A new simulation technique for RF oscillators

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A new method for simulating radio-frequency (RF) oscillators is presented. The asymptotic numeric method enables the efficient simulation of the transient response of such oscillators. Simulation results are provided for a practical RF oscillator circuit to validate the approach.

Introduction

The study of phase-noise in oscillators and the design of new circuit topologies necessitates an efficient technique for the simulation of oscillators. While numerous approaches have been developed over the years e.g. [1-3], each has its own merits and demerits. In this contribution, an asymptotic numeric method developed in e.g. [4-5] is applied to the simulation of RF oscillators. The method is closely related to the stroboscopic and high-order averaging method in [6] and the Heterogeneous Multiscale Methods in [7]. The method is advantageous in that the same methodology can be applied for the simulation of general circuit problems involving highly oscillatory ordinary differential equations, partial differential equations and delay differential equations. Furthermore and counter-intuitively, its efficacy improves with increasing frequency, a feature that is very favourable in modern communications systems where operating frequencies are ever rising. Results for a CMOS oscillator will confirm the validity and efficiency of the proposed method.

Methodology

Our concern is with highly oscillatory equations involving frequency ω:

$$\frac{d^n v}{dt^n} + \beta_1 \frac{d^{n-1} v}{dt^{n-1}} \cdots \beta_{n-1} v = f(v) \tag{1}$$

where *n* is the highest order derivative present. In particular, note that the some of the coefficients $\beta_1 \cdots \beta_{n-1}$ will be polynomials in ω. *f* is a nonlinear function.

The method proceeds by employing a Modulated Fourier Expansion for v

$$v = \sum_{s\geq 0} \frac{\psi_s(t)}{\omega^s} = \sum_{s\geq 0} \frac{1}{\omega^s} \sum_{m\in Z} p_{s,m}(t) e^{im\omega t} \tag{2}$$

This is substituted in (1).

Separating orders of magnitude (powers of ω) and then frequencies (values of m) leads either to non-oscillatory ordinary differential equations (ODEs) or recursions for the coefficients $p_{s,m}(t)$. In this manner, the oscillations are removed from the numerical process and only added at the end when assembling the Modulated Fourier Expansion for v. This means that the high-frequency oscillations no longer place impractical restrictions on the step size employed in the numerical integration for the simulation of the oscillator.

Test example: To illustrate the proposed method, a CMOS oscillator similar to that in [8] is chosen and the equation describing its behaviour around an equilibrium point is

$$\frac{d^3 v}{dt^3} + \alpha \frac{d^2 v}{dt^2} + \omega^2 \frac{dv}{dt} = \beta\left(\gamma v + \delta v^2\right) \tag{3}$$

Where $\alpha = \frac{R_s}{L_s}$, $\omega^2 = \frac{1}{L_s C}$, $\beta = -\frac{1}{L_s C_1 C_2}$, $\gamma = 2\sqrt{\frac{k_n'}{2}\frac{W}{L} I_0}$, $\delta = \frac{k_n'}{2}\frac{W}{L}$ and $C^{-1} = C_s^{-1} + C_1^{-1} + C_2^{-1}$. k_n' and W and L are transistor parameters. The parameter values are selected to have a very-high frequency oscillation present and confirm the merits of the proposed method with such oscillators.

Equation (2) is substituted into (3) and the orders of magnitude are compared. The ω^3 terms are compared first as this is the highest order of ω present for this oscillator.

$$p_{0,m}(1-m^2)=0 \tag{4}$$

From (4), the only non-zero $p_{0,m}$ terms are $p_{0,0}, p_{0,1}, p_{0,-1}$.

On comparing the ω^2 terms

$$-3m^2 p'_{0,m} - \alpha m^2 p_{0,m} + p'_{0,m} - im^3 p_{1,m} + imp_{1,m} = 0 \tag{5}$$

Three differential equations result for m=0, ± 1. For $|m|>1$, $p_{1,m}=0$ as $p_{0,m}=0$ when $|m|>1$ from the previous level (i.e. from the results identified in eqn. 4).

The equations may be solved analytically to give

$$p_{0,0}=c_0 \tag{6}$$

where c_0 is a constant determined by the initial conditions and

$$p_{0,\pm1}(t)=e^{-\frac{\alpha}{2}t}p_{0,\pm1}(0) \tag{7}$$

Similarly, on comparing ω^1 terms

$$3imp''_{0,m} - 3m^2 p'_{1,m} - im^3 p_{2,m} + 2im\alpha p'_{0,m} - \alpha m^2 p_{1,m} + p'_{1,m} + imp_{2,m} = 0 \tag{8}$$

Again, three differential equations result for m=0, ± 1. For $|m|>1$, $p_{2,m}=0$ as $p_{0,m}=0$ and $p_{1,m}=0$ when $|m|>1$ from the previous levels.

The analytical solutions are

$$p_{1,0}=c_1 \tag{9}$$

where c_1 is a constant and

$$p_{1,\pm1}(t) = e^{-\frac{\alpha}{2}t} p_{1,\pm1}(0) + e^{-\frac{\alpha}{2}t}\left(\frac{\mp i\alpha^2}{8}\right) t p_{0,\pm1}(0) \qquad (10)$$

For the ω^0 terms

m=0

$$\begin{aligned} & p'''_{0,0} + \alpha p''_{0,0} + p'_{2,0} = \\ & \beta\gamma p_{0,0} + \beta\delta(p^2{}_{0,0} + 2p_{0,1}p_{0,-1}) \end{aligned} \qquad (11)$$

m=± 1

$$\begin{aligned} & p'''_{0,\pm1} \pm 3ip''_{1,\pm1} - 2p'_{2,\pm1} + \alpha p''_{0,\pm1} \pm 2i\alpha p'_{1,\pm1} - \alpha p_{2,\pm1} = \\ & \beta\gamma p_{0,\pm1} + \beta\delta(2p_{0,0}p_{0,\pm1}) \end{aligned} \qquad (12)$$

m=±2

$$p_{3,\pm2}(t) = \mp\frac{\beta\delta p_{0,\pm1}^2}{6i} \qquad (13)$$

In this case, differential equations occur for *m*=0,± 1 and recursive equations for *m*=±2. Again, equations 11-12 may be solved analytically.

The process may be continued by matching decreasing orders of powers of ω to achieve a higher level of accuracy.

For the example oscillator, it was possible to solve the differential equations analytically. However, even if this was not the case, the equations are non-oscillatory and hence, numerical integration with an acceptable step size may be employed.

Results

Fig. 1 shows the result obtained with the proposed method matching powers of ω to the ω^0 level. It is superimposed on the result from an ODE solver in MATLAB. For the ODE solver, the absolute and relative tolerances were set to 1×10^{-16}. Note, from the figure, the accuracy of the proposed method without having to resort to using numerical integration with very small and inefficient stepsizes. Matching additional powers in ω results in greater accuracy.

An important feature of the method is that its accuracy increases with frequency while matching the same number of powers of ω. Fig. 2 shows the result when the oscillation frequency is increased by a factor of ~10. Note the excellent match in the results.

Conclusion

The letter has presented a new technique for the efficient simulation of RF oscillators. The technique involves expansion of the solution in inverse powers of ω together with modulated Fourier expansions. The technique obviates the need for the exceedingly slow and inefficient use of very small step sizes in numerical integration. A further feature of the method is that its performance improves with increasing frequency. Consequently, it is most suited for the simulation of high-frequency oscillators in state-of-the-art communication systems.

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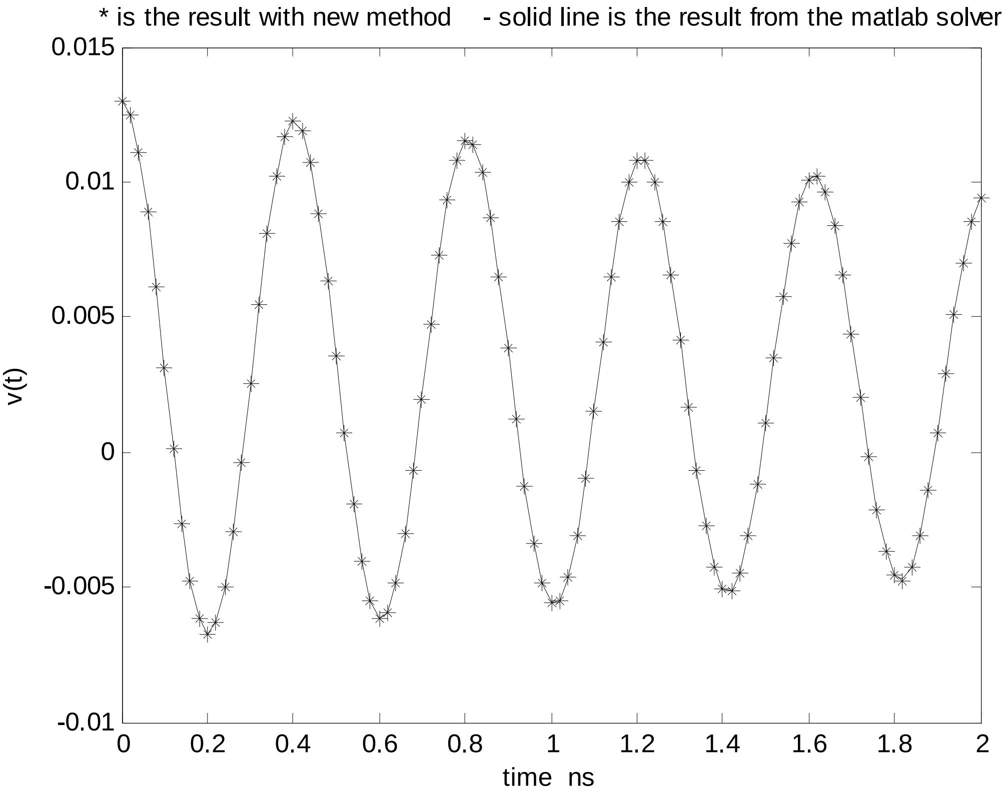
Figure captions:

Figure 1: Comparison between result from MATLAB ODE solver and new method matching terms up to order ω^0

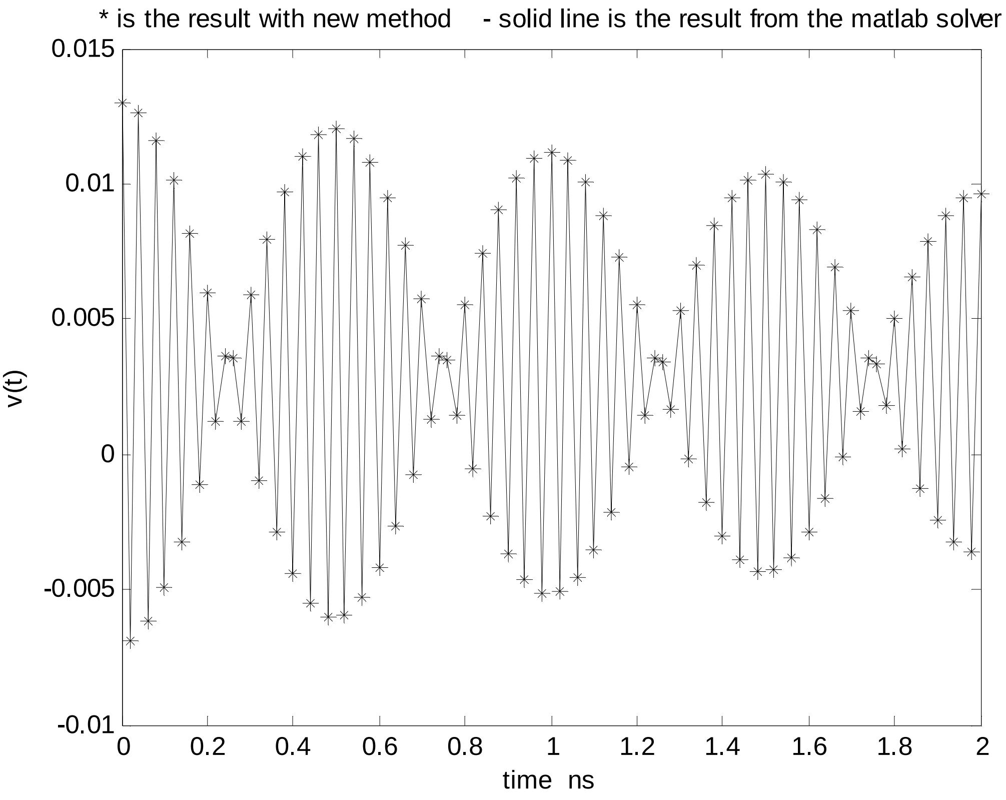
Figure 2: Comparison between result from MATLAB ODE solver and new method matching terms up to order ω^0 when the oscillation frequency is increased by a factor of ~10.

Figure 1



* is the result with new method - solid line is the result from the matlab solver
0.015
0.01
0.005
0
-0.005
-0.01
v(t)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time ns

Figure 2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