A NEW TECHNIQUE FOR THE TRANSIENT SIMULATION OF TRANSMISSION LINES INCLUSIVE OF SKIN EFFECT

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ABSTRACT

A new approach is presented for the transient simulation of lossy transmission lines in high-speed circuits. The approach is based on developing a model for the transmission line which is structured around natural modes of oscillation unlike other transmission-line models which are based on travelling waves. The principal advantage of the new approach is that conversion of frequency-domain prototype models for the transmission lines to the time domain for use in circuit simulators is particularly straightforward and obviates the need for numerical convolution. An illustrative case is given to confirm the efficacy of the new approach.

INTRODUCTION

With the rapid advances in high-speed digital circuits and increasing clock frequencies, accurate circuit simulations are now obliged to incorporate the transmission-line behaviour of interconnects. However, transmission lines present difficulties when including them in a transient simulation in a general-purpose circuit simulator. Circuits containing devices with non-linear or time-dependent characteristics must be characterised in the time domain. However, transmission lines with frequency-dependent characteristics are best described in the frequency domain. Many approaches have been suggested to overcome this problem, e.g. references (1-4). However, these approaches, for the most part, have been based on the conventional travelling-wave solution of the Telegraphers' Equations. The approach presented in this paper solves the Telegraphers' Equations in terms of the natural modes of oscillation on the transmission line and results in a model which allows ease of conversion from the frequency domain to the time domain and with minimal loss in accuracy.

FREQUENCY-DOMAIN MODEL

If TEM (Transverse Electromagnetic) mode of wave propagation is assumed, the behaviour of transmission lines is described in the frequency domain by the familiar Telegraphers' Equations:

$$\frac{dV(x)}{dx} = -Z(x)I(x) \tag{1}$$

$$\frac{dI(x)}{dx} = -Y(x)V(x) \tag{2}$$

Equations (1) and (2) may be solved as follows:

$$V(x) = K_1 e^{-\vartheta(x/L)} + K_2 e^{\vartheta(x/L)} \tag{3}$$

$$I(x) = Y_0\{K_1 e^{-\vartheta(x/L)} - K_2 e^{\vartheta(x/L)}\} \tag{4}$$

where $\vartheta = L\sqrt{ZY}$ and $Y_0 = Z^{-1}\sqrt{ZY}$.

This travelling-wave solution forms the basis of many time-domain models for transmission lines.

The approach presented in the current paper differs from the travelling-wave solution and follows that used in reference (5) for modelling a power cable. Equations (1) and (2) are first integrated to yield:

$$V(x) = V(L) + \int_x^L Z(\tau)I(\tau)d\tau \qquad (5)$$

$$I(x) = I(L) + \int_x^L Y(\tau)V(\tau)d\tau \qquad (6)$$

Owing to the difficulty in solving eqns. (5) and (6) analytically, a numerical solution is necessary. The transmission line is therefore divided into K sections where each section is an exact-π representation. So far as the frequency-domain prototype is concerned, dividing the transmission line into a cascade of sections of the type shown in Fig. 1 does not involve any approximation.

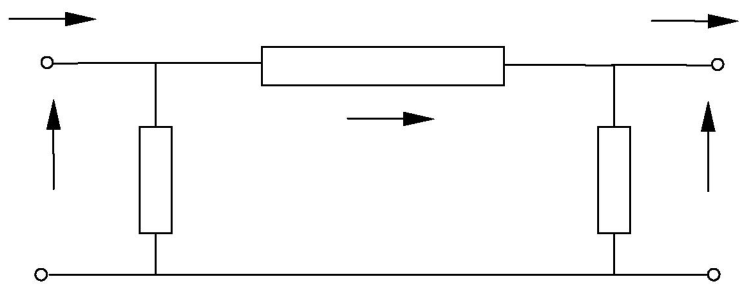


Fig. 1 *Exact-π representation of kth section*

Equations (5) and (6) may now be rewritten as

$$V(k) = V(K) + \sum_{j=k+1}^{K} Z_{aj} I'(j) \qquad k=0,1, \ldots K\text{-}1 \qquad (7)$$

$$I'(k) = I'(K+1) + \sum_{j=k}^{K} (Y_{b,j+1} + Y_{cj})\, V(j) \qquad k=0,1,\ldots K \qquad (8)$$

While it is not a necessary assumption, if the kth section happens to be longitudinally homogeneous then $Z_{ak} = \sinh(l_k\sqrt{ZY})Z_o$ and $Y_{bk} = Y_{ck} = Y_o \tanh\left(\frac{l_k\sqrt{ZY}}{2}\right)$ where l_k is the length of the kth section.

Equations (7) and (8) can be solved as described in reference (5) to obtain the following model for the transmission line:

$$\boldsymbol{I}_B = (\boldsymbol{A} + \boldsymbol{P}\boldsymbol{\zeta}\boldsymbol{g}\boldsymbol{P}_t)\boldsymbol{V}_B \qquad (9)$$

where $\boldsymbol{I}_B = \begin{bmatrix} I_S \\ -I_R \end{bmatrix}$ and $\boldsymbol{V}_B = \begin{bmatrix} V_S \\ V_R \end{bmatrix}$ with $\boldsymbol{A}$, $\boldsymbol{P}$, $\boldsymbol{\zeta}$ and $\boldsymbol{g}$ as defined in reference (5).

The transmission-line model defined by eqn. (9) necessarily yields the same results as the travelling-wave solution when applied in the frequency domain. The advantage of the new structure is that it allows ease of conversion to the time domain and that the bandwidth of the resultant time-domain model is explicit.

The crucial part of the new model is the ***g*** matrix. The elements of ***g*** have the form shown in Fig. 2a. It is seen that up to a specific folding frequency, each element of ***g*** has a single resonant hump. These resonant humps are identified as corresponding to natural *modes of oscillation* within the model. The lowest natural frequency physically corresponds to fundamental natural resonance with common-mode energisation ($V_R = V_S$). The second natural frequency corresponds to second-harmonic natural resonance and so on.

The nature of the elements of $\boldsymbol{\zeta}$ is as shown in Fig. 2b. The first near-singularity defines the folding frequency. This specific folding or Nyquist frequency determines the bandwidth of the time-domain model for the transmission line. All the elements of the ***A***, $\boldsymbol{\zeta}$ and ***g*** matrices are accurately approximated up to this frequency. The ***P*** matrix is a transformation matrix and is independent of frequency and hence translates directly to the time domain. (Note: $\boldsymbol{P}_t$=transpose of ***P***).

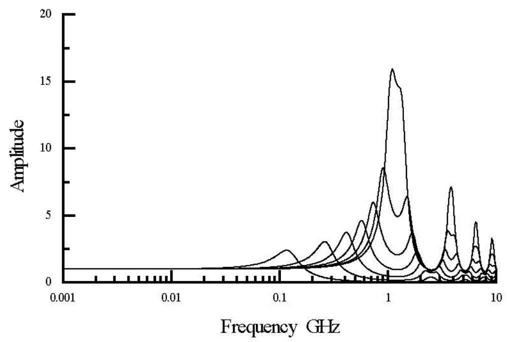


Fig. 2a *Nature of the elements of the* ***g*** *matrix*

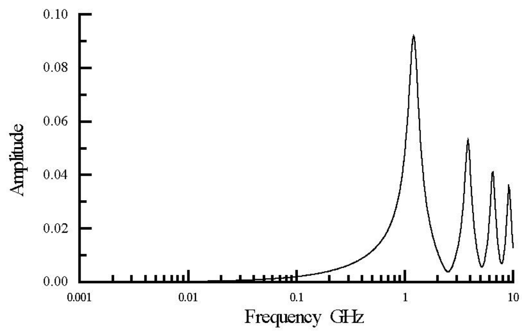


Fig. 2b *Nature of the elements of the* $\boldsymbol{\zeta}$ *matrix*

CONVERSION OF FREQUENCY-DOMAIN MODEL TO THE TIME DOMAIN

For conversion to the time domain, the elements of the diagonal matrices ***g*** and $\boldsymbol{\zeta}$ and the elements of the ***A*** matrix can be approximated using low order *z*-transfer functions. For example, the nature of the elements of the ***g*** and $\boldsymbol{\zeta}$ matrices is such that the following transfer-function forms yield accurate approximations:

$$\bar{g}(z) = \frac{a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}} \qquad \bar{\zeta}(z) = \frac{a_0 + a_1 z^{-1}}{1 + b_1 z^{-1}} \tag{10}$$

The *z*-domain transfer functions translate directly to the time domain and hence no time-consuming convolution is required.

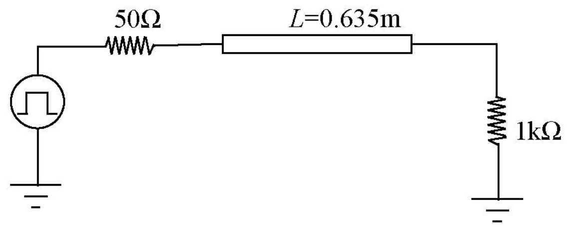
ILLUSTRATIVE CASE

To confirm the efficacy of the technique for the transient simulation of lossy interconnects, a simple illustrative case is considered. The case considered is that of the single transmission line given in Fig. 7 in Kyung Suk Oh (6) and shown in Fig. 3. Skin effect is modelled as in Kyung Suk Oh (6) i.e.

$$R_{skin}(\omega) \approx R_s(1 + j)\sqrt{\omega} \tag{11}$$

The input to the circuit is a digital pulse with a rise time of 1ns and a duration of 20ns as in Kyung Suk Oh (6). An eight-section model was developed for the sample transmission line and converted to the time domain as described above.

Fig. 3 compares the output from the time-domain model with the exact output at the near and far end of the transmission line. As evidenced by the results, a high degree of accuracy is achieved.



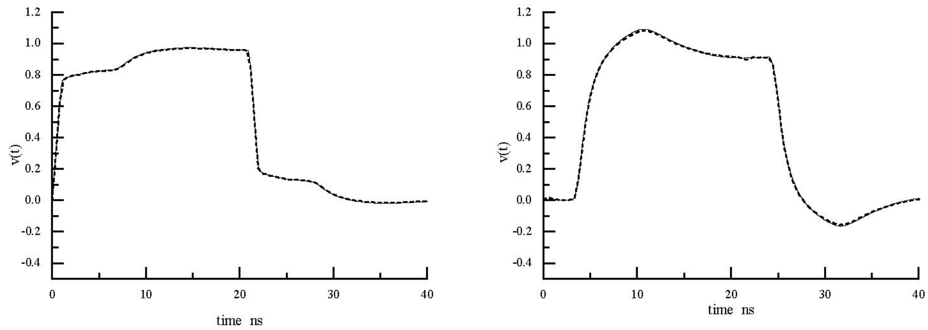


Fig. 3 *Output at near and far end of transmission line with a digital pulse at the input*
—— *Exact result* - - - *New model*

CONCLUSIONS

The paper presents a new approach to the simulation of transmission-line effects in high-speed digital circuits. The approach involves the identification of natural modes of oscillation on the transmission line as opposed to most conventional modelling approaches which are based on a travelling-wave solution. One important advantage of the approach presented in this paper is that the bandwidth of the resultant time-domain model is known. The bandwidth is fixed by the length chosen for the transmission-line sections since this fixes the folding frequency. This allows the computational requirements of the model to be tailored to the accuracy requirements of the simulation. Conversion of the model to the time domain involves z-domain transfer functions which translate directly to the time domain thereby obviating the need for convolution and leading to an efficient time-domain model. An illustrative case confirms the accuracy of the resultant model.

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