## DCU

# The distributions of stellar remnants in disk galaxies 

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## Contents

1 Introduction ..... 1
2 Observing Runaways ..... 4
2.1 Introduction ..... 4
2.2 The Sample ..... 6
2.2.1 Constraints ..... 7
2.2.2 Work on the Sample ..... 8
2.3 Results ..... 16
2.3.1 Edge-on Sample ..... 16
2.3.2 Face-on sample ..... 20
2.4 Discussion ..... 24
3 Modelling Runaways ..... 26
3.1 Introduction ..... 26
3.2 Initial Distribution ..... 27
3.2.1 Distributions within the disk ..... 27
3.2.2 Velocity Kicks ..... 34
3.2.3 Stellar masses ..... 36
3.3 The Potential ..... 37
3.3.1 Description of potential ..... 37
3.4 Runge-Kutta-Fehlberg Method for Stellar Orbits ..... 38
3.4.1 Runge-Kutta Methods ..... 39
3.4.2 Applying methods to Orbits ..... 41
3.4.3 RKF Orbits ..... 43
3.4.4 Example application of an RKF method ..... 43
3.4.5 Testing the potential/method ..... 46
3.5 Results ..... 50
3.5.1 Face-on ..... 50
3.5.2 Edge-on ..... 51
3.6 Discussion ..... 56
4 Modelling the dispersion of HMXBs ..... 58
4.1 Introduction ..... 58
4.2 Population Synthesis ..... 61
4.2.1 The binary population ..... 61
4.2.2 The SN ..... 65
4.2.3 X-ray Luminosity ..... 67
4.3 Output ..... 70
4.3.1 Starburst Model ..... 70
4.3.2 Continuous Starburst Model ..... 71
4.4 The Dynamical Model ..... 73
4.4.1 Constant pattern speed, $\Omega_{p}$ ..... 74
4.4.2 Constant pitch angle, $\alpha$ ..... 89
4.5 Discussion ..... 97
A Runaway Model ..... A1
A. 1 Derivation of $R_{i}$ ..... A1
A. 2 Derivation of $z_{i}$ ..... A3
A. 3 Derivation of $M_{i}$ ..... A4
A. 4 Derivation of Terms from Galactic Potential ..... A5
A.4.1 The Acceleration Due to Gravity ..... A6
A.4.2 The Rotation Curve ..... A7
B HMXB Model ..... B1
B. 1 Dispersions from the spiral arm ..... B1
C SN Sample ..... C1

## List of Figures

2.1 Geometry of galaxies in the edge-on sample ..... 8
2.2 Estimating errors in $z_{S N}$ (a) ..... 11
2.4 Estimating errors in $z_{S N}(\mathrm{~b})$ ..... 12
2.3 SN2007ac in UGC10550 ..... 12
2.5 Correcting for projection effects in inclined galaxies ..... 14
2.6 The scaleheight of CC SNae ..... 16
2.7 SN distances above the plane of their host galaxy ..... 16
2.8 Runaway candidates extracted from the photometric method ..... 18
2.9 SNae in areas of low surface brightness ..... 19
2.10 Surface density distribution of SNae of the face-on sample ..... 20
2.11 Type Ib/c - II host galaxies ..... 21
2.12 Surface density distribution of type Ib/c - II SNae ..... 21
2.13 Shaw effect ..... 22
2.14 Shaw effect for type Ia SNae ..... 23
2.15 Normalised Shaw effect ..... 23
2.16 Type Ib/c/II host galaxies - edge-on sample ..... 24
3.1 The radial distribution of test particles ..... 30
3.2 The height distribution of test particles ..... 31
3.3 3D distribution of test particles ..... 33
3.4 Distribution of kick velocities ..... 35
3.5 The rotation curve due to the potential ..... 39
3.6 A simple circular and elliptical orbit ..... 46
3.7 Orbits in the galactic plane for varying $v_{R}$ ..... 47
3.8 Orbits out of the galactic plane at the solar radius $\left(R_{\odot}=8 \mathrm{kpc}\right)$ ..... 48
3.9 Orbits defined by the parameters in Table 3.4 ..... 49
3.10 Trajectories for a sample of test particles ..... 50
3.11 Final distribution of radial distances of test particles ..... 51
3.12 Final surface density distributions of test particles ..... 52
3.15 The initial/final vertical distributions for various scaleheights ..... 55
3.16 Converging initial/final scaleheights ..... 55
4.1 Logarithmic spiral imposed on M51 ..... 59
4.2 The co-rotation radius ..... 60
4.3 Distribution of primary/secondary stellar masses and mass ratios ..... 62
4.4 Distribution of semi-major axes and eccentricities ..... 63
4.5 Gaussian distribution of pulsar velocity kicks ..... 66
4.6 Disruption rates ..... 71
4.7 Population levels for the various kick distributions ..... 72
4.8 Spiral structure at different epochs for $\alpha=10^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1} 75$
4.9 Spiral structure at different epochs for $\alpha=20^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 76
4.10 Spiral structure at different epochs for $\alpha=30^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 76
4.11 Populations of HMXBs compared to the pre-SN binaries at different epochs (A) $-\alpha=10^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 78
4.12 Populations of HMXBs compared to the pre-SN binaries at different epochs (B) $-\alpha=10^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 79
4.13 Populations of HMXBs compared to the pre-SN binaries at different epochs (A) $-\alpha=20^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 80
4.14 Populations of HMXBs compared to the pre-SN binaries at different epochs (B) $-\alpha=20^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 81
4.15 Populations of HMXBs compared to the pre-SN binaries at different epochs (A) $-\alpha=30^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 82
4.16 Populations of HMXBs compared to the pre-SN binaries at different epochs (B) $-\alpha=30^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 83
4.17 Increased sample of HMXBs for $\alpha=10^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 84
4.18 Increased sample of HMXBs for $\alpha=20^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 85
4.19 Increased sample of HMXBs for $\alpha=30^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 85
4.21 Dispersion in $\phi$ for $\alpha=10^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 88
4.22 Dispersions in $\phi$ for a constant pattern speed ..... 89
4.23 Spiral structure at different epochs for $\alpha=20^{\circ}$ and $\Omega_{p}=10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1} 9$ ..... 90
4.24 Spiral structure at different epochs for $\alpha=20^{\circ}$ and $\Omega_{p}=50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 90
4.25 Populations of HMXBs compared to the pre-SN binaries at different epochs (A) $-\alpha=20^{\circ}$ and $\Omega_{p}=10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 92
4.26 Populations of HMXBs compared to the pre-SN binaries at different epochs (B) $-\alpha=20^{\circ}$ and $\Omega_{p}=10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 93
4.27 Populations of HMXBs compared to the pre-SN binaries at different epochs (A) $-\alpha=20^{\circ}$ and $\Omega_{p}=50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 94
4.28 Populations of HMXBs compared to the pre-SN binaries at different epochs (B) $-\alpha=20^{\circ}$ and $\Omega_{p}=50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 95
4.29 Increased sample of HMXBs for $\alpha=20^{\circ}$ and $\Omega_{p}=10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 96
4.30 Increased sample of HMXBs for $\alpha=20^{\circ}$ and $\Omega_{p}=50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... 96
4.31 Dispersions in $\phi$ for a constant pitch angle ..... 97
B. 2 Dispersion in $\phi$ for $\alpha=20^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... B3
B. 4 Dispersion in $\phi$ for $\alpha=30^{\circ}$ and $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... B5
B. 6 Dispersion in $\phi$ for $\alpha=20^{\circ}$ and $\Omega_{p}=10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... B7
B. 8 Dispersion in $\phi$ for $\alpha=20^{\circ}$ and $\Omega_{p}=50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ..... B9

## List of Tables

2.1 Sample parameters and units ..... 8
2.2 Runaway Candidates ..... 19
3.1 Sample of confirmed BSS runaways (Hoogerwerf et al., 2001) ..... 34
3.2 Parameters of the galactic potential ..... 37
3.3 Butcher tableau ..... 41
3.4 Parameters defining the orbits in Figure 3.9 ..... 47
3.5 Initial/final scaleheights ..... 57
4.1 Co-rotation radii ..... 60
4.2 Varying kick velocity distributions and their corresponding disruption rates ..... 70
C. 1 Sample of SNae for the edge-on study ..... C2
C. 2 Sample of SNae for the face-on study ..... C3


#### Abstract

Data from the Asiago SN catalogue is used to study the distributions of CC SNae about their host galaxy. The vertical distribution of CC SNae in edge-on galaxies is fit with a sech ${ }^{2}$ profile with scaleheight $0.49 \pm 0.08 \mathrm{kpc}$. A photometric method is developed and applied to the sample and a separate sample of six runaway candidates is found. The radial distribution of CC SNae is also studied for the case of face-on disk galaxies. The surface density distribution is fit with and exponential profile with scalelength $0.67 \pm 0.18 R_{\text {Gal }}$. A selection effect is confirmed to be present in the centres of disk galaxies and taking this into account we tentatively fit the surface density distribution with a scalelength of $0.35 \pm 0.05 R_{\text {Gal }}$. The selection effect is shown to increase in strength for ever distant galaxies. The dynamics of massive runaway stars is modelled using a (fifth and sixth order) Runge-Kutta scheme and a galactic potential that produces a flat rotation curve. We find that a velocity kick imparted during the SN of a former binary companion is generally not enough to displace the SN progenitor to distances beyond 2 kpc above the plane of the disk for progenitors witha disk scaleheight of $<0.4 h_{z}$. The dispersion of HMXBs from their birthplace in the spiral arms is also modelled. For this purpose we develop a population synthesis model which includes the effects of mass transfer and tidal interactions to evolve a sample of binary systems. A kick is then applied to the SN remnant of one of the components in the binary. We determine the disruption rate of these binaries for various kick velocity distributions. The undisrupted binaries are then assumed to produce X-ray emission due to a wind fed mechanism and we use these objects to populate a dynamical model. We find that there is no obvious dispersion from the spiral pattern and show that HMXBs are an excellent tracer for recent star formation.


## Chapter 1

## Introduction

Supernova events are important areas of study when looking at many different astrophysical phenomena. They are important for areas such as nucleosynthesis and galactic chemical enrichment, determining the origins of neutron stars and black holes, the study of cosmic rays, the disruption of the interstellar medium (ISM) and subsequent star formation and their use as cosmological standard candles.

They can be set into separate classes and subclasses depending on their spectra (and the temporal evolution of the spectra) and their photometric characteristics. There are two main classes established by Minkowski (1941) which are denoted Type I/II SNae and are based on clear spectral differences. Type I SNae show no obvious Hydrogen in their spectra (although in some cases it may appear to be due to contamination of the spectra). Type II SNae show prominent Hydrogen lines which can vary temporally.

SNae II, Ib and Ic have never been found in elliptical/S0 galaxies and are usually found in HII regions (in or around spiral arms and OB associations) which suggests that their progenitors were massive stars with $\mathrm{M} \gtrsim 8-10 M_{\odot}$ (Huang, 1987). SNae Ia tend to occur in all types of galaxies. They are more common in spirals and are randomly distributed with no particular region having an excess (McMillan and Ciardullo, 1996). Della Valle (1994) showed that the luminosity of SNae Ia increases as one moves from early to late Hubble type galaxies. Studies have shown that the progenitors of SNae Ia are probably massive stars with $M \gtrsim 4-7 \mathrm{M}_{\odot}$ of an intermediate age $\sim 1-5 \times 10^{8}$ Yrs (Oemler and Tinsley, 1979). Woosley (1986) showed that the SNae Ia originate in binary systems with a white dwarf rich in carbon and oxygen. The white dwarf in this case then accreted matter from its companion and then underwent a thermonuclear runaway resulting in the SN explosion.

As mentioned above, the progenitors of $\mathrm{SNae} \mathrm{II} / \mathrm{Ib} / \mathrm{Ic}$ are thought to be more massive stars than the progenitors of SNae Ia that underwent an Iron core collapse (CC) (usually) leaving behind a neutron star or black hole (Brown and Bethe, 1994). Due to the absence of Hydrogen in the spectra of SNae $\mathrm{Ib} / \mathrm{c}$ it is thought that the progenitors of these explosions somehow lost their Hydrogen envelopes either through mass loss in a binary system or through very strong stellar winds (WR stars). The lack of Helium in SNae Ic suggests that the Helium envelope in these stars was also lost at some stage in their stellar evolution. Since stellar winds grow stronger with increasing metallicity a comparison of the distributions of varying types of CC SNae can be argued to trace the metallicity gradient in galaxies (Boissier and Prantzos, 2009).

A SN event in a binary system is one of many mechanisms which can alter the orbital parameters of the system. Others include the development of a common envelope and the interaction of the binary with another massive object. SN events in massive binary systems can eject a significant amount of mass from the system which changes the orbital dynamics. Here we estimate the effects of such a SN event on the disruption rate of massive binaries (Section 4.2). When the SN shell passes the companion star there is a sizeable drop in gravity and depending on the details of the evolution, the eccentricity of the orbit and the velocity kick due to an asymmetric SN, the SN remnant may not stay with the secondary, releasing both as runaway stars. In most cases, not enough mass is released to unbind the companions (Blaauw, 1961) and the runaway is expected to remain a binary, an example of which are High Mass X-ray Binaries (HMXBs) which have velocities of $\sim 50 \mathrm{kms}^{-1}$ (van den Heuvel et al., 2000). It will be shown that if the mass of the ejecta exceeds a certain limit, or if the companion star is small enough, then the binary system can become unbound. It will also be shown that if the SN is asymmetric, the ejecta can impart a very strong kick to the post-SN object which unbinds the system and can cause the binary components to be flung off at very high speeds. These runaway stars have massive peculiar velocities and are the subject of great interest as some appear to be leaving the galaxy altogether (Irrgang et al., 2010).

This thesis has three main Sections. We firstly, in Chapter 2, look at the distributions of CC SNae in disk galaxies. We use a sample of SNae from the Asiago SN database for the case of edge-on and face-on galaxies. In the case of the edge-on sample we investigate the height distribution of SNae above/below the plane of the galaxy and develop a photometric method to extract a sample of runaway candidates. For the face-on sample we use de-projected and normalised radial distances to determine a surface density distribution. In Chapter 3 we model the dynamics of these runaways
by distributing a sample of massive stars according to the observed light profiles of disk galaxies, giving each star a high space velocity and following their trajectories in an axisymmetric disk potential. In Chapter 4 we investigate the dispersion of HMXBs from their birthplace in the spiral arms. For this purpose we develop a population synthesis model in which we evolve massive binaries. These binaries are then used to populate a dynamical model in which they are distributed about a spiral pattern. We then measure the dispersion of the population from the spiral pattern.

## Chapter 2

## Observing Runaways

### 2.1 Introduction

While SNae have been the subject of great interest for most of the past century, studies have mainly focused on the physical processes which govern their evolution. Models have been created to try to explain the enormous release of energy and the spectral features that are associated with SNae (eg. (Thielemann et al., 2004)). However, there have been only a few investigations into the distribution of SNae about their host galaxies and into the study of how many of these may have been ejected from their progenitor's birthplace by either a dynamical event or a SN in a binary system. This can be a fruitful endeavour since SNae can be seen from distant galaxies and their distribution can tell us about star-forming regions in these galaxies. With the flood of high quality data about SNae and their host galaxies becoming available, more accurate distribution studies can be done.

A recent paper by Hakobyan et al. (2009) used SNae and host galaxy data to study the radial distribution of SNae in face-on galaxies. They used the data from the Asiago database to conduct their survey and then compared their results with those that describe the distributions of stars and ionized gases. They found that the distribution of core-collapse SNae is consistent with an exponentially decaying law similar to the distribution of light attributed to the disk structure in edge-on spiral galaxies. They derive an exponential distribution of core-collapse SNae with a scale length of $h_{S N}=0.29 R_{25}$ (where $R_{25}$ is the radius of the galaxy derived from the 25 th B-band magnitude isophotal diameter - here denoted $R_{\text {Gal }}$ ). They also notice that there is a deficit of SNae in the centre of spiral galaxies within $0.2 R_{\text {Gal }}$ and argue that this could be due to extinction in the central bulge regions of spirals or that the
brightness of the central regions hinders the discovery of SNae in those places. They also note that the scale lengths they derive are in agreement with the scale lengths of ionized gas reported by Athanassoula et al. (1993) even though there is a large uncertainty in these values.

With regard to the distribution of secondary SNae (those that have been ejected from their original cluster by a kick imparted by a former binary companion) only one study has investigated candidates in the solar neighbourhood. Hoogerwerf et al. (2001) looked at 56 runaway stars to determine, with accurate proper motions and parallaxes, the cause of their runaway status. They use these data to trace back the path of these runaways and associate them with a parent cluster and then, with the timescales involved, deduce whether the SN was ejected either in a dynamical ejection scenario (DES) or a binary SN scenario (BSS). A DES is a scenario in which a young star undergoes a three- or $n$-body gravitational interaction and is ejected from the parent cluster and would seen as having a large peculiar motion compared to the stars in the parent cluster (Blaauw, 1961). A BSS is a scenario in which a (stable) binary system is disrupted due to the SN explosion of one (in most cases the "primary", the initially most massive star) of the components. The SN explosion ejects matter from the system and in some cases can be asymmetric, resulting in a kick being imparted to the SN remnant. In Section 4.2 we describe the effects of such an occurrence in a massive binary system which can completely disrupt a binary system sending both components out with a high space velocity (Hills, 1983). Out of their sample of 56 runaways they were able to determine the parent cluster and the likely cause of the ejection for 16 objects. Eleven out of the 16 were deemed to have been ejected via a velocity kick imparted by a SN and the rest were ejected in a gravitational interaction. Another candidate had two possible parent clusters but in either case was argued to have been ejected via a BSS. This gives some idea of the statistical properties of both types of runaways (even though the sample is small) suggesting that about two thirds of runaways are ejected via a BSS and a third are ejected via a DES. These estimates can then be used to draw conclusions about the distributions of SNae found in surveys in that it implies that some SNae should occur away from the birthplace of the progenitor. A number of models have also been created to determine the fractions of SNae ejected by DES or BSS (a ratio of 50:50 for O-type stars with B-type stars having a slightly higher chance of being ejected in a BSS (O Maoileidigh, 2009))

Models of high velocity O- and B-stars also shed light on the SN distributions. Dray et al. (2005) ran numerical simulations of binary interactions to estimate the fraction of O-type runaways caused by gravitational or SN interactions. They estimate that the fraction of O-type runaways can be as high as $50 \%$ of the total O-type population
but that, due to the asymmetry of SN explosions, many of these will have a space velocity that is too low for them to be observed as runaways. The percentage of all O stars that are runaways has been observed to lie between $10 \%$ and $30 \%$ (Gies, 1987) while de Wit et al. (2005) suggest that a further $20 \%$ are not resident in a parent cluster but are not identified as runaways, due to their low space velocities. This result correlates well with the model by Dray et al. (2005).

The characteristics of runaway O- and B- stars has been investigated by Portegies Zwart (2000) who ran binary population synthesis calculations with the primary star exploding in a SN and imparting a kick to its companion. The results of his model suggested that in order to maximise the amount of runaways the binary fraction of O and B stars should be approaching $100 \%$ and that the masses of both stars should be roughly equal. He argues that BSS runaways should exhibit a high rotational velocity and lower than usual helium abundances due to accretion of material from the 'preSN' primary and that this would rejuvenate the runaway to the point where it would appear as a "blue straggler" in its parent group. He also points out that the frequency of O type runaways from his model ( $\sim 2 \%$ ) may be much lower than the observed frequency ( $\sim 20 \%$ ) because many young early type stars may be hidden in the clouds in which they are born, increasing the fraction of observed O-star runaways. His model, however, only allows runaways to be produced by a SN which may distort the results since strong gravitational forces, which can also create runaways, could perturb the initial binary fractions and also the mass fractions.

A paper by Kazarian (1997) attempted to link the distribution of SNae with active or star forming galaxies (with a UV excess) but showed no correlation. That study concluded that the rates of SNae are not enhanced in active galaxies.

### 2.2 The Sample

The sample of SNae is taken from the Asiago SN database which is maintained by the Asiago-Padova SN group, is updated regularly with SN discoveries announced in the IAU circulars and supplemented with additional information from the NED and LEDA galaxy databases. The database is easily accessible on a web form hosted by the Vizier service. ${ }^{1}$ As of the 9th of October 2011 the catalogue contained 5,745 instances of recorded SNae. Out of this catalogue we have selected edge-on galaxies as an investigation into the positions of SNae relative to their parent galaxy should

[^0]exhibit more runaway candidates if the host galaxy is viewed edge on since a SN that is above or below the galaxy disk, away from the likely birthplace of its progenitor, would be easier to spot than if looking at the galaxy face-on. We use only CoreCollapse (CC) for this purpose, the progenitors of which are formed primarily in the thin disk of spiral galaxies. A photometric method is defined and employed to separate the SNae that are placed away from an idealised representation of their host galaxy. We also investigate the distributions of CC SNae for a sample of face-on galaxies using de-projected and normalised radial distances. For the face-on sample we also compare the radial distributions of type Ib/c and type II SNae.

### 2.2.1 Constraints

Each SN is required to have a known Position Angle (PA) so that the orientation of the galaxy in the sky could be determined and from that, the position of the SNae in relation to the galactic nucleus. Another required parameter was the redshift, $z$, of the host galaxy which was needed to calculate values for the distance to the galaxy, $D$, the diameter and radius of the host galaxy, $D_{\text {Gal }}$ and $R_{\text {Gal }}$ respectively, and also the real distance, in kpc, of the SNae from the galactic disk and nucleus.

For the case of investigating SNae in edge-on galaxies we restrict the inclination of the host (disk) galaxy to between $80^{\circ}$ and $90^{\circ}$ so as to limit projection effects. Also, since this is an investigation into the distributions of Core-Collapse SN we require the SN to be classified as either a type II, Ib or Ic (or some variant, eg. type IIp or IIl). Applying these restrictions we retrieve a sample for the edge-on investigation of 64 SN, all of which reside in spiral galaxies with redshifts $0.0018<z<0.0383$. For the photometric method outlined below we also require for the edge-on sample a galaxy type code, $T$, which gives an indication of the host galaxy's place along the Hubble sequence. From this it is possible to infer some characteristics of the host galaxy.

For the case of face-on galaxies we restrict the inclination of the host galaxy to below $50^{\circ}$ and in the same way as for the edge-on case, we constrain the sample by SN type. For the face on sample we retrieve a total of 444 SNae which reside in spiral galaxies with redshift $0.0007<z<0.0514$. For both samples it is required that the offset from the host galaxy centre is known as well as a position angle of the host galaxy and also an isophotal diameter of the host galaxy which is used to normalise distances in later parts of the study.

### 2.2.2 Work on the Sample

The following section outlines the work performed on each sample. The parameters used in the calculations are listed in Table 2.1.

| Parameter | Units |
| :---: | :---: |
| Inclination, $i$ | degrees |
| Position Angle, $P A$ | degrees |
| Redshift, $z$ | $\sim$ |
| Isophotal Diameter, $d_{i s o}$ | $\log _{10} d_{i s o}\left[\mathrm{~B}_{25}\right][\mathrm{d}$ in 0.1 arcmins $]$ |
| SN offset in the x-direction, $x$ | $\operatorname{arcsec}$ (East/West) |
| SN offset in the y-direction, $y$ | $\operatorname{arcsec}$ (North/South) |
| Galaxy type code, $T$ | $\sim$ |

Table 2.1: This table lists the parameters used in the following calculations.

## Edge-on



Figure 2.1: This figure shows how the orientation of galaxies is defined for the edge-on sample. The galaxy was taken to be a line through the origin with the position angle ( $P A$ ) determining the slope of the line. The length of the line was set by the isophotal diameter ( $D_{\text {Gal }}$ ) and the position of the $S N$ was given by the offset values. $R_{G a l}=\frac{1}{2} D_{G a l}$.

In order to determine the distances of the SN along the host galaxy, $R_{S N}$, and the height of each SN above the disk of the galaxy, $z_{S N}$ (see Figure 2.1), it is assumed that the host galaxy centre is located at the origin of a Cartesian coordinate system.

The disk of the galaxy is taken to be a line, through the origin, with a slope, $m$, calculated from the position angle, $P A$

$$
\begin{equation*}
y=m x \tag{2.2.1}
\end{equation*}
$$

where

$$
\begin{equation*}
m=\frac{1}{\tan (P A)} \tag{2.2.2}
\end{equation*}
$$

giving an equation for the line representing the disk of the galaxy

$$
\begin{equation*}
y=\left(\frac{1}{\tan (P A)}\right) x \tag{2.2.3}
\end{equation*}
$$

where $x$ is the coordinate in the East/West direction and $y$ the coordinate in the North/South direction. As the centre of the host galaxy is taken to be at the origin the position of the SN is given simply by the offset values ( $x_{S N}$ and $y_{S N}$ ) from which the vertical height of the SN, above/below the plane of the disk, is given by

$$
\begin{equation*}
z_{S N}=\frac{\left|a x_{S N}+b y_{S N}+c\right|}{\sqrt{a^{2}+b^{2}}} \tag{2.2.4}
\end{equation*}
$$

where in this case $a=m, b=-1$ and $c=0$ giving

$$
\begin{equation*}
z_{S N}=\frac{m x_{S N}-y_{S N}}{\sqrt{m^{2}+1}} \tag{2.2.5}
\end{equation*}
$$

The distance of the SN along the galaxy is found by first defining a line, perpendicular to the plane of the galaxy, which intersects the position of the SN. The slope of this line, $m_{\perp}$, is given simply by

$$
\begin{equation*}
m_{\perp}=\frac{-1}{m}=-\tan (P A) \tag{2.2.6}
\end{equation*}
$$

giving an equation for the perpendicular line of

$$
\begin{equation*}
y-y_{S N}=\left(\frac{-1}{m}\right)\left(x-x_{S N}\right) \tag{2.2.7}
\end{equation*}
$$

or

$$
\begin{equation*}
y=m_{\perp} x+c_{\perp} \tag{2.2.8}
\end{equation*}
$$

where

$$
\begin{equation*}
c_{\perp}=\frac{x_{S N}}{m}+y_{S N} \tag{2.2.9}
\end{equation*}
$$

Subsequently, the point of intersection of this perpendicular line, which goes through the position of the SN , with the plane of the galaxy, $x_{i n t}$ and $y_{i n t}$, is then used to
deduce the distance of the SN along the plane of the galaxy, away from the centre.

$$
\begin{gather*}
x_{\text {int }}=\frac{c_{\perp}}{m+\frac{1}{m}}  \tag{2.2.10}\\
y_{\text {int }}=m_{\perp} x_{i n t}+c_{\perp}=\frac{-1}{m}\left[\frac{c_{\perp}}{m+\frac{1}{m}}\right]+c_{\perp} \tag{2.2.11}
\end{gather*}
$$

Giving

$$
\begin{equation*}
R_{S N}=\sqrt{x_{i n t}^{2}+y_{i n t}^{2}} \tag{2.2.12}
\end{equation*}
$$

where $R_{S N}$ is in arcseconds.

An attempt is also made to quantify the possible errors associated with projection effects in inclined galaxies. It is assumed above that the SN lies at a distance which is exactly equal to the distance to the centre of the host galaxy. It is not known whether each of the SN is located on the near or far side of the galaxy - this provides some ambiguity in the true height of the SN above/below the disk of the host galaxy (see Figure 2.4(a)). For this error analysis it is assumed that each of the host galaxies is an infinitely thin circular disk. The error in the height $\left(z_{S N}\right)$ due to this projection effect will be a function of the galaxy's inclination and the distance of the SN along the disk (or, more accurately, the normalised distance $R_{S N} / R_{\text {Gal }}$ where $R_{\text {Gal }}$ is the radius of the disk defined by the isophotal diameter). This error will be at a maximum for those SN with $R_{S N} / R_{G a l}=0$ since the distance variation over the disk (see $y$ in Figure 2.4(b)) will be at a maximum here. As seen in Figure 2.2 the parameters $R_{\text {Gal }}$ and $R_{S N}$ lead to

$$
\begin{equation*}
\alpha=\cos ^{-1}\left(\frac{R_{S N}}{R_{G a l}}\right) \tag{2.2.13}
\end{equation*}
$$

and

$$
\begin{equation*}
y=R_{\text {Gal }} \sin (\alpha) \tag{2.2.14}
\end{equation*}
$$

where $2 y$ is the (real, de-projected) complete distance through the disk from the observers point of view at $R_{S N}$. As can be seen from Figure 2.4, if the SN is at the far side of the galaxy then its true height above the disk will be much smaller than if it were at the near side of the disk. Following the assumptions above, the SNs true height can be anywhere in the range $z_{S N} \pm \epsilon_{z}$ where

$$
\begin{equation*}
\epsilon_{z}=y \sin (\beta) \tag{2.2.15}
\end{equation*}
$$

and $\beta$ is calculated from the inclination of the galaxy, $i$

$$
\begin{equation*}
\beta=90^{\circ}-i \tag{2.2.16}
\end{equation*}
$$



Figure 2.4: (a) Depending on whether the SN lies at the front of the disk or at the back, the value of $z_{S N}$ computed above may be an under- or over-estimate. Assuming that the disk is completely circular, a crude estimation of the error in $z_{S N}, \epsilon_{z}$ may be made. (b) $\beta$ is easily calculated from the inclination $i$ and $y$, the distance through the disk at a given value of $R_{S N}$, follows from known values $\left(R_{S N}\right.$ and $\left.R_{G a l}\right)$ allowing an estimation of the error in $z_{S N}$.

An obvious shortcoming of this error calculation is that it is zero for perfectly edgeon galaxies and fails to take into account the thickness of the galaxy. It does however give at least some idea of the possible true heights of SN above their host galaxy and in certain cases assures the reader that a SN must lie well above the disk of the galaxy (see for example Figure 2.3 which shows the position of SN2007ac - 3.9 kpc above the disk of galaxy UGC10550 which has a recorded inclination of $83^{\circ}$ ).


Figure 2.3: Above shows the position of SN2007ac (black x) which occurred over 3.9kpc above the disk of its host galaxy UGC10550.

For a galaxy with an inclination of $80^{\circ}$ the maximum possible error in $z$ could be as much as $0.17 R_{\text {Gal }}\left(\right.$ At $\left.R_{S N}=0\right)$. Although this error calculation is also a function of $R_{S N}$, the upper limit for this kind of error in a galaxy with $R_{\text {Gal }}=15 \mathrm{kpc}$ is 2.55 kpc which is non-negligible given the dimensions of disk galaxies.

We also introduce a method, based on the light profiles of observed edge-on galaxies, that aims to separate potential runaway candidates from the ever increasing samples of SNae using only the parameters available from the Asiago SN database. We define a synthetic galaxy based on the available parameters of each host galaxy from the edge-on sample. We then calculate the expected surface brightness at the point where the SN in question lies and apply a criterion to determine whether the SN could be considered away from the disk. The method assumes that the distribution of light in edge-on disk galaxies can be adequately described by a function of the form

$$
\begin{equation*}
I(r, z)=I_{0} \exp \left(\frac{-r}{h_{r}}\right) \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{2.2.17}
\end{equation*}
$$

where $I(r, z)$ and $I_{0}$ are the brightness at $(r, z)$ and $(0,0)$ respectively and $h_{r}$ and $h_{z}$ are the scalelength and scaleheight of the brightness distribution respectively. Along with the parameters used above we also use the morphological type code, $T$, of the host galaxy which gives an indication of where the galaxy sits on the Hubble sequence: $1<n<4$ indicates galaxies in the range Sa-Sbc and $n>4$ indicates galaxies of type Sc and later. We use this morphological code to assign each case a scaleheight by noting that early type galaxies $(1<n<4)$ have been shown to have systematically thicker disks than later type galaxies $(n>4)$ (de Grijs, 1998). From a sample of edge-on galaxies Mosenkov et al., (Mosenkov et al., 2010) have shown that scaleheights can lie in the range $0.4-1.2 \mathrm{kpc}{ }^{1}$. We therefore set the scaleheights for our synthetic galaxy profiles as having a simple linear relation to the host galaxy's morphological type code. We set the scalelengths of the synthetic galaxies as a fraction of the isophotal diameter with $h_{r}=0.3 R_{\text {Gal }}$. For this simple analysis we set $I_{0}=1$. We then input the position of each SN $\left(R_{S N}, z_{S N}\right)$ into Equation 2.2.17 along with their respective $h_{r}$ and $h_{z}$ to find $I(r, z)$. As a criterion, we consider a SN to be a runaway candidate if $I(r, z)$ is less than 0.01 .

## Face-on

Here we analyse the positions of SNae for the sample of face-on galaxies. We use a larger range of inclinations for this sample which introduces projection effects due to

[^1]

Figure 2.5: To correct for the projection effects caused by the inclination of host galaxies the SNae are assume to have occurred in the plane of the disk and so a coordinate system, $(U, V)$, in the plane of the disk is defined - $U$ is aligned with the plane of the sky and the long axis of the galaxy and $V$ is aligned with the plane of the disk and the short axis of the galaxy. Here $\Delta \alpha$ and $\Delta \delta$ relate to $x$ and $y$ offsets respectively. (Picture taken from Hakobyan et al., (2009).)
the inclination of the host galaxy. In order to correct for the projection effects caused by inclined galaxies we follow the prescription of Hakobyan et al., (2009). Their method assumes that the SN occur in the plane of the disk, the likely birthplace of their progenitor stars - but as can be seen from the edge-on sample, they may not explode there (see Section 2.3.1). Firstly one defines the Cartesian coordinate system, $(U, V)$ - see Figure 2.5, in the plane of the galaxy disk and relates it to the coordinate system on the plane of the sky - as used for the edge-on sample above. In terms of the $(U, V)$ coordinate system, the SN is at

$$
\begin{align*}
& U=x \sin (P A)+y \cos (P A)  \tag{2.2.18}\\
& V=x \cos (P A)-y \sin (P A) \tag{2.2.19}
\end{align*}
$$

where again $x$ and $y$ are the offsets described in Table 2.1. The $U$-axis is aligned with the plan of the sky and the long axis of the galaxy while the $V$-axis is aligned with the plane and the short axis of the disk. The true radial distance of the SN from the
centre of the host galaxy is then given by

$$
\begin{equation*}
R_{S N}=\sqrt{U^{2}+\left(\frac{V}{\cos i}\right)^{2}} \tag{2.2.20}
\end{equation*}
$$

where $i$ is the inclination of the galaxy and $R_{S N}$ is in arcseconds. A surface density profile is also constructed in order to extract the scalelength of the distribution. This is done by counting the number of SN in concentric annuli in the galaxy disk such that

$$
\begin{equation*}
\Sigma_{j}=n_{j} / \pi\left(r_{j+1}^{2}-r_{j}^{2}\right) \tag{2.2.21}
\end{equation*}
$$

where $n_{j}$ is number of SN in the annulus with inner radius $r_{j}$ and outer radius $r_{j+1}$.

## Distances and Normalisation

The distances, $D$, to the host galaxies are easily found from the redshift, z. Here a value for the Hubble constant of $H_{0}=74.2 \mathrm{kms}^{-1} \mathrm{Mpc}^{-1}$ is used (Riess and Macri, 2007).

$$
\begin{equation*}
v=z c ; \quad D=\frac{v}{H_{0}} \tag{2.2.22}
\end{equation*}
$$

The values of $R_{S N}$ and $z_{S N}$ are converted to kpc using $D$ and the isophotal diameter of the host galaxy $d_{i s o}$. The diameter of the galaxy is

$$
\begin{equation*}
D_{\text {Gal }}=10^{\log \left(d_{\text {iso }}\right)} \text { in units of } 0.1 \text { arcmins } \tag{2.2.23}
\end{equation*}
$$

The radius of the host galaxy, $R_{\text {Gal }}$, is then used to normalise the radial distances of the SNae to the size of their host galaxy. The normalised distances are then denoted

$$
\begin{equation*}
R_{N}=\frac{R_{S N}}{R_{\text {Gal }}} \tag{2.2.24}
\end{equation*}
$$

### 2.3 Results

### 2.3.1 Edge-on Sample

Plotting the positions of the SN about their host galaxy it is easily seen that some do occur well beyond the extent of the galaxy defined by the $\mathrm{B}_{25}$ isophotal diameter. It can be seen from Figures 2.6 and 2.7 that in most cases the SNae lie close to the disk ( $z_{S N}<1 \mathrm{kpc}$ ) but that in some instances they clearly lie above it. The error bars associated with each point refer to the range of possible heights for each case as calculated in Section 2.2.2. The points with no error bars indicate those SNae that resided in perfectly edge-on systems. In order to derive the scaleheight,


Figure 2.6: The distribution of $z_{S N}$ exhibits an sech ${ }^{2}$ decay with a scaleheight of $h_{z}=0.486 \pm 0.081 \mathrm{kpc}$.


Figure 2.7: For the edge-on sample the heights of the SNae above their host disks, $z_{S N}$, are calculated according to the prescription in Section 2.2.2. The error bars show the possible range of $z_{S N}$ due to the inclinations of the host galaxy - those with no error bars occurred in perfectly edge-on galaxies. Figure (a) shows the heights as a function of distance along the plane, $R_{S N}$, in kpc while figure (b) shows the heights as a function of the normalised distance $R_{N}=R_{S N} / R_{\text {Gal }}$.
$h_{z}$, of the density of SNae for this sample we find the populations of SNae in bins of 0.3 kpc and then fit, with a scaled Levenberg-Marquardt algorithm ${ }^{1}$, a sech ${ }^{2}$ decay of

[^2]the form
\[

$$
\begin{equation*}
\Sigma_{z}=\Sigma_{0} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{2.3.1}
\end{equation*}
$$

\]

where $\Sigma_{0}$ is the central surface density. We find a scaleheight of $h_{z}=0.486 \pm 0.081 \mathrm{kpc}$. The distribution (shown in Figure 2.6) does deviate from the sech ${ }^{2}$ fit beyond 1kpc since the SNae in the sample resided in galaxies of many sizes $\left(4.6<R_{\text {Gal }}<35.5 \mathrm{kpc}\right)$. It does show however that over half of the sample SNae lay within 0.6 kpc of the $z=$ 0kpc plane which naturally follows from the concentration of OB stars to the disks of spiral galaxies.

We now look at each member of the sample individually by applying the method outlined above which defines a synthetic brightness distribution based on parameters about the host galaxy of each SN. A criterion $(I(r, z)<0.01)$ is applied to the brightness of the synthetic galaxy at $\left(R_{S N}, z_{S N}\right)$ to see if the SN could be considered a runaway candidate. From this method we extract nine runaway candidates from the sample of 64 . Of the nine candidates, three reside in barred galaxies which deviate from the idealised brightness distributions and are rejected. The remaining six however do represent possible runaway candidates and are shown in Figure 2.8 with their corresponding $I(r, z)$ listed in Table 2.2.


Figure 2.8: From the method outlined above we extract 9 runaway candidates (3 are discarded due to an irregular host galaxy morphology), we include here 4 others that do not meet the criterion but have low $I(r, z)$. The point at which $S N$ occurred is marked with black/white cross. Refer to Table 2.2 for the galaxy/SN designations and the calculated $I(r, z)$ for each case.


Figure 2.9: The positions of the runaway candidates extracted from the photometric method along with four others with low $I(r, z)$ (red dots). Also depicted are the rejected candidates whose host galaxy deviated from the idealised brightness profile due to the presence of a bar (blue squares).

|  | SN | Galaxy | $I(r, z)$ |
| :---: | :---: | :---: | :---: |
| (a) | SN1992N | IC4831 | $0.68 \%$ |
| (b) | SN1988I | LEDA0086944 | $0.59 \%$ |
| (c) | SN2009gc | M-03-28-32 | $0.37 \%$ |
| (d) | SN2001dh* | M-06-44-26 | $2.61 \%$ |
| (e) | SN2005ab | NGC4617 | $0.95 \%$ |
| (f) | SN2003dr* | NGC5714 | $1.28 \%$ |
| (g) | SN1940A* | NGC5907 | $4.88 \%$ |
| (h) | SN2007ac | UGC10550 | $0.00 \%$ |
| (i) | SN2005da | UGC11301 | $0.04 \%$ |
| $(j)$ | SN2004cr* | UGC11603 | $2.74 \%$ |

Table 2.2: Runaway candidates extracted from the photometric method outlined above along with four other candidates (marked with "*") who do not meet the selection criterion but have low $I(r, z)$. The candidates are listed with the designation of their host galaxy and the computed $I(r, z)$ for each case (as a percentage of the central surface brightness of the host galaxy).

We also include in Figure 2.8 four other possible runaways (SN1940A in NGC5907, SN2003dr in NGC5714, SN2001dh in M-06-44-26 and SN2004cr in UGC11603) who do not meet the criterion above but nonetheless have small $I(r, z)$. Plotting the runaway candidates among the whole sample shows how effective this simple method is in separating SNae that are placed well away from the disk of their host galaxy (see Figure 2.9). These runaway candidates account for just over $15 \%$ of the initial sample suggesting that the observed and theoretical runaway populations of OB stars (Dray et al., 2005) has manifested itself in the distribution of CC SNae.

### 2.3.2 Face-on sample



Figure 2.10: A surface density profile is constructed for the face-on sample using the normalised inclination corrected radial distances. The profile exhibits a central deficit (discussed further on) with an exponential decay beyond $R_{N}=0.2$. Including the central deficit the scalelength of the distribution is $h_{R}=0.67 \pm 0.18 R_{\text {Gal }}$. However, as an example we fit an exponentially decaying profile to the outer parts of the distribution with a scalelength in the range $0.3<h_{R}<0.4 R_{\text {Gal }}$.

We construct surface density profiles according to Equation 2.2.21 for the face-on sample, firstly for the whole sample and then separately for the type $\mathrm{Ib} / \mathrm{c}$ and type II SNae. We use the normalised de-projected radial distances and find that there is a clear deficit of SNae in the central regions of the host galaxies (Figure 2.10). The central regions of disk galaxies can be quite bright making the detection of SNae here more difficult since they may be confused with the bright central bulge and not noticed at all. This effect is more pronounced with more distant galaxies so that the more distant a galaxy is, the harder it is to notice a SN occurring in the central regions. An exponentially decaying curve is the fit to the distribution revealing a scalelength in the density distribution for type Ib/c/II SNae of $h_{R}=0.67 \pm 0.18 R_{\text {Gal }}$. Because the host galaxies in this sample vary in size and shape the scalelength calculated here is not representative of any particular distribution of stellar population or brightness. The scalelength is however more representative of type II SNae since they account for over $75 \%$ (336 out of 444) of the sample. In an effort to account for the central deficit we also fit an exponential with a scalelength of $0.35 \pm 0.05 R_{\text {Gal }}$ to the outer section of the sample $\left(R_{N}>0.2\right)$.

Separating the sample into separate groups of type $\mathrm{Ib} / \mathrm{c}$ and II it is shown in Figure 2.12 that SNae of type $\mathrm{Ib} / \mathrm{c}$ tend to be more concentrated towards the centres of their host galaxy. Although the sample contains a host of galaxy morphologies the two groups are distributed similarly in Hubble type (see Figure 2.11) allowing a more direct comparison of their distribution. We fit the surface density distribution of type $\mathrm{Ib} / \mathrm{c}$ SNae with a scalelength of $h_{R, I b / c}=0.43 \pm 0.05 R_{\text {Gal }}$ while the distribution of type II SNae is fit with a more extended scalelength of $h_{R, I I}=0.75 \pm 0.08 R_{\text {Gal }}$


Figure 2.11: Both groups of type Ib/c and II SNae are distributed similarly among their host galaxies. (see Figure 2.12).


Figure 2.12: Here shows the surface density distribution of both groups of SNae (type $I b / c$ and type II). The type $I b / c$ SNae are more centrally concentrated and are fit with an exponential decay with scalelength $h_{R, I b / c}=0.43 \pm 0.05 R_{\text {Gal }}$. The type II SNae display a more extended distribution and is fit with a scalelength $h_{R, I I}=0.75 \pm 0.08 R_{\text {Gal }}$.

## Shaw effect

For the SNae in the face-on sample (both type $\mathrm{Ib} / \mathrm{c}$ and type II) we plot the surface density distributions of their de-projected radial distance for host galaxies that lie in distinct distance bins $0-50 \mathrm{Mpc}, 50-100 \mathrm{Mpc}, 100-150 \mathrm{Mpc}$ and $150 \mathrm{Mpc}+$. The distributions are normalised to the number of SNae in each galaxy distance bin. It is apparent from these plots (Figure 2.13) that the distributions of radial distance for


Figure 2.13: With ever increasing host galaxy distances the central deficit in the distribution of radial distances becomes more pronounced. Shown here is the distribution for host galaxies with distance $D<50 \mathrm{Mpc}$ (circles), $50<D<100 \mathrm{Mpc}$ (diamonds), $100<D<150 \mathrm{Mpc}$ (triangles) and $D>150 \mathrm{Mpc}$ (stars).
ever distant host galaxies exhibit larger deficits of CC SNae near the centres of the host galaxies with increasing galaxy distance. This effect was first shown by Shaw (1979) and is reproduced here with a sample almost twice as large (444 compared to 228). To ensure that this central deficit is not due to a physical lack of CC SN progenitors in the central regions of disks we also plot the surface density distribution of type Ia SNae. Type Ia progenitors should be distributed evenly about the disk and bulge of a galaxy and their abundance in the stellar halo should produce an overdensity in the central regions of disk galaxies since in these regions, assuming a homogeneous distribution throughout the halo, one would be looking through a greater volume of space which would contain a greater number of type Ia progenitors (see for eg. Baade (1944) or Wyse et al., (1997)). Indeed we find the same deficit, which is also more pronounce with host galaxy distance, implying that the selection effect due to the brightness of central bulges is also present in the case of CC SNae. Normalising the radial distances, $R_{S N}$, to the radius of the galaxy defined by the isophotal diameter appears to remove the increasing deficit of SNae with increasing host galaxy distance. This result suggests that the same fractional area of disk galaxies hide SNae due to bright central bulges, irrespective of host galaxy distance. Normalisation of radial distances to the isophotal radius of the host galaxy shows that the amount of disk area that obscures SN , preventing their discovery, is dependant only on the radius of the galaxy. From Figure 2.15 we see that obscuration begins to take effect at radial distances less than $0.4 R_{\text {Gal }}$ implying that the central $16 \%$ of area in disk galaxies provides enough light to hide a SN.


Figure 2.14: In order to support the hypothesis that the deficit of SNae in the central regions of disk galaxies is caused by confusion with the bulge rather than there being a lack of CC SN progenitors in these regions we construct a similar radial distribution as a above for a sample of type Ia SNae. The progenitors of type Ia SNae are known to populate these inner regions yet with increasing galaxy distance the distributions exhibit similarly more pronounced central deficits.


Figure 2.15: Normalising the radial distances shows that, regardless of host galaxy distance, the same fraction of area in the host galaxy hides SN. At radial distances less than $0.4 R_{\text {Gal }}$ SNae are less likely to be noticed.

### 2.4 Discussion

From the Asiago SN database we have extracted a sample of sixty four Core-Collapse SNae that reside in edge-on ( $\mathrm{i}>80^{\circ}$ ) spiral galaxies. We fit the distribution of the heights of the SNae with a sech ${ }^{2}$ profile and a scaleheight of $h_{z}=0.49 \pm 0.08 \mathrm{kpc}$. The host galaxies of the SNae from the sample ranged from Sa to Sd type galaxies with no real preference shown for any particular type although early type spirals are underrepresented (see Figure 2.16). It should also be noted that only ten out of the sixty four SNae were of type Ib/c making the distribution almost entirely representative of type


Figure 2.16: The host galaxies in the edge-on sample exhibit no preference for Hubble type. II SNae. The concentration of SNae to the disk of the host galaxies, with over half lying within 0.6 kpc of the disk, is characteristic of the distribution of CC SN progenitors in galactic disks. The calculated scaleheight agrees well with the scaleheights of edge-on spirals derived from $I$-band photometry of Kregel et al., (2002) and indeed the scaleheight of the thin Milky Way disk of 0.3 kpc (Jurić et al., 2008). Although the derived scaleheight fits well with other quoted values it is clear that in some instances the SNae appear completely detached from the host galaxy disk. It is thought that the progenitors of these wayward SNae had been ejected from their parent stellar cluster, by either a dynamical or SN event, and would be seen as "Runaway" stars with a high space velocity (Poveda et al., 1967; Blaauw, 1961; Hills, 1983). We develop a method, based on an idealised photometric profile galaxy disks, to separate this group of SNae from the others, who lie clearly within the disk of the host galaxy. Using this method we extract six possible candidates, with three rejected due to an odd host galaxy morphology. Another four had low $I(r, z)$ and could also be considered runaway candidates. From Figure 2.9 it can be seen that this method, which uses only readily available archival data, can reliably select SNae that are distinct from their host galaxy and whose progenitors may have been ejected from their likely birthplace in the disk. This method could be used to identify candidates worthy of a detailed spectroscopic follow up to ascertain abundances which may shed light on the binary evolution history of the progenitor.

For the face-on sample we have extracted a total of 444 CC SNae that are hosted in galaxies with inclinations less than $50^{\circ}$. We use the method of Hakobyan et al. (2009) to calculate the de-projected radial distance of the SNae from the centre of their host galaxy. A surface density distribution of the normalised (to the isophotal radius of the host galaxy) radial distances is then fit with an exponential profile with a scalelength of $h_{R}=0.67 \pm 0.18 R_{\text {Gal }}$. The density distribution does show a truncation in the inner regions of the host galaxies $\left(R_{S N}<0.4 R_{G a l}\right)$. A similar distribution for the case of type Ia SNae, the progenitors of which are known to populate the bulges of disk galaxies, implies type Ia SNae in these regions go undetected due to confusion with the bright bulges and that a similar effect also affects the detection of CC SNae. We also fit a scalelength to the outer ( $R_{S N}>0.4 R_{\text {Gal }}$ ) part of the distribution in the range $h_{R}=0.35 \pm 0.5 R_{\text {Gal }}$. The host galaxies, as in the case for the edge-on sample, range in size and shape (see Figure 2.11) and the derived scalelength is not representative of any particular galaxy type. When the sample is split into groups of type $\mathrm{Ib} / \mathrm{c}$ and type II SNae, the density distribution of radial distances shows that the type $\mathrm{Ib} / \mathrm{c}$ SNae are more concentrated towards the centres of their host galaxy. Although the host galaxies vary in Hubble type, the two groups are distributed similarly in $T$ allowing a confident comparison (see Figure 2.12). The type $\mathrm{Ib} / \mathrm{c}$ SNae are fit with a scalelength of $h_{R, I b / c}=0.43 \pm 0.05 R_{\text {Gal }}$ while the type II SNae are fit with a scalelength of $h_{R, I I}=0.75 \pm 0.08 R_{\text {Gal }}$. This difference in scalelengths has been suggested to be due to the radial metallicity gradient in galaxy disks (eg. Boissier and Prantzos (2009)). The metal rich inner disk causes the massive OB stars to generate strong winds compared to the relatively metal poor OB stars in the outer disk. This mass loss removes Hydrogen from the CC SN progenitors resulting in an absence of Hydrogen in their spectra and consequently, their classification as type Ib/c type SNae.

We reproduce the Shaw effect (Shaw, 1979) with a sample twice as large to confirm a selection effect due to distance. As one investigates the radial distribution of SNae for ever distant host galaxies it is clear that the central deficit described above becomes more apparent (see Figure 2.13). This selection effect is due to confusion with bright bulges in spiral galaxies and the decreasing spatial resolution of telescopes with distance. Normalising the radial distances suggests that for all galaxies the same fractional disk area is bright enough to hide SNae, preventing their detection. Noting that in Figure 2.15 the inner truncation in the distribution occurs for galaxies in all distance bins at $<0.4 R_{\text {Gal }}$ we estimate that the inner $16 \%$ of the area in galaxy disks is bright enough to obscure SNae, regardless of host galaxy distance.

## Chapter 3

## Modelling Runaways

### 3.1 Introduction

In this chapter we investigate the distributions of SNae, whose progenitors have been given a kick during the SN explosion of a former binary companion, by placing test particles in an axisymmetric potential and giving them a kick. The test particles are firstly distributed about a galaxy disk following the surface brightness profiles of observed spiral galaxies. These brightness profiles are parameterised by their respective scalelengths. The test particles are given a "kick" velocity which is added to their initial "systematic" velocity which is due to their position in the galactic potential. This kick velocity is based on observations of high velocity, or "runaway", OB stars and pulsars and varies from 30 to $150 \mathrm{kms}^{-1}$. Each test particle is also assigned a mass which determines how long the particle will travel before exploding as a SN. Once the particle has exploded its position is noted and the distribution these positions is then fit with new scalelengths. This allows a measure of the dispersion from the initial distribution.

Also in this chapter we describe the routine with which we evaluate the trajectories of the test particles. We use a $5^{\text {th }}-$ and $6^{\text {th }}-$ order Runge-Kutta scheme, which is tested and evaluated, to reproduce the orbits. The gravitational potential is also described and is shown to produce a flat rotation curve. This aspect of the potential affords the model a certain consistency in that the test particles will, without any alterations, replicate the orbital characteristics of stars in any spiral galaxy.

### 3.2 Initial Distribution

The light profiles of disk galaxies form the basis of the initial distribution of test particles in this model. Along with input from mass models of the inner galactic disk, the light profiles serve to adequately distribute the test particles about an azimuthally homogeneous disk with a central hole. The spherical bulge is not included in our model since its constituents are population II stars and are not connected to the phenomenon being investigated here.

### 3.2.1 Distributions within the disk

Here the initial distribution of test particles is described. We define the distribution separately for each component of the cylindrical-polar coordinate system in which the particles reside. Once each of the $R, z$ and $\phi$ components are populated they are then combined to give the full 3 -dimensional distribution.

## Radial Distribution

While radial light profiles of disk galaxies are well fitted by two component models (a central spheroidal bulge (Sersic, 1968; de Vaucouleurs, 1948) and an exponential disk (Freeman, 1970; Knapen and van der Kruit, 1991)) there still remains an ambiguity about the physical distribution of high mass stars in the inner regions of disk galaxies. Freeman (1970) also describes his Type II disk where the disk is truncated at small $R$ showing a dip in surface brightness outside the central bulge. This inner truncation of galaxy disks has been observed in early type spiral galaxies by (Ohta et al., 1990) and dynamical models of the Milky Way have included disk density profiles described by the difference of two exponentials which creates a distribution with a central hole (Robin et al., 2003; Freudenreich, 1998). This central hole will be included when distributing our test particles as dynamical interactions with the central compact object described in our potential (Equation 3.3.2) may produce high velocity objects which are not the subject of study here ie. hyper velocity stars (Brown et al., 2005). Freudenreich (1998) describes a central hole in the Galactic centre which is truncated at $R=3 \mathrm{kpc}$ but in our case, as we want to investigate all possible cases, we choose that the inner truncation is described by the difference of two exponentials - an exponential disk, with scalelength $h_{r}$, minus an exponential bulge with scalelength $h_{b}$.

Face-on galaxies are commonly modelled with light profiles consisting of two com-
ponents - a spherical bulge described by a $R^{\frac{1}{4}}$ law (de Vaucouleurs, 1948) (or, more generally, a $R^{\frac{1}{n}}$ law where $n$ is the Sérsic index (Sersic, 1968)) and an exponentially decaying disk (Knapen and van der Kruit, 1991). However, while the bulges of spiral galaxies are commonly modelled with Sérsic profiles it should be noted that bulges of spiral galaxies have been successfully described with exponential profiles (Andredakis et al., 1995). We derive $h_{b}$, the scalelength of the bulge, from the $K$-band effective radius ( $r_{e}$, the radius containing half the total light due to the Bulge) observed for Sc-type galaxies by Graham and Worley (2008). We adopt an $r_{e}$ value of 0.9 kpc and a Sérsic index of $n=1$ (both of which lie in the observed ranges - $r_{e}=0.60 \pm_{0.26}^{0.92} \mathrm{kpc}$ and $n=1.78 \pm_{0.79}^{2.18}$ ). The Sérsic profile is parameterised by $r_{e}$ and $n$ and the equivalent exponential profile is parameterised by $h_{b}$ and $n$ and both can be equated as follows (Sersic, 1968);

$$
\begin{equation*}
I(r)=I_{0} \exp \left[\left(\frac{-r}{h_{b}}\right)^{\frac{1}{n}}\right]=I_{e} \exp \left[k\left\{\left(\frac{r}{r_{e}}\right)^{\frac{1}{n}}-1\right\}\right] \tag{3.2.1}
\end{equation*}
$$

where $k$, a normalising factor, has been shown to be well approximated by $k=$ $-(2 n-0.327)$ (see Equation A.1.2); $I(r)$ describes the surface brightness at $r, I_{0}$ is the central surface brightness and $I_{e}$ is the surface brightness at $r_{e}$. It can be seen that for a Sérsic index of 1 , the first equation in 3.2.1 describes a purely exponential falloff and by equating the equations one can deduce the relationship between $h_{b}$ and $r_{e}$ as

$$
\begin{equation*}
\left(\frac{r_{e}}{h_{b}}\right)=(2 n-0.327)^{n} \tag{3.2.2}
\end{equation*}
$$

which gives in the case of $n=1$ : $h_{b}=r_{e} / 1.67=0.538 \mathrm{kpc}$.
We also use the B-band Sc-type disk scalelength, $h_{r}=3.37 \pm_{0.88}^{1.66} \mathrm{kpc}$, from Graham and Worley (2008) which is in good agreement with the median value in a photometric (r-band) study of face-on galaxies by Fathi et al. (2010). They also describe a bimodality in scalelengths with lower mass galaxies $\left(10^{9}-10^{10} M_{\odot}\right)$ having an average scalelength of $1.52 \pm 0.65 \mathrm{kpc}$ and higher mass galaxies $\left(10^{10}-10^{12} M_{\odot}\right)$ having an average scalelength of $5.73 \pm 1.94 \mathrm{kpc}$.

As mentioned above we model the distribution of test particles on the difference of two exponentials - an exponential bulge profile derived from a Sérsic profile taken from an exponential disk profile - to create a central hole in the distribution. This gives a function of the form

$$
\begin{equation*}
I(r)=I_{0}\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)^{\frac{1}{n}}\right] \tag{3.2.3}
\end{equation*}
$$

where $h_{r}$ and $h_{b}$ are the scalelengths of the surface brightness of the disk and bulge respectively and $n$ is the Sérsic index which we take to be equal to 1 . A list of random numbers between 0 and 1 is generated (Set $B_{R}$ containing numbers $B_{R i}$ where $i$ goes from $1-10,000$ ). These numbers relate to the area under the curve described by

$$
\begin{equation*}
A_{R}\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)\right] \tag{3.2.4}
\end{equation*}
$$

such that

$$
\begin{equation*}
B_{R i}=\int_{0}^{R_{i}} A_{R}\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)\right] d r . \tag{3.2.5}
\end{equation*}
$$

In Equation 3.2.4 $A_{R}$ is a constant, fixed so that the area under the curve of 3.2.5, integrated from 0 to $R_{\max }$ is equal to 1 (where $R_{\max }$ is the maximum extent of the disk). We tentatively set the maximum extent of the disk to be 15 kpc - the average galaxy radius (using $\mathrm{B}_{25}$-isophotal radii) in the sample of edge-on and faceon galaxies in Section 2.2 is 16.21 kpc and 14.92 kpc respectively. Kregel et al., (Kregel et al., 2002) argue that the radial truncation in disks does not occur until at least $4 h_{r}$. Unfortunately there is no way to analytically evaluate $R$ from Equation 3.2.5 so another method has been employed to extract the distribution. A simple script was written that implements Newtons method for 50 iterations. A first guess, $R_{n}$, was estimated and following values of $R$ were calculated by

$$
\begin{equation*}
R_{n+1}=R_{n}-\frac{f\left(R_{n}\right)}{f^{\prime}\left(R_{n}\right)} \tag{3.2.6}
\end{equation*}
$$

where $f(R)$ is just Equation 3.2.5 rearranged with one side equal to zero: ie.

$$
\begin{equation*}
f(R)=\left(\int_{0}^{R_{i}} A_{R}\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)\right] d r\right)-B_{R} \tag{3.2.7}
\end{equation*}
$$

and $f^{\prime}(R)$ is the derivative of Equation 3.2.7 with respect to $R$ (a full derivation is outlined in Appendix A.1). This method proved quite successful as can be seen in Figure 3.1. Although quite a high proportion of test particles have a radial distance between 0 kpc and 2 kpc , only a small fraction are situated very close to the central compact object described in Equation 3.3.2 thereby significantly limiting the chance of strong gravitational encounters and consequently the production of hype-velocity stars (Brown et al., 2005).


Figure 3.1: For a model galaxy with a maximum extent of $R_{\max }=15 \mathrm{kpc}$ this is the radial distribution of test progenitors produced by set $B_{R i}$. We subtract an exponential bulge with scalelength $h_{b}=0.538 \mathrm{kpc}$ from an exponential disk scalelength of $h_{r}=3.37 \mathrm{kpc}$. This distribution of values of $R_{i}$ is produced by equation 3.2.5 and is overlain with a smooth plot of the function.

## Azimuthal Distribution

Now that each test particle has an $R$ position, each needs to be distributed evenly around a disk. A set of random numbers (set $B_{\phi}$ containing random numbers between 0 and $1, B_{\phi i}$, where i goes from 1 to 10,000 ) is then generated and used to position the test particles at certain angles, $\phi_{i}$, around the disk. This operation is simply

$$
\phi_{i}=B_{i \phi} \times 2 \pi
$$

## Height Distribution

Height distributions of brightness in disk galaxies have been successfully modelled on sech $^{2}$ profiles of the form

$$
\begin{equation*}
I(z)=I_{0} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{3.2.8}
\end{equation*}
$$

where $I$ is the surface brightness ( $I_{0}$ the central surface brightness), $z$ is the height above the disk, and $h_{z}$ is the scaleheight of the surface brightness (van der Kruit and Searle, 1981; Mosenkov et al., 2010). High mass stars, the progenitors of SNae, are said to be located in the "thin disk" of spiral galaxies as opposed to the "thick disk" which contains older stars. These two disk components can be separated by analysing characteristics such as metallicity and velocity dispersions. Thin disk stars are said to formed by gas that has been accreted on to the galaxy and consequently exhibit similar galactic rotation speeds and abundances while the thick disk stars, said to be accreted on to the galaxy from satellite clusters or dwarf galaxies, exhibit a lower
metallicity, higher $z$-velocity dispersions and tend to lag behind the rotation of thin disk stars (Yoachim and Dalcanton, 2006). This thin disk can be reasonably well described in the outer regions of disks by photometry of external galaxies (Yoachim and Dalcanton, 2006) and also by surveys of stars in the Solar neighbourhood (Jurić et al., 2008). We use the scaleheight, $h_{z}$, from Juric et al. (2008) of 300pc but note that we use a larger scalelength than theirs $(2.6 \mathrm{kpc})$ so as to include the possibility of high mass stars forming at the outer, often bright, regions of disks (Bush et al., 2010).

Another set of numbers, between 0 and 1, is generated (Set $B_{z}$ containing numbers $B_{z i}$ where $i$ goes from 1 to 10,000 ) which corresponds to the height of each test particle in an analogous way to set $B_{R}$. Firstly the distribution is based on

$$
\begin{equation*}
B_{z i}=\int_{z_{\min }}^{z_{i}} A_{z} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) d z \tag{3.2.9}
\end{equation*}
$$

where, in the same way as for the derivation of the values of $R_{i}, A_{z}$ is a constant set so that the area under the curve of Equation 3.2.9 integrated from $z_{\text {min }}$ to $z_{\text {max }}$ is equal to 1. Again, as in the case for the derivation of $R_{i}$, we tentatively set a maximum/minimum height of the disk of 1 kpc (or $3.33 h_{z}$ ) so that


Figure 3.2: The distribution of test particle heights as defined by Equation 3.2.10 overlain with a smooth plot of Equation 3.2.8. A large proportion lie within 500pc from the plane of the disk making it an extremely flat disk when compared to the radial distribution. $z_{\text {min }}=-z_{\text {max }}$. A full derivation is outlined in Section A. 2 and gives a formula to determine the height above/below the disk for each test particle:

$$
\begin{equation*}
z_{i}=\frac{h_{z}}{2} \ln \left[\frac{1+\tanh \left(\frac{z_{\max }}{h_{z}}\right)\left(2 B_{z}-1\right)}{1-\tanh \left(\frac{z_{\max }}{h_{z}}\right)\left(2 B_{z}-1\right)}\right] \tag{3.2.10}
\end{equation*}
$$

In Figure 3.2 the distribution of the values of $z_{i}$ is plotted. It can be seen that the vast majority of test particles have a height between -500 pc and 500 pc . To test the dependence of the final height distribution on the initial scaleheight we also run the model with different initial $h_{z}$. The distributions of these varying heights can be seen in Section 3.5.2.

## Combining the initial distributions

For the case of a disk scalelength of $h_{r}=3.37 \mathrm{kpc}$, a bulge scalelength of $h_{b}=0.538 \mathrm{kpc}$, a maximum radial disk length of $R_{\max }=15 \mathrm{kpc}$, a disk scaleheight of $h_{z}=0.3 \mathrm{kpc}$ and a maximum disk height of $z_{\max }=1 \mathrm{kpc}$ we present a graphical view of the initial distribution of test particles in Figure 3.3. The full initial distribution of test particles about a homogeneous disk is defined by

$$
\begin{equation*}
I(r, z)=A\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)\right] \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{3.2.11}
\end{equation*}
$$




## 3D view of the initial distribution of test particles



Figure 3.3: For scalelengths $h_{r}=3.37 \mathrm{kpc}$ and $h_{b}=0.538 \mathrm{kpc}$ and a scaleheight of $h_{z}=$ 0.3 kpc an initial distribution test particles is shown above. The initial distribution of test particles, viewed edge-on, exhibits the familiar profile of late type disk galaxies.

### 3.2.2 Velocity Kicks

Previous studies of the dynamics of OB populations have shown that the majority of OB stars have motions which follow those of the gas clouds from which they were formed, with a small dispersion $\sim 10 \mathrm{kms}^{-1}$. These surveys use distances and radial/proper motion data, corrected for solar motion and galactic rotation, to discern the space velocities of these high mass stars (Blaauw, 1958; Stone, 1979; Gies, 1987). The distribution of the space velocities of OB stars has also shown an excess of higher velocity stars and when these are taken as a separate group (runaway stars - usually defined to be those with a space velocity $\geq 30 \mathrm{kms}^{-1}$ although some studies include those with a high distance from the galactic plane) they show a much higher velocity dispersion $\sim 30 \mathrm{kms}^{-1}$ (and higher space velocities) (Stone, 1991). These high velocities show that a proportion of these stars ( $\sim 10-30 \%$ of O-stars and $5-10 \%$ of B-stars (Gies, 1987; Stone, 1991)) undergo a process which not only ejects the star from its parent gas cloud but frequently unbinds the star from a previous binary companion (Mason et al., 1998; Gies and Bolton, 1986). There has been much debate about the nature of this mechanism with some suggesting a dynamical (ie. gravitational) three- or n-body origin (Poveda et al., 1967). The ejection mechanism we focus on is the one whereby the primary star in a binary system (the initially most massive) undergoes a SN event and imparts a velocity to the secondary star (the initially least massive), potentially unbinding the binary and sending the secondary out with a high space velocity due to a recoil experienced when SN ejecta leaves the system (a Binary Supernova Scenario, BSS) (Blaauw, 1961; Hills, 1983; Burrows et al.,

| HIP | $v_{\text {space }}\left(\mathrm{kms}^{-1}\right)$ |
| :--- | :---: |
| $14514^{\dagger}$ | 39.4 |
| 18614 | 64.9 |
| 38455 | 41.4 |
| 38518 | 31.1 |
| 42038 | 31.3 |
| 46950 | 32.1 |
| 48943 | 35.2 |
| 49934 | 31.2 |
| 57669 | 31.1 |
| 76013 | 69.0 |
| $81377^{*}$ | 23.5 |
| 82868 | 30.3 |
| $91599^{\dagger}$ | 44.7 |
| 102274 | 46.1 |
| $109556^{*}$ | 74.0 |

Table 3.1: Sample of confirmed BSS runaways reproduced from Hoogerwerf et al. (2001). Those marked with an "*" are known as the classical runaways whose parent cluster were known, those marked with $a$ "" have an uncertainty about their parent cluster which therefore presents an ambiguity as to their ejection mechanism. The average $v_{\text {space }}$ is $41.68 \mathrm{kms}^{-1}$ and this sample is $63.3 \% \pm 4.5 \%$ of the total sample with confirmed ejection mechanism (BSS or $D E S$ ). 1995). We investigate the disruption of such binaries in our population synthesis in Section 4.2. We do not use the population synthesis model to derive velocity kicks as the nature of asymmetric SNae is quite uncertain.

A criterion for the breakup of a system via this mechanism requires that more than half the mass of the system is ejected in the process, however, an asymmetric SN explosion increases the chances of binary separation by lowering the amount of ejected matter needed for disruption (Hills, 1983; Burrows et al., 1995). The binary fraction among runaway OB stars is low ( $\sim 10 \%$ ) compared to the "normal" population of OB stars ( $\geq 50 \%$ ) however an example of SNae in undisrupted binary systems is high mass X-ray binaries (HMXBs) which are often found to have high space velocities $50 \mathrm{kms}^{-1}$ (Kaper et al., 1997; van den Heuvel et al., 2000). Our distribu-

## Distribution of velocity kicks



Figure 3.4: Since the number of space velocities of known BSS runaways is quite low (Hoogerwerf et al., 2001) we use a t-distribution to assign each of our test particles a velocity. We centre the distribution at $90 \mathrm{kms}^{-1}$ with $\sigma=30 \mathrm{kms}^{-1}$ so that a good range of velocities is covered. tion of velocity kicks ( $v_{k i c k}$ ) is partly based on observations of high velocity early-type stars. Hoogerwerf et al. (2001) use milli-arcsecond Hipparcos astronomy to retrace the path of known runaway stars and describe a range of velocities for confirmed BSS runaways of between $30-75 \mathrm{kms}^{-1}$ (A full list of the confirmed BSS runaways is reproduced from Hoogerwerf et al. (2001) in Table 3.1). The velocities of the sample in Table 3.1 only describe a limited selection according to Martin (2006) who suggests that runaways can have ejection velocities much greater than $100 \mathrm{kms}^{-1}$. Asymmetric neutrino emission has been shown to provide a mechanism that could impart a velocity kick to neutron stars of $\approx 1000 \mathrm{kms}^{-1}$ (Kotake et al., 2003; Kusenko and Segrè, 1996; Scheck et al., 2004), other sources of velocity include an electromagnetic rocket effect (Tademaru and Harrison, 1975) and to a lesser extent, convective instabilities in a compact stellar core. The distribution of observed pulsar velocities has traditionally been described as having two Gaussian components centred on $90 \mathrm{kms}^{-1}$ and $500 \mathrm{kms}^{-1}$ (Arzoumanian et al., 2002) but more recently it has been described as a Maxwellian with a mean velocity of $\approx 400 \mathrm{kms}^{-1}$ with $\sigma=265 \mathrm{kms}^{-1}$ (Hobbs et al., 2005). Assuming a neutron star mass of $1.4 \mathrm{M}_{\odot}$ and neglecting the ejection of a SN shell, conservation of momentum would suggest that a $7 \mathrm{M}_{\odot}$ star (a lower limit for CC SN progenitors) would gain a speed of $\approx 200 \mathrm{kms}^{-1}$ if the neutron star receives a natal kick of $\approx 1000 \mathrm{kms}^{-1}$. Since the observed number of runaways with a confirmed ejection mechanism is quite low (with low space velocities) we set the distribution of velocity kicks imparted to our test particles to be centered at $90 \mathrm{kms}^{-1}$ with a dispersion of $\sigma=60 \mathrm{kms}^{-1}$ so as to include a range of possible velocities. The velocity kicks are given a t-distribution (used when there is a only a small sample of known
values). The t-distribution produced in Octave ${ }^{1}$ is centred on 0 and extends from -3.78 to +3.78 (due to the way the program runs). All values are then shifted to the right so that they are positive by adding 3.78. The numbers are then multiplied by $120 / 7.56$ since the width of the velocity dispersion is $120 \mathrm{kms}^{-1}$ (ie. $2 \sigma$ ) and the final step is to then add 30 to each number so that the distribution is centred on $90 \mathrm{kms}^{-1}$ and extends from $30 \mathrm{kms}^{-1}$ to $150 \mathrm{kms}^{-1}$ (Figure 3.4).

### 3.2.3 Stellar masses

We assume that the test particles have undergone a period of mass transfer during their binary evolution and have therefore been "rejuvenated" to a zero age main sequence (ZAMS) star. The masses of the test progenitors, $M_{i}$, range from $M_{\min }=$ 10 to $M_{\max }=100 M_{\odot}$ and their distribution, $\Theta\left(M_{i}\right)$, follows the formulation of Bethe \& Brown (1998) such that

$$
\begin{equation*}
\Theta(M) \propto M_{i}^{-1.5} \tag{3.2.12}
\end{equation*}
$$

A set of random numbers, $B_{M}$, is again generated and each element is equated to the area under the curve of 3.2.12 integrated from 0 to $M_{i}$. This gives a function which determines the mass of each of the test particles:

$$
\begin{equation*}
B_{M i}=A_{M} \int_{M_{\min }}^{M_{i}} M^{-2.5} d M \tag{3.2.13}
\end{equation*}
$$

where $A_{M}$ is a constant which sets Equation 3.2.12, integrated from 10 to $100 M_{\odot}$, equal to one. Following the derivation in Section A. 3 the stellar mass for each test particle is calculated from

$$
\begin{equation*}
M_{i}=\left\{B_{M i}\left[M_{\max }^{-1.5}-M_{\text {min }}^{-1.5}\right]+M_{\text {min }}^{-1.5}\right\}^{\frac{1}{1.5}} \tag{3.2.14}
\end{equation*}
$$

From the stellar masses the main sequence stellar lifetimes, $\tau_{M S}$, are deduced from the formula of Belczyński and Bulik (1999):

$$
\begin{equation*}
\tau_{M S}(M)=20 \times 10^{6}\left(\frac{M}{10 M_{\odot}}\right)^{-2} \mathrm{yr} \tag{3.2.15}
\end{equation*}
$$

where $M$ is in solar masses. We then assume that the duration of the giant phase of the stars evolution is $0.2 \tau_{M S}$. When a period of $1.2 \tau_{M S}$ has elapsed the position of the test particles are noted - these positions make up the final distributions described in Section 3.5.

[^3]
### 3.3 The Potential

In order to examine the dynamics of runaway SNae in a realistic way, the test particles are situated in a gravitational potential that takes into account the well observed kinematics of stars in disk galaxies. The rotation curves of disk galaxies show that stars in the disk orbit the galactic centre at a speed which is almost constant over most of the disk. A simple galactic potential which replicates this behaviour has been described by Flynn et al., (1996) and is adopted for this study. This potential includes components due to the disk, the bulge/stellar halo, a dense central component and a dark halo and is described further on.

### 3.3.1 Description of potential

Flynn et al. (1996) describe a gravitational potential (per unit mass) which corresponds to observed Galaxy parameters such as its rotation curve, local density and disk scalelength. The potential, $\Phi(R, z)$, is composed of three main parts: $\Phi_{H}$ due to the dark halo, $\Phi_{C}$ due to the central component and $\Phi_{D}$ due to the disk. The central component comprises of a term for a dense central object and a bulge/stellar halo. The disk component is made up of three Miyamoto-Nagai potentials (Miyamoto and Nagai, 1975) which together describe a disk (with a central hole) that also satisfy measurements of the local density and reproduces a scalelength that is consistent with observation. The potential due to the dark halo is spherical and is described as

$$
\begin{equation*}
\Phi_{H}=\frac{1}{2} V_{H} \ln \left(R^{2}+z^{2}+r_{0}^{2}\right) \tag{3.3.1}
\end{equation*}
$$

| Parameter | Value |
| :---: | :---: |
| $V_{H}$ | $220 \mathrm{kms}^{-1}$ |
| $r_{0}$ | 8.5 kpc |
| $M_{C 1}$ | $3.0 \times 10^{9} \mathrm{M}_{\odot}$ |
| $M_{C 2}$ | $1.6 \times 10^{10} \mathrm{M}_{\odot}$ |
| $r_{C 1}$ | 2.70 kpc |
| $r_{C 2}$ | 0.42 kpc |
| $M_{D 1}$ | $6.6 \times 10^{10} \mathrm{M}_{\odot}$ |
| $M_{D 2}$ | $-2.9 \times 10^{10} \mathrm{M}_{\odot}$ |
| $M_{D 3}$ | $3.3 \times 10^{9} \mathrm{M}_{\odot}$ |
| $a_{1}$ | 5.81 kpc |
| $a_{2}$ | 17.43 kpc |
| $a_{3}$ | 34.86 kpc |
| $b$ | 0.30 kpc |

Table 3.2: Values of the parameters used in Equations 3.3.1, 3.3.2 and 3.3.3
where $V_{H}$ is the rotational velocity at a large distance from the core radius $r_{0}$ (values shown in Table 3.2). The potential of the central component is described as

$$
\begin{equation*}
\Phi_{C}=\frac{-G M_{C 1}}{\sqrt{\left(R^{2}+z^{2}\right)+r_{c 1}^{2}}}-\frac{G M_{C 2}}{\sqrt{\left(R^{2}+z^{2}\right)+r_{C 2}^{2}}} \tag{3.3.2}
\end{equation*}
$$

where $G$ is the gravitational constant, $M_{C 1}$ and $r_{C 1}$ denote the mass and core radius of the bulge/stellar halo and $M_{C 2}$ and $r_{C 2}$ are the mass and core radius of the compact
central component. The disk is represented by a function of the form

$$
\begin{equation*}
\Phi_{D}=\sum_{i}^{3} \frac{G M_{D i}}{\sqrt{R^{2}+\left[a_{i}+\sqrt{z^{2}+b^{2}}\right]^{2}}} \tag{3.3.3}
\end{equation*}
$$

The constants $a_{i}$ and $b$ relate to the scalelengths and scaleheight of each of the disk components, each having a total mass $M_{D i}$, respectively. The total potential per unit mass is then just the sum of each of the above;

$$
\begin{equation*}
\Phi(R, z)=\Phi_{H}+\Phi_{C}+\Phi_{D} \mathrm{~m}^{2} \mathrm{~s}^{-2} \tag{3.3.4}
\end{equation*}
$$

Together the components of the potential create a flat rotation curve for the model galaxy (Figure 3.5). A circular rotation curve of the galactic plane is constructed by noting that

$$
\begin{equation*}
\nabla \Phi(R)=\ddot{R}-R \dot{\phi}^{2} \tag{3.3.5}
\end{equation*}
$$

giving;

$$
\dot{\phi}=\sqrt{\frac{-\nabla \Phi}{R}}
$$

since for circular rotation $\ddot{R}=0$. Following the derivation in A.4.2 this gives the function for the circular speed, $\dot{\phi}_{c}$

$$
\begin{equation*}
\dot{\phi}_{c}=\left[\frac{V_{H}^{2}}{\left(r^{2}+r_{0}^{2}\right)}+\sum_{i=1}^{2} \frac{G M_{C i}}{\left(r^{2}+r_{C i}^{2}\right)^{\frac{3}{2}}}+\sum_{i=1}^{3} \frac{G M_{D i}}{\left(R^{2}+A_{D i}^{2}\right)^{\frac{3}{2}}}\right]^{1 / 2} \tag{3.3.6}
\end{equation*}
$$

### 3.4 Runge-Kutta-Fehlberg Method for Stellar Orbits

Simple numerical methods like a direct summation need a small timestep to be moderately accurate and are therefore computationally intensive and time consuming. Runge-Kutta methods offer a good solution since they apply a straight-forward procedure, produce an estimation of the local truncation error (at each iteration) and can be integrated into an adaptive method that keeps the local errors within certain predefined limits by altering the step size. In this example we use the RKF (Runge-Kutta-Fehlberg) scheme, which uses a fifth- and sixth-order method to give local


Figure 3.5: The rotation curve, in the $z=0 k p c$ plane, which is determined by the potential 3.3.4. At the solar radius, $R_{\odot}=8 \mathrm{kpc}$, the rotational velocity is $\sim 220 \mathrm{kms}^{-1}$ which is consistent with Galactic mass models (Dehnen and Binney, 1998) but is slightly lower than recent observational measurements (Reid et al., 2009).
truncation errors accurate to a fifth order Taylor expansion, to reproduce a simple orbit. This method is then applied to the main models described in this thesis - the dynamics of high velocity SN progenitors and the dispersion of HMXBs from spiral arms in disk galaxies. We describe here a brief synopsis of the method but refer the reader to Butcher (2003) for an extensive review.

### 3.4.1 Runge-Kutta Methods

Runge-Kutta (RK) methods are used to numerically solve initial value problems of the type

$$
\begin{align*}
y^{\prime} & =f(t, y)  \tag{3.4.1}\\
y\left(t_{0}\right) & =y_{0}
\end{align*}
$$

where $y^{\prime}$, equal to some function $f(t, y)$, is the time derivative of $y$ and $y_{0}$ is the value of $y$ at $t=0$. If a particle has some position $y$, then $y^{\prime}$ is the particles velocity. If the velocity of the particle is some function of time and/or position, $f(t, y)$ (due to some external force eg. gravity), then there is some error when calculating the particles position after some increment of time, $h$, by

$$
\begin{equation*}
y_{1}=y_{0}+h y^{\prime} \tag{3.4.2}
\end{equation*}
$$

since this assumes that the velocity is constant over the time interval $h$. Runge-Kutta methods attempt to reduce this error by using a velocity which is a weighted average of the velocities at different points during the timestep (eg. it could be an average of
the velocities at $h=0, h / 4, h / 2$ and $h)$. The points at which the velocities are worked out and the weightings that each one is given are defined in an array of values known as a Butcher Tableau. A Butcher Tableau can have any size but as more intermediate velocities are used the accuracy of each new position increases. The number of these velocities that are used define the order, $i$, of the RK method. These velocities, which are to be averaged, are represented below as the $k$-coefficients (see Equation 3.4.4) and the weightings they are given in the averaging process are defined by the $b$ values in the table which defines any fourth order RK method below.

$$
\begin{array}{c|cccc}
c_{1} & a_{11} & a_{12} & a_{13} & \\
c_{2} & a_{21} & a_{22} & a_{23} & \\
c_{3} & a_{31} & a_{32} & a_{33} & \\
c_{4} & a_{41} & a_{42} & a_{43} & \\
\hline & b_{1} & b_{2} & b_{3} & b_{4}
\end{array}
$$

The points at which the velocities are worked out are defined by the $a$ and $c$ coefficients such that

$$
\begin{equation*}
k_{i}=f\left(t_{n}+c_{i} h, y_{n}+\sum_{j=1}^{i-1} a_{i j} h k_{j}\right) \tag{3.4.3}
\end{equation*}
$$

which gives for this fourth order method

$$
\begin{align*}
& k_{1}=f\left(t_{n}+c_{1} h, y_{n}\right) \\
& k_{2}=f\left(t_{n}+c_{2} h, y_{n}+a_{21} h k_{1}\right)  \tag{3.4.4}\\
& k_{3}=f\left(t_{n}+c_{3} h, y_{n}+a_{31} h k_{1}+a_{32} h k_{2}\right) \\
& k_{4}=f\left(t_{n}+c_{4} h, y_{n}+a_{41} h k_{1}+a_{42} h k_{2}+a_{43} h k_{3}\right)
\end{align*}
$$

Notice that for the method to be consistent $\sum_{j=1}^{i-1} a_{i j}$ must be equal to $c_{i}$ for $i=2,3,4$ since the function $f(t, y)$ at $t_{n}+c h$ must correspond to the function at $y_{n}+\Sigma a h k$. Once all of the velocities, or $k$-coefficients, have been evaluated then it is possible to to get the weighted average and, in turn, work out the particles position by

$$
\begin{equation*}
y_{n+1}=y_{n}+h \sum^{i} b_{i} k_{i} \tag{3.4.5}
\end{equation*}
$$

which for this fourth order method gives

$$
\begin{equation*}
y_{n+1}=y_{n}+h\left(b_{1} k_{1}+b_{2} k_{2}+b_{3} k_{3}+b_{4} k_{4}\right) \tag{3.4.6}
\end{equation*}
$$

| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 |
| $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
|  | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{6}$ |

Table 3.3: A Butcher tableau for a fourth order RK method.

As an example, a particular fourth-order RK method for the numerical solution to the initial value problem above (Equation 3.4.1) has the tableau which has the solution

$$
\begin{equation*}
y_{n+1}=y_{n}+\frac{h}{6}\left(k_{1}+2 k_{2}+2 k_{3}+k_{4}\right) \tag{3.4.7}
\end{equation*}
$$

and $k$-coefficients

$$
\begin{align*}
k_{1} & =f\left(t_{n}, y_{n}\right) \\
k_{2} & =f\left(t_{n}+\frac{h}{2}, y_{n}+k_{1} \frac{h}{2}\right) \\
k_{3} & =f\left(t_{n}+\frac{h}{2}, y_{n}+k_{2} \frac{h}{2}\right)  \tag{3.4.8}\\
k_{4} & =f\left(t_{n}+h, y_{n}+k_{3} h\right)
\end{align*}
$$

### 3.4.2 Applying methods to Orbits

This method must be slightly changed since numerically solving orbits involves second order differential equations. In this case we use polar coordinates in which the potential is a function of $r$ only, i.e.

$$
\begin{equation*}
\Phi(r)=\frac{G M m}{r} \tag{3.4.9}
\end{equation*}
$$

and the force is

$$
\begin{equation*}
\nabla \Phi(r)=-\frac{G M m}{r^{2}}=\text { mass } \times \text { acceleration } \tag{3.4.10}
\end{equation*}
$$

giving the acceleration in the $r$-direction, $a_{r}$,

$$
\begin{equation*}
a_{r}=-\frac{G M}{r^{2}} \tag{3.4.11}
\end{equation*}
$$

which in polar coordinates is

$$
\begin{equation*}
a_{r}=\ddot{r}-r \dot{\theta^{2}} \tag{3.4.12}
\end{equation*}
$$

This gives

$$
\begin{equation*}
\ddot{r}=-\frac{G M}{r^{2}}+r \dot{\theta^{2}} \tag{3.4.13}
\end{equation*}
$$

This second order differential equation can be expanded into two first order equations that will describe the motion in the potential

$$
\begin{gather*}
\dot{v}_{r}=a_{r}  \tag{3.4.14}\\
\dot{r}=v_{r} \tag{3.4.15}
\end{gather*}
$$

The solutions to these equations follow the same method as above except that the $k$ coefficients for each solution must be evaluated in turn such that the second $k$ coefficient for Equation 3.4.14 $\left(k_{2 v}\right)$ relies on the first $k$ coefficient of Equation 3.4.15 ( $k_{1 r}$ ) and vice-versa. The first coefficients, $k_{1 v}$ and $k_{1 r}$, are simply the initial acceleration and velocities respectively. The $k_{i v}$ coefficients have units of $\mathrm{ms}^{-2}$ and the $k_{i r}$ coefficients have units of $\mathrm{ms}^{-1}$. The solutions for the fourth order RK method can be found below;

$$
\begin{align*}
& v_{n+1}=v_{n}+\frac{h}{6}\left(k_{1 v}+2 k_{2 v}+2 k_{3 v}+k_{4 v}\right) \\
& r_{n+1}=r_{n}+\frac{h}{6}\left(k_{1 r}+2 k_{2 r}+2 k_{3 r}+k_{4 r}\right) \tag{3.4.16}
\end{align*}
$$

where the coefficients are as follows

$$
\begin{align*}
& k_{1 v}=a\left(t_{n}, r_{n}\right)-\text { i.e. the acceleration at } r_{n} \\
& k_{2 v}=a\left(t_{n}+\frac{h}{2}, r_{n}+k_{1 r} \frac{h}{2}\right)-\text { i.e. the acceleration at } r_{n}+k_{1 r} \frac{h}{2} \\
& k_{3 v}=a\left(t_{n}+\frac{h}{2}, r_{n}+k_{2 r} \frac{h}{2}\right) \\
& k_{4 v}=a\left(t_{n}+h, r_{n}+k_{3 r} h\right) \\
& k_{1 r}=v\left(t_{n}, r_{n}\right)-\text { i.e. the velocity at } r_{n} \\
& k_{2 r}=v\left(t_{n}+\frac{h}{2}, r_{n}\right)+k_{1 v} \frac{h}{2}-\text { i.e. the velocity at } r_{n} \text { plus the velocity from } k_{1 v} \frac{h}{2} \\
& k_{3 r}=v\left(t_{n}+\frac{h}{2}, r_{n}\right)+k_{2 v} \frac{h}{2} \\
& k_{4 r}=v\left(t_{n}+h, r_{n}\right)+k_{3 v} h \tag{3.4.17}
\end{align*}
$$

Notice that the coefficients need to be evaluated alternately and that both solutions follow the Butcher Tableau (Table 3.3).

### 3.4.3 RKF Orbits

The method outlined above can be easily implemented for any higher order method once the appropriate Tableau has been obtained. For our simulation of a simple orbit we use the extended Butcher Tableau which has two solutions (two rows of $b$ 's), one of order five (which is accurate to a fourth order Taylor series expansion) and one of order six (which is accurate to a fifth order Taylor series expansion). This allows us to evaluate the local truncation error for each iteration which can be used to to alter the time-step and keep within a predefined error tolerance.

The extended Butcher Tableau for our method was developed by Fehlberg (1969) and is defined by

| 0 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 4$ | $1 / 4$ |  |  |  |  |  |
| $3 / 8$ | $3 / 32$ | $9 / 32$ |  |  |  |  |
| $12 / 13$ | $1932 / 2197$ | $-7200 / 2197$ | $7296 / 2197$ |  |  |  |
| 1 | $439 / 216$ | -8 | $3680 / 513$ | $-845 / 4104$ |  |  |
| $1 / 2$ | $-8 / 27$ | 2 | $-3544 / 2565$ | $1895 / 4104$ | $-11 / 40$ |  |
|  | $25 / 216$ | 0 | $1408 / 2565$ | $2197 / 4104$ | $-1 / 5$ |  |
|  | $16 / 135$ | 0 | $6656 / 12825$ | $28561 / 56430$ | $-9 / 50$ | $2 / 55$ |

where the bottom two lines give the fifth and sixth order results ( $y_{n+1}$ and $y_{n+1}^{\prime}$ ). The local truncation error is then calculated as

$$
\begin{equation*}
\epsilon=\left|y_{n+1}-y_{n+1}^{\prime}\right| \tag{3.4.18}
\end{equation*}
$$

If $\epsilon_{\text {tol }}$ is a desired accuracy and $\beta$ is some safety factor that controls how the time-step changes, then the optimum time-step, $h_{\text {opt }}$, can be evaluated by

$$
h_{\text {opt }}= \begin{cases}\beta h\left(\frac{\epsilon_{\text {tol }}}{\epsilon}\right)^{0.2} & , \epsilon \geq \epsilon_{\text {tol }} \\ \beta h\left(\frac{\epsilon_{\text {tol }}}{\epsilon}\right)^{0.25} & , \epsilon \leq \epsilon_{\text {tol }}\end{cases}
$$

### 3.4.4 Example application of an RKF method

In order to begin the orbit simulation the orbiting body must be given some initial values. These include the distance from the mass about which it orbits, $r_{n}$, the angle of the starting point, $\theta$, which for simplicity is taken to be zero, the velocity in the
$r$-direction, $v_{r}$, and the velocity in the $\theta$-direction, $v_{\theta}$. For an example of a simple circular orbit we take the case of the Earth orbiting the Sun - the radial velocity is zero and the orbital velocity (in rads $^{-1}$ ) is just 365 days (in seconds) divided by $2 \pi$. The radial distance is taken to be $1 \mathrm{AU}\left(1.496 \times 10^{11} \mathrm{~m}\right)$ and from this we can define the initial specific angular momentum, $P$

$$
\begin{equation*}
P=r^{2} v_{\theta, 1} \tag{3.4.19}
\end{equation*}
$$

where subscript 1 refers to the initial velocity (ie. at $r_{n, 1}$ - subsequent subscripts, $2,3+$, refer to the rotational velocity at $r_{n, 2}$ etc.). The first task is to calculate the $k$-values defined above which in this case ultimately depend only on different values of $r$. The first is

$$
\begin{equation*}
k_{1 v}=a\left(r_{n}\right) \tag{3.4.20}
\end{equation*}
$$

which is simply derived from our initial values (see Equation 3.4.13)

$$
\begin{equation*}
k_{1 v}=-\frac{G M}{r_{n}^{2}}+r_{n, 1} v_{\theta, 1}^{2} \tag{3.4.21}
\end{equation*}
$$

We also have $k_{1 r}$ from our initial values which is simply the initial radial velocity

$$
\begin{equation*}
k_{1 r}=v\left(r_{n}\right)=v_{r_{n, 1}}=0 \mathrm{~ms}^{-1} \tag{3.4.22}
\end{equation*}
$$

$k_{1 r}$ can now be used to evaluate $k_{2 v}$, or, more precisely, the radial acceleration at $r_{n, 2}=r_{n}+k_{1 r} \frac{h}{4}$ (or $r_{n}+k_{1 r} h a_{21}$ following the extended Butcher Tableau). For consistency we also evaluate the orbital velocity at this new value of $r, v_{\theta, 2}$ from the angular momentum we defined earlier

$$
\begin{equation*}
v_{\theta, 2}=\frac{P}{\left(r_{n}+k_{1 r} \frac{h}{4}\right)^{2}}=\frac{P}{r_{n, 2}^{2}} \tag{3.4.23}
\end{equation*}
$$

giving

$$
\begin{align*}
k_{2 v} & =-\frac{G M}{\left(r_{n}+k_{1 r} \frac{h}{4}\right)^{2}}+\left(r_{n}+k_{1 r} \frac{h}{4}\right) v_{\theta, 2}^{2} \\
& =-\frac{G M}{r_{n, 2}^{2}}+r_{n, 2} v_{\theta, b}^{2} \tag{3.4.24}
\end{align*}
$$

Now the next coefficient is worked out

$$
\begin{equation*}
k_{2 r}=v_{r_{n}}+k_{1 v} \frac{h}{4} \tag{3.4.25}
\end{equation*}
$$

from which we get the new value of $r$

$$
\begin{equation*}
r_{n, 3}=r_{n}+h\left(k_{1 r} \frac{3}{32}+k_{2 r} \frac{9}{32}\right) \tag{3.4.26}
\end{equation*}
$$

Then

$$
\begin{equation*}
k_{3 v}=-\frac{G M}{r_{n, 3}^{2}}+r_{n, 3} v_{\theta, 3}^{2} \tag{3.4.27}
\end{equation*}
$$

where $v_{\theta, 3}$ has been calculated from the angular momentum and the new $r$. This process is followed until all the required $k$-coefficients have been evaluated and which are then used in the following equations to compute the next values: $r_{n+1}, \theta_{n+1}$, $v_{r}\left(r_{n+1}\right)$ and $v_{\theta}\left(r_{n+1}\right)$.

$$
\begin{aligned}
r_{n+1} & =r_{n}+h\left(k_{1 r} b_{1}+k_{2 r} b_{2}+k_{3 r} b_{3}+k_{4 r} b_{4}+k_{5 r} b_{5}\right) \\
\theta_{n+1} & =\theta_{n}+h\left(v_{\theta, a} b_{1}+v_{\theta, b} b_{2}+v_{\theta, c} b_{3}+v_{\theta, d} b_{4}+v_{\theta, e} b_{5}\right) \\
v_{r_{n+1}} & =v_{r_{n}}+h\left(k_{1 v} b_{1}+k_{2 v} b_{2}+k_{3 v} b_{3}+k_{4 v} b_{4}+k_{5 v} b_{5}\right)
\end{aligned}
$$

and the new orbital velocity is

$$
\begin{equation*}
v_{\theta, n+1}=\frac{P}{r_{n+1}^{2}} \tag{3.4.28}
\end{equation*}
$$

This gives us the next set of values from which we can proceed with the next iteration. In order to evaluate an error for this iteration we calculate the sixth order values of $r_{n+1}, r_{n+1}^{\prime}$, by using the $b^{\prime}$ values from the extended Butcher Tableau (the very bottom line)

$$
\begin{equation*}
r_{n+1}^{\prime}=r_{n}+h\left(k_{1 r} b_{1}^{\prime}+k_{2 r} b_{2}^{\prime}+k_{3 r} b_{3}^{\prime}+k_{4 r} b_{4}^{\prime}+k_{5 r} b_{5}^{\prime}+k_{6 r} b_{6}^{\prime}\right) \tag{3.4.29}
\end{equation*}
$$

which gives a local truncation error of

$$
\begin{equation*}
\epsilon=\left|r_{n+1}-r_{n+1}^{\prime}\right| \tag{3.4.30}
\end{equation*}
$$

The time step can then be altered following the method outlined above.
All of this may seem redundant in the case of a perfectly circular orbit since the only parameter to change is $\theta$, but by altering the initial values of $v_{r}$ or $v_{\theta}$ it is possible to simulate elliptical orbits which obey the laws of orbital mechanics. Once a program has been written it is then a simple task to alter the initial conditions and in order to simulate orbits in a more complex gravitational potential it is required to only change that term. We present an example of the simple circular and elliptical orbit in Figure 3.6. For the elliptical orbit the only change in the initial conditions was that the test particle had an initial radial velocity of $v_{r_{n}}=15 \mathrm{kms}^{-1}$.


Figure 3.6: A simple circular and elliptical orbit using the RKF method outlined above.

### 3.4.5 Testing the potential/method

As can be seen from Figure 3.5 the circular speed in the plane of this potential rises sharply in the inner regions and then falls and levels out with $\dot{\phi}$ almost constant for $R>8 k p c$. This ensures that particles placed in the potential act in a way which is consistent with the observed stellar dynamics in spiral galaxies. It also allows the model to run more freely without the need to include some artificial rotation speed which would in-turn alter the specific angular momentum and therefore the consistency of the model. In order to test for this consistency the model is run for a variety of initial $R$-velocities ( $5 \mathrm{kms}^{-1}, 10 \mathrm{kms}^{-1}, 15 \mathrm{kms}^{-1}$ : with $z=0$ ) which are shown in Figure 3.7. These test are run with a timestep of 1 Myr up to a lifetime of 2.5 Gyr with an initial radius of 5 kpc .

The $z$-dependence is tested by keeping $R$ constant at $R=8 \mathrm{kpc} \approx R_{\odot}$ and varying the initial height of the particle. This test is done for $z=50 \mathrm{pc}, 100 \mathrm{pc}, 300 \mathrm{pc}, 800 \mathrm{pc}$ and 1 kpc , again evaluated with a timestep of 1 Myr up to a lifetime of 2.5 Gyr . The test shows that as the initial height is increased the particle oscillates above and below the plane fewer times - at 50 pc the particle oscillates $\approx 2.5$ times per orbit and decreases to $\approx 1.75$ times for an initial height of 1 kpc (see Figure 3.8).

The model is then run for a variety of initial parameters (shown in Table 3.4) with a timestep of 1 Myr up to a lifetime of 2.5 Gyr . For all initial parameters the orbits are fairly stable but gradually become more unstable due to the inherent errors associated with the model and the large timestep used here ( 1 Myr ) which is apparent in the gradual increase of the local truncation error, $\epsilon_{R}$, in Figures 3.9 b, d and f (see Equation 3.4.18 in Section 3.4.3).

|  | $R$ | $v_{R}$ | $z$ | $v_{z}$ | $\phi$ | $v_{\phi}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Set A | 8 kpc | $-25 \mathrm{kms}^{-1}$ | 50 pc | $30 k m s^{-1}$ | $25^{\circ}$ | $10 \mathrm{kms}^{-1}$ |
| Set B | 5 kpc | $10 \mathrm{kms}^{-1}$ | 200 pc | $7 \mathrm{kms}^{-1}$ | $25^{\circ}$ | $0 k m s^{-1}$ |
| Set C | 8 kpc | $-10 \mathrm{kms}^{-1}$ | -100 pc | $-7 \mathrm{kms}^{-1}$ | $125^{\circ}$ | $5 k m s^{-1}$ |

Table 3.4: The initial parameters which define the orbits plotted in Figure 3.9. Note that the velocity in the $\phi$-direction, $v_{\phi}$, above is added to the circular rotation speed which is defined by equation 3.3.6.


Figure 3.7: Orbits in the galactic plane for different values of $v_{R}\left(5 \mathrm{kms}^{-1}, 10 \mathrm{kms}^{-1}\right.$ and $15 \mathrm{kms}^{-1}$ ). Also shown are how $R, \epsilon_{R}$ (the truncation error at each step, see Equation 3.4.18), $v_{R}$ and $v_{\phi}$ change with time -1 iteration $=1 \mathrm{Myr}$.
z=0.1kpc
z=0.05kpc

(b)

## 덩ㅎㅇ우ㅁㅜㅜ웅



Figure 3.8: Orbits with different initial heights above the galactic plane (at $R_{\odot}=8 k p c$ ) are evaluated so as to test the z-dependence of the potential. Plotted here are orbits with initial height (a)50pc (b) 100pc (c)300 (d)800 (e)1kpc showing that the smaller the initial height, the more
$z=0.3 k p c$
(c)
Figure 3.8: Orbits with different initial heights above the galactic plane (at $R_{\odot}=8 k$ rapid the oscillation above and below the plane as shown in $(f)$ where the vertical line indicates one full orbit.

(c)
(c)
(c)
(c) P

(d)


Figure 3.9: The orbits determined by the parameters in Table 3.4. The figures on the right describe the orbits in the $x-y$ and $x-z$ plane along with the variations of $z, R$ and $\epsilon_{R}$ with respect to time.

### 3.5 Results

We first present some example trajectories for the test particles in the model in Figure 3.10 .



Figure 3.10: From the distributions defined in Section 3.2 we present the trajectories of a sample of the 10,000 test particles.

### 3.5.1 Face-on

Using the initial distribution of radial distances $\left(R_{i}\right)$, heights $\left(z_{i}\right)$ and velocities ( $v_{z}$, $v_{R}$ and $v_{\phi}$ ) defined in Section 3.2 the model is run for a total of 10,000 test particles. The model has little impact on the radial distribution of particles. Out of the 10,000 test particles only $0.3 \%$ end up with a radial distance greater than the maximum imposed when setting the initial distribution - 15 kpc - with only $0.02 \%$ having a final radial distance greater than 17 kpc . This result would suggest that it is extremely
unlikely that a SN will end up beyond the maximum radial extent of the disk due to a kick imparted by a former binary companion. The morphology of the distribution in $R_{S N}$ does however change. We fit the final distribution of radial distances using a Scaled Levenberg-Marquardt algorithm ${ }^{1}$ with a function of the form

$$
\begin{equation*}
\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right) \tag{3.5.1}
\end{equation*}
$$

The resulting fit shows that while the scalelength of the disk, $h_{r}$, has grown the scalelength of the bulge, $h_{b}$, has been reduced.


Figure 3.11: Although there is no obvious deviation of the final distribution of $R_{S N}$ (bottom) from the initial distribution (top) a fit to the profile reveals that the scalelength of the disk, $h_{r}$, has increased from 3.370 to 4.131 kpc while the scalelength of the central deficit, $h_{b}$, has been reduced from 0.538 to $0.322 k p c$. Note also that only a very small fraction of the test particles end up beyond the initial maximum extent of the disk.

We also construct a surface density profile for the final distribution using Equation 2.2.21. As can be seen from Figure 3.12 there is no major deviation from the initial distribution. A surface density profile of the initial distribution is fit with an exponentially decaying profile with a scalelength of 1.196 kpc or $0.080 R_{\text {Gal }}$ (where we take $R_{\text {Gal }}=15 \mathrm{kpc}$ ) while the final distribution has only a very small variation and is fit with a scalelength of 1.291 kpc or $0.086 R_{\text {Gal }}$.

### 3.5.2 Edge-on

For the case of the edge-on distributions the deviation from the initial distribution is much more pronounced than for the radial distribution. The test particles become much more dispersed in the $z$-direction than they do in the $R$-direction, most

[^4]

Figure 3.12: The initial and final surface density distributions of test particles varies only slightly. The initial distribution is fit with an exponentially decaying scalelength of 1.196 kpc while the final distribution is fit with a scalelength of 1.291kpc.
likely because of the finite distances each particle can travel and the extent of the $z$-distribution compared to the $R$-distribution. As mentioned above we test the dependence of the final distribution of heights on the initial scaleheight by running the model with varying $h_{z}$. We fit the final distributions with a profile of the form

$$
\begin{equation*}
\operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{3.5.2}
\end{equation*}
$$

and compare the initial, $h_{z, i}$, and final, $h_{z, f}$, scaleheights. We vary the scaleheights, in steps of 0.1 kpc , from 0.2 to 1.0 kpc each time setting the maximum height of the disk to be $3.33 h_{z}$. The initial and final distributions are plotted together in Figure 3.15 with each plot showing the scalelength of the initial/final distribution. An interesting result of this exercise is that as the initial scaleheight is increased, $h_{z, f}$ tends to converge on $h_{z, i}$. As can be seen from Figure 3.16 the initial and final scalelengths converge at $\sim 0.9 \mathrm{kpc}$ after which the model has a compressing effect on the test particles.




Figure 3.15: The initial (left) and final (right) vertical distributions of test particles from the runaway model are shown above (and on the preceding pages). The initial distributions are characterised by their respective scaleheights, $h_{z, i}$. The smallest $h_{z, i}$ ( $0.2 k p c$ ) leads to the greatest dispersion of test particles with the final distribution fit with a scaleheight of $h_{z, f}=$ 0.489 kpc or $2.445 h_{z, i}$. This dispersion decreases as $h_{z, i}$ is increased until the distribution begins to be compressed at $h_{z, i}>0.9 k p c$.

As a function of the initial scaleheight, the final scaleheights are fit with a linear function:

$$
\begin{equation*}
h_{z, f}=0.517 h_{z, i}+0.43 \mathrm{kpc} \tag{3.5.3}
\end{equation*}
$$

This result suggests that there is a higher chance of seeing a SN away from the disk of a "thin" galaxy if its progenitor had been given a high space velocity due to a kick provided by a former binary companion. Also, it suggests that for "thicker" galaxies the prob-


Figure 3.16: The initial and final scaleheights converge for increasing $h_{z, i}$ at 0.9 kpc . For $h_{z, i}>\sim 0.9 \mathrm{kpc}$ the model tends to compress the test particles. ability of finding a SN above/below the disk is diminished.

### 3.6 Discussion

We define a distribution of OB stars about an azimuthally homogeneous disk with a scalelength of 3.37 kpc and varying scaleheights. The stars are situated in a gravitational potential which conforms to the dynamical characteristics of stellar orbits around a galaxy centre (ie. a flat rotation curve). We impart to each star a kick velocity based on observations of high velocity OB stars and pulsars assuming that the star had previously been a component in a massive binary where the primary component exploded as a SN. Each star also has a defined mass from which we derive a stellar lifetime, after which the star is assumed to have exploded as a SN. The positions at which the stars explode then make up the final distributions. We find that the scalelength of the final distribution of radial distances has increased from $3.37 \mathrm{kpc}\left(0.24 R_{\text {Gal }}\right)$ to $4.131 \mathrm{kpc}\left(0.28 R_{\text {Gal }}\right)$ while the scalelength of the surface density distribution of SNae has increased only slightly from $1.196 \mathrm{kpc}\left(0.08 R_{\text {Gal }}\right)$ to 1.291 kpc $\left(0.09 R_{\text {Gal }}\right)$. Only $0.3 \%$ of the stars end up beyond the initial maximum extent of the galaxy ( 15 kpc ) but note that we have used a minimum mass of $10 M_{\odot}$ which sets a limit on the amount of time a star can travel. These results suggest that the radial distribution of CC SN progenitors does not completely follow the distributions of light in galaxies since the scalelength of the surface density of CC SNae from Chapter 2 exhibits a much more gradual falloff $\left(0.67 R_{\text {Gal }}\right.$ - or $0.35 R_{\text {Gal }}$ excluding the observed central deficit - compared to $0.09 R_{\text {Gal }}$ from the model). The final distribution of heights does however show a marked variation from the initial distribution for the case of a scaleheight of 0.3 kpc . We find that for a scaleheight of 0.3 kpc the final distribution is fit with a sech ${ }^{2}$ profile with a scaleheight of 0.596 kpc , an increase of almost double the initial value. We also find that $13.6 \%$ of the stars end up beyond the maximum height of the initial distribution ( 1 kpc in this case). Applying the same photometric method as for the observed SNae in Section 2.3.1 (see Equation 2.2.17) we construct a light profile of the form

$$
\begin{equation*}
I(r, z)=I_{0} \exp \left(\frac{-r}{h_{r}}\right) \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{3.6.1}
\end{equation*}
$$

and use the final positions $\left(R_{S N}, z_{S N}\right)$ to separate a subgroup of SNae that would be deemed to be positioned away from brightness profile of the host galaxy. From this method we find that at the final positions of $24.7 \%$ of the stars the surface brightness is less than $1 \%$ of the central surface brightness for the case of $h_{z}=0.3 \mathrm{kpc}$. We show in Table 3.5 for differing initial scaleheights the corresponding characteristics of the final height distributions. For smaller initial scaleheights it is clear that it is much easier for a runaway star to distance itself from the host galaxy. This is due

| Initial $h_{z}(\mathrm{kpc})$ | Final $h_{z}(\mathrm{kpc})$ | $z_{S N}>z_{\max }(\%)$ | $I(r, z)<0.1 I(0,0)(\%)$ |
| :---: | :---: | :---: | :---: |
| 0.2 | 0.49 | $20.6 \%$ | $32.7 \%$ |
| 0.3 | 0.60 | $13.6 \%$ | $24.7 \%$ |
| 0.4 | 0.62 | $7.5 \%$ | $16.1 \%$ |
| 0.5 | 0.70 | $5.5 \%$ | $13.4 \%$ |
| 0.6 | 0.71 | $2.9 \%$ | $8.5 \%$ |
| 0.7 | 0.84 | $1.9 \%$ | $7.8 \%$ |
| 0.8 | 0.86 | $1.4 \%$ | $6.3 \%$ |
| 0.9 | 0.88 | $0.5 \%$ | $4.4 \%$ |
| 1.0 | 0.91 | $0.4 \%$ | $3.7 \%$ |

Table 3.5: For each initial scaleheight ( $h_{z}$ ) we show the corresponding final scaleheight, the percentage of stars that end up beyond the initial maximum height of the disk $\left(z_{\max }=\right.$ $3.33 h_{z}$ ) and the percentage of stars whose positions would have less than $1 \%$ of the central surface brightness of the galaxy.
to the finite lifetimes, and the finite distances they can travel, of the SN progenitors. For a star to travel to a away from the disk in a thick galaxy would require either a greater velocity and/or a longer lifetime. The initial and final scalelengths converge at $\sim 0.9 \mathrm{kpc}$ which suggests that the potential of the galaxy acts a s a "high pass filter", allowing only very high velocity stars to escape it. This brings into question the origin of the the examples of CC SNae which occurred well above the disk of their host galaxy from Chapter 2 (see Figure 2.9). A possible explanation for their extraordinary height may be that their progenitors had lower masses than the current model allowed $\left(10 \mathrm{M}_{\odot}\right)$ and therefore longer lifetimes allowing them to travel further. Our model was also limited in the magnitude of the velocity kick imparted to the SN progenitors. Stars may achieve extremely high velocities in encounters with a massive compact object at the centre of a galaxy (Brown et al., 2005). There is also the possibility that these SNae, which are placed well above the disk of their host galaxy, may be tracers of field type O-stars (de Wit et al., 2005). The percentage of SNae that lie in low surface brightness areas from our model compares well with percentage of runaway candidates from Chapter 2 for the cases with initial scaleheight 0.4 or 0.5 kpc . However, the SNae that are observed to be well above the disk cannot be explained by the current model and suggests that their presence indicates either a dynamical event which imparts massive velocities to their progenitors or a population of massive stars which form in isolation, away from the disk. It is interesting to note that for the observed sample of SNae in Chapter 2, those SNae that are recorded to have a vertical height greater than 2 kpc are all type II SNae, indicating a relatively low metallicity (which is common for halo stars) compared to those in the disk.

## Chapter 4

## Modelling the dispersion of HMXBs

### 4.1 Introduction

The nature of the spiral structure observed in many galaxies has been studied for many years with a number of hypotheses emerging about their origin. Among these theories are (i) the Spiral Detonation Wave theory which suggests that star formation is a self renewing action - star formation in the spiral arms trigger more star formation in nearby regions causing a self-propagating wave of stellar births (Mueller and Arnett, 1976) (ii) the formation of spirals due to interactions with a galactic bar - it has been suggested that the spiral structure can be driven by forces exerted by a central bar (Sanders and Huntley, 1976) however this proposal has difficulty explaining spiral structure in galaxies with no bar (iii) Magnetohydrodynamic theories - forces due to magnetic fields have been proposed to have influence on the spiral structure in galaxies but observations of magnetic fields yield field strengths which are much too small to have any significant effect in building and maintaining a spiral (Spitzer, 1978) (iv) Gravitational effects of nearby galaxies - Toomre and Toomre (1972) suggested that gravitational forces from nearby galaxies can introduce a perturbation which manifests itself as a spiral pattern and was shown to have this effect (Toomre, 1981) but are unable to maintain it for long periods of time. There is still much unknown about the nature of spiral patterns in galaxies but a theory accepted by most and which is capable of creating and maintaining spiral structure over long periods of time is the theory put forward by Lin and Shu (1964) in which the spiral pattern is the result of a density wave propagating through the disk. Stars in the galactic
disk have elliptical orbits with varying epicyclic frequencies which naturally lead to a spiral structure which can be sustained over long periods of time.

These spiral structures are host to vigorous star formation and young stellar objects should be tightly correlated with the spiral structure. Maps of $\mathrm{H} \alpha$ emission have been shown to follow this pattern which is a consequence of its origin from the young stellar populations here (Kennicutt, 1998). An example of a short-lived stellar population in galaxies is the High Mass X-ray Binaries (HMXBs). These are binary systems which contain either a neutron star or a black hole. Mass is accreted on to the compact object from a massive companion and powers the X-ray emission. HMXBs have a lifetime in the range $10^{6-7}$ years and are therefore commonly associated with regions of recent star formation (Giacconi and Gursky, 1974). In this chapter we develop a population synthesis code to evolve binary systems and produce a sample of these HMXBs. We then use this sample to populate a dynamical model and measure their dispersion from the spiral arms.

The spiral pattern in most galaxies (unperturbed "grand design" galaxies) can be adequately described by a logarithmic spiral of the form (in polar coordinates)

$$
\begin{equation*}
\phi-\phi_{0}= \pm \frac{\ln \left(\frac{r}{r_{0}}\right)}{\tan (\alpha)} \tag{4.1.1}
\end{equation*}
$$

where $\phi_{0}$ and $r_{0}$ indicate the origin of the spiral arm and $\alpha$ is the angle made between the arm and the tangent to a circle at that point known as the pitch angle. The $\pm$ operator defines the direction that the spiral arms swing - using a minus sign a spiral such as that in Figure 4.1 is produced.


Figure 4.1: This figure shows a logarithmic spiral defined by equation 4.1.1 with a pitch angle of $\alpha=19^{\circ}$ overlain on an image of the spiral galaxy M51. As can be seen, the spiral arms are not perfectly logarithmic but are well approximated by the spiral.

Not to be confused with the rotational velocities of stars in the disk, the spiral pattern moves with its own characteristic pattern speed, $\Omega_{p}$. For our purposes we take it that the pattern speed moves with constant angular velocity. As a starting point we assume the pattern rotates with a speed of $30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ which leads to an angular speed of $\approx 9.72 \times 10^{-16} \mathrm{rads}^{-1}$. At a radial distance of $R=1 \mathrm{kpc}$ the spiral pattern has a rotational velocity, $v_{p}$, of $30 \mathrm{kms}^{-1}$ which is given simply by

$$
\begin{equation*}
v_{p}=R \Omega_{p} \tag{4.1.2}
\end{equation*}
$$



Figure 4.2: Since no analytical solution is available from equating the pattern speed with the equation governing the rotation curve due to the potential we take the co-rotation radius to be the point where the two lines plotted above intersect - 7.2kpc.

Rotating the spiral pattern is done simply by

$$
\begin{equation*}
\phi-\phi_{0} \pm \phi_{p}=\frac{\ln \left(\frac{r}{r_{0}}\right)}{\tan (\alpha)} \tag{4.1.3}
\end{equation*}
$$

where now we have a $\pm$ operator which defines which way the spiral rotates (trailing $(+)$ or leading (-)) and we set

$$
\begin{equation*}
\phi_{p}=\Omega_{p} \times t\left(r a d s^{-1}\right) \tag{4.1.4}
\end{equation*}
$$

where $t$ is the time.
The co-rotation radius, $r_{c}$, is the radius at which the circular speed of the test particles, $\dot{\phi}$ from Equation A.4.11, due to the gravitational potential equals the rotational speed of the spiral pattern, $\Omega_{p}\left(\mathrm{rads}^{-1}\right)$. Within this radius the test particles move, without any "kick", ahead of the spiral pattern due to their higher rotational ve-

| $\Omega_{p}\left(\mathrm{kms}^{-1} \mathrm{kpc}^{-1}\right)$ | $r_{c}(\mathrm{kpc})$ |
| :---: | :---: |
| 10 | 23.4 |
| 30 | 7.3 |
| 50 | 3.9 |

Table 4.1: The various pattern speeds used in the model and their corresponding co-rotation radii. locities. Beyond this radius the test particles lag behind the pattern. Equating the pattern speed with the equation which defines the rotation curve (Equation A.4.11) is a natural first step in trying to evaluate the co-rotation radius. However, due to the nature of the rotation curve equation it is not possible to evaluate analytically the co-rotation radius so here we have plotted both functions and taken the co-rotation radius as the point where both lines intersect. From Figure 4.2 it is clear that they
intersect at around 7.2 kpc . In order to refine the value of the co-rotation radius further we use a minimization script to obtain more accuracy. The script subtracts the value of $\dot{\phi}$ (Equation A.4.11) evaluated at different $R$ from $\Omega_{p}$ until the remainder goes below a certain threshold. From this method we obtain for a pattern speed of $30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ a value of $r_{c}=7.302 \mathrm{kpc}$. We use the same method to determine the co-rotation radii for pattern speeds of $10 \mathrm{kms}^{-1}$ and $50 \mathrm{kms}^{-1}$ which are listed in Table 4.1.

### 4.2 Population Synthesis

A simple population synthesis is developed so that as the model is run the number and nature of the binary systems are consistent with binary evolution theory. We first set up a population synthesis model which has initially thousands of objects with coeval evolution (hereafter the "starburst model") to ensure that the model produces a plausible amount of binary disruptions. For example Dray et al., (2005) estimate from models that the runaway fraction of OB stars is $\sim 50 \%$ of the total population whereas observations by Gies (1987) suggest the runaway population to be $10-30 \%$ with an extra $20 \%$ estimated to be uncounted runaways due to their low space velocities (de Wit et al., 2005). The synthesis is then run in a way in which it is continuously populated at a constant rate (hereafter the "continuous starburst model") to ensure that the population levels reach an equilibrium. Once the population levels agree with previous studies (Meurs and van den Heuvel, 1989) then the synthesis model is used to populate the dynamical model to investigate the motions of such objects and ultimately their final positions in relation to the spiral arms. Stellar parameters are defined according to the fitting functions of Eggleton et al., (1989) but for simplicity we assume no temporal dependence during the main sequence or giant phases of stellar evolution - Eggleton et al., (1989) set the luminosity and radius of their stars as a function of their respective ages.

### 4.2.1 The binary population

The initial masses of the primary stellar population ( $M_{1}$, the primary component being the initially most massive star in the binary) have a distribution, shown in Figure 4.3, following the formulations of Bethe \& Brown (1998) such that

$$
\begin{equation*}
\Theta\left(M_{1}\right) \propto M_{1}^{-1.5} \tag{4.2.1}
\end{equation*}
$$



Figure 4.3: The distribution of primary stellar masses (the primary being the initially most massive star in the binary) is defined by $\Theta\left(M_{1}\right) \propto M^{-1.5}$. The distribution of secondary masses, $\Theta\left(M_{2}\right)$, is then defined by the distribution of mass ratios $\left(q=\frac{M_{2}}{M_{1}}\right), \Gamma(q)=\frac{2}{(1+q)^{2}}$, where $q$ is limited to the range $0.35 \leq q \leq 1.0$.
where we set a lower mass limit of $M_{1}=10 M_{\odot}$ since we require that the primary undergo a SN event to evolve and become a compact object and an upper mass limit of $100 M_{\odot}{ }^{1}$. We follow Portegies Zwart \& Verbunt (1996) and set a mass ratio distribution $\left(\Gamma(q)\right.$, where $\left.q=M_{2} / M_{1}\right)$ according to

$$
\begin{equation*}
\Gamma(q)=\frac{2}{(1+q)^{2}} \tag{4.2.2}
\end{equation*}
$$

$\Gamma(q)$ is limited to the interval $0.35<q<1$ as Equation 4.2 .2 relates only to binaries with massive primaries (see Figure 4.3). The distribution for high mass binaries with a lower mass ratio is uncertain and has been suggested to have a flat distribution (Hogeveen, 1992). We set the distribution of semi-major axes, $\Xi(A)$, to follow a logarithmic distribution (Figure 4.4)

$$
\begin{equation*}
\Xi(A) \propto \frac{1}{A} \tag{4.2.3}
\end{equation*}
$$

with $6.96 \times 10^{10} \mathrm{~m} \leq A \leq 7 \times 10^{13} \mathrm{~m}\left(10 R_{\odot} \leq A \leq 10^{5} R_{\odot}\right)$ where the maximum separation, taken from Belczyński and Bulik (1999) is the point beyond which the binary period becomes extremely large and the likelihood of the system surviving a SN is greatly diminished. The lower limit on the semi-major axis allows the binary components to evolve separately - the primary will have at least a radius of $\sim 3.75 R_{\odot}$ (for a $10 M_{\odot}$ star - see Equation 4.2.4). We again follow Belczyński and Bulik (1999) in their procedure for the more massive binaries. If the sum of the radii in the binary exceeds $A_{\text {min }}=10 R_{\odot}$ the semi-major axis is set as twice the primary radius and the

[^5]

Figure 4.4: The distribution $\Xi(A)$ is logarithmic. We set the minimum orbital separation to be $A_{\text {min }}=10 R_{\odot}$ and, so as to avoid a period of common envelope evolution or merger, if the sum of the radii in the binary exceeds this $A_{\text {min }}$ the semi-major axis is set to twice the radius of the primary (and the orbit is circularised). The maximum is set to be the that radius, beyond which, the binary is unlikely to survive a $S N$ explosion and the period becomes extremely long which we assume to occur at radii approaching $\approx 10^{5} R_{\odot}$. The orbital eccentricities have a distribution between 0 and 1 with $\Pi(e)=2 e$ following Duquennoy \& Mayor (1991).
eccentricity set to zero. This should remove any possibility of a merger. The main sequence (MS) radius is calculated from the fitting formula described by Eggleton et al., (1989)

$$
\begin{equation*}
R_{M S}=\frac{1.968 M^{2.887}-0.7368 M^{1.679}}{1.821 M^{2.337}-1} R_{\odot} \tag{4.2.4}
\end{equation*}
$$

where $M$ is in solar masses.
The eccentricities, $e$, of the binary orbits are distributed according to Duquennoy \& Mayor (1991) (Figure 4.4) such that

$$
\begin{equation*}
\Pi(e)=2 e \tag{4.2.5}
\end{equation*}
$$

where $0 \leq e \leq 1$. We include the effects of tidal interactions on the orbital eccentricity of close massive binaries. Proposed by Portegies Zwart \& Verbunt (1996), if the radius of the primary is greater than 0.2 times the periastron distance, $r_{p}$, then, conserving angular momentum and in one iteration, the orbit is circularised until $r_{p}$ is five times the primary radius or until the orbit is completely circular - ie. $e$ is reduced until either $r_{p}$ is $5 R_{q}$ or $e=0$. This change occurs according to

$$
\begin{equation*}
\left(1-e_{1}^{2}\right) a_{1}=\left(1-e_{0}^{2}\right) a_{0} \tag{4.2.6}
\end{equation*}
$$

where subscript 0 refers to the initial state, subscript 1 to the final state and $r_{p}=$
$a(1-e)$.
We follow Belczyński and Bulik (1999) and determine the main sequence lifetime of each of the stars, $\tau_{M S}$, to be

$$
\begin{equation*}
\tau_{M S}(M)=20 \times 10^{6}\left(\frac{M}{10 M_{\odot}}\right)^{-2} \mathrm{yr} \tag{4.2.7}
\end{equation*}
$$

where $M$ is expressed in solar masses. We also take the duration of the giant phase to be $\tau_{G}=0.2 \tau_{M S}$ giving a total lifetime before a SN event $\tau_{\text {total }}=\tau_{M S}+\tau_{G}$ after which, due to the short timescales after the giant phase, it is assumed that the stellar component evolves instantly to a compact object.

Now that the initial distribution of the parameters is set, the population synthesis is run. If the radius of the primary after it becomes a giant, $R_{1, G}$, (which we assume to be simply twice the MS radius) exceeds the Roche lobe radius, $r_{L}$, then a period of mass transfer ensues. The Roche lobe radius is approximated by the formula of Eggleton (1983)

$$
\begin{equation*}
r_{L}=A \frac{0.49 q^{2 / 3}}{0.6 q^{2 / 3}+\ln \left(1+q^{1 / 3}\right)} R_{\odot} \tag{4.2.8}
\end{equation*}
$$

We assume that during the period of mass transfer the primary loses all of its Hydrogen envelope and all that remains is the Helium core which we take to be 0.3 times the initial mass (Bethe and Brown, 1998) giving a primary mass, after mass transfer, $M_{1, f}$

$$
\begin{equation*}
M_{1, f}=0.3 M_{1} \tag{4.2.9}
\end{equation*}
$$

Only a proportion of the lost mass is given to the secondary, the amount of which is dependent on the mass ratio $q$ (Vrancken et al., 1991; Bethe and Brown, 1998). The mass of the secondary, after mass transfer, $M_{2, f}$, is

$$
\begin{equation*}
M_{2, f}=M_{2}+0.7 q^{2} M_{1}=f \times M_{1} \tag{4.2.10}
\end{equation*}
$$

where

$$
\begin{equation*}
f=q+0.7 q^{2} \tag{4.2.11}
\end{equation*}
$$

and the semi-major axis is then, from Pols and Marinus (1994),

$$
\begin{equation*}
A_{f}=A\left[\left(\frac{M_{1, f}}{M_{1}} \frac{M_{2, f}}{M_{2}}\right)^{-2}\left(\frac{M_{1, f}+M_{2, f}}{M_{1}+M_{2}}\right)^{(2 \beta+1)}\right] \tag{4.2.12}
\end{equation*}
$$

where here we take $\beta=6$ which is an uncertain parameter that takes into account the angular momentum lost by the ejected matter (Belczyński and Bulik, 1999). At this
point the secondary will have gone through a fraction $g(q)$ of its lifetime whereby

$$
\begin{equation*}
g(q)=q^{2}+0.2 f^{2} \tag{4.2.13}
\end{equation*}
$$

since, from Equation 4.2.7, the stellar lifetime is inversely proportional to the square of the mass (Bethe and Brown, 1998). If $g>1.2$ then we assume the secondary has exploded as a SN before the primary (shortly after mass transfer) and it is then considered the primary component for the rest of the synthesis - it is now the stellar component that accretes matter from the secondary (formerly the primary) to produce X-ray emission. If $1<g<1.2$ the the secondary is instantaneously in the giant phase while if $g<1$ then the secondary is still on the main sequence.

For mass transfer to occur in a binary the primary must fill its Roche lobe. This also has the effect of circularising the orbit due to tidal interactions. For this reason it is assumed that those binaries that have undergone mass transfer now have zero orbital eccentricities as in Belczyński and Bulik (1999).

### 4.2.2 The SN

For the population of binaries who have not undergone a period of tidal interaction or mass transfer (ie. their orbital eccentricities are non-zero) a position in orbit is randomly chosen so as to determine the effect of a SN kick on the binary parameters of the remaining system. The position in the stars orbit, defined by its eccentric anomaly - the angular distance from perihelion in an elliptic orbit, does not progress uniformly with time. A fictitious orbit is defined which is circular with a radius and period equal to our "real" stars semi-major axis and period. The position in this idealised orbit, which has constant angular momentum, is defined by the "mean" anomaly. Given a mean anomaly and the (real) systems orbital eccentricity it is possible to randomly choose a position in an eccentric orbit in a way which affords an equal probability of being chosen to all orbital positions. A mean anomaly, $\kappa$, between 0 and $2 \pi$ is chosen randomly since it progresses uniformly with time and gives an instantaneous position in the elliptical orbit. A simple method is used along with the each systems eccentricity to solve Kepler's equation iteratively to find the eccentric anomaly, $\varepsilon$, from which the true anomaly, $\theta$, and radial distance from the focus, $r$, are found:

$$
\begin{gather*}
\cos (\theta)=\frac{\cos (\varepsilon)-e}{1-e \cos (\varepsilon)}  \tag{4.2.14}\\
r=A_{f}[1-e \cos (\varepsilon)] \tag{4.2.15}
\end{gather*}
$$

where we denote $A_{f}$ as the semi-major axis of the system just prior to the SN. The orbital velocity of the SN progenitor just prior to the $\mathrm{SN}, v$, is then given by (Portegies Zwart and Verbunt, 1996)

$$
\begin{equation*}
v^{2}=G\left(M_{1, f}+M_{2, f}\right)\left[\frac{2}{r}-\frac{1}{A}\right] \tag{4.2.16}
\end{equation*}
$$

For the distribution of velocity kicks imparted to the SN remnant, $\Psi\left(v_{k}\right)$, two options are tested. Firstly a simple Gaussian distribution with a mean velocity of $500 \mathrm{kms}^{-1}$ and $\sigma=150 \mathrm{kms}^{-1}$ (Figure 4.5) which we denote $\Psi_{A}\left(v_{k}\right)$ and secondly a flat distribution from $150-200 \mathrm{kms}^{-1}$ which is denoted $\Psi_{B}\left(v_{k}\right)$. There have been many efforts to constrain the distribution of SN kicks, mainly by investigating the distribution of pulsar velocities. It has been shown that many pulsars have a velocity well over $500 \mathrm{kms}^{-1}$ and measurement of offsets from SN remnants indicate that the distribution has a tail of high velocities (Arzoumanian et al., 2002; Frail et al., 1994) - some authors model their kick velocities on Maxwellian, Gaussian or the sum of two Gaussian distributions (Hobbs et al., 2005; Portegies Zwart and Verbunt, 1996). However, the dynamics of neutron stars has also been fit with models that give the remnant a kick of $200 \mathrm{kms}^{-1}$ (Blaauw and Ramachandran, 1998) and kicks in the range $150-200 \mathrm{kms}^{-1}$ (Lipunov et al., 1997).

After the SN the velocity of the remnant, $v_{n}$, is

$$
\begin{equation*}
v_{n}^{2}=v^{2}+v_{k}^{2}+2 v v_{k} \cos (\gamma) \tag{4.2.17}
\end{equation*}
$$

where $\gamma$ is the angle, chosen randomly, between $v$ and $v_{k}$. We assume that all stars which were initially less than $40 M_{\odot}$ evolve after SN to become a neutron star of mass $M_{\text {Rem }}=1.4 M_{\odot}\left(M_{\text {Rem }}\right.$ - remnant mass $)$ and that those larger than this become black holes with a mass according to the following equation (van den Heuvel and Habets, 1984)

$$
\begin{equation*}
M_{R e m}=\left[0.35 M_{1, f}-12\right] M_{\odot} \tag{4.2.18}
\end{equation*}
$$



Figure 4.5: For some of the scenarios described below we model the distribution of velocity kicks which are imparted to the compact object as a simple Gaussian with a mean of $500 \mathrm{kms}^{-1}$ and $\sigma=150 \mathrm{kms}^{-1}$ $\left(\Psi_{A}\left(v_{k}\right)\right)$.
with $M_{1, f}$ in solar masses. Following Hills (1983) we assume that the binary is disso-
ciated if

$$
\begin{equation*}
\frac{M_{M E}}{\left(M_{1, f}+M_{2, f}\right)} \geq \frac{r}{2 A_{f}}\left[1-\left(\frac{v_{k}}{v_{c}}\right)^{2}-2\left(\frac{v}{v_{c}}\right)\left(\frac{v_{k}}{v_{c}}\right) \cos (\gamma)\right] \tag{4.2.19}
\end{equation*}
$$

where $M_{M E}$ is the mass ejected in the SN explosion $\left(M_{M E}=M_{1, f}-M_{R e m}\right)$ and $v_{c}$ is the circular velocity of the orbit (ie. when $r=A_{f}$ ). The new semi-major axis for those systems which have not been disrupted, $A_{n}$, is then found from

$$
\begin{equation*}
v_{n}^{2}=G\left(M_{R e m}+M_{2, f}\right)\left[\frac{2}{r}-\frac{1}{A_{n}}\right] \tag{4.2.20}
\end{equation*}
$$

so that

$$
\begin{equation*}
A_{n}=\left[\frac{2}{r}-\frac{v_{n}^{2}}{G\left(M_{R e m}+M_{2, f}\right)}\right]^{-1} \tag{4.2.21}
\end{equation*}
$$

A disruption rate, $d_{S N}$, is defined from the starburst model which quantifies the percentage of all initial binaries that have been disrupted in the SN

$$
\begin{equation*}
d_{S N}=\frac{\text { number of binaries disrupted }}{\text { total number of initial binaries }} \tag{4.2.22}
\end{equation*}
$$

This rate allows one to determine the effect of altering the distribution of velocity kicks and gives an idea of how many X-ray emitters could populate the dynamical model.

### 4.2.3 X-ray Luminosity

We assume that all of the binaries that remain bound after the SN explosion of the primary component emit X-rays due to the wind fed accretion on to the the compact SN remnant. Depending on the initial conditions the synthesis produces a population of X-ray emitters according to the disruption rate, $d_{S N}$. The X-ray luminosity of the undisrupted population is assumed to be due to a wind-fed mechanism which, for the purposes of this study, is adequately described by the accretion method outlined by Bondi \& Hoyle (1944) along with the assumption that all of the gravitational potential of the accreted material is released as X-rays which follows the work of Iben et al., (1995). The X-ray luminosity is approximated according to Equation 4.2.23:

$$
\begin{equation*}
L_{x}=\frac{4.1 \times 10^{11}}{\alpha_{w}^{4}}\left[\frac{M_{R e m}}{M_{2, f}}\right]^{2}\left[\frac{R_{2}}{A_{n}}\right]^{2} \dot{M}_{w} L_{\odot} \tag{4.2.23}
\end{equation*}
$$

where $M_{\text {Rem }}$ and $M_{2, f}$ are the masses of the compact SN remnant and the donor (secondary) star respectively, $R_{2}$ is the radius of the donor star, $A_{n}$ is the post-SN semi-major axis, subscript " $w$ " indicates that the parameter is associated with the stellar wind and $\dot{M}_{w}$ is in $M_{\odot} y r^{-1}$ (Iben et al., 1995). Equation 4.2.23 also takes into account the kinetic energy of the wind which is parameterised by its velocity. $\alpha_{w}$ is a term which takes into account the velocity of the wind as it is accreted onto the compact object. Stellar winds may not stop accelerating out to great distances meaning that in a close binary system the emitted winds may not have reached their maximum velocity (where $\alpha_{w}=1$ ). We follow the prescription of Waters \& Kerkwijk (1989) and take

$$
\begin{equation*}
\alpha_{w}=1-\frac{R_{2}}{A_{n}} \tag{4.2.24}
\end{equation*}
$$

We take the mass loss rate due to stellar winds, $\dot{M}_{w}$, from the donor star (the secondary) to be given by

$$
\begin{equation*}
\dot{M}_{w}=\frac{L_{O B}}{v_{\infty} c} \tag{4.2.25}
\end{equation*}
$$

where $L_{O B}$ is the luminosity of the donor star. The luminosity depends on whether it is a MS or giant star and is calculated with the fitting functions of Eggleton et al., (1989) (Equations 4.2.27 and 4.2.28). In Equation $4.2 .25 v_{\infty}$ is 3 times the escape velocity of the star (Equation 4.2.26) and $c$ is the speed of light.

$$
\begin{gather*}
v_{\infty}=3 \sqrt{\frac{2 G M_{2, f}}{R_{2}}}  \tag{4.2.26}\\
L_{O B, M S}=\frac{13990 M^{5}}{M^{4}+2151 M^{2}+3908 M+9536} L_{\odot}  \tag{4.2.27}\\
L_{O B, B G B}=\frac{2.15 M^{2}+0.22 M^{5}}{1+\left(1.4 \times 10^{-2}\right) M^{2}+\left(5 \times 10^{-6}\right) M^{4}} L_{\odot} \tag{4.2.28}
\end{gather*}
$$

where $L_{O B, M S}$ is the main sequence luminosity and $L_{O B, B G B}$ is the luminosity at the base of the giant branch and $M$ is in solar units.

For the lifetimes of the X-ray sources we again follow Iben et al., (1995) and use the analytical approximations of Massevitch et al., (1979) for donor stars, with mass $M_{2}$, who are still on the main sequence (ie. $g(q)<1$ ), $\tau_{A}$, and for those who are in the giant phase $(1<g(q)<1.2), \tau_{B}$.

$$
\begin{gather*}
\tau_{A}=1000 M_{2}^{2}\left(1+0.000005 M_{2}^{3.5}\right)^{-1} \mathrm{yr}  \tag{4.2.29}\\
\tau_{B}=14000 M_{2}^{-0.44} \mathrm{yr} \tag{4.2.30}
\end{gather*}
$$

where $M_{2}$ is in solar masses. Following the SN of the primary component the X-ray
phase of the system is ceased when a time defined by either Equation 4.2.29 or 4.2.30 (depending on the donor stars evolutionary stage) has expired.

### 4.3 Output

### 4.3.1 Starburst Model

The starburst model initially has 10,000 binaries which have parameters as set out in Section 4.2. After $\sim 28 \mathrm{Myrs}$ all of the primary components have undergone a SN event which agrees well with the prescription for the stellar lifetimes in Equation 4.2.7. At this point in the model $d_{S N}$ can be extracted. The starburst model is run with different kick velocity distributions; $\Psi_{A}\left(v_{k}\right)$ which is a Gaussian distribution, an example of which is shown in Figure 4.5 , and $\Psi_{B}\left(v_{k}\right)$ which is a flat distribution between 150 and $200 \mathrm{kms}^{-1}$. We also run the model with no kick velocity so that the disruption rate, $d_{S N}$, is completely dependent on the mass lost from the system, $M_{M E}$, the semi-major axis, $A_{f}$, and the orbital velocity at the time of the SN, $v$. As a diagnostic the model is also run for

| $v_{k}$ | Disruption Rate, $d_{S N}$ |
| :---: | :---: |
| $\Psi_{A}\left(v_{k}\right)$ | $87.23 \%$ |
| $\Psi_{B}\left(v_{k}\right)$ | $77.63 \%$ |
| $400 \mathrm{kms}^{-1}$ | $85.79 \%$ |
| $300 \mathrm{kms}^{-1}$ | $82.63 \%$ |
| $200 \mathrm{kms}^{-1}$ | $78.42 \%$ |
| $100 \mathrm{kms}^{-1}$ | $71.70 \%$ |
| No kick | $36.08 \%$ |

Table 4.2: In order to test the dependence of the output of the population synthesis on the magnitude of the velocity kick we varied the distribution. $\Psi_{A}\left(v_{k}\right)$ refers to a velocity kick distribution which is a Gaussian with a mean velocity of $500 \mathrm{kms}^{-1}$ and $\sigma=150 \mathrm{kms}^{-1}$ and $\Psi_{B}\left(v_{k}\right)$ refers to a distribution which is flat from $150-200 \mathrm{kms}^{-1}$. We also tested the model in which all of the remnants of the primary component received no kick and kicks of $100,200,300$ and $400 \mathrm{kms}^{-1}$. the case where all of the SN remnants receive kicks of $100,200,300$ and $400 \mathrm{kms}^{-1}$. From Table 4.2 and Figure 4.6 it is clear that the rate of disruption is dependent the magnitude of the velocity kicks (the kicks are oriented in a random direction). Where the remnant of the primary component receives no kick the disruption rate is relatively low at about $36 \%$. Applying a small kick of $\sim 100 \mathrm{kms}^{-1}$ doubles the disruption rate indicating the strong dependence of $d_{S N}$ on the presence of a kick. For the disruption rates due to the theoretical velocity distributions $\left(\Psi_{A / B}\left(v_{k}\right)\right)$ the kicks can disrupt between $77-88 \%$ of all the binaries in the model. Using a Gaussian distribution which can have velocities up to $\approx 1000 \mathrm{kms}^{-1}$ means that less than $13 \%$ of the binaries remain bound after the primary goes SN . If each of the disrupted binaries produces a runaway star this would lead to a runaway fraction much higher than those suggested by Dray et al., (2005), Gies (1987) and de Wit et al., (2005). However this model doesn't include the populations of stars that evolve in solitude or those that exist in very wide binaries.


Figure 4.6: The disruption rate, $d_{S N}$, is highly dependent on the magnitude of the velocity kick imparted to the remnant of the primary component in a SN. Although disruption can occur solely as the result of the loss of mass during a $S N$ the percentage of disruptions double as a result of even a modest kick (100kms ${ }^{-1}$ ). Here we show the disruption rates, for a number of velocity kick distributions, as a function of the age of the starburst. All the kicks are oriented in a random direction.

### 4.3.2 Continuous Starburst Model

In the continuous starburst model we introduce new binaries at a specific rate. This emulates the situation across a spiral arm where star formation can be assumed to be practically constant. In this model the population of binaries reaches an equilibrium, as does the population of X-ray emitters. This allows the dynamical model to retain a population of binaries and X-ray emitters which is consistent with the binary evolution theory outline above. This model is run with the two theoretical velocity distributions defined above - $\Psi_{A / B}\left(v_{k}\right)$. The rate at which the model is populated is governed by the approximation of the birthrate of binaries by Iben et al., (1995)

$$
\begin{equation*}
d v\left(\mathrm{yr}^{-1}\right)=0.2 d \log (A)\left(\frac{d M_{1}}{M_{1}^{2.5}}\right) d q \tag{4.3.1}
\end{equation*}
$$

where, as in Section 4.2, $A$ is the semi-major axis, $M_{1}$ is the primary mass and $q$ is the mass ratio. Taking $d \log (A)$ to be $4, d q$ to be 0.65 and $M_{1}$ to lie in the range of $10-100 M_{\odot}$ we get a binary birthrate of $1.06 \times 10^{-2} \mathrm{yr}^{-1}$. If the lifetimes of the primary components of the binaries are, on average, in the range $10-20 \mathrm{Myrs}$ the population of binaries in the starburst model will be expected to lie in the range 1.06 $-2.16 \times 10^{5}$. With the duration of the X-ray phase ranging from 2,000 to $100,000 \mathrm{yrs}$ for case A (Equation 4.2.29) and 2,000 to $8,000 \mathrm{yrs}$ for case B (Equation 4.2.30) the expected population of X-ray emitters lies in the range of $\sim 20-1,000$ for the case

A binaries and $\sim 20-90$ for the case B binaries. The continuous starburst model


Figure 4.7: The continuous starburst model is run with different kick velocity distributions - $\Psi_{A}\left(v_{k}\right)$ (top) and $\Psi_{B}\left(v_{k}\right)$ (bottom). In line with expectations, the total binary population (filled circles) remains at $\sim 1.12 \times 10^{5}$ in equilibrium. The total number of $X$-ray emitters (filled squares) for the $\Psi_{A}\left(v_{k}\right)$ velocity distribution levels off at between 30 and 50 but is increased for the $\Psi_{B}\left(v_{k}\right)$ velocity distribution which reflects the different disruption rates, $d_{S N}$, for each case. The lower lines depict the populations of X-ray emitters whose emission exceeds certain thresholds: $L_{X}>500 L_{\odot}$ (diamonds), $L_{X}>1,000 L_{\odot}$ (triangles) and $L_{X}>10,000 L_{\odot}$.
reaches a population equilibrium of about $1.12 \times 10^{5}$ objects at roughly 20 Myr . For kick velocity distribution "A" $\left(\Psi_{A}\left(v_{k}\right)\right.$ - the Gaussian) the total number of X-ray emitters stays in the region of between $30-50$ while kick velocity distribution " B " $\Psi_{B}\left(v_{k}\right)$ allows a higher population - from about 85-110 - reflecting the dependency of the population on the disruption rate. The populations of ever more intense X-ray emitters is also looked at. We show the populations of X-ray emitters whose X-ray luminosity (defined by Equation 4.2.23) exceeds certain thresholds, namely; $500 L_{\odot}=$ $1.9195 \times 10^{36} \mathrm{ergs}^{-1}, 1,000 L_{\odot}=3.839 \times 10^{36} \mathrm{ergs}^{-1}$ and $10,000 L_{\odot}=3.839 \times 10^{37} \mathrm{ergs}^{-1}$.

The number of extremely bright X-ray sources (those above $10,000 L_{\odot}$ ) never exceeds more than ten for both kick velocity distributions.

### 4.4 The Dynamical Model

We use the potential and method described in Section 3.3 as the basis for the dynamical model. As in the case for the runaway model the distribution of test systems is based on the light profiles of disk galaxies. The distribution in $R$ is defined by an exponential function:

$$
\begin{equation*}
I(r)=I_{0} \exp \left(\frac{-r}{h_{r}}\right) \tag{4.4.1}
\end{equation*}
$$

where $I(r)$ is the surface density of systems at $r, I_{0}$ the central surface density and $h_{r}$ is the scalelength of the distribution which we tentatively set at 3.37 kpc as in Section 3.2.1. We do not include a central hole in this distribution but instead set a lower limit on $R$ of 1.5 kpc and a maximum limit of 15 kpc . Since we are investigating the dispersion of HMXBs from the spiral arms their distribution inside 1.5 kpc is not the subject of our study although they do occur there. As can be seen from Figure 4.8 even at 2 kpc binaries from both spiral arms tend to mix together and objects from both arms become inseparable. In their study of the spiral structure of HMXBs in M51 Shtykovskiy and Gilfanov (2007) also omit HMXBs in the galaxy centre (within $2.7 \mathrm{kpc})$. The test systems also have a height distribution as outlined in Section 3.2.1 such that

$$
\begin{equation*}
I(z)=I_{0} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{4.4.2}
\end{equation*}
$$

where $h_{z}$ is the scaleheight of the distribution which we take to be 0.3 kpc . As in Section 3.2.1 the positions $(R, z)$ of the test systems are defined by sets of random numbers ( $B_{R}$ and $B_{z}$ ). The position of the test particles on a spiral pattern is now found from the $R$ - position and the equation which defines a logarithmic spiral:

$$
\begin{equation*}
\phi(R)=-\frac{\ln \left(\frac{R}{r_{0}}\right)}{\tan (\alpha)} \tag{4.4.3}
\end{equation*}
$$

where $r_{0}$ is minimum radius -1.5 kpc and $\alpha$ is the pitch angle of the spiral. We rotate the spiral pattern as time, $t$, progresses by noting the pattern speed, $\Omega_{p}$, in rads ${ }^{-1}$. The model can now be populated in a rotating spiral pattern from the function

$$
\begin{equation*}
\phi(R, t)=-\frac{\ln \left(\frac{R}{r_{0}}\right)}{\tan (\alpha)}+\Omega_{p} t \tag{4.4.4}
\end{equation*}
$$

We randomly assign the test particles to one of two arms for this model by modulating Equation 4.4.4 by $\pi$.

The synthesis model described above is now used to populate the dynamical model at a rate defined by Equation 4.3.1 in order to measure the dispersions of the Xray emitters from the spiral arms. As the dynamical model is populated each new binary system orbits the galaxy centre with a rotation speed defined by the potential (Equation 3.3.6). When the primary component of the binary explodes as a SN the resulting undisrupted systems each receive a kick of $50 \mathrm{kms}^{-1}$ in a random direction the kick can have any 3D orientation. The dispersion is firstly qualitatively analysed on an $R$ - $\phi$ plot (Figure 4.8). We measure the dispersions for varying pitch angles ( $\alpha$ ) and use for this purpose the kick velocity distribution $\Psi_{A}\left(v_{k}\right)$ defined in Section 4.2.2 while holding the spiral pattern speed $\left(\Omega_{p}\right)$ constant. We then measure the dispersion for varying $\Omega_{p}$ while holding $\alpha$ constant.

### 4.4.1 Constant pattern speed, $\Omega_{p}$

For the purposes of investigating the effect of varying pitch angles we hold the pattern speed constant at $30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$.

We show first how the structure of the dynamical model which includes all particles (from newly born systems to evolved X-ray emitters) evolves with time. For the first example (Figure 4.8) we use a pitch angle of $\alpha=10^{\circ}$.

Figure 4.8 shows snapshots of the dynamical model at different epochs - 1, 10 and 100 Myr . From Section 4.3 .2 we know that the model should reach a population equilibrium at roughly 20 Myr . At 1 Myr the model systems are still tightly correlated with the spiral and there is indeed little apparent deviation from this as one progresses to 10 and 100 Myr . In the top panel the 100 Myr "snapshot" does show some distortion from the perfect (infinitesimally thin) spiral arm. The most notable change is the "mixing" of systems close to the galaxy centre where objects from both spiral arms are now inseparable. The extent of the "central mixing" is not obvious in the top 10 Myr snapshot but by inspecting the $R-\phi$ plot for the same epoch it is evident in the spreading out of points at small $R$. In the $R-\phi$ plots each logarithmic line represents a distinct arm and in the 100 Myr plot it is clear that systems from one arm have indeed mixed with the other. There is less deviance from the arms as one moves outwards from the centre and the pattern speed approaches the circular rotation speed of the potential (ie. at the co-rotation radius, 7.3 kpc see Table 4.1).

The systems then move away from the arms again in the very outer parts as they lag behind the spiral arm. The spiral arm progresses with its constant characteristic pattern speed ( rads $^{-1}$ ) while the stellar systems progress according to their rotational velocity $\left(\mathrm{kms}^{-1}\right)$ which is almost constant for all $R$. This means that systems that "lag" behind a spiral arm should do so in a circular arc centred on the galaxy centre. For spirals with a small pitch angle the pattern produced by systems lagging behind (which essentially has a pitch angle of $0^{\circ}$ - ie. circular) is not immediately apparent since it closely resembles the "almost" circular spiral pattern. This dispersion of systems beyond the co-rotation radius is therefore more apparent in spiral systems with greater pitch angles which is evident in Figure 4.9 where the pitch angle has been increased from $10^{\circ}$ to $20^{\circ}$ and even more so in Figure 4.10 where a pitch angle of $30^{\circ}$ has been used.


Figure 4.8: For a pitch angle of $\alpha=10^{\circ}$ and a pattern speed of $\Omega_{p}=30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ the top set of figures shows the structure of the dynamical model at different epochs; 1, 10 and 100Myr. These plots include all systems, from newly added binary systems to HMXBs. The spiral structure is still present long after the model has reached a population equilibrium ( $\sim 20 \mathrm{Myr}$ ) however dispersion from the arms is evident at 100Myr. The most notable feature is the mixing in the centre of the galaxy. The bottom plots show $R$ - $\phi$ plots of the positions of the systems. In these plots the $\phi$ dispersion of the systems from the arms becomes much clearer and the onset of the central mixing is apparent even at 10Myr.

The following pages depict the positions of X-ray emitters (red dots) as defined by the prescription in Section 4.2.3 in relation to the pre-SN binary systems (black dots) for different epochs along with their respective populations. Figures 4.11, 4.13 and 4.15 show how the model begins for pitch angles of 10,20 and $30^{\circ}$ respectively. The


Figure 4.9: While the central mixing is equally as apparent as in Figure 4.8 the dispersion in the outer arms is more noticeable due to the increased pitch angle of $\mathfrak{2 0}$.


Figure 4.10: The dispersion of systems past the co-rotation radius is most apparent for the spiral with $\alpha=30^{\circ}$.
density of systems is higher at small $R$ - a consequence of the the radial distribution function (Equation 4.4.1) - which consequently leads to more X-ray emitters in the central regions as can be seen from the "snapshots" from 0.5-5Myr. Kennicut (1983) describes a star formation rate (SFR) that is is an exponentially decaying function of radial distance while more recently the multi-wavelength study of M33 by Verley et al., (2009) suggests that the SFR of young stars in that galaxy declines with radius (with scalelength 2 kpc ). We note however that the SFR in galaxies has been traditionally parameterised by the surface density of gas (Schmidt, 1959) or the dynamical timescales (Shu, 1973). We therefore suggest that the populations of HMXBs in the outer regions of our model galaxies may be underestimated. As the model reaches equilibrium ( $\sim 20 \mathrm{Myr}$ ) this central concentration is apparent as is the apparent lack of dispersion of X-ray emitters, even with a kick, from the spiral pattern produced by the pre-SN systems (see Figures 4.12, 4.14 and 4.16). For the case of a pitch angle of $10^{\circ}$ we see almost no dispersion of the pre-SN binaries from the thin spiral arms. In some cases a HMXB can be seen separate from the arm (see Figure 4.12 60 Myr ). This suggests that for low pitch angles neither the young stellar or HMXB populations will deviate much from the spiral pattern. For this reason, HMXBs with a large space velocity may travel to areas distinct from arms with a low pitch angle. For the case of a pitch angle of $20^{\circ}$ the pre-SN binaries tend to disperse more from the logarithmic spiral and indeed the HMXBs follow suit. The X-ray sources however, even with a high space velocity, become less separated from the binaries (see Figure 4.14). For even greater pitch angles this effect is compounded (Figure 4.16) suggesting that although the dispersion from an infinitesimally thin spiral is greatest for higher pitch angles, it also becomes less likely to find the HMXBs separated from the light due to young stars (eg. $\mathrm{H} \alpha$ ). It should also be noted that for higher pitch angles the co-rotation radius becomes ever clearer (by the reduction in the width of the arms at this point). This implies that dispersions from a thin spiral in (well behaved) spiral galaxies could provide a diagnostic for the determination of the pattern speed in the galaxy (provided a rotation curve has been defined).

Population levels $\mathbf{0 . 5 - 5 M y r}$
Population levels 0.5-5Myr
Figure 4.11: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $10^{\circ}$ at epochs from 0.5-5Myr.

Figure 4.12: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $10^{\circ}$ at epochs from 10-100Myr.

Population levels $\mathbf{0 . 5 - 5 M y r}$

Figure 4.13: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $20^{\circ}$ at epochs from 0.5-5Myr.



Population levels $\mathbf{1 0 - 1 0 0} \mathbf{M y r}$

Figure 4.14: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $20^{\circ}$ at epochs from 10-10Myr.
 15



(15

Population levels $\mathbf{0 . 5 - 5 M y r}$

Figure 4.15: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $30^{\circ}$ at epochs from 0.5-5Myr.


Figure 4.16: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model for a pitch angle of $30^{\circ}$ at epochs from 10-100Myr.

As the populations of X -ray emitters is quite low $(<1,000)$ we collect the positions of these emitters at 0.5 Myr year intervals up to 100 Myr to create an increased sample of positions. These positions are then modulated (in $\phi$ ) via their epoch and the pattern speed of the spiral so that they occupy the same region of space in the polar coordinate system (ie. if $\tau$ is the epoch of the recorded position of $\phi$ then we subtract $\tau \Omega_{p}$ from $\phi)$. This increased sample allows a better determination of the dispersion of HMXBs from the spiral arms as it includes a range of possible positions. This distribution of HMXB positions then necessarily includes positions of the same HMXBs at different epochs as is evident in the trajectories of HMXBs emanating from the spiral arm (eg. Figure 4.17). We plot the increased sample of HMXB positions (black dots) over the positions of pre-SN systems at 100 Myr (red dots). The positions of pre-SN systems at 100 Myr is taken as a reference since at this point the model is in equilibrium and gives a representative view of the expected distributions of young stars about a spiral arm for each respective pitch angle (and consequently $\mathrm{H} \alpha$ emission). It is apparent, even with this larger sample of HMXB positions, that HMXBs do not tend to travel very far from their birthplace in the spiral arms. In fact, it could be suggested that HMXBs represent a good tracer for star forming regions in spiral galaxies. The dispersion from the spiral arms is most notable in Figure 4.17 as the tracks protruding from the spiral arm (note we switch that colour code in an effort to improve the contrast between the populations: red dots - pre-SN binaries, black dots - HMXBs). The low pitch angle in this case keeps the spiral arms from getting very wide making the dispersion of HMXBs clearer whereas for the higher pitch angles (Figures 4.18 and 4.19) the spiral arms are much thicker, hiding the movement of HMXBs from the arm.


Figure 4.17: For a pitch angle of $10^{\circ}$ an increased sample of HMXB positions (black dots) is plotted over the positions of pre-SN systems at 100Myr (red dots). The dispersion of HMXBs is apparent in the many "tracks" coming out of the spiral arms.


Figure 4.18: Here we plot the increased sample of HMXB positions for the case of a spiral pitch angle of $20^{\circ}$.


Figure 4.19: Here we plot the increased sample of HMXB positions for the case of a spiral pitch angle of $30^{\circ}$.

To quantitatively measure the dispersion of the population from the spiral arms we measure the distribution in $\phi$ for the increased sample of HMXBs (those modulated by $\tau \Omega_{p}$ ) and pre-SN systems (at 100 Myr ) which lie in distinct ranges of $R$. These distributions are then fit with multi-peak Gaussian curves. The variance in these distributions is then plotted as a function of $R$. This gives a measure of how "dispersed" the systems are as one moves out along the spiral arm.

We plot histograms of $\phi$-values for those HMXBs and pre-SN systems in the range $R=1-2 \mathrm{kpc}$. These distributions are then fit with a curve representing the sum of two Gaussian distributions (see Figure 4.21). The variance, $\sigma$, is then a measure of the
dispersion from the spiral arms of the systems within a radial distance of $1-2 \mathrm{kpc}$. This is then done for the systems with a radial distance in the range $2-3 \mathrm{kpc}, 3-4 \mathrm{kpc}$ etc. out to $\sim 15 \mathrm{kpc}$ (the maximum radial distance) so that we end up with a set of variances measuring how the systems become dispersed as a function of radial distance $R$. For the $\alpha=10^{\circ}$ dataset from above we plot the distributions in Figure 4.21. In each plot is listed the variance, $\sigma$, of the Gaussian fit for both spiral arms. For those systems within $1-2 \mathrm{kpc}$ the central mixing manifests itself as one of the spiral arms having a wider distribution than the other. Although this mixing is symmetric the arms originate (at $\tau=0 \mathrm{Myr}$ ) at $\phi=0$ and $\pi$ and consequently the distributions on the $R-\phi$ plot lead to the second arm (which originates at $\pi$ ) "acquiring" binaries from the first. Also, those systems within the range of $2-3 \mathrm{kpc}$ have an even wider distribution in $\phi$ than those in the range $1-2 \mathrm{kpc}$. One would expect that the more central systems would have a wider distribution (due to the increased rotational velocity) but due to the spiral arms having a starting radius of 1.5 kpc the population of systems within 1 - $2 \mathrm{kpc}(16,291)$ is less than two thirds the population between $2-3 \mathrm{kpc}(25,623)$. The variance in the distributions then systematically drops as one moves out to greater $R$. As the effects of mixing are diminished in the outer regions the variance in the distributions from both arms tend to equalise. However, this coherence is marginally lost in the very outer regions as the population levels drop and identical fits become less probable. We make these plots for all three pitch angles (the plots for $\alpha=20^{\circ}$ and $30^{\circ}$ are shown in Figures B. 2 and B. 4 in Appendix B respectively). The dispersion along the arms are then plotted in Figure 4.22. It can be seen that the dispersions in $\phi$ are generally lower for spirals with higher pitch angle as expected but also that the dispersions of HMXBs is tightly correlated with those of the young stellar population for the lowest pitch angle. This correlation is decreased with increasing pitch angle and suggests that, in general, the distribution of HMXBs should be thinner than that of young stars.


Figure 4.21: For systems in distinct radial distance bins (eg. 1-2kpc, 2-3kpc, etc.) the distribution of $\phi$-values for both the increase sample of HMXBs and the pre-SN systems at 100Myr are plotted. These distributions are then fit with a double peaked Gaussian curve. The variance, $\sigma$, for each of the fits is listed (one for each arm - subscript 1 refers to the left arm) and is a measure of the dispersion of the systems from the spiral arm. The dispersion is greatest in the galaxy centre and decreases outwards but at the very centre mixing causes one arm to exhibit a greater dispersion than the other. As this mixing effect decreased as greater radial distances the dispersions in both arms tends to equalise.


Figure 4.22: The dispersions from Figures 4.21, B.2 and B. 4 are gathered here and plotted as a function of radial distance. Increasing pitch angles mean that the dispersions will be generally lower for spirals with a high pitch angle. For the case of a pitch angle of $10^{\circ}$ there is no great difference in the distributions of $H M X B s$ and the pre-SN binaries. This difference increases with increasing pitch angle with HMXBs having a lower dispersion than the pre-SN binaries.

### 4.4.2 Constant pitch angle, $\alpha$

In this section we investigate the dispersions for varying pattern speeds while holding the pitch angle constant at $20^{\circ}$. We use pattern speeds of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ and $50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ for this purpose (Note that the dispersions can be compared with the dispersions from Figure 4.9 which also has a pitch angle of $20^{\circ}$ but a pattern speed of $30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ ). These pattern speeds have co-rotation radii at 23.4 kpc and 3.9 kpc respectively (see Table 4.1). As in the previous section we firstly qualitatively describe the dispersions of all objects in the model from the spiral arm at epochs of $1 \mathrm{Myr}, 10 \mathrm{Myr}$ and 100 Myr (see Figures 4.23 and 4.24 ). The major difference from above is that in the case $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ pattern speed the co-rotation radius lies well beyond the maximum radial extent of the objects in the model. For this reason all of the objects move ahead of the spiral pattern and the thickening of the spiral arm sets in much quicker. In the case of a pattern speed of $50^{-1} \mathrm{kpc}^{-1}$ the co-rotation radius is clearly apparent in the 100 Myr plots and is already seen in the 10 Myr plots making spirals with high pattern speeds a good diagnostic in determining the co-rotation radius. As in the case for the previous section the central mixing of objects is also evidenced in these snapshots.


Figure 4.23: For a pattern speed of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ we show here the progression of the dispersion of objects in the model (both newly formed binaries and HMXBs) from the spiral arms. The co-rotation radius for this case lies well beyond the maximum radial distance in the model at 23.4kpc. Because of this the objects move ahead of the spiral pattern at all radii and results in much thicker arms.


Figure 4.24: As above, here we present the progression of the dispersion of objects in the model with a pattern speed of $50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$. The co-rotation radius in this case is at 3.9 kpc and can be seen already at 10Myr in the apparent thinning of the spiral arms.

We plot on the following pages the positions of the HMXBs compared to the population of pre-SN binaries at different epochs. As in the case for all of the varying pitch angles the population of HMXBs is tightly correlated with the spiral arms up to 5 Myr . However, when the model has reached equilibrium (after 20Myr) we notice that for a low pattern speed (Figure 4.25) the HMXBs are closer to the spiral arms than the population of pre-SN binaries. The orbital rotation of the pre-SN binaries is much greater than the pattern speed for the range of radii here and results in thick arms being formed. HMXBs can be found in any part of the arms but are preferentially placed close to the thin spiral. For this reason we suggest that HMXBs won't, in general, be placed away from the light of young stars in spirals with low pattern speed. On the other hand, for the case of spirals with high pattern speed we see that the thickness of the arms is at a minimum at small galactocentric radii and in these places the high space velocity of the HMXBs allows the to travel appreciably away from the population of pre-SN binaries at the co-rotation radius (see Figure 4.28 $60 / 70 \mathrm{Myr})$.


Figure 4.25: HMXBs (red dots) plotted with the population of pre-SN systems (black dots) from the model with a pattern speed of $10 \mathrm{kms}{ }^{-1} \mathrm{kpp}^{-1}$ at epochs from 0.5-5Myr.
-15 ${ }^{-10}{ }^{-10}{ }^{-5}$
(x) 100 My , 15 $\circ$

0
0
0
 n... on $\frac{0}{1}$








Population levels $10-100 \mathbf{M y r}$

Figure 4.26: HMXBs (red dots) plotted with the population of pre-SN systems (black dots) from the model with a pattern speed of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ at epochs from 10-100Myr.
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Figure 4.27: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model with a pattern speed of $50 \mathrm{~km} s^{-1} \mathrm{kpc}^{-1}$ at epochs from 0.5-5Myr.

Population levels $\mathbf{1 0 - 1 0 0 M y r}$

Figure 4.28: $H M X B s$ (red dots) plotted with the population of pre-SN systems (black dots) from the model with a pattern speed of $50 \mathrm{kms}{ }^{-1} \mathrm{kpc}^{-1}$ at epochs from 10-100Myr.

To get a better sample of HMXB positions we use the same method as above to modulate the $\phi$-positions to the same area in a polar coordinate system. As can be seen for the case of a low pattern speed (Figure 4.29) the thick spiral arms mean that HMXBs won't be able to travel to positions which are distinct from the population of pre-SN binaries. Although the HMXBs can be placed anywhere in the thick arms they are preferentially found in a thinner spiral than their pre-SN counterparts. In contrast to the latter, the HMXBs from the model with a pattern speed of $50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$ can be clearly seen as separate from the spiral arms around the co-rotation radius (Figure 4.30).


Figure 4.29: Here we plot the increased sample of $H M X B$ positions for the case of a spiral with a pattern of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$. The thick arms which are due to the low pattern speed hide instances where the HMXB might have travelled away from the pre-SN population.


Figure 4.30: Here we plot the increased sample of $H M X B$ positions for the case of a spiral with a pattern of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$. The high pattern speed causes the spiral arm to thin at the co-rotation radius ( 3.9 kpc ) allowing the HMXBs to travel away from the population of pre-Sn binaries.


Figure 4.31: The dispersions from Figures 4.21, B.2 and B.4 are gathered here and plotted as a function of radial distance.

Again we quantitatively measure the dispersion in $\phi$ along the spiral arm for both cases. Following the same prescription as above we fit multi-peak Gaussian curves to the distributions in $\phi$ for distinct radial distance bins (see Figures B. 6 and B. 6 in Appendix B). We plot the dispersions as a function of $R$ in Figure 4.31

### 4.5 Discussion

We have constructed a population synthesis program that populates a dynamical model in order to investigate the dispersions of HMXBs from the spiral arms where they originated. The population synthesis creates a sample of high mass binaries which conforms to observed distributions of stellar mass, mass ratios, semi-major axes and orbital eccentricities. We use the fitting functions of Eggleton et al., (1989) to evolve the binaries until one of the components explodes as a SN. We include the effects of mass transfer and tidal interactions in the pre-SN systems. We apply a distribution of kicks to the SN remnants and determine whether the system remains intact or is disrupted following the prescription of Hills (1983). The undisrupted systems, each containing a massive star and a compact object, are then assumed to emit X-rays due to a wind fed accretion onto the compact object. We use the binary birthrate of Iben and Tutukov (1995) to populate the dynamical model with binary systems that evolve according to the above prescription. The binaries are born along a spiral pattern with varying pitch angles and pattern speeds and initially orbit the galaxy centre in accordance with the potential defined in Section 3.3. The undisrupted post-SN binaries are then given a kick of $50 \mathrm{kms}^{-1}$ in a random 3D direction. We then measure the distribution of these X-ray emitters compared to the population of preSN binaries which we assume to adequately represent a population of young stars in
a galaxy and consequently the distribution of $\mathrm{H} \alpha$ emission.
From the population synthesis model we find that the rate of disruption of massive binaries due a SN is strongly sensitive to the presence of of kick velocity which is imparted to the stellar remnant. The presence of even a modest kick can double the disruption rate in our model from $\sim 35 \%$ to $\sim 70 \%$ (disruption with no kick can also occur due to the escape of mass from the system). We find that the theoretical distributions of SN kick velocities (described in Section 4.2.2) can disrupt between $77 \%$ and $88 \%$ of high mass binaries (see Table 4.2 and Figure 4.6).

We find from the dynamical model that the population of HMXBs do not move appreciably far from the population of pre-SN binaries even though they have a high space velocity. In fact, we find that in all cases our HMXBs are tightly correlated to the spiral pattern from which they originate and are only seen distinctly apart from the pre-SN binaries in areas where the spiral arm is thin (Figures 4.17, 4.18, 4.19, 4.29 and 4.30). The thickness of the arms defined by the pre-SN binaries is a direct result of the nature of the spiral arms themselves. For low pitch angles we find that the spiral arms remain thin because the circular motion of the stars in the potential is similar to the almost circular (low $\alpha$ ) spiral pattern (Figure 4.12). For this reason we suggest that HMXBs may be more readily found away from the spiral arms in galaxies with a low pitch angle. For galaxies with increasing pitch angle the circular motion of the stars is more and more at odds with the spiral pattern. This causes a thickening of the arms and although the HMXB distribution follows this thickening they are less likely to be found away from the light due to a young stellar population. The HMXBs tend to have a thinner distribution than the pre-SN binaries and are generally tightly correlated with the spiral pattern making them excellent tracers of recent star formation.

We also find that the pattern speed of the spiral has a noticeable effect on the dispersion of stars from the spiral. For a low pattern speed (and hence large corotation radius) we find that the dispersion creates very thick arms since the stars move much quicker than the spiral pattern and as a consequence the HMXBs are rarely placed away from the young stars in this case (Figure 4.29). Conversely, for a high pattern speed (and small co-rotation radius) we find that the high space velocity of the HMXBs enable them to travel away from the pre-SN binaries near the corotation radius, placing them in a position which would be distinct from the light of the young stellar population (Figure 4.30).

We also measure the dispersion of the pre-SN binaries and HMXBs in $\phi$ for each case as a function of galactocentric radius. We find that for low pitch angles the
dispersion of pre-SN binaries and HMXBs is very similar but as the pitch angle is increased the pre-SN binaries become more dispersed than the HMXBs (Figure 4.22). In an opposite fashion we find that for small pattern speeds the dispersion of pre-SN binaries is greater than that of HMXBs and tends to equalise as the pattern speed is increased (Figure 4.31).

Observations of spiral galaxies have revealed many different populations of X-ray sources. These include the HMXBs, Low Mass X-ray binaries (LMXBs), SN remnants (SNRs) and also Active Galactic Nuclei (AGNs) ${ }^{1}$. These sources represent different populations with HMXBs, and the less luminous SNRs, representing a young population with lifetimes in the range $10^{6-7}$ yrs and should be concentrated towards the regions of recent star formation, namely the spiral arms of galaxies. The LMXBs represent an older population of stars with lifetime in the range $10^{8-9} \mathrm{yrs}$ and should generally be distributed evenly around the disks of spirals although it has to be noted that they may trace out a spiral pattern if the arm/inter-arm stellar density ratio is high. The AGNs represent a population of background sources and have no connection with the spiral arms of a particular galaxy. The LMXBs may be separated from the HMXBs by their soft spectra although (rare) high mass black hole binaries have been suggested to have soft spectra also. For this reason colour-colour diagrams have been used to distinguish X-ray sources. HMXB sources also exhibit variability in their X-ray emission due to the intermittent nature of accretion on to the compact object and so a study of the temporal variation in X-ray luminosities can identify HMXBs. The LMXBs also have higher X-ray luminosities compared to HMXBs. In order to compare our model to observations of X-ray sources in spiral galaxies one would obviously have to use these methods to separate the different classes of sources.

XMM-Newton and Chandra have amassed many extragalactic X-ray sources and a study of their distributions will compliment the results of our model. Chandra's subarcsecond resolution can resolve discrete X-ray sources in galaxies out to a distance of the Virgo cluster and its photometric abilities allow it to distinguish between Xray sources. In general the distributions would have to be corrected for host galaxy inclination and assumptions would have to be made about their vertical distribution in the galaxy. Shtykovskiy and Gilfanov (2007) have already used Chandra data to study the distributions of HMXBs in M51. However, their study reveals that HMXBs have a wider distribution than the observed distribution of $\mathrm{H} \alpha$ emission at odds with the results from our model ${ }^{2}$. We suggest that the major sources of disagreement

[^6]between the studies is that our model does not include a radial velocity dispersion in the pre-SN binaries which could greatly increase dispersion of HMXBs from our model and that M51 does not display a perfectly logarithmic spiral (see Figure 4.1). Further work would include a radial velocity dispersion in the pre-SN binaries and would also investigate varying SFRs along the spiral arms.

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## Appendix A

## Runaway Model

## A. 1 Derivation of $R_{i}$

As described in Section 3.2 above we obtain the radial distribution by defining a function which is the difference of two exponential functions. This gives us a distribution which has a central hole but beyond this, the distribution is exponentially decaying, the rate of which is defined by the scalelength, $h_{r}$. The radial distribution function is

$$
\begin{equation*}
A_{0}\left[\exp \left(\frac{-r}{h_{r}}\right)-C \exp \left(k\left[\left(\frac{r}{r_{e}}\right)^{\frac{1}{4}}-1\right]\right)\right] \tag{A.1.1}
\end{equation*}
$$

which is an exponential profile minus a Sérsic profile. Here $A_{0}$ is a constant which keeps the area under the curve of the above equation equal to one when it is integrated from 0 to $R_{\max }$ (which is set at 15 kpc ). We introduce a constant $C$ which, at $r=0$, sets the Sérsic part of the equation equal to the exponential part of the equation. This means that no test particle will have $R=0 k p c . k$ is a constant which can be evaluated numerically by noting that $r_{e}$, the effective radius which contains half the total light, must satisfy the following equality

$$
\begin{equation*}
2 \int_{0}^{r_{e}} C \exp \left(k\left[\left(\frac{r}{r_{e}}\right)^{\frac{1}{4}}-1\right]\right) \partial r=\int_{0}^{\infty} C \exp \left(k\left[\left(\frac{r}{r_{e}}\right)^{\frac{1}{4}}-1\right]\right) \partial r \tag{A.1.2}
\end{equation*}
$$

Using Equation 3.2.2 it is now possible to express the Sérsic part of Equation A.1.1 as an exponential with scalelength $h_{b}$ such that

$$
\begin{align*}
I(r) & =I_{0} \exp \left[\left(\frac{-r}{h_{b}}\right)^{\frac{1}{n}}\right] \\
& =I_{e} \exp \left[k\left\{\left(\frac{r}{r_{e}}\right)^{\frac{1}{n}}-1\right\}\right] \tag{A.1.3}
\end{align*}
$$

The total distribution function can now be written as the difference of two exponentials, greatly simplifying the derivation of radial distances.

$$
\begin{equation*}
I=I_{0}\left[\exp \left(\frac{-r}{h_{r}}\right)-\exp \left(\frac{-r}{h_{b}}\right)^{\frac{1}{n}}\right] \tag{A.1.4}
\end{equation*}
$$

We then set each of the elements of set $B_{R}$ equal to the area under the curve of equation A.1.4 integrated from 0 to $R_{i}$, the test particles radial distance.

$$
\begin{equation*}
B_{R i}=\int_{0}^{R_{i}} A_{0}\left[\exp \left(\frac{-r}{h_{r}}\right)-C \exp \left(\frac{-r}{h_{b}}\right)^{\frac{1}{n}}\right] \partial r \tag{A.1.5}
\end{equation*}
$$

This gives, taking $n=1$;

$$
\begin{equation*}
B_{R i}=A_{0}\left\{h_{r}-h_{r} \exp \left(\frac{-R_{i}}{h_{r}}\right)+h_{b} \exp \left(\frac{-R_{i}}{h_{b}}\right)-h_{b}\right\} \tag{A.1.6}
\end{equation*}
$$

It is clear from the above equation that there is no analytical solution for $R_{i}$. We now employ Newtons method to solve Equation A.1.5 for the radial distance of each test particle. The method can be applied using the following equation where $f\left(R_{i}\right)$ is a function defining the behaviour of $R_{i}$ and is just Equation A.1.5 rearranged to equal zero.

$$
\begin{align*}
& f\left(R_{n}\right)=0 \\
& f^{\prime}\left(R_{n}\right)=\frac{\partial f\left(R_{n}\right)}{\partial R}  \tag{A.1.7}\\
& R_{n+1}=R_{n}-\frac{f\left(R_{n}\right)}{f^{\prime}\left(R_{n}\right)}
\end{align*}
$$

At each iteration of the above equation $R_{n+1}$ gets ever closer to the real value of $R_{i}$. Of course some initial guess must be inserted for the initial iteration and this can be extracted from the element $B_{R i}$ - if for example $B_{R i}=0.5$ then a rough value can be taken from a quick glance at a plot of the curve, in practice we assign an initial $R$ for $B_{R i}$ values in distinct bins - Figure 3.1. We performed with this method over 50 iterations and extract a reasonable distribution (Figure 3.1) but note that if the
function $f(R)$ traces out multiple peaks this method becomes inefficient. For the above method we set

$$
\begin{equation*}
f\left(R_{1}\right)=A_{0}\left\{h_{r}-h_{r} \exp \left(\frac{-R_{1}}{h_{r}}\right)+h_{b} \exp \left(\frac{-R_{1}}{h_{b}}\right)-h_{b}\right\}-B_{R i} \tag{A.1.8}
\end{equation*}
$$

and $f^{\prime}\left(R_{1}\right)$ is simply

$$
\begin{equation*}
f^{\prime}\left(R_{1}\right)=A_{0}\left[\exp \left(\frac{-R_{1}}{h_{r}}\right)-C \exp \left(\frac{-R_{1}}{h_{b}}\right)^{\frac{1}{n}}\right] \tag{A.1.9}
\end{equation*}
$$

$A_{0}$ can be evaluated by setting $B_{R i}$ in Equation A.1.5 equal to one and integrating from 0 to $R_{\text {max }}$. This gives

$$
\begin{equation*}
A_{0}=\left\{h_{r}-h_{b}-h_{r} \exp \left(\frac{-R_{\max }}{h_{r}}\right)+h_{b} \exp \left(\frac{-R_{\max }}{h_{b}}\right)\right\}^{-1} \tag{A.1.10}
\end{equation*}
$$

Using the parameters $h_{r}=3.37, h_{b}=0.538$ and $R_{\max }=15 A_{0}$ is calculated as 0.3581 .

## A. 2 Derivation of $z_{i}$

The distribution of the heights of the test particles above/below the galaxy disk is based on a function of the form

$$
\begin{equation*}
A_{z} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \tag{A.2.1}
\end{equation*}
$$

where $A_{z}$ is again a number that sets the area under the above curve equal to 1 when integrated from $z_{\min }$ to $z_{\max }$ (where $z_{\min }=-z_{\max }$ ). It is useful to express $A_{z}$ as a function of $h_{z}$ and $z_{\max } . A_{z}$ is found by setting

$$
\begin{align*}
1 & =\int_{z_{\min }}^{z_{\max }} A_{z} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) d z \\
& =\left[A_{z} h_{z} \tanh \left(\frac{z}{h_{z}}\right)\right]_{-z_{\max }}^{z_{\max }} \\
& =A_{z}\left[h_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)-h_{z} \tanh \left(\frac{-z_{\max }}{h_{z}}\right)\right]  \tag{A.2.2}\\
& =A_{z}\left[2 h_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)\right]
\end{align*}
$$

giving

$$
A_{z}=\left[2 h_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)\right]^{-1}
$$

In the same manner as for the derivation of $R_{i}$, the elements of set $B_{z}$ are set equal to the area under the curve of Equation A.2.1 integrated from $-z_{\max }$ to $z_{i}$ such that

$$
\begin{equation*}
B_{z i}=\int_{-z_{\max }}^{z_{i}} A_{z} \operatorname{sech}^{2}\left(\frac{z}{h_{z}}\right) \partial z \tag{A.2.3}
\end{equation*}
$$

This gives

$$
\begin{align*}
B_{z i} & =\left[A_{z} h_{z} \tanh \left(\frac{z}{h_{z}}\right)\right]_{-z_{\max }}^{z_{i}} \\
& =A_{z}\left[h_{z} \tanh \left(\frac{z_{i}}{h_{z}}\right)+h_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)\right] \\
\frac{B_{z i}}{A_{z}} & =2 B_{z} h_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)  \tag{A.2.4}\\
& =h_{z}\left[\tanh \left(\frac{z_{i}}{h_{z}}\right)+\tanh \left(\frac{z_{\max }}{h_{z}}\right)\right]
\end{align*}
$$

so that

$$
2 B_{z} \tanh \left(\frac{z_{\max }}{h_{z}}\right)-\tanh \left(\frac{z_{\max }}{h_{z}}\right)=\tanh \left(\frac{z_{i}}{h_{z}}\right)
$$

Where we have used the relation $\tanh (-x)=-\tanh (x)$. Now we use the relation

$$
\begin{equation*}
\tanh ^{-1}(x)=\frac{1}{2} \ln \left[\frac{1+x}{1-x}\right] \tag{A.2.5}
\end{equation*}
$$

to solve for $z_{i}$ such that

$$
\begin{equation*}
z_{i}=\frac{h_{z}}{2} \ln \left[\frac{1+\tanh \left(\frac{z_{\max }}{h_{z}}\right)\left(2 B_{z}-1\right)}{1-\tanh \left(\frac{z_{\max }}{h_{z}}\right)\left(2 B_{z}-1\right)}\right] \tag{A.2.6}
\end{equation*}
$$

## A. 3 Derivation of $M_{i}$

The masses of the test particles is determined randomly by the following equation:

$$
\begin{equation*}
B_{M i}=A_{M} \int_{M_{\min }}^{M_{i}} M^{\alpha} \partial M \tag{A.3.1}
\end{equation*}
$$

where, as above, $A_{M}$ is a constant which sets the above equation equal to one when integrated from the minimum, $M_{\min }$, to maximum, $M_{\max }$, allowable masses, 10 and
$100 M_{\odot}$ respectively such that

$$
\begin{align*}
1 & =A_{M} \int_{M_{\min }}^{M_{\max }} M^{\alpha} \partial M \\
& =A_{M}\left[\frac{M^{(\alpha+1)}}{\alpha+1}\right]_{M_{\min }}^{M_{\max }}  \tag{A.3.2}\\
A_{M} & =\left[\frac{M_{\max }^{(\alpha+1)}}{\alpha+1}-\frac{M_{\min }^{(\alpha+1)}}{\alpha+1}\right]^{-1}
\end{align*}
$$

Equation A.3.1 is now integrated to find the stellar mass associated with the random number $B_{M i}$ :

$$
\begin{align*}
B_{M i} & =A_{M}\left[\frac{M^{(\alpha+1)}}{\alpha+1}\right]_{M_{\min }}^{M_{i}} \\
\frac{B_{M i}}{A_{M}} & =\frac{M_{i}^{(\alpha+1)}}{\alpha+1}-\frac{M_{\min }^{(\alpha+1)}}{\alpha+1} \tag{A.3.3}
\end{align*}
$$

giving for the stellar masses

$$
M_{i}=\left\{(\alpha+1)\left[\frac{B_{M i}}{A_{M}}+\frac{M_{\min }^{(\alpha+1)}}{\alpha+1}\right]\right\}^{\frac{1}{\alpha+1}}
$$

Using the previous equation for $A_{M}$ the above expression for $M_{i}$ can be simplified as

$$
\begin{equation*}
M_{i}=\left\{(\alpha+1)\left[B_{M i}\left(\frac{M_{\max }^{(\alpha+1)}}{\alpha+1}-\frac{M_{\min }^{(\alpha+1)}}{\alpha+1}\right)+\frac{M_{\min }^{(\alpha+1)}}{\alpha+1}\right]\right\}^{\frac{1}{\alpha+1}} \tag{A.3.4}
\end{equation*}
$$

so that

$$
M_{i}=\left\{B_{M i}\left[M_{\max }^{(\alpha+1)}-M_{\min }^{(\alpha+1)}\right]+M_{\min }^{(\alpha+1)}\right\}^{\frac{1}{\alpha+1}}
$$

## A. 4 Derivation of Terms from Galactic Potential

Here we will derive the terms which describe the rotation curve due to the galactic potential and also the forces exerted on each of the test particles during the simulation. In cylindrical coordinates ( $R, \phi, z$ ) the galactic potential (per unit mass) is defined by Flynn et al., (1996) as

$$
\begin{equation*}
\Phi(R, z)=\frac{1}{2} V_{H}^{2} \ln \left(r^{2}+r_{0}^{2}\right)+\sum_{i=1}^{2} \frac{-G M_{C i}}{\sqrt{r^{2}+r_{C i}^{2}}}+\sum_{i=1}^{3} \frac{-G M_{D i}}{\sqrt{R^{2}+A_{D i}^{2}}} \tag{A.4.1}
\end{equation*}
$$

with

$$
A_{D i}=\left[a_{i}+\sqrt{\left(z^{2}+b^{2}\right)}\right]
$$

where $r$ is the galactocentric distance $\left(r^{2}=\left(R^{2}+z^{2}\right)\right), V_{H}$ is the rotation speed at a $r \gg r_{0}$ ( $r_{0}$ the core radius of the spherical dark halo), $M_{C 1}$ is the mass of the bulge with core radius $r_{C 1}, M_{C 2}$ is the mass of the central compact object with core radius $r_{C 2} . M_{D i}$ are the masses of each of the disk components $(i=1,2,3)$ which have parameters $a_{i}$ that are related to the scalelengths while the parameter $b$ is related to the scaleheight (which is the same for each of the disk components).

## A.4.1 The Acceleration Due to Gravity

Since equation A.4.1 describes the potential per unit mass the acceleration, $a$, of a particle within that potential is

$$
\begin{align*}
a & =-\nabla \Phi(R, z) \\
& =-\left[\frac{\partial}{\partial R}[\Phi(R, z)] \hat{R}+\frac{1}{R} \frac{\partial}{\partial \phi}[\Phi(R, z)] \hat{\phi}+\frac{\partial}{\partial z}[\Phi(R, z)] \hat{z}\right] \tag{A.4.2}
\end{align*}
$$

which, with no $\phi$ dependence, gives

$$
\begin{equation*}
a=-\left[\frac{\partial}{\partial R}[\Phi(R, z)] \hat{R}+\frac{\partial}{\partial z}[\Phi(R, z)] \hat{z}\right] \tag{A.4.3}
\end{equation*}
$$

The acceleration in the $\hat{R}$-direction, $a_{R}$, is

$$
\begin{align*}
a_{R} & =-\frac{\partial}{\partial R}\left[\frac{1}{2} V_{H}^{2} \ln \left(r^{2}+r_{0}^{2}\right)+\sum_{i=1}^{2} \frac{-G M_{C i}}{\left.\sqrt{r^{2}+r_{C i}^{2}}+\sum_{i=1}^{3} \frac{-G M_{D i}}{\sqrt{R^{2}+A_{D i}^{2}}}\right]} \begin{array}{rl} 
& =-\left[\frac{V_{H}^{2} R}{\left(r^{2}+r_{0}^{2}\right)}+\sum_{i=1}^{2} \frac{G M_{C i} R}{\left(r^{2}+r_{C i}^{2}\right)^{\frac{3}{2}}}+\sum_{i=1}^{3} \frac{G M_{D i} R}{\left(R^{2}+A_{D i}^{2}\right)^{\frac{3}{2}}}\right]
\end{array} \$ .\right.
\end{align*}
$$

and the acceleration in the $\hat{z}$-direction, $a_{z}$, is

$$
\begin{equation*}
a_{z}=-\left[\frac{V_{H}^{2} z}{\left(r^{2}+r_{0}^{2}\right)}+\sum_{i=1}^{2} \frac{G M_{C i} z}{\left(r^{2}+r_{C i}^{2}\right)^{\frac{3}{2}}}+\sum_{i=1}^{3} \frac{G M_{D i} A_{D i} z}{\left(R^{2}+A_{D i}^{2}\right)^{\frac{3}{2}}\left(z^{2}+b^{2}\right)^{\frac{1}{2}}}\right] \tag{A.4.5}
\end{equation*}
$$

## A.4.2 The Rotation Curve

To derive the rotation curve due to the potential A.4.1 we first note that the force per unit mass, $F$, on a particle in this potential is given by

$$
\begin{equation*}
F=-\nabla \Phi(R, z) \tag{A.4.6}
\end{equation*}
$$

The acceleration in the $\hat{R}$-direction, $a_{R}$, is then

$$
\begin{equation*}
a_{R}=-\frac{\partial \Phi(R, z)}{\partial R} \tag{A.4.7}
\end{equation*}
$$

which is also given by

$$
\begin{equation*}
a_{R}=\ddot{R}-R \dot{\phi}^{2} \tag{A.4.8}
\end{equation*}
$$

Since we want to find the circular speed of a particle in the plane of this potential we let $\ddot{R}=0, z=0$ and work out $\dot{\phi}$ from the equation

$$
\begin{equation*}
R \dot{\phi}^{2}=\nabla \Phi(R) \tag{A.4.9}
\end{equation*}
$$

giving,

$$
\begin{equation*}
\dot{\phi}=\sqrt{\frac{\nabla \Phi(R)}{R}} \tag{A.4.10}
\end{equation*}
$$

which is just

$$
\begin{equation*}
\dot{\phi}=\sqrt{\frac{-a_{R}}{R}}=\left[\frac{V_{H}^{2}}{\left(r^{2}+r_{0}^{2}\right)}+\sum_{i=1}^{2} \frac{G M_{C i}}{\left(r^{2}+r_{C i}^{2}\right)^{\frac{3}{2}}}+\sum_{i=1}^{3} \frac{G M_{D i}}{\left(R^{2}+A_{D i}^{2}\right)^{\frac{3}{2}}}\right]^{1 / 2} \tag{A.4.11}
\end{equation*}
$$

## Appendix B

## HMXB Model

## B. 1 Dispersions from the spiral arm

Below we plot the dispersions in $\phi$ of HMXBs and pre-SN binaries in distinct radial distance bins with multi-peak Gaussian curves as defined by the prescription in Section 4.4.




Figure B.4: Dispersions for a pitch angle of $30^{\circ}$ and a pattern speed of $30 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$.











Figure B.6: Dispersions for a pitch angle of $20^{\circ}$ and a pattern speed of $10 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$.












Figure B.8: Dispersions for a pitch angle of $20^{\circ}$ and a pattern speed of $50 \mathrm{kms}^{-1} \mathrm{kpc}^{-1}$.

## Appendix C

## SN Sample

In the following tables we list the SNae used for the study in this thesis. We also include some details about the host galaxy of each SN and also some derived properties.

Table C.1: Here we list the sample of SN for the "edge-on" study. The first and second column list the SN designation and also its type, determined by a spectral and photometric analysis. The third, fourth, fifth and sixth columns describe some properties of the host galaxy - the host galaxy designation, the galaxy type, its redshift and the inclination of the galaxy respectively. The seventh, eighth and ninth columns describe some derived properties - the height of the SN above the disk of the host galaxy, the distance along the disk and the normalised distance along the disk. See Section 2.2 for a full description of these parameters.

| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination $\left(i^{\circ}\right)$ | $z_{S N}(\mathrm{kpc})$ | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1989C | II P | M+01-25-25 | SBcd | 0.0063 | 90 | 0.20 | 0.04 | 0.00 |
| 2005dn | II | NGC6861F | SBdm? | 0.0107 | 80 | 1.17 | 0.04 | 0.00 |
| 1983E | II L | NGC3044 | SBc | 0.0043 | 90 | 2.59 | 0.35 | 0.03 |
| 2002E | II | NGC4129 | SBab | 0.0039 | 90 | 0.12 | 0.17 | 0.03 |
| 1995F | Ic | NGC2726 | Sa | 0.0051 | 90 | 0.11 | 0.19 | 0.04 |
| 1999an | II | IC 755 | SBb | 0.0050 | 82 | 0.03 | 0.35 | 0.05 |
| 2003dr | $\mathrm{Ib} / \mathrm{c}$ pec | NGC5714 | Scd: | 0.0075 | 83 | 1.95 | 0.82 | 0.06 |
| 2003dg | $\mathrm{Ib} / \mathrm{c}$ pec | UGC 6934 | Scd: | 0.0186 | 81 | 0.05 | 1.50 | 0.08 |
| 2011cb | IIb | IC5249 | SBd | 0.0078 | 83 | 0.93 | 1.33 | 0.08 |
| 2000G | II | UGC 1773 | SB: | 0.0121 | 80 | 0.28 | 0.86 | 0.08 |
| 2005ae | IIb | E209-G09 | SBcd: | 0.0037 | 82 | 0.39 | 1.35 | 0.10 |
| 1997 dm | II P: | E294-G17 | Sc: | 0.0304 | 85 | 0.74 | 3.68 | 0.11 |
| 2011ef | IIb | UGC12640 | Sdm: | 0.0135 | 83 | 0.01 | 0.79 | 0.11 |
| 2001ci | Ic | NGC3079 | SBc | 0.0037 | 80 | 0.04 | 1.90 | 0.11 |
| 2009au | IIn | E443-G21 | Scd: | 0.0095 | 81 | 0.15 | 1.32 | 0.12 |
| 2008U | II | UGC 8917 | Scd: | 0.0126 | 84 | 0.22 | 1.61 | 0.13 |
| 1988I | IIn | LEDA0086944 | Sc | 0.0383 | 90 | 2.39 | 2.99 | 0.13 |
| 1981A | II | NGC1532 | SBb | 0.0040 | 82 | 3.96 | 3.97 | 0.14 |
| 1986J | II | NGC 891 | Sb | 0.0018 | 90 | 0.14 | 2.23 | 0.16 |
| 1999cr | II | E576-G34 | Scd | 0.0101 | 81 | 1.24 | 1.00 | 0.16 |
| 2007ac | II | UGC10550 | Scd: | 0.0307 | 83 | 3.92 | 3.66 | 0.16 |
| 1994ac | II | M + 00-60-52 | Sc | 0.0179 | 81 | 0.32 | 2.49 | 0.17 |
| 2007av | II P | NGC3279 | Sd | 0.0048 | 83 | 0.17 | 1.50 | 0.18 |
| 2004ay | IIn | UGC11255 | Scd: | 0.0330 | 82 | 0.16 | 5.49 | 0.21 |
| 1961F | II L: | NGC3003 | SBbc | 0.0050 | 83 | 1.00 | 3.59 | 0.21 |
| 2003E | II | M-04-12-04 | Sbc | 0.0148 | 84 | 0.30 | 3.11 | 0.22 |
| 2010gw | II P | IC4992 | SBc | 0.0140 | 84 | 0.58 | 4.43 | 0.24 |
| 2001 cm | II | NGC5965 | Sb | 0.0115 | 82 | 1.96 | 8.74 | 0.25 |
| 2005aw | Ic | IC4837A | Sb : | 0.0134 | 81 | 1.26 | 8.49 | 0.26 |
| 19850 | II | UGC 511 | Sc | 0.0154 | 82 | 1.39 | 4.12 | 0.27 |
| 2000 ez | II | NGC3995 | SBm | 0.0110 | 90 | 2.66 | 4.81 | 0.28 |
| 2001ey | IIn | M-01-57-10 | Sc | 0.0255 | 80 | 0.38 | 5.91 | 0.28 |
| 1937A | II P: | NGC4157 | SBb | 0.0026 | 90 | 1.08 | 2.82 | 0.28 |
| 2001dc | II P | NGC5777 | Sb | 0.0071 | 82 | 0.22 | 3.62 | 0.28 |
| 2006iv | IIb | UGC 6774 | Scd: | 0.0081 | 83 | 0.20 | 3.06 | 0.31 |
| 2003da | II | UGC 4992 | Scd: | 0.0139 | 84 | 0.49 | 3.27 | 0.35 |
| 1995H | II | NGC3526 | Sc | 0.0047 | 84 | 0.48 | 1.88 | 0.35 |
| 2010bj | II P | NGC2357 | Sbc: | 0.0076 | 82 | 0.06 | 5.55 | 0.35 |
| 2003J | II P | NGC4157 | SBb | 0.0026 | 90 | 0.14 | 3.71 | 0.37 |
| 2008eu | II | E289-G10 | SBcd | 0.0101 | 80 | 0.80 | 4.63 | 0.39 |
| 2005aj | Ic | UGC 2411 | S? | 0.0085 | 86 | 0.39 | 8.72 | 0.43 |
| 2005ab | II | NGC4617 | Sb | 0.0155 | 80 | 2.34 | 12.11 | 0.45 |
| 2001ch | Ic | M-01-54-16 | Scd | 0.0098 | 82 | 0.53 | 3.50 | 0.46 |
| 2001ir | IIn | M-02-22-22 | Sd: | 0.0199 | 84 | 1.16 | 10.38 | 0.47 |
| 2007sq | II P | M-03-23-05 | Sbc: | 0.0155 | 82 | 0.07 | 13.00 | 0.47 |
| 1989L | IIn L | NGC7339 | SBbc | 0.0045 | 85 | 0.26 | 3.34 | 0.48 |
| 2007ag | Ib | UGC 5392 | Scd: | 0.0209 | 84 | 0.40 | 6.54 | 0.49 |
| 1998en | II | UGC 3645 | Sbc | 0.0212 | 82 | 0.52 | 8.73 | 0.52 |
| 2010ku | II P | IC 716 | Sbc pec | 0.0183 | 80 | 0.14 | 10.34 | 0.59 |
| 2001dh | II | M-06-44-26 | Sd | 0.0085 | 84 | 0.95 | 6.64 | 0.59 |
| 2010E | II P | E013-G28 | Sbc | 0.0154 | 80 | 0.85 | 10.53 | 0.61 |
| 2004dw | II | UGC11394 | Scd: | 0.0142 | 83 | 0.38 | 11.19 | 0.66 |
| 2010cl | II P | M-02-25-20 | Sd: | 0.0090 | 83 | 0.23 | 5.33 | 0.67 |
| 1990Z | II | $\mathrm{M}+01-57-14$ | Scd: | 0.0250 | 83 | 1.29 | 19.31 | 0.69 |
| 2002bx | II | IC2461 | Sb | 0.0076 | 80 | 0.71 | 8.11 | 0.77 |
| 1986E | II L | NGC4302 | Sc | 0.0037 | 90 | 0.94 | 8.52 | 0.78 |
| 2003ac | IIb: | IC3203 | Sb | 0.0234 | 82 | 0.66 | 17.75 | 0.85 |
| 2003A | $\mathrm{Ib} / \mathrm{c}$ | UGC 5904 | Sb | 0.0221 | 81 | 1.59 | 23.10 | 0.89 |
| 2001ak | II | UGC11188 | Sd | 0.0178 | 82 | 0.40 | 9.35 | 0.89 |
| 2009gc | II: | M-03-28-32 | S? | 0.0270 | 82 | 2.66 | 20.00 | 0.96 |
| 1940A | II L | NGC5907 | Sc | 0.0022 | 90 | 0.30 | 14.60 | 0.96 |
| 1992 N | II | IC4831 | Sab | 0.0146 | 90 | 1.85 | 29.50 | 0.99 |
| 2004cr | II | UGC11603 | S? | 0.0175 | 86 | 0.28 | 16.15 | 1.22 |
| 2005da | Ic pec: | UGC11301 | Sc | 0.0151 | 80 | 1.29 | 33.65 | 1.90 |

Table C.2: Here we list the sample of SN for the "face-on" study. The first and second column list the $S N$ designation and also its type, determined by a spectral and photometric analysis. The third, fourth, fifth and sixth columns describe some properties of the host galaxy - the host galaxy designation, the galaxy type, its redshift and the inclination of the galaxy respectively. The seventh and eighth columns describe some derived properties - the distance along the disk and the normalised distance along the disk. See Section 2.2 for a full description of these parameters.

| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination $\left(i^{\circ}\right.$ ) | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1985F | Ib | NGC4618 | SBd | 0.0018 | 37 | 0.56 | 0.12 |
| 2007Y | Ib | NGC1187 | SBc | 0.0047 | 46 | 14.06 | 0.93 |
| 1996N | Ib | NGC1398 | SBab | 0.005 | 48 | 5.13 | 0.25 |
| 1984L | Ib | NGC 991 | SBc | 0.0051 | 28 | 3.88 | 0.44 |
| 2007C | Ib | NGC4981 | SBbc | 0.0056 | 46 | 3.37 | 0.38 |
| 2004ao | Ib | UGC10862 | SBc | 0.0057 | 24 | 2.78 | 0.30 |
| 2004gq | Ib | NGC1832 | SBbc | 0.0067 | 49 | 4.97 | 0.49 |
| 2000de | Ib | NGC4384 | Sa | 0.008 | 39 | 1.02 | 0.17 |
| 2009jf | Ib | NGC7479 | SBc | 0.008 | 40 | 13.23 | 0.66 |
| 1999dn | Ib | NGC7714 | SBd: pec | 0.0094 | 42 | 3.06 | 0.29 |
| 2006gi | Ib | NGC3147 | Sbc | 0.0094 | 31 | 27.34 | 1.16 |
| 2011gd | Ib | NGC6186 | SBa | 0.0098 | 41 | 0.59 | 0.07 |
| 2009iu | Ib | NGC7329 | SBc | 0.0106 | 47 | 19.21 | 0.79 |
| 2003gk | Ib | NGC7460 | SBb pec: | 0.0108 | 44 | 4.13 | 0.45 |
| 1997 dc | Ib | NGC7678 | Sc | 0.0116 | 45 | 2.95 | 0.18 |
| 2000 dv | Ib | UGC 4671 | Sb | 0.0138 | 32 | 2.75 | 0.26 |
| 2009ha | Ib | M-01-07-24 | Sc pec: | 0.0148 | 42 | 4.64 | 0.42 |
| 2010 kc | Ib | NGC7624 | Scd: | 0.0151 | 42 | 1.05 | 0.12 |
| 2006ep | Ib | NGC 214 | Sc | 0.0151 | 42 | 15.22 | 0.92 |
| 1991ar | Ib | IC 49 | Sc | 0.0153 | 34 | 5.26 | 0.40 |
| 2002cw | Ib | NGC6700 | SBc | 0.0153 | 46 | 8.98 | 0.69 |
| 1999di | Ib | NGC 776 | Sb | 0.0166 | 12 | 5.89 | 0.35 |
| 2010ln | Ib | UGC 2685 | Sb | 0.0171 | 45 | 7.37 | 0.42 |
| 20050 | Ib | NGC3340 | SBbc | 0.019 | 29 | 2.68 | 0.24 |
| 2004 gv | Ib | NGC 856 | S0/a: | 0.0202 | 45 | 7.31 | 0.48 |
| 2004ew | Ib | E153-G17 | Sc | 0.022 | 44 | 7.97 | 0.24 |
| 2006cb | Ib | NGC5541 | Sbc | 0.026 | 46 | 3.25 | 0.27 |
| 2001fx | Ib | IC5345 | Sab | 0.0271 | 45 | 5.94 | 0.50 |
| 2004eh | Ib | UGC 1892 | SBb | 0.0339 | 24 | 9.32 | 0.42 |
| 2010ig | Ib | UGC 1306 | S0 | 0.0385 | 27 | 22.29 | 0.76 |
| 2006jc | Ib pec | UGC 4904 | SBbc | 0.0056 | 49 | 2.18 | 0.76 |
| 2007J | Ib pec | UGC 1778 | Sdm: | 0.0169 | 37 | 5.95 | 0.50 |
| 1982R | Ib: | NGC1187 | SBc | 0.0047 | 46 | 8.66 | 0.57 |
| 2007fo | Ib: | NGC7714 | SBd: pec | 0.0094 | 42 | 2.39 | 0.23 |
| 2011fl | Ib: | IC1584 | SBb | 0.016 | 29 | 5.88 | 0.39 |
| 2011br | Ib: | IC1100 | S? | 0.0221 | 34 | 5.88 | 0.54 |
| 2010if | Ib: | NGC7372 | Sbc | 0.0398 | 27 | 9.64 | 0.28 |
| 2010br | $\mathrm{Ib} / \mathrm{c}$ | NGC4051 | SBbc | 0.0023 | 35 | 1.01 | 0.14 |
| 2003jg | $\mathrm{Ib} / \mathrm{c}$ | NGC2997 | Sc | 0.0036 | 41 | 0.89 | 0.05 |
| 2002ji | $\mathrm{Ib} / \mathrm{c}$ | NGC3655 | Sc: | 0.0049 | 49 | 2.88 | 0.65 |
| 2007rb | $\mathrm{Ib} / \mathrm{c}$ | NGC2889 | Sc | 0.0114 | 29 | 5.98 | 0.41 |
| 2009iz | $\mathrm{Ib} / \mathrm{c}$ | UGC 2175 | Sbc | 0.0143 | 17 | 5.40 | 0.59 |
| 2003ih | $\mathrm{Ib} / \mathrm{c}$ | UGC 2836 | S0 | 0.0167 | 21 | 3.31 | 0.35 |
| 2010 gr | $\mathrm{Ib} / \mathrm{c}$ | UGC 2035 | Sb | 0.0173 | 49 | 5.82 | 0.33 |
| 2002cp | $\mathrm{Ib} / \mathrm{c}$ | NGC3074 | Sc | 0.0173 | 27 | 24.07 | 1.01 |
| Continued on Next Page. . |  |  |  |  |  |  |  |



| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{S N}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2002ap | Ic pec | NGC 628 | Sc | 0.0021 | 24 | 12.01 | 0.93 |
| 2003id | Ic pec | NGC 895 | Scd | 0.0078 | 45 | 6.89 | 0.41 |
| 2003jd | Ic pec | M-01-59-21 | Sm | 0.019 | 41 | 5.13 | 0.33 |
| 2005lr | Ic: | E492-G02 | Sb pec | 0.0087 | 46 | 4.32 | 0.40 |
| 2000F | Ic: | IC 302 | SBbc | 0.0198 | 36 | 9.63 | 0.44 |
| 2004 dx | Ic: | $\mathrm{M}+07-37-36$ | S? | 0.0303 | 32 | 4.61 | 0.33 |
| 2001dq | Ic? | IC1222 | Sc | 0.0312 | 37 | 12.53 | 0.40 |
| 1998dn | II | NGC 337A | Sdm | 0.0013 | 41 | 3.81 | 0.85 |
| 1921B | II | NGC3184 | SBc | 0.002 | 15 | 6.46 | 0.72 |
| 20091s | II | NGC3423 | Scd | 0.0028 | 31 | 1.93 | 0.31 |
| 1994P | II | M + 09-20-51 | SBc | 0.0036 | 47 | 7.90 | 1.08 |
| 1992bd | II | NGC1097 | SBb | 0.0043 | 47 | 0.94 | 0.04 |
| 1947A | II | NGC3177 | Sb | 0.0043 | 42 | 3.86 | 1.01 |
| 1995ag | II | UGC11861 | Sd | 0.005 | 47 | 3.32 | 0.33 |
| 1997 db | II | UGC11861 | Sd | 0.005 | 47 | 5.10 | 0.50 |
| 1995X | II | UGC12160 | Sc | 0.0052 | 37 | 2.57 | 0.40 |
| 1964F | II | NGC4303 | SBbc | 0.0053 | 25 | 3.19 | 0.15 |
| 1961I | II | NGC4303 | SBbc | 0.0053 | 25 | 9.32 | 0.45 |
| 2004G | II | NGC5668 | Scd | 0.0053 | 22 | 4.76 | 0.47 |
| 1996W | II | NGC4027 | SBdm | 0.0055 | 41 | 4.18 | 0.41 |
| 1995ad | II | NGC2139 | Scd | 0.0061 | 42 | 3.99 | 0.42 |
| 2004ep | II | IC2152 | SBa | 0.0063 | 46 | 1.44 | 0.23 |
| 2000cb | II | IC1158 | Sc | 0.0064 | 47 | 4.97 | 0.54 |
| 2002ce | II | NGC2604 | SBcd | 0.0067 | 0 | 2.62 | 0.32 |
| 2005 kh | II | NGC3094 | SBa | 0.0074 | 46 | 7.87 | 0.91 |
| 2004be | II | E499-G34 | SBm | 0.0076 | 27 | 0.72 | 0.14 |
| 2003hr | II | NGC2551 | S0/a | 0.0076 | 47 | 7.95 | 1.07 |
| 1978H | II | NGC3780 | Sc | 0.008 | 38 | 3.39 | 0.24 |
| 2005 dl | II | NGC2276 | SBc | 0.0081 | 21 | 3.05 | 0.23 |
| 1993X | II | NGC2276 | SBc | 0.0081 | 21 | 11.94 | 0.89 |
| 2004 eg | II | UGC 3053 | Scd: | 0.0081 | 29 | 3.63 | 0.90 |
| 2008cn | II | NGC4603 | Sc: | 0.0082 | 44 | 3.93 | 0.24 |
| 1998ce | II | M-04-24-19 | SBdm: | 0.0084 | 37 | 2.48 | 0.25 |
| 1999gk | II | NGC4653 | Scd | 0.0085 | 29 | 8.06 | 0.52 |
| 1997Z | II | NGC3261 | SBb | 0.0087 | 41 | 9.71 | 0.51 |
| 2009af | II | UGC 1551 | SBc | 0.0089 | 32 | 3.28 | 0.23 |
| 1981E | II | NGC5597 | SBc | 0.0089 | 35 | 3.75 | 0.35 |
| 2006ca | II | UGC11214 | Scd: | 0.0089 | 34 | 4.28 | 0.61 |
| 1993K | II | NGC2223 | SBb | 0.0091 | 29 | 8.34 | 0.53 |
| 2002dq | II | NGC7051 | SBa | 0.0091 | 34 | 5.54 | 0.79 |
| 1997ds | II | M-01-57-07 | SBd: | 0.0095 | 27 | 2.91 | 0.27 |
| 2009aj | II | E221-G18 | Sab | 0.0095 | 36 | 3.19 | 0.63 |
| 2004cx | II | NGC7755 | SBc: | 0.0097 | 41 | 5.23 | 0.24 |
| 2007W | II | NGC5105 | SBc | 0.0097 | 42 | 7.97 | 0.70 |
| 1995J | II | NGC4512 | SBd | 0.01 | 47 | 9.66 | 0.82 |
| 2008br | II | IC2522 | SBc pec | 0.0101 | 44 | 5.00 | 0.31 |
| 2000M | II | NGC6389 | Sbc | 0.0103 | 49 | 3.57 | 0.21 |
| 2002gw | II | NGC 922 | SBcd | 0.0103 | 36 | 8.73 | 0.74 |
| 1992ab | II | NGC6389 | Sbc | 0.0103 | 49 | 18.63 | 1.09 |
| 2008fq | II | NGC6907 | SBbc | 0.0106 | 42 | 2.00 | 0.09 |
| 2002ca | II | UGC 8521 | SBab | 0.0109 | 34 | 3.63 | 0.48 |
| 1999dh | II | IC 211 | Scd | 0.0109 | 37 | 6.94 | 1.08 |
| 2005io | II | UGC 3361 | SBdm: | 0.0111 | 34 | 4.57 | 0.77 |
| 1991J | II | NGC5020 | SBbc | 0.0113 | 32 | 16.15 | 0.77 |
| 2007an | II | NGC4017 | Sbc | 0.0114 | 39 | 5.58 | 0.47 |
| 2006st | II | NGC4017 | Sbc | 0.0114 | 39 | 17.52 | 1.47 |
| Continued on Next Page... |  |  |  |  |  |  |  |


| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2003bw | II | IC1077 | SBbc? | 0.0115 | 36 | 2.97 | 0.32 |
| 2006ci | II | E182-G10 | Sc pec | 0.0119 | 42 | 7.41 | 0.57 |
| 1998W | II | NGC3075 | Sc | 0.012 | 48 | 5.68 | 0.67 |
| 2008P | II | NGC2550A | Sc | 0.0122 | 32 | 2.53 | 0.22 |
| 2003bj | II | IC4219 | SBb pec | 0.0122 | 27 | 4.15 | 0.54 |
| 1998ar | II | NGC2916 | Sb | 0.0123 | 47 | 10.15 | 0.57 |
| 2005H | II | NGC 838 | S0: pec | 0.0129 | 41 | 0.60 | 0.07 |
| 2002an | II | NGC2575 | Scd: | 0.013 | 32 | 10.83 | 0.62 |
| 2001J | II | UGC 4729 | SBcd: | 0.0132 | 32 | 5.34 | 0.69 |
| 2008ex | II | UGC11428 | Scd: | 0.0133 | 0 | 2.58 | 0.33 |
| 2000N | II | M-02-34-54 | SBbc | 0.0134 | 45 | 11.36 | 0.81 |
| 1997bn | II | UGC 4329 | Scd | 0.0138 | 41 | 2.44 | 0.15 |
| 2003ef | II | NGC4708 | Sab pec? | 0.0138 | 45 | 4.72 | 0.44 |
| 2007 ct | II | NGC6944A | SBd: pec | 0.0141 | 45 | 6.53 | 0.79 |
| 2003bl | II | NGC5374 | SBbc? | 0.0144 | 29 | 6.32 | 0.44 |
| 1987C | II | M $+09-14-47$ | S pec: | 0.0144 | 29 | 6.46 | 1.03 |
| 2003ho | II | E235-G58 | SBd | 0.0145 | 47 | 7.59 | 0.37 |
| 2004gf | II | UGC11864 | SBdm | 0.0145 | 37 | 13.77 | 0.74 |
| 2008bh | II | NGC2642 | SBbc | 0.0148 | 21 | 8.50 | 0.48 |
| 2004er | II | M-01-07-24 | Sc pec: | 0.0148 | 42 | 7.14 | 0.65 |
| 1999go | II | NGC1376 | Scd | 0.0149 | 29 | 4.97 | 0.28 |
| 2001ee | II | NGC2347 | Sb : | 0.0151 | 45 | 6.41 | 0.41 |
| 2006ee | II | NGC 774 | S0 | 0.0154 | 39 | 4.36 | 0.32 |
| 2006 ms | II | NGC6935 | Sa | 0.0156 | 32 | 5.66 | 0.31 |
| 2011dy | II | UGC12628 | SBc: | 0.0156 | 39 | 10.37 | 0.75 |
| 2001cy | II | UGC11927 | SBb | 0.0158 | 41 | 2.90 | 0.23 |
| 2005 dk | II | IC4882 | SBb: | 0.0159 | 34 | 5.22 | 0.57 |
| 2006 V | II | UGC 6510 | Scd | 0.016 | 21 | 16.83 | 0.94 |
| 2004 dv | II | M-01-06-12 | SBb | 0.0162 | 45 | 9.48 | 0.87 |
| 2001cx | II | UGC12266 | SBab | 0.0163 | 36 | 7.61 | 0.72 |
| 1999 et | II | NGC1643 | SBbc pec: | 0.0164 | 0 | 3.66 | 0.35 |
| 2001cl | II | NGC7260 | Sbc | 0.0165 | 41 | 7.49 | 0.41 |
| 2007gw | II | NGC4161 | Sbc | 0.0166 | 49 | 3.69 | 0.35 |
| 2006 dk | II | NGC4161 | Sbc | 0.0166 | 49 | 4.42 | 0.41 |
| 2001di | II | UGC 3259 | Sd | 0.0166 | 41 | 16.51 | 0.95 |
| 2010id | II | NGC7483 | Sa | 0.0167 | 49 | 38.51 | 2.42 |
| 2010hb | II | UGC 2537 | Scd | 0.0168 | 17 | 10.64 | 0.90 |
| 2001ab | II | NGC6130 | SBbc: | 0.017 | 47 | 5.55 | 0.53 |
| 1998cu | II | IC1525 | SBb | 0.017 | 44 | 19.65 | 1.01 |
| 2003ej | II | UGC 7820 | Scd: | 0.0173 | 29 | 10.91 | 0.58 |
| 2002bh | II | UGC 5286 | Sd | 0.0175 | 47 | 12.19 | 0.58 |
| 2008fc | II | UGC 2883 | Sb | 0.0175 | 41 | 6.39 | 0.62 |
| 1990ah | II | M +02 -02-09 | SBd | 0.0176 | 46 | 4.00 | 0.32 |
| 2004ek | II | $\mathrm{M}+05-03-75$ | S | 0.0176 | 34 | 15.45 | 0.71 |
| 2001H | II | M-01-10-19 | Scd | 0.0177 | 37 | 2.58 | 0.13 |
| 2003C | II | UGC 439 | SBa | 0.0177 | 17 | 3.32 | 0.28 |
| 2011 dk | II | NGC7003 | Sbc | 0.018 | 49 | 1.87 | 0.15 |
| 1996bw | II | NGC 664 | Sb : | 0.0182 | 34 | 7.24 | 0.45 |
| 1997W | II | NGC 664 | Sb : | 0.0182 | 34 | 7.60 | 0.47 |
| 2003 cn | II | IC 849 | Scd? | 0.0182 | 37 | 12.92 | 0.78 |
| 2005I | II | IC 983 | SBbc | 0.0184 | 29 | 18.61 | 0.32 |
| 2007L | II | UGC 466 | SBa | 0.0184 | 17 | 11.44 | 0.78 |
| 2006cx | II | NGC7316 | Sc | 0.0188 | 37 | 1.77 | 0.15 |
| 2008 T | II | UGC 3304 | Sbc | 0.0188 | 0 | 4.49 | 0.35 |
| 1985R | II | IC1809 | SBab | 0.0188 | 44 | 10.83 | 1.03 |
| 2006bx | II | UGC 5434 | Sb | 0.0188 | 41 | 27.21 | 1.91 |
| Continued on Next Page... |  |  |  |  |  |  |  |


| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{S N}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2007fp | II | NGC3340 | SBbc | 0.019 | 29 | 1.52 | 0.14 |
| 1999ge | II | NGC 309 | Sc | 0.019 | 34 | 7.67 | 0.23 |
| 2008bj | II | M + 08-22-20 | Sd | 0.0191 | 47 | 2.61 | 0.53 |
| 2005 dz | II | UGC12717 | Scd: | 0.0191 | 24 | 11.37 | 0.97 |
| 2003jc | II | M-01-58-18 | S? | 0.0194 | 0 | 8.08 | 0.54 |
| 2009jw | II | $\mathrm{M}+07-16-08$ | Sbc | 0.0197 | 45 | 3.51 | 0.25 |
| 2006dp | II | M-01-03-56 | Sc | 0.0197 | 17 | 4.82 | 0.33 |
| 2004 dh | II | M +04-01-48 | Sbc | 0.0197 | 39 | 3.36 | 0.33 |
| 2019:00:00 | II | UGC11585 | Sb | 0.02 | 29 | 14.10 | 0.69 |
| 2000di | II | IC1637 | SBc: | 0.0202 | 41 | 7.75 | 0.38 |
| 2004fb | II | E340-G07 | S? | 0.0206 | 27 | 2.99 | 0.21 |
| 2002eo | II | NGC 710 | Scd: | 0.0206 | 17 | 6.66 | 0.43 |
| 1994ad | II | E152-G26 | SBa | 0.0207 | 49 | 15.86 | 0.72 |
| 2006C | II | UGC 7020 | Sbc | 0.0207 | 29 | 24.54 | 0.94 |
| 2011go | II | $\mathrm{M}+07-15-02$ | S? | 0.0211 | 0 | 3.24 | 0.40 |
| 2009ii | II | UGC 3627 | Sd | 0.0211 | 49 | 13.36 | 0.92 |
| $2008 i l$ | II | E355-G04 | SBb | 0.0213 | 46 | 12.46 | 0.77 |
| 2007il | II | IC1704 | Sbc | 0.0214 | 44 | 5.98 | 0.43 |
| 2005aa | II | M + 05-22-08 | Scd | 0.0216 | 46 | 4.58 | 0.43 |
| 2004 T | II | UGC 6038 | Sb | 0.0217 | 17 | 5.24 | 0.37 |
| 2005 dq | II | UGC12177 | S? | 0.0219 | 24 | 4.10 | 0.32 |
| 2000I | II | NGC2958 | Sbc | 0.0224 | 41 | 4.54 | 0.33 |
| 2009ay | II | NGC6479 | Sc | 0.0224 | 24 | 5.71 | 0.43 |
| 2005 gm | II | NGC1423 | SBa | 0.0224 | 49 | 9.17 | 0.73 |
| 2003 gg | II | IC1321 | SBbc | 0.0225 | 46 | 3.06 | 0.21 |
| 2002as | II | UGC 3418 | SBb | 0.0228 | 17 | 5.48 | 0.39 |
| 2005Q | II | E244-G31 | Sc | 0.0228 | 34 | 14.72 | 0.69 |
| 2005me | II | E244-G31 | Sc | 0.0228 | 34 | 18.36 | 0.86 |
| 2006du | II | IC1529 | S0 | 0.0233 | 17 | 22.03 | 0.97 |
| 2010bs | II | UGC 7416 | SBb | 0.0234 | 32 | 16.83 | 0.79 |
| 2003hk | II | NGC1085 | Sbc: | 0.0236 | 45 | 4.29 | 0.10 |
| 1997co | II | NGC5125 | Sb | 0.0236 | 37 | 7.55 | 0.33 |
| 2001ae | II | IC4229 | SBb: | 0.0236 | 44 | 11.13 | 0.78 |
| 2002C | II | IC3376 | SBa | 0.024 | 42 | 20.93 | 0.87 |
| 2004 gr | II | NGC3678 | Sbc | 0.0243 | 21 | 5.46 | 0.47 |
| 2003at | II | M + 11-20-23 | Sbc | 0.0243 | 29 | 6.63 | 0.81 |
| 2007ab | II | M-01-43-02 | SBbc | 0.0244 | 32 | 24.23 | 0.83 |
| 2001fy | II | UGC11922 | Sc | 0.0245 | 32 | 1.76 | 0.15 |
| 2006ai | II | E005-G09 | SBdm | 0.0251 | 37 | 1.80 | 0.12 |
| 2010D | II | UGC 5714 | SBc | 0.0251 | 37 | 8.60 | 0.61 |
| 1999ew | II | NGC3677 | S0/a: | 0.0253 | 32 | 3.33 | 0.11 |
| 2004ed | II | NGC6786 | SB? | 0.0254 | 34 | 5.98 | 0.37 |
| 2003 eg | II | NGC4727 | SBbc | 0.0258 | 47 | 9.70 | 0.38 |
| 2005 dt | II | M-03-59-06 | SBc: | 0.0259 | 37 | 7.21 | 0.37 |
| 2005 cv | II | UGC 1359 | SB? | 0.026 | 17 | 3.26 | 0.27 |
| 20110 | II | UGC 8829 | SBa | 0.0263 | 36 | 5.43 | 0.34 |
| 2003 kw | II | UGC 6314 | S? | 0.0266 | 34 | 4.74 | 0.39 |
| 2011G | II | UGC 7144 | Sb | 0.0266 | 29 | 16.25 | 1.09 |
| 2001aj | II | UGC10243 | SBc: | 0.0268 | 21 | 8.72 | 0.61 |
| 2004I | II | NGC1072 | SBb | 0.0271 | 19 | 3.47 | 0.15 |
| 2001ag | II | M +08-18-09 | SB | 0.0271 | 44 | 3.61 | 0.33 |
| 2003 db | II | $\mathrm{M}+05-23-21$ | Sc | 0.0272 | 46 | 7.05 | 0.56 |
| 2005kx | II | NGC3197 | Sbc | 0.0273 | 41 | 4.39 | 0.21 |
| 2010ck | II | M + 06-31-61 | SBa | 0.0277 | 47 | 6.54 | 0.45 |
| 2005bt | II | UGC 8205 | Sb | 0.028 | 44 | 2.75 | 0.18 |
| 2005bp | II | UGC10732 | Sbc: | 0.028 | 42 | 8.59 | 0.57 |
| Continued on Next Page... |  |  |  |  |  |  |  |


| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2008 dz | II | NGC5123 | Scd: | 0.028 | 24 | 13.69 | 0.66 |
| 2003T | II | UGC 4864 | Sab | 0.0281 | 0 | 16.02 | 0.51 |
| 1994A | II | UGC 8214 | SBb | 0.0282 | 29 | 5.87 | 0.48 |
| 2005D | II | UGC 3856 | Scd: | 0.0288 | 32 | 25.29 | 1.53 |
| 2000du | II | UGC 3920 | Sb | 0.0289 | 12 | 8.29 | 0.37 |
| 2006ds | II | PGC0070011 | Sa | 0.0295 | 36 | 3.84 | 0.39 |
| 2007G | II | UGC 68 | SBbc | 0.0297 | 36 | 2.11 | 0.13 |
| 2001af | II | M-04-24-01 | SBbc | 0.0298 | 42 | 16.23 | 0.70 |
| 2003ab | II | UGC 4930 | Scd: | 0.0299 | 49 | 7.93 | 0.56 |
| 2001dy | II | M+04-40-16 | Sbc | 0.0304 | 46 | 4.83 | 0.54 |
| 2001W | II | $\mathrm{M}+07-34-134$ | Sc | 0.0305 | 48 | 18.31 | 1.32 |
| 2003la | II | $\mathrm{M}+10-15-89$ | Sab | 0.0313 | 42 | 3.14 | 0.35 |
| 2008ev | II | UGC10155 | Scd: | 0.0314 | 32 | 3.94 | 0.21 |
| 2006dq | II | UGC11089 | Sbc | 0.0316 | 48 | 10.63 | 0.60 |
| 2003hf | II | UGC10586 | Sb | 0.0318 | 27 | 4.00 | 0.13 |
| 2001ea | II | M + 05-54-38 | S? | 0.0324 | 0 | 7.06 | 0.67 |
| 2005cw | II | IC1439 | SBa | 0.0327 | 49 | 13.98 | 0.55 |
| 2005bi | II | $\mathrm{M}+07-34-36$ | Sbc | 0.0336 | 45 | 9.12 | 0.53 |
| 2002en | II | UGC12289 | Sd | 0.0345 | 17 | 10.66 | 0.41 |
| 2003ht | II | UGC 2457 | Scd | 0.0347 | 17 | 4.49 | 0.21 |
| 2001it | II | $\mathrm{M}+09-25-15$ | Sc | 0.0351 | 29 | 5.78 | 0.35 |
| 2003 gv | II | $\mathrm{M}+05-03-66$ | S? | 0.0354 | 0 | 7.70 | 0.67 |
| 2001id | II | UGC12424 | Sb: | 0.036 | 34 | 6.53 | 0.24 |
| 2003hi | II | M + 07-33-16 | S? | 0.0364 | 36 | 7.73 | 0.66 |
| 2006cd | II | IC1179 | SBcd | 0.0376 | 47 | 9.51 | 0.73 |
| 2005es | II | M + 01-59-79 | S? | 0.0385 | 34 | 7.76 | 0.52 |
| 2009hj | II | NGC7372 | Sbc | 0.0398 | 27 | 5.37 | 0.16 |
| 2000dq | II | $\mathrm{M}+00-06-43$ | Sb | 0.0418 | 34 | 10.92 | 0.67 |
| 2003ii | II | $\mathrm{M}+06-06-53$ | Sbc | 0.0457 | 36 | 8.53 | 0.58 |
| 2007iv | II | UGC12917 | SBb | 0.0461 | 42 | 13.67 | 0.46 |
| 2001 dz | II | UGC 471 | Sbc | 0.0506 | 32 | 16.92 | 0.37 |
| 2006aq | II | M + 07-24-32 | SBb | 0.0514 | 41 | 10.23 | 0.55 |
| 1926A | II L | NGC4303 | SBbc | 0.0053 | 25 | 7.27 | 0.35 |
| 1979C | II L | NGC4321 | SBbc | 0.0053 | 36 | 12.79 | 0.54 |
| 2009 kr | II L | NGC1832 | SBbc | 0.0067 | 49 | 4.74 | 0.47 |
| 1996L | II L | E266-G10 | S: | 0.0336 | 37 | 15.14 | 0.94 |
| 2008bk | II P | NGC7793 | Sd | 0.0007 | 47 | 2.54 | 0.66 |
| 1999gq | II P | NGC4523 | Sm | 0.0009 | 21 | 0.82 | 0.76 |
| 1999gi | II P | NGC3184 | SBc | 0.002 | 15 | 2.41 | 0.27 |
| 2003gd | II P | NGC 628 | Sc | 0.0021 | 24 | 6.80 | 0.53 |
| 2003ie | II P | NGC4051 | SBbc | 0.0023 | 35 | 4.49 | 0.63 |
| 1999em | II P | NGC1637 | Sc | 0.0024 | 36 | 1.14 | 0.20 |
| 2006my | II P | NGC4651 | Sc | 0.0027 | 49 | 2.14 | 0.33 |
| 2009N | II P | NGC4487 | Scd | 0.0036 | 47 | 5.92 | 0.67 |
| 1940B | II P | NGC4725 | SBab | 0.004 | 49 | 11.90 | 0.48 |
| 2003B | II P | NGC1097 | SBb | 0.0043 | 47 | 16.58 | 0.70 |
| 2007aa | II P | NGC4030 | Sbc | 0.0048 | 44 | 8.77 | 0.75 |
| 1983K | II P | NGC4699 | SBb | 0.0048 | 43 | 18.61 | 1.70 |
| 2001X | II P | NGC5921 | SBbc | 0.0049 | 36 | 4.22 | 0.30 |
| 2001 fv | II P | NGC3512 | Sc | 0.0049 | 21 | 2.40 | 0.51 |
| 1988A | II P | NGC4579 | SBb | 0.0051 | 38 | 5.83 | 0.33 |
| 1999 gn | II P | NGC4303 | SBbc | 0.0053 | 25 | 5.35 | 0.26 |
| 2006ov | II P | NGC4303 | SBbc | 0.0053 | 25 | 5.42 | 0.26 |
| 2008in | II P | NGC4303 | SBbc | 0.0053 | 25 | 11.94 | 0.58 |
| 2005ad | II P | NGC 941 | Sc | 0.0053 | 42 | 5.97 | 0.73 |
| 2009G | II P | IC4444 | Sbc: | 0.0065 | 34 | 0.64 | 0.10 |
| Continued on Next Page. . |  |  |  |  |  |  |  |


| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2008X | II P | NGC4141 | SBcd: | 0.0066 | 45 | 1.19 | 0.24 |
| 2009E | II P | NGC4141 | SBcd: | 0.0066 | 45 | 2.79 | 0.56 |
| 2009hq | II P | NGC4152 | Sc | 0.0069 | 39 | 4.16 | 0.46 |
| 2003ao | II P | NGC2993 | Sa pec | 0.0075 | 46 | 1.78 | 0.30 |
| 2010F | II P | NGC3120 | Sbc: | 0.0089 | 45 | 6.30 | 0.68 |
| 2002ed | II P | NGC5468 | Scd | 0.0095 | 24 | 10.61 | 0.72 |
| 2008ho | II P | NGC 922 | SBcd | 0.0103 | 36 | 9.28 | 0.79 |
| 2009ga | II P | NGC7678 | Sc | 0.0116 | 45 | 7.40 | 0.46 |
| 2009bu | II P | NGC7408 | SBcd: | 0.0116 | 39 | 5.33 | 0.52 |
| 2009hf | II P | NGC 175 | SBab | 0.0128 | 29 | 10.82 | 0.67 |
| 2010ie | II P | NGC2333 | Sa | 0.0144 | 44 | 10.62 | 1.31 |
| 2009kl | II P | IC2548 | SBb | 0.0149 | 27 | 8.09 | 0.47 |
| 2008ag | II P | IC4729 | Sc | 0.0149 | 36 | 12.04 | 0.87 |
| 2008 ea | II P | NGC7624 | Scd: | 0.0151 | 42 | 5.11 | 0.59 |
| 2006 it | II P | NGC6956 | SBb | 0.0157 | 17 | 11.36 | 0.63 |
| 2007 hv | II P | UGC 2858 | Sdm | 0.017 | 44 | 9.77 | 0.72 |
| 2009es | II P | IC1525 | SBb | 0.017 | 44 | 15.57 | 0.80 |
| 1965 N | II P | NGC3074 | Sc | 0.0173 | 27 | 4.06 | 0.17 |
| 2008F | II P | M-01-08-15 | SBa | 0.0182 | 41 | 11.12 | 0.64 |
| 2008dn | II P | UGC11946 | Sc | 0.0184 | 41 | 11.21 | 0.95 |
| 2009lb | II P | UGC 2944 | S? | 0.0188 | 45 | 4.01 | 0.23 |
| 2008hg | II P | IC1720 | Sbc | 0.0189 | 43 | 7.78 | 0.60 |
| 2008W | II P | M-03-22-07 | Sc | 0.0193 | 29 | 6.21 | 0.61 |
| 2007ah | II P | UGC 2931 | Sc | 0.0194 | 24 | 1.89 | 0.17 |
| 2010 hm | II P | UGC12687 | SBbc | 0.0206 | 41 | 7.56 | 0.36 |
| 2008cy | II P | M-02-39-16 | Sd | 0.0208 | 36 | 5.62 | 0.40 |
| 2007aq | II P | IC2409 | Sa | 0.0212 | 41 | 23.38 | 1.88 |
| 2010aj | II P | M-01-32-35 | Sbc: | 0.0214 | 49 | 7.62 | 0.57 |
| 2008 gr | II P | IC1579 | SBbc: | 0.0231 | 49 | 11.27 | 0.89 |
| 2010kx | II P | NGC7645 | SBc | 0.0232 | 32 | 6.14 | 0.31 |
| 2010jc | II P | NGC1033 | Sc: | 0.0245 | 29 | 18.06 | 0.95 |
| 2009 dm | II P | $\mathrm{M}+07-24-16$ | Sc | 0.0247 | 44 | 7.74 | 0.84 |
| 2011 cl | II P | IC2373 | Sc | 0.0254 | 0 | 18.47 | 1.15 |
| 20091x | II P | $\mathrm{M}+01-30-08$ | SBb: | 0.0272 | 39 | 9.62 | 0.69 |
| 2007 ck | II P | $\mathrm{M}+05-43-16$ | S? | 0.0273 | 36 | 11.97 | 0.84 |
| 2011bi | II P | $\mathrm{M}+07-35-37$ | Sc | 0.0281 | 27 | 4.71 | 0.38 |
| 2006bs | II P | M $+00-27-14$ | Sc | 0.0303 | 43 | 7.52 | 0.61 |
| 2008fe | II P | UGC 9578 | SBb | 0.0314 | 17 | 16.08 | 0.83 |
| 200911 | II P | E122-G04 | SBab | 0.0456 | 29 | 13.07 | 0.46 |
| 1999eu | II P pec | NGC1097 | SBb | 0.0043 | 47 | 16.41 | 0.70 |
| 1937F | II P: | NGC3184 | SBc | 0.002 | 15 | 5.94 | 0.67 |
| 1992bt | II P: | NGC3780 | Sc | 0.008 | 38 | 3.45 | 0.24 |
| 2010ct | II P: | NGC3362 | Sc | 0.0282 | 39 | 18.54 | 0.79 |
| 1997D | II pec | NGC1536 | SBc pec: | 0.0052 | 44 | 4.59 | 0.75 |
| 2011cq | II pec | M $+00-31-44$ | Sbc: | 0.0173 | 34 | 11.30 | 0.79 |
| 2001dj | II pec | NGC 180 | SBbc | 0.0177 | 36 | 33.99 | 1.36 |
| 2003ka | II pec | $\mathrm{M}+06-50-20$ | S: | 0.0195 | 0 | 4.11 | 0.41 |
| 2004gg | II pec | UGC 5234 | Sc | 0.0203 | 46 | 5.71 | 0.30 |
| 2001Y | II pec | NGC3362 | Sc | 0.0282 | 39 | 10.08 | 0.43 |
| 2009jq | II pec | UGC 1919 | SBb | 0.0366 | 36 | 22.44 | 0.77 |
| 1966 E | II: | NGC4189 | SBc | 0.0071 | 47 | 8.02 | 0.80 |
| 1968 V | II: | NGC2276 | SBc | 0.0081 | 21 | 8.43 | 0.63 |
| 1991au | II: | UGC11616 | Sc | 0.0088 | 36 | 2.45 | 0.47 |
| 1999A | II: | NGC5874 | Sc | 0.0105 | 46 | 8.82 | 0.62 |
| 2003ja | II: | NGC 846 | SBab | 0.0165 | 29 | 4.85 | 0.26 |
| 2005lx | II: | IC 221 | Sc | 0.0168 | 42 | 11.70 | 0.71 |
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| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift (z) | Inclination ( $i^{\circ}$ ) | $R_{\text {SN }}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2003iy | II: | NGC6143 | Sbc: | 0.0176 | 17 | 3.12 | 0.32 |
| 2003 gu | II: | UGC12331 | S? | 0.0195 | 47 | 7.62 | 0.75 |
| 2002at | II: | NGC3720 | Sa: | 0.0201 | 24 | 3.23 | 0.29 |
| 2003gw | II: | UGC 3252 | Sc | 0.0206 | 34 | 6.63 | 0.23 |
| 2009 hz | II: | UGC11499 | Sdm: | 0.0256 | 24 | 11.17 | 0.85 |
| 2004gx | II: | UGC12663 | S? | 0.0269 | 24 | 4.77 | 0.29 |
| 2003 gm | II? | NGC5334 | SBc: | 0.0046 | 44 | 5.32 | 0.47 |
| 1971K | II* P | NGC3811 | SBc | 0.0104 | 42 | 9.39 | 0.69 |
| 2006G | II/IIb | NGC 521 | SBbc | 0.0169 | 25 | 17.92 | 0.58 |
| 2011dh | IIb | NGC5194 | Sbc | 0.0015 | 48 | 6.41 | 0.73 |
| 2001ig | IIb | NGC7424 | Scd | 0.0031 | 32 | 11.86 | 0.68 |
| 2001ad | IIb | NGC6373 | Sc | 0.0111 | 39 | 9.82 | 1.14 |
| 2006ss | IIb | NGC5579 | Scd | 0.0121 | 44 | 7.26 | 0.54 |
| 2006qp | IIb | NGC5735 | SBbc | 0.0126 | 37 | 9.75 | 0.55 |
| 2008ie | IIb | NGC1070 | Sb | 0.0138 | 34 | 7.88 | 0.42 |
| 2007ay | IIb | UGC 4310 | Sm | 0.0146 | 21 | 5.93 | 0.56 |
| 2004ex | IIb | NGC 182 | Sa pec: | 0.0176 | 34 | 14.80 | 0.72 |
| 2008cx | IIb | NGC 309 | Sc | 0.019 | 34 | 24.95 | 0.74 |
| 2009fz | IIb | NGC6209 | Sbc | 0.0198 | 39 | 21.88 | 0.92 |
| 2001cf | IIb | UGC 7020 | Sbc | 0.0207 | 29 | 23.68 | 0.91 |
| 2006 dl | IIb | M + 04-31-05 | Sbc | 0.0221 | 39 | 5.07 | 0.65 |
| 1987K | IIb: | NGC4651 | Sc | 0.0027 | 49 | 1.31 | 0.20 |
| 2008ay | IIb: | UGC 8050 | SBbc | 0.0353 | 17 | 12.66 | 0.48 |
| 2010ad | IIb: | M + 03-41-52 | S0/a | 0.0449 | 45 | 6.44 | 0.40 |
| 1999 el | IIn | NGC6951 | Sbc | 0.0044 | 34 | 2.33 | 0.23 |
| 2011fh | IIn | NGC4806 | SBc | 0.0081 | 34 | 4.09 | 0.73 |
| 1994Y | IIn | NGC5371 | SBbc | 0.0086 | 37 | 7.50 | 0.36 |
| 2011A | IIn | NGC4902 | SBb | 0.0088 | 25 | 9.72 | 0.68 |
| 2003G | IIn | IC 208 | Sbc | 0.0116 | 12 | 2.64 | 0.21 |
| 2005aq | IIn | NGC1599 | SBc pec: | 0.0135 | 24 | 2.92 | 0.43 |
| 2002fj | IIn | NGC2642 | SBbc | 0.0148 | 21 | 7.16 | 0.40 |
| 2003lo | IIn | NGC1376 | Scd | 0.0149 | 29 | 6.28 | 0.36 |
| 2005 db | IIn | NGC 214 | Sc | 0.0151 | 42 | 5.74 | 0.35 |
| 2006 bo | IIn | UGC11578 | Sdm | 0.0155 | 43 | 7.44 | 0.53 |
| 2009 kn | IIn | M-03-21-06 | Sb : | 0.0159 | 41 | 5.40 | 0.63 |
| 1995G | IIn | NGC1643 | SBbc pec: | 0.0164 | 0 | 4.91 | 0.47 |
| 2001I | IIn | UGC 2836 | S0 | 0.0167 | 21 | 1.65 | 0.17 |
| 1999gb | IIn | NGC2532 | Sc | 0.0173 | 34 | 7.99 | 0.36 |
| 2004gd | IIn | NGC2341 | Sc | 0.0176 | 17 | 1.32 | 0.15 |
| 2001fa | IIn | NGC 673 | Sc | 0.0176 | 39 | 6.29 | 0.28 |
| 2004F | IIn | NGC1285 | SBb pec | 0.0177 | 46 | 5.28 | 0.34 |
| 1999 eb | IIn | NGC 664 | Sb : | 0.0182 | 34 | 2.64 | 0.16 |
| 2006jd | IIn | UGC 4179 | SBb | 0.0187 | 34 | 9.45 | 0.73 |
| 2008B | IIn | NGC5829 | Sc | 0.0192 | 29 | 9.43 | 0.47 |
| 2007 bb | IIn | UGC 3627 | Sd | 0.0211 | 49 | 13.75 | 0.94 |
| 2007K | IIn | M +06 -20-50 | SBa | 0.0219 | 0 | 3.48 | 0.31 |
| 2005 cp | IIn | UGC12886 | Sbc | 0.0224 | 41 | 2.72 | 0.13 |
| 2007rt | IIn | UGC 6109 | Sc | 0.0226 | 0 | 3.02 | 0.19 |
| 2003as | IIn | M $+08-10-07$ | Sbc | 0.0235 | 32 | 5.09 | 0.51 |
| 2011ap | IIn | IC1277 | Scd: | 0.0239 | 27 | 12.68 | 0.57 |
| 2008aj | IIn | $\mathrm{M}+06-30-34$ | SBc | 0.0252 | 34 | 3.95 | 0.37 |
| 2005cl | IIn | M-01-53-20 | SBb | 0.0261 | 27 | 12.77 | 0.60 |
| 2002bv | IIn | UGC 4042 | SBb: | 0.0281 | 34 | 9.13 | 0.62 |
| 2002cb | IIn | M +08 -24-34 | SBc | 0.03 | 21 | 7.12 | 0.69 |
| 2008be | IIn | NGC5671 | SBb | 0.0305 | 46 | 24.08 | 0.79 |
| 2005cy | IIn | UGC11241 | Sb | 0.0337 | 27 | 8.72 | 0.44 |
| Continued on Next Page... |  |  |  |  |  |  |  |


| SN Name | SN Type | Host Galaxy | Galaxy Type | Redshift $(z)$ | Inclination $\left(i^{\circ}\right)$ | $R_{S N}(\mathrm{kpc})$ | $R_{N}\left(\frac{R_{S N}}{R_{G a l}}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2008 en | IIn | UGC 564 | Sbc | 0.0375 | 45 | 17.32 | 0.86 |
| $2005 R$ | IIn | UGC 6274 | Scd: | 0.0378 | 29 | 2.50 | 0.13 |
| 2005 gn | IIn | E488-G30 | Sc | 0.0405 | 49 | 8.55 | 0.34 |
| 2004 ec | IIn | UGC10717 | Sd | 0.0429 | 37 | 1.98 | 0.09 |
| 1987 B | IIn L | NGC5850 | SBb | 0.0086 | 36 | 32.65 | 1.48 |
| 2007 pk | IIn pec | NGC 579 | Scd | 0.0169 | 29 | 2.84 | 0.25 |
| 2002 bu | IIn? | NGC4242 | Sdm | 0.0022 | 41 | 5.33 | 0.82 |


[^0]:    ${ }^{1}$ The database is available at http://vizier.u-strasbg.fr/viz-bin/VizieR by searching for "Asiago".

[^1]:    ${ }^{1}$ They also describe a "tail" of large scaleheights in their distribution.

[^2]:    ${ }^{1}$ This is implemented within the GNU QTIplot program

[^3]:    ${ }^{1}$ A high level interpreted language similar to MatLab - see http://www.gnu.org/software/octave/

[^4]:    ${ }^{1}$ Implemented within the GNU QTIplot program.

[^5]:    ${ }^{1}$ (Portegies Zwart \& Verbunt (1996) set their distribution proportional to $M_{1}^{-1.7}$ )

[^6]:    ${ }^{1}$ Other low luminosity sources may include accreting white dwarfs and old SNRs
    ${ }^{2}$ Due to the low counts of X-ray sources their conclusion does not have a high statistical significance

