Computation of Electromagnetic Fields inside Three Dimensional Inhomogeneous Dielectrics Using a Buffered Block Forward Backward Algorithm

Marissa Condon* and Bijilash Babu
RINCE, School of Electronic Engineering,
Dublin City University, Dublin 9, Ireland
*Email: marissa.condon@dcu.ie

Abstract:
The paper is concerned with the electromagnetic scattering from a three-dimensional inhomogeneous dielectric object. In particular, the paper compares the use of a Buffered Block Forward Backward (BBFB) algorithm to the use of the commonly employed weak form of the CG-FFT method for the numerical solution of the resultant Electric Field Integral Equation (EFIE). The BBFB method is based on the spatial segmentation of the dielectric into smaller pieces. Results are shown which illustrate the convergence of the algorithm and its superior performance to the CG-FFT.

1. Introduction

The paper is concerned with electromagnetic scattering from an inhomogeneous dielectric. Because of the inhomogeneous nature of the body, the surface equivalence principle cannot be applied and hence the problem is formulated in the form of a volume integral equation. For two-dimensional problems, the solution methods involve the use of the Method of Moments with pulse expansion functions and point matching [1-2]. While initial solution approaches involved the computational and memory intensive numerical inversion of a matrix, more recent work has employed the significantly more efficient iterative techniques such as the Conjugate-Gradient Fast-Fourier Transform (CG-FFT)[3]. For three-dimensional (3D) scatterers, however, the use of the basic Method of Moment method with pulse expansions functions is inappropriate [4]. Hence, approaches such as the weak form of the conjugate gradient method or the Pre-corrected-FFT technique have been proposed [5-7]. In this paper, a physically inspired iterative solution technique is implemented. This method is the Buffered Block Forward Backward (BBFB) iterative method that was introduced and developed in [8-9]. However, in that work, only conducting bodies were examined. The technique may be used in conjunction with any initial formulation - the weak form of the integral equation as proposed in Zwamborn [5] or with the tetrahedral modelling method as proposed in Schaubert et al. [12] and [7]. In this contribution, the method proposed by Zwamborn [5] for discretisation is employed.

2. Formulation

Consider a 3D inhomogeneous dielectric medium illuminated by a monochromatic source. The scattering problem is formulated as a domain integral equation over the object domain. The vectorial position in the three dimensional space is denoted by $\boldsymbol{x} = (x, y, z)$. The complex permittivity of the dielectric at any point is given by:

$$\varepsilon(\boldsymbol{x}) = \varepsilon_r(\boldsymbol{x})\varepsilon_0 + i\frac{\sigma(\boldsymbol{x})}{\omega} \tag{1}$$

Here, ε_r is the relative permittivity of the dielectric with respect to the homogeneous embedding of permittivity ε_0 and σ is the electric conductivity. ω is the angular frequency. The incident electric field is denoted as $\boldsymbol{E}^i = (E^i_x, E^i_y, E^i_z)$.

$$\boldsymbol{E}^i(\boldsymbol{x}) = \varepsilon(\boldsymbol{x})\exp(ik_0\boldsymbol{\theta}.\boldsymbol{x}) \tag{2}$$

The total field in the scattering domain is the sum of the incident and the scattered electric fields. $k_o = \omega\sqrt{(\varepsilon_0\mu_0)}$.

The scattering problem is formulated as a domain integral equation for the unknown electric flux density $\boldsymbol{D} = (D_x, D_y, D_z)$

$$\boldsymbol{E}^i(\boldsymbol{x}) = \frac{\boldsymbol{D}(\boldsymbol{x})}{\varepsilon(\boldsymbol{x})} - (k_0^2 - \nabla\nabla.)\boldsymbol{A}(\boldsymbol{x}) \tag{3}$$

The vector potential $\boldsymbol{A}$ is expressed as:

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{\varepsilon_0}\int_{\boldsymbol{x}'\in v} G(\boldsymbol{x}-\boldsymbol{x}')\chi(\boldsymbol{x}')\boldsymbol{D}(\boldsymbol{x}')d\boldsymbol{x}' \tag{4}$$

where G is the three-dimensional Green's function given by :

$$G(\boldsymbol{x}) = \frac{\exp(ik_0|\boldsymbol{x}|)}{4\pi|\boldsymbol{x}|} \tag{5}$$

χ is the normalised contrast function defined as :

$$\chi(x) = \frac{\varepsilon(x) - \varepsilon_0}{\varepsilon(x)} \tag{6}$$

The domain integral equation (3) is in its strong form and is discretized and converted to its weak form using the testing and expansion procedure described in [5]. A uniform mesh is utilised with grid widths of $\Delta x, \Delta y, \Delta z$ in the x, y, and z directions, respectively. M_m, N_n and P_p are the total number of subdivsions that are chosen in the x, y, and z directions, respectively. The result from discretisation and conversion to weak form is a matrix equation of the form:

$$\boldsymbol{e}^i = \boldsymbol{Ld} \tag{7}$$

where $\boldsymbol{e}^i$ follows from $\boldsymbol{E}^i$ and $\boldsymbol{d}$ follows from $\boldsymbol{D}$ in eqn. (3). $\boldsymbol{L}$ is the matrix relating the quantities $\boldsymbol{e}^i$ and $\boldsymbol{d}$.

3. CG-FFT Solution technique

In theory, the matrix equation in (7) may be solved by inversion of $\boldsymbol{L}$ matrix. However, this is only practical if the dimension of $\boldsymbol{L}$ is small. Thus, this caveat limits its application. To overcome this limitation, iterative methods such as the conjugate gradient method (CGM) [10] or the more advanced CG-FFT method and its various forms [11] have been applied. These methods reduce the computation time and memory requirements significantly. However, there is as yet no specific computational tool deemed to surpass all others. In this paper, the Buffered Block Forward Backward algorithm proposed in [8] and furthered developed in [9] is employed. Results highlight its efficiency, accuracy and ease of use.

4. Buffered Block Forward Backward (BBFB) algorithm.

The essence of the basic BBFB algorithm is to split the scatterer into smaller blocks or subregions numbered from $1{:}M$ or to decompose the $\boldsymbol{L}$ matrix into submatrices, denoted by $\tilde{\boldsymbol{L}}_{ij}$. The submatrices give the interactions between subblocks i and j. A single iteration of the Backward/Forward algorithm involves solving the following two equations:

$$\tilde{\boldsymbol{L}}_{ii}\tilde{\boldsymbol{d}}_{i(k)} = \tilde{\boldsymbol{e}}_i^i - \sum_{j=1}^{i-1}\tilde{\boldsymbol{L}}_{ij}\tilde{\boldsymbol{d}}_{j(k)} - \sum_{j=i+1}^{M}\tilde{\boldsymbol{L}}_{ij}\tilde{\boldsymbol{d}}_{j(k-1)} \tag{8}$$

$$\tilde{\boldsymbol{L}}_{ii}\tilde{\boldsymbol{d}}_{i(k+1)} = \tilde{\boldsymbol{e}}_i^i - \sum_{j=1}^{i-1}\tilde{\boldsymbol{L}}_{ij}\tilde{\boldsymbol{d}}_{j(k)} - \sum_{j=i+1}^{M}\tilde{\boldsymbol{L}}_{ij}\tilde{\boldsymbol{d}}_{j(k+1)} \tag{9}$$

$\tilde{\boldsymbol{e}}^i$ and $\tilde{\boldsymbol{d}}$ are subvectors of $\boldsymbol{e}^i$ and $\boldsymbol{d}$. $\boldsymbol{d}_k$ refers to the kth estimate of $\boldsymbol{d}$.

Equation (8) is termed a forward sweep. Equation (9) is termed a backward sweep. Note that the right hand side of equations (8) and (9) is modified to include the effects of the most up to date estimates of $\tilde{\boldsymbol{d}}_j$.

Now, for 3D analysis, equations (8) and (9) may have to be modified to avoid the inaccuracies that occur due to spurious edge effects [8]. These edge effects arise due to the treatment of each subblock as a physically isolated scatterer. Thus unless the edge of the subblock coincides with an edge of the overall scatterer, edge effects occur. The effects would propagate and consequently, distort the computation if not suppressed. To eliminate this problem, buffer regions immediately adjacent to the subregion are identified. The inclusion of buffer regions suppresses the spurious edge effects that occur with 3D scatterers if the standard form of the forward backward method (eqns. 8 and 9) is employed and thus results in greater accuracy. The revised forward and backward sweeps are:

$$\bar{\boldsymbol{L}}_{ii}\bar{\boldsymbol{d}}_{i(k)} = \bar{\boldsymbol{e}}_i^i - \sum_{j=1}^{i-1}\bar{\boldsymbol{L}}_{ij}\bar{\boldsymbol{d}}_{j(k)} - \sum_{j=i+1,\, j\notin b(i)}^{M}\bar{\boldsymbol{L}}_{ij}\bar{\boldsymbol{d}}_{j(k-1)} \tag{10}$$

$$\bar{\boldsymbol{L}}_{ii}\bar{\boldsymbol{d}}_{i(k+1)} = \bar{\boldsymbol{e}}_i^i - \sum_{j=1,\, j\notin b(i)}^{i-1}\bar{\boldsymbol{L}}_{ij}\bar{\boldsymbol{d}}_{j(k)} - \sum_{j=i+1}^{M}\bar{\boldsymbol{L}}_{ij}\bar{\boldsymbol{d}}_{j(k+1)} \tag{11}$$

where $b(i)$ denotes the buffer region immediately adjacent to the boundary of the subregion, i.

$$\bar{\boldsymbol{L}}_{ii} = \begin{bmatrix} \tilde{\boldsymbol{L}}_{ii} & \tilde{\boldsymbol{L}}_{i,b(i)} \\ \tilde{\boldsymbol{L}}_{b(i),i} & \tilde{\boldsymbol{L}}_{b(i),b(i)} \end{bmatrix} \quad \bar{\boldsymbol{d}}_i = \begin{bmatrix} \tilde{\boldsymbol{d}}_i \\ \tilde{\boldsymbol{d}}_{b(i)} \end{bmatrix} \quad \bar{\boldsymbol{e}}_i^i = \begin{bmatrix} \tilde{\boldsymbol{e}}_i^i \\ \tilde{\boldsymbol{e}}_{b(i)}^i \end{bmatrix}$$

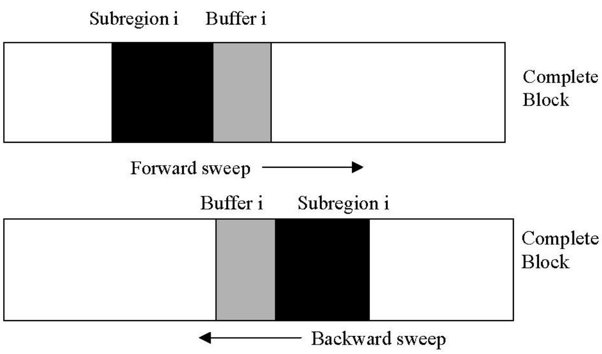

Fig. 1. *The Buffered Block Forward Backward Method.*

Fig. 1 shows the forward backward scheme inclusive of buffer regions. The black regions denote the selected subblock regions during a forward and backward sweep while the grey regions denote the buffer zones.

While the quantity $\tilde{\boldsymbol{d}}_{b(i)}$ is redundant, the minimal amount of increased computational overload results in an accurate algorithm and a far faster level of convergence. Each sweep of the BBFB is solved using the CG method.

5. Recursive implementation

It is possible to apply the BBFB method as just described in a recursive manner to enable efficiency to be maintained as the scatterer size increases. For example, consider a 3D cube (see Appendix). The cube may be considered as a stack of planes or slices. The BBFB method may proceed on a slice by slice basis (ie one slice or plane is chosen as one subregion). However, the computational requirements associated with the solution of eqns. 10 and 11 with this selection may become quite onerous as the size increases and buffers are included. For this reason, the forward backward sweeps in eqns. 10 and 11 may each be treated as local problems. Each slice is divided into strips. The local problem is then solved by a series of forward and backward sweeps across the set of strips in the slice.

6. Convergence Analysis

In order to ascertain if the buffered block forward method will converge for a specific type of segmentation, it is necessary to perform a convergency test. Suppose the BBFB algorithm is applied to a complete block either the complete scatterer (eg. 3D cube) if no recursive step is included or a specific subregion, (e.g. slice of the cube) if recursive steps are required. The block consists of m subblocks. The ith subblock is of size n_i. The forward sweep of the BBFB method may be written as the forward sweep of an **unbuffered** block forward backward sweep as follows where the subscript b refers to the buffer quantities. $\tilde{L}_{ii}$ is an $n_i \times n_i$ matrix.

$$\begin{bmatrix} \tilde{L}_{11} & \tilde{L}_{12} & 0 & 0 & \tilde{L}_{13} & 0 & \cdots & \tilde{L}_{1m} \\ \tilde{L}_{21} & \tilde{L}_{22} & & & & & & \\ \tilde{L}_{21} & & \ddots & & & & & \\ \tilde{L}_{31} & & & & & & & \vdots \\ \vdots & & & & \ddots & & & \\ & & & & & & \ddots & \\ \tilde{L}_{m1} & & & \cdots & & & & \tilde{L}_{mm} \end{bmatrix} \begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_{b1} \\ \tilde{d}_2 \\ \tilde{d}_{b2} \\ \vdots \\ \\ \tilde{d}_m \end{bmatrix} = \begin{bmatrix} \tilde{e}_1^i \\ \tilde{e}_{b1}^i \\ \tilde{e}_2^i \\ \tilde{e}_{b2}^i \\ \vdots \\ \\ \tilde{e}_m^i \end{bmatrix}$$

$$\text{or} \qquad \boldsymbol{L_f d_f} = \boldsymbol{e_f^i} \qquad (12)$$

This forward sweep is that of a **block** forward method. The subscript f denotes the forward sweep. However, it is equivalent to the forward sweep of a ***standard point to point*** forward backward method applied to a preconditioned version of eqn. 12. The preconditioner matrix, $\boldsymbol{P_f}$, is a block diagonal matrix whose elements are formed by taking the inverses of the diagonal blocks in eqn. 12. Performing this preconditioning operation results in the following matrix equation for $\boldsymbol{d}$:

$$\boldsymbol{P_f L_f d} = \boldsymbol{P_f e_f^i} \quad \text{or} \quad \boldsymbol{Yd} = \boldsymbol{A} \qquad (13)$$

Similarly, the backward sweep of the block FB algorithm is equivalent to the backward sweep of a ***standard point to point*** forward backward method applied to a matrix equation of the form:

$$\boldsymbol{Wd} = \boldsymbol{B} \qquad (14)$$

Based on eqns. 13 and 14, the iteration matrix for the BBFB method may be defined as:

$$\boldsymbol{M} = (\boldsymbol{D_W} + \boldsymbol{U_W})^{-1}\boldsymbol{L_W}(\boldsymbol{D_Y} + \boldsymbol{L_Y})^{-1}\boldsymbol{U_Y} \qquad (15)$$

$\boldsymbol{D_W}$, $\boldsymbol{U_W}$ $\boldsymbol{L_W}$ $\boldsymbol{D_Y}$ $\boldsymbol{U_Y}$ $\boldsymbol{L_Y}$ are the diagonal, upper triangular and lower triangular parts of the $\boldsymbol{W}$ and $\boldsymbol{Y}$ matrices, respectively. The error after the complete kth BBFB sweep is $\varepsilon^{(k)} = \boldsymbol{d}^* - \boldsymbol{d}^{(k)}$ where $\boldsymbol{d}^*$ is the exact solution. Hence, the relationship between progressive errors is:

$$\varepsilon^{(k+1)} = \boldsymbol{M}\varepsilon^{(k)} \qquad (16)$$

Consequently, the BBFB method applied to a specific segmentation will converge if the absolute value of the eigenvalues of $\boldsymbol{M}$ is less than 1.

7. Summary of BBFB Algorithm

1. Decide if the scatter size necessitates a recursive implementation.
2. Select the appropriate initial selection of subblocks and associated buffers.
2. Confirm convergence for the selection of subblocks.
3. Apply BBFB as in eqns. 10 and 11.
4. Repeat application of the backward forward sweeps until the tolerance criterion is met.

8. Test Cases and Results

The first test case is a double layered inhomogeneous spherical dielectric with its origin at the center of the scattering domain. The radii of the inner and outer spheres are taken to be $k_0a_1 = 0.163$ and $k_0a_2 = 0.314$, respectively. The relative permittivities and conductivities are taken to be $\varepsilon_{r1} = 72$, $\varepsilon_{r2} = 7.5$ and $\sigma_1 = 0.9$ S/m and $\sigma_2 = 0.05$ S/m respectively. The frequency of operation is taken to be 100 MHz. The incident field is a uniform plane with parameters:

$$\varepsilon_x = 1V/m, \quad \varepsilon_y = 0, \quad \varepsilon_z = 0 \qquad (17)$$

$$\theta_x = 0, \; \theta_y = 0, \; \theta_z = -1 \qquad (18)$$

The second test case is a homogeneous lossless dielectric cube with its origin at the center of the scattering domain and of side length $2a = 0.2\,\lambda_0$. The relative permittivity is taken as $\varepsilon_r = 9.0$. The frequency of operation is 100 MHz. The parameters of the incident wave are:

$$\varepsilon_x = 1V/m, \quad \varepsilon_y = 0, \quad \varepsilon_z = 0 \qquad (19)$$

$$\theta_x = 0, \; \theta_y = 0, \; \theta_z = 1 \qquad (20)$$

The matrix equation in (7) is solved initially for both test cases using the weak CG-FFT method described in [5-6]. The subdivision size or mesh size is 7×7×7. The numerical convergence is measured by finding the normalized root mean square error in satisfying equation (7):

$$Err = \frac{\| \boldsymbol{r(n)} \|}{\| \boldsymbol{r(0)} \|} \qquad (21)$$

where $\boldsymbol{r(n)} = \boldsymbol{e}^{\boldsymbol{i}} - \boldsymbol{Ld_n}$. As stated in [6], this is a global error quantity. (n denotes the nth iteration).

The BBFB method is then applied to both test cases. To compare the methods, a specific level of accuracy is set and the time taken by each method to achieve this is given in Table 1. All computations were performed on a Workstation running with Intel Xeon processors of 3.9 GHz.

The BBFB method is seen to be significantly faster than the CG-FFT Technique.

Table 1. Computation time for test cases

Test Case	Accuracy	Iteration time Needed in CG FFT(Sec)	Iteration time needed in BBFB (Sec)
1	10^{-05}	2	1
2	10^{-05}	10	2

9. Conclusions

The paper has presented a Buffered Block Forward Backward algorithm that is applicable for scattering problems involving 3D inhomogeneous dielectrics. Two test cases are considered - an inhomogeneous lossy dielectric sphere and a homogeneous dielectric cube. Results confirm that a greater level of efficiency for a comparable level of accuracy is achieved using the buffered BBFB method

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Appendix

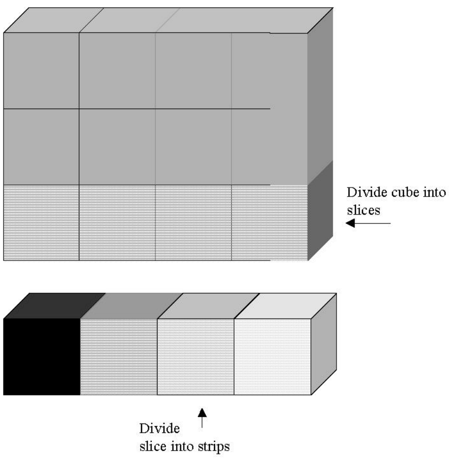

Fig. A1 *Further division of subregions to facilitate greater efficiency*