

Compact Models for Wireless Systems

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Abstract—For the design and analysis of wireless systems, complex simulations are required and performed. Model order reduction techniques enable greater efficiencies to be achieved and concomitantly, a reduction in memory-resource usage. However, maintaining a certain level of accuracy is paramount. In this contribution, two techniques are combined to enable the formation of a compact model of a high-order system, structure or component. The first is a Krylov subspace method which reduces the original model to a moderate size and the second is a Fourier series expansion method that enables speed and ease of determination of the time-domain responses of the system to arbitrary inputs.

I. INTRODUCTION

The technological advances in the circuitry involved in wireless systems are such that there is an ongoing need for efficient and effective model-order reduction techniques to counter the ever-increasing complexity of simulations. Many different techniques have been proposed, for example in reference [1 and all references therein]. In this contribution, two different methods are combined to form a compact model of a large high-order system thereby enabling fast and accurate repeated simulations.

The first method considered is the Krylov subspace method e.g. [2]. This method can handle very large systems and is numerically efficient. However, there is no error bound for the method and it can generate non-optimal models [3]. In general, the reduced models formed from Krylov methods contain information that is not necessary for a good approximation.

The second method addressed is based on a Fourier Series Expansion (FSE) [4]. The medium-size model obtained after application of the Krylov method is simulated and time-domain responses can be explicitly obtained in a simple form for an arbitrary input using only a compact set of FSE coefficients. Guaranteed stability and causality is assured with this model.

The proposed combined method is applied to a coplanar waveguide and results will highlight the efficacy of the proposed method.

II. KRYLOV SUBSPACE METHODS

Consider a linear system with a state-space representation as:

$$\begin{aligned} E \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t), \end{aligned} \quad (1)$$

where $x(t)$ is the state-space of the system of dimension n , $u(t)$ is the input and $y(t)$ is the output of the system. Here A , B , C , D and E are the corresponding state-space matrices. Eqn. (1) may be transformed to the Laplace Domain and a transfer function $H(s)$ relating the input $U(s)$ to the output $Y(s)$ obtained:

$$H(s) = C(sE - A)^{-1}B + D. \quad (2)$$

There are several variations of Krylov methods but the PRIMA [5] Krylov method is considered here. The Krylov space is then defined as:

$$K_q(\hat{A}, \hat{B}) = [\hat{B}, \hat{A}\hat{B}, \dots, \hat{A}^{q-1}\hat{B}], \quad (3)$$

where $\hat{A} = (A - s_0E)^{-1}E$ and $\hat{B} = (s_0E - A)^{-1}B$ and q is the dimension of the reduced system. An orthonormal basis, V , is formed for K_q . The reduced system is then formed as:

$$\begin{aligned} \tilde{E} \frac{d\tilde{x}(t)}{dt} &= \tilde{A}\tilde{x}(t) + \tilde{B}u(t) \\ \tilde{y}(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}u(t), \end{aligned} \quad (4)$$

with $\tilde{E} = V^T E V$, $\tilde{A} = V^T A V$, $\tilde{B} = V^T B$, $\tilde{C} = C V$ and $\tilde{D} = D$.

The reduced system obtained with PRIMA matches q moments of the original transfer function (2) and stability and passivity are preserved as proven in [5].

However, it is well-known that Krylov methods generate models which are, in general, larger than required. Several remedies for this redundancy have been proposed [3, 6-7]. In this contribution, the Fourier Series Expansion is proposed as an alternative post-processing step to cure the problem.

III. FOURIER SERIES EXPANSION

The Fourier Series Expansion was first introduced in [4] and is summarized here for completeness. Let $\tilde{H}(\omega)$ be the transfer function of the reduced system obtained from application of the PRIMA algorithm of Section II. Suppose $\tilde{H}(\omega)$ is nonzero for $|\omega| \in [0, \omega_m]$ where ω_m is assumed to be large, but finite. Also, assume $\tilde{H}(\omega) = \tilde{H}^*(-\omega)$. Then $\tilde{H}(\omega)$ may be expanded in a Fourier Series as follows, bearing in mind that it must be an even function of frequency.

$$\text{Re } \tilde{H}(\omega) = \sum_{k=0}^{\infty} a_k \cos k\omega, \quad (5)$$

where $\tilde{\omega} = \pi\omega/\omega_m$. The expression in (5) describes an even function, defined for $\omega \in [-\omega_m, \omega_m]$ (i.e. $\tilde{\omega} \in [-\pi, \pi]$). To enforce causality, the expression for $\text{Im} \tilde{H}(\omega)$ may be obtained from (5) via the Kramers-Kronig relations (Hilbert transform) [8]:

$$\text{Im} \tilde{H}(\omega) = - \sum_{k=0}^{\infty} a_k \sin k\tilde{\omega}, \quad (6)$$

From (5) and (6), it follows that

$$\tilde{H}(\omega) = \sum_{k=0}^{\infty} a_k e^{jk\tilde{\omega}}, \quad (7)$$

for $\omega \in [-\omega_m, \omega_m]$.

The representation of the output in the time domain may be obtained by an Inverse Fourier Transform. The output caused by an (arbitrary) input $x(t)$ defined for $t > 0$, (i.e. input signal $x(t)\theta(t)$ with Fourier image $X(\omega)$, where $\theta(t)$ is the unit step-function), is:

$$\begin{aligned} y(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} Y(\omega) d\omega \\ &= \frac{1}{2\pi} \sum_{k=0}^{\infty} a_k \int_{-\infty}^{\infty} e^{j(t-\tilde{k})\omega} Y(\omega) d\omega \\ &= \sum_{k=0}^{\infty} a_k x(t-\tilde{k})\theta(t-\tilde{k}), \end{aligned} \quad (8)$$

where $\tilde{k} = \pi k/\omega_m$.

Therefore, once the set of FSE coefficients is obtained from the frequency-domain simulations, then the response for an arbitrary input may be readily determined from (8).

To determine the set of FSE coefficients, let $\tilde{H}(\omega)$ be obtained at a number of points, ω_i :

$$F_i^{(1)} = \text{Re} \tilde{H}(\omega), \quad i = 1, 2, \dots, N_1; \quad (9)$$

$$F_i^{(2)} = \text{Im} \tilde{H}(\omega), \quad i = 1, 2, \dots, N_2, \quad (10)$$

where N_1 is the number of real parts of the data points and N_2 is the corresponding number imaginary parts of the data points.

Then let a be the set of real coefficients $a = [a_0, a_1, \dots, a_N]^T$. Introduce $M_{ik}^{(1)} = \cos k\tilde{\omega}_i$, $M_{ik}^{(2)} = -\sin k\tilde{\omega}_i$, and $\tilde{\omega}_i = \pi\omega_i/\omega_m$, where $k = 1, \dots, N$. Then from (9) and (10):

$$F^{(1)} = M^{(1)}a + E^{(1)}; \quad (11)$$

$$F^{(2)} = M^{(2)}a + E^{(2)}. \quad (12)$$

Here $E^{(1,2)}$ represent the errors that arise due to limiting the summation in (5)-(8) to a finite number of terms, N ; (11) and (12) may be merged to yield:

$$F = Ma + E \quad (13)$$

with

$$F = \begin{bmatrix} F^{(1)} \\ F^{(2)} \end{bmatrix}, \quad M = \begin{bmatrix} M^{(1)} \\ M^{(2)} \end{bmatrix}, \quad E = \begin{bmatrix} E^{(1)} \\ E^{(2)} \end{bmatrix}$$

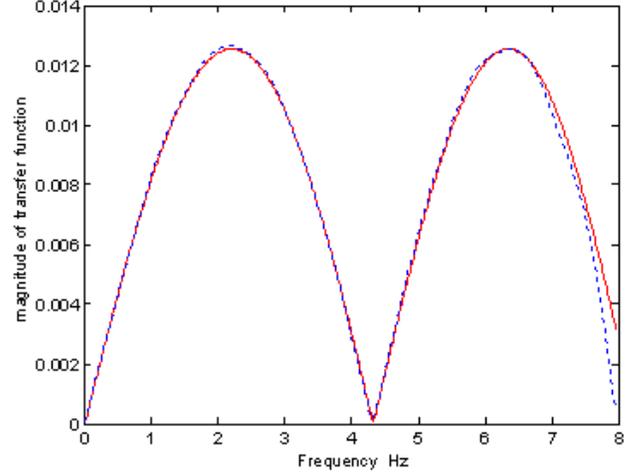


Fig. 1. Comparison of frequency responses for the coplanar waveguide – red solid line is full model, blue dashed line is reduced model.

and the minimal error, $E^T E$, for (13) is achieved with:

$$a = (M^T M)^{-1} M^T F. \quad (14)$$

The number of points N determines the size of this reduced system. By selecting N as small as possible to obtain the required error, any redundancy in the system in (4) can be removed.

IV. EXAMPLE

The example considered is that of a coplanar waveguide. The original model is described by a state-space representation of the order of 300. The Krylov method is used to reduce this dimension to 200. The Fourier Series Method is then applied and the result obtained with $N = 20$ is shown in Fig. 1. It is superimposed on the frequency domain response of the original full model.

V. CONCLUSION

The paper has proposed a two-stage method for forming a reduced-order model of large-scale systems. The method combines two techniques, a Krylov method and a Fourier Series approach. The method achieves a high degree of accuracy and results in a compact model that enables ease of determination of the time-domain responses.

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