Chapter 1

A Risk Management Paradigm for Monetary Policy.

'If there is anything about which modern macroeconomics is clear however – and on which there is substantial consensus – it is that policy rules have major advantages over discretion in improving economic performance. Hence, it is important to preserve the concept of a policy rule even in an environment where it is practically impossible to follow mechanically the algebraic formulas economists write down to describe their preferred policy rules.' (John B. Taylor, 1993, p.197)

1.1. Introduction

A considerable literature has emerged, since the mid 1990s, employing Taylor Rules to examine the conduct of monetary policy: Judd and Rudebusch (1998), Batini and Haldane (1999), Clarida, Gali and Gertler (1999), Nelson (2000), Svensson (2003) and Kuttner (2004). More generally, the Taylor Rule and Inflation targeting frameworks are routinely invoked when analysing how central banks discharge their responsibilities concerning price stability and output.[1] These frameworks draw heavily on contributions from Kydland and Prescott (1977), Barro and Gordon (1983) and Blanchard and Fischer (1989) and are largely the product of the 'rules versus discretion' debate. This debate also represents a useful starting point for understanding what Bernanke (2004) refers to as 'Greenspan's risk management approach'.

The common championing of rules in the literature has had a number of motivations. Kydland and Prescott (1977) stressed that the credibility obtained from committing to a future policy path, helped reduce current inflation with less cost. Their analysis showed that the stagflation experience of the Great Inflation was not necessarily attributable to irrational policy decisions, but rather to an unwillingness to maintain consistency. This observation helped redirect the focus of scholarly activity to the design of institutions that lessen the time inconsistency problem. It may furthermore have contributed to the reform of central banks since the 1990s.[2]

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[1] A significant advance in the development of these rules can be attributed to the policy regime evaluation project published by the Brookings Institution. Bryant, Hooper and Mann (1993) attempted to uncover simple reactive interest rate rules that would produce sound economic outcomes in terms of price stability and output and could be shown to be robust across a range of alternative estimated models.

[2] See Vickers (1998) footnote 3.

The desire to make policy more rule-based, in part, reflects the desire to make rate decisions more transparent. Mishkin (1999) points out that a key advantage of an inflation targeting framework relates to the greater accountability implied by announcing publicly the central bank's intended aims. Increased transparency should have the effect of lessening the time-inconsistency problem and defuses political pressure to navigate an electoral calendar. In what follows, the 'rules versus discretion' literature is examined with a view to understand nonlinear policy rate adjustment. The opportunistic approach developed by Orphanides and Wilcox (1996, 2002), Orphanides, Small, Wieland and Wilcox (1997) and Aksoy, Orphanides, Small, Wieland and Wilcox (2006) is examined here using an added perspective; that policy makers in the United States wanted to work within a rules framework, while not wanting to be overly restrained by those same set of rules. The internal FOMC construal of opportunistic policy was defined at the December 1995 meeting by Don Kohn. In his briefing, he used a policy matrix identifying the policy implications of both the deliberative and opportunistic approaches.[3]

Preserving flexibility implied central bankers, by default, subscribed to a contingency rule i.e. an evolving unannounced zone target rather than to a given point target. Committee dynamics also contributed to a zone of inaction.[4] That is, central bankers tend to implement rate changes that are designed to manage expected or future inflation. Greenspan (2003, 2004) points out that future outcomes are essentially unknown and monetary policy, as a consequence, is heavily dependent on risk management. Zone targeting and opportunistic frameworks imply policy makers respond to the likelihood of inflation breaching an upper and lower bound. This chapter develops the upper bound as being equivalent to the strike price of a call option. Using Monte Carlo and non-numerical techniques, a variety of option pricing approaches are developed to investigate how policy makers respond to expected inflation in a forward looking risk management context.

[3] In the appendix to the December 1995 FOMC meeting p. 10 – 11, Don Kohn outlined how the opportunistic monetary policy would be useful for interfacing with other branches of government. Strategic considerations for developing the opportunistic approach were prompted by political developments, largely linked to the Mack Bill. This is developed more in chapter 3. The FOMC transcripts are downloadable from the Federal Reserve.

[4] This is developed in chapter 2.

1.2. A History of Rules and Contingency – The Gold Standard

Prominent contributions that set out contingency rules for central bankers have included Bagehot (1873). The Bank Act of 1844 imposed a very stringent regime limiting the Bank of England's authority to issue money. This constituted an earlier form of rules-based stabilisation.[5] The Act prevented the issuance of new notes that were not matched by an increase in its gold reserve. This, in effect, consolidated the workings of the Gold Standard. From a conventional perspective, it also afforded a *de facto* monetary policy by explicitly containing currency issue and by implicitly containing inflation.[6] It can be argued that significant parallels exist between elements of modern inflation targeting and 'the rules of the game' associated with the Gold Standard. The Classical Analysis configures the role of central banks as exerting a moderating influence. Inflation targeting, as implemented across a number of jurisdictions, is similar to the Taylor Rule in terms of crafting rate decisions.[7] The parameter weights associated with Taylor (1993) despite generally having their origins linked to the Greenspan Fed, can be used to gauge the tenor of policy even when no official monetary policy agency has existed.[8] Taylor (1998, 2007) used his benchmark rule to identify 'policy mistakes' that occurred during a number of historical episodes. Taylor (1998) pointed out that the specie flow (or transfer of gold) associated with the international Gold Standard implied a form of policy rule not unlike that suggested by modern policy design. The counter cyclical nature of much of contemporary monetary policy is sometimes neatly summarised as 'leaning against the wind'. Equally, central bankers have coined the phrase: 'removing the punchbowl before the party gets going', to denote the pre-emptive stance that they are obliged to adopt.[9] Since Kydland and Prescott (1977), establishing a rules-based framework has increasingly been seen to be indispensable to appropriately executing policy. Of

[5] It is open to debate how activist or complete this type of stabilisation was. The Gold Standard was set in place when the accepted norm of public policy was decidedly *laissez-faire*.

[6] Bordo and Kydland (1995) however interpreted the gold standard as a contingent rule in so much as convertibility of the currency was maintained, except during prolonged military interventions.

[7] Particularly, in the absence of supply shocks.

[8] Ironically, explicit Federal Funds targeting was only gradually acknowledged as being the operating procedure used by the Federal Reserve. Thornton (2004) maintained that it was not until the December 21, 1999, that the FOMC's policy statement finally made unequivocal the fact that the FOMC targeted the Federal Funds rate.

[9] This phrase is generally attributed to William McChesney Martin.

course, the advocacy of rules has often predated the formal setting up of central banks and explicit pursuit of economic stabilisation.[10]

Bordo (1981) maintained that the Bank of England (BOE) played by the ‘rules of the game’ over much of the period between 1870 and 1914. Whenever Great Britain was confronted by a balance-of-payments deficit with a corresponding decline in gold reserves, the BOE raised its "bank rate" (discount rate). Interest rates more generally in the United Kingdom would rise. As a consequence, investment and spending would decrease ultimately leading to a fall in the price level. At the same time, the increase in the bank rate would stem any short-term capital outflow and draw funds from abroad. Bordo (1981) pointed out that the moderating influence of restricting money supply via the Gold Standard delivered an era of low inflation well into the twentieth century. *Per contra*, the international Gold Standard may have been less good in terms of stabilising output (the other gap incorporated into the Taylor Rule). According to the Classical Analysis, the inherent logic of the Gold Standard is not inconsistent with implementing a policy rule.[11] The systematising effects exerted on market interest rates are similar to the recommendations or imperatives associated with Taylor (1993) or inflation targeting.[12] Taylor (1998) found however that interest rates were not sufficiently mobilised to keep output in check during the international Gold Standard era, nor inflation in check prior to the 1980s.[13]

Simons (1936), p.13-14, maintained that rules make for good monetary policy even if the underlying instrument can sometimes be extremely blunt. He asserted that monetary rules once established ‘should work mechanically, with the chips falling where they may’. This is a somewhat recurrent theme in the literature and it is clear that there has always existed a tension between playing by the ‘rules of the game’ while also accommodating contingencies. Whittlesey (1968) maintained that most monetary rules have largely culminated in failure. Instances of successful rules are

[10] The Federal Reserve was established in 1913 long after the International Gold Standard came into being.
[11] Goodhart (1972) questioned a number of the hypotheses contained in the Classical Analysis.
[12] McCallum (1999a) makes the point that the Taylor Rule and McCallum’s own monetary base rule have historically recommended similar policy prescriptions.
[13] Taylor (1998) retrospectively identified ‘policy mistakes’ as deviations historically from his benchmark rule. He argued that an adherence to the International Gold Standard implied that interest rates adjusted to contain excesses in demand and prices.

difficult to reliably identify. Referring to the Bank Act of 1844, Whittlesey (1968), p. 259, pointed out:

> "…that the adoption of a strict monetary rule led to a financial crisis, which was resolved by discretionary action suspending the rule. The source of the difficulty was a Rule; relief lay in the exercise of Discretion. The rule resulted in rigidity; flexibility was achieved through discretion. ... In the end we were left with a government of men after all."

Bordo and Kydland (1995) argued that the Gold Standard represented a monetary policy rule but also was sufficiently flexible to accommodate contingencies such as wartime so that the commitment mechanism allowed for monetary constraint and temporary relaxations. Significantly, they argued convertibility at the original price of gold would be restored when a given crisis lifted. This, in effect, would imply that the Gold Standard constituted a *contingent rule* which essentially defines the tree or trajectory to be taken but not always in an exact manner. This is similar to the view expressed by Taylor (1993) that no algebraic rule can be mechanically implemented, yet policy should be principled.[14] In a more modern setting, the task of navigating between the implementation of systematised and *ad hoc* policy falls ultimately to a monetary policy committee. Policy makers expend much political capital in elaborating a rules-framework despite the fact that virtually all rules are re-interpreted, circumnavigated, suspended and often abandoned.[15] The desire to maintain escape clauses while publicly subscribing to a particular code of conduct is not surprising given the extended history of contingencies. The notion of 'opportunistically' implementing policy as observed during the 1990s naturally has appeal, given its scope for a more flexible interpretation of the rules. In prioritising the rules based type policy, it is nevertheless difficult to conceive that one could permanently remove all the escape clauses. Opportunistic disinflation represented a relatively new explanation of strategy in the 1990s and is explored here from the perspective that it constituted internally a contingent rule for the FOMC.[16]

[14] Von Hagen (1999) identified that the form of monetary targeting as implemented by the Bundesbank seemed also to adopt the practice of permitting breaches. In this regard monetary targeting, as implemented in Germany, conceivably also constituted a contingent rule.

[15] The shadowing of the DM and subsequent withdrawal from the E.R.M. was a recent example for the U.K.

[16] Advocates of opportunism would not perceive it as the softer option. On the whole, inflation is targeted with equal force regardless of whether employing an opportunistic or deliberative type strategy. See President Boehne, (FOMC transcripts, p.47, December 1995).

1.2.1 More recent views from the coalface

The advocacy of contingent rules or constrained discretion would seem to accord with Chairman Greenspan's risk management approach:

> 'To be sure, sensible policymaking can be accomplished only with the aid of a rigorous analytic structure. A rule does provide a benchmark against which to assess emerging developments. However, any rule capable of encompassing every possible contingency would lose a key aspect of its attractiveness: simplicity. On the other hand, no *simple* rule could possibly describe the policy action to be taken in every contingency and thus provide a satisfactory substitute for an approach based on the principles of risk management.' Remarks by Chairman Alan Greenspan, At the Meetings of the American Economic Association, San Diego, California, January 3, 2004. 'Risk and Uncertainty in Monetary Policy.'

A recurrent theme that has emerged from observing the history of monetary policy institutions relates to whether central banks followed the rules of the game? This research describes how monetary policy makers balance the need to configure simple, fixed and easy to understand rules that perennially fail or are abandoned by their authors, against advancing more elastic rules that bend to contingency, but are less spectacularly successful in anchoring expectations. Goodhart (1989) points out that from a central banker's perspective this ongoing friction is difficult to resolve:

> 'The more the authorities seek room to adjust for contingencies, the more the resulting policy tends to mimic pure discretion. Whereas most economists now accept that in *some* contingencies rules would have to be relaxed, there remains a tension between those whose preferences and priors would cause them to advocate simple, tight rules with little adjustment for contingencies and those who would prefer a more elastic response to contingent developments.' (p.369)

Not surprisingly, this potentially leads to strain between members of that committee which can impinge on policy decisions. The extent to which policy bends may also in part be underscored by the legislative framework that defines monetary policy. The tension between balancing outcomes may be less pronounced should policy be governed by a cogent framework. In the United Kingdom, the mandate from government is clearly defined by the Chancellor of the Exchequer requiring the Monetary Policy Committee (MPC) to maintain the official inflation target. Goodhart (2003) in examining the statistical properties of the MPC's inflation forecasts finds that:

> "…the MPC has indeed aimed to drive the inflation forecast into line with target at a two-year horizon, with this latter horizon being well determined

> empirically. The Orphanides-Wieland-Wilcox 'opportunistic loss function' does not hold in the United Kingdom." (p. 167)

This view may not be too surprising given the more tightly defined mandate imposed on central bankers in the United Kingdom. An important distinction should be made between policy implemented under certainty and uncertainty. This is examined in chapter 2 where the time value and intrinsic value of an option portfolio are considered. A key intricacy with the Orphanides-Wieland-Wilcox opportunistic loss function relates to how it purportedly tilts the policy regime toward discretion.[17] In this regard, the perception holds that the opportunistic disinflation approach creates latitude (wiggle) for officials to achieve economic objectives over time.[18] In chapter 2, the nonlinearity associated with opportunistic policy is examined from the perspective of how the FOMC shapes the zone target.[19] Risk management type strategies also offer policy makers scope for discretion in so much that a pre-emptive stance involves identifying a particular forecast e.g. stressing deflationary concerns. The absence of formal legislation identifying an inflation target in the United States implies that there exists more scope for central bankers to define policy when compared to the United Kingdom.[20] It also implies that the escape clauses once explained by the anomalous behaviour of monetary aggregates may during the Greenspan incumbency have been elaborated in terms of opportunistic policy and implicit zone targeting.

During the Greenspan years, the capacity to act without reference to an explicit inflation targeting framework was preserved intact. Initially, policy was elaborated in terms of an intermediate monetary target. Subsequently, this seemed no longer practicable to some members of the FOMC, of whom a number began to endorse an

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[17] See Orphanides and Wilcox (2002).

[18] In chapter 2, President Corrigan's concept of maintaining 'wiggle' is examined from the perspective of safeguarding the Federal Reserve's credibility.

[19] This forward looking (expected) inflation targeting approach adopted by Bank of England has a number of parallels. In chapters 2 and 3, the risk management approach however can be seen to remove some of the nonlinearity created by the zone target.

[20] The current mandate set out by the Federal Reserve Act (Section 2A) relating to monetary policy objectives, states that: 'The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the economy's long run potential to increase production, so as to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.' [12 USC 225a. As added by act of November 16, 1977 (91 Stat. 1387) and amended by acts of October 27, 1978 (92 Stat. 1897); Aug. 23, 1988 (102 Stat. 1375); and Dec. 27, 2000 (114 Stat. 3028).]
Poole (2005), p.3, maintained that the FOMC interpreted its objective as the responsibility to achieve price stability to promote maximum sustainable economic growth.

opportunistic disinflation strategy. Thornton (2004) traces out the nebulous path, policy makers had to negotiate over the period 1982 to 1997; moving from an operating procedure that privileged monetary aggregates to a more transparent operating procedure which was expressed more clearly in terms of the Federal Funds rate. The opportunistic strategy is much like the inflation zone targeting practiced in many jurisdictions but a key difference relates to the absence of a very precise inflation goal. From Section 2A of the Federal Reserve Act, the board has the capacity to define a short-run inflation target that ultimately leads to a price stability objective. In contrast, the Bank of England is set a very specific inflation goal which if breached requires an open letter to be sent to the Chancellor of the Exchequer. The independence of the Bank of England is undoubted and has been a cornerstone of its inflation targeting architecture. The goal of inflation containment has nevertheless been very clearly pronounced by government.[21] In the United States, the desire to bend to circumstance, has manifested itself in terms of what some policy makers assert as implementing 'opportunistic disinflation'. In chapter 2, nonlinear inflation targeting frameworks are examined from the perspective of institutional and committee dynamics. Despite the seeming differences between the United Kingdom and United States, it is possible to illustrate that both the nominally rules-based and discretionary frameworks are nevertheless unified from a risk management viewpoint. Flexibility has taken the shape of not announcing the inflation target. This pragmatic approach largely accords with 'weak form' inflation targeting (IT), as characterised by Kuttner (2004). Ambiguity in terms of announcing the target has frequently been criticised. Svensson (2004) maintained that the Greenspan FOMC has sought to maintain maximum discretion by avoiding 'commitment, transparency and accountability'. For some critics this has the potential to promote an inflation-bias of the kind identified by Kydland and Prescott (1977) and Barro and Gordon (1983).[22]

[21] The MPC's objective is determined each year by the Chancellor of the Exchequer. From 1997 to 2003, the inflation target was 2.5% p.a., measured in terms of the RPIX: the retail price index excluding interest payments on mortgages. From 2004 this changed to be 2% p.a. measured by the Harmonised Index of Consumer Prices (HICP).

[22] The conventional wisdom is that Greenspan enjoyed a good deal of success in containing price increases. Estimates of reaction functions for the period since the 1960s generally posit the last two decades of the twentieth century as being appropriately responsive to inflation, especially when contrasted with the preceding two decades. Taylor (2007) however identified fragilities in monetary policy from 2001.

This may also be regarded as a substantive indirect criticism of the opportunistic strategy.[23]

1.2.2 Leaning against the wind – rules and discretion

The Taylor Rule has been a useful construct because it explains simply what policy makers do without diverting attention away from perhaps insightful but commonly obscuring nuances. Mervyn King cited by Nelson (2000) made the following observation:

> '…[T]he Taylor Rule is no more in a sense than a restatement of the obvious, which is that if inflation looks to be higher, either now or in prospect, than the target, then you're likely to want to raise interest rates, and if it looks as if it's falling, and is likely to be lower than the target, then you'll cut interest rates. It's common sense, but that's why probably most central banks that have been successful appear ex post to have been following a Taylor Rule, even if they'd never heard of that concept when they were actually making the decisions'. (Nelson, 2000, p.27)

To examine its motivation, it is worth relating post war influences. At the advent of the Great Depression, Keynes (1930) expressed disquiet that the appropriate monetary policy response would not be supplied and that the greatest danger to economic progress was linked to the unwillingness of the Central Banks of the world to allow the market rate of interest to fall fast enough. In the United States, the Employment Act of 1946 and Treasury Accord may have attempted to redress some of these purported deficiencies. Policy makers embraced more activist stabilisation and exuded greater confidence in terms of perceived ability to master their own destinies. The logic of a dual objective seemed particularly present in the legislative mandate and this also seemed to endorse a somewhat more Keynesian ideal.[24] Orphanides (2003) identified the following Congressional Hearing (1957) statement as capturing the spirit of the 1946 Act.

> 'The objective of the System is always the same - to promote monetary and credit conditions that will foster sustained economic growth together with stability in the value of the dollar.' (cited Orphanides, 2003, p. 7)

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[23] Bomfim and Rudebusch (1997) maintain that the absence of transparency and decisive action tends to undermine the credibility of the opportunistic approach.

[24] Meyer (2004) identifies a more contemporary classification that posits policy as being either dual or hierarchical. If policy prioritises inflation outcomes over the real economy, it is said to be hierarchical.

Macroeconomic outcomes, as the 1970s approached, became ever more frustrating for policy makers following the perceived triumph of stabilisation policy, during the post-war period. Despite the hitherto perceived advancements in macroeconomic theory, academics started to question basic design flaws.[25] Friedman (1968) and Phelps (1968) disputed the assumption that the central bank exercises control over real variables, by virtue that the central bank only holds sway over monetary base creation. They assumed in their model of the natural rate hypothesis that the correlations between unemployment and inflation arise from unanticipated monetary shocks. Similarly, Lucas (1973) and Sargent and Wallace (1975), in a rational expectations model of the natural rate hypothesis, maintained that monetary policy was ineffective. In theory, the predictable component of a given monetary policy rule would not impact upon real variables implying that an activist type framework would frustrate policy makers by its inability to systematically lower excess capacity.

The relative success of the Bundesbank during the 1970s, may have encouraged policy makers elsewhere to foster monetary control type strategies. Von Hagen (1999) suggested that the monetary growth targeting afforded the Bundesbank a policy regime that had political appeal in that it subtly permitted agents to implement initiatives that otherwise were less palatable.[26] It also had the benefit that it precluded the sharing of responsibility for quantitative loan limits with the Finance ministry in 1973.[27] Although successful in containing inflation, the paradigm of monetary control was less startlingly successful at hitting its own intermediate target. In the United States, targeting non-borrowed reserves had the advantage of delegating the responsibility of higher interest rates to the money markets when perhaps policy makers were unwilling or unable to tolerate public hostility to higher interest rates. The Federal Reserve continued to announce monetary targets even after it had switched to a funds rate targeting procedure in 1982. This plausibly was intended to maintain consistency with a policy rule that had hitherto managed to achieve price stability and had earned the FOMC some reputational capital. Using the language of monetary targeting after monetary targets were abandoned seems to support the

[25] See Figures 1.2a for a portrayal of the distribution of inflation rates over the period 1958 – 2003. See Figure 1.2b to compare rate setting against inflation backdrop over the same period.
[26] The interwar experience of hyperinflation assisted policy makers to explain the rationale for controlling i.e. raising interest rates.
[27] An alternative proposal for containing credit creation.

contention that policymakers did not want to be openly associated with rule-breaking. That is, policy makers go to great lengths to preserve reputational capital. Opponents to a rules-based approach contend that central bankers would be incapable of reacting appropriately to each destabilising event if they were hampered *a priori* in their deliberations by a given rule.[28] A simple observation would suggest that if a rule were inherently good then a discretionary policy maker could elect to pursue it. In contrast, a policymaker who was locked in by pre-commitments inevitably from time to time diverges from the best course of action. Rules, accordingly, would seem to unnecessarily burden the strategy. The persuasiveness of this contention diminished somewhat once the concept of dynamic inconsistency was proposed by Kydland and Prescott (1977) and later developed by Barro and Gordon (1983). They illustrated that the mere absence of commitment to a low inflation policy could precipitate higher inflation.

A central bank that credibly commits to a plan to curb inflation going forward may be capable of reducing current inflation with a smaller cost in terms of output reduction. So long as the public anticipate inflation to be contained and low, policy makers have an incentive to implement expansionary policy to effect higher output at the cost of marginally higher inflation. When the public discern the ‘ruse’ they adjust their inflation expectations upwards and policy makers relinquish their ability to push output higher. This process ultimately culminates in higher inflation without any appreciable change in output. It also appears to offer a viable explanation of the stagflation experienced in a number of jurisdictions during the 1970s.[29] In this regard, the requirement to work within a Humphrey-Hawkins framework seemed to offer benefits to policy-makers even when the framework itself may have been felt to be outdated.[30] In practice, implementing monetary or reserve aggregate targets meant

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[28] This would seem to be particularly relevant in the event of a major stock market crash or banking default.

[29] In three of the better known works Kydland and Prescott (1977), Barro and Gordon (1983) and Blanchard and Fisher (1989) identified a policy rule respectively as being the ‘optimal’, ’rules’ and ‘pre-committed’ solution in a dynamic optimisation problem. Discretionary policy was conversely described respectively as being ‘inconsistent’, ‘cheating’ and ‘short-sighted’. (Taylor (1993), p.198). The consensus over this period seems to be weighted in favour of a rules-based strategy in implementing monetary policy as opposed to applying discretion.

[30] The Humphrey-Hawkins Act or the Full Employment and Balanced Growth Act of 1978 required the Federal Government to promote full employment and production, increased real income, balanced growth, a balanced Federal budget, adequate productivity growth, proper attention to national priorities, achievement of an improved trade balance and reasonable price stability. The FOMC

that the presumed stability of the quantity theory broke down, leaving the Federal Reserve without a clear compass to discern policy tightness.[31] By attempting to adhere to more rule-like behavior, the board paradoxically came to incorporate important elements of discretion into policy.

1.3. The role of committee dynamics in interpreting the policy rule

Friedman (1962) explained that the freedom of speech imperative that informed most aspects of law and was taken as a given, had the benefit of precluding case-by-case judgement. Friedman (1962, p.241) maintained that:

> 'Exactly the same considerations apply in the monetary area. If each case is considered on its individual merits, the wrong decision is likely to be made in a large fraction of cases because the decision-makers are…not taking into account the cumulative consequences of the policy as a whole. On the other hand, if a general rule is adopted for a group of cases as a bundle, the existence of that rule has favourable effects on people's attitudes…and expectations that would not follow even from the discretionary adoption of precisely the same actions on a series of separate occasions.' (cited McCallum 1999, p.1488, Handbook of Macroeconomics, Edited by J.B. Taylor and M. Woodford.)

Importantly however there was an escape clause pertaining to 'false alarms' signalled in crowded theatres. Of course, this caveat to freedom of speech would seem to provide for a fairly isolated set of circumstances. Stating and observing a rule creates tensions within a committee, particularly if the rule is based on a monetary aggregate that is difficult to control. This is evident from the viewpoints expressed by members of the Federal Reserve Board during the early part of the Volcker chairmanship. From the FOMC transcripts during 1982, there seems to have been a good deal of debate regarding how monetary aggregates should be best targeted.[32] The debate sharpened when the Fed Funds rate exceeded for a time 15%. The following passage from the June/July (1982) FOMC meeting, page 44 of the transcripts, suggests commitment to a monetary aggregate target framework created tension within the committee, concerned that the Fed Funds rate was moving outside an acceptable trajectory:

> Mr. Partee: So I would say around 9 percent [for M2]. And then it seems to me that we ought to reestablish 15 percent as the upper limit on the funds rate —

specifically had to set targets for monetary and credit aggregates that accomplished these aims. The monetary and credit targets may have appeared to have been necessary after the removal of Bretton Woods (1973) then considered to have been an important policy anchor.

[31] Goodhart's Law.

[32] The FOMC transcripts are available from the Board of Governor's website.

not as an indication to consult or anything like that. I ‘d say we would seek growth in the area of about 5 to 6 percent for M1 and about 9 percent for M2 provided that does not drive the funds rate above 15 percent. That’s a really radical change compared to what we’ve done before but it seems to me that the threat of higher interest rates is so great now that we can’t tolerate it and we have to put that in as a limit.

Mr. Roos: Chuck, may I inquire, sir: If we make a strong statement in reference to interest rates, doesn’t that imply a significant departure and doesn’t that signal that once again we are trying to balance interest rates and aggregate growth?

Mr. Partee: I guess what I’m saying is that I will accept any aggregate growth to keep the funds rate below 15 percent.

Mr. Roos: So you are placing primary emphasis on

Mr. Partee: On that upper end of the funds rate range for the time being.

Mr. Ford: On a weekly, daily, or monthly basis? Well, you are proposing a significant change in policy.

Mr. Partee: I would want to say so long as the funds rate does not move rather consistently above 15 percent. I don’t mean daily: I’m not even sure I mean weekly, Bill--maybe biweekly or something like that. But I don’t think we can tolerate the effect on the market of a funds rate higher than 15 percent which is a little higher than where it has been. It gives us some [upward leeway] but I just think we need to draw the line now.

Mr. Black: Mr. Chairman, just so I don’t appear to be too much of a Simon Legree, may I say that I ’m just as interested in getting rates down as anybody else but I differ on the method for doing it.

Mr. Partee: Well, one way to do it is to crash the economy.

Mr. Black: Well that is not the method I am suggesting. We may have been a little too tight last year [unintelligible]. (Federal Reserve Board, June/July meeting, 1982, p. 44)

This was one of a number of seminal moments in mid - 1982 when a rules-based strategy conflicted with the urge to impose a discretionary shift. Over the next two decades monetary aggregates became increasingly de-emphasised.[33] This episode also suggested that a contingency rule or framework was necessitated to implement policy.

[33] Thornton (2004) maintained that the persistence of monetary targeting in the Federal Reserve lexicon and published documents continued long after the adherence to a monetary target was discontinued.

An important observation here is that much of recent economic history has been framed around the cut and thrust of Open Market Committee debate. The freewheeling exchange seems to produce an important process of discovery. The dynamics of securing agreement within the committee structure is pivotal in terms of shaping not just rate adjustments but also in terms of defining and reinterpreting strategy. Moreover, the role that is accorded to the chairman is significant in terms of consensus fixing. In reality, it may not be practical to impose a binding commitment to a rule by virtue that not enough is known regarding the structure of the economy. A rules-framework might also be seen as a way to ensure policy makers agree and in so doing, thwart the potential stalemate that could arise in the absence of a generally accepted *modus operandi*. In the early 1980s, both the authorities in the United Kingdom and the United States experienced difficulties in adhering to monetary targets, fulfilling the prophecy of Goodhart's Law.

Von Hagen (1999) asserted that central banks are not unified actors. Interpreting the rules and what constitutes appropriate contingencies is very obviously contentious. In this regard, the role of the committee is enormously important. A nontrivial consideration relates to the appointment of the committee's chairman. When President Carter decided to select Paul Volcker as Federal Reserve chief, the appointment very likely appeared highly unusual given the dissimilarities in respective profiles.[34] Rogoff (1985), a staff economist at the Board of Governors illustrated analytically that a President who appeared soft on inflation may benefit from appointing a reputed "inflation hawk" as chairman to the central bank. The augmented inflation-fighting credibility that such an appointment delivers, permits the monetary authorities to realise low inflation at a smaller cost than a non-credible central bank. The public are also more disposed to believe an inflation hawk when he commits to contain inflation. They appreciate that an appointee who possessed a greater distaste for inflation than the societal norm, is less likely to renege on his commitment.[35]

Taylor (1993) expresses the view that modern macroeconomics posits clear advantages to policy rules over discretion in terms of enhancing economic

[34] See Bernanke (2004c)

[35] One might argue of course, that Arthur Burns also possessed a greater distaste for inflation than the societal norm.

performance. Indeed, Taylor (1993) attempted to combine aspects of both discretionary and rules based behaviour in to a single construction.[36] A key objective of his influential paper was to preserve the sense of a unified or systematic strategy where it was near impracticable to observe rules mechanically. At the same time policy would not be crafted in a casual or random fashion. Interestingly, what subsequently became known as the Taylor Rule did not start life explicitly as an appendage of any particular school of thought. In fact, Taylor (1993) stressed that his policy rule was very much the product of empirical and econometric evaluation.[37]

1.3.1 Committee dynamics and preserving the escape clauses (wiggle)

The absence of unanimity within the committee has implied that the chairman has to keep policy options open. Having examined the transcripts of FOMC meetings, Thorton (2004) concluded that the Board commenced targeting the Federal Funds rate when non-borrowed reserves were de-emphasised as the operating procedure in 1982. Some members of the board may have wished not to target interest rates explicitly, but were constrained given the purportedly uncharacteristic behavior of M1. At the behest of Chairman Volcker, in 1982, borrowed reserves were nominated the operating objective. While some members of the Committee advocated simply acknowledging that the board was targeting the funds rate, this was resisted by Chairman Volcker who maintained that the borrowed reserves operating procedure was distinct from Federal Funds targeting. This remained the official position despite challenges internally. The FOMC targeted borrowed reserves from September 1982 and then subsequently switched to targeting the Federal Funds rate. The latter move was not announced sparking some debate as to precisely when the operating procedure focused on the policy rate. With the advent of a new Fed Chairman, tension continued between the board’s nominal and effective operating procedure. Thornton (2004) points out that the FOMC did not explicitly target the funds rate once chairman Greenspan was appointed and only in a gradual manner acquiesced to officially

[36] The prescriptions of interest rate rules are best read as useful adjuncts to policy implementation. A deliberative strategy would stick firmly to the policy rule. President Boehne, at the December, 1995 FOMC meeting, p.47 of the transcripts, used the term ‘deliberative’ to describe conventional linear policy. Don Kohn in the appendix to the December 1995 FOMC meeting contrasted the ‘deliberate strategy’ to the ‘opportunistic strategy’ by implementing a monetary policy matrix.

[37] By stressing the econometric basis of this type of analysis Taylor (1993) avoided fostering exclusively a Keynesian perspective. The Taylor Rule could also loosely be viewed as an extension of the Friedman money supply rule. Blinder and Reis (2005) contend that the lack of a formal model may have reflected Chairman Greenspan’s own position.

acknowledging the effective operating procedure. This may have reflected the fact that the committee disagreed internally. Given the exchanges observed from the FOMC transcripts, Thornton (2004) contended that the switch to a funds rate target occurred shortly after September 1982.[38] Greenspan (1997) maintained that the FOMC set the Fed Funds rate and this practice was increasingly implemented since 1982. The slow recognition was desirable in that it permitted the FOMC to maintain a significant degree of latitude in driving policy.[39] It also allowed the board some space to maintain the façade of adhering to a consistent rule-based architecture. An early example of internal strain was evident at the February 1988 meeting of the FOMC: Governor Seger concluded that policy would be directed at maintaining the Federal Funds rate between 6.25 – 6.5 percent:

> Governor Seger: '6-1/4 to 6-1/2 percent.'
> Vice-Chairman Corrigan: 'You're calling a spade a spade.'
> President Boehne: 'You at least would pass a lie detector.'(Federal Reserve Board, February meeting, 1988, p. 73)

Only in a very piecemeal fashion over the period did the Federal Reserve adopt a language consistent with its effective operating procedure. August 1997 marked a switch in the policy directive prior to which the directive did not explicitly refer to the FOMC's target for the Federal Funds rate and was written exclusively in terms of the desired degree of restraint on reserve positions. At the August board meeting, the wording changed: 'In the implementation of policy for the immediate future the Committee seeks conditions in reserve markets consistent with maintaining the federal funds rate at an average of around 5½%.' (p.79). Previously, the policy directive did not explicitly make reference to target the Federal Funds rate.[40] The 1997 FOMC July transcripts carried the following directive wording (p.127): 'In the implementation of policy for the immediate future, the Committee seeks to maintain the existing degree of pressure on reserve positions.'

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[38] Notwithstanding the fact that the Greenspan chairmanship has always been considered to have been relatively open to financial markets and press.

[39] By proffering a monetary target the FOMC could distance itself from interest rate hikes and accordingly from political censure.

[40] Commencing in 1994, at Chairman Greenspan's behest, the FOMC initiated a procedure of stating policy actions upon making them. Previous to that, Fed watchers would have had to await the publication of minutes at the next FOMC meeting. The press release did implicitly acknowledge the Fed Funds significance by stating that "the action was expected to be associated with a small increase in short-term money market interest rates."

1.3.2 Institutional dynamics and preserving escape clauses (wiggle)

One explanation offered for preserving the choreography of reserve targeting for so long is related to a desire to safeguard room for manoeuvre relative to political institutions. An attractive feature of reserve targeting was the capacity of policy makers to point to the market as being the real arbiter of interest rates. Plausibly, as the Chairman grew in confidence, he was more willing to accept that the FOMC would be perceived as setting interest rates. This change in language may have also brought about or was accompanied by a desire to elaborate a systematised nonlinear strategy. The lack of transparency up to August 1997 may, in part, be explained by a desire not to render the Federal Reserve vulnerable to political rebuke. In this sense, nominally implementing monetary targeting offered some protection to policy makers against criticism should interest rates have to rise. It also lessened the need to burden the FOMC with an inflation target that could easily have been missed or would have potentially divided the committee.[41] The political process within which monetary policy is framed can explain why nonlinearities are relevant. Consider the following remarks made by Chairman Greenspan at the FOMC in December 1989.

> ‘I would like just basically to raise the question of how we develop political support to do what it is we perceive is necessary for a stable economy and sound monetary policy. If there were a [law] out there, which legally required us to do something very specific about inflation or the money supply, I suspect we’d all applaud that meaning, in effect, that we would be required to do something independent of the secondary consequences on the grounds that some other institution or some other policy instrument would pick that up. There is no way that’s going to happen, as I’m sure we are all. We all have to live with the fact that the Federal Reserve is going to be in the eye of the political system increasingly [unintelligible]. (Federal Reserve Board, December meeting, 1989, p. 43)

This view is re-iterated at the December 1995 meeting, (p.58), when Chairman Greenspan pointed out that even if Congress overwhelmingly endorsed a policy rule to contain inflation, afterwards this would not automatically marshal support as any given rule started to bite. Herein, lies one candidate explanation for nonlinearity or opportunism: it was feared that a mechanical pursuit of inflation containment would not be tolerated by political institutions even if the legislature had originally endorsed a given time table for price stability. Thus proportionate increases of the policy rate relative to inflation were difficult to implement. A Taylor type rule may have to be

[41] See the Babe Ruth Analogy explained in chapter 2.

amended for local conditions.[42] In this sense, progress to a given target has to be gradual to take account of political sensitivities. As a consequence, policy makers like President Corrigan perhaps supported the looser definition of price stability.

> ‘The idea would be that the stated policy of the Committee would be couched in terms of a goal of price behavior that would be broadly compatible with what we had, say, in the ‘50s and early ‘60s. In other words, we wouldn’t get hung up with one [indicator such as the] CPI or deflator, but we’d state a goal in terms of trying to return to a pattern that had the characteristics of that [earlier period] and we could say that we were going to try to achieve that in the time frame of the mid- ‘90s. So, it would not be all that specific in terms of a particular price index and it would allow for some wiggle for shocks.’ (Federal Reserve Board, December meeting, 1989, p. 30)[43]

This is not unlike the definition of price stability that is commonly attributed to both Chairmen Volcker and Greenspan: that is inflation is contained when it stops being a routine consideration in day-to-day decision-making for all economic agents. Understandably no measure of price change can be perfect in all circumstances. In this regard, an important aspiration of policy is to convince agents that they do not need to presume prices are going to alter when they are making their investment and spending decisions.[44] In chapter 2, an insightful analogy is developed with reference to the baseball legend Babe Ruth and issues of credibility. By identifying an inflation target it was feared policy makers were offering critics ammunition. Meyer (2004) makes the point that during his tenure at the FOMC, only when severely pressed did chairman Greenspan once temporarily identify a working definition of price stability, and then subsequently quickly withdrew it.

Plausibly, some members of the Federal Reserve saw that the opportunistic approach as affording the possibility of conveying nonlinearities to a political audience while still preserving the benefits of operating within a rules framework. Don Kohn outlined

[42] More recently, the Balance of Risks assessment issued in the FOMC press release allows the committee to convey the direction of future rate moves. Upside and downside risks replaced the previous ‘tilt’ language which was deemed useful in describing how far the policy rate was removed from neutrality.
[43] This definition of price stability implies making progress over a longer time horizon of one or two years. The gradual approach is somewhat akin to the intermediate targeting of inflation as described by Orphanides and Wilcox (1996) and Orphanides et al. (1997).
[44] It is worth remembering that this definition of the inflation goal predates the successful setting up of most of the inflation targeting regimes.

to the FOMC a policy matrix at the December 1995 meeting.[45] This was calculated to assist in explaining rate decisions, should the Mack Bill ever become law. Implicit in establishing the matrix was a desire to communicate and justify nonlinearity. Just as monetary targeting had afforded wiggle, the policy matrix was intended to systematise escape clauses that could be explained to a broad church cogently without the inference of rules being broken. In this regard, some members of the Federal Reserve were preserving a strong semblance of a rules based framework even though implicit in that arrangement, rules could be potentially bent. More recently, Greenspan (2003) has expounded the risk management approach as affording crucial insights into implementing monetary policy.[46] Both the opportunistic policy and the risk management paradigm were intended to offer policy makers latitude for exercising judgement by adhering to a contingency rule:

> 'Some critics have argued that such an approach (the risk management approach) to policy is too undisciplined--judgmental, seemingly discretionary, and difficult to explain. The Federal Reserve should, some conclude, attempt to be more formal in its operations by tying its actions solely to the prescriptions of a formal policy rule. That any approach along these lines would lead to an improvement in economic performance, however, is highly doubtful. Our problem is not the complexity of our models but the far greater complexity of a world economy whose underlying linkages appear to be in a continual state of flux.
>
> Rules by their nature are simple, and when significant and shifting uncertainties exist in the economic environment, they cannot substitute for risk-management paradigms, which are far better suited to policymaking. Were we to introduce an interest rate rule, how would we judge the meaning of a rule that posits a rate far above or below the current rate? Should policymakers adjust the current rate to that suggested by the rule? Should we conclude that this deviation is normal variance and disregard the signal? Or should we assume that the parameters of the rule are misspecified and adjust them to fit the current rate? Given errors in our underlying data, coupled with normal variance, we might not know the correct course of action for a considerable time. Partly for these reasons, the prescriptions of formal interest

[45] The Opportunistic strategy could be used to explain how the actual policy differed from deliberative implementation.

[46] After the January 2000 meeting, the FOMC employed 'balance of risks' as opposed to the 'tilt' or 'bias' language which previously had been seen to be helpful in attaining consensus when rate decisions were being made. The substitution to a balance of risks statement was intended to furnish insights into the board's perception of future real growth and inflation without providing full blown forecasts as the BOE. The change in language was made transparent by a press release which would after every FOMC meeting be made available so that market participants could form a view of the committee's opinion concerning risks to the policy objectives going forward. The Press Release of June 30, 2005 carried the following statement that: 'The committee perceives that, with appropriate monetary policy action, the upside and downside risks to attainment of both sustainable growth and price stability should be kept roughly equal.'

> rate rules are best viewed only as helpful adjuncts to policy, as indeed many proponents of policy rules have suggested.' (Remarks by Chairman Alan Greenspan, at a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 29, 2003.)

The risk management approach as outlined by chairman Greenspan implies implementing rate changes that orbit around a given imperative or standard. Much of the literature to date has stressed the Taylor Rule, even though it is clear that Taylor (1993) p.197, concedes that 'it is practically impossible to follow mechanically any particular algebraic formula'. In what follows, the theoretical rationale for implementing a Taylor Rule is examined. In addition, a risk management paradigm is set out. An option-based construct is implemented to examine the effects of uncertainty. First, the theoretical basis for directing monetary policy using a simple policy rule is considered.

1.4. A New Keynesian Perspective on Taylor Rules: the application of science

The Taylor Rule (1993) is consistent with an activist stabilisation framework which assumes that monetary policy can achieve real economy outcomes. This arguably implies that some form of nominal rigidity exists, although Taylor (1993) initially advanced his rule on the basis of empirical work.[47] The New Keynesian framework relies on nominal price rigidities to give the central banks non-neutral effects. Methodological advances in macro-economic modelling that stressed price frictions assisted in providing a conceptual structure for monetary policy to be relevant. Following a decade of heavy emphasis on the role of non-monetary factors in the business cycle, a series of empirical research seemed to indicate that monetary policy impacted on short-run real economy outcomes. Romer and Romer (1990) and Bernanke and Blinder (1992) stressed that it was no longer appropriate to relegate monetary policy as being peripheral. McCallum (1999a) points out that there exists a convergence of thought in terms of motivating the policy rule as being driven by developments in inflation π_t and the output gap y_t. This format is similar to what McCallum (1999a, 2002) describes as a nearly standard framework employed at

[47] In this sense, his rule might be described as being not particularly reliant on any school of thought. Analogously, Blinder and Reis (2005) refer to Greenspan's 'non-model'.

NBER and Riksbanks conferences in 1998.[48] The first component is an IS-type relation (or set of relations) that specifies how interest rate movements affect aggregate demand and output. The second constituent, Phillips curve, comprises a price adjustment equation that specifies how inflation behaves in response to the output gap and to expectations of future inflation. Clarida, Gali and Gertler (1999) apply this sort of construction. Their stylised baseline framework, in part, represents a *post hoc* elaboration of how, from a purely theoretical perspective, a Taylor type policy rule can emerge. To chart out the key influences, it is necessary to describe a particular form of economic process.

$$y_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t y_{t+1} + g_t \qquad (1.1)$$

$$\pi_t = \lambda y_t + \beta E_t\pi_{t+1} + u_t \qquad (1.2)$$

(1.1) represents an IS curve that relates the current output gap, y_t, inversely to the real interest rate and positively to expected developments in the real economy. i_t is the nominal interest rate. This rate is used by the central bank to implement monetary policy. The current expectation of inflation for the next time period is denoted by $E_t\pi_{t+1}$. (1.2) explains inflation in terms of a positive relationship with the output gap and future expected inflation. g_t and u_t are given here as disturbance terms.

$$g_t = \mu g_{t-1} + \hat{g}_t$$

$$u_t = \chi u_{t-1} + \hat{u}_t$$

where $0 \le \mu$, $\chi \le 1$ and where both $\hat{g}_t$ and $\hat{u}_t$ are i.i.d. random variables. This type of construction characterises the application of a number of broad principles that underscore the basis of optimal policy administration.

The preferences of central bankers are frequently represented by assuming that monetary policy attempts to minimise volatility in the output gap and inflation rate. The central bank objective function reads the target variables into a measure of welfare that shapes monetary policy. Consequently, the following construct emerges:

$$\text{Max} \quad -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\psi y_{t+i}^2 + \pi_{t+i}^2\right]\right\} \qquad (1.3)$$

[48] McCallum (2002) stressed that there seemed to have been consensus between academic and central bank economists both in terms of concerns and techniques. This convergence of opinion conceivably encouraged Clarida, Gali and Gertler (1999) to entitle their paper: '*The Science of Monetary Policy: A New Keynesian Perspective*'.

The β weight, if less than one, can capture a form of time decay where distant observations are less significant and decline in importance as the horizon becomes more distant. Implied in the loss function are targets for zero inflation and to align output with its potential level. This representation of policy identifies a point target.[49]

In deriving the now near standard policy rule Clarida, Gali and Gertler (1999) simplify the algebra by assuming that the central banks exert no influence over expected values.[50] In other words expected values are given. This implies the loss function reduces to:

$$-\frac{1}{2}\left[\psi y_t^2 + \pi_t^2\right] + F_t \tag{1.4}$$

where the β term is subsumed into F_t. F_t isolates the future observations of output and inflation. (1.4) is optimised subject to

$$\pi_t = \lambda y_t + f_t \tag{1.5}$$

where

$$F_t \equiv 1/2\, E_t\left\{\sum\nolimits_{i=1}^{\infty} \beta^i \left[\psi y_{t+i}^2 + \pi_{t+i}^2\right]\right\} \tag{1.6}$$

and

$$f_t \equiv \beta E_t \pi_{t+1} + u_t\,. \tag{1.7}$$

The optimal policy rule is obtained from (1.8).

$$L_t = -\frac{1}{2}\left[\psi y_t^2 + \pi_t^2\right] + F_t + \phi\left[\pi_t - \lambda y_t - f_t\right] \tag{1.8}$$

Alternatively, a number of central banks have explicitly described policy objectives as maintaining inflation within a comfort zone. Point targeting conceptually could lead to excessive policy activism. Opportunistic policy might be thought of as attempting to maintain inflation within an acceptable band. Unlike (1.8), this might warrant a non-quadratic construction of the loss function. Using their notation, Orphanides and Wilcox (1996, 2002) suggest the following as the starting-point:

[49] This is a little different to the more common practice of targeting an inflation zone. Issing (2004) maintained that the Governing Council clarified its' inflation objective in May, 2003 stating that the ECB would endeavour to maintain inflation rates 'below but close to 2 percent' (p.175). The Bank of England, as do a number of other explicit inflation targeters, specifies a zone.

[50] Clarida, Gali and Gertler (1999) maintain that the approach and principles they invoke in terms of deriving an optimal policy rule are sufficiently proven and generalised to warrant the term 'science' being applied. This bolder lexicon conceivably denotes a growing consensus associated with the then New Keynesian methodology.

$$L_A = (\pi - \tilde{\pi})^2 + \gamma y^2 + \psi|y| \qquad (1.9)$$

The notation is similar to (1.8) where y still denotes the output gap and π denotes the inflation rate. The inflation target $\tilde{\pi}$, can assume alternative magnitudes including zero percent. The parameters ψ and γ are the weights attributed to output deviations from the natural rate. The key difference between (1.8) and (1.9) relates to the inclusion of the non-quadratic term $\psi|y|$ for the output gap. This permits policy to be implemented opportunistically or nonlinearly.[51] In contrast, the linear solution that satisfies the first order conditions for (1.8) suggested by Clarida, Gali and Gertler (1999) gives:

$$y_t = -\frac{\lambda}{\psi}\pi_t \qquad (1.10)$$

Clarida, Gali and Gertler (1999) describe (1.10) as implying that the optimal condition enshrines a 'lean against the wind' approach to administer monetary policy. Policy should operate so as to redress the effect of economic activity on the price level. The nature of the Phillips curve is important in that regard. The responsiveness of inflation to changes in output governs the degree to which monetary authorities apply counter-cyclical rate cuts or increases. Optimal policy should operate in a manner that contracts the output gap when inflation rises and vice-versa. The optimal policy rule is obtained by substituting for y_t in the IS curve (1.1) to give:

$$i_t = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t \qquad (1.11)$$

where

$$\gamma_\pi = 1 + \frac{(1-\chi)\lambda}{\chi\varphi\psi} > 1 \qquad (1.12)$$

(1.11) constitutes the optimal policy rule given by the parsimonious Clarida, Gali and Gertler (1999) framework. One key finding of this constrained optimisation exercise, identifies the following significant benchmark for policy activism.[52]

> 'Under the optimal policy, in response to a rise in expected inflation, nominal rates should rise sufficiently to increase real rates. Put differently, in the

[51] Later, in chapter 2 the notion of committee voting stalemate is used to rationalise nonlinearity in the reaction function.

[52] This policy recommendation is referred to as Result 3. The benchmark spells out optimal policy to be designed so as to contract aggregate demand, by raising the real Federal Funds rate, when inflation is above target and *vice-versa*. Result 3 provides a neat summary for what is now referred to as the 'Taylor Principle.'

> optimal rule for the nominal rate, the coefficient on the expected inflation should exceed unity.' (Clarida, Gali and Gertler, 1999, p. 1674)

Notably, this benchmark accords with Taylor (1993) who maintained that the policy response (the Fed Funds rate), i is given by:

$$i = 1 + 1.5\pi + 0.5y \tag{1.13}$$

where π is the rate of inflation over the previous four quarters and y is the percent deviation of real GDP from a target. Issing (2004), by way of a critique, has pointed out that most advocates of inflation targeting almost invariably depend on a parsimonious economic framework that is configured by no more than three elegant equations that sideline the role of monetary aggregates. This omission he regarded as being extreme and 'a central bank can legitimately question the usefulness of [such] a model for monetary policy setting.' (p.174). Blinder (1998) p.22 also questions the realism of positing the decision making in terms of maximising utility:

> 'My experience as a member of the FOMC left me with a strong feeling that the theoretical fiction that monetary policy is made by a single individual maximizing a well-defined preference function misses something important. In my view, monetary theorists should start paying attention to the nature of decision making by committee, which is rarely mentioned in the academic literature.'

Subsequently, Blinder and Reis (2005), p.10 state:

> 'All economists cut their teeth on optimization techniques and feel most at home in that framework. However, Greenspan has suggested a different methodological paradigm for monetary policy – that of risk management.'

1.5. The forward-looking policy framework and lags in the transmission mechanism

A common modification applied to the Taylor Rule, or (1.13), incorporates a role for the target variable forecast of inflation as (1.11), so that π_t becomes $E_t\pi_{t+1}$. The effect of lags in the economy points to using a forward-looking policy rule. Clarida, Gali and Gertler (1998, 1999, and 2000) formulate the reaction function as being forward-looking on the grounds of plausibility.[53] A policy maker can not reduce the

[53] Svensson (1999) footnote 35, attributes the first printed expression of central bankers asserting that the inflation forecast represented an intermediate target, to King (1994): 'The use of an inflation target does not mean that there is no intermediate target. Rather, the intermediate target is the expected level of inflation at some future date chosen to allow for the lag between changes in interest rates and the resulting changes in inflation. In practice, we use a forecasting horizon of two years.'

current inflation rate because this, in effect, is already an historic event. Central bankers are accordingly obliged to set interest rates to contain future imbalances. It is generally understood that monetary policy requires a forward-looking dimension. In the presence of transmission lags, delivering inflation back to target, immediately after a shock, may necessitate incurring significant costs. Instantaneously offsetting the inflationary shocks may demand substantial movements in the policy instrument, precipitating unduly large output losses. One plausible approach to mitigate this type of dislocation is to try to anticipate inflationary events prior to them materializing.[54]

Reacting in a more pre-emptive fashion allows central banks to contain these losses by reducing the extent to which the instrument has to be moved in the short run. Policy makers may avoid 'falling behind the curve'. Many central banks, including those of Australia, Canada, New Zealand, Sweden and the United Kingdom, adhere to an explicit inflation target. In practical terms, this usually involves 'targeting' the conditional forecast of inflation - the inflation rate expected to prevail in the future given presently available information. The Bank of England, for instance, embraces this framework in the design of their forecast-based policy rules. According to Batini and Haldane (1999), this approach confers tangible benefits given the extent to which transmission lags exist in the U.K. economy. The forecast targeting approach also permits the practical inclusion of all information relevant to predicting future inflation. This may appear trivial but importantly it allows a large information set to be incorporated into rate decisions. Batini and Haldane (1999) maintained that an inflation forecast horizon of three to six quarters delivered the best outcomes by virtue that imposing shorter horizons risked increasing both output and inflation variability. Conversely, longer horizons risked macroeconomic instability. Rudebusch and Svensson (1998) found when comparing a variety of different operational techniques in the context of a two-equation model of the U.S. economy, and varying alternative versions of their assumed loss function that forecast targeting rules performed strongly. Although Rudebusch and Svensson (1998) were not principally concerned with detecting the optimal horizon length, the implication of their results suggest that,

[54] Of course the timing of policy initiatives can be affected by other factors. Central bankers may refrain from altering rates of interest during periods when their likely impact could be construed to contain political bias. The pre-emptiveness of policy initiatives may be implemented so as to maintain political neutrality.

in aggregate, economic (adjustment) costs were mitigated if the horizon for the inflation target were greater than two years.

Imposing a simple forward-looking specification in the reaction function however would suggest that central bankers are also inevitably working within a risk management framework, given the increased uncertainty of managing future events. In the United Kingdom, the Bank of England targets inflation at a horizon of two years. The highly publicised fan chart, released by the Bank of England, has become a standard tool to convey policy maker expectations, the uncertainty associated with these expectations and the expected trajectory associated with reversing inflation shocks. Similarly, as part of the jargon used to denote the pre-emptive nature of Fed Funds setting, policy makers in the United States frequently refer to buying insurance or staying ahead of the curve.[55] The sense that policy is forward looking has been proffered by a number of central bankers. An important insight as to why policy makers hedge their position on expected inflation and output has been suggested by chairman Greenspan (1999).

> 'For monetary policy to foster maximum sustainable economic growth, it is useful to pre-empt forces of imbalance before they threaten economic stability. But this may not be possible - the future at times can be too opaque to penetrate. When we can be pre-emptive, we should be, because modest pre-emptive actions can obviate more drastic actions at a later date that would destabilise the economy.' (Cited Orphanides, 2003, p.5)

1.5.1 Opportunism and uncertainty – applying a risk management framework

The opportunistic approach to policy has been, over the 1990s, advocated by a number of key policy makers in the United States. As a disinflation strategy, it specifies both an interim and a long-run objective for price stability. Proponents of this policy framework maintain that when inflation remains within a zone of comfort, central banks should concentrate more on stabilising output and employment.[56] In describing the FOMC's strategy in the 1990's Blinder (1997) maintained that:

> "Under certain circumstances, the optimal disinflation strategy is asymmetric in the following specific way: you guard vigorously against any rise in

[55] See Orphanides (2003) for forecast-based variants of the classic rule.

[56] Previously, the zone of comfort was motivated as a zone of stalemate. The two appear contradictory. The latter stresses disagreement between policy makers on the board or between the board and the legislature. The former posits that the central bank behaves as a unified actor and the loss function is non-quadratic. In both cases, policy is found to be nonlinear and operate within zones. In chapter 2, committee dynamics and stalemate are mainly stressed when developing a nonlinear reaction function.

> inflation, but wait patiently for the next favourable inflation shock to bring inflation down. The opportunistic strategy makes the time needed to approach the ultimate inflation target a random variable. When I was the Vice Chairman of the Fed, I often put it this way: the United States is 'one recession away from price stability'." (cited Minford and Srinivasan, 2003, p.2)

In what follows, like Orphanides, Small, Wieland and Wilcox (1997) and Aksoy, Orphanides, Small, Wieland and Wilcox (2006), this thesis uses a definition of opportunism that is akin to inflation zone targeting.[57] Zone targeting is motivated, in this thesis, by appealing to voting stalemate. The width of this zone may be indicative as to what level of accord exists between members of the board or between the board and Congress. As illustrated in chapter 2, a greater level of discord is associated with a wider region of inactivity. The opportunistic approach would suggest that stalemate, of itself, is not necessarily undesirable in that disinflation can be induced either by letting a recession occur via policy inaction or by refraining from stimulating the economy when positive supply shocks materialise.[58] In theory, the opportunistic approach has the benefit of permitting policy makers to refrain from immediately imposing tough economic medicine of disinflation. A board chairman may be in favour of exploiting the economy's own counter cyclicality when rate decisions are contentious, internally at board level or externally with other government agencies. From the FOMC transcripts, it would appear that important elements of risk management feature in policy making. In what follows, portfolio option theory is used to investigate the linkages between opportunistic policy and risk management.[59] In chapters 2 and 3, 'the Greeks': delta, gamma, vega and theta are developed to examine more formally the extent to which policy rate setting is sensitive to parameters such as the volatility of the target variable, the band width and the targeting horizon employed to manage the target variable.[60]

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[57] Orphanides, Small, Wieland and Wilcox (1997) motivated opportunism with a non-quadratic loss function. The intrinsic payoff from an option is used to portray the extent to which this approach offers policy makers discretion.

[58] Disagreement between policy makers does not preclude the possibility of harnessing positive supply shocks or negative demand shocks in chipping away at inflation.

[59] Revealing the rule however may not be desirable for a policy maker who wishes to maximise latitude for judgement by asserting a quasi-rules based formulation. Chairman Greenspan may have privileged a risk management approach over others because it appeared to offer more escape clauses. Blinder and Reis (2005) point out that Greenspan never fully spelled out exactly what his risk management approach entailed. This is not to say that Greenspan Fed was not forward looking or pre-emptive.

[60] See Abbreviations for individual explanation of terms.

1.5.2 Opportunism and the hockey stick diagrams

The starting point here is to examine rate setting with regard to some rudimentary elements of option pricing. The basis of implementing disinflation policy as set out here, involves raising the Fed Funds rate when expected inflation $E(\pi_T)$ exceeds a given target, *k*. Conversely, if inflation is below a particular tolerance level or upper bound, monetary policy consequently abstains from contracting demand. One way of representing this type of policy is to consider Figure 1.1a, (end of chapter). Ignoring standard risk neutrality arguments, the policy reaction to inflation may be parsimoniously represented as:

$$c = E[\max(\pi_T - k, 0)]$$

Similarly, Aksoy, Orphanides, Small, Wieland and Wilcox (2006) describe the opportunistic central banker as combating inflation when inflation is high, but concentrates attention on stabilising output when inflation is low. Adjusting the upper tolerance, *k*, and inflation forecast offers scope for discretion. Some of the basic insights relating to the effects of uncertainty on opportunistic monetary policy can be found by comparing Figures 1.1a and 1.1b.[61] Under uncertainty, the policy response denoted by *c* must respond to $g(\pi)$, the probability density function of π:

$$c = E[max(\pi_T - k, 0)] = \int_k^{\infty} (\pi_T - k) g(\pi) d(\pi)$$

This typically would imply policy responses are better described by using a time value parabola, similar to Figure 1.1b, as opposed to the intrinsic value. In this regard, central bankers when preemptive, exercise less scope for discretion because monetary policy is forced to react even when inflation is below the upper tolerance, *k*. Bernanke (2004) noted that the Federal Reserve was not just concerned with the average or most probable outcome but also with the entire distribution of feasible inflation outcomes. This seems in line with Greenspan (2003). Portfolio option theory helps to systematise aspects of policy.[62] Portfolio option theory is useful for examining the linkages that exist between opportunistic policy and risk management. When policy is opportunistic and preemptive, it is possible to consider the circumstances that permit interest rates to rise prior to inflation, π prospectively exceeding a given tolerance

[61] In chapter 2, a more rigorous justification is offered in rationalising the application of option theory employing a basic put-call parity relationship.

[62] Systematising policy and elaborating it in terms of an explicit rule would not necessarily be a desirable end for a policy maker who wished to maximise room for manoeuvre or discretion.

level, k. A key insight is that the risk that inflation will exceed a given magnitude k, produces a policy reaction that resembles the mapping out of varying time valuations on an inflation call or caplet. The time valuation map of an option is consistent with policy acting pre-emptively when the upside risk to inflation breaching k becomes pronounced. So long as there is a calculable probability that inflation will breach a tolerance threshold, k, policy responds.[63] From the perspective of a policy maker, even if inflation currently resides nominally within a zone of comfort, policy can still respond, so long as there is a chance that inflation at the future targeting horizon date will exceed an upper acceptable critical level. One way of understanding this construction is to consider how monetary policy is communicated by the Bank of England via fan charts. In attempting to keep inflation within a zone, policy makers consider the full spectrum of plausible inflation outcomes. To do this policy makers specify the varying moments and the assumed probability distribution of expected inflation extending over a given forecasting horizon. In the case of the Bank of England, this happens to be a two piece normal distribution where the principal moments of that distribution are made public. In using this construction, upside and downside risks to inflation can be gauged by the level of volatility and skew reported for the following eight quarters. Similarly, after each FOMC meeting the policy statement, made available, also indicates the balance of inflation risks, (up to November, 2007, economic forecasts were made available only twice a year).[64] In just considering the upper bound of the Bank of England's price stability mandate, here given as k, (the exercise on an option), it is possible to map out a time value parabola as illustrated in Figure 1.1b. Even when inflation resides below the upper bound, policy makers must allow for the fraction of inflation outcomes at the end of the following two years or targeting horizon that will exceed k. In this regard, when setting the short term rate, central bankers respond to the proportion of total outcomes that randomly breach their inflation mandate. The two piece normal distribution

[63] Movements in inflation both above and below the comfort zone are considered in chapter 2.

[64] The FOMC introduced six enhancements to the publication of its economic projections in November, 2007. First, in order to provide more timely information, the projections are now published four times a year rather than twice a year, as before. Second, the forecast horizon now stretches to approximately three years. Third, the forecast includes not only core price inflation, but in addition, overall inflation. Fourth, a more exhaustive discussion of the key influences shaping the FOMC outlook is furnished. Fifth, that forecast discussion also incorporates the FOMC member's qualitative assessments of the level and balance of uncertainty relating to their respective economic outlook. Sixth, the dispersion of forecasts among the FOMC participants is made available in more detail.

employed by the Bank of England effectively assumes that the random process is Gaussian. The Gaussian family is also widely used in option pricing and in what follows, both a lognormal and non-standard distributions of inflation are developed using Monte Carlo analysis. Monte Carlo has the advantage that it provides flexibility in terms of not privileging any particular distribution. It also constitutes a discrete estimator given that it can specify a number of steps that in turn are iterated. The Heston (1993) model is used to extend the Black-Scholes model. Importantly, Heston (1993) permits varying measures of skewness and kurtosis. The Heston model reveals how pricing and indirectly monetary policy reactions can be affected by the third and fourth moments.[65] This is relevant because varying distributional forms can be incorporated into the analysis. Both positive and negative skewness in the inflation forecast can be accommodated.

1.6. Risk neutrality: from Black-Scholes (1973) to Derman and Taleb (2005)

The major insight of Black and Scholes (1973) relates to forming a riskless portfolio that can be dynamically hedged. Their key contribution was to show that it was in fact not necessary to use any risk premium when valuing an option subject to the assumptions (A.1) – (A.6) given below. This permits the derivation of a theoretical valuation formula using a risk free discount rate, r i.e. risk neutrality can be established. The concept of market neutral delta hedging can be traced back to Higgins (1902) and Nelson (1904), but is generally understood to have been rigorously developed by Black and Scholes (1973). Thorp and Kaussof (1967) and Thorp (1969) outlined how risk neutrality could be established and implemented a practical working formula for pricing although this was largely considered to have been *ad hoc*. Black and Scholes (1973) pointed out that an option could be hedged by applying a continuous dynamic stream of hedging positions in the underlying. This necessitated assuming that the underlying conformed to Geometric Brownian motion (GBm). In practice, this is impossible, although delta hedging type strategies have enjoyed some degree of popularity. The significance of Derman and Taleb (2005) relates to establishing risk neutrality when a number of the traditional assumptions break down including GBm. For the purposes of developing an option's framework for monetary policy and elaborating a forward-looking opportunistic reaction

[65] Black-Scholes is also adapted using the Gram-Charlier model to take account explicitly of skew and kurtosis in inflation return.

function, it is worth considering varying frameworks. Three possible approaches are considered here:

(1) Disregard risk neutrality by setting arbitrarily the discount rate, *r*, in the valuation formula (1.25) equal to zero. This might be termed a 'zero time decay' approach where central bankers regard future inflation with the same disdain as current inflation i.e. there is no benefit to postponing inflation. This approach would imply side-tracking in particular assumption (A.2) and much of the mathematical development set out below. The 'zero time decay approach' focuses primarily on the collar option construction, developed in Appendix A.2.1, which produces (a.2.1.16) – the forward-looking opportunistic reaction function.

(2) Establish risk neutrality using the Black-Scholes (1973) approach.

(3) Establish risk neutrality using the Derman-Taleb (2005) approach.

It is worth considering approaches (2) and (3) in order to preserve flexibility. The logic here is simple. By developing explicitly risk neutral conditions using (2) and (3), it opens the possibility to use market data on inflation options as an important reference for policy. Despite rapid growth however inflation markets are perhaps not sufficiently liquid or transparent to provide reliable signals. In future however this is less likely to be the case. With this in mind, the analysis is kept sufficiently broad to accommodate market approaches (2) and (3). Risk neutrality is considered here using both Black-Scholes (1973) and Derman and Taleb (2005). Of the two, the latter is substantially less restrictive in particular with regard to assumptions (A.2) and (A.5). Setting out a market approach initially entails examining the Black and Scholes (1973) assumptions which lay down the basis for continuous time dynamic replication of a European option:

(A.1) The short-term discount rate is known and constant.

(A.2) The underlying asset adheres to Geometric Brownian motion (GBm).

(A.3) The variance rate on the underlying is proportional to the square of the underlying asset value.

(A.4) The underlying asset pays no dividends i.e. has non-negative drift.

(A.5) Markets are frictionless i.e. transaction costs are zero.

(A.6) The variance rate of the return on the underlying is constant.

When considering monetary policy, the applicability of Geometric Brownian motion to inflation is particularly difficult given that inflation does not trade in a liquid spot market.[66] [67] Jarrow and Yildirim (2003) and Korn and Kruse (2004) however assume that the inflation index follows Geometric Brownian Motion.[68] This would be attractive from the perspective of imposing a workable theory but may lack intuition given that the Consumer Price Index is a discrete time series.[69] The customary requirement to continuously dynamically hedge a portfolio consisting of an option and its' underlying would seem especially difficult where the underlying is inflation. The classic risk neutral arguments only exist in a very virtual or idealised world which is not easily attained in the absence of liquid market conditions. To understand this and many other nuances that relate to the proposed market option pricing model, it is useful to set out how the Black (1976) differential equation might apply to expected inflation.[70] Initially, it assumed that (A.1) – (A.6) hold.[71] The idealised stochastic differential equation would represent the change in the underlying asset which is given as expected inflation, $E(\pi)$:

$$dE(\pi) = \alpha_S E(\pi)dt + \sigma\, E(\pi)dz \tag{1.14}$$

where dz denotes a Wiener process and the volatility, σ is constant. Over an infinitesimally small period, dt, the change in asset price, $dE(\pi)$, equals the product of an expected drift rate of the asset α_S, multiplied by both the asset price $E(\pi)$ and dt plus a random magnitude proportional to the instantaneous standard deviation of the rate of change in the asset price σ, multiplied by the asset price. It can be illustrated by applying Ito's lemma that a derivative contract, f written on the underlying, $E(\pi)$ follows the process:

$$df = \left(\frac{\partial f}{\partial E(\pi)} \alpha_S E(\pi) + \frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial E(\pi)^2} \sigma^2 E(\pi)^2 \right) dt + \frac{\partial f}{\partial E(\pi)} \sigma E(\pi) dz \tag{1.15}$$

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[66] Formalising previous work by Thorp (1969). See MacKenzie (2003) and Derman and Taleb (2005).

[67] Indexed linked instruments however do trade in a growing number of markets with ever increasing liquidity. These instruments relative to nominal instruments capture the expectation of inflation.

[68] This would imply that the logarithm of the inflation index is normally distributed.

[69] A forward price based on inflation that would trade in a liquid market should provide a stronger basis for asserting risk neutral conditions than using purely current inflation. The TIPS market implicitly provides measures of inflation expectations and plausibly provides a means to hedge inflation options. Some effort has been made recently to produce a real time price index. (www.inflacionverdadera.com)

[70] Alternative risk neutral conditions using static portfolio replication are proposed by Derman and Taleb (2005).

[71] This construction including the stochastic differential equation (1.14) implies stipulating both GBm and frictionless markets to establish a dynamic hedging portfolio and ultimately risk neutral conditions. Given the real world absence of GBm for all underlying, risk neutral conditions may be difficult to justify. Derman and Taleb (2005) provide a more robust derivation using put-call parity so that dependence on GBm can be relaxed.

As with the Black-Scholes/Merton model the basis of the Black differential equation is that a riskless portfolio can be created by assuming a long position in the option and a stream of short positions in the underlying with the same expiration date.[72] The equity of the combined portfolio, V is just the value of the option, by virtue that the value of a futures at inception is always zero. In what follows, the term Δ denotes the change that occurs in time t. In discrete time (1.14) becomes:[73]

$$\Delta E(\pi) = \alpha_S E(\pi)\Delta t + \sigma\, E(\pi)\Delta z \tag{1.16}$$

and similarly that

$$\Delta f = \left(\frac{\partial f}{\partial E(\pi)} \alpha_S E(\pi) + \frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial E(\pi)^2} \sigma^2 E(\pi)^2 \right)\Delta t + \frac{\partial f}{\partial E(\pi)} \sigma E(\pi)\Delta z \tag{1.17}$$

By developing the BSM insight, it is possible to illustrate that a risk free hedge can be obtained by combining a long position of $\partial f / \partial E(\pi)$ in the expected inflation, $E(\pi)$, with a short position in the derivative asset f.[74] [75] In discrete time the initial value of the portfolio is given by:

$$V = -f + \frac{\partial f}{\partial E(\pi)} E(\pi) \tag{1.18}$$

However for a futures contract the value of the portfolio is:

$$V = -f$$

given that it costs nothing to initially take a position in a futures contract. The change in the value of the portfolio in a discrete time period Δt can be shown to be:

$$\Delta V = -\Delta f + \frac{\partial f}{\partial E(\pi)} \Delta E(\pi) \tag{1.19}$$

By substituting the discrete versions into the above it is possible to attain:

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[72] In what follows, expected inflation is modelled as a futures price, in part reflecting market developments. Cash settled CPI futures with a notional principal of $1 million began to trade electronically on the Globex platform of the Chicago Mercantile Exchange in February 2004. Their design was similar to the better known CME Eurodollars futures contract. Eurozone HICP futures contracts started trading on the CME Globex platform in September, 2005. Market expectations for future inflation over the following year, as implied by the Eurozone HICP futures contract, are reported by the CME. Perhaps more significantly, expected inflation can be read from TIPS instruments. The Federal Reserve of Cleveland use TIPSs' prices to estimate expected inflation.

[73] When applying Monte Carlo techniques the discreteness of the underlying process is made explicit.

[74] $\partial f/\partial E(\pi)$ is the measure used to dynamically hedge. This however is dependent upon assuming that the underlying adheres to Geometric Brownian Motion. Derman and Taleb (2005) illustrate how risk neutrality can be developed using a static replicating portfolio. $\partial f/\partial E(\pi)$ is later employed in chapter 2 to motivate interest rate smoothing in an innovative way. See Propositions 2.3 – 2.3d.

[75] The BSM insight however is not completely intuitive and provides a highly idealised justification for risk neutrality when applied to inflation.

$$\Delta V = -\left(\frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial E(\pi)^2} \sigma^2 E(\pi)^2 \right) \Delta t \qquad (1.20)$$

In a world where arbitrage cannot persist and adapting the major insight of the Black-Scholes/Merton model, it follows that the portfolio can only earn the risk free rate:

$$\Delta V = r\,V \Delta t \qquad (1.21)$$

Thus, by substituting for ΔV

$$-\left(\frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial E(\pi)^2} \sigma^2 E(\pi)^2 \right) \Delta t = r(-\,f\,)\Delta t \qquad (1.22)$$

and by rearranging (1.22):

$$\left(\frac{\partial f}{\partial t} + ½ \frac{\partial^2 f}{\partial E(\pi)^2} \sigma^2 E(\pi)^2 \right) = r(f\,) \qquad (1.23)$$

This is similar to the original Black-Scholes/Merton differential equation. The $[\partial f / \partial E(\pi)] r E(\pi)$ term is omitted from the left hand side of (1.23) by virtue that expected inflation is treated like a futures contract. As developed by Black (1976), the cost of entering into a futures contract is zero hence the value of the portfolio is initially the value of the option. The differential equation has a number of solutions contingent on the boundary conditions that are applied. For a European call option, *c* where the underlying is expected inflation the boundary condition is defined by:

$$c = e^{-r(T-t)} \hat{E}[\max(\pi_T - k, 0)] \qquad (1.24)$$

Where $\hat{E}$ denotes the expectation in a risk-neutral world. Adhering to the literature the time value of the call can be written as:

$$c = e^{-r(T-t)} [E(\pi) N(d_1) - kN(d_2)] \qquad (1.25)$$

where

$$d_1 = \frac{ln(E(\pi)/k) + (0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \qquad (1.26)$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

N(.) denotes the cumulative probability distribution of the standardised normal variable inside the parentheses. Replacing the derivative f by c

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 (E(\pi))^2 \frac{\partial^2 c}{\partial E(\pi)^2} = rc \qquad (1.27)$$

It will be illustrated, in appendices B.3 and C.3 of chapter 3, how the individual terms in (1.28) can be obtained.[76] (1.27) is expanded by differentiating the varying terms to give:

$$\frac{\partial c}{\partial t}+\frac{1}{2}\sigma^2\left(E(\pi)\right)^2\frac{\partial^2 c}{\partial E(\pi)^2}=-e^{-r(T-t)}\left[E(\pi)N'\left(d_1\right)\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right]$$
$$+re^{-r(T-t)}\left[E(\pi)N\left(d_1\right)-kN\left(d_2\right)\right]+\frac{1}{2}\sigma^2\left(E(\pi)\right)^2\left\{e^{-r(T-t)}\left[N'\left(d_1\right)\left(\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right)\right]\right\} \tag{1.28}$$

Where $N'(.)$ represents the normal probability density function of the value inside the parentheses. The following equality is found to exist:

$$\frac{1}{2}\sigma^2\left(E(\pi)\right)^2e^{-r(T-t)}\left[N'\left(d_1\right)\left(\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right)\right]=e^{-r(T-t)}\left[E(\pi)N'\left(d_1\right)\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right]$$

consequently (1.28) reduces to:

$$re^{-r(T-t)}\left[E(\pi)N\left(d_1\right)-kN\left(d_2\right)\right]=rc \tag{1.29}$$

The formula for the call is seen to satisfy the Black (1976) differential equation. By positing expected inflation as a futures price, it is possible to apply the same logic as proposed by Black (1976). In a monetary policy risk management context, the Black (1976) and Black-Scholes (1973) construction is difficult to apply directly when inflation evolves discretely. More generally, the BSM assertion that dynamic continuous replication is possible outside a purely idealised world may not be tenable when markets trade infrequently. The BSM risk neutral argument depends on the feasibility of constructing a riskless portfolio and this is created by holding a long position in the option and a short position, in this instance, in expected inflation. Moreover as time passes it must be possible to dynamically replicate the riskless portfolio in continuous time. In reality, this would appear to be difficult and a more tailored approach when establishing risk neutrality is worth identifying. Here, the analysis is extended to take account of discrete time using Derman and Taleb (2005) who provide a less onerous rationale for establishing risk neutrality. Their approach

[76] The mathematical notation is standard in the literature. The subscripts somewhat exceptionally however read differently. The *g1* and *g2* subscripts found in the appendices are intended to assign the strikes associated with different players in the policy game. Here the strike or exercise is denoted by *k*, subsequently the strike is interpreted as a level of tolerance. π^*_{g1} and π^*_{g2} are developed in chapter 2 to denote the tolerance levels for a hawk and dove who collectively set the policy rate.

permits a derivation for option pricing that does not necessitate reliance on GBm and frictionless markets. This is consistent with the following two propositions:

Proposition 1.1: *The familiar Black-Scholes (1973) formulae for valuing options can be extended to a range of underlying that violate assumptions (A.2) and (A.5), as outlined by Black Scholes (1973), see p.31. The well-known no-arbitrage relationship, implied by put-call parity can be used to establish risk neutrality.*

Proposition 1.2: *Risk neutral conditions are attainable without resorting to dynamic and continuous replication. The Derman and Taleb (2005) consistency argument, (DT), posits that the time decay (or discount factor), which applies to a forward position, will also apply to an option portfolio based on the same underlying. A central banker who is indifferent between the intertemporal choice of inflation today and inflation in the future applies a zero discount rate, (a 'zero time decay'), to the payoff from a forward position. As a corollary of (DT), the same central banker would also apply a zero discount rate when valuing an option portfolio.*

Derman and Taleb (2005) exploit a relationship known in finance as put-call parity that was observed by Nelson, as early as 1904. If this relationship did not hold an arbitrage would be possible. Fundamentally:

$$Call - Put \; = \; PV(Forward \;\; Price - Strike \; Price) \qquad (1.30)$$

A portfolio composed of a long position in a call and a short position in a put with the same strike k has exactly the same payoff as a forward contract with expiration time, T and delivery price, k. If the forward price is configured as the expectation of future inflation then the value of a forward contract F_v:

$$F_v = c - p = PV\left[E(\pi) - k\right] \qquad (1.31)$$

One European call option minus one European put option on expected inflation with an exercise, k, equates to the difference between a discounted forward expected inflation rate, $E(\pi)e^{-rT}$ and the discounted exercise, ke^{-rT}. This can be generalised to give:

$$F_v = c - p = e^{-r(T-t)}\left[E(S) - k\right] \qquad (1.32)$$

The risk free discount rate is given by r. S might be thought of as representing any asset class and lower case, t denotes the current time period.[77] [78] The 'time honoured actuarial' way to estimate the value of a European call where S can denote any asset class including the present value of expected inflation is given as:

$$c(S,t)=e^{-\alpha_o(T-t)}\left(E[S-k]_+\right)=e^{-\alpha_o(T-t)}\left\{Se^{\alpha_s(T-t)}N(d_1)-kN(d_2)\right\} \quad (1.33)$$

and analogously for a put

$$p(S,t)=e^{-\alpha_o(T-t)}\left(E[k-S]_+\right)=e^{-\alpha_o(T-t)}\left\{kN(-d_2)-Se^{\alpha_s(T-t)}N(-d_1)\right\} \quad (1.34)$$

The pricing formula for the European call and put, $c(S,t)$ and $p(S,t)$, preceded the Black and Scholes (1973) representation. Sprenkle (1961) and Samuelson (1965) in the same vein both employed lognormality to model the behaviour of stocks. Whaley (1997) described the Sprenkle-Samuelson formulae for call option pricing as assuming the form:

$$c=exp[-\alpha_o(T-t)](S\,exp[\alpha_S(T-t)]N(d_1)-kN(d_2)) \quad (1.35)$$

where

$$d_1=\frac{ln(S/k)+\left(\alpha_S+0.5\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad (1.36)$$

and

$$d_2=d_1-\sigma\sqrt{T-t} \quad (1.37)$$

α_S and α_o are the expected risk adjusted rates of price appreciation for the respective underlying and option assets. Lower case t indicates the current time period. $N(.)$ denotes the cumulative standard normal probability of the variable inside the parentheses. To establish risk neutrality, it is essential to demonstrate that $\alpha_S = \alpha_o = r$. The major contribution of Black and Scholes (1973) was to show that risk neutrality could be obtained by creating a dynamic replicating portfolio. To extend this framework beyond the idealised world assumed by Black and Scholes (1973), Derman and Taleb (2005) intuit a formula that exploits static as opposed to dynamic replication. By combining (1.33) and (1.34) the following emerges:

$$c(S,t)-p(S,t)=e^{-\alpha_o(T-t)}\left\{Se^{\alpha_s(T-t)}N(d_1)-kN(d_2)\right\} \quad (1.38)$$
$$-e^{-\alpha_o(T-t)}\left\{kN(-d_2)-Se^{\alpha_s(T-t)}N(-d_1)\right\}$$

---

[77] The generic notation for the exercise is given as k. Later π^* is used to indicate the upper and lower bounds of inflation zone targeting.

[78] Unlike inflation, expected inflation can be seen as a traded asset now, given the availability of inflation indexed instruments in financial markets.

$$c(S,t) - p(S,t) = e^{-\alpha_o(T-t)}\left\{Se^{\alpha_s(T-t)}N(d_1) - kN(d_2)\right\} \quad (1.39)$$
$$-e^{-\alpha_o(T-t)}\left\{k[1-N(d_2)] - Se^{\alpha_s(T-t)}[1-N(d_1)]\right\}$$

By cancelling:

$$c(S,t) - p(S,t) = e^{-\alpha_o(T-t)}\{0\} - e^{-\alpha_o(T-t)}\left\{k[1] - Se^{\alpha_s(T-t)}[1]\right\} \quad (1.40)$$
$$c(S,t) - p(S,t) = e^{-\alpha_o(T-t)}\left\{Se^{\alpha_s(T-t)} - k\right\}$$

But this must be equal to:

$$F_v = c - p = e^{-r(T-t)}[E(S) - k] \quad (1.40a)$$

Derman and Taleb (2005) illustrate that when static replication of the put and call using the put-call parity relationship holds, then:

$$F_v = e^{-r(T-t)}[E(S) - k] = e^{-\alpha_o(T-t)}\left\{Se^{\alpha_s(T-t)} - k\right\} \quad (1.40b)$$

The fact that both must equal the value of the forward implies that both α_S and α_o must equal the discount rate, *r*. This implies that the formulae given by (1.33) and (1.34) are equivalent to the Black-Scholes pricing formulae. The time decay or discount factor that applies to the underlying also applies to the option. From a policy perspective, a central banker who is indifferent to the timing of inflation will apply a 'zero time decay' or a zero discount factor to both the option and the underlying. This consistency argument holds even when assumption (A.2), (i.e. GBm) is violated. It is still possible to apply the traditional 'actuarial' formulae (1.25)-(1.26). The Derman-Taleb (2005) put-call parity specification provides a more general construction to ensure risk neutrality.[79] Assets that do not trade continuously and can not be replicated dynamically are, in particular, difficult to price in a conventional option's framework. The static hedge proposed by Derman and Taleb (2005) establishes risk neutrality for a wider class of underlying. It implies that the option's framework for

[79] In reply to a number of questions marked in bold, Emanuel Derman outlined his views via email:
In reply: <emanuel.derman@mac.com> Tue, 10 Jul 2007 12:36:11 -0400, Subject: Re: Illusions
On Jun 22, 2007, at 11:03 AM, Brian Byrne wrote:
Dear Emanuel,
I have read your work: 'The illusions of dynamic replication'. Does this imply when pricing an option:
(1) that it is not necessary to assume that a stock or underlying adheres to Geometric Brownian motion
It's a fact that GBm doesn't hold, so one has to make adjustments somehow beyond BS.
(2) that it is not necessary that asset prices exist in continuous time.
That's what we were trying to argue
The main point is that our paper, as I see it, pointed out the difficulties with traditional options valuation and how well/badly it works, and then made an attempt to intuit a formula for option valuation in more difficult circumstances.

considering inflation risk is robust even when it is not possible to verify that inflation adheres to GBm. Alternatively, ignoring risk neutral conditions altogether, the discount rate *r* in (1.25) could be set arbitrarily to zero implying 'zero time decay' i.e. central bankers are intertemporally indifferent to inflation, whether it occurs today or in the future. This is equivalent to making assumption (A.1) more restrictive so that the short-term discount rate is known and constant at zero. The effect of zero time decay however may be largely academic; as will be observed from (a.2.1.16), p. 183, where *r* is found to cancel out anyway when considering the forward-looking opportunistic reaction function. In effect, all three approaches: Black-Scholes (1973), Derman and Taleb (2005) and the 'zero time decay would appear consistent and produce the same reaction function.

1.7 The Fan Charts and risk management of monetary policy decisions

A number of inflation targeting central banks describe monetary policy as responding to future inflation and to the distribution of expected inflation outcomes. By doing so, they convey not only their forecast of central tendency but also convey anticipated risks or uncertainty. If policy makers simply respond to that proportion of anticipated outcomes that exceed an upper bound *k*, it then becomes possible to describe policy as responding to:

$$E[max(\pi_T - k, 0)] = \int_k^{\infty} (\pi_T - k) g(\pi) d(\pi) \qquad (1.41)$$

where the probability density function of inflation is defined as $g(\pi)$. No official target for inflation has been made explicit in the United States to date. In the United Kingdom, an upper and lower bound for inflation has existed for some time. If the probability density function is known then it is possible using Monte Carlo to value *c*.

$$c = e^{-r(T-t)} \hat{E}[max(\pi_T - k, 0)] \qquad (1.42)$$

Just as it was necessary to establish risk neutrality to implement Black-Scholes, equally Monte Carlo uses the risk free rate to discount future payoffs. The Derman and Taleb (2005) approach is advanced here to justify risk neutrality. Monte Carlo analysis is developed here to show that the option's framework and inflation fan charts construction are consistent. The Bank of England employs a two piece Gaussian normal distribution to generate the fan charts, published in its Inflation Report. When inflation risk is symmetric the two piece distribution produces a consistency in terms of the mean, median and mode CPI inflation projection. This was the case in 2007 Q2, where the following five quarter projections had zero skew i.e. the risks to inflation were considered by the MPC to be roughly balanced.[80] The standard distributions used for pricing options are also usually Gaussian. This may reflect the popularity of the Black and Black-Scholes models which assume lognormality.[81] However as will be shown later, the option's framework can be adapted to accommodate many different types of distribution. Backus, Foresi and Wu (2004), for instance, provide a closed-form solution for estimating the value of an option that takes account of skewness and kurtosis in the underlying asset's return. The Heston (1993) model is developed using Monte Carlo to flexibly incorporate departures from Gaussian distributions when pricing inflation options and also to generate inflation fan charts that embed upside, downside and symmetric risks. See Figures 1.8a – 1.8h.

1.7.1 Advantages of the lognormal model when applying a risk management paradigm to monetary policy

The lognormal distribution, when used to evaluate (1.41), has a number of attractions in describing inflation for policy purposes. The first relates to inflation as not violating non-negativity; the second configures the underlying as adhering to a random walk. These specifications are relaxed, in chapter 2, to incorporate mean reversion and deflation. In this chapter, Heston (1993) is used to extend the lognormal model by taking account of both negative and positive skew in the underlying, (in monetary policy parlance, upside and downside risks). Specifying lognormality is conceptually useful when setting out a risk management framework, in part because this provides a non-negative baseline. Consider the general case where $S_{t,}$ implying any asset price, respects the following no-arbitrage condition:

$$E(S_T) = S_t e^{(r-d)(T-t)} \tag{1.43}$$

[80] The FOMC do not produce a consensus probability distribution. They nevertheless indicate in their policy statement, following scheduled FOMC meetings, whether risks are upside, downside or symmetric. This has the effect of communicating to the public much of what is conveyed in the fan chart that is published by the Bank of England. This concern for risk provides the basis for considering an option's framework. Upside and downside risks can be portrayed using varying Heston generated inflation charts. See Figures 1.8a – 1.8h.

[81] Models that have a normal distribution e.g. Vasicek and the extended Vasicek models are commonly specified for interest rate options.

The notation is the same as before except d constitutes a dividend yield obtained from holding the underlying.[82] The trend growth rate in the underlying declines as d increases. This expression can be easily expanded to give:

$$E(S_T) = S_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)} e^{\frac{\sigma^2}{2}(T-t)} \qquad (1.44)$$

From lognormal properties it is known that:

$$E\left(e^{\sigma\sqrt{T-t}z}\right) = e^{\frac{\sigma^2}{2}(T-t)} \qquad (1.45)$$

where $z \sim N(0,1)$, represents a standard normal random variable and will have a mean of zero and standard deviation of one. Given the above, it is possible to rearrange:

$$E(S_T) = S_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)} e^{\sigma\sqrt{T-t}z} = S_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} \qquad (1.46)$$

This relationship is used extensively for valuation. It could be motivated by applying a no-arbitrage condition. It would imply instantaneous adjustment in the price level so that the current inflation rate reflects expectations. Unlike the standard Phillips curve, the output gap is not accommodated. Inflation spot markets do not exist in the conventional sense and an instantaneous no-arbitrage relationship is perhaps unlikely or even inappropriate. Spot indexed linked markets however would permit expected inflation to trade and this would suggest that the no-arbitrage relationship, as spelled out by (1.46), is tenable. Alternative asset price processes to (1.46) could incorporate mean reversion particularly if central bankers felt they could forecast future inflation. (1.46) sets out inflation as being exogenous to monetary policy. Inflation could be made endogenous to a given monetary policy regime by co-opting, for instance, a Vasicek framework for option pricing.[83] (1.46) could also be set out so that a forward price of inflation is used instead of a spot price. This could possibly be inferred from inflation indexed linked markets which are becoming increasingly established. A minor modification permits the following to emerge:

$$E(S_T) = S_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} = F_{t,T} e^{\left(-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} \qquad (1.47)$$

$F_{t,T}$ denotes here the forward or futures price of inflation quoted in the current time period, t for the period T. When d and r are set equal to each other, the drift in the

[82] *Ceteris Paribus*, the dividend yield results in a decline in the value of the underlying asset.
[83] This is considered in chapter 2. The degree to which inflation mean reverts could be made a function of policy. This endogeneity could be defined by both the speed by which inflation mean reverts and the long term mean which inflation mean reverts to.

asset becomes zero (otherwise referred to as driftless growth). From a policy maker perspective, targeting a forward price of inflation derived from government treasury instrument such as TIPS is not unlike targeting an expectation of inflation.[84]

Proposition 1.3: *It is possible to simulate expected inflation by discretising the stochastic process set out in (A.1) – (A.6). Only non-negative inflation outcomes are considered when applying the lognormal random walk process. This, nevertheless, provides a baseline analytic tool for central bankers who wish to hypothesise the risks to price stability from deflation as being minimal, yet do not want to rule out the risk of hyperinflation for policy purposes. The variance rate of the return on the underlying is constant.*

Three features are worth considering: (1) Lognormality rules out any violation of non-negativity. (2) The lognormal random walk permits hyperinflation when expected inflation is given as the underlying. (3) The constant variance rate implies that the magnitude of inflation volatility grows with inflation. This parsimonious baseline is useful when economists differ in terms of what they understand to be the precise structure of the economy but nevertheless want to focus on risk management issues.[85] (1), (2) and (3) are later relaxed when using the Vasicek model. The imposition of non-negativity is conceptually attractive, in large measure because policy makers often feel confident that they can reverse deflation. Given the experience of Japan, this would seem to rule out a major macroeconomic risk which is not just academic. While deflation can never be excluded as a temporary phenomenon and the Japanese experience should herald a note of warning, Goodhart and Hofmann (2003) nevertheless point out that theoretically governments can resort to the printing presses:

> 'It is remarkable that, under a fiat money system, there should be any worry about deflation at all. Under this system the authorities can in principle, create an unlimited amount of (base) money by buying <u>anything</u> that they choose. So, <u>unwanted deflation should be inconceivable under such a system</u>…'
> (Goodhart and Hofmann, 2003, p.11)

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[84] Policy makers would generally want to use a wider data set. The Chicago Mercantile Exchange (CME) currently reports the implied expectation of HICP inflation up to one year ahead, derived from futures trading.

[85] Economists may not agree on the appropriateness of a given economic forecasting model yet may nevertheless agree on what constitutes a worst case scenario.

Bernanke (2002) similarly contended that deflation is reversible under a fiat money system.[86] When policy makers are assumed to have imperfect knowledge of the precise structure of the economy then the lognormal random walk serves as a baseline for risk analysis.[87] The lognormal random walk model is fitting because it permits central bankers, perhaps counterfactually, to preserve intact the risk of hyperinflation when evaluating (1.41).[88] It also constitutes the most intellectually modest stance that a central banker can assume: what if her best forecasting model were no better at predicting than tossing a coin?[89] It may be impractical to assume inflation mean reverts of its own accord to a knowable long term mean. Policy makers must consider how inflation would behave in the absence of their policy actions. In this regard, the imperative 'to keep plunging the stake' suggests that the view via the *ex-ante* lens differs from the historical experience of mean reversion to a low and stable inflation rate. The qualitative features associated with the random walk offer a powerful analytical tool, not because there is a firm consensus that the random walk mirrors the actual empirical experience with inflation, in the United States or beyond, but rather because it permits policy makers to consider hyper-inflation as a real possibility while not privileging any structured forecasting model. Central bankers may be conscious

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[86] Bernanke (2002) pointed out that acting together government agencies can always remove deflation by simply printing additional money. Equally, equating a money-financed tax cut with Milton Friedman's "helicopter drop" of money suggests that chairman Bernanke believes there are a great many policy options open to prevent a liquidity trap. Advocacy of a number of strategies that also promote the Federal Reserve purchasing varying assets, in the event of falling prices, has earned him the unflattering nickname "Helicopter Ben". Greenspan (2003) however indicated that the risks of deflation were significant in explaining the conduct of monetary policy. In fact, the risk management approach to monetary policy has been expounded as a response to deflation. The purported need to take preemptive measures against deflation however may be inconsistent with the view that deflation is easily reversible. Using the lognormal random walk would rule out deflation. This may not necessarily be a weakness. The Vasicek model is developed, in chapter 2, to address explicitly the risk of deflation within the option's framework.

[87] D'Agostino, Giannone and Surico (2006) contend that the there has been a major decline in the predictive accuracy of institutional forecasters, such as the Federal Reserve's Greenbook and the Survey of Professional Forecasters (SPF) and for methods based on large information sets. Their results implied that the informational advantage of the Federal Reserve and professional forecasters is, in fact, limited to the 1970s and to the beginning of the 1980s. They found that no forecasting model, during the last two decades, has been better than *tossing a coin* beyond the first quarter horizon in predicting inflation and other macro-economic variables for the United States. Stock and Watson (2007) also find that over the same period inflation has become much more difficult to forecast, but only in the sense of providing value added beyond a univariate model.

[88] If inflation is modelled as mean reverting this drastically diminishes the risk of hyperinflation.

[89] Atkeson and Ohanian (2001) find that since the mid 1980s, economists have not produced a version of the Phillips curve that makes more accurate inflation forecasts than those from a naive model that presumes inflation over the next four quarters will be equal to inflation over the last four quarters i.e. the likelihood of accurately predicting a change in the inflation rate from a number of models including one from the Federal Reserve Board is no better than the likelihood of accurately predicting a change based on a coin flip.

that their capacity to predict inflation at any given point will not consistently outperform a naïve forecasting model. The option's framework can also be generalised to accommodate other facets of policy making. A number of inflation targeting regimes, for example, attempt to make their forecasts of inflation as transparent, as possible, by reporting the anticipated levels of skew and uncertainty that they perceive. The Black-Scholes framework, as set out here, can be extended using Heston (1993) to incorporate not just the first and second moments, but also the third and fourth moments, i.e. skew and kurtosis. Within the option's framework many possible processes can be considered. For instance, mean reversion can be incorporated by using the Vasicek model. This is applied in chapter 2.

1.7.2 Lognormality, confidence intervals and Inflation Fan charts

It is worth considering how a series of lognormal random walk processes can be generated using Monte Carlo simulation. The range of simulated paths developed iteratively using (1.48) could, in principle set out a distribution of inflation outcomes. The visual mapping out of these paths is useful in much the same way as the Bank of England conveys it's uncertainty regarding forecasts via fan charts. To extend this analysis to emphasise risk management features, it is worth viewing how confidence intervals for $E(\pi_T)$ can be developed using the lognormal distribution[90]:

$$E(\pi_T) = \pi_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} \tag{1.48}$$

Arnold and Henry (2003) state that by iterating simulation paths using (1.48), it is possible to generate the probability distribution function of future asset prices. The frequency of final asset prices within the simulation data can be used to construct confidence intervals. Integrating all future outcomes weighted by their associated probability (i.e. the generated expectation of future outcomes) is akin to taking the integral of a cumulative standard normal distribution. In a risk neutral world that models inflation as adhering to the Black-Scholes assumptions (A.1) – (A.6), the probability that π_T will exceed a critical level k is calculable. This is consistent with $ln(E(\pi_T)/\pi_t)$ being normally distributed with a mean of $(r - d - \sigma^2/2)(T-t)$ and a standard deviation of $\sigma(T-t)^{0.5}$.[91] A normal variable z can be standardised so that:

[90] The relevance is considered here to be general to all monetary policy.
[91] Later, it will be illustrated that this probability is related closely to the delta of an option.

$$z = \frac{ln(\pi_T) - ln(\pi_t) - (r - d - 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}} \tag{1.49}$$

From (1.49) it is possible to determine the probability of future inflation, at a given point exceeding a critical level, k, by calculating:

$$Pr\,ob(\pi_T > k) = 1 - N\left[\frac{ln(k) - ln(\pi_t) - (r - d - 0.5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}\right]$$

$$Pr\,ob(\pi_T > k) = 1 - N[-d_2] = N[d_2]$$

The term $N[d_2]$ is the same as that identified in the Black-Scholes formula. Assuming a lognormal random walk, it is possible to generate confidence intervals, akin to the fan chart, simply by attributing varying values of z for specified time periods that correspond to given levels of probability. For instance, if the probability level associated with $E(\pi_T)$ was set at 95% then z, defined by lower and upper intervals, would be given by $N^{1}(0.025) = -1.96$ and $N^{1}(0.975) = 1.96$. By substituting these values for z in (1.48), given the appropriate sequence of time periods, it becomes possible to trace out a lognormal fan chart. Monte Carlo simulation could also be used to generate confidence intervals and equivalently fan charts. This would require generating a normally distributed z randomly, over a great many sample paths and then ordering those paths values at given time horizons. Fink and Fink (2006) state that Monte Carlo analysis is useful because it permits many different types of simulation to occur that engender varying types of risk and asymmetry. This type of analysis is presented using Figures 1.8a - 1.8h. The Heston Monte Carlo approach, as developed in the following section, has the benefit of being flexible and incorporates a wide range of stochastic processes. These can be presented as fan charts that exhibit varying levels of skew and kurtosis. Monte Carlo is initially used here to consider the lognormal random walk in discrete time. That is, discrete time steps can be made explicit. Monte Carlo is used here to build, from the ground up, varying approaches that permit the valuation of inflation options implicit in monetary policy under uncertainty - as set out by the broken red parabola of Figure 1.1b. Monte Carlo estimation stresses the discreteness of the underlying variable and permits assumption (A.2) to be relaxed i.e. reliance on continuous time is unnecessary. The Derman-Taleb (2005) static replication is invoked at each discrete step in order to establish risk neutrality. In developing the varying Monte Carlo valuation models, call option time values are investigated. From Figures 1.5a and 1.5b, it is found that time values are equal to or exceed intrinsic values, i.e. uncertainty increases the value that can be attributed to options. This leads to the following observation:

Proposition 1.4: *Under uncertainty, Central Bankers can increase the policy rate even when expected inflation is below the upper bound of an opportunistic or zone targeting monetary policy framework. Once the probability distribution of inflation is made explicit, the policy response can be represented by the option's time value developed using (1.35)-(1.41). This is consistent with the parabola representation given in Figures 1.1b and with the Black-Scholes (1973)/Monte Carlo estimations outlined in Figures 1.5a and 1.5b. It will be observed that greater uncertainty tends to produce a more preemptive policy and limits the extent to which discretion, defined by the upper tolerance, moderates policy activism.*

The impact of uncertainty is examined, more formally in chapter 3, using parameter sensitivities known as 'the Greeks'. Much of the dynamics associated with conventional inflation zone targeting and opportunistic policy are encompassed in Proposition 1.4. Initially, expected inflation is assumed to conform to (A.1) – (A.6), although the process is permitted here to be discrete. In the United States, no explicit target, (official upper bound), has existed during the Greenspan chairmanship. Nevertheless, it would seem that the opportunistic framework as elaborated by a number of policy makers shares a number of parallels with inflation zone targeting. In both instances, the same analysis can be applied with the same conclusion being drawn. Under uncertainty, whether defined by fan charts or a balance of risk statement, policy makers are unlikely to remain inactive within the zone of defined price stability.

Boyle (1977) pioneered Monte Carlo for option pricing, identifying its key advantage as being flexibility. Monte Carlo is sufficiently adaptable to simulate a diverse range of stochastic processes. The simulated paths that are mapped out in Figures 1.3a - 1.3g provide a valuable optic when comparisons with the Bank of England's fan charts are made. If risk neutrality can be established, the value of any security paying some amount, at date, T, can be discounted using the risk free rate r. Calculating the time value of an instrument using Monte Carlo, implies simulating a sample of values for the random variable, π and forming the expectation by averaging the sample values. The sample generated should be representative of the population; otherwise Monte Carlo fails to provide an accurate estimation. The simplest or most vanilla option value that might be ascribed to inflation is European.[92] Before fully implementing the option's framework, it is worth comparing a sample of 50 simulated paths generated by (1.48). Each path is divided into discrete time steps so that:

$$\Delta t = T / m$$

where the time period T = 2 years and m = 24. π_t is the initial inflation rate set here arbitrarily at 2%. With each successive j a new inflation rate emerges until the terminal value is reached producing $j = m$:

$$E(\pi_T) = \pi_t e^{\sum_{j=1}^{m} \left(r_j - \frac{\sigma_j^2}{2} \right) \Delta t + \sigma_j \sqrt{\Delta t} z_j} \tag{1.50}$$

The level of growth r is deterministically set here at 5% for the simulation. This was close to the average one-year yield on Treasury Bills over the Greenspan period. d is set equal to zero.[93] σ is assumed to be constant at 25%. Constant volatility is consistent with the Black-Scholes model. Later, this assumption is relaxed, permitting central bankers to take into account volatility changing. The volatility does not change here going from one time step, j to the next. This is relaxed when the Monte Carlo Heston model is applied.[94] The magnitude of 25% represented the lower range of annual volatility estimated by a recursive GARCH(1,1) forecast for the Greenspan tenure.[95] (1.50) by itself, generates only a single path for the terminal inflation level. This would yield a single $E(\pi_T)$, at year 2. A monetary authority that responded only to values of $E(\pi_T)$ in excess of k can be interpreted as reacting to a call option where:

$$c = e^{-r(T-t)} \hat{E}[max(\pi_T - k, 0)] = e^{-r(T-t)} max\left[\pi_t e^{\sum_{j=1}^{m} \left(r_j - \frac{\sigma_j^2}{2} \right) \Delta t + \sigma_j \sqrt{\Delta t} z_j} - k, 0 \right] \tag{1.51}$$

To generate a sample that more closely corresponds to the population, it is necessary to consider many more paths. A simulation generated using n = 50 paths, would generalise to give:

[92] See Hull (1996) or McDonald (2003) for an explanation of this term.
[93] When d is set equal to r, the Black Scholes model time values are equivalent to the Black model time values.
[94] Likewise a Heston (1993) type model is developed to take account of changing volatility.
[95] See chapter 2 for the GARCH appendix A.2.3. σ denotes the standard deviation of the return on inflation. In this instance, the volatility is given as being constant.

$$c = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} max\left[\pi_{t,i} e^{\sum_{j=1}^{m}\left(r_{j,i}-\frac{\sigma_{j\,i}^{2}}{2}\right)\Delta t+\sigma_{j\,i}\sqrt{\Delta t}z_{j\,i}} - k, 0\right] \qquad (1.52)$$

Each *i* denotes an individual terminal value of expected inflation. A limited sample of 50 terminal values would not consistently generate accurate time values.[96] Nevertheless, this small sample is useful for the purposes of illustration. Only the terminal values in excess of *k* are used to calculate (1.52). When the terminal values are less than *k,* from a policy perspective, this would imply that the terminal inflation rate is within the band of tolerance. Terminal values less than *k* are arbitrarily put to zero by virtue that they elicit no monetary policy response. The mean of the 50 is discounted using *r*.[97] Terminal values of inflation less than *k* would produce a negative difference. However, the effect of imposing always a value of zero or otherwise the positive difference, implies that the time value equivalent to a call emerges. Central banks that respond to that proportion of expected inflation outcomes which are in excess of *k,* implicitly are responding to the value of a conventional call option. To see this, and the link to setting out confidence intervals, compare the simulated paths and the fan chart in Figure 1.3a. The former generates 50 separate paths as described by (1.50). The latter is a Bank of England fan chart as reported in May, 2005. The dotted line in the fan chart identifies the two-year horizon forecast with confidence intervals that cover 90% of expected inflation outcomes. If we remove the five most extreme observations from the sample paths in Figure 1.3a, it would appear that both graphs are projecting outcomes that are roughly in the same range. The dark red interval, of course, meanders and this reflects the best judgements made by members of the MPC. If *k* is set equal to 3%, the value of *c* calculated using (1.52) is found to be equal to 0.056%. This calculation can be performed manually taking the terminal values in Tables 1.1. From a policy perspective this implies that even if the initial inflation is well below the upper bound of 3%, policy would not necessarily remain inactive, although it is clear the magnitude of reaction could be tiny. While quantitatively small (and likely incorrect) given the small sample size, the positive magnitude of 0.056% and subsequent estimates with larger sample sizes, (reported in Table 1.4), provide results consistent with Proposition 1.4.

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[96] See Table 1.1 and Figure 3.1a.

[97] This is assuming risk neutral conditions hold. In Appendix A.2.1, chapter 2, it will be argued that the discount term, *r*, is not required. A 'zero time decay' approach could also be adopted.

Proposition 1.5: A*s the number of sample paths for the lognormal random walk simulation increase, the Monte Carlo time valuations converge to the Black-Scholes (or Black) time values when option pricing. The discrete-time model converges to the continuous-time Black-Scholes model valuations.*

Using 50,000 paths instead of 50 and the same parameters (i.e. $\pi_t = 0.02$, $k = 0.03$, $r = 0.05$, $\sigma = 0.25$, $T = 2$, $n = 50{,}000$) a more computationally intensive estimation for (1.52) produces a value of c equal to 0.0875%. By increasing the number of simulations, accuracy increases. Using the Black and Scholes (1973) formula to calculate c, the same result of 0.0875% emerges. In effect, as the number of simulations multiplies, the value of the Monte Carlo estimation converges with Black-Scholes. In this regard, the parsimoniously specified lognormal random process (1.50) computed intensively using 50,000 simulations provides the same quantitative results as the Black Scholes model, supporting proposition 1.5. The discrete model provides estimates of the call's time value that are consistent with a continuous time model. Figure 1.3b traces out 50 inflation paths using the same parameter values but starting with the initial inflation value of 3%. The values of these paths and associated time periods are reported in Table 1.2. In responding to inflation opportunistically or otherwise targeting a zone, the relevance of the k magnitude relative to the current inflation expectation is enormously significant. This will determine the likelihood of policy responding to expected inflation outcomes, as one might also expect with Black-Scholes. It is clear from Figures 1.3b, c and d that as k increases, the proportion of outcomes, (i.e. terminal values), that fall beyond the zone of tolerance declines. From a central banker's perspective: the probability of monetary policy responding preemptively would seem to decline as a central banker's tolerance for inflation increases. Using (1.52), the time values fall, as k increases.

Figures 1.3e, 1.3f and 1.3g each illustrate 50 sample paths of simulated inflation where three volatility measures are used, $\sigma_1 = 0.25$, $\sigma_2 = 0.35$ and $\sigma_3 = 0.15$. The parameter values are otherwise the same as before. Similar to the fan chart construction, it is easily observed that as volatility increases the dispersion of terminal values for inflation also increases. Using the sample paths from Table 1.3, it is possible to calculate time values for a one-year and two-year forecasting horizon. It is found that raising the level of volatility and forecasting horizon engineers a greater likelihood of breaching the upper bound, k. Using (1.52), it is clear that increasing volatility, σ, from 0.15 to 0.25 to 0.35, produces successively a higher time valuation. Likewise, as the forecasting horizon is extended the resultant greater inflation dispersion precipitates marginally higher time valuations. These observations would seem consistent from the fan chart construction where increasing values for uncertainty, σ, and extending the forecasting horizon have the effect of ultimately widening the confidence intervals, *ceteris paribus*. One drawback associated with using samples of just 50 simulated inflation paths relates to the possibility that the sample does not reflect fully the population. The variance of time values generated by (1.52) can be reduced by increasing the number of simulated paths. One way to gauge the level of accuracy is to compare the Monte Carlo calculation directly against the Black-Scholes or Black model (1.25). Consider the parameter inputs: $\pi_t = 0.03$, $k = 0.03$, $r = 0.05$, $\sigma = 0.25$ and $T = 2$ years. These are the same as those used for the first time value calculation of Table 1.3. Figure 1.4 demonstrates that as the number of simulations multiplies the Monte Carlo valuation converges increasingly to Black-Scholes. By increasing the number of simulations to 50,000 the level of variance is seen to fall and accuracy improves.

Table 1.4 reports call time values using both Monte Carlo and the Black-Scholes (or the Black) formula. The underlying expected inflation spanned the range of 0.25% to 8% in intervals of 0.25%. The Black formula provided identical time valuations as the Black-Scholes formula where the dividend yield was set equal to the risk free rate. Each is generated with the Black-Scholes parameter values where: π_t = 0.25% to 8%, $k = 0.45$, $T = 1$, $r = 0.05$, $d = 0.05$, and $\sigma = 0.25$ (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.25% to 8%, $k = 0.045$, $T = 1$, $r = 0.05$ and $\sigma = 0.25$). The Monte Carlo simulation employed 50,000 simulations using the Black-Scholes parameters. Figures 1.5a and 1.5b trace out the call time value parabolas where the time horizons are respectively 12 months and 24 months. This implies the simulations used respectively 12 time steps and 24 time steps which were, in turn, iterated 50,000 times. Only minor differences are observed between Monte Carlo and Black-Scholes (or Black) valuations. This suggests that convergence between the Black-Scholes model and

Monte Carlo is quite robust with 50,000 simulated inflation paths. A key point to note relates to the intrinsic value of the option and the time value. Under conditions of certainty, policy makers respond to expected inflation only when it exceeds the upper bound i.e. the policy response mimics the intrinsic valuation. If Central Bankers realise that inflation outcomes can differ from their original forecast, then policy makers will not remain unresponsive to expected inflation even when expected inflation is below the upper bound of a zone targeting framework. Using an option's framework, the time value is positive even when expected inflation remains below its exercise implying that *Proposition 1.4* is best understood from a risk management perspective.

The Monte Carlo approach discretises the diffusion process defined by (1.21). Significantly, simulated discrete (monthly) observations of expected inflation can be used to implement Black-Scholes equivalent time valuations. That is, option valuations can be performed without a strict adherence to Geometric Brownian motion. (A key conclusion provided by Derman and Taleb (2005)). To implement Black-Scholes, it is sufficient to assume that the underlying adheres to a lognormal random walk. Later in chapter 2, using the Vasicek model, the option's framework is adapted to incorporate mean reversion. This model extends the option's framework to a policy context where central bankers can predict inflation outcomes better than a coin toss. It also provides a means to endogenise the drift term.[98] In this chapter, a Heston Monte Carlo model is also employed to extend Black-Scholes so as to incorporate stochastic volatility. This, in turn, provides a means by which to account for the likely effects of varying levels of skew and kurtosis. A key disadvantage to using Monte Carlo relates to it being computationally expensive. The Black and Black-Scholes models conversely are convenient in that they provide instantaneous output and this, in part, explains the popularity of these models when pricing generally.[99] The most established segment of interest rate options includes over-the-

[98] In chapter 2, it is shown that the option's framework can be developed so as to take into account the effect of policy on inflation. Mean reversion models would also imply that the risk of hyperinflation is minimised.

[99] This author contends that the Black-Scholes and Black models offer central bankers analytical tools to interpret inflation risk, particularly where it is difficult to implement reliable forecasting models and where central bankers are forced to consider counterfactually what would happen in absence of appropriate monetary policy.

counter caps/floors and swaptions and bond options.[100] The standard market models for valuing these instruments are versions of Black's (1976) model. This model was initially used for valuing options on commodity futures, but has evolved to have many other applications. When Black's model is used to value a caplet (one constituent of an interest rate cap), the underlying interest rate is assumed to conform to Geometric Brownian motion. Traders, nevertheless, have come to be comfortable with these assumptions, in large measure, because the Black, or more correctly Black-Scholes construction, is often used as an interpolation tool. From widely available broker quotes of implied volatilities a trader can verify that an option is valued in line with market prices of other actively traded instruments, i.e. the option is smile-consistent. This particular application of the Black-Scholes model helps explain the conventional heavy emphasis on volatility surfaces. Of course, inflation options to date are not traded with the same liquidity. Calibration may be difficult by virtue that markets are not extremely liquid. Backus, Foresi and Wu (2004) identify that systematic biases which occur in Black-Scholes can be accounted for by incorporating deviations from the lognormal distribution model. This could be important in a risk management context particularly when inflation targeting central bankers provide explicit estimates of volatility and skew. This is often formalised in the form of a fan chart that is periodically published. In this regard, it is worth considering pricing models that take into account the likely views of central bankers in terms of not just the first and second moments but also take into account the skew and kurtosis of expected inflation.

1.7.3 Adjustments for stochastic volatility

Stochastic Volatility models help describe in a self-consistent way how options, possessing varying exercises, have different implied Black-Scholes volatilities. The volatility smile often observed in option markets, reflects departures of the underlying security from the lognormal distribution. Similarly, a key feature of conventional fan charts, as published by central banks, relates to the direction of risk, i.e. whether expected inflation is negatively or positively skewed. In what follows, the effects of

[100] According to Deacon, Derry and Mirfendereski (2004) inflation derivatives contracts have developed in a similar fashion to interest rate derivative contracts.

skew and kurtosis are examined using the Heston model.[101] Monetary policy decisions examined using the option's framework can incorporate varying levels of skew and kurtosis by allowing the assumption of constant volatility to be violated.

Kruse (2007) constructed several closed-form solutions using Heston (1993) to price inflation options where the underlying was the Consumer Price Index. Here rather than using the Consumer Price Index as the underlying, the actual inflation rate is used as the underlying.[102] The approach adopted here also relies on Monte Carlo simulation.[103] Gatheral (2006) maintains that stochastic volatility models such as Heston (1993) are employed widely to price options. A key advantage to using Heston (1993) relates to its ability to incorporate stochastic behaviour that can be associated with varying measures of skew and kurtosis. This implies that the options framework developed here can be made robust to violations of the lognormal distribution. In effect, upside and downside risks and fat tails can be incorporated into the analysis. The Black-Scholes model assumes volatility to be constant. In contrast, the Heston model, which is a generalisation of Black-Scholes, specifies the underlying security's volatility to be a random process. This process is dictated by state variables such as the price level of the underlying, the tendency of volatility to mean revert, and the volatility of the variance process itself. Stochastic volatility models address many of the simplifications associated with classic option pricing models that assume the underlying security's volatility to be constant up to expiration. Black-Scholes type models are conventionally understood to produce anomalies such as volatility smiles and surfaces. The Black model which is referred to as the 'market model' for pricing interest rate options is commonly reversed engineered so that quotes are given by brokers in terms of implied volatility. By assuming that the volatility of the underlying price is a random process rather than constant, it becomes possible to widen the analysis to incorporate asymmetries that are not strictly lognormal. This leads to the following propositions:

[101] Two motivations are advanced for undertaking such an exercise. (1) The Bank of England formally considers distributions that have varying population moments. (2) The option's framework is sufficiently flexible to accommodate differently specified distributions.

[102] By setting inflation as being the underlying in this extended Black-Scholes framework, the Heston model is useful for developing a number of risk management insights that permit skew and kurtosis. This approach also preserves the non-negativity of inflation consistent with Proposition 1.3. This is equivalent to assuming that central bankers feel confident that they could easily reverse deflation yet do not dispel the prospect of hyperinflation.

[103] Time values are verified against a closed-form solution.

Proposition 1.6a*: Departures of expected inflation from lognormality can be captured within the option's framework. Monetary policy decisions that apply a risk management paradigm can incorporate varying levels of skew and kurtosis into forecasts by allowing assumption (A.6) to be violated.*

Proposition 1.6b**:** *By permitting volatility to be stochastic, it is possible to capture the effects of altering the higher moments of expected inflation. The option's framework, as applied to monetary policy, can account for a wide range of distributional asymmetries. Upside and downside risks, as outlined in Figures 1.8a – 1.8h, can be represented using Heston Inflation Fan charts. The effects of these asymmetries on preemptive zone targeting can be made explicit and do not prejudice* Proposition 1.4.

A number of central banks target expected inflation identifying explicit forecast measures of volatility and skew. Departures from the Gaussian distributions constitute part of the formal analysis now regularly reported by a number of central banks.[104] To understand how this risk analysis may influence policy, option pricing models would appear to offer a variety of approaches to link rate decisions to asymmetric forecast distributions.[105] The techniques developed to make pricing smile-consistent and which focus heavily on biases that are understood to exist for the Black-Scholes time values, would appear also potentially useful for appraising the effects of skew and kurtosis of expected inflation on monetary policy. The Heston (1993) model permits many probability distributions, dependent on the assumed magnitude of parameter values. Heston (1993) adapts the lognormal model to take account of stochastic volatility. The lognormal model, consistent with Black-Scholes, was given before as:

$$E(S_T) = S_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} \tag{1.53}$$

To make volatility stochastic it will be necessary to permit volatility to change between time steps. As before, it is possible to calculate the terminal value of each inflation path using the simulation:

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[104] The Bank of England publishes five numerical parameters related to the annual rate of CPI inflation (mode, median, mean, uncertainty and skew) for the MPC's projected probability distributions. Fan charts are now regarded as a standard feature for communicating policy.

[105] Upside and downside inflation risk.

$$E(\pi_T) = \pi_t e^{\left(r-d-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-t}z} \qquad (1.54)$$

Expressed as the log difference of inflation in discrete time, the simulation path could be generated from either Black-Scholes or Heston (1993) that share the same stochastic process:

$$d\ log\ \pi = \left(r - d - \frac{\sigma^2}{2}\right)dt + \sigma\sqrt{dt}\,z \qquad (1.55)$$

Heston's result can also take account of the presence of a constant dividend yield, *d*.[106] The Heston (1993) model extends the Black-Scholes model so that volatility, σ is allowed to adjust. It evolves through time implying that, the volatility measure $\sigma(t)$ fluctuates so that:

$$dv(t) = \kappa[\theta - v(t)]dt + \gamma\sqrt{v(t)}\sqrt{dt}\,z^* \qquad (1.56)$$

where

$$\sigma(t) = \sqrt{v(t)} \quad \text{and} \quad z = z_1,\ z^* = \rho z_1 + \sqrt{1-\rho^2}\,z_2$$

The random variables $z_1 \sim N(0,1)$ and $z_2 \sim N(0,1)$ are standard normal. z^* and z have a constant correlation, ρ. The variance process $v(t)$, given by (1.56) is similar to a Cox, Ingersoll and Ross (1985) process. $\kappa > 0$, $\theta > 0$ and $\gamma > 0$. κ is a mean reversion parameter. θ is the long run mean of the variance. γ is the volatility of the variance. The mean reversion term $\kappa(\theta - v)$ will be negative when $v > \theta$ and positive when $v < \theta$. The variance, $v(t)$, will tend to migrate towards θ. The speed of this migration is dictated by κ.[107] The lower the value of κ, the longer a given shock to volatility is likely to persist. The correlation ρ between shocks to the inflation level and shocks to the variance can be negative or positive. A positive value for ρ would precipitate positive skewness in the underlying inflation distribution, implying that a positive shock to inflation will help precipitate a positive shock to its variance. A positive inflation shock, induces an increase in the variance; predisposing inflation to increase even more. From a policy perspective, ρ is useful in that it can be manipulated so that option valuations can be investigated for varying levels of skew. The fan chart construction, set out by the Bank of England, identifies levels of skew and volatility

[106] When *r* is set equal to *d*, this produces a zero drift.
[107] To see the link between the Heston model and GARCH fat-tailed distributions, see Back (2005) and Hull (2003).

for varying horizons, implying inflation risks can be to the upside or downside. The Heston (1993) model extends the Black-Scholes model so that these asymmetric types of risk can be examined within the options framework.

The Heston (1993) model is popular amongst market practitioners, in part, because it represents a generalisation of the well-understood Black-Scholes (1973) model. To apply Monte Carlo valuation, the same framework as before is developed. The key innovation relates to updating volatility each successive period. Volatility is adjusted at each time step over a horizon of 24 months. To implement the simulation it is necessary to discretise the diffusion processes given by (1.55) and (1.56), so that:

$$log\, \pi(t_{i+1}) = log\, \pi(t_i) + \left(r - d - \frac{\sigma(t_i)^2}{2} \right)\Delta t + \sqrt{v(t_i)}\sqrt{\Delta t}\, z_1 \tag{1.57}$$

$$v(t_{i+1}) = v(t_i) + \kappa[\theta - v(t_i)]\Delta t + \gamma\sqrt{v(t_i)}\sqrt{\Delta t}\left(\rho z_1 + \sqrt{1-\rho^2}\, z_2\right) \tag{1.58}$$

Here the random shock, z_1, influences both the underlying, π and the variance, v. z_2 only influences v. Taking the natural logarithm of (1.53) produces an equivalent form, except here volatility changes going from one period to the next. $z_2 \sim N(0,1)$ creates the possibility of a negative variance process. To ensure that (1.58) remains non-negative, practitioners often adopt the absorbing assumption that: if $v < 0$ then $v = 0$. This leads to the following condition being imposed:

$$v(t_{i+1}) = max\left\lfloor v(t_i) + \kappa[\theta - v(t_i)]\Delta t + \gamma\sqrt{v(t_i)}\sqrt{\Delta t}\left(\rho z_1 + \sqrt{1-\rho^2}\, z_2\right), 0\right\rfloor \tag{1.59}$$

This implies that the simulation never incorporates a negative variance. To calculate the time value of a call, it is necessary to calculate a large number of inflation paths, n, each possessing 24 time steps, i.e. m = 24. The number of iterations, n, is set at 50,000. The value of the call can be calculated by averaging the terminal values in the usual way:[108]

$$c = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} max\left[\pi_{t,i} e^{\sum_{j=1}^{m}\left(r_{j,i} - \frac{\sigma_{j,i}^2}{2}\right)\Delta t + \sigma_{j,i}\sqrt{\Delta t}\, z_{1(j,i)}} - k, 0 \right] \tag{1.52a}$$

As will be observed later, relatively small differences ultimately emerge between Black-Scholes and many of the Heston time valuations in Tables 1.5a and 1.5b. This is true for a variety of parameter inputs presented in the sensitivity analysis proposed

[108] The call time values are also verified against a closed form solution.

in Tables 1.6 and 1.7. Of course, in a monetary policy context the appropriate parameter values to select for Heston time valuations depend on how central bankers view the future. This may not be directly observable, as historical data points and *ex ante* impressions can be very different.[109] In other words, a predominantly positive skew in historical inflation rates should not preclude the existence of negatively skewed inflation forecasts. Historic values for these variables only offer a starting point. Some sensitivity analysis is useful to investigate the effects of altering γ and ρ on the generated moments of expected inflation. By altering γ and ρ, varying Heston Fan Charts can be generated. Figures 1.8a - 1.8h present both the histograms associated with terminal values of expected inflation and the full two year fan charts for a variety of proposed distributions. Tables 1.6 and 1.7 use Monte Carlo simulated paths to calculate descriptive statistics for a range of selected parameter values. It is possible to recover the generated moments from distinctive distributions generated from each of the Heston models and make comparisons against past inflation data.

To implement the Heston model, it is worth noting that to increase kurtosis, (the likelihood of more extreme events), higher values of γ are used. Equally, to reduce positive skew, smaller values of ρ are necessary. As a baseline and consistent with Heston (1993), the value of ρ examined here will be given initially as both positive and negative 0.5 and zero. See Figures 1.6a and 1.6b. It is demonstrated in Figure 1.6c that as the value of ρ approaches zero, the Heston time valuation gets closer to Black-Scholes time valuation.[110] To make the analysis comparable to Black-Scholes, the variance terms v and θ are approximately given by squaring σ_{BS}; the Black-Scholes volatility.[111] This implies that v and θ are given, at first, as 0.0625. In Tables 1.5b and 1.7 a variance of 0.1225 is considered.[112] κ represents the speed with which variance reverts to its long term mean. For policy analysis, it is worth imposing a value that is low. Persistence in the variance implies that central bankers cannot depend on accelerating inflation to immediately correct of its own accord, once deviations from the long-term mean occur. Imposing a low value for κ implies that without policy

[109] This would be less true for the Bank of England where the fan chart makes explicit a number of parameters including the mean, uncertainty and skew of forecasted inflation.
[110] The degree of convergence also depends on the magnitude of gamma. See Table 1.5a and 1.5b.
[111] The volatility and variances are roughly matched for the Black-Scholes and Heston models.
[112] This is equivalent to a Black-Scholes volatility p.a. of 0.35.

intervention, deviations from the long term variance, θ,□ are reversed but only slowly. Again, from a central bank risk perspective, this prudential specification seems sensible in that inflation volatility would not be quickly contained, left to its own devices. By setting κ to a mean reversion speed of 0.01, policy makers are accepting that shocks can persist, although not indefinitely. Further analysis, of course could be used to examine this counterfactually and simulate paths with faster adjustment.

The U.S. Consumer Price Index (Series i.d.: CPIAUCSL) was downloaded from the Federal Reserve Bank of St. Louis to calculate the volatility of inflation return variance, γ, over the period 1958:7 to 2007:5, which was slightly less than 0.1.[113] By lowering or increasing this magnitude, the likely effect of kurtosis on policy reactions can be gauged in Tables 1.5a and 1.5b. Figures 1.6d and 1.6e both illustrate that *ceteris paribus*, an increase in kurtosis produces a greater divergence from Black Scholes.[114] Parameter values for γ and ρ however are probably best considered jointly. A key advantage to using Monte Carlo simulation relates to it being capable of generating a sample estimate for skew and kurtosis for the underlying inflation paths when values for γ and ρ have already been selected. Equally, fan charts can be constructed. By altering γ and ρ, Heston simulation permits additional analysis to verify the effects on the first, second, third and fourth moments of the distribution. In using Heston Monte Carlo, it is possible to transpose parameters inputs into distributional outcomes for both time horizons of one year and two years. Tables 1.6 indicates that the ranges of generated skew and kurtosis implied by ρ and γ in models 1.2(T.1) and 1.3(T.1) seem sufficient to reflect the actual historical norm over the period 1989:12 – 2007:5. Tables 1.6 and 1.7 outline these descriptive statistics generated using a varyingly specified Heston model. These models with differing combinations of parameters are classified as model 1.1, model 1.2….model 1.8 and as model 2.1, model 2.2….model 2.8. Descriptive statistics are calculated from the 1-year (T.1) and 2-year (T.2) forecasts of the simulations where the initial value is set at π = 3%. These descriptive statistics are referred to as the generated moments. Histograms for the terminal values for model 1 are mapped out in Figures 1.8a to

[113] The GARCH appendix in chapter 2 uses quarterly data. Here monthly data was used.
[114] Assuming that under uncertainty the Black/Black-Scholes model captures policy responses to inflation breaching an upper bound.

1.8h. Inflation Fan Charts are included with confidence intervals estimated from the Monte Carlo forecasts stretching over 8 quarters. These are useful for considering varying types of risk and asymmetry and the quarterly observations were constructed from monthly time steps. From Table 1.6 and Figures 1.8 a – h, it is possible to verify the effect of changing ρ and γ over varying forecasting horizons. Tables 1.6 and 1.7 provide some sensitivity analysis in particular for changes in ρ and γ. The moments which are generated using Heston simulation, can be compared against the historical measures of mean inflation, standard deviation, skew and kurtosis. Three periods are selected: 1958:7 - 2007:5, 1958:7 - 1979:8 and 1989:12 - 2007.5. Respectively, these are the full sample period, the pre-Volcker period and the 'risk management / opportunistic' period of Greenspan-Bernanke. The starting date of 1989:12 is used because it coincides with an important debate sponsored by chairman Greenspan relating to a proposed implementation of an inflation target. This theme is further developed in chapter 2.[115] The debate that ensued constituted a seminal discussion of the 'opportunistic' approach, although the early part of this period predates the actual use of this term by central bankers.[116] All three periods display a good deal of variation in terms of the moments implying that the selection of parameter inputs for the Heston model can be largely an exercise in judgement when considering monetary policy responses.[117] The parameter inputs of the Heston model (1993) can be selected to produce an enormous range of probability distributions. Using market data is also possible but somewhat problematic, by virtue that the calibration of parameters would be limited by the absence of a liquid inflation options market.[118] To get around this the Backus, Foresi and Wu (2004) model is considered below. This permits the incorporation of historical measures of skew and kurtosis directly into a modified Black-Scholes option's framework.[119] In contrast, the Heston parameters used here are intuited by working in reverse.

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[115] At the 1989 FOMC December meeting chairman Greenspan posed the question: 'are we looking for zero inflation or are we willing to accept, say 4-½ percent?' FOMC Transcripts, Federal Reserve Board, 1989, p.28.

[116] Kohn however dates the genesis of the opportunistic approach back to the Volcker period. The term 'opportunistic disinflation' was only popularised in the mid 1990s.

[117] Indeed the fan charts as set out by the Bank of England are based on the MPC's best collective judgement about the most likely paths for inflation, and the assumed uncertainties surrounding those central projections. Not all members will even agree on each of these assumptions.

[118] With financial market innovation, this may be less the case in the future.

[119] The Backus, Foresi and Wu (2004) model however uses the moments of inflation return.

Table 1.5a and 1.5b report time values using Heston simulation and the parameter values associated with Heston models 1.1 – 1.8(T.2) and Heston models 2.1 – 2.8 (T.2) respectively. The parameter input values for these models can be obtained in Tables 1.6 and 1.7. Black-Scholes closed-form estimates of the time values are also given for inflation varying from 0.25% to 8%, in intervals of 0.25%. The Black-Scholes parameter values are: π_t = 0.25% to 8%, k = 0.03, T = 2, r = 0.05, d = 0.05, and σ = 0.25 for Table 1.6 and σ = 0.35 for Table 1.7, (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.25% to 8%, k = 0.03, T = 2, r = 0.05 and (σ= 0.25 for Table 1.6 and σ = 0.35 for Table 1.7). Significantly, relatively small differences seem to apply when the time valuations are compared directly, for the full range of Heston and the Black-Scholes models. Figures 1.6a and 1.6b map out these differences, (i.e. the Heston time values minus the Black-Scholes time value), as estimated in Heston Model 1.2 (T.2) and Model 1.3 (T.2) respectively. It is apparent that as the correlation, ρ, becomes increasingly negative (i.e. as the left tail of the expected inflation distribution thickens), the Heston Monte Carlo simulation produces time values that fall relative to the Black-Scholes valuation when the call option is out-of-the-money. The Heston call time values increase vis-à-vis the Black-Scholes call time value when the option is generally in-the-money. These differences from the Black-Scholes model can be modest. When all the models given in Table 1.5a and 1.5b are compared, it would appear from Figures 1.6d and 1.6e that the differences do not exceed 8 basis points. The magnitudes of these discrepancies suggest that by altering the third and fourth moments that the effects on monetary policy would appear to be marginal, (although cumulatively this may be less true).[120] In this regard, the mean value of the distribution would seem to have a greater effect on the valuation than the third or fourth moment of the distribution.[121]

Tables 1.5 a and b, illustrate that the Heston models 1.2(T.2) and model 1.3(T.2) produce broadly similar call time values to Black-Scholes when ρ is given as 0.5 or -

[120] The discrepancies that exist between Black-Scholes and the selected Heston model time values would appear to be, on the whole, inferior to 8 basis points. The monetary policy response discrepancies mapped out by (1.41) for the varying models may well be small. This would suggest that the Black-Scholes framework represents a relatively robust baseline however for larger absolute values for skew and kurtosis greater discrepancies arise. These discrepancies may also be more apparent when a collar portfolio is considered.
[121] See Tables 1.6 and 1.7.

0.5 and γ is equal to 0.1.[122] Table 1.6 illustrates that for parameter values given by Model 1.2 and Model 1.3, the generated moments for the one-year inflation forecast reflect the historic moments over the period 1989:12 – 2007:5. Figures 1.6a and 1.6b trace out the differences in time values as estimated for these models. At most, the discrepancies between Black-Scholes and Heston time values are small, 1.5 to 2 basis points. By comparison, policy interest rate changes typically are in the order of 25 to 50 basis points. This would suggest that over the 1989:12 – 2007:5 period, the Black-Scholes model constitutes a relatively robust baseline when considering monetary policy responses. Of course, *ex ante* estimates of skewness and kurtosis may not always reflect the historical norm. In addition, the higher long-term historical measures of the moments may have coloured the thinking of policy makers, implying that upside and downside risks may have been more prominent in the minds of central bankers. Statements regarding upside and downside risks are probably most important because of their anticipated effects on the mean level of expected inflation. Table 1.5a provides estimates of Black-Scholes time values and the Heston Model 1.8(T.2). It is clear that there is an increase in the divergence between Black-Scholes and the Heston model; when the absolute values of γ and ρ increase. From Figures 1.7a, the differences appear to be small. Figures 1.7a maps out time values for the Black-Scholes model and the Heston model using the parameter inputs of Model 1.8(T.2). The discrepancies between the two time value parabolas are found to be small although more substantial than those with smaller absolute values of skew. The Heston inflation paths and descriptive statistics reported in Table 1.6 for model 1.8(T.1) suggest that even when the generated kurtosis and skew are more negative vis-à-vis the historic norm for the period 1989:12 - 2007:5 and the full sample period, the Heston model, which corrects bias for skewness and kurtosis, appears to provide time values that are quantitatively similar to the Black-Scholes model. This would suggest that Proposition 1.4 is robust to departures from lognormality.

1.7.4 Using empirical estimates of skewness and kurtosis

In setting out a risk management paradigm, the sensitivity analysis of Tables 1.6 and 1.7 would suggest that incorporating skewness and kurtosis would not seem to compromise materially Propositions 1.3 and 1.4, when understanding the effect of

[122] The time values graphs for models 1.2(T.2) and 1.3(T.2) were not included because visually the Black-Scholes and Heston parabolas are not easily distinguishable for standard scaling.

uncertainty on rate decisions. In so much as historical estimates of skew and kurtosis can inform us of how central bankers can perceive upside and downside risks to inflation, a more direct approach relative to Heston (1993) was outlined by Backus, Foresi and Wu (2004).[123] The latter however implement estimations using the one period return: $x_{t+1} = logS_{t+1} - logS_t$ where S_t is the value of the underlying security. The skewness and kurtosis can be defined in terms of cumulants, κ_j of a random variable x. Backus, Foresi and Wu (2004) use the following: the first moment is $\kappa_1 = E(x)$, the second moment is $\kappa_2 = E(x-\kappa_1)^2$, the third moment is $\kappa_3 = E(x-\kappa_1)^3$ and the fourth is $\kappa_4 = E(x-\kappa_1)^4 - 3(\kappa_2)^2$. If the returns are i.i.d., then the T-period return has the mean and variance given by:

$$\mu_T = T\mu$$

and

$$\sigma_T^2 = T\sigma^2$$

Backus, Foresi and Wu (2004) set out the one-period skewness and kurtosis as:

$$\gamma_1 = E\left[\left(\frac{x_{t+1}-\mu}{\sigma}\right)^3\right] = \frac{\kappa_3}{\kappa_2^{3/2}} \qquad (1.60)$$

and

$$\gamma_2 = E\left[\left(\frac{x_{t+1}-\mu}{\sigma}\right)^4\right] - 3 = \frac{\kappa_4}{(\kappa_2)^2} \qquad (1.61)$$

The T-period skewness and kurtosis are given respectively by:

$$\gamma_{1T} = \gamma_1 / \sqrt{T} \qquad (1.62)$$

and

$$\gamma_{2T} = \gamma_2 / T \qquad (1.63)$$

Using a Gram-Charlier expansion up to the fourth order in the distribution of returns of the underlying security, Backus, Foresi and Wu (2004) show that a call option pricing formula can be given approximately as:

$$c \cong S_t e^{-d_{nt}^* T} N(d_1) - K e^{-r_{nt} T} N(d_2) + S_t e^{-d_{nt}^* T} N'(d_1)\sigma_T \left[\frac{\gamma_{1T}}{3!}(2\sigma_T - d_1) - \frac{\gamma_{2T}}{4!}\left(1 - d_1^2 + 3d_1\sigma_T - 3\sigma_T^2\right)\right] \qquad (1.64)$$

The notation here is the same as before. A key advantage of using the Backus, Foresi and Wu (2004) approach relates to being able to estimate directly historical measures

[123] Unlike Heston (1993), Backus, Foresi and Wu (2004) assume that volatility to be constant. The Backus, Foresi and Wu (2004) approach however is useful because it permits the direct incorporation of historical estimates of skewness and kurtosis into the calculation of option time values.

of skewness and kurtosis of inflation return and input these into the calculation of the option time value.

The risk free rate and dividend yield are quoted per period and are given respectively as r_{nt} and d^*_{nt}. The density of the standard normal distribution is given as $N'(d_1) = exp(-d_1^2/2)/(2\pi)^{0.5}$. d_1 and d_2 are identical to Black-Scholes, although these are implemented on a month-to-month period basis. Here the periodicity is taken as monthly, as this coincides with the release of CPI data. Using Haug (2007), (1.63) is generalised so that by setting the cost of carry equal to zero, (i.e. $r_{nt} - d^*_{nt} = 0$), the model can be applied directly to price options on futures (or expected values). Monthly CPI data was downloaded from the Federal Reserve Bank of St Louis (series id: *CPIAUCSL*). This was used to calculate inflation and monthly inflation return over the Greenspan incumbency 1987:9 – 2006:1. To implement (1.63), it is necessary to calculate a number of parameter inputs: the monthly volatility of inflation return, the monthly skewness of inflation return and the monthly kurtosis of inflation return. These are given respectively as the one-period volatility $\sigma = 0.1038$ (i.e. the annualised $\sigma_{BS} = 0.359574$), the one-period skewness, γ_1 was found to be equal to 0.632169 and the one-period kurtosis, γ_2 was found to be equal to 1.542505. Using these, a series of call values are calculated over the range of inflation 0.25% to 8% in intervals of 0.25%. To implement (1.63) and (1.25), the following Black-Scholes parameter values are used: π_t = 0.25% to 8%, $k = 0.03$, $T = 2$ (or 24 months), $r = 0$, $d = 0$, and $\sigma_{BS} = 0.359574$. The time values for both Black-Scholes and Backus, Foresi and Wu (2004) are mapped out in Figure 1.9a. Only minor discrepancies can be detected between the two. The difference is more clearly discernible in Figure 1.9b. It would appear that using historical measures of skewness and kurtosis, calculated over the Greenspan incumbency as a whole, produces relatively small differences that remain, on the whole, inferior to 1.5 basis points. More importantly, the key insight of Proposition 1.4, that policy remains activist when inflation remains below the upper target, is preserved.[124]

[124] The Greenspan chairmanship coincided with the Great moderation. The *ex-ante* estimated magnitudes of these moments were subject to change going one period to the next from 1987 to 2006. This would imply that larger differences, in-sample for the Greenspan tenure, were possible.

1.8. Conclusion - The Central Bank as a Risk Manager

Some historical perspectives concerning rules-based type policy were examined in this chapter. As already explained, a large swathe of the literature has extolled the virtues of committing to a given policy rule. Given the practical difficulties associated with adhering to a strict rule, policy makers are often obliged to implement a contingent rule. On the whole, policy makers publicly tend to describe policy from the perspective of a given strategy, but this can be complex. The Volcker-Greenspan incumbencies seem to fit the description of advocating rules but preserving room for 'wiggle'. Central bankers have appeared to be conflicted in terms of elaborating and implementing policy by virtue of internal dissension (committee dynamics) and by virtue of potentially external opposition (institutional dynamics). Opportunistic monetary policy has been described by Svensson (1999) as being opaque and *ad hoc*. It is asserted here that opportunistic policy attempted to reconcile some of the contradictory imperatives forced upon policy makers who manoeuvre between rules and discretion. Central bankers frequently resolve to implement a contingency rule. In the United States, this has assumed the form of subscribing to an evolving unannounced zone target or band.[125] As Figure 1.1a makes clear, the discretion that policy makers are accorded by using this framework, may be represented as the intrinsic payoff to an option. Central bankers, nevertheless, both at the Federal Reserve and Bank of England, have stressed that rate changes are designed to respond to *expected* inflation outcomes. Greenspan (2003, 2004) maintained that future outcomes are essentially unknown and monetary policy, as a consequence, is reliant on risk management.[126] The fan charts, as published by the Bank of England, go some way to provide formal measures of this uncertainty.[127] Monte Carlo inflation paths were used to simulate the effects of changing the value of different population moments and to construct inflation fan charts that were consistent with pricing options. Aksoy, Orphanides, Small, Wieland and Wilcox (2006) describe the opportunistic central banker as fighting inflation when inflation is high, but focuses

[125] Of course, opportunistic policy has never been formally endorsed by the Federal Reserve Board. This approach also provided policy makers with greater scope for discretion. Later, committee dynamics are used to rationalise zone targeting.
[126] The risk management strategies advocated by chairman Greenspan are interpreted here as strategies that respond to the uncertainty of future inflation outcomes.
[127] The Federal Reserve also has come some distance in offering a communication device to the public. Since November, 2007, the Federal Reserve has published an enhanced range of forecasts that are similar in substance to the fan charts.

on stabilising output when inflation is low. The implied policy rule is nonlinear, similar to the intrinsic payoff to a call. In a world that is forward looking, opportunistic central banks respond to the likelihood of expected inflation breaching an upper bound. The time valuation parabola is appropriate where policy makers are responsive to expected inflation even when inflation is below the upper bound. Consistent with Proposition 1.4, this implies that uncertainty would limit the extent to which policy, defined by the upper tolerance, can be discretionary. The payoff from a call option or from a portfolio of options constitutes a useful policy variable in construing nonlinear reaction function dynamics and the effects of uncertainty. This conceptual framework is developed more in chapter 2.

Portfolio option theory permits the risk management approach to monetary policy to be structured more formally. Options are priced varyingly as either intrinsic or time valuations depending on whether outcomes are certain or uncertain. Monetary policy, when described as responding to the certain positive difference in inflation over the upper bound *k*, may be represented as the intrinsic valuation: $c = E[\max(\pi_T - k, 0)]$. When inflation outcomes are unknown, then the policy variable is better described by (1.41). Using Black (1976) or Derman and Taleb (2005), it is possible to establish risk neutral conditions, although the latter would appear more robust. A ‘zero time decay’ approach could also be applied where central bankers are indifferent between inflation today and inflation in the future. The continuous time closed-form solutions and discrete Monte Carlo techniques are found to produce time values that converge for a large number of iterations. The lognormal random walk assumptions, associated with Black (1976), are initially used as a baseline to examine policy reactions in the absence of a reliable forecasting model. Both the lognormality and random walk assumptions however can be relaxed. The Black-Scholes framework is extended, using Heston (1993), to incorporate the effects of distributional asymmetries. This is useful where central bankers commonly refer to upside and downside risks. Importantly, these asymmetries can be made explicit in the option’s framework without prejudicing Proposition 1.4. Using Backus, Foresi and Wu (2004), it would appear that the estimated skewness and kurtosis for the Greenspan period failed to produce large discrepancies relative to the Black-Scholes model.

Time	Inf 1	Inf 2	Inf 3	Inf 3	Inf 4	Inf 5	Inf 6	Inf 7	Inf 8	Inf 9	Inf 10	Inf 11	Inf 12	Inf 13	Inf 14	Inf 15	Inf 16	Inf 17	Inf 18	Inf 19	Inf 20	Inf 21	Inf 22	Inf 23	Inf 24	Inf 25
0.00	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.08	0.019	0.021	0.024	0.019	0.021	0.020	0.021	0.024	0.019	0.021	0.019	0.018	0.019	0.019	0.020	0.019	0.018	0.021	0.020	0.018	0.020	0.019	0.019	0.020	0.021	0.024
0.17	0.017	0.021	0.025	0.017	0.021	0.021	0.021	0.025	0.017	0.021	0.022	0.017	0.018	0.020	0.020	0.019	0.017	0.020	0.022	0.021	0.020	0.020	0.019	0.019	0.023	0.023
0.25	0.016	0.021	0.025	0.016	0.020	0.026	0.021	0.025	0.016	0.020	0.020	0.016	0.018	0.021	0.021	0.017	0.015	0.020	0.023	0.022	0.020	0.020	0.018	0.022	0.022	0.021
0.33	0.016	0.021	0.027	0.016	0.022	0.023	0.021	0.027	0.016	0.022	0.019	0.015	0.017	0.022	0.023	0.017	0.016	0.019	0.023	0.023	0.021	0.021	0.017	0.022	0.022	0.022
0.42	0.016	0.020	0.026	0.016	0.023	0.023	0.020	0.026	0.016	0.023	0.019	0.012	0.019	0.024	0.024	0.016	0.016	0.022	0.024	0.023	0.022	0.019	0.018	0.018	0.022	0.019
0.50	0.017	0.021	0.024	0.017	0.023	0.021	0.021	0.024	0.017	0.023	0.019	0.012	0.017	0.024	0.025	0.017	0.016	0.020	0.021	0.019	0.024	0.020	0.017	0.018	0.023	0.018
0.58	0.020	0.018	0.027	0.020	0.023	0.021	0.018	0.027	0.020	0.023	0.018	0.012	0.017	0.024	0.030	0.016	0.015	0.021	0.022	0.018	0.024	0.022	0.017	0.018	0.024	0.019
0.67	0.020	0.019	0.027	0.020	0.021	0.020	0.019	0.027	0.020	0.021	0.018	0.012	0.016	0.022	0.029	0.018	0.015	0.020	0.023	0.017	0.024	0.021	0.019	0.018	0.022	0.017
0.75	0.019	0.020	0.028	0.019	0.021	0.020	0.020	0.028	0.019	0.021	0.017	0.013	0.018	0.022	0.029	0.018	0.016	0.018	0.023	0.017	0.026	0.024	0.018	0.020	0.021	0.016
0.83	0.018	0.021	0.029	0.018	0.022	0.020	0.021	0.029	0.018	0.022	0.017	0.012	0.019	0.022	0.029	0.017	0.017	0.017	0.022	0.017	0.024	0.022	0.020	0.022	0.021	0.015
0.92	0.017	0.019	0.028	0.017	0.024	0.023	0.019	0.028	0.017	0.024	0.017	0.011	0.018	0.025	0.030	0.017	0.017	0.016	0.021	0.015	0.023	0.026	0.022	0.020	0.021	0.015
1.00	0.017	0.018	0.032	0.017	0.026	0.024	0.018	0.032	0.017	0.026	0.016	0.011	0.018	0.023	0.028	0.018	0.018	0.015	0.020	0.015	0.022	0.026	0.022	0.019	0.020	0.015
1.08	0.017	0.020	0.032	0.017	0.023	0.025	0.020	0.032	0.017	0.023	0.018	0.011	0.018	0.025	0.028	0.019	0.019	0.018	0.021	0.015	0.021	0.027	0.022	0.016	0.020	0.015
1.17	0.017	0.021	0.030	0.017	0.023	0.024	0.021	0.030	0.017	0.023	0.019	0.011	0.016	0.026	0.028	0.020	0.017	0.018	0.019	0.016	0.017	0.030	0.023	0.015	0.020	0.014
1.25	0.017	0.021	0.029	0.017	0.025	0.026	0.021	0.029	0.017	0.025	0.019	0.010	0.014	0.025	0.030	0.018	0.018	0.018	0.019	0.016	0.018	0.030	0.025	0.014	0.021	0.015
1.33	0.017	0.024	0.030	0.017	0.025	0.028	0.024	0.030	0.017	0.025	0.018	0.011	0.014	0.024	0.030	0.020	0.019	0.020	0.020	0.015	0.020	0.029	0.024	0.013	0.018	0.015
1.42	0.017	0.026	0.033	0.017	0.026	0.027	0.026	0.033	0.017	0.026	0.017	0.012	0.014	0.023	0.030	0.019	0.016	0.019	0.021	0.017	0.019	0.031	0.023	0.016	0.018	0.014
1.50	0.016	0.023	0.031	0.016	0.026	0.030	0.023	0.031	0.016	0.026	0.018	0.012	0.014	0.022	0.030	0.021	0.016	0.019	0.021	0.016	0.020	0.028	0.022	0.017	0.017	0.014
1.58	0.015	0.026	0.034	0.015	0.026	0.030	0.026	0.034	0.015	0.026	0.018	0.011	0.013	0.027	0.029	0.019	0.016	0.019	0.024	0.016	0.019	0.028	0.022	0.017	0.020	0.015
1.67	0.016	0.025	0.035	0.016	0.026	0.028	0.025	0.035	0.016	0.026	0.019	0.012	0.013	0.025	0.029	0.017	0.015	0.018	0.025	0.017	0.021	0.029	0.020	0.018	0.022	0.015
1.75	0.014	0.025	0.038	0.014	0.025	0.031	0.025	0.038	0.014	0.025	0.019	0.011	0.013	0.022	0.029	0.018	0.018	0.018	0.026	0.016	0.022	0.028	0.021	0.017	0.026	0.016
1.83	0.014	0.027	0.040	0.014	0.025	0.031	0.027	0.040	0.014	0.025	0.020	0.011	0.012	0.021	0.030	0.018	0.019	0.017	0.029	0.016	0.021	0.027	0.019	0.016	0.028	0.016
1.92	0.015	0.024	0.039	0.015	0.024	0.036	0.024	0.039	0.015	0.024	0.018	0.011	0.011	0.021	0.033	0.018	0.019	0.017	0.031	0.015	0.019	0.027	0.018	0.014	0.028	0.016
2.00	0.015	0.024	0.039	0.015	0.021	0.031	0.024	0.039	0.015	0.021	0.017	0.011	0.011	0.023	0.032	0.016	0.019	0.019	0.031	0.016	0.019	0.027	0.019	0.015	0.030	0.018

Time	Inf 26	Inf 27	Inf 28	Inf 29	Inf 30	Inf 31	Inf 32	Inf 33	Inf 34	Inf 35	Inf 36	Inf 37	Inf 38	Inf 39	Inf 40	Inf 41	Inf 42	Inf 43	Inf 44	Inf 45	Inf 46	Inf 47	Inf 48	Inf 49	Inf 50
0.00	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.08	0.021	0.020	0.020	0.021	0.021	0.021	0.018	0.019	0.019	0.021	0.018	0.022	0.020	0.017	0.021	0.020	0.019	0.021	0.021	0.021	0.020	0.019	0.019	0.018	0.024
0.17	0.021	0.018	0.021	0.018	0.022	0.022	0.018	0.017	0.019	0.020	0.017	0.022	0.021	0.018	0.021	0.020	0.020	0.019	0.021	0.026	0.019	0.018	0.017	0.017	0.025
0.25	0.020	0.018	0.021	0.018	0.022	0.025	0.020	0.017	0.019	0.017	0.020	0.022	0.020	0.018	0.021	0.018	0.019	0.019	0.020	0.025	0.019	0.020	0.019	0.018	0.025
0.33	0.020	0.018	0.020	0.021	0.023	0.023	0.019	0.017	0.020	0.019	0.019	0.018	0.021	0.018	0.019	0.018	0.017	0.016	0.020	0.026	0.019	0.021	0.019	0.017	0.027
0.42	0.021	0.018	0.022	0.018	0.024	0.023	0.018	0.018	0.019	0.017	0.019	0.017	0.023	0.016	0.022	0.017	0.015	0.017	0.020	0.026	0.018	0.023	0.021	0.017	0.026
0.50	0.021	0.018	0.024	0.020	0.025	0.023	0.018	0.015	0.019	0.017	0.019	0.016	0.022	0.017	0.020	0.020	0.016	0.018	0.020	0.030	0.018	0.021	0.022	0.018	0.024
0.58	0.023	0.020	0.024	0.018	0.026	0.022	0.019	0.015	0.018	0.018	0.019	0.016	0.026	0.016	0.021	0.019	0.017	0.018	0.021	0.030	0.017	0.020	0.024	0.017	0.027
0.67	0.022	0.024	0.024	0.017	0.025	0.021	0.018	0.014	0.021	0.018	0.018	0.019	0.029	0.019	0.022	0.018	0.017	0.017	0.022	0.027	0.019	0.019	0.026	0.020	0.027
0.75	0.023	0.024	0.025	0.018	0.026	0.021	0.018	0.013	0.021	0.020	0.017	0.018	0.032	0.018	0.020	0.018	0.018	0.019	0.020	0.029	0.018	0.016	0.023	0.020	0.028
0.83	0.025	0.024	0.023	0.019	0.023	0.022	0.019	0.014	0.021	0.019	0.016	0.017	0.032	0.017	0.020	0.017	0.021	0.018	0.018	0.027	0.016	0.016	0.025	0.021	0.029
0.92	0.024	0.025	0.021	0.017	0.022	0.020	0.019	0.014	0.021	0.019	0.016	0.019	0.033	0.017	0.021	0.017	0.022	0.018	0.017	0.026	0.018	0.017	0.024	0.022	0.028
1.00	0.023	0.023	0.020	0.015	0.025	0.018	0.019	0.016	0.019	0.020	0.016	0.018	0.033	0.017	0.023	0.017	0.019	0.017	0.015	0.026	0.018	0.017	0.023	0.023	0.032
1.08	0.020	0.021	0.021	0.015	0.027	0.019	0.019	0.018	0.018	0.022	0.016	0.018	0.030	0.016	0.022	0.016	0.020	0.017	0.015	0.026	0.017	0.017	0.020	0.022	0.032
1.17	0.019	0.021	0.024	0.014	0.026	0.020	0.018	0.018	0.017	0.021	0.016	0.018	0.031	0.016	0.020	0.014	0.019	0.019	0.017	0.026	0.018	0.019	0.022	0.023	0.030
1.25	0.023	0.018	0.023	0.014	0.024	0.021	0.018	0.018	0.017	0.021	0.016	0.019	0.032	0.017	0.020	0.015	0.019	0.020	0.016	0.029	0.020	0.017	0.026	0.023	0.029
1.33	0.024	0.016	0.024	0.014	0.023	0.020	0.016	0.017	0.016	0.021	0.017	0.019	0.035	0.016	0.021	0.015	0.019	0.020	0.016	0.029	0.023	0.018	0.025	0.024	0.030
1.42	0.024	0.016	0.025	0.015	0.025	0.019	0.018	0.019	0.015	0.020	0.015	0.021	0.034	0.016	0.018	0.015	0.019	0.018	0.017	0.033	0.023	0.019	0.025	0.021	0.033
1.50	0.025	0.016	0.022	0.015	0.024	0.019	0.018	0.018	0.015	0.021	0.013	0.019	0.030	0.016	0.018	0.016	0.019	0.020	0.017	0.035	0.022	0.018	0.025	0.023	0.031
1.58	0.024	0.017	0.024	0.014	0.023	0.022	0.018	0.019	0.014	0.021	0.012	0.019	0.030	0.020	0.017	0.016	0.017	0.018	0.016	0.034	0.025	0.017	0.023	0.020	0.034
1.67	0.024	0.018	0.024	0.013	0.024	0.023	0.018	0.019	0.014	0.021	0.012	0.020	0.027	0.022	0.015	0.016	0.019	0.019	0.018	0.036	0.027	0.017	0.022	0.019	0.035
1.75	0.025	0.016	0.022	0.013	0.025	0.023	0.019	0.020	0.014	0.022	0.012	0.019	0.026	0.021	0.015	0.016	0.018	0.020	0.016	0.037	0.028	0.015	0.020	0.018	0.038
1.83	0.027	0.015	0.023	0.013	0.026	0.023	0.018	0.021	0.014	0.020	0.012	0.020	0.026	0.022	0.018	0.014	0.017	0.019	0.015	0.036	0.030	0.016	0.021	0.018	0.040
1.92	0.025	0.014	0.025	0.014	0.028	0.023	0.019	0.022	0.014	0.021	0.011	0.019	0.025	0.023	0.018	0.013	0.018	0.021	0.017	0.032	0.030	0.016	0.021	0.018	0.039
2.00	0.025	0.013	0.028	0.014	0.027	0.024	0.018	0.023	0.014	0.020	0.012	0.021	0.029	0.023	0.017	0.012	0.018	0.021	0.018	0.028	0.030	0.018	0.023	0.019	0.039

Table 1.1: reports 50 randomly simulated paths for inflation using (1.50). Parameter values are π_t = 0.02 (i.e. 2%), T = 2 years, r = 0.05, d = 0, σ = 0.25. Each path possesses 24 steps. Terminal values are given in Time period 2.00, (the 24$^{\text{rth}}$ step). These 50 simulated paths are traced out in Figure 1.3a.

Time	Inf 1	Inf 2	Inf 3	Inf 4	Inf 5	Inf 6	Inf 7	Inf 8	Inf 9	Inf 10	Inf 11	Inf 12	Inf 13	Inf 14	Inf 15	Inf 16	Inf 17	Inf 18	Inf 19	Inf 20	Inf 21	Inf 22	Inf 23	Inf 24	Inf 25
0.00	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
0.08	0.0328	0.0358	0.0306	0.0296	0.0306	0.0278	0.0298	0.033	0.0346	0.0298	0.0284	0.0251	0.03	0.0295	0.0293	0.0323	0.0308	0.0297	0.0293	0.0294	0.0324	0.0269	0.031	0.0283	0.0307
0.17	0.0328	0.0363	0.0276	0.0284	0.0297	0.0287	0.0303	0.0313	0.0341	0.0299	0.0291	0.0248	0.0289	0.0307	0.031	0.0337	0.0315	0.032	0.03	0.0296	0.0359	0.027	0.033	0.0294	0.0331
0.25	0.0383	0.0302	0.0253	0.0316	0.03	0.0265	0.0293	0.0348	0.0353	0.0286	0.0274	0.0237	0.0296	0.0313	0.0352	0.0338	0.0317	0.0312	0.0324	0.0317	0.0372	0.0286	0.0295	0.0289	0.0317
0.33	0.0335	0.0309	0.0286	0.0329	0.0261	0.0217	0.0275	0.0334	0.0359	0.0294	0.0281	0.0273	0.0253	0.0343	0.0362	0.0332	0.0329	0.0285	0.0329	0.0284	0.0421	0.0291	0.0311	0.0273	0.0293
0.42	0.0385	0.0325	0.0273	0.0368	0.0284	0.0209	0.0249	0.0334	0.0352	0.0285	0.0281	0.0306	0.0251	0.0364	0.0361	0.0341	0.0296	0.0293	0.0376	0.0296	0.0395	0.0319	0.0351	0.0279	0.031
0.50	0.0415	0.0319	0.0297	0.0367	0.0282	0.0213	0.0266	0.0286	0.0352	0.0293	0.0268	0.0308	0.0228	0.0374	0.0346	0.0312	0.0286	0.0303	0.0435	0.0306	0.0378	0.0341	0.0349	0.0313	0.0277
0.58	0.0399	0.0323	0.0302	0.0356	0.0286	0.0207	0.0265	0.0265	0.0369	0.0314	0.0246	0.0303	0.023	0.0392	0.0329	0.0296	0.0295	0.0321	0.044	0.0292	0.041	0.0354	0.0371	0.0312	0.0296
0.67	0.0377	0.0302	0.0284	0.0379	0.0267	0.0215	0.0263	0.0241	0.0382	0.0305	0.0255	0.029	0.0234	0.0452	0.0345	0.0313	0.0309	0.0318	0.0436	0.029	0.0467	0.036	0.0382	0.0328	0.0274
0.75	0.0374	0.0312	0.0308	0.034	0.0251	0.0217	0.0256	0.027	0.0384	0.0292	0.0237	0.0259	0.0258	0.0433	0.0362	0.028	0.0332	0.0282	0.0445	0.0307	0.0491	0.0322	0.0359	0.0373	0.0251
0.83	0.0387	0.0295	0.0292	0.031	0.0256	0.0209	0.0228	0.0256	0.0317	0.0269	0.0221	0.0259	0.0258	0.0418	0.0338	0.0275	0.0341	0.028	0.0456	0.033	0.0439	0.0297	0.0334	0.0344	0.0245
0.92	0.0372	0.03	0.0282	0.0323	0.023	0.0197	0.026	0.0235	0.0327	0.0287	0.0192	0.0283	0.0246	0.0426	0.0351	0.0241	0.0306	0.0297	0.0432	0.0362	0.0427	0.0304	0.0356	0.0344	0.0235
1.00	0.0425	0.0324	0.025	0.0306	0.0274	0.0196	0.0265	0.0242	0.0357	0.028	0.018	0.0331	0.0312	0.0481	0.0327	0.0251	0.0317	0.0301	0.0467	0.0359	0.0406	0.0306	0.0365	0.0327	0.0209
1.08	0.0399	0.0328	0.0248	0.0299	0.0307	0.0194	0.0254	0.0221	0.0347	0.0284	0.0171	0.0341	0.0302	0.0541	0.0313	0.0251	0.0349	0.0302	0.0457	0.0343	0.0356	0.0291	0.0366	0.0306	0.0217
1.17	0.0411	0.0375	0.0241	0.0316	0.0335	0.0189	0.0253	0.0225	0.0345	0.0271	0.0171	0.0356	0.03	0.0564	0.0312	0.0262	0.0399	0.0325	0.0461	0.0334	0.0325	0.0304	0.0386	0.0324	0.0242
1.25	0.0365	0.0367	0.0247	0.0299	0.0362	0.0221	0.0246	0.0232	0.0379	0.0267	0.0176	0.0358	0.0334	0.0548	0.0302	0.026	0.041	0.0326	0.0384	0.0352	0.0342	0.0314	0.0384	0.0358	0.0291
1.33	0.0398	0.0376	0.0241	0.029	0.0408	0.0215	0.0264	0.0227	0.0382	0.0292	0.0184	0.0354	0.0331	0.0504	0.0329	0.0256	0.0414	0.0354	0.0357	0.0382	0.0354	0.0368	0.0375	0.0327	0.0302
1.42	0.0355	0.0348	0.027	0.0279	0.04	0.021	0.0275	0.0212	0.038	0.0283	0.0183	0.035	0.0349	0.0505	0.0341	0.0272	0.0437	0.0347	0.0381	0.0357	0.0391	0.0361	0.0365	0.0324	0.0313
1.50	0.0355	0.0365	0.0282	0.0278	0.0431	0.0242	0.0301	0.0234	0.0357	0.0269	0.0178	0.034	0.0322	0.0459	0.0352	0.0288	0.0415	0.0323	0.0379	0.0352	0.0361	0.0311	0.0355	0.0333	0.0307
1.58	0.0353	0.0381	0.0253	0.0318	0.0423	0.0233	0.0284	0.0231	0.0362	0.0243	0.0186	0.038	0.0306	0.0441	0.038	0.0292	0.0425	0.0297	0.0402	0.0341	0.0399	0.0332	0.0374	0.0335	0.03
1.67	0.0324	0.0414	0.0246	0.0328	0.042	0.0223	0.026	0.0209	0.0344	0.0235	0.0224	0.0391	0.0326	0.0429	0.0369	0.0286	0.0425	0.031	0.0401	0.0319	0.0412	0.0351	0.0394	0.0359	0.0309
1.75	0.0296	0.0443	0.0242	0.0352	0.0452	0.0212	0.0259	0.0201	0.0345	0.0264	0.0223	0.0411	0.0295	0.0475	0.0359	0.026	0.0427	0.0286	0.0362	0.0328	0.0436	0.0354	0.0404	0.0394	0.0333
1.83	0.0289	0.0458	0.0245	0.0351	0.0483	0.0219	0.0239	0.0224	0.0363	0.0231	0.0209	0.0426	0.028	0.0514	0.0389	0.0263	0.0417	0.0272	0.0344	0.0267	0.0416	0.0338	0.0397	0.0404	0.0382
1.92	0.0258	0.0439	0.0234	0.034	0.0496	0.022	0.0223	0.0216	0.0323	0.0258	0.0222	0.0405	0.0249	0.0486	0.0417	0.0261	0.0405	0.0247	0.0301	0.0281	0.0426	0.0325	0.0362	0.0425	0.0394
2.00	0.0293	0.0436	0.0261	0.0369	0.0489	0.0246	0.0223	0.0237	0.0327	0.0243	0.0208	0.0354	0.0237	0.044	0.0447	0.0255	0.039	0.0244	0.0288	0.0282	0.0434	0.0296	0.0354	0.0468	0.0443

Time	Inf 26	Inf 27	Inf 28	Inf 29	Inf 30	Inf 31	Inf 32	Inf 33	Inf 34	Inf 35	Inf 36	Inf 37	Inf 38	Inf 39	Inf 40	Inf 41	Inf 42	Inf 43	Inf 44	Inf 45	Inf 46	Inf 47	Inf 48	Inf 49	Inf 50
0.00	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
0.08	0.0314	0.0257	0.0321	0.0295	0.033	0.0274	0.031	0.031	0.0268	0.0285	0.0329	0.0309	0.0296	0.0354	0.0303	0.0281	0.0291	0.0294	0.0272	0.027	0.0293	0.0331	0.0357	0.03216	0.025127
0.17	0.0302	0.0276	0.0338	0.0335	0.0351	0.0271	0.0302	0.0337	0.0274	0.0254	0.0307	0.0301	0.0292	0.0351	0.0299	0.0285	0.0251	0.0275	0.0261	0.0298	0.0294	0.0348	0.0342	0.035678	0.024842
0.25	0.0274	0.026	0.0337	0.0337	0.035	0.0265	0.0287	0.0346	0.0256	0.0281	0.0333	0.0296	0.0299	0.0368	0.0295	0.0278	0.0266	0.0303	0.0261	0.0287	0.0298	0.0359	0.0332	0.035139	0.023686
0.33	0.0286	0.0232	0.0313	0.0343	0.0374	0.0252	0.0274	0.0357	0.0244	0.0303	0.0345	0.0287	0.0326	0.0354	0.0244	0.0276	0.0265	0.0332	0.0299	0.0317	0.0293	0.0317	0.0368	0.031706	0.027252
0.42	0.0285	0.0228	0.0305	0.0366	0.0365	0.0242	0.03	0.0341	0.0246	0.0285	0.0313	0.0325	0.0304	0.0335	0.0233	0.0281	0.0245	0.0375	0.0315	0.0312	0.0288	0.0309	0.0372	0.031117	0.030552
0.50	0.0268	0.0232	0.035	0.0386	0.0401	0.0224	0.0325	0.0341	0.0207	0.0297	0.0302	0.0333	0.0273	0.0343	0.0226	0.0302	0.0265	0.0387	0.0325	0.0328	0.0336	0.0351	0.0363	0.030833	0.030841
0.58	0.0251	0.0237	0.036	0.037	0.0455	0.0244	0.0323	0.0381	0.0218	0.0307	0.0304	0.0321	0.026	0.039	0.0239	0.029	0.0284	0.0348	0.0319	0.0325	0.0345	0.0366	0.0328	0.030776	0.030342
0.67	0.0239	0.0226	0.036	0.0332	0.0433	0.0292	0.0323	0.039	0.0217	0.0315	0.0312	0.0335	0.026	0.0407	0.0269	0.0315	0.0311	0.0358	0.0314	0.0341	0.0375	0.0345	0.0356	0.02909	0.029034
0.75	0.0253	0.0241	0.0326	0.0328	0.0448	0.0288	0.0331	0.0359	0.0205	0.0272	0.0325	0.0362	0.0244	0.0431	0.0276	0.0286	0.0308	0.0348	0.0305	0.0333	0.0357	0.0325	0.0346	0.0271	0.025875
0.83	0.0281	0.0241	0.0352	0.0297	0.0446	0.0304	0.0337	0.039	0.0199	0.0311	0.0313	0.033	0.0258	0.0435	0.0311	0.028	0.031	0.0387	0.0303	0.0352	0.0393	0.0339	0.0338	0.02921	0.025932
0.92	0.0271	0.0252	0.0311	0.0271	0.039	0.0261	0.0357	0.0344	0.0177	0.0341	0.0302	0.0336	0.0234	0.0447	0.0284	0.0301	0.0295	0.0366	0.0312	0.0308	0.0415	0.0363	0.0319	0.028656	0.02831
1.00	0.0256	0.0256	0.0306	0.027	0.044	0.0275	0.0343	0.0366	0.016	0.0324	0.0297	0.0376	0.0227	0.0456	0.0293	0.0328	0.0303	0.0381	0.0337	0.0294	0.0374	0.0402	0.0311	0.028103	0.033117
1.08	0.0232	0.0231	0.0289	0.0286	0.0387	0.0289	0.0332	0.0357	0.0143	0.0325	0.029	0.0345	0.0237	0.0472	0.0306	0.0308	0.0305	0.0414	0.0335	0.0279	0.0351	0.0339	0.0313	0.026556	0.03407
1.17	0.0232	0.0226	0.0265	0.0291	0.0363	0.026	0.033	0.0328	0.0159	0.0353	0.0297	0.0368	0.022	0.0472	0.0292	0.0341	0.031	0.0405	0.0293	0.0272	0.0368	0.0348	0.0284	0.025127	0.035553
1.25	0.0232	0.0216	0.027	0.0263	0.0378	0.0264	0.0332	0.0343	0.0141	0.0351	0.0307	0.0349	0.0215	0.0423	0.0297	0.0384	0.0306	0.0368	0.0279	0.0307	0.0374	0.0332	0.0275	0.028295	0.035839
1.33	0.0224	0.0226	0.0312	0.0259	0.0393	0.023	0.0322	0.0357	0.0162	0.0422	0.0301	0.0419	0.022	0.0454	0.0289	0.0392	0.0293	0.038	0.0273	0.0316	0.0412	0.0324	0.0268	0.031754	0.035354
1.42	0.0193	0.0224	0.0279	0.0263	0.0412	0.0229	0.0322	0.0445	0.0139	0.0401	0.03	0.0406	0.0232	0.0474	0.0286	0.036	0.0282	0.0377	0.0268	0.0344	0.036	0.035	0.0283	0.028586	0.034985
1.50	0.0185	0.0242	0.0252	0.0268	0.0414	0.0237	0.031	0.0437	0.0135	0.0436	0.029	0.0394	0.023	0.0468	0.0335	0.0328	0.0286	0.038	0.0319	0.0325	0.035	0.0375	0.0316	0.02752	0.034013
1.58	0.0188	0.0235	0.0248	0.0251	0.0426	0.0246	0.0292	0.0443	0.0128	0.0399	0.0306	0.0396	0.0225	0.0457	0.0385	0.0347	0.0306	0.0355	0.0315	0.0306	0.0356	0.0394	0.0307	0.022409	0.038037
1.67	0.0203	0.0222	0.0257	0.0267	0.0412	0.0227	0.0263	0.0439	0.0128	0.0388	0.0342	0.0413	0.0212	0.0495	0.0395	0.0337	0.0298	0.0399	0.0339	0.0319	0.0372	0.0444	0.0342	0.021006	0.03907
1.75	0.0187	0.021	0.0274	0.0277	0.0442	0.0202	0.0255	0.0421	0.0132	0.0385	0.0346	0.0418	0.0208	0.0456	0.0398	0.0338	0.0301	0.0387	0.0315	0.0318	0.0363	0.039	0.0336	0.020696	0.041127
1.83	0.0161	0.0203	0.0276	0.0284	0.0452	0.0202	0.0275	0.0407	0.0122	0.0394	0.032	0.0433	0.0227	0.0516	0.037	0.0328	0.0298	0.0438	0.0315	0.03	0.0352	0.0407	0.0319	0.020229	0.042646
1.92	0.0161	0.022	0.0255	0.0261	0.045	0.021	0.0281	0.0428	0.0134	0.044	0.0305	0.0437	0.0224	0.0549	0.0362	0.0296	0.0304	0.0489	0.0336	0.0285	0.0402	0.0362	0.0306	0.021192	0.040489
2.00	0.0178	0.0193	0.0249	0.0247	0.0472	0.0228	0.0296	0.0408	0.0132	0.0485	0.0333	0.0459	0.0204	0.0567	0.041	0.0304	0.0275	0.0533	0.0326	0.0311	0.0426	0.0355	0.0327	0.020676	0.035375

Table 1.2: reports 50 randomly simulated paths for inflation using (1.50). Parameter values are π_t = 0.03 (i.e. 3%), T = 2, r = 0.05, d = 0, σ = 0.25. Terminal values are given in the Time period 2.00. Time period 1.00 can also be used as a benchmark. These 50 simulated paths are traced out in Figure 1.3b.

Table 1.3		σ = 0.25	Difference	σ = 0.35	Difference	σ = 0.15	Difference
***k* = 0.03**	**2 years**	0.00528		0.00602		0.00456	
***k* = 0.03**	**1 year**	0.00340	0.00188	0.00372	0.00230	0.00270	0.00185
***k* = 0.04**	**2 years**	0.00166		0.00303		0.00080	
***k* = 0.04**	**1 year**	0.00053	0.00113	0.00140	0.00162	0.00000	0.00080
***k* = 0.05**	**2 years**	0.00018		0.00148		0.00000	
***k* = 0.05**	**1 year**	0.00000	0.00018	0.00050	0.00098	0.00000	0.00000

Table 1.3: reports 3 sets of call time values calculated using (1.52) for three upper bounds: k_1 = 0.03, k_2 = 0.04 and k_3 = 0.05. These estimates are only approximate, given the small sample size and the analysis provided here is largely intuitive. More formal measures of parameter sensitivity are developed in chapters 2 and 3. Each sequence of 50 sample paths is generated using parameter values: π_t = 0.03 and r = 0.05. Two time periods: T_1 = 2 years, T_2 = 1 year and three standard deviations: σ_1 = 0.25, σ_2 = 0.35 and σ_3 = 0.15 are used. Figures 1.3e, 1.3f and 1.3g outline the underlying samples and terminal values of inflation which are used to calculate the time values here. It is found that as volatility and the forecasting horizon (i.e. uncertainty) increases, the value generated by (1.52) correspondingly increases. In other words, as the parameters are engineered to produce a greater likelihood of terminal inflation breaching *k*, the value of the Monte Carlo call increases. Expanding the forecasting horizon from one year to two years has the greatest effect on (1.52) when the initial inflation rate is closest to *k*. In market parlance, this is the at-the-money (ATM) point. Parameter sensitivities are later developed in chapter 3, in the shape of ‘the Greeks’. These can be used to investigate the effects of the volatility of inflation and the time horizon of target on rate decisions.

$$c = \frac{e^{-r(T-t)}}{n}\sum\nolimits_{i=1}^{n} max\left[\pi_{t,i} e^{\sum_{j=1}^{m}\left(r_{j,i}-\frac{\sigma_{j,i}^2}{2}\right)\Delta t+\sigma_{j,i}\sqrt{\Delta t} z_{j,i}} - k, 0\right] \qquad (1.52)$$

Table 1.4

Expected Inflation $E(\pi)$	*MC* MC	*Black Model* BS
0.0025	0.0000	0.0000
0.0050	0.0000	0.0000
0.0075	0.0000	0.0000
0.0100	0.0000	0.0000
0.0125	0.0000	0.0000
0.0150	0.0000	0.0000
0.0175	0.0000	0.0000
0.0200	0.0000	0.0000
0.0225	0.0000	0.0000
0.0250	0.0000	0.0000
0.0275	0.0001	0.0001
0.0300	0.0002	0.0002
0.0325	0.0004	0.0004
0.0350	0.0008	0.0008
0.0375	0.0013	0.0013
0.0400	0.0021	0.0021
0.0425	0.0031	0.0031
0.0450	0.0042	0.0043
0.0475	0.0056	0.0057
0.0500	0.0073	0.0073
0.0525	0.0090	0.0090
0.0550	0.0109	0.0109
0.0575	0.0129	0.0129
0.0600	0.0151	0.0150
0.0625	0.0172	0.0172
0.0650	0.0194	0.0194
0.0675	0.0217	0.0217
0.0700	0.0240	0.0240
0.0725	0.0262	0.0263
0.0750	0.0286	0.0286
0.0775	0.0309	0.0310
0.0800	0.0333	0.0333

Table 1.4

Table 1.4: reports call time values calculated using both Monte Carlo and the Black formula. Each is generated with the parameter values given as: π_t = 0.25% to 8%, k = 0.045, T = 1, r = 0.05 and σ = 0.25. Monte Carlo uses 50,000 simulations. Only minor differences are observed between Monte Carlo and Black-Scholes (Black) valuations. See Figure 1.5 for a graph of the output.

Black-Scholes parameter values: π_t = 0.25% to 8%, k = 0.045, T = 1, r = 0.05, d = 0.05, and σ = 0.25 (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.03, k = 0.045, T = 1, r = 0.05 and σ = 0.25 produce equivalent time values).

$$c = E[max(\pi_T - k,0)] = \int_k^\infty (\pi_T - k) g(\pi) d(\pi)$$

Expected Inflation $E(\pi)$	***Black Model BS***	***Heston 1.1 (T.2)***	***Heston 1.2 (T.2)***	***Heston 1.3 (T.2)***	***Heston 1.4 (T.2)***	***Heston 1.5 (T.2)***	***Heston 1.6 (T.2)***	***Heston 1.7 (T.2)***	***Heston 1.8 (T.2)***
0.0025	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0050	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0075	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000
0.0100	0.00000	0.00000	0.00001	0.00000	0.00000	0.00001	0.00004	0.00000	0.00000
0.0125	0.00001	0.00002	0.00005	0.00000	0.00000	0.00003	0.00010	0.00000	0.00000
0.0150	0.00006	0.00008	0.00014	0.00003	0.00000	0.00010	0.00022	0.00002	0.00000
0.0175	0.00020	0.00022	0.00032	0.00011	0.00003	0.00024	0.00043	0.00009	0.00000
0.0200	0.00048	0.00049	0.00063	0.00034	0.00021	0.00050	0.00075	0.00025	0.00003
0.0225	0.00097	0.00096	0.00111	0.00079	0.00063	0.00094	0.00121	0.00061	0.00024
0.0250	0.00168	0.00165	0.00179	0.00150	0.00137	0.00160	0.00185	0.00126	0.00094
0.0275	0.00263	0.00258	0.00269	0.00248	0.00240	0.00250	0.00267	0.00224	0.00206
0.0300	0.00381	0.00376	0.00381	0.00371	0.00369	0.00366	0.00370	0.00352	0.00347
0.0325	0.00520	0.00515	0.00514	0.00517	0.00518	0.00506	0.00496	0.00505	0.00509
0.0350	0.00677	0.00673	0.00666	0.00680	0.00685	0.00666	0.00645	0.00675	0.00684
0.0375	0.00849	0.00846	0.00834	0.00857	0.00864	0.00842	0.00814	0.00859	0.00871
0.0400	0.01034	0.01032	0.01017	0.01045	0.01054	0.01030	0.00999	0.01052	0.01066
0.0425	0.01228	0.01228	0.01211	0.01242	0.01252	0.01228	0.01197	0.01252	0.01267
0.0450	0.01430	0.01431	0.01414	0.01445	0.01455	0.01433	0.01403	0.01457	0.01473
0.0475	0.01638	0.01640	0.01623	0.01654	0.01664	0.01643	0.01616	0.01667	0.01683
0.0500	0.01850	0.01853	0.01837	0.01866	0.01876	0.01857	0.01833	0.01880	0.01895
0.0525	0.02066	0.02069	0.02055	0.02081	0.02090	0.02073	0.02053	0.02096	0.02110
0.0550	0.02285	0.02288	0.02276	0.02299	0.02307	0.02292	0.02274	0.02313	0.02327
0.0575	0.02505	0.02508	0.02498	0.02518	0.02526	0.02512	0.02497	0.02532	0.02545
0.0600	0.02727	0.02730	0.02721	0.02739	0.02746	0.02734	0.02721	0.02752	0.02764
0.0625	0.02950	0.02953	0.02945	0.02961	0.02967	0.02957	0.02946	0.02973	0.02985
0.0650	0.03174	0.03176	0.03170	0.03184	0.03189	0.03180	0.03171	0.03195	0.03206
0.0675	0.03398	0.03401	0.03395	0.03407	0.03412	0.03404	0.03396	0.03418	0.03428
0.0700	0.03623	0.03625	0.03621	0.03631	0.03636	0.03628	0.03622	0.03641	0.03651
0.0725	0.03849	0.03850	0.03847	0.03855	0.03860	0.03853	0.03847	0.03865	0.03874
0.0750	0.04074	0.04075	0.04073	0.04080	0.04084	0.04078	0.04073	0.04089	0.04097
0.0775	0.04300	0.04301	0.04299	0.04305	0.04308	0.04303	0.04299	0.04313	0.04321
0.0800	0.04525	0.04527	0.04525	0.04530	0.04533	0.04529	0.04525	0.04538	0.04545

Table 1.5a

Table 1.5a: reports call time values using both the Black-Scholes formula and Monte Carlo Heston (1993). The Monte Carlo estimates uses the parameter inputs associated with Model 1.2 – 1.8 (T.2) reported in table 1.6. The Black-Scholes parameter values are: π_t = 0.25% to 8%, k = 0.03, T = 2, r = 0.05, d = 0.05, and σ = 0.25 (alternatively the Black parameters are: $E(\pi_{t+1})$ = 0.25% to 8%, k = 0.03, T = 2, r = 0.05 and σ = 0.25). Monte Carlo Heston uses 50,000 simulations and 24 time steps. Here, the variance parameter is given as the square of σ_{BS}. The long run mean of variance, θ, is also set equal to the initial variance, v = 0.0625. κ is set equal to 0.01. Gamma is estimated to be approximately 0.1. Figure 1.6a traces out the difference between the Heston 1.2(T.2) time value and the Black-Scholes time value. A closed-form solution was used to verify results.

Expected	***Black***	***Heston***	***Heston***	***Heston***	***Heston***	***Heston***	***Heston***	***Heston***	***Heston***
Inflation	***Model***	***2.1 (T.2)***	***2.2 (T.2)***	***2.3 (T.2)***	***2.4 (T.2)***	***2.5 (T.2)***	***2.6 (T.2)***	***2.7 (T.2)***	***2.8 (T.2)***
E(π ***)***	***BS***								
0.0025	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0050	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000
0.0075	0.00000	0.00001	0.00002	0.00000	0.00000	0.00001	0.00004	0.00000	0.00000
0.0100	0.00003	0.00004	0.00008	0.00001	0.00000	0.00005	0.00013	0.00001	0.00000
0.0125	0.00013	0.00014	0.00021	0.00007	0.00003	0.00016	0.00030	0.00005	0.00000
0.0150	0.00034	0.00035	0.00046	0.00024	0.00014	0.00036	0.00058	0.00017	0.00002
0.0175	0.00071	0.00071	0.00085	0.00056	0.00043	0.00071	0.00098	0.00043	0.00016
0.0200	0.00125	0.00124	0.00140	0.00108	0.00094	0.00123	0.00152	0.00089	0.00057
0.0225	0.00199	0.00197	0.00213	0.00181	0.00168	0.00194	0.00223	0.00159	0.00129
0.0250	0.00292	0.00289	0.00303	0.00276	0.00264	0.00284	0.00309	0.00253	0.00230
0.0275	0.00403	0.00400	0.00410	0.00389	0.00382	0.00393	0.00411	0.00369	0.00354
0.0300	0.00531	0.00527	0.00534	0.00521	0.00516	0.00520	0.00529	0.00504	0.00497
0.0325	0.00673	0.00669	0.00672	0.00667	0.00666	0.00662	0.00663	0.00655	0.00654
0.0350	0.00828	0.00825	0.00823	0.00827	0.00828	0.00818	0.00810	0.00819	0.00823
0.0375	0.00994	0.00991	0.00986	0.00997	0.01001	0.00986	0.00970	0.00994	0.01002
0.0400	0.01170	0.01168	0.01159	0.01176	0.01182	0.01164	0.01141	0.01177	0.01188
0.0425	0.01354	0.01352	0.01341	0.01363	0.01371	0.01349	0.01322	0.01368	0.01381
0.0450	0.01545	0.01544	0.01530	0.01556	0.01565	0.01542	0.01512	0.01564	0.01578
0.0475	0.01742	0.01741	0.01726	0.01755	0.01764	0.01740	0.01708	0.01764	0.01780
0.0500	0.01943	0.01943	0.01927	0.01957	0.01967	0.01943	0.01910	0.01969	0.01986
0.0525	0.02148	0.02149	0.02132	0.02164	0.02174	0.02150	0.02117	0.02177	0.02194
0.0550	0.02357	0.02358	0.02341	0.02373	0.02384	0.02360	0.02328	0.02387	0.02404
0.0575	0.02569	0.02570	0.02553	0.02585	0.02595	0.02573	0.02542	0.02599	0.02617
0.0600	0.02783	0.02784	0.02768	0.02799	0.02809	0.02787	0.02758	0.02814	0.02831
0.0625	0.02999	0.03000	0.02985	0.03014	0.03025	0.03004	0.02976	0.03030	0.03047
0.0650	0.03216	0.03218	0.03203	0.03232	0.03242	0.03222	0.03196	0.03247	0.03263
0.0675	0.03435	0.03437	0.03423	0.03450	0.03460	0.03441	0.03417	0.03465	0.03481
0.0700	0.03655	0.03657	0.03644	0.03670	0.03679	0.03661	0.03639	0.03685	0.03700
0.0725	0.03876	0.03878	0.03866	0.03890	0.03899	0.03882	0.03862	0.03905	0.03920
0.0750	0.04098	0.04100	0.04088	0.04111	0.04120	0.04104	0.04085	0.04126	0.04140
0.0775	0.04320	0.04323	0.04312	0.04333	0.04341	0.04326	0.04309	0.04347	0.04361
0.0800	0.04543	0.04545	0.04536	0.04555	0.04563	0.04549	0.04534	0.04569	0.04583

Table 1.5b

Table 1.5b: reports time values for the Heston and Black-Scholes calls using the same parameter values as Table 1.5a except here the value of the variance parameter is given as the square of $\sigma_{BS} = 0.35$. The long run mean of variance, θ, is also set equal to the initial variance, v = 0.1225. A closed form solution was used to verify the Monte Carlo estimates.

Black-Scholes parameter values: π_t = 0.25% to 8%, k = 0.03, T = 2, r = 0.05, d = 0.05, and σ = 0.35 (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.25% to 8%, k = 0.03, T = 2, r = 0.05 and σ = 0.35).

$$c = \frac{e^{-r(T-t)}}{n} \sum_{i=1}^{n} max \left[\pi_{t,i} e^{\sum_{j=1}^{m} \left(r_{j,i} - \frac{\sigma_{j,i}^2}{2} \right) \Delta t + \sigma_{j,i} \sqrt{\Delta t} z_{1(j,i)}} - k, 0 \right]$$

Model	*Heston 1.1*	*Heston 1.2*	*Heston 1.3*	*Heston 1.4*	*Heston 1.5*	*Heston 1.6*	*Heston 1.7*	*Heston 1.8*
Initial Inflation Rate	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
v	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625
Risk Free Rate	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Dividend Yield	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Number of Steps	24	24	24	24	24	24	24	24
Number of Simulations	10000	10000	10000	10000	10000	10000	10000	10000
θ	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625
κ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
γ	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
ρ	0	0.5	-0.5	-0.9	0	0.5	-0.5	-0.9
Forecasting Horizon Yrs	1	1	1	1	1	1	1	1

Descriptive Statistics for terminal inflation generated using Monte Carlo Heston Simulation. Heston input parameters are given above.

Model	*Heston 1.1(T.1)*	*Heston 1.2(T.1)*	*Heston 1.3(T.1)*	*Heston 1.4(T.1)*	*Heston 1.5(T.1)*	*Heston 1.6(T.1)*	*Heston 1.7(T.1)*	*Heston 1.8(T.1)*
Mean	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
Median	0.029	0.029	0.029	0.030	0.029	0.029	0.030	0.030
Standard Deviation	0.008	0.008	0.008	0.007	0.008	0.008	0.007	0.007
Kurtosis	1.258	1.936	0.262	-0.127	2.317	4.350	0.479	-0.413
Skewness	0.810	1.051	0.466	0.234	0.899	1.516	0.286	-0.249
Range	0.065	0.063	0.054	0.050	0.085	0.074	0.071	0.047
Minimum	0.010	0.013	0.010	0.009	0.010	0.012	0.008	0.003
Maximum	0.075	0.076	0.064	0.059	0.095	0.086	0.079	0.051
Count	10000	10000	10000	10000	10000	10000	10000	10000

Forecasting Horizon Yrs	2	2	2	2	2	2	2	2

Descriptive Statistics for terminal inflation generated using Monte Carlo Heston Simulation. Heston input parameters are given above.

Model	*Heston 1.1(T2)*	*Heston 1.2(T.2)*	*Heston 1.3(T.2)*	*Heston 1.4(T.2)*	*Heston 1.5(T.2)*	*Heston 1.6(T.2)*	*Heston 1.7(T.2)*	*Heston 1.8(T.2)*
Mean	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
Median	0.028	0.028	0.029	0.030	0.028	0.027	0.030	0.031
Standard Deviation	0.011	0.011	0.010	0.010	0.011	0.012	0.010	0.009
Kurtosis	4.079	5.404	1.441	-0.343	5.479	19.803	0.977	-0.611
Skewness	1.350	1.704	0.765	0.240	1.487	3.022	0.434	-0.353
Range	0.115	0.115	0.108	0.063	0.126	0.181	0.093	0.056
Minimum	0.006	0.009	0.003	0.004	0.004	0.009	0.003	0.002
Maximum	0.121	0.124	0.111	0.067	0.130	0.190	0.096	0.058
Count	10000	10000	10000	10000	10000	10000	10000	10000

Descriptive Statistics using historical monthly observations of annual inflation data for varying periods

	1958:7 - 2007:5	*1958:7 - 1979:8*	*1989:12 - 2007:5*
Mean	0.041	0.044	0.029
Median	0.032	0.036	0.028
Standard Deviation	0.029	0.031	0.010
Kurtosis	1.984	-0.308	1.529
Skewness	1.523	0.778	1.027
Range	0.142	0.119	0.053
Minimum	0.003	0.003	0.011
Maximum	0.146	0.122	0.064
Count	587	254	210

Table 1.6

Table 1.6: a variety of Heston parameter values outlined in (1.57) and (1.58) are used to generate 8 Heston models each implementing 10,000 simulation paths. For each Heston model 1.1, 1.2…1.8, descriptive statistics (i.e. the generated moments) are calculated at the one-year forecasting horizon and at the two-year forecasting horizon, Heston (T.1) and Heston (T.2). Descriptive statistics (i.e. the historic moments) for actual inflation data are also provided for varying time periods since 1958:7. The U.S. Consumer Price Index (Series i.d.: CPIAUCSL) was used to calculate inflation.

Model	***Heston 2.1***	***Heston 2.2***	***Heston 2.3***	***Heston 2.4***	***Heston 2.5***	***Heston 2.6***	***Heston 2.7***	***Heston 2.8***
Initial Inflation Rate	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
v	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225
Risk Free Rate	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Dividend Yield	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Forecasting Horizon Yrs	2	2	2	2	2	2	2	2
Number of Steps	24	24	24	24	24	24	24	24
Number of Simulations	10000	10000	10000	10000	10000	10000	10000	10000
θ	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225	0.1225
κ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
γ	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
ρ	0	0.5	-0.5	-0.9	0	0.5	-0.5	-0.9

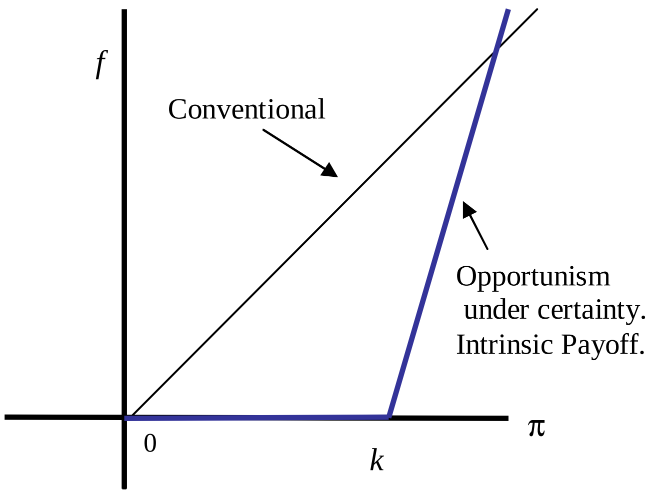
Forecasting Horizon Yrs	1	1	1	1	1	1	1	1
Descriptive Statistics for terminal inflation generated using Monte Carlo Heston Simulation. Heston input parameters are								
Model	***Heston 2.1(T.1)***	***Heston 2.2(T.1)***	***Heston 2.3(T.1)***	***Heston 2.4(T.1)***	***Heston 2.5(T.1)***	***Heston 2.6(T.1)***	***Heston 2.7(T.1)***	***Heston 2.8(T.1)***
Mean	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
Median	0.028	0.028	0.028	0.029	0.028	0.028	0.029	0.029
Standard Deviation	0.011	0.011	0.011	0.010	0.011	0.012	0.010	0.010
Kurtosis	3.556	3.696	0.954	0.483	3.024	13.509	0.947	-0.320
Skewness	1.244	1.421	0.838	0.664	1.186	2.188	0.696	0.281
Range	0.137	0.105	0.075	0.074	0.093	0.212	0.101	0.060
Minimum	0.007	0.007	0.007	0.005	0.006	0.008	0.004	0.005
Maximum	0.144	0.113	0.082	0.078	0.099	0.220	0.104	0.065
Count	10000	10000	10000	10000	10000	10000	10000	10000

Forecasting Horizon Yrs	2	2	2	2	2	2	2	2
Descriptive Statistics for terminal inflation generated using Monte Carlo Heston Simulation. Heston input parameters are								
Model	***Heston 2.1(T2)***	***Heston 2.2(T.2)***	***Heston 2.3(T.2)***	***Heston 2.4(T.2)***	***Heston 2.5(T.2)***	***Heston 2.6(T.2)***	***Heston 2.7(T.2)***	***Heston 2.8(T.2)***
Mean	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030
Median	0.027	0.026	0.027	0.028	0.027	0.026	0.028	0.029
Standard Deviation	0.016	0.016	0.015	0.014	0.016	0.018	0.014	0.013
Kurtosis	6.244	7.901	2.534	0.812	11.640	23.770	2.198	-0.555
Skewness	1.802	2.121	1.258	0.836	2.209	3.403	1.005	0.276
Range	0.187	0.175	0.131	0.114	0.217	0.302	0.134	0.075
Minimum	0.003	0.005	0.002	0.003	0.003	0.004	0.001	0.001
Maximum	0.190	0.180	0.134	0.116	0.220	0.306	0.135	0.076
Count	10000	10000	10000	10000	10000	10000	10000	10000

Table 1.7

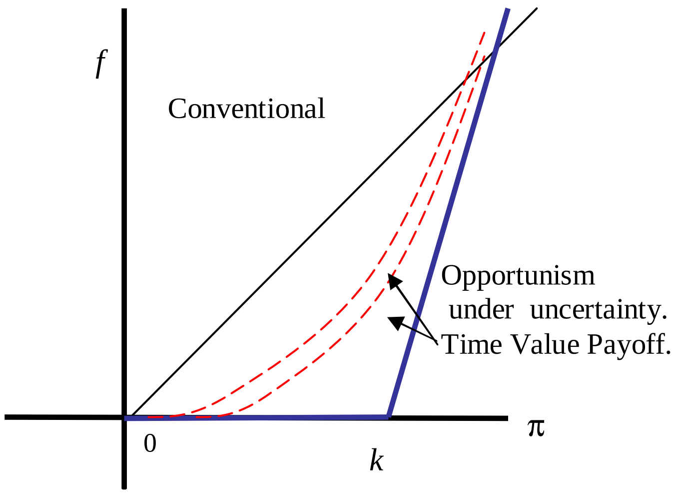
Table 1.7: the generated moments are calculated in the same way as Table 1.6 except a higher value for v and θ is used. Here they are set equal to 0.1225 (i.e. the square of 0.35).

Figure 1.1a



$$f = [\max(\pi_T - k, 0)] \qquad \text{Intrinsic Payoff}$$

Figure 1.1b



$$f = E[max(\pi_T - k, 0)] = \int_k^\infty (\pi_T - k) g(\pi) d(\pi) \qquad \text{Time Value Payoff}$$

Figure 1.1a illustrates how the policy rate (or penalty rate), *f* may be adjusted in an 'opportunistic' framework with regard to inflation, π. Only after a given tolerance level of inflation, *k* is breached will monetary policy respond. The hockey stick configuration mimics the payoff from an 'intrinsic' call option where the strike is set at *k*. The intrinsic value given by $f = [\max(\pi_T - k, 0)]$ implies that policy remains unresponsive to increases in inflation so long as inflation remains within the target zone. The true extent to which policy can be allowed to be unresponsive however is more curtailed given that central banks tend to target a forecast or an expected value. The effects of uncertainty might be best described in Figure 1.1b, by the 'time value' of the call option. The broken curve depicts how the interest rate decision is affected by uncertainty. The longer the targeting horizon (and/or the higher the level of volatility for the underlying observed target variable), the closer the opportunistic approach resembles the conventional approach, (i.e. the less scope there exists for policy makers to remain inactive.) The conventional approach is given plausibly by a linear Taylor type rule or inflation point targeting rule. Importantly, the parabola construction indicates that policy makers are not unresponsive to expected inflation even when inflation is below the lower bound. This is consistent with Proposition 1.4. It is clear that uncertainty limits the extent to which policy makers can use discretion when working within a zone targeting or opportunistic framework.

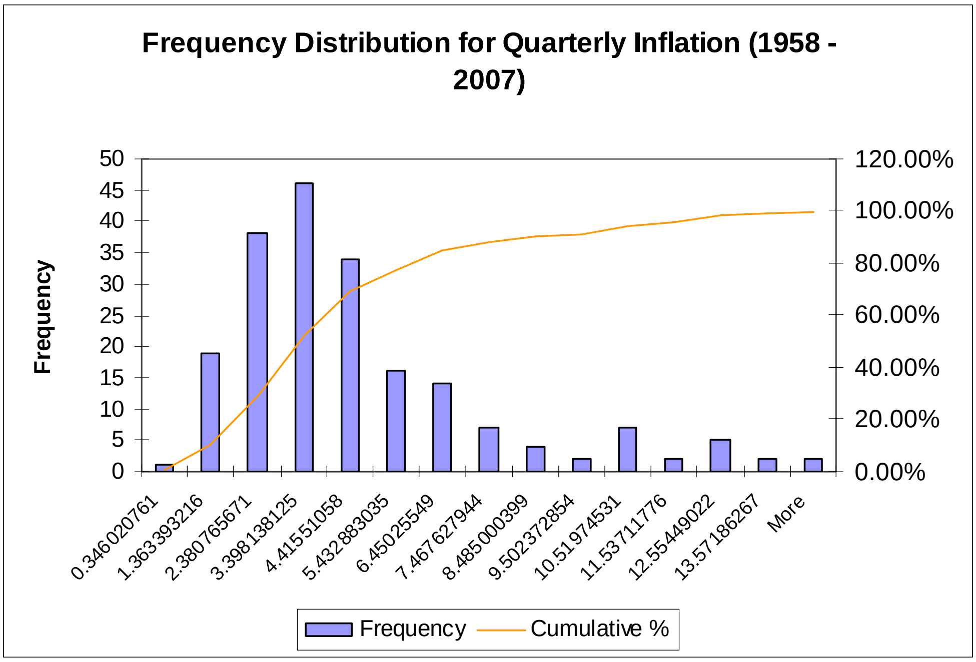


Figure 1.2a: sets out a histogram that illustrates the incidence of percentage inflation rates expressed in decimals for year on year quarterly changes in the CPI. See table 3.1 in chapter 3 for data sources. Figure 1.8i sets out a histogram using monthly observations of inflation.

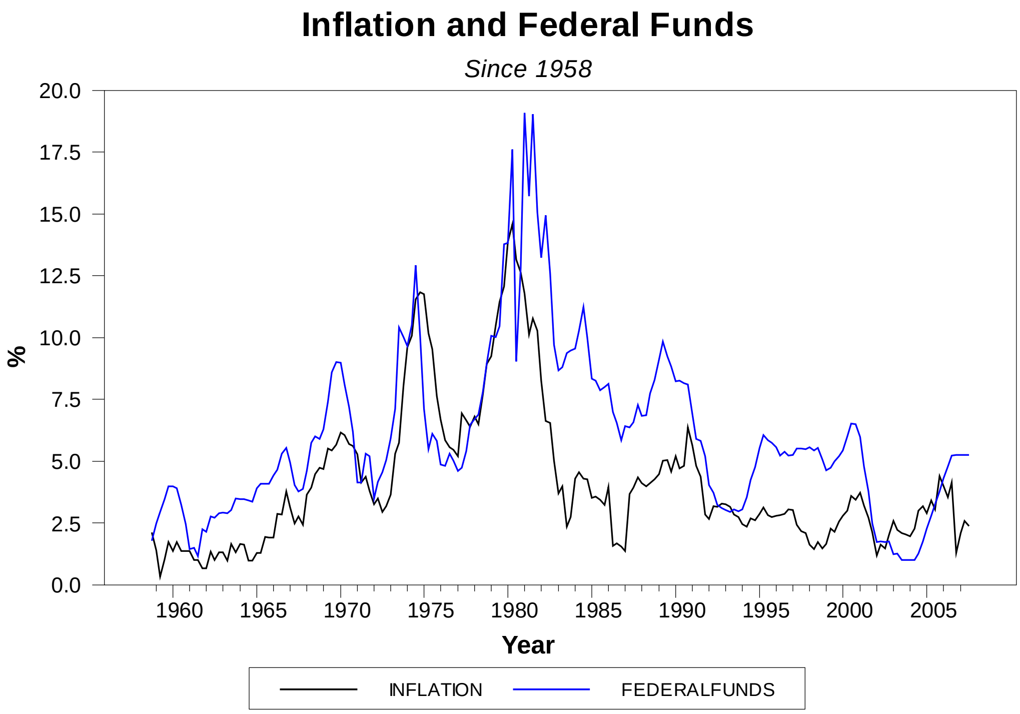
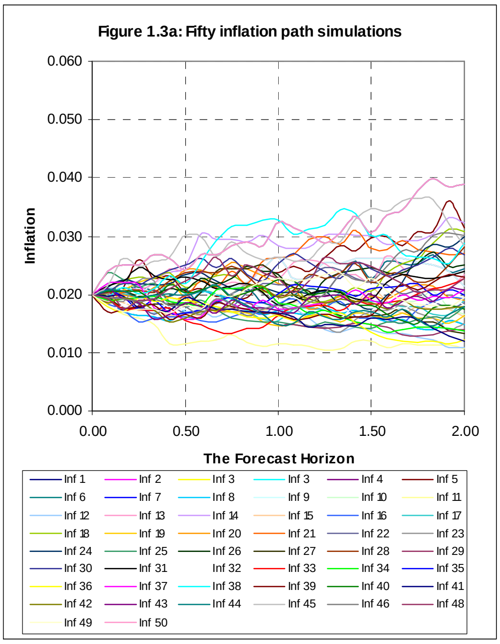


Figure 1.2b: maps out the quarterly year on year inflation rate and Federal Funds rate over the period 1958 – 2007. All data series were downloaded from the Federal Reserve Bank of St. Louis.



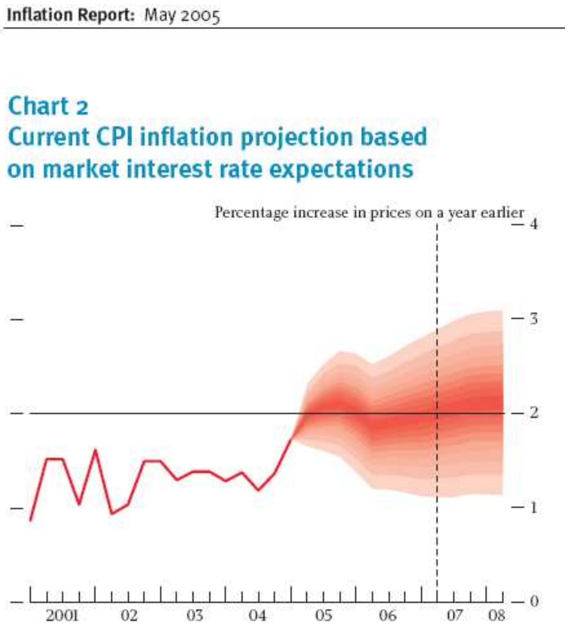
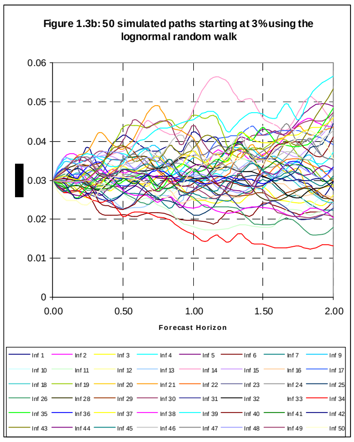
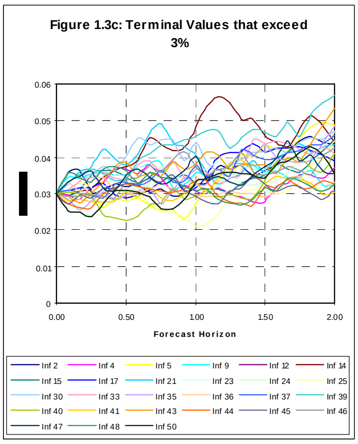
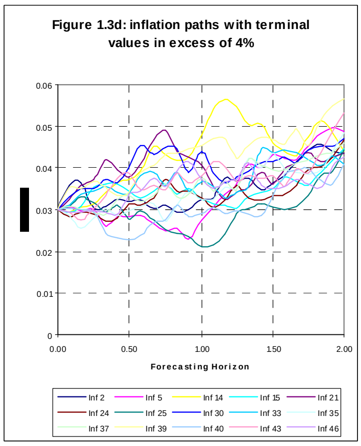


Figure 1.3a: compares 50 inflation paths (starting at 2%) generated by the lognormal random walk with 24 steps against the May, 2005 fan chart for CPI inflation. Table 1.1 reports the numerical values for Figure 1.3a. $\pi_t = 0.02$, $T = 2$, $r = 0.05$, $\sigma = 0.25$.







Figures 1.3 b, c and d: present the simulated inflation paths starting with inflation equal to 3%. All the other parameter values are the same as before. $\pi_t = 0.03$, $T = 2$, $r = 0.05$, and $\sigma = 0.25$. Each path is divided into discrete time periods of 24 steps, corresponding to 24 months over two years. Figure 1.3 b presents all 50 paths. Figure c presents all those paths with terminal values in excess of 3%. Figure d presents all those paths with terminal values in excess of 4%. It is clear that as the upper bound, *k*, increases, the probability of terminal inflation being in excess of *k*, declines. If central bankers respond to that proportion of outcomes that exceed *k*, then as *k* increases the probability of a pre-emptive policy response declines. (All other parameters being equal).

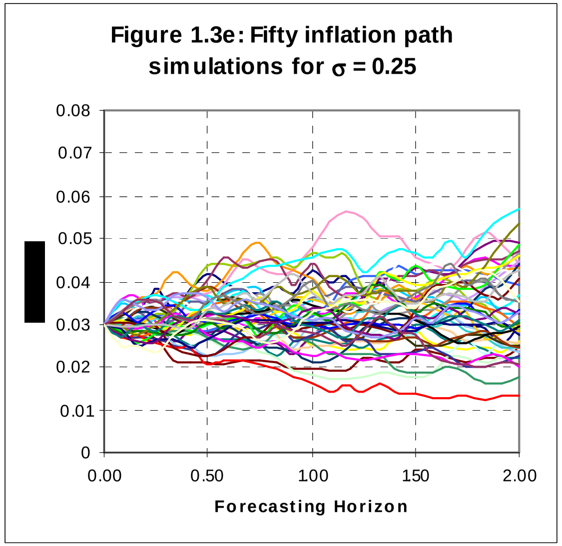


Figure 1.3e: Fifty inflation path simulations for σ = 0.25

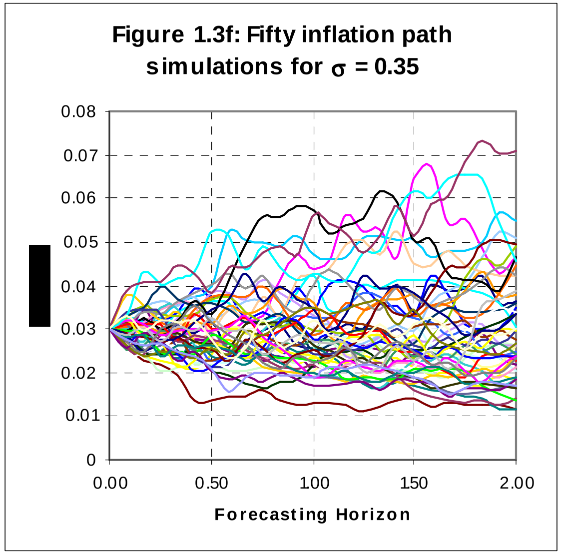


Figure 1.3f: Fifty inflation path simulations for σ = 0.35

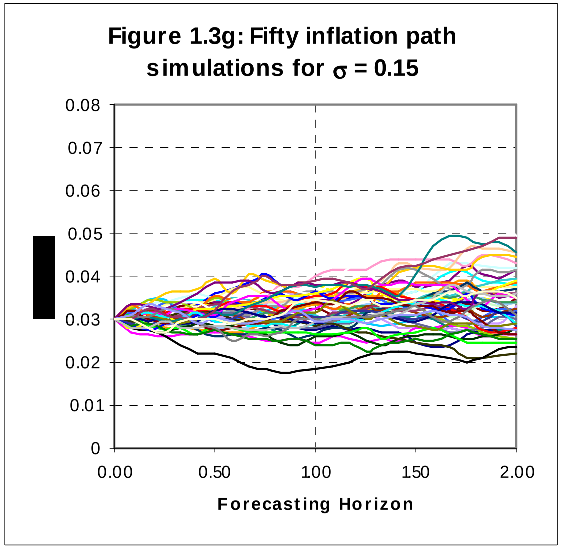


Figure 1.3g: Fifty inflation path simulations for σ = 0.15

Figure 1.3 e, f and g illustrate the importance of volatility, σ, in determining the spread of the random inflation paths. The sample paths are generated using parameter values: $\pi_t = 0.03$ and $r = 0.05$. Three standard deviations: $\sigma_1 = 0.25$, $\sigma_2 = 0.35$ and $\sigma_3 = 0.15$ are used. Clearly within the 2 years forecasting horizon, the one year horizon can easily be determined. The key insight here is: as volatility increases, terminal inflation outcomes become more dispersed frequently presaging a greater likelihood of breaching an upper bound, *k*. Likewise as the forecasting horizon is extended the dispersion of outcome also increases. Targeting inflation over a longer time frame may obviate the need to move interest rates dramatically further into the future but this would seem to precipitate bigger rate movements today. See Table 1.3 for time values using small sample sizes of fifty inflation paths and fifty terminal inflation values for these distributions.

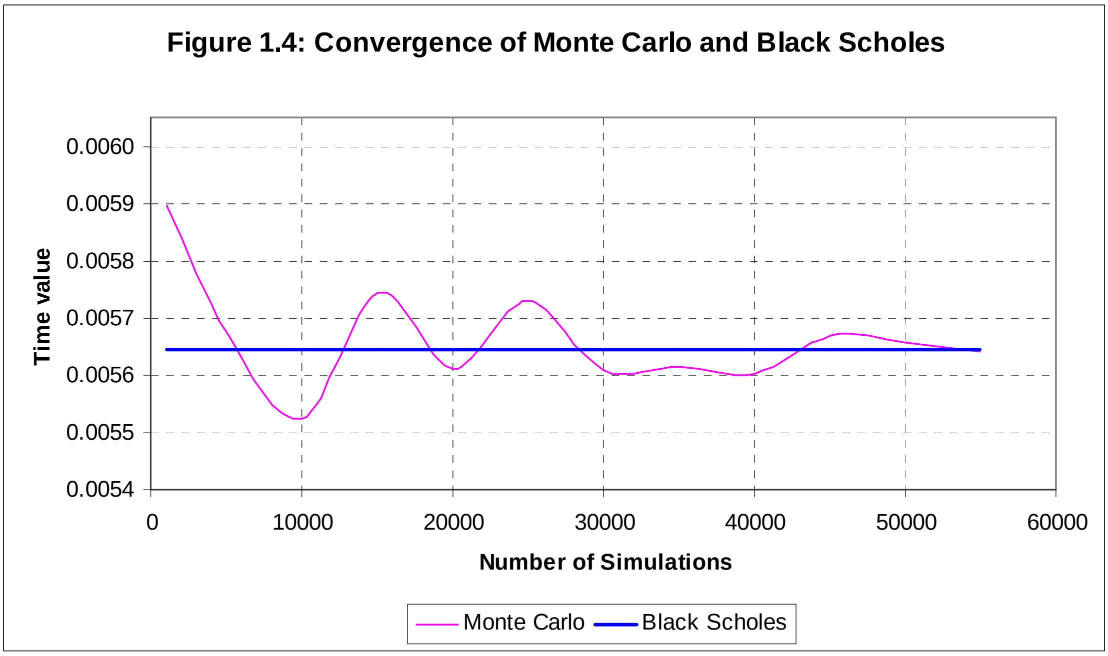


Figure 1.4: illustrates that as the number of simulations increase, the continuous time Black-Scholes model and the lognormal random walk model, estimated using discrete time Monte Carlo, converge. Parameter values are $\pi_t = 0.03$, $k = 0.03$, $T = 2$, $r = 0.05$, $d = 0$, $\sigma = 0.25$, n = 1,000, 5,000, 10,000, 15,000, 20,000, 25,000, 30,000, 35,000, 40,000, 45,000, 50,000 and 55,000.

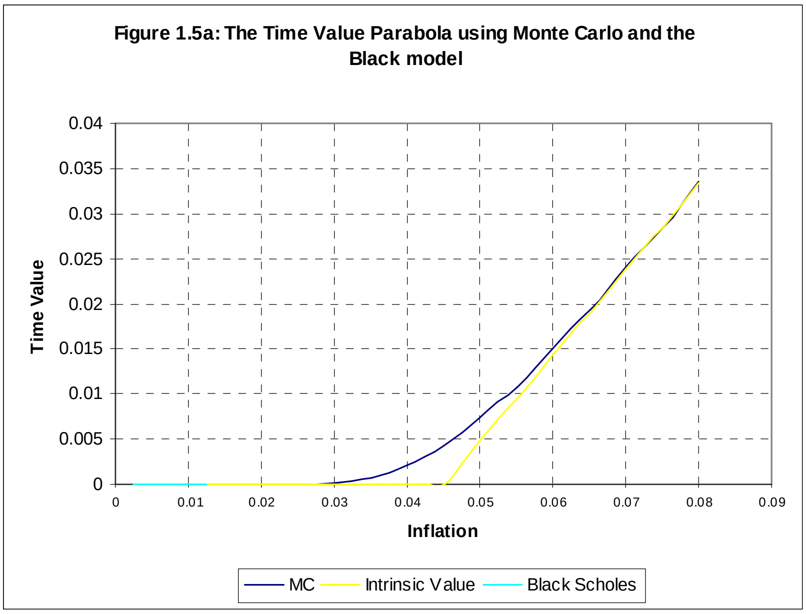


Figure 1.5a: generates call time values using both Monte Carlo and the BS formula. Each is generated with the following Black-Scholes parameter values: π_t = 0.25% to 8%, k = 0.045, T = 1, r = 0.05, d = 0.05, and σ = 0.25 (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.25% to 8%, k = 0.045, T = 1, r = 0.05 and σ = 0.25). Monte Carlo uses 50,000 simulations. No clear visible difference is observed between Monte Carlo and the Black-Scholes time valuation. See Table 1.4 for numerical values.

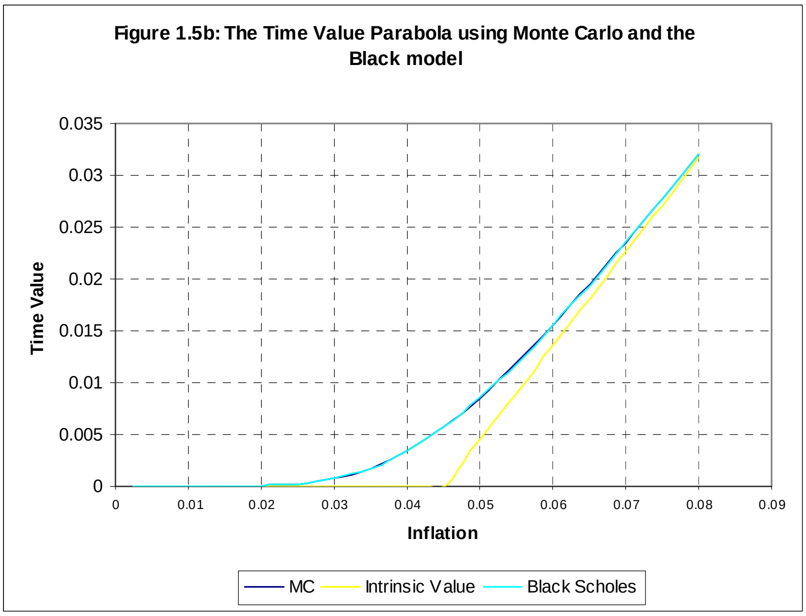
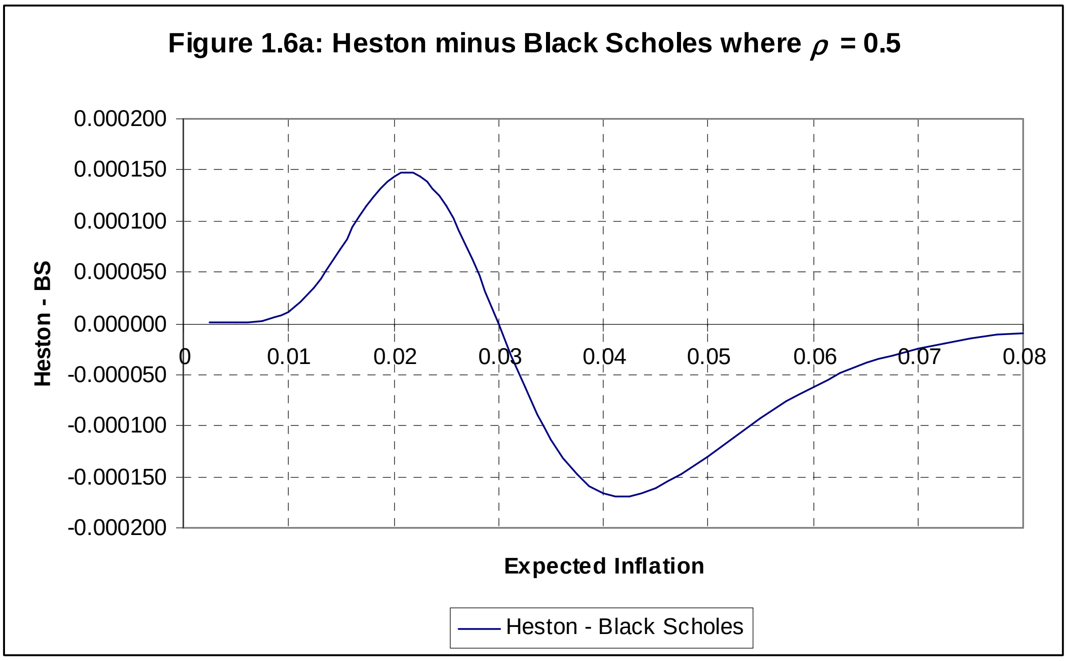
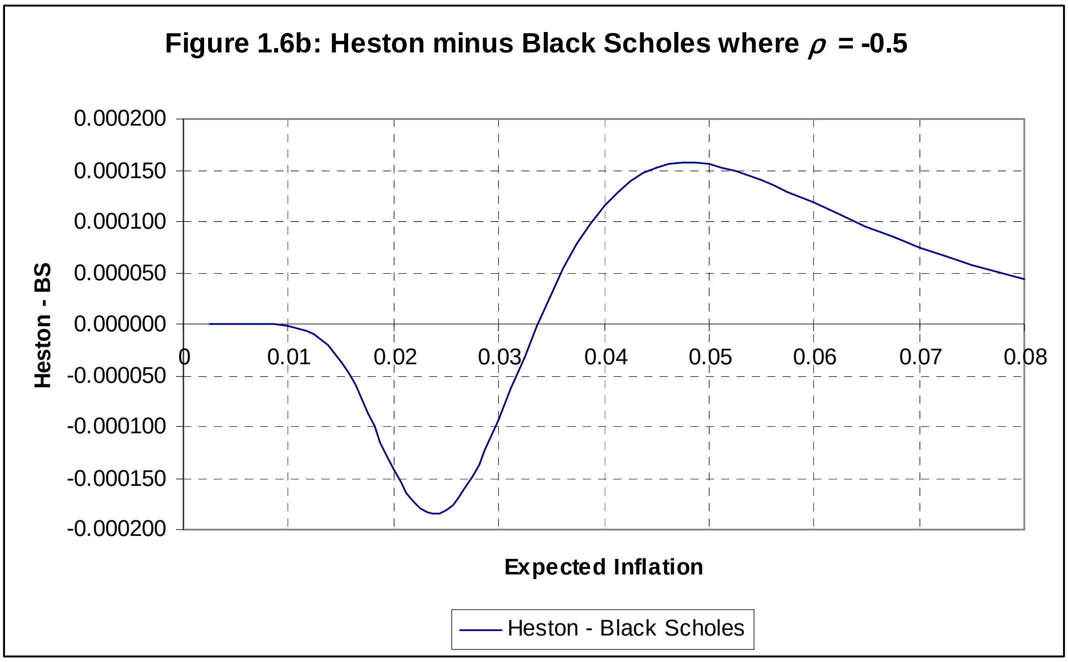


Figure 1.5b: generates call time values using both Monte Carlo and the Black-Scholes (Black) formula. The same parameters values are used, except $\boldsymbol{T}$ **= 2**. The call valuations are equal using either numerical or closed form approach.





Figures 1.6a and 1.6b trace out the differences in call time values as estimated by Model 1.2 (T.2) and Model 1.3 (T.2) respectively minus the Black-Scholes. It is clear that as the correlation, ρ, becomes increasingly negative (i.e. as the level of skew becomes more negative), the Heston Monte Carlo simulation produces time values that fall relative to Black-Scholes valuation when the option is out-of-the-money and increase when the option is in-the-money. Significantly, these discrepancies between Black-Scholes and Heston are here relatively small, 1.5 to 2 basis points, using the parameters in Table 1.5a. The policy interest rate changes, in contrast, are generally of the order of 25 to 50 basis points. The impact of the discrepancies would seem quite moderate.

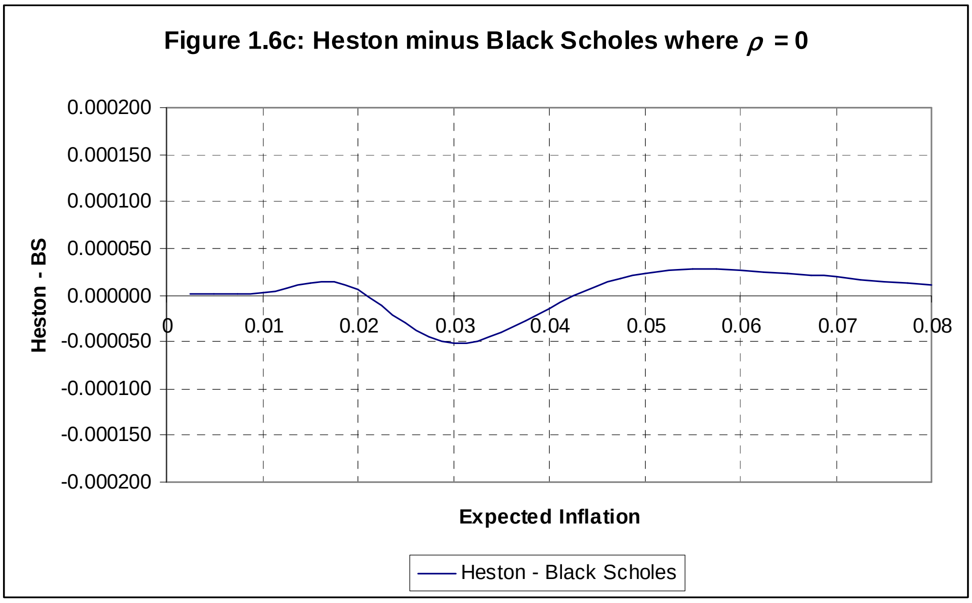
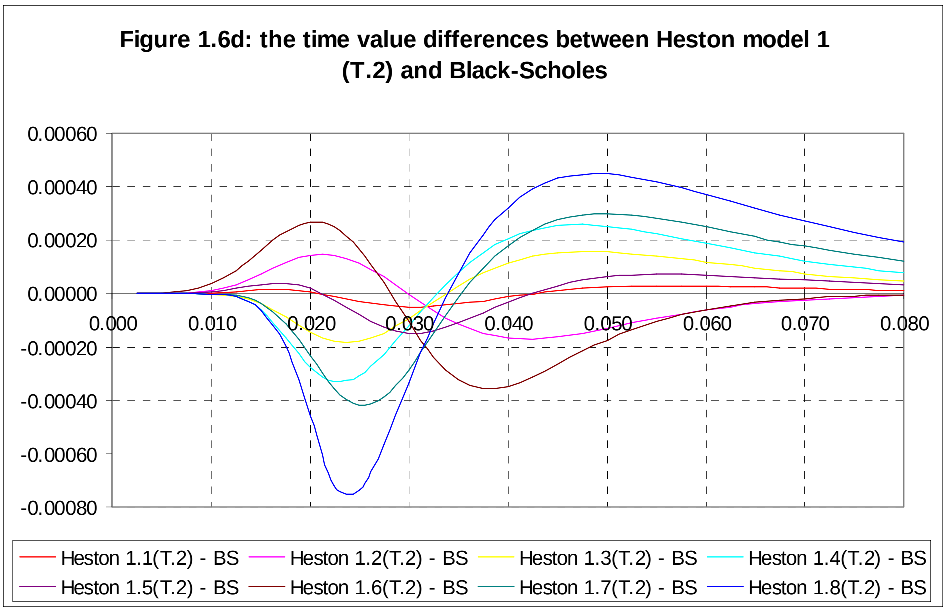
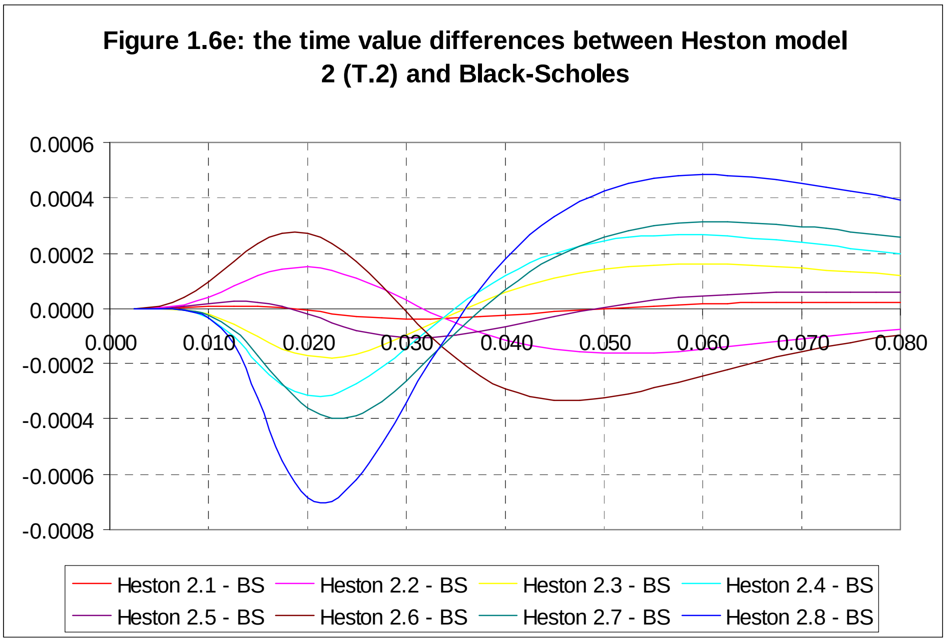
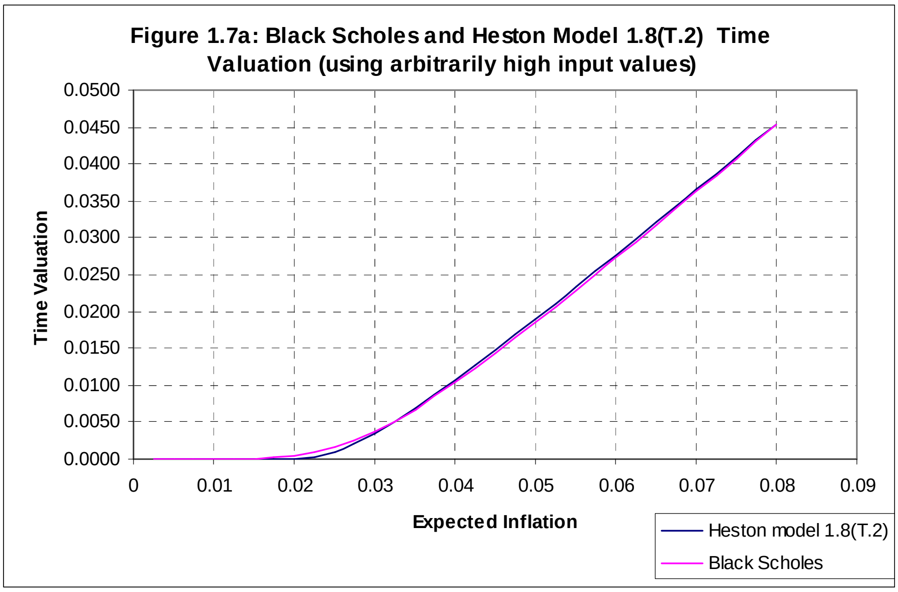


Figure 1.6c reports call time values using both the Black Scholes formula and Monte Carlo Heston (1993). The parameter inputs associated with Heston Model 1.1 (T.2) are used. The Black Scholes parameter values are: π_t = 0.25% to 8%, k = 0.03, T = 2, r = 0.05, d = 0.05, and σ = 0.25 (alternatively the Black parameters: $E(\pi_{t+1})$ = 0.25% to 8%, k = 0.03, T = 2, r = 0.05 and σ = 0.25). ρ is set at an initial baseline of 0. It is clear that when the level of correlation, ρ, is set equal to zero, the differences between the Heston time values and Black-Scholes time values decline markedly.





Figures 1.6d and 1.6e trace out the differences between the time values for each of the Heston models and the Black-Scholes models where the volatility magnitude was matched. The parameter inputs for each of these models were specified respectively in Tables 1.6 and 1.7. The time values that are used to calculate the Heston models minus the matched Black-Scholes models were given, respectively in Tables 1.5a and 1.5b.



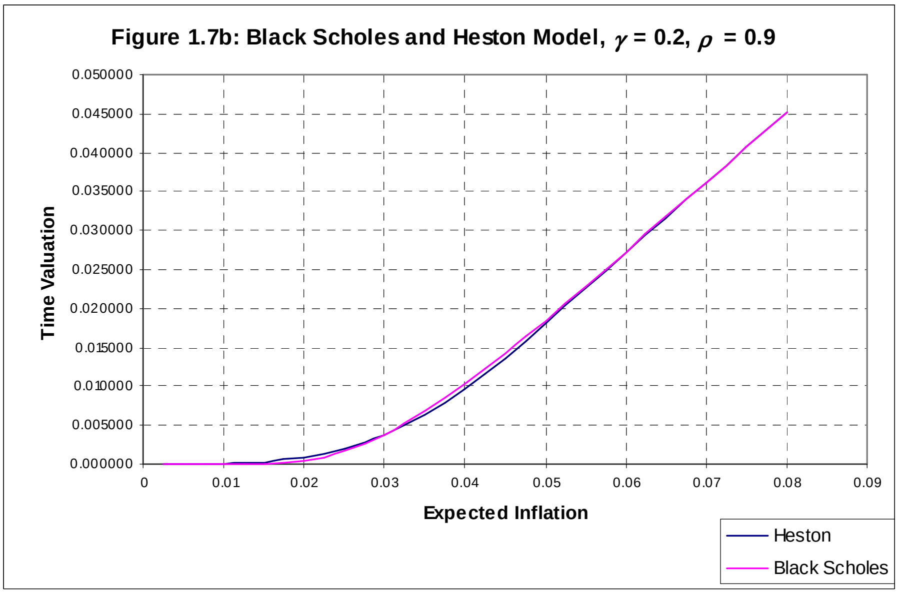
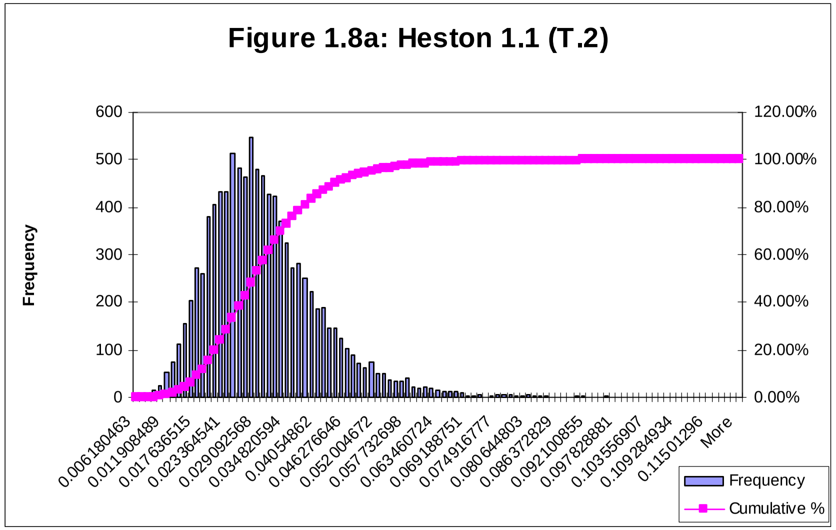
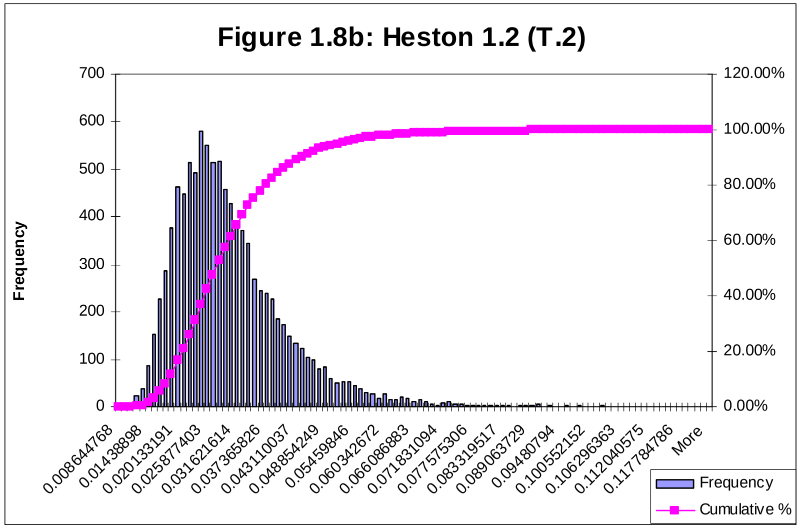
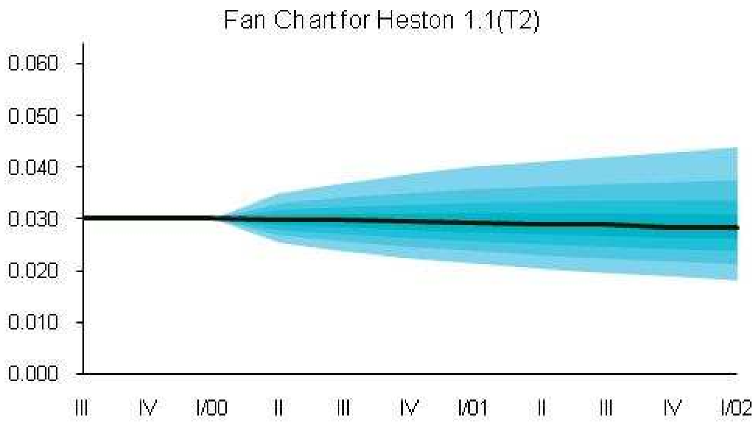


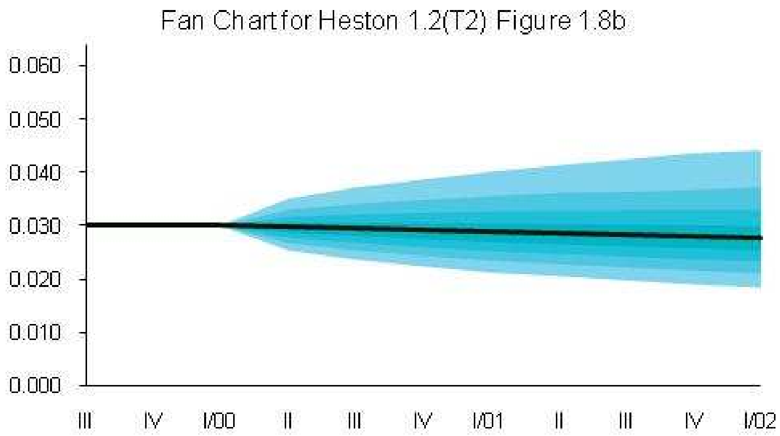
Figure 1.7a presents time values for the Black-Scholes model and the Heston model that use the same parameter inputs as Model 1.8(T.2). Relatively small discrepancies emerge between the two time value parabolas. The Heston inflation paths and descriptive statistics reported in Table 1.6 for model 1.8(T.1) suggest that even when kurtosis and skew are high relative to the historic norm suggested by period 1989:12 - 2007:5, the Black-Scholes model appears to provide time values that are close to Heston. Figure 1.7b also produces Heston and Black-Scholes time values that are relatively close.

$$Heston\ c = \frac{e^{-r(T-t)}}{n}\sum_{i=1}^{n} max\left[\pi_{t,i}e^{\sum_{j=1}^{m}\left(r_{j,i}-\frac{\sigma_{j,i}^2}{2}\right)\Delta t+\sigma_{j,i}\sqrt{\Delta t}z_{1(j,i)}}-k,0\right] \qquad (1.52)$$









Figures 1.8 a – 1.8 h provide an overview of the effects of altering parameter inputs γ and ρ on the probability distribution of expected inflation. Using varying combinations of parameters values, presented as Heston models 1.1 (T.2) – 1.8 (T.2) in Table 1.6, 8 pairs of graphs are produced outlining both a histogram and corresponding fan chart for each model. The histograms are generated by taking terminal values from the

Figure 1.8c: Heston 1.3 (T.2)

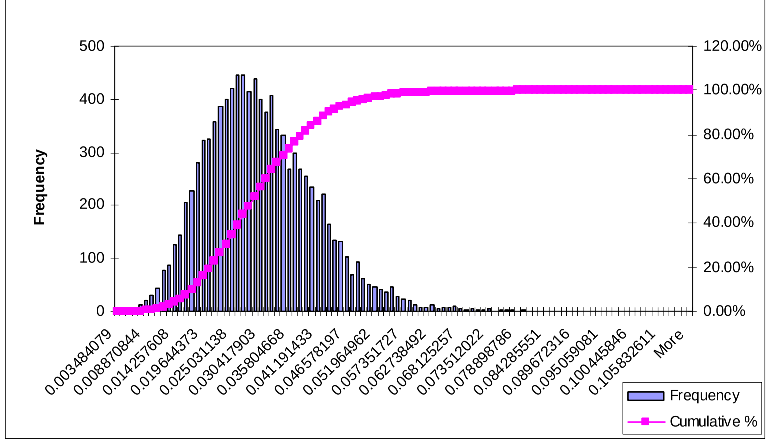
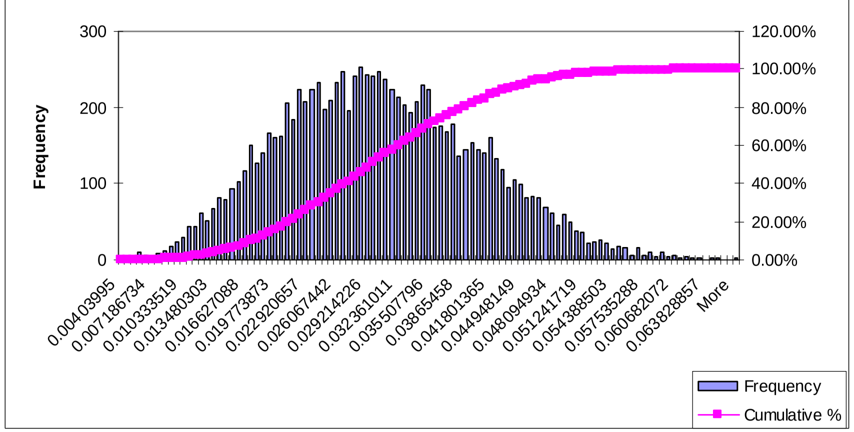
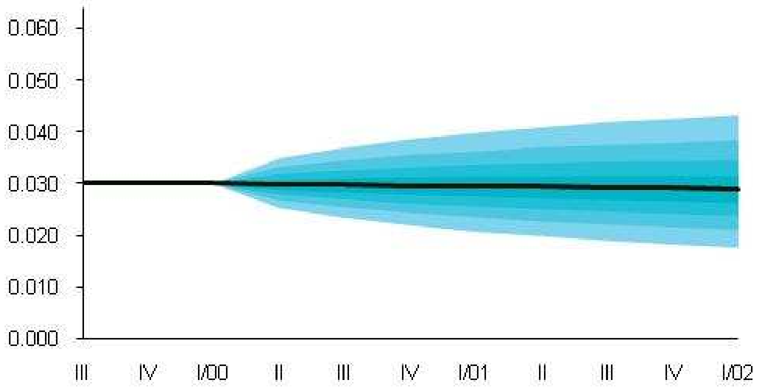


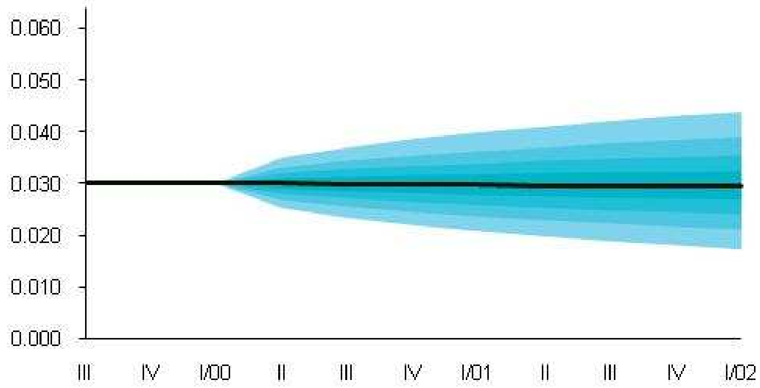
Figure 1.8d: Heston 1.4 (T.2)



Fan Chart for Heston 1.3(T2) Figure 1.8c



Fan Chart for Heston 1.4(T2) Figure 1.8d



Heston 1.1 – 1.8 (T.2) Monte Carlo simulations. 10,000 paths were used in each two-year simulation. The moments of these distributions are given in Table 1.6 and are referred to as the generated moments: that is the mean, standard deviation, skew and kurtosis of expected inflation. In addition, a corresponding inflation fan chart is generated for each of the simulations using quarterly observations. (Not just the terminal values from the simulation.) Varying distributional asymmetries can be demonstrated.

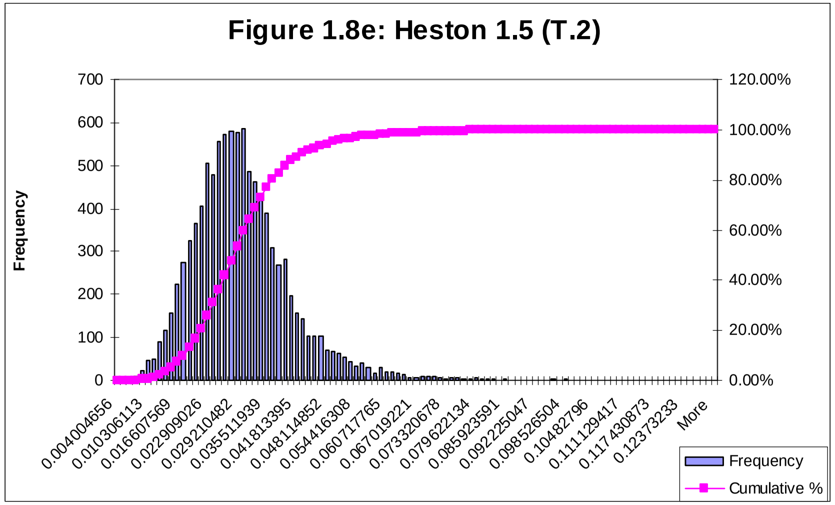


Figure 1.8e: Heston 1.5 (T.2)

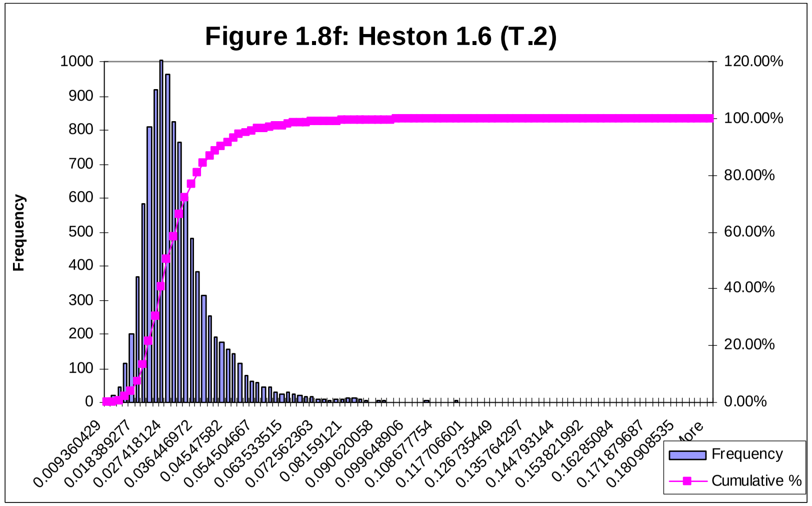
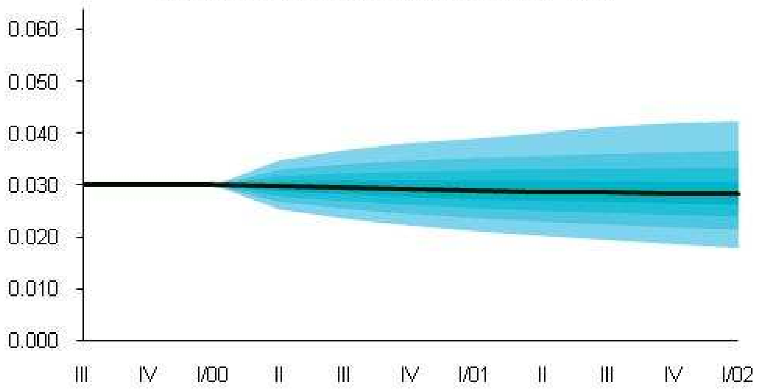
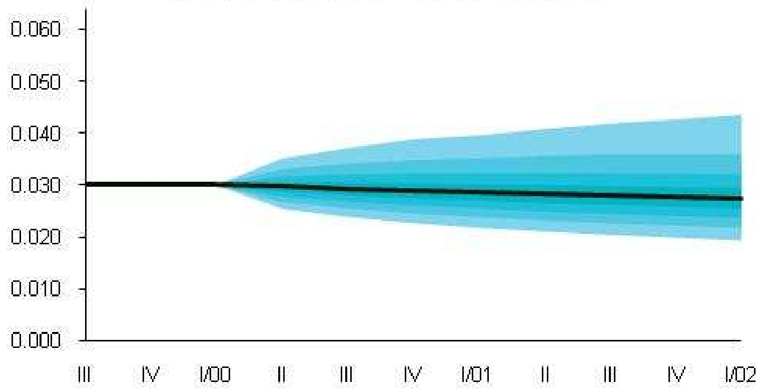


Figure 1.8f: Heston 1.6 (T.2)

Fan Chart for Heston 1.5(T2) Figure 1.8e



Fan Chart for Heston 1.6(T2) Figure 1.8f



Upside and downside risks to inflation can be made explicit in the forecasts, with confidence intervals being identified by the band shading. The fan chart captures 90% of all the simulation values. At each time step or period the values that are produced by the simulation are ordered. Each band captures 10% of observations. Hence, the two darkest bands, at the centre, capture 20% of the ordered observations. This type of confidence

Figure 1.8g: Heston 1.7 (T.2)

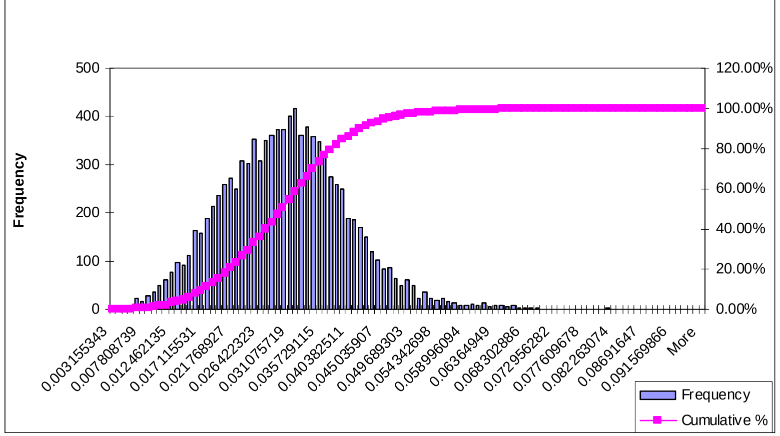
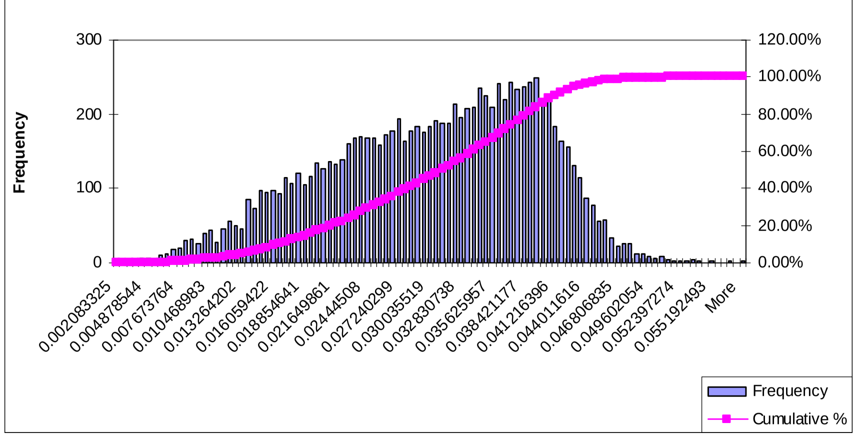
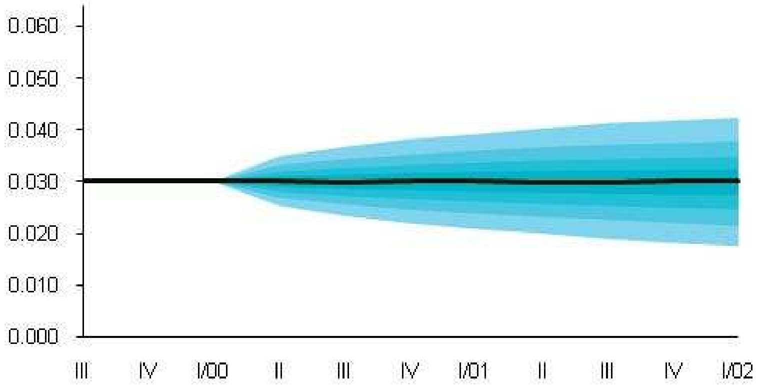


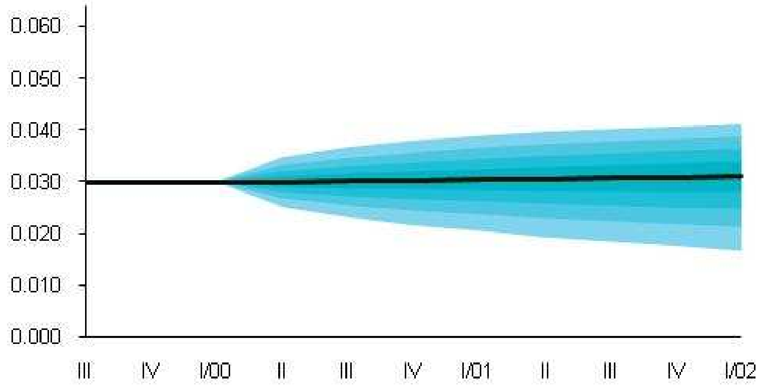
Figure 1.8h: Heston 1.8 (T.2)



Fan Chart for Heston 1.7(T2) Figure 1.8g



Fan Chart for Heston 1.8(T2) Figure 1.8h



interval constructions makes explicit the distributional asymmetries that are associated with parameter input combination for each of the Heston models. Equally, it can be said that the option's framework can be made sufficiently flexible to incorporate upside and downside risks. Table 1.6 outlines the parameter inputs associated with each of the Heston models.

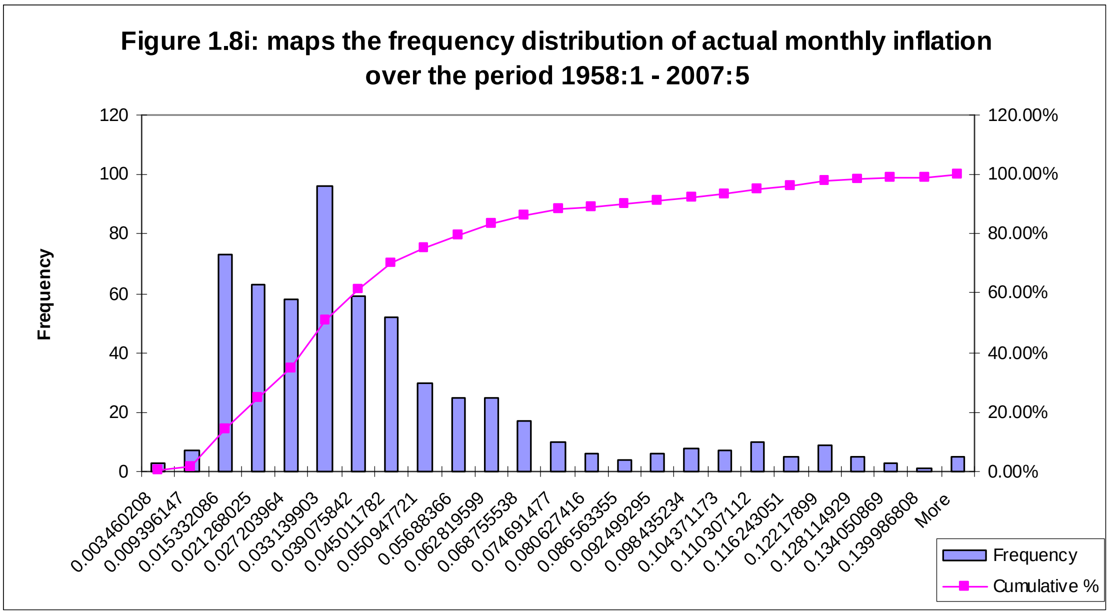
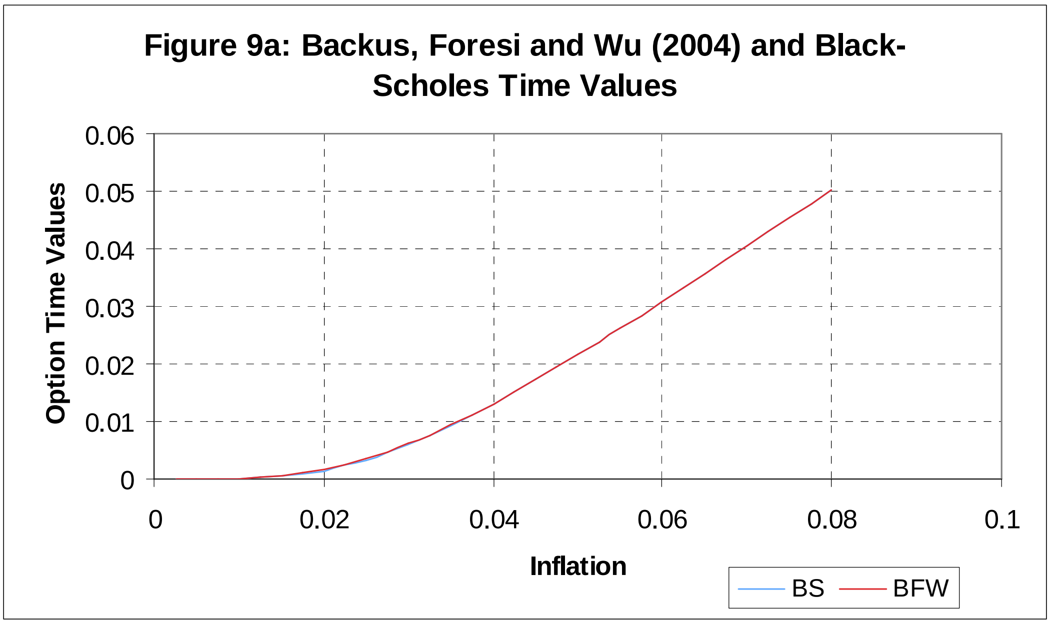
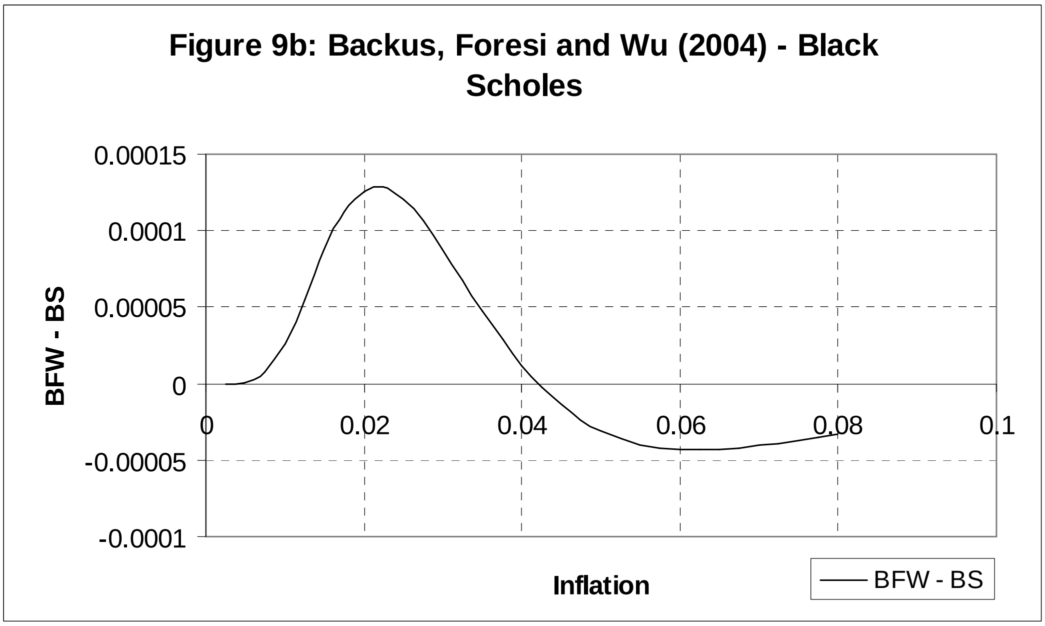


Figure 1.8i traces out the frequency distribution of actual monthly inflation over the period 1958:1 – 2007:5. The U.S. Consumer Price Index (Series i.d.: CPIAUCSL) was downloaded from the Federal Reserve Bank of St. Louis, from which the percentage annual change was calculated.





Figures 1.9a and 1.9b illustrate that only minor differences would appear to emerge between Black-Scholes and Backus, Foresi and Wu (2004) time values. The skewness and kurtosis parameter inputs were estimated on inflation return over the Greenspan incumbency. These historical estimates however may have been different from *ex ante* estimates. The inflation returns were calculated from the same data reported in Figure 1.8i.

Chapter 2

Opportunistic Policy under uncertainty.

‘Uncertainty is not just an important feature of the monetary policy landscape; it is the defining characteristic of that landscape. As a consequence, the conduct of monetary policy in the United States at its core involves crucial elements of risk management, a process that requires an understanding of the many sources of risk and uncertainty that policymakers face and quantifying of those risks when possible. It also entails devising, in light of those risks, a strategy for policy directed at maximizing the probabilities of achieving over time our goal of price stability and the maximum sustainable economic growth that we associate with it.’ (Remarks by Chairman Alan Greenspan, at a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 29, 2003.)

2.1. Introduction

The proliferation of financial derivatives since the 1970s has been unprecedented. The expansion in broker and exchange traded contracts that embed greater complexity has brought many welcome and unwelcome developments that central bankers are now forced to understand and deal with. From the remarks made by key central bankers, it would appear that risk management techniques have coloured the thinking and vocabulary of pivotal policy makers for several years. What precise elements of risk management influenced monetary policy has however never been clearly enunciated. In this chapter, the Black (1976) model is used largely as an interpolation tool to consider opportunistic policy under uncertainty. This approach, developed in chapter 1, makes explicit the relevance of parameters that normally the literature has ignored when analysing rate decisions. The volatility of the underlying inflation, the targeting horizon of policy and the ease with which consensus can be achieved by rate setting committees are examined by applying a standard option’s framework.

The advocacy of the ‘opportunistic’ strategy by ‘Fed insiders’ and the explicit adoption of zone targeting elsewhere would suggest that monetary policy is nonlinear, at least in conception. The opportunistic model developed by Aksoy, Orphanides, Small, Wieland, and Wilcox (2006) is extended in this chapter by applying portfolio option theory to consider the effects of uncertainty. The opportunistic construction, set out in (2.5), is investigated using an option's framework and from what Federal Reserve officials have expressed at the policy table. FOMC transcripts from the late 1980s are examined to see how central bankers devise and implement monetary policy in the context of moderate and stable levels of inflation.[128] Several officials have explained quite publicly and at FOMC meetings that interest rate decisions were the product of an opportunistic approach. This would imply that ignoring all other factors, policy was nonlinear.[129]

In Section 2.2 the early evolution of the opportunistic strategy is traced out and its impact on the policy debate, during the initial phase of the Greenspan chairmanship is considered. Option pricing theory is used in Section 2.2 and 2.3 to exploit further the opportunistic models proposed by Orphanides and Wilcox (2002) and Aksoy et al. (2006). Option theory is developed to examine the effects of uncertainty on monetary policy. A combination of two put-call parity relationships is exploited to express inflation in terms of long calls and short puts. It is shown that if policy is inactive over a given range of inflation, the two put-call parity relationships can be aggregated to form a long position with a call, with a higher exercise, and a short position with a put, with a lower exercise on inflation. The option portfolio that underpins this collar construction is useful in describing inflation zone targeting when monetary policy is subject to uncertainty.[130] A number of nonlinearities, that are found to exist in policy, are explained using option theory. Different option pricing models are also proposed to take account of varying inflation behaviour.

In Section 2.3 monetary policy decision making that conceptually apply a risk management approach is developed using portfolio option theory. The Black model is used to estimate the time value of the collar and the likely policy response under uncertainty. Importantly, the long-established delta, associated frequently with hedging, is calculable for this portfolio of options. Delta is one of a number of 'Greeks' that is used innovatively to show

[128] The transcripts are available in the public domain from the Board of Governors website.

[129] Of course the term opportunism is relatively new and only has gained common parlance since the mid 1990s. The earlier FOMC transcripts during the Greenspan tenure make no explicit reference to opportunism or use the term opportunistic. Here, it is assumed to mean implicit flexible zone targeting.

[130] See McDonald (2003) for definition. Here, the collar refers to the purchasing (longing) of a call option and selling (shorting) of a put option on inflation with a lower strike (exercise). The collar width is the difference between the call and put strikes. The use of the term collar is not meant to imply that inflation can not exceed or fall below the designated strikes.

that interest smoothing or inertia is dynamic and is dependent on a number of factors including the width of the upper and lower target bounds. The delta metric is also used to illustrate that as the target zone for inflation widens, monetary policy becomes increasingly nonlinear. Section 2.4 draws on the transcripts to identify to what extent, this type of risk management reflects the broad thrust of comments made by central bankers. In part, this is intended to demonstrate the effects of policy makers disagreeing. This leads to considering how divergent opinions amongst voting committee members curtail precipitated interest rate moves. Section 2.5 widens the analysis to consider how the risk management framework is affected when inflation is considered to mean revert. Using the Vasicek model, it is possible to incorporate both mean reversion and deflation. The asymmetries associated with the Black model are also viewed against a number of metrics; including a Vasicek delta. In Section 2.6, several nonlinear reaction functions are set out and estimated. A key result relates to how well nonlinear rules that operate lower bounds (i.e. with a 1 – 3% target range), serve to improve upon the standard Taylor Rule appraisal of rate setting.

2.2. The Federal Reserve and the advent of Opportunism

Opportunistic monetary policy was largely conceived within the Federal Reserve as a framework for Fed Funds rate setting during the period that is referred to as the Great Moderation. From the mid 1980s as inflation stabilised, policy makers were confronted with continuing the disinflation process while not aggravating losses in output in the United States. The Federal Open Market Committee (FOMC) had accumulated a considerable measure of credibility in reducing inflation during the Volcker years. At the outset of the Greenspan tenure, inflation was moderate relative to the previous decade. Any immediate future triumph in containing prices would be naturally deemed modest by comparison with what went before.

It may have been felt that to elicit further support from political institutions to carry on pushing inflation down would be problematic, particularly, if it implied a short-term opportunity cost in terms of output and jobs. In addition, monetary aggregates were gradually being de-emphasised so the capacity to find political cover became increasingly constrained.[131] Conceivably, the lack of consensus at committee level and institutionally, discouraged robust monetary intervention, although previously during the Volcker incumbency robust monetary policy was judged necessary and beneficial. A number of studies suggested that preferences could be alternatively defined by a non-quadratic loss function. Orphanides and Wilcox (1996, 2002) and Aksoy, Orphanides, Small, Wieland and Wilcox (1997, 2006) pointed out that it is possible to motivate the central bank's policy rule by a non-quadratic loss function. This type of innovative specification configured the central banker as an agent who is more preoccupied with small departures of output from target and less so relatively with small incipient departures of price level from stability. *Per contra*, the marginal trade-off between inflation and output, abstracted from a quadratic loss function, were found to be linear. Only increases in inflation over a critical range/point spur policy makers to supply robust rate changes. The appeal of this type of central banker profile is that it makes her less a creature of habit, in so much as, she opportunistically responds in a nonlinear fashion to changes in inflation.[132]

It could also be asserted that when inflation fell to quite moderate levels, relative to the early 1980s in the United States, the FOMC members became less decidedly resolute in continuing to push inflation down. Attempts to continue lowering inflation were made all the more difficult when it was inevitably acknowledged that the operating procedure had switched to targeting the Federal Funds rate. Kohn (1996) pointed out that once inflation had fallen to moderate single digits, policy makers responded differently. While the gains from containing inflation during the Volcker years were clearly apparent and *post hoc* generally applauded, the exercise of increasing unemployment above NAIRU was considerably more contentious when inflation was less than 4% or 5% per annum. The evolution of policy-making culture that occurred from the mid to late 1980s feasibly saw politicians and society generally less willing to accept a restrictive monetary policy.[133]

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[131] See Thornton (2004).
[132] See the policy debate as recorded in the December, 1989, FOMC transcripts, p.18.
[133] See President Forrestal's comments at the FOMC December meeting 1989, page 14. 'The parallel to 1979-80 time frame, it seems to me, is not quite applicable because we were coming from double digit inflation, and I think people clearly recognise that that was a terribly insidious thing that was happening…….'

2.2.1 Defining Opportunism

During his tenure at the FOMC, Alan Blinder submitted that if inflation is close to an acceptable optimum, policy should not forcefully squeeze demand to contain prices. Instead it should adopt what has subsequently come to be described as the 'opportunistic' approach. One of the more striking features of the recent debate is the notable number of central bankers who perceive this approach as being what policy makers collectively instigated and implemented. Don Kohn (1996) for example, previously secretary and long standing economist to the FOMC and subsequently a member of the Board of Governors, characterised the nonlinear reaction function as involving:

> "different modes of behaviour on the part of the central bank depending on the prevailing level of inflation. When inflation is high, an opportunistic policy maker would actively seek to bring it down. The period of 1979 to 1982 is an example of this sort of situation. Inflation in 1979 was clearly too high, and the Federal Reserve fought it, opening an output gap. On the other hand when inflation is already low or moderate, the opportunistic policymaker does not take active measures to reduce it further. Once inflation had fallen into the 3.5 to 4.5 percent range in the mid-1980s, people observing the Federal Reserve thought they could not detect steps to lower it more." (Kohn, 1996, p.303)

This describes one of the principal elements of opportunistic interest rate setting, that is, policy is path dependent. The concept however also embraced a somewhat broader remit where the objective of policy is not just the performance of inflation today but examines the behaviour of inflation over the economic cycle.

> "…it [policy] leans very hard against increases in inflation. Examples of this in recent years would include the tightenings of 1984, 1988-89, and 1994. In these cases when inflation threatened to exceed its previous range the Federal Reserve firmed policy to prevent the uptick or bring inflation back into the range again." (Kohn, 1996, p.304)

This implied that the target range for inflation might have evolved through time and over the economic cycle. It also suggested that if a given threshold were breached, policy accommodation would be removed at an accelerated pace. The approach described by Kohn (1996) seemed qualitatively very different to the conventional Taylor Rule or inflation point targeting strategies.[134] A number of FOMC members have used the term 'opportunistic disinflation' to describe the strategy employed by the Federal Reserve during

[134] Bomfim and Rudebusch (1997) contrast the opportunistic approach to the conventional monetary policy advocated by Leiderman and Svensson (1995) and Haldane (1995).

the early 1980s and 1990s.[135] In retrospect, early proponents of the opportunistic approach advocated, in the late 1980s that while inflation was not exceeding 4½% but still perhaps above the long-run target, the monetary policy committee should refrain from aggressively fighting price increases. A longer-term perspective envisaged either favourable supply shocks or inevitably recession to deliver progress toward the desired target.[136] If inflation were increased above a given threshold level of tolerance, the opportunistic approach would only then attempt to curtail price rises even if this involved a sacrifice of output and jobs.

This, it was felt, also constituted a strategy for disinflation because it exploited the next recession or positive supply shock to lower inflation over time. The timeframe envisaged was ultimately governed by the business cycle. Proponents of this strategy sometimes have described the approach as reducing inflation by moving from cycle-to-cycle. A common interpretation fostered the notion of disinflating gradually or being one recession removed from price stability. Opportunistic policy nevertheless purportedly defines aggressive policy responses to any acceleration beyond an acceptable zone of inflation. The central banker also is seen as being capable of capitalising on the likely disinflation that would occur, going from peak to trough. In the appendix to the December 1995 FOMC meeting p. 10 – 11, Don Kohn spelled out how the opportunistic monetary policy would be constructive when dealing with other branches of government. A political gain secured by implementing opportunistic strategy related to the central bankers' capacity to be less evidently tarnished when removing the proverbial 'punch bowl'. Recession would shoulder more of the blame. This would have been made all the more welcome given the gradual de-emphasis of monetary aggregates in operationalising policy over the Greenspan incumbency. Arguably, this reading of policy is more in line with what some policy makers have said in the U.S., rather than deduced from an explicit mandate or from legislation pertaining to the Federal Reserve. Interestingly, the contemporaneous Canadian experience where an explicit strategy has been implemented since 1991, suggested that range targeting

[135] See Remarks made by Governor Laurence H. Meyer, At the National Association of Business Economists 38th Annual Meeting, Boston, Massachusetts, September 8, 1996.
[136] The intermediate target can only be defined for a given period. The term opportunism was not employed to describe nonlinearity until the mid 1990s.

became the basis for formulating monetary policy.[137] The definition of opportunism developed in this chapter resembles inflation zone targeting. The Federal Reserve however never clearly spelled out an inflation target during the Greenspan years.

2.2.2 The Taylor Rule and Opportunism

The Kydland-Prescott and Barro-Gordon literature identified the importance of commitment to a given policy rule. Taylor (1993) provided a benchmark that spelled out a form of conventional deliberative monetary policy that seemed in line with the Greenspan chairmanship. As a primer to understanding opportunistic policy setting, the Taylor Rule represents an intelligible grid reference for describing point targeting or targeting without zones. In this regard, it can be seen as a normative guide to appraise decision-making. In empirical work, the Taylor Rule is frequently nested in a forward-looking reaction function. A key advantage of this specification relates to the Taylor Rule being measurable through time and across regimes. The Taylor rule maps out deliberative systematic patterns in terms of implementation. In this respect, outlining the Taylor rule is a useful starting point when considering the opportunistic reaction function. This is the approach adopted by Aksoy et al. (2006).

A generalised Taylor Rule that does not of itself specify weights for output and inflation gaps but captures the broad thrust of linear policy is given by:

$$r_t = \left(r^* + \pi_t\right) + \omega_1\left(\pi_t - \pi^*\right) + \omega_2\left(y_t\right) \qquad (2.1)$$

In a forward looking context this becomes:

$$r_t = r^* + E_t\left(\pi_{t+1}\right) + \omega_1\left(E_t\left(\pi_{t+1}\right) - \pi^*\right) + \omega_2\left(y_t\right) \qquad (2.1a)$$

Alternatively,

[137] In 1991, the Bank of Canada targeted inflation for a five-year period. The inflation rate then was 5.9%. The original goal was to reduce inflation to gradually lower levels, first to 3%, later to 2.5% and then to 2%. By December 1993, inflation had fallen to 2% and the central bank extended the period of inflation-control target range to 1998. The target range was expressed as a band from 1% to 3%. In the same year the target range was extended to 2001. In 2001, the 1% to 3% range was imposed again and made applicable to the end of 2006. Monetary policy was intended to be aimed at keeping inflation at the 2% target midpoint. Bomfim and Rudebusch (1997) however contrast the Federal Reserve's opportunistic approach with the Bank of Canada's implementation of inflation targeting. Thornton (2007), using FOMC transcripts and minutes, conjectured that that the targeting zone ranged between 1% and 3.5% with a 2.25% mid-point.

$$r_t = r^* + \pi^* + (1+\omega_1)\left(E_t(\pi_{t+1}) - \pi^*\right) + \omega_2(y_t) \quad (2.1b)$$

The policy stance spelled out by (2.1), (2.1a) and (2.1b) is independent of the inflation path in that the objective of policy ignores the historical movements of the consumer price index and of the real economy. The parameter weights are constant and independent of the inflation level. The short-term interest rate r_t is determined by an equilibrium interest rate r^* and the contemporaneous inflation rate π_t or its expectation, $E_t(\pi_t)$. The weights ω_1 and ω_2 respectively, are applied on both the contemporaneous inflation gap, $\pi_t - \pi^*$ and output gap, y_t. Taylor (1993) arbitrarily applied the weight of 0.5 on each of the gaps in the classic rule.[138] π_t represents the rate of inflation over the previous four quarters, and π^* represents the inflation target. By relaxing these weights broader interpretations can emerge permitting wider latitude for describing different policy stances.[139]

Significant advantages of the rule relate to its simplicity and robustness. Clarida, Gali and Gertler (2000), for example, estimate the rule to gauge policy activism for pre- and post-1979 Federal Reserve regimes and Clarida, Gali and Gertler (1998) estimate the rule that applied in several countries.[140] Their analysis suggests that (2.1) is not only normative, it also furnishes a means to measure policy in very different contexts. This flexibility is important given that the Federal Reserve has at its disposal open market operations, the discount window and reserve requirements to implement monetary policy. It has not always been clear which lever the FOMC has been using for which end. Historically, by exerting varying levels of pressure on bank reserve positions, the FOMC exercises control of the Fed Funds rate. Broadly speaking the Federal Reserve is capable of assuming one of two fundamental postures. It can control the price of money or the quantity (in terms of reserves) but not both simultaneously. The Federal Reserve has adopted both perspectives of policy over the past three decades.[141] Targeting the Federal Funds rate however has been more usual. Importantly in terms of robustness, even when policy is designed to control

[138] These magnitudes were representative of a number of studies Taylor had reviewed in the early 1990s.
[139] If the weight on the output gap fell to zero for example, (2.1) could be construed to define a pure inflation targeting regime.
[140] The G7 minus Canada.
[141] Goodfriend (1991) maintained that even during the early Volcker years there was an unarticulated or implicit Fed Funds target. This accords with Thornton (2004) who analysed FOMC transcripts.

money supply there is by default an implied rate of interest.[142] In this regard, the formulation specified in (2.1) is flexible and constitutes a useful point of reference in understanding the conduct of monetary policy.

Interestingly while (2.1) has provided a good description of policy behaviour, particularly in the early Greenspan years, it does not accord entirely with what policy makers say they have actually put into practice. Kohn (1996) explicitly depicts Fed Funds setting as being conditioned on wider inflation path dynamics. This implies that the policy rule is nonlinear/dynamic and that is somewhat out of kilter with the classic Taylor Rule. Aksoy et al. (2006) for instance amend the Taylor Rule to take account of opportunism.[143]

$$r_t = r^* + \pi_t + f\left(\pi_t - \pi_t^*\right) + \omega_2 y_t \tag{2.2}$$

where

$$\pi_t^* = (1-\lambda)\pi^{**} + \lambda \pi_t^h \tag{2.3}$$

In this formulation π_t^* constitutes an intermediate target which is computed as a weighted average of the long-run target π^{**} and of the inherited inflation rate, π_t^h, representing a retrospective moving average of actual inflation. A simpler formulation originally posited by Orphanides and Wilcox (1996, 2002) has the long-term target potentially equal to zero.[144] This makes the intermediate target a function of past inflation and might be thought of as a special case of (2.3). In light of more recent analysis, this might be considered overly restrictive in terms of setting policy.[145] The magnitude of λ, lies between zero and unity implying that progress towards the inflation target can be gradual. This configuration also permits the target rate to rise:

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[142] The corollary is that even if a central bank explicitly uses money as the nominal anchor of monetary policy, its stance can still be evaluated using a Taylor type reaction function. Clarida, Gali and Gertler (1998) estimate the same policy rule for the Greenspan years as the Volcker years, despite the fact that operating procedures changed from targeting non-borrowed reserves to targeting the Fed Funds rate.

[143] Meyer (1996) compares both the Taylor Rule and Opportunistic policy and makes the point that the two work in tandem. 'This [opportunistic policy] is just another rule, though a more complicated one than the simple Taylor Rule.'

[144] See the explanation given regarding equations (1.8) and (1.9).

[145] The Boskin Commission (1996) generally has advocated for a bit more slack for policy makers in that, the CPI may not fully reflect tangible quality and technological improvements. Changes in the CPI have historically tended to overstate changes in the true cost of living. One implication of this type of analysis is that the Federal Reserve accordingly may find it desirable to have a target inflation rate slightly in excess of zero. This suggests that (2.4) is perhaps overly restrictive and (2.3) provides a more credible description of how the inflation target evolves.

$$\pi_t^* = \lambda \pi_t^h \qquad (2.4)$$

Policy makers may be conscious of varying biases that occur with regard to CPI measurement and consequently perceive price stability to be in a region higher than the zero percent point target. This may provide some evidence supporting the contention that $\pi^{**} > 0$. Aksoy et al. (2006) propose the following relationship where the interest rate penalty is defined by:

$$f\left(\pi_t - \pi_t^*\right) = \begin{cases} \kappa\left(\pi_t - \pi_t^* - \delta\right) \text{ if } \pi - \pi_t^* > \delta \\ 0 \qquad\qquad \text{if } \delta \geq \pi_t - \pi_t^* \geq -\delta \\ \kappa\left(\pi_t - \pi_t^* + \delta\right) \text{ if } \pi - \pi_t^* < -\delta \end{cases} \qquad (2.5)$$

κ represents the parameter value applied to the inflation gap when inflation moves outside the threshold band.[146] The policy rule spelled out by (2.1) differs from (2.2) in that the former is independent of inflation path dynamics. The latter however implies the policy rule is contingent upon the inflation context. If the difference between the inflation rate and its target is less than or equal to δ, (a given level of tolerance) the short-term policy rate component, derived from the inflation gap, remains unchanged. In other words, if inflation does not move outside a given defined interval then the fraction of r_t owing to the inflation gap (i.e. the inflation penalty)[147] in (2.2) does not actively drive inflation closer to its long run target. A larger magnitude of δ, permits the monetary policy committee to refrain from intervention for longer. This may have an advantage in that a wider target zone allows inflation to rise without the board having to resort mechanically to restraining the economy. Conversely, a wide band may erode the central bank's credibility in that policy reactions may be perceived to be sluggish. A narrowing of the band would also seem to signal a greater capacity for rate adjustment. The central banker may set δ so that the appropriate balance is struck between choosing a band width that effectively stabilises inflation expectations but not so narrow as to precipitate undue levels of policy intervention. Later, δ is seen as a measure of disagreement on the Monetary Policy Committee.[148]

[146] This portrayal of policy posits the central bank as a unified actor. Later in this chapter, the policy decision is described in terms of a committee that can disagree producing occasionally stalemate - when it is difficult to marshal a definite consensus for an increase or decrease in the policy rate.

[147] This is the term used by Orphanides and Wilcox (1996).

[148] The width of such a band may vary, as heterogeneity of opinion varies. A supply side shock is likely to produce greater disagreement internally amongst policy makers than a demand side shock. The effect of this

Conceivably, the zone width that was explicitly implemented by the Bank of Canada may have reflected practices employed elsewhere. Aksoy et al. (2006) used a magnitude of 1% for δ, which is consistent with the 2% bandwidth employed by the Bank of Canada since 1991. The magnitude of δ for convenience may be taken to be constant. A supply shock could plausibly widen the zone simply because policy makers would find it more difficult to agree. The effect of widening is examined in Proposition 2.3c. Heterogeneity of opinion can vary in degrees of intensity as macroeconomic events unfold. Kohn (1996) asserted that when inflation had fallen to the range 3½% to 4½% in the mid-1980s, Fed watchers could not detect additional resolve to lower inflation further. This suggests that alternative bandwidths were practicable and δ could have conceivably been as low as ½%. Figure 2.1 juxtaposes the inflation response associated with the opportunistic rule against the conventional linear inflation gap response.[149] The interest penalty $f\left(\pi_t - \pi_t^*\right)$ increases in a nonlinear fashion. Only when inflation passes through $\left(\bar{\pi} + \delta\right)$ will the interest penalty take effect.

It is noteworthy, that Kohn's explanations of the Fed's policy actions can be only understood as arising from nonlinear reactions to inflation. Opportunistic disinflation is described as representing an accurate account of FOMC interest rate determination during the earlier years of the Greenspan tenure. It is clear from Kohn (1996) that his understanding of policy was very much conditioned on these wider price dynamics, represented in Figure 2.1. In the following section, the opportunistic policy rule is motivated differently. Nonlinearity which is a theme developed in a broader literature is examined here where disagreement between committee-voting members is used as a motivational device. It is found that by incorporating an option portfolio, it is possible to describe policy in a manner akin to Aksoy et al. (2006). It is also found that the effect of uncertainty can be gauged using time valuations of this portfolio. The policy inaction or

would be to produce periodically contracting and expanding target bands, as the nature of economic shocks change.

[149] Figure 2.1 describes the representation of opportunistic policy as offered by Orphanides et al. (1997) and Aksoy et al. (2006). This constitutes a form of zone targeting, not unlike the Bank of Canada's approach, where an intermediate target evolves within a successive set of downward floating bands. The Federal Reserve has never explicitly stated an inflation target.

stalemate feature associated with the nonlinear approach is analogous to other types of nonlinearity found in the literature. For example, in the Purchasing Power Parity (PPP) debate, Taylor and Taylor (2004, p. 147) point out that[150]:

> "The qualitative effect of such frictions is similar in all of the proposed models: the lack of arbitrage arising from transaction costs such as shipping costs creates a 'band of inaction' within which price dynamics in the two locations are essentially disconnected."

While transaction cost models have been used to explain the PPP puzzle, the 'band of inaction' has less formally been motivated by Kilian and Troy (2003) suggesting that nonlinearity emerges from the lack of agreement amongst traders. The policy stalemate as described below regarding interest rate decisions accords with the rationale advanced to explain nonlinearity in the International Economics literature. Heterogeneity of opinion in the foreign exchange market implies that only extreme values of the nominal exchange rate generates a sufficient degree of consensus amongst traders concerning the appropriate future direction regarding exchange rates. Here, the impact of stalemate is considered also when inflation outcomes are uncertain. This provides the basis for looking at policy using an option's framework.

2.2.3 The Put-Call Parity Relationships

Proposition 2.1: *Under opportunistic interest rate setting, the monetary policy response to inflation* $\left(E_t\left(\pi_{t+1}\right)-\pi_t^*\right)$ *can be modelled as the intrinsic payoff from a portfolio of options, when future inflation is known with certainty.*

Put-call parity constitutes one of the most significant defining relationships in option theory. A synthetic forward with inflation as the underlying can be developed from a short European put and long European call on expected inflation where the strike is the same for both contracts. The difference between the value of a call and put must be equivalent to the present value of the difference between the forward price (expected inflation) and the exercise. From chapter 1, the following relationship was identified:

[150] Taylor and Taylor (2004) point out that Heckscher (1916) allowed for the possibility that absolute PPP did not always hold by virtue of transaction costs in international transactions. Nonlinearities can arise because it is not worth arbitraging and correcting price differences if the anticipated profits do not exceed the costs of shipping goods and administration between the two locations.

$$Call - Put \ = \ PV(Forward \ \ Price - Strike \ Price) \qquad (1.30)$$

Put-call parity opens the possibility that an option portfolio can be nested in the reaction function. By combining two parity relationships, expected inflation can be shown to be modelled similar to a collar construction.[151] In this instance, the collar is comprised of a single call and a single put with initially the same strike. Consider the following two portfolios:

Portfolio A: one European put option on inflation with an exercise π_1^* plus the discounted forward or expected inflation rate, $E(\pi)e^{-rT}$.

Portfolio B: one European call option on inflation with the same exercise, π_1^*, plus the discounted exercise $\pi_1^* e^{-rT}$. Both are worth $\max(\pi, \pi_1^*)$ at expiration. Using put-call parity the portfolios A and B must have identical values given that the strikes and time periods of the contracts are the same. Thus:

$$c_1 + \pi_1^* e^{-rT} = p_1 + E(\pi)e^{-rT} \qquad (2.6)$$

Likewise consider two similar portfolios but with a *higher* exercise than observed in (2.6):

Portfolio C: one European put option on inflation with an exercise π_2^* plus the discounted expected inflation rate, $E(\pi)e^{-rT}$.

Portfolio D: one European call option on inflation with an exercise, π_2^*, plus the discounted exercise, $\pi_2^* e^{-rT}$. From put-call parity it is possible to establish the identity:

$$c_2 + \pi_2^* e^{-rT} = p_2 + E(\pi)e^{-rT} \qquad (2.7)$$

From chapter 1 the following general relationship was found to exist:

$$c - p = e^{-r(T-t)}\left[E(S) - k\right] \qquad (1.32)$$

Where S constituted the underlying asset and k denoted the exercise or strike. This implies that the following must hold:

$$\left[E(\pi) - \pi_1^*\right] = [c_1 - p_1]e^{rT} \qquad (2.8)$$

[151] Deacon, Derry and Mirfendereski (2004) have documented the rapid growth of inflation derivatives. In the U.K., inflation collars are common in the pension funds industry by virtue of a regulatory quirk: Limited Price Indexation (LPI). Pensions paid by an occupational pension schemes, and protected rights paid by personal pension schemes must increase by at least a given rate each year. This rate is the lesser of the two: five per cent or the increase in the Retail Price Index. Pension payouts are related to the Limited Price Index (LPI), which has a floor of 0% and a cap of 5%. It should be noted that when nesting a collar in the reaction function here, it is not implied that inflation can not move beyond the upper and lower bound.

$$\left[E(\pi)-\pi_2^*\right]=\left[c_2-p_2\right]e^{rT} \qquad (2.9)$$

It is clear from (2.1a) and (2.1b) that inclusion of the option portfolio $\left[c_2-p_2\right]$ into the Taylor Rule is possible.

Proposition 2.1a: *Under opportunistic or zone targeting interest rate setting, the monetary policy response to inflation given by (2.5), $f\left(\pi_t-\pi_t^*\right)$, can also be modelled as the payoff from an option's framework.*[152]

Proposition 2.1b: *Even when individual members of the board or rate setting committee have individually adhered to linear rate setting, the dynamics of majority voting, when policy makers disagree, permit their collective behaviour to be characterised by $f\left(\pi_t-\pi_t^*\right)$.*

Proposition 2.1c: *When expected inflation is known with certainty* then $f\left(\pi_t-\pi_t^*\right)$ *can be modelled as the intrinsic payoff from the option's portfolio.*

Propositions 2.1b - 2.1c imply that disagreement between policy makers, precipitates a policy stalemate, over a range of expected inflation. Additional rate decisions can only be implemented when a majority of votes are achieved for a given policy action. This will be characterised below as expected inflation moving outside the zone defined by the upper and lower exercise. Disagreement may not be solely present at the committee, wider institutional disagreement with political agencies may also produce stalemate.

Proposition 2.1d: *If expected inflation is not known with certainty, policy makers can invoke risk management principles, developed from appendix A.2.1. This permits the time valuation of the option's portfolio payoff to be applied to $f\left(\pi_t-\pi_t^*\right)$. As a result, the pre-emptive raising and lowering of the policy rate when expected inflation resides within the upper and lower bounds can be explained using standard option theory.*

[152] It is assumed that (2.5) can be made forward looking.

Later, it will be shown that increased uncertainty makes policy increasingly linear. Time valuation can be used to examine the effects of uncertainty and gauge the level of policy activism when standard risk management principles are employed. When applying the option's framework to $f\left(\pi_t - \pi_t^*\right)$, the portfolio is composed of a long call and short put where the exercise on the former is specified to exceed the latter. The higher exercise corresponds to the target rate of expected inflation associated with the marginal hawk whose voting intentions secure an outright majority for raising the policy rate. The lower exercise is the target rate of expected inflation, associated with the marginal dove whose voting intentions can secure an outright majority for lowering the policy rate. This analysis can be applied to both the nominal and real policy rate:

Proposition 2.1e: *The zone of stalemate (or disagreement) can be varyingly specified so that the nominal rate or real policy rate responds to* (*2.5*).

Proposition 2.1e permits alternative rate dynamics to be considered below respectively, in scenario one and scenario two. From (2.5) it is clear that the opportunistic term, $f\left(\pi_t - \pi_t^*\right)$ constitutes a policy behaviour which pivots around a zone of inaction. Aksoy et al. (2006) motivate this from non quadratic preferences. This zone of inaction is alternatively derived from the observation that rate setting is achieved frequently in the absence of total agreement between policy makers. This can be significant when a clear majority can not be achieved to back a given rate change. Policy stalemate arises over a zone of inflation where policy makers have disparate views at committee level or across institutions regarding the appropriateness of policy actions and inflation targets. In the absence of formal upper and lower bounds, monetary policy must then be shaped via the interaction of committee members who often disagree and who do not work with firm explicit inflation targets that would assist in attaining consensus.[153] In this regard, the FOMC have collectively negotiated the upper and lower bounds in a framework similar to zone targeting without any disclosure of inflation targets. As rate decisions are dependent on majority voting, this

[153] At least, this was the case during the Greenspan incumbency. The FOMC transcripts are later examined to provide some archival evidence for this observation. That is, heterogeneity of opinion, internally and externally, was important in shaping the policy rate decisions.

would imply a degree of nonlinearity. Initially, two scenarios related to voting practices are defined and considered, namely: nominal and real stalemate. Using a dual put-call parity framework, rate decisions can be appraised using an option's construction and subsequently the effect of uncertainty and inflation risk can be gauged when time values are considered.

2.2.4 Scenario One: Nominal Stalemate

Nominal stalemate implies that under conditions of certainty the policy rate will not adjust to changes in expected inflation so long as expected inflation resides within the zone of disagreement i.e. when no clear majority decisions can be achieved. Consider a Monetary Policy committee that is composed of two members who implement similarly deliberative/conventional forward-looking Taylor rules.[154] If their inflation targets differ in magnitude, it can be shown that rate setting behaviour is similarly nonlinear in the manner described by Aksoy et al. (2006). The reaction functions may be represented by:

$$r_{1t} = r^* + \pi^*_{g1} + (1+\omega_1)\left(E_t(\pi_{t+1}) - \pi^*_{g1}\right) + \omega_2(y_t) \qquad (2.10)$$

for Governor (1) and by:

$$r_{2t} = r^* + \pi^*_{g2} + (1+\omega_1)\left(E_t(\pi_{t+1}) - \pi^*_{g2}\right) + \omega_2(y_t) \qquad (2.11)$$

for Governor (2) or Congressman (1). Another way of profiling this scenario is to characterise rate setting as being implemented by a hawk and a dove for Governor (1) and Governor (2), respectively.[155] [156] Using the result obtained from Proposition 2.1 the inflation gap is reconfigured in the form of an option portfolio.

$$\left[E_t(\pi_{t+1}) - \pi^*_{g1}\right]e^{-rT} = c_{g1} - p_{g1} \qquad (2.12)$$

and

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[154] This analysis could easily be extended to a *n*-member committee. It can also be extended to take account of both committee and institutional dynamics where the upper target is influenced by external political agencies.

[155] With $\pi^*_{g1} < \pi^*_{g2}$. If the zone of stalemate is influenced by political agencies, the upper threshold may come to be defined by Congressman (1) instead of Governor (2), assuming that political agencies have a higher tolerance for inflation.

[156] Profiling policy makers using these labels may constitute an over-simplification of actual rate setting behaviour. Goodhart (1999a) suggests that the media caricatures of policy makers with these labels may lead to confusion and distort the true picture. The behaviour of policy makers can be more random and they may not faithfully belong to either supposed camp. In the analysis provided here, it is possible that roles become reversed. It is sufficient to motivate nonlinearity arising because consensus can not be permanently attained over a given range of expected inflation.

$$\left[E_t(\pi_{t+1})-\pi^*_{g2}\right]e^{-rT}=c_{g2}-p_{g2} \qquad (2.13)$$

If we take the average of the two Taylor Rule rates:

$$\frac{r_{1t}+r_{2t}}{2}=\frac{r^*+\pi^*_{g1}+r^*+\pi^*_{g2}}{2}+(1+\omega_1)\frac{\left[(c_{g1}-p_{g1})+(c_{g2}-p_{g2})\right]}{2}e^{rT}+\frac{\omega_2(y_t)+\omega_2(y_t)}{2} \qquad (2.14)$$

The average of the two rules is meant to reflect the effect of a chairman steering a middle course where agreement can be secured.[157] Here it might be thought of as the chair posing the question should the strategy hinge on 0% or 4 ½% and hoping the committee finds an acceptable middle ground. (2.14) can be re-expressed to give:

$$\frac{r_{1t}+r_{2t}}{2}=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+(1+\omega_1)\frac{\left[(c_{g2}-p_{g1})+(c_{g1}-p_{g2})\right]}{2}e^{rT}+\omega_2(y_t) \qquad (2.15)$$

For a forward-looking policy rule the collar can be valued intrinsically when expected inflation is known with certainty. However time valuations using standard pricing formulae can be implemented when expected inflation is not known with certainty. By invoking the following stalemate condition:[158]

$$\begin{aligned} c_{g1}&=0\ \textbf{\textit{when}}\ \pi^*_{g1}\le E(\pi)\le\pi^*_{g2}\Rightarrow c_{g1}=c_{g2}\\ p_{g2}&=0\ \textbf{\textit{when}}\ \pi^*_{g1}\le E(\pi)\le\pi^*_{g2}\Rightarrow p_{g2}=p_{g1}\end{aligned} \qquad (2.16)$$

the following emerges:

$$\frac{r_{1t}+r_{2t}}{2}=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+(1+\omega_1)(c_{g2}-p_{g1})e^{rT}+\omega_2(y_t) \qquad (2.17)$$

By virtue of not achieving consensus when inflation falls between the strikes, the policy board does not respond by altering the nominal interest rate.[159] This implies that c_{g1} and

[157] The chairman here is not allocated a casting vote. This assumption can be easily relaxed if the committee is extended to a more general case of *n* members.

[158] The key restrictions set c_{g1} to be equal to c_{g2} and p_{g2} to be equal to p_{g1}. These are imposed to capture the behaviour of the 'collectively' opportunistic or zone targeting central bankers. If interest rates are not moved immediately higher as inflation increases above the lower strike and not moved immediately lower below the upper strike of the collar, then policy assumes the attributes of opportunistic rate setting. This provides the basis for propositions 2.1a and 2.1b, where certainty regarding expected inflation applies.

[159] Of course when nominal stalemate occurs the real rate adjusts in line with changes in the inflation rate. This is somewhat different to the nonlinearity proposed in (2.2). It also suggests that if the policy impasse

p_{g2} assumes a form given by the collar: $(c_{g2} - p_{g1})$. In a world of certainty, they can only intrinsically procure a payoff when inflation resides outside the zone of disagreement, as outlined by Proposition 2.1c and demonstrated in figure 2.1. When this occurs, c_{g1} and p_{g2} replicate the payoffs from c_{g2} and p_{g1}, respectively. The immediate effect of stalemate, when inflation outcomes are known with certainty, could be to postpone the interest rate decision until the next meeting or more importantly until the inflation forecast lies either above the upper target or below the lower target of the respective board members. When inflation outcomes are uncertain, policy makers can be seen to act pre-emptively. Proposition 2.1d provides the basis for considering the distinction between intrinsic and time valuations. (2.17) implies that, even if individuals on a committee consider themselves to be deliberative and on an individual basis act to set rates in accordance with a conventional Taylor-type Rule, *collective* behaviour can be incidentally opportunistic. If interest rate decisions can only be achieved by consensus, the different inflation targets imply that there exists a zone of inaction when agreement cannot be secured.[160] Scenario two below considers how stalemate affects the real rate as distinct from the nominal rate.

2.2.5 Scenario two: Real Stalemate

If a monetary policy board or committee focussed on the real interest rate, as opposed to the nominal rate, this would imply the nominal rate would always adjust automatically to take account of inflation developments. This would mean that an increase in inflation would mechanically lead to an increase in the nominal rate. It would also imply that policy inaction would only apply to the real rate of interest. To illustrate this, it is necessary to consider a Taylor Rule formulation closer to (2.1a) than to (2.1b). Real stalemate is a less restrictive form of policy inaction in that it still permits nominal adjustments to occur, commensurate with fluctuations in purchasing power. Changes to the real policy rate can

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permits inflation to rise, without nominal adjustments the board are perversely stimulating the economy by virtue of the inaction.

[160] See Proposition 2.1b. Conceivably, if a central bank attains independence from the legislature, there is a greater likelihood that the monetary policy committee achieves consensus more easily. There is less 'second guessing' as to what the political masters would prefer, perhaps implying some compression in the tolerance levels. See also President Forrestal's comments available in the December 1989, FOMC transcripts, p.14. Independence can also be related to the dynamic reaction to inflation shocks. A central bank gaining independence might be modelled here as the bandwidth contracting (i.e. the differential between the strikes falls).

however be curbed by a lack of agreement between members of the board. This implies that, at all times, policy makers adjust the policy rate so that increases or decreases in inflation cause proportionate offsetting changes in the nominal policy rate. In this type of construction, members of the board only wrangle over adjustments to the real fed funds rate but acquiesce to adjustments to the nominal rate to counter inflation. This depiction of policy suggests that nominal fine-tuning is automatic but real rate changes depend on where expected inflation is relative to the respective inflation tolerances.[161] The Taylor Rule given by (2.1b) can be re-written so that:

$$r_t = r^* + E_t(\pi_{t+1}) + \omega_1\left(E_t(\pi_{t+1}) - \pi^*\right) + \omega_2 y_t \tag{2.18}$$

Note here that the nominal rate, r_t, adjusts automatically to take account of $E_t(\pi_{t+1})$. Using (2.18), it is implicitly assumed that policy disagreements only influence adjustments to the real rate.[162] Members of the board are agreed that the nominal rate adjusts mechanically in line with inflation but changes to the real rate are subject to differences of opinion that can emerge regarding the appropriate inflation tolerance. Again in a world of a two-member board, stalemate occurs only when the real Fed Funds rate is being adjusted. As before Governors (1) and (2) have different tolerances for inflation:

$$r_{g1t} = r^* + E_t(\pi_{t+1}) + \omega_1\left(E_t(\pi_{t+1}) - \pi^*_{g1}\right) + \omega_2(y_t) \tag{2.19}$$

and

$$r_{g2t} = r^* + E_t(\pi_{t+1}) + (\omega_1)\left(E_t(\pi_{t+1}) - \pi^*_{g2}\right) + \omega_2(y_t) \tag{2.20}$$

From (2.12) and (2.13) it can be shown that the average of the two reaction functions becomes:

$$\frac{r_{g1t} + r_{g2t}}{2} = \frac{r^* + E_t(\pi_{t+1}) + r^* + E_t(\pi_{t+1})}{2} + (\omega_1)\frac{\left[(c_{g1} - p_{g1}) + (c_{g2} - p_{g2})\right]}{2}e^{rT} + \frac{\omega_2(y_t) + \omega_2(y_t)}{2} \tag{2.21}$$

The average of the two rates is intended to capture the effect of a non-voting chairman weighting the committee members equally. In a world of certainty, stalemate occurs when policy makers differ. Discord between board members is likely when expected inflation remains between the upper and lower targets. Conceivably, the impact of not immediately

[161] This reflects more the opportunistic policy or inflation zone targeting as described by Aksoy et al. (2006).
[162] The rationale is that if policy makers passively permit the nominal rate to remain unchanged in the event of not responding to increases in inflation, they are in effect stimulating the economy despite the fact the nominal rate has been locked in stalemate.

securing agreement leads to stalemate in which the interest rate adjustment is postponed until the next meeting. Policy is more likely to react in real terms as the inflation forecast moves out of tolerance. Invoking Proposition 2.1a and (2.16), (2.21) can be re-expressed to give:

$$\bar{r}_t = r^* + E_t(\pi_{t+1}) + \omega_1(c_{g2} - p_{g1})e^{rT} + \omega_2(y_t) \tag{2.22}$$

The committee-agreed nominal policy rate, $\bar{r}_t$ when expressed as an expected Fed Funds real rate becomes:

$$r\bar{r}_t = \bar{r}_t - E_t(\pi_{t+1}) = r^* + \omega_1(c_{g2} - p_{g1})e^{rT} + \omega_2(y_t) \tag{2.23}$$

This representation of policy hypothesises that the real policy rate as adjusting to an option portfolio. The real case is less restrictive than the nominal stalemate equivalent. Policy decisions are agreed to adjust automatically the nominal Fed Funds rate even when agreement can not be secured to adjust the real Fed Funds rate. The tendency for the board to be depicted as a unified actor is understandable because, in public, policy makers are perceived to be mostly in agreement. In terms of what board officials convey, however, via the FOMC transcripts, there seems to be compelling evidence that quite discernible differences have existed on an on-going basis regarding what constitutes an appropriate inflation target.[163]

2.2.6 Certainty and Uncertainty: Intrinsic and Time Valuations

The analysis in (2.17) and (2.23) can be extended to capture the effects of uncertainty on the policy decision.[164] By applying time valuation to the collar option portfolio, $(c_{g2} - p_{g1})$, it becomes possible to take account of both the level of inflation and inflation risk.[165] Should expected inflation stand marginally below the upper threshold, policy would not in practice, remain inactive. The intrinsic value of the collar provides the basis to

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[163] Meade (2005) found that the official dissent rate in voting was much lower than actual differences of opinion when surveyed via discussions in the FOMC transcripts.

[164] Orphanides and Wilcox (1996, 2002) model uncertainty using probability distributions for inflation shocks and aggregate demand shocks. A key advantage of using a collar option portfolio to describe nonlinearity relates to the very natural distinction between intrinsic and time valuations which in turn allow policy to be examined under conditions of certainty and uncertainty.

[165] The inflation risks might be summarised by a fan chart or Monte Carlo analysis proposed in chapter 1, where the risk to inflation is seen as the proportion of forecasted inflation paths that produce terminal values outside a specified inflation targeting zone.

analyse policy reaction under conditions of certainty. Time valuation for option pricing furnishes a more general framework that permits inflation risk also to be considered. Thus, the probability density function of inflation then can be made a defining driver of policy in a similar way as Black-Scholes permits contingency payments to be priced.

When the collar's time value is considered using a standard option pricing model, many diverse aspects of monetary policy can be subsumed into an integrated risk analysis.[166] Option theory provides a unified framework to examine inflation tolerances, uncertainty and parameter sensitivities.[167] The volatility of the underlying and the time horizon over which the inflation target must be preserved can be factored into understanding the policy decision. Parameter sensitivities related to inflation volatility and targeting horizon can be elaborated in much the same way as are conventionally applied in terms of risk management. In fact, the well-established 'Greeks' in the finance literature, can be measured and provide additional insights into policy when implemented under uncertainty for the collar option portfolio. In this regard, the risk management framework implied by option theory may prove beneficial in gauging how policy makers respond to expected inflation. As inflation changes relative to the strikes, central bankers can be seen to adjust the Fed Funds rate in a dynamic fashion. To appreciate the impact on policy of forecast targeting, the next section considers the time value of the collar in conjunction with the uncertainty regarding expected inflation outcome.

2.3. Risk Management and Opportunism

This section provides an innovative and flexible approach to modelling the policy rate response to inflation under uncertainty. Implicitly a reaction function is set up, that nests a collar portfolio in the classic Taylor Rule. The approach incorporates risk from the perspective of a central banker seeking *ex ante* to realise price stability goals. Rather than use inflation directly in the reaction function, a proxy is modelled using a portfolio of options as described before. In essence, the existing analysis provided in Proposition 2.1,

[166] Different option pricing models will be considered.

[167] The typical parameters used by Black-Scholes relates to the spot, the strike, the risk free rate, the volatility of the underlying and the maturity of the option contract. All these parameters have the capacity to be interpreted in terms of policy rate setting.

2.1a, 2.1b and 2.1c is extended to take into account the effects of uncertainty. To this end, the time valuation of $\left(c_{g2} - p_{g1}\right)$ is initially calculated using the standard Black (1976) model. Mean reversion is considered later by co-opting the Vasicek model. The choice of option pricing model, in large part, will be determined by how central bankers wish to characterise inflation behaviour and the nature of uncertainty they must deal with.

As a starting point; the Black model is attractive in that it assumes that the underlying adheres to a lognormal random walk. From chapter 1, this was observed to be useful because it allowed policy makers to preserve intact the risk of hyperinflation, while minimising the risks of deflation.[168] It also constituted an intellectually modest stance that central banker could assume, i.e. what if the best forecasting models were no better at forecasting inflation than a naïve model?[169] From chapter 1, it was found that by co-opting stochastic volatility into the behaviour of the underlying, it was possible to consider many different types of distributions. Heston Monte Carlo was found, for a given range of skew and kurtosis, to produce relatively small departures from Black time valuations. More importantly these departures do not prejudice Proposition 1.4. Hull and White (1990) also found that the Black model can still be used when the underlying mean reverts. So long as an adjustment to the volatility input is made, the Black model can be used for pricing options. It can be shown that as the speed of mean reversion to a given long term inflation mean increases, the appropriate volatility input into the closed form solution correspondingly declines.[170] Time valuations are implemented initially using the closed form Black model, and then the Vasicek (1977) model is computed numerically to examine the effects of relaxing some of the Black assumptions using inflation data from the Greenspan incumbency. Of course, by using alternative option pricing models that are now well established in the literature, central bankers could adopt time valuation to deal with a wide variety of inflation behaviour and risks. The Vasicek (1977) model for instance

[168] The Bachelier model could be used if policy makers wished to attribute the same level of risk to deflation as inflation.
[169] This may be particularly desirable in a world of Knigthian uncertainty.
[170] This would suggest that the Black model is quite robust for varying inflation behaviour. The option's framework developed here however does not preclude the application of other option pricing models. In fact, this is probably how best to approach zone targeting expected inflation when central bankers are not even sure which type of risk they confront.

permits the analysis to be extended to take into account the following: (1) deflation, (2) predictability or mean reversion and (3) normality of the underlying, as opposed to lognormality. Relaxing lognormality and random walk assumptions permits a more varied risk analysis of inflation.

Using the Black model, it can be shown that the policy rate adjusts dynamically as inflation changes. The time value of the long call and short put is sensitive to the relative proximity of inflation to the inflation thresholds as described in Figure 2.1.[171] In addition, volatility and timing are important parameters inputs that are easily interpreted in a monetary policy context. A delta measure on the collar portfolio can be estimated and this offers an additional metric to gauge how policy theoretically responds to a path dependent inflation variable. Implementing the risk analysis necessitates first computing the time values.

2.3.1 The Black Model, Time Valuation and Uncertainty

Proposition 2.2: *Using the Derman-Taleb (2005) result reported in chapter 1 and applying a standard derivation of the Black model as developed in appendix A.2.1, the time values of the call option and put option on expected inflation can be estimated using the standard Black (1976) formula.*

Proposition 2.2a: *A risk management approach to monetary policy implies that rate decisions are not only influenced by the proximity of expected inflation to a given target but also by the volatility of the underlying target variable and the time horizon over which price stability must be contained.*

Both the volatility and time horizon are necessary parameter inputs into the Black (1976) model and into most other time valuation models. Time valuation using a parsimonious Black (1976) model leads to:

$$c_{g2} = e^{-rT}\left[\hat{E}(\pi)N\left(d_{1g2}\right) - \pi^{*}_{g2}N\left(d_{2g2}\right)\right] \tag{2.24}$$

[171] The inflation thresholds values become the default strikes for the collar valuation.

where π^*_{g2} represents the upper tolerance for inflation that triggers a monetary policy response and $\hat{E}(\pi)$ denotes the expectation of inflation in a Derman-Taleb risk neutral world. The difference in squared brackets is discounted to allow for time decay. *T* denotes the time horizon over which the inflation target is intended to be preserved.[172] In a forward-looking world Proposition 2.2a invokes the Black model to gauge the effect of risk and uncertainty. In so doing, the standard notation applies. *r* represents the continuously compounded risk-free yield on a government treasury bill with the same maturity as defined by the expiration date of the collar.[173] The risk neutral probabilities are given by:

$$\begin{aligned} N(d_{1g2}) &= N\left(\frac{ln(E(\pi)/\pi^*_{g2}) + \sigma^2 T/2}{\sigma\sqrt{\boldsymbol{T}}}\right) \\ N(d_{2g2}) &= N\left(\frac{ln(E(\pi)/\pi^*_{g2}) - \sigma^2 T/2}{\sigma\sqrt{\boldsymbol{T}}}\right) \end{aligned} \tag{2.25}$$

N(*x*) here represents the cumulative probability for a standardised normal distribution. σ measures the standard deviation per annum of expected inflation return.[174] This notation is standard for the Black model and standard for valuing interest rates caps and floors. The time value of the call option is given by (2.24). The forward-looking specification is consistent with the literature. Clarida, Gali and Gertler (1998) configured the central banker as targeting a future rate of inflation. The forward looking nature of recent Taylor Rule specifications makes the time value option construction amenable to risk analysis given the uncertainty implicit in targeting a forecasted variable. Bernanke (2004a) compared ‘forecast-based policies’ and ‘simple feedback policies’. He asserted that the Federal Reserve depends primarily on the forecast-based approach for setting policy.

Proposition 2.2b: *The time value of the put option on inflation can be estimated using the Black model. Using put-call parity it can be shown that:*

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[172] See Appendix A.2.1 for motivation of formulae from the perspective of a central banker implementing inflation forecast targeting.

[173] The inclusion of *r* is not strictly necessary, but is preserved here for the sake of maintaining consistency with the standard Black model. The interest rate, *r* could be ignored because ultimately it does not appear on the r.h.s. of the reaction function. See (a.2.1.16), Appendix A.2.1.

[174] See chapter 1 for more discussion of these inputs and implementation of valuations, using closed-form and numerical estimation.

$$p_{g1} = e^{-rT}\left[\pi^*_{g1}N\left(-d_{2g1}\right) - \hat{E}(\pi)N\left(-d_{1g1}\right)\right] \quad (2.26)$$

where π^*_{g1} represents the lower tolerance for inflation that triggers a monetary policy response. In effect, as expected inflation falls, monetary policy is driven by the relative proximity to this lower threshold. The usual notation applies:

$$N\left(-d_{1g1}\right) = 1 - N\left(d_{1g1}\right) \text{ and } N\left(-d_{2g1}\right) = 1 - N\left(d_{2g1}\right) \quad (2.27)$$

For the put option the risk neutral probabilities are calculated with a different magnitude for the lower strike, π^*_{g1}:

$$\begin{aligned} N\left(d_{1g1}\right) &= N\left(\frac{ln\left(E(\pi)/\pi^*_{g1}\right) + \sigma^2 T/2}{\sigma\sqrt{\boldsymbol{T}}}\right) \\ N\left(d_{2g1}\right) &= N\left(\frac{ln\left(E(\pi)/\pi^*_{g1}\right) - \sigma^2 T/2}{\sigma\sqrt{\boldsymbol{T}}}\right) \end{aligned} \quad (2.28)$$

An important distinction lies between the time value and intrinsic value of the collar. The payoff configuration from a collar that matures imminently, or is at expiration, is akin to the hockey stick shapes defined in Figure 2.1. This type of payoff or monetary policy response is ascribed to the interest rate determination under certainty. If, however, the central banker is assumed to adjust current interest rates to respond to expected future developments in inflation, it makes more sense to estimate the value of the collar with a time horizon extending one or two years. The shape of the policy response changes considerably when compared to Figure 2.1. Figure 2.2 demonstrates the contrast between the intrinsic and time values of the collar. The intrinsic value is computed by taking the difference between the intrinsic value of the long call and short put with the respective exercises at 4.5% and 2.5%:

$$\left(c_{g2}^{\pi^*=0.045} \overset{T=0}{-} p_{g1}^{\pi^*=0.025}\right) \quad (2.29)$$

When expected inflation is uncertain, the collar is estimated using the Black Formula:

$$\left(c_{g2}^{\pi^*=0.045} \overset{T=1,\sigma=0.25}{-} p_{g1}^{\pi^*=0.025}\right) \quad (2.30)$$

The time valuations of a collar agreement based on different levels of expected inflation are estimated using the Black formula as described by (2.24) – (2.27) and then by aggregating into the portfolio $\left(c_2 - p_1\right)$. Taking expected inflation starting at 0.5% and increasing to 7%

in increments of 0.25%, the combined value of the long call and short put are calculated using the Black model as set out in Propositions 2.2a and 2.2b, see table 2.1. The parameter values are set at r = 5%, (r could be set arbitrarily to zero although here is set close to the average one year yield on Treasury Bills over the Greenspan period), $\sigma = 0.25$ (the lower range of volatility estimated using a recursive GARCH(1,1) forecast for the Greenspan tenure)[175], T = 1 year (the forward looking specification used by Clarida et al. (1998)), $\pi^*_{g1} = 2.5\%$, and $\pi^*_{g2} = 4.5\%$.[176] Table 2.1 presents the time and intrinsic values of both the call and put, including risk neutral probabilities. By combining these into portfolios, it is possible to calculate the time value of the payoff.

A key insight from Figure 2.2 is that under uncertainty policy moves very gradually as expected inflation remains inside the strikes. It does not however remain dormant as depicted initially in Figure 2.1.[177] Generally, as the expected value of inflation moves outside the thresholds, the policy response appears to become increasingly more pronounced. This portrayal of policy suggests that under conditions of uncertainty, distinguishing between the deliberative and opportunistic rate setting may not be straightforward since policy makers do not wait for expected inflation to breach the thresholds before adjusting the policy rate. This result mirrors quite closely the finding of Orphanides and Wieland (2000a) when describing zone targeting:

> 'In a world with uncertainty due to unforeseen shocks there is always some probability that a shock pushes inflation outside the range of inflation gaps over which the policymaker with zone-quadratic preferences perceives no relevant welfare loss. To reduce the likelihood of inflation falling outside this zone, the policymaker is willing to open up small output gaps even though inflation is still inside the zone. This result is very much consistent with the practice of central banks, who target ranges but typically emphasise that these ranges are not implemented in a mechanical manner.' (Orphanides and Wieland, 1999, p.29)

[175] The calculation relating to the GARCH(1,1) recursive forecast of inflation volatility is described in Appendix A.2.3.

[176] The upper tolerance appears in the comments made by a number of Federal Reserve officials. See p.14 of the December 1989 FOMC transcripts. The lower tolerance is given here as being 2% lower. This is presented as a baseline but other magnitudes are also considered. Aksoy el al (2006) maintained the width of the zone of inaction fixed at 2%. Other specifications are, of course, possible.

[177] Orphanides and Wilcox (1996) also argued that the opportunist central banker, when facing uncertainty, would not be entirely inactive between the inflation thresholds. See Orphanides et al. (1997) page 6.

Consistent with Proposition 1.4, policy under uncertainty is unlikely to remain dormant when expected inflation resides inside the comfort zone. In this regard, the nonlinear/opportunistic representation as set out in Figure 2.1 can only be regarded as an extreme case where inflation outcomes are known with certainty. The collar construction when estimated using time valuation is very similar to hedging or purchasing insurance. Indeed that phrase is commonly used in the FOMC transcripts.[178] A decrease or increase in inflation does not immediately precipitate a commensurate move in the policy rate. Uncertainty, here, is thought to increase as the volatility of inflation increases and as the targeting horizon extends. Portfolio option theory would suggest that uncertainty would reduce nonlinearity and the existence of nonlinearity in the reaction function is shaped by the ebb and flow of inflation volatility and targeting horizons. Equally, policy is likely to migrate between varying levels of nonlinearity.

2.3.2 Expressions of nonlinearity inside and outside the FOMC

The public utterances of a number of policy makers suggest that for a number of central bankers, rate setting is path dependent.[179] The opportunistic approach gradually targets price stability once inflation falls within a zone of tolerance. The distinction between policy being set under conditions of uncertainty or certainty is then clear. Figure 2.2 suggests that under uncertainty (where uncertainty relates solely to not knowing inflation outcomes in advance) nonlinearity is mitigated. One important insight here is that inflation risk helps shape the extent to which policy implementation is graduated. Inertia in a Taylor type rule is often incorporated by adding a lagged dependent variable. Smoothing is generally denoted in estimations by a coefficient value near unity for the lag that is constant over a given timeframe. Option theory as described by applying the collar option portfolio, permits the inertia to be dynamic. Figure 2.2 is useful in communicating how policy activism evolves as the underlying expected inflation and inflation risk evolve.[180] This

[178] The notion of gradually moving the policy rate is analogous to hedging when conditions are uncertain. The metaphorical reference to buying insurance was frequently used in a number of FOMC meetings. See President Forrestal comments on p. 77 of the December 1989 FOMC transcripts. Likewise, Governor Kelley p. 87, President Boehne p. 92, President Syron p. 95 of the same meeting.
[179] See Kohn (1996).
[180] Policy activism is meant to imply how quickly policy tightening/accommodation is added/removed.

sketch of monetary policy is similar to the views expressed by Governor Johnson at the December 1989 FOMC meeting, (p. 44).

> 'So, my view is that we have to be sensitive to the real economy. We have to be patient enough to pursue our goals consistent with avoiding recession unless inflation accelerates. If inflation starts to accelerate, we don't have any choice.' (Federal Reserve Board, 1989, p.44)

The nonlinearity appears to arise from policy makers wanting to protect jobs and output when inflation is moderate, yet being prepared to switch gear when inflation threatens to rise above a given acceptable threshold.[181] The opportunistic approach is closely linked to interest rate inertia, in that central bankers do not move to squeeze inflation immediately while in the zone. This also suggests that smoothing ought to be dynamic. The notion of 'chipping away' at the inflation objective implies that the ultimate objective is cumulatively much greater than the intermediary steps. The opportunistic approach however does not necessarily sanction a diminished concern for inflation but rather influences the timing of rate adjustments. President Boehne, at the December 1989 FOMC meeting, (p.18), outlined this relationship between opportunism and gradualism:

> 'One thing that I get out of this is that we get into inflation and we tend to get out of inflation not so much in a straight line route but over a period of time over different cycles. Some one made the point earlier that inflation has built up over the 15-year period because it would peak out in a subsequent cycle at a higher rate than the previous inflationary peak and it wouldn't drop as low. And I wonder if that is not instructive in terms of how one gets out of it. (Federal Reserve Board, 1989, p.18)

In the opportunistic strategy, the Fed Funds adjustments are applied incrementally. When inflation is moderate, policy moves in a gradual fashion. The measured adjustments to curb rising inflation indicate that the opportunistic approach can not be fully described by Figure 2.1. Portfolio Option Theory, when applied, would suggest that opportunistic central bankers can indeed respond to inflation within the tolerance levels. The effect of uncertainty is to place the policy rate along a continuum of activism.[182] In an uncertain world, policy makers may be prepared to effect precautionary changes to thwart incipient

[181] An important consideration here seems to be the perceived acceptability in the public eyes of different courses of action.
[182] Policy activism may be thought of as the speed with which policy becomes accommodative or tight. If policy activism is high, this implies there is less interest rate inertia.

moves in inflation even though inflation remains within the thresholds of tolerance.[183] It should be kept in mind that in the late 1980s opportunistic policy had not yet been fully or formally elaborated as a strategy for containing price increases. The views expressed by these Fed insiders plausibly reflected their practical experience. President Boehne and Governor Johnson seemed, nevertheless, to be providing a relatively complete description of the then embryonic opportunistic approach.[184]

Bernanke and Mishkin (1997) refer to a strategy of “inflation zone targeting” which shares much in common with Aksoy et al. (2006). This advocates keeping inflation within a given range rather than specifying an exact point target.[185] The Federal Reserve did not during the Greenspan chairmanship have an explicit target that is analogous to those attributable to the monetary policy regimes in the U.K., Canada, or EMU. Much of the Humphrey-Hawkins Act however did stress credit and monetary aggregates. Thornton (2004) in reviewing the FOMC transcripts has found that the board, in effect, targeted the Federal Funds rate since 1982, but has allowed the fiction of monetary/reserve targeting to persist since it expediently provided ‘political cover’ to officials when wanting to raise interest rates. In discussions, the FOMC preserved the lexicon of the borrowed reserves operating procedure until January 1991. Even when monetary policy discussions de-emphasised reserves and the board was internally aware that they were targeting the funds rate, they did not fully describe the funds rate targeting operating procedure in official announcements. Thornton (2004) maintained that the Greenspan FOMC did not clearly acknowledge an explicit target for the Federal Funds rate in the policy directive until December 21, 1999. This lack of

[183] The strike levels may reflect the heterogeneous beliefs of policy makers, inside the committee and across institutions. Bandwidth is also plausibly influenced by the chairman’s capacity to achieve consensus and by the nature of any economic shock. For instance, the inflation thresholds could potentially widen in the event of a supply shock by virtue that consensus may not be so easily attained. If policy makers were divided between those who subscribed to a hierarchical mandate and those who attributed equal importance to both gaps: inflation and output, this could thwart reaching agreement easily.

[184] Compare President Boehne’s view here, in December 1989 with that of Don Kohn’s appended comments to the September 1995 FOMC meeting. ‘….the Federal Reserve does not seek to raise the unemployment rate above the natural rate, but effectively leans harder against shocks to the economy that would increase inflation than those that would decrease it. The resulting pattern would be successively lower inflation rates at cycle peaks and troughs.’ (Federal Reserve Board, FOMC, September, 1995, p.6, Appendix)

[185] This is not for the want of trying. Proposed legislation: the Neal Amendment and Connie Mack Bill failed to culminate in explicit targets being set for the Fed.

clarity may have reflected, at a deeper level, a political unwillingness to tackle economic imbalances precipitately.

Most explicit inflation targeting central banks provide some room for manoeuvre in achieving price stability.[186] In this sense, the opportunistic or zone targeting regime provides for a dynamic set of responses. It is possible that a significant motivation for the Federal Reserve over the 1990s, in developing the opportunistic approach, was to enable Fed officials to describe their actions, in the absence of an explicit inflation targeting mandate, in a language accessible to other public institutions. It constituted a form of contingency rule where transitory departures from a linear progression towards price stability could be explained in the context of preserving short-term output objectives. Inflation zone targeting, as implemented in other countries, contains many elements that can be observed from Figure 2.2. It effectively implies targeting inflation more aggressively when inflation moves outside a given target range. Figure 2.2 summarises some of the dynamics described by central bank officials from other countries. The New Zealand Reserve Bank widened the target zone for inflation from 0% to 2% to 0% to 3% at the end of 1996 to reduce the need for activist policy responses. In 1997, Governor Brash pointed out:

> "The tension is between, on the one hand, choosing a target range which effectively anchors inflation expectations at a low level but which is so narrow that it provokes excessive policy activism and risks loss of credibility by being frequently exceeded;

[186] Following the United Kingdom's departure from the Exchange Rate Mechanism in October 1992 the Bank of England switched to a new policy of inflation targeting. The adoption of an inflation target in the U.K. was accompanied by the publication of an Inflation Report. In May 1997, the Bank of England was granted operational independence. Originally the United Kingdom set out a target range for inflation. Since receiving independence the target has been conveyed as a point but with 1% symmetric thresholds. From 1997 to 2003, the inflation target was 2.5% per annum, measured using the retail prices index excluding mortgage interest payments (RPIX). Thereafter, the target was amended by the Chancellor to be the Harmonised Consumer Prices Index, (HICP), and set at 2% again retaining the 1% symmetry.
The Bank of Canada adopted inflation targeting in 1991. Its target definition is stated as a range. A target range of +/- 1 % was set around a midpoint of 3% at the end of 1992, thereafter 2.5% by mid-1994 and 2% since the end of 1995. In November 2006, the 1 to 3 per cent target range was renewed to the end of 2011. Monetary policy will continue to be aimed at keeping inflation at the 2 per cent target midpoint. The rate of change in the CPI represents the official target but "underlying CPI", excluding food, energy, and the contribution from changes in indirect taxes is used as an operational objective.
The Governing Council of the ECB targets "the two pillars": money supply and inflation. The Council decided to specify a quantitative range for price stability in the Euro area as being: 'a year-on-year increase in the Harmonised Index of Consumer Prices of below 2 percent'. The word *increase* is used advisedly in that the term excludes negative rates of change in the price index. See Diewert (2002).

> and on the other, a target range which does a less effective job of anchoring inflation expectations, but requires less policy activism and protects credibility by being rarely breached" (cited in Bernanke et al, 1999, p. 113)

There is an obvious parallel with the opportunistic approach in that the inflation thresholds give the monetary authority a wider span for manoeuvre. In both cases, there is an inverse relationship between the width of the policy bands and policy activism. This can be illustrated using a delta metric developed below. Opportunistic policy and zone targeting both imply that central bankers are not compelled to be unrelenting when inflation remains within tolerable thresholds. This paradoxically can help maintain credibility in that it permits inflation shocks to be accommodated without precipitating automatic sacrifices in output. Zone targeting permits central banks to balance competing demands. Activist stabilisation is managed in a manner that permits some political economy in achieving long term institutional objectives.

2.3.3 Policy inertia and activism using delta

Proposition 2.3: *Delta is the partial derivative of the time value of the option with respect to expected inflation. This partial derivative represents an innovative metric that captures monetary policy inertia/activism.*

Proposition 2.3a: *Policy inertia/activism can be gauged by estimating the Black delta where the option portfolio is given by:* $\left(c_{g2} - p_{g1}\right)$.

Proposition 2.3b: *The portfolio delta is normally estimated to be minimised, when expected inflation resides within the strikes. This is consistent with policy being least active or most graduated when expected inflation falls between the upper and lower target.*[187]

Proposition 2.3c: *A widening of the zone between the exercise rates (i.e. greater disagreement between policy makers), precipitates greater nonlinearity.*

[187] When volatility is elevated this tends to push the point of minimum policy activism to a lower level of expected inflation. This asymmetry is described later in propositions (2.4) and (2.5).

Proposition 2.3d: *Policy inertia/activism can be otherwise gauged by estimating the Vasicek or Heston deltas using the option portfolio based on inflation:* $\left(c_{g2} - p_{g1}\right)$*. This implies that the delta framework can be extended to consider a large variety of policy contexts where the lognormal random walk model is thought to be less appropriate or where central bankers feel confident they can make an argument that privileges mean reversion and departures from lognormality.*

Consider the time value of the call:

$$c_{g2} = e^{-r(T\text{-}t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi_2^* N\left(d_{2g2}\right)\right] \tag{2.31}$$

If this is differentiated with respect to expected inflation, this yields a measure for the call's delta[188]

$$\frac{\partial c_{g2}}{\partial E(\pi)} = e^{-r(T\text{-}t)}\left[N\left(d_{1g2}\right) + E(\pi)N'\left(d_{1g2}\right)\frac{\partial d_{1g2}}{\partial E(\pi)} - \pi_{g2}^* N'\left(d_{2g2}\right)\frac{\partial d_{2g2}}{\partial E(\pi)}\right] \tag{2.32}$$

From the appendix C.3 in chapter 3 it can be shown that:

$$E(\pi)N'\left(d_{1g2}\right) = \pi_{g2}^* N'\left(d_{2g2}\right) \tag{2.33}$$

and since the standard normal density function is an even function:

$$\frac{\partial d_{1g2}}{\partial E(\pi)} = \frac{\partial d_{2g2}}{\partial E(\pi)} \tag{2.34}$$

The implies that the delta, for the call option in the portfolio can be estimated to give:

$$\frac{\partial c_{g2}}{\partial E(\pi)} = e^{-r(T\text{-}t)}N\left(d_{1g2}\right) \tag{2.35}$$

Likewise

$$p_{g1} = e^{-r(T-t)}\left[\pi_{g1}^* N\left(-d_{2g1}\right) - E(\pi)N\left(-d_{1g1}\right)\right] \tag{2.36}$$

When differentiated with respect to expected inflation (2.37) emerges:

[188] Note that the normal cumulative probability of *x* occurring is given by *N(x)*. *N'(x)* denotes the standard normal probability density function

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \quad N(x) = \int N'(x)$$

$$\frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T\text{-}t)}\left[\pi^*_{g1}N'\left(-d_{2g1}\right)\frac{\partial\left(-d_{2g1}\right)}{\partial E(\pi)} - E(\pi)N'\left(-d_{1g1}\right)\frac{\partial\left(-d_{1g1}\right)}{\partial E(\pi)} - N\left(-d_{1g1}\right)\right]$$

(2.37)

From (2.19) and (2.20) it can be shown that:

$$\frac{\partial p_{g1}}{\partial E(\pi)} = -e^{-r(T\text{-}t)}N\left(-d_{1g1}\right) = e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right)-1\right] \quad (2.38)$$

The risk neutral probabilities for the put and call have different strikes. The delta of the collar portfolio can be calculated from the deltas of the individual options in the portfolio.[189]

$$\frac{\partial c_{g2}}{\partial E(\pi)} - \frac{\partial p_{g1}}{\partial E(\pi)} \quad (2.39)$$

The collar represents a portfolio of a long call and a short put on the same underlying. The delta of the portfolio is obtained by taking a weighted sum of the deltas of the individual positions. (2.39) calculates the delta of the collar by subtracting the delta of the short put from the delta of the long call. The same range of expected inflation values as Table 2.1 is considered when computing the portfolios' deltas. The portfolio delta:

$$\frac{\partial\left(\overset{T=1,\sigma=0.25}{c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}}\right)}{\partial E(\pi)} = e^{-r(T\text{-}t)}N\left(d_{1g2}\right) - e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right)-1\right] \quad (2.40)$$

represents the slope on the collar, $\left(c_{g2} - p_{g1}\right)$ at varying levels of expected inflation. It measures the rate of change in the monetary policy response with respect to a change in expected inflation, where the collar captures policy behaviour. The delta curve has the capacity to map out policy responsiveness over a specified range of expected inflation. Figure 2.3 shows that as expected inflation moves outside the policy bands 2.5% to 4.5% delta rises. This confirms the economic intuition provided before. As inflation moves beyond the thresholds, policy becomes more robust. Figure 2.3 maps out the delta curve for the portfolio. It is lowest when expected inflation lies inside π^*_{g1} and π^*_{g2}, the inflation

[189] The respective $N(d_1)$s for the put and for the call are not equal given that the strikes π^*_{g1} and π^*_{g2} for both options are different.

thresholds. Likewise, as the underlying departs from the band defined by π^*_{g1} and π^*_{g2}, the value of delta rises close to 1.

Advocates of opportunism hold that while expected inflation remains within tolerable levels, the monetary policy makers should refrain from unremittingly containing price increases. This version of events seems to accord well with the risk analysis implied by the delta curve of Figure 2.3. That is, policy makers react to inflation in a more attenuated fashion between the strikes. In this sense, the delta of the collar traces out policy activism. If policy was conventional or deliberative this would lead to a constant delta.[190] Figure 2.4 shows that if the band is widened for the same option portfolio the delta drops further. This seems to be consistent with the assertion made by Governor Brash that narrowing the band has the capacity to provoke additional policy activism (i.e. the speed at which the output gap is opened is accelerated). A very narrowly defined zone target may test more severely the reputational capital of policy makers who are forced to more hastily sacrifice jobs to preserve price stability when compared with central bankers who more loosely define the zone target.[191] A very widely defined zone however may undermine credibility by virtue that policy makers are seen to respond to inflation developments too sluggishly. Figure 2.5 provides a more complete overview of six zone targeting bands that abound 3.5%. Delta is calculated using the collar. 0.0025 corresponds to the inflation band going between 3.25% and 3.75%, i.e. 0.25% either side of 3.5%. 0.0125 corresponds to the inflation band 2.25% and 4.75% etc.. In all cases the arithmetic mean of the upper and lower band is 3.5%. It is clear that delta declines as the bands widen. That is, there would appear to be greater smoothing. There would also appear to be asymmetry in each of the delta curves.

2.3.4 Asymmetry in the Delta curve

A key feature of the Black delta curves is that they are asymmetric, i.e. the expected inflation rate associated with the minimum delta value lies below the mid point or

[190] This is illustrated later in Proposition (2.4) and (2.5).

[191] Ultimately, policy opens the output gap to contain inflation. The delta describes the pace with which that output gap is opened. As the zone target widens this implies that output gap is opened at a more gradual rate, all else being equal.

arithmetic mean of the two inflation thresholds. To illustrate this, it is useful to employ the gamma of the collar portfolio. This leads to the following propositions:

Proposition 2.4: *The gamma of the collar portfolio can be used to establish the level of expected inflation at which policy is least responsive to a change in expected inflation or where the policy rate adjustment is most graduated. Using the Black model, it is found that the expected inflation rate, associated with the lowest delta, is inferior to the arithmetic average of the thresholds. This implies that the delta curve for the Black model is* ***asymmetric****.*

Proposition 2.5: *If the upper and lower thresholds* (*strikes*) *are equal, this implies gamma is zero and delta is constant for all levels of expected inflation. That is, using gamma it is possible to show that an agreed inflation point target, produces a linear policy.*

To estimate gamma, Γ, it is first of all necessary to determine its algebraic form:

$$\Gamma = \frac{\partial^2 c_{g2}}{\partial E(\pi)^2} - \frac{\partial^2 p_{g1}}{\partial E(\pi)^2} \tag{2.41}$$

This in effect, implies taking the second derivative of the difference between the call and put with respect to expected inflation.

$$N\left(d_{1g2}\right) = N\left(\frac{\ln\left(E(\pi)/\pi^*_{g2}\right) + \sigma^2 T/2}{\sigma\sqrt{\mathrm{T - t}}}\right) = N\left(\frac{\ln\left(E(\pi)\right) - \ln\left(\pi^*_{g2}\right) + \sigma^2 T/2}{\sigma\sqrt{\mathrm{T - t}}}\right) \tag{2.42}$$

When d_{1g2} is differentiated with respect to expected inflation the expression in (2.43) is obtained.

$$\frac{\partial d_{1g2}}{\partial E(\pi)} = \frac{1}{E(\pi)\sigma\sqrt{T-t}} \tag{2.43}$$

This can be substituted into (2.44)

$$\frac{\partial^2 c_{g2}}{\partial E(\pi)^2} = e^{-r(T\text{-}t)}\left[N'\left(d_{1g2}\right)\frac{\partial d_{1g2}}{\partial E(\pi)}\right] \tag{2.44}$$

to become

$$\frac{\partial^2 c_{g2}}{\partial E(\pi)^2} = e^{-r(T\text{-}t)}\left[N'\left(d_{1g2}\right)\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right] \tag{2.45}$$

(2.45) gives the gamma expression for the call. Similarly, it can be shown that the put's gamma is equal to:

$$\frac{\partial^2 p_{g1}}{\partial E(\pi)^2} = e^{-r(T\text{-}t)}\left[N'\left(d_{1g1}\right)\frac{\partial d_{1g1}}{\partial E(\pi)}\right] = e^{-r(T\text{-}t)}\left[N'\left(d_{1g1}\right)\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right] \tag{2.46}$$

where

$$\frac{\partial d_{1g1}}{\partial E(\pi)} = \frac{1}{E(\pi)\sigma\sqrt{T-t}}$$

Combining the short put and long call, the portfolio's gamma, Γ, is obtained by taking the difference.

$$\frac{\partial^2 c_{g2}}{\partial E(\pi)^2} - \frac{\partial^2 p_{g1}}{\partial E(\pi)^2} = \Gamma = e^{-r(T\text{-}t)}\left[N'\left(d_{1g2}\right) - N'\left(d_{1g1}\right)\right]\frac{1}{E(\pi)\sigma\sqrt{T-t}} \tag{2.47}$$

By setting (2.47) equal to zero and solving for expected inflation, the delta is optimised. In this instance, delta is being minimised. Given (2.2) – (2.5) and Proposition 2.2, policy is least activist when delta reaches its minimum. This occurs when Γ is equal to zero. By taking (2.47) and setting out the first order conditions for attaining the minimum delta:

$$e^{-r(T\text{-}t)}\left[N'\left(d_{1g2}\right)\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right] - e^{-r(T\text{-}t)}\left[N'\left(d_{1g1}\right)\frac{1}{E(\pi)\sigma\sqrt{T-t}}\right] = 0 \tag{2.48}$$

By collapsing (2.48), two possible outcomes emerge because taking the square root implies there is both a positive and negative root. This can be illustrated by observing that:

$$\left\lfloor N'\left(d_{1g2}\right)\right\rfloor = \left\lfloor N'\left(d_{1g1}\right)\right\rfloor \tag{2.49}$$

The first outcome assumes that both roots simultaneously are positive or negative. The delta is optimised when:

$$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(\frac{\ln\left(E(\pi)/\pi_{g2}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)^{2}/2\sigma^{2}\right]$$
$$=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(\frac{\ln\left(E(\pi)/\pi_{g1}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)^{2}/2\sigma^{2}\right]$$
$$\Rightarrow\left(\frac{\ln\left(E(\pi)/\pi_{g2}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)^{2}=\left(\frac{\ln\left(E(\pi)/\pi_{g1}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)^{2}$$

Taking the square root of both sides it is found that:

$$\left(\frac{\ln\left(E(\pi)/\pi_{g2}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)=\left(\frac{\ln\left(E(\pi)/\pi_{g1}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) \qquad (2.50)$$

The second outcome assumes that both roots have different signs. Again delta is optimised when:

$$\left(\frac{\ln\left(E(\pi)/\pi_{g2}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right)=-\left(\frac{\ln\left(E(\pi)/\pi_{g1}^{*}\right)+\sigma^{2}(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) \qquad (2.51)$$

(2.50) holds when the upper threshold and lower threshold of the inflation targeting zone are equal. This is a special limiting case.[192] When the upper and lower thresholds are equal the delta is constant, so no single minimum delta exists. In other words, the delta is optimised for the entire range of expected inflation.

$$\begin{aligned}&\ln\left(E(\pi)/\pi_{g2}^{*}\right)=\ln\left(E(\pi)/\pi_{g1}^{*}\right)\\&\Rightarrow\pi_{g2}^{*}=\pi_{g1}^{*}\end{aligned} \qquad (2.52)$$

When the strikes are the same it is found that the delta is constant.[193] Correspondingly, for the entire range of expected inflation, gamma is equal to zero when the strikes equate. The delta magnitude remains constant implying no single minimum or maximum value can be identified. This is consistent with Proposition 2.5. When the upper and lower thresholds converge, the monetary policy response is consequently no longer path dependent. This might be thought of as an important limiting case of opportunistic smoothing. If alternatively, $E(\pi)$ is derived from square roots that have different signs as described by (2.51) the following emerges:

[192] When policy makers agree this implies policy is linear and the delta as a consequence is constant.
[193] This would imply policy makers are fully agreed on the inflation target.

$$E(\pi)=\left[\frac{\pi^*_{g2}.\pi^*_{g1}}{e^{\sigma^2(T-t)}}\right]^{0.5} \tag{2.53}$$

and this outlines a geometric mean type formula that permits the calculation of an expected inflation level that minimises delta.[194] Once the inflation thresholds, time horizon for targeting inflation and volatility of the underlying are known, it is possible to deduce the extent to which policy is most graduated or least policy activist due to the uncertainty regarding the underlying.[195] It is clear from (2.53), that the delta curve is asymmetric given that:

$$E(\pi)=\left[\frac{\pi^*_{g2}.\pi^*_{g1}}{e^{\sigma^2(T-t)}}\right]^{0.5} < \frac{\pi^*_{g1}+\pi^*_{g2}}{2} \text{ where } \pi^*_{g2} \neq \pi^*_{g1} \tag{2.54}$$

It is also clear that as the volatility and maturity (i.e. targeting horizon) increase, the delta curve becomes more asymmetric for a given set of thresholds. If the product of the variance and time go to zero, the expected inflation rate associated with the minimum delta converges to the geometric mean of the threshold. This implies that the expected inflation rate associated with the minimum delta will *always* be less than the arithmetic mean. This asymmetry is a key feature of the Black/Black-Scholes model. From the price behaviour implicit in (1.14), it is clear that as expected inflation increases the effect of volatility, σ, is greater. If central bankers accepted that inflation volatility increased as inflation increased, this would imply that policy makers using this type of risk analysis would respond more aggressively to inflation as inflation increased.[196] In this regard, the Black model has built in more upside risk than downside risk. This feature is particularly useful to inflation risk analysis that weights the uncertainty of accelerating inflation differently to disinflation. The asymmetry implicit in the Black model is examined in more detail when the Vasicek model is developed in Section 2.5.

[194] See appendix A.2.2 for derivation.
[195] The application of (2.53) would appear to have a wide applicability.
[196] This would also imply that a policy maker, who perceived the risks of deflation as being minimal, would better describe their policy responses as being consistent with the time valuations associated with Black model.

2.4 Explicit zone targeting and the desire for 'wiggle': arguments for an opportunistic FOMC policy response

During the early discussions of monetary policy strategy at the Greenspan FOMC, members frequently identify the need to preserve latitude. This notion of providing leeway was significantly developed by President Corrigan in the December debate (FOMC transcripts, p.30, 1989) which also proved important in terms of understanding the origins of the opportunistic debate. President Corrigan described leaving some 'wiggle for shocks' in much the same way as Governor Brash described preserving a sufficiently wide target band. Providing a timetable seemed to be difficult given the array of possible exogenous shocks that could derail policy actions. President Corrigan indicated an unwillingness to gamble the Feds hard won reputational capital by too precisely flagging the Federal Reserve's target for inflation. If the inflation target was very precisely signalled and that target subsequently was not met, it was feared that the Federal Reserve would loose credibility by virtue that a highly activist policy was not politically feasible. In the context of both explicit 'inflation zone targeting' and the lesser well defined opportunistic disinflation strategies, Propositions 2.3 – 2.3c provide an innovative approach to understand how policy becomes more graduated over a contentious range of expected inflation. By avoiding excessive policy activism, a robust political economy stance may be assumed. Figure 2.4 shows that widening the target zone is associated with the delta declining. This is consistent with providing extra 'wiggle' room for rate setters. It also provides some insight into understanding why central bankers may advance a somewhat fuzzy or vague inflation target. Finding the optimal bandwidth marries two institutional requirements of (a) preserving discretion over policy and (b) maintaining a sufficiently tight grip on inflation expectations. The following exchange between chairman Greenspan and Governor Laware at the December 1989 FOMC meeting, p. 29-30 of the transcripts, identifies the pitfalls of too precisely identifying a point target.

> Mr. Laware: 'If Babe Ruth had hit that home run in the 1932 World Series, whether he pointed to the center field stands or not wouldn't have made any difference. But, [after pointing to the stands], if he hadn't hit it he'd have been seen as a fool.'
>
> Chairman Greenspan 'No, if he hadn't hit it, he never would have been seen as the ball game's….'

> Mr. Laware: 'But having pointed, I think we run the distinct danger of [losing] credibility as well confidence and then we get into the position, politically, where we as an institution become more vulnerable.'(Federal Reserve Board, 1989, p.29-30)

The Babe Ruth analogy demonstrates an understandable reluctance by central bankers to construct a benchmark against which policy could be routinely criticised. By not positing a point target, policy makers had more scope for 'wiggle'.[197] That 'wiggle' of course has become a point of criticism for a number of economists. Svensson (2004) maintained that Greenspan has always sought to maintain maximum discretion. Conversely, Aksoy et al. (2006) maintain that a central bank dealing with moderately accelerating inflation may not be able to freely impose a conventional anti-inflation policy. The likely political acrimony occasioned by lost output may thwart central bankers in their efforts to control inflation.

So the Greenspan FOMC, at least with regard to some members, interpreted policy as nonlinear. There implicitly existed discernible elements of opportunism even though the term had not yet been coined. The desire for 'wiggle' constituted a form of pragmatic central banking in so far as policy makers were sensitive to reconciling short-term objectives. Portfolio option theory provides the means to describe opportunism under uncertainty using delta and other measures of risk. President Boehne's words at the December meeting portray a significant element of dynamic behaviour that seems to accord with the delta framework:

> 'I think we have to be careful here that we don't let perfection become the enemy of improvement. I would be happy to see us pursue a goal of disinflation over time and not necessarily in a straight line.' (Federal Reserve Board, 1989, p.33).

Importantly, opportunistic policy indeed afforded room for manoeuvre in terms of how policy could be orchestrated and articulated. The desire to avoid pointing, to continue the Babe Ruth analogy, suggests that the origins of opportunism were tied to retaining some discretion over balancing short-run objectives.

[197] The wiggle might be thought of as a discretionary zone target that policy makers can collectively change as need be and do not disclose to the public.

2.4.1 Heterogeneity of opinion: a zone of stalemate

Reading the FOMC transcripts, shows that in committee discussions, policy makers rarely achieve complete agreement. There are time-honoured controversies on the role of money, the inflation target and on the appropriate inflation targeting strategy. Members of the FOMC frequently hold patently divergent views.[198] The role of the chairman is important in securing accord wherever possible, and this may cause incumbents to lean towards a more vague definition of price stability even in the absence of a formal zone target. Attaining agreement often implies steering a middle course that the majority of members can accept as being reasonable and for which they would endorse when voting for changes in the policy rate. Paradoxically, this task may be made easier when the future path of inflation is uncertain.[199]

In the absence of an agreed target, policy maker differences may affect the dynamics of implementing rate decisions. Without explicitly defining the mechanics of inflation targeting, policy can be significantly influenced by personality, suggesting that consensus fixing depends enormously on the skills of the board's chairman.[200] Agreement internally and externally is important if policy is to minimise stalemate, particularly where rate decisions comes down to majority voting. The delta curve and the associated risk analysis shows however how disagreement dynamically influences rate decisions.

2.4.2 Heterogeneity of opinion: committee and institutional dynamics

A number of Fed insiders have maintained that the Federal Reserve has implemented policy within an opportunistic framework over the Greenspan incumbency.[201] The conceptual development of opportunism can be traced back to the late 1980s when inflation

[198] In so much as policy makers are willing to buy insurance and increase the policy rate even when expected inflation remains within the thresholds.

[199] By increasing the volatility of inflation, it is possible to illustrate that policy becomes increasingly linear.

[200] Meltzer (2005) makes the following observation: 'You can not end inflation (i) if you don't agree on how to do it, (ii) if you and the public think it is less costly to let it continue, and (iii) if you are overly influenced by politics. The Federal Reserve was better able to control inflation when the President was named Eisenhower or Reagan instead of Johnson, Carter or Nixon.' (p. 172)

[201] A number of former and current policy makers including Alan Blinder, former vice chairman of the Board of Governors (BOG), Edward Boehne, former President of the Philadelphia Federal Reserve Bank, former Governor Lawrence Meyer and Governor Donald Kohn have set out arguments in favour of the opportunistic approach.

moderated to levels where policy makers turned increasingly their attention to containing unemployment. Central bankers may target inflation less aggressively during periods of modest price rises simply because of difficulties encountered in marshalling any form of sustained consensus amongst members of the Board. Meade (2005), using a dataset constructed from the FOMC transcripts, found that despite only an official dissent rate of 7.5 percent, the level of discord was of the magnitude of 30 percent for opinions expressed regarding the Fed Funds rate in internal discussions. The effect of committee dynamics may be more significant than that suggested purely by voting patterns. Blinder, Goodhart, Hildebrand, Lipton and Wyplosz (2001) maintained that members of the committee do not invariably vote their true preference. Meade (2005) makes the point that during discussions, opinions are much more diverse and that, as a tool for consensus building, the chairman has been able to use the bias statement in the policy directive.

The chairman may also feel obliged to ensure that policy should not elicit political censure. Divergent opinions not just between members of the Board but also between the Board and the legislature influence substantially the delivery of policy decisions. The FOMC transcripts from the late 1980s tend to corroborate the view that the incumbents of the board sought not to markedly antagonise their political peers. Institutional dynamics may also be used to motivate a zone of policy stalemate. President Forrestal of the Federal Reserve Bank of Atlanta outlined one key stumbling block in reducing inflation rapidly to its desired level, during the FOMC meeting in December, 1989:

> ‘There is an acceptance now-- rightly or wrongly, and I think it’s wrongly—that 4-1/2 percent inflation is not all that bad. As inflation goes up, there comes a point where people get concerned about it; I think people would be willing to suffer some sacrifice to go from, say a 7 to 8 percent rate of inflation to something lower than that. But to go from 4-1/2 to zero, I think raises a question about the political consequences of getting from where we are in 1989 to 1995. I’m not saying that I disagree with the concept of moving in that direction. But I think a question that we need to ask ourselves is whether 7 percent unemployment will be accepted by the public at large and, particularly, by the Congress.’(Federal Reserve Board, December, 1989, p.14)[202]

[202] President Forrestal’s view did not seem overly extreme by reference to comments that followed. Interestingly Chairman Greenspan’s immediate response to President Forrestal short discourse was that “I (the chairman) think that is a crucial question and it’s obviously implicit in everything we do”.

President Forrestal is clearly indicating that institutional or political dynamics influence the policy response to the inflation gap and this varies over different levels of inflation. The delta curve, as outlined by Figure 2.3, provides a means by which to capture this nonlinearity. President Forrestal comments would seem to suggest that there exists an asymmetry in monetary policy, in that increasing levels of inflation warrant disproportionately greater levels of concern. Note that the definition of opportunism, proffered here, should not be confused with the 'political opportunism' spelled out by Persson and Tabellini (1999). This latter envisages monetary policy being crafted to deliver economic growth and low interest rates just before elections so as to cast political incumbents in a more favourable light. Opportunistic policy, as elaborated in this chapter, is meant to convey policy responses that are inflation path dependent. This may have a political dimension but not to the extent that the FOMC sets interest rates to alter the outcomes of elections. Opportunism does not explicitly enshrine an electoral cycle in inflation.

2.4.3 More on committee dynamics: early attempts at defining the upper threshold

Two years into the Greenspan incumbency, a cognitive upper threshold seems to be 4 ½%. No theoretical argument was advanced to explain why this level of inflation is significant from the public or policy maker perspective. One way of viewing this 4½% level, is that it represents the sum of the intermediate target π_t^* and the tolerance level δ. Interestingly, in what follows, this magnitude appears to be a recurring threshold level, at least, in what has been said by a number of policymakers.[203] When investigating the notion of opportunism, the December FOMC transcripts of 1989 provide an interesting point of departure, namely because Chairman Greenspan (p. 28) explicitly posed the question: how should central bankers respond to inflation when price increases are generally moderate? The resulting discourse, revealed a somewhat divided committee.[204] The December 1989 meeting was also significant in that it pre-dated a formal definition of opportunism and its inclusion in the literature. From the transcripts the approach appeared to evolve over time and probably culminated with a reasonably clear exposition in December 1995. The December 1989

[203] It may have also reflected where inflation stood at the time of the December 1989 meeting.
[204] The relevance of this division was developed in Propositions 2.1a – 2.1e.

meeting however was representative of a number of FOMC discussions where the policy makers were seen as grappling with how to discern an appropriate course of setting interest rates when inflation was already moderate. The strategies considered for attaining price stability reflected the views of individual members of the committee. Nonlinearities can be detected in the way the chairman defines with the committee the region of price stability. The debate on this occasion was substantively motivated by two questions posed by Chairman Greenspan (in the December FOMC transcripts):

> 'Do the committee members believe that there are significant advantages in targeting stability in the general price level as opposed to seeking to establish a steady low rate of inflation? That is, are we looking for zero inflation or are we willing to accept, say 4-½ percent? (Federal Reserve Board, 1989, p.28)

and

> Is a precise timetable for moving to the ultimate objective important either as a self discipline or for expectational reasons or would it be sufficient simply to focus on maintaining progress in the disinflationary direction? (Federal Reserve Board, 1989, p.27-28)

The discussion that ensued from the chairman's questions is instructive in that a number of the committee members interpret monetary policy as being much more subtle than a unified conventional linear rule would imply.[205]

The discussion seemed to lay the foundations for adapting a particular strategy to curb inflation when price level changes are modest. One recurring theme that comes through the December discussion is that objectives are attained not always in a straight line. Not all around the table shared President Forrestal's opinion. The transcripts reveal a mosaic of views and deliberative anti-inflationary rhetoric was by no means absent. Governor Laware pointed out that he was not prepared to accept even transitory departures from the goal of price stability in the December FOMC transcripts.

> 'I have been repeatedly shocked, or guess dismayed, by the level of nonchalance evidenced by some of my colleagues in my previous incarnation on how they felt about the current level of inflation. We have sat here at the Federal Advisory

[205] Cukierman (2000) suggests that political institutions tend to interpret the costs incurred by contraction as exceeding the benefits that flow from economic expansion. In a representative democracy autonomous yet accountable central bankers cannot be totally removed from the wishes of their political peers.

> Council meetings and talked about the economy and almost had to drag out of them some level of concern about inflation.' (Federal Reserve Board 1989, p. 29)

Views differed substantially in terms of what policy makers regarded as constituting the correct approach to implementing monetary policy. Equation (2.1) with a low inflation target, probably better describes Governor Laware's desire to restore price stability as a matter of urgency. This impetus to lower inflation is manifestly not conditioned on levels of tolerance around an intermediate target. In this regard, the type of policy one would suspect Governor Laware would advocate would be strongly deliberative. In addition, his immediate inflation target seems to have been much lower than a number of his peers. Likewise, Governor Angell at the December 1989 meeting makes the point that the resolute anti-inflation type stance is the most 'honest' approach.

> 'Finally, it seems to me that there's basic integrity involved. I just don't understand why anyone would want to say they wanted to participate in a lack of integrity, meaning we're just making promises. It's our job to make promises in regard to the purchasing power of U.S. dollars. To me it's a moral question of integrity. And I cannot participate – I cannot serve on a Board and an FOMC that doesn't have this integrity. Excuse me for being so extreme! But I don't know how else to deal with it.' (Federal Reserve Board 1989, p. 35)

This position would certainly seem at odds with the views expressed previously by a number of his colleagues. There appears to be a conflict between adhering to a rule of systematically lowering inflation or alternatively taking on board other considerations. A logical question to ask then is, given the prominence of formidable deliberative policy makers, could policy ever be feasibly described as being opportunistic?

The nominal and real stalemate scenarios as proposed previously, suggest that the opportunistic approach can be viewed as the default mode for implementing policy when no defined target for inflation is legally mandated, rendering the target vague. So, even when two deliberative policy makers set interest rates linearly but have very different inflation targets, policy can become *collectively* opportunistic.[206] Here, disagreement in voting intentions between policy makers is used to explain stalemate. Heterogeneity of

[206] No explicit inflation target was spelled out by the Federal Reserve during the Greenspan incumbency. Opportunistic policy (or flexible inflation zone targeting) is developed here from the perspective that no single durable point target was shared by a majority of FOMC members.

opinion is used as a motivational tool, just as Taylor and Taylor (2004) use a 'band of inaction' to explain anomalies in the Purchasing Power Parity (PPP) relationship. Kilian and Troy (2003) also explain nonlinearity in PPP as being a consequence of a lack of agreement amongst foreign exchange traders.

2.5 The Vasicek Model: deflation, mean reversion and normality

In chapter 1, the lognormal random walk was found to be a useful analytical tool when examining inflation risk.[207] The options framework as already set out can borrow techniques that are now well established in pricing and appraising risks. Central Bankers may wish to incorporate alternative features into the risk analysis. The Heston (1993) model was used to take account of varying measures of skewness and kurtosis for the underlying asset price. The Vasicek (1977) model which is commonly used for the time valuation of interest rate derivatives will be used here to investigate the effects of: (1) negative inflation outcomes and (2) predictability or mean reversion in the underlying behaviour and (3) normality. The parameter values for the Vasicek model can also be selected so as to render expected inflation to be an endogenous variable as opposed to being exogenous. The application of other time valuation models to $\left(c_{g2} - p_{g1}\right)$ suggests that the analysis offered in (2.17) can be made extremely general and flexible to accommodate a wide spectrum of behaviours and assumptions.

Proposition 2.6: *The option's framework can be extended to incorporate a wider set of inflation behaviours than that suggested by Black-Scholes. The parameter values of the Vasicek (1977) model can be selected to reflect varying degrees of central banker uncertainty and the speed of adjustment to a long term mean inflation rate given that a particular regime is understood to exist.*

Proposition 2.6a: *The option's framework as set out by Propositions 2.1a – 2.1d, can be extended to a policy context where expected inflation is considered to be predictable.*

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[207] Proposition 1.3.

Proposition 2.6b: *The option's framework as set out by Propositions 2.1a – 2.1d, can be extended to incorporate negative inflation.*

Unlike many of the standard models, the Vasicek model opens the possibility that the underlying asset price becomes negative.[208] For most applications of option pricing, this would be perceived as a major disadvantage. In a monetary policy context considering negative values (i.e. deflation), could constitute a practical counterfactual analysis. If central bankers believe that they can use existing patterns in the data to forecast future outcomes, then their risk analysis may be enhanced by also incorporating mean reversion, which is common in many interest rate option pricing models. Alternatively, they may be satisfied that they know, (given their proprietary information), the effect that a given policy stance will have on future inflation and that this effect can be accurately estimated. Another advantage of the Vasicek model is that it can be used to make expected inflation endogenous. The Vasicek model when applied to inflation takes the form:

$$d\pi(t) = a\left[b - \pi(t)\right]dt + \sigma\sqrt{dt}z \qquad (2.55)$$

where the change in inflation, $d\pi(t)$, is dictated by *a*, *b* and σ, which are constants. The current change in inflation in this simple model is influenced by the distance the current inflation rate is from its long-term mean *b*. The speed of adjustment is given by *a*. Super-imposed upon this adjustment is a normally distributed stochastic term, $\sigma\sqrt{dt}z$. Note that the level of inflation does not impact on the behaviour of volatility. This is quite different to the Black/Black-Scholes model of asset price behaviour outlined in chapter 1. In the Vasicek model, when $\pi(t) > b$, the drift term will be negative and will drive $\pi(t)$ down towards *b*. When $\pi(t) < b$, the drift will be positive and will push $\pi(t)$ up in the direction of *b*. Importantly, it should be noted that the volatility term here, σ, denotes the volatility of $d\pi$ as opposed to the more customary volatility of $d\pi/\pi$ used in the Black model.

These parameter values *a*, *b* and σ are generally obtained using the term structure of interest rates or alternatively by making use of time series estimation. In a monetary policy

[208] The Bachelier model could also be used to achieve this end.

risk management context, it maybe more natural to intuit these parameters from a view regarding the appropriateness of policy, the effectiveness of a given regime and from an appreciation of how successful a given policy will ultimately be.[209] When implementing the Black model, it is commonly assumed that the current spot price constitutes a good forecast of the future price.[210] The Vasicek model permits more subtle forms of behaviour where the asset price, over the life of the option, can migrate along a given path. In a monetary policy context this enables the forecast to depend upon the policy instrument and perhaps on the confidence policy makers have that an agreed strategy can push inflation in a particular direction. When using an option's framework, it is not necessary to assume that the inflation process is exogenous to the policy instrument, although this is what the Black model approach would normally imply.[211] In this regard, if central bankers believe that their policies will drive inflation in a given direction and are confident that this progression can be correctly calibrated and parameterised, then the Vasicek model offers a means by which to implement these additional features in the risk analysis. This leads to:

Proposition (2.7): *The Vasicek model permits the drift of the inflation process to be dependent on the policy instrument. In other words, expected inflation can be made to be endogenous to the rate decision.*

2.5.1 Estimating the parameters of the Vasicek model

Typically, to estimate interest rate behaviour using the Vasicek parameters, a, b and σ are inferred from market bond prices. Alternatively, as illustrated by Back (2005) it is possible to estimate the parameters using Ordinary Least Squares. The linear regression used here is given as:

[209] The Vasicek parameters are likely to change depending on macro-economic conditions.
[210] This arbitrage condition can however be relaxed.
[211] A more ambitious approach links the behaviour of inflation to the policy instrument. This goes to the heart of a key debate in monetary economics: what proportion of the recent moderation in inflation and inflation volatility can be ascribed to better monetary policy or alternatively to favourable trends such as globalisation? The Vasicek model has the benefit of not imposing a Krugman (1991) type relationship, where in an exchange rate setting, the value of a currency is obstructed from moving outside a target band using an option's framework. The Vasicek model, in contrast, permits a relatively large proportion of inflation paths to exit outside the zone target, although it acts like a loaded dice in that a given inflation trajectory is privileged. This could be a useful device when policy makers feel confident in their forecasting abilities and sanguine regarding the risk of hyperinflation.

$$\pi_t - \pi_{t-1} = \alpha + \beta\pi_{t-1} + \varepsilon \tag{2.56}$$

Where ε is a normally distributed random variable. Once α, β and ε are estimated, it is possible to infer a, b and σ from the following equations:

$$\alpha = \left(1 - e^{-a\Delta t}\right)b \tag{2.57}$$

$$\beta = -\left(1 - e^{-a\Delta t}\right) \tag{2.58}$$

$$\mathrm{var}(\varepsilon) = \frac{\sigma^2\left(1 - e^{-2a\Delta t}\right)}{2a} \tag{2.59}$$

Monthly CPI data downloaded from the Federal Reserve Bank of St Louis (series id: *CPIAUCSL*). This was used to calculate inflation over the Greenspan incumbency 1987:9 – 2006:1. Ho and Lee (2005) illustrated how (2.55) can also be calculated directly using Maximum Likelihood estimation. Using both Maximum Likelihood and Ordinary Least Square approaches, outlined respectively by Ho and Lee (2003) and Back (2005), yield the same parameter values for a, b and σ. The linear regression model of (2.56), estimated using OLS, had however a R^2 of just 13%. Both α and β estimates were statistically significant at 5%.[212] Using (2.57), (2.58) and (2.59) it is found that a = 0.03636, b = 0.03052 and the Vasicek volatility, σ_{vas} = 0.00286. These monthly parameters are annualised before being used numerically to calculate the time value of the following collar option portfolio: $\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=2,\sigma_{VAS}=0.00286}$.

2.5.2 The Monte Carlo Vasicek model

Monte Carlo is used to calculate the value of the option portfolio similarly to what was outlined in chapter 1. A path is first defined by a mean reverting process given by (2.55). Each path is then divided into discrete time steps so that:

$$\Delta t = T / m$$

where the time period T = 2 years and m = 24. π_t denotes the initial inflation rate. Here a series of rates are considered ranging from - 5.75% up to 12% in intervals of 0.25%. It

[212]The low R^2 would suggest that a mean reverting model, such as that given, may not reliably predict future changes in the inflation rate. This is not to rule out the possibility that central bankers could not have proprietary information that would permit a better configuring of α, β and σ, when compared to a historical estimation.

should be noted that the paths should converge in the direction of *b*, the long-term mean. These produce average Monte Carlo terminal values ranging from -0.43% to 6.6%. The mean terminal values are necessary if the Vasicek model is to be compared directly with the Black model, by virtue that the appropriate input into the Black model is the expectation of inflation. With each successive *j* a new inflation rate emerges until the terminal value, π_T is reached producing $j = m$:

$$\pi_T = \pi_t + \sum_{j=1}^{m} a\left(b - \pi_{t+j}\right)\Delta t + \sigma_{vas}\sqrt{\Delta t} z_j \qquad (2.60)$$

$z \sim N(0,1)$, represents a standard normal random variable and will have a mean of zero and standard deviation of one. The parameter estimates, already given for *a*, *b* and σ, are necessary to carry out the calculation. This defined path would yield just a single terminal π_T. For many different *n* paths, i.e. $n = 50{,}000$, an improved mean estimate of π_T, $E(\pi_T)$ can be made so that:

$$E\left(\pi_T\right) = \frac{1}{n}\sum_{i=1}^{n}\pi_{T,i} \qquad (2.61)$$

$E(\pi_T)$ constitutes a forecast of future inflation. *n* simulations are also run to calculate the value of the call and the value of a put for a given π_t. To calculate the call, this implies taking the terminal values, subtracting the exercise k_i^c given here as 4.5% and then summing only the positive values. This is then divided by $n = 50{,}000$. The continuously compounded rate is given here at 5%.

$$c = e^{-r(T-t)}\frac{1}{n}\sum_{1=1}^{n}\left[\max\left(\pi_{T,i} - k_i^c, 0\right)\right] \qquad (1.51)$$

Likewise, the value of a put can be calculated simultaneously so that:

$$p = e^{-r(T-t)}\frac{1}{n}\sum_{1=1}^{n}\left[\max\left(k_i^p - \pi_{T,i}, 0\right)\right]$$

where $k_i^p = 0.025$.

Figure 2.6a maps out the time valuations of the Vasicek collar using the parameter values as described above. A key point to note here is that to make the Vasicek collar comparable with Black time valuations, it is necessary to infer the average terminal values of the inflation paths from the Monte Carlo estimations. This has the effect of compressing the range on the horizontal axis around *b*. Then using an iterative technique, that minimises the RMSE between the Vasicek and Black collar time values, a Black volatility is found. This is computed to be close to 22%, over the range of expected inflation considered in Figure 2.6a. Significantly, this volatility estimate is much smaller than what would normally have been considered the mean Black volatility estimate over the Greenspan incumbency, which was closer to 35%.[213] One consequence of imposing mean reversion is to mitigate the effect of volatility when pricing an option contract. This is consistent with the closed form solution derived by Hull and White (1990) for the extended Vasicek model. They showed that as mean reversion increased, the appropriate volatility parameter input declined.[214] [215] Figure 2.6b compares the results demonstrated in Figure 2.6a against a set of Black collar time valuations having the same parameter inputs as Table 2.1, the exception being that volatility is set at 22% and the maturity is extended to two years. Interestingly, it is found that the collar valuations differ. In particular, for the call option, the Black model attributes a higher value than for the put where the underlying is in-the-money and commensurately different from the exercise. The Vasicek model would appear to provide approximately equal time valuations when considering the inverse positions of the call and put. This would imply that the Vasicek model produced a more symmetric collar.

2.5.3 The Vasicek model and asymmetry

The difference in Black time valuations of the call vis-à-vis the put is reflected by the asymmetry observed in the Black delta curve. This is especially apparent when compared to the Vasicek delta curve, illustrated in Figure 2.7.

Proposition 2.8: *The Black model engenders a proportionately greater policy response as the underlying expected inflation rate increases. This can be benchmarked against the Vasicek model where policy responses defined by the delta are found to be symmetric.*

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[213] Keeping in mind, that the Vasicek parameter inputs were estimated over the Greenspan chairmanship.

[214] Jamshidian (1989) also found a corresponding result in developing a closed form analytic pricing formula for bond options, using the Vasicek term structure model.

[215] Heuristically, Lo and Wang (1995) found that predictability, in general, decreases option prices. An increase in predictability in some cases is equivalent to a reduction in the asset's residual uncertainty or prediction-error variance. Option prices monotonically increase with the volatility of residual uncertainty in the Black-Scholes case. Reducing this uncertainty, should in turn reduce the option's premium.

The asymmetric Black model as developed before in Proposition 2.4 becomes more apparent when the Black delta is compared to the Vasicek delta curve. This is illustrated in Figure 2.7. The Vasicek delta curve was computed numerically for each expected inflation rate, given in Figure 2.6a, where $E(\pi)_u > E(\pi) > E(\pi)_d$, and $E(\pi)_u - E(\pi) = E(\pi) - E(\pi)_d = 0.0025$. The numerical delta can be estimated using:

$$\frac{\partial\left(c_{g2} - p_{g1}\right)}{\partial E(\pi)} = \frac{Collar_u - Collar_d}{E(\pi)_u - E(\pi)_d} \tag{2.62}$$

where the $Collar_u$ denotes the value of the collar when expected inflation is $E(\pi)_u$ and $Collar_d$ gives the value of the collar when expected inflation is $E(\pi)_d$. The Black delta curve is computed from the collar parameter inputs associated with Figure 2.6b. The Black parameters are as follows: $E(\pi_{t+1})$ = 0.25% to 6.5%, $\pi^*_{g2} = 0.045$, $\pi^*_{g1} = 0.025$, $T = 2$, $r = 0.05$ and $\sigma = 0.22$. The usual analytical delta formula is used:

$$\frac{\partial c_{g2}}{\partial E(\pi)} - \frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T-t)} N\left(d_{1g2}\right) - e^{-r(T-t)}\left[N\left(d_{1g1}\right) - 1\right]$$

It is clear that the Black delta curve is less symmetric than the Vasicek delta curve in Figure 2.7. This is discernible given the relative positions of the respective expected inflation rates associated with the minimum delta values. The expected inflation rate associated with the minimum Vasicek delta, is 3.5%. Whereas the expected inflation rate, associated with the minimum Black delta, is less than 3.5%, implying an asymmetry.[216] A key difference relates to the stochastic behaviour implicit in each model. The Black model posits that as the level of the underlying increases, the volatility of the underlying increases correspondingly. This specification may be useful to central bankers who wish to characterise the upside risks of inflation as being greater than the down side risks to inflation.[217] This would imply a Black model risk analysis would precipitate a more aggressive policy response when expected inflation is rising above the upper threshold. The Vasicek model characterises volatility as remaining invariant to the level of expected inflation. From Figure 2.6b, it is clear that when valuing the options portfolio, the Black model is asymmetric. Other things being equal, should central bankers possess the insight

[216] Proposition 2.4.

[217] Particularly, if policy makers believe that inflation can get close to the zone of falling prices but not go into that zone.

that volatility increases as inflation increases; this view would be better co-opted into the risk analysis by using the Black model. In the following section, the time valuation of $\left(c_{g2} - p_{g1}\right)$, estimated by the Black model, is investigated from the perspective of offering a better empirical description of monetary policy over conventional linear reaction function estimation.

2.6 The nonlinear reaction function

From Figures 2.8 and 2.9 it is clear that Fed Funds rate setting behaviour was subject to change, particularly after 2002. This would suggest that it is worth initially providing an analysis that considers varying specifications for the policy rule which are both linear and nonlinear. The latter would have likely entailed an evolving zone target, given the extent to which inflation had fallen over the Greenspan incumbency. Initially, a linear Taylor rule is estimated in Table 2.3a and this is then compared against several zone targeting rules, reported in Table 2.3 b – f, using the option's framework and Black model as outlined before.

2.6.1 Estimating the linear and nonlinear reaction functions

Table 2.3a estimates the Taylor Rule in the usual way using OLS, where a constant, inflation and the growth gap are independent variables.[218] Both independent variables appear to be statistically significant and their coefficients are comparable to the classic rule. The constant is estimated to be slightly less than 0.01 or 1%.[219] These parameter weights will be used in both Figures 2.10a and 2.10b to trace out the estimated linear rule. In addition, nonlinear rules are initially considered with varying Black volatility parameter inputs. To make the graphical analysis comparable, the growth gap in each rule is assumed to be in balance at 0% when tracing the scatter plots. This permits a two dimensional representation to describe each rule. The Residual Sum of Squares, (RSS), reported in the Tables, will be used initially to compare goodness of fit. This preliminary analysis is intended to detect how best the option portfolio might help explain rate setting over the

[218] See Appendix A.2.4 for estimation of growth.
[219] The fed funds, growth gap and inflation rates are initially expressed here in decimal form.

Greenspan period. More formal estimates of the rule are provided later. Applying the usual notation the parameters α_0, β_1 and β_2 are calculated using OLS for the linear rule:

$$r_t = \omega_0 + (1+\omega_1)(\pi_t) + \omega_2(y_t) \tag{2.63}$$

where

$$\alpha_0 = \omega_0 = (r^* - \omega_1\pi^*),\ \beta_1 = 1+\omega_1,\ \beta_2 = \omega_2 \tag{2.64}$$

Likewise, the parameters estimates for α_0, β_1 and β_2 are calculated using OLS for the nonlinear rules:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25} e^{rT} + \beta_2(y_t) \tag{2.65}$$

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) \tag{2.66}$$

The Collar time values $\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25}$, $\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.35}$ are calculated using the Black model, as outlined before. From the GARCH appendix A.2.3, it would appear that the historic norm for volatility was closer to 0.35 than 0.25 over the Greenspan incumbency. From Tables 2.3b and 2.3c the estimated constant term α_0, (given in square brackets below), tends to be higher for the nonlinear specification. This should be expected given that α_0 represents the arithmetic mean of the inflation targets plus the equilibrium interest rate[220]:

$$r_t = \left[r^* + \frac{\pi_{g1}^* + \pi_{g2}^*}{2}\right] + (1+\omega_1)(c_{g2} - p_{g1})e^{rT} + \omega_2(y_t) \tag{2.67}$$

where

$$\alpha_0 = r^* + \frac{\pi_{g1}^* + \pi_{g2}^*}{2}, \beta_1 = 1+\omega_1 \text{ and } \beta_2 = \omega_2 \tag{2.68}$$

It is found that as σ increases, the nonlinearity of the policy rule is reduced. This is illustrated in Figure 2.10a and 2.10b where the parameter estimates, once obtained, are used to map the policy rules over the range of expected inflation spanning 0.01 to 0.065, in intervals of 0.0025.[221] These minimum and maximum rates were close to the historic high and low over the period, 1987:4 – 2007:3. The Residual Sum of Squares (RSS), reported for the linear rule in

[220] The intercept will increase as both the upper and lower bounds increase.
[221] Inflation and the fed funds rate are initially expressed in decimal form.

Table 2.3a, is less than the respective Residual Sum of Squares reported in Tables 2.3b and 2.3c, implying that the zone specification of 0.045 – 0.025, provided a poorer historical fit for rate setting when compared to just applying the linear Taylor type rule. Figure 2.10b superimposes a scatter plot where inflation is compared directly against the contemporaneous fed funds rate for the same period. This is intended to provide some historical backdrop, in that, the quarterly fed funds rate is viewed against the quarterly inflation rate. It would seem clear from Figure 2.10b that using the 0.045 – 0.025 specification, does not visibly yield any improvement over the linear specification for the full period: 1987:4 – 2007:3.

Using OLS, Table 2.3d provides estimates for the parameters, α_0, β_1 and β_2 where the zone target and nonlinear policy rule are specified differently as:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.04} - p_{g1}^{\pi^*=0.02}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) \tag{2.69}$$

Likewise, Table 2.3e reports estimates for α_0, β_1 and β_2 where:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.035} - p_{g1}^{\pi^*=0.015}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) \tag{2.70}$$

As the upper and lower bounds are lowered from 4% - 2% to 3.5% - 1.5% the Residual Sum of Squares would also seem to decline. Tables 2.3f supplies parameter estimates for:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) \tag{2.71}$$

Again, it is noticeable that as the upper and lower bounds fall in magnitude, the nonlinear specification provides an improved fit, in particular over the linear specification. The RSS, reported in Table 2.3f, would appear to fall substantially when the upper bound is 3%, suggesting that policy makers in general responded more aggressively to inflation as it threatened to exceed 3% over the 1987:4 – 2007:3 period. One might expect however that the zone evolved over time, as was the case in Canada where the zone was publicly announced.[222] The parameter weights estimated in Table 2.3f are used in Figure 2.11 to map out the nonlinear policy rule. The same scatter plot as before is super-imposed to

[222] This is consistent with opportunistic theory where an intermediary inflation target is initially proposed and declines, as inflation declines. From (2.3), it is clear that opportunistic policy spells out an intermediary target.

provide some historical background. The nonlinear rule that applied the 0.03 – 0.01 zone specification would appear visibly, to provide the best fit. Significantly, the Residual Sum of Squares (RSS) reported in the tables for the linear rule and other nonlinear rules were higher. Figure 2.11 combines the policy fit for the linear rule, the nonlinear rule and a disaggregated scatter plot of the fed funds rate relative to inflation. The scatter plot is made to be time-discriminating and this seems to capture the policy accommodation that was forthcoming over the period 2001:1–2005:4.[223] From Figures 2.8 and 2.9, it is clear that rates were cut from 2000:4, when the effective fed funds rate had hit a high of approximately 6.5%. In setting out varying time frames, the 2001:1 quarter is a useful starting point for tentatively identifying the period when monetary policy became increasingly accommodative. It is apparent from Figures 2.8 and 2.9 that over the period 2001:1 – 2005:4, the federal funds rates were depressed relative to both inflation and the growth gap. This may have reflected concerns that Greenspan (2003) expressed regarding deflation.

2.6.2 Capturing the effect of policy accommodation using dummies

To measure the extent to which policy was relaxed, Table 2.4a reports estimates of the nonlinear rule above with the addition of a dummy variable over the period 2001:1 – 2005:4. Tables 2.4a reports OLS parameters estimates for α_0, β_1, β_2 and β_3, using:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2001:1-2005:4}\right) \qquad (2.72)$$

where D denotes a dummy variable having a value of 1 from 2001:1 to 2005:4 or otherwise 0 over the complete period. β_3 was found to be negative; supporting the hypothesis that policy was more accommodative over the 2001:1 – 2005:4 period, when compared to the rest of Greenspan and Bernanke incumbencies. Figure 2.12a maps out the effect of the dummy used in (2.72). The period from 2001:1 is associated with sharp rate cuts. Figures 2.8 and 2.9 also provide evidence that policy accommodation was supplied in an unparalleled fashion deep into 2004 and then was substantially removed prior to the appointment of chairman Bernanke. The dates for the dummy are refined in Table 2.4b to produce:

[223] Different episodes are denoted by varying colours. See legends.

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2002:1-2006:2}\right) \tag{2.73}$$

The dates for the dummy, $D^{2002:1-2006:2}$, were obtained by iteratively estimating a series of regressions and then calculating the RSS. Figure 2.12b illustrates the effect of the dummy, given in (2.73). The easing of policy would appear to be very significant. The dates associated with the dummy that produced the lowest RSS were used to implement the estimations in Table 2.4b. These dates closely correspond with a period of policy accommodation, identified by Taylor (2007). The negative co-efficient estimate for the dummy variable supports again the view that monetary policy was more relaxed over the 2001:1 – 2005:4 period. Taylor (2007) p.2 found that:

> "…during the period from 2003 to 2006 the federal funds rate was well below what experience during the previous two decades of good economic performance - the Great Moderation - would have predicted. Policy rule guidelines showed this clearly."

This may have purportedly reflected fears concerning deflation.[224] As it turned out, inflation did not ultimately move into the zone of falling prices and indeed, re-ignited partially on the back of global commodity prices. Given the significance of the dummy and the observable effects of policy accommodation, the results obtained in Tables 2.3 a – f are now tested for the shorter time frame that leads up to 2002:1. In particular, the nonlinear specification is compared against a linear rule. It is found that the nonlinear specification that contains 1 – 3% bounds provides a better description of policy.

2.6.3 Nonlinearity with a non-constant volatility.

In making additional estimations that focus on the earlier period, the option's framework is refined to take account of other factors. The constant Black volatility input parameter is relaxed and is permitted to vary over the period of estimation by incorporating a recursively estimated GARCH(1,1) model. This provides a successively updated volatility forecast of inflation, using only past observations of inflation.[225] Tables 2.5 a – e provide estimates for the Greenpsan incumbency prior to 2002:1. The options framework, here, would also

[224] Other central bankers however have not shared the same concern regarding deflation particularly given that most modern economies are based on fiat systems of money.
[225] See appendix A.2.3 for explanation of GARCH estimates.

appear to be useful in extending and improving upon the monetary policy analysis. The comparison with a linear rule, for the same period, is particularly telling. In providing additional estimations in Tables 2.5 a – e, it is found that while parameter estimates may have changed when the results are compared to Tables 2.3 a – f, the specification of a 3% upper bound remained most significant regardless of whichever time frame or methodology were being applied.[226]

Given the quite visible change in fed funds rate setting behaviour from 2002, Tables 2.3 a, b, c, d, e and f are re-estimated excluding the period from 2002:1 onwards. In addition, the assumption of a constant volatility level is relaxed so that an updating GARCH(1,1) volatility estimate can be used. This is explained in more detail in appendix A.2.3. A Taylor Rule is initially estimated in Table 2.5a with a marked improvement in fit when the R-square is compared to Table 2.3a. This would further suggest that treating the post-2002:1 period differently helps to describe aspects of the Greenspan monetary policy regime. The nonlinear rules, examined in Tables 2.5 b – e, regress a constant, an inflation collar and an output gap on to the effective Federal Funds rate in much the same way as before except inflation volatility is not assumed to be constant.[227] Each of the independent and dependent variables is expressed in percentage terms. Newey-White robust estimation is implemented to correct the t-statistics and standard errors for positive serial correlation and, as a consequence, estimates of these should not be compared directly with Tables 2.3 b - f. Table 2.5a reports estimates for a Taylor type rule similar to Table 2.3a, however the time period relates only to 1987:4 – 2002:1. Both OLS and Nonlinear Least Squares (NLS) produced identical results for parameter estimates for Tables 2.5 a - e. This is true for both the linear and varyingly specified nonlinear estimated rules.

Parameter estimates for (2.74) are presented in Table 2.5b,

$$r_t = \alpha_0 + (\beta_1)\overset{T=1,\sigma=GARCH}{\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)}e^{rT} + \beta_2(y_t) \qquad (2.74)$$

[226] In comparing Tables 2.3a - 2.3f and Tables 2.5a - 2.5e it is found that the inflation option portfolio with the 3% upper bound specification has the highest *t-stat*.
[227] The estimation covers the Greenspan incumbency up to the end of quarter 1, 2002. This date was selected on the basis of best fit offered by a varyingly specified dummy.

where the upper bound/strike is equal to 4.5% and the lower bound/strike is equal to 2.5%. These exercise rates are assumed to be unchanged over the period considered.[228]

The dynamic zone targeting typically associated with inflation targeting frameworks may provide some insight regarding the Federal Reserve's own strategies. The advocacy of an opportunistic strategy by a number of Fed officials may similarly shed some light. In practice, the 4.5% threshold as suggested by Greenspan, in the December 1989 FOMC transcripts, may have reflected an international norm for the upper tolerance at that time. From policy statements of several inflation targeting central banks, it would appear that the upper thresholds would have progressively declined as inflation declined. Thresholds/boundaries other than those that would just incorporate 4.5%, would appear to be worth contemplating. For practical reasons, these thresholds may be difficult to pin down exactly, given that the upper and lower bounds in an opportunistic framework are largely discretionary. Using the *collectively opportunistic* model, they are arrived at, in the absence of agreement and are likely to be complex given that the bounds would intermittently converge and diverge.[229] The regression estimates developed in the Tables ignore this nuance but are nevertheless still insightful. The collars are uniformly assumed to conform to a 2% zone gap over the entire Greenspan incumbency. This was unlikely, but nevertheless the explanatory power of the nonlinear option's framework is greater than a linear rule. Several nonlinear formulations are considered with varying upper and lower bounds being implemented:

$$r_t = \alpha_0 + \beta_1 \left(c_{g2}^{\pi^*=0.04} - p_{g1}^{\pi^*=0.02} \right)^{T=1,\sigma=GARCH} e^{rT} + \beta_2 (y_t) \qquad (2.75)$$

[228] Orphanides and Wieland (2000) point out that during the 1990s as inflation targeting frameworks were being established in a number of countries, target ranges were generally more common than point targets. New Zealand's first Policy Target Agreement (PTA) spanned 3% - 5% inflation in 1990. In December 1991, the upper and lower bands of the PTA fell to 2.5% - 4.5%. Ultimately, this declined to a hard floor and ceiling of 0% - 2%. A similar pattern of evolving targets occurred in the United Kingdom, Canada and Sweden. While the Federal Reserve has never explicitly indicated it employed inflation targets, it sometimes has been depicted as being an 'implicit' inflation targeter. Thornton (2007), using the FOMC minutes and transcripts, has identified a lower bound of 1% for inflation, with an upper bound likely to be between 3.1% and 3.75%. These bounds have never been formalised and indeed lack specificity in terms of which measure of inflation is guiding policy.

[229] Before, it was suggested that the inflation bounds are likely to diverge when confronting a supply-side shock.

$$r_t = \alpha_0 + \beta_1 \left(c_{g2}^{\pi^*=0.035} - p_{g1}^{\pi^*=0.015} \right)_{T=1,\sigma=GARCH} e^{rT} + \beta_2 (y_t) \qquad (2.76)$$

$$r_t = \alpha_0 + \beta_1 \left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01} \right)_{T=1,\sigma=GARCH} e^{rT} + \beta_2 (y_t) \qquad (2.77)$$

Tables 2.5 a – e illustrate that the nonlinear specifications particularly those with the least elevated bounds have the greatest statistical significance.[230] It is noticeable that as the strikes fall the statistical significance improves. The consistent improvement in the results over varying time frames when compared to linear estimations implies that the collar construction provides a significant reference in understanding monetary policy. How might this result be changed by applying real time forecast data? In what follows the real time forecasts of the GDP deflator and GDP growth are used to determine whether the linear or nonlinear construction provides a better description of policy.

Table 2.6a reports estimates of the Taylor type reaction function similar to Table 2.5a using the same time period as before 1987.4 - 2002:1. (2.78) differs however by making the forecast of inflation and growth gap explicit.

$$r_t = \omega_0 + (1 + \omega_1) E_t (\pi_{t+4}) + \omega_2 E_t (y_t) \qquad (2.78)$$

Greenbook projections for both inflation (4 quarters ahead) and growth (the current projection) were used instead of revised data used hitherto. The real time forecasts are worth examining because they help to capture elements of policy that are essentially forward-looking. The Greenbook projections of real output growth and inflation are available at the Federal Reserve of Philadelphia. The Greenbook projections are released with a lag of 5 years and differ in a number of respects from the Federal Reserve of St. Louis data used in previous estimations. As a consequence, direct comparisons with the results obtained in (Tables 2.3a – 2.5e) might not be straightforward to make with Tables 2.6a and 2.6b. More explanation of this data can be found at:
http://www.philadelphiafed.org/econ/forecast/greenbook-data/phila-data-set.cfm.
The growth gap was obtained using the trend growth methodology described in Appendix A.2.4. (2.78) is estimated using Ordinary Least Squares. A Newey-White correction to the standard errors with a lag value of 4 is applied. It is found that the statistical significance

[230] The t-stats for β_1 improve as the bounds fall. See Tables 2.5b, 2.5c, 2.5d and 2.5e.

for the parameter values, reported in Table 2.6a, falls when compared against Table 2.5a, suggesting that these particular measures of inflation and growth provide a poorer description. The inflation forecast and growth gap data are used again to calculate parameter values for (2.79), reported in Table 2.6b. An inflation collar is calculated as before using the recursive GARCH estimate for volatility, outlined in Appendix A.2.3. The parameter values are estimated using OLS, with a Newey-White correction being applied.

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=GARCH} e^{rT} + \beta_2 E_t(y_t) \qquad (2.79)$$

Table 2.6b produces a small deterioration in fit when compared against Table 2.5e. Interestingly, the nonlinear construction reported in Table 2.6b would appear to offer some improvement over the linear model, reported in Table 2.6a. Using either the real time forecast data set from the Federal Reserve of Philadelphia or the revised data set from the Federal Reserve of St. Louis tends to produce results which favour policy being described using the collar structure to replace inflation in the reaction function.

These results provide evidence that taking the entire Greenspan incumbency or in part up to 2002, monetary policy responded more robustly when inflation threatened to exceed three percent. The 1% – 3% targeting specification appears to offer an improvement over the linear representations of policy for real time forecasts and historic data sets that were subject to subsequent revision. This nonlinearity may be largely symptomatic of inflation generally declining during the Greenspan incumbency. As inflation moderated over the period, the 3% upper strike may have served as a significant conceptual threshold for FOMC members. This is mirrored in the results obtained in Tables 2.3a to 2.6b. These results would also appear to be robust for the post-2002 period when policy accommodation was vigorously supplied. The upper threshold may however have been much higher in early stages of the Greenspan incumbency, perhaps as much as 4 1/2 % when the December 1989 FOMC transcripts are taken into account.

2.7 Conclusion

In this chapter, option theory was exploited to advance a theoretical framework that brings together both risk management and opportunistic policy perspectives. The analysis presented here borrows heavily from a conceptual framework that emphasises inflation risk. It was also shown that it was not necessary to attribute any special preferences to individual central bankers in order to observe opportunistic behaviour. Even, in the presence of linear rate setting members, committee and institutional dynamics can serve to produce opportunistic type policy responses. If policy can only be implemented by first achieving consensus, where opinions differ regarding what constitutes an appropriate inflation target then rate setting can still be 'collectively' opportunistic.

A number of propositions were advanced in this chapter and were largely motivated by viewpoints expressed by policy makers at the FOMC table. In particular, the December 1989 meeting was examined from the context of offering tentative first steps in setting out realistic policy maker objectives. The put-call parity relationship and voting dynamics were exploited so as to nest an option portfolio in the reaction function. This was contingent on policy being inactive within thresholds over a given range of expected inflation. Using the standard Black formula for option valuation, it was also possible to gauge policy reactions under uncertainty. If the reaction function were stipulated to be forward looking, then portfolio option theory becomes particularly instructive. Variables such as volatility, time horizon and the relativities of differing inflation targets that are generally ignored in conventional monetary policy analysis can be incorporated into the risk management framework.

The genesis of opportunistic strategy was mapped out in this chapter. Option theory was applied in Section 2.2 and 2.3 to develop key insights of opportunistic disinflation as outlined by Orphanides and Wilcox (2002) and Aksoy et al. (2006). An option's framework was employed to investigate the impact of uncertainty on monetary policy. A combination of two put-call parity relationships was developed and combined with a zone of disagreement to form a long position with a call with a higher strike and a short position with a put with a lower strike on expected inflation. This option portfolio captured key aspects of inflation zone targeting when monetary policy is subject to both known inflation outcomes and to unknown inflation outcomes. A number of nonlinearities were examined using option theory by applying standard parameter sensitivities such as delta and gamma. A major attraction of using option theory relates to the availability of proven models that can feasibly handle varying conceptual inflation behaviour(s).

The Black model was used in Section 2.3 to estimate the time value of the collar and the likely monetary policy response under uncertainty. The Black ‘Greeks’ can be estimated for this portfolio. Delta was used to demonstrate how policy becomes less active when inflation resides between the upper and lower target bounds. It was also found that as the target zone expands; monetary policy becomes increasingly nonlinear or less activist for that range. This helps to establish the link between interest rate inertia and opportunistic policy. The transcripts were used in Section 2.4 to illustrate how risk management permeated FOMC policy discussions. Disagreement between policy makers were used to explain how divergent opinions thwart precipitated interest rate moves. This analysis was extended in Section 2.5 to take into account how alternative inflation behaviour can influence risk management. The Vasicek model was found to be useful for incorporating mean reversion and deflation. The asymmetries, characteristic of the Black model, were also judged against a number of metrics; including a Vasicek delta curve. Several nonlinear reaction functions were estimated in Section 2.6. Reduced bounds specifying the 1 – 3% target range, served to improve upon the standard linear Taylor Rule construal of rate setting.

Table 2.1: Presents estimates of the option portfolio for varying levels of expected inflation.

Expected Inflation	d1	d2	N(d1)	N(-d1)	N(d2)	N(-d2)	Call Time Value Rk =0.025	Put Time Value Rk =0.025	Intrinsic Value of Call Rk =0.025	Intrinsic Value of Put Rk =0.025	Collar Call(0.045) - Put(0.025)
0.0050	-6.313	-6.563	0.000	1.000	0.000	1.000	0.0000	0.0190	0.0000	0.0200	-0.01902
0.0075	-4.691	-4.941	0.000	1.000	0.000	1.000	0.0000	0.0166	0.0000	0.0175	-0.01665
0.0100	-3.540	-3.790	0.000	1.000	0.000	1.000	0.0000	0.0143	0.0000	0.0150	-0.01427
0.0125	-2.648	-2.898	0.004	0.996	0.002	0.998	0.0000	0.0119	0.0000	0.0125	-0.01189
0.0150	-1.918	-2.168	0.028	0.972	0.015	0.985	0.0000	0.0095	0.0000	0.0100	-0.00955
0.0175	-1.302	-1.552	0.097	0.903	0.060	0.940	0.0002	0.0073	0.0000	0.0075	-0.00731
0.0200	-0.768	-1.018	0.221	0.779	0.154	0.846	0.0005	0.0053	0.0000	0.0050	-0.00529
0.0225	-0.296	-0.546	0.383	0.617	0.292	0.708	0.0013	0.0036	0.0000	0.0025	-0.00363
0.0250	0.125	-0.125	0.550	0.450	0.450	0.550	0.0024	0.0024	0.0000	0.0000	-0.00234
0.0275	0.506	0.256	0.694	0.306	0.601	0.399	0.0039	0.0015	0.0025	0.0000	-0.00140
0.0300	0.854	0.604	0.804	0.196	0.727	0.273	0.0056	0.0009	0.0050	0.0000	-0.00069
0.0325	1.174	0.924	0.880	0.120	0.822	0.178	0.0076	0.0005	0.0075	0.0000	-0.00010
0.0350	1.471	1.221	0.929	0.071	0.889	0.111	0.0098	0.0003	0.0100	0.0000	0.00049
0.0375	1.747	1.497	0.960	0.040	0.933	0.067	0.0121	0.0002	0.0125	0.0000	0.00116
0.0400	2.005	1.755	0.978	0.022	0.960	0.040	0.0144	0.0001	0.0150	0.0000	0.00199
0.0425	2.248	1.998	0.988	0.012	0.977	0.023	0.0167	0.0000	0.0175	0.0000	0.00301
0.0450	2.476	2.226	0.993	0.007	0.987	0.013	0.0190	0.0000	0.0200	0.0000	0.00423
0.0475	2.692	2.442	0.996	0.004	0.993	0.007	0.0214	0.0000	0.0225	0.0000	0.00565
0.0500	2.898	2.648	0.998	0.002	0.996	0.004	0.0238	0.0000	0.0250	0.0000	0.00726
0.0525	3.093	2.843	0.999	0.001	0.998	0.002	0.0262	0.0000	0.0275	0.0000	0.00902
0.0550	3.279	3.029	0.999	0.001	0.999	0.001	0.0285	0.0000	0.0300	0.0000	0.01092
0.0575	3.457	3.207	1.000	0.000	0.999	0.001	0.0309	0.0000	0.0325	0.0000	0.01293
0.0600	3.627	3.377	1.000	0.000	1.000	0.000	0.0333	0.0000	0.0350	0.0000	0.01503
0.0625	3.790	3.540	1.000	0.000	1.000	0.000	0.0357	0.0000	0.0375	0.0000	0.01720
0.0650	3.947	3.697	1.000	0.000	1.000	0.000	0.0380	0.0000	0.0400	0.0000	0.01942
0.0675	4.098	3.848	1.000	0.000	1.000	0.000	0.0404	0.0000	0.0425	0.0000	0.02169
0.0700	4.243	3.993	1.000	0.000	1.000	0.000	0.0428	0.0000	0.0450	0.0000	0.02399

Expected Inflation	d1	d2	N(d1)	N(-d1)	N(d2)	N(-d2)	Call Time Value Rk =0.045	Put Time Value Rk =0.045	Intrinsic Value of Call Rk=0.045	Intrinsic Value of Put Rk=0.045
0.0050	-8.664	-8.914	0.000	1.000	0.000	1.000	0.0000	0.0380	0.0000	0.0400
0.0075	-7.042	-7.292	0.000	1.000	0.000	1.000	0.0000	0.0357	0.0000	0.0375
0.0100	-5.891	-6.141	0.000	1.000	0.000	1.000	0.0000	0.0333	0.0000	0.0350
0.0125	-4.999	-5.249	0.000	1.000	0.000	1.000	0.0000	0.0309	0.0000	0.0325
0.0150	-4.269	-4.519	0.000	1.000	0.000	1.000	0.0000	0.0285	0.0000	0.0300
0.0175	-3.653	-3.903	0.000	1.000	0.000	1.000	0.0000	0.0262	0.0000	0.0275
0.0200	-3.119	-3.369	0.001	0.999	0.000	1.000	0.0000	0.0238	0.0000	0.0250
0.0225	-2.648	-2.898	0.004	0.996	0.002	0.998	0.0000	0.0214	0.0000	0.0225
0.0250	-2.226	-2.476	0.013	0.987	0.007	0.993	0.0000	0.0190	0.0000	0.0200
0.0275	-1.845	-2.095	0.033	0.967	0.018	0.982	0.0001	0.0167	0.0000	0.0175
0.0300	-1.497	-1.747	0.067	0.933	0.040	0.960	0.0002	0.0145	0.0000	0.0150
0.0325	-1.177	-1.427	0.120	0.880	0.077	0.923	0.0004	0.0123	0.0000	0.0125
0.0350	-0.880	-1.130	0.189	0.811	0.129	0.871	0.0008	0.0103	0.0000	0.0100
0.0375	-0.604	-0.854	0.273	0.727	0.196	0.804	0.0013	0.0085	0.0000	0.0075
0.0400	-0.346	-0.596	0.365	0.635	0.276	0.724	0.0021	0.0068	0.0000	0.0050
0.0425	-0.104	-0.354	0.459	0.541	0.362	0.638	0.0031	0.0054	0.0000	0.0025
0.0450	0.125	-0.125	0.550	0.450	0.450	0.550	0.0043	0.0043	0.0000	0.0000
0.0475	0.341	0.091	0.634	0.366	0.536	0.464	0.0057	0.0033	0.0025	0.0000
0.0500	0.546	0.296	0.708	0.292	0.617	0.383	0.0073	0.0025	0.0050	0.0000
0.0525	0.742	0.492	0.771	0.229	0.688	0.312	0.0090	0.0019	0.0075	0.0000
0.0550	0.928	0.678	0.823	0.177	0.751	0.249	0.0109	0.0014	0.0100	0.0000
0.0575	1.105	0.855	0.866	0.134	0.804	0.196	0.0129	0.0010	0.0125	0.0000
0.0600	1.276	1.026	0.899	0.101	0.847	0.153	0.0150	0.0008	0.0150	0.0000
0.0625	1.439	1.189	0.925	0.075	0.883	0.117	0.0172	0.0006	0.0175	0.0000
0.0650	1.596	1.346	0.945	0.055	0.911	0.089	0.0194	0.0004	0.0200	0.0000
0.0675	1.747	1.497	0.960	0.040	0.933	0.067	0.0217	0.0003	0.0225	0.0000
0.0700	1.892	1.642	0.971	0.029	0.950	0.050	0.0240	0.0002	0.0250	0.0000

Table 2.1 : The Black model is used to calculate the time value of of the call and put where expected inflation is the underlying. Intrinsic values are also calculated. Expected Inflation ranges between 0.5% and 7.0%. The strikes Rk are equal to 2.5% and 4.5%, producing two sets of valuations respectively. The parameter values are explained in more detail in the text. tk denotes maturity and is expressed in years but set at 1. The Volatility input constitutes the annual standard deviation of the return on inflation, set at 0.25. The C rate and P rate give the time values for each of the options. Otherwise the notation is standard. The Collar valuation denotes the difference between the call and put which has respective strikes of 0.045 and 0.025. The risk-free rate is set at 5%.

Table 2.2: Data Summary and Sources.

	Real Gross Domestic Product	***Consumer Price Index***	***Federal Funds Rate %***	***One year Treasury Bill rate %***	***One year Treasury Bill Rate %***	***Inflation (generated from CPIAUCSL)***	***Growth Gap (generated from GDPC96) %***
Periodicity	Quarterly	Quarterly	Quarterly	Monthly	Quarterly	Quarterly	Quarterly
Start Date	1957 Q1	1957 Q1	1957 Q1	1957 M1	1987 Q4	1987 Q4	1987 Q4
End Date	2007 Q3	2007 Q3	2007 Q3	2007 M5	2007 Q3	2007 Q3	2007 Q3
Series ID	GDPC96	CPIAUCSL	FEDFUNDS	TRS1Y			
Source	*Federal Reserve Bank of St. Louis*	*Federal Reserve Bank of St. Louis*	*Federal Reserve Bank of St. Louis*	*Federal Reserve Bank of St. Louis*	*Federal Reserve Bank of St. Louis*		

Table 2.2 provides an overview of data and the source for each of the series. See Figures 2.8 and 2.9 for a graphical illustration of the Fed Funds rate, the Inflation rate and the Growth Gap over the period 1987:4 – 2007:3.

Table 2.3a: OLS estimates of Taylor Rule 1987:4 - 2007:3				
	Coefficient	***St. Error***	***T-Stat***	***Significance***
$\alpha 0$	*0.688*	*0.557*	*1.24*	*0.22*
$\beta 1$	*1.401*	*0.176*	*7.97*	*0.00*
$\beta 2$	*0.380*	*0.144*	*2.65*	*0.01*
R-Squared	*0.453*			
RSS	*205.168*			
D.F.	*77*			
DW	*0.232*			

Table 2.3a estimates the Taylor Rule co-efficients using inflation and the growth gap as independent variables. This preliminary analysis would suggest that both inflation and output gap parameters are statistically significant. The estimated co-efficient values would appear reasonably faithful to the original Taylor Rule parameterisation but slightly less than 1.5 and 0.5. The constant estimate is slightly less than 1% suggesting that the inflation target may have exceeded 2%, as was originally developed in the classic Taylor Rule. These parameter weights are subsequently used in Figures 2.10a and 2.10b to map out the estimated linear rule although the growth gap is assumed to be in balance at 0%, for purposes of comparison. The Residual Sum of Squares is given by RSS.

$$r_t = \omega_0 + (1+\omega_1)(\pi_t) + \omega_2(y_t)$$

where

$$\alpha_0 = \omega_0 = \left(r^* - \omega_1 \pi^*\right),\ \beta_1 = 1 + \omega_1,\ \beta_2 = \omega_2$$

Table 2.3b: OLS estimates of Policy Rule 1987:4 - 2007:3 using Ck = 4.5% and Pk = 2.5%, σ = 0.25, r = 0.05, T = 1 year				
	Coefficient	**St. Error**	**T-Stat**	**Significance**
α_0	5.221	0.203	24.63	0.00
β_1	2.781	0.407	6.83	0.00
β_2	0.361	0.153	2.35	0.02
R-Squared	0.378			
RSS	233.137			
D.F.	77			
DW	0.254			

Table 2.3c: OLS estimates of Policy Rule 1987:4 - 2007:3 using Ck = 4.5% and Pk = 2.5%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	**St. Error**	**T-Stat**	**Significance**
α_0	5.200	0.203	25.59	0.00
β_1	2.616	0.351	7.46	0.00
β_2	0.378	0.148	2.55	0.01
R-Squared	0.421			
RSS	217.230			
D.F.	77			
DW	0.254			

Table 2.3b: calculate the parameters: α_0, β_1 and β_2 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25} e^{rT} + \beta_2(y_t)$$

Table 2.3c: calculate the parameters: α_0, β_1 and β_2 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

Pk and *Ck* denote the lower bound and the upper bound respectively, i.e. π^*_{g1} and π^*_{g2}. The Collar time values are calculated using the Black model. The constant term tends to be higher than for the Taylor Rule estimates by virtue that the relationship is given as:

$$r_t = \left[r^* + \frac{\pi^*_{g1} + \pi^*_{g2}}{2}\right] + (1+\omega_1)(c_{g2} - p_{g1})e^{rT} + \omega_2(y_t)$$

where

$$\alpha_0 = r^* + \frac{\pi^*_{g1} + \pi^*_{g2}}{2}, \beta_1 = 1+\omega_1 \text{ and } \beta_2 = \omega_2$$

It is found that as σ increases, the nonlinearity of the policy rule is mitigated. This is confirmed in Figure 2.10a and 2.10b where the parameter estimates obtained here are used to map the policy rules over the range of expected inflation 0.01 to 0.065 (or 1% to 6.5%) in intervals of 0.0025. These rates were close to the historic high and lows over the period. Notice that the RSS on the linear rule is less than that of the nonlinear rule implying that the linear rule captured better policy behaviour.

Table 2.3d: OLS estimates of Policy Rule 1987:4 - 2007:3 using Ck = 4% and Pk = 2%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	**St. Error**	**T-Stat**	**Significance**
α_0	4.510	0.196	23.01	0.00
β_1	2.852	0.354	8.05	0.00
β_2	0.423	0.144	2.93	0.00
R-Squared	0.458			
RSS	203.240			
D.F.	77			
DW	0.287			

Table 2.3e: OLS estimates of Policy Rule 1987:4-2007:3 using Ck = 3.5% and Pk = 1.5%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	**St. Error**	**T-Stat**	**Significance**
α_0	3.945	0.213	18.48	0.00
β_1	2.757	0.321	8.59	0.00
β_2	0.455	0.141	3.23	0.00
R-Squared	0.490			
RSS	191.151			
D.F.	77			
DW	0.297			

Table 2.3d: calculate the parameters: α_0, β_1 and β_2 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.04} - p_{g1}^{\pi^*=0.02}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

Table 2.3e: calculate the parameters: α_0, β_1 and β_2 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.035} - p_{g1}^{\pi^*=0.015}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

As the upper and lower bounds are decreased from 4% - 2% to 3.5% - 1.5% the Residual Sum of Squares declines marginally.

Table 2.3f: OLS estimates of Policy Rule 1987:4-2007:3 using Ck = 3% and Pk = 1%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	***St. Error***	***T-Stat***	***Significance***
α_0	*3.597*	*0.232*	*15.53*	*0.00*
β_1	*2.397*	*0.271*	*8.83*	*0.00*
β_2	*0.460*	*0.139*	*3.31*	*0.00*
R-Squared	*0.504*			
RSS	*185.946*			
D.F.	*77*			
DW	*0.287*			

Table 2.3f: calculate the parameters: α_0, β_1 and β_2 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

As the upper and lower bounds decrease, there appears to be an improvement in the explanatory power of the relationship, suggesting that policy may be better described as being nonlinear. The RSS falls as the bounds are reduced, suggesting that over the period policy makers applied greater force to policy as inflation threatened to exceed 3%. The parameter weights estimated here are used in Figure 2.11 to map out the nonlinear policy rule.

Table 2.4a: OLS estimates of Policy Rule (1987:4-2007:3) with dummy (2001:1-2005:4) using Ck = 3% and Pk = 1%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	***St. Error***	***T-Stat***	***Significance***
α_0	*4.463*	*0.210*	*21.297*	*0.000*
β_1	*1.789*	*0.221*	*8.091*	*0.000*
β_2	*0.146*	*0.113*	*1.291*	*0.201*
β_3	*-2.566*	*0.338*	*-7.593*	*0.000*
R-Squared	*0.718*			
RSS	*105.739*			
D.F.	*76*			
DW	*0.344*			

Table 2.4a: reports the parameters: α_0, β_1, β_2 and β_3 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2001:1-2005:4}\right)$$

where D denotes a dummy variable having a value of 1 from 2001:1 to 2005:4 or otherwise 0 over the complete period. These dates coincide with the period of rate cuts that occurred from 2001:1 up to the beginning of the Bernanke incumbency when much of the policy accommodation was removed. See Figures 2.8 and 2.9. The dates are refined in the following Table 2.4b. Of interest here is the co-efficient estimate for the dummy variable, which is significantly negative, supporting evidence that monetary policy was more accommodative over the 2001:1 – 2005:4 period when compared to the rest of the Greenspan and Bernanke incumbency. This may have reflected fears concerning deflation. It may have also been consistent with a greater tolerance for inflation over the period. The parameter estimates given above were used to trace out the policy rule fit in Figure 2.12a.

Table 2.4b: OLS estimates of Policy Rule (1987:4-2007:3) with dummy (2002:1-2006:2) using Ck = 3% and Pk = 1%, σ = 0.35, r = 0.05, T = 1 year				
	Coefficient	**St. Error**	**T-Stat**	**Significance**
$\alpha 0$	*4.431*	*0.176*	*25.115*	*0.000*
$\beta 1$	*1.972*	*0.186*	*10.582*	*0.000*
$\beta 2$	*0.343*	*0.093*	*3.669*	*0.000*
$\beta 3$	*-2.812*	*0.286*	*-9.830*	*0.000*
R-Squared	*0.782*			
RSS	*81.863*			
D.F.	*76*			
DW	*0.627*			

Table 2.4b: reports the parameters: α_0, β_1, β_2 and β_3 using OLS:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2002:1-2006:2}\right)$$

where D denotes a dummy variable having a value of 1. The dates for the dummy, $D^{2002:1-2006:2}$, were searched iteratively until a minimised Residual Sum of Squares emerged from running a series of regressions. The dates also closely correspond with Taylor (2007) who identified a period of rate relaxation. The results reported above support the view that the Fed acted to contain inflation when inflation threatened to exceed 3% but policy was, nevertheless, more accommodative over the period 2002:1 – 2006:2 when compared with other periods. The dummy and other parameters estimated here were used to produce the policy rules in Figure 2.12b. The inclusion of the dummy for the period 2002:1 – 2006:2 helps to provide a better explanation of events when comparisons are made with Table 2.4a. The parameter coefficients estimated here are subsequently used to map out the policy rule fit in Figure 2.12b.

Table 2.5a: OLS estimates of Taylor Rule 1987:4 - 2002:1				
	Coefficient	St. Error	T-Stat	Significance
α_0	1.293	0.956	1.35	0.17
β_1	1.372	0.261	5.26	0.00
β_2	0.426	0.082	5.17	0.00
R-Squared	0.611			
RSS	74.954			
D.W.	0.311			
D.F.	55			

Table 2.5a estimates the Taylor rule as before in Table 2.3a. The final date is however restricted to 2002:1 and the fed funds rate, inflation rate and growth gap are expressed as percentages. The same parameter estimates are obtained using both Linear and Nonlinear Least Squares. A Newey-White correction to the standard errors was applied using a lag value of 4. Both inflation and output gap parameters remain statistically significant.

$$r_t = \omega_0 + (1+\omega_1)(\pi_t) + \omega_2(y_t)$$

Where

$$\alpha_0 = \omega_0 = (r^* - \omega_1\pi^*),\ \beta_1 = 1+\omega_1,\ \beta_2 = \omega_2$$

Table 2.5b: OLS estimates of Policy Rule 1987:4 - 2002:1 using $Ck = 4.5\%$ and $Pk = 2.5\%$, σ = GARCH, T = 1 year

	Coefficient	St. Error	T-Stat	Significance
α_0	5.688	0.384	14.80	0.00
β_1	2.567	0.544	4.72	0.00
β_2	0.410	0.091	4.50	0.00
R-Squared	0.566			
RSS	83.573			
D.W.	0.353			
D.F.	55			

Table 2.5c: OLS estimates of Policy Rule 1987:4 - 2002:1 using $Ck = 4\%$ and $Pk = 2\%$, σ = GARCH, T = 1 year

	Coefficient	St. Error	T-Stat	Significance
α_0	5.048	0.362	13.96	0.00
β_1	2.735	0.471	5.80	0.00
β_2	0.440	0.088	5.01	0.00
R-Squared	0.617			
RSS	73.6570			
D.W.	0.457			
D.F.	55			

Table 2.5d: OLS estimates of Policy Rule 1987:4 - 2002:1 using $Ck = 3.5\%$ and $Pk = 1.5\%$, σ = GARCH, T = 1 year

	Coefficient	St. Error	T-Stat	Significance
α_0	4.577	0.376	12.18	0.00
β_1	2.568	0.418	6.15	0.00
β_2	0.461	0.086	5.37	0.00
R-Squared	0.650			
RSS	67.337			
D.W.	0.508			
D.F.	55			

Table 2.5e: OLS estimates of Policy Rule 1987:4 - 2002:1 using Ck = 3% and Pk = 1%, σ = GARCH, T = 1 year				
	Coefficient	*St. Error*	*T-Stat*	*Significance*
α_0	4.283	0.401	10.69	0.00
β_1	2.219	0.336	6.61	0.00
β_2	0.472	0.087	5.46	0.00
R-Squared	0.657			
RSS	66.069			
D.W.	0.500			
D.F.	55			

Tables 2.5b – 2.5e: report parameters for α_0, β_1 and β_2.

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2} - p_{g1}\right)^{T=1,\sigma=GARCH} e^{rT} + \beta_2 (y_t)$$

A recursive GARCH(1,1) volatility estimate is used in the calculation of the time value of the collar. See GARCH appendix (A.2.3). The updating GARCH(1,1) forecast input is more realistic than assuming a constant value and attempts to capture the *ex ante* nature of volatility - typically used in options frameworks. *Ck* and *Pk* denote the upper and lower bounds respectively. As before the nonlinear rule can be expressed as:

$$\bar{r}_t = \left[r^* + \frac{\pi^*_{g1} + \pi^*_{g2}}{2} \right] + (1 + \omega_1)(c_{g2} - p_{g1})e^{rT} + \omega_2 (y_t)$$

where

$$\alpha_0 = r^* + \frac{\pi^*_{g1} + \pi^*_{g2}}{2}, \beta_1 = 1 + \omega_1 \text{ and } \beta_2 = \omega_2$$

Nonlinear and Ordinary Least squares provide the same parameter estimates. A Newey-White correction is applied using a lag value of 4 to adjust for serial correlation. The Fed Funds rate, the inflation rate and the growth gap are each expressed in percentage terms. Consistent with Tables 2.3c - f, it is found that as the bounds fall, the explanatory power of the nonlinear policy rule improves.

Table 2.6a: OLS estimates of Taylor Rule 1987:4 - 2002:1				
	Coefficient	St. Error	T-Stat	Significance
α_0	1.670	0.986	1.69	0.09
β_1	1.508	0.301	5.01	0.00
β_2	0.115	0.078	1.47	0.14
R-Squared	0.552			
RSS	86.170			
D.W.	0.432			
D.F.	55			

Table 2.6a reports Ordinary Least Squares estimates of the Taylor type reaction function (2.78) for the time period 1987.4 - 2002:1.

$$r_t = \omega_0 + (1+\omega_1)E_t(\pi_{t+4}) + \omega_2 E_t(y_t) \qquad (2.78)$$

where $\alpha_0 = \omega_0 = (r^* - \omega_1 \pi^*),\ \beta_1 = 1+\omega_1,\ \beta_2 = \omega_2$

Greenbook projections for both inflation (4 quarters ahead) and growth (the current projection) were used. The projections of real output growth and inflation are available at the Federal Reserve of Philadelphia and are released with a lag of 5 years. The growth gap was obtained using the trend growth methodology described in Appendix A.2.4. A Newey-White correction to the standard errors with a lag value of 4 is applied when estimating (2.78). The statistical significance for the parameter values, falls relative to Table 2.5a.

Table 2.6b: OLS estimates of Policy Rule 1987:4 - 2002:1 using Ck = 3% and Pk = 1%, σ = GARCH, T = 1 year				
	Coefficient	St. Error	T-Stat	Significance
α_0	4.594	0.412	11.15	0.00
β_1	2.848	0.391	7.29	0.00
β_2	0.111	0.097	1.13	0.26
R-Squared	0.611			
RSS	74.946			
D.W.	0.559			
D.F.	55			

Table 2.6b reports OLS estimates for (2.79). An inflation collar with strikes of 1% and 3% is calculated as before but using the real time Greenbook inflation forecast in conjunction with the recursive GARCH estimate for volatility, outlined in Appendix A.2.3. The same growth gap that is used in (2.78) and Table 2.6a is used here. The OLS parameter values are estimated with a Newey-White correction having 4 lags:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=GARCH} e^{rT} + \beta_2 E_t(y_t) \qquad (2.79)$$

Table 2.6b produces a small deterioration in fit when compared against Table 2.5e. Nevertheless Table 2.6b would appear to offer some improvement over the linear model, reported in Table 2.6a. These results favour policy being described as nonlinear and favour the collar structure replacing inflation in the reaction function.

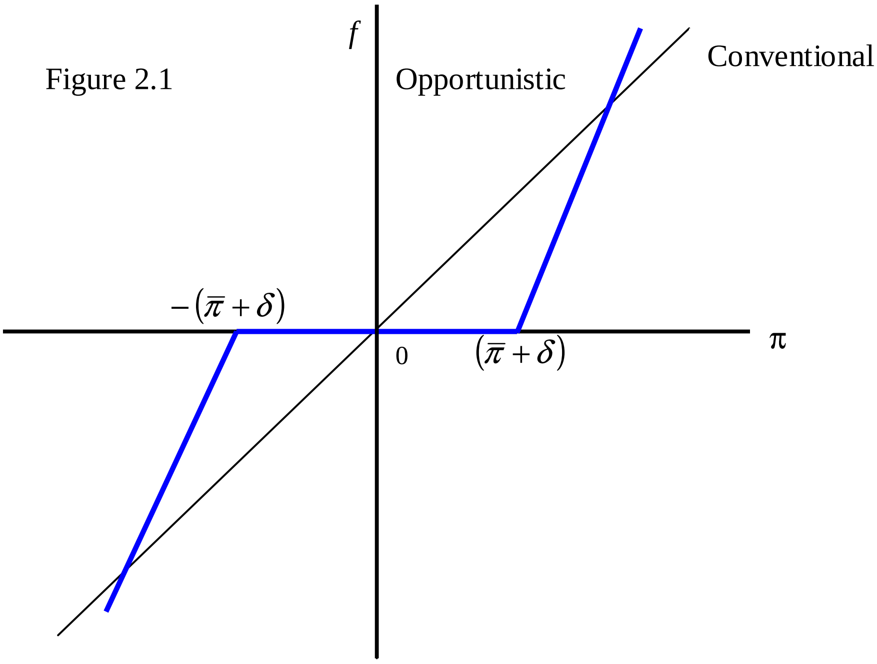


Figure 2.1 illustrates how an opportunistic central banker operates under certainty. Only when inflation exceeds a given threshold does the central banker activate the interest rate penalty. See Aksoy et al (2006) for different levels of $\bar{\pi}$. Figure 2.1 corresponds closely to what Kohn (1996) has described as representing Federal Reserve policy, importantly this depiction of policy would attribute a substantial measure of incisiveness to a forward looking rate setting committee where the underlying variable was expected inflation.

Figure 2.2: The Collar valuation under certainty and uncertainty

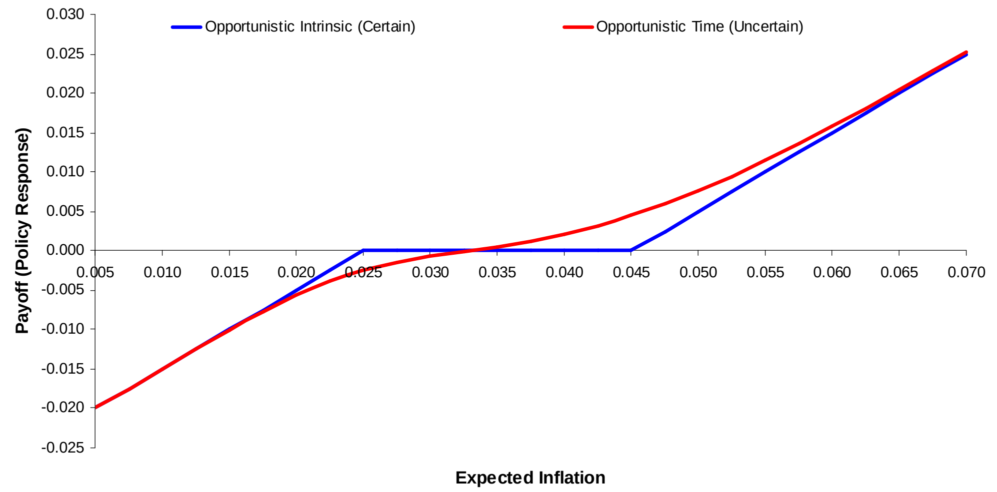


Figure 2.2: compares the intrinsic and time values of the collar. It is observed that opportunistic policy under uncertainty does not remain completely inactive between the threshold levels of inflation. That is, policy in general moves gradually as expected inflation resides inside the strikes and the policy reactions become increasingly more pronounced as inflation moves outside the bands of tolerance. Under certainty the policy response resembles the collar construction given in Figure 2.1. Here the intrinsic value can be obtained by calculating the difference between the intrinsic value of the long call and short put where the upper inflation target is 4.5% and the lower bound is 2.5%:

$$\left(c_{g2}^{\pi^*=0.045} \overset{T=0}{-} p_{g1}^{\pi^*=0.025}\right)$$

When expected inflation is uncertain, the collar:

$$\left(c_{g2}^{\pi^*=0.045} \overset{T=1,\sigma=0.25}{-} p_{g1}^{\pi^*=0.025}\right)$$

is valued using the Black Formula. To calculate the time value it is necessary to include an estimate for volatility, σ and the expiration, T. It is found that under uncertainty, policy does not remain inactive between the thresholds.

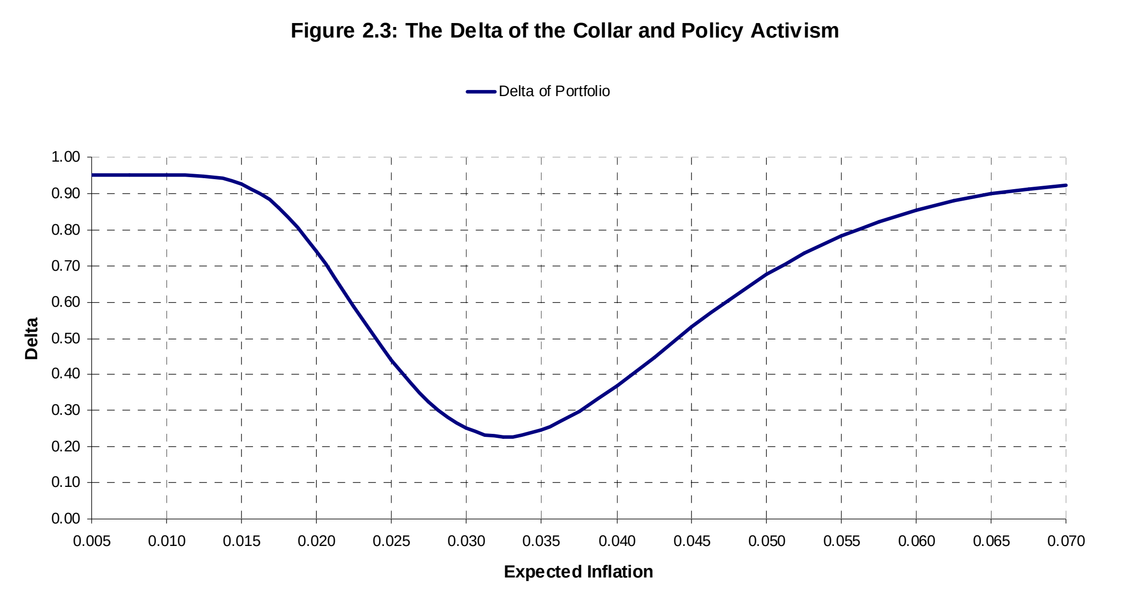


Figure 2.3 illustrates that as Expected Inflation exits the policy bands 2.5% to 4.5% the delta measure:

$$\frac{\partial\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)} = e^{-r(T-t)}N\left(d_{1g2}\right) - e^{-r(T-t)}\left[N\left(d_{1g1}\right)-1\right]$$

increases. This bears out the economic rationale described before: opportunistic monetary policy generally entails a more elevated response to rates of inflation that moves outside the inflation band. Figure 2.3 traces out the delta for the long call and short put. It is most conspicuously minimised when expected inflation resides inside the inflation band defined by π_{g1}^* and π_{g2}^*. The delta is not minimised however at the arithmetic mean of the thresholds, which would be here 3.5%. The Black model attributes higher levels of risk to higher inflation. The Black model may be useful then in capturing asymmetric aspects of monetary policy where the upside risk to inflation generates a bigger policy response than downside risks to inflation.

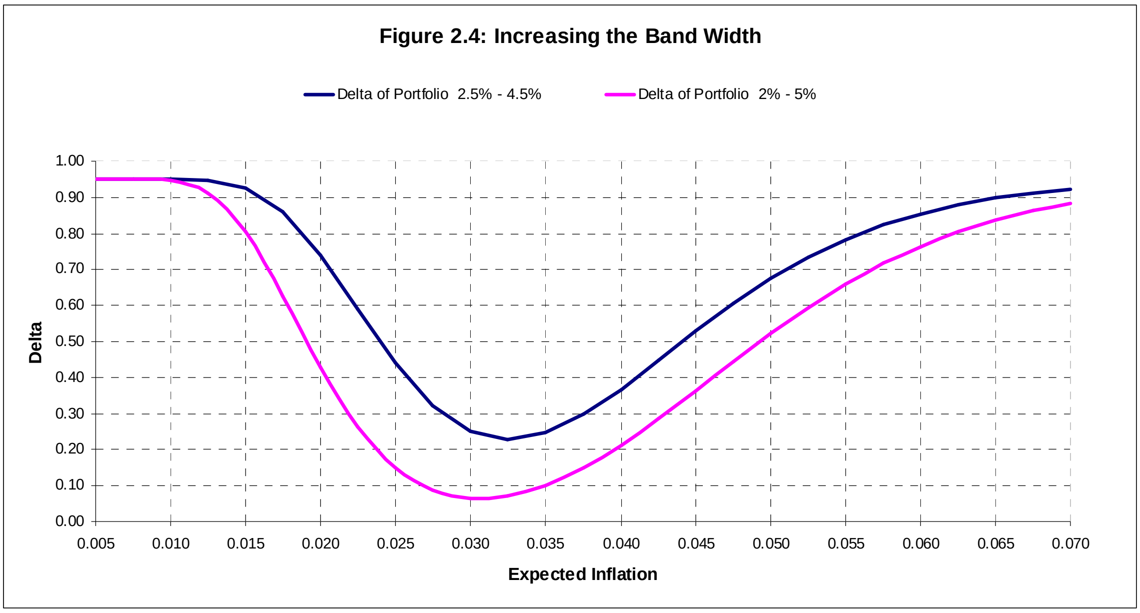


Figure 2.4: maps for varying strikes the following delta values:

$$\frac{\partial\left(c_{g2}^{\pi^*=0.045}-p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)} \text{ and } \frac{\partial\left(c_{g2}^{\pi^*=0.05}-p_{g1}^{\pi^*=0.02}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)}$$

The figure shows that as the bandwidth increases from 2.5% - 4.5% to 2% - 5% the delta measure falls for nearly the entire range. Inflation zone targeting and opportunism both appear to be broadly similar in this regard. If increased uncertainty results in an increase in the zone of inactivity (i.e. the inflation threshold band widens) policy becomes more inactive.

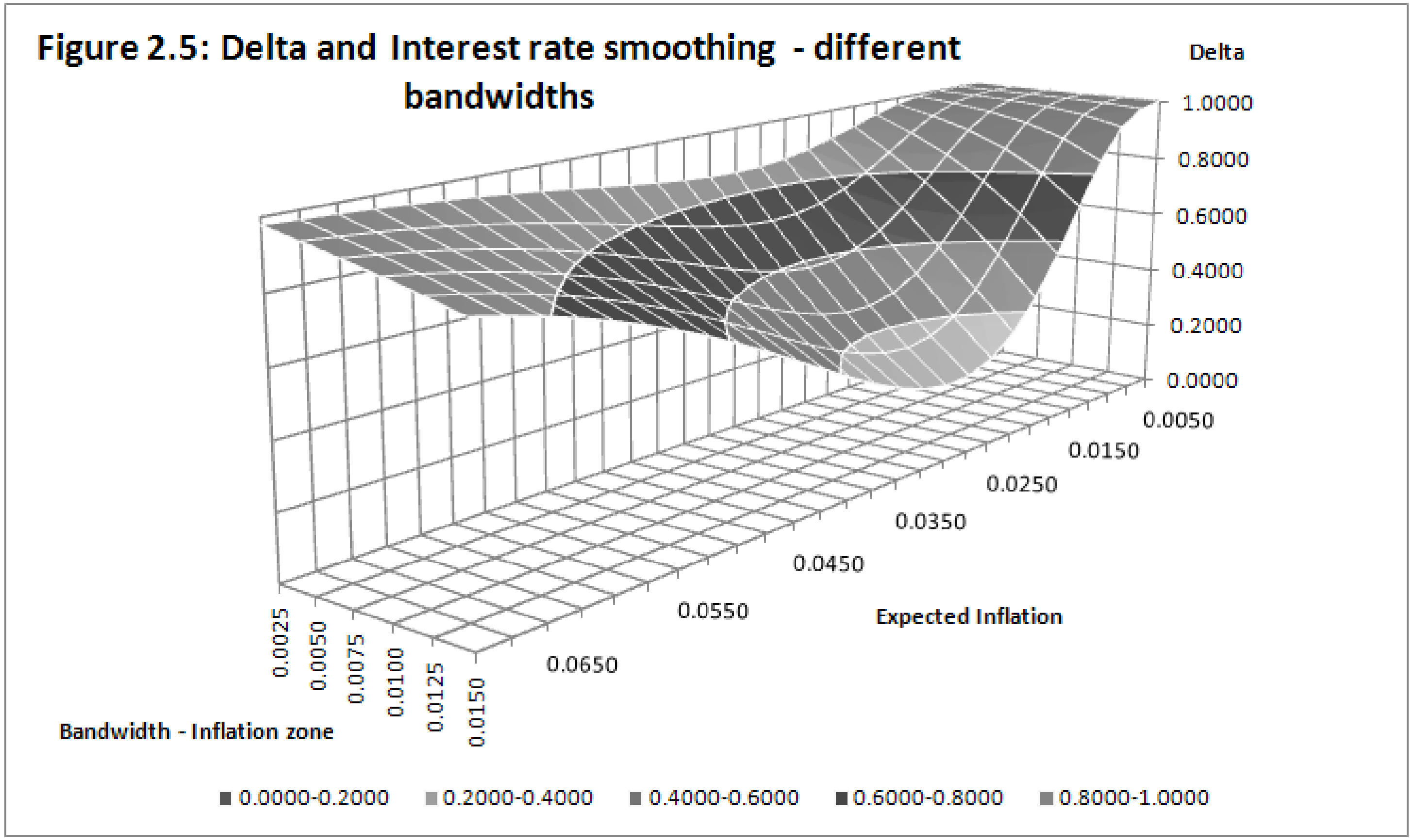
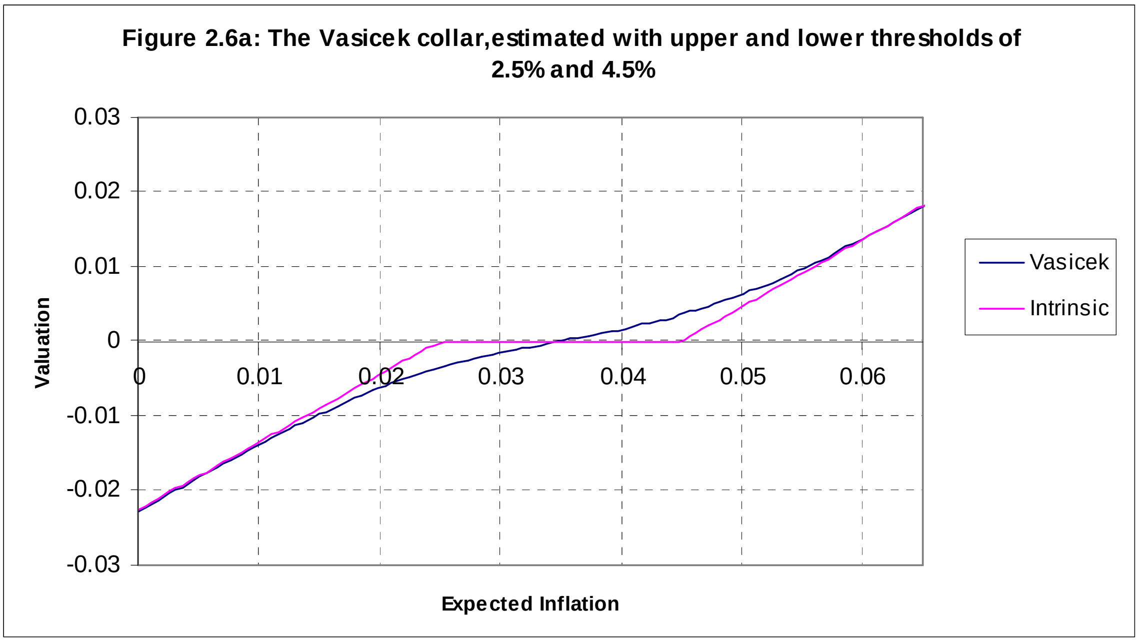


Figure 2.5: Considers six bands or inflation thresholds that abound 3.5%. The same expected inflation range as before is considered. Delta is calculated using (2.24). Here 6 bands of inflation tolerances are used to calculate the delta. 0.0025 corresponds to the inflation band going between 3.25% and 3.75%, i.e. 0.25% either side of 3.5%. 0.0125 corresponds to the inflation band 2.25% and 4.75% and so on. In all cases the mid-point of the band is 3.5%. It can be observed from the graph that as the inflation band widens policy inactivity increases. As the band widens there appears to be greater smoothing.



Figures 2.6a: maps out the time values of the following collar for values of expected inflation ranging from 0% to 6.6%. Monte Carlo simulation was used to generate both the time values and the expected inflation range. The Vasicek model incorporates mean reversion behaviour so that an underlying can be seen to be predictable:

$$d\pi(t) = a\left[b - \pi(t)\right]dt + \sigma\sqrt{dt}z$$

a, *b* and σ can be estimated using regression. Here, they were estimated empirically for the Greenspan incumbency. a = 0.03636, b = 0.03052 and the Vasicek volatility, σ_{vas} = 0.00286. (They might also be inferred from a given policy stance. For instance, the risk analysis could be extended to incorporate the speed of mean reversion to a long run mean. The Vasicek approach provides scope to permit the drift of inflation to be made to be dependent on the policy instrument. This however may be very ambitious if forecasting accuracy is limited.) The other parameter values were given as r = 5%, T = 2, π^*_{g2} = 0.045, π^*_{g1} =0.025, the Vasicek σ = 0.00286.

$$\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=2,\sigma_{VAS}=0.00286}$$

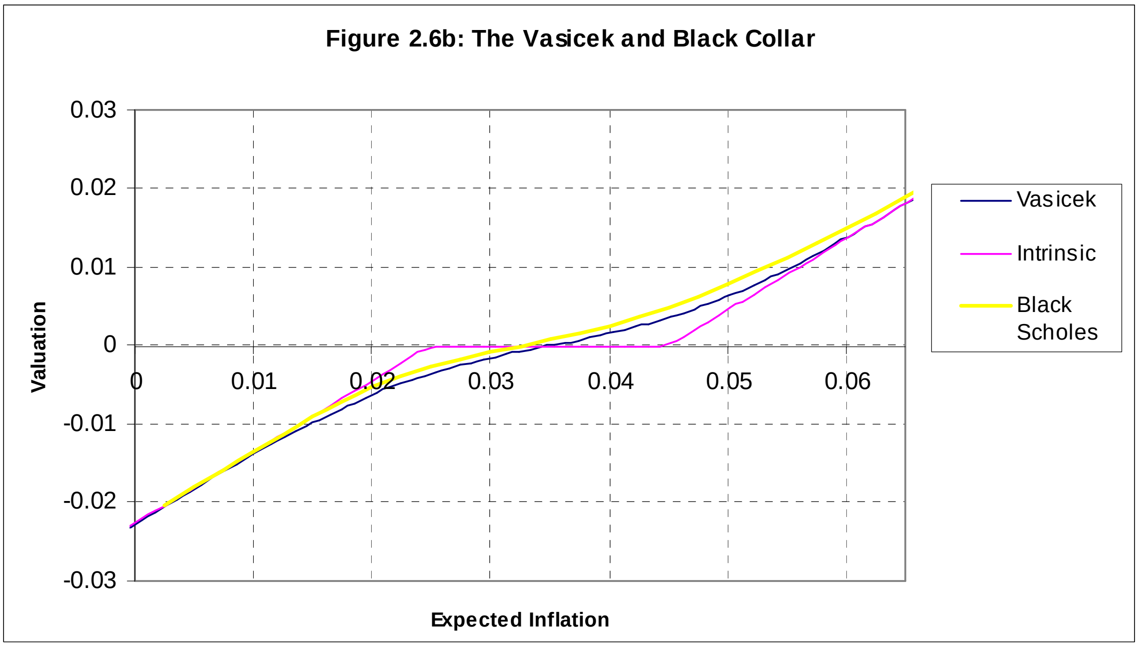


Figure 2.6b: superimposes the Black time valuations for the collar as described before, using:

$$\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.22}$$

The parameter inputs are the same as before however the Black model does not use *a* or *b* as inputs. The volatility input is estimated differently. For the Greenspan incumbency, the Black volatility was close to 0.35 whereas, the Vasicek volatility was just 0.00286. To make both comparable, a grid search was used to find the Black volatility that minimised the root-mean-squared-error (RMSE). This was found to be close to 0.22, well below 0.35. A key feature of Vasicek models is that as the speed of mean reversion increases volatility declines. In the monetary policy context this implies that as predictability increase volatility declines. Another feature worth noting relates to the asymmetry associated with the Black model. It is clear here that as expected inflation increases the policy response increases in magnitude. To see this more clearly, it is necessary to consider the deltas examined in Figure 2.7.

Figure 2.7: The Vasicek Delta and the Black Delta

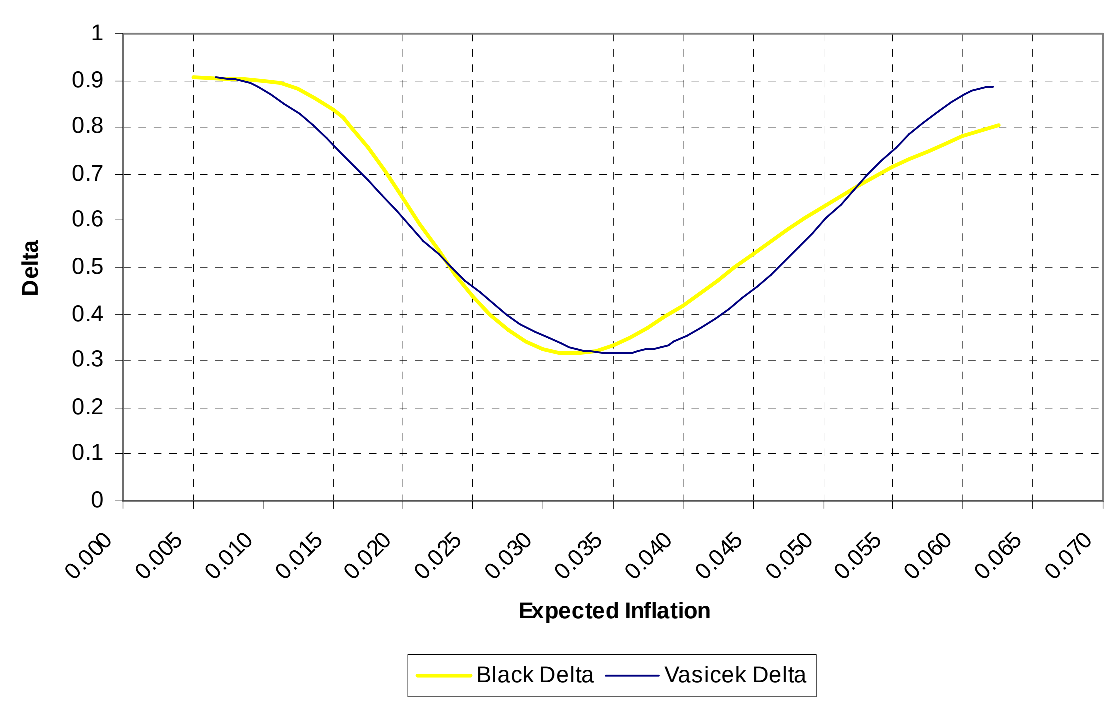


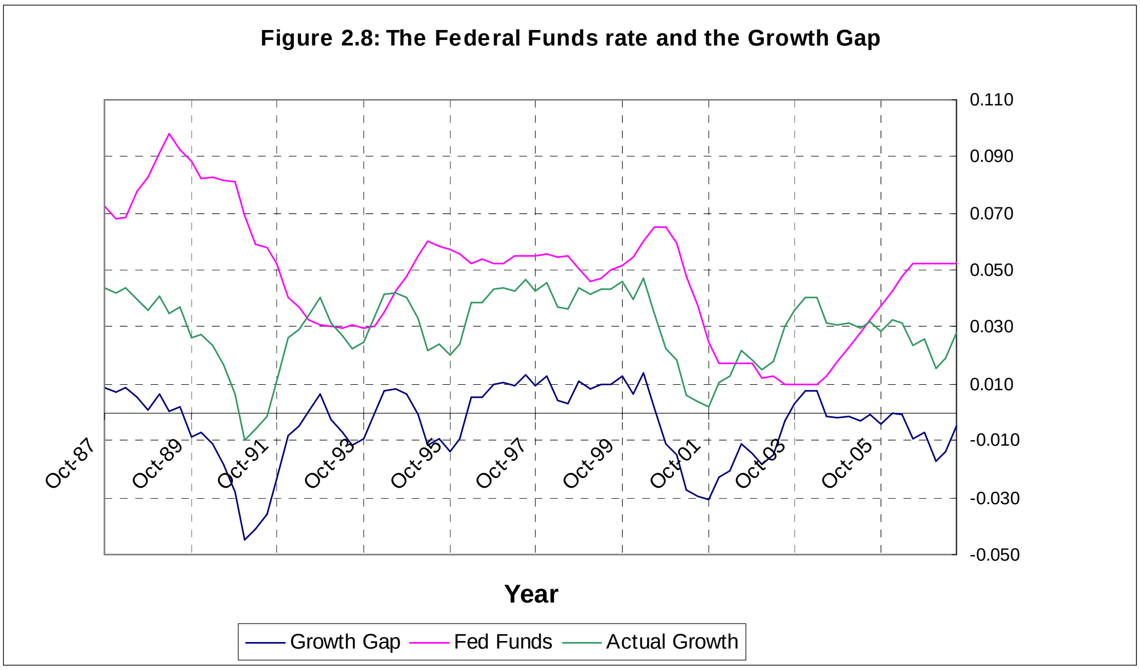
Figure 2.7: illustrates the asymmetry of the Black model. This is particularly evident when comparisons between the Black and Vasicek deltas are made directly. It is noticeable that the expected inflation rate associated with the minimum delta is not at 0.035. In fact, delta is computed using:

$$\frac{\partial\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.22}}{\partial E(\pi)} = e^{-r(T-t)}N\left(d_{1g2}\right) - e^{-r(T-t)}\left[N\left(d_{1g1}\right)-1\right]$$

is lowest when expected inflation is equal to:

$$E(\pi) = \left[\frac{\pi_{g2}^{*}.\pi_{g1}^{*}}{e^{\sigma^2(T-t)}}\right]^{0.5} = \left[\frac{(0.045).(0.025)}{e^{0.22^2(2)}}\right]^{0.5} = 0.031956294$$

This magnitude is lower than the arithmetic mean of the upper and lower bound: 0.045 and 0.025.



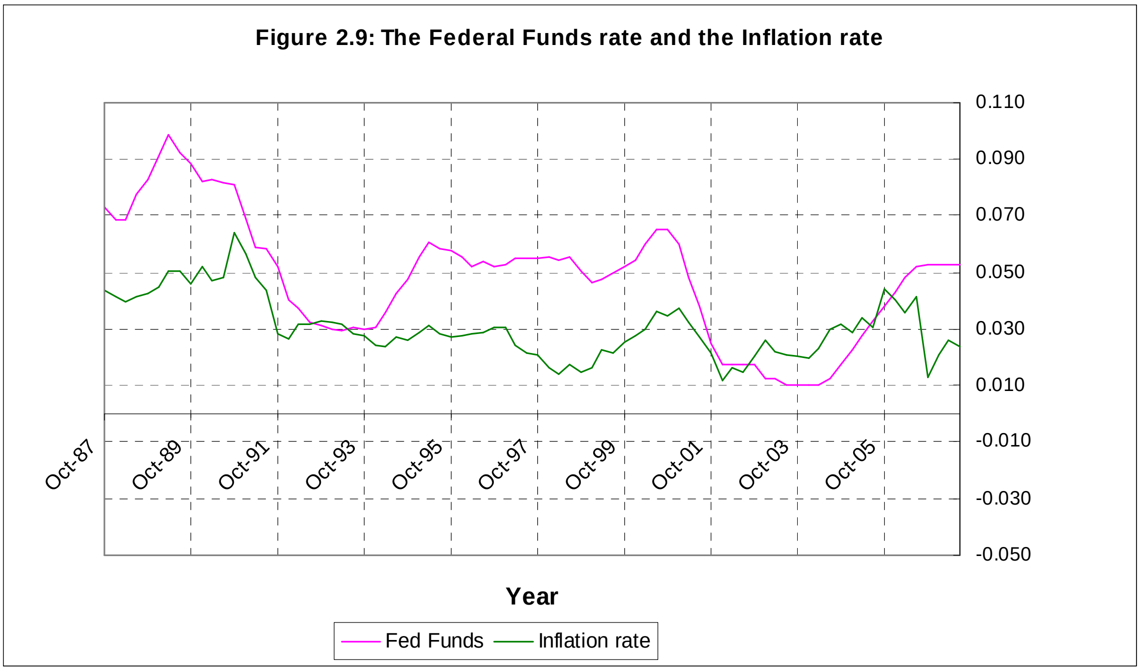
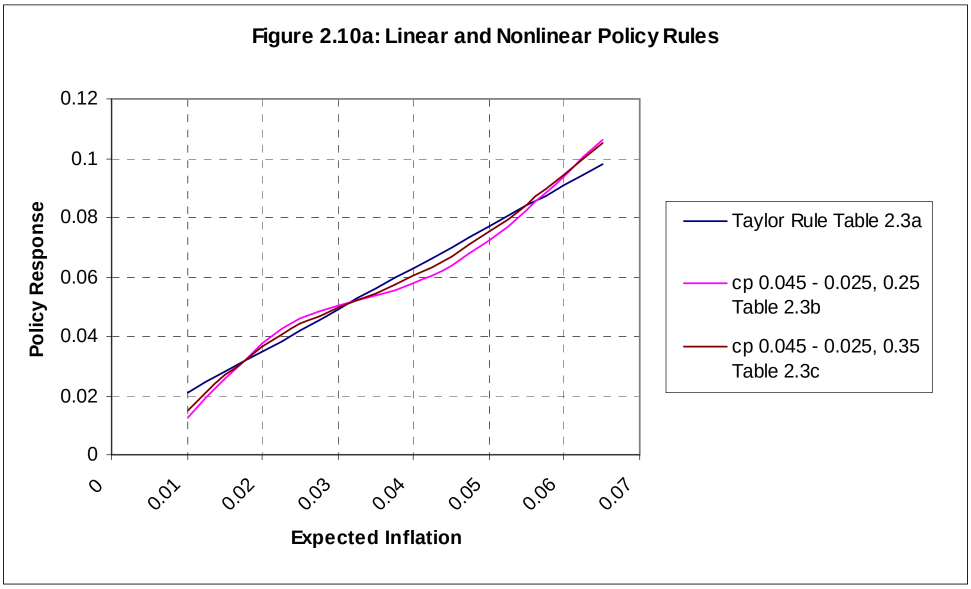


Figure 2.8 and Figure 2.9 provide an overview of the Fed Funds rate, the inflation rate and growth gap (see Appendix A.2.4**)** for the Greenspan and Bernanke incumbencies. From Figure 2.8, it is clear that the recursively estimated trend growth does not substantially affect the variability of the growth gap when compared against the original annual growth rate.



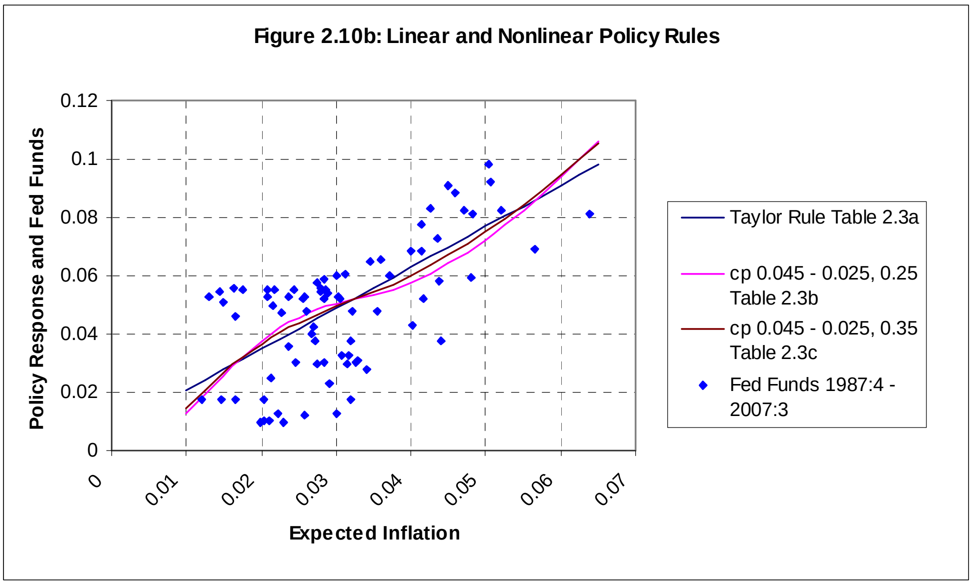


Figure 2.10a maps out a linear rule using the estimated weights obtained in Table 2.3a. The growth gap is set to zero for purposes of illustrations. The range of expected inflation considered is from 0.01 to 0.065, close to the historic minimum and maximum over the 1987:4 – 2007:3 period. In addition, the estimated weights obtained for the nonlinear rules (given below), obtained in Tables 2.3b and 2.3c, are also used to map the nonlinear rules. Figure 2.10b superimposes a scatter plot for inflation and Fed Funds quarterly rates for the same period 1987:4 – 2007:3. It was found that the nonlinear policy rules underperformed relative to the linear rule.

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.25} e^{rT} + \beta_2(y_t)$$

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.045} - p_{g1}^{\pi^*=0.025}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

Figure 2.11: The Historical Fed Funds Rate for the Greenspan and Bernanke Incumbencies

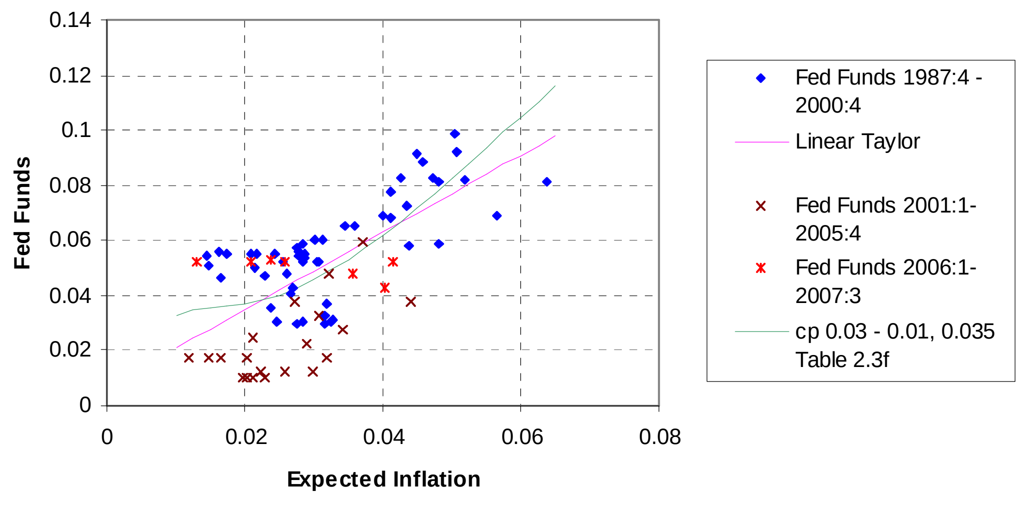


Figure 2.11 maps out the nonlinear rule using weights estimated from Table 2.3f. The same linear rule as before is mapped out using parameter estimates from Table 2.3a. The nonlinear rule:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t)$$

outperforms the linear rule and other nonlinear rules for the whole period 1987:4 - 2007:3 based on the Residual Sum of the Squares (RSS). The disaggregated scatter plot of the fed funds rate relative to inflation suggests that rate setting behaviour changed from 2001:1. Over the period 2001:1 – 2005:4, the Feds Funds rate appears depressed relative to inflation when compared to the other periods between 1987:4 – 2007:3. This episode coincides with the series of rate cuts apparent from Figures 2.8 and 2.9. This may have reflected concerns regarding deflation during this period that ultimately did not materialise. Table 2.4a estimates the nonlinear rule with the addition of a dummy variable that specified 1 over the period 2001:1 – 2005:4 and otherwise zero. This dummy was found to have been negative supporting evidence that policy was more accommodative relative to what had been implemented over the previous decade and a half. It also suggests that there is some empirical basis for disaggregating the fed funds time series.

Figure 2.12a: The Policy Rule with a dummy 2001:1 - 2005:4

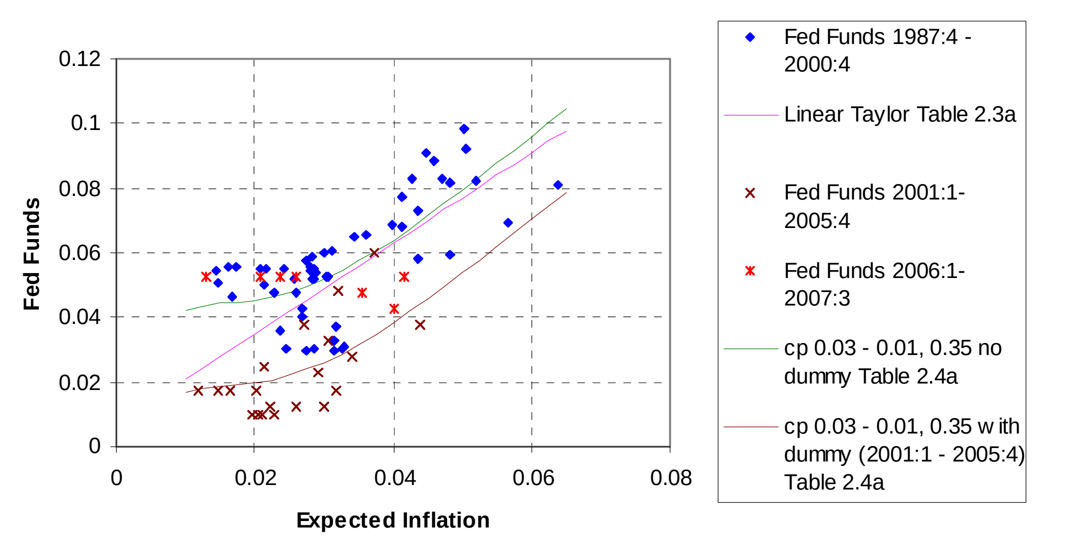


Figure 2.12a uses the parameters: α_0, β_1, β_2 and β_3,estimated in Table 2.4a, to produce the policy fit r_t:

$$r_t = \alpha_0 + (\beta_1)\overset{T=1,\sigma=0.35}{\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2001:1-2005:4}\right)$$

where D denotes a dummy variable having a value of 1 from 2001:1 to 2005:4 or otherwise 0 over the remaining period. These dates tentatively capture the period of policy relaxation discernible from Figures 2.8 and 2.9. The negative co-efficient estimate for the dummy variable, from Table 2.4a, confirms that monetary policy was more relaxed over the 2001:1 – 2005:4 period. This construction permits prospective breaches of the upper threshold of 3% inflation to trigger a nonlinear reaction as described by the collar, even with policy accommodation. The lower threshold/bound of 1% was never violated implying that the put is less visibly evident in the options framework. The dis-aggregation of the fed funds rate, (i.e. the application of varying legends for corresponding time periods), would appear to help explain the statistical improvement evident in Table 2.4a. Fears concerning deflation, not realised in the data, may help explain policy behaviour over the period.

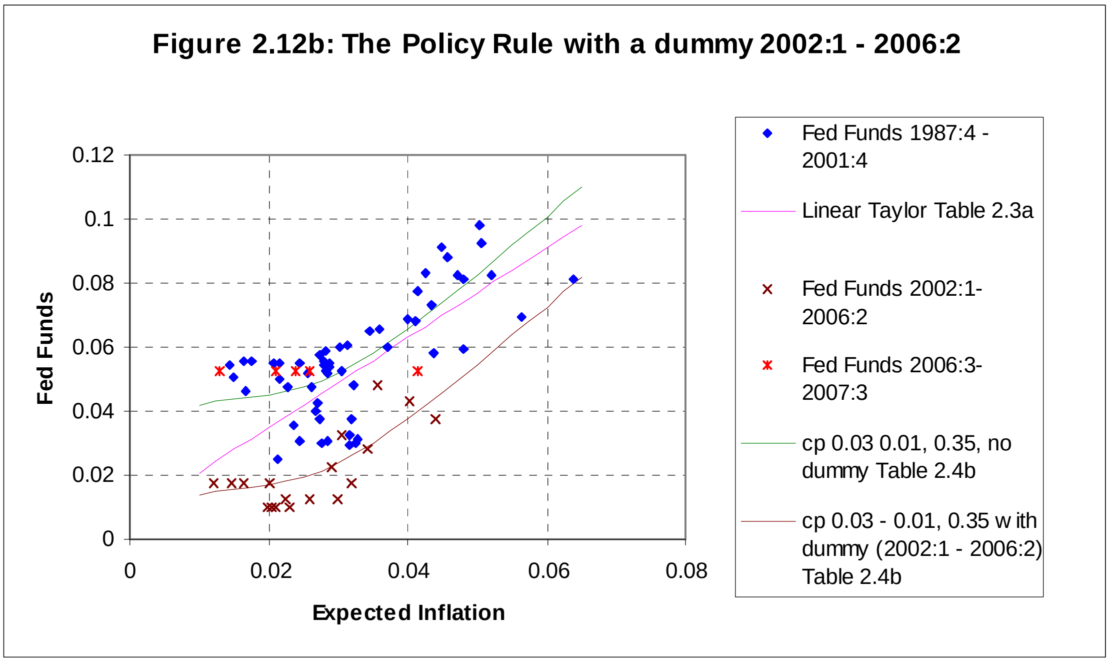


Figure 2.12b: uses the parameters: α_0, β_1, β_2 and β_3 from Table 2.4b to trace out the nonlinear policy rule given by r_t:

$$r_t = \alpha_0 + (\beta_1)\left(c_{g2}^{\pi^*=0.03} - p_{g1}^{\pi^*=0.01}\right)^{T=1,\sigma=0.35} e^{rT} + \beta_2(y_t) + \beta_3\left(D^{2002:1-2006:2}\right)$$

where D denotes a dummy variable having a value of 1. The dates for the dummy, $D^{2002:1-2006:2}$, were changed from Table 2.4b and Figure 2.12a. The criteria applied to select the dummy dates were based on minimisation of the Residual Sum of Squares for the specification above, starting 2001:1. The statistical significance and improved fit offered by the dummy reported in Table 2.4b and illustrated here suggest that the Fed became more accommodative over the period 2002:1 – 2006:2 when compared with other periods. These dates would seem to broadly coalesce with Taylor (2007) who uses a counterfactual policy rule to make a similar comparison. Taylor (2007) maintained that the federal funds rate, from 2003 to 2006, was well below what one would have predicted, given successful policy reactions during the two previous decades. The inclusion of the dummy for the period 2002:1 – 2006:2 helps to capture this easing in policy, while also preserving nonlinearity in the policy rule.

Appendix A.2.1: Interest rate setting under uncertainty

Proposition 2.2a developed the monetary policy response to inflation as an option time valuation where expected inflation was not known with certainty. The configuration of the option's framework is based on the monetary policy reaction, outlined in Figure 2.1. This construction can be motivated by Aksoy et al. (2006) or alternatively by considering committee and institutional dynamics. Here, a risk management approach is adopted by equating the opportunistic policy response to the intrinsic value of an option's portfolio, when future outcomes are certain. If however, the reaction function were set, such that interest rates were determined today in response to future levels of inflation that are uncertain, the exercise of implementing policy, changes fundamentally. It is generally accepted that economic policy operates in a domain that is forward-looking so that policy makers try to discern the emergence of pricing pressures, in advance, and react in a pre-emptive manner. Clarida, Gali and Gertler (1998, 1999, 2000) formulate the reaction function as being forward looking.[231] Indeed, as part of the jargon used to denote the pre-emptive nature of Fed Funds setting, policy makers frequently refer to buying insurance or staying ahead of the curve.[232] In this regard, the upper strike of the collar relative to inflation does not constitute the only determinant that drives policy. The probability of inflation exceeding an acceptable tolerance is also fundamental to precipitating a policy response. This implies that even when the current expectation of inflation resides within a comfort zone, policy makers will not necessarily remain inactive. So long as there is a probable inflation outcome that exceeds an acceptable limit, the board will, at the margin, be prepared to adjust the policy rate.[233] Previously, it was noted that traditional Black-Scholes (1973) assumptions have involved stipulating; that the short-term rate is known and constant, that the underlying asset follows a random walk in continuous time with a variance rate

[231] The Bank of England publishes quarterly *the Inflation Report* which sets out detailed inflation projections. In theory, any deviation between the projection and inflation target constitutes the primary basis for the Bank's rate setting decision. A similar approach is adopted at the Bank of Canada, the Reserve Bank of New Zealand and other central banks. The Federal Reserve since November, 2007 also publishes forecasts on a quarterly basis. Enhanced FOMC economic projections were released with the FOMC minutes of the October 30-31 meeting in November, 2007. Four panels displayed the Q4-to-Q4 percent changes in real output (measured by real GDP), the Q4-to-Q4 percent changes in overall and core prices (measured by the price index for personal consumption expenditures), and the Q4 level of the unemployment rate. Central tendency projections that defined the range of FOMC participants' projections were also innovatively provided.

[232] See Orphanides (2003) for forecast-based variants of the classic rule.

[233] This appendix appeals to a widely understood body of knowledge in finance. In this regard, Neftci (1996), Hull (2003), McDonald (2003) and Whaley (2003) provide useful background to the Black-Scholes/Merton model. The derivation proposed by Hull (2003) is adapted here specifically for its application to monetary policy.

proportional to the square of the underlying asset value, and that markets are frictionless. In chapter 1, static replication, proposed by Derman-Taleb (2005), was applied - permitting (A.2), and (A.5) to be relaxed.[234] Using these, the policy response is set out as:

$$E_t\left[max\left(\pi_{t+1} - \pi^*_{g2}, 0\right)\right] \qquad \text{(a.2.1.1)}$$

where π^*_{g2} denotes the upper tolerance for inflation which if exceeded, triggers a monetary policy response. E_t denotes the current expectation.[235] It will be assumed that expected inflation is lognormally distributed and the standard deviation of $\ln(\pi)$ is s. Given that policy is forward-looking, a probability density function (pdf) of inflation can be defined as $g(\pi)$. The pdf construal is not dissimilar to how the MPC communicates its inflation expectations using a fan chart. The BOE has published a probability distribution for future inflation outcomes up to two and three years ahead in its *Inflation Report* since February, 1996. Prior to this, the *Inflation Report* described in text, the risks associated with the forecast distribution.[236] One would expect that any deviation of the forecast from the mandated target would trigger a policy response.[237] To keep the analysis consistent with a baseline option pricing model, it is assumed here that expected inflation is lognormally distributed.[238] As outlined in chapter 1, the lognormal random walk neatly captures a number of risk management perspectives. To this end a probability density function can be employed to describe the likelihood of inflation exceeding a given threshold, π^*_{g2}, with these risk management perspectives taken into account. When considering only the upper bound, the following would apply:

$$E\left[max\left(\pi_{t+1} - \pi^*_{g2}, 0\right)\right] = \int_{\pi^*_2}^{\infty} \left(\pi_{t+1} - \pi^*_{g2}\right) g(\pi) d(\pi) \qquad \text{(a.2.1.2)}$$

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[234] Otherwise, assumptions (A.1) – (A.6) set out in Section (1.6) still apply. As before, it is still assumed however that the variance rate is proportional to the square of the underlying asset value. The Vasicek model permits the asset's variance to be constant.

[235] Goodhart (2003) allows for the possibility that the Bank of England targeted the forecast mode. If the distribution is skewed, it then becomes a consideration whether policy should target a measure of central tendency other than the mean expectation. The Heston (1993) and Backus, Foresi and Wu (2004) models, developed in chapter 1, permit considerations relating to the higher moments to be taken into account.

[236] Hence the fan charts represent an important innovation.

[237] The practice of peering into the future using pdfs, explicitly or implicitly, provides a useful starting point for considering the option's framework. Central banks do this in different ways although parallels would appear evident. The Federal Reserve releases its own inflation staff forecast but only after a lag of five years. The information contained in the Greenbook is supplemented however by ranges for the forecasts. These were released twice a year in the Monetary Policy Report to the Congress. In 2007, they started to be released quarterly.

[238] Other stochastic processes were considered in chapter 1.

Under conditions of certainty, the opportunistic central bankers, as described before, only moves to increase the policy rate should inflation breach a given threshold, otherwise policy remains inactive. In this regard, the payoff defined by the call is analogous to the opportunist's or zone targeter's position. The natural logarithm variable $\ln E(\pi)$ is normally distributed. From the properties associated with the lognormal distribution, the mean of $\ln(\pi)$ can be expressed as m.

$$m = \ln[E_t(\pi_{t+1})] - s^2/2 \qquad \text{(a.2.1.3)}$$

By taking the natural log of expected inflation the new variable becomes normally distributed. This can be standardised by subtracting the mean and dividing by the standard deviation to compute (a.2.1.4)

$$z = \frac{\ln(\pi) - m}{s} \qquad \text{(a.2.1.4)}$$

With a mean of zero and standard deviation of one, the probability distribution of *z* can be read directly from the standardised normal cumulative distribution. This implies (a.2.1.2) can be re-expressed to give:

$$E[\max(\pi\text{-}\pi^*_{g2},0)] = \int_{(\ln(\pi^*_{g2})-m)/s}^{\infty} (e^{zs+m}) h(z) dz\text{-}\pi^*_{g2} \int_{(\ln(\pi^*_{g2})-m)/s}^{\infty} h(z) dz \qquad \text{(a.2.1.5)}$$

where $h(z)$ denotes the normal density function for z. If it can be shown that $\pi = e^{z(s)+m}$, then (a.2.5) becomes:

$$E[\max(\pi\text{-}\pi^*_{g2},0)] = e^{m+s^2/2} \int_{(\ln(\pi^*_{g2})-m)/s}^{\infty} h(z-s) dz\text{-}\pi^*_{g2} \int_{(\ln(\pi^*_{g2})-m)/s}^{\infty} h(z) dz \qquad \text{(a.2.1.6)}$$

(a.2.1.6) displays the main features associated with the traditional Black and Black-Scholes formulae. The economic interpretation is that as expected inflation rises relative to the boundary, the probabilities associated with both integrals increase. Option pricing provides the valuable insight that as the underlying increases the normal cumulative probabilities, denoted below by $N(d_{1g2})$ and $N(d_{1g2})$ also increase. $N(d_{1g2})$ is the cumulative probability that a variable, that possesses a standardised normal distribution, will be less than d_{1g2}.[239] $N'(d_{1g2})$ represents the probability density function for a standardised normal distribution. In other words:

$$N'(d_{1g2}) = \frac{1}{\sqrt{2(3.141593)}} e^{-\frac{d_{1g2}{}^2}{2}}$$

[239] In considering the probability of expected inflation being in excess of a given threshold, it becomes necessary to calculate *1-N(d₁g₂)*.

and

$$N(d_{1g2}) = \int N'(d_{1g2})$$

The fan chart and Monte Carlo analysis considered in chapter 1, provides some intuition. Both here and in the Monte Carlo analysis, it is found that as the expectation of inflation rises above a given level in this instance, π^*_{g2}, the probability associated with a given monetary policy response also rises. By substituting for m, the first integral of (a.2.1.6) can be written as $N(d_{1g2})$:

$$\int_{(ln(\pi^*_{g2})-m)/s}^{\infty} h(z-s)dz = 1 - N\left(\frac{(ln(\pi^*_{g2})-m)}{s} - s\right) = N\left(\frac{(ln(E(\pi)/\pi^*_{g2}) + s^2/2)}{s}\right) = N(d_{1g2}) \quad \text{(a.2.1.7)}$$

Likewise, the second integral associated with upper tolerance in (a.2.1.6) can be re-expressed to give:

$$N(d_{2g2}) = N\left(\frac{ln[E(\pi)/\pi^*_{g2}] - s^2/2}{s}\right) \quad \text{(a.2.1.8)}$$

By applying a risk neutral valuation (a.2.1.6) can be rewritten.

$$\hat{E}[max(\pi\text{-}\pi^*_{g2},0)] = e^{m+s^2/2}N(d_{1g2}) - \pi^*_{g2}N(d_{2g2}) \quad \text{(a.2.1.9)}$$

The analysis assumes the familiar shape of a call option by substituting for *m*. (a.2.1.9) can be re-written as:

$$c_{g2} = e^{-rT}\hat{E}[max(\pi_{t+1} - \pi^*_{g2},0)] = e^{-rT}[E(\pi)N(d_{1g2}) - \pi^*_{g2}N(d_{2g2})] \quad \text{(a.2.1.10)}$$

By substituting for m, the expectation that inflation will exceed a given level of tolerance can be calibrated if a mean and standard deviation are ascribed. $\hat{E}$ denotes the expectation in a risk neutral world. This implies that the European call can be computed by discounting with e^{-rT} where r denotes the risk free rate and T denotes the forward-lookingness of the central bank.[240] If the standard deviation of the natural logarithm is

[240] By invoking the BSM insight, it is possible to show that a risk free hedge can be created by combining a dynamic long position of $\partial c/\partial E(\pi)$ in the expected inflation, $E(\pi)$, with a short position in the derivative asset *c*. Alternatively, it is possible to establish a risk neutral position using the static hedge identified by Derman and Taleb (2005). A third possibility here is to allow *r* to be arbitrarily set equal to zero. This could be justified by simply proffering that central bankers are indifferent to the timing of inflation. That is, central bankers are equally hostile to inflation whenever it occurs along the time line. They would not seek to trade off today's inflation for future inflation and they would not apply time decay nor risk premia. This would eliminate the need to establish risk neutrality either using the classic Black-Scholes dynamic replication approach or using the static hedge proposed by Derman and Taleb (2005). The same actuarial formulae however would still apply.

taken to be equal to $\sigma\sqrt{T}$ then the call structure assumes a form that resembles the Black model. More specifically:

$$N\left(d_{1g2}\right)=N\left(\frac{ln(\hat{E}(\pi)/\pi^*_{g2})+\sigma^2T/2}{\sigma\sqrt{T}}\right) \qquad \text{(a.2.1.11)}$$

and

$$N\left(d_{2g2}\right)=N\left(\frac{ln(\hat{E}(\pi)/\pi^*_{g2})-\sigma^2T/2}{\sigma\sqrt{T}}\right) \qquad \text{(a.2.1.12)}$$

Using put-call parity it can be shown that the put can be expressed as:

$$p_{g2}=e^{-rT}\left[\pi^*_{g2}N\left(-d_{2g2}\right)-\hat{E}(\pi)N\left(-d_{1g2}\right)\right] \qquad \text{(a.2.1.13)}$$

or

$$p_{g1}=e^{-rT}\left[\pi^*_{g1}N\left(-d_{2g1}\right)-\hat{E}(\pi)N\left(-d_{1g1}\right)\right] \qquad \text{(a.2.1.14)}$$

where the lower threshold level of inflation, π^*_{g1} is substituted for π^*_{g2}. This constitutes an ‘actuarial’ derivation of option pricing formulae. By applying Derman and Taleb (2005), risk neutrality can be established without assuming Geometric Brownian Motion or frictionless markets. By setting the strike, π^*_{g1}, to be lower than π^*_{g2}, it is possible to engineer a collar structure (i.e. a long call and short put on inflation) to capture some of the nonlinearity of opportunistic monetary policy. From (2.17) the reaction function was given as:

$$\frac{r_{1t}+r_{2t}}{2}=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+\left(1+\omega_1\right)\left(c_{g2}-p_{g1}\right)e^{rT}+\omega_2\left(y_t\right) \qquad (2.17)$$

or

$$\overline{r}_t=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+\left(1+\omega_1\right)\left(c_{g2}-p_{g1}\right)e^{rT}+\omega_2\left(y_t\right)$$

By incorporating (a.2.1.10) and (a.2.1.14) the following emerges:

$$\overline{r}_t=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+\left(1+\omega_1\right)\left\{e^{-rT}\left[E(\pi)N\left(d_{1g2}\right)-\pi^*_{g2}N\left(d_{2g2}\right)\right]-e^{-rT}\left[\pi^*_{g1}N\left(-d_{2g1}\right)-E(\pi)N\left(-d_{1g1}\right)\right]\right\}e^{rT}+\omega_2\left(y_t\right)$$

(a.2.1.15)

By collecting the exponential terms it is possible to remove any reference to the risk free rate, *r*, on the r.h.s.. This produces the forward-looking opportunistic reaction function:

$$\overline{r}_t = \left[r^* + \frac{\pi_{g1}^* + \pi_{g2}^*}{2} \right] + (1+\omega_1)\left[E(\pi)\left(N\left(d_{1g2}\right) + N\left(-d_{1g1}\right)\right) - \pi_{g2}^* N\left(d_{2g2}\right) - \pi_{g1}^* N\left(-d_{2g1}\right)\right] + \omega_2\left(y_t\right)$$

(a.2.1.16)

This result implies that the risk free rate, *r*, can be eliminated. This has the effect of simplifying and removing some circularity in the relationship as spelled out by (2.17). The opportunistic reaction function, that makes uncertainty explicit, can be established without any dependence on the risk free rate. The risk management paradigm is adapted here to monetary policy and made practicable using both (a.2.1.16) and the forward-looking opportunistic framework. (a.2.1.16) could also be obtained more simply by ignoring risk neutrality and stipulating that *r* is arbitrarily set to zero from the outset. As explained in Section 1.6, the 'zero time decay' approach could be asserted by imposing the assumption that central bankers are indifferent to the timing of inflation i.e. they do not extract any pecuniary or reputational gain by postponing inflation or trading future inflation for current growth. In chapter 3 however the discount rate, *r*, is preserved in the analysis to maintain the standard notation. While a 'zero time decay' approach is attractive and would simplify the algebra (i.e. it avoids having to establish risk neutral conditions), it would also limit the analysis. Although equivalent to (a.2.1.16), the estimated forward-looking opportunistic reaction functions (2.65) – (2.79) made explicit *r*. This perhaps was not necessary as (a.2.1.16) provides a more general template. The application of portfolio option theory to monetary policy was developed here using the classic Black-Scholes framework and more robustly by using Derman and Taleb (2005). The advantage of applying these market frameworks largely relates to preserving the flexibility to use market traded inflation options when considering (a.2.1.16). In this regard, market variables and market uncertainty may be useful pointers for setting out policy. The analysis developed here however can and should be applied independent of market variables, especially when markets are illiquid. By preserving *r* in the analysis, presented in both chapter 2 and 3, it nevertheless remains possible to consider the likely effects of central bankers falling short of the zero time decay bar (i.e. the risk management paradigm can be made more flexible, general and market oriented). This would suggest that the Derman and Taleb (2005) approach is best placed to offer a greater range of applications with less taxing assumptions. In the following chapter, the option's framework is considered largely from the perspective of how monetary policy can assume both linear and nonlinear forms. This theoretical framework illustrates the conditions that lead nominally nonlinear policy to be linear (or almost linear).

Appendix A.2.2: the level of expected inflation at which policy activism is minimised

As illustrated in the text, two scenarios are plausible in the event that:

$$\lfloor N'(d_{1g2})\rfloor = \lfloor N'(d_{1g1})\rfloor \qquad \text{(a.2.2.1)}$$

The first scenario optimises the delta where agreement is attained. This implies the upper and lower strikes are found to be equal. The more general case is considered here.[241] If the square roots have different signs the following holds:

$$\left(\frac{\ln\left(E(\pi)/\pi^*_{g2}\right)+\sigma^2(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) = -\left(\frac{\ln\left(E(\pi)/\pi^*_{g1}\right)+\sigma^2(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) \qquad \text{(a.2.2.2)}$$

By taking the sum and setting equal to zero the equality becomes:

$$\left(\frac{\ln\left(E(\pi)/\pi^*_{g2}\right)+\sigma^2(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) + \left(\frac{\ln\left(E(\pi)/\pi^*_{g1}\right)+\sigma^2(T-t)/2}{\sigma\sqrt{\mathrm{T-t}}}\right) = 0 \qquad \text{(a.2.2.3)}$$

By multiplying both sides by the denominator the equality reduces to:

$$\ln\left(E(\pi)/\pi^*_{g2}\right)+\ln\left(E(\pi)/\pi^*_{g1}\right)+\sigma^2(T-t)=0 \qquad \text{(a.2.2.4)}$$

Express each of the terms in logs and re-arrange to give:

$$\begin{aligned} 2\ln E(\pi) - \ln\pi^*_{g2} - \ln\pi^*_{g1} + \ln\left\lfloor e^{\sigma^2(T-t)}\right\rfloor &= 0 \\ 2\ln E(\pi) = \ln\pi^*_{g2} + \ln\pi^*_{g1} - \ln\left\lfloor e^{\sigma^2(T-t)}\right\rfloor & \end{aligned} \qquad \text{(a.2.2.5)}$$

To obtain (5.17), implement the following algebraic manipulation so that the delta is optimised:

$$\begin{aligned} 2\ln E(\pi) &= \ln\left\lfloor \pi^*_{g2}.\pi^*_{g1}\right\rfloor - \ln\left\lfloor e^{\sigma^2(T-t)}\right\rfloor \\ \ln E(\pi)^2 &= \ln\left[\frac{\pi^*_{g2}.\pi^*_{g1}}{e^{\sigma^2(T-t)}}\right] \end{aligned} \qquad \text{(a.2.2.6)}$$

$$E(\pi) = \left[\frac{\pi^*_{g2}.\pi^*_{g1}}{e^{\sigma^2(T-t)}}\right]^{0.5} \qquad \text{(a.2.2.7)}$$

This expression denotes the level of expected inflation at which policy activism is minimised.

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[241] Noting that: $N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

Appendix A.2.3: Using GARCH(1,1) to update the Black volatility parameter input

A key input to pricing an option is the forecast of the annualised standard deviation of the return on the underlying. Equally, the estimate of expected annualised volatility is critical in determining the value of the inflation collar. Modelling and forecasting the volatility of returns on financial time series has been the subject of a large and growing literature. Engle (1982) produced an autoregressive conditional heteroscedastic framework used to estimate conditional variance. His seminal paper incorporated a model of U.K. inflation. Bollerslev (1986) pointed out that 'estimating totally free lag distribution often will lead to violation of the non-negativity constraints.' To avoid going back a large number of lags of ε and prevent negative volatilities from emerging, Bollerslev (1986) suggests an alternative process: GARCH (Generalised Autoregressive Conditional Heteroskedasticity) and applied this to quarterly inflation data over the period 1948-1983 period. The simplest formulation is given by GARCH(1,1) which takes the form:

$$h_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 h_{t-1}^2 \qquad \text{(a.2.3.1)}$$

The variance of the error term can be described as having three constituents: a constant, last period's volatility and the previous period's variance. In attempting to estimate the volatility or forecasted volatility that permits clustering, the GARCH and ARCH processes are particularly useful. To value the inflation collar, it is necessary to estimate the volatility of the annualised percentage return of inflation. This can be accomplished by taking:

$$\rho_t = \ln(\pi_t) - \ln(\pi_{t-4}) \qquad \text{(a.2.3.2)}$$

Subsequently, equations for the mean and variance are specified:

$$\rho_t = \mu + \varepsilon_t, \varepsilon_t \sim N(0, h_t^2)$$

$$h_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 h_{t-1}^2 \qquad \text{(a.2.3.3)}$$

A log-likelihood function (*LLF*) is established:

$$L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(h_t^2) - \frac{1}{2}\sum_{t=1}^{T}(\rho_t - \mu)^2 / h_t^2 \qquad \text{(a.2.3.4)}$$

and maximised using the Berndt, Hall, Hall and Hausman (1974) BHHH algorithm under a normality assumption for the disturbances. This model is useful here because it can be used to describe the volatility of a series through time and more important from the perspective of option valuation, can be used to forecast volatility.[242] It is possible to calibrate one of the key Black formula inputs by estimating parameters: ϕ_0, ϕ_1 and ϕ_2. Thus the forecast:

$$h_{t+1}^{2\ F} = \phi_0 + \phi_1 \varepsilon_t^2 + \phi_2 h_t^2 \qquad \text{(a.2.3.5)}$$

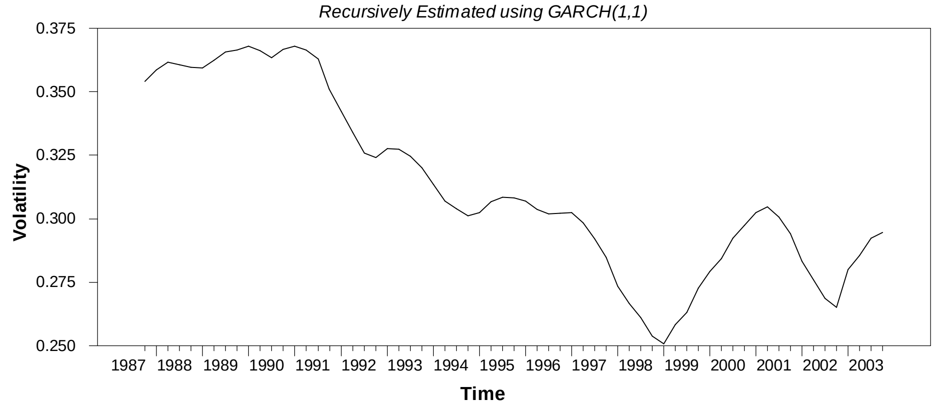
can be deduced for the upcoming period. Although, the ARCH and GARCH processes were initially conceived to deal with economic time series, much of the subsequent applications have been directed towards Value at Risk and option volatility modelling. The anticipated annualised standard deviation can be estimated by taking the square root.

$$h_{t+1}^{\ F} = \sqrt{\phi_0 + \phi_1 \varepsilon_t^2 + \phi_2 h_t^2} \qquad \text{(a.2.3.6)}$$

The forecast, $h_{t+1}^{\ F}$ then represents the annualised volatility estimate used to input into the Black formulae so σ = GARCH(1,1), when estimating the collars for (2.74), (2.75), (2.76), (2.77) and (2.79). To avoid the end-of-sample problem a recursively estimated volatility is computed by updating the GARCH(1,1) parameters each period. The GARCH model is estimated from 1957 up to 1987. Therein the re-parameterisation occurs each successive quarter. This avoids including lead values that policy makers would not normally have known in an *ex ante* context. Figure A2.1 outlines an updating forecast of volatility over the Greenspan incumbency.

Figure A2.1: Ex ante Volatilty 1987:4 - 2003:4

Recursively Estimated using GARCH(1,1)



[242] The GARCH(1,1) model is intended to provide estimates for the volatility of the annualised percentage return of inflation. An exponentially weighted moving average (EWMA) model could also have been used. The GARCH construction recognises that volatility is not constant and permits a systematisation of how expectations regarding volatility were formed by the Open Market Committee.

Appendix A.2.4: Estimation of the Growth Gap

The output gap is estimated atheoretically using GDP data downloaded from the Federal Reserve Bank of St. Louis. The y_t, given in (2.63), is generated by using the seasonally adjusted real GDP (rebased with 1996 prices to 100) and computing the quarter on quarter growth rate.[243] This is annualised and compared against actual annual growth. The time series commences in Q1, 1957. This data set is used to generate a chain of successive geometric growth rates (one for each new quarter). Central bankers are posited as taking a long-term trend into account when assessing the extent to which potential is available in the economy. The geometric mean growth rate is calculated recursively by regressing, in succession, a constant and time, t on the natural logarithm of the GDP using OLS.

$$\begin{aligned} \ln(GDP_t) &= \alpha + \beta_t^{GDP}(t) + \varepsilon_t \\ \ln(GDP_{t+1}) &= \alpha + \beta_{t+1}^{GDP}(t+1) + \varepsilon_{t+1} \\ &. \qquad\qquad\qquad\qquad\qquad\qquad (a.2.4.1) \\ &. \\ \ln(GDP_{t+n}) &= \alpha + \beta_{t+n}^{GDP}(t+n) + \varepsilon_{t+n} \end{aligned}$$

where

$$\alpha = \ln(GDP_0)$$

The coefficient estimates on time produce a series of continuous end of sample growth rates for each quarter over three decades. The intuition advanced to explain this approach is that the initial observation of real GDP_0 if compounded by the correctly estimated continuous growth rate β_t^{GDP}, will yield GDP_{t+i}. The chain of regressions given by (a.2.4.1), are calculated recursively for each quarter up to 2007:3. The first estimate is made for Q1, 1967 using 10 years of data. Each successive estimation produces a new end of sample continuous growth rate: β_{t+i}^{GDP}, for each subsequent quarter, (the start date of the data being 1957:1 is fixed). A vector of continuous growth rates: **B** is converted to discrete annual growth rates by sequentially quadrupling, β_{t+i}^{GDP} and then calculating the exponential function of the product. One is then subtracted from each observation to make comparable to a discrete annual GDP percentage growth rate. For the Greenspan years the trend growth rates may be represented by (a.2.4.2)

[243] See the series GDPC96 given in Table 2.2.

$$\begin{bmatrix} G^{T}_{1987Q4} \\ . \\ . \\ G^{T}_{2003Q4} \end{bmatrix} = \left\{ \begin{bmatrix} \exp\left(\beta^{GDP}_{1987Q4} \times 4\right) \\ . \\ . \\ \exp\left(\beta^{GDP}_{2003Q4} \times 4\right) \end{bmatrix} - \begin{bmatrix} 1 \\ . \\ . \\ 1 \end{bmatrix} \right\} \times 100 \qquad \text{(a.2.4.2)}$$

The trend growth, denoted by superscript *T*, is meant to represent a historic norm that the central banker recursively estimates. Thus the trend is atheoretical. The actual year on year GDP growth rate can be estimated using (a.2.4.3)

$$\begin{aligned} G_t &= \left[\frac{GDP_t - GDP_{t-4}}{GDP_{t-4}} \right] \times 100 \\ G_{t+1} &= \left[\frac{GDP_{t+1} - GDP_{t-3}}{GDP_{t-3}} \right] \times 100 \\ &. \\ &. \\ G_{t+n} &= \left[\frac{GDP_{t+n} - GDP_{t+n-4}}{GDP_{t+n-4}} \right] \times 100 \end{aligned} \qquad \text{(a.2.4.3)}$$

(a.2.4.3) constitutes the magnitude of annual increase, observed on a quarterly basis. (a.2.4.2) is successively subtracted from the year on year percentage GDP growth rate obtained in (a.2.4.3). This provides an end-of-sample growth gap measure: y_t. In this regard, it consistently uses only lagged and contemporaneous information. The difference between the two rates is given by (a.2.4.4):

$$y_t = G_t - G_t^T \qquad \text{(a.2.4.4)}$$

Figure 2.8 illustrates the growth gap and the Fed Funds rate over the Greenspan years.

Chapter 3

Reverse-engineering the Taylor Rule.

‘I would like to take the opportunity to talk a bit about monetary policy strategy and in particular about a memo that we all get called “Monetary Policy Rules”, which I hardly ever hear referred to. It relates to the Taylor Rule, which is actually quite a favorite among the academic economists. I have been looking at that rather hard this year and, in my view, it has not been that helpful. For most of the year, if one looked at its various predictions, it could have been interpreted as arguing for higher or lower interest rates, depending on how one estimates the equation and some other technicalities. Now it seems to be saying that we should be raising interest rates when one could make a case that interest rates are about right. I think the problem here is the output gap term. And the deeper problem is that to apply this rule, we must, have point estimates of our targets for both inflation and unemployment. At the very best I think we have bands; we do not have point estimates. As one listens to the way all of you talk about monetary policy, you seem to have different approaches to how to think about it. Suppose for the sake of argument that inflation and unemployment are reasonably within their target bands if not at one’s point estimates. As long as inflation is neither accelerating nor decelerating, we seem to be striving to maintain existing conditions. Partly this involves watchful waiting on acceleration or deceleration, not necessarily on inflation as such but on leading indicators of inflation such as those on the output side. And in part this involves aiming policy so that future growth in aggregate demand equals the trend growth in aggregate supply, which is roughly 2% percent under most models. At the last meeting, I said that the trend growth in aggregate demand was too low and that the economy needed some further stimulus. At this meeting it looks about right--at least to me, maybe not to some others. But I think that most of us have this more informal way of keeping things on an even keel for stable noninflationary growth. I believe this is what most of us do and I think it is working. Thank you.’ (Governor Gramlich, Federal Reserve Board, FOMC Transcripts, December, 1998, p.45)

3.1 Introduction

Increasingly during the 1990s central bankers at the US Federal Reserve appear to be describing more coherently what was understood by the ‘opportunistic’ strategy. Orphanides and Wilcox (1996) explored the theoretical foundations of the opportunistic approach. Their paper accompanied an internal debate that had asserted a nonlinear stance and arguably captured some pragmatic aspects of rate setting. Many of the earlier expressions that endorsed nonlinearity since the late 1980s, as outlined before in chapter 2, were tentatively developed into a more lucid *modus operandi* by December 1995.[244] The Greenspan Federal Reserve, to paraphrase its chairman, adopted a price stability

[244] In the accompanying appendix to the December 1995 FOMC transcripts, Don Kohn delineates using a policy matrix the implications of employing alternatively the deliberative and opportunistic approaches.

objective that was intended to lower inflation over each successive cycle. Significantly, the then secretary and economist to the FOMC Don Kohn, at the December 1995 meeting, submitted to members an opportunistic blueprint strategy. Not unrelated, the FOMC from January 2000 started to elaborate its policy actions in terms of a 'balance of risks' assessment that the committee could perceive to be upside or downside in attaining the goals of both sustainable growth and price stability.

The political/social context within which such monetary policy is formulated is important. Board dynamics in securing consensus, as conveyed by Governor Gramlich at the December 1998 FOMC meeting, suggests that the band structure was relevant in terms of how policy was implemented.[245] Orphanides and Wilcox (1996, 2002) suggested that a non quadratic loss function provided a theoretical basis for opportunism. Nonlinearity can also result from heterogeneity of opinion within the board or between the board and the legislature as outlined in chapter 2. Since policy makers operate with different models and preferences, consensus may not always be easily attained. Meade (2005) makes precisely this point. In Chapter 1, it was suggested that the opportunistic disinflation strategy also furnished Fed officials with an alternative way to communicate nonlinearities, in a policy environment where monetary aggregates were being slowly de-emphasised. The latter motivation is somewhat tactical, in that target bands afford, as President Corrigan put it, some 'wiggle'.

Ironically, given the broad endorsement of nonlinearity especially by Board members, no significant empirical evidence has emerged to confirm that monetary policy is opportunistic, or path dependent. Kim, Osborn and Sensier (2002), for example, find little evidence to support nonlinearity in the post 1979 period for the policy rule. Equally, the linear estimations of the type set out by Taylor (1993), Clarida, Gali and Gertler (1998) and Orphanides (2002) tend to define the status quo or standard approach.[246] One possible explanation is that the linear Taylor Rule and opportunistic policy outcomes are similar when one takes into account the targeting horizon, volatility of the underlying and bandwidth within which the risk management framework operates.

[245] See p. 48 of the December, 1998, FOMC transcripts.
[246] Conversely, Dolado, Pedrero and Ruge-Murcia (2004) find that estimates indicate that US monetary policy can be characterised by a nonlinear policy rule after 1983, but not before 1979.

In this chapter, the importance central bankers attribute to political institutions when framing policy, is investigated. Changes in the inflation target bandwidth, in the volatility of inflation and in time horizon over which an inflation target is applied, are examined via portfolio option theory. Risk management issues are used to explain the factors that influence the linearity of the reaction function. A number of propositions using the delta, theta and vega of an option portfolio based on inflation, are put forward to illustrate that opportunistic/zone targeting policy makers can routinely behave in a linear fashion when setting the Fed Funds rate despite the rhetoric endorsing nonlinearity.[247] Likewise, it can be illustrated that policy becomes increasingly linear as agreement between central bankers and with other executive branches of government is attained. An ESTAR or Exponential Smooth Transition Autoregressive model is used in this chapter to track the autoregressive structure of residuals obtained by subtracting the linear Taylor Rule rate from the actual fed funds rate over the Greenspan years. It is found that there is some empirical evidence to support the contention that policy was nonlinear.

3.2. The Taylor Rule Supreme

The opportunistic approach is not generally used in empirically gauging the tenor of policy. Typically, there is little empirical work in the literature that identifies anything but the deliberative policy for interest rate determination. Examples of Federal Reserve economists who have used linear constructions to estimate the reaction function include Judd and Rudebusch (1998), Hetzel (2000), Kuttner (2004) and importantly Orphanides (2004). At first glance, the prevalence of the linear estimation approach might not be expected given that many policy makers have been prepared to publicly endorse the opportunistic policy as an integral part of the rate setting process.[248]

Even though policy is described as path dependent, interest rate determination is generally treated as linear for estimation purposes. Attempts to detect nonlinearity in the reaction function during the post 1979 period have generally not been successful.

[247] These measures are conventionally used in risk management to cope with uncertainty. The vega of a portfolio of derivatives is the rate of change of the value of the portfolio with respect to, (w.r.t.), the volatility of the underlying asset with all else remaining the same. The theta of a portfolio of options is the rate of change of the value of the portfolio w.r.t. the passing of time, *ceteris paribus.*

[248] FOMC members however have never described the opportunistic policy as being the official policy of the Federal Reserve.

Likewise, the continuing ability of the fairly minimalist Taylor Rule to describe monetary policy has helped contribute to its pre-eminence over other more elaborate specifications. In what follows, the approach developed by Taylor (1993) will be treated as a limiting case of a more generally specified rule that can incorporate inflation target thresholds, timing adjustments and the second moment of inflation. The collar structure is sufficiently flexible to permit both linear and nonlinear policy to co-exist. This way of presenting the policy rule may help resolve a number of inconsistencies between what some central bankers describe and the linear form empirical estimation tends to assume.

3.2.1 Institutional dynamics and tactical considerations at the Greenspan Fed

The opportunistic strategy has been described by Don Kohn at the September 1995 FOMC meeting as where:

> '..the Federal Reserve does not seek to raise the unemployment rate above the natural rate, but effectively leans harder against shocks to the economy that would increase inflation than those that would decrease it. The resulting pattern would be one of successively lower inflation rates at cycle peaks and trough.' (Appendix September, 1995, p.6)

An important hallmark of this kind of policy is to opportunely profit from the usual reversals in economic growth or from favourable supply shocks to contain inflation. For instance:

> 'A drop in oil prices, for example, may be used to move to lower inflation under an opportunistic strategy, but it could be just as well be taken in the form of a transitory gain in output, leaving inflation where it was.' (Appendix September, 1995, p.7)[249]

These comments are akin to the approach elaborated by Orphanides and Wilcox (1996, 2002). That is, within defined thresholds policy becomes less aggressive and inflation targeting becomes more gradual.[250] This type of policy structure was echoed in the comments of Alan Greenspan, at the previous January 31st, 1995 meeting where he outlined how the then proposed legislation to give the Federal Reserve an explicit target for inflation was somewhat dual edged. While Fed officials welcomed the clarity that a legislative mandate offered, there appeared also some scepticism regarding the

[249] The term 'Opportunistic' is employed more generally in policy discussions. While the language used to express the nonlinear stance is similar to that used at the 1989 December FOMC meeting: the term 'opportunistic' is still relatively new. It is frequently used at the September 1995 meeting and this suggests that the conceptual framework that underpins a nonlinear policy was understood and had gained currency, at least in parlance, amongst a number of policy makers.

[250] See Vice - Chairman McDonough's endorsement of the opportunistic strategy at the December 1995 meeting p.8

steadfastness with which this type of policy rule would remain supported by the then members of Congress.[251] Implicitly, there appeared to be an acceptance that the inflation objective could not be perpetually implemented in an unrestrained linear fashion because political institutions would not steadfastly preserve such a mandate, even if initially they had conceived and put into law an explicit inflation target.[252]

Pages 57 to 59 of the January 1995 transcripts see chairman Greenspan concluding a debate by members of the FOMC regarding the then proposed legislation to set an inflation goal. Two key points emerge from the chairman's discussion. First, the actions of central bankers were understood to be circumscribed by the political and social environment within which they operate. Second, targeting inflation could not be implemented without reference to the short-run considerations of output and employment. In that regard, at moderate levels of inflation, short-term objectives are attributed greater importance than allowed for in a purely inflation targeting framework. This inevitably prejudiced a linear progression to a particular price goal.

> 'What strikes me about where we are is that even though the Federal Reserve is an independent institution in the legal sense, meaning that our decisions are not subject to further evaluation by other authorities we are in fact very dependent on the culture and the philosophy of the society in which we function.' (Federal Reserve Board, January, 1995, p.57)

The Fed chairman, p.57 – 59 of the same transcripts, went on to make reference to policy failures in the past and explained that central bankers are as much a product of the cultural/social milieu within which they operate, as their political masters. Meltzer (2005) pointed out that a key weakness during the 1960s related to the excessive emphasis chairman Martin attributed to achieving consensus within the FOMC prior to implementing determined dis-inflationary action.[253] In addition, the presence of

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[251] Meyer (2004) made the point that the specific Bills introduced in Congress to move the Fed in the direction of inflation targeting, in fact, ultimately reflected a minority position. Legislation of this kind would be most likely to succeed, only after a period of elevated inflation or following a period when monetary policy had been insufficiently disciplined. In his opinion there was no chance that 'Congress would accept a regime with a hierarchical mandate that raised the profile of price stability and diminished the responsibility of the FOMC for stabilization policy.' p. 154. Bernanke (2004b) conversely maintained that Congress would accept a Federal Reserve espousal of inflation targeting as being within the remit of existing legislation.

[252] The extracts that follow come from pages 57 to 59 of the January 1995 meeting transcripts. From the perspective of understanding where committee members sit in the policy game, an opportunistic type stance seems to provide latitude or 'wiggle'. The comments made by chairman Greenspan strongly suggest that central bankers are obliged to respond flexibly to a somewhat capricious political juggernaut.

[253] This would seem to point further to the importance of committee dynamics when making rate decisions. Equally, other branches of government can not be discounted.

institutional arrangements that promoted policy co-ordination between fiscal and monetary policy weakened significant aspects of Federal Reserve independence during the 1960s and 1970s. Political factors, according to Meltzer, hindered the Federal Reserve from moving promptly to contain prices. The Greenspan view of political dependence seemed to apply to chairman Martin, who believed that the Federal Reserve could not unilaterally negate the effects of budget deficits that were proposed and congressionally approved by an elected administration. Similarly, the same cultural and political realities confronted Arthur Burns. At the January 1995 meeting, Chairman Greenspan, made the point that central bankers, regardless of how averse they are personally to inflation, can only be as effective as the political context permits them to be. The ease with which the inflation target can be achieved changes through time. Learning from policy mistakes may have constituted an important catalyst in triggering the determined action of the Volcker years. If central bankers and politicians are unable to resolve policy differences, this can lead to inaction.[254] Chairman Greenspan pointed out how very pronounced anti-inflation instincts can be thwarted:

> 'I remember Arthur Burns, with whom I used to visit quite often and whom I had known from graduate school would speak against inflation like none of us here is used to hearing. If one looks at what the Federal Reserve did in that period that anti-inflation attitude is scarcely to be seen.' (Federal Reserve Board, January, 1995, p.57)

The change in psychology that had occurred in the political arena by the mid 1990s was very stark. A younger generation of elected politician, sought to legislate for ever lower inflation. The world had moved on from the 1970's but this *volte-face* was not regarded, by the chairman, as being entirely etched in stone. There was recognition that the then current policy framework was less predisposed to exploiting the traditional trade-offs but there was also a sense that this consensus between the different branches of government might be fragile and likely to be reversed given a change in economic fortunes. The lessons that had been learned from the turbulence of the 1970s were not complete safeguards and the FOMC and the legislature could diverge in the future in terms of identifying an appropriate inflation targeting framework. The zeal to curb inflation might not persist. Chairman Greenspan had some reservations about the permanence of any congressional endorsement for a specific inflation objective:

[254] This sense of tension is not confined solely to the United States. Goodhart (2005) offers the following insight: 'Combine slower growth with perhaps a mistake in judging the transmission mechanism, and it is easy to see how a populist politician might choose to run against central bank independence.' (pp. 301 – 2).

> 'I would not even take as a given, if the Congress gave us authority to have an explicit goal, that we really would be able to adhere to it …. I tell you that if we could get 80 percent of the Congress to vote for that goal, 95 percent would take a different position when the world changes.' (Federal Reserve Board, January, 1995, p.58)

This is consistent with comments made by chairman Greenspan 5 years previously at the December 1989 meeting, p. 43. The secondary consequences of monetary policy would not be handled by fiscal policy given that fiscal policy had already 'fumbled the ball'. Regardless of the legislated mandate, the Federal Reserve would be held to account for the other economic ramifications of adhering to a price stability rule. Greenspan proposed a policy approach:

> '… where the inflation rate is going to be lower at each progressive cyclical peak and lower at each cyclical low. But that objective is not being implemented in a straight line because we have recognised, and I think correctly, that the Congress would not give us a mandate to do that.' (Federal Reserve Board, January, 1995, p.58)

Greenspan's success over a long period and with differing administrations, in no small way, reflects a tactical *savoir-faire* given the institutional dynamics in place.[255]

> 'There still is a short term Phillips curve. People respond to it; they are aware of these trade-offs, and to deny them, I think, is a misunderstanding of how our political system works.' (Federal Reserve Board, January, 1995, p.58)[256]

This frank statement concluded a FOMC discussion on the proposed inflation targeting legislation then under consideration in 1995. The committee was somewhat divided whether an explicit inflation mandate should be adopted by Congress. Some members felt it contravened the 1946 Employment Act. Others felt it simplified the task of the Federal Reserve in achieving its ultimate objective of attaining price stability. In light of this and possibly to defuse tension, the chairman suggested that the benefits of inflation

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[255] From the FOMC transcripts, it appears that analyses of the political milieu absorb the committee from time to time. Chairman Greenspan is not unique in considering how political institutions impact on the economy and other government institutions. Governor Lindsey, p. 21 of the 1995 August meeting gives an intriguing commentary regarding presidential candidates and budget deficits. Governor Blinder p. 30 in the September meeting of the same year makes a similar argument to the chairman that interest rate decisions have to be politically and socially acceptable. Vice Chairman McDonough of the July 1997 meeting p. 68-69 discusses the effects of political censure on the Federal Reserve.

[256] Goodhart (2005) makes a similar point: 'The analytical concept of the vertical Phillips curve is not one that lends itself easily to the public imagination. The idea that an increase in interest rates to safeguard price stability may be the best way to maintain long-run growth is not self-evidently obvious, especially to indebted business men.' (p. 301)

targets may have been somewhat illusory in that, political support for any proposed course of action could waver through time.[257]

The chairman seemed to adopt a strategy of taking advantage of the economic cycle's natural rhythm and this seems intimately linked to the political backdrop. No matter how strong the impetus was to contain prices, there appeared to be some degree of scepticism as to how far the FOMC could test the then proposed mandate going forward. For a number of policy makers containing inflation was conceivably best achieved when the opportunity presented itself and the approach envisaged by some, spelled out a stance to chip away at the inflation goal while preserving other short run objectives. These extracts illustrate that part of the momentum generated for opportunistic policy incorporated consensus fixing and tactical responses to maintaining price stability.[258] If political institutions possessed a very different inflation target to the FOMC and the committee is answerable to the elected branch of government, then it is understandable that policy inevitably moves cautiously.[259]

3.2.2 Tactical considerations: a more formal evaluation of opportunism by Fed insiders

The December FOMC meeting of 1995, in contrast to some of the earlier meetings, explicitly alluded to the opportunistic strategy.[260] Opportunism at this meeting was clearly defined by Don Kohn.[261] In the briefing (which is included in the accompanying appendix to the December 1995 meeting), he spelled out, using a policy matrix, the implications both the deliberative and opportunistic approaches would have in shaping

[257] Of course, had Babe Ruth not got the home run after pointing, he may have been viewed to have failed in achieving his objective, despite what otherwise may have been considered a stellar performance.
[258] Von Hagen (1999) argues that the adoption of any particular policy stance is frequently related to how central bankers perceive they can best play the policy game. Many of the observations made by Von Hagen with regard to the Bundesbank's adoption of monetary targets have parallels with how Chairman Greenspan espoused strategy. Indeed, as mentioned previously, preserving the language of monetary targeting in policy directives long after this operating procedure became ineffective plausibly constitutes a case in point.
[259] Navigating between different points of view provides an institutional motivation for graduating the Federal Funds rate adjustments.
[260] Alan Blinder joined the Board of Governors in 1994, as vice chairman. In his testimony to the Senate Committee prior to taking office Blinder outlined an opportunistic type strategy to implement disinflation.
[261] In the appendix to the December 1995 meeting p. 10 – 11, Don Kohn describes how the opportunistic monetary policy feasibly could be implemented.

monetary policy.[262] The briefing was meant to clarify ambiguities that had arisen in the previous November meeting relating to varying interpretations of what opportunistic disinflation constituted. To some degree this briefing constituted a synthesis of a number of long-term strategy discussions and comments previously made by members of the FOMC.[263] Significantly, the FOMC secretary and economist meticulously spelled out a thoroughly delineated set of proposals for interpreting monetary policy. He juxtaposed conventional and opportunistic strategies in terms of their respective impact on the economy. The briefing clarified views held by some policy makers into a form capable of becoming a monetary policy game-plan.[264]

Both strategies, as Kohn saw it, presupposed that price stability was the correct long-term goal of monetary policy. In this regard, for a given range of expected inflation there was parity in terms of force applied whether linearly or nonlinearly implemented. The opportunistic strategy was defined in terms of a tactical need to communicate with political agencies, particularly as the cover or shield afforded by monetary aggregate targeting was being slowly removed. Don Kohn identified the opportunistic strategy as a tool to communicate given the institutional dynamics in play:

> If the Mack bill ever becomes law, the Committee will need to confront these issues: Why are you doing this deliberately? Why are you not doing this deliberately? Why do you have the opportunistic approach? Is it worth going to price stability? Why not get there? This question of justifying this opportunistic strategy in a fundamental underlying sense of society's utility will, I think it will be very much on the table if we have to confront that particular bill. (Federal Reserve Board, December, 1995, p.35)

Indeed the path dependency implicit in Greenspan's January 1995 comments, correspond closely to Kohn's views. Kohn appeared to furnish a mechanism by which the committee could define itself relative to other policy players and that the strategy of nonlinear type responses would not blur the goal of attaining price stability. In this

[262] The focus on strategy at this point, is to some degree prompted by political developments. Just as the impending legislation triggered some soul searching at the 1989 December meeting, the Mack Bill similarly was forcing policy makers to ask the same questions.

[263] See comments made by Presidents Stern and Melzer, pages 49 and 51 respectively of the November 1995 meeting. President Stern at the September 1995 meeting p. 45 indicated that in setting out a time horizon to achieve a price stability target, the opportunistic strategy afforded a useful alternative means to explain the Fed's position to outsiders. This is reminiscent of Von Hagen (1999) where the merits of the Bundesbanks' pursuit of monetary targeting related largely to the institution's need to communicate policy to outside players.

[264] Not all policy makers perceived themselves to be opportunistic.

regard, the policy matrix was meant to assist policy makers to explain what President Boehne at the previous September meeting in 1995, described as latitude:

> ‘In other words, whatever we do in the short run ought to be consistent with this longer-run objective, and I think the Mack bill does give us that latitude. It does open up the question of what the strategy is. It is easier on paper to explain that we are going to move toward stable prices year after year until we get there. In practice, I think we have to take into account the business cycle in getting there. We got ourselves into this inflationary problem cycle to cycle. We have gotten out of it marginally cycle to cycle, taking advantage of cyclical developments. So, as we go forward, if this bill becomes legislation, we need to take into account the cycle-to-cycle progress even though it can be more difficult to explain. (Federal Reserve Board, September, 1995, p.44)

Orphanides and Wilcox (1996) attributed different types of central bank responses to different levels of inflation, to produce a nonlinear reaction function. The inflation path dependency of policy can be demonstrated by Figure 3.1. This depiction of rate setting that is conveyed in Figure 3.1, seems to accord better with what chairman Greenspan enunciated at the January 1995 FOMC meeting and with Governor Gramlich’s December 1998 comments than the simple Taylor Rule linear specification.

Depending on where inflation was relative to critical thresholds, varying reactions were envisaged. As in, the policy matrix set out by Don Kohn in the December 1995 briefing page 10 – 11, both the comments of the chairman in January 1995 (and the policy matrix as developed by Federal Reserve economists) foster responses that are largely defined by the inflation context.

3.2.3 The gulf between what central bankers say and what economists estimate

There is empirical evidence available to question nonlinearity. It would appear that discerning nonlinearity is not straight-forward. Kim, Osborn and Sensier (2002) find that while there is evidence that the Fed operated a nonlinear monetary policy rule during the pre-Volcker period (1960-1979), the same is not true during the Volcker-Greenspan era.[265] Perhaps linear and nonlinear stances co-exist, even within the time-

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265 In reply to some questions via email Denise Osborn offered the following insights:
(1) Were you somewhat surprised by the lack of nonlinearity in the policy rule during the Greenspan years given ostensibly the evidence in the transcripts?
Yes, we were. I have also spoken at various times to economists working in various Fed offices, and their feeling supports the hypothesis that the Fed's policy is nonlinear.

(2) More generally, are you aware that other people have expressed surprise/intrigue at the paucity of results supporting nonlinearity?

frame of a single Chairman. If this is true, it is then critical to tease out the circumstances under which both linear and nonlinear policy can be mutually relevant. This leads to a key research question: how can the opportunistic model produce linear monetary policy responses? In what follows using portfolio option theory, I show that monetary policy can appear to be operating linearly and nonlinearly within the same framework. Alternatively one might describe this as permitting, across a whole spectrum of possible opportunistic policy rules, a linear rate setting policy subset.[266] To see this, it is first necessary to see how by adjusting the parameters used in option pricing, the baseline Black formula can produce linear and near linear reactions. In the remainder of this chapter, the factors that drive policy to assume varyingly linear or near linear forms will be discussed.

3.3. Resolving the gulf: when opportunistic policy becomes linear

Three candidate explanations are submitted here to resolve the gulf that seems to exist between the rhetoric and econometric evidence. These relate to the inflation target horizon, volatility of the underlying and to factors that determine threshold inflation rates, as described in Figure 3.1. Each of these taken individually could lead an opportunistic policy rule to assume a more deliberative form. The Greek measures: theta and vega that are used conventionally to manage portfolio option risk, can be also used to examine the effects on pricing of time horizon and volatility. The Taylor reaction function (1993) offers a good point of reference for understanding the conduct of

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Yes, though we know that nonlinearity can be rather subtle.

(3) In your estimation technique if, let us say, that the policy rule was linear from time to time but subsequently nonlinear would your empirical tests have picked this up? In other words if the true policy rule was dynamic moving from being linear or nonlinear within the Greenspan years would you have picked this up in your tests?
The Hamilton technique used in the paper with Dong Heon Kim and Marianne Sensier implicitly assumes that the equation is stable (though nonlinear) over time.

(4) Do you think there can be found any empirical evidence to support the hypothesis of nonlinearity /opportunism during the Greenspan years?
In the attached paper (with Marianne Sensier & a recent PhD student of mine, Mehtap Kesriyeli) we look a bit more at the time change issue. There we also find nonlinearity, but the US model is not entirely satisfactory. I do think that there is more to uncover about the Greenspan years, and my hunch is that getting a better handle on time change will be an important part of that.

[266] The family of policy rules is determined here principally by the thresholds, volatility and timing using portfolio option theory.

monetary policy. Its appeal is not just simplicity: it also seems to track the Fed Funds rate quite well since the late 1980s.[267]

3.3.1 Opportunism when policy makers agree

Proposition 3.1: *If policy makers agree the target level of inflation, then the policy rule becomes linear. Greater agreement produces a more conventional response both under conditions of certainty and uncertainty.*

To see this, consider the original Taylor Rule given by (3.1). This can be rearranged to give:

$$r_t = r^* + \pi^* + (1+\omega_1)(\pi_t - \pi^*) + \omega_2(y_t) \qquad (3.1)$$

The same notation as before is used. As usual

$$r_{1t} = r^* + \pi^*_{g1} + (1+\omega_1)(E_t(\pi_{t+1}) - \pi^*_{g1}) + \omega_2(y_t) \qquad (3.2)$$

represents Governor (1) and

$$r_{2t} = r^* + \pi^*_{g2} + (1+\omega_1)(E_t(\pi_{t+1}) - \pi^*_{g2}) + \omega_2(y_t) \qquad (3.3)$$

represents Governor (2)/Congressman(1).[268] Alternatively we might think of Governor (2) as reflecting the views of an external political agency that tolerates a higher inflation target. The rationale for incorporating an elected political chamber's tastes reflects the mutual pressures that different branches of government experience.[269] Institutional and committee dynamics have up to now implied two competing inflation targets simultaneously define the boundary of stalemate.[270]

A portfolio of options can be constructed to proxy inflation. A long call and short put on inflation that share the same exercise or strike rate can proxy the inflation gap. The inflation targets have the same magnitudes as the strikes. Options that have their

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[267] So much so, that it might be thought of as a form of Greenspan rule, although no claims of ownership have ever been made.

[268] Kutz and Taylor (2003) similarly motivate nonlinearity in exchange rates using the dynamic behaviour of currency traders. They suggest that nonlinear adjustment of exchange rates can arise from heterogeneity of opinion or beliefs between traders concerning the equilibrium level of the nominal exchange rate. However, as the nominal rate assumes more extreme values, a consensus between traders may be more easily formed regarding the appropriate direction of exchange rate moves.

[269] In the words of Charles Goodhart (1999), p.110 'The point I would like to make here is that such pressures [electoral] affect central bankers, and even independent members of MPCs, in exactly the same kind of way, even if not to the same extent, that they affect politicians.'

[270] A strict zone of inactivity only applies when expected inflation is known with certainty. Otherwise between the thresholds policy becomes less active. If policy makers agree (i.e. institutional and committee dynamics conspire to produce the same target for the upper and lower thresholds), then policy is found to become linear.

subscript denoted by $g1$, share the same lower exercise price/rate. Likewise, options that have their subscript denoted by $g2$, share the same higher exercise price/rate.

$$\left[E_t(\pi_{t+1})-\pi^*_{g1}\right]e^{-rT}=c_{g1}-p_{g1} \qquad (3.4)$$

and

$$\left[E_t(\pi_{t+1})-\pi^*_{g2}\right]e^{-rT}=c_{g2}-p_{g2} \qquad (3.5)$$

c_{g2} and p_{g2} share a superior strike when compared to c_{g1} and p_{g1}. The disparity in terms of the inflation target may reflect differences that exist between the hawks and doves on the committee.[271] (3.4) and (3.5) can be justified by appealing to put-call parity, alternatively it can also be deduced from positing the difference between a call and put that share the same exercise.[272] The chairman's role is negotiated by taking the average of the two Taylor Rule rates for the respective Governors. The chairman's role is set out in terms of steering a consensual path.[273] Both rates are defined by different inflation targets given that it is assumed all the other parameters are shared. The average rate is given by:

$$\frac{r_{1t}+r_{2t}}{2}=\frac{r^*+\pi^*_{g1}+r^*+\pi^*_{g2}}{2}+(1+\omega_1)\frac{\left[(c_{g1}-p_{g1})+(c_{g2}-p_{g2})\right]}{2}e^{rT}+\frac{\omega_2(y_t)+\omega_2(y_t)}{2} \qquad (3.6)$$

The rationale for taking the arithmetic mean is based on an assumption that policy makers are accorded by the chairman equal status in the interest rate decision. The policy rate represents the combination of two collar agreements where the strikes for the long call and short put positions are equal. (3.6) can be re-expressed to give:

$$\frac{r_{1t}+r_{2t}}{2}=\left[r^*+\frac{\pi^*_{g1}+\pi^*_{g2}}{2}\right]+(1+\omega_1)\frac{\left[(c_{g2}-p_{g1})+(c_{g1}-p_{g2})\right]}{2}e^{rT}+\omega_2(y_t) \qquad (3.7)$$

If no agreement can be reached then no interest rate change is implemented while expected inflation lies within the two inflation targets. Policy becomes nonlinear.[274] If alternatively there is agreement and a common strike (i.e. inflation target) is achieved

[271] Alternatively, it may correspond to the appointment of a Rogoff central banker who has a greater distaste for inflation than the rest of society. If the central bank were a unified actor and the policy rate had to be agreed with another executive branch of government with a higher inflation target, heterogeneity of opinion could also feasibly arise and precipitate a path dependent policy.
[272] See appendix A.3.
[273] In reality, the chairman votes and in addition exerts considerable moral suasion over the board. Meade (2005) maintained that members of the board generally voted with the chairman unless there was an unacceptable difference in opinion.
[274] This is most evident when expected inflation is known with certainty.

policy becomes deliberative. Complete consensus can be achieved if both members of the committee agree a policy rate. This can be measured by degree, in that the closer the targets converge, the more linear policy becomes. Complete agreement implies that the strikes are equal. This has the effect of simplifying the algebra above, so that convergence, here at the arithmetic mean, can be re-engineered back to the Taylor Rule. Using the arithmetic mean as a device to convey convergence or consensus between policy makers produces the following:

$$\frac{\pi^*_{g1} + \pi^*_{g2}}{2} = \pi^* \qquad (3.8)$$

This implies the collars reduce to a single collar to give:

$$\frac{\left[(c_{g2} - p_{g1}) + (c_{g1} - p_{g2})\right]}{2} = c^* - p^* \qquad (3.9)$$

By virtue that agreement is achieved in terms of the setting the inflation target:

$$\pi^*_{g1} = \pi^*_{g2}$$

Policy maker agreement is denoted by simply taking the arithmetic $(c_{g2} + c_{g1})/2 = c^*$ and $(p_{g2} + p_{g1})/2 = p^*$. The short put and the long call position on inflation basically mimic the underlying minus the inflation target. From appendix A.3, it is found that if a collar portfolio is constructed where the strikes are equal, then the payoff becomes a linear function of expected inflation. For the purposes of implementing monetary policy, it is illustrated that the Taylor Rule can be thought to constitute an important limiting case when the varying inflation targets of policy makers converge.[275] Significantly, if the collar has a common strike for the long call and short put, the payoff from the portfolio replicates the payoff from the inflation gap.

$$\bar{r}_t = \left[r^* + \pi^*\right] + (1 + \omega_1)(c^* - p^*)e^{rT} + \omega_2(y_t) \qquad (3.10)$$

Consider the time value payoff from the portfolio $\left(c^* - p^*\right)$. This produces a linear payoff just as the inflation gap in the original Taylor Rule generated a linear policy rate given that:

$$\left[E_t(\pi_{t+1}) - \pi^*\right]e^{-rT} = c^* - p^*$$

and (3.10) reverts to the original deliberative Taylor Rule:

$$\bar{r}_t = \left[r^* + \pi^*\right] + (1 + \omega_1)\left[E(\pi_{t+1}) - \pi^*\right] + \omega_2(y_t)$$

[275] This can be observed also from figure 3.2a.

The time value of the collar may be considered as the monetary policy response under conditions of uncertainty. The intrinsic value of the portfolio describes decision making under certainty. The aggregation of the long call and short put positions, that share the same exercise, is found to produce a linear payoff. Linearity is thus accomplished regardless of certainty or uncertainty so long as the inflation targets of the policy committee converge.

Increased agreement by policy makers may result from institutional change. Granting independence to a central bank could lead to a reduction in the differences between the strikes. Of course, this would only occur if independence signalled the actual removal of political interference from the rate setting process. The views expressed within the board then became more homogenous.[276] Institutional change may serve to influence committee dynamics. The inflation targeting framework, in effect, could be seen as a form of agreed social contract where the rules of the game are put in place. This may still not deliver point targeting, given that a number of inflation targeting central banks, as outlined in chapter 1, frequently employ bands or apply different weights to the output gap. It may nevertheless be associated with a contraction of the bandwidth.[277] The nature of an economic shock may also influence how policy makers react. A supply shock may be associated with greater disagreement between central bankers as to how, for instance, oil price increases should be handled.

3.3.2 Observing the effects of agreement using portfolio option theory

Table 3.1 provides the estimates of the time values and deltas of two collars for each of the underlying expected inflation values ranging from 0.5% to 7%. The delta of the collar portfolio can be calculated from the deltas of the individual options in the portfolio. This has been illustrated in chapter 2. The two collars given in Table 3.1 are the same in all respects other than the strikes are different. The parameter values that are given in the first four columns are explained below. The time values of the collar are calculated by subtracting the short put position:

$$p_{g1} = e^{-rT}\left[\pi^*_{g1}N\left(-d_{2g1}\right) - \hat{E}(\pi)N\left(-d_{1g1}\right)\right]$$

[276] The Bank of England received operational independence in May 1997. It is not unreasonable to contend that this may have had the effect of removing some heterogeneity given the more clearly defined remit set out by government to tackle inflation.
[277] The social contract, of course, could have a longer maturity.

from the long call:

$$c_{g2} = e^{-rT}\left[\hat{E}(\pi)N\left(d_{1g2}\right) - \pi^{*}_{g2}N\left(d_{2g2}\right)\right]$$

The collar labelled as 2.5% - 4.5% in Table 3.1 estimates:

$$\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}$$

The collar given as 3.5% - 3.5% implies that the strikes are equal yielding:

$$\left(c_{g2}^{0.035} - p_{g1}^{k=0.035}\right)^{T=1,\sigma=0.25}$$

The value of the collar is seen to rise with the underlying expected inflation; however the rate of change in the rise adjusts as the underlying adjusts. This is reflected by the change observed in the delta magnitude as alternative bands are considered. Here the 2.5 – 4.5% band is compared to the 3.5 – 3.5% band. Delta for the call is calculated by using:

$$\frac{\partial c_{g2}}{\partial E(\pi)} = e^{-r(T\text{-}t)}N\left(d_{1g2}\right)$$

where

$$N\left(d_{1g2}\right) = N\left(\frac{\ln\left(E(\pi)/\pi^{*}_{g2}\right) + \sigma^2 T/2}{\sigma\sqrt{\mathrm{T}}}\right), N\left(d_{2g2}\right) = N\left(\frac{\ln\left(E(\pi)/\pi^{*}_{g2}\right) - \sigma^2 T/2}{\sigma\sqrt{\mathrm{T}}}\right)$$

Delta for the put is calculated by using:

$$\frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right) - 1\right]$$

where

$$N\left(d_{1g1}\right) = N\left(\frac{\ln\left(E(\pi)/\pi^{*}_{g1}\right) + \sigma^2 T/2}{\sigma\sqrt{\mathrm{T}}}\right), N\left(d_{2g1}\right) = N\left(\frac{\ln\left(E(\pi)/\pi^{*}_{g1}\right) - \sigma^2 T/2}{\sigma\sqrt{\mathrm{T}}}\right)$$

Table 3.1 provides the basis to examine the effect of narrowing the inflation targets. By taking the delta of the collar:

$$\frac{\partial c_{g2}}{\partial E(\pi)} - \frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T\text{-}t)}N\left(d_{1g2}\right) - e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right) - 1\right]$$

it is possible to gauge how the monetary policy response alters, as expected inflation alters.[278] The policy responsiveness is given by the portfolios delta multiplied by the

[278] See Appendix B.3.

number of collar portfolios there are.[279] One of the more striking features in Table 3.1 is that the delta for the second collar remains constant. If the put-call parity relationship is differentiated w.r.t. $E(\pi)$, the following emerges:

$$\frac{\partial c}{\partial E(\pi)} - \frac{\partial p}{\partial E(\pi)} = e^{-rT} \text{ consistent with } \frac{\partial \left(c_{g2}^{k=0.035} - p_{g1}^{k=0.035} \right)^{T=1,\sigma=0.25}}{\partial E(\pi)} = e^{-rT}$$ [280]

When the strikes are equal and agreement between board members is secured, policy is linear. Policy responsiveness is constant, unlike when the strikes were different:

$$\frac{\partial \left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025} \right)^{T=1,\sigma=0.25}}{\partial E(\pi)}$$

Monetary policy becomes linear when convergence is achieved. Table 3.1 provides the raw data for Figure 3.2a. The underlying is expected inflation and the time value is calculated from the Black formula. It is clear from Figure 3.2a that if agreement can be achieved, i.e. a common inflation target, then the payoff from the portfolio consequently becomes linear. In other words, the closer the inflation targets pertaining to different groupings within the policy committee converge, the more linear interest rate determination becomes.[281] The option model spelled out by the collar construction posits that the monetary policy becomes linear when a common inflation target is accepted. Theoretically, it is possible that a collectively opportunistic committee may much of the time be pursuing, by default a linear policy rule. As policy makers ever increasingly accede to sharing a common inflation target, policy becomes increasingly linear. The Taylor Rule therefore could be viewed as representing an important limiting case. Conceivably, in the absence of a government mandate, committee thresholds are much of the time converging and diverging, so that monetary policy can at times be assuming different states of linearity/nonlinearity through time. This type of alternating policy rule has not generally been considered in the empirical literature.[282] It may also

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[279] The number of collars is estimated by calculating the coefficient on the collar in the policy rule given by (3.10). Here it is assumed for simplicity that the coefficient is unity. This then implies the analysis looks purely at the portfolio's time valuation and delta estimate.

[280] This is a general result for a portfolio where the options have the same exercise. By differentiating put-call parity w.r.t. expected inflation, the delta of a European call and the delta of a European put can be shown to have this general relationship. Table 3.1 assumes that there is only one call and one put in the portfolio. If the r is set equal to zero the portfolio collar is set equal to unity. Equally, when the collar delta is multiplied by e^{rT}, as set by 3.10, the policy response can also be seen to be unity.

[281] Of course, one of these targets may be influenced by an external political agency.

[282] In this regard, any policy rule whether monetary/inflation targeting, linear/nonlinear are marginal rules in that any one operating procedure can produce ostensibly similar outcomes.

account for the diverse and perhaps conflicting perspectives that central bankers have expressed regarding rate setting.

The parameter values outlined in Table 3.1 for the option collar portfolio are set so as to reflect likely policy concerns: the maturity of the portfolio is equal to one year, the risk free rate is fixed at 5% and annual volatility of the underlying equals 0.25.[283] The one year time horizon has previously been used by Clarida, Gali and Gertler (1998) in their forward looking reaction function. The annualised standard deviation of expected inflation magnitude is 0.25. This would be at the lower end of annual inflation volatility, as estimated recursively in Appendix A.2.3 by using a GARCH(1,1). The GARCH(1,1) model describes the behaviour of forecasted volatility over the Greenspan years and provides a benchmark against which to compare the 0.25 used here. The risk-free rate is slightly below the mean of the one-year Treasury bill rate over the Greenspan tenure. The exercise or strike of 4.5 % is somewhat arbitrary but seems to be a rate that feasibly represented an upper tolerance for inflation during the early tenure of chairman Greenspan. These parameters are used in the following exercises to compute the time value of the collar and a number of other parameter values. Similarly to Figure 3.2a, Figure 3.2b illustrates that if the time values of a short put and long call are estimated using the Black model then a linear payoff from the collar is generated by the time values. The inflation threshold is set at 3.5%. The parameter values otherwise are the same as before given in Table 2.1. When a consensus is secured in terms of agreeing a common inflation target, monetary policy is observed not to be path dependent. Additionally, the aggregation of the time values produces a linear monetary policy response under conditions of uncertainty.

If agreement can be achieved within this nominally nonlinear construction, policy can in fact respond linearly to inflation. Figure 3.2a demonstrates that when policy is constrained by varying states of stalemate, rate setting can migrate between shades of linearity and nonlinearity. Plausibly if such conditions exist, empirical estimation becomes somewhat more subtle in terms of registering monetary policy as being linear or nonlinear. That is, policymakers who adopt an opportunistic or zone targeting

[283] These are the same as those given for Table 2.1. The risk free rate is somewhat notional given that it is not strictly required to operationalise the policy rule. This was made explicit in (a.2.1.16) where the discount rate, r can be shown to be eliminated.

strategy may in fact respond linearly to expected inflation. In this regard, the Taylor Rule represents one of a range of viable policy rules within the entire family of plausible opportunistic rules. The Taylor reaction function (1993) has remained a robust benchmark against which to gauge the tenor of interest rate determination. Policy makers that are modelled explicitly as being opportunistic can, perhaps paradoxically, display a default Taylor Rule. That is, a conventional/deliberative monetary response can emerge around an agreed inflation point target.[284] The deltas derived for the Black model provide a measure of the responsiveness of the value of the collar to changes in expected inflation. The delta estimates can be used to measure policy activism. In the opportunistic approach, one would expect that policy activism would be depressed during intervals when inflation remained between the thresholds - that is when there was disagreement. Figure 3.3 demonstrates that when policy makers can not agree to implement policy, that is, when inflation falls between 2.5% and 4.5% the delta declines very substantially in this range. The contrast is quite stark. The delta subsides significantly when the inflation targets diverge.

Figure 3.3 demonstrates more clearly the result obtained in table 3.1. If consensus can be achieved between policy makers on an agreed target, the responsiveness of monetary decisions remains constant over an entire range of the underlying expected inflation. The monetary policy response, all else being equal, also remains more elevated over a substantial part of that range. In this instance, the arithmetic mean of the two thresholds: 2.5% and 4.5%, might be used approximately to denote the target inflation rate that an effective chair might conceivably be able to accomplish.[285] The delta value obtained when an agreed target is attained is constant over the entire range of inflation. There is no diminution in policy activism over the full range of the underlying. If the delta is constant, that implies there is no change in the slope of the collar over the range of underlying expected inflation. In other words, the default policy rule is linear. Equally figure 3.4 shows if the range of disagreement contracts, for example, from the 2.5% to 4.5% down to 3% to 4% the delta increases in magnitude across nearly the entire range. Policy, in effect, becomes more active within the zone of disagreement. The slopes of

[284] Of course this is not the only reason why monetary policy assumes a more linear form.
[285] An additional consideration relates to the form of disagreement encountered. If the level of stalemate that occurs relates to real interest rates, this implies that policy would correspondingly be more linear than the case of nominal stalemate.

both collars are not constant but importantly as the thresholds converge interest rate determination becomes increasingly linear.

3.3.3 A forward-looking policy rule – a model of uncertainty

A number of economists assert that monetary policy incorporates a forward-looking dimension. Svensson (1997) has advocated that the authorities target the inflation forecast. By default, making forecasts of inflation implies using an extensive information set. This forward-lookingness of central bankers is motivated by the view that monetary policy only exerts an influence on the economy with a lag. Clarida, Gali and Gertler (1998) maintain that central bankers are not unduly concerned by short-term developments in inflation and instead are more concerned by medium to long-term trends.[286] They selected a horizon of twelve months and justified this on the grounds of plausibility. The transmission mechanism is understood to be effective only with a lag.[287] New Zealand, Canada and the United Kingdom explicitly link monetary policy to inflation forecasts.

Batini and Haldane (1999) maintain that in other inflation targeting countries, inflation forecasts are less overtly related to inflation forecasts but even here they are fundamental to understanding central bank behaviour. One device used by the Bank of England to implement this type of strategy is to publish an inflation forecast up to two years ahead. This has the benefit of improving self discipline and transparency. In the words of Charles Goodhart (1999), (p. 8):

> '..it is extremely difficult to publish an inflation forecast without adjusting interest rates to show publicly that the target should be approximately achieved, given our best assessment of the future evolution of all other economic factors.'

Central bankers, of course, do not have a crystal ball. Their forecasts must be ascribed a margin of error. The analysis offered by option theory does not prescribe an optimal

[286] The longer targeting horizon for expected inflation, the greater the level of uncertainty engendered in the interest rate decision.

[287] Alan Greenspan's Humphrey-Hawkins Testimony in 1994 endorsed the forward-looking specification proffered by economists: 'The challenge of monetary policy is to interpret current data on the economy and financial markets with an eye to anticipating future inflationary forces and to countering them by taking action in advance' (cited in Batini and Haldane (1999), p. 7). Equally, Governor Ben Bernanke (2004b) proffered that 'policy involves lags and thus must of necessity be based on forecasts. As we look ahead, core inflation appears likely to remain in the zone of price stability during the remainder of 2004 and into 2005.'

time horizon for targeting expected inflation.[288] The interest rate decision is considered here from the standpoint of not knowing fully the economy.[289]

> ‘A policy action that is calculated to be optimal based on a simulation of one particular model may not, in fact, be optimal once the full extent of uncertainty in the policymaking environment is taken into account. In general, it is entirely possible that different policies will exhibit different degrees of robustness with respect to the true underlying structure of the economy. For example, policy A might be judged as best advancing the policymakers' objectives, conditional on a particular model of the economy, but might also be seen as having relatively severe adverse consequences if the true structure of the economy turns out to be other than the one assumed. On the other hand, policy B might be somewhat less effective in advancing the policy objectives under the assumed baseline model but might be relatively benign in the event that the structure of the economy turns out to differ from the baseline.’
> (Remarks made by Chairman Alan Greenspan. At a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 29, 2003. ‘Monetary Policy under Uncertainty.’)

Inevitably every model is flawed. The role of central bankers relates to managing varying risks; including how changing the time horizon for achieving price stability affects the chances of keeping inflation within target for a given period.

A natural measure to convey this risk is theta, in that it calibrates the extent to which the value of an option portfolio changes with respect to time (or more correctly the decline in maturity). Earlier, the time value of the collar portfolio was seen as providing a useful proxy for gauging monetary policy. Theta here is seen to offer a metric that measures how policy responds when the schedule for maintaining the inflation target is changed. In addition to theta, the effect of time on the deltas and portfolio values offer important additional insights into how this type of uncertainty shapes the committee’s interest rate decision. The theta of the collar is employed to infer the possible effects of changing the central bank’s forward-lookingness. The theta measure, developed more generally in risk management, estimates the rate of change in the portfolios’ value with respect to time. Options generally, but not always, become less valuable as time to maturity

[288] Batini and Haldane (1999) suggest that targeting three to six quarters ahead delivers the best outcome but this is underpinned by a specific economic model and contingent upon the degree of forward lookingness by the private sector. The theta measure here is meant to offer additional insights, which are generally not considered, into how policy changes as greater uncertainty arises by virtue of decreasing/increasing the time horizon during which price stability is preserved.

[289] A naïve model, i.e. the Black model, may well be desirable because it does not privilege any particular view of how the economy works except to assume that inflation adheres to a lognormal random walk.

decreases. For an at-the-money option, time decay is most pronounced.[290] This implies that the most discernible effect of changing the time horizon of monetary policy is observed when expected inflation is close to the upper and lower bands of tolerance. This aspect of uncertainty is investigated by looking at the effect of changing the time horizon on the value of the inflation collar, by considering both the theta and also the magnitude of delta associated with the collar.

3.3.4 The effect of uncertainty caused by a change in the targeting horizon

Proposition 3.2: *If the interest rate decision is designed to preserve price stability over a longer term, uncertainty regarding the future lessens the non-linearity of a path-dependent policy. By applying a number of benchmark measures from portfolio option theory, it is found that as the time horizon for containing inflation increases the monetary policy response becomes increasingly linear.*

A number of inflation targeting regimes make the time frame transparent, over which policy is designed to preserve the objective of price stability.[291] The Bank of England sets the short-term rate so that the forecast of inflation spanning a two-year horizon remains within target.[292] The Federal Reserve has never explicitly defined such an inflation target and timing of an implicit target has never been formalised. Policy makers at the FOMC have nevertheless discussed an appropriate interval within which price stability could be attained.[293] This approach generally is consistent with having a less onerous intermediate target in the intervening years and permits a greater measure of latitude in accomplishing price stability.

The maturity of an option contract is analogous to the time period associated with preserving a particular target or keeping inflation within a zone of tolerance. Policy, to a large degree, is inevitably forward-looking given that it operates with lags. A question

[290] The time decay of the option should not be confused with the 'zero time decay' approach referred to when considering the risk neutral conditions in chapter 1 and in Appendix A.2.1.

[291] There is a subtle but important distinction between stating 'the time period within which or by which price stability will be attained' and 'the time period over which price stability will be preserved'. The latter represents a more taxing commitment in that it implies no intermediate target is acceptable other than the ultimate target. From the point of view of applying theta the relevant statement is 'the time period over which price stability will be preserved'. The theta measures the effect of a fall in maturity.

[292] The Federal Reserve has never formally endorsed a given inflation point or zone target. This may reflect a desire to maintain 'wiggle' and a wish not to be seen to fall short of a given stated objective.

[293] A number of references have been made to this in chapter 1.

frequently posed relates to finding the appropriate horizon or optimal time frame to contain a given target variable. It is not attempted here to answer that question although a number of economists already cited have done precisely that. The emphasis here pivots around determining the likely effects of altering the targeting horizon on the linearity of policy. In what follows, it is illustrated that as the horizon for controlling inflation expands, policy becomes more linear. Intuitively, by attempting to control events across a wider time frame, policy confronts a greater range of possible inflation outcomes. Time may be thought of as another measure of uncertainty. To avoid the cumulative effects of inflation spiralling away from target, policy feasibly migrates to being less path-dependent as the target horizon expands.[294] Accepting small defeats can cumulatively precipitate significant deviation from the ultimate inflation goal and this may make correction subsequently less secure. Conversely, attaining price stability as opposed to preserving price stability over a longer time horizon implies a less stringent approach with an intermediate target perhaps being put into place.

Pragmatic implementation of policy from the perspective of a central banker involves engineering robust strategies that hold across a wide spectrum of outcomes. By lengthening the time frame, the spectrum consequentially expands. A complicating factor in managing economic outcomes relates to the shifting orientation of policy. This in part, may be occasioned by exogenous factors that dictate that policy migrates to placing a greater weight on delivering shorter or longer-term outcomes. An example could be motivated by observing the behaviour of a policy committee attempting to maintain non-partisanship prior to political elections.[295] This may entail switching the emphasis to preserving longer-term price stability at an earlier juncture.[296] In an attempt to protect itself from accusations of political partisanship, we might consider that policy

[294] Intuitively, setting policy to contain inflation over a longer time period would seem more onerous than setting policy to contain inflation over a short time period. The greater uncertainty associated with an expanded time horizon necessitates weighting more the probability of drifting outside the comfort zone. This helps explain why policy becomes less path-dependent as the maturity of preserving the target lengthens.

[295] The former vice chairman of the BOG, Schultz (2005), p.346, made this comment regarding the August 1980 interest rate adjustment: 'The Federal Reserve is a thoroughly non-political institution. I never heard politics discussed at the Board table while I was there, but we did try to make any moves as far away from an election as possible.'

[296] See Governor Gramlich's comment p. 97 of May 1998 FOMC meeting: 'I think we should recognise that this is a political year, and it might get harder to make changes later in the year. We should be mindful of that and perhaps think hard about it at our next meeting'. This suggests that the timing of interest rate moves evolve if only to preserve political neutrality.

shifts between different timing postures.[297] In what follows, portfolio option theory is used to investigate the likely effects on the shape of policy that are caused by a timing switch. A complicating factor is that a timing switch may also be coupled with an adjustment to the intermediate target.

For a given range of expected inflation, an increase in the collar's maturity tends to increase the time value for the call and the put. If a short position is taken on the put, the effect of increasing the time to expiration is to generally reduce the value of the put (to the writer of the instrument).[298] The time values of the collars can be computed by taking the difference between the long call and short put positions. Using the Black Model as the baseline this can be written as:

$$c_{g2} - p_{g1} = e^{-r(T)}\left[\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right] - \left[\pi^*_{g1}N\left(-d_{2g1}\right) - E(\pi)N\left(-d_{1g1}\right)\right]\right]$$

Three different maturities are used in Figure 3.5a. The parameter values are the same as those used in Table 2.1 but the maturity ranges from six to eighteen months. These maturities might be considered to be alternative forward-looking specifications used by central bankers. It can be observed that as the maturity of the call and put increases the absolute values also increase over most of the range of expected inflation. This is most striking when expected inflation approaches the outer bands of the inflation thresholds. Figure 3.5a shows that as the maturity of the collar increases, the time value of the collar becomes increasingly linear. The largest magnitude of change occurs close to the strikes for a given change in time.[299] The time value of the collar or the payoff is increased when expected inflation is equal to 4.5%. When expected inflation falls to 2.5% the impact of increasing time horizon is to reduce the payoff associated with the collar. When inflation moves beyond the inflation bands this relationship is somewhat reversed. The net effect strongly suggests that as central bankers extend the period over which policy is designed to contain inflation, the change in the monetary policy response varies less discernibly with inflation. In effect, policy is seen to be less path-dependent in a world with a more extended time framework for preserving price stability.

[297] This would tend, *ceteris paribus*, to make policy more linear, but this may be offset by policy makers allowing the inflation target to increase affording greater room for manoeuvre. It is conceivable that lengthening the time horizon may be balanced by a widening of the target zone. This type of dual policy action may not be measurable in that both actions may have the effect of partially offsetting each other. In this regard, the view that nonlinearity can be 'subtle' seems highly pertinent.

[298] Figure 3.5a makes this rationale somewhat clearer.

[299] This becomes more evident when the theta of the collar is considered later.

The impact of a fall in maturity on the option portfolio can be measured by the portfolio's theta: the partial derivative of the collar's payoff taken w.r.t. time. It is worth considering the behaviour of theta to appreciate the likely effects of time on the reaction function and policy activism. If the call option for the Black Model is given by:

$$c_{g2} = e^{-r(T-t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right]$$

then using the standard notation and differentiating the call w.r.t. time, t gives:

$$\frac{\partial c_{g2}}{\partial t} = e^{-r(T-t)}\left[E(\pi)N'\left(d_{1g2}\right)\frac{\partial d_{1g2}}{\partial t} - \pi^*_{g2}N'\left(d_{2g2}\right)\frac{\partial d_{2g2}}{\partial t}\right] + re^{-r(T-t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right] \tag{3.11}$$

Importantly t here denotes time decay (i.e. as t increases the residual maturity falls). It is shown in Appendix C.3 that $E(\pi)N'\left(d_{1g2}\right) = \pi^*_{g2}N'\left(d_{2g2}\right)$ and consequently (3.11) can be re-expressed to give:

$$\frac{\partial c_{g2}}{\partial t} = e^{-r(T-t)}\left[E(\pi)N'\left(d_{1g2}\right)\left(\frac{\partial d_{1g2}}{\partial t} - \frac{\partial d_{2g2}}{\partial t}\right)\right] + re^{-r(T-t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right] \tag{3.12}$$

This in turn can be simplified given that the relationship between d_{1g2} and d_{2g2} can be defined by $d_{1g2} - d_{2g2} = \sigma\sqrt{T-t}$.

$$\Rightarrow \frac{\partial c_{g2}}{\partial t} = -e^{-r(T-t)}\left[E(\pi)N'\left(d_{1g2}\right)\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right] \tag{3.13}$$

The theta that is given in (3.13) measures the effect of a fall in maturity on the time value of the call. Similarly, a corresponding process can be used for differentiating the put. By combining both the call and put, the theta of the collar portfolio can be estimated for the range of expected inflation spanning 0.5% to 7.0%. The lower case t in effect, denotes the life of the contract gone, as opposed to the maturity remaining. Thus, as t increases the remaining maturity falls (or in monetary policy parlance the targeting horizon falls). Hence one would expect given the time values in Figure 3.5a, that the payoff of a long position in a call option decreases as t increases and the value of a short put increases as t increases. The short put option position becomes less negative, as t increases for the range of expected inflation closest to the lower threshold. Conversely, the long call position becomes less positive as t increases, for the range of expected inflation closest to the upper threshold. The behaviour of the theta should ideally conform to the time valuations as outlined in Figure 3.5a. From (3.13), it can be observed that the term inside the first square brackets is positive therefore considering this alone, a fall in maturity (t rises) results in a decline in value of the call.[300] Looking at the second square brackets, if the expected inflation rate, $E(\pi)$, exceeds the exercise, π^*_{g2}, a fall in maturity can culminate in an increase in time value over a certain range. This can be observed from Figures 3.5a and 3.5b. Normally, however a fall in maturity reduces the time value. Applying 'zero time decay' as set out in Appendix A.2.1 implies r would be set to zero and the effect of the second term in square brackets would be neutralised.

To consider the effect of time on the put option the associated theta is derived. The put options time value is taken from the Black model:

$$p_{g1} = e^{-r(T-t)}\left[\pi^*_{g1}N(-d_{2g1}) - E(\pi)N(-d_{1g1})\right] \tag{3.14}$$

If differentiated with respect to time it is possible to show that:

$$\frac{\partial p_{g1}}{\partial t} = -e^{-r(T-t)}\left[\pi^*_{g1}N'(d_{2g1})\frac{\partial d_{2g1}}{\partial t} - E(\pi)N'(d_{1g1})\frac{\partial d_{1g1}}{\partial t}\right] + re^{-r(T-t)}\left[\pi^*_{g1}N(-d_{2g1}) - E(\pi)N(-d_{1g1})\right] \tag{3.15}$$

Again from Appendix B.3, (3.15) can be re-expressed to give the Theta for the long put:

$$\frac{\partial p_{g1}}{\partial t} = -e^{-r(T-t)}\left[E(\pi)N'(d_{1g1})\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[\pi^*_{g1}N(-d_{2g1}) - E(\pi)N(-d_{1g1})\right] \tag{3.16}$$

The effect of the first square brackets in (3.13) is to reduce the value of the option as maturity or time horizon falls. This is similar to (3.16) however for the short position in the put; this is reversed so a broadly positive relationship exists between t, time decay and the value of the short put. This is most apparent when expected inflation is close to the lower threshold of the band. Looking at the second square brackets in (3.16) if $E(\pi)$ is greater than π^*_{g1} an increase in t, that is a fall in maturity tends to reduce the value of the put. Thus it exerts the same impact as the first square brackets. If π^*_{g1} is greater than $E(\pi)$ an increase in t can in fact increase the value of the put. This is

[300] Theta is normally negative for an option.

observable from Figures 3.5a and 3.5b. This is somewhat a simplification by virtue of not taking more precisely into account the cumulative probabilities $N(-d_{1g1})$ and $N(-d_{2g1})$. These magnitudes tend to track each other over the expected inflation range.[301] The Theta in (3.16) relates the time value of the put to time. The theta of the portfolio can be given as the difference:

$$\frac{\partial c_{g2}}{\partial t} - \frac{\partial p_{g1}}{\partial t}$$

so that incorporating the parameter values yields:

$$\overset{T=1,\sigma=0.25,\pi^*_{g2}=0.045,r=0.05}{-e^{-r(T-t)}\left[E(\pi)N'(d_{1g2})\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[E(\pi)N(d_{1g2}) - \pi^*_{g2}N(d_{2g2})\right]}$$

minus

$$\overset{T=1,\sigma=0.25,\pi^*_{g1}=0.025,r=0.05}{-e^{-r(T-t)}\left[E(\pi)N'(d_{1g1})\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[\pi^*_{g1}N(-d_{2g1}) - E(\pi)N(-d_{1g1})\right]}$$

The portfolio theta can be interpreted as meaning that, when a small amount of time (measured in years) passes, the option portfolio's value changes by the computed theta times that amount.[302]

To determine the effect of time on the payoff from the collar, the long call and short put thetas are aggregated by subtracting the latter from the former. Just as for the call, the greatest impact that a change in maturity has on the put's payoff, occurs close to the exercise. The portfolio theta as mapped out by figure 3.5b is obtained by subtracting the theta given by (3.16) from (3.13) for the range of expected inflation 0.5% to 7%. The parameter values are the same as those given in Table 2.1. It is possible to identify that the greatest impact time imparts to the value of the option portfolio arises when expected inflation is close to the upper and lower thresholds of tolerance. This result is apparent in both figure 3.5b and in figure 3.5a.

Figure 3.5a traces out in a similar fashion to figure 3.2 the time value of the collar. The former however incorporates the effect of maturity on the collar, as described in figure 3.2. It is observable that as the time horizon increases the dynamics as described by

[301] $N(-d_{2g1})$ tends to be slightly greater than $N(-d_{1g1})$ over the inflation tolerance band.
[302] See Figure 3.5b.

combining (3.13) and (3.16) are borne out. As the maturity or time horizon increases the value of the collar tends to increase for the call position.[303] The same is true for the short put in absolute terms. The overall effect of increasing maturity then is to make the time value of the collar increasingly linear. One way of confirming this, is to observe the effect on the slope of the collar (the collar's delta), by successively changing the time horizon for targeting expected inflation.

Figure 3.6 illustrates the effect of the change in maturity on the delta over the range of expected inflation 0.5 – 7.0 %. Four delta curves associated with varying time periods for preserving price stability are considered:

$$\frac{\partial\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=0.5,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1.5,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=2,\sigma=0.25}}{\partial E(\pi)}$$

The parameter values as given by Table 2.1 apply here with only the time period being adjusted. Note that as time to expiration increases; delta generally falls as expected inflation moves outside the tolerance band. The reverse occurs as expected inflation moves inside the bands.[304] As expiration increases there is an increased likelihood that the out-of-the-money option will become in-the-money and vice versa. From a central banker's point of view this is equivalent to saying that as the time horizon for targeting inflation increases, the range of possible future inflation paths expands implying that monetary policy can not be so precisely path dependent. The delta curve provides a measure of policy activism. As the time horizon increases, there appears to be a tightening of the delta curve. As the range of delta values contracts for longer expirations, this implies that the policy rule becomes more linear. It is observable that the deltas take on a smaller range of values as the time horizon extends. This implies that there is much less variation in the slope. From the perspective of understanding monetary policy, path dependency declines as the horizon for targeting inflation increases. Figure 3.7 provides an interpolated view of the effect of the time horizon on policy activism. Seven time horizons are considered ranging from 0.5 to 3.5 years. The

[303] For the higher values of expected inflation, this is less true. An intuitive explanation relates to the fact that by increasing the time horizon more outcomes are possible, including expected inflation falling below the threshold level. The effect of this is to slightly diminish the value of the call.
[304] For the parameter values given.

delta curves are calculated in an identical manner to Figure 3.6 and the parameter values are the same as those used in Table 2.1.

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=0.5,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1.5,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=2,\sigma=0.25}}{\partial E(\pi)},$$

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=2.5,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=3,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=3.5,\sigma=0.25}}{\partial E(\pi)}$$

The collar thresholds of the upper and lower strikes are 2.5% and 4.5% respectively. Figure 3.7 illustrates that as the time horizon increases the delta curves tend to flatten. Policy activism is less defined by whether expected inflation is inside or outside the bands as expiration increases.

Modelling the interest rate decision with option theory is useful in that it provides a reasonably simple way of understanding how uncertainty influences policymaking. One insight here is that the potential nonlinearities that exist in interest rate determination can be varyingly moderated when uncertainty impinges on the committee's deliberations. Here, the context was defined in terms of timing, but using portfolio option theory this can be easily extended to investigate how the volatility of the underlying inflation rate alters policymaking. This portrayal of the decision process seems to square with the view previously expressed by Chairman Greenspan that ultimately the hypothetical choice between policy A or policy B is inextricably linked to what we do not know as much as to what we think we know. This consideration may provide an explanation why the straightforward Taylor Rule represents a robust work-horse for interest rate analysis, in that much of the time the nonlinear rate setter faced by high levels of risk adopts a linear rule, albeit reverse-engineered. This may also explain how central bankers, despite differences in stated policy strategy, much of the time can respond in a similar fashion to macroeconomic events.[305]

3.3.5 Some caveats to the Brainard conservation principle

Brainard (1967) found that uncertainty about the effect of the policy instrument leads the central banker to apply the instrument less aggressively. The dynamic structure of

[305] This can apply whether policy makers are hawks, doves, opportunists, zone targeters or point targeters.

the economy and related parameter uncertainties are understood generally to deter decisive rate setting initiatives. This restrained approach to implementing monetary policy permits the central banker to take stock and accords with how policy makers are seen to make rate changes. Orphanides (2003) makes the point that a cautious central banker should acknowledge that the quality of information is noisy and should as a result avoid overreaction. Naïve policy that fails to take this on board may become itself a source of instability. Parameter uncertainty can decrease the monetary policy response and thus rationalises a smoother path for interest rate setting than in conditions of certainty. Clarida, Gali and Gertler (1999) illustrated that parameter uncertainty may diminish the response of monetary policy to economic shocks and consequently interest rate adjustments are smaller.[306] The portfolio option framework suggests that a refinement to this result is possible. Policy activism can both increase and decrease as uncertainty measured by maturity and volatility increase. Portfolio option theory accommodates a caveat to the traditional type of conservative response and spells out the circumstances by which policy can become pre-emptive, see also Söderström (2002) who maintained that uncertainty regarding structural parameters does not necessarily lead to a more cautious monetary policy.

Risks to hitting a target can stem from a number of sources. An obvious measure of uncertainty would be the volatility of the underlying inflation. If the standard deviation of expected inflation rises, forecasting price increases becomes ever more tenuous. That is, increased volatility would tend to increase the probability that inflation would breach the thresholds when inflation currently lies inside the thresholds. This type of response would also seem consistent with the effects of increasing the time frame as measured by, T.[307] The tightening behaviour of the delta curves in Figure 3.6 reflected how path dependency tended to diminish as uncertainty with time grew. Over the range of expected inflation 0.5% to 7%, delta values tend to contract into a narrower corridor producing a smaller gap between the minimum and maximum observations. So the effect of uncertainty is dual edged, that is the delta, (a proposed measure of policy activism), increased and decreased as the targeting horizon became longer. Applying portfolio option theory to the policy rate decision can refine the received wisdom.

[306] *Result 11*. CGG (1999).
[307] See Figures 1.3a – 1.3g. It is clear that as volatility increases and maturity increases, terminal inflation values become more dispersed.

The resultant retreat to linearity then can bring about both a more gradualist policy response and a more pre-emptive type policy response. A risk management perspective would suggest that a zone targeting committee, part of the time at least, can behave in a deliberative manner despite having a nonlinear rhetoric. This may explain why it is difficult, at times, to detect nonlinearity in the policy rule.[308] Uncertainty is generally considered to warrant additional caution on the part of central bankers. This is consistent with Brainard (1967). Portfolio option theory would suggest that the nature of the uncertainty confronting central bankers will also influence the timing and pattern of rate decisions. The FOMC may undertake a relatively more aggressive strategy when the risk of effecting too small a change may exceed the risk of a larger intervention. A similar view was elaborated by Bernanke (2004):

> 'The interaction of uncertainty and concerns about undershooting may well have affected Fed policy during the easing cycle that began in 2001. During that cycle, the FOMC faced a worrisome trend of disinflation, a trend that if left unchecked might have brought the economy close to the zone of falling prices, or deflation. The FOMC had two options during that episode: gradual easing, which some observers advocated as a way of saving the remaining "interest rate ammunition"; or a more pre-emptive approach, to try to nip in the bud any further decline of inflation toward the deflation boundary. In this particular episode, the risk of doing too little appeared to exceed the risk of doing too much, and the FOMC undertook a relatively aggressive strategy of rate cuts, as I mentioned in the introduction. Similar considerations presumably played a role during the 1994-95 tightening cycle, when concerns that inflation might rise significantly induced a relatively more rapid tightening.' (Remarks made by Governor Ben S. Bernanke, At an economics luncheon co-sponsored by the Federal Reserve Bank of San Francisco (Seattle Branch) and the University of Washington, Seattle, Washington. May 20, 2004)

The tension between acting decisively and waiting-to-see is finely balanced. Refinements to the Brainard conservation principle seem in line with the comments of central bankers. Conventional analysis suggests that uncertainty results in a more subdued policy response. Orphanides (2003) and Rudebusch (2000) for example, argue that noisy information induces a more cautious reaction. Onatski and Stock (2002) alternatively argue that the policy response in an uncertain world should be more

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[308] In this regard, uncertainty is best not thought of as being uni-dimensional. The effects of uncertainty are multi-faceted. Additional complications relate to the interaction between timing, volatility, and the bandwidth. The respective inflation targets are conceivably influenced by all other factors. If uncertainty transmits to shifting the respective tolerance levels, gradualist and pre-emptive regimes are equally feasible. If uncertainty sharpens the dove-hawk divisions within the committee, then a more path dependent policy could arise. Portfolio option theory as developed here does not spell out how wide these thresholds can go or how frequently they diverge.

aggressive. Analogously, Söderström (2002) has maintained that parameter uncertainty accompanied by persistence in inflation should precipitate a more aggressive policy stance. If central bankers are concerned that inflation may be hard to keep in check once it has breached a given threshold, a more aggressive response to containing inflation could be adopted.

The measures of theta and vega, time valuations and delta all illustrate how monetary policy may adjust to take account of the changing time horizon for targeting inflation and of inflation volatility. The ostensibly nonlinear policymaker can assume a more linear type reaction function when uncertainty is augmented by timing or volatility. Casting the collar as a proxy for inflation in the policy rule then implies that the rate setting environment is dynamic. At any one time, policy is best described by the degree to which it is deliberative or opportunistic and one is seen as the limiting case of the other. This implies the Taylor Rule can represent a convenient default policy rule and can be adopted to take account of issues that are more generally regarded as being important in a risk management framework. The gulf that sometimes exists between what central bankers say and do, may in part, be explained by a risk management paradigm. The Proposition below develops this theme further in applying portfolio option theory to show the likely effects of an increase in the annual standard deviation of inflation return on the policy rule.

3.3.6 The effects of uncertainty generated by inflation volatility

Proposition 3.3: *As inflation volatility increases the monetary policy response becomes more linear and less path dependent.*

If the time value of the call option is differentiated with respect to the volatility of inflation it can be shown that Vega is given by:

$$\frac{\partial c_{g2}}{\partial \sigma} = e^{-r(T\text{-}t)}\left[E(\pi)N'\left(d_{1g2}\right)\frac{\partial d_{1g2}}{\partial \sigma} - \pi^{*}_{g2}N'\left(d_{2g2}\right)\frac{\partial d_{2g2}}{\partial \sigma}\right] \qquad (3.17)$$

From Appendix C.3, (3.17) can be factored to give:

$$\frac{\partial c_{g2}}{\partial \sigma} = e^{-r(T\text{-}t)}E(\pi)N'\left(d_{1g2}\right)\left[\frac{\partial d_{1g2}}{\partial \sigma} - \frac{\partial d_{2g2}}{\partial \sigma}\right] \qquad (3.18)$$

The term in square brackets is equal to the square root of the maturity on the collar so (3.18) becomes:

$$\frac{\partial c_{g2}}{\partial \sigma} = e^{-r(T\text{-}t)} E(\pi) N'\left(d_{1g2}\right)\sqrt{\mathrm{T - t}} \qquad (3.19)$$

Using put-call parity, it is found that the Vega of the put is similarly equal to:

$$\frac{\partial p_{g1}}{\partial \sigma} = e^{-r(T\text{-}t)} E(\pi) N'\left(d_{1g1}\right)\sqrt{\mathrm{T - t}} \qquad (3.20)$$

The vega of the portfolio can be obtained by taking the difference:

$$\frac{\partial c_{g2}}{\partial \sigma} - \frac{\partial p_{g1}}{\partial \sigma} = e^{-r(T\text{-}t)} E(\pi)\sqrt{\mathrm{T - t}}\left[N'\left(d_{1g2}\right) - N'\left(d_{1g1}\right)\right]$$

The vega of a portfolio is the rate of change of the value of the portfolio with respect to the volatility of the underlying expected inflation. Vega can be interpreted as meaning that, when the volatility (measured in decimal form) increases by a small amount, the option portfolio's value changes by the computed vega times that amount. In Figure 3.8b, (where the parameter values are consistent with those of Table 2.1), the vega:

$$\frac{\partial c_{g2}}{\partial \sigma} - \frac{\partial p_{g1}}{\partial \sigma} = e^{-r(T\text{-}t)} E(\pi)\sqrt{\mathrm{T - t}}\left[N'\left(d_{1g2}\right) - N'\left(d_{1g1}\right)\right]^{T=1,\, \sigma=0.25,\, \pi^*_{g1}=0.025,\, \pi^*_{g2}=0.045,\, r=0.05}$$

is found to be most elevated around the inflation thresholds π^*_{g1} and π^*_{g2}. So policy becomes more sensitive to an acceleration/deceleration in inflation the closer the target variable gets to the upper and lower strikes.

From Figure 3.8a, it is found that as volatility increases, the time value of the put and the call in absolute terms both increase. This is somewhat more clear-cut than the previous case where the time values can be differently affected by time to expiration. The value of the short position in the put falls as volatility increases (i.e. it becomes more negative). The writer of the instrument gains as volatility falls. The time value of a series of collars for different levels of volatility is mapped out in Figure 3.8a. The portfolio's worth is computed in the same manner, as given in Table 2.1, however the volatility term is adjusted from 0.25 to 0.5 and then to 0.75. Figure 3.8a illustrates that as volatility increases the time value of the collar becomes more linear as a function of expected inflation and this is consistent with (3.19) and (3.20). The reaction of central bankers to an increase in volatility is to increase the magnitude of the response over the region defined by the inflation bands. The impact of this change may be to make opportunistic policy largely indistinguishable from the deliberative policy implementation. Even while the forecasted mean inflation rate remains within the bands, if its' variance increases, the chances of breaching the band also increases. If expected inflation is marginally below the upper band and then the volatility of the underlying increases, this feasibly drives the policy maker to buy insurance.

In Figures 3.9 and 3.10, the delta curves take on a smaller range of values as volatility increases in magnitude. As the annualised forecast of volatility increases the range of delta values contracts. This compression of the delta range implies that policy assumes a more linear form. Consequently, there is much less variation in the slope of the collar. From the perspective of the central banker implementing monetary policy, path dependency declines as the level of volatility rises. Figure 3.9 illustrates the effects of changing the volatility on the delta curves. The delta is calculated in the same way as Table 3.1, but with three different volatilities used so that:

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.5}}{\partial E(\pi)}\text{ and }\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.75}}{\partial E(\pi)}$$

are considered. The exercise values are also the same as the initial strikes considered in Table 2.1. The delta of the portfolio is computed by taking the difference of the delta of the call and the delta of the put. The volatility level ranging from 0.25 to 0.35 is representative of magnitudes estimable during the Greenspan FOMC. An *ex ante* recursively forecasted GARCH(1,1) was used previously.[309] Figure 3.9 shows that as volatility increases there is a flattening of the values, delta takes on. The delta range tends to contract over the given set of expected inflation values. This is consistent with stating the policy rule becomes increasingly linear under conditions of uncertainty. In this instance, uncertainty is measured by the standard deviation of inflation return. It also furnishes a significant rationale to advance the Taylor Rule as an important limiting case. Figure 3.10 interpolates the delta curves over a wider range of volatility values ranging from 0.1 to 0.8 in intervals of 0.1 so that:

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.1}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.2}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.3}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.4}}{\partial E(\pi)}$$

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.5}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.6}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.7}}{\partial E(\pi)},\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.8}}{\partial E(\pi)}$$

[309] Details are presented in Appendix A.2.3, chapter 2.

The parameter values are otherwise the same as those given by Table 2.1.[310] Again it is clear that as volatility increases, the delta curves compress into a narrower range and implying that policy becomes increasingly linear. In the next section, an exponential smooth transition autoregressive (ESTAR) model is employed to investigate the extent to which policy is mean reverting to the Taylor Rule.

3.4. Nonlinearity – some empirical evidence using ESTAR

In the introduction to this chapter, Governor Gramlich's appraisal of *Monetary Policy Rules* is noteworthy not so much because the Taylor Rule is criticised for falling short but rather because there is an implicit acceptance that it is used as an important reference. The fact that it is '*quite a favorite among the academic economists*' and despite being looked at rather hard had '*not been that helpful*', reveals it to be not an incidental piece of information. The emphasis nevertheless seems to stress policy as being nonlinear. Both rhetoric and actions are compatible if policy is seen through the prism of migrating between different states. In this section, empirical evidence is provided to support the view that nonlinearity exists however the Taylor Rule constitutes an important limiting case. Teräsvirta (1994) outlined a parsimonious parametric time series model with nonlinear mean reversion commonly termed the smooth transition autoregressive (STAR) model. Granger and Teräsvirta (1993) developed STAR models so as to incorporate exponential transition functions. The resultant ESTAR model is particularly useful for investigating how an underlying variable can change between different patterns of behaviour. Here, an exponential smooth transition autoregressive function of order q, ESTAR(q), is developed to capture the effect of policy migrating between different states. The STAR family of models have a particular advantage in terms of examining gradual rather than discreet changes in regime. A TAR model would suggest that the passing through of a given threshold would cause an abrupt change.[311] Here an Exponential Smooth Transition Autoregressive model of order q ESTAR(q) is developed so as to capture the graduated effect of policy migrating between different states.

$$\varepsilon_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + \left[\alpha_0^* + \sum_{i=1}^{q} \alpha_i^* \varepsilon_{t-i} \right] \left[1 - \exp\left\{ -\gamma^* \left(\varepsilon_{t-g} - c \right)^2 \right\} \right] + u_t \quad (3.21)$$

[310] Only σ is adjusted.
[311] The ESTAR model can be viewed as a TAR with an infinite number of regimes.

γ^* constitutes a measure of mean reversion and is assumed to be greater than zero. In effect, mean reversion increases as the scale of deviation from equilibrium or neutrality increases. The u_t is a stochastic disturbance term. ε_t represents the difference between the actual Fed Funds rate r_t and the rate prescribed by the Taylor Rule r_t^{Taylor} over the Greenspan chairmanship.

$$\varepsilon_t = r_t - \left[\left(r^* + \pi_t \right) + \omega_1 \left(\pi_t - \pi^* \right) + \omega_2 \left(y_t \right) \right] \qquad (3.22)$$

where

$$r_t^{Taylor} = \left(r^* + \pi_t \right) + \omega_1 \left(\pi_t - \pi^* \right) + \omega_2 \left(y_t \right) \qquad (3.23)$$

the Taylor Rule rate is calculated using the same notation and methodology as before incorporating the recursive growth gap for y_t, outlined previously in chapter 2.[312] [313] Otherwise, the same approach as prescribed by Taylor (1993) is used to fix the policy rate. Figure 3.11 provides an overview of ε_t. Negative values correspond to periods of policy accommodation most discernible around 1992 and 2003. Positive values constitute periods of policy tightening.

A smooth regime change would appear more appropriate than an abrupt innovation in regime. The autoregressive process for the residual of order q is augmented by an exponential transition function:

$$F\left(\varepsilon_{t-g} \right) = 1 - \exp\left[-\gamma^* \left(\varepsilon_{t-g} - c^* \right)^2 \right] \qquad (3.24)$$

which assumes a U-shaped form when estimated and is bounded between unity and zero. c^* denotes a threshold parameter and sets out the band for the policy deviation term ε such that whenever the difference between the actual Fed Funds rate and the Taylor Rule rate exceeds c^* in both positive and negative directions, the transition function will assume values in the neighbourhood of the outer regime. Whenever the deviation is within the band the transition function will assume values in the neighbourhood of the inner regime. The smoothness parameter is denoted by γ^* and this measures the speed of transition between extreme regimes. When F assumes the

[312] Although, the data here is calculated in percentage form, as opposed to the decimal form used before in chapter 2.

[313] The reported standard errors for the ESTAR model are distorted by use of generated regressors. In this instance, however the growth gap retains much of the original variability of the primitive growth series. Figure 2.8 illustrates that the generated growth gap mirrors quite closely patterns of variation in the original growth.

value of zero, the autoregressive function remains linear so the AR(q) can be described in the form:

$$\varepsilon_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + u_t \tag{3.25}$$

The inner (middle) regime occurs when $F(\varepsilon_{t-g}) = 0$, thus $\varepsilon_{t-g} = c^*$ implying that the exponential term inside the brackets goes to unity. The outer regime conversely is achieved when $F(\varepsilon_{t-g}) = 1$. As the limit:

$$\lim \left| \varepsilon_{t-g} - c^* \right| \to \infty \tag{3.26}$$

the negative exponential term goes to zero implying that the weight of unity is applied to the transition function. The AR(q) process is then augmented to become:

$$\varepsilon_t = (\alpha_0 + \alpha_0^*) + \sum_{i=1}^{q} (\alpha_i + \alpha_i^*) \varepsilon_{t-i} + u_t \tag{3.27}$$

Weights between unity and zero can be attributed to the transition function so that the process is dynamic. Global stability necessitates that:

$$\sum_{i=1}^{q} (\alpha_i + \alpha_i^*) < 1 \tag{3.28}$$

Should

$$\sum_{i=1}^{q} (\alpha_i + \alpha_i^*) < \sum_{i=1}^{q} (\alpha_i) \tag{3.29}$$

then this implies that the level of mean reversion increases as the deviation from the Taylor Rule rate grows. The intuition is that as the path of interest rate setting deviates from the Taylor Rule rate, the error terms tend to ‘correct’. In effect, large errors tend to be reversed supplying a testable hypothesis that policy migrates between varying states. Mean reversion would suggest that large deviations from the Taylor Rule produce increasing pressure to return to neutrality. If policy deviates from the Taylor rule there exists a counteracting tendency to ultimately recoil towards deliberative policy. This is consistent with the migration hypothesis as suggested by zone targeting and portfolio option theory. If the estimated value of

$$\sum_{i=1}^{q} (\alpha_i^*) < 0 \tag{3.30}$$

then policy would appear to be anchored by a linear policy rule. The assumption that key variables incorporated into the reaction function are stationary has formed the basis of much research in recent years, often assumed on the grounds of plausibility, even though Augmented Dickey-Fuller (ADF) tests on ψ of the type:

$$\Delta\varepsilon_t = \psi\varepsilon_{t-1} + \sum_{t=1}^{p}\alpha_i\Delta\varepsilon_{t-i} + u_t$$

do not always strictly bear this out.[314] Using varying lags it is found here that ADF tests do not consistently reject the null of nonstationarity for ε_t. The first-order autocorrelation values for ε_t are close to one suggesting the presence of a unit root. Some evidence is offered here to support the view that the difference between the Taylor Rule Rate and the actual Fed Funds rate is mean reverting when using ESTAR(q). This finding would suggest that although individual variables such as inflation and the short-term rate are frequently found to be *I*(*1*) they may be thought to be nonlinearly cointegrated should (3.28) and (3.29) hold. Failure to reject the unit root hypothesis using a standard linear ADF does not, however, necessarily imply that ε_t is not mean reverting, if one holds open the possibility that the variables in the Taylor Rule are nonlinearly cointegrated. From an econometric perspective, this may lessen the charge that reaction functions regress one random walk against another. The smooth transition approach to modelling ε_t is consistent with the hypothesis that policy responds nonlinearly to changes in inflation.

Teräsvirta (1994) advocates testing linearity against nonlinearity by initially outlining the appropriate magnitude of q. To apply the nonlinear structure, it is first necessary to test the relevance of nonlinearity. Granger and Teräsvirta (1993) and Teräsvirta (1994) outlined the following artificial regression:

$$\varepsilon_t = \beta_{00} + \sum_{i=1}^{q}\left(\beta_{1i}\varepsilon_{t-i} + \beta_{2i}\varepsilon_{t-i}\varepsilon_{t-g} + \beta_{3i}\varepsilon_{t-i}\varepsilon_{t-g}^2 + \beta_{4i}\varepsilon_{t-i}\varepsilon_{t-g}^3\right) + v_t \qquad (3.31)$$

to test the null hypothesis of linearity against an alternative hypothesis of nonlinear adjustment. A simple F-test can be used to determine the plausibility of nonlinearity:

$$H_{01} : \beta_{2i} = \beta_{3i} = \beta_{4i} = 0 \qquad (3.32)$$

[314] Clarida, Gali and Gertler (1998) make the point that 'our econometric approach relies on the assumption that, within our short samples, short-term interest rates and inflation are *I*(0). Standard Dickey Fuller tests of the null that inflation in G3 countries is *I*(1) is rejected in favour of the alternative of stationarity. Also for Germany, we reject that the short-term interest rate is *I*(1). For the US and Japan there is less evidence against the null that short term interest rates are *I*(1). However, we know that the Dickey Fuller test has low power against the alternative of stationarity for the short sample we are studying.' (Footnote 9, p. 1039).

for

$i = 1,........, q$

against the alternative hypothesis that H_{01} is not valid. The relevant Wald Statistic is denoted by W_1. The third order terms in the artificial regression disappear if the transition function is exponential in nature. The rationale advanced broadly relates to the view that the exponential transition function is U-shaped with regard to ε_{t-g}, effectively, that a quadratic as opposed to a cubic relationship is deemed more appropriate. If the Taylor Rule rate over the sample period averaged close to the Fed Funds rate, it would be expected that the ESTAR model would satisfy $\alpha_0^* = c^* = 0$, this implies that $\beta_{2i} = 0$. These observations suggest the following series of tests:

$$H_{04} : \beta_{4i} = 0,\ i = 1,......q \qquad (3.33)$$

$$H_{03} : \beta_{3i} = 0 \mid \beta_{4i} = 0,\ i = 1,......q \qquad (3.34)$$

$$H_{02} : \beta_{2i} = 0 \mid \beta_{4i} = 0\ \ i = 1,......q \qquad (3.35)$$

where "$\mid$" denotes conditional on. Accordingly, the corresponding relevant Wald Statistics are given by W_4, W_3 and W_2. It is expected that H_{04} and H_{02} are not rejected but H_{03} is rejected. This helps to assess the appropriateness of ESTAR. To check the pertinent value of *g*, Granger and Teräsvirta (1993) and Teräsvirta (1994) recommend investigating the values of *g* using the Wald Statistic for the null H_{01} so that g = 1,2….G are considered.

3.5. Empirical Results

The ACF of the dependent variable shows significant correlations up to four lags. The artificial regression is used to check linearity with q being set to four. Table 3.2 provides some discernible evidence of nonlinearity from the artificial regression. The delay parameter *g* was selected on the basis of the most appropriate *p*-value attained in table 3.2. W_1 rejects linearity below the 5% level for *g* when set equal to 6. The tests for W_4, W_3 and W_2 are consistent with an ESTAR(4) model. The test statistics illustrates that $\beta_{4i} = 0$ cannot be rejected at the 5% level. This is consistent with the observations noted from portfolio option theory. The ESTAR(4) model is first estimated. The least statistically significant regressors are dropped and a parsimonious model is then recalculated providing the following parameter estimations:

$$\varepsilon_t = -0.688 + 0.226\varepsilon_{t-1} + 0.779\varepsilon_{t-2} + [0.713 + 1.053\varepsilon_{t-1} - 1.182\varepsilon_{t-2}]$$
$$(0.336) \quad (0.192)) \qquad (0.273) \qquad (0.377) \quad (0.201) \qquad (0.215)$$
$$[\text{-}2.049] \quad [1.176] \qquad [2.859] \qquad [1.891] \quad [5.225] \qquad [\text{-}5.509]$$

$$*\left[1 - \exp\left\{-\left(8.804/\sigma_\varepsilon^2\right)\left(\varepsilon_{t-6} - 0.461\right)^2\right\}\right] + u_t \tag{3.36}$$
$$(5.004) \qquad (0.034)$$
$$[1.759] \qquad [13.493]$$

$\overline{R}^2 = 0.79$	$\text{Q(0-24)} = 34.87\{0.07\}$	$\sigma_\varepsilon^2 = 1.814$
$\text{LR(4)} = 16.58\ \{0.0008\}$	$\text{SSE} = 22.55$	$\text{VR} = 0.77$

The figures in parentheses are standard errors. The figures in brackets are the corresponding t-ratios. The adjusted co-efficient of determination, $\overline{R}^2$, suggests the model has reasonable explanatory power. The basic linear AR(4) model has a lower $\overline{R}^2$ of 0.74. The linear restriction implies:

$$\sum_{i=1}^{q}\left(\alpha_0^* + \alpha_i^*\right) = 0$$

The sum of squared residuals, SSE, is 22.25 compared to 29.34 for the restricted AR(4) model. VR is the ratio of the residual variance from the estimated ESTAR model to the residual variance from alternative linear configuration. Since it is less than unity, it suggests that the nonlinear construction represents an improvement. The Q statistic, reported in braces, denotes the marginal significance. The null hypothesis that all the first 24 autocorrelation coefficients are zero cannot be rejected. For the linear alternative equivalent AR(4) model the marginal significance was 0.0009 implying a stronger presence of autocorrelation in the restricted model. The individual t-statistics are significant within the 10% level, other than the parameter value for ε_{t-1}. The γ^* term has a p-value close to 7%. Importantly this term is positive implying that for a large deviation that is for a large ε, the exponential transition function goes to unity. In the outer regime, the full force of the exponential transition function is applied to the autoregressive process.

LR(4) denotes a likelihood ratio statistic for the four restrictions implicit in the estimated AR(4) as opposed to the parsimonious ESTAR(4) model given by (3.36). Both provide tests for the null hypothesis given by the linear AR(4) model. In each case, it is found that using the chi-square distribution the restricted model can be rejected at the 1% level. The scatter graph that maps the estimated transition function against $(\varepsilon_{t-6} - 0.461)$, is illustrated in Figure 3.12. The transition function is estimated for values of ε_{t-6} that lie between positive and negative 2. For large deviations there is adjustment back towards a long run level. The plot tends to confirm the view that the degree of mean reversion is gradual, given the magnitude of γ^*, supporting the view that switching is not instantaneous. Global stability is achieved since:

$$\sum_{i=1}^{q}\left(\alpha_i + \alpha_i^*\right) < 1$$

obtains. The level of mean reversion increases as the deviation from the Taylor Rule rate grows. The policy rule may be thought to be consistent with the Fed Funds rate and Taylor Rule rate being nonlinearly co-integrated. This finding is consistent with the migration hypothesis that policy incorporates varying responses. The residual from the Taylor Rule reverts in a fashion consistent with ESTAR. This also supports the view that monetary policy is nonlinear. By using option theory and the propositions above however, it is possible to illustrate how the nonlinear stance assumed by policy makers can be made less obvious when other conditions are met.

3.6. Conclusion

Several ways whereby monetary policy could be nonlinear and linear were investigated in this chapter. In the United States, monetary policy is formulated where both the views of individual members of the FOMC and political agencies are taken on board. This implies that the heterogeneous beliefs influence policy via committee and institutional dynamics. Tactical considerations in dealing with the legislature seem also to suggest that policy is nonlinear in conception, plausibly because as chairman Greenspan put it: that there is still a short term Phillips curve trade-off, and to ignore this, is a misreading of how the political system works. During the 1990s, the opportunistic strategy may have represented an attempt to provide policy makers with a means to explain how policy was implemented nonlinearly given that monetary aggregates were being slowly de-emphasised. More recently, the Federal Reserve has explained implementation in terms of balancing risks. In this chapter, it was explained how policy, despite the apparent rhetoric, can often behave in a linear manner not unlike that conventionally suggested by the Taylor Rule.

Portfolio option theory was used in this chapter to illustrate how agreement between the central bank and political institutions leads to a more linear policy response. The level of agreement was measured by bandwidth within a collar construction. If policy makers were completely agreed and the upper and lower tolerances converged then policy can be seen to be linear. The ability of the collar model to adopt different stances is appealing given that monetary policy at any time might experience varying levels of nonlinearity. It may also explain why policy makers can paradoxically talk nonlinearly but walk linearly.

This chapter also exploited portfolio option theory to examine the effects of volatility and timing on policy rate sensitivities. The traditional measures of delta, theta and vega were used to examine the likely effects on the rate decision as parameter values changed. Portfolio option theory can provide a number of insights that help explain the gulf that exists between what central bankers say and what central bankers appear to do. Despite the prevalence of views that seem to support nonlinearity, most empirical investigation starts from the premise that policy is linear. This anomaly is consistent with the opportunistic approach, imparting varying levels of linearity. From portfolio option theory, the policy rule can be seen as continually migrating across different states. So a policy rule, as defined by an option collar, has some advantage in terms of flexibility, by varying over different forms that are not exclusively linear or nonlinear during the life of the policy rule.

An ESTAR(q) model provides evidence that policy is dynamic and is capable of varying responses. By using the difference between the actual Fed Funds rate and the Taylor Rule, one can investigate the probability of stationarity. Mean reversion appears to increase, as the deviation from the Taylor Rule rate increases. Nonlinear mean reversion provides some evidence to support the migration hypothesis. It also supports the hypothesis that monetary policy can assume many forms including the Taylor Rule. This representation of policy is useful because it develops the Taylor Rule as being an important reference for monetary policy, yet also permits monetary policy to be conceived of, in nonlinear terms. This may help reconcile the gulf between what policy makers often say they implement, and what is seen to be implemented.

Appendix A.3: When policy makers agree, the policy response to expected inflation is a linear function of expected inflation

If policy makers agree to a common target for inflation this is equivalent to setting:

$$\pi^*_{g1} = \pi^*_{g2} \qquad \text{(a.3.1)}$$

Thus the collar portfolio:

$$c_{g2} - p_{g1} = c_{g1} - p_{g1} \qquad \text{(a.3.2)}$$

or

$$c_{g2} - p_{g1} = c_{g2} - p_{g2} \qquad \text{(a.3.3)}$$

By taking the difference between a call and put where the notation remains the same as before, the portfolio can be written using the former case:

$$c_{g1} - p_{g1} = e^{-rT}\left\{\left[E(\pi)N\left(d_{1g1}\right) - \pi^*_{g1}N\left(d_{2g1}\right)\right] - \left[\pi^*_{g1}N\left(-d_{2g1}\right) - E(\pi)N\left(-d_{1g1}\right)\right]\right\} \qquad \text{(a.3.4)}$$

If the risk neutral probabilities are re-expressed the following emerges:

$$c_{g1} - p_{g1} = e^{-rT}\left\{\left[E(\pi)N\left(d_{1g1}\right) - \pi^*_{g1}N\left(d_{2g1}\right)\right] - \left[\pi^*_{g1}\left(1 - N\left(d_{2g1}\right)\right) - E(\pi)\left(1 - N\left(d_{1g1}\right)\right)\right]\right\} \qquad \text{(a.3.5)}$$

By multiplying the strike and expected inflation by the respective risk neutral probabilities in the second square brackets it can be shown that:

$$c_{g1} - p_{g1} = e^{-rT}\left\{\left[E(\pi)N\left(d_{1g1}\right) - \pi^*_{g1}N\left(d_{2g1}\right)\right] - \left[\left(\pi^*_{g1} - \pi^*_{g1}N\left(d_{2g1}\right)\right) - \left(E(\pi) - E(\pi)N\left(d_{1g1}\right)\right)\right]\right\} \qquad \text{(a.3.6)}$$

By collecting the terms and cancelling

$$c_{g1} - p_{g1} = e^{-rT}\left\{-\left[\left(\pi^*_{g1}\right) - \left(E(\pi)\right)\right]\right\} \qquad \text{(a.3.7)}$$

The time value of the portfolio collar can be illustrated to have a linear relationship with expected inflation:

$$c_{g1} - p_{g1} = \left[E(\pi) - \pi^*_{g1}\right]e^{-rT} \qquad \text{(a.3.8)}$$

Likewise:

$$c_{g2} - p_{g2} = \left[E(\pi) - \pi^*_{g2}\right]e^{-rT} \qquad \text{(a.3.9)}$$

Appendix B.3: Determining the delta of the rate-setting board

The delta of the collar can be used innovatively to deduce a measure for policy activism. The following appendices however appeal to a broadly understood and accepted body of knowledge used extensively in portfolio option theory. To obtain the portfolio's delta it is necessary to calculate the constituent deltas of the call and put. Policy differences occur between board members and this leads to an upper threshold and lower threshold that reflect the position of (g1) the hawk and (g2) the dove. Thus the call position:

$$c_{g2} = e^{-r(T\text{-}t)}\left[E(\pi)N\left(d_{1g2}\right) - \pi^*_{g2}N\left(d_{2g2}\right)\right] \qquad \text{(b.3.1)}$$

which when differentiated w.r.t. expected inflation gives:

$$\frac{\partial c_{g2}}{\partial E(\pi)} = e^{-r(T\text{-}t)}\left[N\left(d_{1g2}\right) + E(\pi)N'\left(d_{1g2}\right)\frac{\partial d_{1g2}}{\partial E(\pi)} - \pi^*_{g2}N'\left(d_{2g2}\right)\frac{\partial d_{2g2}}{\partial E(\pi)}\right]$$

(b.3.2)

since the following holds:

$$E(\pi)N'\left(d_{1g2}\right) = \pi^*_{g2}N'\left(d_{2g2}\right) \qquad \text{see Appendix C.3} \qquad \text{(b.3.3)}$$

and because $d_{2g2} = d_{1g2} - \sigma\sqrt{T-t}$ then

$$\frac{\partial d_{1g2}}{\partial E(\pi)} = \frac{\partial d_{2g2}}{\partial E(\pi)} \qquad \text{(b.3.4)}$$

The delta of the call can be written as:

$$\frac{\partial c_{g2}}{\partial E(\pi)} = e^{-r(T\text{-}t)}N\left(d_{1g2}\right) \qquad \text{(b.3.5)}$$

The other constituent of the collar is the short put. If the lower threshold is given as π^*_{g1} then:

$$p_{g1} = e^{-rT}\left[\pi^*_{g1}N\left(-d_{2g1}\right) - E(\pi)N\left(-d_{1g1}\right)\right] \qquad \text{(b.3.6)}$$

As before differentiating with respect to expected inflation gives:

$$\frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T\text{-}t)}\left[\pi^*_{g1}N'\left(-d_{2g1}\right)\frac{\partial\left(-d_{2g1}\right)}{dE(\pi)} - E(\pi)N'\left(-d_{1g1}\right)\frac{\partial\left(-d_{1g1}\right)}{\partial E(\pi)} - N\left(-d_{1g1}\right)\right]$$

(b.3.7)

Since the standard normal density function is an even function:

$$E(\pi)N'\left(-d_{1g1}\right) = \pi^*_{g1}N'\left(-d_{2g1}\right) \qquad \text{(b.3.8)}$$

and

$$\frac{-\partial d_{1g1}}{\partial E(\pi)} = \frac{-\partial d_{2g1}}{\partial E(\pi)} \qquad \text{(b.3.9)}$$

$$\Rightarrow \frac{\partial p_{g1}}{\partial E(\pi)} = -e^{-r(T\text{-}t)} N\left(-d_{1g1}\right) = e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right) - 1\right] \qquad \text{(b.3.10)}$$

The delta of the portfolio is obtained by taking the difference:

$$\frac{\partial c_{g2}}{\partial E(\pi)} - \frac{\partial p_{g1}}{\partial E(\pi)} = e^{-r(T\text{-}t)} N\left(d_{1g2}\right) - e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right) - 1\right] \qquad \text{(b.3.11)}$$

Appendix C.3

To show that (c3.1) holds:

$$E(\pi)N'\left(d_{1g2}\right)=\pi^{*}_{g2}N^{'}\left(d_{2g2}\right) \qquad \text{(c.3.1)}$$

where the standard normal density function:

$$N'\left(x\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

a standard approach adheres to the following construction:[315]

$$N'\left(d_{2g2}\right)=N'\left(d_{1g2}-\sigma\sqrt{T-t}\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{\left(d_{1g2}-\sigma\sqrt{T-t}\right)^2}{2}} \qquad \text{(c.3.2)}$$

$$N'\left(d_{2g2}\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{\left(d^2_{1g2}-2d_{1g2}\sigma\sqrt{T-t}+\sigma^2(T-t)\right)}{2}} \qquad \text{(c.3.3)}$$

where the usual applies

$$d_{1g2}=\frac{\ln\left(E(\pi)/\pi^{*}_{g2}\right)+\left(\sigma^2/2\right)\left(T-t\right)}{\sigma\sqrt{T-t}}$$

and

$$d_{2g2}=\frac{\ln\left(E(\pi)/\pi^{*}_{g2}\right)+\left(-\sigma^2/2\right)\left(T-t\right)}{\sigma\sqrt{T-t}}$$

(c3.3) can be re-expressed

$$N'\left(d_{2g2}\right)=N'\left(d_{1g2}\right).e^{\left(d_{1g2}\sigma\sqrt{T-t}-\sigma^2/2(T-t)\right)} \qquad \text{(c.3.4)}$$

Because the following must hold:

$$d_{1g2}\sigma\sqrt{T-t}=ln\left(E\left(\pi\right)/\pi^{*}_{g2}\right)+\sigma^2/2\left(T-t\right) \qquad \text{(c.3.5)}$$

(c3.4) can be re-expressed to give:

$$N'\left(d_{2g2}\right)=N'\left(d_{1g2}\right).e^{\left(ln\left(E(\pi)/\pi^{*}_{g2}\right)\right)}=N'\left(d_{1g2}\right).E\left(\pi\right)/\pi^{*}_{g2} \qquad \text{(c.3.6)}$$

$$\Rightarrow N'\left(d_{2g2}\right).\pi^{*}_{g2}=N'\left(d_{1g2}\right).E\left(\pi\right) \qquad \text{(c.3.7)}$$

or

$$E(\pi)N'\left(d_{1g2}\right)=\pi^{*}_{g2}N'\left(d_{2g2}\right) \qquad \text{(c.3.9)}$$

[315] Hull (2003) and Neftci (1996) provide a good overview the mathematical techniques required to elaborate option pricing and parameter sensitivities.

Expected Inflation	Delta of Collar 2.5% - 4.5%	Time Value of Collar 2.5% - 4.5%	Delta of Collar 3.5% - 3.5%	Time Value of Collar 3.5% - 3.5%
0.0050	0.95123	-0.01902	0.95123	-0.02854
0.0075	0.95123	-0.01665	0.95123	-0.02616
0.0100	0.95104	-0.01427	0.95123	-0.02378
0.0125	0.94737	-0.01189	0.95123	-0.02140
0.0150	0.92505	-0.00955	0.95123	-0.01902
0.0175	0.85955	-0.00731	0.95123	-0.01665
0.0200	0.74152	-0.00529	0.95123	-0.01427
0.0225	0.59034	-0.00363	0.95123	-0.01189
0.0250	0.44067	-0.00234	0.95123	-0.00951
0.0275	0.32234	-0.00140	0.95123	-0.00713
0.0300	0.25083	-0.00069	0.95123	-0.00476
0.0325	0.22807	-0.00010	0.95123	-0.00238
0.0350	0.24734	0.00049	0.95123	0.00000
0.0375	0.29788	0.00116	0.95123	0.00238
0.0400	0.36822	0.00199	0.95123	0.00476
0.0425	0.44806	0.00301	0.95123	0.00713
0.0450	0.52924	0.00423	0.95123	0.00951
0.0475	0.60602	0.00565	0.95123	0.01189
0.0500	0.67490	0.00726	0.95123	0.01427
0.0525	0.73419	0.00902	0.95123	0.01665
0.0550	0.78356	0.01092	0.95123	0.01902
0.0575	0.82357	0.01293	0.95123	0.02140
0.0600	0.85527	0.01503	0.95123	0.02378
0.0625	0.87989	0.01720	0.95123	0.02616
0.0650	0.89871	0.01942	0.95123	0.02854
0.0675	0.91289	0.02169	0.95123	0.03091
0.0700	0.92344	0.02399	0.95123	0.03329

Table 3.1: The Black Model is used to estimate the time values and deltas of two collars for each of the underlying expected inflation values. The two collars are identical except the strikes set on the short put and long call are different. The parameter values are given as 5% for the risk free rate, the time horizon is 1 year and annual Black volatility is 25%. The collar denoted 2.5% - 4.5% is given to mean the collar with these strikes on the respective positions. Likewise the collar given as 3.5% - 3.5% is calculated for where the strikes are equal. Note than when the strikes are equal the delta is constant.

	W1	W4	W3	W2
g = 2	0.5430	0.3502	0.7386	0.3550
g = 3	0.7663	0.8618	0.3423	0.6368
g = 4	0.0824	0.0463	0.6477	0.1430
g = 5	0.6578	0.9910	0.4973	0.1732
g = 6	0.0288	0.6043	0.0945	0.0105
g = 7	0.2132	0.1898	0.2183	0.4880
g = 8	0.6360	0.5965	0.6453	0.3244

Table 3.2: p-Values for the Linearity Tests. The Wald tests are based on q set equal to 4. The sample period runs from 1987:4 to 2003:4 using quarterly observations.

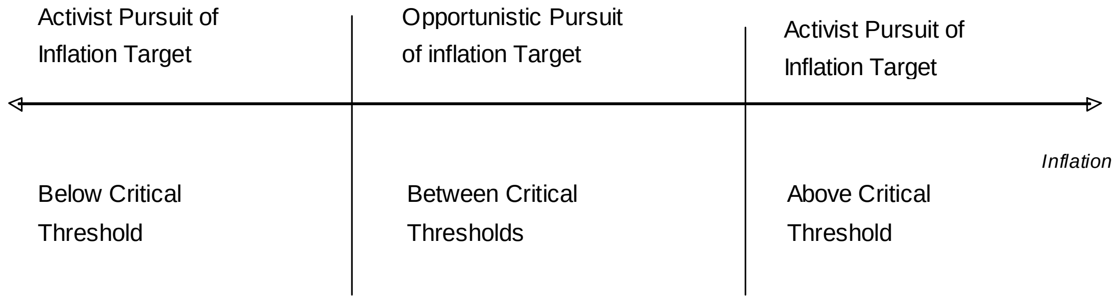

Figure 3.1

Figure 3.1 presents policy reactions for varying regions that define nonlinearity.

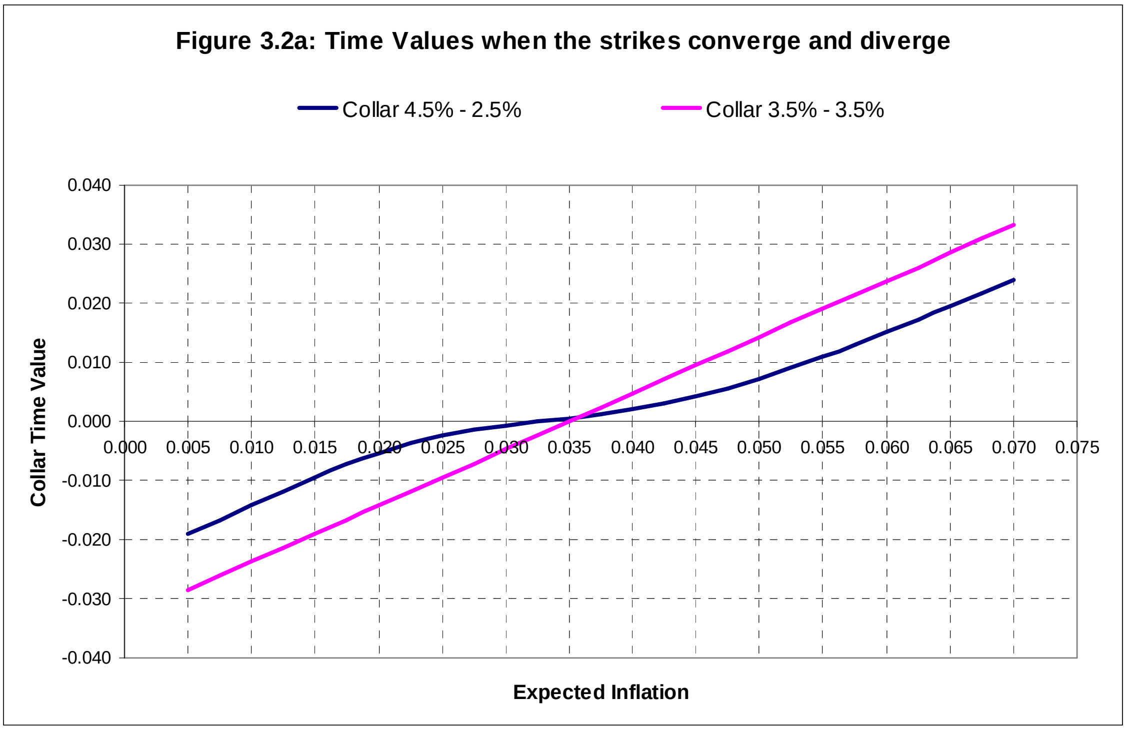

Figure 3.2a illustrates the time value payoff on a collar for varying levels of expected inflation ranging from 0.5% to 7%. The collar is composed of one long call and one short put. The standard Black model is used to determine the time value of the collar where the strikes are set at 2.5% for the short put and 4.5% for the long call:

$$\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}$$

The collars' time value is also estimated with the same strike set at 3.5%:

$$\left(c_{g2}^{0.035} - p_{g1}^{k=0.035}\right)^{T=1,\sigma=0.25}$$

Figure 3.2a demonstrates that if agreement can be achieved by the monetary policy committee as regarding the appropriate target the response to inflation becomes linear. The monetary policy response is seen to be linear when the target set by the doves and hawks converge.

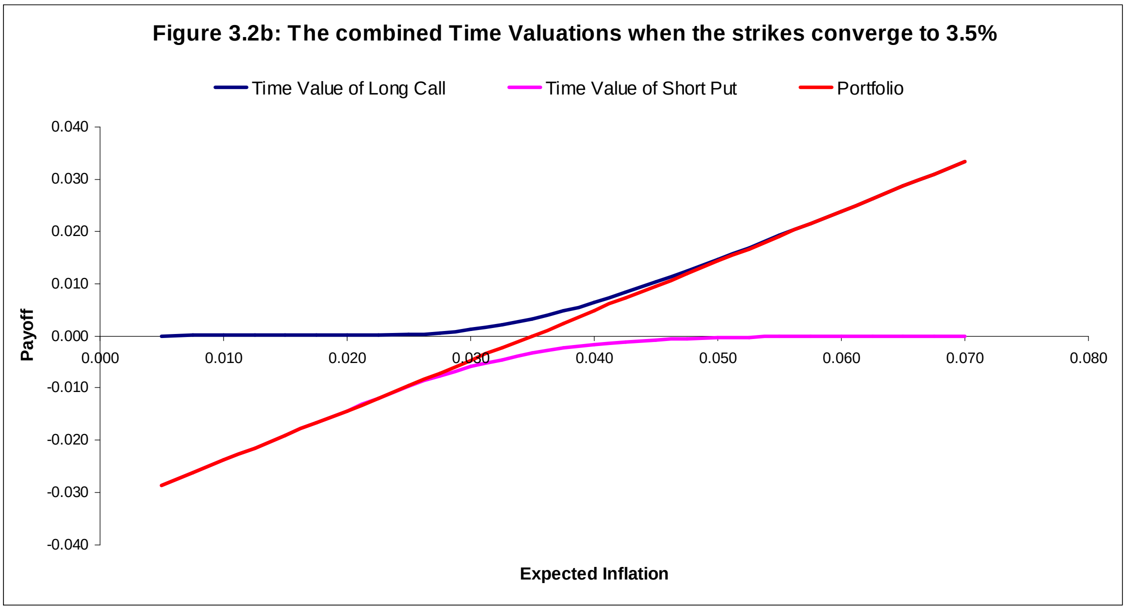

Figure 3.2b: shows that if the combined time values of a short put and long call are computed for a collar where the strikes are equal then a linear payoff is yielded by the portfolio. This representation is consistent with the result obtained in the Appendix A.4. Here the strike or inflation threshold is arbitrarily set at 3.5%. The other parameter values are the same as table 4.1. It is clear that once the strikes are set equal to each other, the combined payoff from the long call and short put moves proportionately with the underlying expected inflation. This implies that the monetary policy response is not path dependent when agreement regarding the inflation target can be secured by the rate setting committee. It also illustrates that the aggregation of the time values can produce a linear monetary policy response both under conditions of certainty and uncertainty.

Figure 3.3: Deltas for Collars with equal and unequal strikes

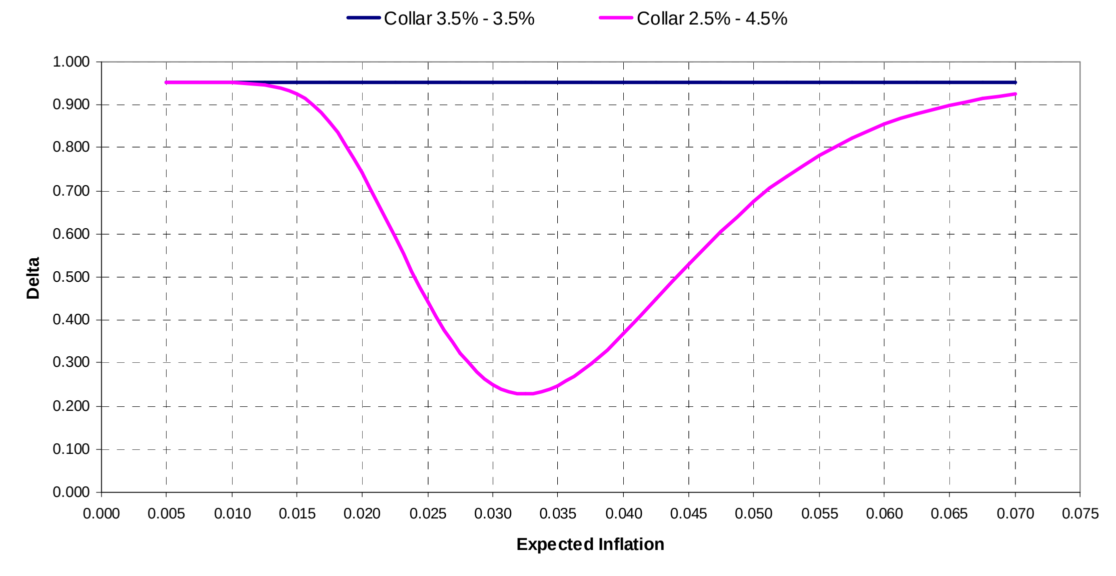

Figure 3.3 illustrates graphically the findings achieved in table 3.1. If policy makers agree a common inflation target the responsiveness of monetary policy does not decline over any part of the entire range of the underlying expected inflation. The delta curves are estimated using the Black model:

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,,\sigma=0.25}}{\partial E(\pi)} \text{ and } \frac{\partial\left(c_{g2}^{k=0.035}-p_{g1}^{k=0.035}\right)^{T=1,,\sigma=0.25}}{\partial E(\pi)}$$

When both strikes are equal, there is no attenuation in the monetary policy response over the full range of the underlying expected inflation. The associated constant delta implies that no variation in the slope occurs. By contrast when the interest rate voting committee is divided, implying that the long call and short put have different strikes, the policy response changes with expected inflation. When the strikes are equal, the constant slope can be shown trivially using put-call parity.

$$p+E(\pi)e^{-rT}=c+Xe^{-rT}$$

$$\textit{differentiating w.r.t. } E(\pi)$$

$$\frac{\partial c}{\partial E(\pi)}-\frac{\partial p}{\partial E(\pi)}=e^{-rT}$$

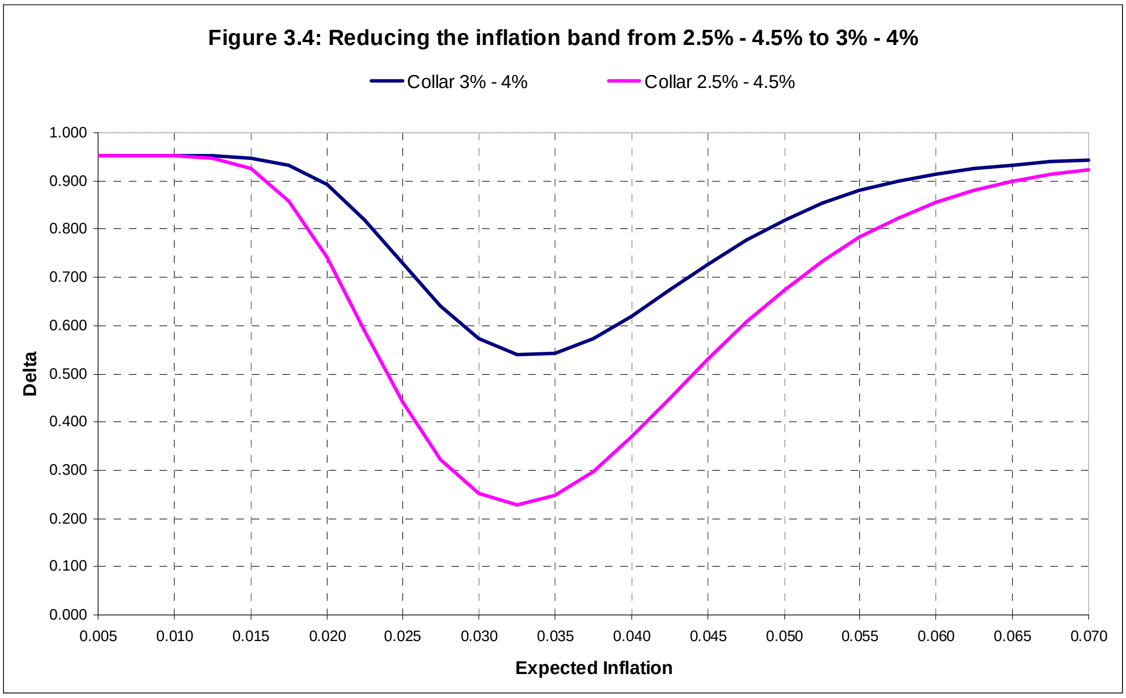

Figure 3.4 demonstrates the effect of the range of disagreement falling from 2.5% - 4.5% down to 3% - 4%:

$$\frac{\partial\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)} \text{ and } \frac{\partial\left(c_{g2}^{k=0.04} - p_{g1}^{k=0.03}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)}$$

The delta, *ceteris paribus*, tends to increase in magnitude when the thresholds converge. The responsiveness of the policy rate to changes in expected inflation increases as ‘agreement’ becomes easier to attain. The relative position of the deltas illustrate that a successful consensus-fixing chairman can produce a greater degree of policy responsiveness. The delta construction reveals varying degrees of linearity and that opportunistic policy can assume a linear or a near linear policy stance.

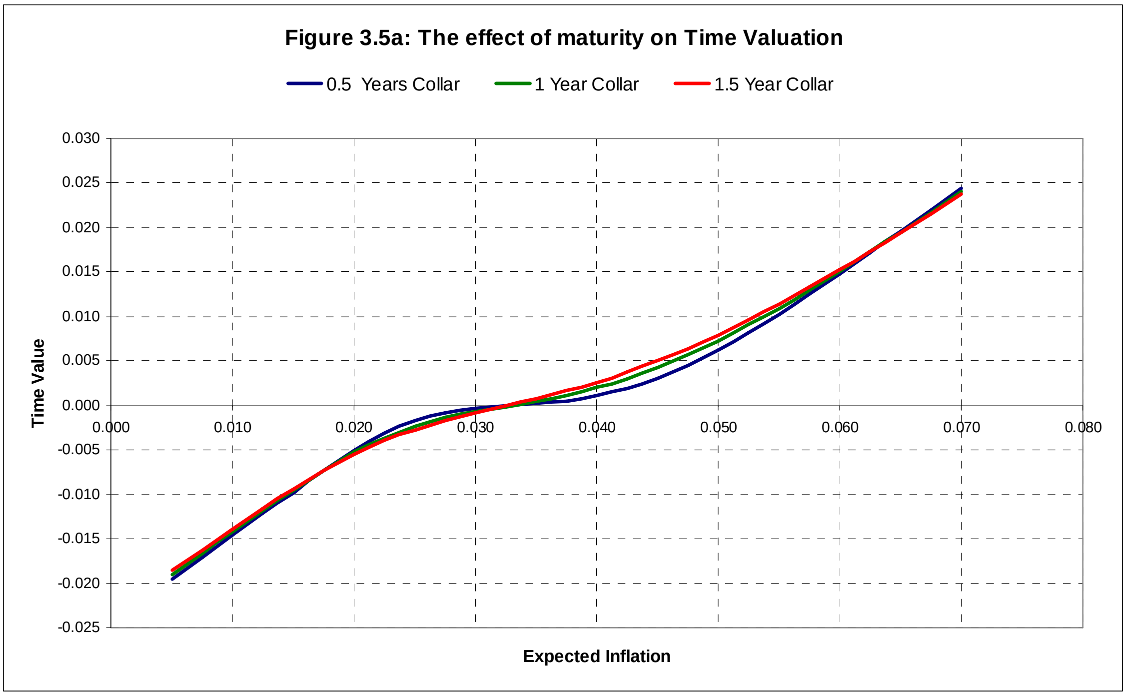

Figure 3.5a: illustrates how the time value of the collar changes as the time to expiration is changed to produce three different maturities:

$$\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=0.5,\sigma=0.25}, \left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25} \text{ and } \left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1.5,\sigma=0.25}$$

The parameter values are the same as those used in table 3.1 but the initial maturity on the contracts ranges from six to eighteen months including twelve months. The exercise values are 2.5% and 4.5% for the short put and long call, respectively. These maturities can be thought of as the forward looking specifications associated with different policy rules. It is noticeable that as the maturity of the call and put increases the absolute values of the contracts also increase over practically the entire range of expected inflation. This seems most conspicuous when expected inflation nears the upper and lower inflation thresholds. (Some decrease can also be detected.) Figure 3.5a demonstrates that as the maturity of the collar increases, the collar itself becomes progressively more linear. Significantly, as the central banker extends the period by which policy is designed to target inflation, the policy in effect is seen to be less path-dependent.

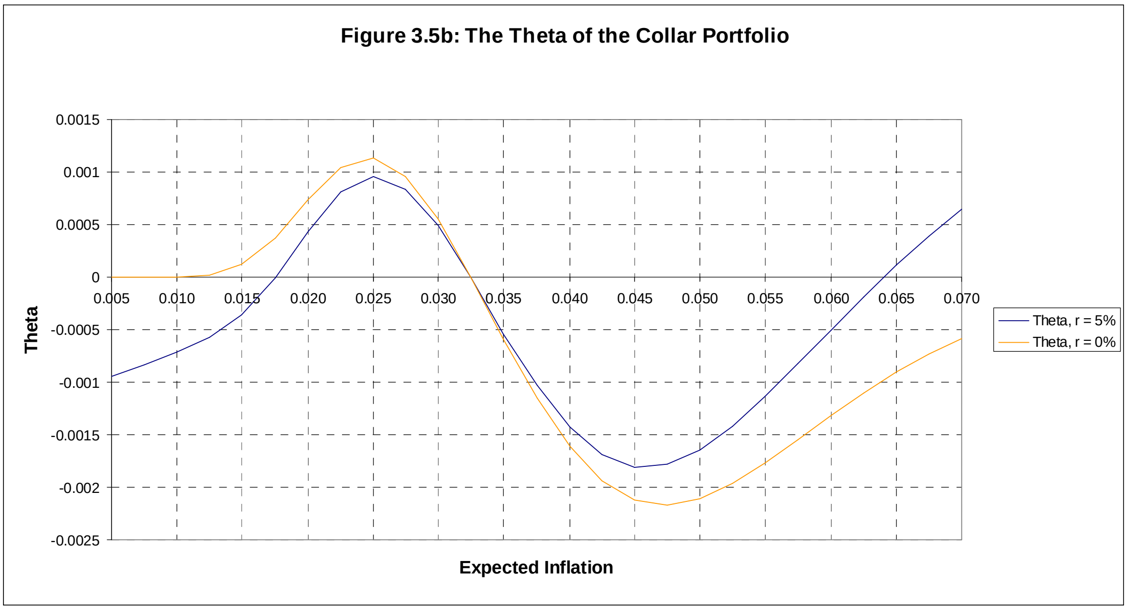

Figure 3.5b: calculates the theta using the following: $\frac{\partial c_{g2}}{\partial t} - \frac{\partial p_{g1}}{\partial t}$ for the range of expected inflation 0.5% to 7%:

$$T=1, \sigma=0.25, \pi^*_{g2}=0.045, r=0.05, 0.00$$

$$\frac{\partial c_{g2}}{\partial t} = -e^{-r(T-t)}\left[E(\pi)N'(d_{1g2})\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[E(\pi)N(d_{1g2}) - \pi^*_{g2}N(d_{2g2})\right]$$

minus

$$T=1, \sigma=0.25, \pi^*_{g1}=0.025, r=0.05, 0.00$$

$$\frac{\partial p_{g1}}{\partial t} = -e^{-r(T-t)}\left[E(\pi)N'(d_{1g1})\left(\frac{\sigma}{2\sqrt{T-t}}\right)\right] + re^{-r(T-t)}\left[\pi^*_{g1}N(-d_{2g1}) - E(\pi)N(-d_{1g1})\right]$$

The parameter values used are the same as those given in Table 2.1 except r = 0% is also considered. The strikes are taken to be the same as the initial values presented: 2.5% - 4.5%. The theta of the option portfolio provides a measure of time decay of the option. Here, it is used to investigate the policy responsiveness as the targeting horizon changes by one year. It is noticeable that the greatest effect of changing the expiration occurs when expected inflation appears close to the upper and lower thresholds of tolerance. This implies that the tempo of policy should increase more dramatically when expected inflation approaches the upper or lower bound of the target zone for a given change in the timing of the target. As before, asymmetry results from using a lognormal model. It might also be noted that if r were set arbitrarily to zero then the effect of the second term in square brackets for the call and put thetas would disappear altogether. The theta curve would converge to zero both above the upper strike and below the lower strike. The relevance of omitting r can be observed from (a.2.1.16).

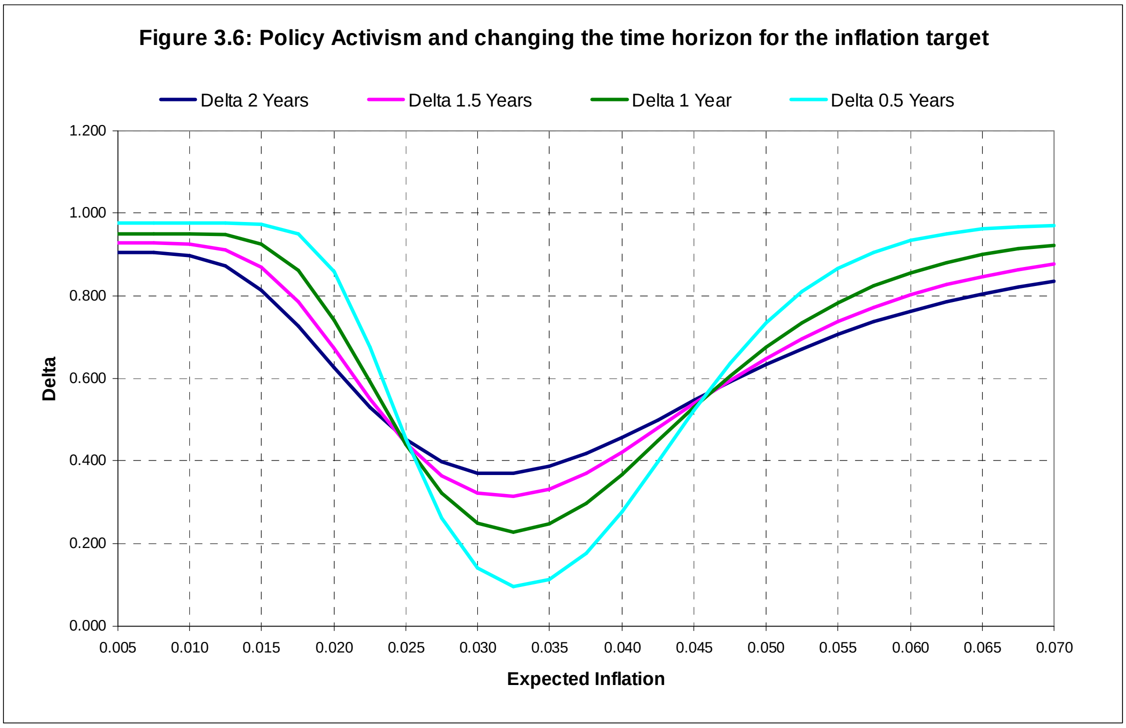

Figure 3.6: illustrates the effect of the change in maturity on the delta. The delta is calculated using the same parameter values as table 3.1 but with four different time horizons considered:

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=0.5,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1.5,\sigma=0.25}}{\partial E(\pi)} \text{ and } \frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=2,\sigma=0.25}}{\partial E(\pi)}$$

The strikes are given as 2.5% for the short put and 4.5% for the long call. The Delta of the portfolio is calculated by subtracting the delta of the put from the delta of the call where:

$$\frac{\partial c_{g2}}{\partial E(\pi)}-\frac{\partial p_{g1}}{\partial E(\pi)}=e^{-r(T-t)}N\left(d_{1g2}\right)-e^{-r(T-t)}\left[N\left(d_{1g1}\right)-1\right]$$

All else being equal, extending the targeting horizon would have the effect of making policy less path dependent. The delta provides a measure of policy activism (or inversely policy gradualism). As the maturity increases, there results a contraction of the delta range precipitating a more linear policy reaction. If r is set equal to zero the in-the-money deltas tend to converge.

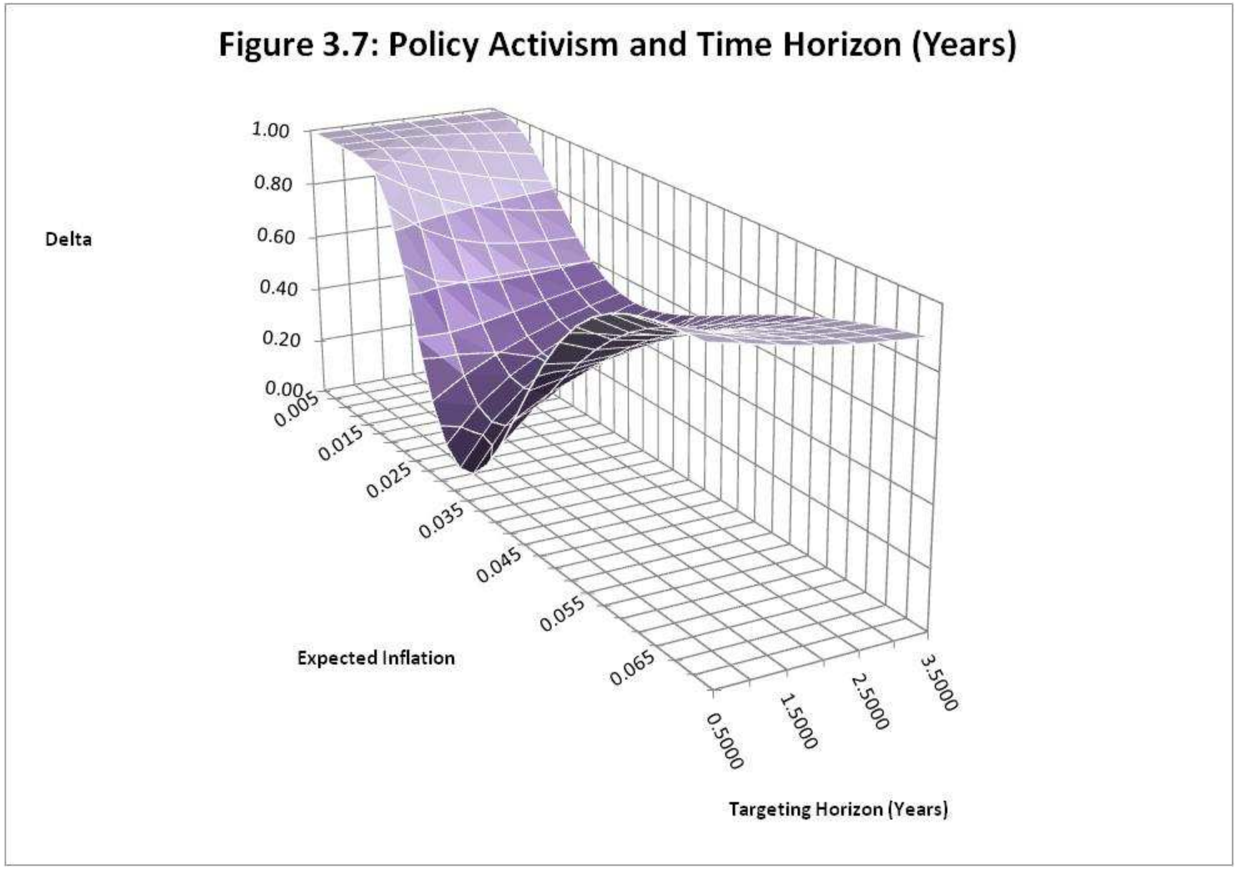

Figure 3.7: provides an interpolated view of the effect of the time horizon on policy activism. Seven time delta curves are computed ranging from 0.5 to 3.5 years. The delta values are calculated in the same way, as outlined by figure 3.6 and the parameter values, (barring maturity), are the same as those as given in Table 2.1. The zone thresholds are 2.5% and 4.5%. Figure 3.7 demonstrates more comprehensively that as the expiration increases the delta ranges tends to contract. Policy activism and consequently policy inertia is less well defined by the given magnitude of expected inflation relative to the bands as expiration increases.

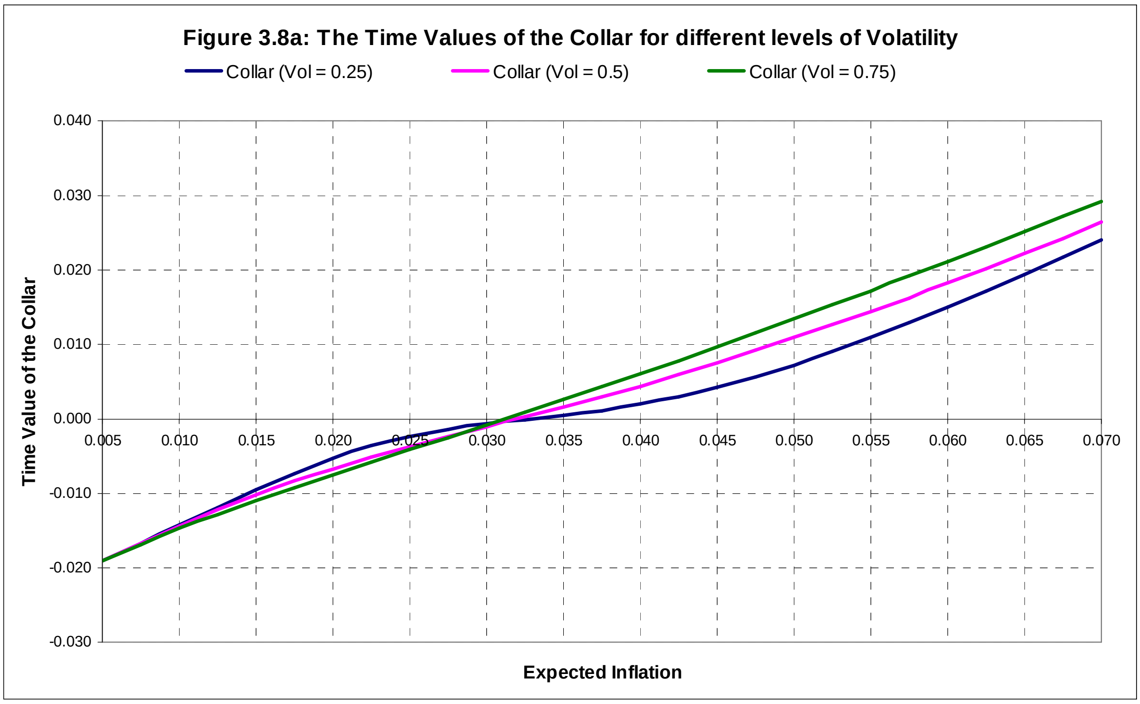

Figure 3.8a demonstrates that the time value of a collar for varying levels of volatility. The collar's time value is computed in the same way as table 3.1 except that the volatility term is adjusted from 0.25 to 0.5 and then to 0.75:

$$\left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}, \left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.5} \text{ and } \left(c_{g2}^{k=0.045} - p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.75}$$

The strikes are given as 2.5% for the short put and 4.5% for the long call. Figure 3.8a shows that as volatility increases the time value of the collar assumes a more linear form. This seems consistent with the formulae derived for the vegas. From the perspective of a central banker setting monetary policy, an increase in volatility precipitates an increase in the magnitude of the response when expected inflation is close to the inflation zone. The effect of this change renders the task of distinguishing between the opportunistic and deliberative policy maker more challenging. When the expected inflation rate resides within or close to a pre-specified zone of tolerance and the risk of breaching the band commensurately increases with volatility, then policy activism will increase, mitigating the effect of nonlinearity.

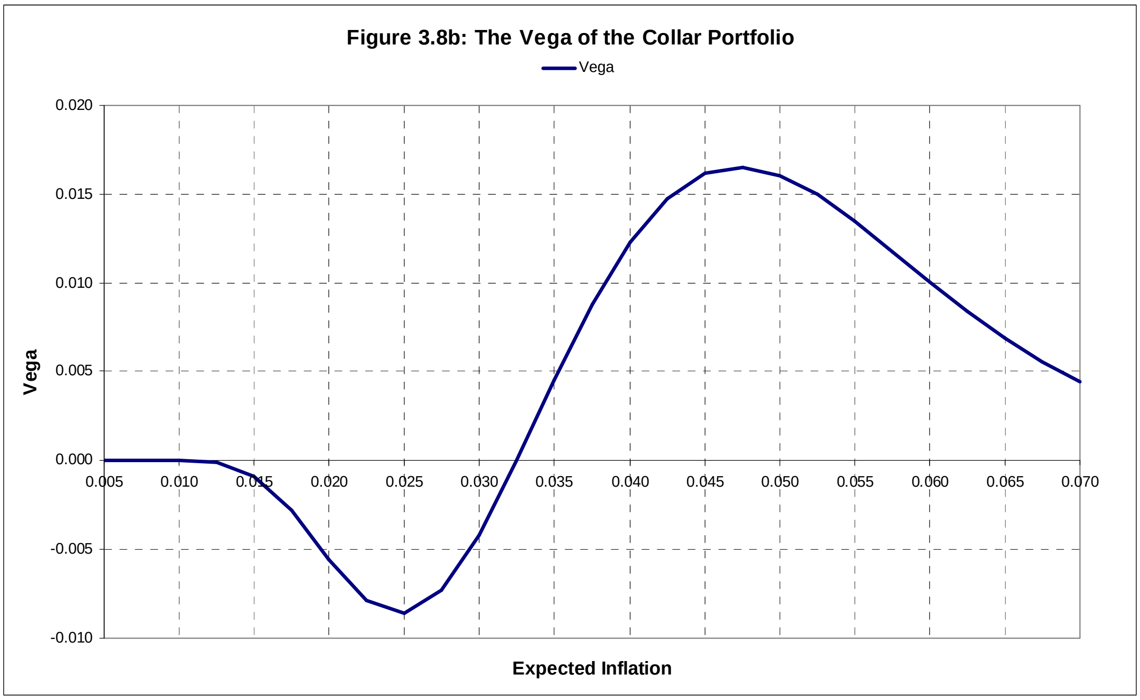

Figure 3.8b calculates the vega of the portfolio by subtracting the vega of the short put position from the vega of long call position where the parameter values are the same as for table 2.1. The strikes are given as 2.5% for the short put and 4.5% for the long call. The vega of the portfolio is given by:

$$\frac{\partial c_{g2}}{\partial \sigma} - \frac{\partial p_{g1}}{\partial \sigma} \stackrel{T=1,\,\sigma=0.25,\,\pi^*_{g1}=0.025,\,\pi^*_{g2}=0.045,\,r=0.05}{=} e^{-r(T-t)} E(\pi)\sqrt{\mathrm{T-t}}\left[N'\left(d_{1g2}\right) - N'\left(d_{1g1}\right)\right]$$

for the put. It is observed that for a given change in volatility the largest impact on the time value of the collar occurs close to the upper and lower thresholds. This implies monetary policy is most likely to be affected when volatility increases or decreases when expected inflation is close to the margins of the targeting zone. As before asymmetry results from using the lognormal model.

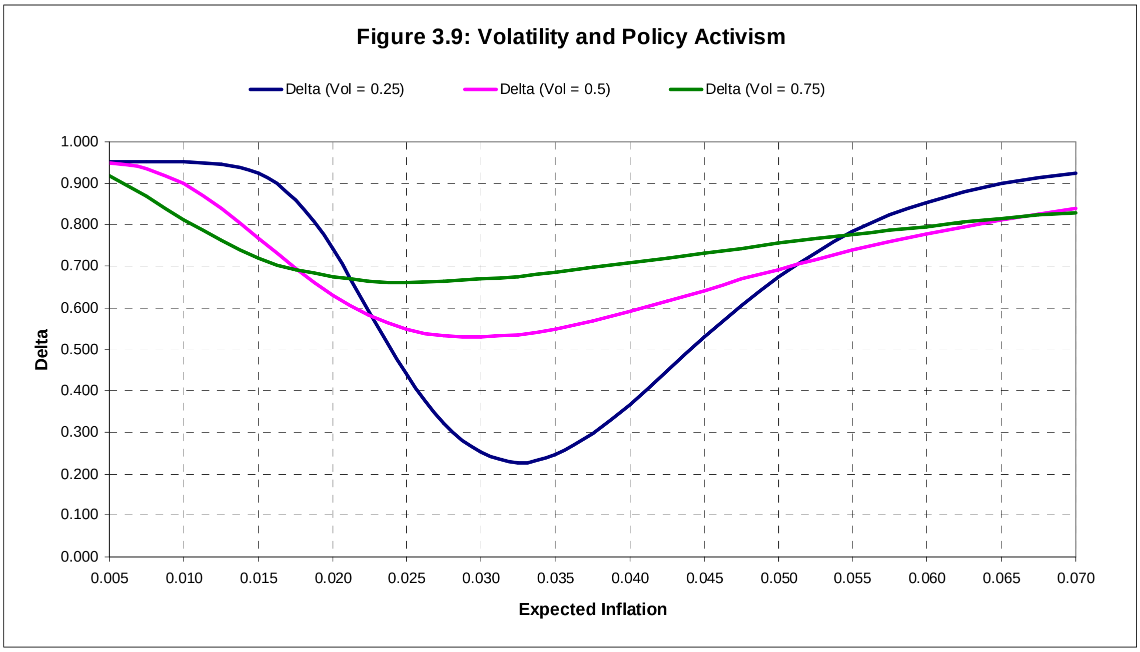

Figure 3.9 illustrates the impact of the changing volatility on the delta curve. The delta is computed using the same parameter values as table 2.1, but with three different volatilities. The strikes are given as 2.5% for the short put and 4.5% for the long call. The delta curves are calculated in the same way as before. The only adjustment relates to altering the standard deviation which varies from 0.25 to 0.5 to 0.75.

$$\frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.25}}{\partial E(\pi)}, \frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.5}}{\partial E(\pi)} \text{ and } \frac{\partial\left(c_{g2}^{k=0.045}-p_{g1}^{k=0.025}\right)^{T=1,\sigma=0.75}}{\partial E(\pi)}$$

σ = 0.5 and σ = 0.75 are values outside the range obtained by using a recursive GARCH (1,1) model, given in appendix A.2.3. The Delta of the portfolio is calculated by subtracting the delta of the put from the delta of the call:

$$\frac{\partial c_{g2}}{\partial E(\pi)}-\frac{\partial p_{g1}}{\partial E(\pi)}=e^{-r(T\text{-}t)}N\left(d_{1g2}\right)-e^{-r(T\text{-}t)}\left[N\left(d_{1g1}\right)-1\right]$$

It is visible that as volatility increases, that the delta values become somewhat compressed. The out-of-the money expected inflation range seems to experience an increase in delta when volatility increases. Likewise a significant part of the in-the-money expected range is associated with a fall in the delta. In terms of monetary policy, this suggests that the policy rule assumes a more linear form, as this type of volatility increases.

Figure 3.10: Policy Activism for given levels of Volatility

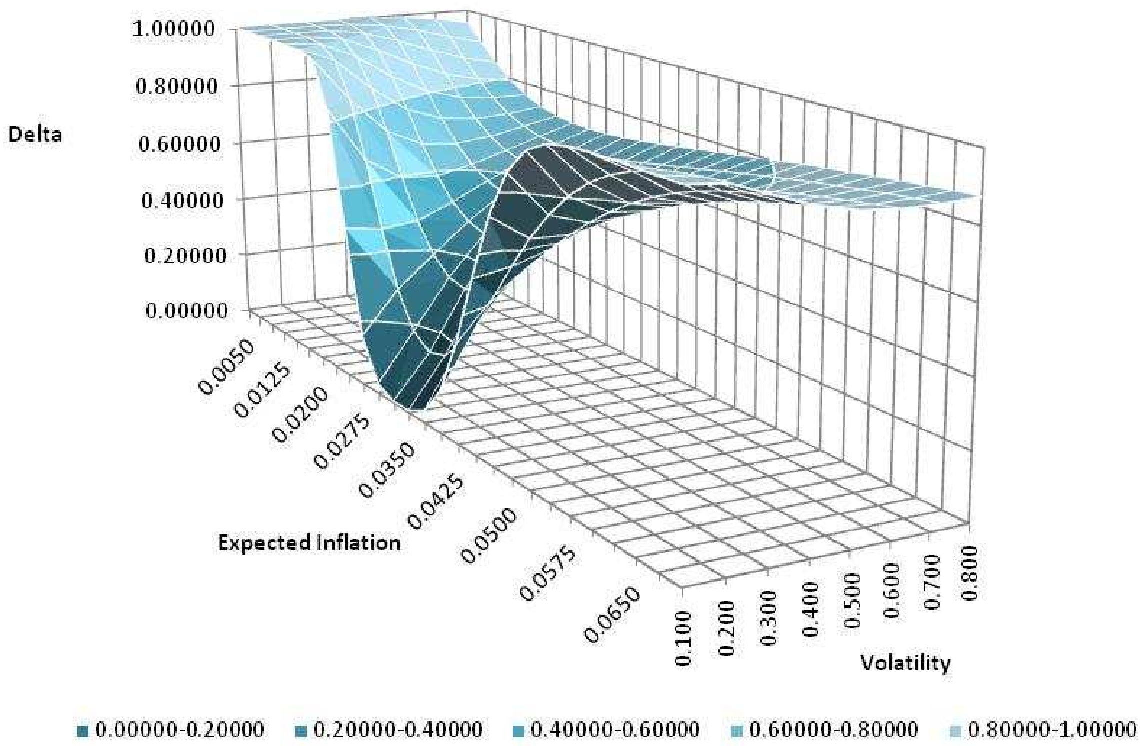

Figure 3.10 illustrates the delta curves over a wider range of standard deviations. Values range from 0.1 to 0.8 in intervals of 0.1. The deltas are calculated in the same manner as outlined in Table 3.1 and Figure 3.9. It is conspicuous that as the volatility of inflation increases, the deltas tend to compress into a narrower range. By adopting a risk management perspective it would seem that policy becomes increasingly linear when uncertainty increases. In this regard, the Taylor Rule appears to be an important limiting case in that it feasibly can materialise (i) by a convergence between the upper and lower inflation targets, (ii) from an increase in time horizon over which price stability must be preserved and (iii) from a rise in inflation volatility (as outlined above).

Figure 3.11 The actual Fed Funds rate minus the Taylor Rule rate

A historical perspective of the Greenspan years

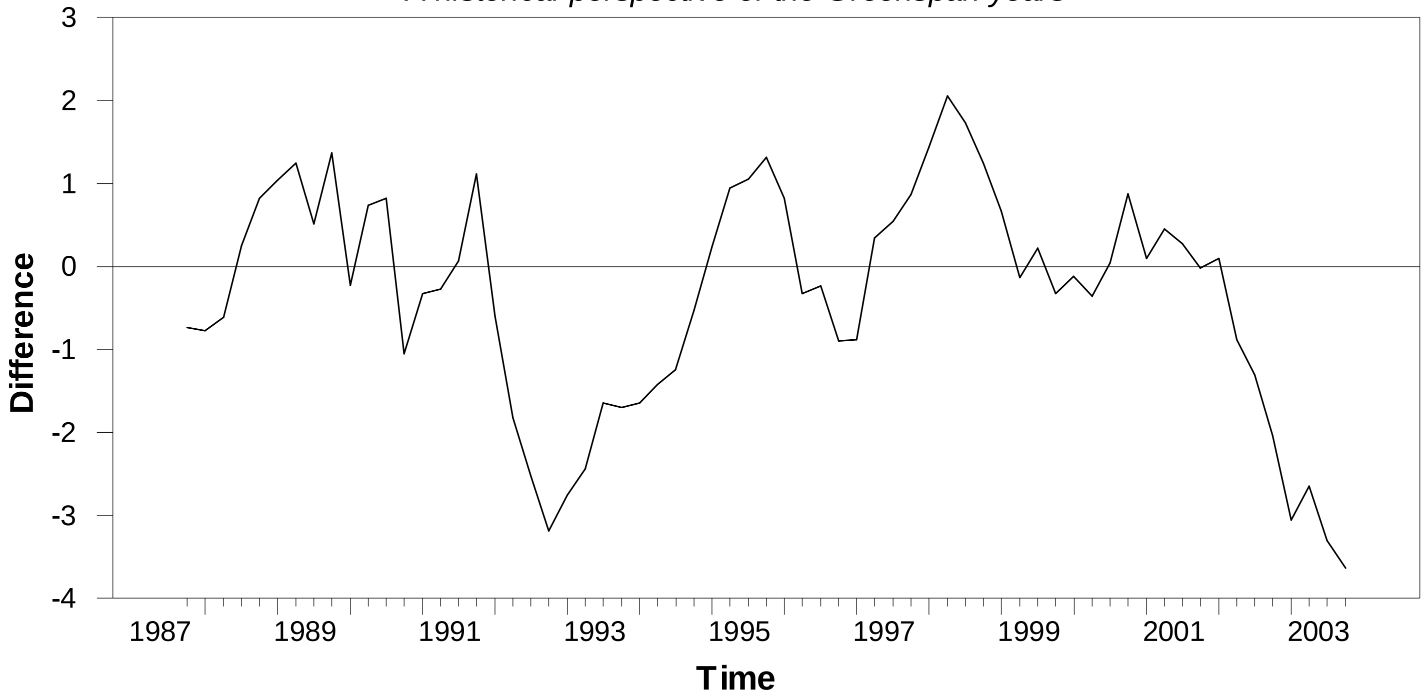

Figure 3.11 Maps out the difference between the actual Fed Funds rate and the Taylor Rule over the period indicated. The difference is given here in percentage form. The difference is denoted by ε_t:

$$\varepsilon_t = r_t - r_t^{Taylor}$$

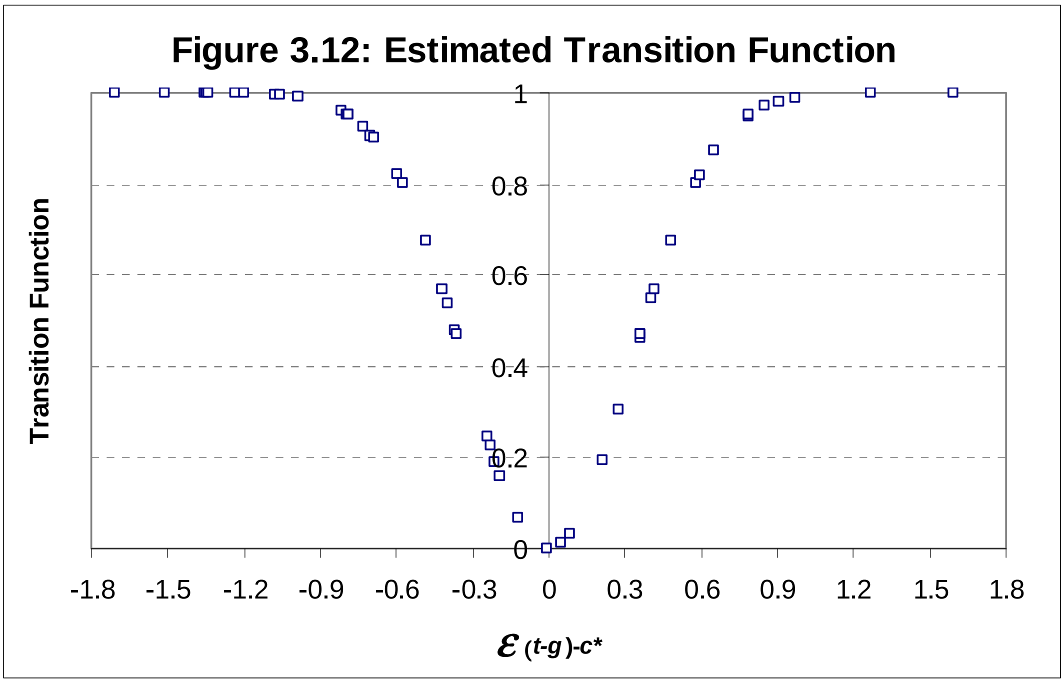

Figure 3.12: illustrates that for large deviations from equilibrium the error term tends to correct. Ultimately, mean reversion is imposed consistent with a migration hypothesis. The error term, in this sense, may be considered to be nonlinearly stationary.

4 Conclusion

The opportunistic approach to disinflation has never been formally endorsed by Fed officials despite pronouncements by individual members from the FOMC who characterise policy as being nonlinear. More recently, risk management concepts seem to have coloured the language of central bankers. This would appear particularly true of chairman Greenspan during the final years of his tenure at the Federal Reserve. Chairman Bernanke at his Nomination Hearing before the Senate Committee on Banking, Housing, and Urban Affairs, identified that the Greenspan risk-management approach would be maintained in ongoing successful monetary policymaking.[316] The risk analysis, now customarily referred to by the FOMC, is comparable to the more transparent expressions of this type of strategy from the Bank of England (BOE). The fan charts, currently employed by a number of central banks, provide a significant conduit through which to convey expectations and uncertainty concerning the development of key macroeconomic variables. In this thesis, a framework was set out to establish the links between risk management and opportunistic monetary policy. The genesis of the opportunistic approach was examined in chapter 1, using historical perspectives relating to tensions thrown up by conforming to a rules-based type policy. Much of the literature to date has praised the benefits of adhering to a particular policy rule. Policy makers profess rules often in a bid to leverage up some reputational capital. Given the intricacies associated with committing to a strict rule, policy makers, by default, are often forced to implement a contingent rule. In chapter 1, the Volcker-Greenspan tenures largely were characterised as advocating rules while always preserving the necessary scope for discretion or 'wiggle'. It is argued in this thesis that opportunistic policy has been the product of these contradictory imperatives, imposed on and by policy makers themselves. In their attempts to resolve tension between rules and discretion, policy becomes opportunistic. In chapter 1, this theme was explored with reference to the varying monetary targeting regimes proposed by the Federal Reserve since 1979. In the United States, during the Greenspan period, this ultimately has meant subscribing to an evolving unannounced zone target for

[316] November 15, 2005.

inflation. In tandem, central bankers commonly stress that monetary policy is designed to contain future developments. Greenspan (2003, 2004) has pointed out that forward variables are essentially unknown and policy makers, as a consequence, are reliant on risk management. The fan charts, as published by the Bank of England, go some way to making explicit this uncertainty. Similarly, the Federal Reserve since November, 2007, has published an enhanced range of forecasts that are reminiscent of these fan charts.

The Aksoy, Orphanides, Small, Wieland and Wilcox (2006) profile of the opportunistic central banker was used extensively throughout the thesis, see equations (2.1) – (2.5), Section 2.2.2. This, at its core, defined policy makers as fighting inflation when inflation is high, but concentrated more on stabilising output when inflation is low. The implied policy rule was found to be nonlinear. Figure 1.1a depicts this type of rate adjustment by equating the upper tolerance of inflation as being the exercise on a call option. Risk management features can be added to this analysis by considering future inflation, as outlined in Proposition 1.4. In a world that is forward looking, opportunistic central banks respond to the likelihood of expected inflation breaching an upper bound. This has been presented in chapter 1, using standard option theory and also by applying Monte Carlo simulation. The time valuation parabolas developed using Black Scholes (1973) can be extended by using Heston Monte Carlo and Backus, Foresi and Wu (2004) to take account of skew and kurtosis.

In setting out an option's framework a key question involves asking: can inflation be technically treated as the 'underlying' in an option's framework? Applying standard option theory to inflation poses a number of difficulties. In particular, inflation is generally viewed as being reported discretely.[317] While stocks and other traded assets also necessarily trade in discrete time, it is clear that the prices of stocks are generally updated on the whole more than once a month.[318] Stocks that trade in real time still present difficulties for modellers, not least because continuous dynamic hedging is not

[317] This is less true for expected inflation read from TIPS instruments that trade in liquid markets and also enjoy a substantial issuance. The TIPS market can, in principal, provide a means to gauge inflation expectations and to hedge inflation option exposures consistent with establishing risk neutral conditions.
[318] This is the periodicity generally associated with the publication of CPI data. Real time inflation data in the future would feasible given the information available to nationwide supermarket chains.

entirely feasible. In the absence of Geometric Brownian Motion (GBm) and continuous time trading, how well does conventional option theory actually work? A key difficulty relates to how dependent is option pricing on the notion of dynamic replication. Does dynamic replication work as standard theories imply? Is there a simpler approach that allows robust option pricing in the absence of GBm? Can risk neutral conditions be established, when GBm can not be verified?[319]

Derman and Taleb (2005) point out that risk neutral conditions can be attained without necessitating any reliance on dynamic delta hedging. Their paper developed a framework for option valuation when GBm did not hold. This was considered, in Section 1.6, where a static hedge framework using put-call parity was discussed and compared against the original Black-Scholes derivation.[320] Proposition 1.1 and Proposition 1.2, p. 36, applied the Derman-Taleb (2005) results where asset prices are considered to move discontinuously. They find that the original actuarial formulae, as set out by Black-Scholes and Black can be recovered, and are made robust for circumstances that are clearly less idealised than originally presented. Proposition 2.2, p. 112, developed the conditions necessary to establish the technical validity of applying the Black (or Black Scholes) model. In effect, the application of Black-Scholes formulae is not dependent on first establishing that the underlying variable's client path can be described as adhering to GBm. This is a key result and serves to establish risk neutral conditions when applying a closed-form solution.[321] Appendix A.2.1, p.178 – 183, also established that the risk free rate can be ultimately eliminated from the collar construction. This finding is useful because it implies that dependence on the risk free rate, r, can in due course be removed.[322] [323] Appendix A.2.1 also set out the basis for the 'time-honoured actuarial

[319] Jarrow and Yildirim (2003) and Korn and Kruse (2004) simply assume the inflation index adheres to GBm.
[320] In fact, the centrepiece of this analysis rests on put-call parity, which is fully set out in chapter 1, p. 35. Given that put-call parity can always be assured, the results that emerge are robust.
[321] See Appendix A.2.1, p.178.
[322] See (a.2.1.16), p. 183.
[323] One possible approach is that the discount rate, r and risk premia are set equal to zero. That is, central bankers are equally hostile to inflation whether experienced today or at some point in the future. This might be termed the 'zero time decay approach'. A very natural interpretation for r is that it measures the extent to which future inflation is less significant to central bankers than current inflation. This posits r as a measure of time decay. A positive r would imply central bankers would see future inflation as involving a

formulae' used when pricing options, as described by Derman and Taleb (2005). These can be applied to many different types of underlying. This framework was adopted for monetary policy risk analysis.

In addition, Monte Carlo analysis was used to build from the ground up a range of alternative approaches that again are not dependent on GBm or continuous time.[324] In fact, the discrete time nature of inflation can be made explicit.[325] A key advantage of this numerical approach relates to its' flexibility. By modifying a number of specifications, it was found that many different types of inflation behaviour can be considered. The option's framework was shown to be sufficiently flexible to incorporate a variety of central banker concerns including inflation skewness and kurtosis.[326] Both numerical and closed-form solutions can be developed to examine the potential effects of departures from lognormality and normality.[327]

The Derman-Taleb (2005) results provide the basis to consider the Black-Scholes/Black framework when prices move discontinuously.[328] The put call parity argument made by Derman and Taleb (2005) opens up the possibility to examine a number of specifications regarding monetary policy risk management. The most parsimonious perhaps is linked to Proposition 1.3, p. 42, which developed a lognormal random walk as being an analytical baseline. This would imply using a naïve model. The lognormal random walk does nevertheless permit central bankers to assume the risks to price stability as minimising deflation risks while never eliminating the risk of hyperinflation.[329] Simulations, using

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smaller loss of utility than experiencing the same magnitude of current inflation. Using this approach would disregard having to establish risk neutral conditions.

[324] Monte Carlo can be implemented explicitly as a discrete time estimator.

[325] Note current inflation is reported monthly which can be modelled using Monte Carlo. The expectation of inflation in contrast can be deduced in real time from indexed-linked bonds that trade.

[326] The fan charts build in formal estimates for skewness.

[327] This would seem relevant given the extent to which upside and downside risks are now stressed by central bankers. See p. 60 – 62 for application of skewness and kurtosis to option pricing.

[328] The original Black-Scholes derivation implied that the underlying adhered to continuous adjustment.

[329] These qualities are useful because they characterise the central banker as being vigilant against accelerating positive inflation. Adopting the lognormal random walk process, implies that policy makers never remove the risk of hyperinflation from their analysis and are prepared 'to plunge the stake' over and over, without being preoccupied by deflation. Of course, this would be somewhat at odds with the policy concerns expressed by chairman Greenspan. Importantly, the lognormal random walk model configures the risks of expected inflation to be less tame than the risks presented by a mean reverting process. If central

this model, were mapped out in Figures 1.3a - 1.3g, p. 76 – 77. They revealed the effects of varying the volatility of inflation and the targeting horizon. It is obvious from these representations that as the volatility and targeting horizon are increased; the terminal inflation values become ever more dispersed. The implications of this for monetary policy were examined in chapter 1 and developed more formally in chapter 3, using 'the Greeks' from the Black model. It was also found that discrete time Monte Carlo valuations, based on the lognormal random walk, converge to the continuous-time Black-Scholes (or Black) valuations.[330] Using the Monte Carlo approach, it is again clear that Black-Scholes valuation can be recovered without dependence on GBm.

Different approaches are possible here. The approach favoured in this thesis was to develop a market model as outlined by Derman-Taleb (2005) that was found to be robust even in the presence of likely market frictions. This could then be used for policy analysis but could also be seen as a reasonable methodology for pricing options. The main effect of using a 'zero time decay' approach would be that the process of justifying an option's framework is enormously simplified. The downside is that the implementation of such a 'virtual' approach is restricted merely to policy analysis and would not warrant making use of inflation option market prices. By setting out the theoretical basis for the options' framework it is then possible to consider both intrinsic and time valuations. Monetary policy can, as a result, be gauged respectively under conditions of both certainty and uncertainty. Applying Black (1976) and Derman and Taleb (2005), it was possible to infer the central banker's policy response, or the time valuation parabola, using closed form and Monte Carlo techniques. These techniques were found to produce equivalent time values. In chapter 2, option theory was used to advance a theoretical framework that brings together both risk management and opportunistic policy perspectives. It was also

bankers believed that inflation could stabilise to some long term mean, of its own accord, then the urgency to respond to rising inflation would naturally be diminished. Mean reversion implies inflation is not likely to run out of control, particularly if the long term mean is thought to be low and stable. This is not to say that *ex poste* when considering the data, that inflation can not mean revert. In fact, one would hope that it does and this could be viewed as a tribute to successful monetary policy (or as a tribute to policy makers acting pre-emptively). The lognormal random walk is a useful benchmark model, precisely because it biases the conceptual inflation risks to be to the upside. In light of the recent financial turmoil the policy bias to respond to positive inflation, embedded in the lognormal model, may prove to be useful. Policy rate adjustments designed to pre-empt deflation would now appear to be problematic.

[330] That is Proposition 1.5, page 47.

shown that it was not necessary to attribute any special preferences to individual members of central bank boards in order to motivate opportunistic behaviour.[331] This can be observed from Propositions 2.1, 2.1a, 2.1b, 2.1c and 2.1d, in Section 2.2.2. Even when individual members sought to implement linear policy, committee and institutional dynamics conspire to produce opportunistic type policy responses. If policy is contingent on achieving consensus when opinions differ regarding the appropriate inflation target, then rate setting can still be 'collectively' opportunistic. Option theory was applied in Section 2.2 and 2.3 to extend the opportunistic policy response, as outlined by Orphanides and Wilcox (2002) and Aksoy et al. (2006). A combination of two put-call parity relationships is set out using a zone of disagreement. This is used to form a long position with a call with a higher strike and a short position with a put with a lower strike on expected inflation. Propositions 2.1 – 2.1d were largely motivated by viewpoints expressed by policy makers at the FOMC table. The December 1989 meeting, in particular, was examined in chapter 2 from the context of understanding opportunistic policy. The put-call parity relationship and voting dynamics were applied innovatively so as to nest an option portfolio in the reaction function in order to capture opportunistic policy responses. Using the standard Black formula for option valuation, developed in Appendix A.2.1 and Propositions 2.2 – 2.2b, p. 112 – 115, it was possible to trace out opportunistic policy reactions under uncertainty. If the reaction function were specified to be forward-looking, then portfolio option theory serves to unearth a number of key insights. In particular, variables such as volatility, time horizon and the relativities of differing inflation targets can be incorporated innovatively into analysing monetary policy. These factors are generally considered independently of each other in the literature. A number of nonlinearities are examined using portfolio option theory by applying standard parameter sensitivities such as delta and gamma. Propositions 2.3a – 2.3d, in Section 2.3.3, develop these parameter sensitivities. Gamma was applied using Proposition 2.4.

[331] Although, the ground work presented by Aksoy et al. (2006) can be used directly to motivate the option's framework when extending their opportunistic reaction function to incorporate uncertainty and risk management.

Delta was used to show how policy becomes less activist, (i.e. more interest rate smoothing), when expected inflation resides between the upper and lower target bounds. It was also found that as the target zone expands; monetary policy becomes increasingly nonlinear or less activist for that range. In Chapter 2, the Vasicek model was used to show how the drift of the inflation process could be made to depend on the policy instrument. Proposition 2.3d, submits that a Vasicek delta can be developed to take into account policy inertia when mean reversion is a feature of the inflation process. Propositions 2.6 and 2.6a extended the Vasicek framework to allow for inflation to be predictable. The Vasicek model permits the drift of the inflation process to be dependent on the policy instrument. Historical parameter estimates for the Vasicek model were computed in Section 2.5.1 and 2.5.2. Alternatively, counterfactual parameter estimates could be used to examine different inflation behaviour and this would permit the drift term to be made endogenous. The parameter values could be selected on the basis of how inflation would be expected to respond to a given policy regime.[332] The asymmetries characteristic of the Black model, were considered in chapter 2 by comparing the Black delta curve against the Vasicek delta. This was presented in Figure 2.6. Several nonlinear reaction functions were estimated in Section 2.6 using revised data from the Federal Reserve of St. Louis. The forward-looking opportunistic reaction function with reduced bounds, specifying the 1% – 3% target range, was found to offer a better empirical fit when compared to the estimated linear Taylor Rule. This result was robust for real time data obtained from the Federal Reserve of Philadelphia. It was also robust for varying periods. The empirical estimates were reported in Tables 2.3a - 2.6b.

Several ways whereby monetary policy could be nonlinear and linear were investigated in chapter 3. In the United States, monetary policy is formulated where both the views of individual members of the FOMC and political agencies are taken into account. This has the effect of drawing together heterogeneous beliefs via committee and institutional dynamics. In chapter 3, tactical considerations in dealing with the legislature were identified from the FOMC transcript. Central bankers when brokering agreement with

[332] It would seem natural that higher interest rates would generally have the effect of lowering expected inflation. By selecting appropriate values for a, b and σ, it is possible capture this type of behaviour.

other branches of government are obliged to be politically astute, if policy is to be implemented. This is consistent with chairman Greenspan's comment: that there is still a short term Phillips curve trade-off, and to ignore this, is a misreading of how the political system works. See p. 195. It was argued that during the 1990s, the opportunistic strategy may have embodied efforts to provide policy makers with a means to explain how policy was implemented nonlinearly given that monetary aggregates were being slowly de-emphasised. Don Kohn's description of the opportunistic policy matrix, referred to in chapter 3, p. 196, reveals how internally policy tactics had to be altered to deal with the transition to making more explicit the fed funds target. Federal Reserve officials have also described policy implementation in terms of balancing risks. Surprisingly, in spite of much policy rhetoric, policy responses are often presumed by academics to be exclusively linear.

This leads to the question: what is the practical difference between the linear Taylor Rule and nonlinearity introduced by the collar construction. In practical terms, the difference will vary and is best viewed by considering the Greeks.[333] When differences occur, they can be reversed. The conditions under which nominally declared nonlinear policy can become linear were set out in chapter 3. The effects of a change in levels of agreement, volatility or time horizon were explained using the traditional 'Greeks'. Figures 3.3 – 3.10, p. 241 – 250, provided an overview of the effect of these factors. The nonlinear and linear responses were not structured in this thesis as being mutually exclusive. Anecdotal and archival evidence would suggest that policy was often conceived in terms of being purely nonlinear during the Greenspan tenure. When option theory is applied it was possible to describe the factors that lessened this nonlinearity. The option's framework, developed in chapter 3, was used primarily to set out the conditions that precipitate a more linear type response. Propositions 3.1, 3.2 and 3.3 formalised many of the observations, made in chapter 1 when considering the simulated lognormal paths. It is possible to think of the Taylor Rule as being reversed engineered. The Gamma metric, explained in Proposition 2.5, was used to describe how consensus amongst policy makers

[333] The difference is a function of the proximity of expected inflation to target bounds, the targeting horizon and of levels of volatility.

leads to a more linear type response, similar to what would be conveyed by the Taylor Rule. An ESTAR(q) model, developed in chapter 3, provided evidence that policy was dynamic and was capable of varying responses. By using the difference between the actual Fed Funds rate and the Taylor Rule, one can investigate the probability of stationarity. Mean reversion appeared to increase, as the deviation from the Taylor Rule rate increased. Nonlinear mean reversion supports the contention that a Taylor type policy response is central to understanding monetary policy. This representation of policy developed the Taylor Rule as being an important reference for monetary policy, yet also permits monetary policy to be crafted in nonlinear terms.

So how can such a construction aid in implementing monetary policy? The option's framework is largely an attempt to formalise aspects of what policy makers have described in the FOMC transcripts and elsewhere.[334] The framework is flexible and robust to a number of stochastic processes that do not adhere to the classic Black-Scholes assumptions. In effect, monetary policy risk management would appear largely amenable to investigation using concepts that have been developed elsewhere in finance over the past four decades.[335] It was found that the put-call parity construction does not refute the Taylor Rule. It does nevertheless help explain; how the opportunistic approach and interest rate smoothing are related, how uncertainty regarding expected inflation influences policy, how the targeting horizon for implementing policy influences policy and how the volatility of expected inflation can influence policy responsiveness. The option's framework also provides an innovative approach to understanding policy reactions as being both linear and nonlinear. These themes are generally discussed but are treated separately in the literature. The option's framework draws them together into an integrated analysis.

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[334] Opportunistic monetary policy has not been explicitly endorsed by the Federal Reserve. Risk management strategies, while never clearly prescribed, have nevertheless been publicly endorsed by both chairmen Greenspan and Bernanke.

[335] Monetary policy announcements have over the past number of years employed a language that is incontrovertibly linked to risk management. What still remains unclear is precisely which risk management paradigm, policy makers have in mind. In this thesis, I describe the effects of uncertainty on actual policy deliberation, when brokering rate decisions, which involve committee and institutional dynamics. I have also identified a number of important linkages that exist between opportunistic policy, risk management and interest rate smoothing.

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