An efficient nonlinear circuit simulation technique

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Abstract - **The paper proposes a new and efficient approach for the analysis and simulation of circuits subject to input signals with widely separated rates of variation. Such signals arise in communication circuits when an RF carrier is modulated by a low-frequency information signal. The proposed technique will initially involve converting the ordinary differential equation system that describes the nonlinear circuit to a partial differential equation system. The resultant system is then semidiscretised using a multiresolution collocation scheme involving cubic spline wavelet decomposition. A reduced equation system is subsequently formed using a nonlinear model reduction strategy. This enables an efficient solution process using trapezoidal numerical integration. Results will highlight the efficacy of the proposed approach.**

I. INTRODUCTION

Harmonic balance [1-3] and Time-Domain Integration [4] are the two most widely employed circuit simulation techniques in circuit simulators for the analysis of high-frequency nonlinear circuits. Harmonic balance is most effective for periodic or quasi-periodic steady-state analysis of mildly nonlinear circuits and thus is of limited use for the complex modulation formats encountered in today's high-speed systems or for systems involving strong nonlinearities. Time-domain integration on the otherhand is only practical for baseband systems. For the simulation of circuits with digitally modulated high-frequency carriers with long bit sequences, time-domain integration is excessively slow. As a result, there is a need for some form of general purpose technique which can simulate state-of-the-art systems which are subject to transient high-frequency signals or complex modulated RF carriers.

Several envelope transient analysis approaches have recently been proposed whereby a mixed-mode technique is implemented [5-6]. The essence of these approaches is that the slowly-varying envelope of a signal is treated by time-domain integration and that the high-frequency carrier is treated by Harmonic Balance. However, existing techniques have limitations, for example, restrictions in the bandwidth of the excitation signal [5] and the limitations of harmonic balance with respect to strong nonlinearities. Roychowdhury in [7] proposes converting the differential-algebraic equations that describe the circuit to multi-time partial differential equations and applying time-domain methods directly to solve the resultant systems. Pedro [8] also employs a multi-time partial differential equation approach but uses a combination of Harmonic Balance and Time-Domain Integration to solve the resultant system.

The approach presented in this paper also employs the multi-time partial differential equation approach. The proposed technique is a variation and improvement of that presented in [9]. A modification of the wavelet-based collocation approach proposed by Cai and Wang in [10] forms the core of the technique and unlike Christoffersen and Steer [11], the cubic spline wavelet basis is employed to solve the multi-time partial differential equation representation of the system rather than the original ordinary differential equation representation. However, the technique presented in [9] is greatly enhanced in this paper in that a nonlinear model reduction strategy similar to that in [12] is employed within the proposed envelope simulation technique to obtain very high efficiencies. The paper proceeds to compare, for a test system, the result that is obtained with a full wavelet system as is employed in [9] to the result which is obtained when the model reduction strategy is utilised. The accuracy will be seen to be excellent while significant gains in computational speed are achieved. A result is also given when a lower-order wavelet scheme is employed. This result will confirm that for comparable computation time significant gains in accuracy may be achieved by employing the approach proposed in this paper as opposed to simply using a lower-order full wavelet scheme. The particular circuit selected is deliberately chosen as it is strongly nonlinear in nature. The ability to simulate the behaviour of this circuit with such accuracy will give further recommendation for employing a wavelet-based simulation technique as opposed to a Harmonic-Balance based approach.

II. MULTITIME PARTIAL DIFFERENTIAL EQUATION APPROACH

Consider a signal $x(t)$ which is composed of a carrier modulated by an envelope where the envelope signal is

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2004 IEEE Radio Frequency Integrated Circuits Symposium

assumed to be uncorrelated with the carrier. The signal may be represented in two independent time variables as follows:

$$x(t) = \hat{x}(t_1, t_2) \qquad (1)$$

t_1 relates to the low-frequency envelope and t_2 relates to the high-frequency carrier.

Now consider a general nonlinear circuit described by:

$$\dot{x}(t) = f(x(t)) + b(t) \qquad (2)$$

where $b(t)$ is the excitation vector, $x(t)$ are the state variables and f is a nonlinear function.

The corresponding multitime partial differential equation system can be written as shown in [7] as:

$$\frac{\partial \hat{x}}{\partial t_1} + \frac{\partial \hat{x}}{\partial t_2} = f(\hat{x}(t_1, t_2)) + \hat{b}(t_1, t_2) \qquad (3)$$

This multitime partial differential equation can be solved using exclusively time-domain approaches as employed by Roychowdhury [7] or using a combination of time-step integration for the envelope and Harmonic Balance for the carrier as in [8]. However, for strongly nonlinear circuits, the use of harmonic balance for the inner loop can prove limited. In this paper, the multitime partial differential equation system is solved using a pseudo-wavelet collocation method derived from that proposed by Cai and Wang [10]. However, the crucial step introduced in this contribution is the application of a nonlinear model reduction process within the simulation technique. This leads to very significant gains in efficiency but without a complementary loss in accuracy.

III. WAVELET COLLOCATION SCHEME

The technique involves approximating the unknown function $\hat{x}(t_1, t_2)$ with a wavelet series in the t_2 dimension where t_2 is scaled such that $t_2 \in [0, L]$, L>4. I.e.

$$\begin{aligned}
\hat{x}_J(t_1, t_2) &= \bar{x}_{-1,-3}(t_1)\eta_1(t_2) + \bar{x}_{-1,-2}(t_1)\eta_2(t_2) \\
&+ \bar{x}_{-1,-1}(t_1)\phi_b(t_2) + \\
&\sum_{k=0}^{L-4} \bar{x}_{-1,k}(t_1)\phi_k(t_2) + \bar{x}_{-1,L-3}(t_1)\phi_b(L - t_2) \\
&+ \sum_{j=0}^{J-1}\sum_{k=-1}^{n_j-2} \bar{x}_{j,k}(t_1)\psi_{j,k}(t_2) \\
&+ \bar{x}_{-1,L-2}(t_1)\eta_2(L - t_2) + \bar{x}_{-1,L-1}(t_1)\eta_1(L - t_2) \\
&= \sum_{k=1}^{N} \bar{x}_k(t_1)\Psi_k(t_2)
\end{aligned} \qquad (4)$$

where $\phi(t)$ and $\psi(t)$ are scaling and wavelet functions respectively and $\eta(t)$ are spline functions introduced to approximate boundary-nonhomogeneities as described in [10]. $\bar{x}(t_1)$ are the unknown coefficients which are a function of t_1 only. The total number of unknown coefficients is $N = 2^J L + 3$ where J determines the level of wavelet coefficients taken into account when approximating $\hat{x}(t_1, t_2)$. From this point forward, $\Psi_k(t)$ shall be referred to as wavelets where it is understood that these comprise the scaling functions, $\phi(t)$, the wavelet functions, $\psi(t)$ and the nonhomogeneity functions, $\eta(t)$.

Eqn. 3 is then collocated on collocation points in t_2 to result in a semidiscretised equation system. The interpolation points are those as chosen in [10]. At this juncture, the technique differs significantly from that presented in [9]. Instead of directly solving for the unknown state-variables and output $y(t)$ at each time-step in t_1, a nonlinear model reduction strategy is employed. The particular model reduction strategy chosen is based on that presented by Gunupudi and Nakhla in [12]. Thus instead of solving an N^{th} order system at each time-step to obtain the unknown state-variables and the output quantity $y(t)$, a reduced-order system of transformed coefficients is solved. Once the transformed coefficients are determined for the entire time range of interest, the original N coefficients, $\bar{x}(t_1)$ and consequently, the value of the state variables and the output quantity $y(t)$ may be obtained in one single post-processing step. The entire solution process is described in detail in the following section.

IV. MODEL REDUCTION TECHNIQUE

The expression in (4), if written for all collocation points in t_2, may be expressed as follows at a specific point in time t_1:

$$\hat{x}_{JN}(t_1) = \boldsymbol{E}\bar{x}(t_1) \qquad (5)$$

where $\boldsymbol{E}$ is a constant N-dimensional square matrix whose columns comprise the values of the N wavelet functions, $\Psi_k(t_2)$, at N collocation points. The matrix is evaluated once at the outset of the algorithm. $\hat{x}_{JN}(t_1)$ is an N-dimensional column vector of the unknown state-variables and $\bar{x}(t_1)$ is an N-dimensional column vector of the unknown wavelet coefficients at the collocation points in t_2 at a specific instant in t_1.

Substitution of (4) and (5) in (3) yields:

$$\boldsymbol{E}\frac{d\bar{x}}{dt_1} = -\boldsymbol{D}\bar{x} + \boldsymbol{f}_N(\bar{x}) + \boldsymbol{b}_N \qquad (6)$$

where $\boldsymbol{D}$ is an N dimensional matrix whose columns are formed from the derivatives of the wavelet functions in (4) evaluated at each of the N collocation points in t_2. Again, $\boldsymbol{D}$ is evaluated only once at the outset of the algorithm. $\boldsymbol{f}_N$ and $\boldsymbol{b}_N$ are column vectors comprising the values of f and b at the collocation points.

At this point, the vector of coefficients, $\bar{x}(t_l)$, is expanded in a Taylor series as follows:

$$\bar{x}(t_l) = \sum_{i=0}^{\infty} \boldsymbol{a}_i (t_l - t_l^0)^i \tag{7}$$

where t_l^0 is the initial time and where the coefficients, $\boldsymbol{a}_i$, may be computed recursively as in [12].

A Krylov space is formed for $\boldsymbol{a}_i$:

$$\boldsymbol{K} = [\boldsymbol{a}_0 \quad \boldsymbol{a}_1 \quad \cdots \quad \boldsymbol{a}_q] \tag{8}$$

where q is the order of the reduced system and is significantly less than N.
An orthogonal decomposition of $\boldsymbol{K}$ results in:

$$\boldsymbol{K} = \boldsymbol{QR} \tag{9}$$

where $\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}_q$. $\boldsymbol{I}_q$ is the q dimensional identity matrix. $\boldsymbol{Q}$ is then employed to perform a congruent transformation of (5):

$$\bar{x} = \boldsymbol{Q}\hat{\bar{x}} \tag{10}$$

Thus a new reduced equation system is formed as:

$$\boldsymbol{Q}^T\boldsymbol{EQ}\frac{d\hat{\bar{x}}}{dt_l} = -\boldsymbol{Q}^T\boldsymbol{DQ}\hat{\bar{x}} + \boldsymbol{Q}^T\boldsymbol{f}_N(\boldsymbol{Q}\hat{\bar{x}}) + \boldsymbol{Q}^T\boldsymbol{b}_N$$

or
$$\hat{\boldsymbol{E}}\frac{d\hat{\bar{x}}}{dt_l} = -\hat{\boldsymbol{D}}\hat{\bar{x}} + \boldsymbol{Q}^T\boldsymbol{f}_N(\boldsymbol{Q}\hat{\bar{x}}) + \hat{\boldsymbol{b}}_N \tag{11}$$

where $\hat{\boldsymbol{E}} = \boldsymbol{Q}^T\boldsymbol{EQ}$, $\hat{\boldsymbol{D}} = \boldsymbol{Q}^T\boldsymbol{DQ}$ and $\hat{\boldsymbol{b}}_N = \boldsymbol{Q}^T\boldsymbol{b}_N$.

This new system, (11), of dimension q may then be solved to determine $\hat{\bar{x}}$ over the entire time domain of interest. A trapezoidal-rule integration scheme is employed because of its superior stability qualities. Once the q coefficients, $\hat{\bar{x}}$, have been determined, $\bar{x}(t_l)$ and consequently, $\hat{x}_{JN}(t_1) = \boldsymbol{E}\bar{x}(t_1)$, may be obtained in one single post-processing step. The above solution process is thus significantly more efficient than solving directly for $\hat{x}_{JN}(t_l)$ at each time step as was done in [9].

V. SAMPLE SYSTEM

The example considered consists of the highly nonlinear diode rectifier circuit as shown in Fig. 1.
The source is:

$$b(t) = \sin(\frac{2\pi t}{T_1})\sin(\frac{2\pi t}{T_2}) \tag{11}$$

where T_1 corresponds to the envelope period and T_2 corresponds to the carrier period.

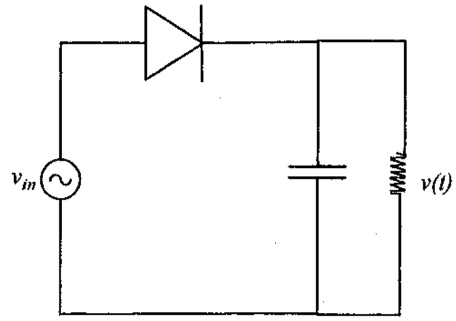

Fig. 1 Diode rectifier circuit

Fig. 2a shows the output from an ordinary differential equation solver with a very short time step in order to obtain a highly accurate version of the output voltage to act as a benchmark for the purposes of confirming the accuracy of the proposed new simulation technique. Fig. 2b shows the output from the wavelet scheme where no model reduction is performed. For this case, L=20 and J=2 in eqn. 4. This results in a system for which N=83. This is obviously a very high-order wavelet scheme and thus is very accurate. Fig. 2c shows the corresponding result with the same wavelet scheme but in this case the model reduction step in included. The system is reduced with q, the reduction level, set equal to 5. The accuracy is seen to be excellent while concomitantly achieving major computational savings. In particular, a 5th order system is now solved instead of an 83rd order system. Finally Fig. 2d shows the result when a lower order full wavelet scheme is employed. In this case, L=5 and J=0 in eqn. 4. This results in an N=8th order system of equations which has similar computational requirements to reduced wavelet scheme with q=5. As can be seen from Fig. 2d, there is a significant loss in accuracy. This result clearly confirms that the approach presented in this paper is significantly better than simply employing a full lower-order wavelet scheme especially when circumstances require high computational efficiency.

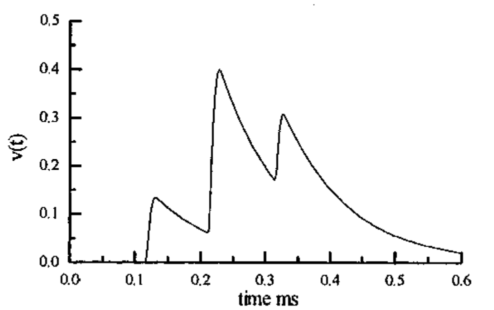

Fig.2a Result from ODE solver with a very short timestep

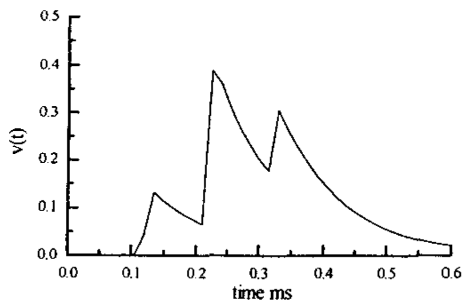

Fig. 2b Result from a full wavelet scheme

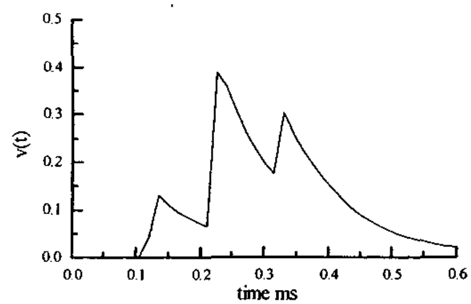

Fig. 2c Result from the proposed new technique

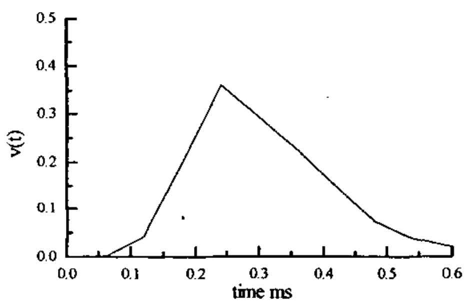

Fig. 2d Result with lower-order full wavelet scheme

VI. CONCLUSION

The paper has presented a highly efficient wavelet based simulation technique for high-frequency circuits. The scheme involves converting the ordinary differential equation system describing the nonlinear circuit to a partial differential equation system. This system is solved using a combination of a nonlinear model reduction strategy and a cubic spline wavelet collocation scheme. The results from a highly nonlinear system indicate the efficiency and accuracy of the proposed approach. The important feature of this method is that the desired trade-off between accuracy and efficiency can be achieved by simply adjusting the wavelet level J and reduction level q.

ACKNOWLEDGMENT

The authors wish to acknowledge the support of IBM for this work.

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