Transactions Briefs

Sensitivity Properties of SC Filters Derived from *LC* Ladder Prototypes

THOMAS CURRAN AND MARTIN COLLIER

***Abstract*— The sensitivity properties of a class of switched-capacitor LDI ladder filters is investigated. It is shown that, because of the frequency variation of the terminations in the equivalent circuit of such filters, the low passband sensitivity property of the *LC* prototype filter is not retained. A modification to the input section of the filter is shown to result in low sensitivity.**

I. Introduction

Doubly-terminated passive lossless two-ports can be designed to achieve low sensitivity of the passband loss to component variations [1]. Hence active filter designs are often modeled on such networks, typically on *LC* ladder filters, in an attempt to obtain low sensitivity. In this letter the passband sensitivity of one such filter, the SC LDI ladder filter [2], is investigated. Such a filter uses integrators of the type shown in Fig. 1(a). Filters of this type have been analyzed exactly by Scanlan, who has shown that they can be represented (in the case of low-pass filters of odd order) by the equivalent circuits of Fig. 2 [2]. Here, the two frequency variables used are $\mu = \cosh(sT/2)$ and $\gamma = \sinh(sT/2)$ where $f_s = 1/T$ is the filter sampling rate and s is the Laplace transform frequency variable.

Because of the frequency variation of the terminations in the circuit of Fig. 2(a), the inductor and capacitor values cannot be obtained using classical methods, unless the circuit operation is to be restricted to frequencies much less than the sampling rate, in which case $\mu \simeq 1$. It shall now be demonstrated, by means of a particular example, that the frequency variation of the terminations also affects the sensitivity properties of the filter.

II. SC LDI Ladder Sensitivity

Consider a ninth-order filter, designed to have an equiripple (1.0 dB) passband response, with a cutoff frequency of $f_0 = 0.25 f_s$. Such a filter, designed using a computer program based on the algorithm of [3], has an equivalent circuit with the capacitor and inductor values listed in Table I(a), with $R_S = R_L = 1.0$. Fig. 3 shows its passband (amplitude-squared) response $|H_{21}|^2$ and the sensitivity of $|H_{21}|^2$ to variations in the value of C_2, where the following definition of sensitivity has been used:

$$S_{C_2}^{|H_{21}|^2} = \frac{C_2}{|H_{21}|^2}\frac{\partial |H_{21}|^2}{\partial C_2}. \tag{1}$$

A doubly-terminated *LC* ladder filter, properly designed to the same filter specifications, possesses zero sensitivity to variations in capacitor and inductor values at frequencies where there is zero insertion loss. Fig. 3 shows that this property is not shared by the filter of Fig. 2. The relatively poor sensitivity properties of this filter will now be explained.

Zero sensitivity to variations in element values (other than the terminations) is expected in doubly-terminated lossless reciprocal two-ports at frequencies where the transducer power gain

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The authors are with the School of Electronic Engineering, Dublin City University, Glasnevin, Dublin 9, Ireland.
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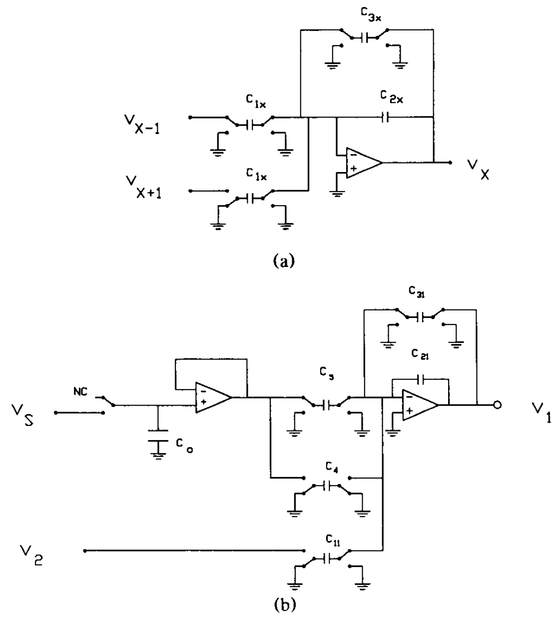

Fig. 1. (a) Integrator used in conventional SC LDI ladder filters. The xth integrator is shown ($1 \leqslant x \leqslant n$). C_{3X} is present only in the first and last integrators. V_{n+1} is zero, V_n is the output voltage and V_0 is the input voltage. Odd- and even-numbered integrators are on opposite phases. (b) Modification to first integrator.

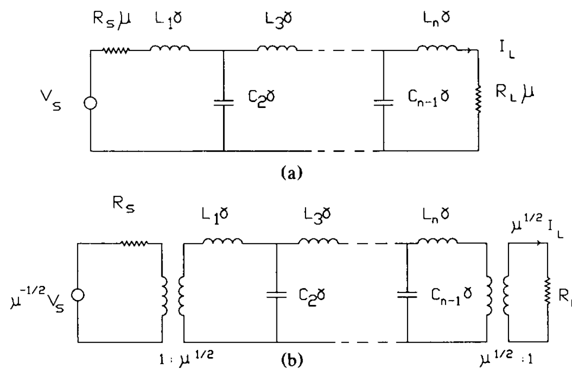

Fig. 2. Equivalent circuits of SC ladder of odd order. (a) Frequency-variable terminations. (b) Frequency-independent terminations.

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TABLE I
EQUIVALENT CIRCUIT ELEMENT VALUES

	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9
(a) Fig. 2:	5.09	1.51	4.48	1.69	4.33	1.77	3.96	1.87	2.08
(b) Fig. 4:	2.89	1.64	4.45	1.69	4.50	1.69	4.45	1.64	2.89

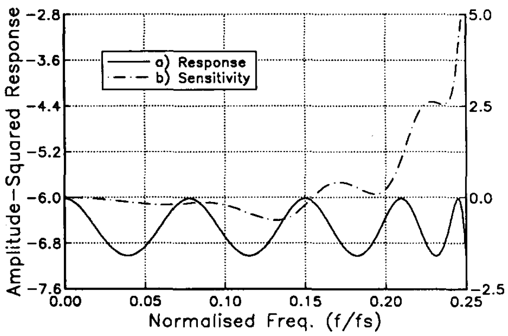

Fig. 3. (a) Equiripple passband response of ninth order filter. (b) Sensitivity of response to value of C_2 in Fig. 2.

$|S_{21}|^2$ is unity, since this value cannot be exceeded. Thus, a ladder filter features low passband sensitivity if designed so that $|S_{21}|^2 = 1.0$ at a number of frequencies in the passband. This can readily be achieved for classical LC ladder filters, by designing for an equiripple passband loss, since $|H_{21}|^2$ is directly proportional to $|S_{21}|^2$ for such a filter. However, for the ladder circuit of Fig. 2, the transfer function of interest is given by $H_{21} = I_1 / V_s$ [2]. Because of the frequency variation of the terminations, it follows that [2]

$$|S_{21}|^2 = 4R_S R_L \cos^2\theta |H_{21}|^2 \tag{2}$$

where $\theta = \pi f / f_s$ and $\cos\theta$ corresponds to the frequency variable μ evaluated for $s = j\omega$. If the filter is designed to achieve an equiripple $|H_{21}|^2$ in the passband, using the appropriate expression presented in [2], then, because of the monotonic decrease in the value of $\cos^2\theta$ as frequency is increased from dc to the Nyquist rate, maximum transducer power gain will be achievable only at zero frequency. This explains why the sensitivity plot in Fig. 3 does not pass through zero at frequencies corresponding to the maxima in the transfer function.

III. DESIGNING FOR LOW SENSITIVITY

The filter of Fig. 2 features low sensitivity if designed for equiripple transducer power gain in the passband. This can be done, using the concept of the auxiliary network, as defined in [2]. The auxiliary network of the ladder in Fig. 2 has a transfer function H'_{21} of the form $H'_{21} = (4R_S R_L)^{-1/2} S_{21}$ and has the same element values, and the same transducer power gain, as the circuit of Fig. 2. It is equivalent (for a ninth-order filter) to a cascade of eight unit elements, followed by a series short-circuited stub [2]. For such a network, an equiripple passband response can be obtained using the expression

$$|S_{21}(\theta)|^2 = \left[1 + \epsilon^2 F_n^2(\theta)\right]^{-1} \tag{3}$$

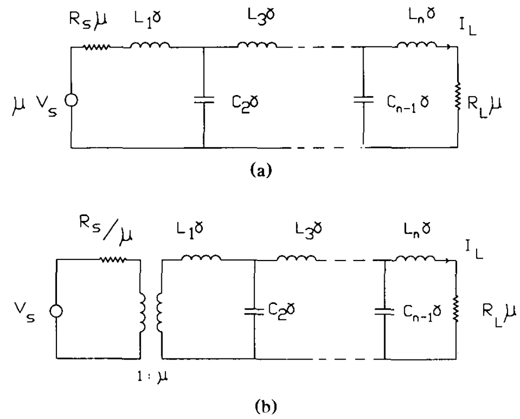

Fig. 4. Equivalent circuits of the modified filter. (a) With prefilter of transfer function μ. (b) As doubly-terminated two-port.

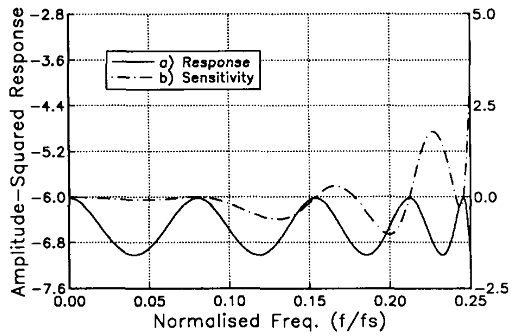

Fig. 5. (a) Equiripple passband response of modified ninth-order filter. (b) Sensitivity of response to value of C_2 in Fig. 4.

where

$$F_n(\theta) = \cosh(n-1)\Phi \cosh\eta + \sinh(n-1)\Phi \sinh\eta \tag{4}$$

with $\cosh\Phi = \sin\theta / \sin\theta_0$, $\cosh\eta = \tan\theta / \tan\theta_0$ and $\theta_0 = \pi f_0 / f_s$ [4]. It follows that, by synthesizing this auxiliary network, and hence obtaining values for the elements in Fig. 2, a circuit with equiripple $|S_{21}|^2$ can be obtained.

The frequency response of this filter must now be corrected so as to yield an equiripple $|H_{21}|^2$. To do this, the input voltage is multiplied by $\cos\theta$ before being applied to the circuit of Fig. 2, i.e., the filter is now of the form shown in Fig. 4(a). An alternative representation of this filter is that shown in Fig. 4(b). The transducer power gain of this filter is

$$|S_{21}|^2 = P_0 / P_{in}(\max) = 4R_S R_L |H_{21}|^2 \tag{5}$$

since the power delivered to the load and the maximum power which can be supplied in the source in this circuit are, respectively,

$$P_0 = |I_L|^2 R_L \cos\theta; \quad P_{in}(\max) = |V_S|^2 / (4R_S / \cos\theta).$$

Hence, the new filter has $|H_{21}|^2$ directly proportional to $|S_{21}|^2$, as required to achieve low sensitivity. Synthesizing such a filter, for the same specifications as chosen earlier, results in the element values of Table I(b). Fig. 5 shows the passband response of the filter, and the sensitivity of the passband response to variations in the value of C_2. It can be seen that this filter possesses lower passband sensitivity to the value of C_2 than the earlier design, and this can be shown to be true for the remaining inductors and capacitors.

To simulate the filter of Fig. 4 requires only a simple change to the original SC filter, namely the addition of an extra switched capacitor (and a sample/hold at the input) to the first integrator as shown in Fig. 1(b). In fact, the filter is now of the form considered in [5]. Since the sensitivity of the transfer function with respect to the capacitance ratios in, for example, the second integrator, is related to the sensitivity with respect to C_2 by the equivalence $C_2 = 2C_{22}/C_{12}$, it follows that setting $C_{12} = 1$ results in

$$\frac{\partial |H_{21}|^2}{\partial C_{22}} = 2\frac{\partial |H_{21}|^2}{\partial C_2} \tag{6}$$

so that a low-sensitivity equivalent circuit implies a low-sensitivity SC realization. An extra capacitance ratio C_5/C_4 is introduced in Fig. 1(b), since these two capacitors replace a single capacitor of value C_{11} in Fig. 1(a). The term μ is realized in Fig. 1(b) by the expression

$$T(z) = C_4/C_{11}\left(z^{1/2} + C_5/C_4 z^{-1/2}\right)$$

with C_4(nominal) = C_5(nominal) = $1/2C_{11}$. The transfer function from V_S to V_1 in Fig. 1(b) equals $T(z)$ times the corresponding transfer function in Fig. 1(a). It can be shown that

$$S^{|H_{21}|^2}_{(C_5/C_4)} = 1 \tag{7}$$

for all frequencies. Thus to a first-order approximation, a mismatch in the values of C_4 and C_5 affects only the gain level of the filter, and not the frequency response.

VI. Conclusions

This paper has shown why a class of SC LDI ladder filter does not retain the low-sensitivity property of the prototype LC ladder. Only the low-pass case has been considered, although, since the sensitivity appears to deteriorate as frequency increases towards the Nyquist limit, this phenomenon is expected to be of greater significance in bandpass and highpass filter designs. It has been demonstrated how the introduction of an extra switched capacitor at the input can result in lower passband sensitivities to the capacitance ratios, including the newly introduced capacitance ratio.

References

[1] H. Orchard, "Inductorless filters," *Electron. Lett.*, vol. 2, pp. 224–225, June 1966.

[2] S. O. Scanlan, "Analysis and synthesis of switched-capacitor state-variable filters," *IEEE Trans. Circuits Syst.*, vol. CAS-28, pp. 85–93, Feb. 1981.

[3] H. Baher and S. O. Scanlan, "Stability and exact synthesis of low-pass switched-capacitor filters," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 488–492, July 1982.

[4] H. J. Carlin, "Distributed circuit design with transmission line elements," *Proc. IEEE*, vol. 59, pp. 1059–1081, July 1971.

[5] R. B. Datar and A. S. Sedra, "Exact design of strays-insensitive switched-capacitor ladder filters," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 888–898, Dec. 1983.

The Analytic Signal in the Hartley Transform Domain

SOO-CHANG PEI AND SY-BEEN JAW

Abstract —**Recently, the Hartley transform has been considered as an interesting alternative to the Fourier transform for spectrum analysis and convolution of real signals. A new analytic signal in the Hartley transform domain can be similiarly defined as the Fourier analytic signal. This analytic signal Hartley spectrum is simpler and much more appropriate for signal analysis than the conventional one.**

I. Introduction

Fourier analytic signal is useful in many signal processing and communication applications such as bandpass sampling, narrow-band modulation, envelope detection, generation of one-sided frequency spectra, etc. In this paper, a new analytic signal is proposed in the Hartley transform domain. It's more closely related to the Fourier transform form, and much more appropriate for signal analysis than the conventional Hartley spectrum.

II. The Hartley Analytic Signal-Continuous Case

For a real signal $v(t)$, the well-known Fourier transform of $v(t)$ is

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft}\, dt. \tag{1}$$

Recently, the Hartley transform [1]–[3] has been studied as an another alternative for spectrum analysis, and is defined as below:

$$H_v(f) = H\{v(t)\} = \int_{-\infty}^{\infty} v(t)[\cos 2\pi ft + \sin 2\pi ft]\, dt \tag{2a}$$

$$= V_{\mathrm{Re}}(f) - V_{\mathrm{Im}}(f) \tag{2b}$$

where $V_{\mathrm{Re}}(f)$ and $V_{\mathrm{Im}}(f)$ are the real and imaginary parts of the Fourier transform $V(f)$, respectively.

Assume $\hat{v}(t)$ is the Hilbert transform of $v(t)$. The analytic signal $z(t)$ associated with the real signal $v(t)$ is expressed as follows:

$$z(t) = v(t) + j\hat{v}(t). \tag{3}$$

It's a well-known effect that the Fourier spectra of the analytic signal $z(t)$ are zero for negative frequencies, i.e.,

$$\begin{aligned} Z(f) &= 2\cdot V(f), && f>0 \\ &= V(f), && f=0 \\ &= 0, && f<0. \end{aligned} \tag{4}$$

Also, the Fourier transform of the Hilbert transform $\hat{v}(t)$ of $v(t)$ is

$$\begin{aligned} \hat{V}(f) &= j\,\mathrm{sgn}(f)\cdot V(f) = -jV(f), && f>0 \\ &= 0, && f=0 \\ &= jV(f), && f<0 \end{aligned} \tag{5}$$

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The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

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