. Equally, even noise added at $10 \%$ level of the original values does not affect relative values, so that, perturbations in the data have small effect on weights assigned.

Similar to results shown in Table 5.5, for the Stegmaier dataset, average absolute variation in edge weights is $\sim 0.26 \%$ for an added noise level of $10 \%$ (not shown), while denoting $10 \%$ of the dataset as 'missing', gives average absolute variation in values $\sim 0.3 \%$, while $99.5 \%$ of weights are stable. For the Cho dataset, the corresponding values for $10 \%$ noise added were: $\sim 0.22 \%$ (absolute variation) and $\sim 99.65 \%$ (stable weights); and for missing values at $10 \%$ was: $\sim 0.15 \%$ (absolute variation) and $\sim 99.69 \%$ (stable weights).

Using the Tanay weighting scheme, perturbations in the data have very little effect on weights derived: (similar results for all tested datasets). We were surprised by this result and tested missing values up to a level of $80 \%$ however the effect was still minimal ( $0.01 \%$, average variations and $99.99 \%$ stable weights). It may be

| \% Noise level | 1.5 | 2.5 | 5 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| Empirical Based |  |  |  |  |
| \% Average Absolute variation | $6 \times 10^{-2}$ | $2 \times 10^{-2}$ | $3 \times 10^{-2}$ | $5 \times 10^{-2}$ |
| \% "stable" weights | 99.66 | 99.68 | 99.68 | 99.65 |
| Tanay Scheme |  |  |  |  |
| \% Average Absolute variation | $2.7 \times 10^{-3}$ | $2.6 \times 10^{-3}$ | $2.3 \times 10^{-3}$ | $3.2 \times 10^{-3}$ |
| \% "stable" weights | 99.98 | 99.98 | 99.99 | 99.99 |
| \%missing values | 1.5 | 2.5 | 5 | 10 |
| Empirical Scheme |  |  |  |  |
| \% Average absolute variation | 0.04 | 0.04 | 0.09 | 0.16 |
| \% of "stable" weights | 99.70 | 99.72 | 99.64 | 99.62 |
| Tanay Scheme |  |  |  |  |
| \% Average absolute variation | 0.0023 | 0.0027 | 0.0028 | 0.0069 |
| \% of "stable" weights | 99.98 | 99.98 | 99.99 | 99.99 |

Table 5.5: Influence of noise level and missing values on weights assigned
the case that this scheme identifies 'interesting' gene-sample couples, even if none exist, due to the 'hard' threshold nature of the scheme and its reliance on a ranking system. If $10 \%$ noise is added to the dataset, thresholds still depend on ranking and not a calculated mean level, thus approximately the same gene-sample couples are selected as interesting as the ranked position is not changed. Missing values also have little effect, as these are replaced by the mean, and the same thresholds used so that decisions are more conservative if anything.

### 5.4.4 Discrimination

For this analysis, a 'random graph' refers to a graph created from a random dataset, as described in Section 4.5.2.

From the threshold analysis for the Empirical scheme, described above, maximum discrimination between empirical and random graphs is obtained. As expected, the largest number of gene-sample couples falls in the weak response cat-
egory. Discriminating between gene responses clearly depends on the category thresholds used and, while threshold derivation described is based on statistical considerations, this can obviously be augmented by biological information. From Table 5.6, for strong and moderate response, the probability of an edge existing between a gene and sample node in the real graph is greater than that for the random graph, indicating that significant structure is present. For a weak response, the ratio of probabilities is smaller and it is less convincing that real differences exist. Nevertheless, an examination of the average degree of sample nodes in the real graphs indicates that the average number of genes responding is higher than expected. For example, for the weak repression sub-category, a sample node is, on average, connected to $\sim 3.2 \%$ of gene nodes compared to $\sim 1.2 \%$ in the random graph, $\left(d_{\perp} / n_{\top}\right)$. The average degree of a sample node in the real graph is much higher than expected, ( $>m / n_{\perp}$ with degree $\geq 0=96$ ). This suggests that, although the ratio of edge probabilities in the weak response category compared to the random graph is not high, some pattern structure is present and the method is capable of identifying 'indicative' gene-sample couples, even in this less-reactive category.

From our analysis of the Tanay et al. scheme (Table 5.7), we observed that (a) a smaller number of total positive weights were identified compared to those identified from corresponding graphs generated from a random dataset (Section 5.2.2), (b) the number of positive weights in each category is roughly equivalent to that for the random case(the exceptions are Stegmaier Moderate and Strongly repressed categories). Observations (a) and (b) imply that the amount of edge 'sharing' of gene-sample couples in the real dataset is considerable, (i.e. gene-sample couples having a positive edge in the weakly induced category, also feature in the strongly induced category) - thus categories are not mutually exclusive. Since 'hard' thresh-

|  | Repres |  |  | Induc |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wk | Mod. | Str. | Wk. | Mod. | Str. |
| $n_{\top}$ | 1889 | 296 | 63 | 2088 | 476 | 209 |
|  | $(2004)$ | $(6)$ | $(5)$ | $(2030)$ | $(4)$ | $(2)$ |
| $n_{\perp}$ | 53 | 13 | 16 | 79 | 43 | 46 |
|  | $(82)$ | $(2)$ | $(2)$ | $(85)$ | $(2)$ | $(2)$ |
| $m$ | 3227 | 327 | 64 | 3713 | 550 | 219 |
|  | $(2754.04)$ | $(6.01)$ | $(2.92)$ | $(2779.27)$ | $(5.57)$ | $(2.89)$ |
| $d_{\top}$ | 1.7 | 1.1 | 1.01 | 1.77 | 1.15 | 1.04 |
|  | $(1.4)$ | $(1)$ | $(1)$ | $(0.98)$ | $(0.79)$ | $\left(6.6 e^{-4}\right)$ |
| $d_{\perp}$ | 60.89 | 25.15 | 4 | 47 | 12.79 | 4.7 |
|  | $(35.23)$ | $(1.02)$ | $(1)$ | $(34.95)$ | $(2)$ | $(1)$ |
| $\delta$ | 0.01 | 0.001 | $2.2 e^{-4}$ | 0.012 | 0.002 | $7.5 e^{-4}$ |
|  | $(0.003)$ | $\left(1.7 e^{-5}\right)$ | $\left(1 e^{-5}\right)$ | $(0.009)$ | $\left(2 e^{-5}\right)$ | $\left(1 e^{-5}\right)$ |

Table 5.6: Categories for Alizadeth data, created by cut off thresholds 0.20 (weak induction/repression), 0.10 (moderate induction/repression), and 0.06 (strong induction/repression). $n_{\top}=$ the active set of genes (gene nodes with degree $\geq 1$ ), $n_{\perp}$ = active set of samples, $m=$ number of edges, $d_{\top}=$ average degree of active set of genes, $d_{\perp}=$ average degree of active set of samples, $\delta=$ bi-partite density i.e the fraction of existing links with respect to possible ones (i.e. gene nodes with degree $\geq 0 \times$ sample nodes with degree $\geq 0$ ). Numbers in brackets indicate corresponding values for random graphs.

| Datasets | Aliz. | Cho. | Steg. |
| :---: | :---: | :---: | :---: |
| \% total edges | 2.7 | 3.34 | 2.13 |
|  | $(2.9)$ | $(4.3)$ | $(3.6)$ |
| Induced |  |  |  |
| \% strong | 1.2 | 1.1 | 0.8 |
|  | $(1.1)$ | $(1.2)$ | $(0.8)$ |
| \% moderate | 3.2 | 4.1 | 1.0 |
|  | $(3.1)$ | $(3.9)$ | $(1.0)$ |
| \% weak | 3.9 | 5.0 | 1.1 |
|  | $(3.9)$ | $(4.8)$ | $(1.1)$ |
| Repressed |  |  |  |
| \% strong | 1.2 | 1.1 | 1.3 |
|  | $(1.1)$ | $(1.2)$ | $(1.4)$ |
| \% moderate | 3.2 | 3.8 | 3.8 |
|  | $(3.1)$ | $(3.9)$ | $(3.4)$ |
| \% weak | 4.0 | 4.7 | 4.7 |
|  | $(3.8)$ | $(4.8)$ | $(4.1)$ |

Table 5.7: Tanay scheme - percentage of total possible edges is taken as Number of Positive Edges $\backslash n \times p \times 6$ categories whereas percentage of edges in each category is expressed in terms of the total possible edges in that category i.e. Number of positive Edges $\backslash n \times p$, $(n=$ number of genes, $p=$ number of samples). Bracketed values represent results from random graphs.
olds between categories were used and arbitrarily chosen, the order of the number of edges in each category is Strong $<$ Moderate $<$ Weak. Note also that those gene-sample couples, evaluated as strongly reacting, will have a magnified impact on any clustering procedure for the resulting graph, due to the overlap between categories, (i.e. weak influences are a subset of strong influences).

### 5.5 Summary

We have presented an empirical-based method for the extraction of a bi-partite graph from a gene expression dataset. This method is important because it builds the gene expression graph in a data dependent manner. Analysing gene expression
datasets is important as it allows for fine grained analysis of the reactivity of genes at various response levels. The scheme for constructing graphs presented here results in independent non-overlapping sub-graphs, each representing a strength/type of response category. These can then be used to construct, what we have called an "all-in-one" graph, where the interactions and reinforcement of response groupings from combined categories can be analysed. This can be used to determine subtle coherence in patterns of co-expression. Using tools and notation for classic network analysis for extraction, identification and analysis, we have uncovered organisational structure in graphs, constructed with this scheme. To our knowledge, this is the first time basic network analysis techniques for bi-partite graphs have been applied to the analysis of gene expression. Clustering coefficients, $c c$, were obtained for gene and sample node sets individually with that for gene nodes found, unsurprisingly, to be much larger than that for sample nodes, due to the large disparity in average degree between sets. In the latter, the $c c$ were dominated by sample nodes of high degree. Consideration of the minimum clustering coefficient measure removes large neighbourhood bias, and reveals there more subtle, local interactions in the data. We examined the size of the neighbourhood intersections for genes, and found that, genes react more coherently in larger neighbourhoods than expected by chance. An examination of the gene node degree distribution of the extracted graphs suggests that has some Poisson characteristics, but suggests that for large gene sets is well-approximated a Normal distribution. However, the size of the sample node set for most datasets is relatively low, so that degree distribution is less easy to establish and appears to depend on the observed data itself.

The issue of data-dependent threshold estimation is addressed in the empiricalscheme presented, but is non-trivial, as numerous thresholds need to be assessed, which is computationally expensive. The scheme presented here is more specific in
that fewer gene-sample couples are identified than in the Tanay et al. scheme. For example, for the Alizadeth dataset, our scheme extracts $\sim 3 \%$ edges ( at optimal threshold levels). The Tanay scheme extracts $\sim 16 \%$ for the same dataset. In real terms this means that the denser graphs created with the Tanay scheme, could contain more irrelevant gene-sample couples, while our scheme can more precisely target the gene set of interest.

We have presented a novel edge-weighting scheme for gene expression bipartite graphs. From the investigations, presented in this chapter, it is clear that interpretations of edge weights in graphical gene expression schemes can be difficult and a comparative analysis with the well-known Tanay et al. (2002) scheme is also presented. This analysis was carried out w.r.t four major properties: reusability, parameter influence, robustness and discrimination. Both schemes result in positive weights for interesting gene-sample couples. Our new weighting scheme, for a particular sample $j$, determines affected genes relative to other gene expression values for that sample $j$. This is an important feature, as absolute level of gene expression is not directly accounted for, but rather the fact that change occurs, together with the significance of this change relative to the majority of genes. Relative evaluation is also an intrinsic feature of the Tanay scheme as the initial probability $\phi(i, j)$, Eq 3.10 is based on ranks. However, the selection of a pre-determined thresholds (between ranks) with the Tanay scheme has a large effect on robustness and discrimintation as it is not data dependent. Overall, therefore, weights resulting from the Tanay scheme seem little affected by noise and missing values, which indicates that this scheme could assign high weights to gene-sample couples, even if none are present. Our contention, therefore, is that this ignores the subtleties in the data, and selects 'interesting' gene-sample couples based on absolute values, (including noise levels). With our empirical scheme, however, small sample size (number of
microarray experiments), the performance deteriorates with respect to the random graph comparison basis, (difficult to estimate $\mu$ and $s d$ for each gene variable), and the thresholds between categories become increasingly difficult to identify.

We have also demonstrated that edge-weighting schemes should be considered to be independent of the subsequent clustering procedure in the sense that they should satisfy intrinsic requirements and be internally consistent. Alternative edgeweight derivation can be seen as providing different probes for data interrogation leading to complimentary interpretations. This type of assessment framework for weighting is not unique to gene expression data, but is also crucial for other applications, generating large, complex datasets.

Using graphical techniques to extract meaningful information from biological data is both intuitive and valuable. In this chapter, we have limited our investigation to bi-partite graphs, a representation which captures essential properties of gene expression datasets and allows for the extraction of suitable bi-clusters. In the next chapter we investigate how information from these bi-partite graphs can be used to find important gene sets.

## CHAPTER 6

## Partitioning Gene Expression

## Graphs of Local Interactions

In Chapter 5, a framework for creating weighted bi-partite gene expression graphs and highlighting interesting properties of these was outlined in Chapter 5. Here, a method is presented for the construction of a one-mode gene expression network which specifically focuses on local interactions of genes, (i.e. across a subset of samples). This approach permits use of classical network analysis tools and adds to previously published work in the area, (Stuart et al., 2003; Carter et al., 2004; Bergmann et al., 2004; Zhang and Horvath, 2005). Specifically, no developed framework exists to date for the construction and examination of a one-mode gene expression network which captures local structures of a dataset. In what follows we present our analysis, and validation, of such a framework. We present a method to extract cliques from the network of local interactions, and provide compelling evidence that cliques extracted from a suitably constructed graph, represents meaningful biological structures.

### 6.1 Introduction

Gene co-expression networks provide a straightforward mechanism to explore systemwide functionality of genes, by using intuitive network concepts to analyse complex interactions. However, it is essential that relationships between genes in the network are captured in a meaningful manner. We address this issue by constructing a gene expression network from the underlying bi-partite graph. Nodes are connected in a one-mode graph, if the corresponding genes are significantly co-expressed across a subset of samples in the bi-partite graph, reflecting local gene interactions. However, in the one-mode projection information, such as the number of samples which a gene shows similar expression can be lost (Chapter 5). This is overcome by using a weighting scheme to capture this information, where edges are weighted in the one-mode graph to reflect the size and significance of the intersection in the bi-partite graph.

This represents a further development and improvement on previous gene expression network analysis, (Stuart et al., 2003; Carter et al., 2004; Bergmann et al., 2004). In their work each node in the network represents an expression profile of a given gene, and an edge represents a significant pairwise expression profile association across all samples. Zhang and Horvath (2005) also use an adjacency function to weight the edges, which assumes that connections between nodes approximate a scale free topology. The framework presented here does not require such assumptions and node connections are more biologically plausible as these represent significant pairwise gene expression across a subset of samples (i.e. identifies gene groups with similar expression for this set of samples and divergent expression otherwise). We describe an approach where the graph encoding is built from the bi-partite version and designed to reveal local interactions in the data, hence the no-
tion of similarity, (weighted edges), is significantly different from previous work. This leads us to define edge weights in the one-mode graph, based on information obtained from the bi-partite graph, and an "intersection" score.

The main purpose of the network analysis is to use gene connectivity information to group genes according to function and to relate to external gene information. For each graph constructed, gene subgroups are identified which show similar expression across a subset of samples. Many graphical clustering procedure have been proposed, Kernighan-Lin algorithm (Kernighan and Lin, 1970), Simulated Annealing (Johnson et al., 1989), Path Optimization (Berry and Goldberg, 1995), Genetic algorithms (Bui and Moon, 1996), MinMax clustering (Ding et al., 2001), heuristic search using hash tables (Tanay et al., 2002) and others. We use a number of graph properties in a clustering procedure to identify biologically meaningful groups.

### 6.2 Graph Properties

A one-mode graph, $G=(U, E, W)$, is constructed, where $U$ is the set of gene nodes and $n=|U|, e_{i j} \in E$ is the set of edges - an edge exists between two gene nodes $u_{i}$ and $u_{j}(i \neq j)$, if they show similar co-expression across at least two samples (i.e. their intersection neighbourhood in the underlying all-in-one bi-partite graph is $\geq 2) .{ }^{1}, w_{i j} \in W$ is the set of edge weights associated with the edge between nodes $u_{i}$ and $u_{j}, \forall e_{i j} \in E, i \neq j$.

Once the network has been constructed several biologically important network concepts can be identified which relate connectivity information to external gene information. Descriptive statistics of the one-mode graphs, extracted for the test datasets, are given in Table 6.1.

[^22]|  | $n$ | $m$ | $d$ | density | cc | $c c_{\text {min }}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aliz | 2349 | 90624 | 77.15 | 0.03 | 0.66 | 0.69 | $1.1(2.09$ tail $)$ |
| Alon | 1205 | 19404 | 32.2 | 0.03 | 0.72 | 0.72 | $1.2(2.33$ tail $)$ |
| Cho | 27 | 91 | 6.7 | 0.26 | 1 | 0.84 | - |
| Gasch | 3086 | 1348783 | 874.13 | 0.28 | 0.69 | 0.77 | $0.76(1$ tail $)$ |
| Golub | 1974 | 20931 | 21.2 | 0.01 | 0.68 | 0.68 | $1.26(2.45$ tail $)$ |
| Hsiao | 1163 | 42243 | 72.64 | 0.06 | 0.84 | 0.88 | $1.18(1.12$ tail $)$ |
| Spell | 2736 | 176248 | 122.25 | 0.04 | 0.62 | 0.67 | $1.89(2.47$ tail $)$ |
| Steg | 549 | 14189 | 51.69 | 0.09 | 0.94 | 0.89 | $1.08(2.06$ tail $)$ |
| West | 1556 | 102462 | 131.69 | 0.08 | 0.86 | 0.87 | $1.09(1.58$ tail $)$ |
| Random Graphs |  |  |  |  |  |  |  |
| Aliz | 2392 | 16747 | 117.58 | 0.005 | 0.16 | 0.20 | $1(1.6$ tail $)$ |
| Alon | 1296 | 32093 | 49.52 | 0.03 | 0.13 | 0.18 | $1.1(2.15$ tail $)$ |
| Cho | 36 | 56 | 3.11 | 0.08 | 0 | 0 | - |
| Gasch | 3094 | 3002425 | 970.40 | 0.62 | 0.19 | 0.25 | $0.75(1$ tail $)$ |
| Golub | 2052 | 30457 | 29.69 | 0.02 | 0.05 | 0.10 | $1.19(2.65$ tail $)$ |
| Hsiao | 1199 | 37893 | 63.43 | 0.05 | 0.20 | 0.26 | $1.12(1.12$ tail $)$ |
| Spell | 2763 | 275068 | 199.10 | 0.07 | 0.18 | 0.20 | $0.92(1.67$ tail $)$ |
| Steg | 585 | 10915 | 37.31 | 0.06 | 0.18 | 0.25 | $1.16(2.28$ tail $)$ |
| West | 1631 | 149530 | 183.36 | 0.11 | 0.33 | 0.40 | $0.98(1.56$ tail $)$ |

Table 6.1: one-mode Graph Properties: For each real dataset: $n$, the number of active genes, i.e. $U^{\prime} \in U$ with $d \geq 1, m$, the number of edges between gene nodes, $d$, the average degree of the graph, $c c$, the unweighted clustering coefficient for the graph, $\gamma$ the value of the exponent of the power law that best fits its degree distribution based on maximum likelihood estimation. The value in brackets refers to the power law estimate for values $\log (d)>1$

## Descriptive Statisitics:

For these gene expression test datasets, the number of gene nodes, $n$, is small compared to typical real world complex networks, (see Guillaume and Latapy (2004) for examples). It is clear that the size of the one-mode graph (nodes and number of edges) is dependent on the number of sample nodes in the underlying graph, Table 6.1, Fig. 6.1. Not typical of most real world complex networks is the observation that the average node degree, $d$, of each graph is quite high compared to $n$, and varies depending on the dataset under observation. (The Cho dataset is indeterminate, which has low $n$ and $d$, thus has limited information encoded.) The average degree of gene nodes is independent of the number of sample nodes in the underlying graph. For example, for the Alon data, genes are connected on average to $3 \%$ of the nodes, while for the West data genes connect on average to $8 \%$ of the nodes the sample node set size in the underlying graphs are 57 and 48 respectively. There are, on average, more nodes in the random graphs ${ }^{2}$, which suggests that there is less overlap in sample node neighbourhoods in the underlying random bi-partite graph compared to real, Fig. 6.2.

The density of all graphs is quite low, with the exception of the Gasch and Cho dataset. These datasets represent the maximum and minimum size graphs in terms of number of nodes. For the Cho dataset, $n$ is small and, as noted, the intersection neighbourhoods in the underlying bi-partite graph are snot extensive, resulting in a larger number of neighbours in the one-mode projection. The Gasch dataset contains a large number samples from experiments under extreme conditions, resulting in a large subset of genes responding. If the number of samples for which

[^23]two genes have similar expression is increased to qualify for an edge in one-mode graph (eliminating spurious measurements), to, for e.g., 5 , the density of the Gasch graph reduces to 0.03 .


Figure 6.1: The size of the one-mode graph (number of nodes and edges) is dependent on the number of samples in the underlying bi-partite graph.

## Clustering:

The clustering co-efficient, $c c$, in the one-mode gene expression graphs are again high, compared to the corresponding random graphs (Table 6.1), despite most nodes not begin linked (lower density), i.e. this implies that graphs have locally dense structures. Random networks (both the Erdös-Rényi model and the BarabasiAlbert model), result in a small clustering coefficient, which corresponds to $c c$ values generated for the random case. This indicates that there is a non-trivial level of organisation in the graph, such that, if two genes are connected, they are more than likely to have similar neighbourhoods.


Figure 6.2: There are smaller intersection neighbourhoods in the underlying random graphs compared to real, resulting in a higher average number of nodes.

As suggested in Chapter 5, cc can be dominated by the gene node with the larger neighbourhood. Thus we examine the minimum clustering coefficient, $c c_{\text {min }}$, (Eq. 5.5, (Latapy et al., 2006)). This measure does not deviate much from $c c$, suggesting that high degree nodes do not dominate. To further investigate this, the relationship between the average clustering coefficient for a node and its degree was analysed. The clustering coefficients, both $c c$ and $c c_{\text {min }}$, have a large spread for gene nodes of small degree, which are the majority, Figures 6.3, 6.4. For the fewer nodes of high degree, $c c$ decreases, while $c c_{\text {min }}$ increases (as its denominator is the minimum neighbourhood).

For all graphs, the values for the random graphs are well below the values for the test datasets. This shows that the values of $c c$ and $c c_{\text {min }}$ are larger in real gene expression datasets and the difference is substantial. There is an inverse relationship between $c c$ and node degree in the real datasets, while it remains roughly constant in the random case. This was also found by Ravasz et al. (2002) in a study of metabolic networks, who suggested that it was due to a hierarchical structure within the network. In the case of $c c_{\text {min }}$ there is a positive relationship with node
degree for both the real and the random graphs.


Figure 6.3: Average Clustering Coefficient of Nodes Vs. Degree

In Fig. 6.5 the cumulative distribution of the clustering coefficients for four test datasets are shown. In the random case the value of $c c_{\text {min }}$ grows very quickly and are close to unity at low values of $c c_{\text {min }}$. This means that $c c_{\text {min }}$ are very small for most nodes. That is, the intersection of the neighbourhoods is quite small compared to the minimum neighbourhood size. The value of $c c_{\min }$ grows much less quickly for the real datasets and remain lower than unity for a long time. This means that for an important number of nodes, $c c_{\text {min }}$ is large, closer to one in most cases - the neighbourhoods of many nodes significantly or completely overlap with other node


Figure 6.4: Average Clustering Coefficient of Nodes Vs. Degree.
neighbourhoods.

## Degree Distribution:

For $\sim \log (d)>1$, (where $d=$ node degree), approximate power-law behaviour is observed for the node degree distributions, implying that its heterogeneous nature is non-trivial, i.e. the networks are scale-free, Figures C. 26 - C. 27 Appendix C. This implies that the node degree distribution exhibits greater heterogeneity for one-mode compared to bi-partite graphs, with most genes having small degree and only few having high degree. Exceptions include the Gasch and Hsiao datasets (Figures C.26d,C.27b, Appendix C), for all $d$. Recall that an edge in the one-mode


Figure 6.5: Cumlative distribution of the minimum clustering coefficient. For each value on the x -axis the probability of all the nodes having lower than $x$ for $c c_{\text {min }}$
graph results when two genes show similar expression across samples. As with the density information, if the condition of number of samples for which gene expression is similar is increased to qualify for an edge in the one-mode graph (eliminating spurious measurements) a power-law distribution emerges, Fig. 6.6. Interestingly, Gasch and Hsiao are the datasets with near normal behaviour in the gene node degree distributions in the underlying bi-partite graph, suggesting that the near-normal behaviour of these underlying datasets is the limiting case, and increased sample intersection size requirements need to be applied when creating these graphs.

From this investigation, it is clear that there is structure in the constructed gene


Figure 6.6: As the requirement for intersection neighbourhood increases, the degree distribution conforms to a power law distribution
expression graph, that is not typical of a random network, and projecting into onemode does reveal new information. There is considerable sharing among neighbourhoods, (high clustering coefficients), indicating that these are co-active in similar samples. Although the underlying gene node distributions in the bi-partite graph were approximated by the Poisson, the node degrees in the corresponding one-mode graph are more closely modelled by a power law. This indicates that the topology of the network is dominated by a few highly connected genes which link the rest of the less connected genes to the network, i.e. genes are preferentially attached to genes of high degree.

### 6.2.1 Edge Weights in one-mode Graph

We have already discussed how two genes are linked in a one-mode projection of a bi-partite graph if they have a sample neighbour in common. If two genes were truly co-expressed, they would be co-expressed in more than one sample i.e. the intersection neighbourhood of the genes would be large. Here, we define the weighted strength of this interconnection as the "interconnection coefficient", Eq. 6.1.

$$
\begin{equation*}
c c_{i n t e r}(u, v)=\frac{\sum_{j \in\{N(u) \cap N(v)\}} w_{u j} w_{v j}}{|N(u) \cap N(v)|} \tag{6.1}
\end{equation*}
$$

where $w_{u j}$ and $w_{v j}$ is the weight of edges $(u, j)$ and $(v, j)$ in the bi-partite graph respectively, and $N(u)$ and $N(v)$ are the neighbourhoods of gene nodes $u$ and $v$ in the bi-partite graph respectively. This quantifies the level of confidence in the co-expression of two genes, defined through edge weights in the bi-partite graph. For example, if two genes have a high significance of expression(i.e. large edge weights) under three samples, the interconnection coefficient will be close to one, on the other hand if one gene has a high significance and the other a low significance under three samples it will be closer to 0.5 , and if both genes have a low significance under three sample the interconnection coefficient will be close to 0 . Hence, the weight of a link between two genes $u$ and $v$ in the one-mode projection captures the weight of the intersection of their neighbourhoods. This weight corresponds to a similarity measure as it is non-negative and symmetric.

### 6.3 Detecting High Scoring Coherent Modules GraphCreate

A principal objective is to detect meaningful subsets of genes which are tightly connected to each other across a subset of samples, i.e. to detect modules in the one-mode graph and to develop a scoring scheme which takes into account the samples for which these genes show similar expression. Here we introduce a new method, GraphCreate, designed to detect high scoring modules. A module of genes is defined by considering nodes with high neighbourhood overlap. We have already shown that if two genes have a neighbour in common, they are more likely to have similar neighbourhoods when compared to a random graph. All potential modules, $1 \ldots L$ are identified by considering, for each gene, $u$, neighbours of $u$ which have an intersection neighbourhood greater than a predefined threshold ( $I$ ) with $u$, i.e.

$$
\begin{equation*}
L=\{N(u) \cap N(j)\}>t_{\forall j \in N(u), L=1 \ldots n} \tag{6.2}
\end{equation*}
$$

This identifies a maximum of $n$, (= number of gene nodes) potential modules. Modules are processed to remove those with a high degree of overlap between their nodes.

Which, or the number of, samples gene nodes exhibit co-activity under is considered by a bit string associated with each gene node, $u$, in the one-mode graph. The bit string has length equal to the number of samples nodes in the bi-partite graph, where a 1 at position $s$ indicates that $u$ shows response in sample $s$. For each bicluster formed by gene $u$ define:

$$
\begin{equation*}
M_{\ell}=\max [\operatorname{sum}\{b s(i) \wedge b s(j)\}]_{\forall i, j \in \ell, \ell=1 \ldots L} \tag{6.3}
\end{equation*}
$$

and

$$
m_{i j \ell}=\frac{b s(i) \wedge b s(j)}{M_{\ell}}
$$

where $b s(\cdot)$ is the bit string associated with the node $(\cdot), \wedge=$ logical AND of each position in the bit string. (Note: $b s(i) \wedge b s(j)$ is the intersection size of $N(i)$ and $N(j)$ sample neighbourhoods in the bi-partite graph.) $M_{\ell}$ is the maximum intersection for the bicluster, $\ell$, formed by nodes in the module. It is a factor which scales the weight of an edge between two gene nodes based all gene nodes in the modules, see Fig. 6.7. This scaling factor for an edge weight will change between modules, depending on its membership, i.e. the edge weight will have greater or lesser significance depending on which genes it is grouped with. Thus the weight of a module can be found by:

$$
\begin{equation*}
W_{\text {bicluster }}=\sum_{i, j \in \ell} w_{i j} m_{i j} \tag{6.4}
\end{equation*}
$$

where $w_{i j}$ is weight of the edge between gene's $i$ and $j$ in the one mode projection, defined in Eq. 6.1. This weighting scheme reflects the intersection of the two genes and the samples for which genes show similar expression.

### 6.4 Representative Modules Found

In practice, the search space is restricted by searching only gene nodes with a minimum degree $>d_{t}$. Thus, there are two parameters affecting the number and size of gene groups found by GraphCreate: the size of the intersection neighbourhood, $I$, and $d_{t}$. Fig. 6.8 illustrates how these parameters affect the number, size and weight of modules found in the Hsiao dataset. Similar results, found for other datasets, are not given here. In general, the number of modules, $K$, decreases as $d_{t}$ and $I$


Figure 6.7: Finding groups in the data. Neighbour of A whose neighbourhood intersects with $\mathrm{A} \geq t$ are found. The bit string AND operation is then used to weight the edges according to the number of samples the group are expressed similarly in the bi-partite graph.
increase (Fig. 6.8a), while the average weight of the modules increases, (Fig. 6.8b). This is due to the additive nature of the scoring scheme, since as the requirement for $d_{t}$ and $I$ increases, more edges are included in the module. The average size (i.e. number of gene nodes) in a module tends to increase with $d_{t}$, but not strictly so, indicating that as more edges are required the module does not necessarily acquire more nodes.


Figure 6.8: Effects on the Number of Clusters ( $K$ ), Average Size (S) and Average Weight (W) as input parameters ( $I=$ Intersection Neighbourhood Size, $M=d_{t}$ $=$ minimum degree of nodes) change for the Hsiao dataset. Similar results were obtained for all datasets.

| Dataset | K | Avg. Size | Min. Size | Max. Size | Avg. Score | Min. Score | Max Score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aliz | 21 | 275.29 | 106 | 566 | 1457.02 | 247.61 | 3348.63 |
| Alon | 7 | 159.57 | 102 | 264 | 425.00 | 217.32 | 666.50 |
| Gasch | 4 | 280.25 | 141 | 446 | 4081.05 | 1455.76 | 7065.32 |
| Golub | 6 | 106 | 84 | 139 | 303.04 | 230.97 | 437.57 |
| Hsiao | 4 | 177 | 118 | 232 | 2665.96 | 1674.98 | 3477.03 |
| Steg. | 3 | 130 | 99 | 178 | 1636.66 | 1161.44 | 2401.49 |
| Spell | 46 | 285.30 | 108 | 728 | 1801.884 | 329.63 | 4990.37 |
| West | 11 | 352.90 | 135 | 721 | 3623.64 | 637.48 | 8458.10 |

Table 6.2: Descriptive statistics of modules found. The minimum and maximum values are presented to illustrate the range of modules found.

Table 6.2 provides descriptive statistics for modules (i.e. sets of genes exhibiting coherent activity), found in all datasets ${ }^{3}$. The average weight of the modules varies between graphs, e.g. Alon and Hsiao modules have very different average weight although the size of the modules are approximately the same. Unsurprisingly, the density of the Hsiao one-mode graph is much greater than the density of the Alon one-mode graph. Indeed, in all cases, higher weight modules tend to be associated with one-mode graphs of higher density. Therefore, although within graphs higher weighted modules tend to be associated with larger size, this is not the case between graphs.

One prediction of GraphCreate is that groups of high degree genes will be repeated across modules for a particular dataset, while genes appearing solely in one module are less reactive (have smaller degree). This arises due to the power-law distribution of the gene nodes whereby a few genes are connected to many nodes and many nodes are connected to a few genes. Modules found in all datasets are indeed hierarchical in nature (i.e. there is gene overlap between modules). Gene node memberships of modules found in the Hsiao dataset, are given in Fig. 6.9, which illustrates these overlaps e.g. module 3 is completely formed from subgroups of nodes found in modules 1 and 2, Fig.6.9a. If we examine the degree of gene nodes within a module, $x$ say, it is clear that those gene nodes which overlap with other modules have higher connectivity within module $x$ (i.e. considering only these connections within $x$ ), than those gene nodes which appear solely in $x$ and nowhere other than $x$, Fig. 6.10. This indicates that genes found in the overlaps of the modules are highly reactive, either affecting or affected by genes which appear in one

[^24]module only.


Figure 6.9: Overlap between modules in the Hsiao dataset.

Groups of genes forming modules in the Hsiao dataset were compared with those gene clusters found by the original authors, Hsiao et al. (2001), who categorised these according to involvement in house-keeping functions (HK), tissue selectivity (TS) or tissue variance (TV). It is clear, (where genes in modules found by GraphCreate are annotated by original authors), that overlaps occur for TV and TS categories. Modules 1 and 3 (Fig ??) have a significant overlap of 127 TS genes


Figure 6.10: Schematic depicting the degree of modules found in Hsiao dataset. In each instance, "modules $x$ and $y$ ", blue represents the average degree of the gene nodes only found in x , while red represents the average degree of the nodes which are co-operating in module y. For e.g. "Modules 3 and 4", the average degree of gene nodes found solely in 3 is 111.15 , while those co-operating with module 4 also is 139.03 . Similarly, the average degree of gene nodes found solely in module 4 is 124.21, while those co-operating with module 3 also is 131.67 . In each instance, there is a higher average node degree with the co-operating nodes, (except modules 2 and 3, as module 3 also has large commonality with module 1)
(hypergeometric test ${ }^{4} p=9.3 e^{-33}$ ). Similarly modules 1 and 4 have a TS overlap of $121\left(p=6.7 e^{-33}\right)$, modules 2 and 3 of $85\left(p=6.5 e^{-28}\right)$, and finally modules 3 and 4 of 76 ( $p=1^{e}-22$ ). These TS genes forming commonalities across modules have the highest variance across samples in the original dataset, and therefore have a higher degree in the underlying bi-partite graph, (and consequently a higher degree in the one-mode graph).

We continue our investigation of modules found by GraphCreate by considering the cancer and yeast datasets separately. From our set of test datasets, four relate to cancer experiments and two relate to experiments on yeast, Table 6.3.

| Cancer | Alon <br> (Colon) | Golub <br> (Leukemia) | Stegmaier <br> (Leukemia) | West <br> (Breast) |
| :---: | :---: | :---: | :---: | :---: |
| Yeast | Gasch | Spellman <br> (Stress Response) <br> (Cell Cycle) |  |  |
|  |  |  |  |  |

Table 6.3: Datasets used for analysis

### 6.4.1 Cancer Datasets

A hypergeometric test was used to find groups of over-represented genes in each module ${ }^{5}$, (Falcon and Gentleman, 2007). For these tests, the entire set of genes in each dataset was used as the pool to draw from (i.e. the gene universe). Genes that mapped to more than one entrezID were removed to avoid Gene Ontology

[^25]HK = House Keeping, TS = Tissue Selective, TV = Tissue Variant


Figure 6.11: Hsiao module comparison with author's annotation.
(GO) categories being counted twice. Only annotated genes in each module were considered for this analysis. Fig. 6.12 shows the most significant GO ontology terms associated with groups of over-represented genes, (using hypergeomtric tests described above), for the Golub dataset. All ontology terms selected had an associated $p$-value $<0.001$. In each of the modules (Fig. 6.12a-6.12f), it may be seen that the percentage of genes within a module associated with a particular term is greater than the percentage of genes in the 'gene universe' associated with the same term. This illustrates graphically the idea that there is a higher representation of terms in the modules, or gene subsets, than would be expected by chance given the number of terms in the gene universe.

Although a few of the modules extracted from the Golub dataset have a diverse categorisation of genes, other are quite specific. For instance, Module 5 quite clearly contains genes associated with changing the state of a cell as a result of a stimulus. Module 4 on the other hand contains genes involved in transport of substances, biosynthetic processes (reactions resulting from the formation of substances), and erythrocyte (red blood cell) development. There is overlap among modules for genes involved in RNA processing events and pathways involving ATP (a universal coenzyme and enzyme regulator). Not all genes found in each module were associated with the most significant GO terms. Table 6.4 illustrates the percentage of annotated genes in each module which were associated with the most significant GO terms.

| Module | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\%$ | $55 \%$ | $54 \%$ | $54 \%$ | $57 \%$ | $65 \%$ | $56 \%$ |

Table 6.4: Percentage of genes annotated in associated with the most significant GO categories for each module in the Golub dataset


Figure 6.12: GO annotations of modules found for Golub Dataset, using Biological Process Ontology of GO database. GO categories with $p$-value $<0.001$ where chosen.

Associated GO categories of over-represented genes for the remaining three cancer test datasets are shown in Table 6.5. Clearly, there are more genes with associated significant GO annotations in the Stegmaier dataset compared to both the West and Alon datasets. In the Stegmaier dataset, many of the genes are associated with cell-cycle ontology terms (this dataset consists of leukemia samples obtained at 6 hr and 24 hr time points). The edge set from the underlying bi-partite graph (used to generate the one-mode graph) can be used to determine to which samples the genes in a module, $M$, are coherently responding, i.e. $E^{\prime} \subset E,(E=$ edge set in bi-partite graph), where $(x, y) \in E^{\prime}$ iff $x \in M$, then $y \in Y$ ( $=$ subset of interesting samples). For example, it was found that in Module 2 of the Steigmaier dataset all genes had an edge to sample 8, 9 and 10 only (Kasumi cell line, Genetifib treated at 24 hrs ) in the underlying bi-partite graph, (Fig. 6.13), indicating a significant alteration in expression of all genes in module 2 at this time point.

Modules in the West dataset have quite a low \% of annotated genes associated with the most significant GO terms (Module 11 has an insignificant amount, hence not shown). However, a noteworthy significant identification is the RAS protein signal transduction, identified in modules 6,9 , and 10 . This forms part of the MAPK (mitogen-activated protein kinase) pathway, the activity of which was found to be high in breast cancers (Maemura et al., 1999), and which is correlated to the degree of RAS activation (von Lintig et al., 2004). RAS protein signal transduction is hyper-activated in breast cancer by overexpression of growth factor receptors which signal through it. RAS involvement in breast cancer has been well documented (von Lintig et al., 2004; McGlynn et al., 2009), and, although not mutated itself, is abnormally activated in breast cancers overexpressing the ErbB-2 receptor, (von Lintig et al., 2004). ErbB2 was also identified in modules 6, 9 and 10, as was GPR30, (G protein-coupled estrogen receptor 1). The fact that these

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| West |  |  |  |  |  |  |  |  |  |  |
| Cell Proliferation | 19 |  | 49 |  |  |  | 28 |  |  | 52 |
| Cell localization | 13 |  |  |  |  |  |  |  |  |  |
| Intracellular Transport | 11 |  |  | 20 |  |  |  |  |  |  |
| Biopolymer metabolic process |  | 251 | 147 |  | 102 |  | 97 |  |  | 185 |
| Nucleoside/tide/base |  | 189 |  |  |  | 136 | 70 | 25 | 63 | 137 |
| Transcription |  | 127 |  |  |  | 91 |  | 25 | 65 | 90 |
| RAS protein signal transduction |  |  |  |  |  | 14 |  |  | 12 | 12 |
| Regulation of biological processes |  |  |  |  |  | 184 |  |  |  |  |
| Regulation of apoptsis |  |  |  |  |  |  |  | 11 |  |  |
| \% | 28\% | 88\% | 49\% | 7\% | 40\% | 73\% | 86\% | 31\% | 23\% | 85\% |
| Stegmaier |  |  |  |  |  |  |  |  |  |  |
| Biopolymer metabolic process | 81 | 57 | 51 |  |  |  |  |  |  |  |
| Cell Cycle | 42 | 33 | 28 |  |  |  |  |  |  |  |
| Regulation of apoptsis | 14 |  | 9 |  |  |  |  |  |  |  |
| Response to Stress | 22 |  |  |  |  |  |  |  |  |  |
| Nucleoside/tide/base metabolic process |  | 48 | 44 |  |  |  |  |  |  |  |
| \% | 93\% | 91\% | 90\% |  |  |  |  |  |  |  |
| Alon |  |  |  |  |  |  |  |  |  |  |
| Celular Macromolecule metabolic process | 30 | 31 | 42 | 39 | 56 | 46 | 34 |  |  |  |
| Regulation of biological quality |  |  | 10 | 9 |  |  | 8 |  |  |  |
| Organelle Organization and biogenesis |  |  |  | 13 | 22 | 16 | 12 |  |  |  |
| Prgrammed cell death |  |  |  |  | 19 |  |  |  |  |  |
| Regulation of apoptsis |  |  |  |  | 15 |  |  |  |  |  |
| Cell developement |  |  |  |  | 22 |  |  |  |  |  |
| Negative regulation of programmed cell death |  |  |  |  |  | 7 |  |  |  |  |
| \% | 68\% | 53\% | 68\% | 59\% | 57\% | 43\% | 54\% |  |  |  |



Figure 6.13: Genes in module 2 of Stegmaier dataset where induced or repressed in samples 8,9 and 10 , found through an examination of the edges in the bi-partite graph. These correspond to genetifib treated Kasumi cell line sample at 24 hrs .
genes where identified indicates proliferation and migration of the breast cancer cells (Pandey et al., 2009). Other genes involved in MAPK pathway (i.e. MAPK, MAP2K, MAPKAPK2, MAPK14 (p38 isoform)) which play a central role in invasive breast cancer were also found in modules 6, 9 and 10 (Maemura et al., 1999; Han et al., 2002). These genes were not identified in modules that did not have genes associated with the RAS signal transduction.

The gene nodes involved in the RAS signal transduction pathway (RAS genes) also contribute heavily to the weight of the modules. For example, the 14 nodes associated with RAS genes in module 6 make up $2.8 \%$ of the total gene nodes of the module, however the weight of the RAS gene nodes (i.e. weight of all edges incident to these nodes) make up $40 \%$ of the total weight of the module. The average degree of RAS gene nodes is higher than the rest of the gene nodes in module 6 (1354.64:1333.614, RAS:non-RAS).

West et al. (2001), (the authors of the original analysis of this dataset), made available the top 100 genes found to be most discriminatory (DS genes) between ER+ and ER- ${ }^{6}$ (based on a Bayesian regression model). Of these, five DS genes were found in modules 6 and 9, while seven DS genes where found in Module 10. From an analysis of the edges in the bi-partite graph, the RAS genes in Module 6 were responsive in either ER+LN- or ER-LN+ samples ${ }^{7}$. Fig. 6.14 shows the pattern of expression of RAS genes across the subset of samples (ER+LN- or ER$\mathrm{LN}+$ ) in Module 6. The genes in these modules did not discriminate between ER+ and ER- tumors, (because the technique was applied to a graph which contained edges for both induction and repression).

[^26]

Figure 6.14: Pattern of coherent expression (i.e. induced or repressed coherently) of genes involved in RAS protein signalling pathway in module 6 found in the West dataset, across a subset of 37 samples in which change of expression was identified. Each of the 37 samples were either ER+LN- or ER-LN+.

### 6.4.2 Yeast Datasets

There are a core set of genes in yeast that are transcribed under numerous stressful conditions, representing a general yeast response set (Mager and Kruijff, 1995; Ruis and Schuller, 1995). The Gasch dataset represents a compendium of gene response in yeast under a variety of stressful conditions ${ }^{8}$. The structure of the modules found in this dataset is presented in Fig. 6.15. Again, a hierarchical organisation is indicated, where Module 2 is a super-module, containing nodes from all other modules, i.e. the general stress response. The genes in Module 2 where found to be responsive under a wide variety of samples.

The most significant GO term associations for genes in modules found in the Gasch dataset are given in Table 6.6 (all terms identified with $p<0.01$ ). The wide range of GO terms found illustrates the large scale effect of stress conditions on gene expression in yeast cells. (Note that although Module 2 is a superset of all other modules significant GO terms can alter between module 2 and other modules found, due to the nature of the hypergeometric test.) Genes in Module 1, with known function, are mainly involved in fatty acid metabolism, while genes in Module 4 are associated both with these and also with cell wall organisation and modifications. GO terms identified in Module 3 largely intersect those of Module 2 , however a large percentage of genes are also significantly over-represented in cellular macromolecule and protein metabolic processes.

Genes which are responsive to stress can also be isolated to a particular stressful condition. For instance, 22 genes were found to be responsive in the heat-shock ( $25^{\circ}$ to $30^{\circ}$ ) samples of the Gasch dataset (HS genes). All of these genes where members

[^27]

Figure 6.15: Overlap between modules in the Gasch dataset.

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Lipid Metabolic Proces | 15 |  |  | 20 |
| Fatty Acid Metabolic Process | 6 |  |  | 7 |
| Cell Wall Organisation and Biogenesis |  |  |  | 17 |
| Ergosterol Biosynthetic Process |  |  | 20 | 6 |
| Carboxylic acid metabolic process | 20 |  |  |  |
| tRNA aminoacylation for protein translation | 5 | 12 | 11 |  |
| Secretion |  | 38 | 29 |  |
| Secretory pathway |  | 13 | 11 |  |
| Intracellular transport |  | 62 | 38 |  |
| Cellular localisation |  | 67 | 53 |  |
| Amino acid activation |  | 12 | 11 |  |
| Ergosterol biosynthetic process |  | 12 | 10 |  |
| Vesicle-mediated transport |  | 35 |  |  |
| Steroid biosynthetic process | 5 | 11 | 10 | 6 |
| Sterol metabolic process |  | 12 | 10 |  |
| Post-Golgi vesicle-mediated transport |  | 12 |  |  |
| ER to Golgi vesicle-mediated transport |  | 12 |  |  |
| External encapsulating structure organization and biogenesis |  | 26 |  |  |
| Protein amino acid glycosylation |  | 11 |  |  |
| Glycoprotein metabolic process |  | 11 |  |  |
| Macromolecule localization |  | 37 | 30 |  |
| Protein import |  | 13 |  |  |
| Nitrogen compound metabolic process |  | 38 |  |  |
| Cellullar lipid metabolic process |  |  | 24 |  |
| Vaculor Transport |  |  | 10 |  |
| Macromolecule biosynthetic process |  |  | 49 |  |
| Cellular macromolecule metabolic process |  |  | 76 |  |
| Protein metaboic process |  |  | 74 |  |
| \% | 25\% | 48\% | 64\% | 21\% |

Table 6.6: Significant GO associations in modules found in the Gasch dataset. These where identified via a hypergeometric test. All annotations have $p$-value $<0.01$
of Module 3 (subset of Module 2) only. Notably, the application of sudden heat shock elicited large and rapid alterations in the expression of these genes before they returned to a steady state, Fig 6.16. From an examination of the underlying bipartite graph, the HS genes where identified in samples 2, 3, 4 and 6 ( 10 mins, 15 mins, 20 mins and 40 mins respectively) of the heat shock samples. Thus, this rapid alteration in gene expression is captured in the bi-partite graph, translated to the one-mode graph and captured in modules by GraphCreate. Notably, the HS genes are also a subset of genes found to be activated in the stationary phase, Fig. 6.17. Interestingly, the HS genes were not identified as significantly responding under any other conditions. This relationship between HS and stationary phase genes was also identified by Gasch et al. (2000).

As a second example from the Gasch dataset, 130 genes which are responsive under nitrogen depletion were found in Module 4 (ND genes). From an examination of the underlying bi-partite graph, the ND genes were identified across all time points in the experiment ( $1 \mathrm{hr}, 2 \mathrm{hr}$, 4 hr , 8 hr , 12hr, 1day, 2 day, 3 day, 5 day), Fig. 6.18. Notably, this condition elicited a large response in the later stages of nitrogen depletion, Fig. 6.18.

The most significant GO terms associated with the modules in the Spellman (Yeast Cell Cycle) dataset are given in Table 6.7, (all 41 modules not shown). Clearly, there are distinct modules associated with (i) rRNA processing and ribosome assembly, and (ii) protein catabolic processes, (a similar distinction between rRNA processing and protein catabolic processes was found in a study by Carlson et al. (2006) of yeast cell cycle data). These processes are fundamental processes in the general function of the cell. The pattern of GO terms associated with many of the modules is unique, indicating a unique relationship between genes in individual modules. For instance, Modules 2 and 34 both have strong associations with rRNA


Figure 6.16: Responsive genes to heat shock ( $25^{\circ}$ to $30^{\circ}$ ) in the Gasch dataset. These genes showed a rapid change in expression in the early stages, which was captured in the bi-partite graph, projected into one-mode graph and grouped into module 3 by GraphCreate.


Figure 6.17: Responsive HS genes in stationary phase in the Gasch dataset. Genes which showed a rapid change in expression in heat shock where also identified as a subset of those changing in stationary phase.
processing and ribosome assembly. However, Module 34 also has a strong association with Organelle ATP synthesis coupled electron transport, Metabolic process and Alcohol metabolic process suggesting a possible relationship between genes involved in these and those involved in rRNA processing and ribosome assembly.

Patterns of gene expression are shown in Fig. 6.19, for selected modules. In these examples, Module 22 is the smallest, with genes coherently expressed in 22 samples (out of a possible 82), 12 of which are from the cdc15 strained based time course experiment ${ }^{9}$. (There are 37 cdc 15 time-points in the Spellman dataset in total.) Module 12 is the largest with genes showing coherent expression in 47 samples (out of a possible 82). Again, 29 of the samples were cdc 15 based, which repre-

[^28]

Figure 6.18: Responsive genes to nitrogen depletion found in module 4 in the Gasch dataset. These genes showed a rapid change in expression in the later stages of the time series.

| Module $\rightarrow$ | 2 | 10 | 11 | 12 | 14 | 16 | 22 | 23 | 31 | 32 | 34 | 36 | 37 | 39 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| rRNA processing | 60 |  | 21 |  | 25 |  | 28 | 60 | 25 |  | 53 | 15 | 18 | 20 |
| RNA metabolic process | 104 |  |  |  |  |  |  | 108 |  |  | 86 |  |  |  |
| Maturation of SSU-rRNA from tricistronic rRNA transcript | 13 |  |  |  |  |  |  | 13 |  |  |  |  |  |  |
| Cellular component organization and biogenesis | 215 |  |  |  | 80 |  |  | 196 |  |  | 183 |  | 56 |  |
| Organelle organization and biogenesis |  |  |  |  |  |  |  |  |  |  |  | 41 |  |  |
| Ribosome biogenesis and assembly | 29 |  |  |  | 12 |  |  | 28 |  |  | 21 | 9 |  | 11 |
| Ribosomal large subunit biogenesis and assembly | 10 |  |  |  |  |  |  | 10 |  |  | 10 |  |  |  |
| Ribosomal large subunit assembly and maintenance | 16 |  |  |  | 10 |  | 9 | 16 |  |  | 15 |  |  |  |
| Ribosome assembly | 21 |  |  |  |  |  | 10 | 22 | 31 |  | 20 |  |  |  |
| Oxidative phosphorylation | 10 |  |  |  |  |  |  |  |  |  | 10 |  |  |  |
| Ribonucleoprotein complex biogenesis and assembly |  |  | 37 |  |  |  |  |  |  |  |  | 18 |  |  |
| Proteolysis involved in cellular protein catabolic process |  | 20 |  | 18 |  | 22 |  |  |  | 21 |  |  |  |  |
| Protein catabolic process |  | 22 |  | 21 |  | 24 |  |  |  | 23 |  |  |  |  |
| Ubiquitin-dependent protein catabolic process |  | 19 |  | 17 |  | 21 |  |  |  | 20 |  |  |  |  |
| Protein folding |  |  |  |  |  |  |  |  |  |  |  |  |  | 14 |
| Modification-dependent macromolecule catabolic process |  | 19 |  |  |  | 21 |  |  |  |  |  |  |  |  |
| Cellular protein metabolic process |  | 74 |  | 59 |  | 80 |  |  |  | 97 |  |  |  |  |
| Steroid biosynthetic process |  |  | 7 |  |  |  |  |  |  |  |  |  |  |  |
| Ergosterol biosynthetic process |  |  | 6 |  |  |  |  |  |  |  |  |  |  |  |
| Glycolysis |  |  | 7 |  | 7 |  | 8 |  |  |  |  | 6 | 6 |  |
| Modification-dependent macromolecule catabolic process |  |  |  | 17 |  |  |  |  |  |  |  |  |  |  |
| Macromolecule catabolic process |  |  |  | 28 |  |  |  |  |  |  |  |  |  |  |
| Hexose catabolic process |  |  |  |  |  |  | 9 |  |  |  |  | 6 | 7 |  |
| Alcohol catabolic process |  |  |  |  |  |  | 9 | 15 |  |  |  |  | 7 |  |
| Alcohol metabolic process |  |  |  |  |  |  |  |  |  |  | 28 |  |  |  |
| Metabolic process |  |  |  |  |  |  |  | 17 |  |  | 237 |  |  |  |
| Vacuolar transport |  |  |  |  |  |  |  |  |  | 14 |  |  |  |  |
| Organelle ATP synthesis coupled electron transport |  |  |  |  |  |  |  |  |  |  | 8 |  |  |  |
| Tricarboxylic acid cycle |  |  |  |  |  |  |  |  |  |  | 6 |  |  |  |
| \% | 87 | 41 | 25 | 40 | 65 | 30 | 75 | 85 | 19 | 33 | 90 | 61 | 70 | 43 |

Table 6.7: Significant GO term associations for modules found in the Spellman dataset. All GO categories were chosen with a $p$-value $<0.001$. Light grey columns represent rRNA processing and ribosome assembly modules, while dark grey modules represent protein catabolism modules.
sents the majority. However, the entire elutriation-chamber time course was represented in this module, (there are 10 elutriation-chamber experiment time-points in the Spellman dataset in total). Additionally, 24 of the 33 samples in Module 37 are cdc 15 based, while Module 14 has representative samples from all time-course experiments.

### 6.4.3 Overall Summary of Modules Found

It is clear from the evaluation of modules found in the test datasets that significant groups of genes are found by the GraphCreate algorithm. To avoid the misleading results arising from overlaps of associations at different levels of the GO ontology tree, a 'conditional' hypergeometric test was used in all cases, whereby the leaves of the GO graph were tested first, before terms with children previously tested with all genes annotated at significant children removed from the parent's gene list, (Falcon and Gentleman, 2007). Regardless, strong associations were found in each dataset, which agreed with published findings, e.g. RAS signal transduction pathway in West dataset. When examining the Gasch dataset, it was insightful to observe how each specific stressful condition affected groups of genes found by GraphCreate, and how the patterns of expression for these genes where repeated across stressful conditions.

A drawback of projecting a gene expression bi-partite graph into one-mode is of course the loss of sample information. However, once a module of genes is discovered to be important, it is trivial to search the bi-partite edge-set to highlight those samples for which these gene nodes are coherently expressed. This analysis was carried out on one-mode graphs, projected from bi-partite graphs that contained both induced and repressed genes, with the intention of capturing groups of


Figure 6.19: Expression pattern of Spellman dataset modules 12, 14, 22 and 37, captured by GraphCreate.
coherently expressed genes. This could easily be adapted to analyse induced or repressed genes, simply by projecting the appropriate induced or repressed bi-partite graph (see Chapter 5).

There is a hierarchical structure to the groups found (overlap of modules), which was also suggested in work by Ravasz et al. (2002) on metabolic networks. This structure arises from the power-law distribution of the gene nodes, and it was observed that gene nodes involved in overlaps of modules have a higher degree within the modules than those nodes not not included in overlaps. All this suggests that genes which 'connect' distinct modules either affect or are affected by a number of sub-groups of genes with membership in one module only.

### 6.5 Summary

In this chapter, we developed a general technique for weighted gene co-expression network construction that can be applied to gene expression datasets based on a bi-partite model, developed in Chapter 5. Microarray-based experiments of gene expression allow for a detailed survey of gene expression across a wide range of experimental samples. An advantage of using the bi-partite model as a basis include the fact that relationships in the gene co-expression network are based on a subset of these samples.

A thorough investigation of the gene co-expression network was carried out using important network concepts. This investigation uncovered many important organisational properties in these network not evident in the corresponding random networks of similar size and degree distribution. Chief amongst these was the increased probability that high density cliques, (measured through the clustering coefficients), in the gene co-expression networks together with the finding that the
node degree followed a power-law distribution.
An algorithm, GraphCreate, was developed for identification of modules of coherently expressed genes, based on considering nodes with high neighbourhood overlap, (our investigations also indicate that this is more likely in real gene coexpression networks). A module of gene nodes identifies those genes which form a common functional group. A scoring scheme for modules was developed, whereby the weight of a module is a direct consequence of the significance of expression of all genes in that module, (i.e. the weights of the edges are scaled based on other genes in the module). This innovative scoring scheme takes into account the samples under which the genes are co-regulated.

GraphCreate was used to find modules of gene nodes in gene co-expression graphs constructed. Our analysis demonstrates that significant functional groups of genes, which are co-active across a subset of experiments, are identified within the gene co-expression networks by GraphCreate. These results were corroborated from various published papers.

We carried out this analysis on a one-mode graph, obtained from a bi-partite all-in-one graph, containing information on both induced and repressed genes, to identify genes which were co-activated coherently. For if induced or repressed gene expression patterns are only required, this investigation can easily be adapted by projecting only the bi-partite subgraph which encodes the required category, (see Chapter 5). For instance, if only strongly induced genes where desired, the strongly-induced bi-partite graph would be projected. This is a powerful method of analysis which allows the researcher to analyse patterns of gene expression at different granularities if desired.

## CHAPTER 7

## Overall Discussion and Future

## Work

### 7.1 Goals of this Thesis

Understanding the interactions of genes expressed in a cell is critical to elucidating how biological organisms function. In this work efforts were concentrated on identifying these interactions through clustering (unsupervised) techniques to highlight meaningful patterns of gene co-regulation. The hypothesis is that genes which show co-regulated will be grouped together, indicating shared functionality, hence clusters defined at gene level represent biological modules.

The main goals of this thesis have been to: (i) investigate common unsupervised clustering methods and their applicability to gene expression data, (ii) extensively examine the properties of the gene expression data (iii) develop a robust solution to the computational analysis of gene expression data, and (iv) test the solution proposed for diverse datasets.

The view that clustering methods are universally applicable is a common misconception and has provoked controversy among practitioners, (Levsky and Singer, 2003; Shendure, 2008). While traditional global clustering techniques are popular,
biological theory supports the view that bi-clustering methods offer better interpretation in terms of data features and local structure. Limitations of commonlyused algorithms are well documented in the literature, while adoption of new (and hybrid) techniques has been slow among practitioners of microarray experiments and would be catalysed by transparent guidelines and increased availability in specialised software and public dataset repositories.

This work attempts to provide a new framework for analysis of gene expression analysis, which include:

- An numerical assessment of the performance of common unsupervised clustering methods and applicability of associated assessment measures.
- Adoption of graphical and statistical theory concepts to achieve an extensive examination of the properties of gene interactions over a diverse range of datasets, chosen reflect a range of experimental designs, platforms, sizes and objectives.
- Drawing on biological theory to develop a data dependent probabilistic model for weighting gene-sample interactions in bi-partite graph structures.
- Construction of a gene expression network from an underlying bi-partite graph, whereby nodes are connected in the gene expression network, if the corresponding genes are significantly co-expressed across a subset of samples in the bi-partite graph, reflecting local gene interactions.
- Using information of properties of gene interactions, development of GraphCreate, which identifies local structures of interaction among genes in a gene coexpression network. Development of a scoring scheme whereby the weight
of each module is a consequence of the significance of expression among all genes in that module.


### 7.2 Summary and Conclusions

We summarise a number of findings related to this work as follows:

1. The set of assessment measures for biological data is incomplete, with omissions for assessment metrics for overlapping local structures. Careful consideration of the compatibility of a particular assessment measure and clustering algorithm is required. Many assessment measures exhibit biases towards a particular algorithm or number of clusters. Internal measures by themselves may not be suitable for gene expression data, and validation through external measures, (although continued development of public annotation databases and metrics is required), is optimal.
2. Using classical network analysis tools, organisational structure was found in bi-partite graphs. To our knowledge, this is the first time basic network analysis techniques for bi-partite graphs have been applied to the analysis of gene expression. Clustering coefficients, $c c$, were obtained for gene and sample node sets individually with $c c$ of gene nodes found, unsurprisingly, to be much larger than that for sample nodes, due to the large disparity in average degree between sets. Consideration of the minimum clustering coefficient measure removes the bias of nodes with larger neighbourhood, and reveals there more subtle, local interactions in the data. We examined the size of the neighbourhood intersections for genes, and found that, genes react more coherently in larger neighbourhoods than expected by chance. An examination
of the gene node degree distribution of the extracted graphs suggests that, for large gene sets, it is well-approximated a Normal distribution. However, the size of the sample node set for most datasets is relatively low, so that degree distribution is less easy to establish and appears to depend on the observed data itself.
3. The issue of data-dependent threshold estimation for the identification of edges is non-trivial, as numerous thresholds need to be assessed, which is computationally expensive.This work is important as it directly addresses the problem transformation of gene-sample couples into edges of a graph, and associated edge-weights, and has shown that it is critical to successful analysis of gene expression and extraction of meaningful patterns. This transformation process has received little attention in the literature.
4. Investigations into weighting schemes of gene expression networks is often overlooked. Comparison analysis of the empirical-based scheme with the well know Tanay et al. (2002) scheme, has shown that the edge weighting scheme itself deserves careful consideration. Presented an analysis under four major properties, parameter influence, robustness, reuseability and discrimination and found that our new empirical based scheme outperforms the Tanay scheme, capturing subtleties in the data, by selecting 'interesting' gene-sample couples in a data dependent manner and based on relative values. The new empirical based scheme presented is more specific in that fewer gene-sample couples are identified than in the Tanay scheme.
5. This investigation one-mode gene co-expression networks uncovered many important organisational properties in these network not evident in corresponding random networks of similar size and degree distribution. Amongst
these was the increased probability that high density cliques, (measured through the clustering coefficients), in the gene co-expression networks. Although gene-node degree distributions in the underlying bi-partite graph are well approximated by the Normal distribution, node degree in the gene co-expression networks more closely follow a power-law distribution, suggesting there a few 'hub' genes which connect to many other genes in the network.
6. Our analysis demonstrates that significant functional groups of genes, which are co-active across a subset of experiments, are identified within gene coexpression networks by GraphCreate, and these results were corroborated from various published papers.

Gene expression analysis represents only one parameter by which cells or tissues may be characterised. While clusters found at the transcript level represent potential shared function, the ability to combine RNA and protein expression data to comprehensively profile both transcriptional and post-transcriptional changes is particularly appealing, and will inevitably provide a more complete picture cell function. Although it is more difficult to identify proteins that are differentially expressed, advances in techniques for rapid and reproducible two-dimensional gel protein separation and mass spectrometry-based protein identification make high throughput proteomics feasible as an adjunct to microarray gene expression analysis, (Bowtell, 1999). Consequently, it is important that accurate algorithms and computing techniques are developed to measure and understand gene expression data before integration with other levels.

### 7.3 Future Work

Analysis in Chapter 6 was carried out on a one-mode graph, obtained from a bipartite all-in-one graph, containing information on both induced and repressed genes. Future work will involve further investigation of the dynamics of gene interaction at various levels of granularity and the framework presented in Chapter 6 can easily be adapted by projecting only the bi-partite subgraph which encodes the required category, (for e.g., if only strongly induced genes where desired, the strongly-induced bi-partite graph would be projected). This method of analysis has enormous potential which allows the researcher to systematically analyse patterns of gene expression at different granularities if desired.

Throughout this thesis, care was taken to implement efficient and robust algorithms. This was achieved through the R and Bioconductor platform, calling external C functions for computationally demanding tasks. However, as with all open-source packages, R is continously improving. There are ongoing projects developing packages for parallel computing and message passing (e.g. Rmpi (Yu, 2009)). As biological data is increasingly complicated, important packages like these are vital to continue to successfully use this platform for Bioinformatics tasks. This work would benefit for these ongoing projects, enabling a larger scale analysis and yet even more thorough investigations.

Providing code developed in this work as an open-source package to the Bioconductor community would ensure that the algorithms and techniques would be thoroughly examined and tested. Although implementations of popular algorithms are sometimes available as standalone implementations, they often have clunky GUI interfaces and particular presentation of results which cannot be directly used as input to other statistical methods. For this reason, we have carried out this analy-
sis using, and developed algorithms which use, command line function calls in R. However, although a GUI has its disadvantages, (clunky and platform specific), it also has benefits for dissemination of the technique to practitioners of microarray experiments, and hence future work would involve a development of user interface for this package. Continued development and addition of algorithms to this package is part of future work.

Gene expression in the genome of a cell an extremely complex and does not act as a predetermined system, but as a system that responds to chemical changes in its environment. Therefore, although it is very important to study the transcriptome in isolation, it can only be understood by taking the complete state of the cell into consideration. The development of Bioinformatics/data-mining tools that span different levels of "omics", and which consider sequence similarity in promoter regions, is a crucial next step in the investigation of gene expression and its role in cell function.

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## Glossary

## Chapter 2

Array
See microarray

## Base-pairing

The process by which two nucleotides become connected by hydrogen bonds. The only permissible pairs are the nitrogenous base Adenine(A) pairing with the nitrogenous base Thymine(T), similar Guanine(G) pairing with Cytosine(C).

## Coding-RNA (non-coding RNA)

Coding RNA refers to RNA which will be translated into a protein. Non-coding RNA refers to RNA which carries out various essential functions in the cell.

## Complementary Strand

In double stranded DNA or RNA structures, each strand is complementary to the other in that base pairing occurs between the nitrogenous bases on each strand. Since since each base can only pair with one other possible type of base, the complementary strand can be reconstructed from any single strand.

## DNA (Deoxyribonucleic Acid)

The collective term for polymers nucleotides. Each nucleotide consists of a deoxyribose sugar, a phosphate group and one of four nitrogenous bases; Adenine,

Guanine, Cytosine and Thymine .

## End Modifications

This refers to 5 ' capping, 3 ' polyadenylation which are modifications made to the mRNA strand (referred to as pre-mRNA before end-modifications) before it is translated.

Exon
DNA sequences with a gene which code for a protein.

## Expression

This refers to the process of converting the information encoded in a gene sequence to information encoded in an RNA sequence.

Gene
Sequence of DNA in the genome of a cell, which is transcribed into RNA.

Intron
DNA sequences within a gene which do not code for a protein.

## Microarray

A technology which consists of glass or silicon chip with thousands of DNA oligonucleotides or cDNA sequences immobilised at distinct known positions.

Polymerase (RNA Polymerase)
An enzyme involved in DNA replication. RNA Polymerase is an enzyme involved in the transcription of RNA from DNA.

## Probes

DNA RNA segments, immobilized at distinct positions, which 'probe' a sample if interest, i.e. bind to a complementary strand in a sample of interest. See also: Target

## RNA (Ribonucleic Acid)

The collective term for polymers nucleotides. Each nucleotide consists of a ribose sugar, a phosphate group and one of four nitrogenous bases; Adenine, Guanine, Cytosine and Uracil .

## Splicing

Alternative removal of mRNA coded for by introns, resulting in alternative protein products.

## Target

DNA RNA extracted from a sample of interest which is to be analysed.

## Transcription

The process of realisation of the RNA coded for in the genome.

## Translation

The process of realisation of the proteins coded for in the mRNA strands.

## Chapter 3

## Bi-clusters

A cluster defined over a subset of attributes and subset of samples.

## Cluster

A group of object which are more similar in some properties compared to groups in other clusters.

## Complete clustering

A cluster structure in which every variable and attribute are in some cluster.

## Crisp Membership

An object is assigned membership of a cluster with a certainty of 1 . In the case of overlapping clusters, a cluster can be a member of $\geq 1$ cluster with a membership of 1 .

Exclusive (hard) clusters
Each object is a member of only one cluster.

## Feature extraction

Identifying latent features in the dataset which can differentiate groups.

## Feature selection

Selecting features of the variables most distinguishable in grouping objects.

Fuzzy Membership
An object is assigned to a cluster with a membership value which indicates the associativity with that cluster.

## Global clusters

Structure in the dataset defined over all attributes and all samples.

## Local structure

Structure in the dataset defined over a subset of attributes (samples) and variables (genes).

## Overlapping clusters

Two or more clusters which contain common objects.

## Partial clustering

A clustering structure found in the dataset where every variable and attribute are in a cluster.

## Chapter 4

## Assessment

Formative in nature which is ongoing and used to improve process. It is also diagnostic, i.e. identify areas for improvement.

## Cluster Validation

The process of determining the correctness of grouping genes in a cluster, and of the cluster structure overall.

## Cophenetic Distance

This refers to how similar two objects have to be in order to be grouped together in the same cluster. The cophenetic correlation measure, measures the correlation between the cophenetic distance matrix and the distance matrix.

Evaluation
Summative in nature which is final and used to gauge quality. Judgemental i.e. arrive at an overall grade/score.

## Validation

The process of evaluating techniques and determining suitability based on statistical evidence and/or meeting user needs and requirements.

## Chapter 5

## All In One Graph

A bipartite graph, $G=(T, \perp, E)$ which contains independent subgraphs from all subcategories, i.e. weak repressed $G_{w k R}=\left(\top_{w k R}, \perp_{w k R}, E_{w k R}\right)$, moderate repressed $G_{\text {mod } R}=\left(\top_{\bmod R}, \perp_{\bmod R}, E_{\text {mod } R}\right)$, strong repressed $G_{s t r R}=\left(\top_{s t r R}, \perp_{s t r R}, E_{s t r R}\right)$, weak induced $G_{w k I}=\left(\top_{w k I}, \perp_{w k I}, E_{w k I}\right)$, moderate induced $G_{\text {mod } I}=\left(\top_{\operatorname{modI} I}, \perp_{\text {modI }}, E_{\text {mod } I}\right)$,
strong induced $G_{s t r I}=\left(\top_{s t r I}, \perp_{s t r I}, E_{s t r I}\right)$, such that $\{\top\}=\left\{\top_{w k R} \cup \top_{\operatorname{modR}} \cup\right.$ $\left.\top_{s t r R} \cup \top_{w k I} \cup \top_{\text {mod } I} \cup \top_{s t r I}\right\},\{\perp\}=\left\{\perp_{w k R} \cup \perp_{\text {modR }} \cup \perp_{s t r R} \cup \perp_{w k I} \cup \perp_{\text {modI }} \cup\right.$ $\left.\perp_{\text {strI }}\right\},\{E\}=\left\{E_{w k R} \cup E_{\text {mod } R} \cup E_{\text {str } R} \cup E_{w k I} \cup E_{\text {mod } I} \cup E_{\text {strI }}\right\}$.

## Bipartite Graph

A graph, $G=(\top, \perp, E)$, is considered to be bipartite if there are two disjoint subsets of vertices, $\top, \perp$, and there is no edge between two vertices in the same subset. A gene expression dataset can be modelled in this way, where $T$ vertices represent genes and $\perp$ vertices represent samples. An edge $w_{i j} \in W$ is the weight matrix, were $w_{i j} \neq 0$ if there is an edge between $i \in T$ and $j \in \perp$.

## Clustering Coefficient

Clustering Coefficient refers to a measure of the degree of sharing of neighbourhoods among nodes.

Degree
Degree of a node refers to how many edges are incident to the node.
Density
Density of a graph refers to the number of edges in the graph proportional to the total possible number of edges.

Edge

An edge connects two nodes/vertices in a graph. Each pair node can be connected by an edge, referred to as a complete graph, or a subset of nodes can be connected referred to as a partial graph.

## Edge-Weight

Value assigned to an edge in a graph which can have various interpretations depending on the problem, e.g. distance, cost, length, capacity. In the problems represented in this thesis the weights represent significance of expression.

## Node

Graphs comprise of a set of fundamental units, referred to as nodes. These nodes are the connecting points for edges in the graph. Nodes of the graph have various interpretations depending on the problem. In this thesis nodes represent genes and/or samples in the gene expression dataset under consideration.

## Node Neighbourhood

The neighbourhood of a node, $i$, refers to the set of nodes an $i$ has a connection to.

## One-mode Graph

A graph, $G=(V, E)$, is considered to be in one mode if an edge can exist between any two vertices, $v \in V$. A gene expression dataset can be modelled in this way where vertices in the graph represent genes and an edge exists between two gene vertices if they show common expression (e.g. measured as a distance function).

## One-Mode Projection

A bipartite graph can be projected to a one-mode graph. Two nodes in a bi-partite graph are linked in a one mode projection if they have a sample neighbour in common, A bipartite graph is either projected with the $T$ set of nodes, or the $\perp$ set of nodes.

Random Graph

A random graph is a graph in which properties such as the number of vertices, edges and/or connections between are determined randomly.

Vertex
Also known as /textitnode, see above

## ApPENDIX A

## Distance and Assessment

## Metrics

## Distance Formulas

## Minkowski Distance

This is a measure of distance in euclidean space. The Minkowski distance of order $p$ between two points of $n$ dimension, $\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ and $\left(y_{1}, y_{2}, \ldots, y_{n}\right)$, is:

$$
\begin{equation*}
\mathrm{p} \text { norm dis }=\left(\sum_{i=1}^{n}\left|x_{i}-y_{i}\right|\right)^{\frac{1}{p}} \tag{A.1}
\end{equation*}
$$

Of special interest arise when $p=1$ (Manhattan distance), $p=2$, (Euclidean distance), and $p=\infty$, (Chebychev distance).

## Correlation distance

Correlation measures the linear relationship between two variables. Pearson's Correlation is defined as:

$$
\begin{equation*}
r_{x y}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{(n-1) s_{x} s_{y}} \tag{A.2}
\end{equation*}
$$

where:

- x and y are two data points of n -dimension.
- $s_{x}$ and $s_{y}$ are the standard deviations of data points $\mathbf{x}$ and y respectively.
- $\bar{x}$ and $\bar{y}$ are the mean values of x and y respectively.

Spearman's correlation is a non-parametric correlation measure, where $x_{i}$ and $y_{i}$ are converted to rankings. It is defined as:

$$
\begin{equation*}
\rho=1-\frac{6 \sum_{i=1}^{n} d^{2} i}{n\left(n^{2}-1\right)} \tag{A.3}
\end{equation*}
$$

where:

- $d_{i}=x_{i}-y_{i}=$ the difference in ranks.

An alternative non-parametric measure is Kendal's tau:

$$
\begin{equation*}
\tau=\frac{n_{c}-n_{d}}{n(n-1) / 2} \tag{A.4}
\end{equation*}
$$

where:

- $n_{c}$ is the number of concordant pairs, i.e. pairs ordered the same way
- $n_{d}$ is the number of discordant pairs i.e. pairs ordered differently


## Assessment measures

## Average Distance (AD)

$$
\begin{equation*}
A D=\frac{1}{N P} \sum_{i=1}^{N} \sum_{\ell=1}^{P} \frac{1}{n\left(K_{i, 0}\right) n\left(K_{i \ell}\right)}\left(\sum_{i \in K_{i, 0, j \in K_{i, \ell}}} d i s t(i, j)\right) \tag{A.5}
\end{equation*}
$$

where:

- $K_{i, 0}$ represents the cluster containing observation $i$ using the original clustering (based on all available data),
- $K_{i, \ell}$ represents the cluster containing observation $i$ where the clustering is based on the dataset with column $\ell$ removed.


## Average Distance between Means (ADM)

$$
\begin{equation*}
A D M=\frac{1}{N P} \sum_{i=1}^{N} \sum_{\ell=1}^{P}\left(\operatorname{dist}\left(x_{K_{i, \ell},}, x_{K_{i, 0}, 0}\right)\right. \tag{A.6}
\end{equation*}
$$

where:

- $x_{K_{i, 0}}$ is the mean of the observations in the cluster which contain observation $i$, when clustering is based on the full data,
- $x_{K_{i, \ell}}$ is the mean of the observations in the cluster containing observation $i$ where the clustering is based on the dataset with column $\ell$ removed


## Average Proportion of non-overlap (APN)

$$
\begin{equation*}
\left.A P N=\frac{1}{N P} \sum_{i=1}^{N} \sum_{\ell=1}^{P}\left(1-\frac{n\left(K_{i, \ell} \in K_{i, 0}\right)}{n\left(K_{i, 0}\right)}\right)\right) \tag{A.7}
\end{equation*}
$$

where:

- $K_{i, 0}$ represents the cluster containing observation $i$ using the original clustering (based on all available data),
- $K_{i, \ell}$ represents the cluster containing observation $i$ where the clustering is based on the dataset with column removed.


## C-Index, (Hubert and Schultz, 1976)

$$
C I=\frac{S-S_{\min }}{S_{\max }-S_{\min }}
$$

where $S=$ the sum of the distances over all pairs of patterns from the same cluster. Let that number of patterns in a cluster $=l$, then $S_{\text {min }}$ is the sum of the $l$ smallest distances if all pairs of distances are considered, similarly $S_{\text {max }}$ is the sum of the $l$ largest distances. Hence, a small CI indicates a good clustering.

## Connectivity, (Handl et al., 2005)

$$
\begin{equation*}
\text { Conn }=\sum_{i=1}^{N} \sum_{j=1}^{L} x_{i, n n_{i(j)}} \tag{A.8}
\end{equation*}
$$

where:

- $n n_{i(j)}$ is the $j^{\text {th }}$ neighbour of observation $i$.
- $x_{i, n n_{i(j)}}$ is 0 is both $i$ and $n n_{i(j)}$ are in the same cluster, otherwise $x_{i, n n_{i(j)}}$ is $1 / j$.
- $L$ is an input parameter which determines the number of neighbours that contribute to the measure.


## Cophentic correlation

$$
\begin{equation*}
C O P H=\frac{\sum_{i<j}\left(x_{i j}-x\right)\left(z_{i j}-z\right)}{\sqrt{\sum_{i<j}\left(x_{i j}-x\right)^{2} \sum_{i<j}\left(z_{i j}-z\right)^{2}}} \tag{A.9}
\end{equation*}
$$

where:

- Z is a symmetric matrix of size $N \times N$, where, for a hierarchical clustering, each entry $\left(z_{i j}\right)$ of the matrix indicates the level at which genes $i$ and $j$ where put into a cluster together.
- X is the original distance matrix


## Davies-Bouldin Index, (Davies and Bouldin, 1979)

$$
\begin{equation*}
D B=\frac{1}{K} \sum_{i=1}^{K} \frac{\max _{i \neq j}\left(\operatorname{diam}\left(K_{i}\right)+\operatorname{diam}\left(K_{j}\right)\right)}{\operatorname{dist}\left(K_{i}, K_{j}\right)} \tag{A.10}
\end{equation*}
$$

where:

- $K=$ the number of clusters.
- $\operatorname{diam}\left(K_{x}\right)$, is the diameter of cluster $x$.
- $\operatorname{dist}\left(K_{i}, K_{j}\right)$ is the distance between clusters $K_{i}$ and $K_{j}$.


## Dunn Index, (Dunn, 1974)

$$
\begin{equation*}
\text { Dunn }=\min _{1 \leq i \leq K}\left\{\min _{1 \leq j \leq K, j \neq i}\left(\frac{\operatorname{dist}\left(X_{i}, X_{j}\right)}{\max _{1 \leq c \leq K} \operatorname{diam}\left(X_{c}\right)}\right)\right\} \tag{A.11}
\end{equation*}
$$

where

$$
\begin{align*}
\operatorname{diam}\left(X_{i}\right) & =\max _{x, y \in X_{i}}\{d(x, y)\} \quad \text { and } \\
\operatorname{dist}\left(X_{i}, X_{j}\right) & =\min _{x \in X_{i}, y \in Y_{i}}\{d(x, y)\} \tag{A.12}
\end{align*}
$$

Diam and dist can be severely affected by noisy values.

## Figure of Merit

This is a measure of the predictive power of the algorithm. This measure assesses predictive power of each sample.

$$
\begin{equation*}
F O M(e, k)=\sqrt{\frac{1}{n} \times \sum_{i=1}^{K} \sum_{x \in K_{i}}\left(R(x, e)-\mu_{K_{i}}(e)\right)^{2}} \tag{A.13}
\end{equation*}
$$

where:

- $\mathrm{e}=$ the sample that is being assessed for predictiveness
- $\mathrm{K}=$ the number of clusters
- $\mathrm{n}=$ the number of genes
- $K_{i}=$ the $i^{\text {th }}$ cluster
- $\mathrm{R}(\mathrm{x}, \mathrm{e})=$ the expression level of gene x in sample i .
- $\mu_{K_{i}}(e)=$ the average expression level in sample e of genes in cluster $K_{i}$.

The aggregate figure of merit assesses the total predictive power of the algorithm over all the samples for $K$ clusters.

$$
\begin{equation*}
F O M(k)=\sum_{e=1}^{p} F O M(e, k) \tag{A.14}
\end{equation*}
$$

This figure is biased towards larger number of clusters, due to the fact that (a) smaller clusters will tend to be more homogenous and (b) increasing the number of clusters will decrease the $\mathrm{FOM}(\mathrm{e}, \mathrm{k})$ equation. The FOM equation can be adjusted to account for (b), see Yeung et al. (2001) for details.

$$
\begin{equation*}
F O M(e, k)_{\text {adjusted }}=F O M(e, k) / \sqrt{\frac{n-K}{n}} \tag{A.15}
\end{equation*}
$$

## Rand Index

The rand index is the proportion of concordant gene pairs in two partitions of a gene expression matrix, (two genes are concordant if they appear in the same cluster in both partitions or different clusters in both partitions) Rand (1971).

$$
\begin{equation*}
\operatorname{Rand}_{1}=\frac{a+d}{a+b+c+d} \tag{A.16}
\end{equation*}
$$

where:

- $a=$ the number of pairs of genes which are in the same cluster in both partitions
- $b=$ the number of pairs of genes which are in different clusters in both partitions
- $\mathrm{c}=$ the number of pairs of genes which are in the same cluster in partition 1 and different clusters in partition 2
- $\mathrm{d}=$ the number of pairs of genes which are in different clusters in partition 1 and different clusters in partition 2

This can be standardised to an expected value of zero if the partitions are randomly generated and takes a maximum value of 1 if the partitions are perfectly correlated (Rand ${ }_{\text {adjusted }}$ ), Hubert and Arabie (1985).

## SD-Validity Index

This index is composed of the average Scattering of the clustering (measures the variance/compactness of the clusters)and the total Separation of the clustering (distance between cluster centres).

Average scattering of the clustering is given by:

$$
\begin{equation*}
\text { Scatt }=\frac{1}{n_{c}} \sum_{i=1}^{n_{c}}\left\|\frac{\sigma\left(v_{i}\right)}{\sigma\left(v_{x}\right)}\right\| \tag{A.17}
\end{equation*}
$$

and Separation is given by:

$$
\begin{equation*}
D i s=\frac{\max _{i, j=1 \ldots n_{c}}\left(\left\|v_{j}-v_{i}\right\|\right)}{\min _{i, j=1 \ldots n_{c}}\left(\left\|v_{j}-v_{i}\right\|\right)} \sum_{k=1}^{n_{c}}\left(\sum_{j=1, k \neq j}^{n_{c}}\left\|v_{j}-v_{k}\right\|\right)^{-1} \tag{A.18}
\end{equation*}
$$

and the SD-Validity index is defined to be:

$$
\begin{equation*}
S D=\alpha \cdot S c a t t+D i s \tag{A.19}
\end{equation*}
$$

where:

- $\alpha$ is a weighting factor which is equal to the total separation of the maximum number of input clusters.


## Silhouette Width

For each observation $i$, the Silhouette Value is:

$$
\begin{equation*}
S(i)=\frac{b_{i}-a_{i}}{\max \left(b_{i}, a_{i}\right)} \tag{A.20}
\end{equation*}
$$

where:

- $a_{i}$ is the average distance between $i$ and all other observations in the same cluster.
- $b_{i}$ is the average distance between $i$ and all observations in the nearest neighbouring cluster.

The Silhouette Width of a cluster is the average of the silhouette values of each observation.

$$
\begin{equation*}
S i l=\frac{1}{N} \sum_{i=1}^{N} S(i) \tag{A.21}
\end{equation*}
$$

Xie-Beni Index, (Xie and Beni, 1991)

$$
\begin{equation*}
X B=\frac{\sum_{i \in N} \sum_{j \in K} u_{i j}^{2} \cdot d^{2}\left(X_{i}, K_{j}\right)}{n \cdot \min _{i, j} d^{2}\left(K_{i}, K_{j}\right)} \tag{A.22}
\end{equation*}
$$

where:

- $u_{i j}^{2}$ is the membership of gene $i$ to cluster $j$.
- $d^{2}\left(X_{i}, K_{j}\right)$ is the distance between $i$ and cluster $j$ centroid.


## APPENDIX B

## DATASETS

## Alizadeth - Lymphoma data

The dataset downloaded from http://llmpp.nih.gov/lymphoma/data. shtml, and contains data pertaining to the seminal paper of Alizadeh et al. (2000).

The dataset consists of 4026 genes and 96 samples. It represents data from a two-colour spotted array. The available values are ratio values that were logtransformed (base 2). The available data were centered by subtracting (in log space) the median observed value for each gene, (Alizadeh et al., 2000)

## Alon - Colon Cancer

The dataset was downloaded from http://microarray.princeton.edu/ oncology/affydata/index.html, which contains data pertaining to a colon cancer study by Alon et al. (1999).

The dataset contains measurements for 2000 genes across 62 samples ( 40 tumor samples, 22 normal tissue samples). An oligonucleotide array was used and the available data represented raw intensity values (i.e. unprocessed). The data was log transformed and scaled to have mean 0 and standard deviation of one for each
sample.

## Cho - Yeast Cell Cycle

Data was downloaded from: http://genomics.stanford.edu/yeast_ cell_cycle/full_data.html. The purpose of this experiment data was the characterization of mRNA levels during the cell cycle of the yeast Saccharomyces cerevisiae.

The dataset contains 6601 genes and across 17 time points. The downloaded dataset was normalized between timepoints with respect to each other, and represents information from 4 chips. The provided values for each of the 17 time points data for each gene are the normalized fluorescence between 0 and 160 minutes after cell cycle initiation from time 0 . Data was normalized similar to the technique used in: http://www.nature.com/ng/journal/v22/n3/full/ ng0799_281.html.

## Gasch Dataset - Yeast Stress

Dataset was retrieved from http://genome-www.stanford.edu/yeast_ stress/data.shtml and contains measurements of mRNA from an experiment monitoring yeast expression under various stressful conditions.

The data contains 6,152 genes under 173 samples. The available data represents normalized, background-corrected $\log 2$ values of the Red/Green ratios measured on spotted DNA microarrays. Details of materials and methods can be found at http:

```
//genome-www.stanford.edu/yeast_stress/materials.pdf
```


## Golub - Leukemia data

The dataset was downloaded from http://www.broad.mit.edu/cgi-bin/ cancer/publications/pub_paper.cgi?mode=view\&paper_id=43. and contains data pertaining to an experiment to monitor human acute leukemia's by Golub et al. (1999).

The dataset consists of 7129 genes across 72 samples, and represents unprocessed data. The data was processed according to technique outlined in Dudoit et al. (2002)

## Hsiao data - Human tissue

Data was downloaded from http://www.biotechnologycenter.org/hio/ databases/index.html, (HuGE Index) and represents a compendium of gene expression data for normal human tissues, (Hsiao et al., 2001).

The dataset represents 7,070 genes over 59 samples. As the data is from a collection of sources the units of expression level are arbitrary. All data was processed by the curators identically so that data can be compared across samples and tissues, (see http://www.biotechnologycenter.org/hio/faq/ index.html).

## Spellman - Yeast Cell cycle.

Data downloaded from http://genome-www.stanford.edu/cellcycle/ data / rawdata/, complementing the work of Spellman et al. (1998). The dataset represents processed measurements from various experiments, with the aim to to identify all genes whose mRNA levels are regulated by the cell cycle in the yeast

## Saccharomyces cerevisiae.

The dataset contains 6,178 genes across 82 sample points ( and includes analysis of Cho et al. (1998) data). Spotted two cDNA arrays were used and details of materials and methods can be found at: http://www.molbiolcell.org/ cgi/content/full/9/12/3273\#MaterialsMethods.

## Stegamaier - Kasumi data

Raw data was downloaded from http://www.broad.mit.edu/cgi-bin/ cancer/publications/pub_paper.cgi?mode=view\&paper_id=117. and pertains to myeloid differentiation in acute leukemia. The data represents HL60 and Kasumi- 1 cells treated in replicates of 3 with getinib at $10 \mu M$ or DMSO, and RNA was prepared at 6 and 24 hours for hybridization to Affymetrix U133A microarrays. (Stegmaier et al., 2005)

The dataset contains 22,283 probes over 22 samples and represented unprocessed values. The data was $\log$ transformed and scaled to have mean 0 and standard deviation of 1 across samples. Detail of materials and methods can be found at above url.

## West - Breast Cancer

The data was downloaded from http://data.cgt.duke.edu/west.php. It contains data resulting from an experiment to analyse gene expression assays from breast cancer tissue, with the aim of identifying potential prognostic and/or predictive factors.

It contains 7,129 probes for 49 samples. The collection of tumors for RNA
extraction consisted of 13 estrogen receptor (ER) + lymph node $(\mathrm{LN})+$ tumors, $12 \mathrm{ER}-\mathrm{LN}+$ tumors, $12 \mathrm{ER}-\mathrm{LN}-$ tumors and $12 \mathrm{ER}+\mathrm{LN}-$ tumors. The level of RNA transcripts was measured using oligonucleotide array's (HuGeneFL Genechip array). The data was processed by scaling to have mean 0 and standard deviation of one. The data was $\log$ transformed.

## Synthetic Dataset - Repeated Measures

The following functions were used to create the 11 groups in the synthetic repeated measures dataset. A $12^{\text {th }}$ group containing random data was also created.

| 1 | Treatment effect on group 1, group 2 and group 3 similarly but to different levels. |
| :---: | :---: |
| 2 | Affect first individual in each group differently from others individuals |
| 3 | Affects each individual in similar way across treatment groups, apart from 3rd treatment group |
| 4 | Affects treatment groups 1 and 3 similarly, has no effect on 2nd treatment group (random numbers) |
| 5 | Affects treatment groups 1 and 3 in opposite way, has no effect on 2nd treatment group (random numbers) |
| 6 | Affects 4 individual in treatment groups 1 and 2 and 3 individuals in group 3. Affects each treatment of an individual equally not the same across treatment groups were it affects the last treatment of each individual differently. |
| 7 | Affects 4 individuals in groups 1 and 2; 2 individuals in group 3. Affects each treatment of individuals differently. |
| 8 | Affects only individuals in group 1, affects each treatment of an individual equally. |
| 9 | Affects first individual in each treatment group only, affects each treatment equally. |
| 10 | Affect each treatment, affects all individuals of each treatment equally. |
| 11 | Linear combination of groups 2 and 10 , such that $j=\left(g 2^{3}+g 10\right) / 10$ |
| Noise | Random |

## Synthetic Dataset - Time Series

The following functions were used to create the 11 groups in the synthetic time series dataset. For sine wave functions, $y(t)=A \cdot \sin (\omega \times t+\theta), \omega \times t=$ frequency, $\theta=$ phase shift. $t 1=$ time-series one, $t 2=$ time-series two. A $12^{\text {th }}$ group containing random data was also created.

|  | T1 | T2 |
| :---: | :---: | :---: |
| 1 | $\begin{aligned} & \omega=2 \times p i / 5, \theta=-10 \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{5}-10\right) \end{aligned}$ | $\begin{aligned} & \omega=2 \times p i / 10, \theta=-10 \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{10}-10\right) \end{aligned}$ |
| 2 | $\begin{aligned} & \omega=2 \times p i / 5, \theta=+10, \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{5}+10\right) \end{aligned}$ | $\begin{aligned} & A=1, \omega=2 \times p i / 10, \theta=+10 \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{10}+10\right) \end{aligned}$ |
| 3 | group $1+$ group 2 |  |
| 4 | $\begin{aligned} & A=0.4, \omega=2 \times p i / 20, \theta=+0 \\ & 0.4 \times \sin \left(2 \times 3.14159 \times \frac{1: 30}{20}\right)+0.6 \end{aligned}$ | random |
| 5 | $\begin{gathered} A=1.5, \omega=2 \times p i / 40, \theta=+10 \\ 1.5 \times \sin \left(2 \times 3.14159 \times \frac{1: 30}{20}+10\right) \end{gathered}$ |  |
| 6 | $\begin{aligned} & A=1, \omega=2 \times p i / 40, \theta=+10 \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{40}+10\right) \end{aligned}$ | random |
| 7 | $\begin{aligned} & A=0.8, \omega=2 \times p i / 10, \theta=+0 \\ & 0.8 \times \sin \left(2 \times 3.14159 \times \frac{1: 30}{10}\right) \end{aligned}$ | $\begin{aligned} & A=1, \omega=2 \times p i / 3, \theta=+0 \\ & \sin \left(2 \times 3.14159 \times \frac{1: 30}{3}\right) \end{aligned}$ |
| 8 | $(\mathrm{i}, \mathrm{j}) \rightarrow \mathrm{i}=\frac{\mathrm{j}}{15}$ | $(\mathrm{i}, \mathrm{j}) \rightarrow \mathrm{i}=\frac{-j}{15}$ |
| 9 | $(\mathrm{i}, \mathrm{j}) \rightarrow \mathrm{i}=\frac{j^{2}}{1000}$ | $(\mathrm{i}, \mathrm{j}) \rightarrow 0.5$ |
| 10 | random | $\begin{aligned} & A=2, \omega=2 \times p i / 40, \theta=+20 \\ & 2 \times \sin (2 \times p i \times(1: 30) / 40+20)+0.5 \end{aligned}$ |
| 11 | $($ group $7+$ group 8)/10 |  |
| Noise |  | ndom |

## Appendix C

Graphs, Chapters 4, 5, AND 6

## Chapter 4 - Method Analysis

## Hierarchical Clustering - Internal Analysis



Figure C.1: Hierarchical Assessment (Single) using Internal measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. The majority of indices for 'real' datasets remain constant across all $K$, however the Cho and Stegmaier datasets reveal information.


Figure C.2: Hierarchical Assessment (Average) using Internal measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. Similarly to all linkage methods, the Silhouette index is $<0$ for real datasets, indicating a bad clustering.


Figure C.3: Hierarchical Assessment (Complete) using Internal measures. The xaxis indicates the cluster number while the $y$-axis indicates the score achieved obtained. The Dunn index indicates a better clustering in Synthetic datasets compared to 'real' datasets.


Figure C.4: Hierarchical Assessment (Ward) using Internal measures. The x -axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. As with all linkage methods, the SD-Validity index increases with $K$, (small values indicate a better clusterings.)


Figure C.5: Hierarchical Assessment using C-Index. The $x$-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. Contrary to other assessment metrics, this index indicates a better structure in the 'real' datasets compared to Synthetic (small values indicate ${ }^{2} 0$ better clustering).

## Hierarchical Clustering - Stability Analysis



Figure C.6: Hierarchical Assessment (single linkage) using AD,ADM, APN and FOM stability Measures. The $x$-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.


Figure C.7: Hierarchical Assessment (average linkage) using AD, ADM, APN and FOM stability Measures. The $x$-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. The APN index reveals a marked deterioration in stability with the Cho dataset as the value of $K$ increases. AD and FOM indices reveal little information.


Figure C.8: Hierarchical Assessment (complete linkage) using AD, ADM, APN and FOM stability Measures. The x -axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.


Figure C.9: Hierarchical Assessment (Ward linkage) using AD, ADM, APN and FOM stability Measures. The $x$-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. The APN values in Ward and Complete linkage increase exponentially with $K$, compared to a linear increase observed with Single and Average linkage.

## Hierarchical Clustering - External Analysis



Figure C.10: Hierarchical Assessment (Single and Complete linkage) using Biological Homogeneity and Biological Stability Measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.


Figure C.11: Hierarchical Assessment (Average and Ward linkage) using Biological Homogeneity and Biological Stability Measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. The BSI stability indices exponentially decrease in Ward and Complete linkage, while the linearly decrease with Single and Average linkage.

## KMeans - Internal Analysis



Figure C.12: K-means Assessment using Internal measures. The x -axis indicates the cluster number while the $y$-axis indicates the score achieved obtained. Better clustering are obtained for the Synthetic datasets when assessed with Dunn metric (larger values preferred). Again, SD-Validity increases with $K$ (smaller values preferred).


Figure C.13: K-means Assessment using Internal measures c-index and connectivity. The x -axis indicates the cluster number while the y -axis indicates the score achieved obtained.

## KMeans - Stability Analysis



Figure C.14: K-means Assessment using Stability measures. The x -axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.

## SOTA - Internal Analysis



Figure C.15: SOTA Assessment using Internal measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.


Figure C.16: SOTA Assessment using Internal measure - c-index. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.

## SOTA - Stability Analysis



Figure C.17: SOTA Assessment using Stability measures. The x-axis indicates the cluster number while the $y$-axis indicates the score achieved obtained.

## Chapter 5

## Node Degree -Bipartite SubGraphs



Figure C.18: Gene Degree Distribution of Alizadeth, Alon, Cho and Gasch subgraphs


Figure C.19: Gene Degree Distribution of Golub, Hsiao, Spellman, Stegmaier and West subgraphs


Figure C.20: Sample Degree Distribution of Alizadeth, Alon, Cho and Gasch subgraphs


Figure C.21: Sample Degree Distribution of Golub, Hsiao, Spellman, Stegmaier and West subgraphs

Node Degree Distributions -Bipartite All In One Graphs


Figure C.22: Degree Distribution of Alizadeth, Alon, Cho and Gasch datasets


Figure C.23: Degree Distribution of Golub, Hsiao, Spellman, Stegmaier and West datasets

## Chapter 6

## Node Degree Distributions



Figure C.24: Degree Distribution of Alizadeth, Alon, Cho and Gasch datasets


Figure C.25: Degree Distribution of Golub, Hsiao, Spellman, Stegmaier and West datasets

## Degree Distribution One Mode



Figure C.26: Degree Distribution of Alizadeth, Alon, Cho and Gasch datasets


Figure C.27: Degree Distribution of Golub, Hsiao, Spellman, Stegmaier and West datasets

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## Computers in Biolog and Medicin

# Techniques for clustering gene expression data 

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## Abstract

Many clustering techniques have been proposed for the analysis of gene expression data obtained from microarray experiments. However hoice of suitable method(s) for a given experimental dataset is not straightforward. Common approaches do not translate well and fail to take of the data profile. This review paper surveys state of the art applications which steh, it provides a framework for the evaluation of clustering in ge © 2007 Elsevier Ltd. All rights reserved.

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## 1. Introduction

Searching for meaningful information patterns and dependencies in gene expression (GE) data, to provide a basis fo hypothesis testing, is non-trivial. An initial step is to cluster or "group" genes, with similar changes in expression. Lack a priori knowledge means that unsupervised clustering tech niques, where data are unlabeled (un-annotated), are commo in GE work. These are an exploratory techniques and assum that there is an unknown mapping that assigns a group "label to each gene, where the goal is to estimate this mapping. How ever, common clustering approaches do not always translat well to GE data, and may fail significantly to account for data profile.

Many excellent reviews of GE analysis, using clustering tech niques, are available. Asyali et al. [1] provide a synopsis of class prediction and discovery (respectively, supervised pattern recognition and clustering), while Pham et al. [2] provide a comprehensive titerature review of the various stages of dat analysis durng a microaray expermen. In a lan mak paper Jain et al. [3] provide a thorough introduction to clustering, and

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give a taxonomy of clustering algorithms (used in this review) Reviewing the state of the art in GE analysis is complicat by the high level of interest in the field, and the many tech niques available. This review aims to evaluate modification to currently used techniques which address shortcomings of conventional approaches and special properties of GE data
GE data are typically presented as a real-valued matrix, with row objects corresponding to GE measurements over a number of experiments, and columns corresponding to the patter of expression of all genes for a given microarray experiment Each entry, $x_{i j}$, is the measured expression of gene $i$ in ex periment $j$. Dimensionality of a gene refers to the number its expression values recorded (number of matrix columns). gene/gene profile $x$ is a single data item (row) consisting $d$ measurements, $x=\left(x_{1}, x_{2}, \ldots, x_{d}\right)$. An experiment/sample $y$ is a single microarray experiment corresponding to a single column in the GE matrix, $y=\left(x_{1}, x_{2}, \ldots, x_{n}\right)^{\mathrm{T}}$ where $n$ is the number of genes in the dataset.
Accuracy of GE data strongly depends on experimental design and minimisation of technical variation, whether due to instruments, observer or pre-processing [4]. Image corruption and/or slide impurities may lead to incomplete data [5]. Many clustering algorithms require a complete matrix of input values, so imputation (missing data estimation) techniques nee to be considered before clustering. GE data are intrinsically
noisy, resulting in outliers, typically managed by: (i) robust statistical estimation/testing (when extreme values are not of primary interest) or (ii) identification (when outlier information is of intrinsic importance [6]. As cluster analysis is usually exploratory, lack of a priori knowledge on gene groups or their number, $K$, is common. Arbitrary selection of this number may undesirably bias the search, as pattern elements may be illdefined unless signals are strong. Meta-data can guide choice of correct $K$, e.g. genes with common promoter sequence are likely to be expressed together and thus are likely to be placed in the same group. Methods for determining optimal number of groups, $K$, are discussed in [7,8].
Clustering a GE matrix can be achieved in two ways: (i) genes can form a group which show similar expression across conditions and (ii) samples can form a group which show simila expression across all genes. Both (i) and (ii) lead to global clusters, where a gene or sample is grouped across all dimensions. rither gin she che
 are defined over a subset of genes and a subset or samples thus of biclutering as cellular proses a ie understod to rely on of bi-clustering as cellular processes are understood to rely on der certain conditions and behave independently under others [91. Jutifibly, this aproach ha been ining mod inters
 of late. For an excellent see [10]
Additionally, clustering can be complete or partial, where he former assigns each gene to a cluster, and the latter does
 (i) "noisy genes" to be left out, with correspondingly less impact an the outcome and (ii) genes to belong to no cluster-omitting large number of irrelevant contributions. This is important as microarrays measure expression for the entire genome in one experiment, but genes may change expression independent of experimental condition (e. $g$ due to stage in the cell cycle), Forced inclusion (as demanded by complete clustering) in well defined but inappropriate groups may impact final structure found for the data. Partial clustering thus avoids the situation here an interesting sub-group in a cluster is obscured through forcing membership of unrelated genes.
Finally, clustering can be categorised as exclusive (hard) or Verlapping. Exclusive clustering requires each gene to belong to a single cluster, whereas overlapping clusters permit genes simultaneously to be members of numerous clusters. An additional qualification is crisp and fuzzy membership. Crisp membership is boolean-either the gene belongs to a group or not In the case of fuzzy membership, each gene belongs to a cluster with a membership weight between 0 (definitely excluded) and 1 (definitely included). Clustering algorithms, which permit genes to belong to more than one cluster are typically more applicable to GE since: (i) impact of "noise" is reduced-the assumption is that "noisy" genes are unlikely to belong to any one cluster but are equally likely to be members of several; (ii) this supports the underlying principle that genes, with similar change in expression for a set of samples, are involved in
a similar biological function. Typically, gene products that are volved in several such biological functions and groups need not be co-active under all conditions. Thus gene groups are fluid and constraining a gene to a single group (hard cluster) is ounter-intuitive.
Cluster analysis includes several basic steps [3]. Initially, the ata matrix is represented by number, type, dimension and scale of the GE profiles. Some features are set experimentally, others are controllable (e.g. scaling, imputation, normalisation, etc.). An optional step of feature selection or feature extraction may also be carried out. The former refers to selecting, from the riginal features, a subset, which is most effective for clustering, while the latter refers to transformation of the input features to form a new set that may be more discriminatory in clustering, .g. through principal component analysis.
Pattern proximity assessment is needed, usually provided by a "distance" measure between pairs of genes. (Alternatively, conceptual measures can be used to characterise similarity of gene profiles, e.g. Mean Residue Score of Cheng and Church (see Section 2)). The next step is to apply a clustering algorim detenme stur in dhe dase. egorised according to taxonomy [3]
hose structures are then described by data abstraction. For able therpretation by a human, o abstracion should idealy be straghtorward (for of luster, through pithe rest s, hers cannot reasonably be achieved by chance or as an artefact of e clustering algorithm. Validation requires formal statistical esting, and can be categorised as: (i) internal, (ii) external (iii) relative. The fous here is on proximity mas and clustering algorithms, within the wider analysis context.

## 2. Clustering methods

Analysis of large GE datasets is a relatively new task, although pattern recognition of complex data is well established in a number of fields. Many common generic algorithms have, in consequence, been adopted for GE data (e.g. hierarchical [11], SOMs [12] and others), but not all perform well. A good method must deal with noisy high dimensional data, be inensitive to the order of input, have moderate time and space complexity (i.e. allow increased data load without breakdown or requirement of major changes), require few input paramers, incorporate meta-data knowledge (an extended range of biological context.
2.1. Pattern proximity measures

The choice of proximity measure, needed to evaluate degree of expression coherence in a group of gene vectors, is as important as choice of clustering algorithm, and is based on data type and context of the clustering. Many clustering algorithms either employ a proximity matrix directly (e.g. hierarchical clustering) or use one to evaluate clusters during execution
(e.g. $K$-Means). Proximity measures are calculated betwee pairs (e.g. Euclidean distance) or groups of genes (e.g. Mean Residue Error).
Distance functions between two vectors include the so-called Minkowski measures (Euclidean, Manhattan, Chebyshev [13]) useful when searching for exact matches between two profiles in the dataset. These tend to find globular structures and work well when these are compact and isolated. A drawback is th the largest feature dominates, so measures are sensitive to out liers [3]. However, more sophisticated variants, such as Maha lanobis distance, also account for correlations in the dataset and are scale-invariant [13]. Different distance measures produce clusters of different shape (e.g. Euclidean are spherical, while Mahalanobis are ellipsoidal). Alternatively Kim et al. [14 describe an adaptive distance norm (the Gaustafson-Kesse mede. Here covainces are estimated for he dan in each
 Dituser bed created using a uique distance measure. Distances based on correlations reflect degree of similarity without regard to scale. For sample, if, for les $X$ is up $X$. ples, gene $X$ is up-regulated, and gene $Y$ is down-regulated would clearly not be if Minkowski distaces were used sine the average absolute distance between the points would be large. Correlation coefficients include both parametric (stan dard Pearson, cosine) and non-parametric (Spearma, and Kendall's $\tau$ ) the latter used when outliers and noise ar present [13] In geral distance $=1-$ correlation $^{2}$, if sign is present [13]. It unimportant
As an alternative to measures of distance, "conceptual" measures of similarity can be used. Models are based on constan [10] (Fig. 1). A "good fit" indicates high correlation within a sub-matrix (thus a possible cluster). These models are com mon to several clustering algorithms. For example, Cheng and Church [15] and Flexible Overlapped biclustering (FLOC) [16] use the additive model (Fig. 1(C)), to evaluate bi-clusters ob tained by determining the Mean Residue Score Given a GE matrix $A$, the residue of an element $a_{i j}$ in a sub-matrix ( $I J$ is given by the difference $r_{i j}=\left(a_{i j}-a_{i J}-a_{I j}+a_{I J}\right)$, where $a_{i j}, a_{i J}, a_{I j}$ and $a_{I J}$ are the sub-matrix value, the row, column and group mean, respectively. The " $H$-score" of the sub-matrix is then the sum of the residues, given by
$H(I, J)=\frac{1}{|I||J|} \sum_{i \in I j \in J}\left(r_{i j}\right)^{2}$.
A perfect bi-cluster gives a $H$-score equal to zero (correspondgen data, with constant additive matrix rows and

The Plaid Model [17] bi-cluster variant builds the GE matrix as a sum of layers, where each layer corresponds to a bi-cluste Each value $a_{i j}$ is modelled by $a_{i j}=\sum_{k=0}^{K} \theta_{i j k} \rho_{i k} \kappa_{j k}$, where $K$ is the layer (bi-cluster) number, and $\rho_{i k}$ and $\kappa_{j k}$ are binary variables representing membership of row $i$ and column $j$ in layer $k$. The value of an element in the GE matrix can be modelled as a linear function of the contributions of the different
bi-clusters to which the row $i$ and the column $j$ belong (Fig. 2) [17]. For layer $k$, expression level $\theta_{i j k}$ can be estimated using the general additive model, $\theta_{i j k}=\mu_{k}+\alpha_{i k}+\beta_{j k}$ in layer $k$ (Fig. 1(C)).
For the coherent evolution model the exact values of $x_{i j}$ are not directly taken into account, but a cluster is evaluated to see if it shows coherent patterns of expression. In simplest model form, each GE value can have three states: up-regulation, downregulation and no change. Thresholds between states are crucial and additional complexity results from extending model definitions to include further states such as "slightly" up-regulated, "strongly" up-regulated and so on, e.g. Statistical Algorithmic Method for Bi-Cluster Analysis (SAMBA) [18].
Other measures used to evaluate coherency of a group of genes include conditional entropy: $H(C \mid X)=-\int \sum_{j=1}^{m}$ $p\left(c_{j} \mid x\right) \log p\left(c_{j} \mid x\right) p(x) \mathrm{d} x$ (the average uncertainty of the random variable $C$ (cluster category), when a random variable $X$ (GE profile) is known). The optimal partition of the pe dataset is obtained when wis entopy is mimised, i.e. a partition where each gene is assigned with a high probability
 a posteriori probabilities $p\left(c_{j} \mid x\right)$, usually by non-parametric underlying GE data [19]. rancing GE data [19].
Pattern proximity measures described so far make no dis tinction between time-series data and those obtained from express to sion time series have non-uniform intervals and are usually very shot ( $4-20$ smples while clasically even 50 observations are low for statistical inference), futher data are not indepa dently identically distributed Similarity in time series should deny, identialy dstrbated. Similary in ime series should change across time points, while robust measures must allow change across tine por whity for non-une (internal structure of clusters) [20] and (inthe sturture of clusters) [20].
Eace of proximity meas one choice of proximity measure and inherits the limitations of that choice.

### 2.2. Agglomerative clustering

All agglomerative techniques naturally form a hierarchical cluster structure in which genes have a crisp membership. Eisen et al. [11] studied GE in the budding yeast, Saccha-
rmyces cerevisiae, using hierarchical methods which have been popularised due to ease of implementation, visualisation capability and availability. Methods vary with respect to choice of distance metric, decision on cluster merging, (linkage), as well as parameter selection affecting structure and relationship between clusters. Options include: single linkage (cluster separation as distance between two nearest objects), complete linkage (as previously, but between two furthest objects), average linkage (average distance between all pairs), centroid (distance between centroid's of each cluster) and Ward's method (which minimises ANOVA Sum of Squared Errors between two clusters) [21].

A


C

| $\left[\begin{array}{cccc}1 & 4 & 8 & 9 \\ 3 & 6 & 10 & 11 \\ 6 & 9 & 1 & 14 \\ 9 & 21 & 14\end{array}\right]$ |  |
| :---: | :---: | :---: | :---: |
| $\left.\qquad \begin{array}{llll}1 & 7 & 7\end{array}\right]$ |  |
| $\left.\begin{array}{llll}1+0+0 & 1+0+3 & 1+0+7 & 1+0+8 \\ 1+2+0 & 1+2+3 & 1+2+7 & 1+2+8 \\ 1+6+0 & 1+6+3 & 1+6+7 & 1+6+8 \\ +8+0 & 1+8+3 & 1+8+7 & 1+8+8\end{array}\right]$ |  | Fig. 1. Models for bi-clusters. (A) Bi-cluster with constant rows. Each row is obtained from a typical value $\mu$ and row offset $\alpha_{i}$. (B) Constant columns. Each

value is obtained from a typical value $\mu$ and column offset $\beta_{j}$. (C) Additive model. Each value is predicted from $\mu$, and a row and column offset, $\alpha_{i}+\beta_{j}$. Similar model constructs apply for the multiplicative case with ( $\mathrm{A}(\mathrm{i})) \mu \times \alpha_{i}$, (B(i)) $\mu \times \beta_{j}$ and (C(i)) $\mu \times \alpha_{i} \times \beta_{j}$.


Fig. 2. Plaid Model GE values at overlaps are seen as a linear function of different bi-clusters

Distance and linkage determine level of sensitivity to noise: Ward's and Complete method are particularly affected (due to the ANOVA basis and outlier importance respectively, since clustering decisions depend on maximum distance between two genes). Single linkage forces cluster merger, based on minimum distance, regardless of other gene contributions to the cluster, so noisy or outlying values are among the last to be considered. Consequently, the "chaining phenomenon" may arise [13]. For commonly used average and centroid linking this problem is clusters are no special consideration
Results for agglomerative clustering may be intuitively presented by dendograms but there are $2^{n-1}$ different linear orderings consistent with tree structure, so care is needed in pruning. Dendrogram analysis, based on gene class information from specialised databases is presented in [22], where optimal correlations are obtained between gene classes and used to form clusters from different branch lengths. In [23] authors present
an agglomerative technique for which each internal node has most $N$ children, allowing up to $N$ genes (or clusters) to be directly connected (extending traditional hierarchical concepts and reducing the effects of noise). Permutation is used to decide on the number of nodes $(\max N)$ to merge, based on a similarity threshold. Heuristically, algorithm complexity is comparable to traditional hierarchical clustering [23], although the authors also present a "divide and conquer" approach for optimal leaf ordering for small $N$, which has implications of increased time and space complexity
It should be stated that such methods cannot, in general, compensate for the greedy nature of the traditional algorithm, where misclustering at the beginning cannot be corrected at a ater stage and are magnified as the process continues. Further, Yeung et al. [24] and Gibbons and Roth [25] note that hierarchical clustering performance is close to random, despite its popularity and is poorer than other common techniques such as $K$-means and self-organising maps (SOM).

### 2.3. Partitive techniques

Partitive clustering divides data by similarity measure, wher typical methods measure distance from a gene vector to a prototype vector representing the cluster, and intra-cluster/inter cluster distance are, respectively, maximised and minimised. A major drawback is the need to specify the number of cluste in advance. Table 2 summarises algorithms discussed here.
$K$-means produces crisp clusters with no structural relation ship between these [26]. It deals poorly with noise, since outliers must belong to a cluster and this distort the means. Equally cluster inclusion is dependent on the cumulative values of gene already present, so order matters. Results are dependent on initial cluster prototype (which varies between clustering at tempts), this leads to imstabily and, frequeny, to a local minimum solution. Incremental approaches to refine local minim Golutions $K$ cose towards a global solution include he Modifie partitions of the data using $k-1$ clusters from previous itera pions A of data number of clusters indirectly, and with regular $K$ mean return sphercal cluster. For he, 10 whed [27], the MGKM algorithm showed slight improvemt over $K$-means, MGKM algorithm showed sligh inprent but at higher computational time cost.
he prevalence of local minima for $K$-means is linked to ini ther (GAs), as an evolu tonary approach, work well for small datasets (less hin time genstraints for anything larger so are less desirable for GE anal ysis. Although GAs find the global optimum, they are senitive to user defined input parameters and must be fine tued for each specific problem. Studies which have combined $K$-means an
 [28]. This is a hybrid approach which converges to a globa [28]. This is a hybrid approach which converges to a globa ity to initialisation prototypes. The fitness function for the GA is based on Total Within Cluster Variance (TWCV), while the basis of the algorithm is to cluster centroids incrementally, us ing a standard similarity measure The GA method requires the number of output clusters, $K$, to be specified, but is further complicated by inherent GA parameters (mutation probability rate number of generations, size of the chromosome populations, etc.), which influence time taken by the algorithm to converg to a global optimum.
Fuzzy modifications of $K$-means include Fuzzy $C$-Means (FCM) [29] and Fuzzy clustering by Local Approximations of MEmberships (FLAME) [30]. In both, genes are assigned a cluster membership degree indicating percentage association with that cluster, but the two algorithms differ in the weighting scheme used to determine gene contribution to the mean For a given gene, FCM membership value of a set of cluster is proportional to its similarity to cluster mean. The contribu tion of each gene to the mean of a cluster is weighted, base on its membership grade. Membership values are adjusted it eratively until the variance of the system falls below a threshold. These calculations require the specification of a degree of fuzziness parameter which is problem specific [29]. As with
$K$-means, clusters are unstable, and considerably influenced by nitial parameter values, while $K$, the number of clusters, mu be specified a priori. In contrast FLAME requires member ship of a cluster, $i$, to be determined by the weighted similarit of the gene to its $K$-nearest neighbours, and their membership of cluster $i$. This density-based approach further reduces noise impact, since genes with a density lower that a pre-define threshold are categorised as outliers, and grouped with a ded icated "outlier" cluster. FLAME produces stable clusters, but the size of the neighbourhood and the weighting scheme used affect $K$ (as above) and clustering achieved. For both FCM and FLAME, genes may have multiple and varied degrees of menbershe, bat interpretaion diffes. F and FLAME veraging, where each gene contributes to the calculation or (ier cenroid, and is overall membership value set sums 1 (i.e. gene-chuster probability). Thus strong membership for given gene does not indicate it to be more typical of the cusser Table 1 illu stan
hle 1 . published yeast genomic expression data [32], resuls are avail Mertp: cluster 21, although both a mex the centrid of cluter Similaly genes C id have
 typical (closer to the centroid) than gene D. With similar cen tid distes, mer 1
 arise from the mernip gene membership in one cluster to increase it in another List ing genes in a cluster based on membership values is therefore counter-intuitive and does not reflect their compatibility with the cluster, but rather how they are shared between clusters. Similarly for FLAME as the memberships are weighted rela tive to the $K$-nearest neighbours, a low membership value in dicates a high degree of cluster sharing among these and not a more typical value of a given cluster. This interpretative flaw was recognised by Cano et al. [33], who developed the pos sibilistic biclustering algorithm, which removes the sum rule restriction. The authors used spectral clustering principles [34] to create from the original GE matrix, a partition matrix, Z, to which possibilistic clustering is applied. The resulting clusters were evaluated using the $H$-score (Eq. (1)), and improved on traditional techniques. The algorithm requires, inter alia, two specific parameters, namely cut-off membership for (i) gene inclusion and (ii) a sample inclusion in a cluster. In this case, these cut-offs are intuitively reasonable as membership does indicate how typical a gene/sample is to a defined cluster, and not the degree to which it is shared between clusters (Table 2).
2.4. Neural networks

Neural networks (NN), loosely based on the biological parallel, can be modelled as a collection of nodes with weighted interconnections. Only numerical vectors are processed so meta-information cannot be included in the clustering

Membership of a gene and distance to cluster centroid, as calculated by Euclidean distance

| GID | Cluster 4 |  | Cluster 21 |  | Cluster 46 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Centroid dist. | Mem. | Centroid dist. | Mem. | Centroid dist. | Mem. |
| A | 10.691 | 0.002575 | 8.476 | 0.002002 | 3.864 | 0.482479 |
| в | 6.723 | 0.009766 | 3.855 | 0.009341 | 6.33 | 0.007381 |
| C | 6.719 | 0.007653 | 5.29 | 0.00515 | 8.024 | 0.005724 |
| D | 7.725 | 0.007609 | 3.869 | 0.01782 | 6.279 | 0.010249 |

Table 2
Summary of partitive techniques

|  | Cluster mem. | Input | Proximity | Other |
| :---: | :---: | :---: | :---: | :---: |
| $K$-means | Hard | Starting prototypes, <br> stopping <br> threshold, $K$ | Pairwise distance | Very sensitive to input parameters and order of input |
| MGKM | Hard | Tolerance threshold | Pairwise distance | Not as sensitive to starting prototypes $K$ specified through tolerance threshold |
| IGKA | Hard | $K$, mutation prob. generation number, population size | TWCV | Time taken to converge to global influenced by parameters |
| FCM | Fuzzy | Degree of fuzziness starting prototypes, stop threshold, $K$ | Pairwise distance | Careful interpretation of membership values. Sensitive to input parametres and order of input |
| flame | Fuzzy | $K_{\mathrm{nn}}-\text { number of }$ neighbours | Pairwise distance to $K_{\text {nn }}$ neighbours | Careful interpretation of membership values. Output determined by $K_{\mathrm{nn}}$ |
| Possibilistic biclustering | Fuzzy | Cut-off memberships, max. residue, number of rows and number of columns | H-score | Number of bi-clusters determined when quality function peaks by re-running for different numbers of eignevalues |

rocedure. Interconnection weights are adaptively learned, i.e features are selected by appropriate assignment of weights. In particular, SOMs, a type of NN, have proved popular for GE $[12,35,36]$. A kernel function, which defines the region of influence (neighbourhood) for an input gene, distinguishes SOM from $K$-means. Updating the kernel function causes the output node and its neighbours to track towards the gene vector. The network is trained (adjusting strengths of interconnections) from a random sample of the dataset. Once training is complete, all genes in the dataset are then applied to the SOM. Cluster members, represented by output node $i$, are the set of genes causing $i$ to "fire" (hard clustering).

SOMs are robust to noise and outliers, dependent on distance metric and neighbourhood function used. As for $K$-means, an SOM produces a sub-optimal solution if the initial weights for the interconnections are not chosen properly. Convergence is controlled by problem-specific parameters such as learning rate and neighbourhood function. A particular input pattern can fire different output nodes at different iterations (while this
be overcome by gradually reducing the learning rate to ero during training, it can result in over fitting, which leads to poor performance for new data). In specifying $K$, based on the number of output nodes, it should be noted that too few output nodes in the SOM gives large within-cluster distance, while too many results in meaningless diffusion.
The Self-Organising Tree Algorithm (SOTA) [37], Dynamically Growing Self-Organising Tree (DGSOT) Algorithm [38] and, more recently, Growing Hierarchical Tree SOM (GHTSOM) [39] were developed to combine strengths of NN (i.e. speed, robustness to noise) and hierarchical clustering (i.e. tree structure output, minimum a priori requirement for number of lusters specification and training) to deal with properties of GE ata. Here the SOM network is a tree structure, trained by comparing only leaf nodes to input GE profiles (each graph node represents a cluster). SOTA and DGSOT result in a binary and $n$-tree structure, respectively, while in GHTSOM, each node is triangular SOM (three neurons, fully connected), each having three daughter nodes (also triangular SOMs) (Fig. 3). Tree

A
B
C


Fig. 3. Self-rganising tree structures: (A) SOTA. A binary tree structure. Neighbourhood of adaption indicated for (i) node with sibling, (ii) node with no
sibling. (B) DGSOT. N-ary tree structure. Neighbourhood of adaption indicated when $L=2$. (C) GHTSOM. Each node represented by triangular SOM. Each layer indicated with line styles (three layers shown).

Summary of neural network techniques presented

|  | Structure | Proximity | Input | Other |
| :---: | :---: | :---: | :---: | :---: |
| Som | None | Distance | Number of output neurons, learning rate | Careful consideration of initialisation weights |
| SOTA | Binary tree | Distance | Threshold $\beta$ |  |
| DGSOT | N -ary tree | Distance | Thresholds $\beta, \alpha$ and $L$. | Corrects for misclusterings |
| GHTSOM | Each node <br> triangular SOM, arranged in tree structure | Distance | Minimal requirement - learning rate |  |

replacement will interfere with the discovery of future biclusters [16]. To overcome this problem of random interference, flexible overlapped biclustering (FLOC) was developed-a generalised model of Cheng and Church incorporating null values [16]. FLOC constrains the clusters to both a low mean residue score and a minimum occupancy threshold of $\alpha$, $0 \leqslant \alpha \leqslant 1$ (user defined). Note: this method does not require pre-processing for imputation of missing values. Both these bi-clustering algorithms find coherent groups (Section 2.1) in the data and permit overlapping.
The Plaid Model [17] (Section 2.1) assumes that bi-clusters can be generated using a statistical model and aims to identify the parameter distribution that best fit the available data, by minimising the error sum of squares for the $k$ th bi-cluster assuming that $k-1$ bi-clusters have already been identified. Explicitly, it seeks to minimise for the whole matrix: $Q=$ $\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m}\left(Z_{i j}-\theta_{i j k} \rho_{i k} \kappa_{j k}\right)^{2}$, where $Z_{i j}$ is the residual after deducting $k-1$ previous layers ( $Z_{i j}=a_{i j}-\sum_{k=0}^{K-1} \theta_{i j k} \rho_{i k} \kappa_{j k}$ ). Parameters $\theta_{i j k}, \rho_{i k}$ and $\kappa_{j k}$ are estimated for each layer and for each value in the matrix, and are updated iteratively, providing refined estimates of $\mu_{k}, \alpha_{i k}$ and $\beta_{j k}$ (Fig: 1(C)) and $\rho_{i k}$ and $\kappa_{j k}$ to minimise $Q[17]$ (Table 4).

The importance of a layer is defined by $\delta_{k}^{2}=\sum_{i=1}^{n} \sum_{j=1}^{p}$ $\rho_{i k} \kappa_{j k} \theta_{i j k}^{2}$. To evaluate the significance of the residual matrix, $Z$ is randomly permuted and tested for importance. If $\delta_{k}^{2}$ is significantly better than $\delta_{\text {random }}^{2}, k$ is reported as a bi-cluster. The algorithm stops when the residual matrix $Z$ retains only noise, with the advantage that the user does not need to specify the number of clusters beforehand.
2.6. Graph theoretic methods

Graph theoretic approaches have recently gained ground in analysing large complex datasets. The Cluster Affinity Search Technique (CAST) [43] models data as an undirected graph, $G=(V, E)$, where $\{V, E\}$ is the set of $\{$ vertices, edges $\}$ representing \{genes, similar expression\}. The model assumes that there is an ideal clique graph (a disjoint union of complete sub-graphs), $H=(U, E)$, which represents the ideal input GE dataset, while data to be clustered are a "contamination" of the ideal graph $H$ by random errors. In a clique graph each clique
represents a cluster. For a pair of genes in $G$, the model asumes that an edge/non-edge was assigned incorrectly, with a probability of $\alpha$. The true clustering of $G$ is assumed to be that which requires fewer edge changes to generate $H$. CAST uses an affinity (similarity) measure, either binary or real valued, to assign a vertex to a cluster. Affinity to a cluster must be above a threshold, $t$ (user defined which determines size and number of clusters). The affinity of a vertex $v$ to a cluster is the sum of affinities over all objects currently in the cluster, so $v$ has high ffinity with $i$ if affinity $(x)>t|i|$, and low affinity otherwise. The CAST algorithm alternates between adding high affinity elements and removing low affinity elements, finding clusters one at a time. The result is dependent on the order of input as once initial cluster structure is obtained, a vertex $v$ is moved to hat cluster for which it has a higher affinity value.
CLICK [44], builds on the work of Hartuv et al. [45], which uses a probabilistic model for edge weighting. Pairwise similarity measures between genes are assumed to be normally distributed: between "mates" $\left(N\left(\mu_{T}, \sigma_{T}^{2}\right)\right)$, and between "non-mates" $\left(N\left(\mu_{F}, \sigma_{F}^{2}\right)\right)$, where $\mu_{T}>\mu_{F}$. These parameters can be estimated via Expectation Maximisation methods [46]. The weight of an edge is derived from the similarity measure between the two gene vectors, and reflects the probability that $i(\in V)$ and $j(\epsilon V)$ are mates, specifically that: $w_{i j}=\log \frac{p_{\text {mates }} \sigma_{F}}{\left(1-p_{\text {mases }} \sigma_{T}\right.}+\frac{\left(S_{i j}-\mu_{\mu}\right)^{2}}{2 \sigma_{T}^{2}}-\frac{\left(S_{i j}-\mu_{T}\right)^{2}}{2 \sigma_{T}^{2}}$. Edges with weights below a user defined non-negative threshold are omited from the graph. The graph is partitioned using a minimum weight cut algorithm [45].
The SAMBA method finds bi-clusters based on the coherent evolution model (Section 2.1) [18]. Firstly, the GE matrix is modelled as a bipartite graph, $G=(U, V, E)$, where $U$ is he set of sample vertices, $U \cap V=\varnothing$ and an edge ( $u, v$ ) only exists between $v \in V$ and $u \in U$ iff there is a signifiant change in expression level of gene $v$, w.r.t. to its normal evel, in sample $u$. Key to SAMBA is the scoring scheme for a bi-cluster, corresponding to its statistical significance, where a weight is assigned to a given edge $(u, v)$ based on the loglikelihood of getting that weight by chance [18], $\left(\log \frac{P_{c}}{P_{c}}>0\right.$ for edges and $\log \frac{\left(1-P_{c}\right)}{\left(1-P_{(u, v)}\right)}<0$ for non-edges). The probability $P_{(u, v)}$ is the fraction of random bipartite graphs, with degree sequence identical to $G$ that contain edge ( $u, v$ ) (and can be

Table 4
Summary of search-based techniques presented

|  | Proximity | Deterministic/stochastic | Clusters | Other |
| :---: | :---: | :---: | :---: | :---: |
| SA | Depends on application | Stochastic | Depends on application | Specification of annealing Schedule |
| CC | Additive model | Deterministic | Overlapping, partial bi-clusters | $\delta$, random interference |
| floc | Additive model | Deterministic | Overlapping, partial bi-clusters | $\alpha$ and $\delta$ <br> to specify. Overcomes random interference, allows missing values |
| Plaid | Additive model | Deterministic | Overlapping, partial bi-clusters | Values seen as sum of contributions to bi-clusters |

Table 5

|  | Mode | Proximity | Search | Other |
| :---: | :---: | :---: | :---: | :---: |
| CAST | One mode | Similarity | $\begin{aligned} & \text { Clique } \\ & \text { graph } \end{aligned}$ | Parameters $\alpha$ and $t$. Finds global, complete, crisp clusters |
| CLICK | One mode | Distribution based on distance | Minimum weight cut | Stat. sig. of clusters. EM to estimate parameters. Finds global, partial, crisp clusters |
| SAMBA | Bi-partite | Probability | Heuristic search of neighbours | Stat. sig. of clusters. Input $P_{\mathrm{c}}$ difficult to define. Finds partial overlapping bi-clusters |

estimated using Monte Carlo methods). $P_{\mathrm{c}}$ is a constant prob estimited $\rightarrow$ max $P_{\text {a }}$. Assigning these weights to the edges and non-edges in the graph, the statistical significance edges and non-edges in the graph, the statistical significance weight) sub-graphs for each vertex in $G$ found. The authors [18] present two ways to calculate the weight of the resulting sub graph. In the simpler model, bi-clusters, which reflect changes relative to normal expression level, without considering direc tion of change are sought. The second model focuses on con sistent bi-cliques, targeting those samples which have the same or opposite effect on each of the genes (Table 5)

## 3. Discussion

Despite shortcomings, application of clustering methods to GE data has proven to be of immense value, providing in sight into cell regulation, as well as into disease characterisation. Nevertheless, not all clustering methods are equally valuable for high dimensional GE data. Recognition that wellknown, simple clustering techniques, such as $K$-means and hierarchical clustering, do not capture complex local structure, has led to investigation of other options. In particular bi-clustering has gained considerable recent popularity. Indications to date are that these methods provide increased sensitivity at local structure level in discovery of meaningful biological patterns.
An inherent problem with exploratory clustering is $a b$ initio knowledge of $K$, the number of clusters. Consequently, those methods for GE analysis which do not need $K$ specified $a b$ initio have an advantage. Most algorithms seek empirically to
determine this at run time, but derive complicated thresholds that may not make sense in the context of GE data. There is a risk that determination of these thresholds is not a onestep process but requires testing and validation of clusters produced. While space limits a comprehensive survey of robust cluster validation and evaluation methods here, their importance is clear: (see [47] for a comprehensive review). A discipline of information-driven clustering is emerging, which integrates of information-driven clustering is emerging, which integrates
cluster and meta-information, [48-52]. These provide a basis for validation, independent of the current problem and simplify interpretation of clustering results.

## 4. Conclusion

Cluster analysis applied to GE data aims to highlight meaningful patterns for gene co-regulation. The evidence suggests that, while commonly applied, agglomerative and partitive techniques are insufficiently powerful given the high dimensionality and nature of the data. While further testing on non-standard and diverse data sets is required, comparative assessment and numerical evidence, to date, support the view that bi-clustering methods, although computationally expensive, offer better inthe limitations of commata features and local sere well documented in the literature, adoption by the bioinformatic community of new (and hybrid) techniques developed specifically for GE analysis has been slow, mainly due to the increased algorithmic complexity required. This would be catalysed by more transparent guidelines and increased availability in specialised software and public dataset repositories.

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# Chapter III Pattern Discovery in Gene Expression Data 

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## ABSTRACT

Microarray technology ${ }^{l}$ provides an opportunity to monitor mRNA levels of expression of thousands of genes simultaneously in a single experiment. The enormous amount of data produced by this high throughput approach presents a challenge for data analysis: to extract meaningful patterns, to evaluate its quality, and to interpret the results. The most commonly used method of identifying such patterns is cluster analysis. Common and sufficient approaches to many data-mining problems, for example, Hierarchical, K-means, do not address well the properties of "typical" gene expression data and fail, in significant ways, to account for its profile. This chapter clarifies some of the issues and provides a framework to evaluate clustering in gene expression analysis. Methods are categorised explicitly in the context of application to data of this type, providing a basis for reverse engineering of gene regulation networks. Finally, areas for possible future development are highlighted.

## INTRODUCTION

A fundamental factor of function in a living cell is the abundance of proteins present at a molecular
level, that is, its proteome. The variation between proteomes of different cells is often used to explain differences in phenotype and cell function. Crucially, gene expression is the set of reactions
that controls the level of messenger RNA (mRNA) in the transcriptome, which in turn maintains the proteome of a given cell. The transcriptome is never synthesized de novo; instead, it is maintained by gene expression replacing mRNAs that have been degraded, with changes in composition brought about by switching different sets of genes on and off. To understand the mechanisms of cells, involved in a given biological process, it is necessary to measure and compare gene expression levels in different biological phases, body tissues, clinical conditions, and organisms. Information on the set of genes expressed, in a particular biological process, can be used to characterise unknown gene function, identify targets for drug treatments, determine effects of treatment on cell function, and understand molecular mechanisms involved.
DNA microarray technology has advanced rapidly over the past decade, although the concept itself is not new (Friemert, Erfle, \& Strauss, 1989; Gress, Hoheisel, Sehetner, \& Leahrach 1992). It is now possible to measure the expression of an entire genome simultaneously, (equivalent to the collection and examination of data from thousands of single gene experiments). Components of the system technology can be divided into: (1) Sample preparation, (2) Array generation
and sample analysis, and (3) Data handling and interpretation. The focus of this chapter is on the third of these.
Microarray technology utilises base-pairing hybridisation properties of nucleic acids, whereby one of the four base nucleotides (A, T, $\mathrm{G}, \mathrm{C}$ ) will bind with only one of the four base ribonucleotides (A, U, G, C: pairing $=\mathrm{A}-\mathrm{U}$, $\mathrm{T}-\mathrm{A}, \mathrm{C}-\mathrm{G}, \mathrm{G}-\mathrm{C}$ ). Thus, a unique sequence of DNA that characterises a gene will bind to a unique mRNA sequence. Synthesized DNA molecules, complementary to known mRNA, are attached to a solid surface, referred to as probes. These are used to measure the quantity of specific RNA of interest that is present in a sample (the arget). The molecules in the target are labelled, and a specialised scanner is used to measure the amount of hybridisation (intensity) of the target at each probe. Gene intensity values are recorded or a number of microarray experiments typically carried out for targets derived under various experimental conditions (Figure 1). Secondary variables (covariates) that affect the relationship between the dependent variable (experimental condition) and independent variables of primary interest (gene expression) include, for example, age, disease, and geography among others, and can also be measured.

Figure 1. mRNA is extracted from a transcriptome of interest, (derived from cells grown under precise experimental conditions). Each mRNA sample is hybridised to a reference microarray. The gene intensity


An initial cluster analysis step is applied to gene expression data to search for meaningful informative patterns and dependencies among genes. These provide a basis for hypothesis testing - the basic assumption is that genes, showing similar patterns of expression across experimental conditions, may be involved in the same underlying cellular mechanism. For example, Alizadeh, Eisen, Davis, Ma, Lossos, Rosenwald, Boldrick, Sabet, Tran, Yu, Powell, Yang, Marti, Moore, Hudson Jr, Lu, Lewis, Tibshirani, Sherlock, Chan, Greiner, Weisenburger, Armitage, Warnke, Levy, Wilson, Grever, Byrd, Botstein, Brown, and Staudt (2000) used a hierarchical clustering technique, applied to gene expression data derived from dif fuse large B-cell lymphomas (DLBCL), to identify two molecularly distinct subtypes. These had gene expression patterns, indicative of different stages of B-cell differentiation-germinal centre B-like DLBCL and activated B-like DLBCL. Findings suggested that patients, with germinal centre Blike DLBCL, had a significantly better overall survival rate than those with activated B-like DLBCL. This work indicated a significant methodology shift towards characterisation of cancers based on gene expression, rather than morphological, clinical and molecular variables.

## BACKGROUND

## The Gene Expression Dataset

Data are typically presented as a real-valued matrix, with rows representing the expression of a gene over a number of experiments, and columns representing the pattern of expression of all genes for a given microarray experiment. Each entry $x$ is the measured expression of a gene $i$ in experiment $j$, (Figure 1). The following terms and notations are used throughout this chapter:

- A gene/gene profile $x$ is a single data item (feature vector) used by the clustering algorithm. It consists of $d$ measurements, $x$ $=\left(x_{p}, x_{2}, \ldots x_{d}\right)$.
- A condition $y$ is a single microarray experiment corresponding to a single column in the gene expression matrix, $y=\left(x_{1}, x_{2}\right.$, $\left.x_{j}\right)^{T}$, where $n$ is the number of genes in the dataset.
- The individual scalar components of each gene vector $x_{i j}$ represent the measured expression of gene $i$ under experimental condition $j$.

Table 1. Selection of publicly available dataset repositories

| Database | Description | URL |
| :---: | :---: | :---: |
| ArrayExpress | Gene expression and hybridisation array data repository | http:/www.ebi.ac.uk/arrayexpress//ae-main[0] |
| CellMiner | Data from 60 cancer cell lines based on Affymetrix and cDNA microarray data | http://discover.nci.nih.gov/cellminer |
| ExpressDB | Collection of E. Coli and Yeast RNA expression datasets | htpp://arep.med.harvard.edu/ExpressDB/ |
| GEO | Gene expression and hybridisation array data repository | http://www.ncbi.nlm.gov/geo/ |
| RAD | Gene expression and hybridisation array data repository | http:/www.cbil.upenn.edu/RAD/ |
| SMD | Extensive collection of microarray data | http://genome-www.stanford.edu/microarray |

There are a number of publicly available dataset repositories, which contain a wealth of microarray datasets ${ }^{2}$ : Table 1 provides a sample of these. Typically, these repositories store data using the 'Minimum Information About Microarray Experiment' (MIAME) standard (Brazma, Hingamp, Quackenbush, Sherlock, Spellman, Stoeckert, Aach, Ansorge, Ball, Causton, Gaasterland, Glenisson, Holstege, Kim, Markowitz, Matese, Parkinson, Robinson, Sarkans, Schulze-Kremer, Stewart, Taylor, Vilo, \& Vingron, 2001), which allow researchers to replicate the experiments. This allows analysts to compare gene expression data from different laboratories effectively, based on information about the microarrays used in experiments, how these were produced, samples obtained and mRNA extracted and labelled. Additional information is also recorded on methods used to hybridise the sample, scan the image and normalise the data.

## Characteristics of the Gene

 Expression DatasetChoice of the appropriate clustering technique relies on the amount of information on the particular properties of gene expression data available to the analyst, and hence the likely underlying structure. The following data characteristics are typical of the gene expression dataset:

Measurement accuracy of mRNA expression levels depends on the experimental design and rigour. While design of experiments is not a specific focus of this chapter, a good design minimises variation and has a focused objective (Kerr \& Churchill, 2001). Technical variation between microarray slides depends on numerous factors including experimental technique, instrument accuracy for detecting signals, and observer bias. Biological variation may arise due to differences in the internal states of a population of cells, either from predictable processes, such as cell cycle progression, or from random processes such as partitioning of mitochondria during cell
division, variation due to subtle environmental differences, or ongoing genetic mutation (Raser \& O'Shea, 2005). Pre-processing techniques attempt to remove technical variation while maintaining interesting biological variation.
Many variables, both random and fixed, (biological and technical), are associated with microarray measurements. Data is thus intrinsically noisy and outliers in the dataset need to be identified and managed effectively. This usually takes one of two forms, (i) outlier accommodation; uses a variety of statistical estimation or testing procedures, which are robust against outliers, (ii) dentification and decision on inclusion/exclusion, used when outliers may contain key information Liu, Cheng, \& Wu, 2002). Normalisation procedures applied to gene expression data (Bolstad, Irizarry, Astrand, \& Speed, 2003), aim at minimising the effect of outliers (assuming these to be due o experimental variation and thus undesirable). Most manufacturers of microarrays, aware of effects of optical noise and non-specific binding, include features in their arrays to measure these directly: these measurements can be used in the normalisation procedures. Note: although prerocessing methods attempt to remove all noise these may be only partially successful.

Missing values are common to microarray data, and can be caused by insufficient resolution in image analysis, image corruption, dust or scratches on the slide, robotic method used to reate the slide, and so on, (Troyanskaya, Cantor, Sherlock, Brown, Hastie, Tibshirani, Botstein, \& Altman, 2001). In general, the number of missing values increases with the number of genes being measured. Many clustering algorithms, used for ene expression data, require a complete matrix of input values. Consequently, imputation or missing ata estimation techniques need to be considered in advance of clustering. The effect of missing data on pattern information can be minimised hrough pre-processing.

Commonly, missing values in the gene expression matrix are replaced by zeroes or by an
average expression level of the gene, (or "row average"). Such methods do not, however, take into account the correlation structure of the data and more sophisticated options include K-Nearest Neighbour (KNN) and Support Vector Decomposition type methods. Troyanskaya et al. (2001) note that KNN and SVD-based methods are more effective than traditional methods of replacement, with KNN being more robust as the number of missing values increases.

Clustering algorithms that permit overlap (probabilistic or fuzzy clusters) are typically more applicable to gene expression data since: (i) the impact of noisy data on clusters obtained is a fundamental consideration in algorithm choice. (The assumption is that "noisy genes" are unlikely to belong to any one cluster, but are equally likely to be members of several clusters): (ii) the underlying principal of clustering gene expression data, is that genes with similar change in expression for set of conditions are involved, together, in a similar biological function. Typically, gene products (mRNA) are involved in several such biological functions and groups need not be co-active under all conditions. This gives rise to high variability in the gene groups and/or some overlap between them. For these reasons, constraining a gene to a single cluster (hard clustering) is counter-intuitive with respect to natural behaviour.

Additionally, methods that aim at a partial clustering tend to be more suited to expression data, with some genes or conditions not members of any cluster (Maderia \& Oliveira, 2000). Clustering the microarray dataset can be viewed in two ways: (i) genes can form a group which show similar expression across conditions, (ii) conditions can form a group which show similar gene expression across all genes. It is this interplay of conditions and genes that gives rise to bi-clusters, whereby conditions and genes are simultaneously grouped. Such partial clusterings, (or bi-clusters), are defined over a subset of conditions and a subset of genes, thus capturing local structure in the dataset. Clearly, this allows: (i) "noisy genes"
to be left out, with correspondingly less impact on the final outcome, (ii) genes belonging to no cluster-omitting a large number of irrelevant contributions, (iii) genes not belonging to welldefined groups. (Microarrays measure expression for the entire genome in one experiment, but genes may change expression, independent of the experimental condition, [e.g. due to stage in cell cycle]. Forced inclusion of such genes in well-defined but inappropriate groups may impact the final structures found for the data).

## Methods of Identifying Groups of Related Genes

Cluster definition is dependent on clearly defined metrics, which must be chosen to reflect the data basis. Metric categories include:

## Similarity-Based

The cluster is defined to be the set of objects in which each object is closer, (or more similar), to a prototype that defines that cluster as opposed to any other cluster prototype. A typical gene expression cluster prototype is often the average or centroid of all gene vectors in the cluster. The similarity metric used affects the cluster produced. Common measures include: (i) Euclidean distance, (ii) Manhattan distance, and (iii) Squared Pearson correlation distance (Quakenbush, 2001), with the last being the most popular as it captures gene expression "shape" without regard to the magnitude ofthe measurements. However, this distancemeaof the measurenents. However, his distance measurement is quite sensitive to outliers, although, correlation, rather than "distance," is inherently
more important for gene expression data. Take, for example, two gene vectors $X_{1}=(1,2,3,4,5)$ and $X_{2}=(3,6,9,12,15)$. These two profiles result in a Euclidean distance of 14.8323 and a Manhattan distance of 30. The Pearson correlation distance however is 0 , reflecting the fact that the two genes are showing the same patterns of expression.

## Density-Based

Clusters, in this instance, are based on dense regions of genes, surrounded by less-dense regions. Such methods are often employed when clusters are irregular or intertwined, and when noise and outliers are present (Sander, Ester, Kriegel, \& Xu, 1998). However, as each cluster is assumed to have a uniform density, the method is not readily applicable to gene expression data, as some biological functions involve more gene products than others. The high dimensionality also means that density thresholds can be difficult to define and expensive to compute.

## Model-Based

Despite the convenience of similarity-based measures, it can be biologically meaningless to characterise a cluster through a cluster prototype, such as the mean or centroid, as these may be poorly representative of the cluster elements as a whole. As a typical gene expression dataset is large, noisy distortion of these prototypes may be considerable, resulting in relatively uninformative structures. In contrast, model-based techniques, applied to expression space, consider the "fit" of genes in a given cluster to the "ideal" cluster. Concentrating on the strengths of the bi-clustering approach, and following notation from Maderia and Oliveira (2004), four types of model can be identified:
(i) Bi-clusters with constant values. A perfect cluster is a sub-matrix $(I, J)$ of the gene expression matrix $(N, D)$, with all values equal, $x_{i j}=\mu$. The ideal bi-cluster is, of course, rarely found in noisy gene expression data.
(ii) Bi-clusters with constant values on rows or columns. A subset of the "ideal" or constant bi-cluster model, and one which is more realistic for gene expression data is a
sub-matrix with constant rows or columns. For the former, rows have constant value in a sub-matrix $(I, J)$ given by $a_{i j}=\mu+\alpha_{i}$ or $a$ $=\mu \times \alpha$, where $\mu$ is the "typical" bi-cluster value and $\alpha$ is the row offset for $i \in I$. Similarly, perfect bi-clusters with constant columns can be obtained for $a_{i j}=\mu+\beta_{j}$ or $a_{i j}=\mu \times \beta_{j}$, where $j \in J$.
(iii) Bi-clusters with coherent values. From iii, a combined additive model can be derived. In this framework, a bi-cluster is a sub-matrix $(I, J)$, with coherent ${ }^{3}$ values, based on the model:
$a_{i j}=\mu+\alpha_{i}+\beta$
Eq. 1
(where $\mu, \alpha_{i}$ and $\beta_{j}$ are as for (iii). Similarly, the multiplicative model assumes that a perfect bi-cluster could be identified using $a_{i j}=\mu^{\prime} \times \alpha_{i}^{\prime} \times$ $\beta^{\prime}$. Note: the additive form clearly follows for $\mu$ $=\log \left(\mu^{\prime}\right), \alpha_{i}=\log \left(\alpha_{i}^{\prime}\right)$ and $\beta_{j}=\log \left(\beta_{i}^{\prime}\right)$.

The artificial example in Figure 2(a) and (b) illustrates this point. The sub-matrix is an ideal bi-cluster found in a fictional dataset, where $\mu=1$, the offset for row 1 to 3 is $\alpha_{1}=0, \alpha_{2}=2, \alpha_{3}=4$ respectively, and the offset for columns 1 to 6 is $\beta_{1}=0, \beta_{2}=1, \beta_{3}=2, \beta_{4}=4, \beta_{5}=1, \beta_{6}=-1$ respectively. The expression levels can be obtained from Eq. 1. Of course, when searching the dataset for a fit to this model, the mean and offset parameters are unknown and must be estimated from the data. The schematic illustrates the coherent expression profile over the six conditions. Similarly for the multiplicative model where $\mu=1, \alpha_{1}=1, \alpha_{2}=2, \alpha_{3}=5$, $\beta_{1}=1, \beta_{2}=2, \beta_{3}=4, \beta_{4}=6, \beta_{5}=3, \beta_{6}=1.5$.
In reality, these "perfect" bi-clusters are, of course, unlikely to occur, so each entry in the sub-matrix can be regarded as having a residue component (Cheng \& Church, 2000):
${ }_{i j}=\mu+\alpha_{i}+\beta_{i}-\alpha_{i j}$.
Eq. 2

Figure 2. Models in gene expression datasets. The matrix gives clusters found, where rows are gene expression values across 6 experimental conditions (columns). X-axis indicates experimental condition or time point, $y$-axis indicates gene expression level. Model forms are (a) Additive for rows and columns, (b) Multiplicative for rows and columns and (c) Coherent evolution.


## CLUSTER ANALYSIS

## Current Methods

With extensive choice of metric, structure, completeness etc. in cluster analysis it is useful to consider a framework (Table 2) for performance comparison. The taxonomy used is due to Jains, Murty, and Flynn (1999).

## Hierarchical Methods:

Ever since the landmark paper of Eisen et al. (1998), numerous clustering algorithms have been applied to gene expression data. Predominantly these have been hierarchical methods, (Higgins, Shinghal, Gill, Reese, Terris, Cohen, Fero, Pollack, van de Rijn, \& Brooks, 2003; Khodursky, Peter, Cozzarelli, Botstein, Brown, \& Yanofsky, 2000; Makretsov, Huntsman, Nielsen, Yorida Peacock, Cheang, Dunn, Hayes, van de Rijn, Bajdik, \& Gilks, 2004; Wen, Fuhrman, Michaels, Carr, Smith, Barker, \& Somogyi, 1998), due mainly to ease of implementation, visualisation capability and general availability.

The basic steps of a hierarchical clustering algorithm include: (i) computation of the proximity matrix of distances between each gene, (initially

Table 2. Popular clustering techniques applied to gene expression data. Partial (overlapping) clusters are more relevant in this context.

| Common Clustering techniques |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Gene } \\ \text { Membership } \end{gathered}$ | Cluster Structure | Cluster Type | Complete/Partial |
| Hierarchical (Eisen, Spellman, Brown, \& Botstein, 1998) | Hard | $\begin{gathered} \text { Hierarchical } \\ \text { (nested) } \end{gathered}$ | Similarity-Based | Complete |
| K-Means (Tavazoie, Hughes, Campbell, Cho, \& Church, 1999) | Hard | $\begin{gathered} \text { No struc- } \\ \text { ture } \end{gathered}$ | Similarity-Based | Complete |
| $\underset{ }{\text { FCM (Gasch \& Eisen, }}$ | Fuzzy | $\begin{aligned} & \text { No struc- } \\ & \text { ture- } \end{aligned}$ | Similarity-Based | Complete |
| SOM (Golub, Slonim, Tamayo, Huard, Gaasenbeek, Mesirov, Coller, Loh, Downing, Caligiuri, Bloomfield, \& Lander, 1999) | Hard | Topological Structure | Similarity and Neighbourhood kernal functionbased | Complete |
| Delta clusters (Cheng \& Church, 2000) | Shared | Overlap | $\underset{\substack{\text { Based on } \\ \text { Coherent Additive } \\ \text { Model }}}{ }$ | Partial |
| FLOC (Yang, Wang, Wang, \& Yu, 2003) | Shared | Overlap | Based on Coherent Additive Model | Partial |
| SAMBA (Tanay, Sharan, \& Shamir, 2002) | Shared | Overlap | Based on Coherent Evolution Model | Partial |

each is in a unique cluster of size one),(ii) searching the proximity matrix for the two closest clusters, (iii) merging these two clusters and updating the proximity matrix, and (iv) repeating steps two and three until all genes are in one cluster.
Such agglomerative clustering techniques vary with respect to the (i) distance metric used and the decision on cluster merger (that is linkage choice as single, complete, average or centroid; see Quackenbush [2001]). Typically output of a hierarchical clustering algorithm is a dendogram, representing nested patterns in the data and the similarity level at which clusters are merged. The choice of parameters affects both structure of, and relationship between the clusters. Hierarchical cluster structure works well for situations where membership is crisp, but, despite their popularity these methods may not be appropriate to capture natural structures in gene expression data.

Nevertheless, some successes of clustering conditions based on gene expression have been eported. For example, Makrestov et al. (2004) used gene expression profiles, to determine wether sub-types of invasive beast cancercould be identified, with a view to improving patient prognosis. Hierarchical clustering successfully identified three cluster groups with significant differences in clinical outcome. Similarly, astudy on renal cell carcinoma, Higgins et al. (2003), found that hierarchical clustering led to segregation of "histologically distinct tumour types solely based on their gene expression patterns" p. 925). These studies indicate that characterisation of tumours is potentially viable from gene expression profiling.
Hierarchical clustering algorithm properties include location of complete clusters, forced membership and large time-space complexity,
but inclusion of "noisy genes" in the cluster can affect the final grouping, (depending to a greater affect the final grouping, (depending to a greater
or lesser extent on the linkage method and the or lesser extent on the linkage method and the
distance measure used). As algorithms are prodistance measure used). As algorithms are pro-
totype-based, further iterations exacerbate noise totype-based, further iterations exacerbate noise
effects. Given the distance metric basis, hierareffects. Given the distance metric basis, hierar-
chical techniques also tend to produce globular structures.

## Partitive Methods

In contrast to hierarchical algorithms, which create clusters in a bottom up fashion resulting in nested levels of clustering, partitive methods optimise a function of given criteria, partitioning the entire dataset and obtaining one cluster structure.

Partitive K-Means clustering (MacQueen, 1967) produces hard clustering with no structural relationship between the individual clusters. The main steps of the K -means algorithm are: (i) Identification $K$ prototype vectors for $K$ clusters in the dataset. (ii) Assignment of each gene to a cluster based on its similarity to the cluster prototype, (iii) computation of cluster prototypes based on current genes in the cluster, (iv) repeating steps two and three until convergence criteria are satisfied. These may be for example no (or minimal) reassignment of genes to new clusters or for example minimal improvement in optimisation of the criteria function. A typical optimisation approach is to minimise the squared error within a cluster:
$C=\sum_{j=1}^{k} \sum_{i=1}^{n} y_{i j} d\left(x_{i}, q_{j}\right)$
Eq. 3
where $q_{j}$ is the vector representing the mean of the cluster, $x_{i}$ is the vector representing the gene, $d\left(x_{i} q_{j}\right)$ is a distance measure and $y_{i j}$ is a partition element. Here $y_{i j} \in\{0,1\}$, and $y_{i j}=1$, indicates that gene $i$ is assigned to cluster $j$.

An example of use of the K -means method is discussed in Tavazoie et al. (1999), and is based on a yeast time-course gene expression dataset, containing profiles for more than 6000 genes,
with 15 time points (at 10 minute intervals-over nearly two cell cycles), (Cho, Campbell, Winzeler, Steinmetz, Conway, Wodicka, Wolfsberg, Gabrielian, Landsman, Lockhart, \& Davis,, 1998). (This work succeeded in identify \& regulated genes in yeast). Unfortunately, initial prototype vectors in K-Means usually have a large impact on the data structures found. Prototype vectors are often genes selected at random from the dataset. Alternatively, Principal Component Analysis can be used to project the data to a lower dimensional sub-space and K-means is then applied to the subspace (Zha, Ding, Gu, He, \& Simon, 2002). Whichever method is used in practice to select prototype vectors, it is usually practice to select prototype vectors, it is usually
the case that different initial prototypes are investigated to assess stability of the results, with the bestconfiguration, (according to the optimisation criteria), used as output clusters.

## Fuzzy Methods:

As observed,(Section Characteristics of the Gene Expression Datast) mutipleclesterm is more appropriate for gene expression data. The Fuzzy C-Means (FCM) algorithm extends the standard K-means algorithm, to the case where each gene has a membership degree indicating its "fuzzy" or percentage association with the centroid of a given cluster. Typically, each gene has a total membership value of 1 , which is divided proportionally between clusters according to its similarity with the cluster means. A fuzzy partition matrix $Y$, (of dimension $N K$, where $K$ is the number of clusters and $N$ is the number of genes), is created, where each element $y_{i j}$ is the membership grade of gene $i$ in cluster $j$ and a weighted version of Eq. 3 applies. At each iteration, the membership value, $y_{i j}$, and the cluster center, $k_{p}$, is updated by:
$y_{i j}=1 / \sum_{c=1}^{k}\left(\frac{d\left(x_{i}-k_{j}\right)}{d\left(x_{i}-k_{c}\right)}\right)^{\frac{2}{m-1}}$

where $m>1$ denotes the degree of fuzziness, (everything else is as forEq. 3). The iterations stop when $\max \left|j_{i j}^{k+1}-j_{i j}^{k}\right|<\varepsilon$, where $\varepsilon$ is a termination criterion with value between 0 and 1 , and $k$ is the number of iterations.

Given the usual constraint that membership values of a gene must sum to unity, these values should be interpreted with care. A large "membership value" does not indicate "strength of expression" but rather reduced co-membership across several clusters (Krishnapuram \& Keller, 1993). Table 3 illustrates this idea for three clusters. FCM was carried out on published yeast genomic expression data (Gasch \& Eisen, 2002; results available athttp://rana.lbl.gov/FuzzyK/data.html). The membership values for gene B and gene D are very different for cluster 21 , although they are approximately equidistant from the centroid of the cluster. Similarly, gene C and gene D have comparable membership values for cluster 4. However, gene C is more "typical" than gene D. With similar centroid distance measures, membership value for gene B in cluster 21 is smaller than membership value of gene A in cluster 46 . These values arise from the constraint that membership values must sum to unity across all clusters, forcing a gene to give up some of its membership in one
cluster to increase it in another. Listing the genes of a cluster, based on membership values alone is somewhat non-intuitive as it is not a measure of their compatibility with the cluster. However, if interpretation of the list in terms of degree of sharing between clusters is of value.
The work of Gasch and Eisen (2002) on the use of FCM in analysing microarray data looked at clustered responses of yeast genes to environmental changes. Groups of known functionally coregulated genes, and novel groups of co-regulated genes, were found by this method, although missed b wiarchical an K-means methods.

## Artificial Neural Networks

Artificial neural networks (ANN) mimic the idea ofbiological neural networks, where links between various neurons (nodes) can be strengthened or weakened through learning. A number of ANN ypes have been explored, with Self-Organising Maps (SOM) (Kohonen, 1990) proving popular for the analysis of gene expression, as these provide a fast method of visualising and interpreting high dimensional data. The network maps the igh-dimension input gene vector into a lower dimensional space. A SOM is formed by an input ayer of $D$ nodes, (where $D$ is the gene vector dimension), and an output layer of neurons arranged in a regular grid (usually of 1 or 2 dimensions). vector, of the same dimension as the input gene, references each node in the output layer.

Table 3. Difficulties interpreting membership values for FCM. GENE1649X (Gene A), GENE6076X (Gene B), GENE5290X (Gene C) and GENE2382X (Gene D). The table highlights distance to cluster centroid, in terms of Euclidean distance, and the associated membership values of the gene.

| GID | Cluster 4 |  | Cluster 21 |  | Cluster 46 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Centroid Dist. | Mem. | Centroid Dist. | Mem. | Centroid Dist. | Mem. |
| GENE1649X | 10.691 | 0.002575 | 8.476 | 0.002002 | 3.864 | 0.482479 |
| GENE6076X | 6.723 | 0.009766 | 3.855 | 0.009341 | 6.33 | 0.007381 |
| GENE5290X | 6.719 | 0.007653 | 5.29 | 0.00515 | 8.024 | 0.005724 |
| GENE2382X | 7.725 | 0.007609 | 3.869 | 0.01782 | 6.279 | 0.010249 |

Briefly, the mechanism involves: (i) initialisation of the prototype vectors of the output nodes, (ii) training the network to find clusters in the data (Genes are selected at random from the datase and the closest output neuron is identified by its prototype vector. Once an output neuron is identified, its topological neighbours are update to reflect this. Training continues until the refer ence vectors satisfy a stopping criterion), and (ii1) Sequential application of all gene vectors to the SOM, where only one output neuron "fires" upon receiving an input gene vector. "Members" of a cluster, represented by output neuron $i$, are the set of genes, applied to the input neurons, causing output neuron $i$ to fire.

ANN techniques have been used in a numbe of gene expression studies, including Tamayo Slonim, Mesirov, Zhu, Kitareewan, Dmitrovsky Lan-der, and Golub (1999) (to analyse haematopoietic differentiation); Toronen, Kolehmainen, Wong, \& Castren (1999) (to analyse yeast gene expression data); and Golub et al. (1999) (to cluster acute lymphoblastic leukaemia [ALL] and acute myeloid leukaemia [AML]). The stopping crite rion of the SOM is crucial, since over-fitting to the training dataset is a risk. A further disadvantage is the amount of prior information needed. SOM requires input parameters such as learning rate, neighbourhood size and kernel function, as well as topology of the map, (typically hexagonal or square). The stability of results is also an issue, as a particular gene vector can be found to cause different outputnodes to fire at different iterations, (Jains et al. 1999). Furthermore, clusters produced by the SOM are sensitive to choice of initial vec tors for the output neurons, and a sub-optimal structure may result from a poor selection.

## Search Based Methods:

While methods considered so far focus on finding global structures in the data, local structures are frequently of great interest. Cheng and Church (2000) adapted work of Hartigan (1972) for gene
expression data, producing simultaneous clusters of genes and conditions and an overall partial clustering of the data. From Eq. 1 each value $a$ of a sub-matrix can be defined from the typical value within the bi-cluster $a_{I J}$, plus the offsets for the row mean, $a_{i J}-a_{I J}$ and column mean $a_{I J}$ $a_{I J}$. Thus, each value in the sub-matrix should (ideally) be:
$\mathrm{a}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{ij}}-\mathrm{a}_{\mathrm{ij}}-\mathrm{a}_{\mathrm{ij}}+\mathrm{a}_{\mathrm{iJ}}$
Eq. 6
The Cheng and Church technique defines the Mean Residue Score (H) of a sub-matrix, based on Eq. 2 and Eq. 6, such that:
$H(I, J)=\frac{1}{|I \| J|} \sum_{i \in I, j \in J}\left(r_{i j}\right)^{2}$
Eq. 7
The algorithm carries out greedy iterative searches for sub-matrices $(I, J)$, which minimise this function (Eq. 7), generating a large time cost, as each row and column of the dataset must be tested for deletion. (A sub-matrix is considered a bi-cluster if its Mean Residue Score falls below user specified threshold). A further overhead a user specified threshold). A further overhead is the masking of a bi-cluster with random numbers once it is found, to prevent finding the same clusters repeatedly on successive iterations. There is, nevertheless, high probability that this replacement with random numbers affects the discovery of further bi-clusters. To overcome this random "interference" Yang et al. (2003) developed Flexible Overlapped bi-Clustering (FLOC), generalising the model of Cheng and Church to incorporate null values.
For both the Cheng and Church (2000) algorithm and the FLOC generalisation, $K$ can be specified to be much larger than the desired number of groups, without affecting the outcome, as each row and column in the expression matrix can belong to more than one bi-cluster, (Cheng \& Church, 2000; Yang et al., 2003). Selecting $K$ then reduces to selecting the percentage of the bi-clusters with the best Mean Residue-score (Eq. 7). The cost is increased computation time - the

Figure 3. Bipartite graph representing expression for seven genes under five conditions-edges indicate a change in expression

## Conditions

Genes

Cheng and Church algorithm finds one bi-cluster at a time while FLOC finds all simultaneously However, a major additional strength of the biclustering techniques is the minimal requirement for domain knowledge. Also, as FLOC accounts for null values in the dataset, the preliminary imputation of missing values is not necessary.

## Graph theoretic Methods:

A further approach, which is proving useful in the analysis of large complex biological datasets, is that of graph theory, (Aiello, Chun, \& Lu, 2000; Aittokallio \& Schwitowski, 2006; Guillaume \& Latapy, 2006; Maslov, Sneppen, \& Zaliznyak, 2004). A given gene expression dataset can be viewed as a weighted bipartite graph, $G=(V, U$, $E$ ), where $V$ is the set of gene vertices, $U$ is the set of condition vertices, with $V \cap U=\Phi$, and $E$ is the set of edges, with $(u, v) \in E$ having a weight $a$ proportional to the strength of the expression of gene $v$ under condition $u$ (Figure 3).
Analysis of the models involved focuses on identification of similarly or densely connected sub graphs of nodes and, of course, relies greatly on the method used to define the edge weights. Clustering by graph network leads to similar issues as before; (i) results are highly sensitive to data quality and input parameters, (ii) predicted clusters can vary from one method of graph clustering to another. Clusters that share nodes and edges of a graph networks, clearly "overlap" and as noted in the section titled Characteristics of the Gene Expression Dataset, are desirable for
ene expression interpretation.
The "Statistical Algorithmic Method for Bicluster Analysis" (SAMBA) (Tanay et al., 2000), uses a graphical approach and, unlike previous bi-clustering techniques, finds coherentevolution in the data. An edge is defined to exist between a gene node $u$ and a condition node $v$ if there is significant change in expression of gene $u$ under condition $v$, relative to the genes normal level (a non-edge exists if $u$ does not change expression under $v$ ). Each edge and non-edge is then weighted, based on a $\log$ likelihood model, with weights:
$\log \frac{p_{c}}{P_{(u, v)}}>0$ for edges, and $\log \frac{\left(1-P_{c}\right)}{\left(1-P_{(u, v)}\right)}<0$
for non-edges.
Eq. 8
Here, $P_{(u, v)}$ is the fraction of random bipartite graphs, with degree sequence identical to $G$, that contain edge $(u, v)$, and $P_{c}$ is a constant probability assigned by the user. For $P_{c}>\max _{(u, v) \in U \times V} P_{(u, v)}$, edges are taken to occur in a bi-cluster with equal probability. Weights as determined by Eq. 8 are assigned to the edges and non-edges in the graph. major strength of this method is that statistical significance of any sub graph is then simply determined by its weight.

## Cluster Evaluation and Comparison

Evaluation is not particularly well developed for clustering analyses applied to gene expression data, as very little may be known about the daaset beforehand. Many clustering algorithms are designed to be exploratory; producing different clusters according to given classification criteria and will discover a structure, meaningful in that context, which may yet fail to be optimal or even iologically realistic. For example, for K-Means the "best" structure is one that minimises the um of squared errors (MacQueen, 1967) while, or the Cheng and Church algorithm (Cheng \& Church, 2000), it is that which minimises of the

Figure 4. Gene expression profiles for two equivalent clusters; cluster in (B) has Profile 1 scaled down by one third


Mean Residue-Score (Eq. 7). The two may not be directly comparable, as the former highlights global patterns in the data and the latter local global patterns in the data and the latter local
patterns. While larger deviations from the mean patterns. While larger deviations from the mean will not always be the case. For example, Figure 4 highlights a simple situation with three genes in a cluster. According to the K-means criterion, the within cluster distance is approximately 11.02 , based on Euclidean distance and centroid of the cluster. The Mean Residue Score is 0 . Reducing the scale of profile 1 by one third, (Figure 4(b)), decreases the within-cluster distance to 7.91, while increasing the Mean Residue Score slightly to 0.0168 . Both (a) and (b) are roughly equivalent. Consequently, interpretation of cluster results relies on some level of subjectivity as well as independent validation and integration of findings. Subjective evaluation, even for low dimensional data, is non-trivial at best, but becomes increasingly difficult for high dimensional gene expression data. Clearly, each technique will find patterns even if these are not meaningful in a biological context.

The benefits of the individual techniques, as applied to gene expression data, were highlighted in the last section. This section aims at providing navigational guidelines for some degree of objective evaluation and comparison.


## Determining the Correct Number of

 ClustersCluster number, in the absence of prior knowledge is determined by whether a non-random structure is determined by whether a non-random structure
exists in the data. Limiting to a specific number of groups will bias the search, as patterns tend to be well-defined only for the strongest signals. Commonly, statistical tests for spatial randomness test if a non-random structure exists, but identification of small, biologically meaningful clusters remains non-trivial. This is particularly true of standard methods, which find global structure, but lack fine-tuning to distinguish local structures. Selection of the correct number of clusters $(K)$ thus is inherently iterative. Near optimal $K$ should clearly minimise heterogeneity between groups, while maximising homogeneity within groups, but determining the number of significant clusters relies, not only on direct extraction (or assessment) but also on appropriate hypothesis testing. Direct methods are based on various of criteria ${ }^{4}$. Nevertheless, improvement in terms of identification of local clusters is slight. Specific tests include Gap Statistic (Tibshirani, Walther, \& Hastie, 2001), Weighted Discrepant Pairs (WADP) (Bittner, Meltzer, Chen, Jiang, Seftor, Hendrix, (Bittner, Meltzer, Chen, Jiang, Seftor, Hendrix, Dougherty, Wang, Marincola, Gooden, Lueders, Glatfelter, Pollock, Carpten, Gillanders, Leja, Dietrich, Beaudry, Berens, Alberts, \& Sondak,
2000), and a variety of permutation methods (Bittner et al., 2000; Fridlyand \& Dudoit, 2001). Since most involve bootstrapping, these methods can be computationally very expensive. Comparison of methods for selecting the number of groups is discussed by Milligan and Cooper (1985) and, more recently, by Fridlyand and Dudoit (2001), who note that no existing tests are optimal for gene expression data.

Comparing Results from Clustering Algorithms

Numerical measures of cluster "goodness" include cluster cohesion (compactness or tightness), that is how closely related genes in a cluster are, while measures of cluster separation (isolation), determine how distinct each cluster is. The Group Homogeneity Function, is often used to measure the association (distinctiveness) of genes within and between groups.

## Comparison with Metadata

Including biological function information in the Including biological function information in the
gene list for each cluster inevitably provides a more complete picture of the dataset and of the success of the technique. This information can be used to validate the clusters produced, and a number of functional annotation databases are available. The Gene Ontology database (Ashburner, Ball, Blake, Botstein, Butler, Cherry, Davies, Dolinski, Dwight, Epping, Harris, Hill, Issel-Tarver, Kasarskis, Lewis, Matese, Richardson, Ringwald, Rubin, \& Sherlock, 2000) for example, provides a structured vocabulary that describes the role of genes and proteins in all organisms. The database is organised into three ontologies: biological process, molecular function, and cellular component. Several tools ${ }^{5}$ have been developed for batch retrieval of GO annotations for a list of genes. Statistically relevant GO terms can be used to investigate the properties shared by a set of genes. Such tools facilitate the transi-
tion from data collection to biological meaning by providing a template of relevant biological patterns in gene lists.

## FUTURE TRENDS

Despite the shortcomings, the application of clustering methods to gene expression data has proven to be of immense value, providing insight on cell regulation, as well as on disease characterisation. Nevertheless, not all clustering methods are equally valuable in the context of figh dimensional gene expression. Recognition hat well-known, simple clustering techniques, uch as K-Means and Hierarchical clustering, o not capture more complex local structures in the data, has led to bi-clustering methods, in particular, gaining considerable recent popularty, (Ben-Dor, Chor, Karp, \& Yakhini, 2002, Busygin, Jacobsen, \& Kramer, 2002; Califano, Stolovitzky, \& Tu, 2000; Cheng \& Church, 2000; Getz, Levine, \& Domany, 2000; Kluger, Basri, Chang, \& Gerstein, 2003; Lazzeroni \& Owen, 2002; Liu \& Wang, 2003; Segal, Taskar, Gasch, Friedman, \& Koller, 2003; Sheng, Moreau, \& De Moor, 2003; Tanay et al., 2002; Yang et al., 2003;). Indications to date are that these methods provide increased sensitivity at local structure level for iscovery of meaningful biological patterns
Achieving full potential of clustering methods is constrained at present by the lack of robust validation techniques, based on external resources, uch as the GO database. Standardisation of gene annotation methods across publicly available databases is needed before validation techniques can be successfully integrated with clustering formation found from datasets
The "Central Dogma" that "DNA makes mRNA makes proteins" that comprise the proteome is overly simple. A single gene does not ranslate into one protein and protein abundance epends not only on transcription rates of genes butalso on additional control mechanisms, such as
mRNA stability ${ }^{6}$, regulation of the translation of mRNA to proteins ${ }^{8}$ and protein degradation ${ }^{8}$. Proteins also can be modified by post-translation ac tivity ${ }^{9}$ (Brown, 2002a). The study of proteomic and transcription data investigates the way in which transcription data investigates the way in which
changes connect gene expression to the physical chemistry of the cell. Integration and merger of proteomic and transcription data sources across platforms is needed, together with development of automated high-throughput comparisons methods if detailed understanding of cell mechanisms is to be achieved. To this end, a standard method of gene and protein annotation across databases is overdue (Waters, 2006). The development of Bioinformatics/data-mining tools that span dif ferent levels of "omics" is a necessary next step in the investigation of cell function

## CONCLUSION

Clustering gene expression data is non-trivial and selection of appropriate algorithms is vital if meaningful interpretation of the data is to be achieved. Successful analysis has profound implications for knowledge of gene function, diagnosis, and for targeted drug developmen amongst others. The evidence to date is that methods, which determine global structure, are insufficiently powerful given the complexity of the data. Bi-clustering methods offer interpretability of data features and structure to a degree not possible with standard methods. However, even though less sophisticated algorithms such as K -means are achieving some success and while bi-clustering methods seem promising these are the first steps only to analysing cellula mechanisms and obstacles remain substantial. A significant barrier to the integration of genomi and proteomic platforms and understanding cel lular mechanisms is the lack of standardisation. Integration of heterogeneous datasets must be addressed before analysis of gene expression data comes of age.

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## ENDNOTES

1 Microarray developmenttimeline: 1989-development of world's first microarray; 1991 - Photolithographic printing technique developed by Affymetrix; 1993 - Microarray containing over 1 million DNA sequences developed; 1994 - First cDNA collections developed by Stanford; 1995-Quantitative monitoring of gene expression patterns with cDNA microarray; 1996 - Commercialisa tion of arrays (Affymetrix); 1997-Genome wide expression monitoring in Yeast; 2000 - Portraits/Signatures of gene expression in cancer identified; 2002-Genechip ${ }^{\circledR}$ Huma Genome two array set developed for analysi of over 33,000 genes from public databases; 2003 - Microarray technology introduce to clinical practices; 2004 - Whole human genome on one microarray

The two most popular array platforms are complementary DNA (cDNA) and oligonucleotide microarrays. The former contains cDNA probes that are products synthesized from polymerase chain reactions generated from cDNA and clone libraries, the latter from CDNA and clone libraries, the latter probes (prefect match and mismatch) generated directly from sequence data. A key difference between the two platforms is the manner in which the data is presented for analysis. Intensity measurements for cDNA arrays are the result of competitive hybri disation, (where two transcription samples finterest (labelled with two different dyes) are hybridised to the same array), resulting in a measurement of the ratio of transcript levels for each gene, (usually reported as a log ratio). Oligonucleotide arrays, on the other hand, results from non-competitive hybridisation (where one transcription sample is hybridised to a array and difference in expression levels between two samples are mpared acrossays Here mear are level for a gene is presented as the average easurement of all probes representing the gene (depending on pre-processing echnique this may have mismatch probes subtracted first). See Schulze and Downward (2001) for a review.

Gene expression patterns with similar frequency and phase.
4 These include likelihood ratios (Scott and Symons, 1971), cluster sums of squares (Milligan and Cooper, 1985), average silhouette (Kaufmann and Rousseeuw, 1990) or mean split silhouette (van der Laan and Pollard, 2001).

5 Tools important for the management and understanding of large scale gene expression data: FatiGo (Al-Shahrour et al., 2003), GoMiner(Zeebergetal., 2003), OntoExpress (Draghici et al., 2003), EASE (Hosack et al., 2003), DAVID Gene classification tool (Dennis et al., 2003).
. Sequences ofmRNA may vary considerably in stability. The balance between mRNA degradation and mRNA synthesis determines the level of mRNA in the cell.
2 The mechanisms, including regulatory proteins, which dictates which genes are expressed and at what level.

The method and rate at which protein is broken down in the body.
Before taking on a functional role in the cell an amino acid sequence must fold into its correct tertiary structure. Additional post-processing events may occur, such as proteolytic cleavage, chemical modifications, intein splicing. (Brown, 2002(b)
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## EDGE WEIGHTING OF GENE EXPRESSION GRAPHS

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In recent years, considerable research efforts have been directed to microarray technologies and their role in providing simultaneous information on expression profiles for
thousands of genes. These data, when subjected to clustering and classification dures, can assist in identifying patterns and providing insight on biological processes To understand the properties of complex gene expression datasets, graphical representa tions can be used. Intuitively, the data can be represented in terms of a bipartite graph, with weighted edges corresponding to gene-sample node couples in the dataset. Bio logically meaningful subgraphs can be sought, but performance can be influenced both
by the search algorithm, and, by the graph weighting scheme and both merit vigorous investigation. In this paper we focus on edge-weighting schemes for bipartite graphical representation of gene expression. Two novel methods are presented: the first is based on empirical evidence; the second on a geometric distribution. The schemes are compared for several real datasets, assessing efficiency of performance based on four influence on scheme efficiency and reusability. Recommendations and limitations are briefly discussed.
Keywords: Edge Weighting; Weighted Graphs; Gene Expression; Bi-clustering.

## 1. Introduction

Advances towards a better description of the dynamics of complex systems often rely on a detailed analysis of large datasets. Financial modelling is an obvious example: to understand the interactions between worldwide markets, it is crucial to extract information for large time-series datasets, (see e.g. [1]). Graph theoretical modinformation for large time-series datasets, (see e.g. [1]). Graph theoretical mod-
elling is proving a useful tool in the analysis of large complex biological datasets. For instance, protein-protein interactions can be modelled by undirected graphs, [2], where nodes represent proteins and an edge connects two nodes if the proteins physically combine. Transcriptional factor binding sites can be identified through the use of undirected weighted graphs, where weights of edges capture the similarity

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between aligned nucleotides in an input set of promoters, [3]. Further, metabolic networks can be represented as bipartite graphs. In this case an edge connects a reaction node to a compound node, representing either substrate or product relationships, [4]. Gene expression can also be modelled as a weighted bipartite graph, here two node types are used to represent genes and samples and an edge exists between a gene and a sample node. In the pioneering work of Tanay et al. [5, 6], the weight of an edge, $e_{i j}$ was designed to incorporate the probabilities of an effect of experimental condition $j$ on the expression of gene $i$, and of $e_{i j}$ existing in the graph.
Clustering techniques are fundamental for the exploration of gene expression data, $[7-9,5]$. Gene profiles are grouped into $K$ subsets (clusters), with $K$ not necessarily known a priori, such that elements in the same subset are associated with one another. The fundamental biological premise underlying these approaches is that genes, which display similar expression patterns, are co-regulated and may share a common function or contribute to a common pathway. Identification of patterns is made more difficult by the fact that association (through similarity patterns is made more dificult by the fact that association (through similarity determined by a subset of samples giving rise to biclusters, [10]. Moreover, genes may determined by a subset of samples giving rise to biclusters, [10]. Moreover, genes may
belong to a number of biclusters, with varying degrees of membership, [11]. This, and belong to a number of biclusters, with varying degrees of membership, $[11]$. This, and
the fact that gene expression profiles often originate from very noisy experimental the fact that gene expression profiles often originate from very noisy experimental
measurements, makes computational solutions to the clustering problem difficult and patterns difficult to interpret
Graphical clustering techniques involve identification of similar, (e.g densely connected), subgraphs, where usual limitations (such as NP-completeness) apply. A variety of evolutionary and heuristic algorithms can be used to find subgraphs and include Genetic algorithms, [12], Simulated Annealing, [13], the MinCut algorithm, 14], CLICK, [15] and Samba, [5]. These algorithms all rely on an objective function to evaluate the sub-graphs found. Fundamentally, this function relies on the concept of an edge weight and its derivation.

Substantial efforts have been devoted to the development of weighted graphbased clustering methods for biological tasks, (see references above). However, it is mportant to recognise the essential role a weighting scheme itself plays on cluster determination, and thus to investigate the intrinsic nature of these schemes in isolation. To this end, we introduce and compare two new weighting schemes: (i) a distribution-based method, (where edge weights are estimated from a pre-defined probability distribution); (ii) an empirical-based method, (where edge weights are determined by the experimental data). We also discuss performance measures for these (and the Tanay) edge weighting schemes, and present a comparative validation based on application to several real datasets

Edge Weighting of Gene Expression Graphs 3

## 2. Clustering Approach and Evaluation Framework

### 2.1. Gene Expression as a Bi-Partite Graph

Gene expression data is typically presented as a matrix, where rows $(i=1 \ldots n)$ correspond to gene vectors, $g_{i}=\left(x_{i 1}, x_{i 2}, \ldots, x_{i p}\right)$, which record expression values for gene $i$ across $p$ experiments. A bipartite graph is a triplet $G=(\tau, \perp, E)$, where or gene $\imath$ across $p$ experiments. A bipartite graph is a triplet $G=(\top, \perp, E)$, where of edges, which links top and bottom nodes. $T \cap \perp=\emptyset$, is a defining property of of edges, which links top and bottom nodes. $\mathrm{T} \cap \perp=\emptyset$, is a defining property of
bipartite graphs. We define a weighted bipartite graph to model gene expression bipartite graphs. We define a weighted bipartite graph to model gene expression
under a number of experiments as, $G=(\top, \perp, E, W)$, where $T$ corresponds to the under a number of experiments as, $G=(\top, \perp, E, W)$, where T corresponds to the
set of genes, $|\top|=n, \perp$ to the set of samples, $|\perp|=p, E$ is the set of edges between genes and samples, and $W=\left(w_{i j}\right)$ where $w_{i j} \in \Re$ denotes the weight of the edge $e_{i j}$ between 'gene' node $i$ and 'sample'a node $j$. If every gene sample couple is connected by an edge, (i.e. $|E|=p \times n$ ), then the gene expression network is said to be fully connected, otherwise it is partially connected.
As mentioned previously, essential properties of the dataset can be transformed to bi-partite graph properties by appropriate representation. However, care must be taken not to lose important information in the transformation process. This allows us to study the gene-sample inter-activity using the powerful tools and notions provided for classical networks. Here, we are concerned with what constitutes an (important) edge in the context of gene expression and how this edge is weighted.

### 2.2. Definition of Assessment Properties

It is important, both for the evaluation of any clustering technique and for its reusability, that weighting schemes are validated independently of the subsequent network analysis. A clustering technique should not be considered as a "black box", and assessment of its core features is crucial to improve the overall performance. For a given graph-clustering technique reliable results reflect a well-designed weighting scheme, together with a reasonably robust and efficient search algorithm, and both aspects are susceptible to refinement. Consequently, we propose a graphical edge weighting assessment procedure for gene-sample networks, based upon four properties, as detailed below.

### 2.2.1. Discrimination:

Ability of the method to "rate" highly those gene-sample couples which contribute to a cluster. The range and distribution of edge weights establish how well a given scheme distinguishes between relevant and irrelevant gene-sample couples.
${ }^{\text {a }}$ A sample refers to a microarray experiment carried out on mRNA extracted from biological samples; each microarray experiment is a column in the gene expression dataset

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2.2.2. Reusability:

Independence of the proposed scheme and the subsequent clustering technique. This deals with how the weighting scheme must change to reflect additional layers of analysis.
2.2.3. Robustness:

Ability of a given weighting scheme to deal with noise and missing values. This involves investigation of distortion of weights caused by different levels of noise and missing values.
Noise and missing values were included in the dataset in the tests reported here in order to mimic measurement error of different amounts. Noise was randomly "added/subtracted" to each value in the dataset as a percentage (up to $10 \%$ ) of the original value. To replace data with missing values, up to $10 \%$ of expression values from the original dataset were randomly selected and removed. Commonly, missing values in the experimental gene expression matrix are replaced by zeroes or by an average expression level of the gene, ("row average"). Such methods, however, do not take into account the correlation structure of the data, and more sophisticated options include methods of K-Nearest Neighbour (KNN) and Support Vector Decomposition type, [16]. Missing value estimation methods have generated a considerable literature in their own right, so to test our weighting schemes the common practice of replacing missing values by the row mean was adopted.
For this analysis, we defined "Average Absolute Variation" as the average difference in edge weights, compared to $0 \%$ noise/missing values and "Stable weights" to be those for which the variation is less than the level of noise/missing values added.

### 2.2.4. Parameter Influence:

Any weighting scheme ideally requires minimal specification of input parameters. We thus examined input parameter influence on discrimination and robustness, as well as on the distribution of weights themselves.

### 2.3. Datasets

The three weighting schemes were tested on three datasets. (i) The Yeast Cell Cycle Data, provided by [17], contains time-course expression profiles for more than 6000 enes, with 17 time points for each gene (taken at 10-min intervals) and covers nearly two yeast cell cycles $(160 \mathrm{~min})$. The raw gene expression profiles were downloaded two yeast cell cycles ( 160 min ). The raw gene expression profiles were downloaded
from http://genomics.stanford.edu. (ii) A Lymphoma Dataset (downloaded from supplementary web by [7], http://llmpp.nih.gov/lymphoma/index.shtml), relates to an experiment to characterize gene expression in Diffuse Large B-cell lymphoma (DLBCL). The complete dataset contains expression levels for 4,026 genes and 96 samples. Finally, (iii) Gefitinib Treated Kasumi Cell Line dataset. Here

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Kasumi cells were treated with gefitinib or dimethyl sulfoxide (DMSO) control in duplicate for 6 hours and in triplicate for 24 hours. This results in a dataset of 22283 genes and 10 samples [18], and is available from the MIT Broad Institute website, (http://www.broad.mit.edu/cgi-bin/cancer/datasets.cgi).

## 3. Edge Weighting Schemes

### 3.1. Empirical-based Weighting

The empirical-based weighting scheme is data-driven in the sense that it is determined from direct analysis of the dataset, i.e. based on observations obtained. Fundamental to the method proposed here is that genes which, for a given sample, have either high or low expression (equivalent to induction or repression) are more, likely to contribute to a function, or have a functional response, than for those for which expression values remain unaffected. Affected genes can thus be extracted for further analysis.
A high/low expression value for gene, $i$ under sample $j$, is determined, relative to other expression values in gene vector $i$, i.e. across rather than within samples. The motivation for this is that, with microarray technology, direct comparison of expression measures within arrays is problematic, because fluorescent intensities are not the same across genes. The measured intensities are roughly proportional to mRNA abundance but the proportionality factor is different for each gene. Specifically, this means that between-sample, within-gene comparisons are appropriate, but within-sample, between-gene comparisons are not straightforward ${ }^{\mathrm{b}}[19]$.
Under this weighting scheme, an edge $e_{i j}$ exists when the $i^{\text {th }}$ gene shows "significant" induction or repression, relative to its mean level of expression, for sample $j$. It makes use of Chebyshev's inequality [20], as the distributional form of expression values for each gene is not assumed. Chebyshev's inequality, for any real number $k>0$, can be written:

$$
\begin{equation*}
\operatorname{Pr}(|X-\mu| \geq \kappa \sigma) \leq \frac{1}{\kappa^{2}} \tag{1}
\end{equation*}
$$

with random variable $X, \mu$ the expected value of $X$ and $\sigma^{2}$ the variance. This scheme uses a two step process (Figure 1). The first step involves identification of those expression values $X=x_{i j},(i=1 \ldots n, j=1 \ldots p)$, of interest which, for a given sample $j$, are $\geq \kappa \sigma$ from the mean expression of gene vector $i$. From Eq. 1, for example, the associated probability of an expression value $\geq 3.16 \sigma$
${ }^{\mathrm{b}}$ For example, say, genes $a$ and $b$, have measured expression of 100 and 200 respectively, in sample $j 1$. These observed data do not reflect the real relative abundance of mRNA for these two genes.
There could in fact be more mRNA for gene $a$. On the other hand, if in a second sample, $j$, gene has an expression measure of 200, we could conclude that the abundance of mRNA for $a$ in sample $j 2$ is likely to be higher than that observed in sample $j 1,[19]$

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Fig. 1. Two-step process of empirical scheme. Step 1: A univariate analysis of each gene vector is carried out to determine strength of response. Step 2: A univariate analysis of each sample vector is performed and used to order gene response.
from the mean of gene $i$ is less than 0.10 . Expression values $\geq 3.16 \sigma$ from the mean would indicate a strong response of gene $i$ to sample $j^{\text {c }}$
Clearly, categories can be established to highlight those expression values which indicate a weak response, moderate response and strong response, where $\kappa$ indicates the threshold between categories. For example, expression values which are $\geq 2.58 \sigma$, $\geq 3.16 \sigma$ and $\geq 4.47 \sigma$ from $\mu$ fall into non-overlapping categories of weak, moderate and strong respectively, (i.e. a gene-sample couple will not be categorised as moderately and strongly responsive), corresponding to probabilities $=0.15,0.10$ and 0.05 . Obviously the number of categories can be extended for fine-grained response, while thresholds between categories can also be adjusted. For this analysis, the categories weak, moderate and strong, as defined above and in Tanay et al [5], were used. Thresholds chosen are discussed further in Section 3.1.1.
In the second step, a univariate analysis of expression values within each sample vector is carried out and used to order gene response. For each sample vector, $j$, expression values $x_{i j}$ which indicate strong response of the $i^{\text {th }}$ gene under $j$, (as determined in step one) are selected. Similarly, genes which show moderate and weak response under $j$ can be identified. For each of the three "strength of response" categories, a gene may be repressed or induced; ( $X-\mu<0$, or $X-\mu>0$ respectively), giving six sub-categories in total. For each sub-category, $C_{s}, s=1 \ldots 6$ and for each sample variable, $j=1 \ldots p$, the empirical probability of $x_{i j} \in C_{s}$ is calculated as $\left|x_{i j} \geq x_{v j}\right| /\left|C_{s}\right|, x_{v j} \in C_{s}, i \neq v$, (probability $=1$ if $\left|C_{s}\right|=1$ ) and hence the edge weight in the bi-partite gene expression graph is obtained.
The weight is thus a direct reflection of obtaining a given expression level in an induced/repressed response category for a particular sample. So, if many genes react strongly in the sample, the weight is smaller, while if only a few react strongly, the weight will be larger. Note that, with this weighting scheme, a given sample (experiment) may also have no reacting genes.

Note that this method assumes that for the majority of samples, the expression of gene $i$ will not be affected

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The weighted graph can be broken down into an independent subgraph for each sub-category. Gene-sample edges in these different response groups may well have similar weight "values" but are distinguished, in terms of absolute levels of expression ${ }^{\text {d }}$, by the category into which they fall. Hence, the analysis does not depend directly on levels of expression but rather strength of response. Within a category, the weight can nevertheless be interpreted directly in terms of the relative probability of gene-sample response. Thus the higher the weight, the more confidence that a relationship exists between gene and sample in that category.

### 3.1.1. Threshold Estimation

This scheme requires specification of number of categories and threshold values for each. To decide on thresholds between categories, graphs from real datasets, $G=(\mathrm{T}, \perp, E)$ are compared to graphs from random datasets, $G_{\text {Rand }}=$ ( $\left.\top_{\text {Rand }}, \perp_{\text {Rand }}, E_{\text {Rand }}\right)$, for a range of thresholds, (Fig. 2). A random dataset of the same dimension as the input dataset was created, were for each row (gene) $i$, random numbers were selected from a Normal distribution of mean, $\mu_{i} R_{d}=\mu_{i}$ Re dom numbers were selected from a Normal distribution of mean, $\mu_{i}$ Rand $=\mu_{i}$ Real, , to any sample, expression would be relatively constant with no deviation from Norto any sample, expression would be relatively constant with no deviation from Nor-
mal to indicated response). For each threshold choice in this analysis, 100 random mal to indicated response). For each threshold choice in this analysis, 100 random
datasets were created to estimate cut-offs, with comparisons based on averaging datasets we
over these.
A null model assumes each edge in the graph to be created with a probability $=\left(\left|E_{\text {Rand }}\right| /\right.$ Number of possible edges $)$, while an alternative assumes an edge was created with probability $=(|E| /$ Number of possible edges $)$. The level at which the $\log$ of the ratio of these two probabilities is maximised is taken to be optimal in terms of real effect observed. Clearly, one criterion for definition of maximum threshold is that at least one gene-sample couple must be identified in the real and random graph. Thresholds are then tested in probability increments of 0.02 , to determine percentage inclusion of expression values. Once the "strong response" threshold is found, moderate and weak response thresholds can be established similarly.

### 3.2. Distribution-based Weighting

The motivation for the distribution-based scheme, proposed here, is that it is difficult, if not impossible, to give an absolute characterisation of an important genesample couple. An absolute expression level, on its own, means very little, as noted in Section 3.1. On the other hand, the study of the expression level of a given couple, relative to that for other couples can provide interesting insight on functionality.
${ }^{\mathrm{d}}$ If the dataset was not categorised, a weak response and a strong response gene-sample couple would have very different weights, with the consequence
dominate the analysis and obscure more subtle patterns.

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Fig. 2. Threshold Analysis of Lymphoma data. X-axis indicates the probability threshold $\frac{1}{k^{2}}$, the $y$-axis indicates the number of edges selected.

An 'interesting couple' is loosely one where we can highlight a significant effect for a given gene responding to a sample

This implies an expression level, differing notably from the 'average' expression observed for this gene across all samples, (and used by empirical-based approach). It also implies some deviation from the effect this sample has on the whole set of genes: if a certain sample leads to over-expression of virtually all the genes, it might be of limited interest to consider its effect on one, as this would not be in any way unique.
The dataset is considered as a matrix containing only positive values ${ }^{\mathrm{e}}$, the expression levels. Each gene vector is scaled to have a mean of 1 , by dividing the expression level, $x_{i j}$, by the row mean for gene $i$. Thus, genes $i \in I$ with expression value $x_{i j} \leq 1$ are considered repressed and $x_{i j}>1$ are considered induced in sample $j$. For a given gene, we therefore obtain a series of positive values, of average 1, with values $x$, s.t. $0 \leq x<1$ when the gene is under-expressed in a certain sample, and s.t. $x>1$ for a gene over-expressed in the sample.

For each sample, in order to differentiate between genes that show specific behaviour and those that react similarly to other genes, a geometric series, $\left(a, a R, a R^{2}, a R^{3}, \ldots\right)$ is used to create tightly defined categories for values close to 1 (= mean expression level for a gene across all samples, as above), and broader categories as expression deviates further from this value. Such series are used because of the skewness of the data: many genes show very little response to a given
eSome microarray datasets are only made available after they are transformed into log space, (a esult of the normalisation processs), thus leading to some negative values for low expression levels. positive values.

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sample, having an expression value equal to 1 . Two series are used: one for values eater than 1 , and the other for valler than 1 . $A$ ratio $R$ for each geomet ric series is calculated using Eq. 2, where $N_{c}$ is the number of categories needed, and Min and Max are, respectively, the smallest and largest value of the partial set considered for the series. For the $[1 ; \infty)$ part of the dataset, $\operatorname{Min}=1$, and the category boundaries are, therefore, $1, R, R^{2}$, etc., with $R \geq 1$. For the $[0 ; 1]$ part of the dataset, $\operatorname{Max}=1$, and the category boundaries are, therefore, $1, \frac{1}{R}, \frac{1}{R^{2}}$, etc., with $R \geq 1$. For this part of the dataset, there can be a problem if $\operatorname{Min}=0$, since $R$ cannot be calculated in such cases. This is typically due to missing values. One solution here is to use the average value of $R$, observed for samples where Min $\neq 0$, ensuring that the distribution obtained is consistent with the data. Alternatively, the smallest value larger than zero could be used to calculate $R$.

$$
\begin{equation*}
R=\left(\frac{M a x}{M i n}\right)^{1 / N_{c}} \tag{2}
\end{equation*}
$$

Once the categories have been created and populated by expression values of the dataset, weights are assigned to gene-sample couples, depending on the size of the category to which each belongs. As most optimization techniques traditionally minimize a given objective function, we want to have negative weights for "interesting" couples, and positive otherwise. To obtain size-dependent weights, the average population, (calculated over all categories) is subtracted from the population of that category to which the couple belongs (Eq. 3): hence "small" categories are negatively weighted, while larger ones have positive values. To avoid extreme weight values for datasets with very large number of genes, $n$, weights are normalised by dividing by $n$

$$
\begin{equation*}
w_{i j}=\left(\left|N_{\in(i, j)}\right|-\frac{\sum_{i=1: c}\left|N_{i}\right|}{N_{c}}\right) / n \tag{3}
\end{equation*}
$$

These weights should be interpreted as follows: the lower the weight $w_{i j}$ of edge $e_{i j}$, the more significantly the expression level of gene $i$ deviates from what is observed for the majority of genes for sample $j$.

### 3.3. Tanay Scheme

This method is incorporated into the SAMBA clustering algorithm [5] and is available with the EXPANDER software suite [21], (which also offers a number of other clustering options and is available from: http://www.cs.tau.ac.il/~rshamir/ expander/expander.html). As this software does not need to provide output on the weight of each gene-sample couple in the dataset, completion of our analysis required us to recode this scheme to allow access to the weight information during the algorithmic process. The information for this scheme was taken from [5] and [22].

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Note that the bipartite representation of the data in the SAMBA algorithm, which uses this weighting scheme, has node sets $\{$ genes $\}$ and $\{$ properties $\}$. The property set includes ${ }^{\mathrm{f}}$ nodes for properties of strong, moderate and weak induction/repression for each particular sample $j$. Thus $\mid\{$ properties $\} \mid=6 \times p$, where $p=$ number of samples. A gene $i$ is defined to be: weakly induced in sample vector $j$ if the expression value $x_{i j}$ is ranked $\geq x$ $\qquad$ in $j$; moderately induced if it is ranked $\geq x$ $\qquad$ ${ }_{\mathrm{d}}$ in $j$; and in $j$. Similarly it is: weakly repressed if it is ranked $\leq y$ $\qquad$ ed $\geq x_{\text {strong Induced }}$
in $j$; moderately repressed if it is ranked $\leq y$ $\qquad$ in $j$; and strongly repressed if it is ranked $\leq y_{\text {strong repressed }}$ in $j$. These thresholds are arbitrarily chosen and are fixed for all samples. For this analysis we used thresholds of $0.97,0.90,0.87,0.03,0.10,0.13$, for each of the values above respectively.

Briefly, with this method let $\phi(i, j)$ be the probability that gene $i$ has property (i.e. induced/repressed in sample $j)^{8}$, (see [5,22] for more details on this calculation). The majority of gene sample couples will have $\phi=0$. This probability is assigned as the weight of a given edge, $e_{i j}$, scaled with the log-likelihood of getting that edge by chance. Thus each gene-sample couple in the dataset has weight Eq.4:

$$
\begin{equation*}
w_{i j}=\left(\phi(i, j) \times \log \frac{P_{c}}{P_{i, j}}+(1-\phi(i, j)) \times \log \frac{1-P_{c}}{1-P_{i, j}}\right) \tag{4}
\end{equation*}
$$

The probability $P_{(i, j)}$ is the fraction of random bipartite graphs, with degree sequence identical to $G$, that contain $e_{i j}$ (and can be estimated using Monte-Carlo methods). $P_{c}$ is based on the assumption that an interesting edge occurs with a constant probability $>\max _{(i, j) \in I x J} P_{(i, j)}$. For this work a $P_{c}$ value of 0.9 was used [5].

## Scheme evaluation

In this Section, we use the 4 -point framework introduced in Section 2.2 to analyse our two novel weighting schemes, and to compare them with the scheme introduced by Tanay et al. [5].

### 4.1. Reusability

The empirical weighting scheme results in a partially connected graph for each subcategory, since genes which do not show a significant change for a given sample do not generate an edge. Subsequent clustering techniques would need to allow for this non-edge set, by optimisation of the objective function excluding nodes not connected by an edge. This scheme requires a dedicated algorithm for subsequent

[^30] probability, it may also be strongly induced, albeit with a smaller probability.

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biclustering, which maintains the lists of gene-sample couples in each category, (i.e. strongly induced, moderately induced, etc.).

The distribution-based scheme is independent of subsequent clustering approaches, since it was designed without an explicit approach in mind. It is versatile and can be used with most clustering techniques and with other types of large datasets, biological or otherwise.

Likewise, the Tanay scheme is independent of the subsequent clustering technique, as it results in positive edges for interesting gene-sample couples and negative edges for non-interesting gene-sample couples. Indeed, the scheme was specifically designed for an additive scoring system, where the sum of the edge weights in a subgraph corresponds to its statistical significance, (see [22] for more details). This scheme has also been applied to a compendium of information, and not just to gene expression data, [6].

### 4.2. Parameter influence

The distribution and empirical weighting schemes under consideration are controlled by a single parameter and, therefore, easily configurable while offering some flexibility.

The distribution-based scheme is controlled by the number of categories, $N_{c}$. The influence of this parameter is assessed through examination of the robustness and discrimination achieved, as detailed below.

With respect to the empirical scheme the parameter that influences results is $\kappa$, (Eq. 1), which determines thresholds between categories. Table 1 illustrates the results of the threshold analysis for the Lymphoma dataset. The maximum threshold for which any gene-sample couple was identified in the real dataset was $\kappa=7.07$ (probability $\leq 0.02$, (Eq. 1)). Thresholds of $\kappa=5,4.08,3.58$ and 3.162, (i.e. proba(probability $\leq 0.02$, (Eq. 1 )). Thresholds of $\kappa=5,4.08,3.58$ and 3.162, (i.e. proba-
bilities $\left(\frac{1}{2^{2}}\right) \leq 0.04,0.06,0.08$ and 0.10 respectively) were then tested. The log ratio bilities $\left(\frac{1}{\kappa^{2}}\right) \leq 0.04,0.06,0.08$ and 0.10 respectively) were then tested. The log ratio
of the probabilities is maximised at $\kappa=4.08$, and this was taken to be the strong of the probabilities is maximised at $\kappa=4.08$, and this was taken to be the strong
response threshold. Thresholds for moderate and weak response were then deduced response threshold. Thresholds for moderate and weak response were then deduc
to be $\kappa=3.162$ and $\kappa=2.23$, respectively. to be $\kappa=3.162$ and $\kappa=2.23$, respectively.

Table 2 provides results of threshold analysis for all three test datasets..
The main parameters affecting the Tanay weighting scheme are, again, thresholds between categories, and $P_{c}$, (Eq. 4). Thresholds between categories are arbitrarily chosen, based on normalized ranked values within each sample and are not data dependent. This 'hard thresholding' has consequences for the deterioration of the scheme when noise and missing values are added to the data. As the threshold parameter is lowered, a higher percentage of edges will be identified, even if none exist. For an analysis of parameter $P_{c}$ see [5].

### 4.3. Robustness

The influence of noise and missing values are summarised respectively in Table 3 for the Lymphoma dataset. Results for other datasets, not displayed here, are

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| Strong |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa$ | 7.07 | 5 | 4.08 | 3.58 | 3.16 |  |
| P | 0.02 | 0.04 | 0.06 | 0.08 | 0.10 |  |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e}\right)$ | undef | undef | 3.98 | 2.45 | 1.5 |  |
| Moderate |  |  |  |  |  |  |
| $\kappa$ | 3.162 | 2.88 | 2.67 | 2.5 | 2.35 |  |
| P | 0.10 | 0.12 | 0.14 | 0.16 | 0.18 |  |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e)}\right)$ | 4.35 | 3.9 | 3.54 | 3.31 | 3.20 |  |
| Weak |  |  |  |  |  |  |
| $\kappa$ | 2.23 | 2.13 | 2.04 | 1.96 | 1.88 |  |
| P | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 |  |
| $\log \left(\frac{P(X=E d g e)}{P_{\text {rand }}(X=E d g e)}\right)$ | 0.063 | 0.035 | 0.019 | 0.002 | -0.01 |  |

Table 1. Threshold Analysis, Lymphoma data. $\kappa=$ the number of standard deviations from mean (Eq. 1). $\mathrm{P}=\frac{1}{h}$, is the probability that values are $\kappa \sigma$ from mean. The maximum log-ratio is taken
as the threshold between categories.

|  | Strong | Moderate | Weak |
| :--- | :--- | :--- | :--- |
| Lymphoma | 4.08 | 3.162 | 2.23 |
| Yeast Cell Cycle | 3.16 | 2.88 | 2.23 |
| Kasumi | 2.77 | 2.58 | 2.23 |

Table 2. $\kappa$ thresholds identified for each of the tested datasets.
consistent with these.
4.3.1. Empirical Weighting Scheme

The absolute variation in weights is extremely low for the empirical scheme since the technique examines extreme values, i.e. values which appear in the tail of the distributions of each gene variable. In addition, weights are not based directly on a given expression value, but on that expression value relative to other values in the category for a particular sample (Step 2 of scheme, see Fig. 1). The category is in turn defined relative to expected value of the gene variable (Step 1 of scheme, see Fig. 1). As "missing" values are replaced by the row mean, this does not greatly Fig. 1). As missing values are replaced by
affect extreme values. Equally, even noise added at $10 \%$ level of the original values does not affect relative values, thus, perturbations in the data have small effect on weights assigned.

Similar to results shown in Table 3, for the Kasumi dataset, average absolute variation in edge weights is $\sim 0.26 \%$ for an added noise level of $10 \%$ (data not shown), while denoting $10 \%$ of the dataset as missing values, gives average absolute variation in values $\sim 0.3 \%$, with stable weights accounting for around $99.5 \%$. For the

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| \% Noise level | 1.5 | 2.5 | 5 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| Distribution Based |  |  |  |  |
| \% Average Absolute variation | 3.10 | 4.90 | 9 | 15.80 |
| \% "stable" weights | 83 | 84.5 | 89.1 | 88.6 |
| Empirical Based |  |  |  |  |
| \% Average Absolute variation | 0.06 | 0.02 | 0.03 | 0.05 |
| \% "stable" weights | 99.66 | 99.68 | 99.68 | 99.65 |
| Tanay Scheme |  |  |  |  |
| \% Average Absolute variation | 0.00270 | 0.00260 | 0.00230 | 0.00320 |
| \% "stable" weights | 99.985 | 99.989 | 99.994 | 99.997 |
| \%missing values | 1.5 | 2.5 | 5 | 10 |
| Distribution Scheme |  |  |  |  |
| \% Average absolute variation | 17.70 | 27.30 | 48.86 | 92.00 |
| \% of "stable" weights | 37.40 | 33.40 | 32.50 | 30.20 |
| Empirical Scheme |  |  |  |  |
| \% Average absolute variation | 0.04 | 0.04 | 0.09 | 0.16 |
| \% of "stable" weights | 99.70 | 99.72 | 99.64 | 99.62 |
| Tanay Scheme |  |  |  |  |
| \% Average absolute variation | 0.0023 | 0.0027 | 0.0028 | 0.0069 |
| \% of "stable" weights | 99.9837 | 99.9880 | 99.9936 | 99.9969 |

Table 3. Influence of noise
distribution based scheme)

Yeast Cell Cycle data, the corresponding values for $10 \%$ noise added were: $\sim 0.22 \%$ (absolute variation) and $\sim 99.65 \%$ (stable weights); and for missing values at $10 \%$ was: $\sim 0.15 \%$ (absolute variation) and $\sim 99.69 \%$ (stable weights).

### 4.3.2. Distribution-Based Weighting Scheme

The distribution-based weighting scheme is more generic, as it does not rely as much on the underpinning biological information: as such, it is less robust to noise and missing values, which alter the distribution of expression levels.

Influence on the weights used is "reasonable" for low noise perturbation. Specifically, the percentage of stable weights is a helpful indicator, given the nature of the scheme: when a gene-sample couple falls into a new category due to added noise, scheme: when a gene-sample couple fals into a new category due to added noise,
this changes the weights for all couples in the new category as well as all those in this changes the weights for all couples in the new category as well as all those in
the old one. The \% stable weights value may, therefore, give more insight into the scheme robustness, than using only the average absolute variation of the weights, (Table 3). Clearly, as the number of categories increases, the each category interval decreases, and gene-sample couples are more likely to change categories, leading to a smaller proportion of stable weights 3 .

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Fig. 3. Influence of noise - Proportion of stable weights (\%)


Fig. 4. Influence of missing values - Proportion of stable weights (\%)

With respect to missing values, the scheme is far less robust than with respect to noise perturbations. However, it is important to note that the sign of the remaining weights is not lost: a positive weight does not become negative, (except for cases when more than half the values for a given gene are missing; in those cases, it would almost certainly be excluded from the dataset before the scheme is applied). What is partially lost is the degree of over-expression, (or under-expression), rather than the knowledge that this change of expression occurs. A representation of missing value influence, depending on the number of categories, is given in Figure 4.
As noted previously, the results displayed correspond to untreated data, i.e. where the mean to replace missing values, thus creating larger perturbations in the weights. Future work might reasonably include tests of the effect of the various correction techniques on scheme robustness.
4.3.3. Tanay Scheme

Perturbations in the data have very little effect on weights derived with the Tanay scheme, (similar results for all tested datasets, data not shown for Yeast and Kasumi datasets). We were surprised by this result and tested missing values up to a level

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of $80 \%$ however the effect was still minimal $(0.01 \%$, average variations and $99.99 \%$ stable weights). It may be the case that this scheme identifies 'interesting' genesample couples, even if none exist, due to the 'hard' threshold nature of the scheme. If $10 \%$ noise added to the dataset, thresholds still depend on ranking and not mean level, thus approximately the same gene-sample couples selected as interesting as ranked position is not changed. Missing values has little effect, as the values are replaced by the mean, therefore more extreme values are replaced with less extreme values, however the same thresholds are used.

### 4.4. Discrimination

For this analysis, 'random graph' refers to graphs created from random datasets, as described in Section 3.1.1.
4.4.1. Empirical Weighting Scheme

From the threshold analysis, described above, maximum discrimination between empirical and random graphs is achieved. As expected, the largest number of genesample couples falls in the weak response category. Discriminating between gene responses depends on the category thresholds used. From Table 4, for strong and moderate response, the probability of an edge existing between a gene and sample node in the real graph is greater than that for the random graph, indicating that significant structure is present. For a weak response, the ratio of probabilities is smaller and it is less convincing that real differences exist. Nevertheless, an examination of the average degree of sample nodes in the real graphs indicate that average number of genes responding is higher than expected. For example, for the weak repression sub-category, a sample node is, on average, connected to $\sim 3 \%$ of ene nodes compared to $\sim 1.5 \%$ in the random graph, $\left(d_{\perp} / d_{\top}\right)$. The average degree ene nodes compared to $\sim 1.5 \%$ in the random graph, $\left(d_{\perp} / d_{\top}\right)$. The average degree suggests that, although the ratio of probabilities in weak response is not high, some suggests that, although the ratio of probabilities in weak response is not high, some pattern structure is
gene-sample couples.
4.4.2. Distribution-Based Weighting Scheme

By construction, there are no "damaging" false-positives or false-negatives, at least in theory. In practice, false-positive or false-negative edges may have weights very close to zero, (either positive or negative depending on the number of categories). A significant change in expression patterns can not lead to a positive weight, while A significant change in expression patterns can not lead to a positive weight, while
negative weights are only obtained where there is a significant change. Since absolute, (positive vs. negative), discrimination is guaranteed, the focus here is to assess relative discrimination, i.e. distribution of weight values. Results, of the influence of the number of categories, on this discrimination are displayed in Table 5. A first observation, based on the proportion of negative weights, is that discrimination is

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|  | Repres |  |  | Induc |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Wk | Mod. | Str. | Wk. | Mod. | Str. |
| $n_{\top}$ | 2572 | 456 | 111 | 2829 | 719 | 304 |
| $n_{\perp}$ | 63 | 26 | 28 | 81 | 53 | 54 |
| $m$ | 4324 | 506 | 114 | 4939 | 822 | 316 |
| $d_{\top}$ | 1.69 | 1.11 | 1.03 | 1.74 | 1.148 | 1.04 |
| $d_{\perp}$ | 72.29 | 21.66 | 4.75 | 64.09 | 17.75 | 6.08 |
| $\delta$ | 0.0112 | 0.002 | 0.0003 | 0.0127 | 0.002 | 0.0008 |
| $n_{T_{\text {rand }}}$ | 2774 | 8 | 5 | 2718 | 7 | 3 |
| $n_{\perp \text { rand }}$ | 87 | 7 | 5 | 87 | 7 | 3 |
| $m_{\text {rand }}$ | 4007 | 9 | 5 | 4020 | 8 | 4 |
| $d_{T_{\text {rand }}}$ | 1.477 | 1 | 1 | 1.462 | 1 | 1 |
| $d_{\perp \text { rand }}$ | 46.57 | 1.28 | 1 | 45.875 | 1.14 | 1 |
| $\delta_{\text {rand }}$ | 0.0103 | $2 e^{-5}$ | $1 e^{-5}$ | 0.0104 | $2 e^{-5}$ | $1 e^{-5}$ |

able 4. Categories for Lymphoma data, created by cut off thresholds 0.20 (weak induction/repression), 0.10 (moderate induction/repression), and 0.06 (strong induction/repression). $n_{\top}=$ the active set of genes (gene nodes with degree $\geq 1$ ), $n_{\perp}=$ active set of samples, $m=$ umber of edges, $d_{\top}=$ average degree of active set of genes, $d_{\perp}=$ average degree of active se
good: there are on average, just under $41,019(\sim 11 \%)$ negative weights (out of 386,496), implying that $11 \%$ of gene-sample couples in the dataset are of interest. This value varies between 35,542 ( 30 categories) and 49,242 (10 categories), with a standard deviation of $\sim 3908$. The scheme selected $8.75 \% \pm 0.21 \%$ gene-sample couples from a random dataset, (Section 3.1.1), for $N_{c}=14$. Discrimination is satisfactory for any number of categories, $\left(N_{c}\right)$, in the range tested. Weights obtained with fewer categories appear more discriminatory in general, (apart from $N_{c}=10$, hich indicates that $N_{c}<10$ would be ill-advised). Given that $N_{c}=12$ to 16 cat which indicates that $N_{c}<10$ would be ill-advised). Given that $N_{c}=12$ to 16 catThis recommendation also applies to other datasets, for which results obtained are This rec
similar.
4.4.3. Tanay Scheme

From our analysis of this scheme (Table 6), we observed that (a) a smaller number of total positive weights were identified compared to those selected from corresponding graphs generated from random dataset (Section 3.1.1), and (b) the number of ing graphs generated from random dataset (Section 3.1.1), and (b) the number of ception of Kasumi Moderate and Strongly repressed categories). Observations (a) and (b) imply that the degree 'sharing' of gene-sample couples in the real dataset (i.e. gene-sample couples having a positive edge in the weakly induced category, are also feature in the strongly induced category- categories are not mutually exclusive). Since 'hard' thresholds between categories was used and arbitrarily chosen,

|  | Distribution Weights |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k$ | $\leq-6$ | $[-6 ;-4]$ | $[-4 ;-2]$ | $[-2 ; 0]$ | $[0 ; 2]$ | $[2 ; 4]$ | $\geq 4$ |
| 10 | 14977 | 10207 | 8779 | 15279 | 13337 | 12969 | 310948 |
| 12 | 7708 | 13206 | 10917 | 11324 | 11453 | 17783 | 314105 |
| 14 | 733 | 14252 | 14998 | 13327 | 14293 | 17606 | 311287 |
| 16 | 0 | 9778 | 16293 | 17884 | 14973 | 17616 | 309952 |
| 18 | 0 | 4723 | 19030 | 18740 | 18141 | 17629 | 308233 |
| 20 | 0 | 0 | 20095 | 20116 | 21275 | 20942 | 304068 |
| 22 | 0 | 0 | 17435 | 22792 | 23217 | 19002 | 304050 |
| 24 | 0 | 0 | 14050 | 24584 | 24329 | 24004 | 299529 |
| 26 | 0 | 0 | 11334 | 26286 | 26337 | 28134 | 294405 |
| 28 | 0 | 0 | 8325 | 28679 | 27582 | 32757 | 289153 |
| 30 | 0 | 0 | 4751 | 30791 | 29200 | 35930 | 285824 |

Table 5. Influence of the number of categories on discrimination for Lymphoma data: distribution of weight values. Values in cells represent the number of gene-sample couples in various ranges
the order of the number of edges in each category is Strong $<$ Moderate $<$ Weak. Note also that those gene-sample couples, evaluated as strongly reacting, will have a magnified impact on any clustering procedure on the resulting graph, due to the overlap between categories.

### 4.5. Discussion and Conclusion

From the investigations above, it is clear that interpretations of edge weights in graphical gene expression schemes can differ considerably. Primarily, the empirical and Tanay weighting scheme result in positive weights for "interesting" genesample couples, while the distribution technique leads to significant effects reflected in negative weights. If the optimisation technique uses a minimisation-based objective function, the empirical and Tanay weights could simply be negated, (similarly for distribution weights if a maximisation function is used). Also, as noted previously, the empirical scheme results in a set partially-connected oraphs, while the ristributed and Tany sole results in fully-coneted the the distributed and Tanay scheme results in a fully-connected graph. This has conFor both empirical and distribution schemes presented, edge weights linking effected For both empirical and distribution schemes presented, edge weights linking effected
genes for a particular sample $j$, are defined relative to other gene expression values genes for a particular sample $j$, are defined relative to other gene expression values
for sample $j$. This is an important corollary, as absolute level of gene expression is for sample $j$. This is an important corollary, as absolute level of gene expression is
not directly accounted for, only the fact that it does change and the significance not directly accounted for, only the fact that it does change and the significance
of this change relative to the majority of genes. Relative evaluation is also an inof this change relative to the majority of genes. Relative evaluation is also an in-
trinsic feature of the Tanay based scheme, as the initial probability $\phi(i, j),($ Eq. 4) trinsic feature of the Tanay based scheme, as the initial probability $\phi(i, j)$, (Eq. 4)
is based on ranks. However, the selection of a pre-determined thresholds between is based on ranks. However, the selection of a pre-determined thresholds between dent. Conversely, the issue of data dependent threshold estimation is addressed in

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| Datasets | Lymphoma | Yeast Cell | Kasumi |
| :---: | :---: | :---: | :---: |
| \% total edges | 2.7 | 3.34 | 2.13 |
|  | $(2.9 \pm 0.007)$ | $(4.3 \pm 0.01)$ | $(3.6 \pm 0.02)$ |
| Induced |  |  |  |
| \% strong | 1.2 | 1.1 | 0.8 |
|  | $(1.1 \pm 0.009)$ | $(1.2 \pm 0.02)$ | $(0.8 \pm 0.004)$ |
| \% moderate | 3.2 | 4.1 | 1.0 |
|  | $(3.1 \pm 0.01)$ | $(3.9 \pm 0.03)$ | $(1.0 \pm 0.005)$ |
| \% weak | 3.9 | 5.0 | 1.1 |
|  | $(3.9 \pm 0.02)$ | $(4.8 \pm 0.03)$ | $(1.1 \pm 0.007)$ |
| Repressed |  |  |  |
| \% strong | 1.2 | 1.1 | 1.3 |
|  |  |  |  |
| \% moderate | $(1.1 \pm 0.009)$ | $(1.2 \pm 0.02)$ | $(1.4 \pm 0.06)$ |
|  | 3.2 | 3.8 | 3.8 |
| \% weak | $(3.1 \pm 0.01)$ | $(3.9 \pm 0.02)$ | $(3.4 \pm 0.06)$ |
|  | 4.0 | 4.7 | 4.7 |
|  | $(3.8 \pm 0.01)$ | $(4.8 \pm 0.03)$ | $(4.1 \pm 0.12)$ |

Table 6. Tanay scheme - percentage of total possible edges is taken as $\frac{\text { Number of Positive Edges }}{n \times p \times 6}$ whereas percentage of edges in each category is taken with respect to total possible edges in that values represent results $\begin{aligned} & n \times p \\ & \text { from random graphs. }\end{aligned}$
the two novel schemes presented, but is non-trivial as numerous thresholds need to be assessed, which is computationally expensive.

The empirical-based scheme is more specific, in the sense that fewer gene-sample couples are identified, than the distribution-based and Tanay scheme. For example, for the Lymphoma dataset, the empirical based scheme extracts 11,021 gene-sample couples ( $\sim 3 \%$ at optimal threshold levels). The distribution-based scheme extracts 43,155 couples ( $\sim 11 \%$ ) (for 12 categories), although 11,324 of these are close to ( and hence do not have a major impact on clustering), this still leaves 31,831 ( $\sim 8 \%$ ), "interesting couples". The Tanay scheme extracts $\sim 16 \%$ for the same dataset.

The empirical-based method deteriorates slowly with perturbations in data, hence for data that is known to contain many missing values and/or noise it may be a better choice. Weights overall with the Tanay scheme seem little affected by noise nd missing values, which indicates that this scheme will assign high weights to geneand missing values, which indicates thater even if none are present. If the sample size (i.e. the number of microarray experiments) for each gene is small, the performance of the empirical-based scheme deteriorates with respect to its random graph comparison basis (difficult to estimate $\mu$ and sd of each gene variable) and the thresholds between categories become increasingly difficult to identify. In this situation, the distribution-based
scheme would be the better choice.
Using graphical techniques to extract meaningful information from biological data is an intuitive and popular method. In this paper, we limited our investigation to bi-partite graphs as this representation captures essential properties of the gene expression dataset and allows for the extraction of biclusters most suitable for data in this domain, [11]. Alternatively, a one-mode gene expression graph could have been considered, where $G=(N, E)$, with $N$ a set of gene nodes and $\left(n_{i}, n_{j}\right) \in E$ if gene $i$ and $j$ show similar expression across all samples, (measured e.g by a distance function). This approach was used in [23], although not explicitly to investigate weighted graphs. Analysis of complex weighted networks was considered in [24], although the authors did not investigate the weighting scheme itself.

As such, investigation into weighting of gene expression networks is long overdue. In this paper we proposed and compared two weighting schemes applied to gene expression bi-partite graphs with a view to extracting meaningful biclusters from the data. We also compared the properties of these novel schemes to the innovative work of Tanay et al. [5, 6, 22]. The importance of assessing edge-weighting schemes was highlighted and we demonstrated that edge weights must be considered independently from the clustering procedure, since alternative edge weight derivation can lead to different interpretations of the data. This type of assessment framework for weighting is equally crucial in the context of other types of large dataset, biological or otherwise

Further investigation on extraction of meaningful graphical relationships through choice of edge-weighting schemes should include incorporation of information from related sources, (such as protein interaction information, promoter information etc.), to refine weights and improve handling of missing values. Investigation of automatic threshold estimation for category sub-divisions is also indicated.

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[^0]:    ${ }^{1}$ Genes are said to be expressed when the product they code for is realised, see Chapter 2 for more details.

[^1]:    ${ }^{1}$ Non-coding in the sense that they are not translated into protein, but do carry out other essential functions in the cell
    ${ }^{2}$ Adenines in the DNA template do not specify thymines in the RNA copy as RNA does not contain thymine, instead adenine pairs with uracil in DNA-RNA hybrids

[^2]:    ${ }^{3} \mathrm{~A}$ segment of a gene that contains instructions for making a protein. In many genes the exons are separated by "intervening" segments of DNA, known as introns, which do not code for proteins; these introns are removed by splicing to produce messenger RNA.
    ${ }^{4}$ An alternative form of a protein resulting from differential transcription of the relevant gene either from alternative promoter or alternate splicing.

[^3]:    ${ }^{5}$ Base-pair to form double stranded structure
    ${ }^{6}$ Base-pairs are the units for measuring the length of a nucleic sequence. Each nucleotide in a sequence is one base-pair. $1000 \mathrm{bp}=1 \mathrm{~kb}$ (kilo base-pairs), $1000 \mathrm{~kb}=1 \mathrm{mb}$ (mega base-pairs) etc.

[^4]:    ${ }^{7}$ Percentage of bases which are either guanine or cytosine. GC pairs are more thermostable compared to the alternative Adenine-Thymine pairs. Genes are often characterised by having a higher GC content in contrast to the background GC content for the entire genome. Evidence of GC ratio with that of length of the coding region of a gene have shown that the length of the coding sequence is directly proportional to higher GC content, (Oliver and Marn, 1996)

[^5]:    ${ }^{8}$ determined by probe length and composition

[^6]:    ${ }^{9}$ Two measurements are considered independent if the experimental materials on which the measurements are based receive different treatments and if the materials were handled separately at all stages of the experiment.

[^7]:    ${ }^{10}$ Preprocessing is a processes applied to the raw data, such as standardisations, normalisations to remove noise etc. to produce data that can used as input to another program

[^8]:    ${ }^{11}$ Protein measurement methods include ICAT, MudPIT, 2-DE.

[^9]:    ${ }^{1}$ In computer science there are, of course, alternative methods of recording edge lists such as linked lists etc. However to introduce the concept we use only the idea of an adjacency matrix.
    ${ }^{2}$ the weight of a node can represent such things as importance in hub, e.g. internet networks, telephone networks etc.

[^10]:    ${ }^{3}$ A random graph is a graph in which properties such as the number of nodes, edges and/or connections between are determined randomly.

[^11]:    ${ }^{1}$ Reporting for all algorithms is prohibitively detailed. Our aim is to give a 'flavour' of techniques and their validation, by applying selected algorithms from each group in the Jain et al. (1999) taxonomy.

[^12]:    ${ }^{2}$ Note: these stability measures are extremely time consuming and memory intensive measures to calculate. This is in addition to the fact that they are already assessing a very memory intensive algorithm. For example, to assess the Synthetic dataset A took 344.24 hours of CPU execution time, while analysis of the Golub dataset took 744.56 hours of CPU time. This is a practical consideration of computation time and memory and is obviously a drawback of these stability measures.

[^13]:    ${ }^{3}$ available from http://flame-clustering.googlecode.com/svn/trunk/

[^14]:    ${ }^{4}$ In the next sub-section the SAMBA algorithm, a technique which also finds biclusters in the data, (although categorised here as a graphical application) is examined.

[^15]:    ${ }^{5}$ available for download at http://acgt.cs.tau.ac.il/expander

[^16]:    ${ }^{1}$ A system composed of interconnecting parts that as a whole exhibit complex properties not evident from the properties of the individual parts.

[^17]:    ${ }^{2}$ Defined in Section 3.3.4 to be a graph where an edge can exist between any two nodes

[^18]:    ${ }^{3}$ Random datasets of the same dimension as the input dataset were created, where for each row (gene) $i$, random numbers were selected from a Normal distribution of mean, $\mu_{i \text { Rand }}=\mu_{i \text { Real }}$, and standard deviation, $\sigma_{i \text { Rand }}=\sigma_{i \text { Real }}$ (Note: for a gene which does not respond to any sample, gene

[^19]:    expression is relatively constant.). For each threshold choice in this analysis, 100 random datasets were created to estimate cut-offs, with comparisons based on averaging over these.
    ${ }^{4}$ subjectively chosen from analysis of datasets

[^20]:    ${ }^{5}$ The random graphs were Monte Carlo switching process (Maslov et al., 2004), to create random graphs with the same degree distributions. This proceeds by picking two random edges ( $\mathrm{x}, \mathrm{y}$ ) and ( u , v) uniformly with $x, y, u$, $v$ distinct nodes. If ( $x, u$ ) and ( $y, v$ ) are not edges, then adding the edges $(\mathrm{x}, \mathrm{u}),(\mathrm{y}, \mathrm{v})$ and delete edges $(\mathrm{x}, \mathrm{y}),(\mathrm{u}, \mathrm{v})$. This process is repeated $m \times 100$ times

[^21]:    ${ }^{6}$ If the dataset was not categorised, a weak response and a strong response gene-sample couple would have very different weights, with the consequence that the strong response couple would dominate the analysis and obscure more subtle patterns.

[^22]:    ${ }^{1}$ This could be easily extended to $>2$ sample neighbours. We focus on the all-in-one graph as we want to find coherent bi-clusters i.e. genes which may be alternately expressed in a given sample.

[^23]:    ${ }^{2}$ Random graphs were created by projecting uniformly sampled random bi-partite graphs (created with monte carlo edge switching algorithm) into one-mode.

[^24]:    ${ }^{3}$ Note that, due to small size and lack of information, the Cho dataset is not considered for further analysis. Modules for the Gasch dataset was extracted from a one-mode graph created whereby two gene nodes must have a common neighbourhood of at least 5 sample nodes in the underlying bipartite graph in order to qualify for an edge in the one-mode graph.

[^25]:    ${ }^{4}$ A hypergeometric test was used to find groups of over-represented genes in each module (Falcon and Gentleman, 2007). This describes the probability of number of successes from $n$ trails, using sampling without replacement.
    ${ }^{5}$ In the Gene Ontology (GO) annotation hierarchy, each GO term inherits all annotations from its more specific descendants. An analysis for GO term associations can result in the identification of genes associated with directly related GO terms with considerable overlap. To avoid this problem, when analysing the GO ontology graph, the leaves of the GO graph were tested first (nodes with no children), before testing terms whose children have already been tested, and all genes annotated at significant children are removed from the parent's gene list. This continues until all terms were tested.

[^26]:    ${ }^{6}$ ER + Estrogen Receptor positive samples, which ER- represents Estrogen Receptor negative samples
    ${ }^{7}$ ER + LN- represents Estrogen Receptor positive and Lymph Node negative samples, which ERLN+ represents Estrogen Receptor negative and Lymph Node positive samples

[^27]:    ${ }^{8}$ Heat Shock, Hydrogen Peroxide treatment, Menadione exposure, Diamide treatment, DTT Exposure, Hyper-osmotic shock, Hypo-osmotic shock, Amino acid starvation, Nitrogen depletion, Stationary phase, Steady state growth on alternative carbon sources. Each represent time course experiments.

[^28]:    ${ }^{9}$ The Spellman yeast cell cycle experiment is a compendium of time course experiments based on four synchronisation methods: alpha factor (DBY8724), elutriation-chamber (DBY7286), cdc 15-2 (DBY8728), Cln3 and Clb2 (DBY8725, DBY8726).

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    E-mail address: gkerr@computing.dcu.ie (G. Kerr)

[^30]:    This algorithm was designed to be applied to a compendium of data-sources, not just gene expression data. For this analysis we are restricting it to gene expression data

