The Characterisation of International Stock Markets using Signal Processing Techniques.

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DECLARATION

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed

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Abstract

Investors are constantly asking whether beating the market on a consistent basis is possible. There is probably no definitive answer to the question of how to make a guaranteed profit (or return) because index prices can fluctuate at any time. The aim of most investors, therefore, is to predict the stock market return and the volatility, (a measure of investment risk) and this requires an understanding of stock market behaviour. In this research, different techniques, both previously existing and newly developed here (and associated specifically with the discrete wavelet transform (DWT)), are applied to study the behaviour of global stock market indices We consider type of memory, interrelationships between stock markets, market reaction to crashes and events, and the best indicators of market types (short-term, long-term or mixed).

The unifying aim is to provide a baseline set of characteristic features which typify behaviors of given market types Principal remarks include the fact that the DWT, alone or with other methods, can succeed in providing an in-depth view of these data, in particular when confronted with non-stationary, non-normal and noisy characteristics. The approach provides an important method for the analysis and interpretation of financial market time series. Our principal findings on volatility measures, moreover, show strong evidence of long-term memory effects, which are not evident in the returns themselves. Emerging and Mature markets are found to deal differently with crashes and events with the latter taking a shorter time to recover from crises on average, compared to the former. Furthermore, we conclude that this binary classification is too simple and stock markets can now be demonstrated to fall into more than two groups, with the designation "emerging" ("developing") and "mature" ("developed") proving imprecise. Additionally, and in the context of the global market, from Chapter 5, we note that international co-movements and volatility (*or risk*) have increased markedly since the middle of

the 20^{th} century and that *clockwise transmission* between global stock markets is observed, i.e *from Asia to Europe to America back to Asia*). The combination of internal dependencies and external influences provide the impacts for stock market volatility. The ultimate goal, of course, would be to anticipate these impacts to be able to make the right investment decision.

CHAPTER 1

INTRODUCTION

1.1 Motivation

A stock market (or stock exchange) offers investors the opportunity to buy and sell stocks, bonds and other securities Colloquially, **a bear market** reflects prices which are currently going down, while **a bull market** indicates that prices are rising in value.

The stock market is important, both for individual investors to make profits (or returns) and companies to raise finance through the market by issuing new securities, which traders will be able to buy and sell. In a financial market, underlying assets, such as shares, bonds, commodities and foreign currencies and their derivatives[1] are traded.

Nowadays, more people are interested in financial markets and have money, e.g their savings or retirements, invested, usually through unit trusts such as pension fund, hedge fund or other investment advisers. This makes trillions of dollars available to be invested in stock markets around the world which are subject to stock market fluctuations. The stock market is a complex system and is affected by domestic and global information (positive or negative news). Misunderstanding and

[1] A derivative is a common term for specific types of investments from which payoffs over time are derived from the performance of assets and there are four main types of derivatives, namely Options, Futures, Forwards and Swaps.

misdealing (e.g panic, over-reactions or under-reactions), with this information may cause a market crash, resulting in bankruptcies of some companies, people losing their savings or pensions and increases the market volatility[2] (or risk).

Trading in the stock market is non-trivial because of its unpredictability. A question which investors are constantly asking is whether the market can be "beaten" to make a profit (or gain a return) No definitive answer is possible since the prices can fluctuate at any time. No return can be gained without facing a certain degree of risk, so the overall aim of most traders is the prediction of stock market return and assessment of the volatility. These predictions require an understanding of stock market behaviour. In order to achieve this, two important factors should be taken into consideration. The first is the presence of *memory* in the market and the second is noise or randomness, arising from local and global changes (e.g. news) which influences stock market movements.

1.2 Objectives

The purpose of the research presented here is to examine in detail the signal components which comprise a number of international stock markets and to assess the predictability of their behaviours. The statistical and econophysics techniques employed are combined with the *discrete wavelet transform* (DWT), which as a deconvolution approach offers significant advantages to that of more commonly-used-known approaches in signal processing, such as the Fourier Transform [Lee (2002) and Raihan et al (2005)].

The properties that we wish to investigate using both established and novel methods, are memory types (short, long or mixed), interrelationships between stock markets and market reaction to crashes and events. Ultimately, we aim to find some type of "best measure" for the development of a stock market. In order to achieve

[2]Volatility is a statistical measure of the tendency of a market or security to rise or fall sharply within a period of time and it is used as a measure of investment risk.

this, we set up the following objectives:

1. To examine long-term memory properties by employing the *discrete wavelet transform* **(DWT)** as a *new* investigative method. The DWT is capable of providing time and frequency information together, deals well with non-stationary and noisy time series and provides a clear picture of the movements *(short-term, long-term or mixed)* in the data series.

2. To examine the scaling properties of global stock markets by formalising the generalization of the Detrended Fluctuation Analysis (DFA) to scale and time dependencies, and to study the behaviour of the Hurst exponent (H) in different time periods and scale level

3. To study the evidence of *global* co-movements among worldwide stock markets, in Europe, the Americas and Asia. In particular, we are interested in examining (i) whether co-movements are direct (clockwise only) or indirect, impacting on nearest-neighbour (continental grouping) and (ii) whether there is global absorption of major events or large changes in worldwide markets.

4. To investigate how Emerging and Mature markets deal with different crashes and events and also to study which eigenvalues of the Variance-Covariance matrices of the return series contain useful information about market movements. To achieve this by employing the *discrete wavelet transform* **(DWT)** combined with eigenanalysis to study the behaviour of the **three largest** (λ_1, λ_2, λ_3) eigenvalues of the Variance-Covariance matrices and their ratios. To examine behaviour of these series for sliding windows of equal sizes and investigate whether λ_2 and λ_3 contain useful information in addition to that described by λ_1 alone.

5 To classify as precisely as possible from available data and in addition to the above analyses, the stock market degree of development. This, through introduction of a new algorithm, based on the *discrete wavelet transform (DWT)*

and fractional Gaussian noise[3] (fGn) is assessed for different values of the Hurst exponent[4] H

1.3 Outline of Thesis

This thesis is organized as follows: **Chapter 2** gives the theoretical background to stock market properties studied, while the Methodological approaches, which have been used or developed in this research, are described in **Chapter 3**.

Chapter 4 discusses the investigation of the existence of long-term memory properties in the daily returns of five Irish Stock Exchange (ISEQ) indices and their volatility measures (namely absolute and squared returns) by using a novel approach, based on the discrete wavelet transform, (DWT), and three different established tests, (namely Rescaled Range (R/S), its modified form, and the semi-parametric method (GPH)). We further propose a time-scale extension of Detrended Fluctuation Analysis (TSDFA), to study the Hurst exponent behaviour at different time periods and scale levels for different stock markets.

Chapter 5 reports on an investigation of the price interdependence between seven international stock markets, namely Irish, UK, Portuguese, US, Brazilian, Japanese and Hong Kong, using a new *wavelet-based* testing method, suggested by Lee (2002) It also considers the importance of historical transmissions by studying co-movements between Portuguese and Brazilian markets in three different periods using Lee's method.

In **Chapter 6**, we report on attempts to ascertain whether the subdominant eigenvalues (λ_2, λ_3) hold information on stock market risk and also on the recovery time for the Emerging and Mature stock market classifications. The approach combines the *discrete wavelet transform* with *eigenanalysis* to study the behaviour of the first three eigenvalues (λ_1, λ_2, λ_3) and ratios for covariance matrices of the

[3]See Section 3.1.

[4]The Hurst exponent is used to measure the degree of long-term memory property

return series, (thirteen emerging markets and fourteen mature ones). We also introduce a novel *wavelet-based* algorithm (or indicator) of stock market development to investigate whether i) an emerging market is still evolving , (ii) whether it has achieved mature status, and (iii) the behaviour of different market types for different time intervals with different volatility levels **Chapter 7** provides an overall summary of the work, our conclusions and directions for future research

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Predicting Stock Market behaviour, even one or two time points ahead, is non-trivial due to the fluctuation of stock prices and highly unpredictable direction of their movement. The overall aim of most investors is to forecast where the stock market is going next week, next month, or next year, because stock market predictions are the key to successful investing.

Clearly, all investors would like to make money in the stock markets, but it is impossible to gain a return (or profit) without facing a certain amount of risk, so investment is concerned with about balancing return and risk (measured typically by volatility[1]). The ability to make suitable investment decisions and forecast the stock market requires an understanding of stock market behaviour.

In this chapter, we examine previous work on long-term memory, interrelationships between stock markets, their reaction to major events, e.g. crashes and stock market classification.

[1]See footnote number 2 in page 2

2.2 Importance of Wavelet Transform

For some years the Fourier transform has been the most widely-used approach for many problems in signal processing (see e g. Polikar (1994)), but it runs into problems dealing with signals which are not only localised in frequency but also in time or space or if the time series is non-stationary (see e.g Polikar (1994)) It is for this reason that *time-frequency* representations have been adapted recently as very powerful and useful tools for analysing non-stationary signals (or time series) in many areas, such as engineering, medical sciences, geology, etc. The wavelet transform (WT) is a mathematical tool that has been introduced to solve time-frequency problems [Strang (1993), Polikar (1994) and Tsai (2002)].

The WT has been applied to many applications in signal analysis [e.g. Mallat (1989) and Daubechies (1990), Li (1997) and Bremaud (2002)], image processing [e.g. Antonini et al. (1992), Calway (1993) and Drori and Lischinski (2003)], but, relatively few papers so far have paid any attention to its application (or appropriateness) for financial time series analysis [e g. recently only, see Ramsey and Zhang (1997), Gonghui et al. (1999), Capobianco (2001) and Lee (2002)]. A particular strength of the *discrete* wavelet transform (DWT) is that it splits data series into components of different frequency, so that each component can be studied separately to investigate the data series structure in depth. [For more detail on the wavelet transform see: Hijmans (1993), Bruce and Gao (1996), Jensen (1997) and key details in Section 3.5.1, Chapter 3 of this thesis]

Most stock market data exhibit noise disturbance [e g. Black (1986) and Komáromi (2002)], which may be caused by different factors, such as Crashes and Events, etc. Thus, using noisy series to model stock market movements is highly unreliable. However, using DWT enables us to improve the predictability of the series by building a model based on the true signal, after removal of noise, providing, of course, we can identify the true signal from noise.

2.3 Long-Term Memory

2.3.1 Definition

There is no unique definition of long-term memory processes [first identified by Hurst (1951)], which measure long-range dependence between time series observations. Also described as the "Joseph effect[2]" by Mandelbrot and Wallis (1968), such a process is generally defined as a series having a slowly declining autocorrelation or, equivalently, an infinite spectrum at zero frequency [Granger and Ding (1996)].

Beran (1994) stated that a stationary process with long memory has the following *qualitative* features.

1. Certain *persistence* effects are exhibited. This means that in some periods the observations tend to stay at high levels and low in others.

2. During short time periods, there seem to be periodic cycles in the stationary process. However, over the whole process, no apparent periodic cycles can be identified.

In the time domain, a stationary process $\{y_t\}$ with mean μ is said to have *long memory* if the autocovariance γ_j between y_t and y_{t+j}, $\gamma_j = E[(y_t - \mu)(y_{t+j} - \mu)]$, declines slowly as j increases. More specifically,

$$\gamma_j \approx kj^{2H-2} \quad as \quad j \to \infty \tag{2.3.1}$$

where H is fixed and less than one $(0 < H < 1)$, k a constant and H the so-called Hurst exponent [e.g. Lo (1991), Crato (1994) and Wright (1999)] This leads to three cases

- If $H \in (\frac{1}{2}, 1)$ this strictly represents the long memory case (or persistence)

[2] Joseph is the prophet who foretold of the seven years of plenty followed by the seven years of famine that Egypt was to experience [Lo (1991)]

- If $H < \frac{1}{2}$ the situation is that of the anti-persistent case (or intermediate memory).
- If $H = \frac{1}{2}$ this represents short memory or the weakly-dependent case.

However, in the frequency domain, the process $\{y_t\}$ is said to have *long memory* if the spectral density function $f(\delta)$ can be approximated for a positive constant c as follows:

$$f(\delta) \approx c\delta^{1-2H} \tag{2.3.2}$$

where H takes the same values as above [see e g Lobato and Savin (1998)]

2.3.2 Background

The existence of long-range dependence in financial markets has been an important subject of both theoretical and empirical research (see for example, Mandelbrot (1971), Geweke and Porter-Hudak (1983), Lo (1991) and Ding et al. (1993)). A series displays long-term memory, or long-range dependence, if it exhibits significant autocorrelation between observations widely separated in time. Since these observations are not independent over time, the remote past could, on this basis and in theory, help predict the future.

A number of studies have tested the long-memory hypothesis for stock markets using different methods and found strong evidence of long memory in stock market *returns* [Lee et al. (2001), Sadique and Silvapulle (2001) and Assaf and Cavalcante (2005)]. Others, in contrast, have shown that there is either no evidence or at best weak evidence of long-term dependence [Lo (1991), Cheung and Lai (1995), Jacobsen (1996), Hiemstra and Jones (1997) and Berg and Lyhagen (1998)].

Ding et al (1993) investigated the existence of long memory in daily returns and absolute returns of the S&P500 index. The authors found higher correlation between absolute returns than between the returns themselves, while the power transformation of absolute values also exhibited long memory behaviour Crato

(1994), however, applied the semi-parametric method[3] (GPH), as well as classical and modified Rescaled Range (R/S) methods to investigate the long-range dependence in the international stock index returns of the G-7 countries from the first week of 1950 to the first week of 1998 and found no evidence of long memory in these series with the exception of that for the W. German index. Further, Barkoulas and Baum (1996) applied the Spectral Regression Method to test for long-range dependence in the returns of ten U.S, (i.e. three stock indices, seven sectoral stock indices), as well as returns of thirty firms included in the Dow Jones Industrial Index. Their results showed no evidence of long memory in these returns, as a whole, but some evidence for persistence in five companies and anti-persistence in three others Further to this, Lobato and Savin (1998) used a Lagrange Multiplier (LM) procedure to test for the presence of long memory in the S&P500 index and reported that long memory exists in the squared returns but not in the returns themselves

More recently, Lee et al. (2000) examined the volatility process of the returns for the Korean stock index (KOSPI200) using the FIGARCH[4] approach in order to test for long-range dependence and also to check possible spuriousness of long memory They found that this index has a long memory property and also found that results were neither spurious, nor affected by time aggregation nor by cross-sectional aggregation[5] of data. In addition, Elekdag (2001) applied GPH methods to the index returns and volatility measures (squared and absolute) of a large data set of emerging markets[6] and found strong evidence for long memory in these series. This evidence was robust to various volatility measures, specifically the absolute and modified log-squared returns[7]. Further, Sibbertsen (2002) applied the classical

[3]See Section 3 2.3.

[4]FIGARCH stands for Fractionally Integrated Generalized Autoregressive Conditionally Heteroskedastic and was introduced by Baillie et al. (1996) in order to account for the long-memory effects observed in volatility of most financial time series.

[5]The time aggregation is defined in terms of data type, i.e. daily, weekly, monthly data, etc and the cross-sectional aggregation is defined in terms of different series, i e. market index data and data of different stocks in this index.

[6]They considered a large number of countries, such as Argentina, Australia, Canada, Mexico, Netherlands, Portugal, Russia and Singapore.

[7]Absolute, Squared or Log-squared Returns are used to measure the degree of market volatility

and the tapered GPH-estimators[8] to test for long-term memory in the volatilities, measured by absolute returns, of several German stock returns. Their results showed that there is very significant evidence of long-term memory in all these series.

Several articles, in particular, lend support to the view that *emerging*[9] capital markets are more likely to have long-range memory than mature capital markets [Bekaert and Harvey (1995), Wright (1999), Barkoulas et al. (2000), Nath (2001), Henry (2002) and Tolvi (2003)].

To sum up, from the literature, therefore, it can be seen that many different methods (such as classical and Modified R/S, GPH, LM, the FIGARCH, FARIMA[10] and others) have been used to detect the possibility of long-range dependence in stock market returns and their volatilities and, generally, there is mixed evidence for the presence of long memory in these data.

We think that possible reasons for this controversy may include.

- Testing methods, used by researchers, are generally not able to distinguish properly between long-run and short-run memory.

- The null hypothesis for these tests is that weak dependence or short memory is equivalent to the Hurst exponent (H)= 1/2, while the alternative, strong dependence or long memory is equivalent to $H \neq 1/2$. This is unreliable as a basis for testing as we will show later.

- The nature of stock market data, such as non-stationary, non-normal and noisy series, and also *type of data* or data graining (e.g. intradaily, daily, weekly or monthly), as well as length of time series may affect the test decision. This

(or risk).

[8]A modification of the GPH method, which uses tapered periodogram instead of the standard periodogram for estimating the spectral density, providing more robustness against trend and structural breaks in the data

[9]The International Finance Corporation (IFC) uses income per capita and market capitalization relative to GNP for classifying equity markets If either (1) a market resides in a low- or middle-income economy, or (2) the *ratio of investable market capitalization* to GNP is low, then the IFC classifies the market as emerging otherwise IFC classifies it as mature.

[10]FARIMA stand for Fractionally Autoregressive Integrated Moving Average and was introduced by Beran (1994)

implies inconsistency in test estimators because spurious results can be easily produced by e g non-stationarity, structural changes and aggregation.

2.4 International Co-movements

It is known that stock markets are not only dependent on their own history, but also on external influences from other markets, especially demonstrated e.g after "Black Monday" (date 21^{st} October 1987). Several studies, which have investigated the relationships between international stock markets, have indicated that co-integrations (or co-movements) among stock markets increase the possibility that national markets are influenced by changes in foreign ones. For example, Eun and Shim (1989) investigated relationships among nine major stock markets (Australia, Canada, France, Germany, Hong Kong, Japan, Switzerland, the UK and the US) using the Vector Autoregressive (VAR) model and reported that news *beginning* in the US market has the most influence on the other markets and that most of responses to a shock are completed within two days. Lin et al (1994) studied the interdependence between the returns and volatility of the Japanese and US market indices using daytime (open-to-close) and overnight (close-to-open) returns. The results indicated that daytime returns in each market (US or Japan) are linked with the overnight returns in the other

In addition, Kim and Rogers (1995) used GARCH[11] to study the co-movements between the stock markets of Korea, Japan, and the US and their results indicated that the spillovers from Japan and the US have increased since the Korean market became open for outsiders to own shares. Further, Booth et al. (1997) applied EGARCH[12] to investigate the transmission among four Scandinavian stock markets (Danish, Norwegian, Swedish and Finnish) and reported that significant such effects[13] exist among these markets. Additionally, the Securities and Exchange

[11] Generalized Autoregressive Conditionally Heteroskedastic, Bollerslev (1986)
[12] Exponential Generalized Autoregressive Conditionally Heteroskedastic, Nelson (1991)
[13] It means the influence from one market to another which affect the price dynamics of that

Commission of Brazil, CVM (1998) investigated the existence of an influence from Asian markets on the Brazilian (Bovespa) (as representative of the Latin American region during 1997). They found that there is evidence of impact from Asian markets on the Brazilian during this year, with this spillover effect starting on July 15^{th} with the Thai Baht currency crisis, but not clearly observed until after October 23^{rd} (the Hong Kong crash). In a more recent study, Ng (2000) constructed a volatility spillover model to investigate the return and volatility spillover effects from Japan (regionwide) and the US (worldwide) stock markets on six Pacific-Basin markets, namely those of Hong Kong, Korea, Malaysia, Singapore, Taiwan and Thailand. The author found significant spillover effects from Japan and the US stock markets on the Pacific-Basin markets, implying the importance of both regional and global impacts on these markets.

In order to study international transmission effects of this type, a new testing technique based on the wavelet transform, was developed by Lee (2002) and applied to three developed markets (US, Germany and Japan) and two emerging markets in the MENA[14] region, namely Egypt and Turkey. The author reported that movements from the developed markets affected the developing markets but not vice versa.

In addition, Bessler and Yang (2003) combined an error correction model[15] (ECM) and Directed Acyclic Graphs[16] (DAG) to investigate the interdependence among nine mature markets, namely Japan, US, UK, France, Switzerland, Hong Kong, Germany, Canada and Australia. Their results showed that both changes in European and Hong Kong markets influenced the US market, while this was also affected by internal events. Moreover, Brooks and Negro (2003) studied the relationship between market co-integration and the degree to which companies op-

market

[14]MENA stands for the Middle East and North Africa

[15]An error-correction model is a dynamic model in which the movement of the variables in any periods is related to the previous period's gap from long-run equilibrium

[16]Directed Acyclic Graphs (DAG) are directed graphs with no directed cycles, meaning that, for any vertex v, there is no nonempty directed path starting and ending on v.

erate internationally. They considered three factors (global, country-specific and industry-specific) and found that the importance of the international factor has increased since the 1980's while that of the country-specific factor has decreased on all markets.

Strong evidence of international transmission from the US and Japanese markets to Korean and Thai markets during the late 1990's was presented by Wongswan (2003), while most recently, Antoniou et al. (2003) applied a VAR-EGARCH model to study the relationships among three EU markets namely Germany, France and the UK and the results showed some evidence of co-integration among those countries.

2.5 Reaction to Crashes: Covariance and Correlation

Covariance (or Correlation) matrix[17] of financial asset returns are an important issue for financial risk management, (with a large bibliography on the subject, see for a synopsis, Meric and Meric (1997), Galluccio et al (1998), Laloux et al. (1999), Plerou et al. (1999) and Laloux et al. (2000)). Several studies have applied the concepts and methods of statistics to the areas of economics and finance, particularly to the study of the covariance (or correlation) matrices between price changes (returns) of different stocks.

Meric and Meric (1997), for example, applied the Box M method[18] and Principal Component Analysis (PCA)[19] to test whether or not the correlation matrices before and after the international crash in 1987 were significantly different. This was done in order to investigate the changes in the long-term co-movements of twelve European[20] and US equity markets Their results showed that there are significant alterations in the co-movements of these markets and that the benefits of interna-

[17] Covariance matrix measures the variance of two time series, considering the volatility of these series while, in correlation matrix, the variance (or volatility) is normalised out.

[18] Box's M is the most widely used method for checking equality of covariance matrices.

[19] Principal Component Analysis (PCA) is the technique which can be used to reduce the number of data dimensions, without much loss of information.

[20] The authors considered stock markets of Austria, Belgium, Denmark, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, Switzerland and UK.

tional diversification for the European markets decreased markedly after this crash.

Further, Kwapien et al. (2002) investigated the distribution of eigenvalues of correlation matrices for equally-separated time windows in order to study, quantitatively, the relation between stock price movements and properties of the distribution of the corresponding index movement (w.r.t. German DAX). They reported that the importance of a matrix eigenvalue is related to the correlation strength of different stocks whose weights are given by the corresponding eigenvector, which means that the *more aggregated the market behaviour, the larger the maximum eigenvalue, (λ_1)*

Keogh et al (2003) took as measure of the change in the markets aggregate perception of risk the change in the maximum eigenvalue (λ_1) from day to day, i.e. λ_t/λ_{t-1}. The authors showed that periods exist in market sector data *from the Dow Jones EURO-STOXX index*, which are linear over time. These results supported an implied relationship between volatility and the change in magnitude of the dominant eigenvalue and also showed that *epochs* seem to exist in all market sectors although to different degrees More recently, Kwapien et al. (2004) analysed tick-by-tick returns data ranging from seconds up to 48 hours from the NYSE and the German markets. The authors compared the magnitude of the dominant eigenvalue of the correlation matrices for the same group of securities on various time scales. Their results indicated that *collective market behaviour has appeared at significantly shorter time scales in recent times.* Pafka and Kondor (2002) examined the effect of noisy covariance matrices on the portfolio optimisation problem and found that the *volatility (or risk) of the optimal portfolio*[21] *in the presence of noise in these matrices is 5-15% higher than in the absence of noise*, indicating that the decrease in efficiency of the optimal portfolio is actually much less dramatic.

According to the findings of [Galluccio et al. (1998), Laloux et al. (1999), Plerou et al. (1999), Laloux et al (2000), Plerou et al. (2001), Wilcox and Gebbie (2004)

[21] The increase in this volatility is measured by the ratio of the portfolio variance ($\sum_{i,j=1}^{n} \eta_i \sigma_{ij} \eta_j$) in present of noise to that without noise.

and Sharifi et al. (2004)], the correlation (or covariance) matrices of financial time series, apart from a few large eigenvalues and their corresponding eigenvectors, appear to contain such a large amount of noise that their structure can essentially be regarded as random. This means that only a few of the larger eigenvalues might carry collective information. However, most previous studies [Gopikrishnan et al. (2001), Kwapien et al. (2002) and Kwapien et al. (2004)] have focused on the largest eigenvalue with no attention paid to the others If we are to presume that, as with any PCA analysis of data, there are several principal components that are significant, then it should be worth examining lesser order components to see if they can provide additional information for investment strategies. References in the literature to the role of higher order eigenmodes[22] in investment strategy are scarce, but, recently Wilcox and Gebbie (2004) have examined the composition of all the eigenmodes of ten years of the Johannesburg Stock Exchange using Random Matrix Theory (RMT). The authors concluded that "the leading [i e. *first three*] eigenmodes may be interpreted in terms of independent trading strategies with long range correlations" indicating a role not just for λ_1 but also for a small number of the other dominant eigenvalues.

To date, the magnitude of the *maximum eigenvalue* (λ_1) of the correlation (or variance-covariance) matrices has predominantly been studied with no attention paid to the other eigenvalues and it has been reported that there is relationship between the λ_1 and the market movements However, in this thesis, we will study the behaviours of *three eigenvalues* (λ_1, λ_2 and λ_3) and their ratios[23] in order to investigate whether or not λ_2 and λ_3 carry additional information

[22]The eigenmodes of a dynamical system define a set of independent activity states for the system, if a set of orthonormal solutions to the equations of motion for a system can be found, then observed behaviour can be decomposed into superpositions of these modes.

[23]Normalization methods, such as $\lambda_1/\sum_{i-1}\lambda_i$, $\lambda_2/\sum_{i-1}\lambda_i$ and $\lambda_3/\sum_{i-1}\lambda_i$, produce little effect for mature markets while, in the case of emerging markets, the scale of the additional terms is such that more information is derived from calculating the additional ratio components directly. We have also looked at $\lambda_i/\sum_{i-1}\lambda_i$ but found that the ratios reported here are more informative.

2.6 Difference between Emerging and Mature Markets

After looking at the historical dependence of stock markets on their own behaviour patterns, external influence between global stock markets and internal structure of these markets in Sections (2.3.2, 2 4 and 2.5) respectively, we now need to look at their fundamental type or nature.

Several studies have made comparisons between Emerging and Mature markets, according to different characteristics, and these generally have reported that Emerging markets consistently behave differently to Mature ones with regard to crashes. Patel and Sarker (1998) studied eight mature[24] and ten developing[25] markets from 1970 to 1997 and found important differences in the characteristics of stock market crises between major and emerging markets. They also found that, for emerging markets, the decline in prices, following crises, is larger than that for mature markets, and the recovery time is longer. Further, Fuss (2002) used *Discriminant Analysis* (DA) to investigate if emerging and mature markets behave differently according to different financial aspects (such as market pricing, market size and market activity) and stated that the difference between these two market types has increased since the end of the 1990s. He also stated that the reason for this difference could be found in financial crashes of 1994 in Mexico, 1997/1998 in Asia, 1998 in Russia and 1999 in Brazil, indicating that emerging and mature markets deal differently with crashes and crises.

Recently, Salomons and Grootveld (2002) studied the equity risk premium[26] in thirty-one global stock markets using a standard statistical measures approach, (based on Skewness, Kurtosis, Standard Deviation and Wilcoxon test for medians), and found that emerging markets carry a higher equity risk premium than mature ones, meaning that emerging markets are *riskier* than mature markets. However,

[24]Switzerland, Canada, France, Germany, Italy, UK and US.

[25]Indonesia, South Korea, Malaysia, the Philippines, Taiwan, Thailand, Argentina, Chile, Brazil and Mexico

[26]Equity risk premium is the extra return that the stock market provides over the risk free rate to compensate for market risk.

Wooldridge et al. (2003) considered the changes in the links between emerging and mature markets according to capital flows, the investor base and the changing character of global banks In contrast to Salomons and Grootveld, their results showed that emerging and mature markets are *more integrated* nowadays than before. It is evident that there is less consensus than might be expected and hence a need to carry out further investigations in order to clarify this issue.

2.7 Stock Market Classification

There is no precise set of criteria which clearly distinguishes between different stock market types. Different institutions use different criteria to group countries (or stock markets) by their so-called development level. The World Bank, for example, classifies stock markets into *emerging* and *mature* depending on their national economies using GNP per capita[27]. This classification is, however, unsatisfactory for several reasons, not least the fact that most developed countries are still *undergoing development* and some countries, still considered as "developing" , have *graduated* to a further stage over time

Measurement of the level of development of a stock market There is neither a common concept nor a common indicator agreed by Economists. For example, Demeirgü-Kunt and Levine (1995) compared many different developmental measures, including market size, liquidity, concentration, volatility, institutional development and international integration, across forty-one countries. Their findings on average can be summarized as:

1. Small stock markets are less liquid[28], more volatile and less internationally integrated than larger markets.

[27]The World Bank refer to low-income (GNP per capita of $765 or less) and middle-income (GNP of $9,385 or less) countries as "developing" and to high-income countries ($9,386 or more) as "developed" (Sheram and Soubbotina, 2000)

[28]Market liquidity is an economics term that refers to the ability to easily buy or sell securities without causing a significant movement in the price

2. Richer countries, generally, are more developed than poorer ones. Exceptions include some stock markets defined as "developing" on the basis of national economy, (e.g. those of Republic of Korea, Malaysia and Thailand). These show indications of maturity stronger than many "mature" markets (e.g. those of Australia, Canada and many European countries).

Recently, Di Matteo et al. (2003 and 2005) studied the scaling properties of different global stock market indices by using the generalized Hurst approach. They found in particular that deviations from pure Brownian motion behaviour are associated with the degree of the market's development and also that the generalized Hurst exponent $H(q)$ is a powerful tool in distinguishing between the degree of development of stock markets with emerging and mature markets having $H > 0.5$ and $H < 0.5$ respectively.

Importance of classifying stock markets Investors are interested in knowing the market type in order to make the right investment decisions, because, as we shall show below, *emerging* and *mature* markets consistently behave differently (see Section 2.6). Moreover, for foreign investors, emerging markets are more attractive because of their investment opportunities for making higher returns. For example, there is often less competition for global companies in emerging markets than in mature markets and the expectation of more attractive pricing. However, they are riskier and more volatile due to some of their structural issues, such as foreign debt and political instability, while, mature markets tend to be safer, more solid and more stable. Therefore, the investors' goal is to find a risk-return balance which generates acceptable returns (or profit) with acceptable risk.

2.8 Chapter Summary

In this chapter, we have presented a summary of some important previous work, which has stimulated this research into market characterisation, with several aspects of stock market behaviour requiring classification or more detailed interpretation. We have also endeavoured to indicate the importance to investors of this characterisation and the need to understand what drives market behaviour. In the next Chapter, the methods, which are used, newly implemented, and/or developed to achieve the goals of this work, will be described in detail.

Chapter 3

Methodology

A number of different statistical and econophysics techniques are used to study stock market behaviour in what follows, some well-known (or modified), others newly developed here, and most associated with the *discrete wavelet transform* (DWT) The focus on DWT methods is based on their ability to highlight those specific features of interest, such as long-term memory, international co-movements, reaction to crashes (and events) and stock markets classification, as we discussed in Chapter 2.

3.1 Fractional Gaussian Noise (fGn) and the Hurst Exponent (H)

In order to understand some of the methods which are used or developed in the thesis, we need first to define fractional Gaussian noise (fGn) and the Hurst Exponent H The fGn series $\{X_i, i \geq 1\}$ is a self-similar process[1] that is indexed by the Hurst exponent H (where $0 < H < 1$). It is the increment of fractional Brownian motion (fBm)[2], namely

$$X_i = \mathrm{fBm}_H(i+1) - \mathrm{fBm}_H(i), \quad i \geq 1 \tag{3.1.1}$$

[1]Self-Similar Process is a stochastic process that is invariant in distribution with suitable scaling of time and space. These processes are typically used to model random processes with long range dependence

[2]It exhibits complex but linear long-term dependencies and is characterized by Hurst exponent ($H \in [0, 1]$)

with zero mean, where the auto-covariance function $\gamma_j = E(X_i X_{i+j})$ is given by

$$\gamma_j = 2^{-1}[(j+1)^{2H} - 2j^{2H} + \mid j-1 \mid^{2H}], \quad j \geq 0 \tag{3.1.2}$$

Importantly, γ_j satisfies Equation (2.3.1), where $k = H(2H-1)$ for fractional Gaussian noise. The special case of fGn with $H = \frac{1}{2}$ corresponds to Gaussian white noise, representing randomness and implying that values are uncorrelated. For $H < \frac{1}{2}$, fractional Gaussian noise (fGn) indicates *anti-persistent* behaviour, where implies that if series values are going down in one period then these are more likely to rebound (go up) in the next period For fGn with $H > \frac{1}{2}$, *long memory* or *persistent* behaviour is indicated. If, for example, the values of a series are declining in a certain period, then it is likely that in the next period this behaviour will be sustained. (For more details see, Paxson (1997) and Koutsoyiannis (2002))

3.2 Testing for Long-Term Memory

In this section, we discuss the most widely used methods for testing the presence of long-term memory (introduced in Chapter 2). These are (i) the classical Rescaled Range (R/S) statistic introduced by Hurst (1951) and refined by Mandelbrot (1971), (ii) R/S modification constructed by Lo (1991) and (iii) the Semi-parameteric procedure suggested by Geweke and Porter-Hudak (1983), (hereafter GPH).

3.2.1 The Classical Rescaled Range Method (R/S)

The R/S statistic developed by Hurst (1951), is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Given a series $y_1, y_2, \ldots, y_n$ where n is a sample size, the classical R/S is defined as R_n/S_n where

$$R_n = \max_{1 \le l \le n} \sum_{i=1}^{l} (y_i - \bar{y}) - \min_{1 \le l \le n} \sum_{i=1}^{l} (y_i - \bar{y}) \tag{3 2 1}$$

$$S_n^2 = \frac{1}{n} \sum_{j=1}^{n} (y_j - \bar{y})^2 \tag{3.2 2}$$

Hurst (1951) found that $E[R_n/S_n] = kn^H$, (k a constant and H the so-called Hurst exponent as before), which can be estimated from the following regression by the ordinary least squares (OLS) method

$$Ln(R_n/S_n) = Ln(k) + HLn(n) \tag{3.2.3}$$

where H takes the values mentioned in Section 3.2. This method, established to detect long memory or "strong" correlation, has two disadvantages which are that the distribution of its test statistic is not well-defined and that it is sensitive to short-range dependence and heterogeneities of the data series [see Lo (1991)].

3.2.2 The Modified Rescaled Range Method

Lo (1991) found in addition that the R/S statistic was not well-designed to distinguish between short-term and long-term memory, so suggested a modification to ensure robustness in the former case, deriving its limiting distribution under both long and short memory The modified R/S is given by

$$Q_n(q) = \left[\max_{1 \le l \le n} \sum_{j=1}^{l} (y_j - \bar{y}) - \min_{1 \le l \le n} \sum_{j=1}^{l} (y_j - \bar{y}) \right] / \hat{\sigma}_n(q) \tag{3 2 4}$$

here,

$$
\begin{aligned}
\hat{\sigma}_n^2(q) &= \frac{1}{n}\sum_{j=1}^{n}(y_j - \bar{y})^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left[\sum_{i=j+1}^{n}(y_i - \bar{y})(y_{i-j} - \bar{y})\right] \\
&= \hat{\sigma}_y^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j \qquad (3.2.5)
\end{aligned}
$$

where $\omega_j(q) = 1 - j/(q+1)$, $q < n$ and $\hat{\sigma}_y^2$ and $\hat{\gamma}_j$ are the usual sample variance and autocovariance estimators of y_j respectively The weights $\omega_j(q)$ are those suggested by Newey and West (1987) and always yield a positive $\hat{\sigma}_n^2(q)$. Lo showed that his test was sensitive to the choice of q. Various authors such as Lo and MacKinlay (1989), and Andrew (1991), have used Monte Carlo (MC) studies to show that when q becomes large relative to the sample size n, the finite sampling distribution of the estimator can be radically different from its asymptotic limit. However, q cannot be made too small since the autocovariance, beyond lag q, may be substantial and should be included in the weighted sum Thus, the truncation lag choice is *strongly dependent* on the data available, [as noted, Lo (1991)].

Andrew (1991) suggested a data-dependent formula to enable choice of q where this is given by

$$
q = [A_n], \qquad A_n = (3n/2)^{\frac{1}{3}}\left((2\hat{\rho})/(1-\hat{\rho}^2)\right)^{\frac{2}{3}} \qquad (3.2.6)
$$

where $[A_n]$ denotes the largest integer less than or equal to A_n and $\hat{\rho}$ is the estimate of the first-order autocorrelation coefficient of the data. It is also possible to use the following formula suggested by Schwert (1989) to choose the value of q

$$
q = integer(4 \times (n/100)^{\frac{1}{4}}). \qquad (3.2.7)
$$

3.2.3 The Semi-parametric Method (GPH)

Geweke and Porter-Hudak (1983) suggested a semi-parametric procedure to obtain an estimate of the fractional parameter[3] (d) based on spectral regression. Let $I(\xi)$ be the periodogram of y_t at frequency ξ defined as

$$I(\xi) = \frac{1}{2\pi n}\left[\sum_{t=1}^{n}(y_t - \bar{y})e^{it\xi}\right]^2 \tag{3.2.8}$$

Then the regression equation is given by

$$\ln[I(\xi_j)] = \beta_0 + \beta_1 \ln\left[sin^2(\xi_j/2)\right] + \epsilon_j \qquad ,j = 1, 2, \ldots, m \tag{3.2.9}$$

where $\xi_j = 2\pi j/n$, $[j = 1, 2, \ldots, n-1]$, n is the length of series y_t and $m = g(n) << n$ and the assumptions on m and n are :

1 $\lim_{n\to\infty}[m] = \infty$.

2 $\lim_{n\to\infty}[m/n] = 0$

3 $\lim_{n\to\infty}[ln(n^2)/m] = 0$.

The estimate of d is the negative of the OLS estimate of the slope (β_1) in Equation (3.2.9) assuming regression error variance is $\pi^2/6$. Empirical studies [such as Robinson (1995), Hurvich et al (1998) and Tolvi (2003)] provided suggestions for the following formula to choose m

$$m = n^{\nu} \tag{3 2.10}$$

where $0 < \nu < 1$.

3.2.4 Other Testing Methods for Long-Term Memory

Besides the methods we described previously, there are others which can be used to detect the long-range dependence, such as the Lagrange Multiplier (LM) [e.g. Lobato and Savin (1998) and Zaffaroni (2003)], Tapered Log-Periodogram Regression

[3]The fractional parameter d is equal to H mines half ($d = H - \frac{1}{2}$), where H is the Hurst exponent.

Estimators [e.g. Hurvich and Ray (1995), Velasco (1999) and Sibbertsen (2002)], FIGARCH [e.g. Lee et al. (2000) and Dark (2004)] and FARIMA [e.g Cheung (1993) and Barkoulas and Baum (1996)].

3.3 An Extension of Detrended Fluctuation Analysis.

The Detrended Fluctuation Analysis (DFA) technique has recently become a commonly used tool in analyses of scaling properties of noisy non-stationary time series It consists in dividing a random variable sequence (or time series) $X(t)$, of length n, into n/τ non-overlapping boxes (or windows) of size τ. Then, the linear local trend $z(t) = at + b$ in each box is defined to be the standard linear least-square fit of the data points in that box. The detrended fluctuation function F is then defined by·

$$F^2(\tau) = \frac{1}{\tau} \sum_{t=k\tau+1}^{(k+1)\tau} |X(t) - z(t)|^2, \quad k = 0, ..., \frac{n}{\tau} - 1. \tag{3.3.1}$$

This equation is applied to "disjointed" intervals of size τ; for example, if $n = 6$ and $\tau = 2$ then it will be applied to intervals [1,2], [3,4] and [5,6].

If the observations $X(t)$ are random uncorrelated variables or short-range correlated variables, the behaviour is expected therefore to be a power law

$$\langle F(\tau) \rangle \sim \tau^H$$

Where $\langle F(\tau) \rangle$ is the average of $F(\tau)$ over the n/τ intervals and H is the Hurst exponent taking values as before (Section 3.1).

Instead of calculating a single Hurst exponent for the whole return series of the Portuguese PSI20 index, Matos et al. (2004) applied the DFA to calculate the Hurst exponent, $H(\tau, \theta)$, for this data for sliding windows of size τ, but with fixed scale $\theta = 1$, (θ=number of trading days). The window sizes used were 100, 200 and 400 corresponding roughly to 6 months, one year and two years respectively. Their results showed that the financial data exhibits *multifractal*[4] behaviour and that the Hurst exponent depends on time length (may also depend on scale as well).

3.3.1 Time-Scale of Extension of Detrended Fluctuation Analysis

We propose and develop a *novel* Time-Scale extension of DFA, which we call **TS-DFA**, to compute the Hurst exponent, $H(\tau,\theta)$, for different time period τ and scale (or number of trading days) θ for the interval $[\tau-\frac{\theta}{2},\tau+\frac{\theta}{2}]$. From the condition on time τ and scale θ specified, we know that $\theta/2+1 \leq \tau \leq n-\theta/2$ Realistically, however, the maximum scale in addition that we consider is $\theta = n/4$, as for large scales we are essentially recovering the Hurst exponent for the whole series. A major concern in this modification is to guarantee that exponents obtained through DFA are *meaningful*. We *control the "goodniss" of the linear least squares fits* of the regression coefficients to be near unity for all markets studied to ensure that the fit is significantly linear since, if the regression coefficient is low, the scaling behaviour does not hold

3.3.1.1 Example of TSDFA Method

In order to clarify how τ and θ are chosen, a simple example is given as follows. Let us assume that X(t) is a time series of length six (n=6)

t	date	X(t)
1	01/01/2002	1.23
2	02/01/2002	1.26
3	03/01/2002	1.20
4	04/01/2002	1.25
5	05/01/2002	1.24
6	06/01/2002	1.21

if $\theta = 2$ then $2 \leq \tau \leq 5$, so we have subintervals of size $[\tau-\theta/2,\tau+\theta/2]$

[4]Multifractality means that the fractal dimension of time series is changeable over time

Then for

. $\tau = 2 \Rightarrow$ X[1,3]=1 23, 1 20. . $\tau = 3 \Rightarrow$ X[2,4]=1.26, 1.25.

$\tau = 4 \Rightarrow$ X[3,5]=1.20, 1.24. $\tau = 5 \Rightarrow$ X[4,6]=1.25, 1.21.

3.4 The Variance-Covariance Matrix and its Estimation

It is well-known that high profits cannot be generated without accepting a corresponding high risk, so investors have to balance anticipated profits with estimated risks. Achieving this balance requires examination of the measures of volatility and correlation (or more precisely, the covariance) of the return series, where these reflect risk fluctuation and relationships between series values. Thus, empirical variance-covariance (or correlation) matrices are very important for risk management and understanding the behaviour patterns of assets returns. There are several estimation methods available to compute these matrices, but little agreement among authors on an optimal choice [see e.g Litterman and Winkelmann (1998) and Pafka et al (2004)]. We have chosen the following formula from Litterman and Winkelmann (1998) because it uses weighted historical data to account for the empirical irregularities of financial time series (such as the fact that volatility and correlation vary over time and that these series have "Fat Tail" distributions[5]). Then the covariances can be calculated using

$$\sigma_{ij}^{N}(M) = \left(\sum_{\tau=0}^{N} \omega_{N-\tau} r_{i,N-\tau} r_{j,N-\tau}\right) / \left(\sum_{\tau=0}^{N} \omega_{N-\tau}\right) \tag{3.4.1}$$

where $r_{i,N}$ is the return on the i^{th} market at time N and where ω_N is the weight that is applied at N over horizon M. There are *many* possible choices for the weights ω_N, for example, ω_N can be chosen to be equal for every observation in the sample, $\omega_N = \frac{1}{N+1}$, for all N observations However, in this work, the weights ω_N has been chosen

[5]If the distribution is leptokurtic, it is called *fat-tailed* (or peaked), indicating that it cannot be Normally distributed

to be a declining function of time as suggested by Litterman and Winkelmann. Thus, more recent observations are given more weight than observations that occurred in the more distant past, with 100% weight given to the most recent week and where each week in history receives 90% of the weight of the following week, then $\omega_N = 1.0$, $\omega_{N-1} = 0.90$, $\omega_{N-2} = 0.81$, etc. [For more details see Litterman and Winkelmann (1998)].

3.5 Wavelet-Based Approaches

In Section 2 2, we showed that the wavelet transform is a valuable tool for decomposition of a signal (or time series) into different frequency components, providing time and frequency information *simultaneously*. It is particulary useful in handling a variety of non-stationary and noisy signal processes (or time series), so is particularly appropriate for financial data. Within financial analysis, the WT can also be applied, alone or combined with other methods, to different levels of problem, such as auto-correlation within a market and co-relationship or dependency between different markets, etc.

3.5.1 Definition of Wavelet Transform

The WT is a mathematical tool that has many applications, image analysis, and signal processing (see Section 2.2). In particular, the discrete wavelet transform (DWT) divides the data series into components of different frequency, so that each component can be studied separately to investigate the data series in depth and enable identification of further features. Wavelets have two types, *father wavelets* ϕ and *mother wavelets* ψ where

$$\int \phi(t)dt = 1 \quad \text{and} \quad \int \psi(t)dt = 0$$

The smooth and low-frequency parts of a signal are described by using the father wavelets, while the detail and high-frequency components are described by the mother wavelets. Orthogonal wavelet families are of four different types which are typically applied in practical analysis, namely, the *haar, daublets, symmlets* and *coiflets*, (see Figure (3.1)).

The following is a brief synopsis of their features.

- The *haar* is a square wave and is a compactly supported orthogonal wavelet which, unlike the others is symmetric but which is not continuous.
- The *daublets* are continuous orthogonal wavelets with compact support.
- The *symmlets* and *coiflets* are built to be nearly symmetric and are compactly supported orthogonal wavelets. They are capable of perfect reconstruction.

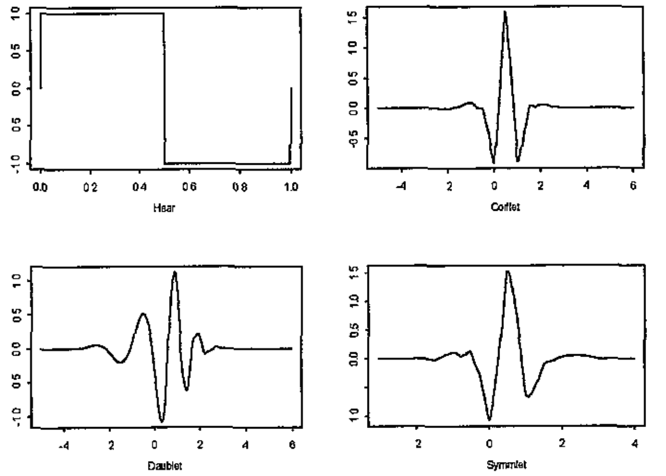

Figure 3 1· Wavelet Families

A two-scale dilation equation[6] used to calculate father $\phi(t)$ and mother $\psi(t)$

[6]Also called two difference equation

wavelets, is defined, respectively by

$$\phi(t) = \sqrt{2}\sum_k \ell_k \phi(2t-k) \tag{3.5.1}$$

$$\psi(t) = \sqrt{2}\sum_k \hbar_k \phi(2t-k) \tag{3.5.2}$$

where ℓ_k and $\hbar_k$ are the low-pass and high-pass coefficients given by

$$\ell_k = \frac{1}{\sqrt{2}}\int \phi(t)\phi(2t-k)dt \tag{3.5.3}$$

$$\hbar_k = \frac{1}{\sqrt{2}}\int \psi(t)\phi(2t-k)dt \tag{3.5.4}$$

The orthogonal wavelet series approximation to a signal $f(t)$ is defined by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \ldots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{3.5.5}$$

where J is the number of multiresolution levels (or scales) and k ranges from 1 to the number of coefficients in the specified components (or crystals). The coefficient $s_{J,k}, d_{J,k}, \ldots, d_{1,k}$ are the wavelet transform coefficients given by

$$s_{J,k} = \int \phi_{J,k}(t)f(t)dt \tag{3.5.6}$$

$$d_{j,k} = \int \psi_{j,k}(t)f(t)dt \qquad (j = 1, 2, \ldots, J) \tag{3.5.7}$$

Their magnitude gives a measure of the contribution of the corresponding wavelet function to the signal. The functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ $[j = 1, 2, \ldots, J]$ are the wavelet functions approximating for the signal and generated from ϕ and ψ through scaling and translation as follows

$$\phi_{J,k}(t) = 2^{\frac{-J}{2}}\phi(2^{-J}t-k) = 2^{\frac{-J}{2}}\phi[(t-2^J k)/2^J] \tag{3.5.8}$$

$$\psi_{j,k}(t) = 2^{\frac{-j}{2}}\psi(2^{-j}t-k) = 2^{\frac{-j}{2}}\psi[(t-2^j k)/2^j] \qquad j = 1, 2, \ldots, J \tag{3.5.9}$$

3.5.1.1 The Discrete Wavelet Transform (DWT)

The DWT is used to compute the coefficient of the wavelet series approximation in Equation (3.5.5) for a discrete signal $f_1, \ldots, f_n$ of finite extent. The DWT maps the vector $f = (f_1, f_2, \ldots, f_n)'$ to a vector of n wavelet coefficients $w = (w_1, w_2, \ldots, w_n)'$ which contains both the smooth coefficient $s_{J,k}$ and the detail coefficients $d_{j,k}$ $[j = 1, 2, \ldots, J]$. The $s_{J,k}$ describe the underlying smooth behaviour of the signal at coarse scale 2^J while the $d_{J,k}$ describe the coarse scale deviations from the smooth behaviour and the $d_{J-1,k}, \ldots, d_{1,k}$ provide progressively finer scale deviations from the smooth behaviour $d^2_{J,k}, \ldots, d^2_{1,k}$ and $s^2_{J,k}$ are the amounts of energy of the original signal which are explained by the detailed and the smooth wavelet components respectively.

For n divisible by 2^J; there are $n/2$ observations in $d_{1,k}$ at the finest scale $2^1 = 2$ and $n/4$ observations in $d_{2,k}$ at the second finest scale $2^2 = 4$. Likewise, there are $n/2^J$ observations in each of $d_{J,k}$ and $s_{J,k}$ where

$$n = n/2 + n/4 + \ldots + n/2^{J-1} + n/2^J + n/2^J$$

The multiresolution decomposition of a signal (or time series) can be defined as follows:

$$S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t) \qquad (3.5.10)$$

$$D_J(t) = \sum_k d_{J,k}\psi_{j,k}(t) \quad \text{for } j = 1, 2, \ldots, J. \qquad (3.5.11)$$

The functions 3.5.10 and 3.5.11 are called the smooth signal and the detail signals respectively, which constitute decomposition of the original signal into different components. Now, the original signal can be expressed in terms of these signals:

$$f(t) = S_J(t) + D_J(t) + D_{J-1}(t) + \ldots + D_1(t) \qquad (3.5.12)$$

3.5.1.2 How The DWT works

The DWT for (3 levels) is applied to a time series with n daily observations and the results are given in Figure 3 2 which shows that the analysis gives four different frequency components d_1, d_2, d_3 and s_3 At the first level, the DWT will smooth a half of the signal (or time series) giving the first detailed component (d_1), (which has $n/2^1$ observations and 2^1 coefficients), and component (s_1), which has $n/2^1$ observations. At the second level, the half of s_1 (or a quarter of the original signal) will be smoothed giving the second detailed component (d_2), (which has $n/2^2$ observations and 2^2 coefficients), and component (s_2), which has $n/2^2$ observations. At the last level (third), half of s_2 will be smoothed giving the third detailed component (d_3), (which has $n/2^3$ observations and 2^3 coefficients), and the smoothest component (s_3), which has $n/2^3$ observations. As the observations of the original series are daily, so the detailed components d_1, d_2 and d_3 represent 2^1 days, 2^2 days and 2^3 days data series respectively.

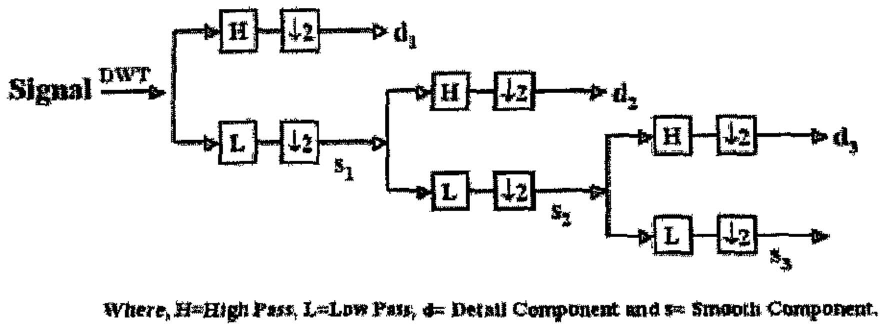

Figure 3.2· Tree showing application of DWT for three levels of decomposition.

3.5.2 A Technique to Test Co-movements

As one of our major interests is the study of inter-dependency in stock markets, we also need to look at predictive or regression type models to investigate international co-movements between these markets. Given that inherent characteristics of financial time series are non-stationarity, non-normality and noisiness, Lee (2002) suggested a new testing method, also based on the DWT, to study these inter-relationships among stock markets. This method (or wavelet-based regression analysis) is described by the following steps:

1. Apply the DWT to divide the return series into different components with different frequencies in order to examine the time-scale properties of the return series.

2. Use the DWT to reconstruct the return series from the first wavelet component (d_1) and from the first two wavelet crystals (d_1 and d_2) together, to examine the relationship between high-frequency fluctuation in these returns.

3. Estimate simple regression and reverse regression models between each pair of stock markets, (estimating $Y = a_0 + a_1 X + \epsilon$ and $X = b_0 + b_1 Y + \varepsilon$ respectively), using three different series, (raw returns series and those rebuilt from d_1 only and from (d_1 & d_2) together), to ensure that we obtain a real indication of the relationships between the returns at different frequency levels.

4. Test the significance of slopes of these models and that of the values of R^2. Thus testing $H_0 : a_1 = 0$ *vs.* $H_1 : a_1 \neq 0$ and $H_0 : b_1 = 0$ *vs.* $H_1 : b_1 \neq 0$, for simple and reverse regression models respectively, leads to rejection of H_0 when the p-value of the slope is greater than 5%. Similarly, the values of R^2 measure the amount of variation in the dependent variable, which is explained by the estimated regression model.

Lee also investigated the effects of using different wavelet families, (e.g. haar and symmlet) and found that choice of wavelet families has no effect on this method.

3.5.3 New Classification Algorithm

Several studies (such as: Di Matteo et al. (2003 and 2005) and Sharkasi et al. (2006)) conclude that emerging and developed markets exhibit *persistent* ($H > 0.5$) and *anti-persistent* ($H < 0.5$) behaviour respectively, indicating that the development of a stock market is associated with the *change* or *crossover* in its behaviour at the *persistence/anti-persistence* threshold ($H = \frac{1}{2}$) Based on this idea, we have developed a novel extended DWT technique, which is described by the following steps:

1. A set of a hundred series, (where choice of 100 ensures sufficient confidence in the corresponding average energy percentages), of fractional Gaussian noise[7] (fGn) is generated for each $H \in \{0.3, 0.4, 0.5, 0.6, 0.7\}$, giving five sets of 100 series.

2. For each set, the DWT is applied to compute the energy percentage explained by each wavelet component for these 100 generated series These percentages are then averaged. The DWT is also used to estimate the energy percentage for the return series of stock market indices.

3. The logarithm[8] to base two of the energy percentages, ($log_2(energy\%)$), explained by the first six wavelet (detailed) components ($d_1 - d_6$), are calculated[9].

4. The linear fit[10] of the return series is compared with that of the fGn series, (for different values of H), in order to group stock markets, based on their degree of development.

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[7]FGn was simulated using the S-plus function $Simulate.FARIMA(0, d, 0)$, where $d = H - \frac{1}{2}$

[8]The base-two logarithm was calculated because there are 2^j coefficients in the j^{th} wavelet component, where $j = 1, 2, \ldots, 6$

[9]Recently, we have come across work by In and Kim (in press) where they have plotted the natural logarithm of variances of wavelet components against the wavelet components and their work is limited to the studying of the correlation between the Australian stock and future markets However, we believe that our method, developed independently, is superior as it serves as a general classifier of stock markets.

[10]We focus on the linear model for the return series in order to ensure clearer comparison The logarithm of all fGn gives straight lines.

3.6 Chapter Summary

This chapter has given details of different statistical and econophysics techniques, some well-known, others newly developed here, that were employed to study different stock market properties in order to understand the dynamic nature of these markets. The results of applying these techniques and the implications of the analyses will be described in the next three chapters for long-term memory in a given market, co-movement between markets and for the reaction of different stock markets to major events respectively.

CHAPTER 4

PRESENCE OF LONG MEMORY

In this chapter, we discuss results for the detection of long-term memory, (slowly decaying autocorrelation or long-range dependence), in stock market returns and volatilities; the Irish (ISEQ) market is taken as an example. Also both time and scale dependency of the Hurst exponent are investigated for individual markets, (at different levels of development), in order to measure their disorder (or entropy[1]).

4.1 Long-Term Memory for the Whole Series

As noted, Chapter 3, many different methods have been used to detect the possibility of long-range dependence in stock market returns, but the evidence in general is mixed for the presence of long memory in these data [Lo (1991), Cheung and Lai (1995), Berg and Lyhagen (1998), Jacobsen (1996), Hiemstra and Jones (1997), Lee et al. (2001), Sadique and Silvapulle (2001) and Assaf and Cavalcante (2005)]. To our knowledge the use of the discrete wavelet transform (DWT) proposed here is novel and has not previously been discussed in the literature. The DWT and three different tests, (namely Rescaled Range (R/S), its modified form, and the semi-parametric method (GPH)), are applied to the daily returns of five Irish Stock Exchanges (ISEQ) indices in order to investigate the long-term memory property.

[1]Entropy is a measure of the disorder or randomness in a system

These example data have been chosen for two reasons: (i) because the Irish market has been one of the most significantly developing markets in the last five years, (ii) to reflect local interest in the research base. These methods have also been applied to the volatility measures (namely absolute and squared returns) in addition to the returns themselves. The case for the existence of long-term memory properties in the Irish data is discussed.

4.1.1 Data Description

In order to investigate the long-term memory property, for the Irish Stock Exchange (ISEQ) example, we considered daily closing prices of five indices, namely Overall, Financial, General (from 4/1/1988 to 30/9/2003), Small Cap (from 4/1/1999 to 30/9/2003) and ITEQ[2] (from 4/1/2000 to 30/9/2003). The daily returns of all these indices are calculated by using the following formula[3]

$$\text{Daily Returns} = r_t = \ln(P_t/P_{t-1}) \tag{4.1.1}$$

where P_t and P_{t-1} are the index prices at time t and $t-1$ respectively.

The statistical summaries of all ISEQ indices are reported in Table (4.1) which shows that the sample means are positive for all indices apart from the ITEQ index. The skewness and kurtosis of all return series are significantly different from zero, meaning that not all series can be regarded as Normally distributed. This is to be expected, as it is well known that return series has a "Fat-Tail" distribution, [as mentioned in, e.g. Mandelbrot (1963), Lux (1998) and Richmond (2001)].

[2]ITEQ is the Technology Market of the Irish Stock Exchange.

[3]The log return, formally the logarithmic return or continuously compounded return, is widely used in financial and economic researches The continuously compounded return is asymmetric thus clearly indicating that up and down returns are not equal.

Table 4.1 Descriptive statistics of the ISEQ indices daily returns series

Index→ Measure↓	Overall	Financial	General	Small Cap	ITEQ
Mean	0.0004	0.0005	0.0003	0.0003	-0.0013
Std.Dev	0 0099	0.0136	0 0100	0.0329	0 0078
Minimum	-0.0757	-0 0843	-0.1075	-0.1776	-0.0634
Maximum	0.0584	0 0711	0.0557	0.1645	0.0607
Skewness	-0 3204**	-0.0145**	-0.8352**	-0.2424**	-0.5955**
Kurtosis	5.1333**	3.7334**	9 9633**	2.7097**	8.1811**

Note: ** denotes statistically significant at 1% level

4.1.2 Results

4.1.2.1 Classical and Modified R/S and GPH

The R/S [Hurst (1951)] and Lo's R/S [Lo (1991)] methods, (described in Section 3.2), are applied to the return series and their volatility measures of ISEQ indices

The V-test [Lo (1991)] is applied to these two methods in order to test the null hypothesis: *The series exhibits short-term memory* (against the alternative one: *It shows long-term memory*). The null hypothesis cannot be rejected at α% level for 5% or 1%, if V-test value lies inside the interval [0.809,1.862] or [0.721,2.098] respectively. From Table (4.2), it can thus be seen that *no evidence* of a long memory property exists in any of *returns series* themselves, since the V-test values for these series lies inside both intervals (from both methods). However, there is agreement between V-test values for both R/S (and the modification of R/S due to Lo (1991)), that *strong evidence* exists for *long-range dependence* in the *absolute* and *squared returns* of Overall, Financial and General indices and in the *absolute returns* of ITEQ index. There is also agreement that no evidence of long memory is found in the Small Cap index. This is due probably to the characteristics of the Small Cap Index which represents an asset class of the smaller companies. This type of class offers potential for growth which may not always be seen in larger entities. It is also more representative of national economic performance compared to the

ISEQ Overall Index and also has a different volatility profile due to both liquidity and constituent weighting. However, for *the squared returns* of ITEQ index classical R/S indicates that there is a *strong evidence of long memory* in this series, whereas, Lo's R/S shows *no such evidence.* This disagreement is found probably because the characteristics of this index do not relate closely to the others and the focus is on the Technology companies of which there are only four in total. We note also that the number of observations is lowest for ITEQ and Small Cap.

Table 4.2: Results of the R/S analysis and Lo's modified R/S test

ISEQ index	Series	No of Observation	V-test of R/S	V-test of Lo's R/S
Overall	Returns	3948	1 7469	1.4776
	Absolute	3948	7 2223**	4 3492**
	Squared	3948	5 0001**	3 2941**
Financial	Returns	3948	1 4493	1 3007
	Absolute	3948	7 8112**	4 6176**
	Squared	3948	5 9294**	3.7573**
General	Returns	3948	1 7150	1 4154
	Absolute	3948	7.5615**	4 8269**
	Squared	3948	4.2129**	3.0913**
Small Cap	Returns	1194	1.3499	1.1666
	Absolute	1194	1 3587	1 1042
	Squared	1194	1 4102	1 0757
ITEQ	Returns	945	1.7038	1.6499
	Absolute	945	2 7325**	1.9878*
	Squared	945	2.1961**	1 6664

Note V-tests are calculated as $V_n = W_n/\sqrt{n}$ where W_n is R/S or Lo's R/S calculated by using Equation(3 2 7) The acceptance or rejection of the null hypothesis at $\alpha\%$ level for 5% or 1% is determined by whether or not V_n is contained in the interval [0 809,1 862] or [0 721,2 098] respectively Thus * and ** indicate statistical significance at the 5% and 1% respectively

The spectral regression procedure (GPH), [see Section 3.2 3 and Geweke and Porter-Hudak (1983)], is also applied to estimation of the fractional differencing parameter (d), to test the null hypothesis: *The series exhibits short-term memory* (or $d = 0$) against the alternative one: *It shows long-term memory* (or $d \neq 0$) for index returns and their volatilities. We report the GPH test for different values of ν

=0.45, 0.50, 0.55, 0.60 in order to measure the sensitivity of this test to the choice of m truncated length of subseries or size of subset [Equation (3.2.10)]. A two-sided t-test is constructed (with the theoretical variance of the spectral regression error equal to $\pi^2/6$), to test the stated hypotheses. The acceptance or rejection of the null hypothesis at 5% or 1% is determined by whether or not the t-statistic is contained in the interval [-1.96,1.96] or [-2.576,2.576] respectively. The results of this calculation are reported in Table (4.3).

From Table (4.3), it can be seen that no evidence for long-term memory exists in any of the returns series, since t-test values for these series are within one of the acceptance intervals {[-1.96,1.96] & [-2.576,2.576]}. It can also be seen that there is strong indication of persistence in the absolute and squared returns of all indices, (except that of Small Cap) and in the squared returns of General index. *In contrast to both R/S tests*, the GPH method shows that the squared returns of the General index have *no long memory* behaviour.

From the previous results, it can be seen that R/S, its modified form and GPH methods sometimes show mixed evidence for the presence of long memory. Different plausible reasons include:

- the nature of the stock market data series; non-stationary, non-normal and noisy series as well as the length of data series
- Insensitivity of the test bases. Modern markets may exhibit different memory types (long, intermediate and short-term), so that it is *unreliable* to test only short memory or weak dependence ($H = \frac{1}{2}$) against long memory or strong dependence ($H \neq \frac{1}{2}$).
- From the results, It can be concluded that methods, such as R/S tests, GPH, etc, are *no longer adequate* to determine memory effects, especially, in rapidly responsive (or less traditional) markets.

Table 4.3: GPH estimation of fractional differencing parameter d for daily returns of Irish Stock Exchange(ISEQ) indices

Index	Series	ν			
		0.45	0.50	0.55	0.60
Overall	Returns	0.0523 (0.452)	0.0519 (0.571)	0.0428 (0.599)	0.1197 (1.697)
	Absolute	0.506 (4.371)**	0.438 (4.810)**	0.400 (5.600)**	0.365 (6.387)**
	Squared	0.3853 (3.256)**	0.3333 (3.664)**	0.2966 (4.150)**	0.2663 (4.665)**
Financial	Returns	0.0019 (0.016)	0.1096 (1.205)	0.1151 (1.309)	0.1289 (1.742)
	Absolute	0.5313 (4.586)**	0.4720 (5.189)**	0.3617 (5.060)**	0.3384 (5.926)**
	Squared	0.3925 (3.388)**	0.3754 (4.127)**	0.3250 (4.548)**	0.3065 (5.368)**
General	Returns	0.028 (0.242)	0.0508 (0.558)	0.0696 (0.974)	0.0972 (1.703)
	Absolute	0.3959 (3.417)**	0.3615 (3.974)**	0.3127 (4.375)**	0.3098 (5.426)**
	Squared	0.2188 (1.889)	0.0969 (1.066)	0.0551 (0.771)	0.0877 (1.537)
Small Cap	Returns	0.0874 (0.543)	0.0787 (0.607)	0.0377 (0.362)	0.1369 (1.611)
	Absolute	0.0189 (0.118)	0.1261 (0.972)	0.1647 (1.579)	0.1586 (1.867)
	Squared	0.1143 (0.710)	0.0209 (0.161)	0.0278 (0.267)	0.0153 (0.180)
ITEQ	Returns	0.1011 (0.576)	0.0609 (0.436)	0.0192 (0.170)	-0.0729 (0.786)
	Absolute	0.5370 (3.061)**	0.4723 (3.371)**	0.4081 (3.621)**	0.3169 (3.413)**
	Squared	0.4161 (2.372)*	0.3411 (2.435)*	0.2989 (2.651)**	0.2316 (2.495)*

Note The d estimates correspond to GPH for level ν (see Equation (3.2.10)). The t-tests of hypothesis [H_0 $d = 0$ vs H_1 $d \neq 0$] are given in parentheses, constructed based on $\pi^2/6$ as the error variance Thus * and ** indicate statistical significance for t-test at the 5% and 1% significance level respectively

4.1.2.2 Results of the Discrete Wavelet Transform (DWT)

While such conventional analyses are useful, serving to contrast the Irish market data with that from other markets, the novel use of the discrete wavelet transform (DWT) lies in its ability to analyse the volatility more directly. To our knowledge, while the DWT has been used to measure the Hurst exponent, [Abry and Veitch (1998) and Simonsen et al. (1998)], it has not been used to study the memory structure itself of the data series In this work, we compute the DWT for 6 levels (scales)[4] for daily returns series and their volatility measures (namely squared and absolute returns) of all Irish indices in order to investigate the memory property of these series. The DWT provides a more detailed breakdown of the contribution to the series energy from the high and low frequencies in the following manner. Table (4.4) (a, b and c) displays the energy percentages for wavelet components (crystals) of the returns, (squared and absolute), of Overall, Financial, General, Small Cap and ITEQ indices respectively. These percentages indicate the amount of variation in these series explained by each wavelet component, Equation (3.5.5).

Table 4 4 Amount of "Energy", explained by each wavelet component (or crystal), for Irish indices (ISEQ). The total energy is equal to one, $\sum_{i=1}^{6} d_i + s_6 = 1$

a: The daily return series.

W. Component→ Index Series↓	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Overall	**0.433**	**0.239**	0.158	0.079	0 036	0 029	**0.026**
Financial	**0.431**	**0.251**	0 163	0 074	0 033	0.024	**0.024**
General	**0.447**	**0.236**	0.138	0 083	0.037	0.029	**0.030**
Small Cap	**0.493**	**0.234**	0 093	0.078	0 045	0 023	**0.035**
ITEQ	**0.476**	**0.210**	0 181	0 055	0.036	0.027	**0.014**

[4]The smoothest component s is obtained at the 6^{th} level, giving six detailed components $d_1, ..., d_6$ and the smoothest one s_6, so it is meaningless to go higher

b: The squared return series.

W. Component→ Index Series↓	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Overall	**0.367**	**0.162**	0.121	0.074	0 046	0 026	**0.205**
Financial	**0.326**	**0.182**	0.115	0.059	0 047	0.021	**0.251**
General	**0.388**	**0.188**	0.116	0.118	0 044	0.024	**0.122**
Small Cap	**0.314**	**0.234**	0.217	0.047	0 042	0.031	**0.116**
ITEQ	**0.317**	**0.185**	0 125	0.069	0 049	0 016	**0.240**

c The absolute return series.

W. Component→ Index Series↓	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Overall	**0.195**	**0.103**	0.060	0.035	0.027	0 015	**0.565**
Financial	**0.183**	0.104	0.063	0.032	0.027	0 013	**0.579**
General	**0.207**	**0.110**	0.062	0.045	.027	0 015	**0.533**
Small Cap	0.194	**0.116**	0.080	0 027	0.022	0 015	**0.546**
ITEQ	0.194	**0.097**	0 069	0 036	0.024	0.008	**0.571**

From Table (4 4) (a), it can be seen that the high-frequency crystals (d_1, d_2, d_3, d_4 and d_5) have much more energy than the lowest frequency one (s_6) and this means that movements in the returns are mainly caused by short-term fluctuations, confirming that there is *little evidence* of long memory in the returns series Table (4.4) (b) shows that the lowest frequency component (s_6) of the *squared* returns of each of the Overall, Financial and ITEQ indices has more energy than the second high-frequency component (or crystal) (d_2) but less energy than the first crystal (d_1), indicating that there is some evidence of long-term memory in these series. This supports the previous analysis, Section 4.1.2.1, on memory but adds further detail, since it provides an in-depth view of ranked contributions to variation in the data series and a real indication of structure in memory effects. These results imply, therefore, that movements in these squared returns are caused by *both* short-term and long-term fluctuations. The lowest frequency component (s_6) of the squared returns of the General index has lower energy [**0.122**] than the second highest frequency (d_2) [**0.188**] but higher energy than that of the third highest component (d_3)

[**0.116**], indicative of a *weak* long memory effect in these squared returns. However, the energy of the lowest frequency component (s_6) [**0.116**] of the squared returns of Small Cap index is even lower than that of the d_3 component [**0.217**] and this clearly implies that the movements of this series are mostly caused by short-term fluctuations with no significant evidence of long-term memory. Table (4.4) (c), in contrast, illustrates a situation where the lowest frequency component (s_6) has much more energy than both the first two high-frequency components (d_1 and d_2) together, which is strong evidence of long-range dependence in the absolute returns series with movements in these series mostly caused by long-term fluctuations. From the overall wavelet analysis, it is clear that frequency patterns are demonstrably different for the respective series where large energy percentages, (associated with high frequency components), imply short-term memory dominance and *vice versa*

4.1.3 Long Memory: In Summary

In this section, the discrete wavelet transform (DWT) was compared to three classical methods to test for the presence of long memory in five Irish Stock Exchange (ISEQ) indices. In agreement with findings for indices derived for other markets, [e.g. Lee et al. (2000), Elekdag (2001) and Sibbertsen (2002)], there is little evidence of long memory for returns series, while for squared and absolute returns, such a property does appear to exist. The exception, interestingly, is the Small Cap index for the Irish data, which shows no significant evidence of long-term dependence for any returns series due presumably to the characteristics of this index. The DWT analysis, however, provides additional insight on the series breakdown, in particular, providing a way to study the sensitivity of the series to increases in amplitude of fluctuations as well as changes in frequency as we can see from the distributions of energy percentage, (Table (4.4)), in relation to actual, squared and absolute returns The results also show that the absolute returns exhibit *higher* degree of long memory property than squared returns, with the indication of such property in the absolute series quite consistent for all methods. This would suggest that the absolute return is a more reliable measure of volatility than the squared. Finally, we can conclude that there is *strong indication for persistence* in the volatilities, (particularly absolute values), of the emerging stock market returns for the Irish data.

4.2 Time-Scale Behaviour of the Hurst Exponent (Multifractality)

From the previous Section, movements in the returns seem to be mainly caused by the short-term fluctuations even though, as has been noted, returns series of stock markets are expected to exhibit multifractal (multiscaling) behaviour [Turiel and Perez-Vicente (2003), Matia et al. (2003), Los and Yalamova (2004) and Oswiecimka et al. (2005)]. In this Section, therefore, we report results for a new Time-Scale extension of Detrended Fluctuation Analysis (TSDFA), (described in Section 3.3), to study the behaviour of the Hurst exponent (H) over different time periods and number of trading days, for stock price fluctuations (or returns) in order to investigate their multifractal property

4.2.1 Data Description

The list of worldwide stock market indices and the statistical summaries of their daily returns are given in Tables (4.5) and (4.6) respectively Table (4.6) shows that the sample means of the returns of all indices are positive except for that of the Nikkei 225 index. We also test the normality of these series by testing whether or not their skewness and kurtosis are different from zero. The results show that these return series are significantly negatively skewed except those for the Hang Sang, IPC, Straits Times and FTSE100 indices, which are not significantly different from zero. However, the returns series of all indices are *leptokurtic* and this means that the daily returns of all these indices can not be regarded as normally distributed.

Table 4.5: List of International Stock Market Indices considered here.

Market	Index Name	Region	Time Period
Australia	All Ordinaries	Asia/Pacifc	1990-2004
Canada	S&P/TSX Composite	America	1990-2004
Germany	DAX	Europe	1991-2004
Hong Kong	Hang Sang	Asia/Pacifc	1990-2004
Ireland	ISEQ Overall	Europe	1990-2004
Japan	Nikkei 225	Asia/Pacifc	1990-2004
Mexico	IPC	America	1992-2004
Portugal	PSI20	Europe	1993-2004
Singapore	Straits Times	Asia/Pacifc	1990-2004
UK	FTSE100	Europe	1990-2004
US	DJI	America	1990-2004

Table 4.6 Descriptive statistics of the daily returns of the stock market index series

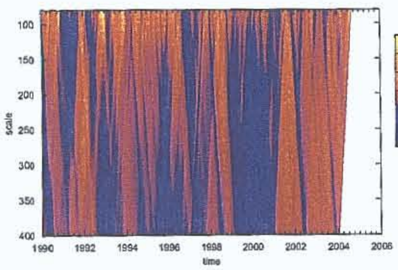
Measure→ Market↓	Min	Max	Mean	Std.Dev	Skewness	Kurtosis
Australia	-0.0745	0.0607	0.00023	0.0078	-0.409**	5.236**
Canada	-0.0846	0.0468	0.00022	0.0087	-0.687**	7.566**
Germany	-0.0987	0.0755	0.00032	0.0146	-0.187**	3.484**
Hong Kong	-0.1473	0.1725	0.00043	0.0165	-0.034	9.413**
Ireland	-0.0756	0.0584	0.00034	0.0099	-0.308**	4.500**
Japan	-0.0723	0.1243	-0.00033	0.0152	0.196**	3.007**
Mexico	-0.1431	0.1215	0.00068	0.0169	-0.016	5.430**
Portugal	-0.1118	0.0694	0.00031	0.0106	-0.942**	7.356**
Singapore	-0.0915	0.1287	0.00009	0.0131	0.096	8.620**
UK	-0.0559	0.0590	0.00018	0.0105	-0.063	2.799**
US	-0.0745	0.0615	0.00036	0.0102	-0.228**	4.449**

Note.** denotes statistical significance at 1% level.

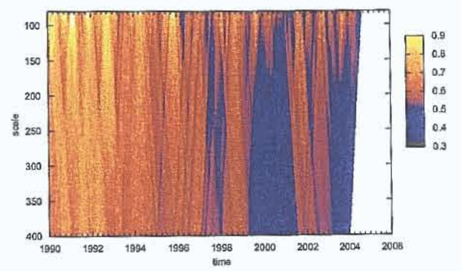
4.2.2 Results

Time-scale extension of Detrended Fluctuation Analysis (**TSDFA**) (Section 3.3) is applied to the daily returns of these stock markets and results are given in Figure 4.1, represented in colour shading for the Hurst exponent $H \in [0.3, 0.9]$, e.g. light yellow for $H = 0.9$, black for $H = 0.3$, etc: the full key is given on the diagram

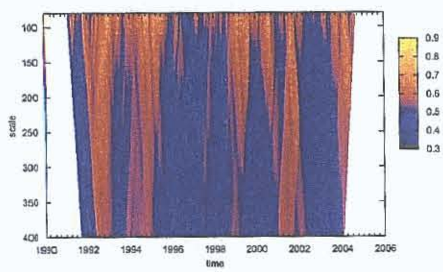
The Australian, (All Ordinaries), market TSDFA, Figure (4.1) (a), displays different memory types that span both persistent ($H > 0.5$) and anti-persistent ($H < 0.5$) behaviour over time For example, the market exhibits anti-persistence, with $H < 0.5$, during the period from 2003 to 2004, (*unexpected* since this market is classified as *mature*). Figure (4.1) (b) shows that the Canadian (S&P/TSX Composite) market also exhibits different memory types, with pre-1997 behaviour persistent ($H > 0.5$), and subsequently mixed, i.e. persistent ($H > 0.5$) for some intervals and anti-persistent ($H < 0.5$) for others, (again *unexpected* from a market, classified by the World Bank as *mature*). Figure (4.1) (c) shows that the German market displays anti-persistence ($H < 0.5$) for most time periods and scale levels (Number of trading days) Figure (4.1) (d and e) give TSDFA for Hong Kong (Hang Sang) and Singapore (Straits Times) markets respectively, showing that these markets exhibit persistent behaviour, with $H > 0.5$, in some time periods and for different number of trading days. In others, notably, they exhibit anti-persistent behaviour ($H < 0.5$) From Figure (4.1) (f, g and h), which give TSDFA for Irish (ISEQ Overall), Portuguese (PSI20) and Mexican (IPC) markets respectively, it can be seen that these markets show signs of persistence ($H > 0.5$) over almost all time periods and scale levels. TSDFA for the Japanese market, (Nikkei 225), is given in Figure (4.1) (i) and indicates that this market exhibits anti-persistence ($H < 0.5$) for most time periods and scale levels: this is as *expected* from a *mature* market. The results for UK, (FTSE100), market are given in Figure (4.1) (j) and it can be seen that the Hurst exponent H has been decreasing over time, with recent values< 0.5. This is what we *expect* from a *mature* market From Figure (4.1) (k), it can be seen that the Hurst exponents for the US market < 0.5 (or *anti-persistence*) for most time periods and scale levels. In contrast, this market shows *persistent* behaviour ($H > 0.5$) in 1997 and 2001 due to the Asian and 9/11 crashes respectively.



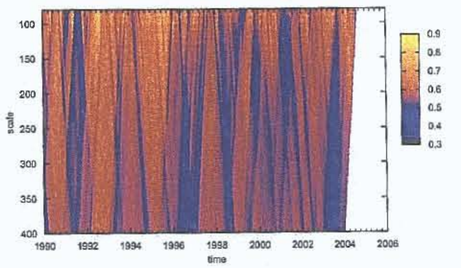
(a) Australian Market.



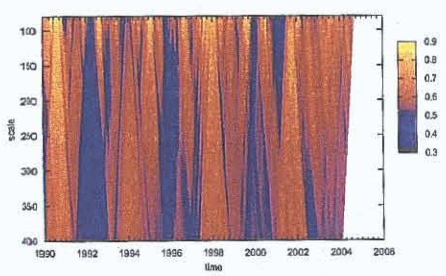
(b) Canadian Market.



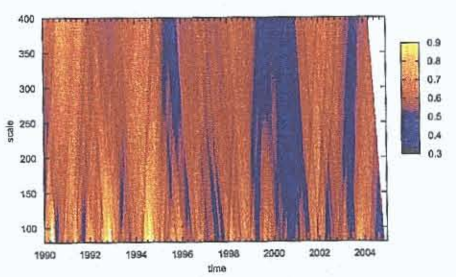
(c) German Market.



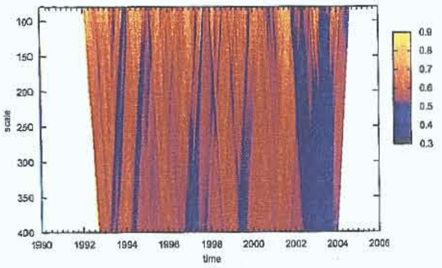
(d) Hong Kong Market.



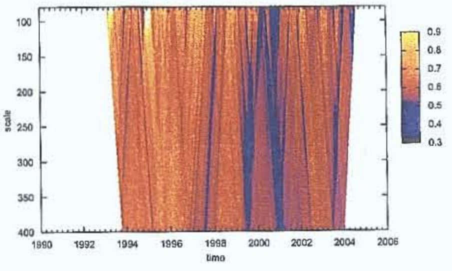
(e) Singapore Market.



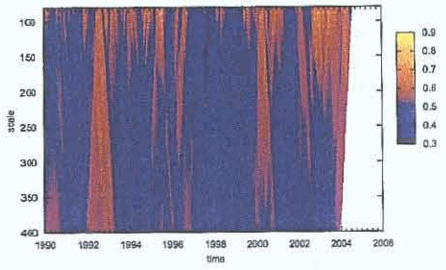
(f) Irish Market.



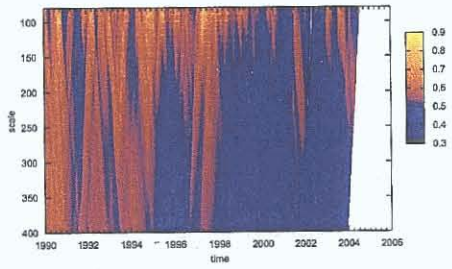
(g) Mexican Market.



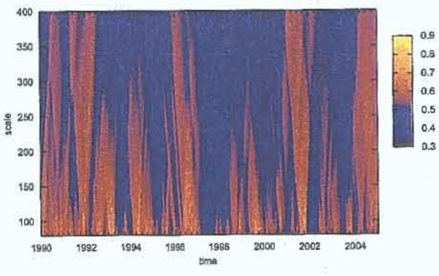
(h) Portuguese Market.



(i) Japanese Market.



(j) UK Market.



(k) US Market.

Figure 4.1: Hurst exponent values over time and for different scale levels calculated by Time-Scale Detrended Fluctuation Analysis (TSDFA).

In summary, Figure (4.1) in particular highlights the following features:

- The Hurst exponent $H(\tau, \theta)$ seems to be dependent on *both* time and scale (number of trading days), indicating that stock markets are *multifractal* in character. This property has been suggested, Turiel and Perez-Vicente (2003), Matia et al. (2003), Los and Yalamova (2004) and Oswiecimka et al. (2005), relatively recently. However, our analysis provides a clear illustration of this and moreover, provides a basis for quantification as well as a useful basis for comparison in the visual sense.

- Classification of global stock markets as either emerging or mature is an over-simplification.

4.2.3 Time-Scale Behaviour: In Summary

Time-Scale extension of Detrended Fluctuation Analysis (**TSDFA**) has been applied to eleven worldwide stock markets and results clearly imply that the behaviour of the Hurst exponent is dependent on both time and scale, indicating that stock market returns show multifractal behaviour. The results obtained empirically also indicate that the designation of the Hurst by $H(\tau, \theta)$ is more appropriate, where differences between global stock markets can *not* be reduced to a distinction between emerging and mature markets only On the contrary, these findings support the theoretical argument for the new method, but also indicate that some markets are inconsistent, i e. switch from one regime (or type) to another frequently or under less stimulus than others In recent years, in particular, the Hurst exponent evidence indicates that markets move to developed positions more rapidly than was traditionally the case. It seems clear that bi-classification of stock markets is inadequate and that a more informative new classification is needed.

CHAPTER 5

INTERNATIONAL CO-MOVEMENTS

5.1 Background

In this Chapter, the investigation of the price interdependence between seven international stock markets, namely Irish, UK, Portuguese, US, Brazilian, Japanese and Hong Kong is discussed. A *wavelet-based* method, Lee (2002), not previously implemented, is used to determine the direction and influence of global changes The new approach is also used to investigate the importance of legacy (or historical) transmissions, by studying the co-movement between Portuguese and Brazilian markets[1] in three different periods.

5.2 Data Description

The data used in investigating the global co-movement consists of the daily prices of stock market indices for seven markets, [Irish (IRL), UK, Portuguese (P), US, Brazilian (BR), Japanese (JP) and Hong Kong (HK)], during the period from January 1993 to September 2003. We considered the indices ISEQ Overall (IRL), FTSE All Share (UK), PSI20 (P), S&P500 (US), Bovespa (BR), Nikkei 225 (JP) and Hang Seng (HK) to be representative of these markets.

[1]Traditionally, these countries have close economic links, with companies listed on both markets.

Note: as each market uses its *local* currency for presenting the index values, we use the daily *returns* instead of using the daily *prices*, where Equation (4.1.1) applies. Some daily observations have been deleted because the markets we studied have different holidays. In other words, if one market closed on a given day, we consider the others to close on the same day as well, (as suggested by Lee (2002)).

Table (5.1) represents the trading hours of each of these markets in **GMT** and shows that the Japanese, together with the Hong Kong market, open first on a given date. The Japanese market closes two hours before the European (i.e. Irish, UK and Portuguese) markets open at 8:00 am, while Hong Kong closes forty-five minutes after the European opening. The last to open are the American (US and Brazilian), two hours prior to closure of the European markets. This implies that the starting point for market opening and closing trading hours is Asia, followed by Europe, then America

Table 5 1: Trading Hours for each market in GMT.

Continental ↓	**Markets** ↓	**Open**	**Close**
Asia	Japanese	0.00 am	6:00 am
	Hong Kong	1:45 am	8:45 am
Europe	UK	8 00 am	4 30 pm
	Irish	7:50 am	4 30 pm
	Portuguese	8:00 am	4 30 pm
America	US	2.30 pm	9:15 pm
	Brazilian	2.00 pm	8 45 pm

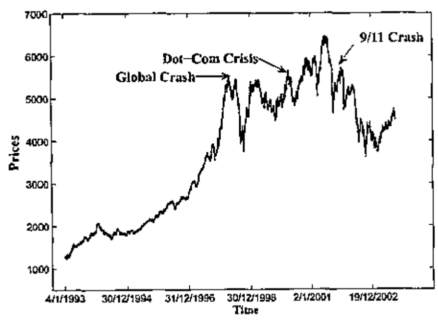
The statistical summaries of the daily returns of all stock market indices are reported in Table (5.2), which shows that the sample means of the returns of all indices are positive except for those of the Nikkei 225 and Hang Seng indices. We test whether or not the skewness and kurtosis of all these series are different from zero in order to test the normality of these series. The results show that the returns series of ISEQ Overall, PSI20 and FTSE All Share indices are significantly negatively skewed. Both Bovespa and Hang Seng indices have significant positive skewness, while S&P500 and Nikkei225 are not significantly different from zero in this sense. However, the returns series of all indices are leptokurtic and this means that the daily returns of all indices can not be regarded as normally distributed.

Table 5.2: Descriptive statistics of the daily returns of the stock market indices

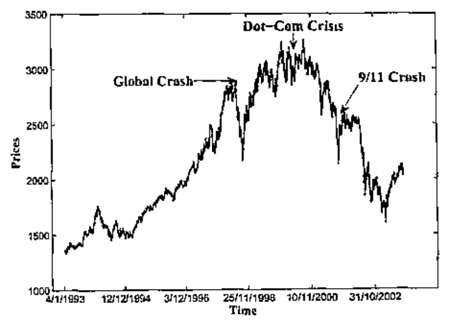
Index→ Measure↓	ISEQ	PSI20	FTSE	S&P500	Bovespa	Nikkei225	Hang Seng
Mean	0 0004	0 0001	0.0003	0 0003	0.0024	-0 0003	-0 0001
Std.Dev	0 0102	0 009	0.00109	0 0112	0.0282	0 0147	0 0179
Minimum	-0 0757	-0 0510	-0.0710	-0.0700	-0 1720	-0 0720	-0 1470
Maximum	0 0483	0 0509	0.0694	0 0557	0 2883	0 0765	0 1725
Skewness	-0 549**	-0 226**	-0.355**	-0.073	0 578**	-0 078	0 176**
Kurtosis	4 465**	2 816**	5 061**	3 072**	8 631**	2 053**	9 240**

Note ** denotes the statistically significant at 1% level

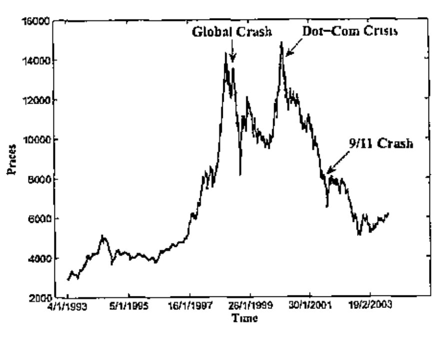
In order to investigate trends in the prices of ISEQ Overall, FTSE All Share, PSI20, S&P500, Nikkei 225 and Hang Seng indices separately, we plot the daily prices of these indices in their local currencies. Figure (5 1) (a) to (e) represent the *daily prices* of ISEQ Overall, FTSE All Share, PSI20 and S&P500 indices respectively It can be seen that the prices of these indices increased until the beginning of 1998, corresponding to a long-term period of growth. After that, the prices became unstable due to international events such as the global crash in October 1998, "dot-com" in March 2000 and September 11^{th}, 2001. While the prices of the Bovespa index show an upward trend until the begin of July 1997, they then became unstable due to the Thailand crisis in July, 15^{th}, 1997 following by the Hong Kong crash in October 23^{th}, 1997 and other global crashes. Key points and features for individual series are flagged in the Figure



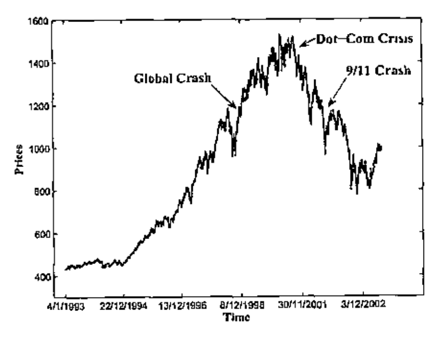
(a) ISEQ Overall ındex



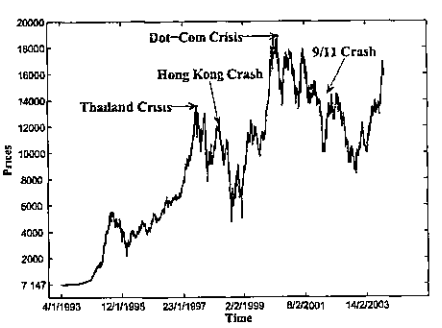
(b) FTSE all share ındex.



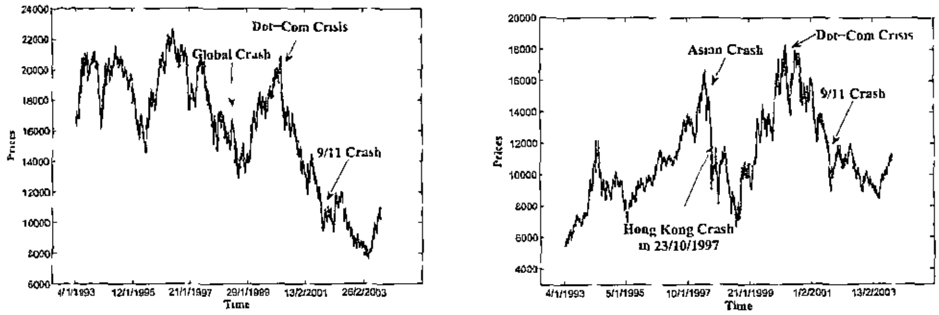
(c) PSI20 index.



(d) S&P500 ındex



(e) Bovespa index



(f) Nikkei 225 index

(g) Hang Seng index

Figure 5.1: Daily prices from May 1^{st}, 1993 to September 30^{th}, 2003.

Figure (5.1) (f and g) represent the prices of Nikkei 225 and Hang Seng indices respectively. These demonstrate that the Japanese market is very sensitive, possibly because companies who have shares in the Japanese stock market tend to be exposed internationally, and so price index levels respond to changes both directly and indirectly.

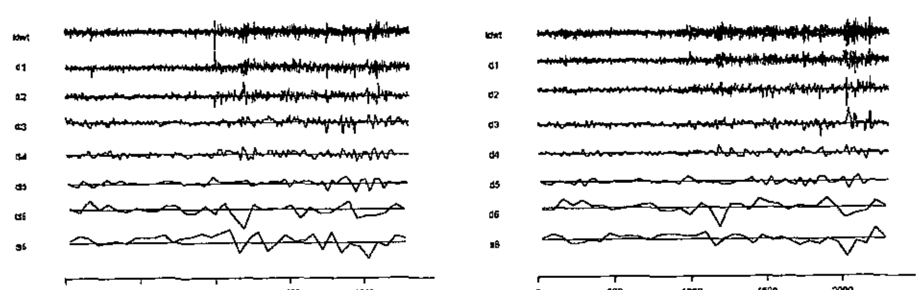
The Hong Kong market is noticeably unstable with a disproportionately large number of regionwide crashes (possible due to serial crises: Bird Flu, SARS, etc) The Asian financial crisis had strong direct effects on the Hong Kong market but, interestingly, affected Japan's economy only weakly since only 40 % of Japan's exports go to Asia In addition, Japan was going through its own ongoing long-term economic difficulties, which seen to have been more dominant in term of affecting the market at that time.

From the above, there are clear indications of influences on international markets from regionwide markets as well as from worldwide markets and this picture become more detailed when we look at the results of the wavelet analysis. For the daily returns of seven market indices, the energy percentages, (which tell us how much variation is carried by each wavelet component), are given in Table (5.3). This shows that more than 65% of variations (or energy) of the daily returns of all these series are explained by the first two high frequency crystals (d_1 & d_2), while the low-frequency component s_6 explains less than 6%. Further, d_1 and d_2 represent short-term variations occurring within ($2^1 = 2$) two and ($2^2 = 4$) four days respectively, implying that movements are mainly caused by ***short-term fluctuations***

Table 5.3: Percentages of energy by wavelet crystals for the daily returns of indices' series

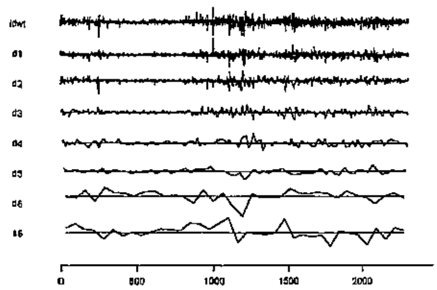
W.Crystals → **Index↓**	d_1	d_2	d_3	d_4	d_5	d_6	s_6
ISEQ	**0.443**	**0.246**	0.145	0.072	0.040	0.031	0.022
FTSE	**0.467**	**0.260**	0.161	0.048	0.032	0.018	0.014
PSI20	**0.440**	**0.262**	0.122	0.081	0.034	0.026	0.035
S&P500	**0.448**	**0.241**	0.161	0.053	0.032	0.013	0.012
Bovespa	**0.476**	**0.234**	0.143	0.046	0.025	0.019	0.057
Nikkei225	**0.534**	**0.240**	0.117	0.51	0.031	0.015	0.013
HSI	**0.515**	**0.230**	0.133	0.055	0.038	0.016	0.014

Figure (5.2) (a) to (g) represents information from the discrete wavelet decompositions of the daily returns of Irish, UK, Portuguese, US, Brazilian, Japanese and the Hong Kong stock market indices respectively corresponding to wavelet components in Table (5.3). The volatility in these markets is clearly shown by the high frequency components, such as d_1, d_2 and d_3 and has increased in the recent years

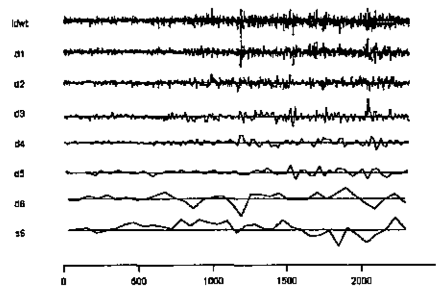


(a) ISEQ Overall index

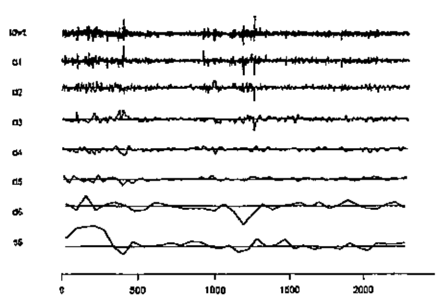
(b) FTSE all share index.



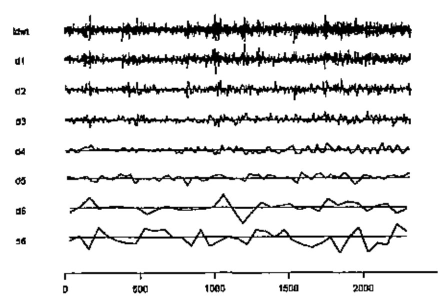
(c) PSI20 index



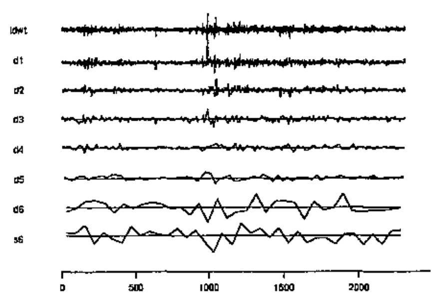
(d) S&P500 index.



(e) Bovespa index



(f) Nikkei 225 index.



(g) HSI index.

Figure 5.2. Discrete wavelet transform (DWT) of daily returns *vs.* Time The top graph in each case (idwt) is the daily return series, which can be reconstructed by using Equation (3 5 12)

5.3 Results

Traditionally, we might expect strong co-movements between nearest-neighbour stock markets, such as those of Ireland and the UK or between Japan and Hong Kong, while also reflecting the still strong historical links among international markets, for example, between Brazilian and Portuguese markets. However, as we shall see below, the results are more complicated than this.

5.3.1 Global Interdependence

In order to determine the dominant factors in such inter-relationships among all seven stock markets and examine if expectations are realised, simple and reverse linear regression models have been used to estimate co-movement between each pair, using three different series: original returns (or raw-returns) series, the returns rebuilt from the first wavelet component (d_1) and those rebuilt from the first two wavelet crystals (d_1 and d_2) together. This means that ordinary least squares fitting is applied twice, interchanging the independent and dependent variables the second time, but not always at the *same calendar day* (due to global time zones) For example, to investigate the co-movement between Irish and US markets, we first start with a simple regression of Irish returns on US returns of *the previous day*[2] Secondly, we preform a simple regression of the US market on the Irish market on *the same calendar day* The reasons for reconstructing returns series are twofold; firstly, to isolate key information that may not observed in the raw data. Secondly, to investigate whether or not co-movement[3] effects are "spurious". Unfortunately, we can not directly apply multiple regression (using forward or backward stepwise) to study the co-movements between the stock markets for two main reasons firstly,

[2]The Irish market closes shortly after US market opens; thus if there is influence from the US market, the Irish market will response to this news on the next trading day.

[3]The influence of X market on Y is inferred by looking at the "slope" and "R^2" values of the regression models of Y on X at three different levels. If all these values are significant; this indicates that X affects Y However, if the estimates are not significant and/or give wrong signs, this demonstrates little or no consistent impact from market X market on market Y.

multicollinearity problems are *expected* due to the relationships between the markets, secondly, we do not know the direction or order of the spillover effects.

The results of a regression analysis between each pair of return series of stock market indices are given **in Table (A.1) in the Appendix**. To take an example (Table (5.4)), from the values of the coefficient of determination (R^2) and P-values of slopes[4], which are given in **bold**, it can be seen that:

- R^2 and highly significant P-values for simple regression of Ireland (*dependent variable*) on the UK (*independent variable*) for three different return series (raw, reconstructed from d_1 and reconstructed from (d_1 & d_2)) , are (0.323, 0.000), (0.222, 0.000) and (0.251, 0.000) respectively. These imply that the regression models of Irish markets on the UK Market for all these different series are significant, meaning that the UK market has impact on the Irish market.
- R^2 and highly significant P-values for regression of UK market (*dependent variable*) on Irish market (*independent variable*) for three different series, are (0.323, 0.000), (0.222, 0.000) and (0.251, 0 000) respectively, indicating that the UK is influenced by Ireland.
- The previous points, taken together, show that there is strong evidence of interrelationship between these two markets.

Where the notations for Tables (5 4) and (5 5) are given as follows:

- P-values of t-tests are given in parentheses.
- Return is an indicator of the raw daily returns series
- Return.D1 and Return.D1.2 are indicators of the returns reconstructed from the first wavelet crystal and those reconstructed from the first and the second wavelet crystals together respectively.

[4] In this analysis, we are interested in investigating whether or not there is significant evidence of the influence from one market to another, but not how much the influence is

Table 5 4· Simple and reverse regression analyses between the daily returns of Irish and UK stock market indices.

Regression→	M_t^{IRL} on M_t^{UK}			M_t^{UK} on M_t^{IRL}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3 80E-04 (0.029)	0 578 (0.000)	**0.323**	-1.74E-04 (0.310)	0.559 (0.000)	**0.323**
Return D1	1.13E-04 (0.391)	0.467 (0.000)	**0.222**	-1.41E-04 (0 289)	0 477 (0.000)	**0.222**
Return.D1 2	1.49E-05 (0.951)	0.495 (0.000)	**0.251**	-4 32E-05 (0.864)	0 508 (0.000)	**0.251**

Overall, from the illustrative table and the complete results (Appendix A.), it can be concluded that there are *strong co-movements* between each two of the Irish, UK and Portuguese markets, while the Irish market is also influenced by the US, Japan and Hong Kong. The UK and Portuguese markets are affected by both Japan and Hong Kong, while these are impacted upon by the US and Brazilian markets Further, the UK and Portuguese markets influence the US and Brazil. Table (A.1) also shows that there is co-movement between US and Brazilian and also between the Japanese and Hong Kong markets *(nearest-neighbours)*. No inter-relationships apparently exist between the Brazilian and either the Irish or Japanese markets, but the Brazilian market itself is significantly affected by that of Hong Kong This implies that there is also an inner loop of "spillover effects" between Asian and American markets within the global circle, (southeast Asia to the Latin Americas). In other words, the US market affects those of Asia, (Japanese and Hong Kong), which in turn impact on Brazil. The co-movement directions are given in Figure (5 3)

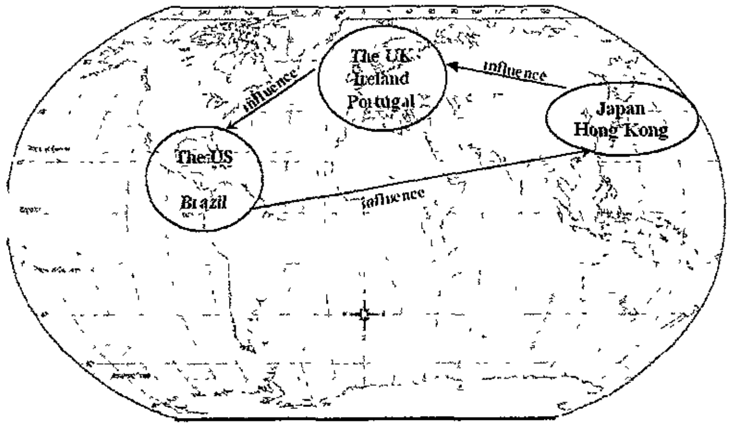


Figure 5 3. Direction of international co-movements (external influence) is indicated by the arrows, where the markets inside each circle have co-movement between each other.

5.3.2 Historical Interrelationship: Case Study, Portugal (PSI20) and Brazil (Bovespa)

In order to get a clear picture of the historical linkage between Portuguese and Brazilian markets, we divided the whole period of the individual series into three sub-periods (1993-1996, 1997-2000 and 2001-2003) and obtained the regression model estimates between these markets using the three different return measures The results are given in Table (5.5) (a, b and c) and Figure (5.4). They show no co-movement between Portugal and Brazil in the first period because PSI20 index (Portugal) was established in 1993. However, there is significant evidence of co-movement between these markets from 1997 to 2000, (i e. supporting historical linkage with markets effectively acting together). However, in the third period, the results show that there are *spillover effects* from the Portuguese market on to the Brazilian market, but not *vice versa*, i e. the implication here is that Portugal is now "leading" (as part of the set of European markets). This implies that the importance of international transmission has been increased while that of historical linkage has been decreased since the last quarter of the 20^{th} century

Table 5.5: Regression Analysis between Portuguese and Brazilian Markets using three different series.

a: From 1993 To 1996

Regression→	$M_t{}^P$ **on** $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ **on** $M_t{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	5.31E-04 (0.000)	1 36E-02 **(0.401)**	**0.001**	2 75E-03 (0.000)	5.54E-02 **(0.430)**	**0.001**
Return.D1	2 04E-07 (0.999)	-4.42E-02 **(0.003)**	**0.010**	-4 14E-07 (0.999)	0 139 **(0.088)**	**0.003**
Return.D1.2	7.95E-07 (0 996)	-6.37E-03 **(0.669)**	**0.001**	1.12E-07 (1.000)	-1 84E-02 **(0.812)**	**0.001**

b: From 1997 to 2000

Regression→	$M_t{}^P$ **on** $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ **on** $M_t{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	5.28E-04 (0 222)	0.272 **(0.000)**	**0.062**	1.91E-04 (0 987)	0 270 **(0.000)**	**0.085**
Return D1	7.95E-07 (0 998)	9 23E-02 **(0.002)**	**0.010**	-4.77E-07 (0 999)	0.136 **(0.000)**	**0.016**
Return.D1.2	1 87E-06 (0 996)	0.181 **(0.000)**	**0.032**	-7.47E-07 (0 998)	0.224 **(0.000)**	**0.054**

c: From 2001 To 2003

Regression→	$M_t{}^P$ **on** $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ **on** $M_t{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-7 35E-04 (0 072)	0.259 **(0.000)**	**0.041**	2.40E-04 (0 455)	0.212 **(0.001)**	**0.070**
Return.D1	-2 16E-06 (0 994)	-1.95E-02 **(0.677)**	**0.000**	-4 42E-06 (0.984)	0.148 **(0.000)**	**0.032**
Return D1.2	1.80E-06 (0 996)	0.164 **(0.001)**	**0.016**	-2.10E-06 (0 994)	0 184 **(0.000)**	**0.048**

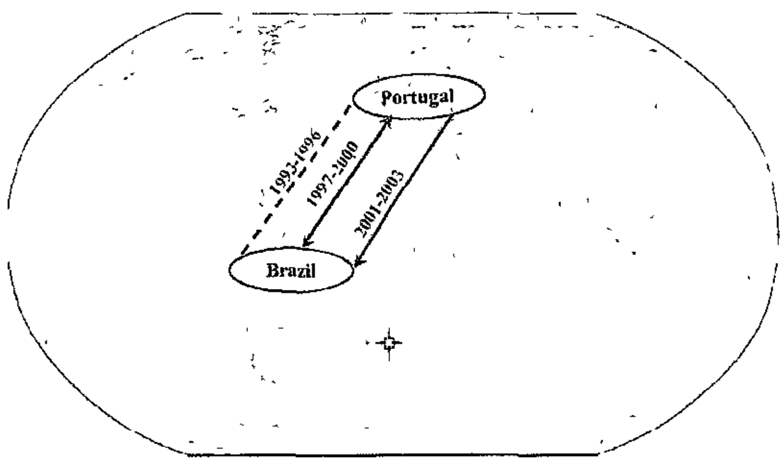


Figure 5.4: Example of historical co-movements. Direction of arrows indicate nature of co-dependence or influence.

Finally, from both pairwise and historical regression analyses reported in this Section, it seems clear that directional influence is *globally clockwise* starting with Asian markets influencing European, European impacting on the Americas and the circle completing with American market changes impacting on those of Asian. Interestingly, only the Japanese market demonstrates *mixed influences*. Possible explanations for these findings on global inter-dependence and circular spillover effects between the stock markets in different Continents, are as follows:

- Many firms with shares in these stock market indices are international investors[5]
- Different time-zones mean that trading is concluded in Asia prior to opening in Europe and similarly for Europe to America and back again to Asia. These

[5] One possibility for future research is evidently to look at the granularity, i.e. to investigate the behaviour of individual stock (or company). In this work, we have focused on the whole market, rather than different firms or company type

spillover effects are noticeable on the markets which open next, but become less-marked for the next global cohort.

5.4 Conclusion

The aim in this Chapter is to look at external effects or influences by investigating the co-movements between seven international stock markets, (namely the Irish, UK, Portuguese, US, Brazilian, Japanese and Hong Kong), based on daily returns. A new testing method suggested by Lee (2002) has been applied and our results show that there are significant co-movements between each European pair separately, between the US and Brazilian markets and also between the Japanese and Hong Kong markets. In addition, the indications are that there are significant spillover effects from the UK and Portuguese markets onto the US and Brazilian markets which themselves in turn, influence the Asian markets. In turn, Japan and Hong Kong impact on Europe. Finally, we can summarize our results by the following statements:

1 There are co-movements between regionwide markets (*nearest-neighbour* or *intra-continental* relationships).

2 There are *clockwise* transmissions between worldwide markets (*inter-continental* relationships).

3 There is an increase in *importance* of global co-movements among worldwide stock markets, in particular since the end of the 20^{th} century. As we have seen, this is demonstrated by studying the historical link between Brazil and Portugal, (Subsection 5 3.2), and is probably due to the advent of modern communications in term of increasing the globalisation of stock markets However, in-depth studies of other historical linkage would also be required to provide further evidence.

Chapter 6

Emerging vs. Mature Markets

In Chapters (4, 5), we have investigated long memory within and co-movements between global stock markets. However, both internal structure and external influences may change and evolve over time, so that market behaviour will also evolve or switch between recognised categories (i.e emerging or mature). Further, in a climate where the volume and rapidity of information exchange is constantly expanding, there is a need to investigate whether these categories are sufficient to characterise observed differences in international stock market behaviour.

6.1 Reaction to Crashes and Events.

It has been known for some time [e g. Meric and Meric (1997), Kwapien et al. (2002), Keogh et al. (2003) and Kwapien et al. (2004)] that, as well as providing information about how assets move with respect to each other, the largest eigenvalue (λ_1) of the covariance matrix of asset measures (e.g. returns) contains information on the *risk* associated with those particular assets. Here we examine whether the *subdominant eigenvalues* (λ_2, λ_3) hold additional information on the stock market risk and also can be used to measure the recovery time from "shocks" for emerging and mature markets. Explicitly, we study the behaviour of the first three eigenvalues

(λ_1, λ_2, λ_3) and their ratios [(λ_1/λ_2), (λ_1/λ_3), (λ_2/λ_3)][1] for the covariance matrices of the original return series, in addition to those rebuilt from wavelet components for emerging and mature markets (Methodology as described in Section 3.4).

6.1.1 Data Description

In this analysis, we use weekly returns[2], calculated using Equation (4 1.1), of a set of thirteen **emerging** market indices and a set of fourteen **developed**[3] market indices during the period from the second week of January 1997 to the third week of March 2004, as given in Table (6.1). The Covariance matrices for these two sets of indices have been calculated (Equation (3.4.1)), where, here, maximum value of $i = 13$ & 14 for emerging and mature markets respectively and N has been chosen to be 20 for both.

Table 6 1 List of Emerging and Mature stock market indices

Mature↓	Index	Region	Emerging↓	Index	Region
Australia	All Ordinaries	Asia/Pacific	Argentina	Mer Val	Americas
Canada	S&P/TEX Composite	Americas	Brazil	Bovsepa	Americas
Denmark	KAXPI	Europe	Ireland	ISEQ Overall	Europe
France	CAC 40	Europe	Korea	KOSPI 200	Asia/Pacific
German	DAX	Europe	Malaysia	KLSE Composite	Asia/Pacific
Hong Kong	Hang Sang	Asia/Pacific	Mexico	IPC	Americas
Italy	MIB 30	Europe	New Zeland	NZSE 50	Asia/Pacific
Japan	Nikkei 225	Asia/Pacific	Norway	BOX	Europe
Netherland	AEX General	Europe	Portugal	PSI20	Europe
Sweden	SAX Allshare	Europe	Russian	Moscow Times	Europe
Switzerland	SSMI	Europe	Singapore	Straits Times	Asia/Pacific
UK	FTSE 100	Europe	Taiwan	Taiwan Weighted	Asia/Pacific
US	NASDAQ	Americas	Turkey	XUTUM	Middle East
US	S&P500	Americas			

[1]We look at the first three eigenvalues to determine if significant absolute contributions are observed in addition to that of the first, which reflects the overall market movement. We also investigate the eigenvalue ratios, to see if relative contributions are important.

[2]The reasons for using weekly data here are twofold (i) the stock market is expected to take longer than a day to respond to major events (ii) Litterman and Winkelmann (1998) found that using k-days returns, e.g. weekly, to estimate the covariance matrix seems to reduce the impact of effects that persist for only a short period, i.e daily returns

[3]Classified in accordance with the International Finance Corporation (IFC) definition footnote number 9, page 11

6.1.2 Results

Note: The aim here is to study the behaviour of eigenvalues of covariance matrices for emerging and mature markets separately, not to compare directly between these two types of markets. The different scales in the Figures in this section are not, therefore, relevant to the discussion.

6.1.2.1 Dealing with crashes and events: Emerging vs. Mature.

Figures (6.1, 6.2, 6.3 and 6.4) show the distributions of eigenvalues (relative size and rank) of the Covariance matrix for overlapping windows[4], before and after the Asian Crisis in July 1997, the Global Crisis in October 1998, the Dot-Com Crash in March 2000 and the September the 11^{th} Crash in 2001, for a group of Emerging markets and a group of Mature ones

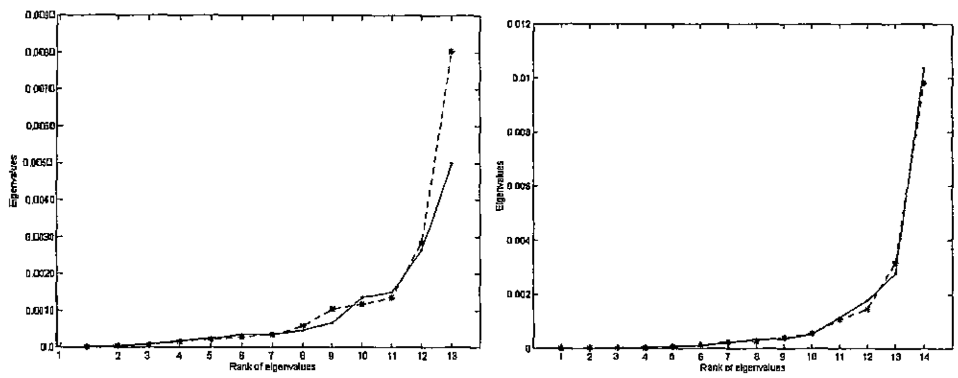
Figure (6.1) (a and b) show that, for markets, classified as emerging by IFC criteria, the magnitude of the maximum eigenvalue λ_1 increased after the Asian Crisis but did not change for developed markets. We take from this that the crisis mainly affected *emerging* markets but *not mature* ones. From Figure (6.2) (a and b), we can see that the Global Crisis in 1998 affected *emerging* and *mature* markets comparably in the same week.

However, Figure (6.3) (a and b) shows that the Dot-Com Crash affected *mature* markets but *not emerging* ones. Figure (6.4) (a and b) show that the value of λ_1 after the September 11^{th} crash, (which was not anticipated, occurring without obvious warning signs), hugely increased for *both* emerging and mature markets. This implies that stock markets around the world were hit very hard and that the markets moved *in coordination* to make a recovery after falling so sharply or being oversold

These remarks also highlight the fact that the nature of the event may influ-

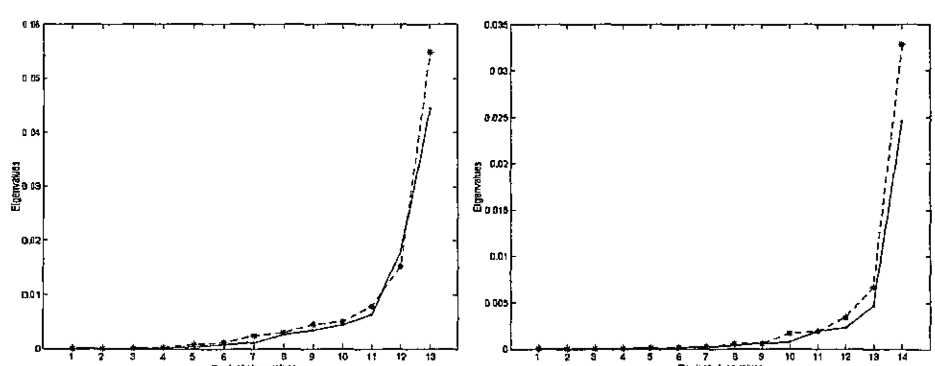
[4]Overlapping here means that elements in common between different periods are shifted along the length of the series. Window numbers are 336 and 345 for emerging and mature groups respectively.

ence how different categories of market respond. It is reasonable to postulate that "predictability" of the event, nature of investment (or sectoral composition) of the market and inner and outer loops of global co-movements may all be factors in response amplitude and time for a specific market.



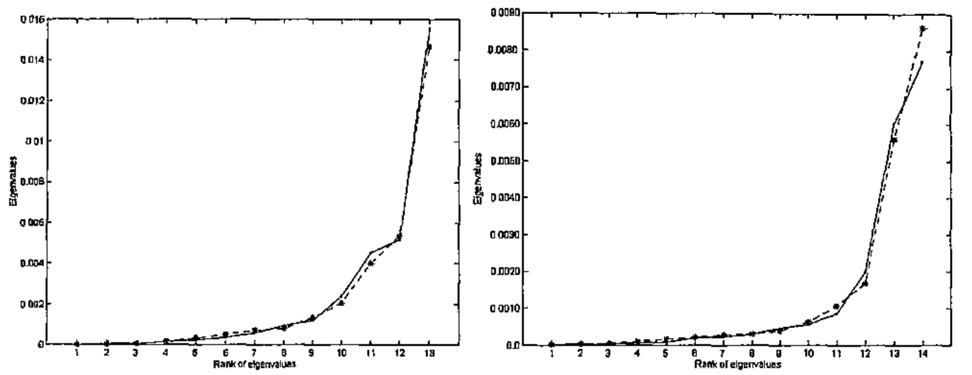
(a) **Emerging markets** (b) **Mature markets**

Figure 6 1. Distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) Asian Crisis, July 1997.



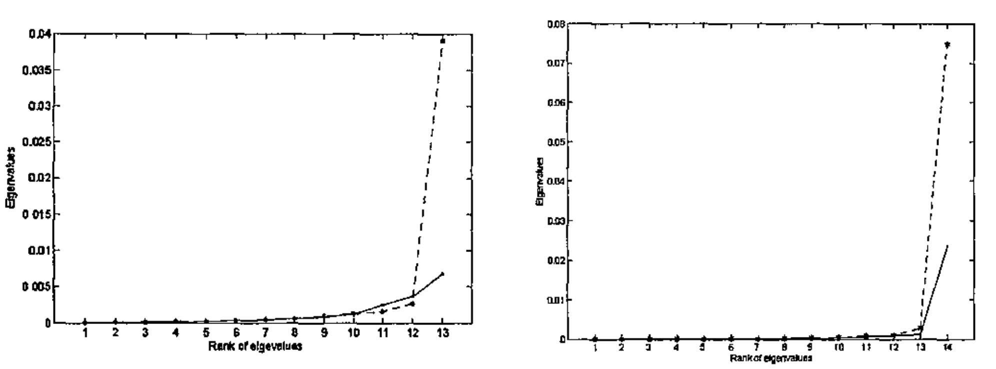
(a) **Emerging markets** (b) **Mature markets**

Figure 6.2. Distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) Global Crisis, October 1998.



(a) **Emerging markets** (b) **Mature markets**

Figure 6.3: Distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) Dot-Com Crash, March 2000



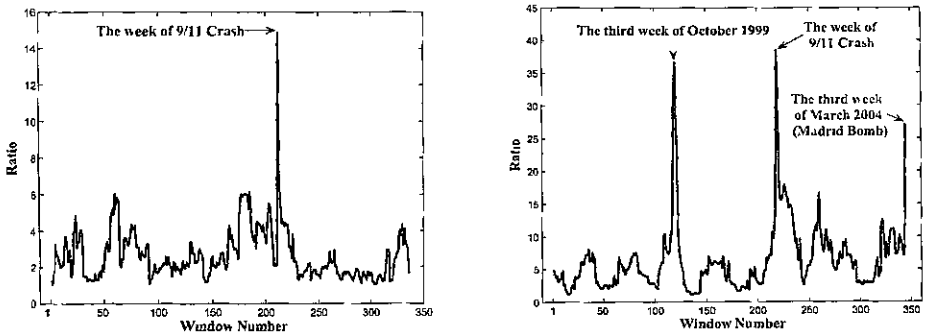
(a) **Emerging markets** (b) **Mature markets**

Figure 6.4. Distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) September the 11^{th} Crash, 2001.

6.1.2.2 Reaction to crashes and events: Emerging vs. Mature.

The variation in the ratio of the two *Largest* eigenvalues (λ_1/λ_2) of the Covariance matrices was examined for *equal period overlapping* time windows of the original returns series for the emerging and mature market groups Results are shown in Figure (6.5) (a and b) respectively and demonstrate a qualitative difference in the way emerging and mature markets deal with crashes and events

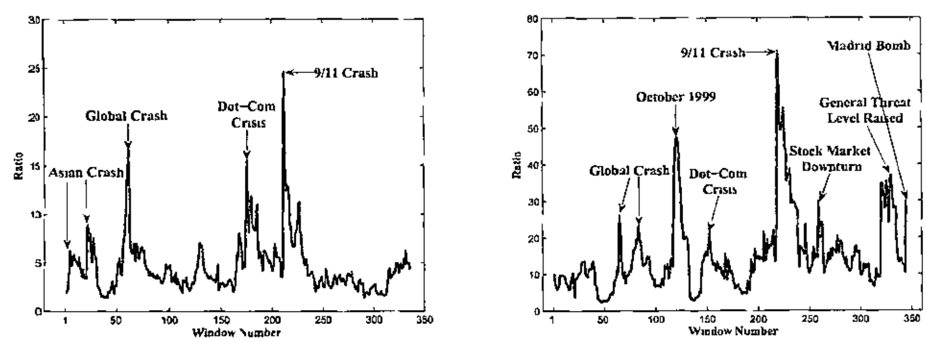
We also plot the ratio of λ_1/λ_3 versus window number to see how pervasive the reaction is to different crashes and events for different market types. The variation in these ratios is plotted in Figure (6 6) (a and b)



(a) Emerging markets.

(b) Mature markets

Figure 6.5 Changes in ratio of *Dominant* (λ_1) to *Subdominant* (λ_2) eigenvalues (λ_1/λ_2) for original return series.



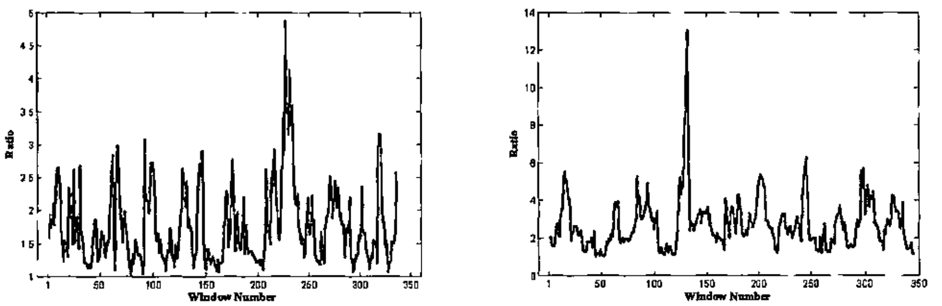
(a) Emerging markets.

(b) Mature markets

Figure 6.6: Changes in ratio of the *First Largest* (λ_1) to the *Third Largest* (λ_3) eigenvalue for original return series.

It can be seen that the mature markets have reacted to events more strongly than emerging markets, especially after the 9/11 crash, it seems, investors in mature markets are happier to presume that a market is oversold than those in emerging markets. This means that mature markets effectively become *anti-persistent*, while emerging markets are *persistent* (Section 4.1) in agreement with the findings of Di Matteo et al. (2003 and 2005) which indicate that emerging markets have Hurst exponent $H > 0.5$, while mature markets have $H < 0.5$.

The complementary ratio λ_2/λ_3, Figure 6.7 (a and b), was also plotted in order to see if the behaviour for λ_2 and λ_3 reflects different contributions for emerging and mature markets.



(a) Emerging markets. (b) Mature markets.

Figure 6.7: Changes in ratio of the *Second Largest* (λ_2) to the *Third Largest* (λ_3) eigenvalue (λ_2/λ_3) for original return series.

We thus investigated whether or not λ_2 carries additional information about these different market types. Figure (6.7) (a and b) suggests that the ratio λ_2/λ_3 for emerging markets is more variable than that for mature markets, leading us to infer that subdominant (λ_2), as well as dominant (λ_1) eigenvalues, do contribute in describing the behaviour of emerging markets while the behaviour of mature markets is described predominantly by λ_1 only, since subsidiary ratios contain little additional information.

In comparing the ratio (λ_1/λ_2), (shown in Figure (6.5) (a and b)), for emerging and mature markets, it can be seen that for the latter, there are three highly significant points in the ratio variation which are for **window numbers 120, 219 and 345** respectively. Window 120 starts from week 120 to week 139 which is the third week of October, 1999 *(the 12^{th} anniversary of October 19, 1987 crash*[5]*)*. The last week in window 219 is week 238 which is the second week of September, 2001 *(9/11 crash)* and window 345 starts from week 345 to week 364 which is the third week of March, 2004 *(Madrid Bomb)*. However, for emerging markets, there is only one highly significant point which is for **window 212**, where the last week in this window is the second week of September, 2001 *(9/11 crash)*. We suggest that these highly significant ratio points have the following implications:

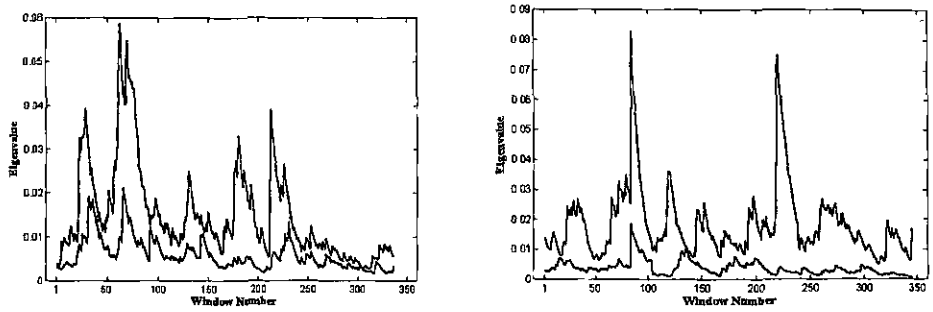
1 Increasing the value of the largest eigenvalue (λ_1) while the second largest eigenvalue (λ_2) remains stable, indicates that λ_1 alone describes the movements of stock markets.

2. Decreasing the value of λ_2 while the value of λ_1 does not change. This implies that only λ_2 explains the behaviour of stock markets, while λ_1 does not carry any information.

3. Increasing the value of λ_1 while decreasing the value of λ_2, (or in other words, λ_1 and λ_2 moving in opposite directions), shows that both λ_1 and λ_2 are important in explaining the behaviour of stock markets.

The absolute changes in λ_1 and λ_2 are plotted in Figure (6.8) (a and b) for emerging and mature markets respectively. For mature markets, (in order to examine likely causes), we compared the values of λ_1 and λ_2 of the covariance matrix for selected windows 120, 219 and 345 with the values of the previous windows, while for emerging markets, we compared the values of λ_1 and λ_2 for window 212 with the values of the previous windows We found that the third (combination) effect above causes peaks in emerging markets while it is the first combination which governs (or influences) movements in the mature markets.

[5]This was the last October in 20^{th} century and October is always hard month for stock markets so, with the end of the century as well, a crash in October was anticipated but did not happen This, not least because, *"The world markets were actually sent into turmoil by a speech by Alan Greenspan, and the Dow Jones for the first time since April 8, 1999 dipped below 10.000 on October 15 and 18, 1999. However, the market did not crash and instead quickly recovered and later started a renewed and strengthened bullish phase"*, (Sornette, 2002)



(a) Emerging markets

(b) Mature markets.

Figure 6.8: Changes in the *Dominant* (λ_1) (Upper line) and the *Subdominant* (λ_2) (Lower line) eigenvalue for original return series.

6.1.2.3 Recovery time from crashes and events: Emerging vs. Mature.

The aim here was to measure the recovery time of emerging and mature markets from crashes and the length of time for which these markets retain information about these events. In order to do this, we introduced *a new approach* based on the discrete wavelet transform (DWT), *together with eigenanalysis.* The steps of this approach are given by: **(i)** Use the DWT to divide the return series of emerging and mature markets into different frequency components. (The DWT then provides a more detailed breakdown of the contribution to the series energy from the high and low frequencies in the following manner) **(ii)** Rebuild the returns using each of these wavelet components, (d_1, d_2, d_3, etc) and **(iii)** Study the distribution of the ratio (λ_1/λ_2) of eigenvalues of the covariance matrices for overlapping windows of size 20 for these series. Tables (6.2) and (6.3) display the energy percentages (or variance) of each wavelet component (or crystal) of the original returns for emerging and mature market indices respectively.

From Tables (6.2) and (6.3), it can be seen in both cases that high-frequency crystals (d_1, d_2 and d_3), which reflect rapid changes over short time periods, have much more energy than the lowest frequency one (s_6) implying that movements in these series are mainly caused by short-term fluctuations.

Table 6.2: Emerging Markets Percentages of energy explained by wavelet components for the original returns series.

W.Crystals→ Market↓	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Argentina (Americas)	**0.415**	**0.203**	**0.192**	0.124	0.034	0.011	0.021
Brazil (Americas)	**0.521**	**0.185**	**0.124**	0.095	0.055	0.002	0.019
Ireland (Europe)	**0.440**	**0.250**	**0.115**	0.104	0.062	0.004	0.025
Korea (Asia)	**0.583**	**0.207**	**0.076**	0.070	0.021	0.020	0.022
Malaysia (Asia)	**0.498**	**0.211**	**0.107**	0.101	0.032	0.016	0.035
Mexico (Americas)	**0.455**	**0.246**	**0.144**	0.074	0.057	0.012	0.013
New Zealand (Pacific)	**0.546**	**0.197**	**0.126**	0.070	0.037	0.019	0.006
Norway (Europe)	**0.469**	**0.247**	**0.109**	0.076	0.059	0.022	0.018
Portugal (Europe)	**0.461**	**0.190**	**0.136**	0.084	0.079	0.020	0.030
Russia (Europe)	**0.434**	**0.239**	**0.126**	0.082	0.063	0.019	0.037
Singapore (Asia)	**0.496**	**0.213**	**0.106**	0.124	0.016	0.020	0.025
Taiwan (Asia)	**0.465**	**0.308**	**0.106**	0.051	0.043	0.009	0.019
Turkey (Middle East)	**0.477**	**0.213**	**0.141**	0.058	0.075	0.014	0.023

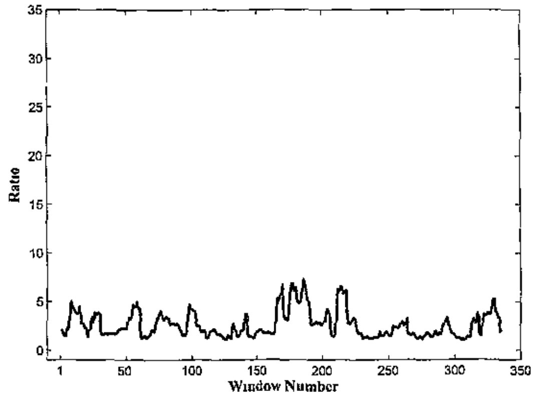
Table 6.3: Mature Markets Percentages of energy explained by wavelet components for the original returns series.

W.Crystals→ Market↓	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Australia (Pacific)	**0.499**	0.226	**0.168**	0.055	0.038	0.010	0.005
Canada (Americas)	**0.552**	**0.202**	**0.104**	0.050	0.054	0.028	0.011
Denmark (Europe)	**0.505**	**0.151**	**0.221**	0.044	0.026	0.033	0.020
France (Europe)	**0.546**	**0.231**	**0.103**	0.055	0.025	0.019	0.022
German (Europe)	**0.594**	**0.214**	**0.128**	0.031	0.023	0.007	0.004
Hong Kong (Asia)	**0.487**	**0.221**	**0.138**	0.100	0.026	0.007	0.021
Italy (Europe)	**0.511**	**0.220**	**0.146**	0.060	0.030	0.014	0.019
Japan (Asia)	**0.557**	**0.213**	**0.123**	0.059	0.020	0.010	0.019
Netherlands (Europe)	**0.390**	**0.418**	**0.064**	0.091	0.010	0.018	0.008
Sweden (Europe)	**0.518**	**0.201**	**0.133**	0.063	0.036	0.026	0.023
Switzerland (Europe)	**0.458**	**0.277**	**0.133**	0.070	0.028	0.015	0.018
UK (Europe)	**0.532**	**0.244**	**0.113**	0.054	0.032	0.011	0.013
US (NASDEQ) (Americas)	**0.531**	**0.233**	**0.121**	0.051	0.023	0.008	0.034
US (S&P500) (Americas)	**0.550**	**0.224**	**0.125**	0.051	0.025	0.009	0.017

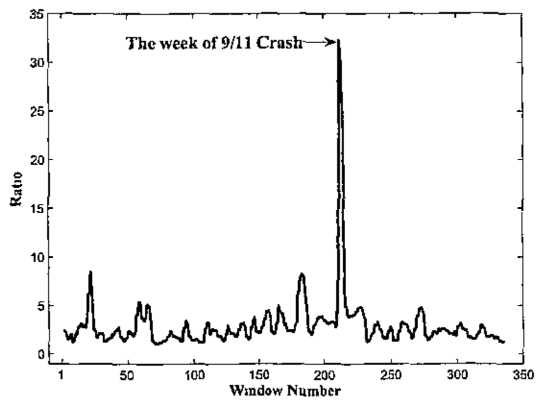
Figures (6.9) (a, b and c) and (6 10) (a, b and c) show for each window the ratio (λ_1/λ_2) from covariance matrices of the return series, rebuilt from d_1, d_2 and d_3, (representing fortnightly, monthly and bi-monthly data respectively), for emerging and mature markets respectively.

Looking at the ratio scales in these Figures, we can clearly see two main features; firstly, for emerging markets, even bi-monthly return series, (rebuilt from d_3), seem to carry information on crashes and events and this seems to imply that emerging markets take up to two months to recover from a crash Secondly, for mature markets, even though the ratios in Figure (6 10) (b and c) show peaks corresponding to additional "possible" crashes, these are wrong in sign (i.e. meaningless and due to large ratio scales, indicating that *neither monthly nor bi-monthly data*, (rebuilt from d_2 and d_3 respectively), reflect information on crises and events. This suggests that mature markets take *less than a month* to recover from crashes

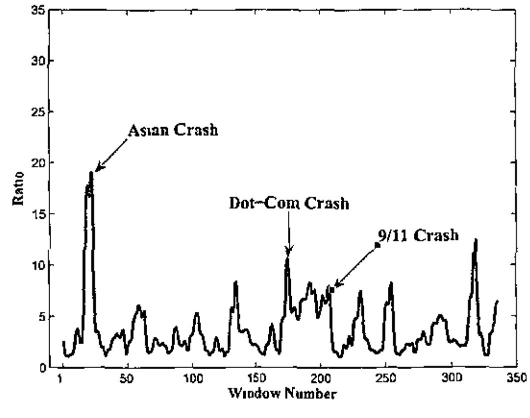
To sum up, we would say that results appear to show that mature and emerging markets exhibit *anti-persistent* and *persistent* behaviour respectively, indicating that mature markets take action more quickly than emerging markets to recover from crashes. In other words, the recovery time from crisis for developed markets (up to two weeks) is shorter than that for developing ones (up to two months).



(a) Rebuilt from first wavelet crystal (d_1) ($\equiv$ fortnightly)

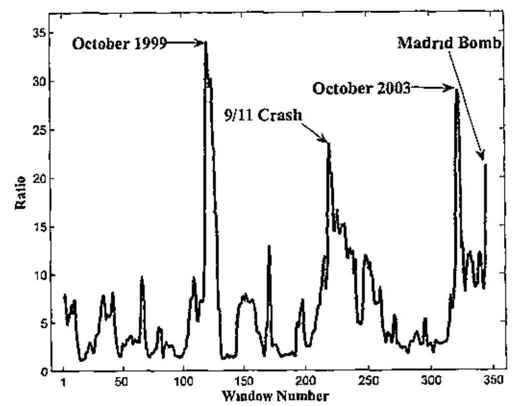


(b) Rebuilt from second wavelet crystal (d_2) ($\equiv$ monthly)

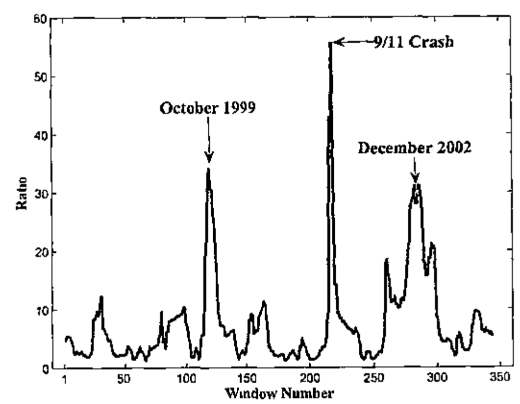


(c) Rebuilt from third wavelet crystal (d_3) ($\equiv$ bi-monthly).

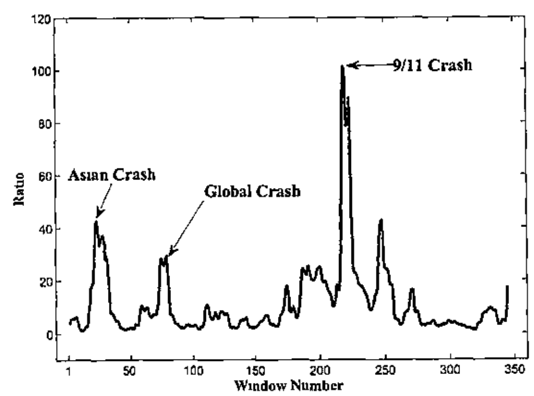
Figure 6.9. Emerging Markets: Changes in ratio of *Dominant* (λ_1) to *Subdominant* (λ_2) eigenvalue of covariance matrices for return series.



(a) Rebuilt from first wavelet crystal (d_1) ($\equiv$ fortnightly)



(b) Rebuilt from second wavelet crystal (d_2) ($\equiv$ monthly)



(c) Rebuilt from third wavelet crystal (d_3) ($\equiv$ bi-monthly).

Figure 6.10: Mature Markets: Changes in ratio of *Dominant* (λ_1) to *Subdominant* (λ_2) eigenvalue of covariance matrices for return series.

6.1.3 Section Summary

Our aims here were threefold; firstly, to study the distribution of the eigenvalues (λ_i's) of covariance matrices of overlapping windows for emerging and mature market groups at known crisis points. These included *the Asian Crisis in July 1997, Global Crisis in October 1998, Dot-Com Crash in March 2000* and *9/11 Crash in September 2001.* Secondly, to study the distribution of the ratios of the first three Largest eigenvalues (λ_1/λ_2, λ_1/λ_3 and λ_2/λ_3) of these covariance matrices for the original return series for emerging and mature markets by plotting these ratios against "windows" number, (where a window looks at consecutive values). Thirdly, we aimed to study the distribution of the ratio λ_1/λ_2 for return series, reconstructed from wavelet components (d_1, d_2 and d_3), in order to isolate signal components.

In summary, we may say that:

1 The Asian Crisis in 1997 disproportionately affected emerging markets compared to the major ones while the Dot-Com Crash influenced major markets but not emerging ones The Global Crisis in 1998 and the September 11^{th} Crash hit both emerging and mature markets equally.

2. Differences exist between emerging and mature markets in dealing with crashes (especially unexpected ones). For major markets, the ratio λ_1/λ_2 is high at three points representing *the* 12^{th} *anniversary of the October 19 stock market crash, 1999*, the *9/11 crash, 2001* and the *Madrid Bomb, March, 2004* respectively. However, for emerging markets, the ratio λ_1/λ_2 is high at one point only, representing the *9/11 crash, 2001.*

3. Using the **discrete wavelet transform** to study the behaviour of stock markets provides a clearer view on the structure and dynamics of the data sets and gives us a good measurement of the recovery time and direction of movements in these markets. It indicates that emerging markets can take up to two months to recover from crashes while mature ones take less than a month

to do so.

4. Both λ_1 and λ_2 are needed to describe the behaviour of emerging markets while λ_1 is adequate alone to describe the behaviour of mature markets. Mature markets move together in the same direction in response to crises. This suggests both that cooperative behaviour applies between such markets, while they also demonstrate reduced entropy[6] (or internal disorder) compared to emerging ones. In other words, shareholders in these markets appear to have similar patterns of selling and buying shares, acting in a fairly coherent fashion. Conversely, emerging markets show more internal variation and thus demonstrate differing views of shareholders, who take different directions in dealing with crashes and unexpected events.

6.2 Stock Market Degree of Development

The results, from the previous section, indicate that stock markets behave differently, especially, at the time of crashes and other events, so that knowledge of market type is important in making the right investment decision. In Section 3 5.3, we introduced and designed a new *wavelet-based* algorithm to classify stock markets based on their degree of development and we now look at applying this.

6.2.1 Data Description

For this investigation, we consider the daily returns of fourteen worldwide market indices, listed in Table (6.4) with their classification by the World Bank.

1. **Emerging.** Argentina (MerVal), Czech Republic (PX50), Ireland (ISEQ Overall), Mexico (IPC), Portugal (PSI20), Russia (Moscow Times) and Singapore (Straits Times).

[6]see footnote 1, page 35

2. **Mature**: Australia (All Ordinaries), Canada (S&P/TSX Composite), Germany (DAX), Hong Kong (Hang Sang), Japan (Nikkei225), the UK (FTSE100) and the US (DJI).

6.2.2 Results

The algorithm of Section 3.5.3 is designed to measure the degree of development of the international markets, (based on their daily return series for the period, January 1993 to December 2004). In brief, the procedure aims to compare behaviour of fractional Gaussian noise[7] (fGn) generated values with that of the return series of stock market indices It consists of the following steps· **(i)** The DWT with symmlet 8 wavelet (s8) was applied to these return series to obtain the energy percentages described by each wavelet component. **(ii)** One hundred series of fGn with $H = 0.3$ were generated and DWT was applied to each of these generated series to compute the energy percentages. These were then averaged. **(iii)** The process was repeated for different values of H=0.4, 0.5, 0.6 and 0 7. The percentages for all the return series and for simulated fGn series with different values of H are given in Table (6.5). **(iv)** The base-two logarithm[8] of the energy percentages ($log_2(energy\%)$), explained by the detailed components ($d_1 - d_6$) [Table (6.5)], were calculated. In order to elaborate on the features illustrated in **Figure B.1 in the Appendix**, we choose three different markets, namely Argentinean, Australian and UK, as examples, illustrated in Figure (6.11) (a, b and c), and aim to explain their behaviour

[7]FGn, corresponding to different values of H (0.3, 0 4, 0 5, 0 6 and 0.7), was simulated using the S-plus function $Simulate.FARIMA(0, d, 0)$, where $d = H - \frac{1}{2}$

[8]The base-two logarithm was calculated because there are 2^j coefficients in the j^{th} wavelet component, where $j = 1, 2, ..., \ell$ and ℓ is scale level

Table 6.4: Classical and New Classification of International Stock Markets.

Market	Index Name	Time Period	No. Observation	Classical Classification [a]	Our Classification
Argentina	MerVal	1997-2004	1971	Emerging	Emerging
Australia	All Ordinaries	1993-2004	3042	Mature	**Intermediate**
Canada	S&P/TSX Composite	1993-2004	3020	Mature	**Intermediate**
Czech Republic	PX50	1998-2004	1739	Emerging	Emerging
Germany	DAX	1993-2004	3027	Mature	Mature
Hong Kong	Hang Sang	1993-2004	2968	Mature	**Intermediate**
Ireland	ISEQ Overall	1993-2004	3012	Emerging	Emerging
Japan	Nikkei 225	1993-2004	2955	Mature	Mature
Mexico	IPC	1993-2004	3000	Emerging	Emerging
Portugal	PSI20	1993-2004	2977	Emerging	Emerging
Russia	Moscow Time	1993-2004	2460	Emerging	Emerging
Singapore	Straits Times	1993-2004	3016	Emerging	**Intermediate**
UK	FTSE100	1993-2004	3031	Mature	Mature
US	DJI	1993-2004	3024	Mature	Mature

[a] This is the World Bank classification, see footnote number 9, page 11

Table 6.5: Percentages of energy explained by wavelet crystals for the daily returns of index series.

W.Crystals→ **Market↓**	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Argentina	0.462	0.248	0.146	0.057	0.046	0.023	0.019
Australia	0.499	0.230	0.149	0.059	0.032	0.016	0.014
Canada	0.437	0.282	0.146	0.057	0.042	0.011	0.024
Czech Republic	0.440	0.259	0.141	0.073	0.038	0.025	0.023
Germany	0.534	0.221	0.129	0.056	0.034	0.009	0.018
Hong Kong	0.503	0.222	0.147	0.063	0.031	0.017	0.018
Ireland	0.423	0.271	0.146	0.074	0.033	0.025	0.029
Japan	0.530	0.246	0.120	0.052	0.027	0.012	0.013
Mexico	0.445	0.263	0.140	0.075	0.034	0.022	0.021
Portugal	0.413	0.278	0.131	0.081	0.036	0.023	0.038
Russia	0.438	0.261	0.139	0.068	0.036	0.031	0.027
Singapore	0.401	0.268	0.179	0.075	0.036	0.015	0.027
The UK	0.499	0.273	0.124	0.057	0.027	0.009	0.011
The US	0.500	0.264	0.129	0.052	0.032	0.010	0.013
fGn with H=0.3	**0.593**	**0.251**	**0.096**	**0.036**	**0.014**	**0.005**	**0.003**
fGn with H=0.4	**0.552**	**0.252**	**0.111**	**0.049**	**0.021**	**0.009**	**0.007**
fGn with H=0.5	**0.500**	**0.250**	**0.125**	**0.063**	**0.031**	**0.016**	**0.016**
fGn with H=0.6	**0.440**	**0.242**	**0.134**	**0.080**	**0.044**	**0.025**	**0.035**
fGn with H=0.7	**0.365**	**0.220**	**0.143**	**0.092**	**0.063**	**0.041**	**0.076**

Where, the averages of the energy percentages over 100 for fGn series are given in bold.

Firstly, we need to understand the following key points:

- *Developing* and *Developed* markets demonstrate *persistent* and *anti-persistent* behaviour respectively (with correspondingly, $H > 0.5$ and $H < 0.5$). The expectation, therefore, is that stock market should move from *persistence* to *anti-persistence* side as it develops

- Our new approach allows for variation in H, (designated Chapter 3, Section 3.2.1 as $H(\tau, \theta)$), but a market will fall on one side or another of the well-defined threshold of $H = \frac{1}{2}$ (Gaussian noise) when it is exhibiting clear persistent or anti-persistent behaviour. (Note that these are the values of $H(\tau, \theta)$, see Section 3.3, with τ =length of series (number of observations) and θ =number of trading days= 1).

- The behaviour of the linear fit of *logarithms* of stock market returns is compared with that of the generated fractional Gaussian noise (fGn) series for different values of H. The straight line fit for the fGn log series versus the wavelet components indicate that the d_1 doublet explains the largest percentage of energy, d_2 the next largest and so on.

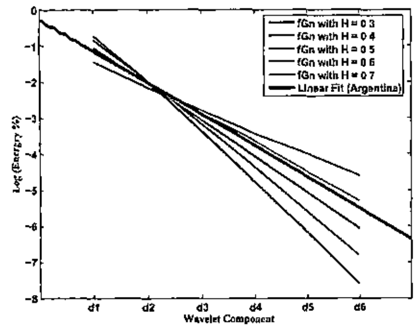
Comparing with the empirical data, we can see from Figure (6.11) (a, b and c) (and **Figure B.1 in the Appendix**) that:

1. The linear fit of the Argentinean market behaves similarly to fGn with $H(\tau, 1) = 0.6$ (*persistent*), indicating that it is essentially an emerging market (*Similarly*, this can be shown for the Czech, Irish, Mexican, Portuguese, Russian markets).

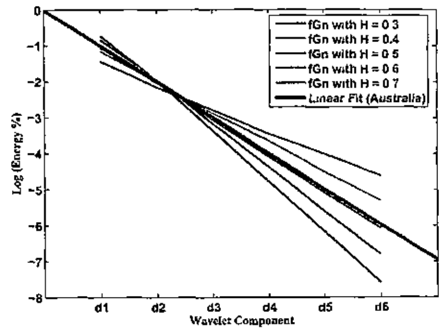
2. The Australian market behaves like to fGn with $H \simeq 0.5$ (or Gaussian noise), meaning that this market has graduated from the emerging group, but is not yet in the mature one (*Similarly*, Canada, Hong Kong and Singapore).

3. However, the UK market fit is close to that for fGn with $H < 0.5$ (*anti-persistent*), indicating that it is a mature market (and the same can be shown for the German, Japan and US markets).

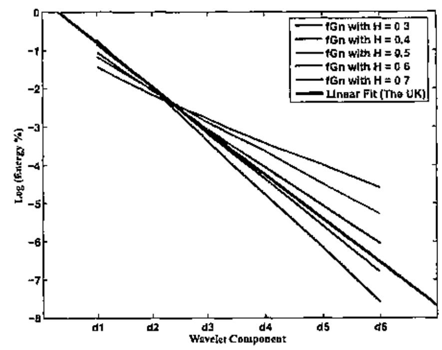
In agreement with other studies, it can be concluded that emerging and mature stock markets behave in a *persistent* (*or long memory*) and *anti-persistent* (*or intermediate memory*) manner respectively. However, our classifier indicated that there are other stock markets which lie outside these two groups and show *short memory* (*or independent*) behaviour. On this basis, we suggest that stock markets should be classified into three different classes or categories, reflecting common characteristic and implying that stock markets bi-classification is inadequate [Table (6.4)].



(a) **Argentinean Market.**



(b) **Australian Market.**



(c) **UK Market.**

Figure 6.11: Logarithm to base two of the energy percentages ($log_2(energy\%)$).

6.2.3 Section Summary

A novel wavelet-based algorithm has been applied to the return series of fourteen stock market indices and the results show that stock market characterisation behaviour (*persistent*, *anti-persistent* or *short-term*) may be determined according to the Hurst exponent associated with its degree of development. This degree of development may be rooted in a number of factors, e.g. market size, liquidity, volatility, global integration, etc. The approach of using fGn and DWT, in particular, allows us to explore the overall behaviour of these markets. Summarising the findings of this preliminary study, it appears therefore that wavelet-based approaches, in regard to stock market evolution/re-classification, also offers considerable potential. The implications of our method and the analysis performed are that stock markets can be grouped into **three categories** designated here as *emerging*, *intermediate (or young mature)* and *mature (or fully mature)* markets. The properties associated with this new classification need to be examined in further detail, but it does seem clear that class 2 is a particularly interesting one due to the possibility of being a new "attractive" stock market type. These markets seems to behave as Gaussian noise (or a pure random walk) indicating that they are less risky on average than emerging markets but also provide more returns than mature ones. Finally, in relation to grouping of the *stocks* themselves, in term of the market composition (Coelho et al. (2006)[9], for example, found that the new clustering, introduced in January 2006, of stocks from the FTSE100 index is more rational than the old one because the stocks from the same group (or sector) are now more connected than those from the old one). This requirement to reclassify is due to rapid changes in individual stocks' behaviour.

6.3 Stock Market Behaviour for Different Time Intervals with Different Volatility Levels

Our previous results show that stock markets show different memory types (persistent, anti-persistent or short-term dependent) for different time intervals and scale levels (number of trading days) They deal differently with major events, with some responding faster than others to these events and taking a shorter time to recover Previous results (Chapter 5) also show that there strong evidences exists for co-movements between stock markets, where these are influenced by both local and global information or news (good or bad), causing different levels of volatility (or variable risk). Our goal in this section is thus to study the *nature of persistence* in different stock market types over different time periods with variable volatility levels

[9] in press

and also to examine *similarity* (or *dissimilarity*) between these markets. In order to do this, we apply our new wavelet-based algorithm, for disjointed[10] windows of size two years from 1993 to 2004, (described in Section 3 5.3), to three different market types, namely Irish (ISEQ Overall), Hong Kong (Hang Seng) and UK (FTSE100) market as representatives of the *emerging*, *intermediate* and *mature* groups respectively. The results of this study are plotted in Figures (6.12, 6.13 and 6.14).

6.3.1 Results

From Figure (6.12), it can be seen that the Irish market (ISEQ Overall) shows persistent behaviour (with $H \simeq 0.6$) in the period between 1993 and 1994, while in the next two years (1995-1996), it develops gradually exhibiting behaviour similar to fGn with $H = 0.5$ However, between 1997 and 1998, the market exhibits longer memory, with $H > 0.6$, (most probably due to the impact of Asian Crash in 1997 following by Global Crash in 1998) After decreasing the influence of these crashes, this market behaves as an intermediate market ($H = 0.5 \Rightarrow$ short-term memory) between 1999 and 2000. In the period 2001-2002, the Irish market exhibits long-range dependency, with $H = 0.6$, due to the influence from US market during the 9/11 crash in 2001. It also shows persistent behaviour ($0.5 < H < 0.6$) between 2003 and 2004 but to a lesser extent, meaning that this market continues to evolve but has long memory features.

The results for Hong Kong Market (Hang Seng) are given in Figure (6.13) which shows that this market shifts from being intermediate ($H \simeq 0.5$) from 1993 to 1994 to mature ($H < 0.5$) between 1995 and 1996. From 1997 to 1998, the market would be expected to display persistent behaviour to a number of crashes that happened in the region and in the market itself. However, it shows *anti-persistence* as we see from Figure (6.13) (c). In fact, what we see is an artificial indication caused by some investors, particularly Hedge fund mangers, who took short (selling) positions to

[10]This means that windows are separated at the joints.

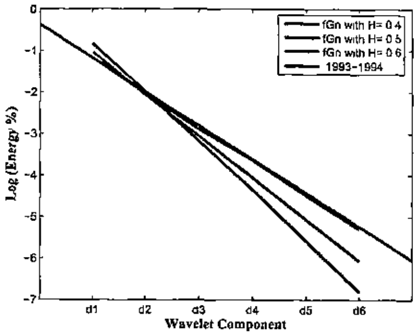
attempt to drive the currency down, forcing the Hong Kong government to increase interest rates, [see Sornette (2002)]. In the periods 1999-2000 and 2001-2002, the Hong Kong market then displayed persistent (or long memory) behaviour, with $0.5 < H < 0.6$, due to influences from US market during the Dot-Com crisis in 2000 and the 9/11 crash in 2001. Between 2003 and 2004, it again behaved as an intermediate market ($H = 0.5$) and this is in general its overall behavior, (as we have seen in Figure (6.11) (g)).

Figure (6.14) gives the results for UK market and shows that the UK market exhibits consistent anti-persistent behaviour with different degree of long memory in all periods, except between 1997 and 1998 where it shows short-term memory ($H = 0.5$), (due to the effects of the Asian Crash in 1997 followed by the Global Crash in 1998 as we have seen in Section 5.3.1, there is influence from Asia on Europe).

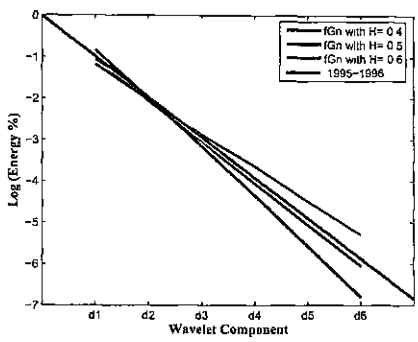
6.3.2 Section Summary

We applied our new wavelet-based algorithm to three different market types, namely, Irish (emerging), Hong Kong (intermediate) and UK (mature), to study the memory type exhibited by a stock market over time, broken down into crude two years intervals[11] with variable volatility levels. The results indicate that stock markets show persistent, anti-persistent or independent behaviour depending on market type and absence or presence of crashes in the period analysed This approach clearly gives another evidence of multifractalty that is exhibited by stock market returns, in agreement with our earlier findings, Section 4.2.

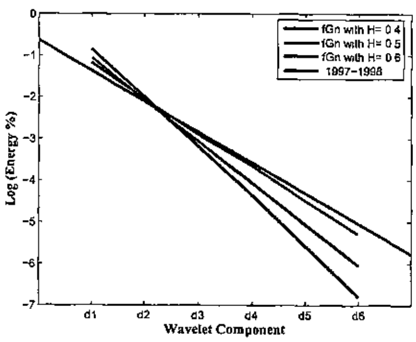
[11]The reason for breaking down the series into two year intervals is that we want to get as short a period as possible; two years is short but still reasonable in term of number of series points



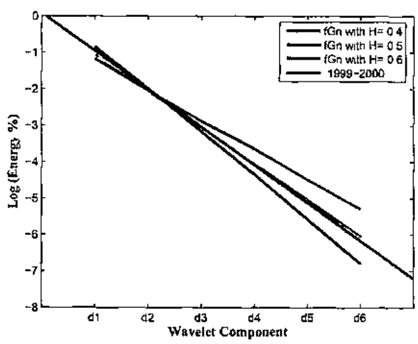
(a) **1993-1994.**



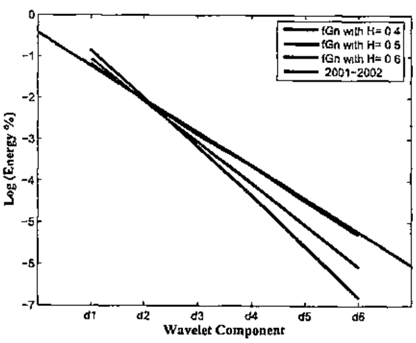
(b) **1995-1996.**



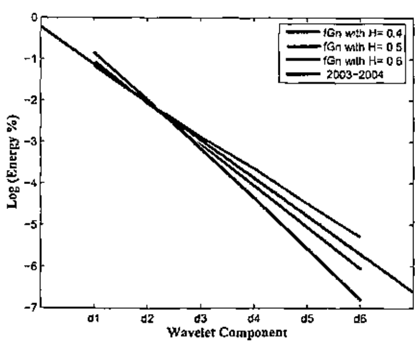
(c) **1997-1998.**



(d) **1999-2000.**

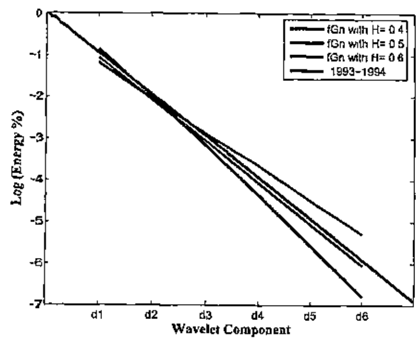


(e) **2001-2002.**

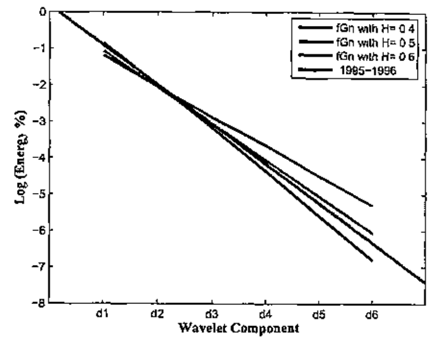


(f) **2003-2004.**

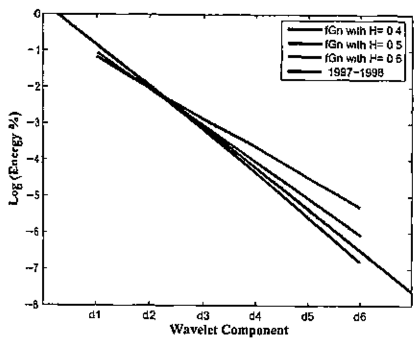
Figure 6.12: Behaviour of Irish market (ISEQ Overall index) over different (two year) time periods with different volatility levels.



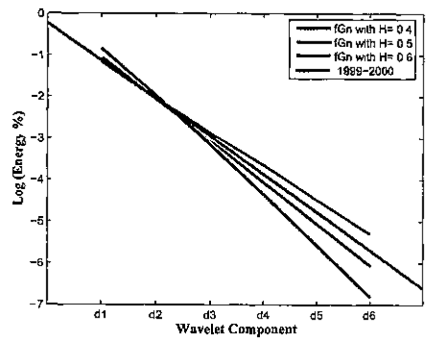
(a) **1993-1994.**



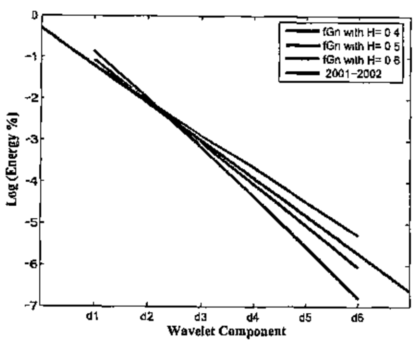
(b) **1995-1996.**



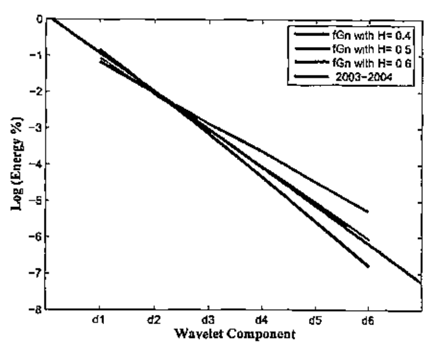
(c) **1997-1998.**



(d) **1999-2000.**

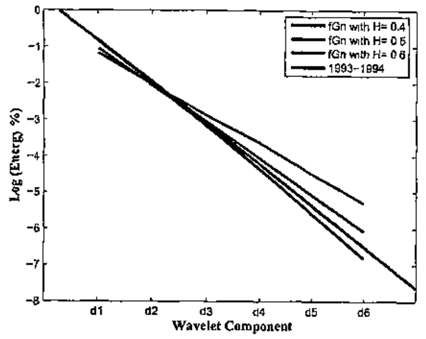


(e) **2001-2002.**

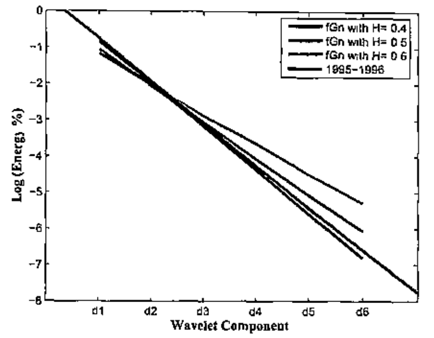


(f) **2003-2004.**

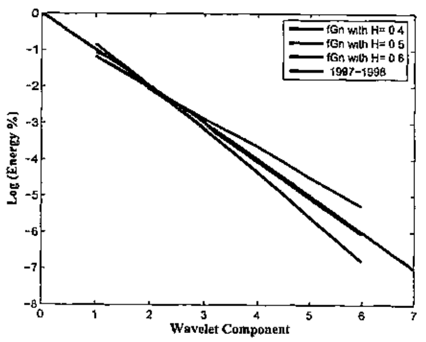
Figure 6.13. Behaviour of Hong Kong market (Hang Sang index) over different (two year) time periods with different volatility levels



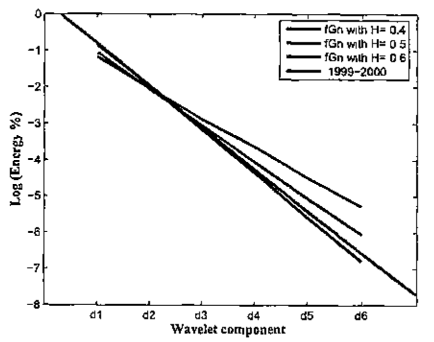
(a) **1993-1994.**



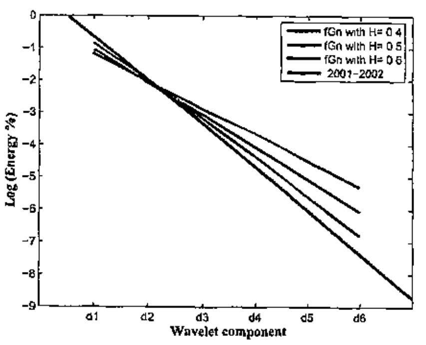
(b) **1995-1996.**



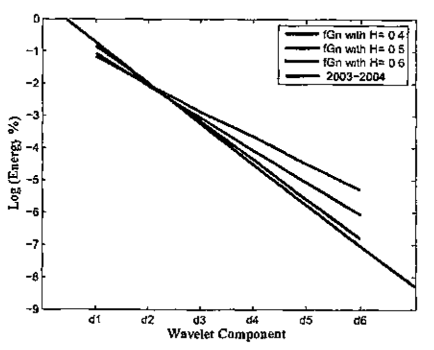
(c) **1997-1998.**



(d) **1999-2000.**



(e) **2001-2002.**



(f) **2003-2004.**

Figure 6.14: Behaviour of UK market (FTSE100 index) over different (two year) time periods with different volatility levels

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Goals of this Thesis

The main goals of this thesis have been to investigate (i) internal effects of markets, (the nature of persistence in different time frames), (ii) external influences (global co-movement) between international stock markets, and (iii) reactions of these markets to major events We also examined the conventional classification of Emerging versus Mature in the light of modern market influences and global membership. This, in particular, in the light of advancing technology and rapid communications.

Techniques used to date have predominantly focused on novel development to exploit the strength of the discrete wavelet transform in terms of frequency (or energy) decomposition and a novel extension of Detrended Fluctuation Analysis (DFA) Frequency decomposition is important because wavelet patterns for long-term and short-term signal repeats can be observed in high amplitude peaks and importantly, can be anticipated for some classes of major events (e.g crashes).

A market crash, which is a sudden dramatic loss of value of shares, could occur in any market causing the evaporation of trillions of dollars and bankruptcy for some companies. This is especially true for unexpected crashes, e.g. September 11^{th} in 2001. Crashes, as we have seen, are driven not just by panic but also by underlying stock market factors, such as autocorrelation, co-movement and individual market characteristics, such as, degree of development. The effects of such shocks now spread rapidly and globally, due to communication advances.

This work attempts to provide new ways of analysing and visualising information on characteristics of international stock markets through investigations which include:

1. An initial long-term memory study: the Irish market is taken as an example, and results from the DWT are compared with other tests, (namely Rescaled Range (R/S), its modified form, and GPH)

2. Exploration of the multifractal property of stock market returns by introducing and applying a *new time-scale extension of Detrended Fluctuation Analysis* (TSDFA) to compute the Hurst exponent (H) in different time periods and scale levels (trading days).

3. Analysis of co-movements among different international stock markets in Asia, Europe and Americas by application of the *wavelet-based* approach suggested by Lee (2002)

4. Examining how stock markets (emerging *vs.* mature) deal with different crashes and events and how long they take to recover. This by investigating the behaviour of the eigenvalues ($\lambda_i's$) of the covariance matrices of return series for emerging and mature markets at times of crisis Also by studying the distribution of the ratio of the first three largest (λ_1/λ_2, λ_1/λ_3 and λ_2/λ_3) of these matrices of the original return series and studying the (noise-free) behaviour of the ratio (λ_1/λ_2) for return series that have been rebuilt from wavelet components for emerging and mature markets separately.

5. Searching for a new meaningful classification of stock markets based on their degree of development, given the inadequacy of the existing bi-classification, (World Bank).

7.2 Summary and Conclusions

We summarise a number of useful findings related to this work as follows:

1. The discrete wavelet transform (DWT) has the additional advantage of providing an in-depth view of the data sets and this gives us a real indication of structure in long memory effects hence enabling the formulation of a clear picture of the movements in the series.

2. The DWT is a strong method for investigation of long memory because it is able to distinguish clearly between memory types (short-term, long-term memory or mixed) and it is not affected by the length of the time series.

3. The DWT alone or with other methods enables examination of the response of financial time series at different resolutions, where these series are well-known to be non-stationary, non-normal and noisy and intractable to standard methods

4. Overall, there is a strong evidence of long-term memory in volatility measures (Absolute and Squared) but not in returns themselves. Also absolute returns exhibit longer memory than squared returns (in agrement with Ding et al. (1993), Lee et al. (2000), Elekdag (2001) and Sibbertsen (2002)) However, the results of applying TSDFA show that return series, exhibits different memory types (short- and long-term), depending on different time periods and scale levels (indicating *multifractal behaviour* of these series)

5. International co-movements of stock markets and corresponding volatility (or risk) have increased since the middle of 20^{th} century, with strong evidences for inter-continental as well as intra-continental price co-movements. There is also evidence of *clockwise* transmissions between worldwide markets.

6. The *second largest* eigenvalue (λ_2) of the covariance matrix (in addition to the *largest* eigenvalue (λ_1)) holds information on the behaviour of stock market

returns, *especially with respect to emerging markets in times of crisis.* Emerging and mature markets deal differently with crashes and events (especially unexpected ones), e.g. emerging markets may take up to two months longer to recover from a crisis while major markets take less than a month to do so [This is in agreement with the findings of Patel and Sarker (1998) and Fuss (2002)].

7. The *new wavelet-based* classifier of stock markets, which we have developed, offers considerable potential and indicates that stock markets can be grouped into **three categories** designated here as *emerging*, *intermediate* (*or young mature*) and *mature* (*or very mature*), supporting the contention that bi-classification of stock markets is no longer sufficient.

8 The efficient market hypothesis[1] (EMH) Peters (1996) has attracted a lot of attention and, to some extent, results presented here can be said to raise questions about its universality In effect, what we have shown is that disparate behaviour can exist in different market types, so that the EMH is essentially a classical or "mean-field" limiting behaviour.

7.3 Future Work

Research on stock market volatility is central for regulation of financial organisations and risk management. There would be huge scope for a model describing the structure of dependence in the time-varying conditional variance of available observations across several series. This would apply to complex temporal systems across many fields, such as finance, traffic networks, biomedical, etc. Researchers

[1]It asserts that financial markets are "efficient", or that prices on traded assets. e.g. stocks, bonds, or property, already reflect all known information and therefore are unbiased in the sense that they reflect the collective beliefs of all investors about future prospects The EMH implies that it is not possible to consistently outperform the market, appropriately adjusted for risk, by using any information that the market already knows, except through luck or obtaining and trading on inside information.

have applied various models to analyse time series and make predictions for future behaviour. However, these models are predominantly based on the primary assumption of stationarity and require transformation of non-stationary and noisy data, (a consequence of combined local and global effects).

As one of most promising applications of wavelet analysis is in the field of prediction, (i.e to forecast an unknown signal from noisy data), the importance of wavelet-based approaches for time series filtering and forecasting has increased in recent years Several authors report that wavelet transforms can be used effectively for noise-filtering and improving the prediction quality in financial time series [e g. Zheng et al. (1999) and Renaud et al. (2005)]. However, different wavelet transforms and models have been used for prediction and there is no single favourable model as yet identified.

Therefore, further investigation is required in order to find these improved models and to develop more efficient techniques to clean or de-noise financial time series with a view to significant improvements in forecasting capability.

Appendix A

Table for Section 5.3, Chapter 5.

The key notations in Table A.1 are given as follows:

- P-values of t-tests are given in parentheses.
- M^{IRL}, M^{UK}, M^{P}, M^{US}, M^{BR}, M^{JP} and M^{HK} are indicators of Irish, UK, Portuguese, US, Brazilian, Japanese and the Hong Kong market indices respectively.
- Return is an indicator of the raw daily returns series.
- Return.D1 is an indicator of the returns series reconstructed by using the first wavelet crystal.
- Return.D1.2 is an indicator of the returns series reconstructed by using the first and the second wavelet crystals together.

Table A.1: Regression Analyses between the daily returns of each pair of the seven stock market indices

1· Ireland vs UK

Regression→	$M_t{}^{IRL}$ on $M_t{}^{UK}$			$M_t{}^{UK}$ on $M_t{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3.80E-04 (0.029)	0 578 **(0.000)**	**0.323**	-1.74E-04 (0 310)	0 559 **(0.000)**	**0.323**
Return.D1	1 13E-04 (0.391)	0 467 **(0.000)**	**0.222**	-1.41E-04 (0.289)	0.477 **(0.000)**	**0.222**
Return.D1 2	1.49E-05 (0 951)	0.495 **(0.000)**	**0.251**	-4.32E-05 (0.864)	0.508 **(0.000)**	**0.251**

2· Ireland vs. Portugal

Regression→	$M_t{}^{IRL}$ on $M_t{}^{P}$			$M_t{}^{P}$ on $M_t{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3 06E-04 (0.119)	0 343 **(0.000)**	**0.135**	1.50E-04 (0.477)	0.394 **(0.000)**	**0.135**
Return.D1	2.64E-05 (0 849)	0.359 **(0.000)**	**0.145**	7.06E-05 (0 631)	0.405 **(0.000)**	**0.145**
Return.D1.2	1.97E-05 (0.940)	0.385 **(0.000)**	**0.165**	-6 98E-05 (0.801)	0 420 **(0.000)**	**0.165**

3· Ireland vs US

Regression→	$M_t{}^{IRL}$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3 11E-04 (0.114)	0.328 **(0.000)**	**0.129**	1.99E-04 (0.378)	0.255 **(0.000)**	**0.053**
Return.D1	7.06E-05 0 632	0.154 **(0.000)**	**0.031**	2 12E-05 (0.900)	7 79E-04 **(0.001)**	**0.004**
Return.D1 2	-2.37E-05 (0.933)	0 217 **(0.000)**	**0.024**	-9.02E-05 (0.785)	7 75E-02 **(0.001)**	**0.004**

4 Ireland vs. Brazil

Regression→	$M_t{}^{IRL}$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	2 79E-04 (0.184)	5 47E-02 **(0.000)**	**0.023**	2.31E-03 (0.000)	0.361 **(0.000)**	**0.016**
Return.D1	6.06E-05 (0.685)	1.67E-02 **(0.036)**	**0.001**	-3.70E-05 (0 928)	0.254 **(0.000)**	**0.008**
Return.D1 2	-8.65E-06 (0.976)	2.69E-02 **(0.023)**	**0.002**	-1 98E-04 (0.692)	0.122 **(0.001)**	**0.004**

5· Ireland vs. Japan

Regression→	$M_t{}^{IRL}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	4.77E-04 (0.019)	0 181 **(0.000)**	**0.068**	-4.06E-04 (0.183)	0.127 **(0.000)**	**0.007**
Return.D1	5.16E-05 (0 723)	0.147 **(0.000)**	**0.052**	6.76E-05 (0.771)	-8.18E-02 **(0.010)**	**0.002**
Return D1.2	-1.18E-05 (0.996)	0 147 **(0.000)**	**0.054**	2 61E-05 (0 954)	-0.115 **(0.024)**	**0.002**

6. Ireland vs. Hong Kong

Regression→	$M_t{}^{IRL}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{IRL}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	4.19E-04 (0 036)	0 183 **(0.000)**	**0.104**	-4.91E-05 (0.895)	3 86E-02 **(0.292)**	**0.000**
Return D1	4.88E-05 (0.731)	0.170 **(0.000)**	**0.097**	9 54E-05 (0.724)	-0 302 **(0.000)**	**0.028**
Return D1.2	-3.13E-06 (0 991)	0.182 **(0.000)**	**0.117**	-1 48E-05 (0 978)	-0 396 **(0.000)**	**0.018**

7. UK vs Portugal

Regression→	$M_t{}^{UK}$ on $M_t{}^{P}$			$M_t{}^{P}$ on $M_t{}^{UK}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-7.95E-05 (0 663)	0.438 **(0.000)**	**0.228**	2 83E-04 (0.155)	0 521 **(0.000)**	**0.228**
Return D1	-1.55E-04 (0.243)	0.451 **(0.000)**	**0.224**	1.51E-04 (0.280)	0.498 **(0.000)**	**0.224**
Return.D1 2	-1.22E-05 (0.961)	0.481 **(0.000)**	**0.252**	-4.86E-05 (0.853)	0.524 **(0.000)**	**0.252**

8· UK vs. US

Regression→	$M_t{}^{UK}$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{UK}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-2 03E-05 (0.919)	0.251 **(0.000)**	**0.078**	2.78E-04 (0.188)	0.473 **(0.000)**	**0.178**
Return D1	-1.12E-04 (0 468)	1 13E-02 **(0.537)**	**0.000**	5.54E-05 (0.736)	0.262 **(0.000)**	**0.054**
Return D1 2	-4.73E-05 (0.871)	-3.46E-03 **(0.906)**	**0.000**	-7.70E-05 (0.810)	0.292 **(0.000)**	**0.065**

9 UK vs. Brazil

Regression→	M_t^{UK} on M_{t-1}^{BR}			M_t^{BR} on M_t^{UK}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-3.38E-05 (0.870)	3.71E-02 **(0.000)**	**0.011**	2.42E-03 (0.000)	0 659 **(0.000)**	**0.054**
Return D1	-1.12E-04 (0.457)	-1 53E-02 **(0.057)**	**0.001**	3 49E-05 (0 931)	0.503 **(0.000)**	**0.034**
Return.D1.2	-4 76E-05 (0 870)	-1.38E-02 **(0.250)**	**0.000**	-1.86E-04 (0.707)	0.265 **(0.000)**	**0.023**

10 UK vs. Japan

Regression→	M_t^{UK} on M_t^{JP}			M_t^{JP} on M_{t-1}^{UK}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	1.21E-04 (0.548)	0.178 **(0.000)**	**0.068**	-3 72E-04 (0 214)	0 292 **(0.000)**	**0.039**
Return.D1	-1 19E-04 (0 423)	0 113 **(0.000)**	**0.030**	5.73E-05 (0 805)	9.20E-02 **(0.003)**	**0.003**
Return.D1.2	-5 03E-05 (0.860)	0.124 **(0.000)**	**0.037**	2.42E-05 (0 957)	0.119 **(0.018)**	**0.002**

11. UK vs Hong Kong

Regression→	M_t^{UK} on M_t^{HK}			M_t^{HK} on M_{t-1}^{UK}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	6.37E-05 (0.745)	0.187 **(0.000)**	**0.112**	-5 60E-05 (0.879)	0 349 **(0.000)**	**0.038**
Return.D1	-1 22E-04 (0 407)	0.133 **(0.000)**	**0.058**	7.07E-05 (0.796)	-4.37E-02 **(0.239)**	**0.000**
Return.D1.2	-4 39E-05 (0.877)	0.122 **(0.000)**	**0.050**	-3 17E-05 (0 953)	-6 80E-02 **(0.257)**	**0.000**

12: Portugal vs. US

Regression→	M_t^{P} on M_{t-1}^{US}			M_t^{US} on M_t^{P}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	2.59E-04 (0.246)	0.174 **(0.000)**	**0.031**	2.21E-04 (0 323)	0.265 **(0.000)**	**0.067**
Return.D1	9.76E-05 (0.539)	3 69E-02 **(0.055)**	**0.001**	9.36E-06 (0.955)	0.174 **(0.000)**	**0.026**
Return.D1.2	-7 64E-05 (0.801)	4.11E-02 **(0.178)**	**0.000**	-7.62E-05 (0.815)	0.199 **(0.000)**	**0.033**

13: Portugal vs. Brazıl

Regression→	$M_t{}^P$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	1.94E-04 (0.391)	4 94E-02 **(0.000)**	**0.015**	2 30E-05 (0 000)	0.489 **(0.000)**	**0.035**
Return D1	9.52E-05 (0.549)	9.93E-03 **(0.240)**	**0.000**	-5.20E-05 (0.899)	0.319 **(0.000)**	**0.015**
Return.D1.2	-7.35E-05 (0.809)	6 73E-03 **(0.590)**	**0.000**	-1 85E-04 (0.770)	0 188 **(0.000)**	**0.013**

14: Portugal vs. Japan

Regression→	$M_t{}^P$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3.60E-04 (0 1060	0.134 **(0.000)**	**0.032**	-4 00E-04 (0 188)	0 150 **(0.000)**	**0.012**
Return.D1	8 83E-05 (0.573)	0 113 **(0.000)**	**0.027**	5.95E-05 (0.798)	4.06E-02 **(0.203)**	**0.000**
Return D1.2	-7 62E-05 (0 798)	0.125 **(0.000)**	**0.034**	1 32E-05 (0 977)	6 50E-02 **(0.174)**	**0.000**

15 Portugal vs. Hong Kong

Regression→	$M_t{}^P$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^P$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3.19E-04 (0.144)	0 168 **(0.000)**	**0.076**	-7.77E-05 (0.834)	0.143 **(0.000)**	**0.007**
Return D1	8.53E-05 (0.560)	0 143 **(0.000)**	**0.060**	7 29E-05 (0.789)	-0.141 **(0.000)**	**0.006**
Return.D1 2	-6.90E-05 (0.814)	0.150 **(0.000)**	**0.070**	-1.34E-02 (0 980)	-0 130 **(0.023)**	**0.002**

16 US vs. Brazil

Regression→	$M_t{}^{US}$ on $M_t{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^{US}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-1.85E-05 (0.933)	0 132 **(0.000)**	**0.110**	2.20E-03 (0 000)	0 841 **(0.000)**	**0.110**
Return.D1	2.84E-05 (0.861)	0.114 **(0.000)**	**0.076**	-3.91E-05 (0 921)	0 674 **(0.000)**	**0.077**
Return.D1 2	-6.03E-05 (0 852)	0 154 **(0.000)**	**0.054**	-1.67E-04 (0 732)	0 351 **(0.000)**	**0.054**

17· US vs. Japan

Regression→	$M_t{}^{US}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{US}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3.31E-04 (0 152)	7.45E-02 **(0.000)**	**0.009**	-4 77E-04 (0.101)	0 401 **(0.000)**	**0.092**
Return D1	2 92E-05 (0.862)	-5.37E-02 **(0.000)**	**0.005**	8.15E-05 (0 718)	0.322 **(0.000)**	**0.056**
Return D1 2	-8.97E-05 (0 991)	-5.10E-02 **(0.786)**	**0.004**	-8.93E-06 (0.994)	0 441 **(0.000)**	**0.041**

18: US vs. Hong Kong

Regression→	$M_t{}^{US}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{US}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3 07E-04 (0.184)	6.78E-02 **(0.000)**	**0.011**	-2.01E-04 (0 566)	0.541 **(0.000)**	**0.114**
Return.D1	2 97E-05 (0 860)	-5 50E-02 **(0.000)**	**0.007**	9.62E-05 (0.719)	0.401 **(0.000)**	**0.063**
Return.D1 2	-9 25E-05 (0.779)	-5.50E-02 **(0.000)**	**0.008**	-7 39E-05 (0 888)	0 630 **(0.000)**	**0.058**

19: Brazil vs. Japan

Regression→	$M_t{}^{BR}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{BR}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-5.34E-04 (0 079)	7.33E-02 **(0.000)**	**0.019**	2 51E-03 (0.000)	0 154 **(0.000)**	**0.006**
Return.D1	6.06E-05 (0.794)	5.79E-02 **(0.000)**	**0.009**	-2.31E-05 (0.955)	2.49E-02 **(0.498)**	**0.000**
Return.D1 2	2 16E-05 (0.962)	7.22E-02 **(0.000)**	**0.006**	-1.99E-04 (0 691)	4.43E-03 **(0.848)**	**0.000**

20: Brazil vs. Hong Kong

Regression→	$M_t{}^{BR}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{BR}$		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-3 30E-04 (0.396)	0 121 **(0.000)**	**0.036**	2.46E-03 (0.000)	0.142 **(0.000)**	**0.008**
Return D1	6.92E-05 (0 798)	9.99E-02 **(0.000)**	**0.020**	-2.03E-05 (0 961)	-1.85E-02 **(0.554)**	**0.000**
Return D1.2	-3 03E-05 (0.955)	0.126 **(0.000)**	**0.014**	-1.99E-04 (0.691)	-9 94E-03 **(0.609)**	**0.000**

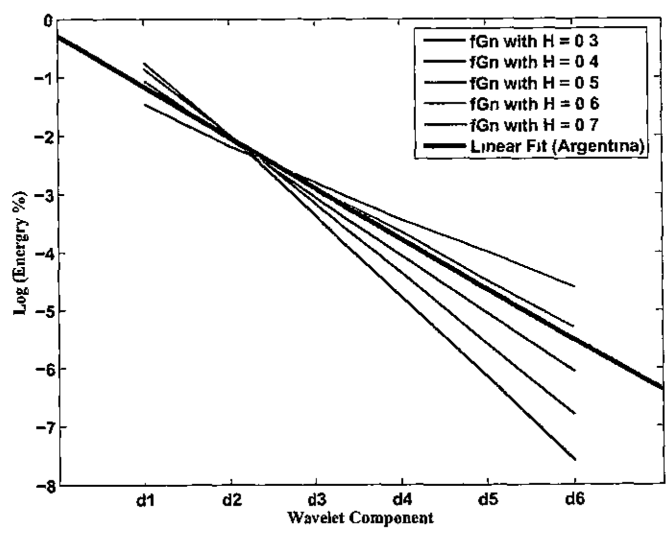
21 Japan vs Hong Kong

Regression→	M_t^{JP} **on** M_t^{HK}			M_t^{HK} **on** M_t^{JP}		
Series↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	-3.44E-04 (0.231)	0.283 **(0.000)**	**0.119**	1.16E-04 (0.741)	0.421 **(0.000)**	**0.119**
Return D1	4.09E-05 (0 852)	0.284 **(0.000)**	**0.111**	4.54E-05 (0.860)	0.393 **(0.000)**	**0.111**
Return D1.2	3.06E-05 (0 942)	0 297 **(0.000)**	**0.126**	-3 94E-05 (0.938)	0.424 **(0.000)**	**0.126**

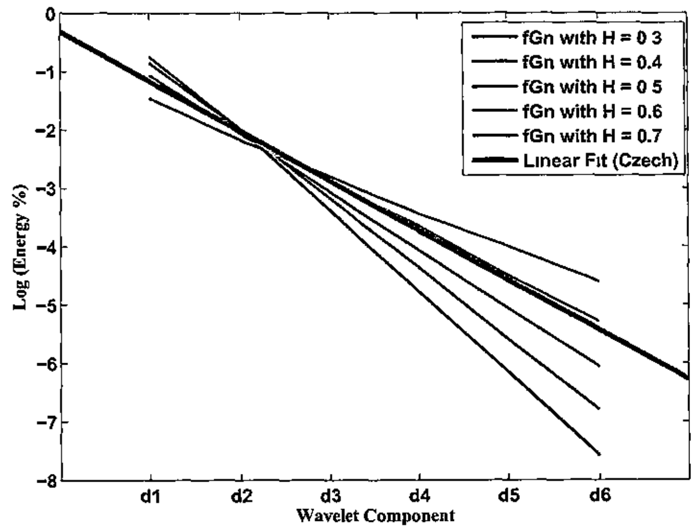
APPENDIX B

FIGURE FOR SECTION 6.2.2, CHAPTER 6.

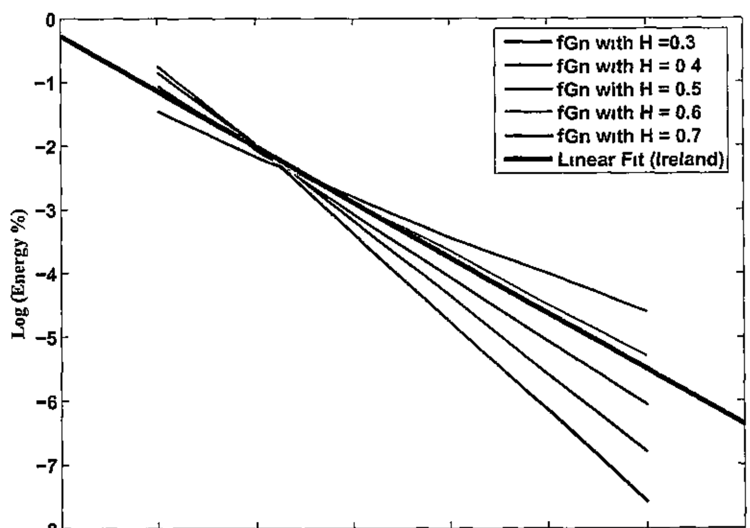
Figure B.1 Logarithm to base two of the energy percentages ($log_2(energy\%)$)



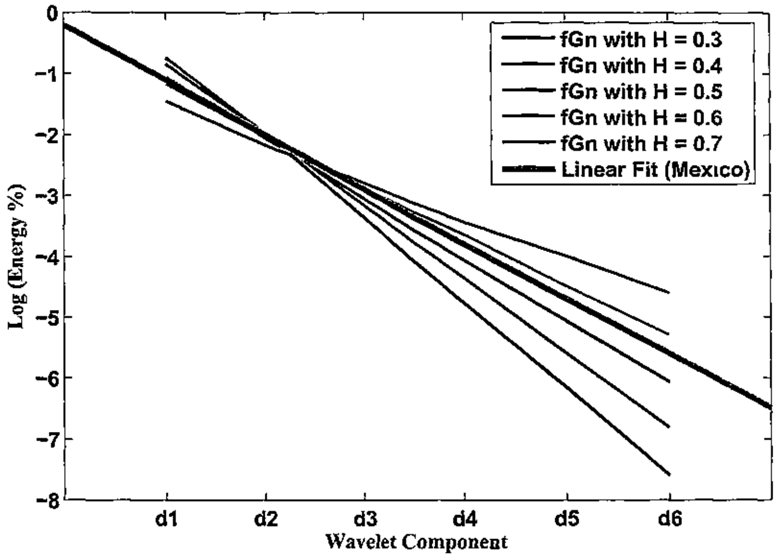
(a) **Argentinean Market.**



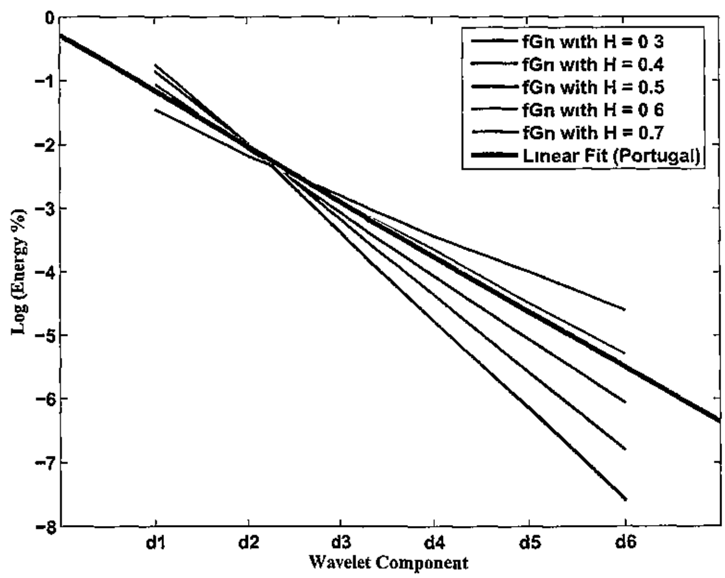
(b) Czech Market.



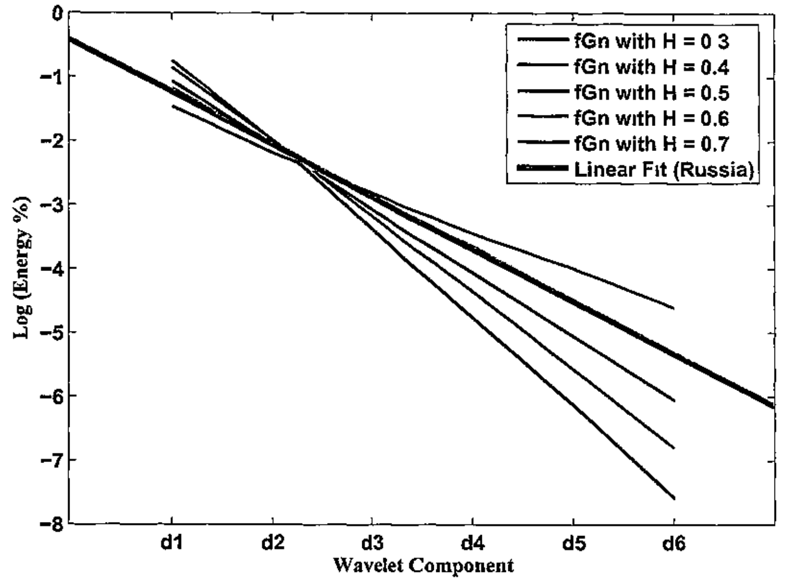
(c) Irish Market.



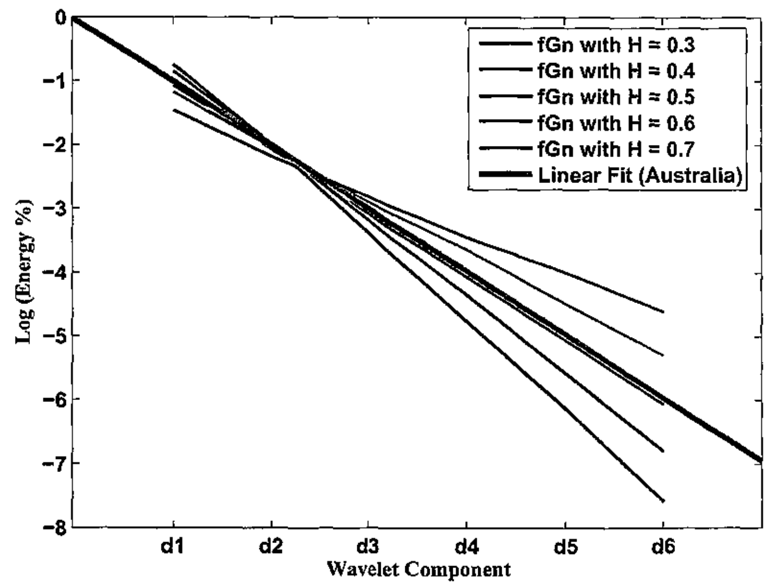
(d) Mexican Market.



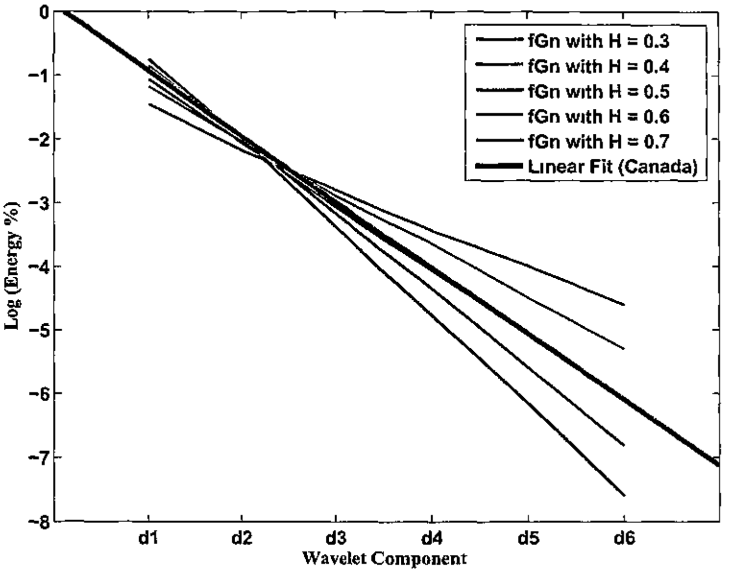
(e) Portuguese Market.



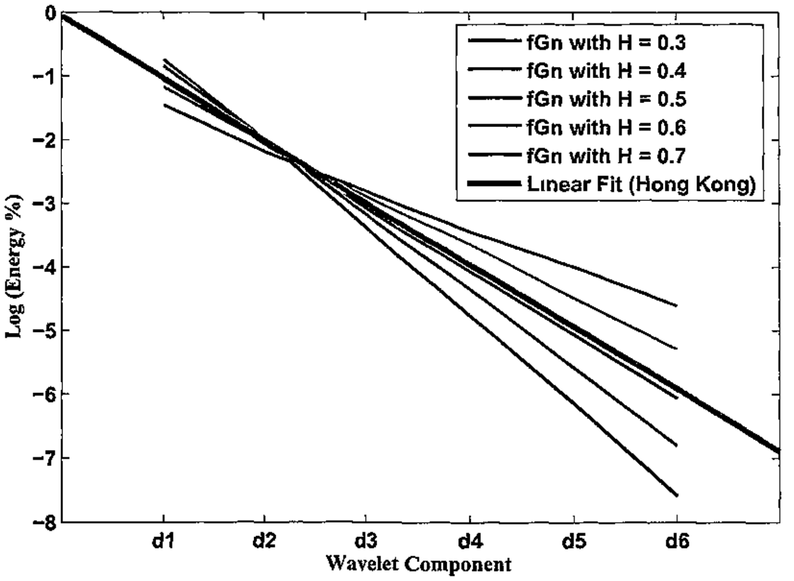
(f) Russian Market.



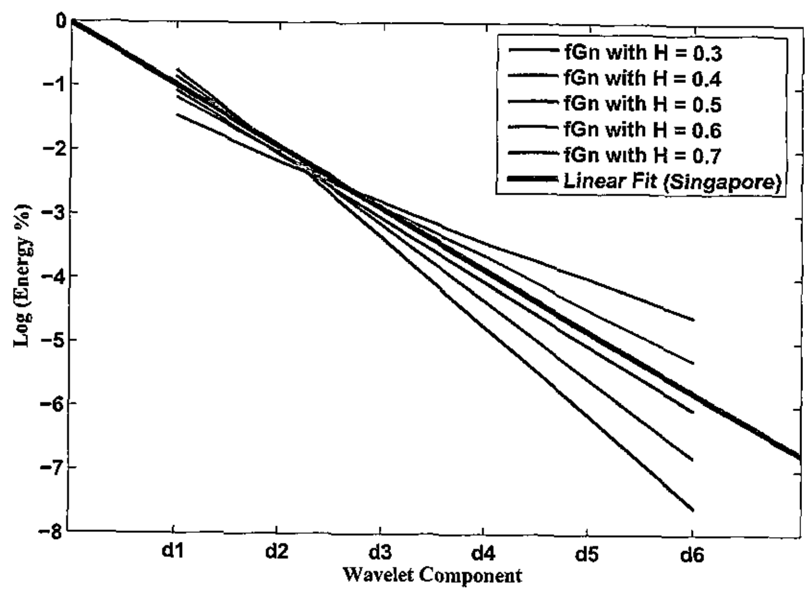
(g) Australian Market.



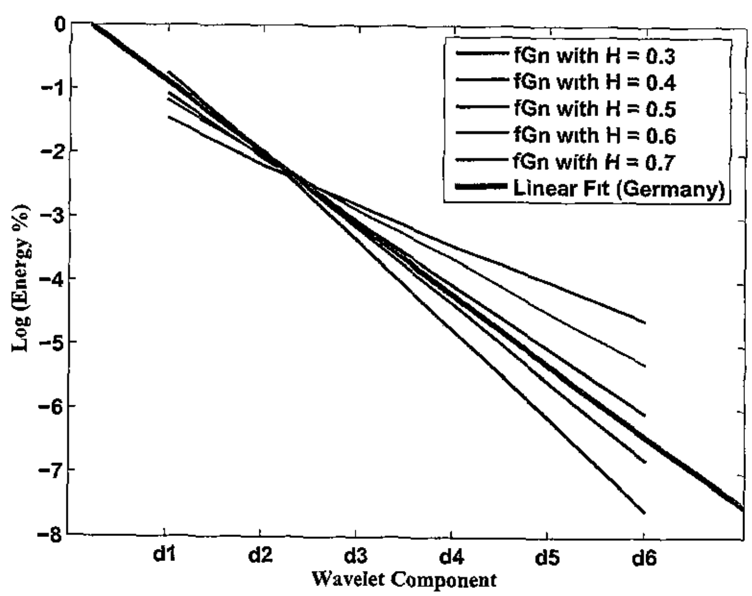
(h) Canadian Market.



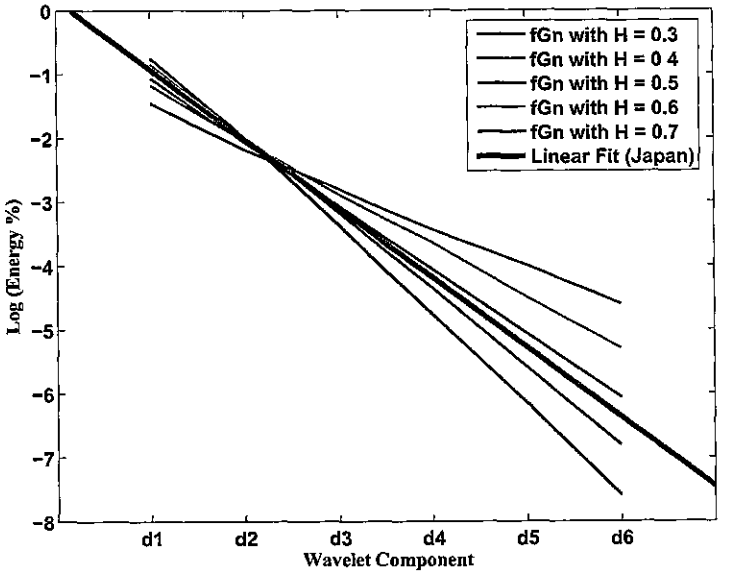
(i) Hong Kong Market.



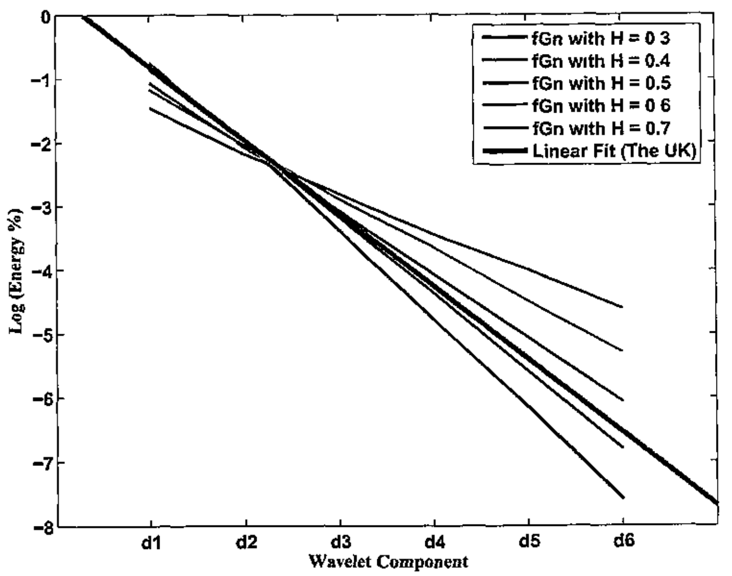
(j) Singapore Market.



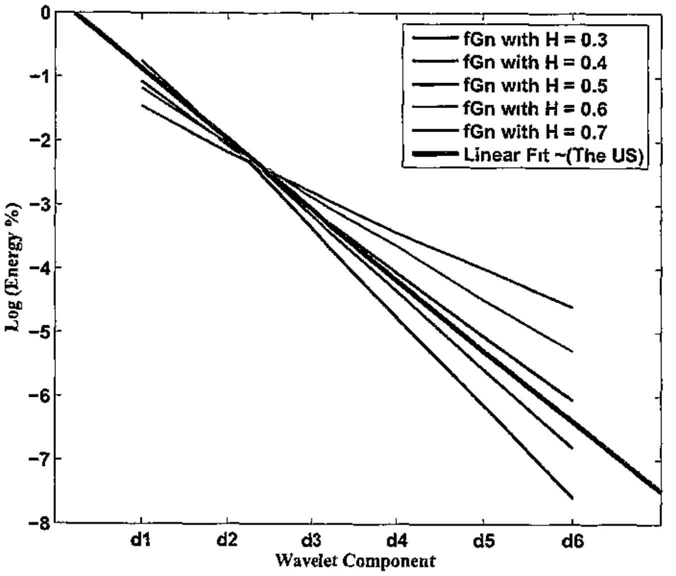
(k) German Market.



(l) Japanese Market.



(m) UK Market.


0
-1
-2
-3
-4
-5
-6
-7
-8
Log (Energy %)
d1
d2
d3
d4
d5
d6
Wavelet Component
fGn with H = 0.3
fGn with H = 0.4
fGn with H = 0.5
fGn with H = 0.6
fGn with H = 0.7
Linear Fit ~(The US)

(n) US Market.

BIBLIOGRAPHY

Abry, P. and Veitch, D. (1998). Wavelet analysis of long-range-dependent traffic *IEEE Transactions on Information Theory*, 44(1):2–15.

Andrew, W. L. (1991). Long-term memory in stock market prices *Econometrica*, 59(5) 1279–1313.

Antonini, M., Barlaud, M., Mathieu, P., and Daubechies, I (1992). Image coding using wavelet transform. *IEEE Transactions on Image Processing*, 1(2):205–220.

Antoniou, A., Pescetto, G , and Violaris, A. (2003). Modelling international price relationships and interdependencies between the stock index and stock index futures markets of three EU countries· A multivariate analysis. *Journal of Business Finance*, 30(5).645–667.

Assaf, A. and Cavalcante, J. (2005) Long range dependence in the returns and volatility of the Brazilian stock market. *European Review of Economics and Finance*, 4(2):3–26.

Baillie, R. T., Bollerslev, T., and Mikkelson, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74:3–30.

Barkoulas, J. T. and Baum, C. F. (1996). Long-term dependence in stock market. *Economics Letters*, 53:253–259.

Barkoulas, J. T., Baum, C. F., and Travlos, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics*, 10(2):177–184.

Bekaert, G. and Harvey, C R. (1995). Time-varying world market integration. *Journal of Finance*, 50:403–444.

Beran, J. (1994). *Statistics for Long Memory Process.* Chapman and Hall Publishing Inc, New York, USA.

Berg, L. and Lyhagen, J. (1998). Short and long run dependence in Swedish stock market. *Applied Financial Economics*, 8(4):435–443.

Bessler, D. A. and Yang, J (2003). The structure of interdependence in international stock markets. *Journal of International Money and Finance*, 22:261–287.

Black, F. (1986). Noise. *Journal of Finance*, 41(3):529–543.

Bollerslev, T. (1986) Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 51:307–327.

Booth, G. G., Mertikainen, T., and Tse, Y. (1997). Price and volatility spillovers in Scandinavian stock markets. *Journal of Banking and Finance*, 21:881–823.

Bremaud, P (2002). *Mathematical Principles of Signal Processing: Fourier and Wavelet Analysis.* Springer-Verlag, New York, USA.

Brooks, R. and Negro, M. D (2003). Firm-level evidence on international stock market movement. Working Paper 2003-8, The Federal Reserve Bank of Atlanta Available at http //ideas.repec.org/p/fip/fedawp/2003-8.html [accessed on 19/06/2006]

Bruce, A. and Gao, H. Y. (1996) *Applied Wavelet Analysis with S-Plus.* Springer-Verlag, New York

Calway, A. D. (1993). Image analysis using a generalised wavelet transform. In *IEE Colloquium on Applications of Wavelet Transforms in Image Processing*, pages 1–8.

Capobianco, E. (2001). Wavelet transform for the statistical analysis of returns generating stochastic processes. *International Journal of Theoretical and Applied Finance*, 4(3):501–534

Cheung, Y. W. (1993) Long memory in foreign-exchange rates. *Journal of Business and Economic Statistics*, 11(1):93–101.

Cheung, Y. W. and Lai, K. S. (1995). A research for long memory in international stock market returns. *Journal of International Money and Finance*, 14(4):597–615.

Coelho, R., Hutzler, S., Repetowicz, P., and Richmond, P. (2006). Sector analysis for a FTSE portfolio of stocks. *Physica A: Statistical Mechanics and its Applications (in press)*.

Crato, N. (1994). Some international evidence regarding the stochastic memory of stock returns *Applied Financial Economics*, 4:33–39

CVM (1998) International transmission of stock market volatility spillover effects on Latin American markets. The conference meeting on management of volatility in turbulent markets, The IOSCSs Emerging Markets Annual Meeting, Kuala Lumpur, Malaysia.

Dark, J. (2004). Long term hedging of the Australian All Ordinaries index using a bivariate error correction FIGARCH model. working paper, MONASH University. Available at http://www.buseco.monash.edu.au/depts/ebs/pubs/wpapers/2004/wp7-04.pdf [accessed on 19/06/2006].

Daubechies, I. (1990). The wavelet transform, time-frequency localization and signal analysis. *IEEE Transactions on Information Theory*, 36(5):961–1005.

Demeirgü-Kunt, A. and Levine, R. (1995) Stock market development and financial intermediaries. *The World Bank, Policy Research Working Paper*, (No 1462).

Di Matteo, T., Aste, T., and Dacorogna, M. M. (2003). Scaling behaviors in differently developed markets. *Physica A: Statistical Mechanica and its Applications*, 324 183–188.

Di Matteo, T., Aste, T., and Dacorogna, M. M (2005) Long-term memories of developed and emerging markets. Using the scaling analysis to characterize their stage of development. *Journal of Banking and Finance*, 29:827–851.

Ding, Z., Granger, C. W., and Engle, R F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1:83–106

Drori, I. and Lischinski, D (2003). Fast multiresolution image operations in the wavelet domain. *IEEE Transactions on Visualization and Computer Graphics*, 9(3).395–412.

Elekdag, S. (2001). Long memory in the volatility of emerging markets. In *International Conference in Economics, The ISE Finance Award Series*. The Middle East Technical University

Eun, C. S. and Shim, S. (1989). International transmission of stock market movements. *Journal of Finance and Quantitative Analysis*, 24(2) 241–256.

Fuss, R. (2002). The financial characteristics between emerging and developed equity markets. In *Proceedings of International Conference on Policy Modeling*, Brussels, Belgium.

Galluccio, S., Bouchaud, J. P., and Potters, M (1998). Rational decisions, random matrices and spin glasses. *Physica A: Statistical Mechanica and its Applications*, 259:449–456

Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long memory time series models *Journal of Time Series*, 4:221–238.

Gonghui, Z., Starck, J. L., Campbell, J., and Murtagh, F. (1999) The wavelet transform for filtering financial data streams. *Journal of Computational Intelligence in Finance*, 7(3):18–35.

Gopikrishnan, P , Rosenow, B., Plerou, V., and Stanley, E. (2001). Quantifying and interpreting collective behaviour in financial markets. *Physical Review E*, 64(035106)

Granger, C. W. and Ding, Z (1996). Varieties of long memory models *Journal of Econometrics*, 73:61–77.

Henry, T. (2002). Long memory in stock returns. Some international evidence. *Applied Financial Economics*, 12:725–729.

Hiemstra, C. and Jones, J. D. (1997). Another look at long memory in common stock returns. *Journal of Empirical Finance.*, 4:373–401.

Hijmans, H. E. (1993). Discrete wavelet and multiresolution analysis. In Koornwinder, T. H., editor, *Wavelets: An Elementry Treatment of Theory and Application*, pages 49–79, Singapore. World Scientific Publishing Co Pte Ltd.

Hurst, H. E. (1951). Long-memory storage of capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, 116:770–799.

Hurvich, C. M , Deo, R S., and Brodsky, J. (1998). The mean squared error of Geweke and Porter-Hudaks estimator of the memory parameter of a long memory time series. *Journal of Time Series Analysis*, 19:19–46.

Hurvich, C. M. and Ray, B. (1995). Estimation of the memory parameter for nonstationary or non-invertible fractionally integrated processes. *Journal of Time Series Analysis*, 16:17–42.

In, F. and Kim, S. (2005). Multiscale hedge ratio between the Australian stock and future markets: Evidence from wavelet analysis. *Journal of Multinational Financial Management (in press).*

Jacobsen, B. (1996). Long-term dependence in stock market. *Journal of Empirical Finance*, 3:393–417.

Jensen, M. J. (1997). Making wavelets in finance. *Financial Engineering News*, 1(1).

Keogh, G., Sharifi, S., Ruskin, H. J., and Crane, M. (2003). Epochs in market sector index data- empirical or optimistic? In Takayasu, H., editor, *Proceedings of the Second Nikkei Econophysics Symposium: Application of Econophysics*, pages 83–89, Tokyo, Japan. Springer

Kim, S and Rogers, J. H. (1995) International stock price spillovers and market liberalization: Evidence from Korea, Japan and the United States. *Journal of Empirical Finance*, 2.117–133.

Komáromi, G. (2002). Why have the stock markets become noisier after the revolution of information and technology?. In *Evolutions of Institutions and the Knowledge Economy Conference Proceedings.* University of Debrecen

Koutsoyiannis, D. (2002). The Hurst phenomenon and fractional Gaussian noise made easy. *Hydrological Sciences Journal*, 47(4):573–595.

Kwapien, J., Drozdz, S., and Speth, J. (2002). Alternation of different scaling regimes in the stock market fluctuations. Available at http://www.kfa-juelich.de/ikp/publications/AR2002/CHAP4/409.pdf [accessed on 19/06/2006].

Kwapien, J., Drozdz, S., and Speth, J. (2004). Time scale involved in emerging market coherence. *Physica A: Statistical Mechanica and its Applications*, 337:231–242.

Laloux, L., Cizeau, P., and Potters, M. (2000). Random matrix theory and financial correlations. *International Journal of Theoretical and Applied Finance*, 3(3):391–397.

Laloux, L., P. Cizeau, J. P. B., and Potters, M. (1999). Noise dressing of financial correlation matrices. *Physical Review Letters*, 83(7)·1467–1470.

Lee, C. F., Chen, G. M., and Rui, O. M. (2001). Stock returns and volatility on China's stock market. *Journal of Financial Research*, 25(4):523–543.

Lee, H S. (2002). International transmission of stock market movements: A wavelet analysis on MENA stock market. In *Proceedings of ERF Eighth Annual Conference: Economic Research Forum,*, Cairo, Egypt.

Lee, J., Kim, T. S., and Lee, H. K. (2000). Long memory in volatility of Korean stock market returns. In *Proceedings of INFORMS/KORMS Seoul 2000 Conference*, Korea, June 2000, pages· 540-546.

Li, J. P (1997). *Wavelet Analysis and Signal Processing: Theory, Applications and Software Implementation.* Chongqing Publishing House, P.R.C.

Lin, W., Engle, R. F., and Ito, T. (1994). Do bulls and bears move across borders? international transmission of stock returns and volatility. *The Review of Financial Studies*, 7(3):507–538.

Litterman, R. and Winkelmann, K. (1998). Estimating covariance matrices. In Krieger, R. A., editor, *Goldman-Sachs Risk Management Series*, page 47. Goldman Sachs & Co.

Lo, A. and MacKinlay, C. (1989). The size and power of the variance ratio test in finite samples: A Monte Carlo investigation. *Journal of Econometrics*, 40:203–238.

Lo, A. W (1991). The long-term memory in stock prices *Econometrica*, 59(4):1279–1313

Lobato, I. N. and Savin, N. E (1998). Real and spurious long memory proerties of stock market data. *Journal of Business and Economic Statistics*, 16:261–267.

Los, C. A. and Yalamova, R. M. (2004). Multi-fractal spectral analysis of the 1987 stock market crash Finance 0409050, Economics Working Paper Archive EconWPA. Available at http://ideas.repec.org/p/wpa/wuwpfi/0409050.html [accesseed in 19/06/2006].

Lux, T. (1998). The socio-economic dynamics of speculative markets: Interacting agents, chaos, and the fat tails of return distributions. *Journal of Economic Behavior and Organization*, 33:143–165

Mallat, S. (1989). A theory for multiresolution signal decomposition. The wavelet representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(7):674–693.

Mandelbrot, B. (1963). The variation of certain speculative prices *Journal of Business*, 36(4):394–419.

Mandelbrot, B. (1971) When can prices be arbitraged efficiently?. A limit to the volatility of the random walk and Martingale models. *Review of Economics and Statistics*, 53:225–236.

Mandelbrot, B. and Wallis, J. (1968). Noah, Joseph, and operational hydrology. *Water Resources Research*, 4:967–988.

Matia, K., Ashkenazy, Y., and Stanley, H. E. (2003) Multifractal properties of price fluctuations of stock and commodities. *Europhysics Letter*, 6(3):422–428.

Matos, J. A. O., Gama, S. M. A., Ruskin, H. J., and Duarte, J. A. M. S. (2004). An econophysics approach to the Portuguese stock index PSI20. *Physica A: Statistical Mechanica and its Applications*, 342(3):665–676.

Meric, I. and Meric, G. (1997). Co-movements of European equity markets before and after the 1987 crash. *Multinational Finance Journal*, 1(2):137–152.

Nath, G. C. (2001). Long memory and Indian stock market: An empirical evidence. In *Proceeding of Seventh Annual Capital Market Conference, Indian Institute of Capital Markets*, Mumbai, India, December 2001.

Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach *Econometrics*, 59(2):347–370.

Newey, W. and West, K. (1987). A simple positive definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica.*, 55:703–705.

Ng, A. (2000). Volatility spillover effect from Japan and the US to the Pacific-Basin. *Journal of International Money and Finance*, 19:207–233.

Oswiecimka, P., Kwapien, J., Drozdz, S., and Rak, R (2005). Investigating multifractality of stock market fluctuation using wavelet and detrending fluctuation methods. *ACTA Physica Polonica B.*, 36(8):2447–2457.

Pafka, S. and Kondor, I. (2002). Noisy covariance matrices and portfolio optimization. *The European Physical Journal B*, 27:277–280.

Pafka, S., Potters, M., and Kondor, I. (2004). Exponential weighting and random-matrix-theory-based filtering of financial covariance matrices for portfolio optimization. Science & Finance (CFM) working paper archive 500050, Science and Finance, Capital Fund Management. Available at http://ideas repec.org/p/sfi/sfiwpa/500050.html [accessed on 19/06/2006].

Patel, S. and Sarker, A. (1998). Stock market crises in developed and emerging stock markets. *Federal Reserve Bank of New York, Research Paper.*, (9809).

Paxson, V. (1997). Fast, approximate synthesis of fractional gaussian noise for generating self-similar network traffic. *ACM SIGCOMM Computer Communication Review*, 27(5):5–18

Peters, E. (1996). *Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility.* John Wiley and Sons, Inc, USA.

Plerou, V., Gopikrishnan, P., and Rosenow, B. (2001). Collective behaviour of stock price movement: A random matrix theory approach. *Physica A: Statistical Mechanica and its Applications*, 299:175–180.

Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L. A N., and Stanley, H. E. (1999) Universal and non-universal properties of cross correlations in financial time series. *Physical Review Letters*, 83(7):1471–1774.

Polikar, R (1994). The wavelet tutorial. Available at http://users.rowan.edu/ polikar/WAVELETS/WTtutorial.html [accessed on 19/06/2006].

Raihan, S. M., Wen, Y., and Zeng, B. (2005). Wavelet A new tool for business cycle analysis. Working Paper Series 2005-050A, The Federal Reserve Bank of St. Louis Available at http://research.stlouisfed.org/wp/2005/2005-050.pdf [accessed on 19/06/2006].

Ramsey, J. B. and Zhang, Z. (1997). The analysis of foreign exchange data using waveform dictionaries *Journal of Empirical Finance*, 4:341–372.

Renaud, O., l. Starck, J., and Murtagh, F. (2005). Wavelet-based combined signal filtering and prediction In *IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetics*, volume 35 (6), pages 1241– 1251. IEEE Systems, Man, and Cybernetics Society.

Richmond, P. (2001). Power law distributions and dynamic behaviour of stock markets. *European Physical Journal B*, 20(4):523–526.

Robinson, P. M (1995). Semi-paremetric estimation of long memory time series *Annals of Statistics*, 22.515–539.

Sadique, S and Silvapulle, P. (2001). Long-term memory in stock market returns· International evidence. *International Journal of Finance and Economics*, 6:59–67

Salomons, R and Grootveld, H. (2002) The equity risk premium: Emerging versus Developed markets. Research Report 02E45, University of Groningen, Research Institute SOM (Systems, Organisations and Management). Available at http://ideas repec.org/p/dgr/rugsom/02e45.html [accessed on 19/06/2006].

Schwert, G W. (1989). Tests for unit roots: Monte Carlo investigation. *Journal of Business and Economic Statistics*, 7:147–159.

Sharifi, S., Crane, M., Shamaie, A , and Ruskin, H. (2004) Random matrix theory for portfolio optimization: A stability approach. *Physica A: Statistical Mechanica and its Applications*, 335(3):629–643.

Sharkasi, A , Crane, M., Ruskin, H J , and Matos, J. A. (2006). The reaction of stock markets to crashes and events: A comparison study between emerging and mature markets using wavelet transforms. *Physica A: Statistical Mechanica and its Applications*, 368(2):511–521.

Sheram, K. and Soubbotina, T. P. (2000). *Beyond Economic Growth: Meeting the Challenges of Global Development.* The World Bank, Washington, D.C, USA.

Sibbertsen, P. (2002). Long memory in volatilities of German stock returns In *Proceedings of International Conference on: Modelling Structural Breaks, Long Memory and Stock Market Volatility*, London, UK. CASS Business School.

Simonsen, I , Hansen, A., and Nes, O. M. (1998). Determination of the Hurst exponent by use of wavelet transforms. *Physical Review E*, 58(3):2779–2787.

Sornette, D. (2002). *Why Stock Markets Crash: Critical Events in Complex Financial Systems.* Princetion University Press, New Jersey, USA.

Strang, G. (1993). Wavelet transforms versus Fourier transforms. *Bulletin of The American Mathematical Society*, 28(2):288–305.

Tolvi, J (2003). Long memory in a small stock market *Economic Bulletin*, 7(3):1–13

Tsai, S. S. (2002). Power transformer partial discharge (PD) acoustic signal detection using fiber sensors and wavelet analysis, modeling, and simulation. Master of Science, Electrical and Computer Engineering, The Faculty of the Virginia Polytechnic Institute and State University. Available at http.//scholar.lib vt edu/theses/available/etd-12062002-152858/ [accessed on 19/06/2006].

Turiel, A. and Perez-Vicente, C J (2003). Multifractal geometry in stock market time series. *Physica A: Statistical Mechanica and its Applications*, 322:629–649.

Velasco, C. (1999). Non-stationary log-periodogram regression. *Journal of Econometrics*, 91:325–371

Wilcox, D. and Gebbie, T. (2004). On the analysis of cross-correlations in South African market data. *Physica A: Statistical Mechanica and its Applications*, 344:394–298.

Wongswan, J. (2003). Transmission of information across international equity markets. International Finance Discussion Papers 759, Board of Governors of the Federal Reserve System (U.S.). Available at http://ideas repec.org/p/fip/fedgif/759.html [accessed on 19/06/2006].

Wooldridge, P. D., Domanski, D., and Cobau, A. (2003). Changing links between mature and emerging financial markets. *BIS Quarterly Review*, pages 45–54.

Wright, J. H. (1999). Long memory in emerging stock market returns. *International Finance Discussion Papers, The Federal Reseve Board*, (650). Available at http://ideas repec.org/p/fip/fedgif/650.html [accessed on 19/06/2006].

Zaffaroni, P. (2003). Gaussian inference on certain long-range dependent volatility models. *Journal of Econometrics*, 115:199–258.

Zheng, G., Starck, J. L., Campbell, J., and Murtagh, F. (1999). The wavelet transform for filtering financial data streams. *Journal of Computational Intelligence in Finance*, 7(3):18–35.

List of Publications

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Sharkasi, A., Crane, M. and Ruskin, H. J., 2006. Apples and Oranges: the Difference Between the Reaction of Emerging and Mature Markets to Crashes. *Proceedings of the Third Nikkei Econophysics Symposium - Practical Fruits of Econophysics*, Tokyo, Japan. Springer-Verlag, Pages 38-42.

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INTERDEPENDENCE BETWEEN EMERGING AND MAJOR MARKETS

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Abstract: In this paper, we investigate the price spillover effects among two developed markets, (the US and the UK), and two developing markets, (Irish and Portuguese), using a new testing method suggested by Lee (2002). We find that there are interrelationships between any two of the Irish, the UK and Portuguese markets and that the co-movements between the emerging markets and the US are statistically significant but weak We also found that the US market is slightly influenced by the UK but not *vice versa*.

1 Introduction

The relationships between international stock markets have been investigated in several articles, especially after "Black Monday", (October 1987). These studies indicated that co-movements among stock markets have increased the possibilities for national markets to be influenced by the changes in international ones ([12], [9], [6], [7] and [13]).

The advantage of global portfolio diversification has been noted in the finance literature for some time. Several studies ([11], [14] and [2]) showed that it is useful to spread content internationally, rather than locally, as stocks in different markets are less correlated than those within the same market. Tang [16] investigated, for instance, Asian emerging and mature markets and reported that an increase in the correlation between worldwide stock markets may cause the reduction of some or all of the diversification benefits and this means that diversification benefits depend upon the degree of the relationships among different stock markets. Tang [17] found that the intertemporal stability of the correlation matrix is important in examining the ex-ante diversification benefits and stock market co-movements The potential diversification effects have decreased and become less important due to increase in the international co-movement among stock markets, especially since the mid 1990's ([15] and [16]).

More recently, Lee [10] developed a new testing technique based on the wavelet transform, in order to study the international transmission effects between three developed markets (the US, Germany and Japan) and two emerging markets in the MENA region, namely Egypt and Turkey. He documented that innovation from the major markets affected the emerging markets but the that opposite was not true

In addition, Bessler and Yang [3] employed an Error Correction Model and Directed Acyclic Graphs (DAG) to study the co-integration among nine major markets namely Japan, the US, the UK, France, Switzerland, Hong Kong, Germany, Canada and Australia. Their results showed that changes in the UK, Switzerland, Hong Kong, France and Germany influenced the US market, while the US market is affected by its own innovation as well. Moreover, Brooks and Negro [4] studied the relationship between market co-integration and the degree to which companies operate internationally. They considered three factors, (global, country-specific and industry-specific), and found that the importance of the international factor has increased since the 1980s while that of the country-specific factor has decreased.

Furthermore, Wongswan [18] found strong evidence of international transmission from the US and Japanese markets to Korean and Thai markets during the late 1990's Most recently, Antoniou et al [1] applied a VAR-EGARCH model to study the relationships among three EU markets namely Germany, France and the UK and their results showed evidence of co-integration among those countries.

Our goal in this article is to study whether or not there is evidence of co-integration between four stock markets (Irish, Portuguese-as developing and the UK and the US-as mature). To examine this, we applied a testing method, (based on the wavelet transform), suggested by Lee [10].

The remainder of this paper is organized as follows· In Section 2, a brief description of the testing method is given The data and empirical results are described in Section 3 and our conclusion is presented in the final section.

2 Brief description of the testing method

With the increase in media coverage of world events and a corresponding increase in access by the wider public to this coverge, global transmissions of information can be expected to be completed within a short period of time. The wavelet analysis and, in particular, the discrete wavelet transform (DWT), is very useful (for more detail see [5]) in splitting data series into different frequency wavelet crystals and high-frequency components which explain the short-term movements in the series . A new testing method based on wavelet analysis was developed by Lee [10] and it can be described as follows:

- Reconstruct the returns series using the first and the second high-frequency wavelet crystals (d_1 & d_2) separately.
- Estimate the simple regression and reverse regression models between each two using three different scales·
  - The row daily returns
  - The returns series rebuilt form d_1.
  - The returns series rebuilt form d_1 plus that rebuilt from d_2
- Test the significant of regression coefficient (slope) and R^2.

3 Data and empirical results

The data used in the following analysis consists of the daily prices of stock market indices for two emerging markets, namely Portuguese and Irish and two major markets, (the US and the UK), during the period from January 1^{st}, 1993 to September 30^{th}, 2003 We considered the indices ISEQ Overall, PSI20, FTSE All Share and S&P500 to be representative of the Irish, Portuguese, UK and US markets respectively.

As these markets use their local currencies for presenting the values of their indices, so we use the daily returns instead of using the daily prices, where the former equal the natural logarithm of the ratio between the closing price of index at time t and that at time $t-1$ Some daily observations have been deleted because the markets we studied have different holidays and closing trading days, (as has been done by e.g. [10]).

Index→ Measure↓	**ISEQ**	**PSI20**	**FTSE**	**S&P500**
No Observations	2556	2556	2556	2556
Mean	0.00052	0.00029	0 00012	0.00033
Std.Dev	0 0104	0.0109	0 0099	0 0111
Minimum	-0.0757	-0.0959	-0.0515	-0.0704
Maximum	0.0584	0 0694	0.0509	0 0557
Skewness	-0.3580**	-0.5760**	-0.1820	-0.021
Kurtosis	4.503**	6 849**	2.794**	3.077**
Jarque-Bera	2203.63**	5109.643**	840 70**	1002.87**

Note.** denotes statistically significant at 1% level

Table 1. Descriptive statistics of the daily returns of the stock markets indices series

Table 1 represents the descriptive statistics of the stock market indices and shows that the sample means of all indices are positive. We test whether or not the skewness and kurtosis of all these series are different from zero. The results show that the returns series of ISEQ and PSI20 indices have significant negative skewness, but those of FTSE and S&P500 are not significantly different from zero. The returns of all indices are leptokurtic and the results of a normal test (Jarque-Bera) also show that all returns series can not be regarded as normally distributed

From Table 2, It can be seen that high-frequency components have more energy than low-frequency ones and this implies that the movements in all index returns are caused by the short-term fluctuations. It also implies that the first "d_1" and the the second "d_2" components of the wavelet transform account for more than 60% of the energy This indicates that there are no long memory effects in the returns series of these indices

In order to study the co-movements among those markets, firstly, we built simple regression models between each of the two European markets on the

Index → Wavelet Crystals↓	ISEQ	PSI20	FTSE	S&P 500
s_6	0.028	0 039	0.014	0.012
d_6	0 023	0 025	0.017	0.012
d_5	0.036	0.042	0 027	0.031
d_4	0.070	0.058	0 047	0 048
d_3	0.155	0.163	0.157	0 145
d_2	0.274	0.267	0 301	0.234
d_1	0.431	0.406	0 436	0.518

Table 2: Peicentages of energy by wavelet crystals for the daily returns of indices series.

same trading day and similarly for each European market on the US market of the *previous* trading day Secondly, we built a simple regression model of the US market on each European maiket on the same trading day and these models are estimated using the three different scales mentioned in Section 2. The results are given in Tables 3(A) to 3(F) for each case and clearly show that there are significant levels of inter-correlation between the Irish and UK markets and also between the Irish and Portuguese. However, the relationship between the Irish and US markets is weak. From Table 3 (D), (E) and (F), we can see that there is significant co-movement between Portuguese and UK markets and there are spillover effects from both Portuguese and UK markets on the US market but not *vice versa.*

Regression→	$M_t{}^{IRL}$ on $M_t{}^{UK}$			$M_t{}^{UK}$ on $M_t{}^{IRL}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	4.46E-04 (0.034)	0 592 (0.000)	0.322	-1.58E-04 (0.328)	0.544 (0.000)	0 322
Retuin D1	-5.85E-07 (0.996)	0.509 (0 000)	0.251	-1 06E-06 (0.992)	0.492 (0.000)	0.251
Return.D1.2	6 18E-08 (1.000)	0 552 (0 000)	0 300	-3.31E-06 (0 981)	0.544 (0.000)	0 300

A ISEQ Overall and FTSE

Regression→	M_t^{IRL} on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{IRL}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Retuin	4 46E-04 (0 034)	0.356 (0 000)	0.145	1 93E-04 (0.365)	0 258 (0.000)	0.057
Return.D1	-1.94E-06 (0.988)	0 172 (0 000)	0.039	-2 65E-06 (0.987)	0.065 (0.007)	0 002
Return.D1 2	-3.41E-06 (0 983)	0 273 (0 000)	0 092	1 26E-06 (0 995)	0.155 (0 000)	0.019

B ISEQ Overall and S&P500

Regression→	$M_t{}^{IRL}$ on $M_t{}^P$			$M_t{}^P$ on $M_t{}^{IRL}$		
Scales↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	4.19E-04 (0.029)	0 340 (0.000)	0 128	9.67E-05 (0.632)	0.378 (0.000)	0 128
Return.D1	-1.28E-06 (0.992)	0.352 (0 000)	0.135	-6 94E-08 (1.000)	0 384 (0 000)	0.135
Return.D1 2	-3 64E-06 (0.995)	0 341 (0 000)	0 135	4.22E-06 (0 370)	0 370 (0.000)	0 126

C ISEQ Overall and PSI20

Regression→	$M_t{}^P$ on $M_t{}^{UK}$			$M_t{}^{UK}$ on $M_t{}^P$		
Scales↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	2 29E-04 (0.230)	0 517 (0 000)	0 221	-1.45E-06 (0 993)	0 428 (0.000)	0.221
Return D1	2.84E-07 (0.998)	0.516 (0 516)	0.236	-1 51E-06 (0 989)	0.459 (0.000)	0 237
Return.D1.2	5 65E-06 (0.971)	0.505 (0.000)	0.231	-6.18E-06 (0 976)	0 458 (0.000)	0.231

D: PSI20 and FTSE

Regression→	$M_t{}^P$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^P$		
Scales↓	**Constant**	**Slope**	R^2	**Constant**	Slope	R^2
Return	2.29E-04 (0 280)	0 196 (0.000)	0 040	2 48E-04 (0.241)	0 266 (0 000)	0.066
Return D1	-7.32E-07 (0 996)	3.41E-02 (0 058)	0.001	-2.62E-06 (0.987)	0.194 (0 000)	0.028
Return.D1.2	2 88E-06 (0.987)	0.122 (0 000)	0.017	1.22E-07 (0 999)	0.228 (0.000)	0.044

E PSI20 and S&P500

Regression→	$M_t{}^{UK}$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{UK}$		
Scales↓	**Constant**	**Slope**	R^2	**Constant**	**Slope**	R^2
Return	3.49E-05 (0.852)	0 272 (0.000)	0.092	2 69E-04 (0.179)	0.471 (0.000)	0.177
Return D1	-1.81E-06 (0.989)	4 46E-03 (0 793)	0.000	-2.20E-06 (0.989)	0.300 (0.000)	0 060
Return D1 2	-5.17E-06 (0 975)	0.151 (0 000)	0 029	2.59E-06 (0.989)	0 368 (0.000)	0.106

F: FTSE and S&P 500

- P-values of t-tests are given in parentheses
- Where subscript refers to the day in question and the superscript indicates the market (e g. IRL, P are the Irish and Portuguese markets respectively)
- Return.D1 is an indicator of the returns series, reconstructed using the first wavelet crystal (d_1).
- Return.D1.2 is an indicator of the returns series, reconstructed using the first and the second wavelet crystals (d_1 & d_2)

Table 3: Regression Analysis between each pair of four stock markets using three different scales

Market Explained↓	Days Ahead	Ireland	Portugal	The UK	The US	OM
Ireland	5	60.77	1.20	26 15	11.88	39 29
	10	60.24	1.31	26.29	12.16	39 76
	15	60 21	1 32	26 30	12.17	39.79
Portugal	5	0 51	77 54	18 40	3 54	22.45
	10	0 83	76.61	18.83	3.73	23.39
	15	0 83	76 52	18.83	3.82	23.48
The UK	5	0 38	0.30	88.77	10.56	11.24
	10	0.59	0 54	87 99	10.87	12.00
	15	0.59	0 55	87.99	10.88	12 02
The US	5	0 37	0.78	19.87	78.98	21 02
	10	0.45	1.16	20.44	77.95	22 05
	15	0.45	1.17	20.45	77.93	22.07

Note: OM denotes the percentage of forecast error variance explained collectively by the other markets

Table 4: The Percentages of error variance of the market in the first column explained by innovation in the market in the first row.

To compare our results with one of the common methods, we estimated the vector autoregressive (VAR) model of order 10 of the daily returns of these markets The percentages of the decomposition of 5-day, 10-day and 15-day ahead forecasts of the returns series have been measured[1] At 15 days ahead, for example, the results, given in Table 4, show that the most of the variance in these markets is explained by their own innovations and that the UK is the most influential market while the Irish is the most influenced market. The UK explains 26.30, 18.83 and 20.45 percent for Irish, Portuguese and the US respectively and the US explains 12 17, 3 82 and 10.88 percent of the variance of Irish, Portuguese and the UK respectively. We also found that the forecast error variance is very sensitive to the order of variables for orthogonalization and to the stability of these series and this suggests that the new technique, based on wavelet analysis, is more reliable than the VAR method

4 Conclusion

Our objective in this paper has been to study the international transmission between four markets namely the Irish, Portuguese, UK and US. A new testing method suggested by Lee [10] has been applied to do so. Our results show that there are significant inter-correlations between each pair of Irish, Portuguese and UK markets separately. In addition, the indications are that the US has insignificant spillover effects from or on to the other markets We can say that the emerging markets have significant spillover effects on each other but there is no co-integration between the major markets.

[1] The orthogonalization is ordered as the UK, Portuguese, the US and Irish.

Wavelet analysis

The Wavelet Transform (**WT**) has been explained in some detail, (particularly in [5] and [10]) and the following offers a brief explanation only The **WT** has two types of wavelets called father and mother wavelets, ϕ and ψ respectively, where $\int \phi(t)dt = 1$ and $\int \psi(t)dt = 0$ These can be computed using the following equations

$$\phi(t) = \sqrt{2}\sum_k \ell_k \phi(2t-k) \tag{1}$$

$$\psi(t) = \sqrt{2}\sum_k \hbar_k \phi(2t-k) \tag{2}$$

The orthogonal wavelet series approximation to a given signal $f(t)$ is defined by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \dots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{3}$$

where J is the number of multiresolution levels, (or crystals), and k ranges from 1 to the number of coefficients in the specified components (or levels). The coefficient $s_{J,k}$, $d_{J,k}$, $\dots$, $d_{1,k}$ are the wavelet transform coefficients given by

$$s_{J,k} = \int \phi_{J,k}(t)f(t)dt \tag{4}$$

$$d_{j,k} = \int \psi_{j,k}(t)f(t)dt \qquad (j = 1, 2, \dots, J) \tag{5}$$

The discrete wavelet transform (**DWT**) computes the coefficient of the wavelet series approximation in Equation(3) for a discrete signal $f_1, \dots, f_n$ of finite extent The DWT maps the vector $f = (f_1, f_2, \dots, f_n)'$ to a vector of n wavelet coefficients $w = (w_1, w_2, \dots, w_n)'$ which contains the "smooth" coefficient $s_{J,k}$ and "detail" coefficients $d_{j,k}$ $[j = 1, 2, \dots, J]$ The $s_{J,k}$ describes the underlying smooth behaviour of the signal at coarse-scale 2^J while $d_{J,k}$ describes the coarse-scale deviations from the smooth behaviour and the $d_{J-1,k}, \dots, d_{1,k}$ provide progressively finer-scale deviations from the smooth behaviour

References

[1] Antoniou A , Pescetto G , Violaris A. (2003) *Modelling international price relationships and interdependencies between the stock index and stock index futures markets of three EU countries: A Multivariate analysis* Journal of Business Finance **30** (5)& (6), 645 - 67

[2] Baily W., Stulz R. (1990). *Benefits of international diversification: the case of Pacific Basin stock markets.* Journal of Portfolio Managment **16** (4), 57–61.

[3] Bessler D.A., Yang J. (2003) *The structure of interdependence in international stock markets* Journal of International Money and Finance **22**, 261–87.
[4] Brook R., Negro M.D (2003). *Firm-level evidence on international stock market co-movement* International Monetary Fund, IMF Working Papers No: **03/55**, Washington, DC, USA.
[5] Bruce A., Gao H. Y. (1996). *Applied wavelet analysis with S-Plus* New York Springer-Verlag
[6] Booth G. G , Martikainen T., Tse Y. (1997). *Price and volatility spillovers in Scandinavian stock market.* Journal of Banking and Finance **21**, 811–823.
[7] CVM (1998). *International transmission of stock market volatility spillover effect on Latin American markets.* the IOSCO's Emerging Markets Annual Meeting, (Conference on Management of Volatility in Turbulent Markets), Kuala Lumpur, Malaysia, May 1998
[8] Francis B.B , Leachman L L. (1998). *Superexogeneity and the dynamic linkages among international equity markets.* Journal of International Money and Finance **17**, 475–92
[9] Kim S W , Rogers J.H. (1995). *International stock price spillovers and market liberalization: evidence from Korea, Japan, and the United States.* Journal of Empirical Finance **2**, 117–33
[10] Lee H S.(2002) *International transmission of stock market movements: A wavelet analysis on MENA stock market.* Economic Research Forum, ERF Eighth Annual Conference, Cairo, Egypt, January 2002.
[11] Levy H., Sarnat M. (1970). *International diversification of investment potfolios* American Economic Review **60**, 668–75.
[12] Lin W., Engle R.F., Ito T. (1994). *Do bulls and bears move across borders? international transmission of stock returns and volatility.* The Review of Financial Studies **7** (3), 507–38.
[13] Ng A. (2000) *Volatility spillover effect from Japan and the US to the Pacific-Basin.* Journal of International Money and Finance **19**, 207–33.
[14] Solnik B H. (1974). *Why not diversify internationally rather than domestically?* Financial Analysis Journal **30** (4), 48–54
[15] Solnik B H. (1990). *Pacifc Basin and international diversification.* In Capital Markets Research **2**, Ghe S. G and Chang R P (ed), Elsevier Sicence Publishers B. V (North-Holland).
[16] Tang G. Y. (1996). *Intervalling effect on intertemporal stability among Asian emerging markets and developed markets* Journal of Business Research **36**, 257–65
[17] Tang G Y. (1998). *The intertemporal stability of the covariance and correlation matrices of Hong Kong stock market.* Applied Financial Economics **8**, 359–65
[18] Wongswan J (2003) *Transmission of information across international equity markets* International Finance Discussion Papers **759**, Board of Governors of the Federal Reserve System, USA

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INTERRELATIONSHIPS AMONG INTERNATIONAL STOCK MARKET INDICES: EUROPE, ASIA AND THE AMERICAS

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In this paper, we investigate the price interdependence between seven international stock markets, namely Irish, UK, Portuguese, US, Brazilian, Japanese and Hong Kong, using a new testing method, based on the wavelet transform to reconstruct the data series, as suggested by Lee [11]. We find evidence of intra-European (Irish, UK and Portuguese) market co-movements with the US market also weakly influencing the Irish market. We also find co-movement between the US and Brazilian markets and similar intra-Asian co-movements (Japanese and Hong Kong). Finally, we conclude that the circle of impact is that of the European markets (Irish, UK and Portuguese) on both American markets (US and Brazilian), with these in turn impacting on the Asian markets (Japanese and Hong Kong) which in turn influence the European markets. In summary, we find evidence for intra-continental relationships and an increase in importance of international spillover effects since the mid 1990s, while the importance of historical transmissions has decreased since the beginning of this century.

Keywords: Simple regression; volatility; wavelet analysis.

1. Introduction

The relationships between international stock markets have been investigated in several articles, especially after "Black Monday" (October 1987). These studies indicated that co-movements among stock markets have increased the possibilities for national markets to be influenced by changes in foreign ones. For example, Eun and Shim [7] investigated the relationships among nine major stock markets (Australia, Canada, France, Germany, Hong Kong, Japan, Switzerland, the UK and the US) using the Vector Autoregressive (VAR) model and reported that news beginning in the US market has the most influence on the other markets. Lin *et al.* [12] studied the interdependence between the returns and volatility of Japan and the US market indices using daytime and overnight returns. The results indicated that daytime returns in each market (US or Japan) are linked with the overnight returns in the other.

In addition, Kim and Rogers [10] used GARCH[1] to study the co-movements between the stock markets of Korea, Japan, and the US and their result indicated that the spillovers from Japan and the US have increased since the Korean market became open for outsiders to own shares. Further, Booths *et al.* [3] reported that there are significant spillover effects among Scandinavian stock markets (Danish, Norwegian, Swedish and Finnish) applying EGARCH.[2] Additionally, CVM[3] [6] investigated the link between the Asian and Brazilian markets as representative of the Latin American region during 1997. They found that the spillover effect started on July 15 with the Thailand currency crisis. However, this spillover was not clearly observed until after October 23 (the Hong Kong crash). In a recent study, Ng [13] found significant spillover effects from Japan and the US stock market on six Pacific-Basin markets, namely those of Hong Kong, Korea, Malaysia, Singapore, Taiwan and Thailand. In order to study international transmission effects of this type, a new testing technique based on the wavelet transform, was developed by Lee [11] and applied to three developed markets (US, Germany and Japan) and two emerging markets in the MENA[4] region, namely Egypt and Turkey. The author reported that movements from the developed markets affected the developing markets but not vice versa.

In addition, Bessler and Yang [2] employed an error correlation model and Directed Acyclic Graphs (DAG) to investigate the interdependence among nine mature markets, namely Japan, US, UK, France, Switzerland, Hong Kong, Germany, Canada and Australia. Their results showed that both changes in European and Hong Kong markets influenced the US market, while this was also affected by internal events. Moreover, Brook and Negro [4] studied the relationship between market co-integration and the degree to which companies operate internationally. They considered three factors (global, country-specific and industry-specific) and found that the importance of the international factor has increased since the 1980s while that of the country-specific factor has decreased on all markets.

Strong evidence of international transmission from the US and Japanese markets to Korean and Thai markets during the late 1990s was presented by Wongswan [14], while most recently, Antoniou *et al.* [1] applied a VAR-EGARCH model to study the relationships among three EU markets namely Germany, France and the UK and the results showed some evidence of co-integration among those countries.

Our goal in this article is to study the evidence of *global* co-movements among seven stock markets, three in Europe (namely Irish, UK, and Portuguese), two in the Americas (namely US, and Brazilian) and two in Asia (namely Japanese and Hong Kong). In particular, we are interested in whether co-movements are direct (clockwise only) or indirect, impacting of nearest-neighbor (continental grouping)

[1] Generalized Autoregressive Conditionally Heteroskedastic.
[2] Exponential Generalized Autoregressive Conditionally Heteroscedastic.
[3] CVM is the Securities and Exchange Commission of Brazil.
[4] MENA stands for the Middle East and North Africa.

and whether there is global absorption of major events or large changes in worldwide markets.

The remainder of this paper is organized as follows. The method due to Lee [11] and based on the wavelet transform is described in Sec. 2, with data and results presented in Sec. 3. Conclusions and remarks form the final section.

2. Wavelet Analysis

The wavelet transform was introduced to solve problems associated with the Fourier transform, when dealing with non-stationary signals, or when dealing with signals which are localized in time or space as well as frequency. The Wavelet Transform (WT) has been explained in more detail, particularly in [5, 8, 9, 11], and we give a brief outline only in the following.

2.1. *Definition of wavelet transform*

The wavelet transform (WT) is a mathematical tool that can be applied to many applications such as image analysis, and signal processing. In particular, the discrete wavelet transform (DWT) is useful in dividing the data series into components of different frequency, so that each component can be studied separately to investigate the data series in depth. The wavelets have two types, *father wavelets* ϕ and *mother wavelets* ψ where

$$\int \phi(t)dt = 1 \quad \text{and} \quad \int \psi(t)dt = 0.$$

The smooth and low-frequency parts of a signal are described by using the father wavelets, while the detail and high-frequency components are described by the mother wavelets. The orthogonal wavelet families have four different types which are typically applied in practical analysis, namely, the *haar, daublets, symmlets* and *coiflets.*

The following is a brief synopsis of their features:

- The haar has compact support and is symmetric but, unlike the others, is not continuous.
- The daublets are continuous orthogonal wavelets with compact support.
- The symmlets have compact support and were built to be as nearly symmetric as possible.
- The coiflets were built to be nearly symmetric.

A two-scale dilation equation used to calculate the wavelets, father $\phi(t)$ and mother $\psi(t)$, is defined respectively by

$$\phi(t) = \sqrt{2}\sum_k \ell_k \phi(2t-k), \tag{2.1}$$

$$\psi(t) = \sqrt{2}\sum_k \hbar_k \phi(2t-k), \tag{2.2}$$

where ℓ_k and $\hbar_k$ are the low-pass and high-pass coefficients given by

$$\ell_k = \frac{1}{\sqrt{2}} \int \phi(t)\phi(2t-k)dt, \tag{2.3}$$

$$\hbar_k = \frac{1}{\sqrt{2}} \int \psi(t)\phi(2t-k)dt. \tag{2.4}$$

The orthogonal wavelet series approximation to a signal $f(t)$ is defined by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t), \tag{2.5}$$

where J is the number of multiresolution levels (or scales) and k ranges from 1 to the number of coefficients in the specified components (or crystals). The coefficient $s_{J,k}, d_{J,k}, \ldots, d_{1,k}$ are the wavelet transform coefficients given by

$$s_{J,k} = \int \phi_{J,k}(t)f(t)dt, \tag{2.6}$$

$$d_{j,k} = \int \psi_{j,k}(t)f(t)dt, \quad j = 1, 2, \ldots, J. \tag{2.7}$$

Their magnitude gives a measure of the contribution of the corresponding wavelet function to the signal. The functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ $[j = 1, 2, \ldots, J]$ are the approximating wavelet functions generated from ϕ and ψ through scaling and translation as follows

$$\phi_{J,k}(t) = 2^{\frac{-J}{2}}\phi(2^{-J}t-k) = 2^{\frac{-J}{2}}\phi[(t-2^J k)/2^J], \tag{2.8}$$

$$\psi_{J,k}(t) = 2^{\frac{-J}{2}}\psi(2^{-J}t-k) = 2^{\frac{-J}{2}}\psi[(t-2^J k)/2^J], \quad j = 1, 2, \ldots, J. \tag{2.9}$$

2.2. *The discrete wavelet transform (DWT)*

The discrete wavelet transform is used to compute the coefficient of the wavelet series approximation in Eq. (2.5) for a discrete signal $f_1, \ldots, f_n$ of finite extent. The DWT maps the vector $f = (f_1, f_2, \ldots, f_n)'$ to a vector of n wavelet coefficients $w = (w_1, w_2, \ldots, w_n)'$ which contains both the smooth coefficient $s_{J,k}$ and the detail coefficients $d_{j,k}$ $[j = 1, 2, \ldots, J]$. The $s_{J,k}$ describe the underlying smooth behavior of the signal at coarse scale 2^J while $d_{J,k}$ describe the coarse scale deviations from the smooth behavior and $d_{J-1,k}, \ldots, d_{1,k}$ provide progressively finer scale deviations from the smooth behavior.

In the case when n divisible by 2^J; there are $n/2$ observations in $d_{1,k}$ at the finest scale $2^1 = 2$ and $n/4$ observations in $d_{2,k}$ at the second finest scale $2^2 = 4$. Likewise, there are $n/2^J$ observations in each of $d_{J,k}$ and $s_{J,k}$ where

$$n = n/2 + n/4 + \cdots + n/2^{J-1} + n/2^J + n/2^J.$$

3. Data and Results

3.1. *Data description*

The data used in the following analysis consists of the daily prices of stock market indices for seven markets, [Irish (IRL), UK, Portuguese (P), US, Brazilian (BR), Japanese (JP) and Hong Kong (HK)], during the period from May 1993 to September 2003. We considered the indices ISEQ Overall (IRL), FTSE All Share (UK), PSI20 (P), S&P500 (US), Bovespa (BR), Nikkei 225 (JP) and Hang Seng (HK) to be representative of these markets.

As each market uses its local currency for presenting the index values, we use the daily returns instead of using the daily prices where the following formula applies:

$$\text{Daily Return} = Ln(P_t/P_{t-1}),$$

where

- P_t is the closing price of the index at time t.
- P_{t-1} is the closing price at time $t-1$.

Note:

1. We use close-to-close returns here because the closing daily price summarizes the local and global changes and influences occurrences within the trading day which strongly affect the market.
2. Some daily observations have been deleted because the markets we studied have different holidays. In other words, if one market closed on a given day, we consider the others to close on the same day as well.

Table 1 represents the trading hours of each of these markets in GMT and shows that the Japanese together with the Hong Kong markets open first. The Japanese market closes two hours before the European (i.e., Irish, UK and Portuguese) markets open at 8:00 am, while Hong Kong closes 45 minutes after the European opening. The last to open are American (US and Brazilian), two hours prior to European markets closure. This implies that the starting point for market opening and closing trading hours is Asia, followed by Europe, then America.

Table 1. Trading hours for each markets in GMT.

Continental ↓	Markets ↓	Open	Close
Asia	Japanese	0:00 am	6:00 am
	Hong Kong	1:45 am	8:45 am
Europe	UK	8:00 am	4.30 pm
	Irish	7 50 am	4:30 pm
	Portuguese	8:00 am	4:30 pm
America	US	2:30 pm	9:15 pm
	Brazilian	2:00 pm	8·45 pm

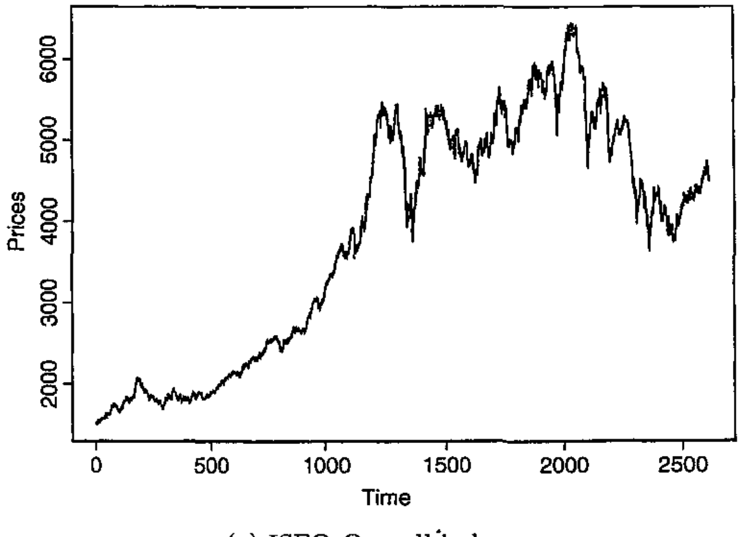
The statistical summaries of the daily returns of all stock market indices are reported in Table 2 which shows that the sample means of the returns of all indices are positive except for those of Nikkei 225 and HSI indices. We test whether or not the skewness and kurtosis of all these series are different from zero and the results show that the returns series of ISEQ, PSI20 and FTSE indices are significantly negatively skewed. Both Bovespa and HSI indices have significant positive skewness, while S&P500 and Nikkei225 are not significantly different from zero in this sense. However, the returns series of all indices are leptokurtic and this means that the daily returns of all indices can not be regarded as normally distributed.

Figures 1(a)–1(e) represent the daily prices of ISEQ Overall, FTSE all shares, PSI20, S&P500 and Bovespa indices respectively. It can be seen that the prices of these indices increased in the first third of the series (1993 to 1996) corresponding to a long-term period of growth. After that, the indices became unstable due to global

Table 2. Descriptive statistics of the daily returns of the stock markets indices series.

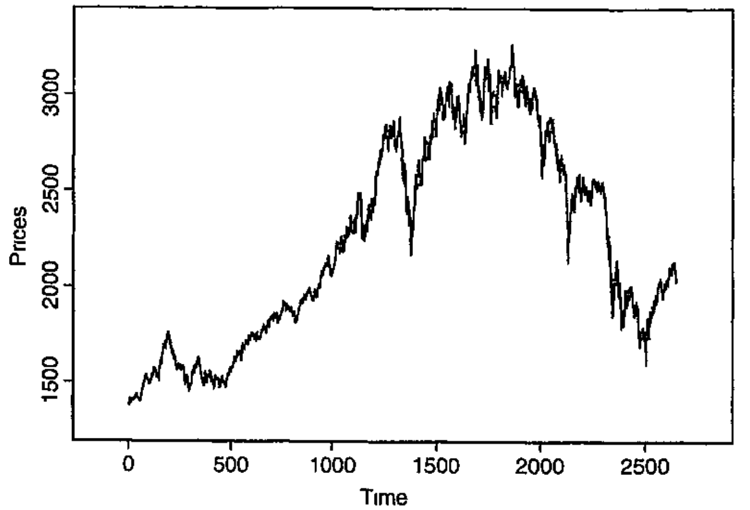
Index→ Measure↓	ISEQ	PSI20	FTSE	S&P500	Bovespa	Nikkei225	HSI
Mean	0.0004	0.0001	0.0003	0.0003	0.0024	−0.0003	−0.0001
Std.Dev	0.0102	0.0099	0 0109	0.0112	0.02823	0.0147	0.0179
Min	−0.0757	−0.051	−0.071	−0.070	−0.172	−0.072	−0.147
Max	0.0483	0.0509	0.0694	0.0557	0.2883	0.0765	0.1725
Skewness	−0.549**	−0.226**	−0.355**	−0.073	0.5780**	0.078	0 176**
Kurtosis	4.465**	2.816**	5.061**	3.072**	8.631**	2.053**	9.242**

Note: **denotes the statistically significant at 1% level.

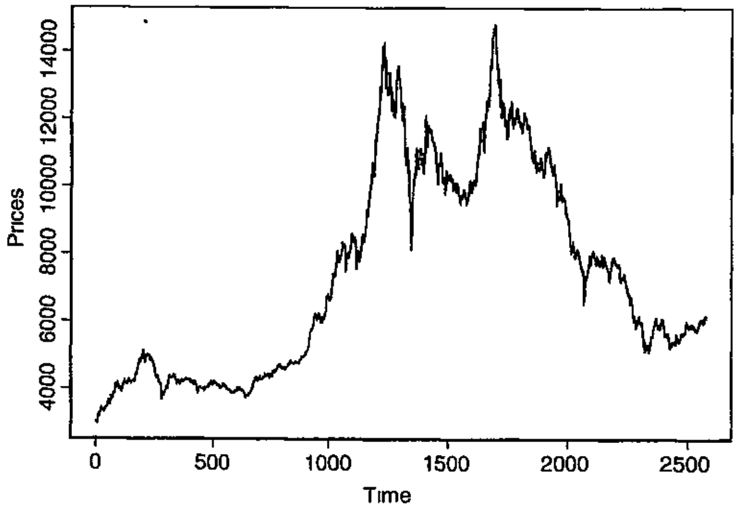


(a) ISEQ Overall index

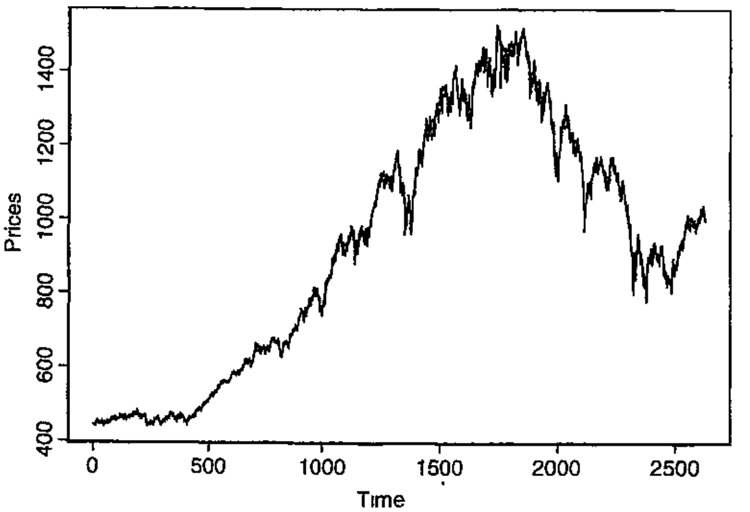
Fig. 1 The daily prices from May 1, 1993 to September 30, 2003.



(b) FTSE all share index

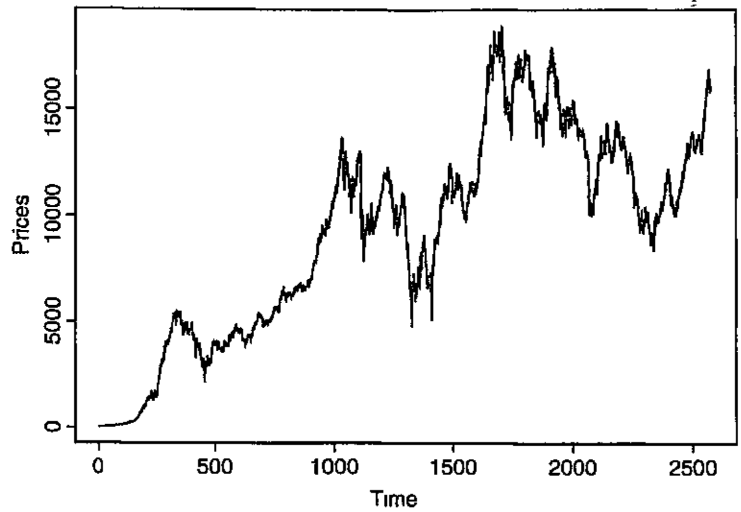


(c) PSI20 index

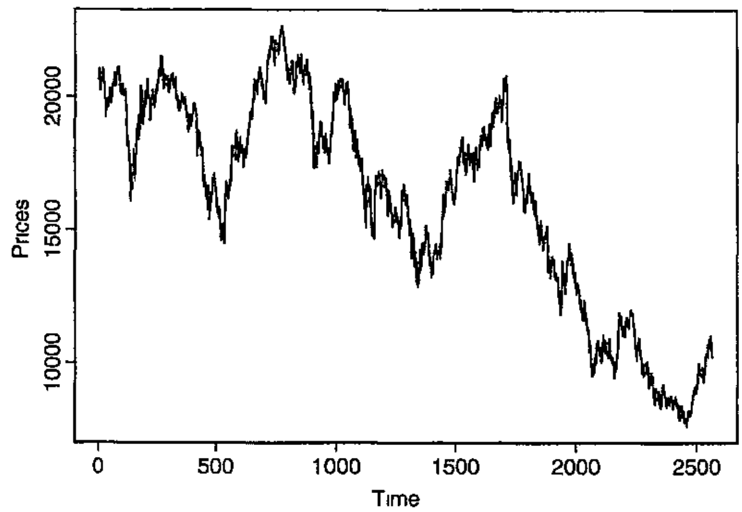


(d) S&P500 index

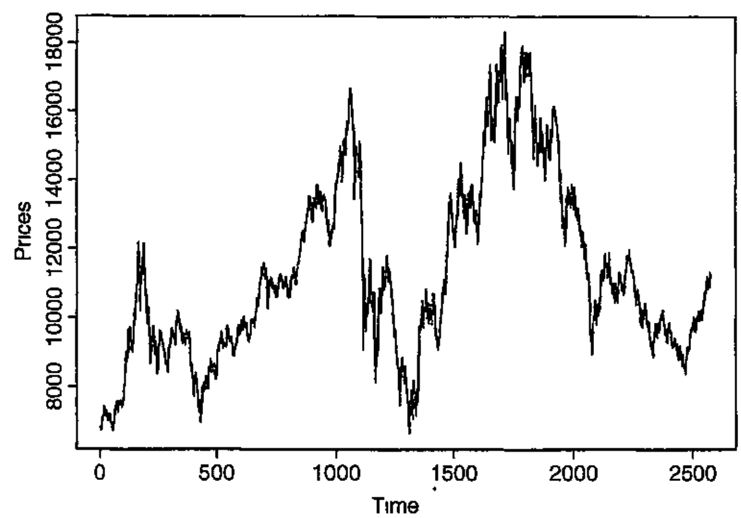
Fig. 1. (*Continued*)



(e) Bovespa index



(f) Nikkei 225 index



(g) HSI index

Fig. 1. (*Continued*)

Table 3. Percentages of energy by wavelet crystals for the daily returns of indices series.

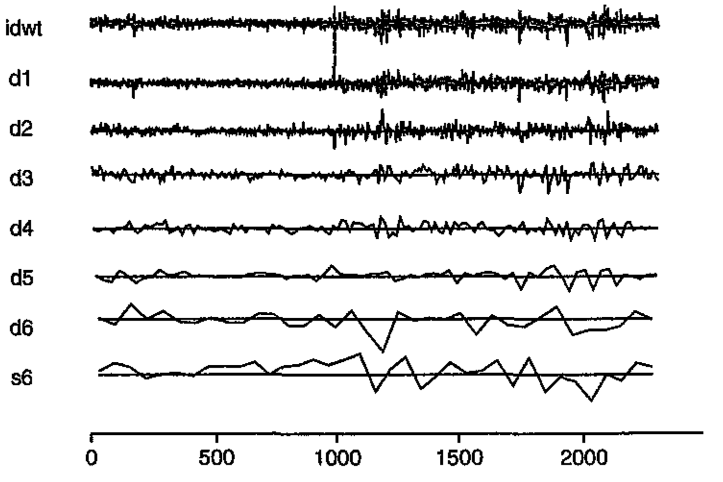
Index → W.Crystals↓	ISEQ	FTSE	PSI20	S&P500	Bovespa	Nikkei225	HSI
d_1	0.443	0.467	0.440	0.448	0.476	0.534	0.515
d_2	0.246	0.260	0.262	0.241	0.234	0.240	0.230
d_3	0.145	0.161	0.122	0.161	0.143	0.117	0.133
d_4	0.072	0.048	0.081	0.053	0.046	0.051	0.055
d_5	0.040	0.032	0.034	0.032	0.025	0.031	0.038
d_6	0.031	0.018	0.026	0.013	0.019	0 015	0.016
s_6	0.022	0.014	0.035	0.012	0.057	0.013	0.014

events such as the Hong Kong crash and Thailand crisis in 1997, "dot-com" in 2000 and September 11, 2001. Figures 1(f) and 1(g) represent the prices of Nikkei 225 and HSI indices respectively. These demonstrate that the Japanese market is the most sensitive, possibly because companies who have shares in the Japanese stock market tend to be exposed internationally and so price index levels respond to changes both directly and indirectly. The Hong Kong market is noticeably unstable with a disproportionately large number of regionwide crashes (possible due to serial crises: Bird Flu, SARS, etc). The Asian financial crisis had strong direct effects on the Hong Kong market but it affected Japan's economy only weakly because only 40% of Japan's exports go to Asia. In addition, Japan was going through its own ongoing long-term economic difficulties.

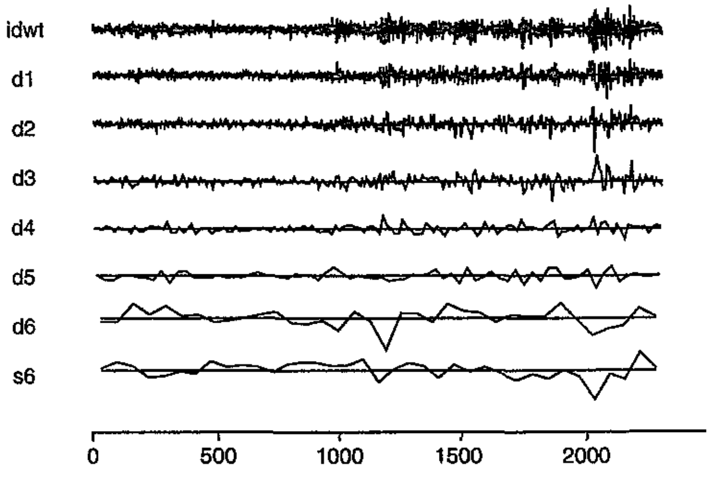
From the above, there are clear indications of effects from regionwide markets as well as from worldwide markets and this picture is more detailed when we look at the results of the wavelet analysis. The energy percentages described by each wavelet component for the daily returns of seven market indices are given in Table 3 which shows that the first two high frequency crystals (d_1 and d_2) explain more than 65% of the energy of these series, implying that movements are mainly caused by short-term fluctuations. Figures 2(a)–2(g) represent the discrete wavelet transform (DWT) for the daily returns of Irish, UK, Portuguese, US, Brazilian, Japanese and the Hong Kong stock market indices respectively. As mentioned, it can be seen that the first and the second wavelet components (d_1 and d_2) together account for most of the variations in the returns series.

3.2. *Empirical results*

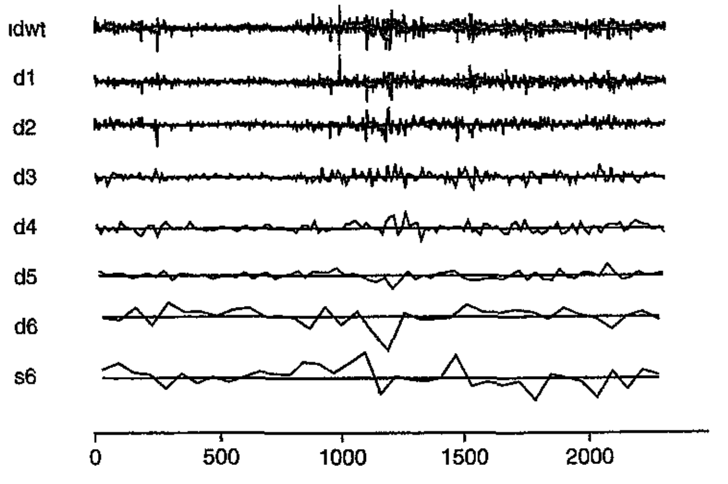
Traditionally, we might expect strong co-movements between nearest-neighbor markets. International stock markets such as those of Ireland and the UK are closely related, while there are strong historical links between Brazilian and Portuguese markets, for example.



(a) ISEQ Overall index

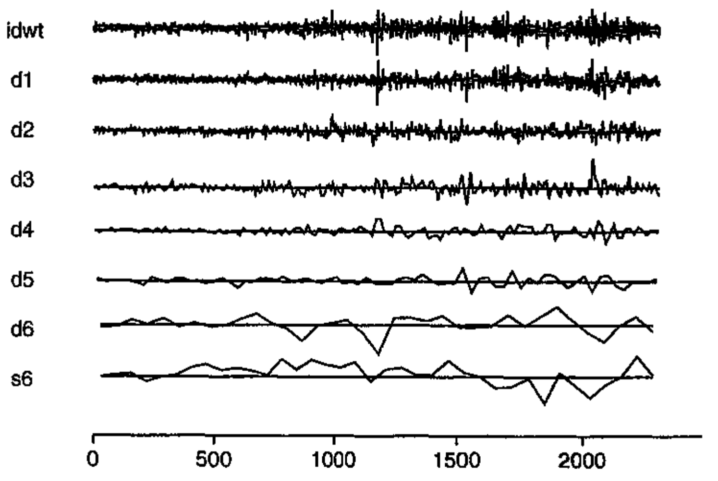


(b) FTSE all share index

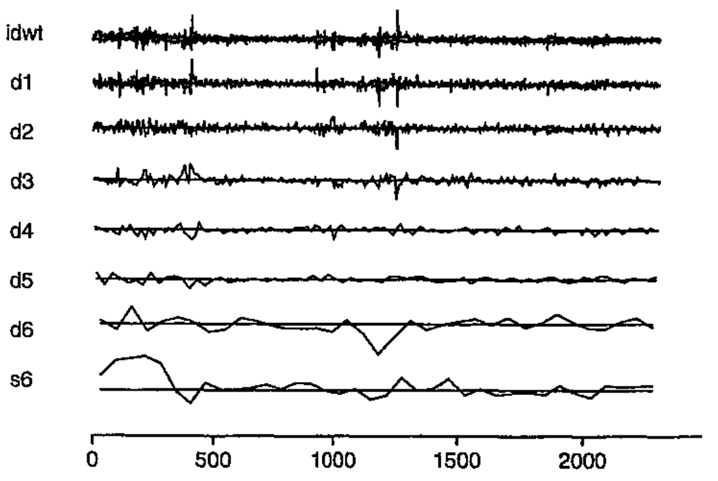


(c) PSI20 index

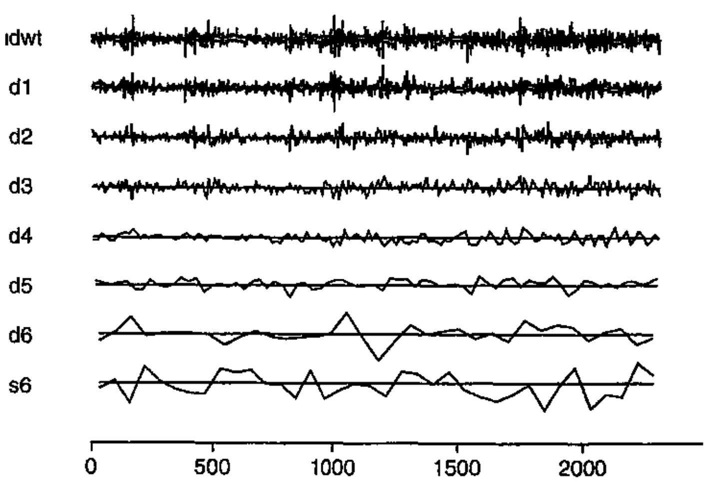
Fig. 2. The discrete wavelet transform (DWT) of daily returns versus time



(d) S&P500 index

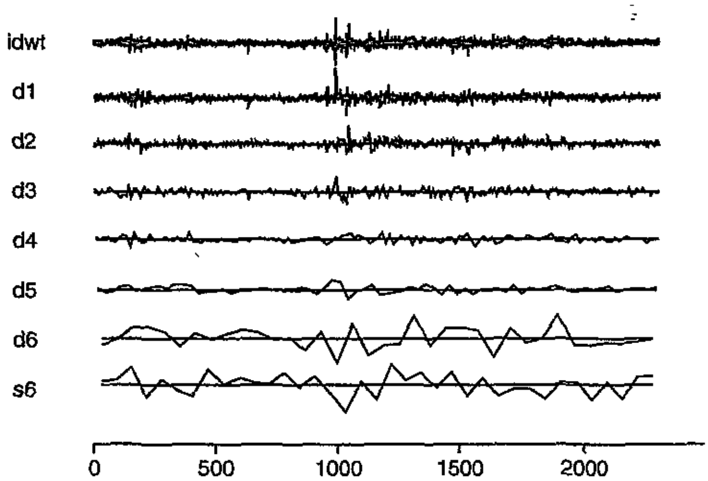


(e) Bovespa index



(f) Nikkei 225 index

Fig. 2. (*Continued*)



(g) HSI index

Fig. 2. (*Continued*)

To investigate the inter-relationships among all seven stock markets, we estimate simple regression and reverse regression models between each pair, using three different scales. These scales are row-returns series, where these are reconstructed from the first wavelet component (d_1) and the returns series, which are rebuilt from the first two wavelet crystals (d_1 and d_2) together. Conversely, we can not apply multiple regression (using forward or backward stepwise) to study the co-movements between the stock markets directly for two main reasons: firstly, multicollinearity problems are to be expected due to the relationships between the markets, secondly, we do not know the direction or order of the spillover effects.

From the results [R^2 and P-values of slopes] in Table 4, it can be seen that there are strong co-movements between each two of the Irish, UK and Portuguese markets, while the Irish market is also influenced by the US, Japan and Hong Kong. The UK and Portuguese markets are affected by both Japan and Hong Kong, while these are impacted upon by the US and Brazilian markets. Further, the UK and Portuguese markets influence the US and Brazil. Table 4 also shows that there is co-movement between US and Brazilian and also between the Japanese and Hong Kong markets (*nearest-neighbors*). No inter-relationships apparently exist between the Brazilian and either the Irish or Japanese markets, but the Brazilian market itself is significantly affected by that of Hong Kong. This implies that there is also an inner loop of "spillover effects" between Asian and American markets within the global circle, (southeast Asia to the Latin Americas). In other words, the US market affects those of Asian (Japanese and Hong Kong), which in turn impact on Brazil.

In order to get a clear picture of the historical linkage between Portuguese and Brazilian markets, we divided the whole period into three sub-periods (1993–1996, 1997–2000 and 2001–2003) and estimated the regression models between these

Table 4. Regression analysis between the daily returns of each pair of the seven stock market indices.

Panel A: IRL vs. UK

Regression→	M_t^{IRL} on M_t^{UK}			M_t^{UK} on M_t^{IRL}		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.80E-04 (0.029)	0.578 (0.000)	0.323	−1.74E-04 (0.310)	0.559 (0.000)	0.323
Return.D1	1.13E-04 (0.391)	0.467 (0.000)	0.222	−1.41E-04 (0.289)	0.477 (0.000)	0.222
Return.D1.2	1.49E-05 (0.951)	0.495 (0.000)	0.251	−4.32E-05 (0.864)	0.508 (0.000)	0.251

Panel B: IRL vs. P

Regression→	M_t^{IRL} on M_t^{P}			M_t^{P} on M_t^{IRL}		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.06E-04 (0.119)	0.343 (0.000)	0.135	1.50E-04 (0.477)	0.394 (0.000)	0.135
Return.D1	2.64E-05 (0.849)	0.359 (0.000)	0.145	7.06E-05 (0.631)	0.405 (0.000)	0.145
Return D1.2	1.97E-05 (0.940)	0.385 (0.000)	0.165	−6.98E-05 (0.801)	0.420 (0.000)	0.165

Panel C. IRL vs. US

Regression→	M_t^{IRL} on M_{t-1}^{US}			M_t^{US} on M_t^{IRL}		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.11E-04 (0.114)	0.328 (0.000)	0.129	1.99E-04 (0.378)	0.255 (0.000)	0.053
Return.D1	7.06E-05 (0.632)	0.154 (0.000)	0.031	2.12E-05 (0.900)	7.79E-04 (0.001)	0.004
Return.D1.2	−2.37E-05 (0.933)	0.217 (0 000)	0.024	−9.02E-05 (0.785)	7.75E-02 (0.001)	0.004

Panel D: IRL vs. BR

Regression→	M_t^{IRL} on M_{t-1}^{BR}			M_t^{BR} on M_t^{IRL}		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	2.79E-04 (0.184)	5.47E-02 (0.000)	0.023	2.31E-03 (0.000)	0.361 (0.000)	0.016
Return.D1	6.06E-05 (0.685)	1.67E-02 (0.036)	0.001	−3.70E-05 (0.928)	0.254 (0.000)	0.008
Return.D1.2	−8 65E-06 (0.976)	2.69E-02 (0.023)	0 002	−1.98E-04 (0.692)	0.122 (0.001)	0.004

Table 4. (*Continued*)

Panel E: IRL vs. JP

Regression→	$M_t{}^{IRL}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{IRL}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	4.77E-04 (0.019)	0.181 (0.000)	0.068	−4.06E-04 (0.183)	0.127 (0.000)	0.007
Return.D1	5.16E-05 (0.723)	0.147 (0.000)	0.052	6.76E-05 (0.771)	−8.18E-02 (0.010)	0.002
Return.D1.2	−1.18E-05 (0.996)	0.147 (0.000)	0.054	2.61E-05 (0.954)	−0.115 (0.024)	0.002

Panel F: IRL vs. HK

Regression→	$M_t{}^{IRL}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{IRL}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	4 19E-04 (0 036)	0.183 (0.000)	0.104	−4.91E-05 (0.895)	3.86E-02 (0.292)	0.000
Return.D1	4.88E-05 (0.731)	0.170 (0.000)	0.097	9.54E-05 (0.724)	−0.302 (0.000)	0.028
Return.D1.2	−3.13E-06 (0.991)	0.182 (0.000)	0.117	−1.48E-05 (0.978)	−0.396 (0.000)	0.018

Panel G. UK vs. P

Regression→	$M_t{}^{UK}$ on $M_t{}^{P}$			$M_t{}^{P}$ on $M_t{}^{UK}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−7.95E-05 (0.663)	0.438 (0.000)	0.228	2.83E-04 (0.155)	0.521 (0.000)	0.228
Return.D1	−1.55E-04 (0.243)	0.451 (0.000)	0.224	1.51E-04 (0.280)	0 498 (0.000)	0.224
Return.D1.2	−1.22E-05 (0.961)	0.481 (0.000)	0.252	−4.86E-05 (0.853)	0.524 (0.000)	0.252

Panel H: UK vs. US

Regression→	$M_t{}^{UK}$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{UK}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−2.03E-05 (0.919)	0.251 (0.000)	0.078	2.78E-04 (0.188)	0.473 (0.000)	0.178
Return.D1	−1.12E-04 (0.468)	1.13E-02 (0.537)	0.000	5.54E-05 (0.736)	0.262 (0.000)	0.054
Return.D1.2	−4.73E-05 (0.871)	−3.46E-03 (0.906)	0.000	−7.70E-05 (0.810)	0.292 (0.000)	0.065

Table 4. (*Continued*)

Panel I: UK vs. BR

Regression→	$M_t{}^{UK}$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^{UK}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−3.38E-05 (0.870)	3.71E-02 (0.000)	0.011	2.42E-03 (0.000)	0.659 (0.000)	0.054
Return.D1	−1.12E-04 (0.457)	−1.53E-02 (0.057)	0.001	3.49E-05 (0.931)	0.503 (0.000)	0.034
Return.D1.2	−4.76E-05 (0.870)	−1.38E-02 (0.250)	0.000	−1.86E-04 (0.707)	0.265 (0.000)	0.023

Panel J: UK vs. JP

Regression→	$M_t{}^{UK}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{UK}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	1.21E-04 (0.548)	0.178 (0.000)	0.068	−3.72E-04 (0.214)	0.292 (0.000)	0.039
Return.D1	−1.19E-04 (0.423)	0.113 (0.000)	0.030	5.73E-05 (0.805)	9.20E-02 (0.003)	0.003
Return.D1.2	−5.03E-05 (0.860)	0.124 (0.000)	0.037	2.42E-05 (0.957)	0.119 (0.018)	0.002

Panel K: UK vs. HK

Regression→	$M_t{}^{UK}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{UK}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	6.37E-05 (0.745)	0.187 (0.000)	0.112	−5.60E-05 (0.879)	0.349 (0.000)	0.038
Return.D1	−1.22E-04 (0.407)	0.133 (0.000)	0.058	7.07E-05 (0.796)	−4.37E-02 (0.239)	0.000
Return.D1.2	−4.39E-05 (0.877)	0.122 (0.000)	0.050	−3.17E-05 (0.953)	−6.80E-02 (0.257)	0.000

Panel L: P vs. US

Regression→	$M_t{}^{P}$ on $M_{t-1}{}^{US}$			$M_t{}^{US}$ on $M_t{}^{P}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	2.59E-04 (0.246)	0.174 (0.000)	0.031	2.21E-04 (0.323)	0.265 (0.000)	0.067
Return D1	9.76E-05 (0.539)	3.69E-02 (0.055)	0.001	9.36E-06 (0.955)	0.174 (0.000)	0.026
Return D1.2	−7.64E-05 (0.801)	4.11E-02 (0.178)	0.000	−7.62E-05 (0.815)	0.199 (0.000)	0.033

Table 4. (*Continued*)

Panel M: P vs. BR

Regression→	$M_t{}^P$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	1.94E-04 (0.391)	4.94E-02 (0.000)	0.015	2.30E-05 (0.000)	0.489 (0.000)	0.035
Return.D1	9.52E-05 (0.549)	9.93E-03 (0.240)	0.000	−5.20E-05 (0.899)	0.319 (0.000)	0.015
Return.D1.2	−7.35E-05 (0.809)	6.73E-03 (0.590)	0.000	−1.85E-04 (0.770)	0.188 (0.000)	0.013

Panel N: P vs. JP

Regression→	$M_t{}^P$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.60E-04 (0.106)	0.134 (0.000)	0.032	−4.00E-04 (0.188)	0.150 (0.000)	0.012
Return.D1	8.83E-05 (0.573)	0.113 (0.000)	0 027	5.95E-05 (0.798)	4.06E-02 (0.203)	0.000
Return.D1.2	−7.62E-05 (0.798)	0.125 (0.000)	0.034	1.32E-05 (0.977)	6.50E-02 (0.174)	0.000

Panel O. P vs. HK

Regression→	$M_t{}^P$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.19E-04 (0.144)	0.168 (0.000)	0.076	−7.77E-05 (0.834)	0.143 (0.000)	0.007
Return.D1	8.53E-05 (0.560)	0.143 (0.000)	0.060	7.29E-05 (0.789)	−0.141 (0.000)	0.006
Return.D1.2	−6.90E-05 (0.814)	0.150 (0.000)	0.070	−1.34E-02 (0.980)	−0.130 (0.023)	0.002

Panel P: US vs. BR

Regression→	$M_t{}^{US}$ on $M_t{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^{BR}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−1.85E-05 (0.933)	0.132 (0.000)	0.110	2.20E-03 (0.000)	0.841 (0.000)	0.110
Return.D1	2.84E-05 (0.861)	0.114 (0.000)	0.076	−3.91E-05 (0.921)	0.674 (0.000)	0.077
Return.D1.2	−6.03E-05 (0.852)	0.154 (0.000)	0.054	−1.67E-04 (0.732)	0.351 (0.000)	0.054

Table 4. (*Continued*)

Panel Q: US vs. JP

Regression→	$M_t{}^{US}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{US}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.31E-04 (0.152)	7.45E-02 (0.000)	0.009	−4.77E-04 (0.101)	0.401 (0.000)	0.092
Return.D1	2.92E-05 (0.862)	−5.37E-02 (0.000)	0.005	8.15E-05 (0.718)	0.322 (0.000)	0.056
Return.D1.2	−8.97E-05 (0.991)	−5.10E-02 (0.786)	0.004	−8.93E-06 (0.994)	0.441 (0.000)	0.041

Panel R. US vs. HK

Regression→	$M_t{}^{US}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{US}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	3.07E-04 (0.184)	6.78E-02 (0.000)	0.011	−2.01E-04 (0.566)	0.541 (0.000)	0.114
Return.D1	2.97E-05 (0.860)	−5.50E-02 (0 000)	0.007	9.62E-05 (0.719)	0.401 (0.000)	0.063
Return.D1.2	−9.25E-05 (0.779)	−5.50E-02 (0.000)	0.008	−7.39E-05 (0.888)	0.630 (0.000)	0.058

Panel S: BR vs. JP

Regression→	$M_t{}^{BR}$ on $M_t{}^{JP}$			$M_t{}^{JP}$ on $M_{t-1}{}^{BR}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−5.34E-04 (0.079)	7.33E-02 (0.000)	0.019	2.51E-03 (0.000)	0.154 (0.000)	0.006
Return.D1	6.06E-05 (0.794)	5.79E-02 (0.000)	0.009	−2.31E-05 (0.955)	2.49E-02 (0.498)	0.000
Return.D1.2	2.16E-05 (0.962)	7.22E-02 (0.000)	0.006	−1.99E-04 (0.691)	4.43E-03 (0.848)	0.000

Panel T: BR vs. HK

Regression→	$M_t{}^{BR}$ on $M_t{}^{HK}$			$M_t{}^{HK}$ on $M_{t-1}{}^{BR}$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−3.30E-04 (0.396)	0.121 (0.000)	0.036	2.46E-03 (0.000)	0.142 (0.000)	0.008
Return.D1	6.92E-05 (0.798)	9.99E-02 (0.000)	0.020	−2.03E-5 (0.961)	−1.85E-2 (0.554)	0.000
Return.D1.2	−3.03E-05 (0.955)	0.126 (0.000)	0.014	−1.99E-04 (0.691)	−9.94E-03 (0.609)	0.000

Table 4. (*Continued*)

Panel U: JP vs. HK

Regression→	M_t^{JP} on M_t^{HK}			M_t^{HK} on M_t^{JP}		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−3.44E-04 (0.231)	0.283 (0.000)	0.119	1.16E-04 (0.741)	0.421 (0.000)	0.119
Return.D1	4.09E-05 (0.852)	0.284 (0.000)	0.111	4.54E-05 (0.860)	0.393 (0.000)	0.111
Return.D1.2	3.06E-05 (0.942)	0.297 (0.000)	0.126	−3.94E-05 (0.938)	0.424 (0.000)	0.126

markets using three different scales. The results are given in Table 5 and show no co-movement between Portugal and Brazil in the first period while there is significant evidence of co-movement between these markets from 1997 to 2000. However, in the third period, the results show that there are spillover effects from the Portuguese market on to the Brazilian market, but not *vice versa*. This appears to provide supporting evidence for an increase in the international transmission mechanism among stock markets.

Finally, it seems clear from the values of the coefficients for each pair of regressions that directional influence is *globally clockwise* starting with Asian markets influencing European, European impacting on the Americas and the circle completing with American market changes impacting on those of Asian. Interestingly, only the Japanese market demonstrates mixed influences. Possible explanations can be put forward for these findings on global inter-dependence and circular spillover effects between the stock markets in different Continents as follow:

- Many firms with shares in these stock market indices are international investors.
- Different time-zones mean that trading is concluded in Asia prior to opening in Europe and similarly for Europe to America and back again to Asia. These spillover effects are noticeable on the markets which open next, but these effects become less-marked for the next global cohort.
- Global investment may imply similar actions on prices throughout.

4. Conclusion

The aim of this work was to investigate the inter-relationships between seven international stock markets namely the Irish, UK, Portuguese, US, Brazilian, Japanese and Hong Kong based on daily returns. A new testing method suggested by Lee [11] has been applied and our results show that there are significant co-movements between each European pair separately, between the US and Brazilian markets and also between the Japanese and Hong Kong markets. In addition, the indications are that there are significant spillover effects from the UK and Portuguese markets onto the US and Brazilian markets which in turn, themselves influence the

Table 5. Regression analysis between Portuguese and Brazilian markets using three different scales.

Panel A. From 1993 to 1996

Regression→	$M_t{}^P$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	5.31E-04	1.36E-02	0.001	2.75E-03	5.54E-02	0.001
	(0.000)	(0.401)		(0.000)	(0.430)	
Return.D1	2.04E-07	−4.42E-02	0.010	−4 14E-07	0.139	0.003
	(0.999)	(0.003)		(0.999)	(0.088)	
Return.D1.2	7.95E-07	−6.37E-03	0.001	1.12E-07	−1.84E-02	0.001
	(0.996)	(0.669)		(1 000)	(0.812)	

Panel B. From 1997 to 2000

Regression→	$M_t{}^P$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	5.28E-04	0.272	0.062	1.91E-04	0.270	0.085
	(0.222)	(0.000)		(0.956)	(0.000)	
Return.D1	7.95E-07	9.23E-02	0.010	−4.77E-07	0.136	0.016
	(0.998)	(0.002)		(0.999)	(0.000)	
Return.D1.2	1.87E-06	0.181	0.032	−7.47E-07	0.224	0.054
	(0.996)	(0.000)		(0.998)	(0.000)	

Panel C. From 2001 to 2003

Regression→	$M_t{}^P$ on $M_{t-1}{}^{BR}$			$M_t{}^{BR}$ on $M_t{}^P$		
Scales↓	Constant	Slope	R^2	Constant	Slope	R^2
Return	−7.35E-04	0.259	0.041	2.40E-04	0.212	0.070
	(0.072)	(0.000)		(0.455)	(0.001)	
Return.D1	−2.16E-06	−1.95E-02	0.000	−4.42E-06	0.148	0.032
	(0.994)	(0.677)		(0.984)	(0.000)	
Return.D1.2	1.80E-06	0.164	0.016	−2.10E-06	0.184	0.048
	(0.996)	(0.001)	(0.994)	(0.000)		

where

- P-values of t-tests are given in parentheses.
- M^{IRL}, M^{UK}, M^P, M^{US}, M^{BR}, M^{JP} and M^{HK} are indicators of Irish, UK, Portuguese, US, Brazilian, Japanese and the Hong Kong market indices respectively.
- Return is an indicator of the row daily returns series.
- Return.D1 is an indicator of the returns series reconstructed by using the first wavelet crystal.
- Return.D1 + D2 is an indicator of the returns series reconstructed by using the first and the second wavelet crystals together.

Asian markets. In turn, Japan and Hong Kong impact the Europe. Finally, we can summarize our results in the following:

1. There are co-movements between regionwide markets (*nearest-neighbor* or *intra-continental* relationships).
2. There are *clockwise* transmissions between worldwide markets.

3. There is an increase in importance of global co-movements among worldwide stock markets, in particular since the end of the 20th century.
4. The effect of the advent of modern communications can be seen since the mid 1990s in term of more rapid response and/or damping of effects on global patterns.

References

[1] A. Antoniou, G. Pescetto and A. Violaris, Modelling international price relationships and interdependencies between the stock index and stock index futures markets of three EU countries: A multivariate analysis, *Journal of Business Finance* **30**(5&6) (2003) 645–667.

[2] D. A. Bessler and J. Yang, The structure of interdependence in international stock markets, *Journal of International Money and Finance* **22** (2003) 261–287.

[3] G. G. Booth, T. Martikainen and Y. Tse, Price and volatility spillovers in Scandinavian stock markets, *Journal of Banking and Finance* **21** (1997) 811–823.

[4] R. Brook and M. D. Negro, Firm-level evidence on international stock market comovement, International Monetary Fund Working Papers, No: 03/55, Washington, DC, USA (2003).

[5] A. Bruce and H. Y. Gao, *Applied Wavelet Analysis with S-Plus* (Springer-Verlag, New York, 1996).

[6] CVM, International transmission of stock market volatility-spillover effect on Latin American markets, the IOSCOs Emerging Markets Annual Meeting on Management of Volatility in Turbulent Markets, Kuala Lumpur, Malaysia (1998).

[7] C. S. Eun and S. Shim, International transmission of stock market movements, *Journal of Finance and Quantitative Analysis* **24**(2) (1989) 241–256.

[8] Z. Gonghui, J. L. Starck, J. Campbell and F. Murtagh, The wavelet transform for filtering financial data stream, available from strule.cs.qub.ac.uk/~gzheng/financial-engineering/finpapermay99.html.

[9] H. E. Hijmans, Discrete wavelet and multiresolution analysis, in *Wavelets: An Elementery Treatment of Theory and Application*, T. H. Koornwinder, ed. (World Scientific Publishing, Singapore, 1993), pp. 49–79.

[10] S. W. Kim and J. H. Rogers, International stock price spillovers and market liberalization: Evidence from Korea, Japan, and the United States, *Journal of Empirical Finance* **2** (1995) 117–133.

[11] H. S. Lee, International transmission of stock market movements: A wavelet analysis on MENA stock market, Economic Research Forum, ERF Eighth Annual Conference, Cairo, Egypt (2002).

[12] W. Lin, R. F. Engle and T. Ito, Do bulls and bears move across borders? International transmission of stock returns and volatility, *Review of Financial Studies* **7**(3) (1994) 507–538.

[13] A. Ng, Volatility spillover effect from Japan and the US to the Pacific-Basin, *Journal of International Money and Finance* **19** (2000) 207–233.

[14] J. Wongswan, Transmission of information across international equity markets, International Finance Discussion Papers, 759, Board of Governors of the Federal Reserve System.

Apples and Oranges: the difference between the Reaction of the Emerging and Mature Markets to Crashes.

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Summary.

We study here the behavior of the eigenvalues of the covariance matrices of returns for emerging and mature markets at times of crises. Our results appear to indicate that mature markets respond to crashes differently to emerging ones and that emerging markets take longer to recover than mature markets. In addition, the results appear to indicate that the *second largest* eigenvalue gives additional information on market movement and that a study of the behavior of the other eigenvalues may provide insight on crash dynamics.

keyword. Covariance Matrix, Eigenvalues and Stock Crashes.

Introduction.

Recently, several studies have applied the concepts and methods of physics to the areas of economics and finance, particularly to the study the covariance (or correlation) between price changes (returns) of different stocks [e.g. Meric and Meric (1997), Kwapien et al. (2002), Keogh et al. (2003) and Kwapien et al. (2004)]. Thus far, the magnitude of the *maximum eigenvalue* of the correlation (or covariance) matrices for different sectors in one stock market index only, has predominantly been studied with no attention paid to the other eigenvalues. The differences in the current work are twofold; firstly, to highlight the information obtained from the subdominant eigenvalue as well as the dominant eigenvalue and study their behaviour. Secondly, to compare this for stock market indices for two different classes, namely emerging and mature markets.

Our objectives in this article are thus; **(a)** To study the distribution of the eigenvalues of the Covariance matrices for equal-interval sliding windows, including the week before the Crisis, together with those of Covariance matrices for windows, including both the week of the Crisis and a week after. This, in order to see the qualitative *difference* between emerging and mature markets to crashes in term of the eigenvalues (the λ's). **(b)** To study the distribution of the ratio of the *largest* to the *second largest* eigenvalue of the Covariance matrices for sliding windows of equal sizes. This, we believe, a measure of the degree of agreement (or coherence) in agent views of the market.

The remainder of this paper is organized as follows: The method of estimating the Covariance matrices is described briefly below (Section 2), with data and results presented in Section 3. Our brief discussion and conclusions form the final section.

Covariance matrix estimation.

The Variance-Covariance matrix can be computed easily, using the following formula, (full details see Litterman and Winkelmann (1998)):

$$\sigma_{ij}^{T}(M) = (\sum_{s=0}^{T} \omega_{T-s} r_{i,T-s} r_{j,T-s}) / (\sum_{s=0}^{T} \omega_{T-s}) \qquad (1)$$

Where $r_{(i,T)}$ is the return on the i^{th} market at date T and ω_T is the weight applied at date T over horizon M. In our study, we use weekly returns of stock market indices (i=13 indices and T=20 for emerging and i=14 indices and T=20 for major markets for our data) and each week, previous to the current, receives 90% of the weight of the following week (where $\omega_T=1$) as suggested in e.g. Litterman and Winkelmann.

Data and Results.

The data used in the following analysis consists of the weekly prices of a set of thirteen **emerging** market indices and a set of fourteen **mature** market indices during the period from the second week of January 1997 to the third week of March 2003. As each market uses its local currency for presenting the index values, we use the weekly *returns* instead of the weekly prices, where the following formula applies: **Weekly Return = $\mathbf{Ln(P_t/P_{t-1})}$,** where P_t and P_{t-1} are the closing prices of the index at week t and t-1 respectively. The Variance-Covariance matrices for overlapping windows of size 20 weeks have been calculated using Equation (1).

Empirical results.

Figures 1 and 2, for the emerging and mature markets respectively, show the distribution of the eigenvalues of the Covariance matrices for overlapping windows of size 20, before and after the Asian Crisis in July 1997, the Global

Crisis in October 1998, the Dot-Com Crash in March 2000 and the September the 11^{th} Crash in 2001.

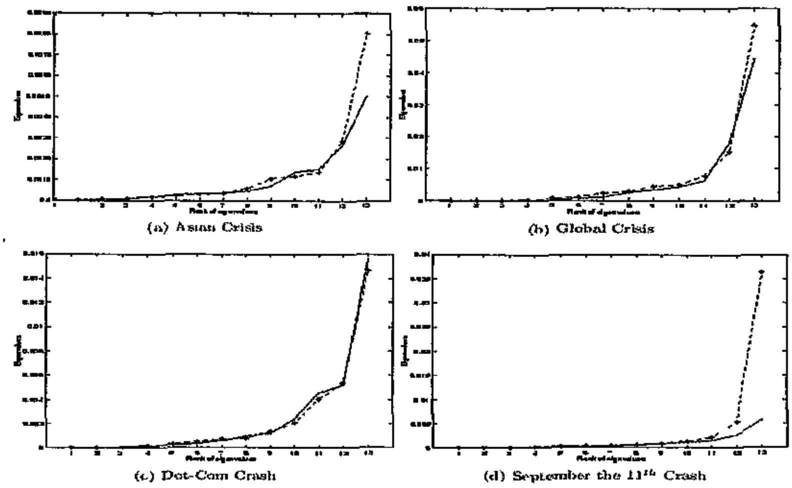


Fig. 1. The distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) the crash for Emerging markets[1].

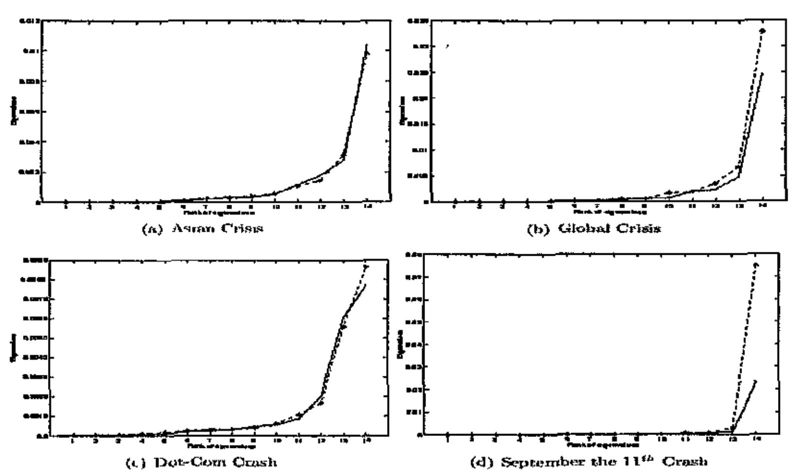


Fig. 2. The distribution of the eigenvalues of the covariance matrices before (**Solid line**) and after (**Dashed line**) the crash for Mature markets[1].

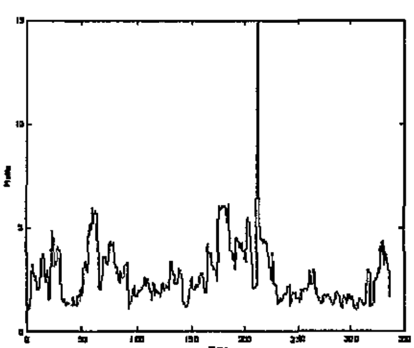
[1] In figures 1 and 2, the **Eigenvalues** are given on the **y-axis** while their **Ranks** are given on the **x-axis**

Figures 1(a) and 2(a) show that the value of the maximum eigenvalue (λ_1) increased, for emerging markets, after the Asian Crisis, which began in July 1997 in Thailand, but did not change markedly for developed markets. This implies that the crisis mainly affected emerging markets but not the mature ones. However, Figures 1(c) and 2(c) show that the Dot-Com Crash influenced major markets but not emerging ones and took longer than a week to show a strong effect.

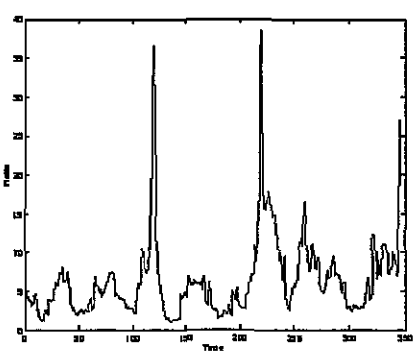
From Figures 1(b) and 2(b), we can see that the Global Crisis in 1998 affected emerging and mature markets comparably in the same week.

Figures 1(d) and 2(d) show that the value of λ_1 after the September 11th crash, which could not have been predicted by most people, hugely increased for **both** emerging and mature markets. This implies that stock markets around the world were hit very hard and that the markets moved in *coordination* to make a recovery after falling so sharply or being oversold.

The ratio of the *Largest* (λ_1) to the *Second Largest* (λ_2) eigenvalues of the Covariance matrices for emerging and mature markets are shown in Figures 3(a) and 3(b) respectively. These show a *qualitative difference* in the way emerging and mature markets deal with crises, (especially unexpected ones). For major markets, there are three highly significant points in the distribution of this ratio representing the third week of October 1999 (the 12th anniversary of the October 19 stock market crash)}, the second week of September 2001 *(9/11 crash)* and the third week of March 2004 *(Madrid Bomb)* respectively. However, for emerging markets, there is only one highly significant point representing the second week of September 2001 *(9/11 crash)*.



(a) Emerging Markets



(b) Major Markets

Fig. 3. The distribution of ratio of *Dominant* (λ_1) to *Subdominant* (λ_2) eigenvalues of covariance matrices for equal overlapping time windows

The results also show that the mature markets move together immediately after the crash to bounce back faster than emerging markets. In other words, the recovery time from crisis for developed markets is shorter than that for developing ones.

Conclusion.

Our aims were to study the distribution of the eigenvalues of covariance matrices for emerging and mature markets at crisis points (namely, the Asian Crisis, Global Crisis, Dot-Com Crash and September the 11^{th} Crash). In particular, we wished to distill the information from the ratio of the Largest to the Second Largest eigenvalues of these covariance matrices. Our findings can be summarized as follows: **(i)** The Asian Crisis in 1997 disproportionately affected the emerging markets compared to the major ones while the Dot-Com Crash influenced major markets but affected emerging ones far less. **(ii)** The Global Crisis in 1998 affected developing markets as much as developed ones in the same week. **(iii)** The September 11^{th} Crash hit both emerging and mature markets very hard because it was totally unpredictable. **(iv)** The distribution of the ratio of λ_1 to λ_2 appears to show that emerging and mature markets deal with crashes differently especially unexpected ones. This means that mature markets move together immediately after the crash to bounce back faster than emerging markets In other words, the recovery time from crisis for emerging markets is longer than that for mature ones.

REFERENCES.

Keogh G, Sharifi S, Ruskin H, Crane M, (2003) Epochs in Market Sector Index Data - Empirical or Optimistic?. Proceedings of the 2nd Nikkei Econophysics Symposium - Application of Econophysics, Takayasu, H. (Eds), Lecture Notes in Computer Science, Springer, November, 2003, pp 83-89, ISBN 4-431-14028-X

Kwapien J, Drozdz S, Speth J (2002) Alternation of Different Scaling Regimes in the Stock Market Fluctuation, (Available from www.fz-juelich.de/ikp/ publications/AR2002/CHAP4/409.pdf}, [Accessed 11 May 2004]

Kwapien J, Drozdz S, Speth J (2004) Time Scale involved in Emergent Market Coherence Physica A, Vol 337, pp 231-242.

Litterman R, Winkelmann K (1998) Estimating Covariance Matrices: in Goldman-Sachs Risk Management Series, Krieger R, (Eds), Goldman, Sachs & Co

Meric I, Meric G. (1997) Co-movements of European Equity Markets before and after the 1987 crash, Multinational Finance Journal, Vol 1(2), pp 137-152.

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The reaction of stock markets to crashes and events: A comparison study between emerging and mature markets using wavelet transforms

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Abstract

We study here the behaviour of the first three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and their ratios $[(\lambda_1/\lambda_2), (\lambda_1/\lambda_3), (\lambda_2/\lambda_3)]$ of the covariance matrices of the original return series and of those rebuilt from wavelet components for emerging and mature markets. It has been known for some time that the largest eigenvalue (λ_1) contains information on the risk associated with the particular assets of which the covariance matrix is comprised. Here, we wish to ascertain whether the subdominant eigenvalues (λ_2, λ_3) hold information on the risk of the stock market and also to measure the recovery time for emerging and mature markets. To do this, we use the *discrete wavelet transform* which gives a clear picture of the movements in the return series by reconstructing them using each wavelet component. Our results appear to indicate that mature markets respond to crashes differently to emerging ones, in that emerging markets may take up to two months to recover while major markets take less than a month to do so. In addition, the results appears to show that the *subdominant* eigenvalues (λ_2, λ_3) give additional information on market movement, especially for emerging markets and that a study of the behaviour of the other eigenvalues may provide insight on crash dynamics.

Keywords Variance–covariance matrix; Eigenvalues and wavelet analysis

1. Introduction

Covariance (Correlation) matrix forecasts of financial asset returns are an important component of current practice in financial risk management with a large bibliography on the subject. Meric and Meric [1], for example, applied the Box M method and principal component analysis (PCA) to test whether or not the correlation matrices before and after the international crash in 1987 were significantly different. This was done in order to investigate the changes in the long-term co-movements of twelve European and US equity markets.

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Their results showed that there are significant alterations in the co-movements of these markets and that the benefits of international diversification for the European markets decreased markedly after this crash.

Further, Kwapien et al. [2] investigated the distribution of eigenvalues of correlation matrices for equally separated time windows in order to study, quantitatively the relation between stock price movements and properties of the distribution of the corresponding index motion (w.r.t. German DAX). They reported that the importance of an eigenvalue is related to the correlation strength of different stocks, which means that the more aggregated the market behaviour, the larger λ_1 (maximum eigenvalue).

Recently, in Keogh et al. [3], we showed that periods in market sector data from the Dow Jones EURO-STOXX index, exist linearly with time. These results supported an implied relationship between volatility and the change in magnitude of the dominant eigenvalue and also showed that *epochs* seem to exist in all market sectors although in different degrees. More recently, Kwapien et al. [4] analysed tick-by-tick returns data ranging from seconds up to 48 h from the NYSE and the German markets. The authors compared the magnitude of the dominant eigenvalue of the correlation matrices for the same group of securities on various time scales. Their results indicated that collective market behaviour appears at significantly shorter time scales in recent times.

Pafka and Kondor [5] examined the effect of noisy covariance matrices on the portfolio optimization problem and found that the risk of the portfolio in the presence of noise in these matrices is 5–15% higher than in the absence of noise, indicating that the decrease in efficiency of the optimal portfolio is actually much less dramatic.

According to the findings of Galluccio et al. [6], Laloux et al. [7], Plerou et al. [8], Laloux [9], Plerou et al. [10], Wilcox and Gebbie [11,12] and Sharifi et al. [13], the correlation (or covariance) matrices of financial time series, apart from a few large eigenvalues and their corresponding eigenvectors, appear to contain such a large amount of noise that their structure can essentially be regarded as random. This means that a few of the larger eigenvalues might carry collective information. However, most previous studies [14] have focused on the largest eigenvalue with no attention paid to the others. If we are to presume that, as with any PCA analysis of data, there are several principal components that are significant, then it should be worth examining lesser order components to see if they can provide additional data for investment strategies. References in the literature to the role of higher-order eigenmodes in investment strategy are scarce, but, recently Wilcox and Gebbie [11,12] have examined the composition of all the eigenmodes of ten years of Johannesburg Stock Exchange using random matrix theory (RMT). The authors concluded that "the leading [i.e., first three] eigenmodes may be interpreted in terms of independent trading strategies with long range correlations" indicating a role not just for λ_1 but also for a small number of the dominant eigenvalues. In the current work we aim, firstly, to highlight the apparent information obtained from the first two subdominant eigenvalues as well as the dominant eigenvalue and study its behaviour and secondly, to compare the behaviour of the second and third eigenvalues for stock market indices for two different classes, namely emerging and mature markets.

Several studies have made comparisons between emerging and mature markets,[1] according to different characteristics, and these generally have reported that emerging markets consistently behave differently from mature ones. Patel and Sarkar [15] studied eight mature and ten developing markets from 1970 to 1997. The authors found important differences in the characteristics of stock market crises between major and emerging markets. They also found that, for emerging markets, the decline in prices following crises is larger than that for mature markets, and the recovery time is longer. Further, Fuss [16] used discriminant analysis to investigate if emerging and mature markets behave differently according to different financial aspects (such as market pricing, market size and market activity) and stated that the difference between these two market types has increased since the end of the 1990s. A reason for this could be found in financial crashes of 1994 in Mexico, 1997/1998 in Asia, 1998 in Russia and 1999 in Brazil, indicating that emerging and mature markets deal differently with crashes and crises.

Recently, Salomons and Grootveld [17] studied the equity risk premium in thirty-one global stock markets using standard statistical approaches and found that emerging markets carry a higher equity risk premium

[1]The International Finance Corporation (IFC) uses income per capita and market capitalization relative to GNP for classifying equity markets If either (1) a market resides in a low- or middle-income economy, or (2) the ratio of investable market capitalization to GNP is low, then the IFC classifies the market as emerging otherwise IFC classifies it as mature

than mature ones indicating that they are perceived to be riskier. More recently, Wooldridge et al. [18] considered the changes in the links between emerging and mature markets according to capital flows, the investor base and the changing character of global banks. Results showed that emerging and mature markets are more integrated nowadays than before. This contravenes with the findings of Patel and Sarkar [15], Fuss [16], Salomons and Grootveld [17] and Wooldridge et al. [18], so it is very important to carry out further investigations in order to clarify the issue.

In Sharkasi et al. [19], we studied the behaviour of eigenvalues of the covariance matrices around crashes and also studied the ratio of the dominant (λ_1) to the subdominant (λ_2) for emerging and mature markets. Our results showed that mature markets react to crashes in a different way than emerging ones which take longer to recover than mature markets. The second largest eigenvalue (λ_2) may thus be expected to provide additional information on market movements.

Our objectives in this article are, therefore as follows:

1. To study the variation of the ratio series of the *largest* (λ_1) to the *second and third largest* (λ_2, λ_3) eigenvalues of the variance–covariance matrices for sliding windows of equal sizes for original return series of stock market indices, thus, in order to compare the behaviour of this ratio across windows with different degrees of risk (or different crashes and events).
2. To study the previous point for return series which have been reconstructed using each wavelet component separately in order to measure how long the markets take to recover and how long these markets retain information about previous crises and events.
3. To study the variation of the *largest* and the *second largest* eigenvalues of these covariance matrices for the original return series, and for those rebuilt from wavelet components, in order to see the direction of the movements in these markets and also to investigate whether λ_2 contains useful information about these movements, in addition to that described by λ_1 alone.

The remainder of this paper is organized as follows: The methodology used here is described briefly below (Section 2), with data and results presented in Section 3. The final section provides discussion and conclusion.

2. Methodology

2.1. How to estimate covariance matrices

There are several methods to compute the variance–covariance matrix but there is no agreement among authors on an optimal one. We have chosen the following formula because it uses weighted historical data to account for the empirical regularities of financial time series (such as the fact that volatility and correlation vary over time and these series have a Fat Tail distribution).

$$\sigma_{ij}^{T}(M) = \left(\sum_{s=0}^{T} \omega_{T-s} r_{i,T-s} r_{j,T-s} \right) \Bigg/ \left(\sum_{s=0}^{T} \omega_{T-s} \right), \tag{1}$$

where $r_{i,T}$ is the return on the ith market at date T and ω_T, which is the weight applied at date T over horizon M, has been chosen to be a declining function of time. The more recent observations are given more weight than observations that occurred in more distant past, where 100% weight is given to the most recent week and each week in history receives 90% of the weight of the following week, then $\omega_T = 1.0$, $\omega_{T-1} = 0.90$, $\omega_{T-2} = 0.81$, etc. (For more details see [20]).

In our study, we use weekly returns of a set of 13 ($i = 13$) emerging indices and a set of 14 ($i = 14$) mature indices and the variance–covariance matrices for overlapping windows of size 20 ($T = 20$) weeks for our data have been calculated using Eq. (1) in order to study the structure change of stock market for different windows with different risk degree (i.e., after including the week of crash).

2.2. Definition of wavelet transform

The wavelet transform (WT) is a mathematical tool that can be applied to many applications such as image analysis, and signal processing. It was introduced to solve problems associated with the Fourier transform, as they occur when dealing with non-stationary signals, or when dealing with signals which are localized in time or space as well as frequency. The wavelet transform has been explained in more detail, particularly by Hijmans [21], Bruce and Gao [22] and Gonghui et al. [23].

In particular, the discrete wavelet transform (DWT) is useful in dividing the data series into components of different frequency, so that each component can be studied separately to investigate the data series in depth. The wavelets have two types, *father wavelets* ϕ and *mother wavelets* ψ, where

$$\int \phi(t)\,\mathrm{d}t = 1 \quad \text{and} \quad \int \psi(t)\,\mathrm{d}t = 0.$$

The smooth and low-frequency parts of a signal are described by using the father wavelets, while the detail and high-frequency components are described by the mother wavelets. The orthogonal wavelet families have four different types which are typically applied in practical analysis, namely, the *haar*, *daublets*, *symmlets* and *coiflets*.

The following brief synopsis of their features is relevant to the analysis reported:

- The haar has compact support and is symmetric but, unlike the others, is not continuous.
- The daublets are continuous orthogonal wavelets with compact support.
- The symmlets have compact support and were built to be as nearly symmetric as possible.
- The coiflets were built to be nearly symmetric.

A two-scale dilation equation, used to calculate the wavelets, father $\phi(t)$ and mother $\psi(t)$, is defined, respectively, by

$$\phi(t) = \sqrt{2}\sum_k \ell_k \phi(2t-k), \tag{2}$$

$$\psi(t) = \sqrt{2}\sum_k \hbar_k \phi(2t-k), \tag{3}$$

where ℓ_k and $\hbar_k$ are the low-pass and high-pass coefficients given by

$$\ell_k = \frac{1}{\sqrt{2}}\int \phi(t)\phi(2t-k)\,\mathrm{d}t, \tag{4}$$

$$\hbar_k = \frac{1}{\sqrt{2}}\int \psi(t)\phi(2t-k)\,\mathrm{d}t. \tag{5}$$

The orthogonal wavelet series approximation to a signal $f(t)$ is defined by

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t), \tag{6}$$

where J is the number of multi-resolution levels (or scales) and k ranges from one to the number of coefficients in the specified components (or crystals). The coefficient $s_{J,k}, d_{J,k}, \ldots, d_{1,k}$ are the wavelet transform coefficients given by

$$s_{J,k} = \int \phi_{J,k}(t) f(t)\,\mathrm{d}t, \tag{7}$$

$$d_{j,k} = \int \psi_{j,k}(t) f(t)\,\mathrm{d}t \quad (j = 1, 2, \ldots, J). \tag{8}$$

Their magnitudes give a measure of the contribution of the corresponding wavelet function to the signal. The functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ $[(j=1,2,\dots,J]$ are the approximating wavelet functions, generated from ϕ and ψ through scaling and translation as follows:

$$\phi_{J,k}(t) = 2^{-J/2}\phi(2^{-J}t-k) = 2^{-J/2}\phi[(t-2^{J}k)/2^{J}], \tag{9}$$

$$\psi_{J,k}(t) = 2^{-J/2}\psi(2^{-J}t-k) = 2^{-J/2}\psi[(t-2^{J}k)/2^{J}] \quad j=1,2,\dots,J. \tag{10}$$

The DWT is used to compute the coefficient of the wavelet series approximation in Eq (6) for a discrete signal $f_1,\dots,f_n$ of finite extent. The DWT maps the vector $f=(f_1,f_2,\dots,f_n)'$ to a vector of n wavelet coefficients $w=(w_1,w_2,\dots,w_n)'$ which contains both the smoothing coefficient $s_{J,k}$ and the detail coefficients $d_{j,k}$ $[j=1,2,\dots,J]$. The $s_{J,k}$ describe the underlying smooth behaviour of the signal at coarse scale 2^J while $d_{J,k}$ describe the coarse scale deviations from the smooth behaviour and $d_{J-1,k},\dots,d_{1,k}$ provide progressively finer scale deviations from the smooth behaviour.

In the case when n is divisible by 2^J; there are $n/2$ observations in $d_{1,k}$ at the finest scale $2^1=2$ and $n/4$ observations in $d_{2,k}$ at the second finest scale $2^2=4$. Likewise, there are $n/2^J$ observations in each of $d_{J,k}$ and $s_{J,k}$ where

$$n = n/2 + n/4 + \cdots + n/2^{J-1} + n/2^J + n/2^J.$$

We apply the discrete wavelet transform to split the weekly return series for emerging and mature market indices into different frequency components to get a clear picture of the movements in these markets. We also wish to rebuild the return series using the first three wavelet components (d_1, d_2 and d_3) which explain more than 80% of energy (or magnitude) of these series[2] (see Tables 1 and 2) in order to study the fortnightly, monthly and bi-monthly data which are represented by d_1, d_2 and d_3, respectively.

3. Data and results

3.1. Data description

The data used in the following analysis consists of the weekly prices of a set of 13 *emerging* market indices and a set of 14 *mature* market indices during the period from the second week of January 1997 to the third week of March 2004. These markets are listed in Tables 1 and 2. As each market uses its local currency for presenting the index values, we use the weekly returns instead of using the weekly prices, where the following formula applies:

$$\text{Weekly return} = Ln(P_t/P_{t-1}) \tag{11}$$

and where

- P_t is the closing price of the index at week t.
- P_{t-1} is the closing price at week $t-1$.

3.2. Empirical results

3.2.1. Eigenanalysis for original return series

The variation of the ratio of the *largest* (λ_1) to the *second largest* (λ_2) eigenvalues of the covariance matrices for *equal overlapping* time windows of the original return series for emerging and mature markets, are shown in Figs. 1(a) and 2(a), respectively. These show a qualitative difference in the way emerging and mature markets deal with crashes and events.

We also plot the ratio of λ_1 to λ_3 to see clearly the reactions of stock markets to different crashes and events. The variation of these ratios is plotted in Figs. 3(a) and (b) (Table 3). It can be seen that the mature markets have reacted to events more strongly than emerging markets, especially after the 9/11 crash, in order

[2]This means that 80% of the time series can be reconstructed by using d_1, d_2 and d_3

Table 1
Emerging markets: percentages of energy explained by wavelet components for the original returns series

Market	W.Crystals						
	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Argentina (Americas)	**0.415**	**0.203**	**0.192**	0.124	0.034	0 011	0.021
Brazil (Americas)	**0.521**	**0.185**	**0.124**	0.095	0.055	0.002	0.019
Ireland (Europe)	**0.440**	**0.250**	**0.115**	0.104	0.062	0.004	0 025
Korea (Asia)	**0.583**	**0.207**	**0.076**	0 070	0 021	0.020	0 022
Malaysia (Asia)	**0.498**	**0.211**	**0.107**	0.101	0 032	0.016	0.035
Mexico (Americas)	**0.455**	**0.246**	**0.144**	0.074	0 057	0.012	0.013
New Zealand (Pacific)	**0.546**	**0.197**	**0.126**	0.070	0.037	0.019	0.006
Norway (Europe)	**0.469**	**0.247**	**0.109**	0 076	0 059	0 022	0.018
Portugal (Europe)	**0.461**	**0.190**	**0.136**	0.084	0 079	0 020	0.030
Russia (Europe)	**0.434**	**0.239**	**0.126**	0 082	0 063	0.019	0.037
Singapore (Asia)	**0.496**	**0.213**	**0.106**	0.124	0.016	0 020	0.025
Taiwan (Asia)	**0.465**	**0.308**	**0.106**	0.051	0.043	0.009	0.019
Turkey (Middle East)	**0.477**	**0.213**	**0.141**	0 058	0.075	0.014	0.023

Table 2
Mature markets: percentages of energy explained by wavelet components for the original returns series

Market	W.Crystals						
	d_1	d_2	d_3	d_4	d_5	d_6	s_6
Australia (Pacific)	**0.499**	**0.226**	**0.168**	0.055	0.038	0.010	0.005
Canada (Americas)	**0.552**	**0.202**	**0.104**	0 050	0.054	0.028	0.011
Denmark (Europe)	**0.505**	**0.151**	**0.221**	0 044	0 026	0.033	0 020
France (Europe)	**0.546**	**0.231**	**0.103**	0 055	0.025	0.019	0 022
German (Europe)	**0.594**	**0.214**	**0.128**	0 031	0.023	0.007	0.004
Hong Kong (Asia)	**0.487**	**0.221**	**0.138**	0.100	0.026	0.007	0.021
Italy (Europe)	**0.511**	**0.220**	**0.146**	0.060	0.030	0 014	0.019
Japan (Asia)	**0.557**	**0.213**	**0.123**	0.059	0.020	0.010	0.019
Netherlands (Europe)	**0.390**	**0.418**	**0.064**	0 091	0 010	0.018	0.008
Sweden (Europe)	**0.518**	**0.201**	**0.133**	0.063	0 036	0.026	0.023
Switzerland (Europe)	**0.458**	**0.277**	**0.133**	0.070	0 028	0.015	0 018
UK (Europe)	**0.532**	**0.244**	**0.113**	0.054	0.032	0.011	0 013
US (NASDEQ) (Americas)	**0.531**	**0.233**	**0.121**	0.051	0.023	0.008	0 034
US (S&P500) (Americas)	**0.550**	**0.224**	**0.125**	0 051	0.025	0.009	0.017

to regain stability and reduce risk to the markets. This means that mature markets effectively became *anti-persistent*, while emerging markets are *persistent* in agreement with the findings of Di Matteo et al. [24,25] which indicate that emerging markets have $H \geqslant 0.5$, while mature markets have $H \leqslant 0.5$ [H is the Hurst exponent].

The ratios of λ_2 to λ_3, Figs. 4(a) and (b), are plotted in order to see if behaviour for λ_2 and λ_3 differs for emerging and mature markets. In other words, we want to investigate whether or not λ_2 carries additional information about these different market types. Figs. 4(a) and (b) suggest that ratios of λ_2 to λ_3 for emerging markets are more variable than those for mature markets, implying that subdominant (λ_2), as well as dominant (λ_1) eigenvalues, do play a part in describing the behaviour of emerging markets while the behaviour of mature markets is described by λ_1 only.

In comparing the ratio (λ_1/λ_2), Figs. 1(a) and 2(a), for emerging and mature markets, it can be seen that for latter, there are three highly significant points in the ratio variation which are for *window numbers* 120, 219 and 345, respectively. Window 120 starts from week 120 to week 139 which is the third week of October 1999

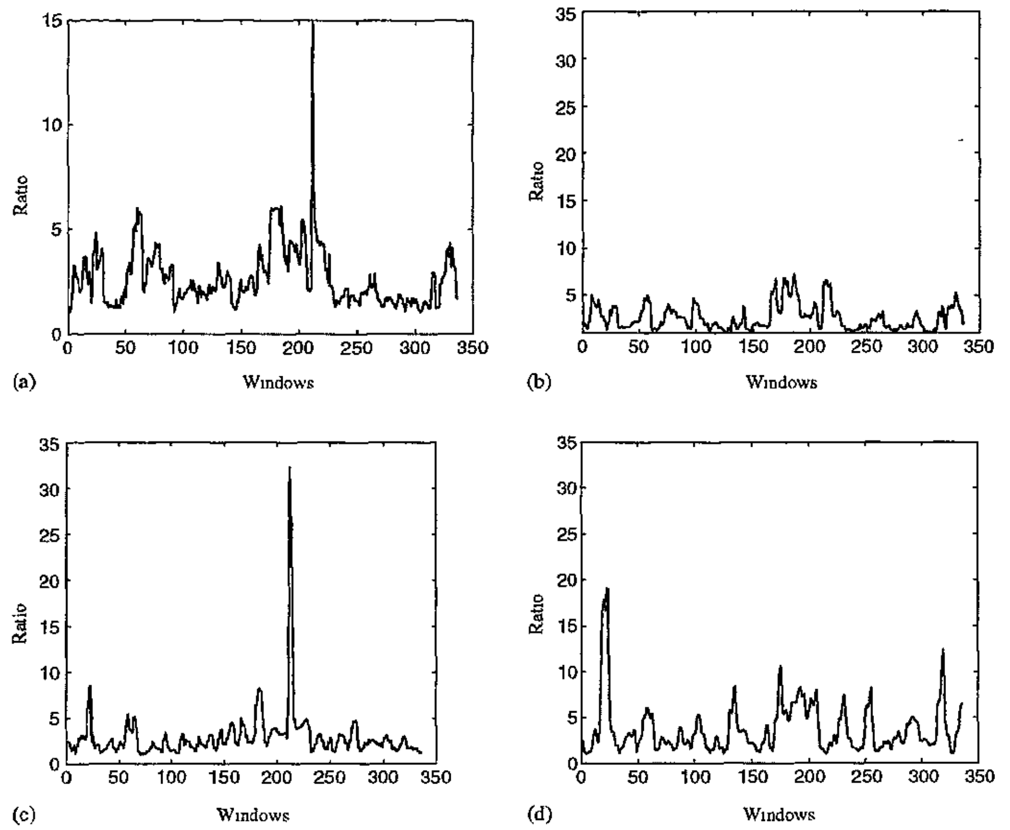

Fig 1 Emerging markets the changes in ratio of *dominant* (λ_1) to *subdominant* (λ_2) eigenvalues (a) original return series, (b) return series rebuilt from first wavelet crystal (d_1), (c) return series rebuilt from second wavelet crystal (d_2) and (d) return series rebuilt from third wavelet crystal (d_3)

(*the* 12th *anniversary of 19 October 1987 crash*[3]). The last week in window 219 is week 238 which is the second week of September 2001 (*9/11 crash*) and window 345 starts from week 345 to week 364 which is the third week of March 2004 (*Madrid Bomb*). However, for emerging markets, there is only one highly significant point which is for *window 212*, where the last week in this window is the second week of September 2001 (*9/11 crash*). We suggest that the cause for these highly significant ratio points is one or more of the following reasons:

1. Increasing the value of the largest eigenvalue (λ_1) while the second largest eigenvalue (λ_2) remains stable.
2. Decreasing the value of λ_2 while the value of λ_1 does not change.
3. Increasing the value of λ_1 while decreasing the value of λ_2, (or in other words, λ_1 and λ_2 moving in opposite directions).

The changes in λ_1 and λ_2 are plotted in Figs. 5(a) and (b) for emerging and mature markets, respectively. For mature markets (in order to examine likely causes), we compared the values of λ_1 and λ_2 of the covariance matrix for windows 120, 219 and 345 with the values of the previous windows, while for

[3]This was the last October in 20th century and October is always hard month for stock markets. So, with the end of the century as well, a crash in October was anticipated but did not happen. This, not least because, "The world markets were actually sent into turmoil by a speech by Alan Greenspan, and the Dow Jones for the first time since 8 April 1999 dipped below 10.000 15 and 18 October 1999. However, the market did not crash and instead quickly recovered and later started a renewed and strengthened bullish phase", Sornette [26].

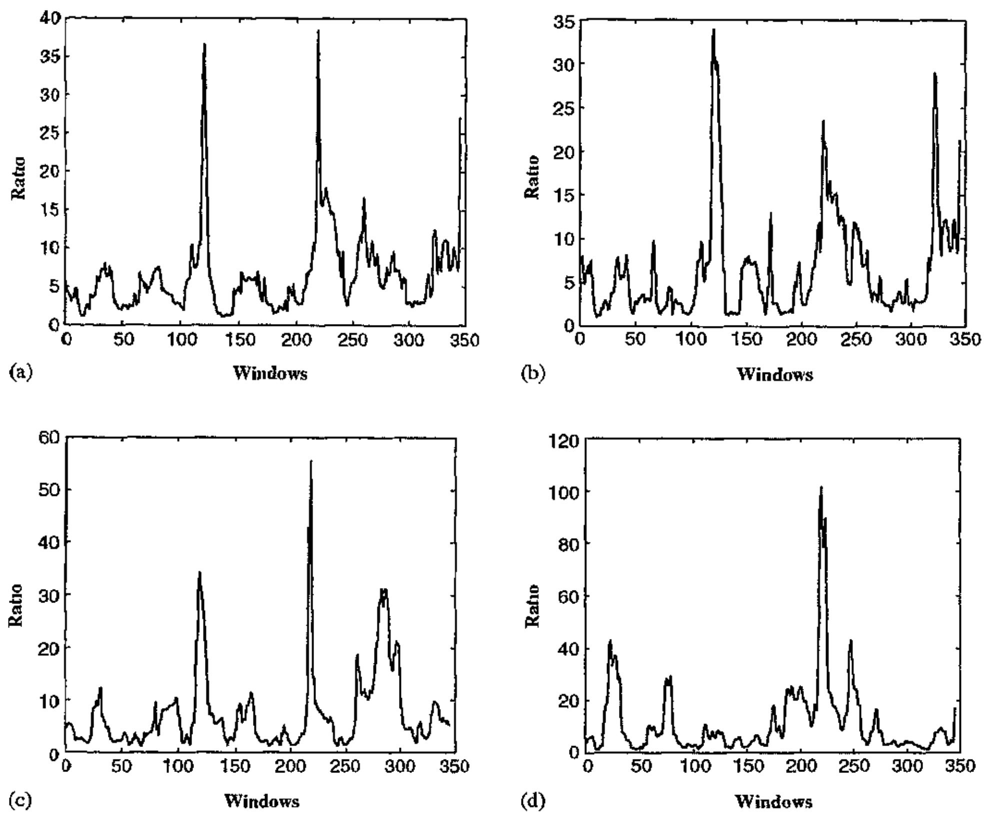

Fig. 2. Mature markets· the changes in ratio of *dominant* (λ_1) to *subdominant* (λ_2) eigenvalues: (a) original return series, (b) return series rebuilt from first wavelet crystal (d_1), (c) return series rebuilt from second wavelet crystal (d_2) and (d) return series rebuilt from third wavelet crystal (d_3)

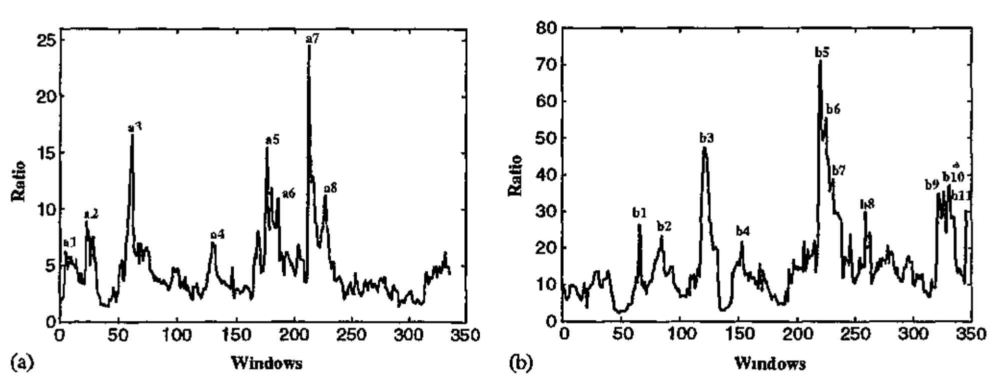

Fig 3 The changes in ratio of the *first largest* (λ_1) to the *third largest* (λ_3) eigenvalues. Original return series for· (a) emerging markets, and (b) mature markets

emerging markets, we compared the values of λ_1 and λ_2 for window 212 with the values of the previous windows. We found that the third reason above causes peaks in emerging markets while it is the first driver for change in the mature markets. This implies that both λ_1 and λ_2 are important in describing the behaviour of emerging markets while λ_1 is sufficient alone to explain the behaviour of mature markets.

Table 3
Description of the marks in the Figs 3(a) and (b)

Mark	Window No.	Last week included	Events
(a) *Emerging markets*			
a1	5	First week of 7/1997	Asian crash
a2	23	Second week of 11/1997	Asian crash
a3	62	Fourth week of 8/1998	Global crash
a4	130	Second week of 1/2000	
a5	176	Second week of 12/2000	Effects of DotCom crash
a6	186	Second week of 3/2001	
a7	212	Second week of 9/2001	September the 11th crash
a8	227	Fourth week of 1/2002	
(b) *Mature markets*			
b1	65	First week of 9/1998	Global crash
b2	84	Fourth week of 12/1998	Global crash
b3	121	Third week of 10/1999	Last October in 20th century
b4	153	Second week of 6/2000	DotCom crash
b5	220	Second week of 9/2001	September the 11th crash
b6	225	First week of 11/2001	Effects of 9/11 crash
b7	231	Second week of 12/2001	Effects of 9/11 crash
b8	259	First week of 5/2002	The stock market downturn
b9	322	First week of 10/2003	
b10	331	First week of 12/2003	General threat level raised
b11	345	Third week of 3/2004	Madrid bomb

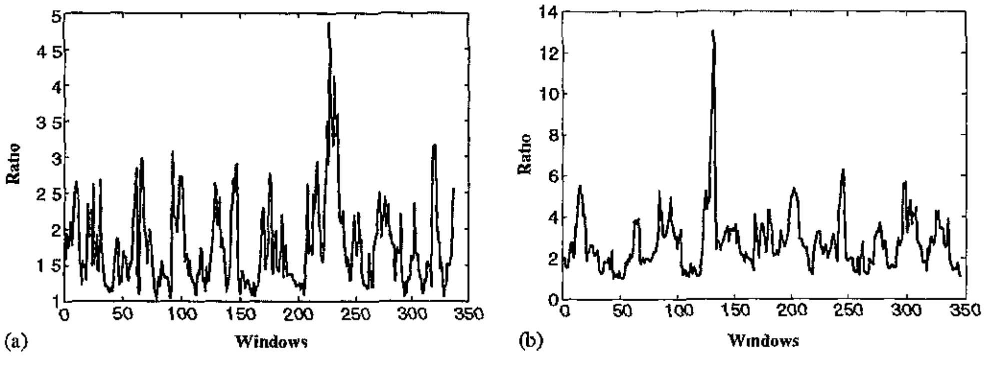

Fig 4. The changes in ratio of the *second largest* (λ_2) to the *third largest* (λ_3) eigenvalues Original return series for. (a) emerging markets, and (b) mature markets.

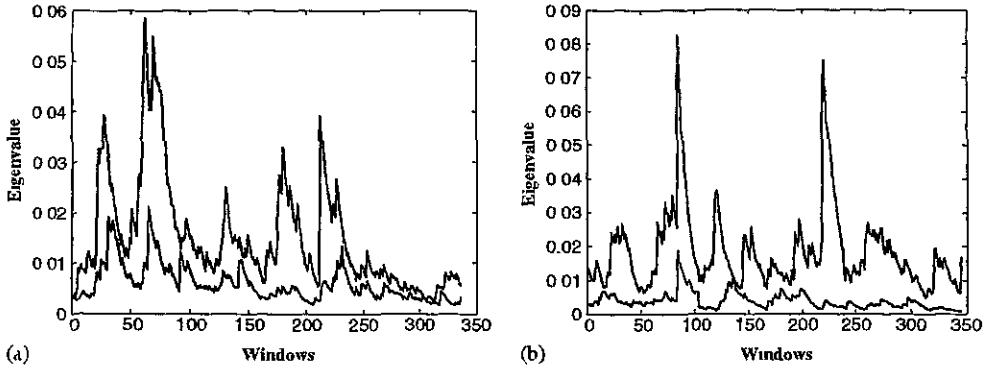

Fig 5 The changes in the *dominant* (λ_1) (upper line) and the *subdominant* (λ_2) (lower line) eigenvalues Original return series for: (a) emerging markets, and (b) mature markets.

3.2 2. Eigenanalysis associated with wavelet transform

The DWT with symmlet 8 wavelet (s_8) for 6 levels (scales) is computed for weekly returns series of all indices for emerging and mature markets. The DWT provides a more detailed breakdown of the contribution to the series energy from the high and low frequencies in the following manner. Tables 1 and 2 display the energy (or magnitude) percentages explained by each wavelet component (crystal) of the original returns for emerging and mature market indices, respectively. From Tables 1 and 2, it can be seen that high-frequency crystals, especially the first three (d_1, d_2 and d_3) have much more energy than the lowest frequency one (s_6) implying that movements in these series are mainly caused by short-term fluctuations.

In order to measure the recovery time of emerging and mature markets from crashes and how long these markets retain information about crashes, we employed the DWT and eigenanalysis. The steps of this process are: (i) Use the DWT to divide the return series of emerging and mature markets into different frequency components. (ii) Rebuild the returns using each wavelet components (d_1, d_2, d_3, etc.) and (iii) Study the distribution of the ratio (λ_1/λ_2) of variance–covariance matrices for overlapping windows of size 20 for these series.

Figs 1(b)–(d) and 2(b)–(d) show the ratio (λ_1/λ_2) from covariance matrices for each window for the return series, which are rebuilt from d_1, d_2 and d_3, (representing fortnightly, monthly and bi-monthly data, respectively), for emerging and mature markets, respectively. Looking at the ratio scales in these figures, we can clearly seen two main features; firstly, for emerging markets, even bi-monthly return series, which are rebuilt from d_3, seem to carry information on crashes and events and this seems to imply that emerging markets take up to two months to recover from a crash. Secondly, for mature markets, the ratio in Figs. 2(c) and (d) are meaningless because the ratio scales are very big and this indicates that neither monthly nor bi-monthly data (rebuilt from d_2 and d_3, respectively), seem to have information on crises and events implying that mature markets take less than a month to recover from crashes.

To sum up, we would say that the results appear to indicate that mature markets take action more quickly than emerging markets to recover from crashes and also that mature markets exhibit anti-persistent behaviour while emerging markets show persistent behaviour. In other words, the recovery time from crisis for developed markets appears to be shorter than that for developing ones.

4. Discussion/Conclusion

The aims of this work were to study the distribution of the largest (λ_1) and the second largest (λ_2) eigenvalues of covariance matrices for emerging and mature markets and also to study the distribution of the ratio of λ_1 to λ_2 for the original return series and for those reconstructed from wavelet components (d_1, d_2 and d_3). The summary of our results is as follows:

1. From studying the original return series, we found that differences exist between emerging and mature markets in dealing with crashes (especially unexpected ones). For major markets, the ratio is high at three points representing *the* 12th *anniversary of the* 19 *October stock market crash*, 1999, *the* 9/11 *crash*, 2001 *and Madrid Bomb*, *March* 2004, respectively. However, for emerging markets, the ratio is only high at one point, representing the 9/11 *crash*, 2001.
2. Using the DWT to study the behaviour of stock markets provides a clearer view on the structure and dynamics of the data sets and gives us a good measurement of the recovery time and direction of movements in these markets. It also indicates that emerging markets take up to two months to recover from crashes while mature ones take less than a month to do so.
3. Both λ_1 and λ_2 are needed to describe the behaviour of emerging markets while λ_1 is adequate alone to describe the behaviour of mature markets.
4. Mature markets move together in the same direction to deal with crises and show little internal variation which suggests that cooperative behaviour applies both within and between such markets. In other words, shareholders in these markets appear to have similar patterns of selling and buying shares. However, emerging markets show more internal variation and thus demonstrate differing views of shareholders in these markets which take different directions in dealing with crashes and unexpected events.

References

[1] I Meric, G Meric, Co-movements of european equity markets before and after the 1987 crash, Multinational Finance J. 1 (2) (1997) 137–152.

[2] J. Kwapien, S Drozdz, J. Speth, Alternation of different scaling regimes in the stock market fluctuation (available from www fz-juelich de/ikp/publications/AR2002/CHAP4/409 pdf) 2002 (accessed 11 May 2004).

[3] G Keogh, S. Sharifi, H Ruskin, M. Crane, Epochs in market sector index data—empirical or optimistic?, in· H. Takayasu, (Ed.), Proceedings of the Second Nikkei Econophysics Symposium—Application of Econophysics, Lecture Notes in Computer Science, Springer, November 2003, pp. 83–89, ISBN 4-431-14028-X

[4] J Kwapien, S Drozdz, J Speth, Time scale involved in emergent market coherence, Physica A 337 (2004) 231–242

[5] S. Pafka, I. Kondor, Noisy covariance matrices and portfolio optimization, Eur Phys J. B 27 (2002) 277–280

[6] S Galluccio, J.P. Bouchaud, M Potters, Rational decisions, random matrices and spin glasses, Physica A 259 (1998) 449–456

[7] L. Laloux, P Cizeau, J P. Bouchaud, M Potters, Noise dressing of financial correlation matrices, Phys Rev Lett 83 (7) (1999) 1467–1470

[8] V Plerou, P. Gopikrishnan, B Rosenow, L.A.N Amaral, H.E Stanley, Universal and non-universal properties of cross correlations in financial time series, Phys Rev. Lett. 83 (7) (1999) 1471–1474

[9] L Laloux, P. Cizeau, M. Potters, Random matrix theory and financial correlations, Int J Theoret. Appl Finance (IJTAF) 3 (3) (2000) 391–397.

[10] V. Plerou, P Gopikrishnan, B. Rosenow, Collective behaviour of stock price movement: a random matrix theory approach, Physica A 299 (2001) 175–180.

[11] D Wilcox, T. Gebbie, On the analysis of cross-correlations in South African market data, Physica A 344 (1–2) (2004) 294–298

[12] D Wilcox, T Gebbie, An analysis of cross-correlations in South African market data, Physica A (2004) e-print http://arxiv org/abs/cond-mat/0402389, submitted for publication.

[13] S Sharifi, M. Crane, A Shamaie, H Ruskin, Random matrix theory for portfolio optimization a stability approach, Physica A 335 (3–4) (2004) 629–643.

[14] P Gopikrishnan, B Rosenow, V. Plerou, E Stanley, Quantifying and interpreting collective behaviour in financial markets, Phys Rev. E 64 (2001) 035106 (1–4).

[15] S Patel, A Sarkar, Stock market crises in developed and emerging stock markets, Federal Reserve Bank of New York, Research Paper, No 9809, 1998.

[16] R Fuss, The financial characteristics between emerging and developed equity markets, Policy Modelling, International Conference Brussels, EcoMod Network, July 2002.

[17] R Salomons, H. Grootveld, The equity risk premium: emerging versus developed markets, University of Groningen SOM Working Paper No 02E45, August 7, 2002

[18] P.D. Wooldridge, D. Domanski, A. Cobau, Changing links between mature and emerging financial markets, BIS Quarterly Review, September 2003, pp. 45–54

[19] A Sharkasi, M Crane, H J. Ruskin, Apples and oranges· the difference between the reaction of emerging and mature markets to crashes, in: H. Takayasu (Ed.), Proceedings of the Third Nikkei Econophysics Research Workshop and Symposium—Practical Fruits of Econophysics, Springer, 2006, ISBN: 104-431-28914-3

[20] R Litterman, K Winkelmann, Estimating covariance matrices, in R A Krieger (Ed.), Goldman–Sachs Risk Management Series, Goldman, Sachs & Co, 1998.

[21] H E Hijmans, Discrete wavelet and multiresolution analysis, in· T.H. Koornwinder (Ed), Wavelets: An Elementary Treatment of Theory and Application, World Scientific, Singapore, 1993, pp 49–79

[22] A. Bruce, H.Y. Gao, Applied Wavelet Analysis with S-Plus, Springer, New York, 1996

[23] Z. Gonghui, J.L. Starck, J Campbell, F. Murtagh, The wavelet transform for filtering financial data stream (available from strule cs qub ac uk/~gzheng/financial-engineering/finpapermay99 html) 1999 (accessed 20 October 2004).

[24] T Di Matteo, T. Aste, M.M Dacorogna, Scaling behaviors in differently developed markets, Physica A 324 (2003) 183–188

[25] T. Di Matteo, T. Aste, M M Dacorogna, Long-term memory of developed and emerging markets· using the scaling analysis to characterize their stage of development, J. Banking Finance 29 (2005) 827–851

[26] D Sornette, Why stock markets crash: critical events in complex financial systems, Princeton University Press, New Jersey, USA, 2002 ISBN 0-691-09630-9