The Valuation of Collateralised Debt Obligations – multi-period modelling in a risk-neutral framework

Name: B.G. O'Kelly, B.Eng., M.B.A., M.Sc.Inv.Tr.

Degree: Ph.D.

University: Dublin City University

Supervisor: Prof. Liam Gallagher

Date: June, 2005

Statement of Originality

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Ph.D. is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed: ______________________

ID No.: 50162217

Date: 1/June/05

Acknowledgements

I wish to thank a number of people who helped me as I prepared this document:

I would especially like to thank my supervisor, Professor Liam Gallagher, for his wise counsel and good humour over the period.

I would also like to thank Professor Emmanuel Buffet for his advice and direction in the early stages of this project.

My friend and mentor, Professor Finbarr Bradley, was a constant source of encouragement throughout this venture. Without his guidance, this thesis would never have been completed. Míle buíochas, Fionbarra – táim faoi chomaoin agat.

I received much encouragement from my colleagues in AIB and benefited greatly from their insights. Two colleagues in particular, Edward Murray and Seán Cooke, were especially helpful. Without the benefit of their knowledge and programming skills, I could not have completed this work. Edward and Seán, sincere thanks.

My own family bore the brunt of this project. A project such as this could not have been undertaken were it not for the good humour and love at home. To Brenda, Orla, Rónán and Aoife, thanks a million – I appreciate very much that you made this possible.

Finally, the resolve to undertake a project such as this is borne out of an environment which affirms and encourages. I have been privileged that my late mother and my father provided such an environment.

To Mom and Dad

Table of Contents – Overview

Table of Contents

List of Figures

List of Tables

Abstract .

For over fifty years, mortgages have been secuntısed by sellıng the nghts to the mortgage cash flows to thırd party ınvestors Over the past ten years or so, a sımılar secuntısatıon process has been undertaken wıth corporate debt The claıms on the cash flowıng from the corporate debt portfolıo are called collateralısed debt oblıgatıons (CDO)

CDO cash flows are dependent on the ınteractıon of a portfolıo of debt secuntıes over many tıme penods They are partıcularly sensıtıve to the correlatıon among the underlyıng secuntıes and to the terms of the ındenture Whıle much progress has been made ın modellıng debt portfolıos over a sıngle penod, there has been a lot less publıshed about the ınteractıon of debt secuntıes m a portfolıo over many penods

Thıs thesıs develops a model for valuıng CDOs usıng a nsk-neutral approach ın a multı-penod settıng A model ıs also developed whıch reproduces Moody's CDO ratıng The Moody's ratıng ıs compared to that whıch ıs ımplıed from applyıng the nsk-neutral model, the dıfferences analysed and the ımplıcatıons for regulatory capıtal for CDOs explored

Executive Summary

In recent years, as liquidity in the credit default swap market has increased for the largest borrowing firms in the market, many debt portfolio-based structured products have been created. For example, single-tranche collateralised debt obligations and n^{th}–to-default swaps are traded in huge volume. The theoretical framework necessary to underpin an analysis of these products has been developed and has gained widespread acceptance. The emphasis has now moved onto the analysis of more complex products referencing the same underlying names for which there is a liquid credit default swap (CDS) available.

By comparison, the market for structured credit products referencing names for which there is no liquid credit default swap market has grown a lot more slowly. This section of the market includes approximately 34,000 publicly quoted firms worldwide compared to the 500 or so for which an active CDS market exists. It is estimated that the debt issuance by these 34,000 firms exceeds that of the top 500 issuers. It comprises, in the main, privately issued debt, provided primarily by banks. Relatively little of this debt is traded; hence price information is lacking.

However, since the firms in this latter category have publicly-quoted equity, it is possible to infer the credit quality of the issuer's debt from the characteristics of its equity and the correlation between the firms' asset returns, as one firm, Moody's KMV (KMV), has done. With this information, the probability distribution of debt portfolio values at a future date can be derived, giving the portfolio owner the information necessary to manage the credit risks presented.

KMV have never publicly disclosed the methodology that they employ in developing their credit portfolio model. Thus, the academic community has largely ignored their approach. But the market has embraced their approach, and their portfolio management product is the clear market leader. This gap between the academic and market approaches is a puzzle.

Credit Portfolio Model Development

This first innovation presented in this thesis is the development of a credit portfolio model in the spirit of KMV. In so doing, it shows for the very first time how the KMV approach sits within current academic thinking. Using only their data and their very limited public references to their methodology, their portfolio modelling approach is re-created and their results replicated It thus provides confirmation that their approach is founded on traditional portfolio management principles identified within the academic literature It is hoped that this will enable a debate to begin in academic circles regarding the merits of their approach, a debate that has not occurred to date because of the lack of understanding of the approach that KMV adopted

Assessing the Marginal Impact of a Loan on a Portfolio

A major disadvantage of current portfolio models is their failure to assess the marginal impact of a proposed new facility on a pre-existing portfolio of credits They are primarily directed at analysing the performance of a portfolio of credit exposures In short, they determine the impact of a facility on the portfolio after the fact While this is clearly important information, it is being delivered too late to give effect to portfolio management action Within most banks, exposures are being written by a large number of credit officers dispersed throughout the organisation while the portfolio is being modelled at periodic intervals by the credit portfolio function

This model enables putative new facilities to be added to the current portfolio and the capital required to support the new facility to be determined immediately Thus, the second innovation of this thesis is the creation of a model which is capable of giving effect to portfolio decisions in real-time since the portfolio impact of potential new facilities can be determined *ex ante* compared to current models which deliver this information *ex post*

Developing a Coherent Measure of Credit Concentration

A primary concern of banks and bank regulators has been with credit risk concentration However, the approaches adopted hitherto have been largely intuitive with very little by way of theoretical underpinning Most banks use simplistic rules of thumb to place limits on acceptable maximum exposures to individual obligors, industries and countries Even under the Basel II proposals, the Bank for International Settlements (BIS) has not attempted to quantify the impact of concentrations on the capital required to support credit portfolios, withdrawing their original proposal for a granularity adjustment in calculating credit portfolio capital

The third innovation presented in this thesis is the development of an approach to measuring the impact of concentrations on credit portfolios In particular, a key issue of concern to the managers of credit portfolios – namely, the *ex ante* assessment of the maximum economic holding of a syndicated loan – is answered The framework provides portfolio managers the basis for measuring the cost of concentrations to favoured relationship clients in a theoretically rigorous manner It also provides a basis for setting limits for clients – a single capital number can become the basis for limits to all customers replacing the qualitative limit framework currently employed in the market Finally, it provides regulators a basis on which to set capital requirements

The analysis also questions the use of contribution to the volatility of portfolio value - the market standard method of allocating portfolio capital among the component securities - for allocating capital in debt portfolios This approach is well suited to allocating capital in portfolios of traded securities whose returns are near-normal over the holding period However, illiquid credit portfolios do not meet these requirements An alternative framework, contribution to Expected Tail Loss, is proposed which is found to give results that accord more closely with intuition In particular, the proposed alternative is shown to be much more sensitive to credit concentrations than the contribution to portfolio volatility framework

Extending the Credit Portfolio Model to a Multi-Period Setting

The credit portfolio models most commonly used in the marketplace are based on a single time period They are ill equipped to assess structured securities such as CDOs that derive their value from cash flows from a credit portfolio over many time periods While the reduced form approach has been applied to value structured securities which reference names for which liquid CDS exist, it is not well suited to modelling portfolios of names which lack the pricing transparency that CDS provide

The fourth innovation presented in this thesis is the development of a multi-period credit portfolio model This model extends current modelling approaches along two dimensions

- It takes the structural model from its single-period frame of reference to the multi-period frame necessary to deal with the complexities of portfolio-based securities The reduced form paradigm is the preferred approach when modelling credit exposures over more than one time period. The author is unaware of any published research that uses the structural model in a multi-period setting.

- It incorporates the complexities of the waterfall, which are central to the structuring of portfolios of cash flow securitisations. By comparison, the market standard for modelling credit portfolios, a copula approach, is primarily geared to modelling credit exposures in synthetic form.

Furthermore, since the model tracks the portfolio of underlying securities over time, it can be adapted to deal with a CDO where the underlying collateral comprises tranches of other CDOs. These CDO-squared securities, as they are called, have never before been analysed using a structural approach to the author's knowledge.

Comparing the Rating Assessments

The rating agencies serve a critical role in assessing the credit risk of firms and securities. As products have become more structured, and hence more complex, market participants have come to rely to an ever greater extent on the rating agency assessment of credit risk. Their credit assessment of collateralised debt obligations will become still more important in the future given the special position granted to them in the revised Basel Accord.

The fifth innovation of this thesis is to compare the rating agency assessment of CDO risk with that of the structural model based on KMV data. This required that the model developed by Moody's for evaluating CDOs - their so-called Binomial Expansion Model - be re-created based on the publications in which they outline their approach. The model is then tested to confirm that it successfully replicates the Moody's rating for a sample of deals in the marketplace. The expected loss under the Binomial Expansion Model, from which Moody's infer their rating, is compared with the expected loss predicted by the multi-period model. This provides a basis for assessing the validity of the binomial approach.

Assessing the Moody's Rating Assessment Methodology

Despite making the rating agencies the sole arbiters of the creditworthiness of CDOs purchased by investors, regulatory doubts about the validity of the rating agency approach remain. In particular, their decision to apply different risk weightings to CDOs and corporate debt of the same rating confirms their unease with the meaning of ratings given to structured securities Equally, the market demands higher spreads for structured products than for corporate debt with the same rating It is unclear whether this additional premium is a charge for the reduced liquidity of CDOs, or compensation for the extra effort in coming to understand the complexity of the product, or a charge for risks not adequately captured in the agency rating process

The sixth innovation is to examine the validity of the agency credit rating approach to structured debt It highlights the shortcoming of expected loss as a measure of CDO risk, ignoring as it does the variability around the average In particular, by ignoring the systematic risk that these products bring to a credit portfolio, it fails to provide the regulator with a coherent basis for setting capital to be assigned to CDOs It confirms that the regulators have good reason for demanding that that more capital is held against subordinated CDO tranches than similarly rated corporate securities

Conclusion

This thesis extends current knowledge as outlined above The approach is new and the results have important implications for investors and regulators In particular, the new model demonstrates that the rating agency approach to grading CDOs under-estimates the embedded risk It suggests that the more onerous risk-weighting of mezzanine CDOs compared with similarly-rated corporate debt under Basel II is justified It also supports the market requirement for wider spreads for these subordinated tranches compared to similarly-rated corporate debt

Chapter 1. Introduction

1.1 Chapter Overview

This chapter

- examines the background to the development of the collateralised debt obligations (CDO) market, explores the reasons for its phenomenal growth and discusses the unique valuation challenges which CDOs present,
- details the objective of this thesis, namely, to create a model which values CDO tranches in a rigorous manner,
- describes the key research issues which are addressed, and
- gives a chapter-by-chapter overview of the research undertaken

1.2 Background

The CDO product will be examined in detail in Chapter 2 However, a brief introduction will be provided here in order to give context to the discussion

A CDO is an asset-backed security (ABS) where the underlying securities are debt instruments The CDO market has grown at a tremendous pace since first introduced ten years ago Many factors have contributed to this growth Some banks have embraced securitisation as a way to manage their regulatory capital requirements Lower quality financial institutions – with a rating of single-A or less – which cannot fund themselves in the inter-bank market at Libor, or which lack a deposit base, have securitised their debt assets as a means to achieving funding at a lower rate Various financial organisations – including investment companies and banks – have seen debt securitisation as an asset management opportunity enabling them to earn fee income managing other people's capital rather than margin income through investing their own capital

However, the new securities that are created through debt securitisation present many new and challenging valuation issues that remain unresolved

- The resulting securities – CDO – are the product of the interaction of a portfolio of debt securities, most of which are unquoted or are highly illiquid

- ❖ The portfolio theory that exists was developed to cater for equities rather than debt. There is very little research published in the academic literature in the area of debt portfolio management. Most of the debate is occurring within the trade literature or at practitioner conferences.

The rating agencies have assumed a central role in creating the framework within which these securities are structured and graded. Without a rating agency grade, a CDO tranche becomes almost unmarketable and, therefore, much of the structuring which takes place and most of the underlying collateral purchase decisions are driven by rating agency requirements.

The practice of modelling debt portfolios in still at an embryonic stage and regulators remain unconvinced by rating agency assessments of the credit risk of structured debt. (BIS 1999a). This is confirmed by the more onerous treatment of lower-rated CDO securities compared to equally rated single name corporate debt securities (BIS 2001a). The CDO rating methodologies employed by the rating agencies were subject to particular criticism in the 2001-2 period due to the significantly higher level of re-rating compared to similarly-rated corporate debt securities.

1.3 Objective

There is an obvious gap between academic scientific research and market practice in relation to CDOs. The rigorous academic approach to the subject fails to address the many critical structural issues. Likewise, the practitioners, while addressing these structural issues, settle for extremely heuristic approaches to many other aspects of the structure.

This thesis attempts to embed academic rigour in a model that incorporates the many complex structural features typical of the CDO product. It develops a new model that transforms the current state-of-the-art portfolio modelling approach from a single time period framework to a multi-period setting and values the CDO tranches created from the credit portfolio in a risk-neutral framework. The results of this model will then be compared to those obtained by one of the rating agencies, Moody's Investors Service (Moody's). In order to do this, it will be necessary to replicate the state-of-the-art credit portfolio model currently employed by KMV (KMV), the leading credit portfolio risk software vendors, in their product, *Portfolio Manager* and the CDO tranche rating methodology employed by Moody's.

It is hoped that the main contribution of this thesis to the literature will be the development of a new model for valuing CDO tranches based on the structural approach, and the comparison of this alternative approach with the rating process currently employed by the rating agencies.

The research proceeds as follows:

A one-time period structural model is developed using KMV's assessment of default probabilities and asset correlations. This enables the value of the individual facilities to be determined, the capital required to support the loan portfolio estimated, and the risk contribution of each facility within the portfolio gauged.

This one time period model is extended to a multi-period setting in order to value CDO tranches. This model comprises two modules: (i) A multi-time step Monte Carlo simulation module to ascertain the behaviour of the credit portfolio between CDO coupon payment dates, and (ii) a cash flow model to disburse the cash flows to the tranches in accordance with the cash flow waterfall.

The Moody's CDO tranche rating model is re-created and the grade of each of the tranches is inferred from this model. This grade is compared to that suggested by application of the Moody's rating process.

In view of the enormous size of the CDO market – in excess of $250 billion are extant - and the controversy which surrounds the rating process, it is suggested that this research is timely and it is hoped it will make a valuable contribution to the literature and help in furthering current understanding of the issues.

1.4 Key Research Issues Addressed

The thesis addresses many significant research issues:

- It examines the nature of the interaction among debt securities in portfolios, a subject about which little has been written in the academic literature.
- It measures the impact of facilities – both new and existing - on a credit portfolio.
- It develops a framework for deciding on the maximum amount of a syndicated loan a bank should purchase.
- It presents a methodology for setting credit limits.
- It takes the structural approach to credit risk from its standard single period framework into a multi-period setting

- It draws on the two main types of rating data – the rating agency letter grade and the probability of default (PD) metric, expected default frequency (EDF), employed by KMV
- It examines actual CDO structures under both approaches

1.5 Overview of Thesis Approach

The thesis proceeds as follows

Chapter 2 introduces the CDO product, discusses the size of the market and its development over the years, and the regulatory attitude to the product

Chapter 3 undertakes a review of the academic literature in the areas of portfolio management, credit default probability assessment and credit portfolio management It examines the two principal competing approaches – the contingent claims approach and the reduced form – and summarises the main strands in the literature It then proceeds to discuss the practical implementation of these approaches It draws on research published in practitioner journals and trade literature published by systems vendors

The state-of-the-art single time period credit portfolio model in use in the market, that employed by KMV and delivered in their *Portfolio Manager* software offering, is re-created in Chapter 4 A new method for determining the impact of a new facility on an existing loan portfolio is also developed in this chapter as well as a framework for determining the cost of portfolio concentration and borrower limit setting

Chapter 5 describes the development of a new model for valuing CDO tranches that is the centrepiece of this thesis This new model takes the state-of-the-art single time period credit portfolio model and converts it into a multi-period model that incorporates the CDO indenture In so doing, it is, to the author's knowledge, the first academic research to take the current market standard credit portfolio paradigm into the structured securities arena

The principal measure of CDO tranche quality quoted in the market is the rating agency-assigned rating Chapter 6 replicates the tranche rating methodology employed by Moody's so that their rating can be attributed to tranches of any deal

Chapter 7 compares the results of the newly developed model with those from Moody's model for a variety of CDO structures The differences between the model-implied ratıng and the Moody's ratıng are explored for each of the tranches The reasons for these dıfferences are examıned and the valıdıty of the competıng approaches ıs assessed

Chapter 8 summarıses the contrıbutıon of thıs thesıs to the lıterature It crıtıcally evaluates the model's assumptıons and makes suggestıons for further research

The Appendıx develops the CredıtMetrıcs approach to credıt portfolıo modellıng and confirms theır publıshed results

1.6 Conclusion

Thıs chapter provıded a brıef overvıew of the CDO market and a summary of the research agenda The next chapter examınes the CDO product ın much greater detaıl and the regulatory approach thereto

Chapter 2. The CDO Market

2.1 Chapter Overview

In this chapter, the market for CDOs is examined and the many product variations are introduced. The nature of the CDO product gives regulators particular cause for concern; the regulatory attitude and response are detailed.

2.2 CDO Market

While the CDO market began in the late 1980s, the market really only became significant in 1996 as reported by Tavakoli (2003) citing Bank of America and Moody's and shown in Figure 2.1 below:

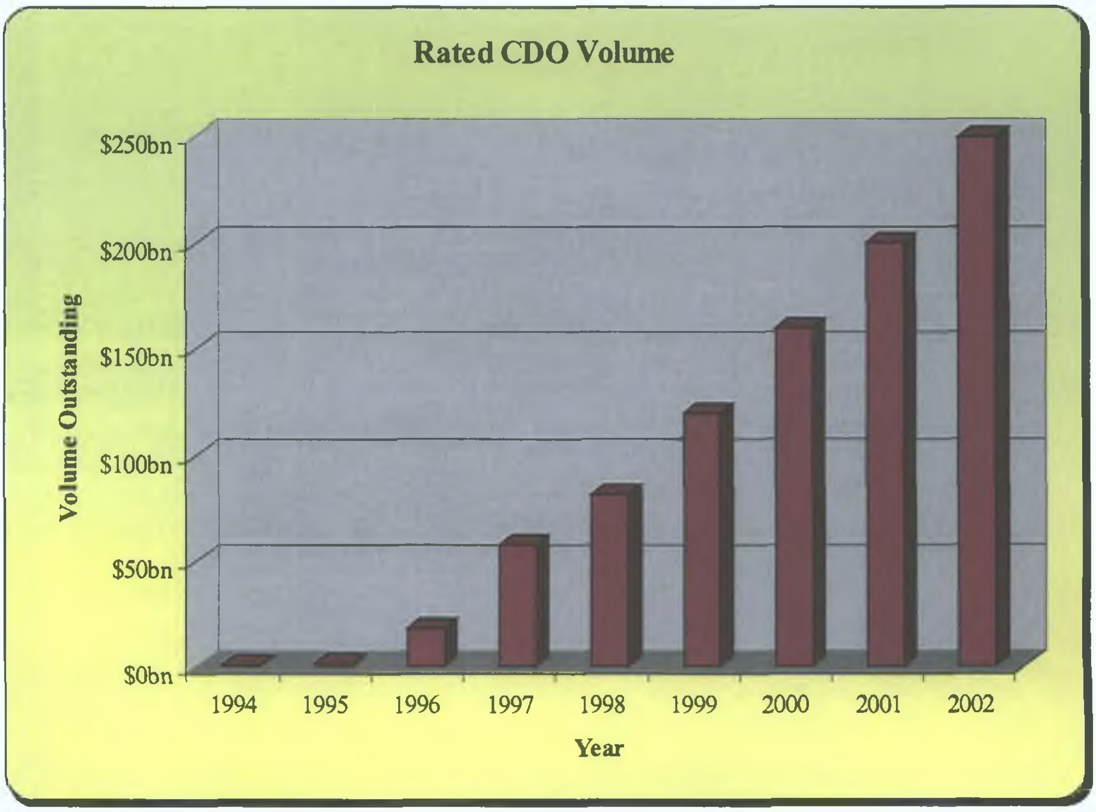

Figure 2.1 CDO Market Size

However, Tavakoli (2003) distinguishes between what she terms 'the old paradigm' of cash flow CDOs which accounted for most of the volume up to 1999 and 'the new paradigm' of synthetic CDOs which accounts for most of the growth since then. "Synthetics facilitate more efficient portfolio ramp-up, synthetics facilitate getting a higher average credit rating, and synthetics facilitate more efficient portfolio dıversıficatıon " (p8) The synthetıc arbıtrage ıs facılıtated by the feasıbılıty of a smaller equıty tranche, whıch creates more leverage The synthetıc arbıtrage gets a further huge boost from the large, ınexpensıve super senıor tranche that makes up the bulk of the synthetıc deal

The synthetıc market accounted for $187 5 bıllıon of the CDOs extant ın 2002 compared to $62 5 bıllıon of cash flow CDOs, Tavakoh notes But she emphasıses that the absolute sıze of synthetıc CDO ıssuance ıs exaggerated by these figures "Assumıng the super senıor tranche makes up 90 per cent on average of the synthetıc CDO, only about $18 75 bıllıon of synthetıc CDO product ıs avaılable to tradıtıonal ınvestors " (p12) She further notes that the super senıor tranche ıs held ın the tradıng book and ıs 'marked to market' ın theory, but not ın practıce

The Moody's revıew of 2001 was the last one to have been reported on a global basıs It reported 277 transactıons coverıng $101bn of tranche ıssuance ın 2001 compared to 189 transactıons ın 2000 and $121bn of ıssuance Theır report on 2003 U S CDO actıvıty reflects sımılar trends but, sınce ıt focuses only on U S tranches whıch they rated, presents somewhat dıfferent numbers Fıgure 2 2 below shows the number of new transactıons and assocıated volume of Moody's-rated CDO tranches year-by-year sınce the CDO market took off ın the mıd-1990s as reported by Gluck (2004)

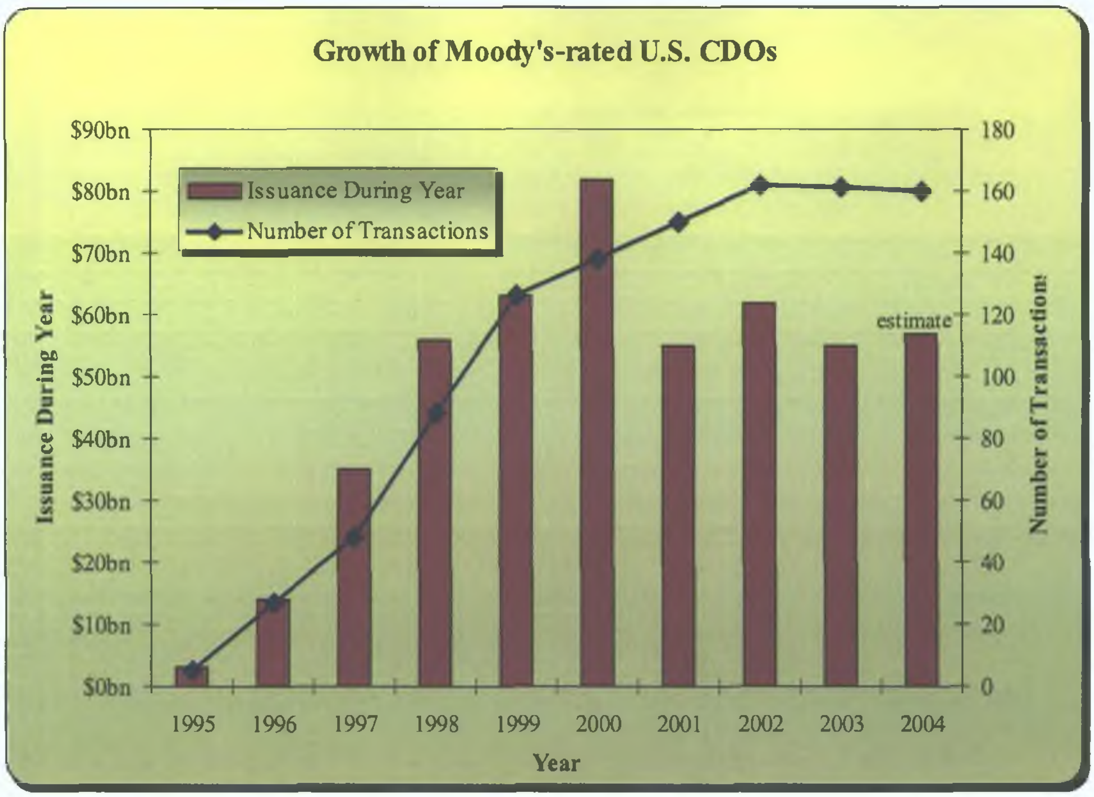

Figure 2.2 Growth of Moody's-Rated U.S. CDOs

The paradigm shift that Tavakoli noted may well have run its course. Commenting on activity in the fourth quarter of 2003, Gluck notes the notable reversal of the long-term trend towards synthetics. Only 12 of the 58 fourth quarter CDOs took synthetic form, or just over 20%. "The pattern shift was entirely due to the lack of arbitrage opportunity in the investment-grade corporate sector." Gluck (2004, p2)

The product mix underlying the CDO market has shifted dramatically in the past few years. The product which was the primary driver of the market in its early years, the high-yield collateralised bond obligation (CBO), was "nearly dormant" in 2003 (p6). High-yield CBOs accounted for 30% of Moody's-rated U.S. CDOs in 2001; by 2003, that had fallen to a mere 2% as shown in Figure 2.3 below. All acronyms are explained in the List of Acronyms below:

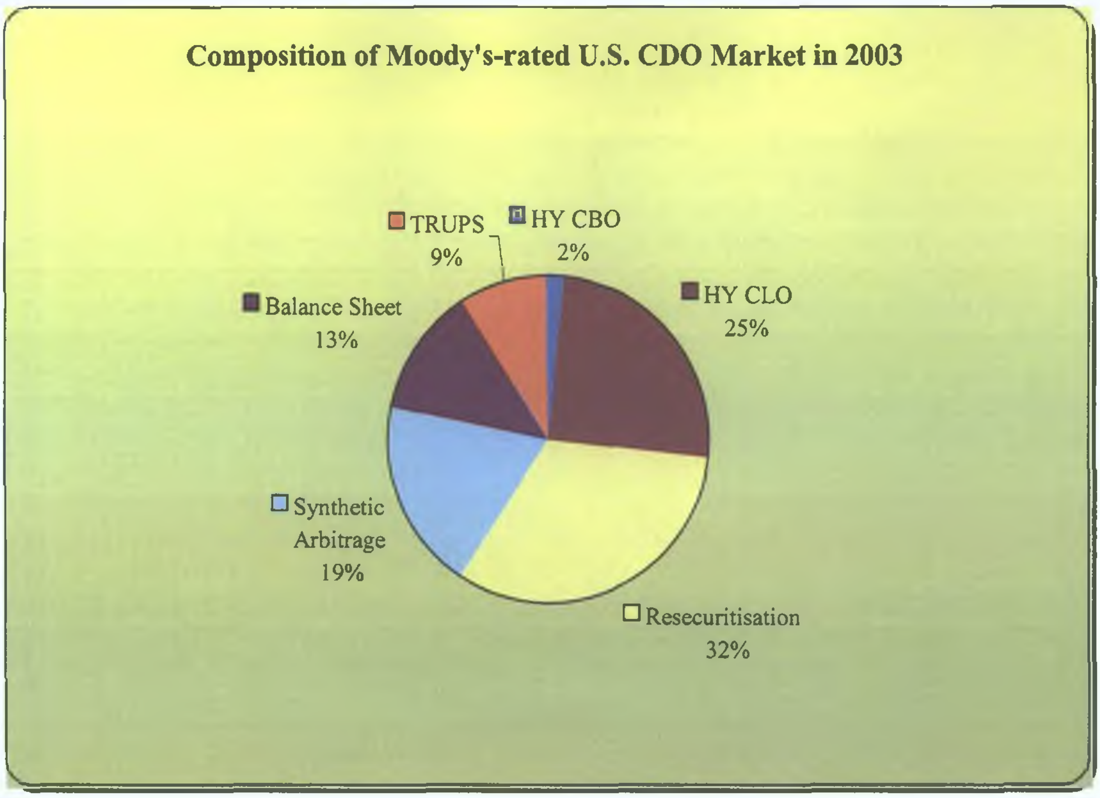

Figure 2.3 Composition of Moody's-rated U.S. CDO Market in 2003

Likewise, there has been a dramatic shift from bonds to structured debt in the collateral underlying the CDO product as evidenced in Table 2-1 below based on Gluck (2004, 2003)

	2003	2002
Corporate Bonds	36%	54%
Loans	24%	17%
Structured Debt	40%	29%

Table 2-1 Collateral Underlying Moody's-rated U.S. CDO Deals

2.3 Overview of the CDO Product

A CDO is an asset-backed security backed by a diversified pool of one or more classes of debt. In a CDO structure, there is an asset manager responsible for managing the portfolio of debt obligations. There are restrictive covenants imposed on what the manager may do and certain tests that must be satisfied for the debt obligations to maintain the credit rating assigned at the time of issuance.

Collateralised debt obligations are structured debt products. They are liabilities of a special purpose vehicle whose only assets are debt securities – either loans or bonds - issued by corporates. These assets by definition, present credit risk. Therefore, the starting point for this research is to gain a thorough understanding of the credit risk of individual firms.

"A *traditional securitisation* is a structure where the cash flow from an underlying pool of exposures is used to service at least two different stratified risk positions or tranches reflecting different degrees of credit risk. Payments to the investors depend upon the performance of the specified underlying exposures, as opposed to being derived from an obligation of the entity originating those exposures." BIS (2003, p100) They suggest the primary difference between the stratified/tranched structures that characterise securitisations and ordinary senior/subordinated debt instruments relates to the cash flow diversion mechanism: junior securitisation tranches can absorb losses without interrupting contractual payments to more senior tranches, whereas subordination in a senior/subordinated debt structure is a matter of priority of rights to the proceeds of a liquidation.

An understanding of the credit risk of the issuing firms individually will not suffice. The value of a CDO is determined by the interaction of many debt securities. Any attempt to value a CDO must seek to gain an in-depth appreciation of the manner in which those individual credit risks behave as a group because the cash flows from the asset pool are channelled through the cash flow waterfall to the individual CDO tranches in order of priority.

2.3.1 CDO Sponsor Motivation

BIS (2001a) notes that banks that securitise assets are able to accomplish several objectives. By securitising rather than holding the originated assets, they suggest banks attain a number of objectives: (i) they can secure a reduction in regulatory capital requirements; (ii) they can tap an additional source of funding, generally at a lower cost; (iii) they can enhance their financial ratios; and (iv) they can manage their portfolio risk by reducing large exposures or sectoral concentrations. By investing in tranches of other banks' securitisations, they suggest banks are able to diversify their portfolios by acquiring different asset types from different geographic areas.

Bluhm (2003) suggests four possible motives for CDO creation:

Spread arbitrage: This occurs where the total spread collected on single credit risky instruments at the asset side of the transaction exceeds the total 'diversified' spread to be paid to investors on the tranched liability side of the structure. Such a mismatch typically creates a significant arbitrage potential which offers an attractive excess spread to the equity or subordinated notes investor.

Regulatory capital relief: The Basel I regulatory capital requirements often exceed the economic capital required given the risks that many loan assets embed. In such circumstances, it is possible for a bank to obtain credit protection for a relatively modest cost once it retains a tranche that absorbs most of the loss which is likely to be experienced. The capital associated with the first-loss piece combined with the 20% risk-weighting of the super-senior credit default swap will necessitate significantly less – 50% or less - regulatory capital than would otherwise be required to hold low-risk assets on balance sheet. As 'opportunity costs' for capital relief, the originating bank has to pay interest to notes investors, a super senior swap premium, upfront costs (rating agencies, lawyers, structuring and underwriting costs) ongoing administration costs and possibly some other expenses. "A full calculation of costs compared to the decline of regulatory capital costs is required to judge about the economics of such transactions." (p7)

Funding: For banks with sub-AA ratings, funding can become too expensive to allow them put high-quality assets on balance sheet. Equally, even highly rated institutions like to have a range of funding sources available should they ever need it. For non-bank institutions, accessing relatively inexpensive funding through securitisation is a key consideration. The advantage of refinancing by means of securitisations is that resulting funding costs are mainly related to the "credit quality of the transferred assets and not so much to the rating of the originator." (p8) He notes, however, that some linkage remains to the originator's rating, if the SPV also enters into a servicer agreement with the originating bank. In such cases, investors and rating agencies will evaluate the servicer risk inherent in the transaction.

Economic risk transfer: The final motivation Bluhm suggests is economic risk reduction. This is a key reason for many regional banks undertaking securitisations. Where they have a strong local franchise but no global presence, concentrations to key customers develop. These concentrations, if held on balance sheet, require significant capital; however, they will not present any concentration risk to investors outside the region and can be supported by substantially less capital Securitising its excess exposures to its best customers is a way of deriving portfolio benefits However, Bluhm, commenting on some transactions undertaken primarily for regulatory or funding reasons, notes that the capitalisation rate for the remaining portfolio can be higher as a result of poor sub-portfolio selection "[S]ecuritising a subportfolio can cause some negative effect on the economic capital of the residual source portfolio" (p9) due to the diversification turn-down caused by taking away a pool of diversifying assets

2 3 2 CDO Market Practice

The 2002 Survey of Credit Portfolio Management Practices undertaken by the International Association of Credit Portfolio Managers asked financial institutions about their use of securitisation The questions and responses are given below

In order to transfer loans from the institution, has your institution issued a CLO – either cash or synthetic?

No	27%
Yes, traditional CLOs	20%
Yes, synthetic CLOs	24%
Yes – both cash and synthetic CLOs	29%

If your institution has issued a CLO, rank these motivations by order of importance (Use 1 to denote the most important and 3 to denote the least important)

Regulatory capital	1 68
Economic capital	2 21
Exposure management (freeing lines)	2 07

Has your institution used a CLO structure as a way of transferring loan exposures into the institution? That is, have you purchased the equity or subordinated tranches of someone else's CLO or have you set up a CLO structure using assets from other originators as a way of importing credit risk?

No	59%
Yes, traditional CLOs	10%
Yes, synthetic CLOs	13%
Yes – both cash and synthetic CLOs	18%

It is clear from these survey results that financial institutions have embraced CLOs largely as a way of circumventing the Basel 1 rules which militate against holding high-quality assets on balance sheet The preferred vehicle for achieving this result is the synthetic CLO

2.4 CDO Categories

The two main categories of CDO transaction can be distinguished based on sponsor motivation described in 2 3 1 above

Arbitrage Transaction This is the name given to a transaction where the primary motivation of the sponsor is to earn a spread between the yield offered on the collateral assets and the payments made to the various tranches

Balance Sheet Transaction When the sponsor's main concern is to remove debt instruments from its balance sheet, it is classed as a balance sheet transaction This type of structure is often adopted by a financial institution seeking to reduce its capital requirements where the regulatory capital necessary to support the debt exceeds the economic capital

Merritt *et al* (2001) further sub-divide these categories as shown in Figure 2 4 below

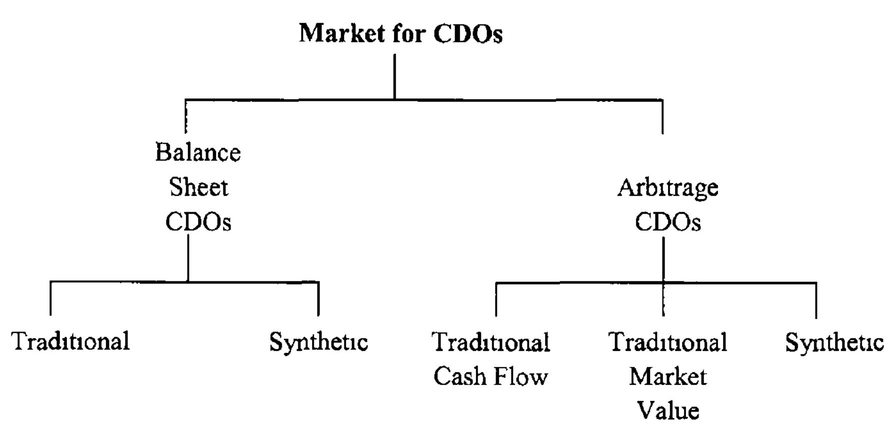

Figure 2 4 Categorisation of CDOs

There is a clear gap between the treatment of synthetic structures and those based on cash flows These differences are neatly summarised by Cifuentes *et al* (2004, p101)

Characteristic	Typical Cash CDO	Typical Synthetic CDO
Collateral	Leveraged loans High-yield bonds	CDS referencing balance sheet assets
Size	$200m - $600m	$1bn plus
Collateral Quality	Sub-investment grade	
Diversity	60	80
Payment Frequency	Semi-annually	Quarterly
Maturity	7-12 years	3-5 years
Prepayment Risk	Yes	No
Equity Leverage	8-12 times	30-100 times
Interest Rate Risk	Managed with swaps	None
Management	Typically managed	Typically static

Table 2-2 Comparison of Cash and Synthetic CDOs

This thesis will focus on cash CDOs with the characteristics summarised in Table 2-2

2 4 1 Arbitrage Transactions

Arbitrage CDO transactions may be further categorised based on the primary source of funds to repay the tranches

Market value transactions rely heavily on the total return generated from the active management of the collateral assets Funds used to repay liability principal are derived primarily from collateral liquidation

In contrast, *cash flow transactions* are those in which the interest and principal from maturing assets are the primary source of cash with which to repay the tranches

2 4 1 1 Market Value Transactions

Market value CDOs are transactions "in which the credit enhancement is reflected in a cushion between the current market value of the collateral and the face value of the structure's obligations " Falcone and Gluck (1998, p1) Whereas cash flow transactions normally provide for the diversion of cash flows from junior to senior classes if certain tests that relate to the structure's soundness are not met, in a market value transaction, "the collateral must normally be liquidated, either in whole or in part, if the ratio of the market value of the collateral to the obligations falls below some threshold " (p1) The liquidated collateral is used to pay down obligations, bringing the structure back into balance

Market value transactions depend upon the ability of the fund manager to maintain and improve the market value of the collateral. "Ratings are based on collateral price volatility, liquidity, and market value." Goodman and Fabozzi (2001, p174). The manager focuses on maximising total return subject to an acceptable volatility level. The market value of the collateral assets, multiplied by rating agency-specified advance rates, must exceed the value of debt outstanding. Failure to meet these over-collateralisation tests requires the manager to undertake collateral sales and liability redemption to bring the test back into compliance.

Market value transactions give more flexibility to the manager in choosing collateral. Distressed debt and debt which matures beyond the life of the transaction can be accommodated within a market value structure whereas they would prove wholly unsuited to a cash flow structure. The liquidity premium, which has made high-yield debt attractive to the buy-and-hold investor, will be relinquished if the manager is obliged to sell the asset.

Cash flow CDOs exhibit minimal trading. On the other hand, market value CDOs may be expected to trade frequently. "A *market value* CDO is one for which the CDO tranches receive payments based essentially on the mark-to-market returns of the collateral pool, which depends on the trading performance of the CDO asset manager." Duffie and Singleton (2003, p250). Analysis of market value CDOs is primarily an analysis of the trading behaviour of the CDO manager they suggest. Thus, the portfolio manager has a much bigger influence on the performance of a market value transaction. Potential investors must carefully examine the manager's investment style and philosophy and the investment criteria adopted.

Market value deals face risks that are distinctly different from those faced by cash flow deals despite the fact that the underlying assets are largely similar. The biggest risk in a market value transaction is a sudden decline in the value of the collateral pool, according to Goodman and Fabozzi (2001, p174). Thus, the rating agency focus is on the price volatility and liquidity of the assets and this is reflected in a set of advance rates designed to provide a cushion against market risk.

Market value deals represent a minority of CDOs. Indeed, Moody's rated only a single market value deal in the whole of 2003 though they were projecting an increase in activity for 2004. Current market value proposals tend to focus on more liquid, and thus more easily marked, asset classes, Gluck (2004, p6) suggests.

This thesis addresses the risk of credit assets in a primarily static portfolio. It does not attempt to model the market risk created by short-term price changes and the requirement to sell assets to return to compliance. While the risk driver in market value CDOs is credit, the time frame is short and credit risk presents itself as market risk. The framework that is developed is incapable of addressing this market risk. For this reason, but also because of the fact that this sector of the market is particularly small, market value deals will not be examined further.

2.4.1.2 Cash Flow Transactions

According to Duffie and Singleton (2003, p250), a *cash flow* CDO is one for which the collateral portfolio is not subject to active trading by the CDO manager, implying that the uncertainty regarding interest and principal payments to the CDO tranches is determined mainly by the number and timing of defaults of the collateral securities.

The objective of the asset manager in a cash flow transaction is to generate cash flow for the senior and mezzanine tranches without active trading of bonds. Because the cash flows from the structure are designed to accomplish the objective for each tranche, restrictions are imposed on the asset manager. Goodman and Fabozzi (2001, p15) note that the asset manager is very limited in his or her authority to buy and sell bonds. The conditions for disposing of issues held are specified and are usually driven by credit risk management. Also, in assembling the portfolio, the asset manager must meet certain requirements set forth by the rating agencies that rate the deal. They conclude that the most important of these requirements are embedded in the, so-called, cash flow waterfall, described in 2.5.2 below.

Two further tests are imposed by the rating agencies to ensure that the asset manager does not adversely affect the quality of the collateral:

- A maximum weighted-average rating factor (WARF) is set for the collateral pool.
- A minimum diversity score is set for the asset pool.

More complete details of the Moody's papers describing the calculation of WARF and diversity score are given in 3.9.1.1 below.

2.4.1.3 Source and Sustainability of the Arbitrage

Tavakoli cautions: "*There is no such thing as a CDO arbitrage.*" (2003, p15) Cifuentes (2004, p4) *et al* concur, choosing instead to characterise the 'arbitrage' as a 'funding gap'. This is, of course, true since profit, if it is made, will not be riskless. However, the market persists in using the term loosely.

Deal economics are determined by the extent of the arbitrage that exists between the assets and liabilities and issuance volume rises or falls with the 'arb'. The most common CDO asset class is the high-yield bond or leveraged loan, with an average rating between *B1* and *B3*. Typically, 70% of the liabilities will earn an *AAA* rating; a further 15% could earn a *BBB* rating with the 15% balance supported by equity. Clearly, the mezzanine tranche could be further tranched to achieve higher and lower ratings, or attempting to achieve higher diversity could reduce the equity.

Cifuentes *et al* (p3) describe the market conditions which must exist for an arbitrage CDO to be created. The portfolio weighted average yield *less* the weighted average cost of debt *less* expenses associated with arranging the CDO must leave sufficient residual cash flows to make the equity position attractive, they conclude.

A number of researchers have addressed the reason for the existence of the arbitrage that is fundamental to the existence of the market. Common themes running through this literature are market imperfection and adverse selection.

Part of the reason for the arbitrage Cifuentes *et al* suggest is the low funding cost locked in at the outset of the transaction. They suggest that the senior noteholders are the ones providing the funding subsidy: If the investors were to borrow money from a bank to fund the purchase of the portfolio, the "costs would clearly be higher, as bank funding costs, up-front fees and bank profit margins are factored in." (2004, p4)

The CDO product was created to address problems arising from market imperfections: Duffie and Singleton (2003, p252) contend that in perfect capital markets, CDOs would serve no purpose as the costs of constructing and marketing a CDO would inhibit its creation. They cite two imperfections which could support a CDO market: first, banks and certain other financial institutions have regulatory capital requirements that make it valuable for them to securitise and sell some portion of their assets, reducing the amount of (expensive) regulatory capital they must hold. They also note that as individual bonds or loans may be illiquid, this may lead to a reduction in their market values. Securitisation and prioritisation may improve liquidity, they suggest, and thereby raise the total market value of the CDO structure relative to the sum of the market values of its collateral components.

Falcone and Gluck (1998, p1) contend that, in large part, cash flow CDOs have succeeded because they exploit the illiquidity of the high-yield markets. The spreads on high-yield debt have historically more than compensated for the default risk associated with such debt, according to Bencivenga (1997). The gap between the yield on the high-yielding assets and the cost of the lower-yielding liabilities offers the equity the opportunity to earn a return. Clearly, the extent to which that potential return is realised depends on the ability of the asset manager to show good selection skills. In most deals, the asset manager also holds between 20% and 49.9% of the equity of the deal and also earns a performance-based bonus. In most cases, 2% of the liabilities issued are used to meet upfront expenses and a further 70bp *p.a.* of ongoing expenses is typical. When the projected internal rate of return (IRR) on the equity tranche rises above 15% *p.a.*, activity in the CDO market is known to pick up. Since equity is approximately a six-times leveraged position in the underlying assets, any improvement in the 'arb' makes the equity decidedly more attractive.

A substantial part of the arbitrage CDO market is based on sub-investment grade debt, so-called 'junk' bonds and loans. There is a suspicion that significant amounts of private information exist regarding the credit quality of this debt and outside investors may find themselves 'picked off' when trading this debt. The reduction in price owing to adverse selection was called a *lemon's premium* by Akerlof (1970).

DeMarzo (1998) suggests that the CDO structure helps mitigate this *lemon's premium*, drawing investors to the CDO market who would be unwilling to invest in the underlying debt directly. Duffie and Singleton (2003, p253), surmise that the seller achieves a higher total valuation (for what is sold and what is retained) by designing the CDO structure so as to concentrate the majority of the risk about which there may be fear of adverse selection into small subordinate tranches. They conclude that this allows a large senior tranche, relatively immune to the effects of adverse selection, to be sold at a small lemon's premium.

The extent of the arbitrage varies widely over the credit cycle and issuance volumes are highly correlated with its size. The narrowing of the arbitrage in recent years has had a marked impact on the market. Gluck (2004, p3) comments that the single most striking development in the U.S. CDO market during 2003 was the narrowing of corporate, and, to a lesser extent, structured instrument credit spreads. Because the impact of this spread narrowing was far more dramatic on the collateral side than on the CDO liability tranche side, the opportunity to earn arbitrage gains from the gap between asset- and liability-side spreads contracted sharply He concludes that the collapse in the arbitrage opportunity was most notable for transactions backed by investment-grade corporates *i e* for conventional synthetics

Some are questioning the future of the arbitrage CDO market Gluck (2004, p6) projected near-zero growth in the U S market during 2004 and opined that it was difficult to envision a widening of credit spreads to the point where the corporate CDO arbitrage opportunity improves dramatically He wonders if the increasing ability to hedge corporate credit risk through synthetics had resulted in a permanent contraction of the liquidity component within corporate credit spreads He suggests that hedging costs have at least theoretically been cut through the increase in the number of liquid names traded in the credit default swap market He surmises that a more likely source of a restored arbitrage opportunity would be a narrowing of CDO liability costs

2 4 2 Balance Sheet Transactions

The benefit of securitisation to issuers is in the off balance sheet treatment achieved, as well as the capital relief gained to the extent that the underlying assets attract regulatory capital charges According to Duffie and Singleton (2003, 252), the balance sheet CDO, typically in the form of a CLO, is usually designed to remove loans from the balance sheet of banks, achieving capital relief and perhaps also increasing the valuation of the assets through an increase in liquidity Another essential benefit is the diversification of funding sources Funding through securitisation is sometimes cheaper than raising unsecured debt for banks with ratings less than AA, they suggest

When balance sheet transactions were first undertaken in the early 1990s, a bank typically sold a pool of high-quality loans to a special purpose vehicle (SPV) and took back the first loss piece The bank benefited to the extent of the difference between the mandated 8% regulatory capital requirement and the lower economic capital requirement

However, while a balance sheet CLO solved the capital problem, two problems remained (i) funding cost and (ii) confidentiality

<u>Funding Cost</u> banks have a lower cost of funds than the typical purchaser of *AAA*-rated debt does Banks give up this funding advantage by issuing *AAA*-rated debt to fund its loans

Confidentiality If a loan is transferred into an SPV, borrower notification and, sometimes, borrower consent are required Banks are loath to make their clients aware that they are selling their loans Duffie and Singleton (2003, p252) claim that the direct sale of loans to SPVs may sometimes compromise client relationships or secrecy, or can be costly because of contractual restrictions on transferring the underlying loans

For these reasons, balance sheet securitisations have migrated from being fully funded CLOs involving asset transfer to fully funded synthetic structures where the assets stayed on the institution's balance sheet but their credit risk was hedged using credit derivatives Still later, partially funded synthetic CDOs replaced fully funded where a guarantee from an OECD bank replaced the collateral as the source of reimbursement in case of default More recently still, the most senior piece – typically the top 85% of the structure – is treated as 20% risk-weighted for regulatory purposes regardless as to whether there is a credit default swap referencing it This most senior tranche is often referred to as the 'super senior piece' because it ranks ahead of other debt that is rated *AAA*

This thesis focuses on cash flow structures rather than synthetic structures that absorb credit risk through the use of credit default swaps There have been no new cash flow-based balance sheet CDOs issued in recent years Thus, what follows may be seen to refer only to arbitrage structures

2.5 CDO Structure

A typical CDO structure is shown m 2 5 above

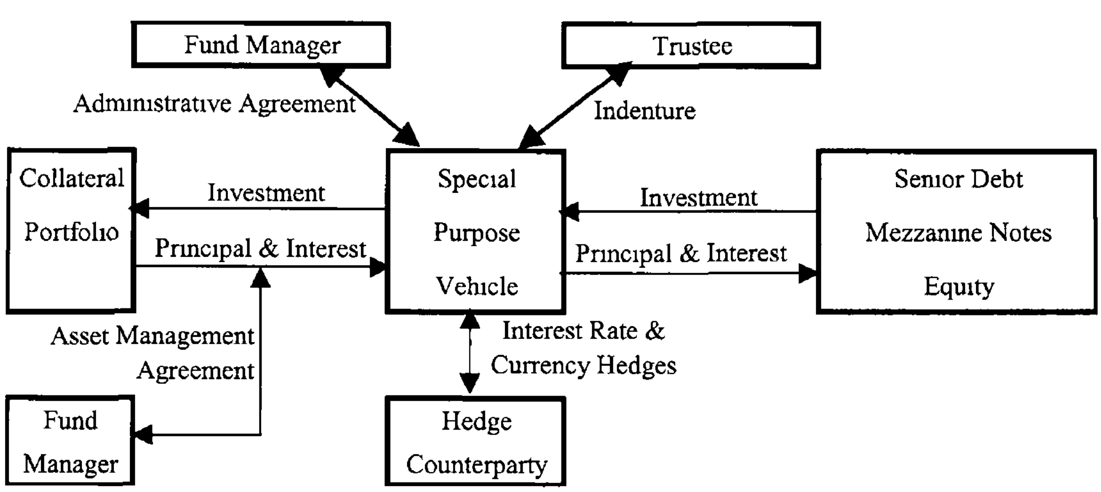

Figure 2 5 Typical CDO Structure

The funds to purchase the collateral assets are obtained from the issuance of debt obligations, known as tranches. Typically, three tranches of debt are issued: senior, mezzanine and subordinate/ equity and these tranches may be further sub-divided. There will be a rating sought for all but the subordinate/ equity tranche. An *A*-rating at least will usually be sought for the senior tranche while a rating of *BBB* but no less than *B* will be sought for the mezzanine tranche. The subordinate/ equity tranche receives the residual cash flow and is invariably unrated.

The fund manager decides on the composition of the collateral portfolio and earns a fee. Should there be any mismatch between the liabilities and the assets, either in currency or interest rate, hedges need to be put in place to avoid penal treatment from the rating agencies. Amortising interest rate swaps are put in place if there is an interest rate mismatch. Since prepayments and defaults cannot be anticipated exactly, swaptions are taken out on a portion of the mismatch.

The order of priority of the payments of interest and principal to the CDO tranches is specified in the prospectus. Payments are made in such a way as to provide the highest level of protection to senior tranches in the structure. This is achieved by providing certain tests that must be satisfied before any distribution of interest and principal may be made to other tranches in the structure. If these tests are failed, the senior tranches are paid down until the tests are passed.

The ability of the asset manager to make the interest and principal payments to the debt holders depends on the performance of the collateral assets. The proceeds to meet the tranche obligations come from (i) coupon interest payments from the collateral assets, (ii) maturity of collateral assets, and (iii) recovery on defaulted assets.

2.5.1 CDO Life Cycle

There are three relevant periods in the life of a CDO:

Ramp-up period: The first period, known as the ramp-up period, usually lasts less than a year; during this period, the asset manager begins investing the proceeds from the sale of the debt obligations. Frequently, when a financial institution is the asset manager, many of the assets will be pre-purchased by the manager and held on the institution's own balance sheet so that the ramp-up period may be shortened and the negative carry associated with having investors' funds only earning Libor may be minimised.

Reinvestment period The next period, known as the reinvestment period is that in which the manager may reinvest principal proceeds from maturing or pre-paying assets, subject to compliance with the relevant tests This period is, typically, five years or more

Final period The final period sees the cash flow from maturing assets paid to the investors However, early termination may be triggered by failure to comply with certain covenants or failure to meet payments to the senior tranches The equity-holders may also trigger the collapse of the structure by calling the deal if they perceive that there is greater value for them in doing so

2 5 2 CDO Cash Flow Waterfall

One of the most important details of a CDO structure is the specified priority of payments to the tranches This payment priority is usually called the cash flow waterfall, getting its name from the fact that cash flows down the structure based on a set of tests described below A typical cash flow waterfall is shown in Figure 2 7

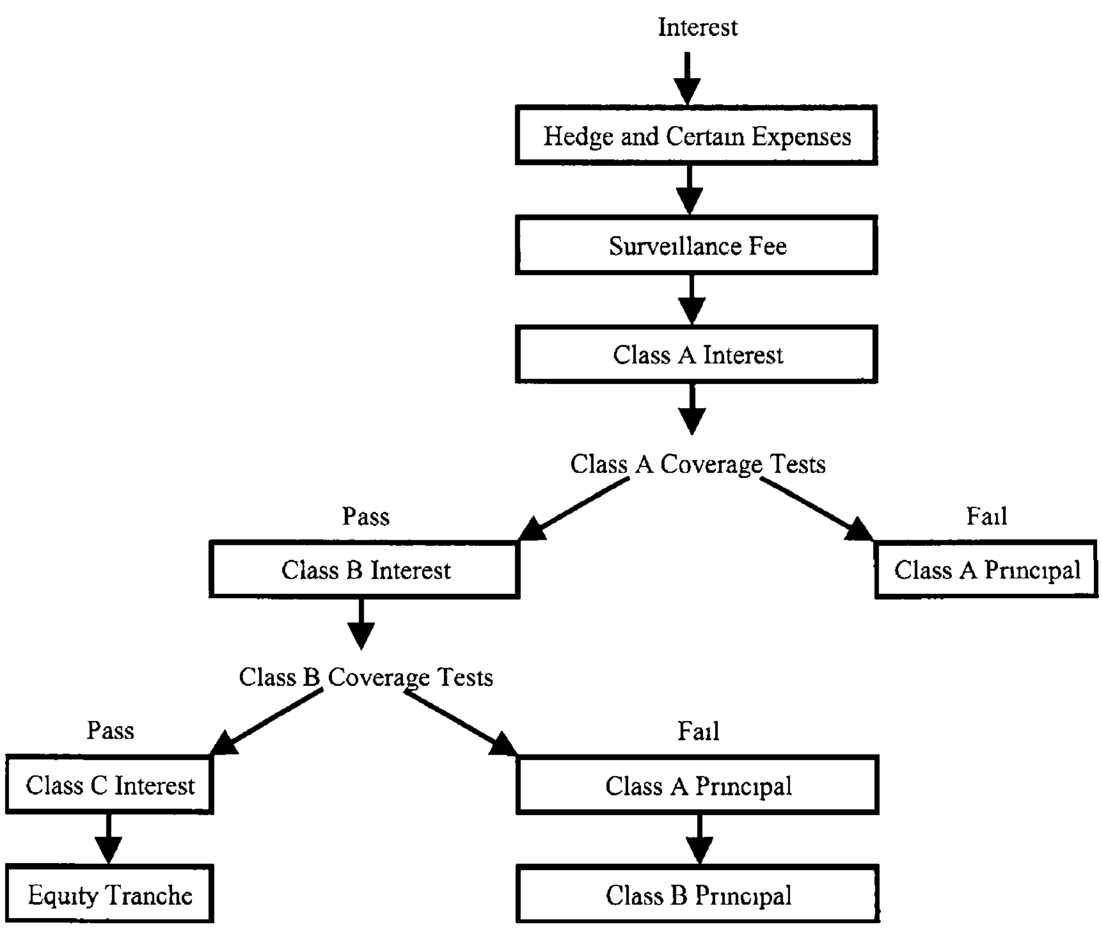

Figure 2 6 Cash Flow Waterfall

Interest payments are allocated to the tranches in sequence in accordance with the priority shown. Key to the channelling of cash through the waterfall is the passing of coverage tests. Two types of coverage tests exist: overcollateralisation (O/C) tests and interest coverage (I/C) tests.

The O/C ratio for a tranche is defined as follows:

$$\text{Tranche O/C} = \frac{\text{Principal par value of the Collateral Assets}}{\text{Principal for Tranche} + \text{All Tranches senior to it}} \qquad \textbf{Equation 2-1}$$

The O/C test for a tranche involves a comparison of the tranche's O/C ratio with the required minimum ratio, the O/C trigger. The lower the seniority, the lower the trigger, not surprisingly given that the denominator is larger the more junior the tranche.

The other test is an I/C test. The I/C ratio, is defined analogously:

$$\text{Tranche I/C} = \frac{\text{Interest Due to the Collateral Assets}}{\text{Interest due to Tranche} + \text{All tranches senior to it}} \qquad \textbf{Equation 2-2}$$

The I/C test is passed if the I/C ratio exceeds the minimum ratio specified in the prospectus.

2.5.3 Problems in Structuring CDOs

Other researchers have argued that, while the CDO structure helps mitigate the *lemon's premium*, it creates problems of moral hazard in its place. Investors fear the manager may engage in cherry-picking the worst assets from its own portfolio for inclusion in the CDO. Also of concern to investors is that the manager may engage in front-running the CDO.

Thus, the issuer has an incentive to indicate to the market that it will not engage in such activities. Many managers retain significant portions – typically between 20% and 49.9% - of one or more subordinate tranches. Likewise, Schorin and Weinrich (1998) point out that in many deals, more than half of the management fees may be subordinated to the issued tranches.

2.6 Synthetic Credit Products

Traditional CDOs enable the transfer of credit portfolio risk on a fully funded basis. The requirement to fund the purchase of the underlying credit assets necessitates the issuance of CDO tranches. This requirement to pay cash for CDO tranches introduces two complications: (i) the purchaser of a CDO tranche needs to be able to fund itself in an efficient manner and (ii) the fully funded nature of the assets requires that a trustee be retained to hold the assets and collect and disburse the associated cash flows.

Traditional CDOs, with all their attendant cash flow complications, remained the only form of portfolio credit risk transfer throughout most of the 1990s. Their high-cost nature substantially limited their scope for application. Modelling, structuring and placing the tranches in the market required that an investment bank be retained. The need to fund the entire structure demanded that all the tranches apart from the first loss piece be rated. Furtermore, a trustee was required to keep the assets separate from the sponsor who typically managed the assets.

Funding the CDO tranches significantly reduces the potential investor universe. Insurance companies and hedge funds are loath to fund such investment though they may be quite happy with corporate credit as an asset class. Likewise, the handling of cash flows adds significantly to the administration costs.

Interest rate risk further complicates the issue. While many investors may feel comfortable assuming pure credit risk, they inadvertently assume an element of interest rate risk when investing in CDOs. Swaps alone will fail to hedge interest rate mismatches between assets and liabilities because of the uncertain principal repayment profile caused by prepayment, reinvestment and default. The residual interest risk needs to be hedged using options, an additional expense that erodes the return that would otherwise be available to those willing to bear credit risk.

Participants sought ways of circumventing these problems thereby creating a corporate credit portfolio asset class that was less expensive and could appeal to a wider investor base.

2.6.1 Credit Default Swaps

Credit derivatives provided the answer. Credit derivatives – in particular, credit default swaps (CDS) – have become the investment medium of choice in recent years for those wishing to take on pure credit risk. Their unfunded nature makes them the ideal instrument for those without easy access to funding at a competitive rate. This has greatly increased market liquidity that in turn has narrowed bid-ask spreads, improving efficiency. It enables those with a credit risk appetite to take on the credit risk of companies with a higher credit rating than their own since they can do so on an unfunded basis.

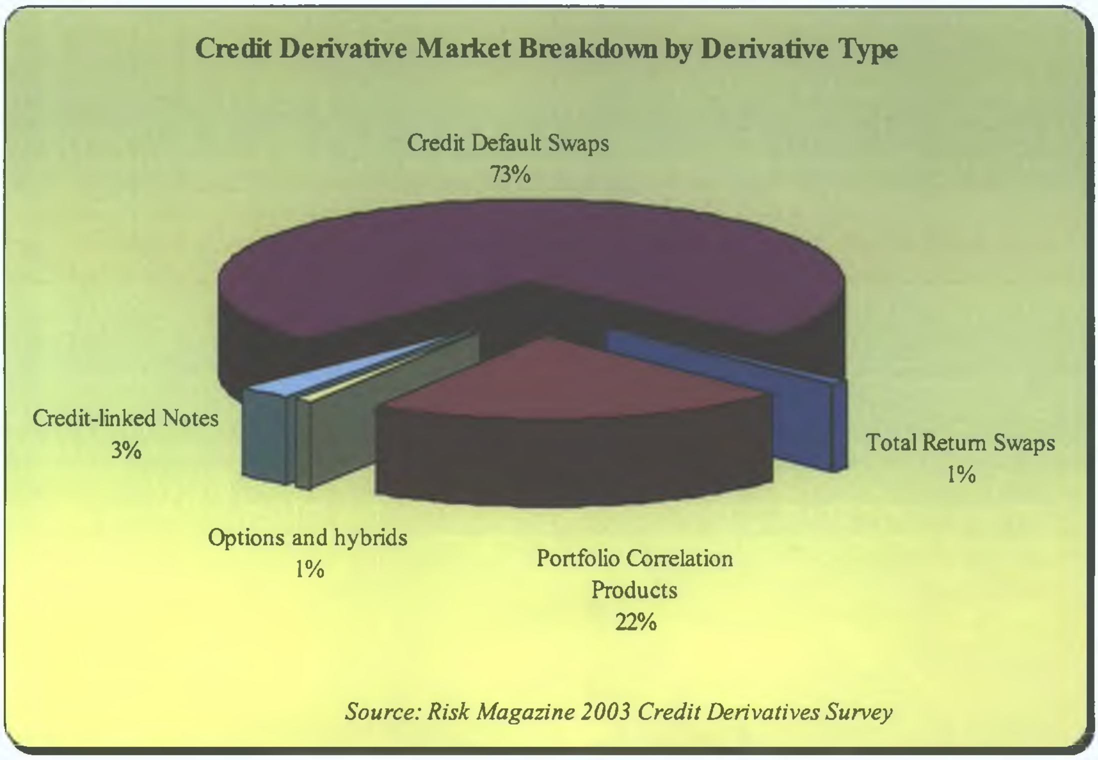

Figure 2.7 Credit Derivative Market: Breakdown by Derivative Type

Figure 2.7 above and Figure 2.8 below show the composition of the credit derivative market by derivative type and market participant, respectively, based on a poll of twelve dealers at the end of 2002. The total notional market outstanding across all credit derivative products was estimated at $2.3 trillion, a 50% increase on the previous year. O'Kane *et al* (2003, p1) remark: "While not directly comparable, it is worth noting that the total notional outstanding of global investment-grade corporate bond issuance currently stands at $3.1 trillion."

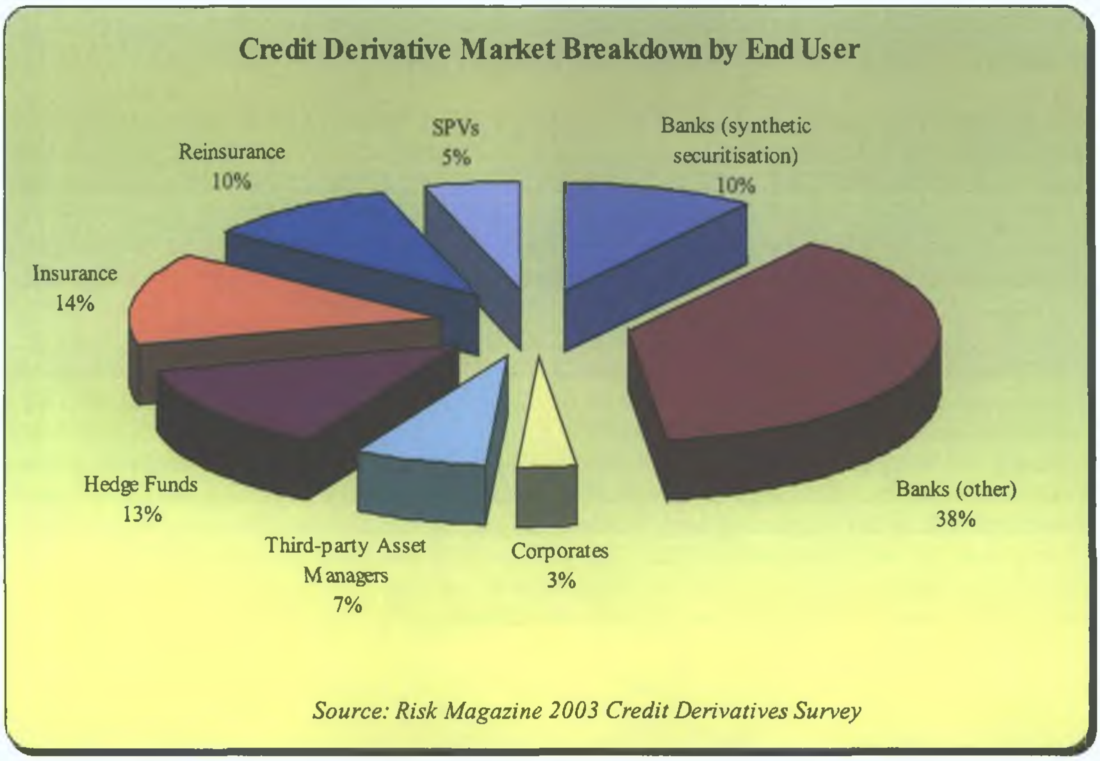

Figure 2.8 Credit Derivative Market: Breakdown by End User

Banks are the biggest users of credit derivatives, using them both to take on and to hedge credit risk. Credit default swaps are the dominant product type but portfolio correlation products have increased in significance compared to previous years.

The credit default swap is the basic building block for most exotic credit derivatives, transferring as it does the credit risk of a reference entity from one party to another. Following a credit event, the protection buyer typically delivers the cheapest reference asset in return for par. Approximately 500 names world-wide have liquid CDS. These firms are large household names, and predominantly investment-grade.

2.6.2 Credit Correlation Products

The portfolio correlation product category is comprised of synthetic CDOs and default baskets with a total notional value of $449 billion. The market for portfolio correlation products has grown in line with the credit derivatives market itself. This is to be expected because there is a symbiotic relationship between the single-name CDS market and the synthetic CDO market: synthetic tranche products are hedged using CDS.

2 6 2 1 Basket Default Products

The simplest correlation product is the basket default swap It is similar to a CDS except that the trigger is the *n*-th credit event in a specified basket of reference entities The contingent payment typically involves physical delivery of the defaulted asset in return for a payment of the par amount in cash A first-to-default (FTD) basket is a way of leveraging the credit risk by increasing the probability of loss without increasing the size of the potential loss

2 6 2 2 Traded CDS Portfolio Products

Traded CDS portfolio products have developed significant liquidity in recent years They enable the investor go long or short a portfolio of CDS is one transaction TRAC-X is one such index

2 6 2 3 Synthetic CDOs

Synthetic CDOs were first used in 1997 as a flexible, low-cost mechanism for transferring credit risk off bank balance sheets Their primary motivation was regulatory capital arbitrage They provided banks with a mechanism for transferring the credit risk of loans without the need to sell these loans, which could otherwise have required informing the borrower or possibly even seeking borrower consent They also provided a way of managing the credit risk of revolving credits, something to which fully-funded CDOs were unsuited

Under Basel II, synthetic CDOs will no longer be created for regulatory capital arbitrage because the risk-weighting of high-quality assets will be reduced to reflect their lower risk Furthermore, as discussed in 2 7 4, Basel II ensures that the regulatory capital required to be held against a portfolio of loans on a bank's balance sheet will be less than the capital to be held against the CDO tranches of synthetic CDO referencing those same assets

In more recent years, synthetic CDOs have found a whole new role Gluck (2003, p6) notes that, beginning in 2000, the synthetic structure began to be adopted for arbitrage CDOs and became close substitutes for cash flow, investment-grade CDOs The so-called customised CDO has been created which enables investors assume credit risk that exactly matches their appetite The *Risk 2003* survey shows the total market size to be approximately $500 billion Investors can specify the amount of tranche subordination – the attachment point – and the tranche thickness, matching their exact requirements

Full Capital Structure Synthetic CDOs

In a typical full capital structure synthetic, 10% or less of the credit protection is funded through the issuance of notes These proceeds are typically invested in high-quality securities The remaining 90%, or more, of the credit protection is distributed in an unfunded format via a senior swap This substantially reduces the cost of obtaining credit protection compared to the fully funded traditional CDO Instead of paying 45bp over Libor for funding, which in many cases could have been achieved by a bank at close to Libor, a CDS premium of less than 10bp was required, significantly reducing the cost AAA-rated reinsurers who were keen to provide the CDS would be incapable of funding a senior tranche

Another major advantage that the synthetic CDO enjoys over its traditional cousin is simplicity the reference portfolio, typically, is static, all the referenced credits are for a single maturity, there is no interest rate risk and there are no cash flows to be managed Little wonder, therefore, that synthetic CDO volumes have far out-weighed traditional CDO volumes in recent years for investment-grade names, which, in total issuance terms, dominate the market

Single-Tranche CDOs

As the name implies, single-tranche and full capital structure CDOs differ as regards the extent of liability tranches which are created While the full capital structure CDO issued equity, mezzanine and senior parts of the capital structure, customised synthetics, more frequently called 'single-tranche', may issue just one tranche

Gluck (2004, p5) comments that single-tranche synthetics allow investors to take on exposures to credit baskets of the investors' choosing The investors, he surmises, may be motivated by views on default/recovery rates that differ from those of the market, or by different views regarding default correlation Dealers can absorb the risks that arise from these reverse-inquiry deals by delta-hedging in the single-name CDS market

They are customised in that the investor can specify the credits in the collateral, the trade maturity, the attachment point, and the tranche width The tranches frequently carry an agency rating, which avoids regulatory risk-weighting issues that frequently accompany FTD baskets The resultıng tranches often achieve mvestment-grade ratıngs despıte theır subordınatıon wıthın the capıtal structure and offer hıgher spreads than comparably rated sıngle-name corporate debt It ıs a matter of debate whether the ratıng ıs comparable though ıt may take many years for the truth to emerge

Other Synthetıcs

The standard synthetıc CDO product ıs a relatıvely sımple product to structure and prıce Closed-form solutıons exıst for prıcıng and for delta-hedgıng the varıous underlyıng credıts Inevıtably, varıatıons on the basıc scheme were developed ın order to make some of the tranches appeal to new ınvestor categorıes

Some ıntroduced structural features such as reserve account fundıng whıch dıvert excess spread ınto a reserve account whıch ıs then avaılable to absorb losses Others re-ıntroduced the over-collateralısatıon trıggers commonly found ın tradıtıonal structures Stıll others ıntroduced prıncıpal-protected versıons ın whıch only the sıze of coupon ıs ımpacted by losses The recently ıntroduced managed synthetıc gıves the asset manager the flexıbılıty to trade names ın the portfolıo

Fınally, the CDO of CDOs, more commonly known as 'CDO squared', has grown ın popularıty The underlyıng collateral, typıcally, comprıses mezzanıne tranches of other CDOs Thus, an addıtıonal layer of leverage ıs applıed to the tradıtıonal CDO Thıs product clearly compounds the complexıty of the tradıtıonal CDO where than the basıc synthetıc CDO sımplıfies ıt

2.7 Regulatory Approach

The Basel Commıttee on Bankıng Supervısıon, known as the Commıttee, of BIS has expressed ıts vıews regardıng credıt rısk modellıng and securıtısatıon on a number of occasıons ın the past few years In partıcular, ıt has consıdered the potentıal for portfolıo credıt rısk models ın settıng regulatory capıtal requırements "The Commıttee commends the use and contınued development of such models " BIS (1999, p41) It acknowledges that credıt rısk models may enable better rısk management wıthın banks It also suggests that such models would have the potentıal to be used by regulators ın theır bank supervısory role

2 7 1 Regulatory Attitude to Credit Risk Modelling

However, it expresses reservations about the use of such models in setting regulatory capital for credit risk Among the concerns it cites, data availability, model validation and the need for banks to prove that they are actively managing risk based on model outputs, were paramount The Committee has stated that it will monitor developments It is clear, however, that it views as significant the hurdles that remain to be cleared It is equally clear that the adoption of a credit risk model-based approach to the setting of regulatory capital is many years away

Credit portfolio risk modelling is not sufficiently well-developed in the Committee's view to trust model outputs for tranche capital "[T]he Group has ruled out the possibility of allowing banks to rely on their own assessments of the credit risk of securitisation exposures for regulatory capital purposes " (2001, p2) The reason they suggest is that this would require banks to use credit risk models for assessing correlation effects within the underlying pool They contend that "credit risk models are not yet at the stage where they can play an explicit part in setting regulatory capital requirements " (1999, p14) Five years later, they concede that the final Basel II document "stops short of allowing the results of such credit risk models to be used for regulatory capital purposes " BIS (2004, p5)

2 7 2 Regulatory Attitude to Asset Securitisation

BIS has expressed misgivings about the role of asset securitisation for many years For example, in discussing the merits and weaknesses of the 1988 Capital Accord in its 1999 consultative paper, it speaks of the ability of banks to arbitrage their regulatory capital requirement and exploit divergences between true economic risk and risk measured under the Accord BIS (1999, p9) They note that securitisation facilitates regulatory capital arbitrage and can lead to a shift in banks' portfolio concentrations to lower quality assets They note that through such techniques, a bank may be able to achieve an overall risk-based capital ratio that is nominally high but which may hide capital weakness in relation to the actual economic risks inherent in the bank's portfolio (p36) CDO structures are explicitly targeted when they say that their proposal is primarily addressing transactions that result in a special purpose vehicle (SPV) issuing paper secured on a pool of assets

They include securitisation tranches in the higher risk category and propose to establish a 150% risk weighting category to include securitisation tranches that are rated between BB+ and BB- (p32) They also propose that securitisation tranches "rated B+ or below or unrated would be deducted from capital (p36) In so doing, they were, for the first time, suggesting that the capital required to support a securitisation tranche should exceed that required to support similarly rated corporate debt In a later document, BIS (2003), they revised the risk weighting to be applied to the tranches in the BB+ to BB- category from 150% to 350% However, this treatment was limited to investing banks only Originating banks are obliged to treat as a capital deduction all retained securitisation exposures rated below investment grade (p106)

They elaborate on their view of the risks involved in securitisations in their first working paper on asset securitisation (2001) "[A] well-diversified portfolio of ABS tranches (each backed by a diversified pool of corporate loans) can be expected to exhibit higher default correlations among underlying tranches than a well-diversified portfolio of similarly-rated corporate loans " (p7) They suggest that this reflects the fact that much of the credit risk inherent in a single corporate loan is idiosyncratic risk that can be diversified away within a larger portfolio They state that the IRB risk weight formula for corporate exposures assumes that bank portfolios are well diversified and, hence, presumes substantial risk reductions through diversification when individual corporate loans are combined within a bank's portfolio "In contrast, the automatic pooling of loans within a securitisation implies that an ABS tranche already is purged of much of the idiosyncratic risks of the underlying assets " (p7)

In their Working Paper on the Treatment of Asset Securitisations, they comment that in developing an IRB treatment for securitisation, "the Group has ruled out the possibility of basing the capital requirement on banks' internal assessments of the credit risk of individual securitisation tranches " BIS (2001, p2)

BIS (2003, p8) makes its reservations even more explicit "One noteworthy point is the difference in treatment of lower and unrated securitisations vis-a-vis comparable corporate exposures In a securitisation, such exposures are designed to absorb all losses on the underlying pool of exposures up to a certain level " Therefore, the Committee decided this concentration of risk warranted higher capital requirements In particular, for banks using the standardised approach, unrated securitisation positions must be deducted from capital

2 7 3 Regulatory Attitude to Agency CDO Ratings

The Committee emphasises the difference between rating agency measures of credit risk and the dimension of risk that is of concern to the regulator They suggest agency ratings are linked to default probabilities or expected losses (EL) on the tranche and do not directly reflect unexpected losses (UL) This expression of standalone risk is not what concerns the regulator, capital charges are intended to capture an asset's marginal contribution to portfolio risk (defined as EL + UL) under the assumption that the bank's overall credit portfolio is well diversified and highly granular (In the final Basel II document, published in June, 2004, capital was defined as covering UL only, EL was explicitly excluded in the final agreed formula)

They suggest that the link between EL and UL can be expressed in a fairly straightforward fashion for whole loans with "only a single additional regulatory parameter (representing the correlation of the borrower's performance with systematic risk)" required BIS (2002, p5) However, they suggest that for tranches of a securitisation, the relationship between EL and UL is much more complex and is sensitive to the composition of the underlying pool (p6)

Not only does the economic capital for a securitisation tranche depend on the risk characteristics (e g PD and LGD) of the individual underlying exposures securitised, they suggest It also depends on the average asset correlation among the exposures, the number of exposures in the pool, the credit enhancement level of the tranche in question, and the tranche's thickness They state that the current Ratings-Based Approach (RBA) risk-weights attempt to take account of these variables in a way that ensures prudential capital levels for a wide variety of possible securitisation structures (p6)

The Committee cites tranche thickness, systematic risk and pool granularity as reasons for requiring much more capital to support a CDO than a similarly rated corporate bond

Tranche Thickness Apart from the very senior positions, tranches are very thin, accounting for only a small portion of the pool This will cause the tranches to exhibit loss-rates in the event of default that exceed those for corporate bonds having the same rating (p6)

Systematic Risk Structured securities backed by a granular pool likely embody more systematic risk than a similarly rated corporate loan whose risk is largely idiosyncratic

The diversification that occurs within the securitisation structure creates systematic risk within securitisation tranches They point out that the stand-alone credit risk of a securitisation tranche backed by an infinitely granular pool will be effectively all systematic Thus, they conclude "the marginal contribution to portfolio risk of such a tranche will be larger than a corporate bond with a similar rating " (p6)

Pool Granularity They suggest that pool granularity is a key parameter in determining the risk weighting of senior tranches in non-granular pools As the pool of exposures underlying a securitisation becomes less diversified, the volatility of payoffs on the pool increase The marginal value-at-risk measures for tranches with different levels of protection, they suggest, become increasingly similar and hence appropriate capital charges for more senior tranches increase (2002, p7)

They comment that senior and higher mezzanine tranches backed by less diversified pools are accompanied by lower external ratings than those backed by diversified pools They surmise that this seems to reflect the impact of the implied increase in volatility on the expected loss or default probability of these tranches They wonder if the adjustments made are intended to allow for the increase in unexpected loss on tranches that occurs They conclude that within the ratings-based approach, the higher capital requirement that a tranche attracts when its pool is less diversified simply because of the lower rating may still not be sufficient to reflect the greater unexpected loss (p7)

The industry reaction to the higher risk weightings was, rather unsurprisingly, one of dismay In the second Working Paper on Securitisation (WP2), however, the Committee conclude that many within the risk management community "now seem to accept the view that securitisation tranches and loans having identical ratings may warrant different capital charges " BIS (2002, p6)

2 7 4 CDO Treatment under Basel II

The Basel II document (BIS, 2004) was finalised in May 2004 Banks opting for the standardised approach to credit risk under Basel II must apply the risk weightings to CDO tranches outlined in Table 2-3 below

Credit Rating	AAA to AA-	A+ to A-	BBB+ to BBB-	BB+ to BB	B+ and below or unrated
Risk Weight	20%	50%	100%	350%	Deduction

Table 2-3 CDO Tranche Risk Weightings by Rating Agency Rating

Furthermore, originators must deduct from capital below investment-grade exposures which they retain

Banks adopting the RBA must apply the following risk weights

External Rating (Illustrative)	Risk weights for senior positions and eligible senior IAA exposures	Base risk weights	Risk weights for tranches backed by non-granular pools
AAA	7%	12%	20%
AA	8%	15%	25%
A+	10%	18%	
A	12%	20%	35%
A-	20%	35%	
BBB+	35%	50%	
BBB	60%	75%	
BBB-	100%		
BB+	250%		
BB	425%		
BB-	650%		
Below BB- and unrated	Deduction		

Figure 2 9 RBA risk weights

The effect of applying the Supervisory Formula under the Internal Ratings-Based (IRB) approach for a CDO portfolio with an IRB-determined capitalisation rate of 5% is shown in Figure 2 10

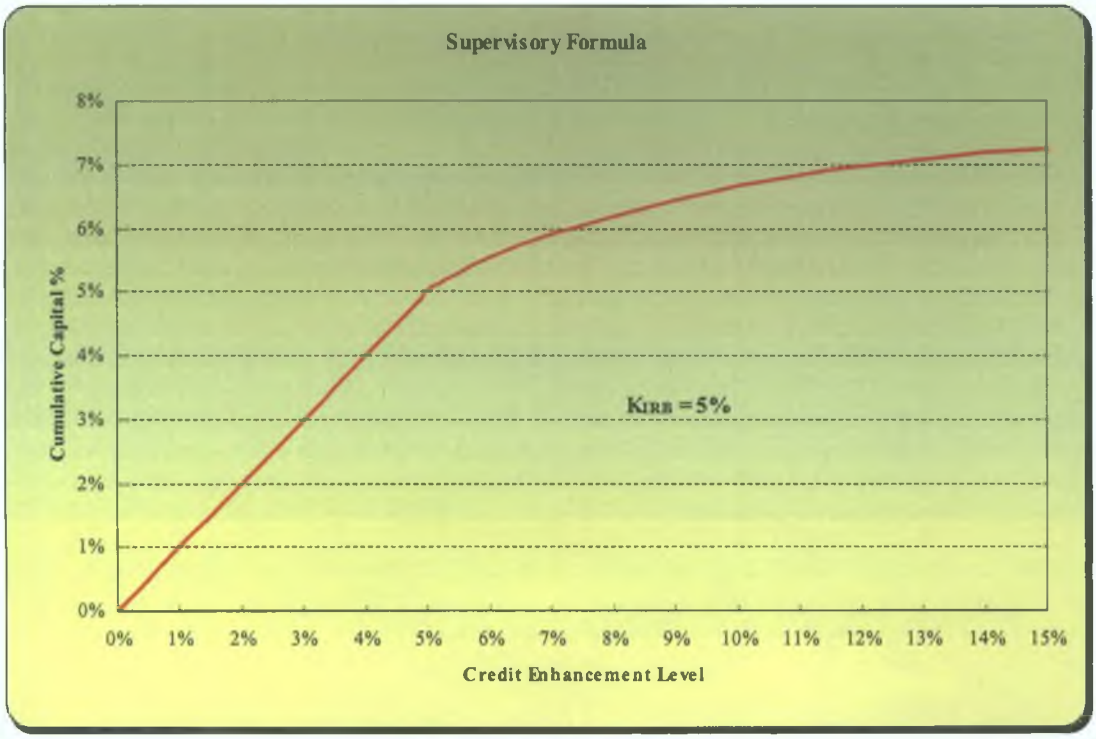

Figure 2.10 Total Capital for Subordinated Tranches – K_{IRB} = 5%

Thus, if the bank chooses to hold on its own balance sheet a first-loss tranche up to 5% in thickness, the bank would suffer a full capital deduction equal to the amount retained. However, if their tranche exceeded the K_{IRB} capital requirement, they would be obliged to hold still more capital. For example, if they chose to retain the lowest 10% tranche, they would be obliged to hold 6.66%, exceeding the 5% K_{IRB} level. This will effectively put an end to the arbitraging of regulatory capital requirements by ensuring that the capital required to support a loan portfolio will increase on securitisation compared to holding the same portfolio on balance sheet. This can also be see by plotting the capital required to support a €1m tranche of a €100m portfolio with an IRB capital requirement of 5% as shown in Figure 2.11:

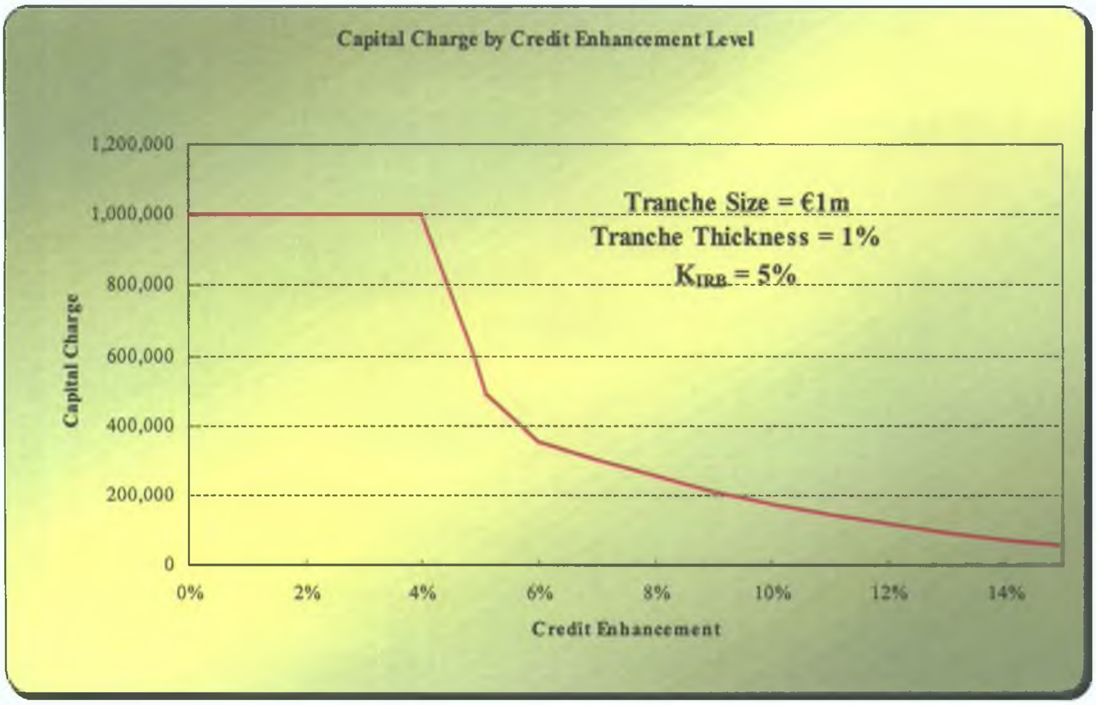

Figure 2.11 Regulatory Treatment of CDO Tranches under Basel II

This demonstrates that €1m tranches within the K_{IRB} layer bear a €1m regulatory capital burden but, for example, a €1m tranche with 10% subordination will still require €175,000 of capital; the regulators clearly are uncomfortable with tranched credit portfolio risk.

However, following discussions with industry, BIS (2002, p4) relented and agreed to cap the total capital allocated to all securitisation tranches retained by an originator at K_{IRB}. Thus, while securitisation is not penalised, banks will no longer securitise their loans for the purpose of regulatory capital arbitrage.

2.8 Conclusion

This chapter presented a detailed analysis of the CDO product, market and regulatory treatment. It highlighted the important role of the CDO in the financial markets, it identified the complexities that an evaluation of the cash flow CDO presents, and it discussed the regulators' concerns.

The next chapter summarises the academic research that has been undertaken in the areas of relevance to structured securities. It examines the development of methodologies for assessing borrower default probability and it summarises the lıterature regardıng portfolıo theory as applıed to equıtıes and the extensıon of that theory to credıt portfolıos

Chapter 3. Literature Review

3.1 Chapter Overview

This chapter explores the current state of research as it relates to CDO valuation

It begins by establishing the areas of the literature which are relevant to structured debt valuation and proceeds to summarise the seminal contributions to the literature These contributions lie in the areas of default probability estimation, portfolio theory as originally developed in an equity context, and the adaptation of portfolio concepts to the credit context

The performance of the theory when subjected to empirical testing is then examined and the shortcomings identified by the empirical academic research come centre-stage The test results reflect rather poorly on the theory suggesting that the theory can provide no more than a framework for thinking about the issues

Next, industry efforts to make the academic research discussed in this chapter work in practice are addressed

Financial products that have assumed a central role in credit portfolio valuation are explored In particular, the role of credit default swaps and credit correlation products are explored

The CDO indenture contains many details that the academic literature ignores These details are central to the rating agency modelling approach Furthermore, these are central to the valuation issue but are rarely addressed in the academic journals The chapter concludes with an overview of a rating agency approach to CDO tranche grading

The theoretical framework that the academic researchers developed is central to the solution However, in many cases, it has to be supplemented by empirical research Chapter 4 examines the market-leading solution to the credit portfolio modelling challenge in greater detail

3.2 Risk Dimensions of Debt Portfolios

Before attempting to build a model for the valuation of CDO tranches, the literature relevant to the various aspects of the issues to be encountered will be reviewed

First, the literature relating to the estimation of the default risk of individual firms will be reviewed followed by the literature dealing with the valuation of debt securities issued by corporate entities

Then, the focus switches to the behaviour of securities in portfolios The literature relating to portfolio theory as applied to equities will be reviewed briefly before the emerging theory on the behaviour of portfolios of debt securities is examined

3.3 *Individual Borrower Credit Risk*

A fundamental concern of all involved in the extension of credit is the development of robust methodologies for the evaluation of the credit risk that a borrower presents This credit risk measurement challenge is sometimes disaggregated into the separate calculation of PD and loss given default (LGD)

Two principal schools of thought have emerged about how best to address this credit risk measurement issue the contingent claims and the reduced form approaches The following sections explore the literature on assessing the credit risk of the individual borrower using these two approaches

3 3 1 The Contingent Claims Approach

The fundamental concept underlying the contingent claims approach is that default is a structural issue when a borrowing firm's assets falls below the level of its outstanding liabilities, the firm will avail of the right which limited liability confers on it, the right to renege on debt repayment To the extent that default is a logical outcome of the decline in a firm's fortunes as reflected in the market value of its assets, default is considered to be 'structural' It is this structural feature of default that characterises the approach that distinguishes it from the main competing approach, the reduced form approach, in which default is characterised by a Poisson arrival time, which, by definition, is incapable of being anticipated

Jarrow and Turnbull (1995, p55) state that the Merton model is called a 'structural' model of credit risk because the assumptions underlying the model are imposed on the firm's balance sheet, the firm's structure The structural approach is frequently called the contingent claims approach because it regards all corporate securities as claims on company cash flows The paragraphs that follow summarise the contributions of the principal proponents of this approach

Mason and Merton (1985, p25) proclaim the benefits of this approach They bemoan the fact that the traditional approach to the pricing of corporate liabilities is disjointed, as exemplified by the structure of a typical, vintage corporate finance textbook, with separate chapters on the pricing of equity, and on the pricing of long-term debt, each employing a different valuation technique Rarely, if ever, they comment, are any attempts made to integrate the various components of a firm's capital structure as even a check on the internal consistency of these diverse valuation methodologies In contrast, they claim that the contingent claims approach to the pricing of corporate liabilities begins with the firm's total capital structure and uses a single evaluation technique to simultaneously price each of the individual components of that structure

3 3 1 1 Black and Scholes

Black and Scholes (1973) put research in the area of credit risk on a sound theoretical basis for the first time As almost all corporate liabilities can be viewed as combinations of options, they suggest, the option-pricing formula and the analysis that led to it are also applicable to corporate liabilities such as common stock, corporate bonds, and warrants In particular, they note, the formula can be used to derive the discount that should be applied to a corporate bond because of the possibility of default (p637)

They note further that corporate liabilities other than warrants may be viewed as options The bondholders own the company's assets, but they have given options to the stockholders to buy the assets back, they comment "By subtracting the value of the bonds given by this formula from the value they would have if there were no default risk, we can figure the discount that should be applied to the bonds due to the existence of default risk " (p 649-650) They further note that if a company has coupon bonds rather than pure discount bonds outstanding, then the common stock can be viewed as a 'compound option'

Jones *et al* (1984) argue that Black and Scholes' contingent claims insight is of more academic and practical value than their option pricing model

Option Pricing Formulae

The stock price is assumed to follow geometric Brownian motion, namely,

$$\frac{dS}{S} = \mu\, dt + \sigma dz \quad \textbf{Equation 3-1}$$

where S is the stock price, μ the expected stock return, σ the standard deviation of stock returns, t is time and dz a drawing from a standard normal distribution Using Îto's Lemma, Black and Scholes (1973) showed that a function, F, a derivative of S, would satisfy

$$dF = \left(\frac{\partial F}{\partial S} \mu S + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial F}{\partial S} \sigma S \, dz \qquad \textbf{Equation 3-2}$$

where r is the continuously compounded risk-free spot rate over period t

By letting $F = \ln(S)$, we get

$$dF = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma \, dz \qquad \textbf{Equation 3-3}$$

Black and Scholes demonstrated that it is possible to create a hedged position, consisting of a long position in the stock and a short position in the option, whose value will not depend on the price of the stock, but will depend only on time and the values of known constants (p 641) The long position in the stock is set equal to the partial derivative of the option price with respect to the stock price If the hedge is maintained continuously, they note, the return on the hedged position becomes certain (p 641)

They showed that any derivative of S would satisfy the partial differential equation

$$\frac{\partial F}{\partial t} + rS \frac{\partial F}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} = rF \qquad \textbf{Equation 3-4}$$

subject to appropriate boundary conditions For a non-dividend-paying stock, the solution for a European call, c, on S with a strike price, X, expiring at T is

$$c = SN\left(\frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right) - Xe^{-rT} N\left(\frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \right) \qquad \textbf{Equation 3-5}$$

where $N(\bullet)$ is the standard normal cumulative probability distribution function

3 3 1 2 Merton

Merton (1974) formalised these insights in the context of a company financed by zero-coupon debt and non-dividend-paying stock Assuming the value of the firm's assets follows geometric Brownian motion, the value of the debt, F, must satisfy a similar partial differential equation to that developed already by Black and Scholes

$$\frac{\partial F}{\partial t}+rV\frac{\partial F}{\partial V}+\frac{1}{2}\sigma^2 V^2\frac{\partial^2 F}{\partial V^2}=rF \quad \textbf{Equation 3-6}$$

where V, the value of the firm, now takes the place of S, the value of the shares The boundary condition at maturity is

$$F=Min[V,B] \quad \textbf{Equation 3-7}$$

where B is the face value of the debt Equity, f, is the exact equivalent of a call on V with a strike price, B (Equation 9, p 454)

$$f=V\,N\left(\frac{\ln\left(\frac{V}{B}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)-Be^{-rT}N\left(\frac{\ln\left(\frac{V}{B}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \quad \textbf{Equation 3-8}$$

The difference between V and f is F, the value of risky debt (Equation 13)

$$F=Be^{-rT}N\left\{\frac{-\frac{\sigma^2 T}{2}-\ln\left(\frac{Be^{-rT}}{V}\right)}{\sigma\sqrt{T}}\right\}+V\,N\left\{\frac{-\frac{\sigma^2 T}{2}+\ln\left(\frac{Be^{-rT}}{V}\right)}{\sigma\sqrt{T}}\right\} \quad \textbf{Equation 3-9}$$

The credit spread, $R(T)-r$, is the difference in yield between the riskless and the risky bonds (Equation 14) where the bond matures at T

$$R(T)-r=-\frac{1}{T}\ln\left\{N\left(\frac{-\left(\frac{\sigma^2 T}{2}+\ln(d)\right)}{\sigma\sqrt{T}}\right)+\frac{V}{Be^{-rT}}N\left(\frac{-\left(\frac{\sigma^2 T}{2}-\ln(d)\right)}{\sigma\sqrt{T}}\right)\right\} \quad \textbf{Equation 3-10}$$

Thus, for a given asset volatility and debt level, the value of the debt and equity can be uniquely determined, as shown in Figure 3.1:

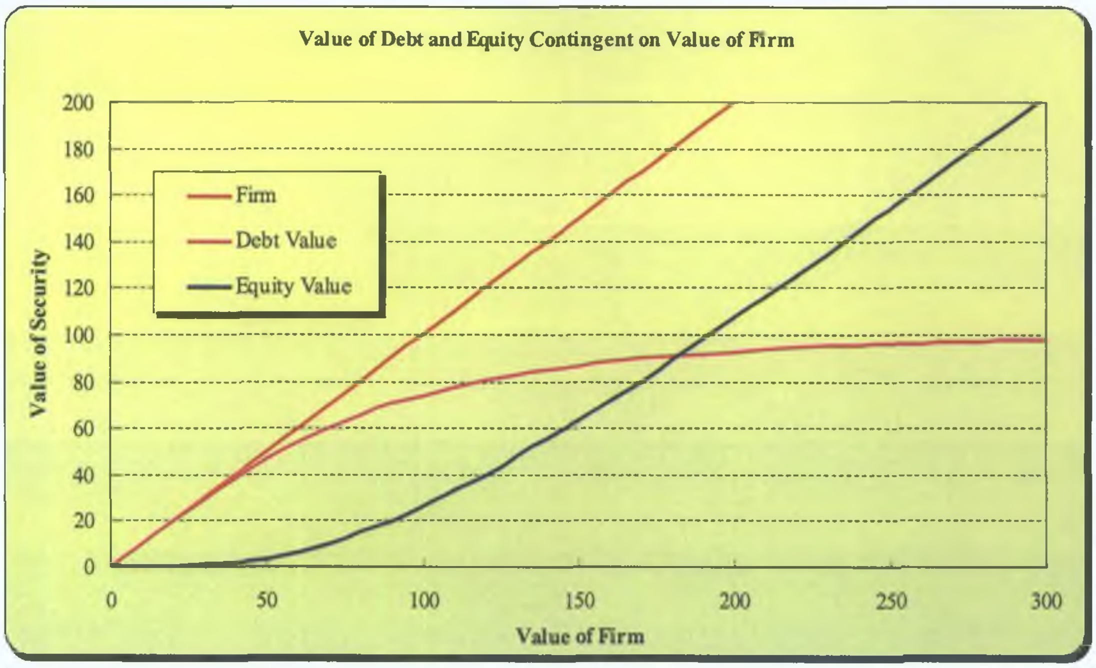

Figure 3.1 Value of Debt and Equity Contingent on Value of Firm

Figure 3.2 below examines the effect of $\frac{V}{B e^{-rT}}$, which Merton calls the leverage ratio, *d*, on the term structure of credit spreads. This is one of the more controversial results of the Merton model. While it indicates a rising term structure of credit spreads for low-leverage firms, it suggests a declining term structure for highly leveraged firms. Sarig and Warga (1989), Franks and Torous (1989) and Wei and Guo (1997) discuss the issue at length. A declining term structure is rarely observed in practice although this may be because many issues contain a prepayment option, a factor that Merton did not consider. Furthermore, it shows the credit spread goes to zero for low leverage firms as they approach maturity. "Empirically we do not observe this to be the case." Jarrow and Turnbull, (1998, p16) These issues will be re-examined in 3.3.3 below.

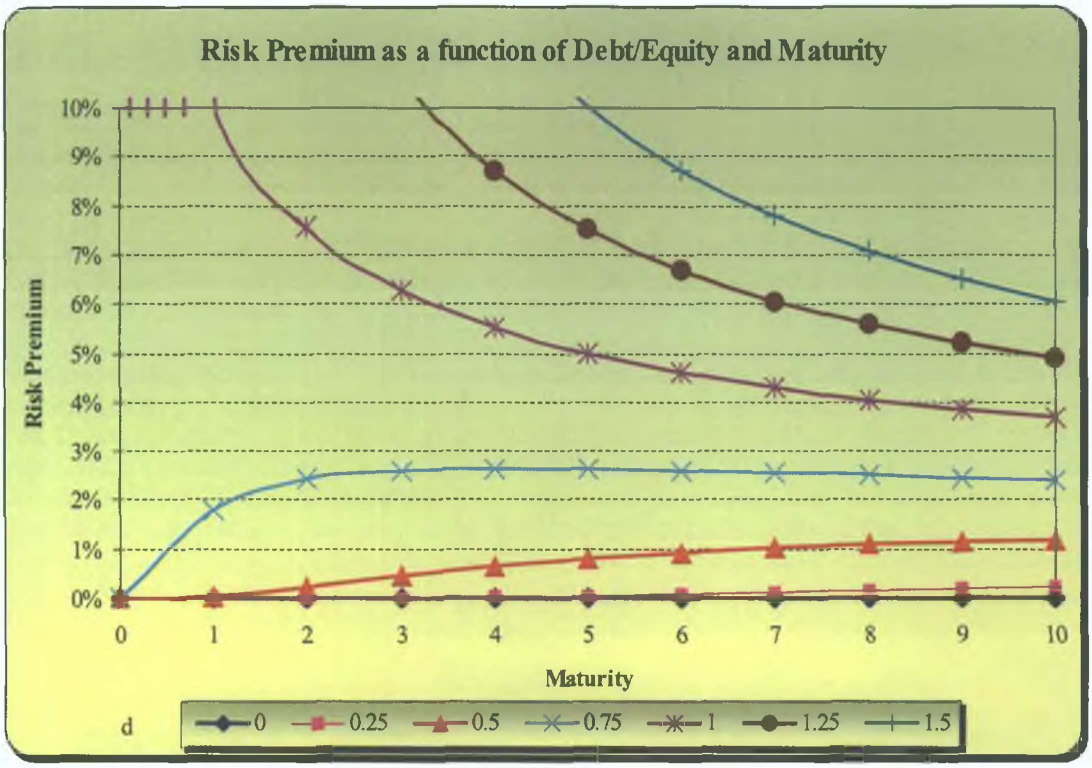

Figure 3.2 The Term Structure of Credit Spreads

Given the unique relationship that both debt and equity have with the asset value for a given asset volatility, they also have a unique relationship with one another, as shown in Figure 3.3:

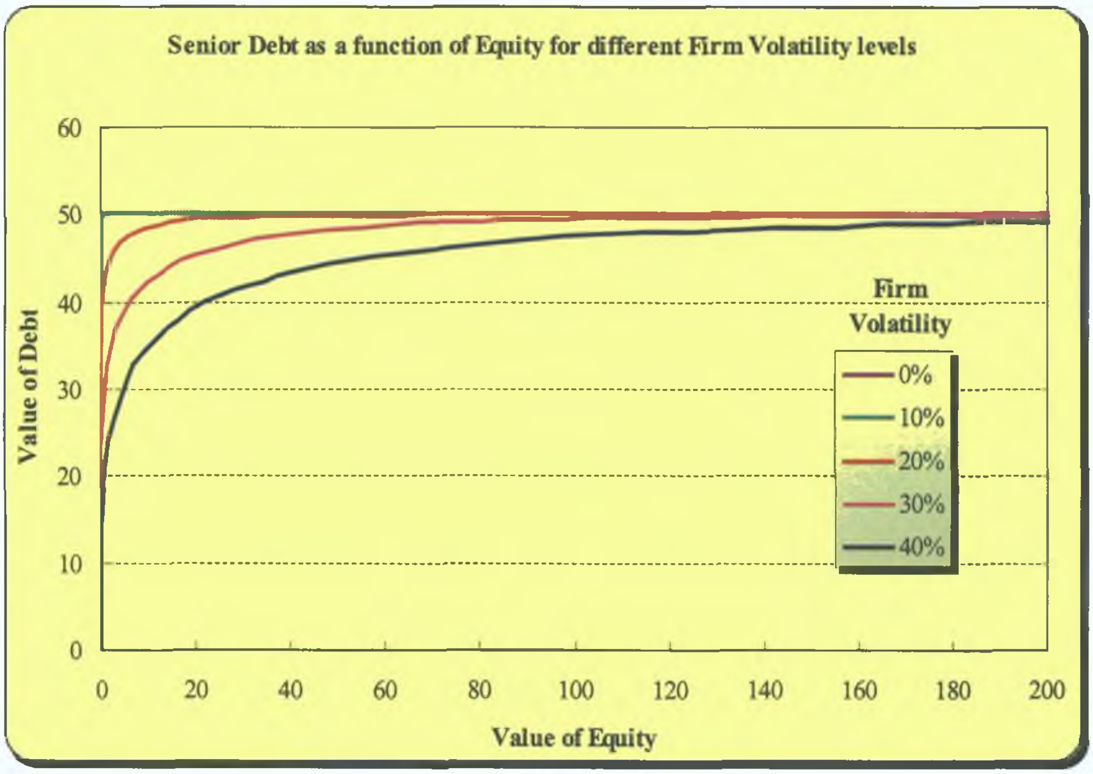

Figure 3.3 Value of Senior Debt vs Value of Equity

The value of the default put option, *p*, or equivalently, the cost of the credit derivative, may be expressed as:

$$p=-V\,N\left(-\frac{\ln\left(\frac{V}{Be^{-rT}}\right)+\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right)+B\,e^{-rT}\,N\left(-\frac{\ln\left(\frac{V}{Be^{-rT}}\right)-\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right) \quad \textbf{Equation 3-11}$$

This cost is a homogeneous function of the leverage ratio, *d*.

The PD in a risk-neutral world is

$$1-N\left(-\frac{\ln\left(\frac{V}{Be^{-rT}}\right)-\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right) \quad \textbf{Equation 3-12}$$

and this enables the value of the credit derivative to be written as:

$$\left[-\frac{N(-d_1)}{N(-d_2)}V+B\,e^{-rT}\right]N(-d_2) \quad \textbf{Equation 3-13}$$

where

$$d_1 = \frac{\ln\left(\frac{V}{B\,e^{-rT}}\right) + rT}{\sigma\sqrt{T}}$$ **Equation 3-14**

and

$$d_2 = \frac{\ln\left(\frac{V}{B\,e^{-rT}}\right) - rT}{\sigma\sqrt{T}}$$ **Equation 3-15**

Crouhy *et al* (2001, p 364) point out

The absolute value of the first term inside the brackets in Equation 3-13 for the credit derivative is the expected discounted recovery value of the loan, conditional on default It represents the risk-neutral expected payment to the bank in the case where the firm defaults The second term inside the brackets is the value of a risk-free bond

Thus, the sum of the two terms inside the brackets is the expected shortfall in present value terms, conditional on the firm being in default at T Multiplying this present value of the expected shortfall by the PD gives the premium for insurance against default The price of a credit derivative (CD) and its component parts – PD, also known as expected default frequency (EDF), and the present value of the LGD - are shown in Figure 3 4 for a loan of 100 to a firm

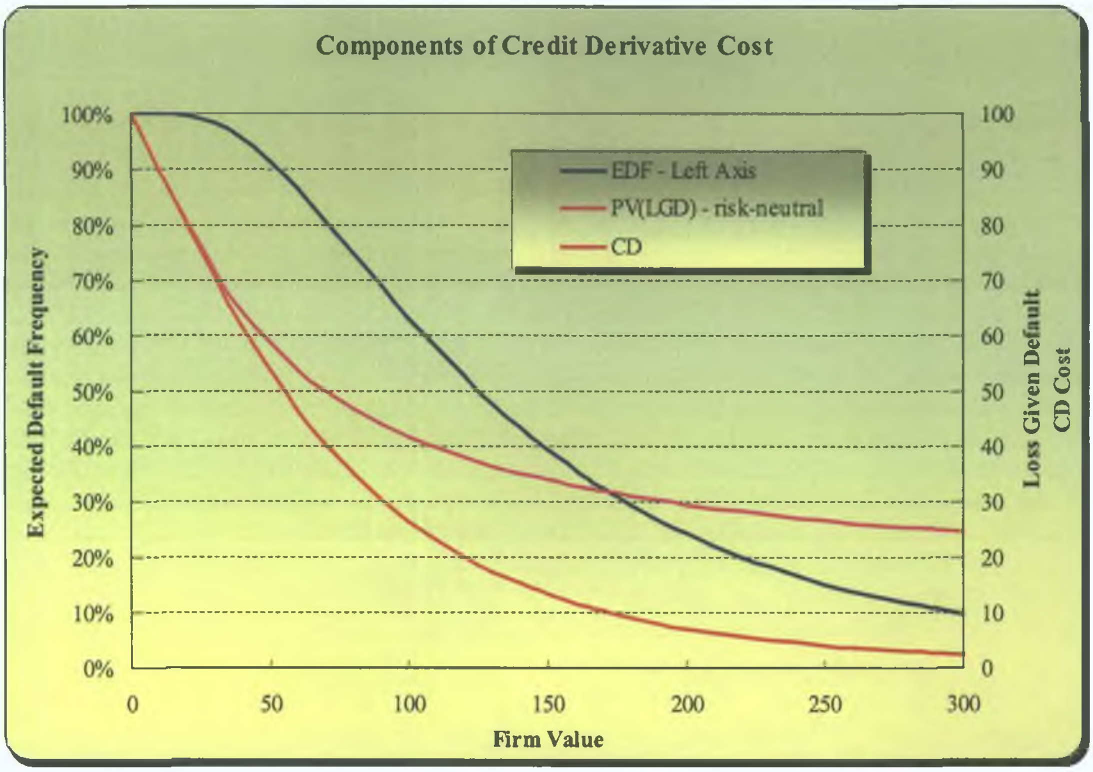

Figure 3.4 Components of Credit Derivative Cost

However, practitioners do not think of LGD in present value terms, and rating agencies quote recovery rates as a percentage of par value. Hence, it is common practice to express LGD in actual monetary terms. Re-arranging the previous equation and expressing it in future value terms, we get:

$$EL_T = B\left(1 - N(d_2) - \frac{N(-d_1)}{d}\right) \qquad \textbf{Equation 3-16}$$

Hence, the expected payoff from the debt at maturity is

$$B - EL_T = B\left(N(d_2) + \frac{N(-d_1)}{d}\right) \qquad \textbf{Equation 3-17}$$

so the expected cost of default, in yield terms, is:

$$-\frac{1}{T}\ln\left(\frac{B\left\{N(d_2) + N(-d_1)\frac{V}{Be^{-rT}}\right\}}{B}\right) \qquad \textbf{Equation 3-18}$$

which is the same as the expression previously derived for the credit spread.

Merton showed how the cost of eliminating credit risk can be derived from the value of the firm's assets, V. However, it is not possible to observe V in most instances – usually only the equity is traded and there are no liquid prices quoted for the other liabilities in the firm's capital structure. As a practical matter, therefore, it is necessary to be able to express a relationship between debt and equity rather than debt and assets as heretofore. If debt is to be hedged and priced, it must be done via the equity.

The value of equity according to Merton (1974) is:

$$f = V\,N(d_1) - F\,e^{-rT}\,N(d_2) \quad \textbf{Equation 3-19}$$

As Crouhy *et al* (2001, p367) note, the equity value is a function of the same parameters as the default put option. They note that a put can be created synthetically by selling short $N(-d_1)$ units of the firm's assets, and buying $B\,e^{-rT}N(-d_2)$ units of government bonds maturing at T, with face value of B. They conclude that by selling short $\frac{N(-d_1)}{N(d_1)}$ units of the stock, f, a short position in the firm's assets of $N(-d_1)$ units is created. Thus, even if V is not directly traded or observed, a put option can be created dynamically by selling short the appropriate number of shares.

Equity is a leveraged position in the asset and its volatility relationship with the assets reflects this leverage:

$$\sigma_f = \eta_{f,V}\,\sigma = \frac{N(d_1)V}{f}\sigma \qquad \textbf{Equation 3-20}$$

where $\eta_{f,V}$ is the instantaneous elasticity of equity with respect to the firm's value, $\frac{\partial f}{\partial V}\frac{V}{f}$. Bensoussan *et al* (1994, 1995) showed that hedge ratio and the formula linking firm volatility and equity volatility above can both be used despite the fact that firm volatility is stochastic, changing with V, which is not theoretically correct if the Black and Scholes model is to be applicable.

3.3.1.3 Vasicek

Vasicek (1997, p1) summarises the contingent claims approach to the measurement of borrower PD. Traditional credit analysis, he notes, involves detailed examination of the company's operations, projection of cash flows, and assessment of the future earning power of the firm But, he contends, such analysis is not necessary, not because future prospects of the firm are not of primary importance – clearly they are - but because an assessment, based on all currently available information on the company's future, has already been made by the aggregate of the market participants, and reflected in the firm's current market value He proceeds to emphasise that this assessment is accurate not in the sense that its implicit forecasts of future prospects will be realised, only that any one person or institution is unlikely to arrive at a superior valuation The challenge, he says, is properly to interpret the changing share prices

He proceeds to extend the contingent claims approach to the valuation of subordinate debt within a capital structure that contains senior debt and equity also He derives the expected loss in a risk-neutral environment, Q, as

$$\begin{aligned}&(D_T + C_T)N\left[\frac{\ln(D_T + C_T) - \ln(A - F) - rT + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right]\\ &-(A - F)e^{rT}N\left[\frac{\ln(D_T + C_T) - \ln(A - F) - rT - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right]\\ &-C_T\,N\left[\frac{\ln(C_T) - \ln(A - F) - rT + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right]\\ &+(A - F)e^{rT}N\left[\frac{\ln(C_T) - \ln(A - F) - rT - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right]\end{aligned}$$

Equation 3-21

where D is the market value of the subordinated debt, D_T is the face value of the subordinated debt, C is the market value of the senior debt, C_T is the face value of the senior debt, A is the market value of the assets, F is the present value of dividends and interest paid over the term of the loan, T, and where r is the risk-free interest rate

The value of the subordinated debt is

$$D = (D_T - Q)e^{-rT}$$

Equation 3-22

The impact of firm value and volatility on the value of the subordinated debt with a risk-free present value of 50 is shown in Figure 3 5 for a range of asset values

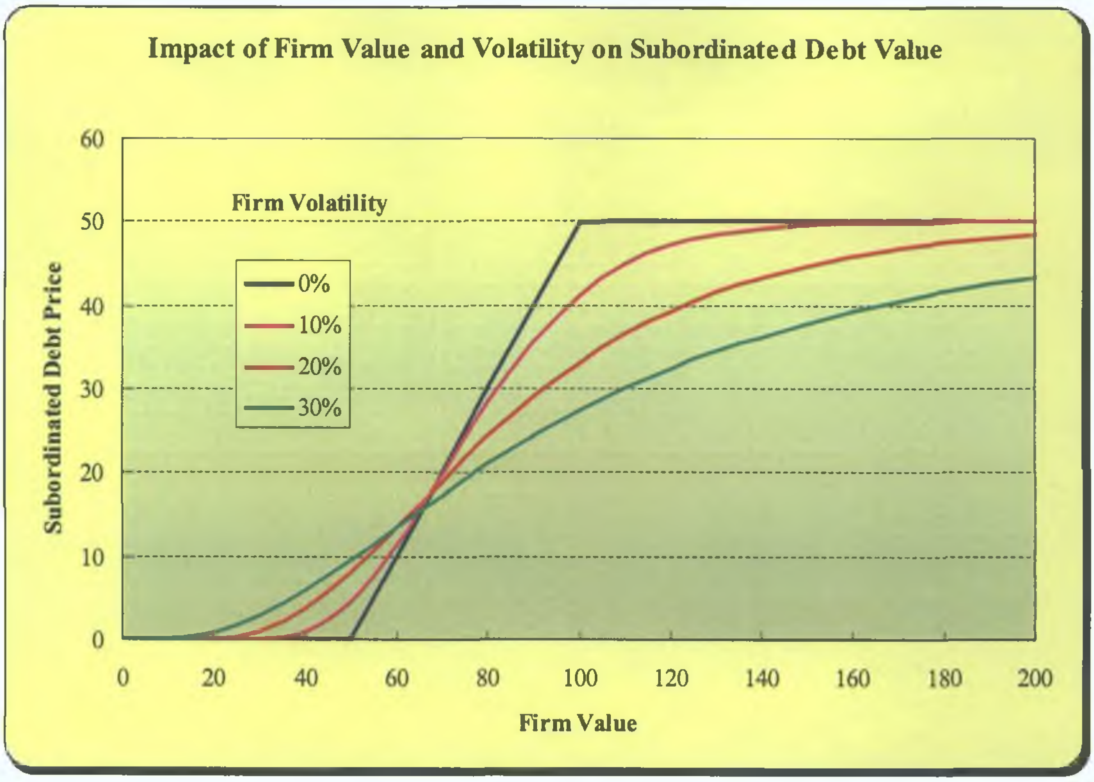

Figure 3.5 Value of Subordinated Debt as a Function of Firm Value

3.3.1.4 Other Contributions to the Contingent Claims Literature

Black and Cox (1976, p351) focus on the assumptions made by Black and Scholes and Merton who had assumed that the bond contract renders the firm's investment, payout, and further financing policies determinate and that the fortunes of the firm may cause its value to dwindle to nearly nothing without any sort of reorganisation occurring in the firm's financial arrangements. (p352) As they point out, in reality, the firm may be reorganised if the asset value reaches upper or lower boundaries. These boundaries, they suggest, may be given exogenously by the contract specifications or determined endogenously as part of the optimal decision problem. (p352)

Black and Cox allow for coupon-paying debt and for default prior to maturity by introducing an exogenously determined lower boundary, which, when crossed, triggers default. In the Merton framework, they note, the time of receipt of each potential payment was known but not the amount which would actually be received. They contrast this with their new approach in which the amount to be received at each boundary is a known function specified by the contract, but the time of receipt is a random variable. (p353) The closed form solution that they develop confirms the benefits to debtholders of covenants that trigger debt repayment in the face of deteriorating company fortunes

Shimko, Tejima and van Deventer (1993) allowed for stochastic interest rates as per Vasicek (1977) in the Merton (1974) framework They conclude that the correlation between interest rate movements and the returns on the underlying asset is an important variable in determining the credit spread on risky debt having shown that for reasonable parameter values, as the correlation increases, the credit spread increases (p64)

Longstaff and Schwartz (1995) address the "clearly unrealistic" assumption in the standard contingent claims approach that default will only occur "when the firm exhausts its assets" (p789) They attempt to extend the Black and Cox (1976) model by incorporating both default and interest rate risk and by explicitly allowing for deviations from strict absolute priority which they do by exogenously imposing a recovery rate for different securities They conclude that "credit spreads are strongly negatively related to the level of interest rates" (p791) and that for investment-grade bonds, "changes in interest rates account for more of the variation in credit spreads than changes in the value of the assets of the firm " (p815)

3 3 1 5 Other Contingent Claims Models

Models based on this contingent-claims approach compare the value of an issuer's assets with the level of debt in the issuer's capital structure to determine the PD Duffie and Singleton (1998) define this modelling framework as the "structural" approach to risky debt valuation Bohn (2000) says that in the Black and Scholes and Merton version of this model, default is assumed to occur when the market value of assets has fallen to a sufficiently low level relative to the issuer's total liabilities Bohn notes that the key characteristic shared by structural models is their reliance on economic arguments for why firms default (p 54)

Lando (1997) demonstrates that the Merton formula for the value of risky debt, F, can be re-cast as the value of a default-free loan of the same amount plus a short position in a put option on the firm's assets with a strike price equal to the debt's face value

$$F = e^{-rT}(B - EL)$$

$$EL = B\ N\left(\frac{-\left[\ln(V) + rT - \frac{1}{2}\sigma^2 T - \ln(B)\right]}{\sigma\sqrt{T}}\right) - \frac{V}{e^{-rT}} N\left(\frac{-\left[\ln(V) + rT + \frac{1}{2}\sigma^2 T - \ln(B)\right]}{\sigma\sqrt{T}}\right)$$

Equation 3-23

Thus, the value of risky debt is the value of otherwise similar, default risk-free debt less the present value of the expected loss, *EL*, given that the company defaults Bohn notes that this expected loss term can be divided into two components The first term, the expected loss on the debt in the case of no recovery, equals the face value of the debt, *B*, multiplied by the risk-neutral PD The second term represents the expected recovery in the event of default

However, Bohn comments the main difficulty with the formulation is empirically finding all the necessary inputs He proposes a simpler characterisation of default as a binary option in which the lender incurs a loss of a fixed amount, *L*, when the borrower defaults, where *L* is fixed as a percentage of face value, and nothing if the borrower does not default The formulation for *EL* above can then be seen as the expected loss with no recovery (*i e* risk-neutral PD times the face value of the debt) less the expected recovery in the event of default

The expected payoff at maturity in a risk-neutral world is the sum of the payoff in the case of no default times the probability of no default plus the payoff in the case of default times the probability of default He calls the risk-neutral PD, *Q*, and derives the following equation

$$F = Be^{-rT}\left[(1 - Q)(1) + Q(1 - L)\right] = Be^{-rT}\left[1 - QL\right] \qquad \textbf{Equation 3-24}$$

Bohn proposes an approach to calculating *Q* by adjusting the actual PD by the market price of risk and a function of time The actual PD, *p*, that the value of the firm's assets will be less than the face value of debt at maturity is

$$p = N\left[\frac{\ln(B) - \ln(V) - \mu T + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right] \qquad \textbf{Equation 3-25}$$

The risk-neutral PD is given by the same formula with the expected asset return, μ, replaced by the default risk-free rate, *r*

$$Q = N\left[\frac{\ln(B) - \ln(V) - rT + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right]$$ **Equation 3-26**

Bohn proposes a factor-pricing framework to formulate a relationship between the expected return on the firm's assets and the overall expected return on the market Using the Capital Asset Pricing Model to describe this relationship

$$\mu - r = \frac{\text{cov}(r_v, r_m)}{\sigma_m}\lambda$$

$$\lambda = \frac{\mu_m - r}{\sigma_m}$$ **Equation 3-27**

where μ_m is the expected return on the market, σ_m is the volatility of market returns and λ is the overall market Sharpe ratio Substituting into the previous equation for p, a formula for Q can be derived

$$Q = N\left(N^{-1}(p) + \rho\lambda\sqrt{T}\right)$$ **Equation 3-28**

In this formulation, ρ is the correlation of the return on the firm's assets, r_v, with the return on the market, r_m Although suggesting the CAPM framework to describe the concept, he proposes the use of a more sophisticated factor model to determine the amount of variation in the return explained by the firm's sensitivity to certain market factors He suggests that the sensitivity parameter, ρ, be set equal to $\sqrt{R^2}$, where R^2 is the coefficient of determination resulting from estimation of a suitable multi-factor model

In this framework, the firm defaults with a probability, Q, and in which case the firm pays $(1 - L)B$ or it does not default with a probability $(1 - Q)$ and pays back B

$$F = e^{-rT}(B - LBQ)$$ **Equation 3-29**

Hence, the term structure of credit spreads is then given by

$$R(T) - r = -\frac{\ln(1 - LQ)}{T}$$ **Equation 3-30**

This valuation framework has been adopted by KMV. They use a 120-factor model in their credit portfolio modelling product, *Portfolio Manager*, described in detail in 4.3.2.1 below.

3.3.2 The Reduced Form Approach

Structural models begin with an economic argument about why a firm defaults. In this framework, default can never occur by surprise. As time to maturity goes to zero, credit spreads also approach zero. However, in practice, non-zero credit spreads are observable in the market regardless of maturity.

By comparison, in the reduced form framework, default is always an unpredictable event governed by an intensity-based or hazard-rate process according to Duffie and Singleton (1998). Reduced form models eliminate the need for an economic explanation of default, comments Bohn (2000, p54). According to Jarrow and van Deventer (1999), the approach is 'reduced form', because the assumptions underlying the model are imposed on the prices of the firm's traded liabilities that can be deduced from the structural models.

Iovino (1999) characterised the difference between the structural and reduced form approaches. She stated that both approaches attempted to model the time a firm defaults. However, while the structural approach addressed the problem by "modelling the time a firm defaults", that is, attaching meaning to each of the underlined words, the reduced form approach instead models *the time a firm defaults*. (p15)

Bohn (2000) formulates the value of a zero-coupon bond issued by a firm with one class of equity as follows:

$$F = B\,e^{-rT}\left(1 - L\,Q(\tau^* < T)\right) \quad \textbf{Equation 3-31}$$

where, as previously, F is the market value of the zero-coupon debt, B is the face value of the debt, L is the LGD expressed as a fraction of face value, r is the risk-free rate and T is the time to maturity.

He states that the difference between the structural characterisation and reduced form characterisation of the model lies in the specification of Q. In the reduced form model, Q indicates the risk-neutral probability the unpredictable event of default occurred at time τ^*, which happened to precede the maturity of the debt. The *time* of default is assumed to follow a stochastic process governed by its own distribution that must be parameterised by an intensity or hazard rate process. "The default or 'stopping' time is inaccessible *i.e.* it jumps out at you (from nowhere)." p.63

Jarrow and Turnbull (1995) developed one of the first reduced form models in which they assumed a constant LGD and an exponentially distributed default-time. They modelled risky bonds as foreign currency bonds denominated in "promised" dollars. The exchange rate is 1 in the absence of default and (1-LGD) if default has occurred. Default is a Poisson arrival. This assumption of constant default intensity is unrealistic, however. For example, in reality, strong firms become weaker over time indicating the necessity for time varying default intensity.

Jarrow, Lando and Turnbull (1997) addressed this weakness. They modelled default as the first time a continuous-time Markov chain with K states hits the absorbing K-th state. States 1 to K are associated with credit ratings where 1 is the strongest rating and the K-th state is default. However, in order to implement such a model, one needs to estimate an entire generator matrix to arrive at transition probabilities for each possible change in state. As a first approximation, they suggest using historical rating agency transition probability matrices.

Duffie and Singleton use reduced form models to value risky debt as if it were default risk-free by replacing the usual short-term default risk-free rate with the default-adjusted short-rate process. They show how to specify a reduced form model in the context of popular default risk-free term structure models such as Heath, Jarrow and Morton (1992):

$$F = E^{Q}\left[e^{-\int_0^T R\,dt} B \right] \qquad R = r_t + h_t L_t + l_t \qquad \textbf{Equation 3-32}$$

where r_t is the default risk-free rate, h_t is the arrival intensity at time t (under Q) of a Poisson process whose first jump occurs at default, L_t is the fractional LGD, and l_t is a variable that is intended to capture liquidity effects. Credit spread data can be used to imply the risk-neutral mean expected loss rate, $h_t L_T$.

Duffie and Lando (1997) show how to formulate a structural model such that it can be represented as a reduced form model in the Duffie and Singleton framework. They begin with a diffusion process for the firm's asset value and a default barrier that marks the asset value at which the firm defaults They then derive a formula for the hazard rate, h_t, in terms of the asset value volatility, the default barrier, and the conditional distribution of asset value given the history of information available to investors The mechanism creating the inaccessible default stopping time is imperfect accounting information With imperfect accounting data, credit spreads remain bounded away from zero even as maturity approaches zero Thus, this version of a structural model is recast in a reduced form framework

3 3 3 Assessment of the Different Modelling Approaches

Next, the success, or otherwise, of the structural and reduced form approaches to evaluating credit risk is examined

3 3 3 1 Evaluating Structural Models

Jones, Mason and Rosenfeld (1983, 1984) found that contingent claims models produced credit spreads which were significantly lower than actual credit spreads They concluded that they produced results which were no more accurate than those obtained by discounting at the risk-free rate in the case of mvestment-grade debt The Contingent Claims Analysis (CCA) model, they assert, is not an improvement over a naive, riskless model for investment grade bonds However, the CCA model "does appear to have incremental explanatory power over the naive model for non-investment-grade bonds " (1984, p624)

The naive model prices are obtained by discounting the promised cash flows at the risk-free rate Based on a sign test, the CCA model outperforms the naive model for 139 of the 176 investment-grade debt and for 117 of the 129 non-investment grade debt All these results are significant at the 95% level However, for pricing, the results were less impressive

Overall results	Number of bonds		Percentage Error		Absolute Percentage Error	
			CCA Model	Naive Model	CCA Model	Naive Model
Entire sample	305	Mean	0 0452	0 0876	0 0845	0 1143
		Standard Deviation	0 1003	0 1441	0 0705	0 1240
Investment grade	176	Mean	0 0047	0 0149	0 0587	0 0574
		Standard Deviation	0 0727	0 0703	0 0432	0 0432
Non-investment grade	129	Mean	0 1005	0 1867	0 1197	0 1919
		Standard Deviation	0 1063	0 1590	0 0840	0 1528

Table 3-1 Pricing comparisons CCA and Naive Models vs Market Prices

Table 3 of their paper, reproduced as Table 3-1 above, shows the difference between the model price and market price expressed as a percentage of the market price The mean pricing error for investment grade debt was small (0 47% of market value), but the mean error of for sub-investment grade debt was large and positive (10 05%) Likewise, fluctuations around the mean errors were large in both sub-samples – the standard deviations were 7 27% and 10 63%, respectively, and the mean absolute errors were 5 87% and 11 97%, respectively

They surmise that the key assumptions which give rise to these negative conclusions include constant asset volatility, the absolute priority rule (APR), perfect asset liquidity enabling firms to sell assets as necessary, Itô dynamics, the frequent requirement to retire bonds via periodic sinking fund provisions, and a non-stochastic term structure and suggest that "introducing stochastic interest rates, as well as taxes, would improve the model's performance " Franks and Torous (1989) concurred with their findings

Sarig and Warga (1989) estimated the term structure of credit spreads using a small number of zero-coupon corporate bonds and zero-coupon treasury bonds They demonstrated curve shapes consistent with contingent claims model predictions, namely, upward sloping for investment grade debt, humped shaped for lower grade debt, and downward sloping for very low grade debt

Delianedis and Geske (1998) used the Black Scholes Merton framework to estimate risk-neutral probabilities of default They found that rating migrations and defaults are detected months before in the equity markets, lending support to the modelling of default as a diffusion process rather than as a Poisson event

Fons (1987) used a risk-neutral model to examine low-grade bonds and concluded either that there is systematic mispricing by investors of low-rated corporate bonds or that the risk-neutral model he derived could not fully capture the market's assessment of the PD of those securities. In a 1994 article, Fons again found his risk-neutral model seriously underestimated the spreads he obtained from fitting linear regressions through data within different classes, but particularly investment grade bonds.

A fundamental assumption of the structural model is that APR, which requires that senior claimants be paid in full before more junior claimants get anything, holds. This is seldom the case in practice. Franks and Torous (1989) found that 21 out of 27 recapitalisations exhibited a violation of APR. In 21 of 30 cases examined by LoPucki and Whitford (1990) in which the total value to be distributed was less than that due to creditors, stockholders received value, averaging 5.6% of the total value of all distributions. Garbade (2001, p104) concludes that "[v]iolation of the absolute priority rule does not come as a surprise to market participants" and that "[s]enior debt is not priced on the assumption that it will be paid in full before subordinated creditors and stockholders get anything."

The structural model assumes that bankruptcy is instantaneous and costless. In practice, it is neither. Warner (1977) found that bankruptcy costs for eleven large railroads averaged 5.3%, ranging between 1.7% and 9.1%. Altman (1984) found bankruptcy costs averaged 6.0% of the debtor's value for a sample of seven industrial firms and twelve retailers.

While acknowledging the APR violation and bankruptcy cost issues, Garbade (2001) suggests that the problem is deeper, as the CCA methodology demands that an analyst recognise not only the existing operating characteristics, capital structure, and contractual obligations of the firm but also the prospect of change attributable to managerial discretion and decision making. (p387) He concludes: "The challenge is to extend the analytical framework to include the role of discretion in the exercise of implicit management options." (p387)

Structural models have been used to value callable debt, with mixed results. Some of the pricing errors can be attributed to a failure to allow for fluctuations in yields on default-proof fixed income securities, Garbade suggests (2001, p114). Vu (1986) found evidence of 'unexpectedly late' redemption of callable bonds that he suggested might be optimal because of the costs associated with calling and refinancing. Likewise, he found evidence of 'unexpectedly early' redemption conjecturing that firms chose to do so in order to eliminate restrictive covenants Asquith and Wizman (1990) found that firms commonly called bonds for early redemption to eliminate restrictive covenants that would otherwise impede a planned buyout However, this type of 'event risk' associated with apparently suboptimal calling of debt cannot be modelled correctly in the structural framework

Ingersoll (1977a) and Brennan and Schwartz (1977, 1980) were among the first to analyse the contingent value of convertible debt However, their models failed to explain corporate behaviour satisfactorily It was Ingersoll (1977b, 463) himself who pointed out that, compared to the conversion behaviour suggested by his structural form model, companies commonly waited "too long" to call their convertible debt Jaffee and Shleifer (1990) suggested that taking proper account of the notice period between announcement of a call and actual redemption could rectify the most egregious failings of the Ingersoll model

Kim, Ramaswamy and Sundaresan (1993) valued coupon-paying debt using the Cox, Ingersoll and Ross (1985) interest rate model and an exogenously defined recovery rate They conclude that their approach is plausible as it generates yield spreads on corporate bonds consistent with those observed m the marketplace They further conclude that stochastic interest rates seem to play an important role in determining the yield differentials between a callable corporate bond and an equivalent government bond "due to the interactions between call provisions and default risk" (p127)

Jarrow and van Deventer (1999) criticise the implicit assumptions embedded in the structural approach regarding corporate capital structure policy The structural approach, they claim, assumes that the corporate capital structure policy is static, with the liability structure fixed and unchanging This assumes that management puts a debt structure in place and leaves it unchanged even if the value of corporate assets has doubled, they note "This is too simplistic to realistically capture management behaviour and the dynamics of bankruptcy " (p302)

Conventional wisdom has it that structural models provide good insights into the cause of default but fail the test of providing good models for valuation However, Bohn (2000) suggests that small sample sizes, doubts about the quality of bond pricing data, and the lack of focus on the appropriate default risk-free rate leave us without conclusive evidence regarding the power of structural models "The resolution of these empirical issues awaits further research ", Bohn comments (p66)

Gemmill (2002) examined the credit spreads on zero-dividend preference shares (ZDP) issued by split-capital trusts in the U K He confirms that two of the biases consistently reported are present in his analysis, namely, that model credit spreads are too small for bonds that are near maturity and also for companies with low leverage and volatility His study once again casts doubt on the appropriateness of the Merton model that assumes the assets follow geometric Brownian motion and, hence, that the spread should reduce to zero as time to maturity and/or volatility goes to zero It is clear the market has not excluded the possibility of a jump in asset value

However, of greater significance is Gemmill's finding that market and model spreads are of similar magnitude and, in line with market practice, the Merton model consistently produces an upward-sloping term-structure (p1)

The standard Merton model suggests a downward-sloping term structure of credit spreads for risky bonds as shown in Figure 3 2 above whereas the observed term structures in the market are invariably rising Gemmill comments "The solution to this conundrum appears to be that the leverage of the companies changes over time, being higher when bonds are issued than when they mature " (p3) The Gemmill model allows for this leverage effect and leads to model term structures of credit risk which are upward-sloping, as empirically observed (p3) He notes his results reinforce the conclusion of Collin-Dufresne and Goldstein (2001) "that it is important to take account of the expected trajectory of leverage when computing credit spreads " (p3) He suggests that firms have a target leverage ratio which they try to maintain by issuing or retiring debt (p4)

3 3 3 2 Evaluating Reduced Form Models

Many reduced form models are parameterised on rating agency ratings and transition data Kealhofer *et al* (1998) take issue with the rating agency approach to grading securities in discrete rating classes In particular they reject the notion that all assets within the rating grade have a single default rate, and the default rate is equal to the historical average default rate (p40)

They assert that even when all loans within a grade have the same default rate, the historical average default rate can deviate significantly from the actual default rate

Similarly, they state that the historical transition probabilities can deviate significantly from the actual transition probabilities

There are substantial differences of default rate within bond rating grade, with some bonds in a higher grade having greater default rates than some bonds in a lower grade, indicating an overlap in default probability ranges The overlap, they suggest, appears to be caused by lack of timeliness in upgrade and downgrade decisions

They claim the range of default rates within a rating grade can cause the mean default rate to significantly exceed the median default rate within a grade They suggest the mean may be almost twice as large as the median, and as many as 75% of the borrowers within a rating grade may have default rates that are less than the mean

They also state that historical default rates are statistics for the mean default rate, and thus may be biased upwards by as much as double from the typical default rate within the grade

They claim that the lack of timeliness in rating changes causes a significant bias in transition probabilities In consequence, the "probability of remaining at the same quality is overstated by about double for most grades, whereas the probabilities for changing to other non-default grades are significantly understated " (p40)

They also take issue with the assumption that ratings and default rates are synonymous They find that the highest EDF within a given grade is in excess of four times the lowest whereas the ratio of mean EDFs from one grade to the next is approximately two to one They conclude that grades overlap significantly They further conclude that the equity markets are faster than the rating agencies in reflecting information on the condition of firms This is borne out by their research, which concludes that "about 70% of apparent spread variation is actually due to EDF variation within a rating grade over time " (p50)

Using non-overlapping ranges of default probabilities, they calculate the one-year transition matrix for 6,000 publicly rated U S firms reproduced in Table 3-2

Initial Rating	Rating at Year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	66 26%	22 22%	7 37%	2 45%	0 86%	0 67%	0 14%	0 02%
AA	21 66%	43 04%	25 83%	6 56%	1 99%	0 68%	0 20%	0 04%
A	2 76%	20 34%	44 19%	22 94%	7 42%	1 97%	0 28%	0 10%
BBB	0 30%	2 80%	22 63%	42 54%	23 52%	6 95%	1 00%	0 26%
BB	0 08%	0 24%	3 69%	22 93%	44 41%	24 53%	3 41%	0 71%
B	0 01%	0 05%	0 39%	3 48%	20 47%	53 00%	20 58%	2 01%
CCC	0 00%	0 01%	0 09%	0 26%	1 79%	17 77%	69 94%	10 13%

Table 3-2 Implied KMV Transition Matrix

The corresponding table, drawn from Standard & Poor's *CreditWeek*, April 15, 1996, and reproduced in Table 3-3, notes the transition of rated firms

Initial Rating	Rating at Year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90 81%	8 33%	0 68%	0 06%	0 12%	0 00%	0 00%	0 00%
AA	0 70%	90 65%	7 79%	0 64%	0 06%	0 14%	0 02%	0 00%
A	0 09%	2 27%	91 05%	5 52%	0 74%	0 26%	0 01%	0 06%
BBB	0 02%	0 33%	5 95%	86 93%	5 30%	1 17%	0 12%	0 18%
BB	0 03%	0 14%	0 67%	7 73%	80 53%	8 84%	1 00%	1 06%
B	0 00%	0 11%	0 24%	0 43%	6 48%	83 46%	4 07%	5 20%
CCC	0 22%	0 00%	0 22%	1 30%	2 38%	11 24%	64 86%	19 79%

Table 3-3 S&P Transition Matrix

The probability of staying in an S&P grade is approximately 90% for most ratings, which is about twice that recorded by KMV Furthermore, the default probability for the lower S&P grades is about twice that in the KMV matrix

In sum, they conclude that ratings-based probabilities will tend to overstate the risk for maturities near to the measurement horizon, due to the overstated default rate, but will tend to understate the risk for longer maturities, due to the overstated probability of credit quality remaining the same (p53)

Duffee (1996) used monthly prices from 1985 to 1994 for the corporate bonds in the Lehman Brothers Bond indexes and the Jarrow, Lando and Turnbull model He finds strong evidence of mis-specification as the model fails to produce the term structures of credit spreads most commonly experienced in the market Although the model fits investment-grade corporate bond prices reasonably well, he concludes that single-factor models of instantaneous default probabilities “face a substantial challenge in matching the dynamic behaviour of corporate bond term structures” (p 26)

As discussed above, the structural model is often criticised for producing a declining term structure for low-grade bonds A similar criticism can be levelled at reduced form models such as that of Jarrow, Lando and Turnbull (1995) and Markov-chain models which rely on rating agency transition matrices If a low-grade firm does not default in the first year, its annual default probability declines in subsequent years

3.4 Portfolio Risk

All research in the area of portfolio theory invariably begins with references to the work of Markowitz (1952, 1959) His work attempts to characterise the interaction of a portfolio of equities and proceeds to calculate the efficient portfolio, that combination of available equities which maximises the mean-variance trade-off

Markowitz assumes that equity returns are normally distributed and that investors' utility functions are quadratic At the end of an investment period, an investor's portfolio has a value $W = W_0(1+\tilde{r}_p)$ Using Taylor's theorem, the utility derived from that portfolio is given by

$$U(\tilde{r}_p) = U\{E(\tilde{r}_p)\} + (r_p - E(\tilde{r}_p))U' + \tfrac{1}{2}(r_p - E(\tilde{r}_p))^2 U'' + \qquad \textbf{Equation 3-33}$$

Taking the expectation of this expression and assuming third- and higher-order terms are of minor importance, Markowitz get the expression

$$E[U(\tilde{r}_p)] = U(E(\tilde{r}_p)) + \tfrac{1}{2}V(\tilde{r}_p)U'' E(\tilde{r}_p) \qquad \textbf{Equation 3-34}$$

where $V(\tilde{r}_p)$ is the variance of the rate of return Under these assumptions, the investor's utility is a function only of the mean and variance of the rate of return on the portfolio

The exclusive focus on the mean and the variance of returns is inappropriate in a credit context for two reasons (i) Credit returns are far from normal, with limited upside potential – the highest price that can be achieved is a risk-free price that will never be much higher than par - whereas the minimum price is zero (ii) Credit portfolios are invariably leveraged Thus, the focus cannot solely rest on the standard deviation The downside tail is of critical importance as the institution that holds the portfolio becomes insolvent when losses exceed its capital This is not a concern for equity portfolios that are mostly unlevered

Thus, credit portfolio research has had to adapt traditional equity portfolio concepts quite significantly to make them applicable to the issues they face

3 4 1 Credit Portfolio Risk

When the asset return distributions for two firms, r_A and r_B are normally distributed with an instantaneous correlation coefficient of ρ between their returns, then their joint returns will be described by the bivariate normal distribution

$$f(r_A, r_B, \rho) = \frac{1}{2\Pi\sqrt{1-\rho^2}} e^{\left\{\frac{-1}{2(1-\rho^2)}\left(r_A^2 - 2\rho r_A r_B + r_B^2\right)\right\}}$$ **Equation 3-35**

The bivariate normal distribution can be used to calculate the joint probability of both borrowers defaulting over a period

$$\Pr(Def_A, Def_B) = \Pr\left(V_A \leq V_{DefA}, V_B \leq V_{DefB}\right) = \Pr\left(r_A \leq d_2^A, r_B \leq d_2^B\right) = N_2\left(-d_2^A, d_2^B, \rho\right)$$

Equation 3-36

where d_2^A and d_2^B are the distances to default for *A* and *B*, respectively

The probability of joint default is, therefore, the volume under the cumulative standard bivariate normal distribution

3 4 1 1 The Normal Inverse Distribution

Vasicek (1997b) explored the situation in which a portfolio of debt securities is financed by equity and notes He developed a methodology for determining the capital necessary to support the desired rating of a lender's notes The credit quality of the lender's notes will depend on the probability that the loss on the portfolio exceeds the equity capital, he suggests The equity capital allocated to the portfolio "must be equal to the ordinate of the distribution of the portfolio loss that corresponds to the desired probability " (p1)

He also examined the probability distribution of portfolio losses He assumed that the portfolio consisted of *n* loans of equal dollar amounts, the PD of any one loan is *p*, the asset returns of the borrowing companies are correlated with a coefficient ρ for any two companies, and all loans had the same term, T

He defines a default indicator L_i to be one if the i-th borrower defaults and zero otherwise Thus, the variable L_i is the gross loss (before recoveries) on the i-th loan and

$$P[L_i = 1] = p \quad P[L_i = 0] = 1 - p \qquad \textbf{Equation 3-37}$$

and the expected value and variance of the loan loss is

$$EL_i = p \quad Var(L_i) = p(1-p) \qquad \textbf{Equation 3-38}$$

He defines $L = \frac{1}{n}\sum_{i=1}^{n} L_i$ to be the portfolio percentage gross loss and he proceeds to calculate the probability distribution of L, that is, the probabilities

$$P_k = P\left[L = {k}/{n}\right],\ k = 0,1,\ \ ,n \qquad \textbf{Equation 3-39}$$

Assuming that all borrowers' assets follow the process

$$dA_i = \mu_i A_i dt + \sigma_i A_i dx_i \qquad \textbf{Equation 3-40}$$

where x_i are correlated joint Wiener processes with correlation ρ

$$E(dx_i)^2 = dt \quad E(dx_i)(dx_j) = \rho\, dt, i \neq j \qquad \textbf{Equation 3-41}$$

The PD is

$$p = P[L_i = 1] = P[A_i(T) < F_i] = P[X_i < c_i] = N\left(\frac{\ln(F_i) - \ln(A_i(0)) - \mu_i T + \frac{1}{2}\sigma^2 T}{\sigma_i\sqrt{T}}\right)$$

Equation 3-42

where A_i is the value of the i-th borrower's assets, F_i is the value of obligations payable, and X_i is defined as

$$X_i = \frac{(x_i(T) - x_i(0))}{\sqrt{T}} \qquad \textbf{Equation 3-43}$$

The variables X_i are jointly normal with equal pairwise correlations ρ and he represents them as

$$X_i = \sqrt{\rho}Y + \sqrt{1-\rho}Z_i \quad \textbf{Equation 3-44}$$

where $Y, Z_1, Z_2, \ldots Z_n$ are mutually independent standard normal variables He interprets the Y variable as a common factor, such as the state of the economy, over the interval (0,T), the term $\sqrt{\rho}\,Y$ as the company exposure to the common factor (the systematic risk) and the term $\sqrt{1-\rho}\,Z_i$ as the company-specific risk

Conditional on Y, the variables L_i are independent equally distributed zero-one variables with the conditional probability

$$p(Y) = P[L_i = 1|Y] = P[X_i < c|Y] = P\left[Z_i < \frac{1}{\sqrt{1-\rho}}\left(c - \sqrt{\rho}\,Y\right)Y\right] = N\left(\frac{1}{\sqrt{1-\rho}}\left(c - \sqrt{\rho}\,Y\right)\right)$$

Equation 3-45

He calculates the portfolio loss distribution as

$$P_k = P\left[L = \frac{k}{n}\right] = \binom{n}{k}\int_{-\infty}^{\infty}(p(Y))^k(1-p(Y))^{n-k}\,dN(Y) \quad \textbf{Equation 3-46}$$

which can be evaluated numerically

He then proceeds to examine the behaviour of this integral as the number of loans in the portfolio increases Because the defaults are not independent, the conditions of the central limit theorem are not satisfied, he argues, and L is not asymptotically normal It turns out, he notes, that the distribution of the portfolio loss does converge to a limiting form, which "can be then conveniently used for large portfolios instead of the integral form " (p5)

He calls the cumulative distribution function

$$Q(x, p, \rho) = N\left(\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}\,N^{-1}(x) - N^{-1}(p)\right)\right) \quad \textbf{Equation 3-47}$$

the *normal inverse distribution*, being the distribution of the variable

$$L = N\left(\frac{1}{\sqrt{\rho}}\left(N^{-1}(p) - \sqrt{\rho}\,X\right)\right)$$ **Equation 3-48**

where X is a standard normal distribution. He derives the density function for this distribution as

$$Q'(x;p,\rho) = \sqrt{\frac{1-\rho}{\rho}}\exp\left(-\frac{1}{2\rho}\left[(1-2\rho)\left(N^{-1}(x)\right)^2 - 2\sqrt{1-\rho}\,N^{-1}(x)N^{-1}(p) + \left(N^{-1}(p)\right)^2\right]\right)$$

Equation 3-49

The mean of the distribution is p and the variance is $N_2\left(N^{-1}(p), N^{-1}(p), \rho\right) - p^2$ while the α -percentile value of L is

$$L_\alpha = Q^{-1}(\alpha\,; p, \rho) = Q(\alpha\,; 1-p, 1-\rho)$$ **Equation 3-50**

"The normal inverse distribution is highly skewed and leptokurtic." (p8) This is observed in Figure 3.6 below in which the loan loss distribution is plotted for a range of asset correlation values:

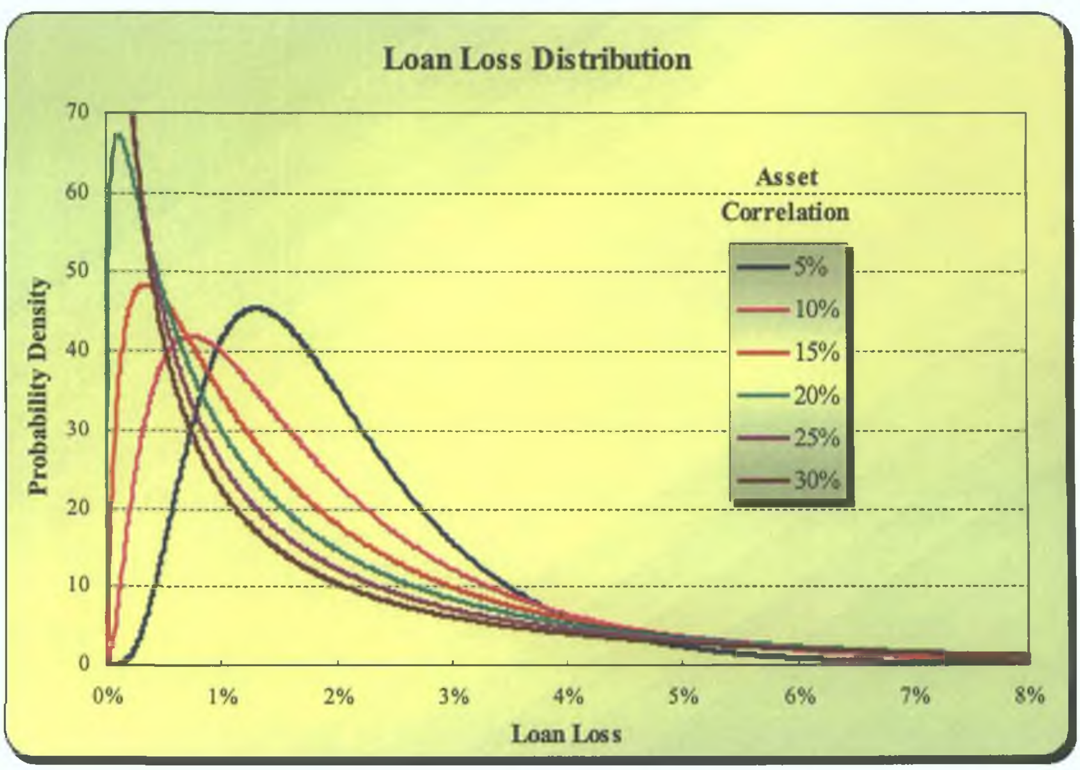

Figure 3.6 Normal Inverse Loan Loss Distribution for Varying Asset Correlation

This family of distributions varies between two extremes: a normal distribution centred on the loan loss probability when the asset correlation is zero, and a binomial distribution exhibiting 0% loss with a probability $(1 - p)$ and 100% loss with a probability p

This, albeit simplistic, characterisation of the loan loss distribution points to the challenge of modelling loan loss portfolios and, consequently, the valuation of derivative products such as CDOs The number of standard deviations from the mean, $\frac{L_\alpha - p}{s}$, for combinations of asset correlation and α -percentile for a portfolio with a portfolio PD of 1% is shown in Table 3-4 below

		Alpha-percentile			
		10%	1%	0 1%	0 01%
	0 1	1 19	3 82	7 01	10 67
Asset	0 2	0 97	4 22	8 77	14 19
Correlation	0 3	0 75	4 41	10 04	16 61
	0 4	0 55	4 51	11 04	18 19
	Normal	1 28	2 33	3 09	3 72

Table 3-4 Standard Deviations Corresponding to Percentiles of the Inverse Normal PD = 1%

However, this situation is further exacerbated if there is low default probability Table 3-5 shows the corresponding table where the borrower default probability is 10bp, typical of an investment-grade portfolio It shows that an institution with a portfolio of loans with average asset correlation of 40% will have to hold capital equal to 13 standard deviations if it wishes to secure a *AA-* rating – a PD of 10bp - for itself

		Alpha-percentile			
		10%	1%	0 1%	0 01%
	0 1	0 98	4 09	8 83	15 37
Asset	0 2	0 60	4 10	11 16	22 39
Correlation	0 3	0 31	3 75	12 45	27 65
	0 4	0 12	3 25	13 18	31 76
	Normal	1 28	2 33	3 09	3 72

Table 3-5 Standard Deviations Corresponding to Percentiles of the Inverse Normal PD = 0 1%

3 4 1 2 Conditional Approaches to Credit Portfolio Loss Distributions

Finger (1999) addressed the issue of credit portfolio loss distributions conditioned on the outcome of a market variable His interest in the issue stemmed from the slow convergence exhibited by Monte Carlo simulation of credit portfolios He noted that reliance on Monte Carlo simulation was unnecessary once account was taken of the assumed correlation structure "[O]nce we condition on the industry factors that drive the model, all defaults and rating changes are independent " (p14)

He adopted the same approach as Vasicek (1997b) outlined in 3 4 1 1 above whereby each normalised asset value change can be expressed as

$$Z_i = wZ + \sqrt{1 - w^2}\,\varepsilon_i \qquad \textbf{Equation 3-51}$$

and each obligor has the same default probability, p, and default threshold, α , where

$$\alpha = \Phi^{-1}(p) \qquad \textbf{Equation 3-52}$$

and all obligors have the same pairwise asset correlation, w^2 Once the market factor, Z, is fixed, he notes, everything else that happens to the obligors is independent, the obligors are conditionally independent given Z (p15) The conditional independence, he contends, proves crucial, as it transforms the complex problem of aggregating correlated exposures into the well understood problem of convolution, or the aggregation of independent exposures

Conditioned on Z, an obligor defaults if

$$\varepsilon_i < \frac{\alpha - wZ}{\sqrt{1 - w^2}} \qquad \textbf{Equation 3-53}$$

and since ε_i is a standard normal deviate, the PD conditioned on Z is

$$p(Z) < \Phi\left(\frac{\alpha - wZ}{\sqrt{1 - w^2}}\right) \qquad \textbf{Equation 3-54}$$

He deduces that the portfolio variance is

$$\left[\Phi_2(\alpha, \alpha, w^2) - p^2\right] + \frac{p - \Phi_2(\alpha, \alpha, w^2)}{N} \qquad \textbf{Equation 3-55}$$

where the first term is the same as Vasicek's and the last term is due to idiosyncratic variance which is not perfectly diversified due to the finite number of obligors

He derives the portfolio distribution as

$$\Pr[V < v] = \int_{-\infty}^{\infty} \phi(z) \Phi\left(\frac{v - (1 - p(z))}{\sqrt{\frac{p(z)(1 - p(z))}{N}}} \right) dz \qquad \textbf{Equation 3-56}$$

While this does not have an analytical solution, a numerical solution can be found substantially more quickly than would the Monte Carlo solution He notes that "the real dimensionality of the problem is not the number of obligors, but the number of market factors" (p33) and suggests that employing this approach will enable the same accuracy as standard Monte Carlo be achieved in a fraction of the time

3 4 1 3 Other Credit Portfolio Models

Koyluoglu and Hickman (1998) showed that despite the apparent differences in approach adopted by the main models used in the marketplace – CreditMetrics, Credit Suisse Financial Products' CreditRisk+, McKinsey's CreditPortfolioView and KMV's Portfolio Manager – the models "in fact represent a remarkable consensus in the underlying framework, differing primarily in calculation procedures and parameters rather than financial intuition " (p29)

All the models, they suggest, fit within a general framework consisting of three components (p32)

Joint default behaviour portfolio correlation is reflected by borrowers' conditional default rates varying together in different states

Conditional distribution of portfolio default rate for a given 'state of the world', the conditional distribution can be calculated, they note, as if borrowers are independent because the joint default behaviour is accounted for in generating conditional default rates

Convolution/Aggregation They comment that the unconditional distribution of portfolio defaults is obtained by combining conditional default rate distributions in each state

They decompose the change in asset value into a set of normally distributed orthogonal systemic factors, x_k, and a normally distributed idiosyncratic component, ε_i

$$\Delta A_i = b_{i1}x_1 + b_{i2}x_2 + \quad + \sqrt{1-\sum_k b_{ik}^2}\varepsilon_i \qquad \textbf{Equation 3-57}$$

where b_{ik} are the factor loadings, and x_k and ε_i are ~ i i d N[0,1]

For a given set of values for the systemic factors, the portfolio default rate can be expressed as

$$p_i|_x = \Phi\left[\frac{c-\sum_k b_{ik}x_k}{\sqrt{1-\sum_k b_{ik}^2}}\right] \qquad \textbf{Equation 3-58}$$

where c is the threshold value of the standard normal variable at which default occurs

For a homogeneous portfolio, they summarise the systemic factors by a single variable, m, simplifying the expression for the portfolio default rate to

$$p|_m = \Phi\left[\frac{c-\sqrt{\rho}m}{\sqrt{1-\rho}}\right] \qquad \textbf{Equation 3-59}$$

where $m \sim N[0,1]$ and $\rho = \sum_k b_k^2$ is the asset correlation

They proceed to derive the probability density function for the default rate, $f(p)$

$$f(p) = \frac{\sqrt{1-\rho}\,\varphi\left[\frac{c-\sqrt{1-\rho}\,\Phi^{-1}(p)}{\sqrt{\rho}}\right]}{\sqrt{\rho}\,\varphi\left(\Phi^{-1}(p)\right)} \qquad \textbf{Equation 3-60}$$

For a homogeneous portfolio of loans with a PD of 1 16%, $c = \Phi^{-1}[1\ 16\%]$ = -2 27, the probability distribution of the default rate is plotted in Figure 3 7

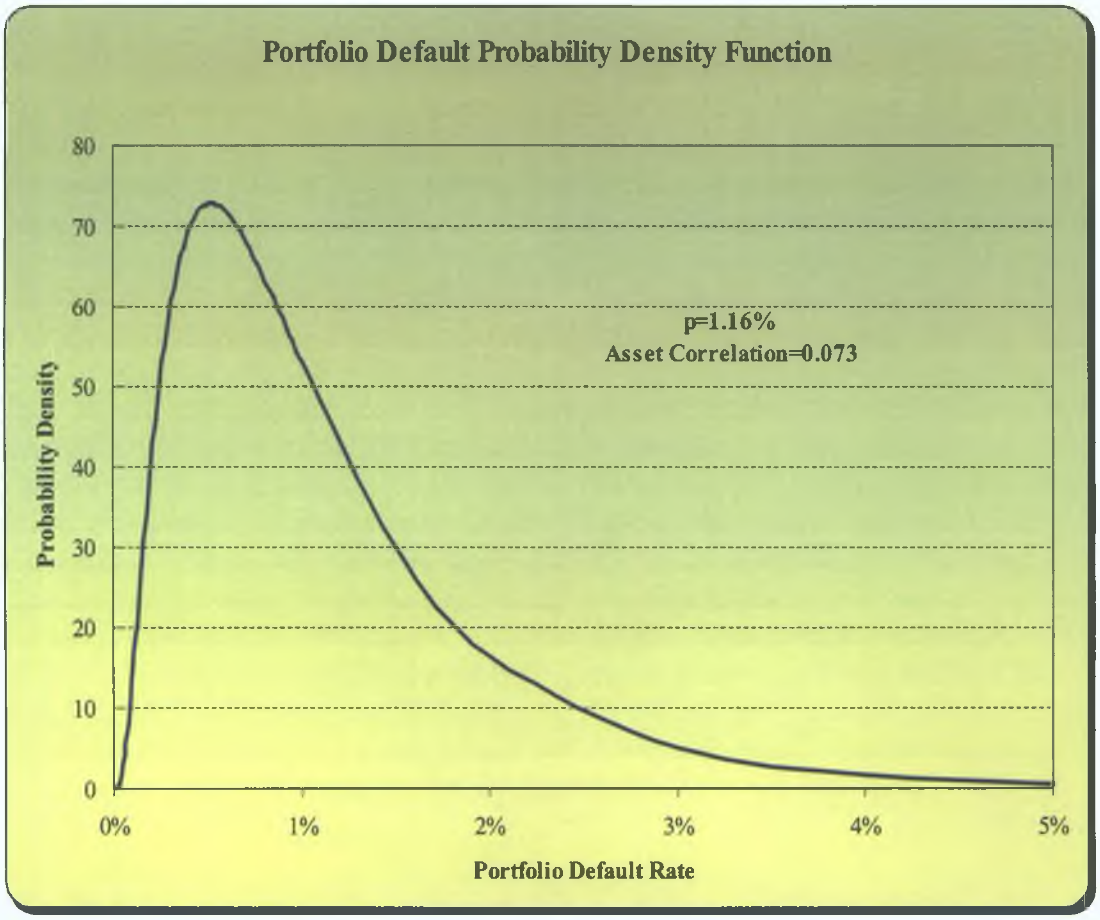

Figure 3.7 Probability Distribution of Portfolio Default Rate

The unconditional probability distribution of portfolio defaults is obtained by combining the conditional distributions across all 'states of the world'. While they point out the similarities in approach to developing the unconditional default probability, they identify differences in the way joint default behaviour is modelled. Whereas the Merton-based models of CreditMetrics and KMV use pairwise asset correlation, the actuarial model employed by CreditRisk+ uses sector weightings and default rate volatilities. They emphasise, however, that despite the parameter differences, "they contain equivalent information to characterise default behaviour." (p35)

They present the generalised framework reproduced as Figure 3.8 below. While their comments are specifically directed at CreditMetrics, they suggest they apply 'reasonably well to *Portfolio Manager*' also:

	CreditMetrics	CreditPortfolioView	CreditRisk+
Joint-default Behaviour	Distribution of systemic factors (normal) Conditional Default Rate: Merton Model	Distribution of systemic factors (normal) Conditional Default Rate: Macroeconomic Regression	Default rate distribution (gamma)
Conditional Default Distribution	Binomial Distribution	Binomial Distribution	Poisson Distribution
Convolution/ Aggregation	Monte Carlo simulation	Monte Carlo simulation	Numeric Algorithm

Figure 3 8 Framework for Comparing Credit Portfolio Models

They conclude "Any significant model differences can be attributed to parameter value estimates that have inconsistent implications for the observable default rate behaviour " (p35)

3.5 Modelling Correlation

Duffie and Singleton (2003, p230), summarise the most popular approaches to modelling correlated defaults

CreditMetrics, they characterise as a method by which ratings transitions for multiple entities can be simulated with the correlation induced by underlying correlated *drivers*, such as asset returns

Doubly stochastic correlated default-intensity processes is, they state, an approach to modelling multi-entity default risk in which "correlation is captured through correlated changes in the default intensities of the entities "

Copulas, they describe as devices that allow entity-by-entity default models "to be linked with auxiliary correlating variables "

Intensity-based models of default with joint credit events can, they suggest, cause multiple issuers to default simultaneously "The simplest example is the multivariate exponential model of default times, which has constant default intensities "

3.5.1 The CreditMetrics Approach

CreditMetrics uses the counterparty's asset returns, X_i, as the driving variable. Each counterparty's asset returns are assumed to be normally distributed, and the asset returns for multiple counterparties, $X_1, X_1, \ldots, X_n$ are assumed to be multivariate normally distributed, with a covariance matrix, Σ. The Cholesky decomposition of Σ is C such that $CC^T = \Sigma$. By simulating independent standard normal variables, $Z_1, Z_1, \ldots, Z_n$, the drivers with the appropriate means and covariances can be simulated by letting $X_i = E(X_i) + C_{i1}Z_1 + C_{i2}Z_2 + \ldots + C_{in}Z_n$

CreditMetrics suggest that the necessary covariance information can be obtained from the volatilities and correlations of equity returns for the n firms in the case of publicly traded firms. Duffie and Singleton (2003, p232), suggest that one could also "take the drivers to be the KMV ... measures of *distance to default*, firm by firm, which is more in the spirit of the asset-return foundation of the CreditMetrics model." They add that time series data on distances to default for pairs of firms could be used to estimate the covariance matrix.

Lando (1998) considered correlation within the framework of finite-state continuous-time Markov chains for each entity's rating. However, the approach proves to be rather intractable because the state space is n^b where there are b borrowers and n rating states.

3.5.2 Copula-Based Correlation Modelling

It was Li (2000) who first applied the copula-based approach to simulating correlated defaults. He began by letting T represent a security's time-until-default, and $F(t)$ denote the distribution function of T,

$$F(t) = \Pr(T \le t), \qquad t \ge 0 \qquad \textbf{Equation 3-61}$$

He defines the probability density function

$$f(t) = F'(t) = \lim_{\Delta \to 0+} \frac{\Pr[t \le T < t + \Delta]}{\Delta} \qquad \textbf{Equation 3-62}$$

and he defines

$$q_x = \Pr[T - x \le 1 \mid T > x] \qquad \textbf{Equation 3-63}$$

as the marginal probability of default in the next year conditional on survival until the beginning of the year "A credit curve is then simply defined as the sequence of $q_0, q_1, \quad , q_n$ in discrete models " (p43)

Next, he introduces the hazard rate function, $h(x)$, as the instantaneous default probability for a security that has attained age x

$$\Pr\left[x < T \le x + \Delta x \middle| T > x\right] = \frac{F(x + \Delta x) - F(x)}{1 - F(x)}$$

$$= \frac{f(x)\Delta x}{1 - F(x)} \qquad \textbf{Equation 3-64}$$

$$= h(x)\Delta x$$

where

$$h(x) = \frac{f(x)}{1 - F(x)} \qquad \textbf{Equation 3-65}$$

The survival function can be defined as

$$S(t) = e^{-\int_0^t h(s)ds} \qquad \textbf{Equation 3-66}$$

so that

$$f(t) = S(t)h(t) \qquad \textbf{Equation 3-67}$$

Li points out that if the typical assumption of a constant hazard rate is made, the density function is

$$f(t) = h\, e^{-ht} \qquad \textbf{Equation 3-68}$$

He states "Modelling a default process is equivalent to modelling a hazard function " (p47) He also notes "[T]here are a lot of similarities between the hazard rate function and the short rate Many modelling techniques for the short rate processes can be readily borrowed to model the hazard rate " (p48) Indeed this is exactly how much credit risk modelling has progressed over the past ten years, and the hazard rate function is called the credit curve because of its similarity to a yield curve

He proceeds to define the joint distributional function for two entities, A and B, as

$$F(s,t)=\Pr[T_A \le s, T_B \le t] \qquad \textbf{Equation 3-69}$$

and defines the survival time correlation as

$$\rho_{AB}=\frac{Cov(T_A,T_B)}{\sqrt{Var(T_A)Var(T_B)}}=\frac{E(T_AT_B)-E(T_A)E(T_B)}{\sqrt{Var(T_A)Var(T_B)}} \qquad \textbf{Equation 3-70}$$

He suggests three methods could be used to extract the term structure of default rates (i) using historical default information from rating agencies, (ii) applying the Merton option theoretical approach, and (iii) taking an implied approach using market prices of defaultable bonds or asset swap spreads

The last approach is the "one used by most credit derivative desks " (p53) Li (1998), demonstrated how to build the credit curve for individual credits from market information based on the Duffie and Singleton (1996) default treatment The challenge, he says, is to create "a joint distribution function with given marginal distributions and a correlation structure " (p9) While it is straightforward to derive the marginal distributions and the correlation structure if the joint distribution is known, creating a joint distribution from a given set of marginals and a correlation structure, he comments, is rather more difficult The copula function is the mechanism he chooses to accomplish this

A copula function links univariate marginal distributions with a joint distribution For given univariate marginal distribution functions, $F_1(x_1), F_2(x_2), \quad , F_m(x_m)$, the function

$$C(F_1(x_1), F_2(x_2), \quad , F_m(x_m)) = F(x_1, x_2, \quad , x_m) \qquad \textbf{Equation 3-71}$$

which is defined using a copula function, C, results in a multivariate distribution with univariate marginal distributions, $F_1(x_1), F_2(x_2), \quad , F_m(x_m)$

Sklar (1959) proved the converse He showed that any multivariate distribution, F, could be written in the form of a copula function He proved that if $F(x_1, x_2, \quad , x_m)$ is a joint multivariate distribution function with univariate marginal distribution functions $F_1(x_1), F_2(x_2), \quad , F_m(x_m)$, then there exists a copula function $C(u_1, u_2, \quad , u_m)$ such that

$$F(x_1, x_2, \quad x_m) = C(F_1(x_1), F_2(x_2), \quad , F_m(x_m)) \qquad \textbf{Equation 3-72}$$

If each F_i is continuous then C is unique

Li showed that the copula function embedded in CreditMetrics is the bivariate normal

$$C(u,v)=\Phi_2\left(\Phi^{-1}(u),\Phi^{-1}(v),\rho\right) \qquad \textbf{Equation 3-73}$$

where ρ is defined as the correlation between the default times of u and v For example, if the one-year default probabilities for two credits, A and B, are q_A and q_B, respectively, the default thresholds are given by

$$q_A=\Pr[Z<Z_A]$$
$$q_B=\Pr[Z<Z_B] \qquad \textbf{Equation 3-74}$$

where Z is a standard normal random variable If ρ is the asset correlation, the joint default probability for credit A and B is

$$\Pr[Z<Z_A\ Z<Z_B]=\int_{-\infty}^{Z_A}\int_{-\infty}^{Z_B}\phi(x,y|\rho)dx\,dy=\Phi_2(Z_A,Z_B,\rho) \qquad \textbf{Equation 3-75}$$

This is graphed in Figure 3 9 for an asset correlation of 0 3 This probability is most easily visualised as the volume under the surface in Figure 3 10 in the bottom left hand corner below the specified y-value and to the left of the specified x-value

He comments that CreditMetrics uses a bivariate normal copula function with the asset correlation as the correlation parameter in the copula function Thus, to generate survival times of two credit risks, a bivariate normal copula function is used with a correlation parameter equal to the CreditMetrics asset correlation (p13)

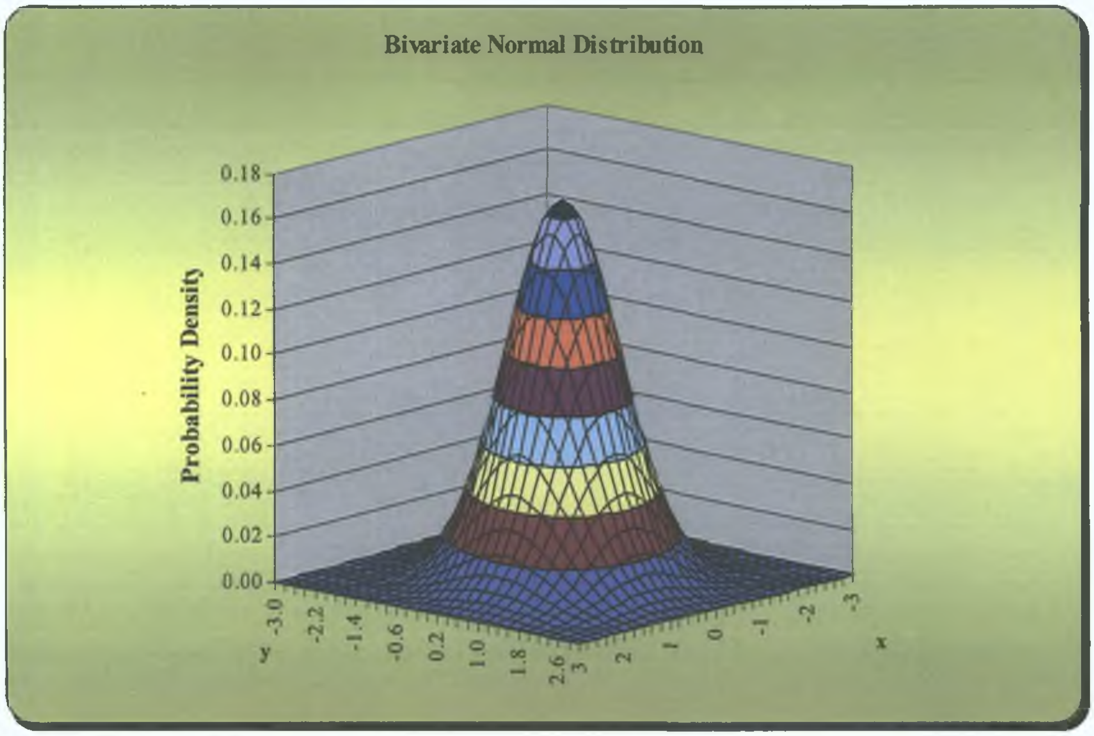

Figure 3.9 Bivariate Normal Distribution

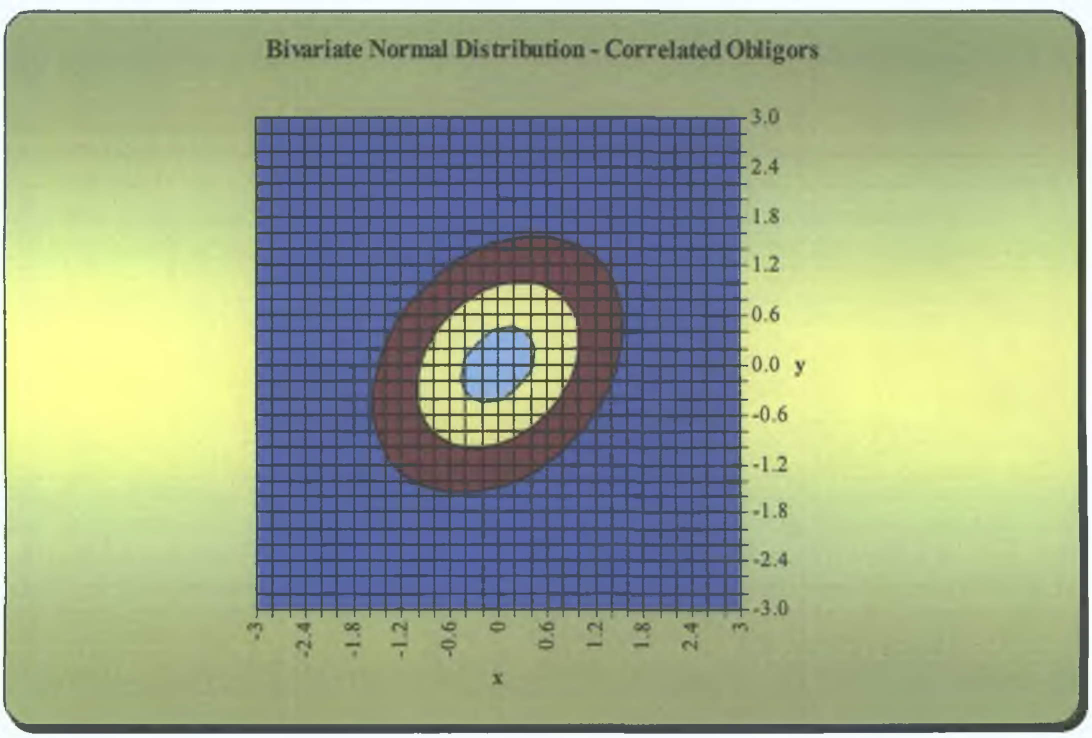

Figure 3.10 Bivariate Normal Distribution – Plan View

He further notes: "Conveniently, the marginal distribution of any subset of an *n* dimensional normal distribution is still a normal distribution. Using asset correlations, we can construct high dimensional normal copula functions to model the credit portfolio of any size " (p13) This has become the standard modelling methodology in the market

O'Kane *et al* (2003) comment that although both the structural and the reduced form approaches can in principle be extended to the multivariate case, structural models calibrated to market-implied default probabilities (often called 'hybrid' models) have gained favour among practitioners because of their tractability in high dimensions "Hybrid models use the dependence among asset returns to generate joint defaults, therefore avoiding the need for a direct estimation of joint default probabilities " (p34)

These hybrid models are Monte Carlo models which generate default paths, where each path is a list of default times for each of the credits in the reference portfolio drawn at random from the joint default distribution Once the time and identity of each default event is known, any credit portfolio product may be valued

The choice of copula is non-trivial as pointed out by Marshal and Naldi (2002) They demonstrate that the Gaussian copula is unable to explain the extreme co-movements that are observed in the market The assumption of normality of asset returns, however, "is certainly not innocuous, since a multivariate normal distribution does not allow for extreme joint events to happen with the frequency that the data suggests " (p41) The multivariate normal distribution exhibits a tail dependence of zero for all correlations less than 1 They propose instead a *t* copula that they suggest is consistent with asset returns being multivariate *t* distributed Their analysis suggests nine as the maximum likelihood degrees of freedom They demonstrate that the impact on a first-to-default credit derivative is to reduce the price by between 5% and 10% compared to the Gaussian copula However, the impact farther out in the tail is quite dramatic they calculate that second-to-default protection will be under-estimated by as much as 58% by using a Gaussian copula

While the copula-based approach to credit correlation modelling has become the market standard for portfolios of debt extended to firms for which there is a liquid CDS market, the approach has limited applicability to the vast majority of firms in order to obtain risk-neutral probabilities of default for a particular company, a precise yield curve specific to Company X debt (or a precise yield curve for debt of other companies that are deemed to be of similar credit risk) is required "Thus it will be difficult to apply reduced form models to middle market companies or illiquid markets " Smithson (2003, p215) concludes

3.6 *Pricing Credit Risk*

Merton's version of the structural approach automatically prices credit risk but it assumes that LGD is endogenous Other variations on this approach assume that default probability only is determined by the asset value process and that recovery is exogenously specified thereby taking account of the fact that the absolute priority rule is frequently violated in practice

Smithson (2003, p209), suggests a 'family tree' of models for pricing default-risky claims as shown in Figure 3 11

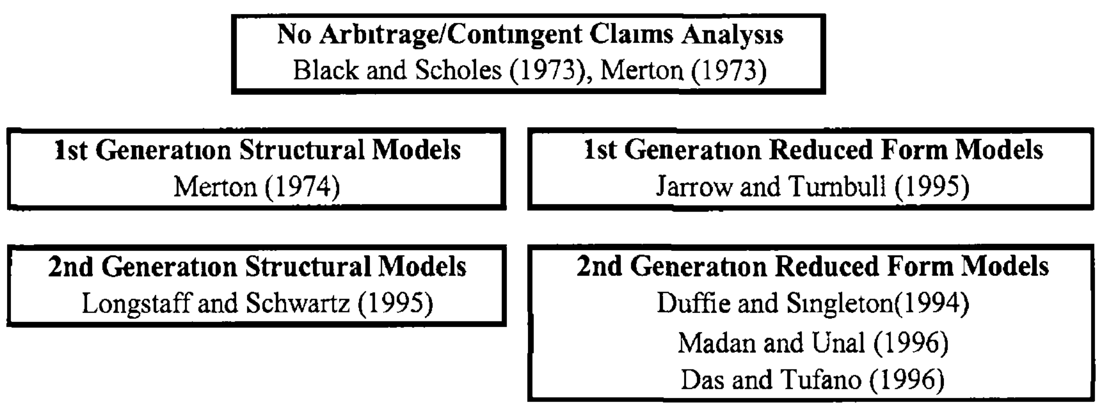

Figure 3 11 A Family Tree of Pricing Models for Default-Risky Claims (after Smithson)

Smithson remarks that at the risk of oversimplifying, credit derivatives and traditional derivatives can all be valued as the present value of their risk-adjusted expected future cash flows He qualifies this remark, however "The bad news is that credit models are much more difficult to implement " He lists three separate areas that cause difficulty

Default definition "Default is an imprecise concept subject to various legal and economic definitions " (p209) He adds that a pricing model will necessarily have to simplify the economics of default or very carefully define the precise conditions being modelled

LGD Pricing models for credit must address the uncertainty in LGD or assume that loss given default is known, he comments

Data on which to parameterise models Data on credit losses are "notoriously limited" and credit spread data are available only for the largest and most liquid markets, he opines (p209)

3 6 1 Pricing Credit Derivatives

This imprecision regarding default definition is evident in the lack of standardisation as regards credit events in credit derivative documentation Despite the improvements in the 2003 ISDA master agreement over its 1999 predecessor, the market remains split between an U S and a non-U S standard In the U S , the market has adopted the Modified Restructuring (Mod-Re) standard while the European market has adopted the Modified-Modified Restructuring (Mod-Mod-re) convention While Duffie (1999) comments that credit default swaps involve some risk of disagreement about whether the event has, in fact, occurred, in his discussion of valuing the credit swap, he ignores issues surrounding documentation and enforceability

Structural models are seldom used to price credit default swaps instead, structural models are generally used to say at what spread corporate bonds should trade based on the internal structure of the company, according to O'Kane *et al* (2003, p5) They state that they require information about the balance sheet of the company and can be used to establish a link between pricing in the equity and debt markets They add that they are limited in a number of ways including the fact that they generally lack the flexibility to fit exactly a given term structure of spreads, and "they cannot be easily extended to price complex credit derivatives " (p5)

The reduced form approach of Jarrow and Turnbull (1995) is the basis for what has become the market standard method of valuing CDS They characterise a credit event as the first event of a Poisson counting process that occurs at time t with a probability defined as

$$\Pr\left[\tau < t + dt \middle| \tau > t\right] = \lambda(t)dt \qquad \textbf{Equation 3-76}$$

Thus, the probability of defaulting in the time interval $[t, t+dt]$ conditional on surviving to time t is proportional to $\lambda(t)$, which they call the hazard rate and the length of the time interval, dt Thus, the survival probability is

$$Q(0,T) = E^{Q}\left[e^{\left(-\int_0^T \lambda(s)ds\right)} \right] \qquad \textbf{Equation 3-77}$$

where the expectation is taken under the risk-neutral measure, Q. The standard market assumption is that the hazard rate process is deterministic, and, therefore, independent of interest rates and recovery rates.

In this framework, the spread is set so that the present values of the premium and protection legs of the CDS contract are equal. If the hazard rate and risk-free rate term structures are flat, and R is the recovery rate, the present value of the protection leg is:

$$(1-R)\int_0^T \lambda e^{-(r+\lambda)s} ds = \frac{\lambda(1-R)\left(1-e^{-(r+\lambda)T}\right)}{r+\lambda}$$ **Equation 3-78**

If the spread, S, on the premium leg is paid continuously, the present value of the premium leg is:

$$S\int_0^T e^{-(r+\lambda)s} ds = \frac{S\left(1-e^{-(r+\lambda)T}\right)}{r+\lambda}$$ **Equation 3-79**

Setting the value of the premium leg equal to that of the protection leg, the value of the spread is extracted:

$$S = \lambda(1-R)$$ **Equation 3-80**

and the risky *PV01*, or *RPV01* as it is known in the market, is:

$$RPV01(t) = \frac{\left(1-e^{-(r+\lambda)(T-t)}\right)}{r+\lambda}$$ **Equation 3-81**

In reality, the interest rate and hazard rate term structures are not flat and it becomes necessary to build a full term structure using bootstrapping techniques.

Duffie (1999) remarks that the term 'swap' applies to credit swaps because they can be viewed under certain ideal conditions as a swap of a default-free note for a defaultable floating-rate note. He proceeds to calculate the at-market annuity premium rate for which the market value of the credit swap is zero at the outset.

3.7 Measuring Probability of Default

There are numerous approaches to the measurement of PD in use in the marketplace. These range from the multivariate approach pioneered by Altman thirty years ago to neural networks. For our purposes, it suffices to examine in greater detail the two methodologies which are most widely used in the market and which are also representative of the two approaches which underpin the models which will be compared later in this thesis The KMV approach is the most widely used default probability measure commercially available while the Moody's rating is the standard on which many market participants still rely These two approaches will be examined in turn

3 7 1 KMV

According to the Merton approach, the PD is a function of (i) the value of the firm's assets, (ii) the volatility of this value, and (iii) the amount of debt in the firm's capital structure KMV have adopted this approach but have adapted it to empirical data to convert the outputs into practical results by developing a measure of risk they call 'distance-to-default'

Crosbie (2002) discusses KMV's understanding of 'default point' He notes that in KMV's studies of defaults, they have found that in general firms do not default when their asset value reaches the book value of their total liabilities He acknowledges that while some firms certainly default at this point, many continue to trade and service their debts The long-term nature of some of their liabilities provides these firms with some breathing space, he surmises KMV found that the default point, the asset value at which the firm will default, lies "somewhere between total liabilities and current, or short-term, liabilities " The relevant net worth of the firm is the market value of the firm's assets minus the firm's default point A firm will default "when its market net worth reaches zero", he notes (p3)

He explains that asset value, business risk and leverage can be combined into a single measure of default risk which compares the market net worth to the size of a one standard deviation move in the asset value KMV refer to this ratio as the *distance-to-default* and it is calculated as

$$\text{Distance to Default} = \frac{\text{Market Value of Assets - Default Point}}{[\text{Market Value of Assets}][\text{Asset Volatility}]}$$

"The default probability can be computed directly from the distance-to-default if the default rate for a given level of distance-to-default is known " (p6)

KMV have compiled a very large database of defaults and have computed the distance-to-default metric for these firms for the years prior to their defaulting A default database is used to derive an empirical distribution relating the distance-to-default to a default probability. "In this way, the relationship between asset value and liabilities can be captured without resorting to a substantially more complex model characterising a firm's liability process." (p7) They call the resulting PD, the expected default frequency, EDF. They have implemented this process in their software product, Credit Edge, and deliver annualised EDF values over 1 to 5 years on a daily basis over the internet on over 35,000 companies. A schematic representation of EDF measurement is shown in Figure 3.12 below:

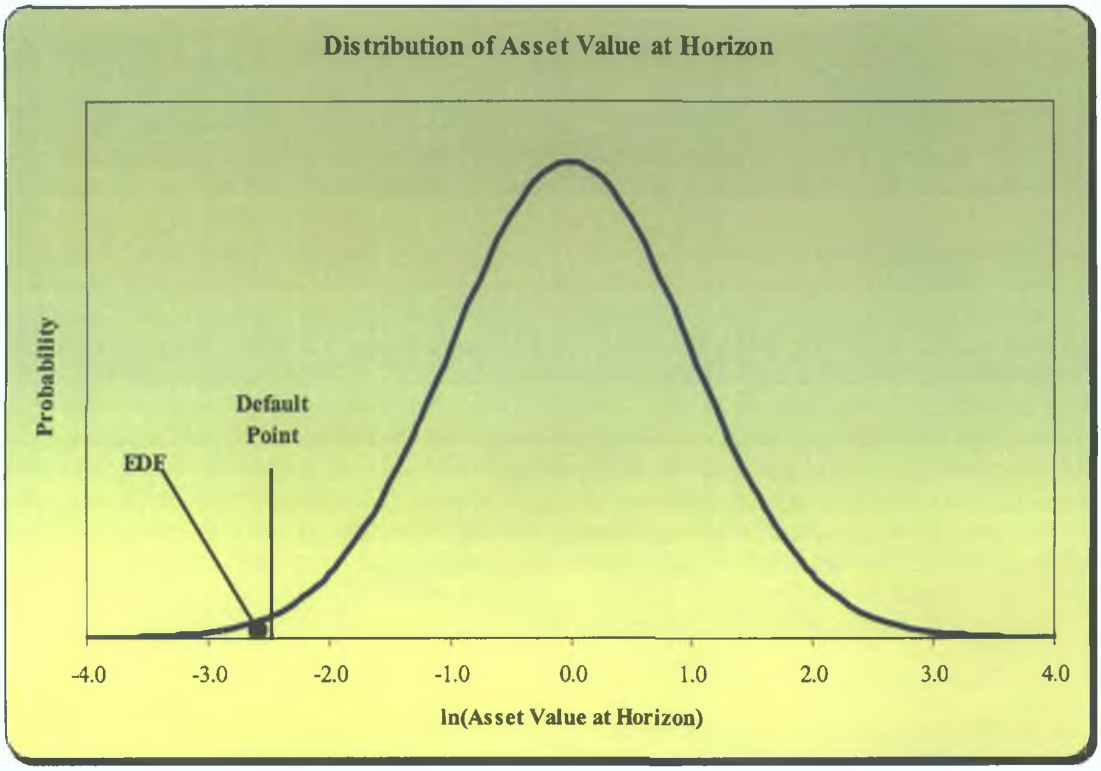

Figure 3.12 Schematic for Estimation of Expected Default Frequency

If the future distribution of the distance-to-default were known, the default probability, EDF, would be the likelihood that the final asset value was below the default point. However, in practice, Crosbie says that the distribution of the distance-to-default is difficult to measure. He states that the usual assumptions of normal or lognormal distributions cannot be used. For default measurement, the likelihood of large adverse changes in the relationship of asset value to the firm's default point is critical to the accurate determination of the default probability, he suggests. These changes may come about from changes in asset value or changes in the firm's leverage. In fact, changes in asset value and changes in leverage may be highly correlated. Consequently, KMV first measures the distance-to-default as the number of standard deviations the asset value is away from default and then uses empirical data to determine the corresponding default probability They obtain the relationship between distance-to-default and default probability from data on historical default and bankruptcy frequencies Their database includes over 250,000 company-years and over 4,700 incidents of default or bankruptcy From this data, a lookup or frequency table is created which generates the likelihood of default to various levels of distance-to-default Thus, if they are interested in determining the default probability over the next year for a firm that is seven standard deviations away from default, they query the default history for the proportion of the firms, seven standard deviations away from default that defaulted over the next year The answer, he says, is about 5bp, 0 05%, or an equivalent rating of AA

The PD by time T, p_T, is the probability that the market value of the firm's assets, V_A^T, will be less than the book value of the firm's liabilities due by time T, X_T, where the asset volatility is σ_A In the Merton framework, the PD is

$$p_T = \Pr\left[\ln(V_A) + \left(\mu - \tfrac{1}{2}\sigma_A^2\right)T + \sigma_A\sqrt{T}\,\varepsilon\right] \le \ln(X_T) \qquad \textbf{Equation 3-82}$$

Rearranging,

$$p_T = \Pr\left[-\frac{\ln\left(\frac{V_A}{X_T}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma_A\sqrt{T}}\right] \ge \varepsilon \qquad \textbf{Equation 3-83}$$

Assuming the asset returns are normally distributed, this probability is

$$p_T = N\left[-\frac{\ln\left(\frac{V_A}{X_T}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma_A\sqrt{T}}\right] = N(-DD) \qquad \textbf{Equation 3-84}$$

In order to calculate these EDF values, KMV must calculate the asset value and asset volatility of the firm from the market value and volatility of equity and the book value of the liabilities They have extended the Merton model into what they call the Vasicek-Kealhofer model to incorporate more complex capital structures including long- and short-term debt, convertible debt and the perpetuity nature of equity Thus, they solve the following two relationships simultaneously

$$\text{Equity Value} = f\left(\text{Asset Value, Asset Volatility, Capital Structure, Interest Rate}\right)$$
$$\text{Equity Volatility} = f\left(\text{Asset Value, Asset Volatility, Capital Structure, Interest Rate}\right)$$

Asset value and volatility are the only unknown quantities in these relationships and thus the two equations can be solved to determine the values implied by the current equity value, volatility and capital structure

KMV start with the Merton relationship

$$V_E = V_A N\left(\frac{\ln\left(\frac{V_A}{X e^{-rT}}\right) + \frac{1}{2}\sigma_A^2 T}{\sigma\sqrt{T}}\right) - X e^{-rt} N\left(\frac{\ln\left(\frac{V_A}{X e^{-rT}}\right) - \frac{1}{2}\sigma_A^2 T}{\sigma\sqrt{T}}\right)$$

Equation 3-85

They then recognise that equity is a leveraged interest in the underlying assets and that its volatility is higher than that of the underlying assets as follows

$$\sigma_A = \frac{V_A}{V_E} \Delta N(d_1) \, where$$
$$d_1 = \frac{\ln\left(\frac{V_A}{X e^{-rT}}\right) + \frac{1}{2}\sigma_A^2 T}{\sigma\sqrt{T}}$$

Equation 3-86

where Δ is the delta of the equity option on the assets

Once again, they need to modify the Merton model to derive meaningful results They adjust the distance-to-default to include not only the increases in asset value given by the rate but also adjust for any cash outflows to service debt, dividends, and so on In addition, they state that the Normal distribution is a very poor choice to define the PD The most important reason they give is the fact that the default point is in reality also a random variable Thus, while they have assumed that the firm's default point is described by the firm's liabilities and amortisation schedule, they acknowledge that this is not true and recognise that firms will often adjust their liabilities as they near default They note that it is common to observe the liabilities of commercial and industrial firms increase as they near default while the liabilities of financial institutions often decrease as they approach default The difference, they suggest, is usually just a reflection of the liquidity in the firm's assets and thus their ability to adjust their leverage as they encounter difficulties However, KMV have found themselves unable to specify *ex ante* the behaviour of the liabilities and thus they must capture the uncertainty in the adjustments in the liabilities elsewhere. They choose to include this uncertainty in the mapping of distance-to-default to the EDF credit measure. They observe that the resulting empirical distribution of default rates has much wider tails than the Normal distribution. "For example, a distance to default of four, four standard deviations, maps to a default rate of 100bp. The equivalent probability from the Normal distribution is essentially zero." Crosbie (2000, p17)

KMV determine the PD of a firm by reference to its distance-to-default. All the firms in its database over many years are categorised based on their distance-to-default and the subsequently realised default rates over one-, two-, three-, four- and five-year periods thereafter. Thus, KMV can extract not just a one-year PD for a firm but the term structure of a firm's PD out to five years.

3.7.2 Rating Agencies

The rating agencies, and indeed, most financial institutions, assign ratings to borrowers based on a variety of financial and non-financial measures. These ratings are merely ordinal rankings and the descriptions of what the ratings mean are extremely general.

In recent years, the rating agencies have responded to market demand for more quantitative risk measures by publishing historical default statistics. Two types of statistics in particular are quoted: (i) historical default rates over various time periods, and (ii) transition matrices which report not just on the frequency of transition from a given grade to default but also on the frequency of transition from a given grade to other non-default grades.

However, two criticisms are frequently levelled at these statistics. The first criticism is that default rates are simply historical and these rates vary significantly from one period to the next. The second is that transition matrices are unconditional averages whereas what the market really needs is a transition matrix that is conditioned to the current state of the credit market.

Despite the criticisms and the many rating errors that the agencies have made – especially notable is their slow reaction to credit deterioration – their rating is still the market benchmark for many companies and debt issues.

3.8 Modelling Credit Portfolios

Crouhy *et al* (2000), cite two additional difficulties in calculating credit VaR compared to market VaR The first difficulty cited is that the portfolio distribution is far from being normal, and the second is that measuring the portfolio effect due to credit diversification is much more complex than for market risk While it was legitimate to assume normality of the portfolio changes due to market risk, they suggest, it is not feasible for credit returns which are by nature highly skewed and fat-tailed There is limited upside to be expected from any improvement in credit quality, they state, while there is substantial downside consecutive to downgrading and default "The percentile levels of the distribution cannot be any longer estimated from the mean and variance only The calculation of VaR for credit risk requires simulating the full distribution of the changes in portfolio value " (p320)

Gupton *et al* (1997) concur with this view Modelling portfolio risk in credit portfolios is neither analytically nor practically easy, they note Fundamental differences between credit risks and equity price risks "make equity portfolio theory problematic when applied to credit portfolios " (p7) They expand on this point by identifying two fundamental problems which credit portfolio modelling presents which are absent when modelling equities The first problem to which they refer is that oft-cited problem that equity returns are relatively symmetric and are well approximated by normal or Gaussian distributions whereas credit returns are highly skewed and fat-tailed Thus, more than just the mean and standard deviation is required fully to understand a credit portfolio's distribution The second problem they cite is the difficulty of modelling correlations For equities, they note, the correlations can be directly estimated by observing high-frequency liquid market prices "For credit quality, the lack of data makes it difficult to estimate any type of credit correlation from history " (p8)

Another important difference between equity portfolio management and credit portfolio management to which they refer is in relation to firm-specific risk Whereas market risks can be diversified with a relatively small portfolio or hedged using liquid instruments, credit risks, they suggest, are more problematic For credit portfolios, simply having many obligors' names represented within a portfolio does not assure good diversification (*e g* they may all be large banks in one country) they state They conclude that when diversification is possible, it is typically achieved by "much larger numbers of exposures than for market portfolios " (p81)

3 8 1 Credıt Portfolıo Models used ın the Market

Accordıng to the *2002 Survey of Credıt Portfolıo Management Practıces* undertaken by the Internatıonal Assocıatıon of Credıt Portfolıo Managers (IACPM), the Internatıonal Swaps and Derıvatıves Assocıatıon (ISDA) and the Rısk Management Assocıatıon (RMA) as reported by Rutter (2003, p161), 85% of the 41 large financıal ınstıtutıons whıch responded stated they used a portfolıo management model Of these, 69% stated they used KMV's *Portfolıo Manager* whıle the CredıtMetrıcs product, *CredıtManager*, was employed by 20% In addıtıon, 17% used an ınternally developed model (Rutter notes that the responses sum to more than 100% as some respondents used more than one model) Neıther McKınsey's *CredıtPortfolıoVıew* nor CSFB's *CredıtRısk+* was used by any of the respondents However, the author ıs aware that many German mortgage banks use *CredıtRısk+* so the results may reflect the survey emphasıs on commercıal and ınvestment banks rather than mortgage/retaıl banks

Therefore, ıt ıs consıdered approprıate to examıne the two credıt portfolıo models that are most frequently used ın ındustry, namely, CredıtMetrıcs' *CredıtManager* and KMV's *Portfolıo Manager*

3 8 1 1 CredıtMetrıcs

In Aprıl 1997, J P Morgan, ın conjunctıon wıth sıx bank sponsors and KMV, launched CredıtMetrıcs It followed the successful launch four years earlıer of RıskMetrıcs, a product for modellıng market rısk

Whıle the products were sımılar, the challenge they faced ın creatıng CredıtMetrıcs was substantıally greater As they acknowledged ın theır technıcal document, (Gupton *et al*, 1994), one major dıfference ın the models was drıven by the dıfference ın avaılable data In RıskMetrıcs, there was an abundance of avaılable daıly lıquıd prıcıng data on whıch to construct a model of condıtıonal volatılıty "In CredıtMetrıcs, we have relatıvely sparse and ınfrequently prıced data on whıch to construct a model of uncondıtıonal volatılıty " (p ııı)

Nor was there any market agreement on the correct modellıng approach Unlıke market rısks where daıly lıquıd prıce observatıons allow a dırect calculatıon of VaR, they state that CredıtMetrıcs seeks to construct what ıt cannot dırectly observe, the volatılıty of value due to credıt qualıty changes Thıs constructıve approach, they claım, makes

CreditMetrics less an exercise in fitting distributions to observed price data, and more an exercise in proposing models which explain the changes in credit related instruments

Furthermore, the nature of credit returns presented special challenges the models that best describe credit risk "do not rely on the assumption that returns are normally distributed, marking a significant departure from the RiskMetrics framework "(p iv)

CreditMetrics' goal, they state, was to estimate the volatility of value due to changes in credit quality, not just the expected loss In their view, as important as default likelihood estimation is, it is only one link in the long chain of modelling and estimation that is necessary fully to assess credit risk within a portfolio "CreditMetrics is a tool for assessing portfolio risk due to changes in debt value caused by changes in obligor credit quality " (p5) They assess risk within the full context of a portfolio, addressing the correlation of credit quality moves across obligors

Their outline of their valuation framework is shown in Figure 3 13

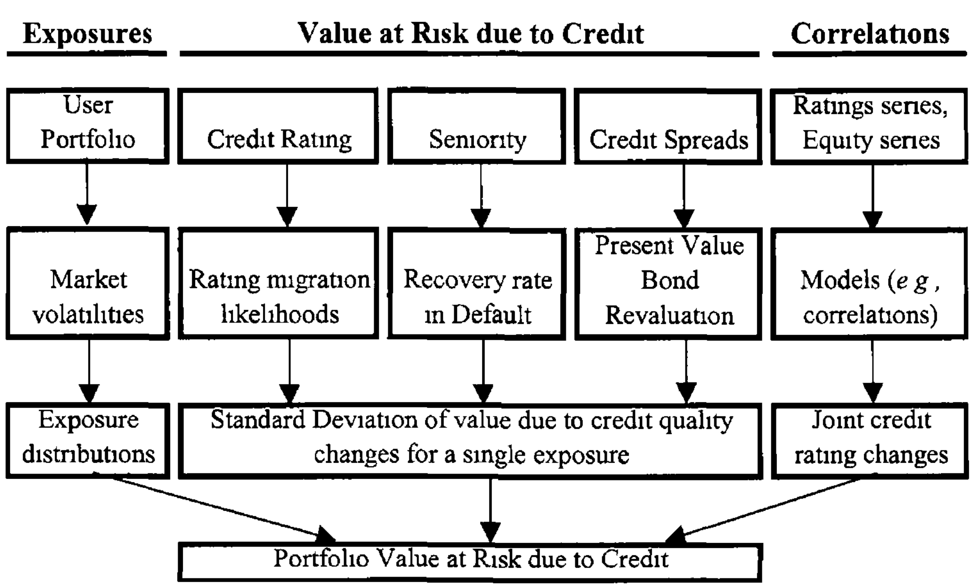

Figure 3 13 CreditMetrics Schematic

Figure 3 13 shows the two main building blocks, namely 'value at risk due to credit' for a single financial instrument and 'portfolio value at risk due to credit' which accounts for portfolio diversification effects 'Correlations' derive the asset return correlations that are used to generate the joint migration probabilities and 'Exposures' produce future exposures of derivatives

CreditMetrics, they state, looks to a horizon and constructs a distribution of historically estimated credit outcomes including rating migrations and, potentially, default Each credit quality migration is weighted by its likelihood using transition matrix analysis Each outcome has an estimate of change in value, given by either credit spreads or studies of recovery rates in default They then aggregate volatilities across the portfolio, applying estimates of correlation CreditMetrics accepts any discrete classification of credit quality and an associated credit migration probability matrix

Gupton *et al* (p6) suggest that the primary reason to have a quantitative portfolio approach to credit risk management is so that "we can more systematically address *concentration risk*" In particular, they suggest that intuitive – but arbitrary – exposure-based credit limits fail to recognise the relationship between risk and return They suggest that their approach allows a portfolio manager to state credit lines and limits in units of marginal portfolio volatility

The decision to take on ever higher exposure to an obligor will meet ever higher marginal risk as risk that grows geometrically with the concentration on that name, they claim They also note how their approach differs markedly from that mandated by the BIS "The BIS risk-based capital guidelines do not distinguish high quality and well-diversified portfolios from low quality and concentrated portfolios" (p6) However, they acknowledge the difficulty in estimating credit correlation because of a lack of data

They state their goal is to estimate portfolio risk due to credit events, the uncertainty in the forward value of the portfolio at the risk horizon caused by the possibility of obligor credit quality changes (p8)

The CreditMetrics Credit Modelling Approach

The starting point for the CreditMetrics model is the transition matrix The CreditMetrics approach is "based on credit migration analysis *i e* estimating the probability of moving from one credit quality to another, including default, within a given time horizon, which is often taken arbitrarily as one year" Crouhy *et al* (2000, p316) CreditMetrics models the full distribution of the values of any bond or loan portfolio, say one year forward, with the changes in values being related to credit migration only, while interest rates are assumed to evolve in a deterministic fashion Credit-VaR of a portfolio is then derived in a similar fashion as for market risk It is simply the percentile of the distribution corresponding to the desired confidence level

CreditMetrics assumes that future migration probabilities are equal to historical rating transition experience They further assume that the value of the facility at the modelling horizon may be obtained by discounting the cash flows using the borrower's forward zero curve, a "curve that is different for each rating category." (p10) Additionally, their framework accommodates variable loss rates assuming a beta distribution based on a mean loss rate and a standard deviation specified by the user based on the seniority of the debt issue.

To measure the effect of portfolio diversification, CreditMetrics need to estimate the correlations in credit quality changes for all pairs of obligors. But these correlations are not directly observable. They base their evaluation on the joint probability of asset returns, "which itself results from strong simplifying assumptions on the capital structure of the obligor, and on the generating process for equity returns." Crouhy *et al* (2000, p321)

CreditMetrics suggest a model of firm value that has the log of each firm's value at horizon described by a normal distribution. The distribution is divided into discrete areas such that the probability of being in a given area corresponds with the probabilities in the transition matrix. Furthermore, all firms may be modelled jointly as multivariate normal based.

Further discussion of the CreditMetrics approach is presented in the Appendix.

Critique of the CreditMetrics Credit Modelling Approach

CreditMetrics make many assumptions that may invalidate their results. They assume (i) that each obligor will migrate to a credit rating at the horizon date based on its senior, unsecured credit rating and the transition matrix of historical migrations, (ii) that all obligors in a given rating category will face the same forward zero curve at the horizon date – in other words, that interest rates will evolve to the forward rates in a deterministic fashion and that the forward credit spreads will be realised at the horizon, (iii) that credit rating distributions are multivariate normal and (iv) that asset correlation may be approximated by equity correlation.

Crouhy *et al* (2000) state that the CreditMetrics approach to measuring credit risk is rather appealing as a methodology, but that "unfortunately it has a major weakness: reliance on ratings transition probabilities that are based on average historical frequencies of default and credit migration." (p357) As a result, they suggest, the accuracy of CreditMetrics calculation depends upon two critical assumptions, namely, that all firms within the same rating class have the same default rate and the same spread curve even when recovery rates differ among obligors, and that the actual default rate is equal to the historical average default rate But this cannot be true because default rates evolve continuously whereas ratings are adjusted in a discrete fashion

3 8 1 2 KMV Portfolio Manager

KMV's approach to modelling debt portfolios is embedded in their software offering, *Portfolio Manager* This product is based on the same structural model underlying their default probability software, *Credit Edge*, and the default probability, their so-called expected default frequency (EDF), is a key input to the portfolio model Furthermore, the asset value time series imputed for each company forms the basis for creating asset value indices by country and industry, from which they extract their factors for the model underlying their correlation estimates

KMV are less transparent about their methodology and their data than CreditMetrics They make their EDF data and their factor sensitivity data available to clients They also explain in high-level terms the approach they adopt However, they avoid explaining their methodology in detail and most of the key steps in the simulation and revaluation elements of the program are not transparent

The CDO model that is developed in this thesis relies on KMV data for its default probabilities and its correlation data In building the model, therefore, the first task was to check that the KMV results could be reproduced to confirm that the data was being interpreted in the appropriate manner A complete description of the tasks undertaken in achieving this goal is given in Chapter 4 and will not, therefore, be repeated here

3 8 1 3 Comparison of the CreditMetrics and KMV Approaches

The most fundamental difference between the CreditMetrics and KMV approaches relates to their default probability estimation methodology Whereas KMV develop an estimate of expected default frequency for each borrower, CreditMetrics rely upon the average historical transition frequencies produced by the rating agencies for each credit class

Another key difference relates to the manner in which they measure correlation CreditMetrics relies on equity values and equity indices for various industry-country combinations in order to imply asset correlation KMV, on the other hand, create asset return series based on all the firms in their database and use principal component analysis to extract orthogonal factors which are then used to drive future asset values

Both approaches rely on the asset value model originally proposed by Merton (1974), but they differ quite substantially in the simplifying assumptions they require in order to facilitate its implementation "How damaging are, in practice, these compromises to a satisfactory capture of the complexity of credit measurement stays an open issue" Crouhy *et al*, (2000, p357)

3 8 1 4 Other Portfolio Models

Credit Suisse Financial Products released CreditRisk+ in 1997 It is a 'default only' model that assumes default for individual loans or bonds follows a Poisson process

McKinsey also has a portfolio model offering which is focused on default risk only It is a discrete time multi-period model, where default probabilities are a function of macro-variables like unemployment, the level of interest rates, and the growth rate in the economy, government expenses and foreign exchange rates, which drive credit cycles

3.9 Moody's Binomial Expansion Technique

Cifuentes and O'Connor (1996) outline Moody's Binomial Expansion Technique (BET) as applied to CDO analysis "Moody's ratings of CBOs and CLOs are ultimately based on the expected loss concept" (p1) They suggest that a number of methods can be used to estimate the expected loss, ranging from Monte Carlo simulation techniques to rather simple single-event models However, they propose an alternative to simulation or single-event models, the so-called BET They suggest it captures the effects of 'tail events', by accounting for all possible default scenarios (p1)

3 9 1 Overview of the BET

The BET is based on the *diversity score* concept "The idea is to use the diversity score to build a hypothetical pool of *uncorrelated* and *homogeneous* assets that will mimic the default behaviour of the original pool" (p2) If D is the diversity score of the collateral portfolio, they suggest the behaviour of the original pool can be modelled using a fictitious portfolio consisting of D bonds, each of which has the same par value (total collateral par value divided by D) They assume that these bonds have the same probability of default (determined by the weighted average probability of default of the original pool) They further assume that the behaviour of this homogeneous pool of D assets can be fully described in terms of D possible scenarios one default, two defaults and so on up to D defaults The probability P_j that scenario j (j defaults) could happen can be computed using the binomial formula where P_j represents the weighted average probability of default of the pool E_j is the term they assign to the loss for the note to be rated under scenario j They calculate the total expected loss, considering all possible loss scenarios, as

$$Expected\ Loss = \sum_{1}^{D} P_j E_j \qquad \textbf{Equation 3-87}$$

3 9 1 1 Weighted Average Rating Factor

Each rating is mapped to a rating factor and the credit quality of the asset pool is determined by the weighted-average rating factor, WARF This is calculated as the par value-weighted average The WARF score measures, in basis points, the Moody's idealised cumulative default rate over ten years The rating factor equivalents from that paper are reproduced in Figure 3 14

APPENDIX D

Table 8
Rating Factor Equivalents

Rating of Debt Security	Rating Factor
Aaa	1
Aa1	10
Aa2	20
Aa3	40
A1	70
A2	120
A3	180
Baa1	260
Baa2	360
Baa3	610
Ba1	940
Ba2	1,350
Ba3	1,780
B1	2,220
B2	2,720
B3	3,490
Caa	6,500
Ca	10,000
C	10,000

Figure 3 14 Moody's Rating Factor Equivalents

3 9 1 2 Diversity Score

The other key attribute of the asset pool is the level of diversification that exists within it To measure this, Moody's have developed the concept of diversity score They look at both the number of firms in the collateral pool and their distribution among industry groups

Diversity Score penalises the structure for having issuers in the same industry For example, Table 4 of their paper, reproduced below as Figure 3 15, shows that the first firm in a particular industry earns the transaction a Diversity Score of 1 The second name in the same industry increases the Diversity Score in that particular industry to 1 5 Subsequent additions from the same industry earn a still lesser addition to diversity score The transaction's total Diversity Score is computed by summing the Diversity Scores of all industries represented in the portfolio

Table 4

Number of Firms in Same Industry	Diversity Score
1	1.00
2	1.50
3	2.00
4	2.33
5	2.67
6	3.00
7	3.25
8	3.50
9	3.75
10	4.00
>10	Evaluated on a case- by-case basis

Figure 3 15 Moody's Diversity Score Measurement Methodology

Moody's 32-industry classification system used to measure intra-industry correlation is described by Backman and O'Connor (1995, p11) Appendix B of that paper is reproduced as Figure 3 16 Since the paper was published, an additional industry, "Broadcasting and Entertainment", has been added to the list

APPENDIX B

Table 6

Moody's Industry Classifications

1 **Aerospace and Defense:** Major Contractor Subsystems Research Aircraft Manufacturing Arms Ammunition
2 **Automobile** Automotive Equipment, Auto-Manufacturing Auto Parts Manufacturing, Personal-Use Trailers Motor Homes Dealers
3 **Banking** Bank Holding, Savings and Loans Consumer Credit, Small Loan Agency Factoring Receivables
4 **Beverage, Food and Tobacco:** Beer and Ale, Distillers, Wines and Liquors Distributors Soft Drink Syrup Bottlers Bakery Mill Sugar Canned Foods Corn Refiners Dairy Products Meat Products Poultry Products Snacks, Packaged Foods Candy Gum Seafood Frozen Food Cigarettes Cigars Leaf/Snuff Vegetable Oil
5 **Buildings and Real Estate** Brick, Cement, Climate Controls Contracting, Engineering Construction Hardware, Forest Products (building-related only), Plumbing Roofing, Wallboard Real Estate, Real Estate Development REITs, Land Development
6 **Chemicals, Plastics, and Rubber** Chemicals (nonagriculture), Industrial Gases Sulphur, Plastics Plastic Products, Abrasives Coatings Paints Varnish, Fabricating
7 **Containers, Packaging, and Glass.** Glass, Fiberglass Containers made of Glass Metal Paper Plastic, Wood or Fiberglass
8 **Personal and Nondurable Consumer Products (Manufacturing Only)** Soaps, Perfumes, Cosmetics, Toiletries Cleaning Supplies School Supplies
9 **Diversified/Conglomerate Manufacturing**
10 **Diversified/Conglomerate Service**
11 **Diversified Natural Resources, Precious Metals, and Minerals.** Fabricating, Distribution Mining and Sales
12 **Ecological:** Pollution Control, Waste Removal Waste Treament Waste Disposal
13 **Electronics** Computer Hardware, Electric Equipment Components Controllers, Motors Household Appliances Information Service Communication Systems, Radios TVs Tape Machines Speakers Printers Drivers, Technology
14 **Finance** Investment Brokerage, Leasing, Syndication Securities
15 **Farming and Agriculture** Livestock Grains Produce Agricultural Chemicals Agricultural Equipment Fertilizers
16 **Grocery** Grocery Stores Convenience Food Stores
17 **Healthcare, Education, and Childcare:** Ethical Drugs Proprietary Drugs Research, Health Care Centers Nursing Homes HMOs Hospitals Hospital Supplies Medical Equipment
18 **Home and Office Furnishings, Housewares, and Durable Consumer Products.** Carpets Floor Coverings Furniture, Cooking, Ranges
19 **Hotels, Motels, Inns, and Gaming**
20 **Insurance** Life, Property and Casualty Broker Agent Surety
21 **Leisure, Amusement, Motion Pictures, Entertainment.** Boating, Bowling, Billiards Musical Instruments, Fishing Photo Equipment, Records Tapes Sports Outdoor Equipment (Camping), Tourism Resorts, Games, Toy Manufacturing Motion Picture Production Theaters Motion Picture Distribution
22 **Machinery (Nonagriculture, Nonconstruction, Nonelectronic)** Industrial Machine Tools, Steam Generators
23 **Mining, Steel, Iron and Nonprecious Metals** Coal Copper, Lead Uranium Zinc Aluminum, Stainless Steel Integrated Steel, Ore Production Refractories, Steel Mill Machinery Mini Mills Fabricating, Distribution and Sales
24 **Oil and Gas** Crude Producer Retailer Well Supply Service and Drilling
25 **Personal, Food, and Miscellaneous Services**
26 **Printing, Publishing, and Broadcasting** Graphic Arts, Paper Paper Products Business Forms Magazines Books, Periodicals Newspapers Textbooks Radio, TV Cable Broadcasting Equipment
27 **Cargo Transport** Rail, Shipping Railroads Railcar Builders, Ship Builders Containers Container Builders, Parts Overnight Mail, Trucking Truck Manufacturing Trailer Manufacturing Air Cargo, Transport
28 **Retail Stores** Apparel Toy, Variety Drug, Department Mail Order Catalog Showroom
29 **Telecommunications.** Local, Long Distance Independent, Telephone Telegraph Satellite Equipment Research Cellular
30 **Textiles and Leather** Producer, Synthetic Fiber Apparel Manufacturer Leather Shoes
31 **Personal Transportation:** Air Bus Rail Car Rental
32 **Utilities** Electric, Water Hydro Power Gas Diversified

Figure 3 16 Moody's Industry Classifications

3 9 2 Applying the BET

The BET attempts to replicate the behaviour of an actual portfolio by modelling an idealised portfolio of assets These idealised assets are all assumed to have the same default probability based on their weighted-average rating factor The diversity score of the asset pool determines the number of idealised assets

3 9 2 1 The BET Approach to Ratıng Senıor Notes

As dıscussed prevıously, the BET ratıng approach was first outlıned ın Cıfuentes and O'Connor (1996) The portfolıo they use to demonstrate the approach ıs shown ın Fıgure 3 17

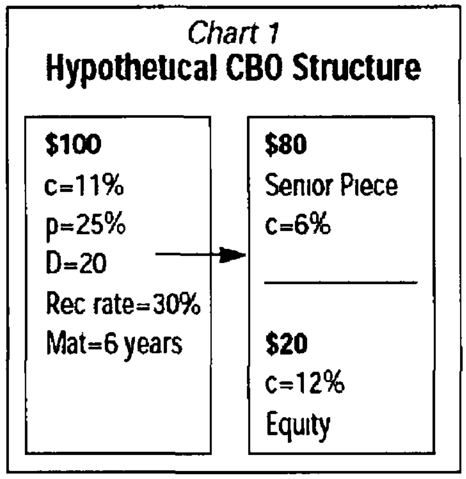

Fıgure 3 17 Moody's Hypothetıcal CBO Structure

They consıder the sımple two-tıer structure depıcted ın Fıgure 3 17 They assume that the collateral pool has a dıversıty score of 20, an average probabılıty of default of 25% (after factorıng ın the stressıng factor), a recovery rate of 30%, a sıx-year tıme to maturıty, and pays an average coupon of 11% They also assume that all bonds have bullet repayments, that there are no overcollateralısatıon or ınterest rate trıggers, and that the excess cash ıs reınvested at 11% per year

The author buılt a model to confirm Moody's results An example of the output ıs shown ın Table 3-6 below when the number of defaults ıs set at 10 In these cırcumstances, the senıor note receıves {2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 2 4, 78 86} The final payment ıs 78 86 ınstead of 82 4 yıeldıng a loss of 3 1%

Collateral	100													
Coupon	11%													
p	25%													
D	20													
Number of Defaults	10													
Recovery	30%													
Maturity	6 yrs													
Senior	80													
Senior coupon	6%													
Equity	20													
Equity coupon	12%													
Year	0	0 5	1	1 5	2	2 5	3	3 5	4	4 5	5	5 5	6	
Percentage of Defaults			50%		10%		10%		10%		10%		10%	100%
Default Distribution		0	5	0	1	0	1	0	1	0	1	0	1	10
Cumulative Defaults		0	5	5	6	6	7	7	8	8	9	9	10	
Defaulted Collateral		0	25	0	5	0	5	0	5	0	5	0	5	50
Cum Def Collateral		0	25	25	30	30	35	35	40	40	45	45	50	
Recoveries		0 0	7 5	0 0	1 5	0 0	1 5	0 0	1 5	0 0	1 5	0 0	1 5	15
Collateral Outstanding	100	100 0	82 5	82 5	79 0	79 0	75 5	75 5	72 0	72 0	68 5	68 5	65 0	
Collateral Coupon		5 5	4 5	4 5	4 3	4 3	4 2	4 2	4 0	4 0	3 8	3 8	3 6	
Surplus Account		0 0	2 0	3 1	4 3	5 3	6 4	7 3	8 3	9 1	10 0	10 7	11 5	
Cash Available		5 5	6 5	7 6	8 6	9 6	10 5	11 4	12 2	13 1	13 8	14 5	15 1	Loss
Payment to Senior		2 4	2 4	2 4	2 4	2 4	2 4	2 4	2 4	2 4	2 4	2 4	78 86	3 10%
Interest to Equity		1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	
Principal to Equity		0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	
Surplus in year		1 9	2 9	4 0	5 0	6 0	6 9	7 8	8 6	9 5	10 2	10 9	0 0	

Table 3-6 Moody's Cash Flow Model

Repeating the analysis for varying numbers of defaults and collating confirms the output in Table 1 of their paper Moody's assume that the portfolio comprises a number of assets equal to the diversity score that default independently Hence the distribution of losses is given by the binomial probability distribution The results are reproduced as Table 3-7 below

Probability	# Defaults	Loss %
0.3171%	0	0.0000%
2.1141%	1	0.0000%
6.6948%	2	0.0000%
13.3896%	3	0.0000%
18.9685%	4	0.0000%
20.2331%	5	0.0000%
16.8609%	6	0.0000%
11.2406%	7	0.0000%
6.0887%	8	0.0000%
2.7061%	9	0.0000%
0.9922%	10	3.1026%
0.3007%	11	7.8958%
0.0752%	12	12.6890%
0.0154%	13	17.4822%
0.0026%	14	22.2754%
0.0003%	15	27.0686%
0.0000%	16	31.5621%
0.0000%	17	34.7819%
0.0000%	18	38.0531%
0.0000%	19	41.6080%
0.0000%	20	45.1629%

Table 3-7 Scenario Default Probability and Senior Tranche Loss Percentage

Repeating the analysis for varying diversity scores confirms the results Cifuentes and O'Connor have graphed in Chart 2 of their paper. Chart 2 is re-created here as Figure 3.18:

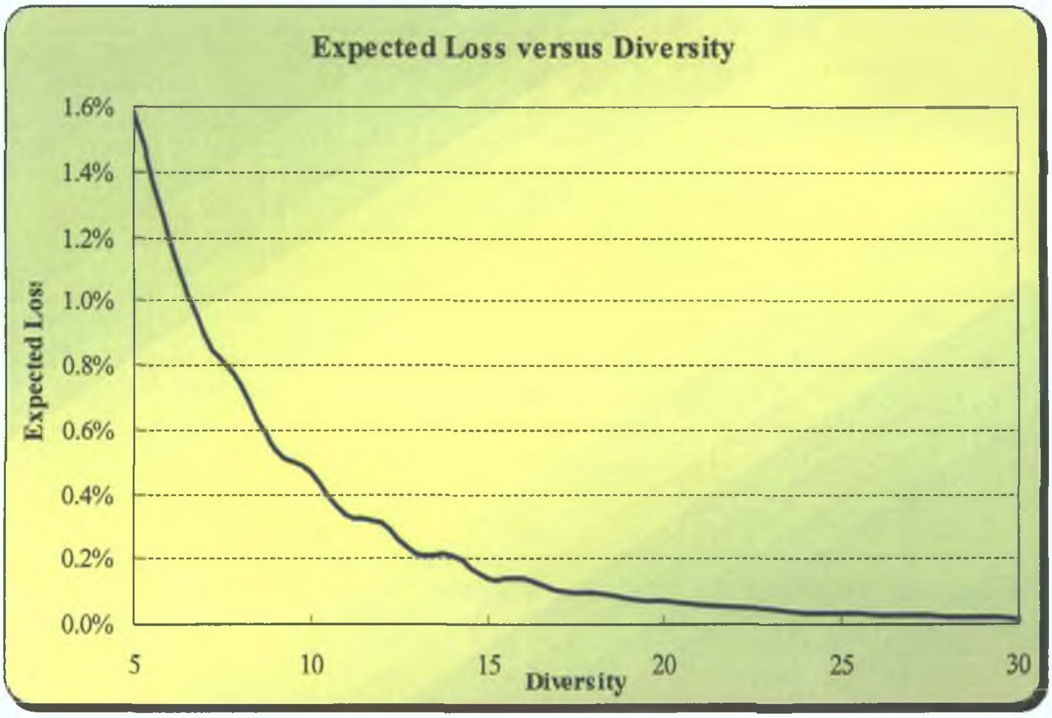

Figure 3.18 Expected Loss for Senior Tranche as a Function of Diversity

Table 3-8 Moody's Idealised Expected Loss Rates by Rating and Debt Maturity

Table 2

Moody's "Idealized" Cumulative Expected Loss Rates (%)

	Year									
Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0 000028	0 00011	0 00039	0 00099	0 00160	0 00220	0 00286	0 00363	0 00451	0 00550
Aa1	0 000314	0 00165	0 00550	0 01155	0 01705	0 02310	0 02970	0 03685	0 04510	0 05500
Aa2	0 000748	0 00440	0 01430	0 02585	0 03740	0 04895	0 06105	0 07425	0 09020	0 11000
Aa3	0 001661	0 01045	0 03245	0 05555	0 07810	0 10065	0 12485	0 14960	0 17985	0 22000
A1	0 003196	0 02035	0 06435	0 10395	0 14355	0 18150	0 22330	0 26400	0 31515	0 38500
A2	0 005979	0 03850	0 12210	0 18975	0 25685	0 32065	0 39050	0 45595	0 54010	0 66000
A3	0 021368	0 08250	0 19800	0 29700	0 40150	0 50050	0 61050	0 71500	0 83600	0 99000
Baa1	0 049500	0 15400	0 30800	0 45650	0 60500	0 75350	0 91850	1 08350	1 24850	1 43000
Baa2	0 093500	0 25850	0 45650	0 66000	0 86900	1 08350	1 32550	1 56750	1 78200	1 98000
Baa3	0 231000	0 57750	0 94050	1 30900	1 67750	2 03500	2 38150	2 73350	3 06350	3 35500
Ba1	0 478500	1 11100	1 72150	2 31000	2 90400	3 43750	3 88300	4 33950	4 77950	5 17000
Ba2	0 858000	1 90850	2 84900	3 74000	4 62550	5 37350	5 88500	6 41300	6 95750	7 42500
Ba3	1 545500	3 03050	4 32850	5 38450	6 52300	7 41950	8 04100	8 64050	9 19050	9 71300
B1	2 574000	4 60900	6 36900	7 61750	8 86600	9 83950	10 52150	11 12650	11 68200	12 21000
B2	3 938000	6 41850	8 55250	9 97150	11 39050	12 45750	13 20550	13 83250	14 42100	14 96000
B3	6 391000	9 13550	11 56650	13 22200	14 87750	16 06000	17 05000	17 91900	18 57900	19 19500
Caa	14 300000	17 87500	21 45000	24 13400	26 81250	28 60000	30 38750	32 17500	33 96250	35 75000

The expected loss suffered by the senior note is 0.067%. Referring to Moody's Idealised Cumulative Expected Loss Rates in Table 2 of that paper, reproduced aboveas Table 3-8, the senior note would be rated Aa3 (the cut-off value is 0.10065% for the Aa3).

3.9.2.2 The BET Approach to Rating Senior/Subordinate Structures

Anderson (1997) describes how diversification of a securitised pool affects Moody's ratings of senior/subordinated structures. The author built a model to replicate the structure he described. The model results are shown in Table 3-9. These concur exactly with the results he reports in this paper.

					Pool	Senior	Mezz.	Junior
Diversity	30	Pool	100					
PD	10%	Senior	60	Probability of Loss	95.8%	0.0%	17.5%	95.8%
LGD	70%	Mezzanine	30	Expected Loss	7.00%	0.00%	1.83%	64.52%
		Junior	10	Loss Given Default	7.31%	3.62%	10.40%	67.38%

# of Defaults	Probability	Cash Flow from Pool	Cash Flow to Senior	Cash Flow to Mezz.	Cash Flow to Junior	Pool Loss %	Senior Loss %	Mezz. Loss %	Junior Loss %
0	4.2%	100.00	60.00	30.00	10.00	0.0%	0.0%	0.0%	0.0%
1	14.1%	97.67	60.00	30.00	7.67	2.3%	0.0%	0.0%	23.3%
2	22.8%	95.33	60.00	30.00	5.33	4.7%	0.0%	0.0%	46.7%
3	23.6%	93.00	60.00	30.00	3.00	7.0%	0.0%	0.0%	70.0%
4	17.7%	90.67	60.00	30.00	0.67	9.3%	0.0%	0.0%	93.3%
5	10.2%	88.33	60.00	28.33	0.00	11.7%	0.0%	5.6%	100.0%
6	4.7%	86.00	60.00	26.00	0.00	14.0%	0.0%	13.3%	100.0%
7	1.8%	83.67	60.00	23.67	0.00	16.3%	0.0%	21.1%	100.0%
8	0.6%	81.33	60.00	21.33	0.00	18.7%	0.0%	28.9%	100.0%
9	0.2%	79.00	60.00	19.00	0.00	21.0%	0.0%	36.7%	100.0%
10	0.0%	76.67	60.00	16.67	0.00	23.3%	0.0%	44.4%	100.0%
11	0.0%	74.33	60.00	14.33	0.00	25.7%	0.0%	52.2%	100.0%
12	0.0%	72.00	60.00	12.00	0.00	28.0%	0.0%	60.0%	100.0%
13	0.0%	69.67	60.00	9.67	0.00	30.3%	0.0%	67.8%	100.0%
14	0.0%	67.33	60.00	7.33	0.00	32.7%	0.0%	75.6%	100.0%
15	0.0%	65.00	60.00	5.00	0.00	35.0%	0.0%	83.3%	100.0%
16	0.0%	62.67	60.00	2.67	0.00	37.3%	0.0%	91.1%	100.0%
17	0.0%	60.33	60.00	0.33	0.00	39.7%	0.0%	98.9%	100.0%
18	0.0%	58.00	58.00	0.00	0.00	42.0%	3.3%	100.0%	100.0%
19	0.0%	55.67	55.67	0.00	0.00	44.3%	7.2%	100.0%	100.0%
20	0.0%	53.33	53.33	0.00	0.00	46.7%	11.1%	100.0%	100.0%
21	0.0%	51.00	51.00	0.00	0.00	49.0%	15.0%	100.0%	100.0%
22	0.0%	48.67	48.67	0.00	0.00	51.3%	18.9%	100.0%	100.0%
23	0.0%	46.33	46.33	0.00	0.00	53.7%	22.8%	100.0%	100.0%
24	0.0%	44.00	44.00	0.00	0.00	56.0%	26.7%	100.0%	100.0%
25	0.0%	41.67	41.67	0.00	0.00	58.3%	30.6%	100.0%	100.0%
26	0.0%	39.33	39.33	0.00	0.00	60.7%	34.4%	100.0%	100.0%
27	0.0%	37.00	37.00	0.00	0.00	63.0%	38.3%	100.0%	100.0%
28	0.0%	34.67	34.67	0.00	0.00	65.3%	42.2%	100.0%	100.0%
29	0.0%	32.33	32.33	0.00	0.00	67.7%	46.1%	100.0%	100.0%
30	0.0%	30.00	30.00	0.00	0.00	70.0%	50.0%	100.0%	100.0%

Table 3-9 Probability of Loss, LGD and EL for Pool and CDO Tranches

The effectiveness of diversification and subordination in protecting the senior notes at the expense of the junior notes is observed by varying the diversity score. Anderson's results, as summarised in Table 1 of his paper, agree exactly with Table 3-10 below. The impact of diversity is best appreciated by graphing these results. Figure 3.19 below plots the model results. Again, these agree exactly with the results depicted in Chart 1 of Anderson's paper: it shows how the expected loss of the pool is increasingly concentrated in the subordinated tranches as diversity increases. Similarly, Figure 3.20 shows how the probability of the senior tranche suffering a loss declines to zero as diversity increases while the probability of the junior notes incurring a loss goes to 100%. Finally, Figure 3.21 plots the model results which agree exactly with Anderson's results shown in Chart 3 of the paper: it shows, as might be expected, that the probability distribution of losses narrows as diversity increases.

	Expected Loss				Probability of Loss			
Diversity	Pool	Senior	Mezz.	Junior	Pool	Senior	Mezz.	Junior
1	7.000%	5.000%	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%
2	7.000%	0.500%	16.000%	19.000%	19.000%	1.000%	19.000%	19.000%
3	7.000%	0.350%	13.600%	27.100%	27.100%	2.800%	27.100%	27.100%
5	7.000%	0.039%	9.604%	40.951%	40.951%	0.856%	40.951%	40.951%
10	7.000%	0.001%	5.496%	53.510%	65.132%	0.015%	26.390%	65.132%
20	7.000%	0.000%	2.758%	61.726%	87.842%	0.000%	32.307%	87.842%
30	7.000%	0.000%	1.826%	64.523%	95.761%	0.000%	17.549%	95.761%
50	7.000%	0.000%	0.938%	67.185%	99.485%	0.000%	12.215%	99.485%
100	7.000%	0.000%	0.304%	69.089%	99.997%	0.000%	7.257%	99.997%

Table 3-10 Expected Loss and Probability of Loss as a Function of Diversity

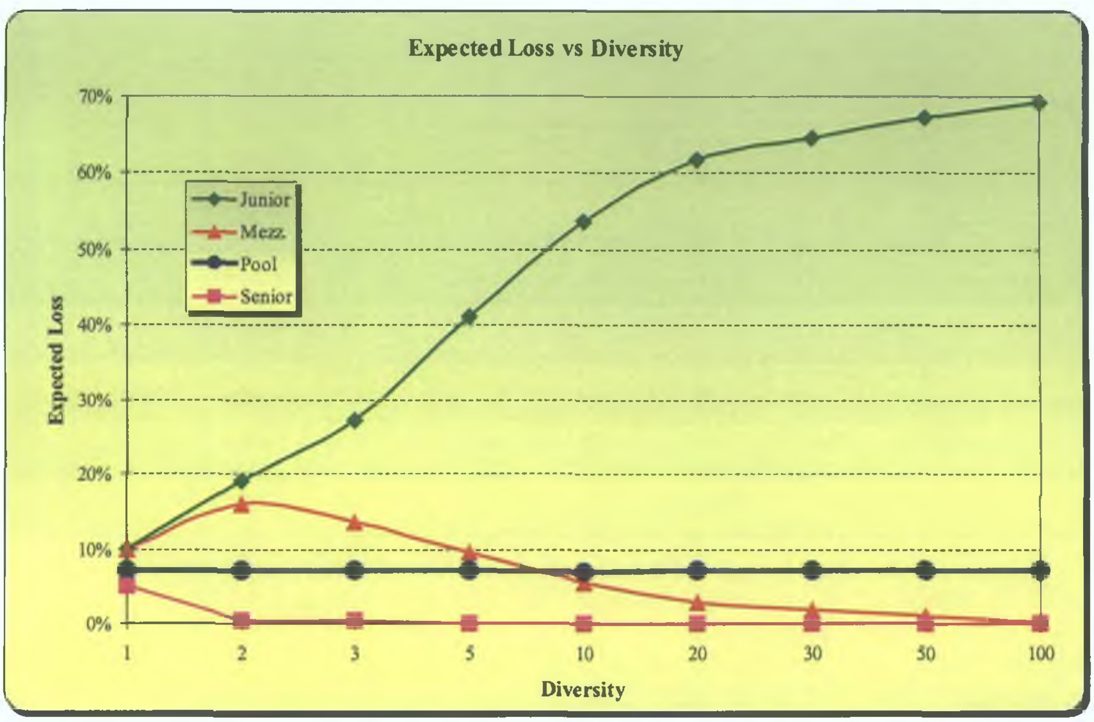

Figure 3.19 Expected Loss vs Diversity for Pool and CDO Tranches

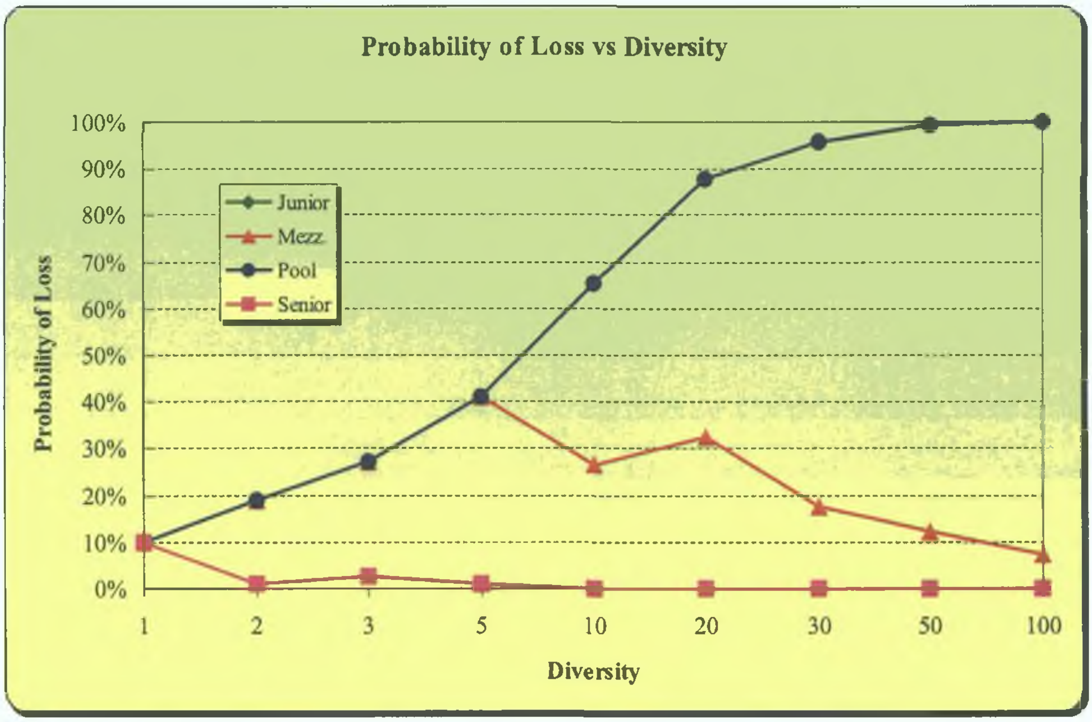

Figure 3.20 Probability of Loss vs Diversity for Pool and CDO Tranches

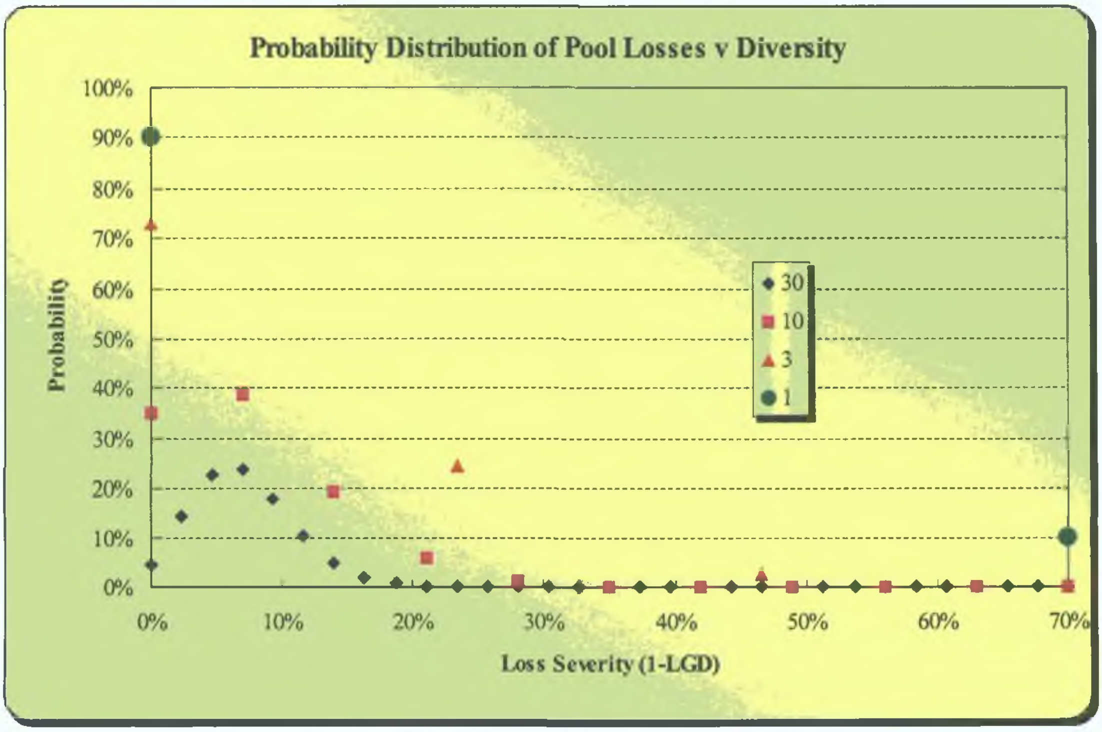

Figure 3.21 Probability Distribution of Pool Losses as a Function of Diversity

3.9.3 Critique of the Moody's Approach

The Moody's approach is extremely heuristic and some would suggest it doesn't merit citing as a portfolio model. The simplistic manner in which the diversity score is calculated is scorned. Likewise, the distillation of a range of credit ratings into a weighted average has been criticised. This latter criticism has prompted them to develop a Double Binomial Method to accommodate portfolios where two groups of assets have distinctly different default probabilities.

Among other criticisms, the role played by the diversity score is often questioned. Greater diversification will always lead to a higher quality senior tranche or will enable the sponsor to achieve a higher percentage of AAA-rated debt. However, in practice, greater diversity has been blamed for some of the problems that have been encountered in the CDO market.

Typical of these comments is that of Goodman and Fabozzi (2001, p34). They claim that a very high diversity score can limit flexibility by requiring an asset manager with broad expertise to invest in an industry he does not like. They suggest that too much diversification is even more of a problem for a smaller asset manager, where the portfolio may have selective strengths in fewer industries. They conclude: "Investors should certainly be wary of deals in which very high diversity scores are achieved by managers straying from their fields of expertise." (p34)

3.10 Modelling Credit Portfolios over Multiple Time Periods

The KMV portfolio model developed in Chapter 4, in common with other major models in the market – CreditMetrics, CreditRisk+ and CreditPortfolioView – are single time period models. The models describe for a specific risk horizon, whether each asset of interest defaults within the horizon. The timing of defaults within the risk horizon is not considered, nor is the possibility of defaults beyond the horizon. "This is not a flaw of the current models, but rather an indication of their genesis as approaches to risk management and capital allocation for a fixed portfolio." Finger (2000, p49)

However, this framework is incapable of dealing with the modelling of CDOs. Finger comments that the performance of a CDO structure depends on the default behaviour of a pool of assets. He notes that the dependence of is not just on whether the assets default over the life of the structure, but also on when the defaults occur. Thus, he concludes that while an "application of the existing models can give a cursory view of the structure, a more rigorous analysis requires a model of the timing of defaults." (p49)

In the paper, he compares the performance of four models – a discrete CreditMetrics extension, a diffusion-driven CreditMetrics extension, a copula approach and the stochastic default intensity approach of Duffie and Gârleanu (2001) Each model is calibrated to the same one-year default rate and single period correlation parameter He modelled a portfolio of 100 obligors under low and high correlation assumptions on the assumption of 40% recovery He then tranched the liabilities into a 10% first loss, a 20% second loss and a senior piece

The discrepancies between the models were not too large for the first loss tranche – the cost of first loss protection was approximately 20% higher for the most expensive compared to the least expensive Likewise, under the high correlation assumption, the variation in the cost of second loss protection was of a similar magnitude However, under the low correlation assumption, the most expensive second loss protection was almost twice that of the least expensive and, for the senior tranche, the subordination necessary to achieve a target rating varied by a factor greater than two At the 30% subordination level, the senior notes were rated Aaa in the most benign model compared to A3 in the most severe

He notes that in the single period case, a number of studies have concluded that the various models do not produce vastly different conclusions when calibrated to the same first and second order information However, in the case of CDOs, "the issue of model choice is much more important, and any analysis of structures over multiple horizons should heed this potential model error " (p64)

However, the actual problem is even greater than Finger suggests, the disparities he has identified remain even after the models have been calibrated to the same input data In fact, the input data is, in most cases, unavailable "Currently the weakest link in the chain of CDO analysis is the limited availability of empirical data bearing on the correlation of default risk " Duffie and Singleton (2003, p252)

3.11 Choosing a Modelling Paradigm

The two market-leading portfolio models – those of CreditMetrics and KMV - have been examined in this chapter Despite the fact that they are both structural models, they adopt very different approaches to the solution of the credit portfolio modelling problem

CreditMetrics relies on credit ratings and rating transition matrices Its most obvious application is to portfolios of publicly rated names for which there is a substantial volume of data on which to build transition matrices While it is possible for any financial institution to build its own transition matrices based on its own internal rating system, few would have a sufficient number of names in the various ratings to enable the extraction of reliable transition probabilities

In contrast, the approach adopted by KMV enables them generate probabilities of default for all publicly quoted firms – some 35,000 at the time of writing - and update these estimates on a daily basis The agency-rated universe, by comparison, is substantially less, numbering less than 5,000 This is a particular problem in Europe where most debt is privately issued and only the largest companies aiming to sell their debt worldwide seek a rating from (predominantly U S) rating agencies Adopting a framework which naturally provides the key input, namely, default probability, greatly expands the universe of firms which are amenable to analysis Many of the so-called 'arbitrage' CDOs purchase the debt of firms which would not normally seek an agency rating because of their smaller size and their lesser creditworthiness, having a ready measure of default probability is critical in these circumstances

The rating agencies' own research has identified the issue of autocorrelation in downgrades CreditMetrics, if it were expanded to a multi-period framework, would ignore this autocorrelation in its Markov modelling framework that assumes grading transition is a Markov process This would systematically under-estimate the tendency for serial downgrades, an issue that is critical to the performance of CDO tranches In contrast, KMV offer a term structure of default probability for each borrower that can inform the evolution of borrower creditworthiness over successive time periods

The transition matrix most frequently used is an average of transitions over a long time period However, the realised transition and default probabilities vary quite substantially over the years depending on whether the economy is in recession or expending "When implementing a model that relies on transition probabilities, one may have to adjust the average historical values to be consistent with one's assessment of the current economic environment " Crouhy *et al* (2001, p325) There is little published in the literature to help inform the user how to condition the transition matrix on the state of the economy Not having a mechanism for conditioning the transition matrix on the stage in the economic cycle is a critical issue militating against using CreditMetrics m a multi-period setting

The grading systems developed by rating agencies and by most financial institutions are long-term ratings That is, they take a 'through-the-cycle' view of credit compared to the KMV view that is characterised as 'point-in-time'. "A strong assumption made by CreditMetrics is that all issuers are credit-homogeneous within the same rating class, with the same transition probabilities and the same default probabilities." Crouhy *et al* (2000, p27) In KMV's approach, in contrast, each issuer is specific and is characterised by its own returns distribution, its own capital structure, and, consequently, its own default probability term structure.

While some agency ratings apply to the issuer, most agency ratings attach to the debt issue. "Bond-rating systems are supposed to rate an individual loan (including its covenants and collateral backing), whereas loan-rating systems are more oriented to rating the overall borrower." Saunders *et al.* (2002, p18) In Moody's words, a rating is, "an opinion on the future ability and legal obligation of an issuer to make timely payments of principal and interest on a specific fixed-income security." Moody's (1998, p4). The following year, they elaborate as follows: "Moody's ratings of industrial and financial companies have primarily reflected default probability, while expected severity of loss in the event of default has played an important secondary role. In the speculative-grade portion of the market, which has been developing into a distinct sector, Moody's ratings place more emphasis on expected loss than on relative default risk."

For these reasons, it was decided to choose the KMV modelling paradigm instead of that adopted by CreditMetrics. Throughout what follows, the default probability estimates used will be KMV's expected default frequencies. Likewise, KMV's asset correlation estimates will be embedded in the portfolio model through the use of the factor sensitivities in their factor model framework. Furthermore, it was decided to adopt Moody's BET approach to CDO tranche rating in preference to those of other rating agencies because of its more widespread usage and acceptance.

3.12 Conclusion

This chapter presented an overview of the principal research strands that are relevant to an analysis of a portfolio of debt securities.

The most fundamental issue in debt valuation is an assessment of the PD of the individual obligors so the chapter began by assessing various approaches to the measurement of default probability proposed in the literature.

The next section undertakes an examination of the literature concerning the interaction of equity securities in a portfolio While portfolios of equities behave in a fundamentally different way from portfolios of debt securities, it is in the area of equity portfolio modelling that the research is most advanced and it is to these approaches those who are faced with modelling debt securities have turned in the first instance

The unique challenges that confront researchers in the area of debt portfolio modelling are explored in the next section Few closed form solutions are available because of the particularly complex nature of the interaction among the securities One formulaic solution to the portfolio value distribution problem for a portfolio of infinite granularity allows us to examine the main attributes of debt portfolios and gives guidance on the challenges facing those who need to model actual portfolios in a more realistic manner

The approach to valuing credit default swaps was then introduced The valuation method of choice for structured products, the copula approach, was also summarised Structured products referencing credits for which a liquid CDS exist are successfully accommodated within this framework

The credit portfolio models which are most used in the market – those of CreditMetrics and KMV - are examined next These models were contrasted and critically assessed The KMV model proved to be particularly suited to the modelling of exposures to sub-investment grade names which are not traded in the market Also, the approach adopted by Moody's to rate CDO tranches, BET, was summarised and critiqued

There is an obvious disconnect between (i) the rating agency approach to CDO tranche grading which takes explicit account of the CDO indenture and the multi-period dimension to the tranche-rating issue but largely ignores the work of academic researchers, and (ii) the more theoretically rigorous academic approaches which have been adopted by the software vendors but which are fundamentally single period models and ignore the CDO indenture which is central to tranche rating and valuation

The challenge that is being taken up in this thesis is to apply the intellectual rigour of the academic approach embedded in one vendor offering to a multi-period framework which takes explicit account of the CDO indenture In undertaking that challenge, the decision was made to adopt the KMV approach in preference to the competing market offerings because of its more robust theoretical foundations and its greater data coverage

As a first step to achieving this goal, the current state-of-the-art – as represented by KMV's credit portfolio modelling approach – will be re-created next in Chapter 4 On completion of the single period model, we will then be ready to take on the challenge of developing a multi-period version This model will incorporate the best features of the single period credit portfolio model and the rating agency model It will model the asset migration over time in an academically rigorous risk-neutral pricing framework and incorporate all the features specified in the CDO indenture in channelling the cash flows to the liabilities The development of this model is the subject of Chapter 5

Chapter 4. Developing a Single Period Credit Portfolio Model

4.1 Chapter Overview

In Chapter 3, the market-leading models were overviewed and the fundamentally different approaches taken by the rating agencies and the portfolio model vendors were noted The differences between the two approaches are such that it is not possible to make meaningful comparison between them

The portfolio models are framed on a single period and are incapable of analysing CDO tranches whose values are fundamentally dependent on the performance of the portfolio over multiple time periods It is only by modelling the underlying debt portfolio over the life of the transaction that the cash flows available to the tranches can be calculated and hence valued Single period models can give the value of the debt portfolio at the period end but are incapable of saying how that value is divided among the different tranches

The rating agency model takes full account of the CDO indenture and explicitly models the cash flow waterfall However, it adopts a heuristic approach to the modelling of correlation and default probability Ignoring all the modelling advances of the past decade is clearly undesirable

The goal of this thesis is to develop a new model that incorporates the best features of the rating agency approach into a multi-period version of the market-leading credit portfolio model A further goal is to compare the rating implied by this new model - which is potentially more rigorous than the current rating agency model – with that assigned by the rating agency model to the various different CDO tranches

In order to achieve these goals, three tasks must be undertaken

1 The KMV approach to modelling credit portfolios in a single period time frame is re-created from public data sources

2 This single period model is expanded into a multi-period setting in order to be capable of valuing CDOs.

3 Moody's BET is replicated, once again by reference to published articles

The completion of the first of these tasks is the subject of this chapter

4.2 *The KMV Modelling Paradigm*

Traditionally, credit risk in a portfolio context was managed qualitatively with stringent underwriting standards, limit enforcement and counterparty monitoring However, such an approach fails because of its inability to measure the correlated nature of credit defaults Furthermore, it fails to inform a bank's efforts to build portfolios with superior return-to-risk characteristics

KMV attempts to replace this qualitative approach with a quantitative one Rather than measure portfolio risk as an exposure-based amount, they focus instead on the amount of economic capital needed to maintain a particular level of risk in the debt issued by the institution holding the portfolio Portfolio performance is based on a comparison between the portfolio's promised return and the capital required to support the portfolio Individual facilities are allocated capital on the basis of their contribution to portfolio variance and individual facility performance can then be measured based on the ratio of the facility's return to this capital

KMV's portfolio management software product, *Portfolio Manager*, is designed to

- Produce a mark-to-model price for credit-risky exposures,
- Characterise the return and risk of exposures in the context of a credit portfolio, and the return and risk of the portfolio as a whole,
- Compute the distribution of portfolio values at a specified horizon date and use this distribution to calculate required economic capital today,
- Determine optimal transactions – buy or sell – for a given set of trading or origination opportunities, and
- Calculate optimal portfolios by rearranging the weights of existing holdings

Whereas performance was traditionally measured in terms of earnings per share or return on equity, the focus within the KMV modelling paradigm is on return on risk-adjusted capital (RORAC) whereby the portfolio return is measured against the capital required to support the portfolio and return per unit of portfolio unexpected loss, the portfolio Sharpe ratio

The key building blocks of the KMV approach are a methodology for measuring default probability and default correlation

4 2 1 1 Measurıng Default Probabılıty

The methodology employed by KMV to calculate the term structure of default probabılıtıes has been descrıbed ın detaıl ın 3 7 1 above The value of the firm's assets ıs ımputed from the value of the firm's equıty, the volatılıty of equıty and the amount of the firm's debt

4 2 1 2 Measurıng Correlatıon

KMV apply the Markowıtz varıance-covarıance approach to determıne portfolıo value varıance They have constructed a factor model to explaın the correlatıon ın the underlyıng asset values of the oblıgors The asset returns are assumed to be multıvarıate normally dıstrıbuted By ısolatıng the systematıc varıatıon ın asset returns, they create a framework wıthın whıch future asset values, and, hence, future credıt exposure values can be modelled

4.3 Re-creating KMV's Portfolio Modelling Approach

Before buıldıng a multı-perıod model to value a CDO usıng KMV EDF and correlatıon data, ıt ıs necessary to buıld a sıngle tıme perıod model usıng these data Developıng a model whıch re-creates the results whıch KMV produce ın theır current portfolıo modellıng product, *Portfolıo Manager*, wıll serve the further purpose of confirmıng the theoretıcal valıdıty of the KMV modellıng paradıgm Such ındependent valıdatıon has not prevıously been avaılable and KMV, presumably for commercıal reasons, have been unwıllıng to dısclose theır methodology

Whıle KMV have not publıshed theır methodology ın detaıl, they have outlıned theır modellıng approach ın varıous artıcles ın academıc and practıtıoner journals See for example, Kealhofer (2003, 2003a), Bohn (2000) and Crosbıe (2002) Furthermore, theır approach has been analysed by varıous commentators Usıng these outlınes and commentarıes and an understandıng of the sımulatıon process, the *Portfolıo Manager* modellıng approach ıs successfully re-created – for the first tıme, to the author's knowledge - as confirmed by the model outputs that replıcate almost exactly those from *Portfolıo Manager*

The task of replıcatıng comprısed the followıng stages

- Each asset ın the portfolıo ıs valued at the outset usıng a rısk-neutral valuatıon approach

- The asset correlation structure is derived using the KMV factor model
- The asset migration process is modelled
- The portfolio value at the horizon date is simulated repeatedly
- The portfolio distribution is derived, and the portfolio parameters – standard deviation, called unexpected loss by KMV, and capital – are calculated
- The capital and unexpected loss are allocated to the component assets and individual asset performance – both Sharpe Ratio and RORAC – are calculated

Each of these stages will now be described in detail A schematic outlining the various tasks undertaken is shown in Figure 4 1 below

4 3 1 Asset Valuation at the Outset

The method that KMV employ for asset valuation was outlined in Bohn (2000) Technical details regarding the manner in which they undertake interpolation of interest rates and cumulative default probabilities are further described in KMV's *Portfolio Manager* product manual, *Modelling Portfolio Risk* (2003) These latter details are of no theoretical importance but it was necessary to account for them correctly if results matching KMV's were to be obtained

As Bohn describes, the KMV so-called risk-comparable valuation method begins by calculating the risk-adjusted PD by adjusting the actual PD to account for the market price of risk in the standard risk-neutral approach outlined m Hull (2003, p203)

4 3 1 1 Calculating Cash Flows

The expected cash flows for an exposure are derived from the reference rate term structure It is assumed that the future spot rate will equal the forward rate Hence, all the expected cash flows are assumed known at the outset There is no modelling of interest rate volatility

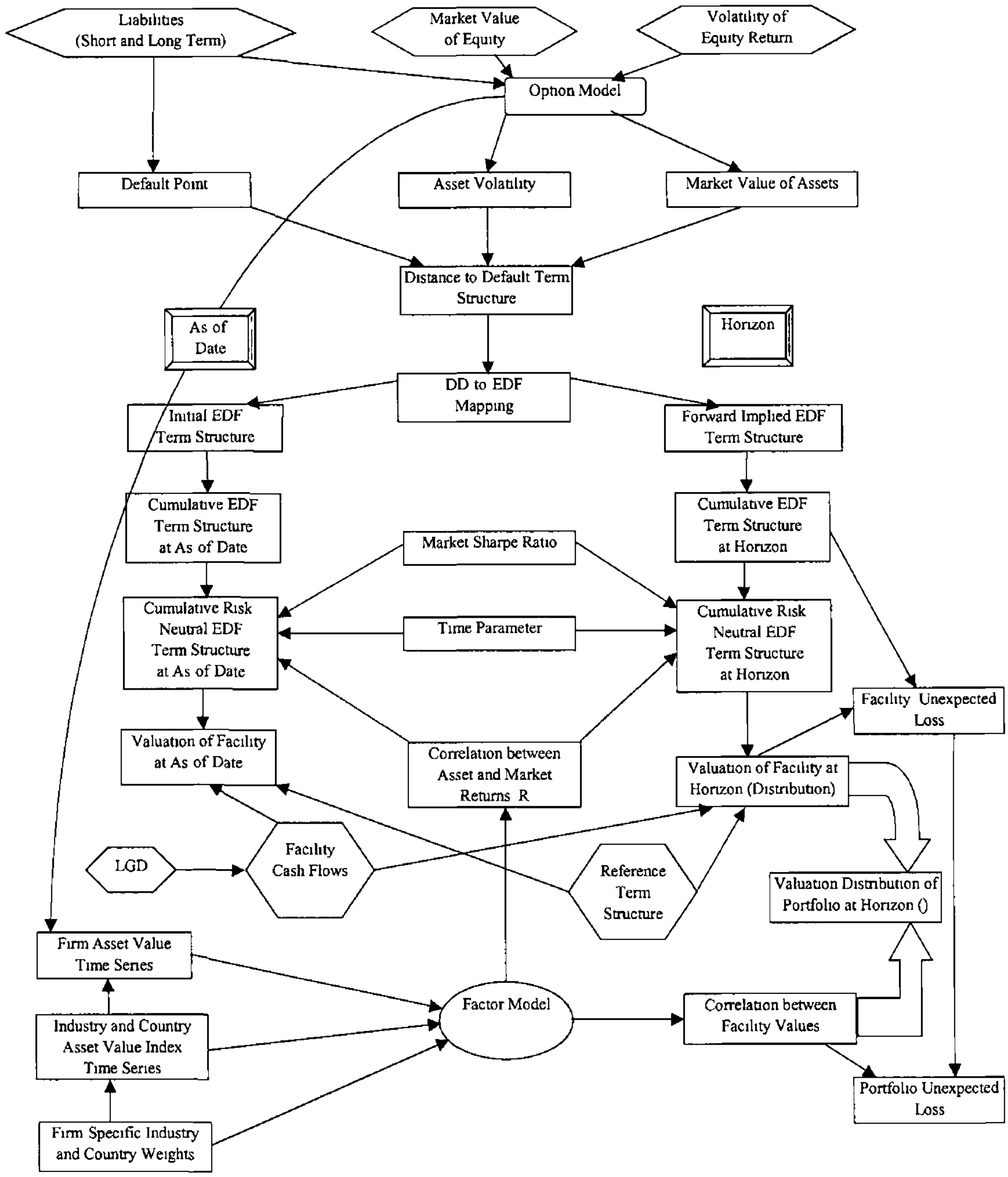

Figure 4 1 Portfolio Manager Outline

4 3 1 2 Cumulative Quasi-Default Probability

The actual cumulative probability of default from 0 to time t, $CEDF_t$, is given by the Merton formula

$$CEDF_t = N\left\{-\frac{\ln(A_0)+\left(\mu-\frac{1}{2}\sigma^2\right)t-\ln(DPT)}{\sigma\sqrt{t}}\right\} \qquad \textbf{Equation 4-1}$$

where

A_0 = market value of the firm's assets at time 0

DPT = default point

σ = volatility of return on the firm's assets, and

μ = drift rate, or expected return, on the firm's assets

However, for valuation purposes, it is the risk-adjusted PD - quasi-probability of default in KMV terminology - that is required The actual drift rate is replaced by r, the continuously compounded risk-free rate to obtain the cumulative quasi-probability of default, $CQDF_t$

$$CQDF_t = N\left\{-\frac{\ln(A_0)+\left(r-\frac{1}{2}\sigma^2\right)t-\ln(DPT)}{\sigma\sqrt{t}}\right\} \qquad \textbf{Equation 4-2}$$

Re-arranging this equation, we get

$$CQDF_t = N\left\{N^{-1}(CEDT_t)+\left(\frac{\mu-r}{\sigma}\right)\sqrt{t}\right\} \qquad \textbf{Equation 4-3}$$

which specifies the cumulative quasi-default probability, $CQDF_t$, as a function of the actual cumulative probability of default, $CEDF_t$ and the Sharpe ratio of the asset KMV (2003, p51) suggest that the expected excess return on the asset is a function of its sensitivity to systematic market risk factors

$$\beta\pi = \mu - r \qquad \textbf{Equation 4-4}$$

where β is the asset's beta to the market and π is the market risk premium, $(\mu_m - r)$ The asset's beta to the market, β, may be re-stated as

$$\beta = \frac{\operatorname{cov}(asset, market)}{\sigma_m^2} = R\frac{\sigma}{\sigma_m} \qquad \textbf{Equation 4-5}$$

where

R = the correlation between the asset return and the market return, and

σ_m = the volatility of the market return

This enables the individual asset's Sharpe ratio to be expressed as a function of the market's Sharpe ratio, λ The market's Sharpe ratio is the market price of risk,

$$\lambda = \frac{(\mu_m - r)}{\sigma_m} \qquad \textbf{Equation 4-6}$$

lınkıng the amount of one year's excess return to annual return volatılıty

KMV (2003, p51) suggest that the market prıce of rısk wıll vary wıth the square root of tıme sınce the excess return wıll be a lınear functıon of tıme whıle volatılıty wıll ıncrease wıth the square root of tıme Furthermore,

$$\frac{(\mu - r)}{\sigma} = R\lambda \qquad \textbf{Equation 4-7}$$

Hence,

$$CQDF_t = N\left\{N^{-1}(CEDF_t) + R\lambda\sqrt{t}\right\} \qquad \textbf{Equation 4-8}$$

KMV suggest an approprıate proxy for the market ıs the custom ındex based on the country and ındustry (or ındustrıes) wıthın whıch the fırm operates (Usıng a multı-factor model, descrıbed ın detaıl later, they create asset value ındıces for ındustrıes and countrıes) They regress the fırm's asset return serıes on the custom ındex to determıne the percentage of the asset return varıabılıty that ıs explaıned by the custom ındex, R^2 The square root of R^2 ıs a measure of the asset correlatıon wıth the market, they suggest based on the assumptıon that the custom ındex proxıes for the market

The market rısk premıum, called the market Sharpe ratıo, measures the requıred return over and above the rısk-free rate for holdıng a unıt of rısk at the aggregate level Research performed at KMV on the rısk premıum ımplıcıt ın credıt spreads for U S corporate bonds "reveals that the market Sharpe Ratıo parameter ıs relatıvely stable over tıme and typıcally around 0 4 " KMV (2003, p19) They contınue "Note that thıs ıs the rısk premıum assocıated wıth the market value of fırm assets and not the value of the fırm's equıty The rısk premıum for equıtıes fluctuates more than that for assets because of the dynamıc nature of fırm leverage " (p19) The value of 0 4 wıll be used throughout the rest of thıs study

4 3 1 3 Valuatıon

A rısky exposure wıll pay (1-LGD) ıf ıt defaults and 1 otherwıse KMV decompose thıs exposure ınto two separate cash flows The fırst wıll pay the recovery amount (1-LGD) whether ıt defaults or not The second pays 0 ın the event of default and LGD otherwıse

The first cash flow has no default risk and can be discounted at the risk-free rate The second cash flow contains credit risk, valuation of this component must include a discount for risk

KMV's risk-comparable valuation (RCV) methodology embeds their risk-neutral pricing technique The RCV value at the modelling date, time 0, is

$$V_0^{RCV} = (1 - LGD)RFV_0 + LGD\,RYV_0 \qquad \textbf{Equation 4-9}$$

where

$V_0^{RCV} \equiv$ Risk-comparable value at time 0

$RFV_0 \equiv$ Risk-free value at time 0 and

$RYV_0 \equiv$ Risk comparable risky value at time 0

The risk-free value at t_0 discounts each cash flow at the risk-free rate

$$RFV_0 = \sum_{t>0}^{M} C_t\, DF_t^{R_f} \qquad \textbf{Equation 4-10}$$

where

M = time to maturity

t = time to payment of a given cash flow

C_t = amount of cash flow at time t

$DF_t^{R_f}$ = risk-free discount factor to time t

The risky value calculation adjusts each cash flow by the quasi-probability and then uses the risk-free discount factor for discounting

$$RYV_0^{RCV} = \sum_{t>0}^{M} (1 - CQDF_t)C_t\, DF_t^{R_f} \qquad \textbf{Equation 4-11}$$

KMV give the term structure of EDF values annually from one to five years Using these values, the cumulative probabilities of default are calculated as follows

$$CEDF_t = 1 - (1 - EDF_t)^t \qquad \textbf{Equation 4-12}$$

Interpolation and extrapolation are based on the following formulae

$$CEDF_t = 1 - \left[(1 - CEDF_1)^t \right] \; for\; t \leq 1$$

$$CEDF_t = 1 - \left[(1 - CEDF_{T_1}) \left(\frac{1 - CEDF_{T_2}}{1 - CEDF_{T_1}} \right)^{\left(\frac{t - T_1}{T_2 - T_1} \right)} \right] \; for\; 1 \leq T_1 \leq t \leq T_2 \leq 5$$

$$CEDF_t = 1 - \left[(1 - CEDF_1)^{\frac{t}{5}} \right] \; for\; t > 5$$

Equation 4-13

As shown earlier, the *CQDF* value may be written in terms of the actual probability of default to time *t*, the market's Sharpe ratio, λ, and the asset's correlation with the market, *R*

$$CQDF_t = N\left\{ N^{-1}(CEDF_t) + R\lambda\sqrt{t} \right\} \qquad \textbf{Equation 4-14}$$

This enables the asset to be valued at time 0

4 3 2 Asset Correlation

The key reason for adopting the KMV-based approach to modelling is the opportunity this affords to use their correlation framework KMV have a substantial database of public companies – approximately 35,000 at the time of writing – for which they have equity price time series over many years and from which they have calculated asset price time series

They aggregate these individual time series to create 61 industry time series and 45 country time series They then use principal components analysis to extract orthogonal factors – two global, five regional and seven sectoral – which may be used to calculate asset correlation between all pairs of obligors The resulting factor sensitivities may be used to model asset migration and hence value a debt portfolio at the modelling horizon KMV update these factor sensitivities on an annual basis and include the file with their monthly updates to obligor EDF values

4 3 2 1 The KMV Factor Structure

The correlation between the market value of a firm's assets and the market embodies the extent to which systematic risk factors in the economy drive the value of the firm's assets While the market, in theory, comprises all available assets, KMV make the assumption that the market becomes "what we can observe that reasonably proxies for the theoretical market" KMV (2003, p61) They define the market as the custom index comprised of the country and industry indexes within which the firm operates

KMV's factor model imposes a structure on the correlation of asset returns, which implies that the correlation between the asset returns of any pair of firms can be explained by the firms' relationships to a set of common factors There are three levels used in KMV's factor structure, KMV (2003, p117) (i) A composite company-specific factor, (ii) country and industry factors, and (iii) global, regional and industrial sector factors

The first level of the structure differentiates between firm-specific and systematic risk Systematic risk is captured by a single common factor This factor is unique to each firm and is a weighted sum of country and industry factors to which the firm has exposure

The country and industry factors at the second level of the factor structure are correlated with each other Therefore, their risk can also be decomposed into systematic and idiosyncratic components The systematic component of the risk is captured by the basic economic factors in the third level of the structure The idiosyncratic risk components of countries and industries are retained as country- and industry-specific factors

KMV emphasise (2003, p118) that the third level of factors is only needed for interpreting the drivers of correlation The actual correlation estimate depends only on the division between the systematic and idiosyncratic parts of the country and industry risks This is shown in schematic format below, adapted from KMV (2003, p118)

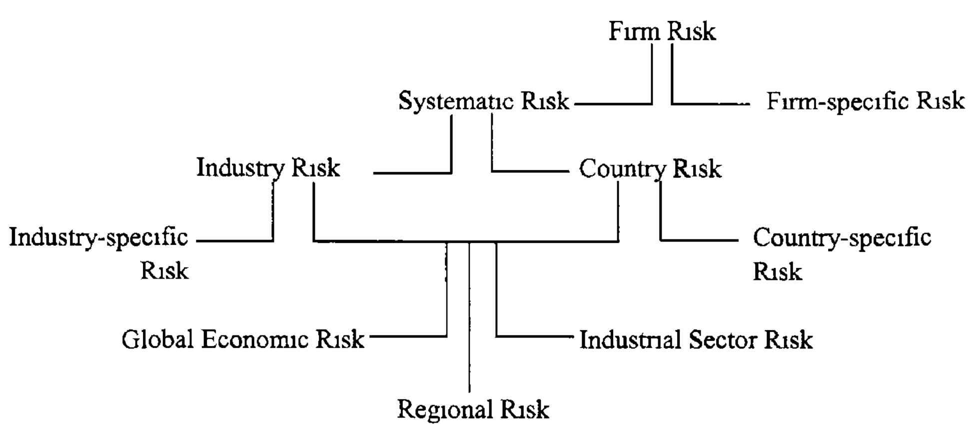

Figure 4 2 KMV's Factor Model Structure

The global, regional and sector factors capture all the common risk between countries and industries That is, they capture all of the correlation between the country and industry factors Likewise, these basic factors also explain all of the common risk between firms in different countries and industries Firms with exposure to the same country or industry also share country- or industry-specific risks

4 3 2 2 Estimating the KMV Factor Model

KMV (2003, p119-122) describe the process whereby they estimate the relevant parameters for their factor model

They construct 14 orthogonal factors from the 106 – 45 countries and 61 industries – indices They regress the 106 country and industry return indices on the 14 orthogonal factors to obtain the country and industry betas on these 14 factors They also obtain the country- and industry-specific risks They regress each firm's return series on its composite index returns to obtain the firm-specific beta and the R^2

Each index is regressed in sequence on the residual of the previous regression so that, by construction, each of the factors is orthogonal This means that the Interest Sensitivity factor, for example, is not the total effect of interest rates but only the portion of that effect that cannot be explained by the global and regional factors

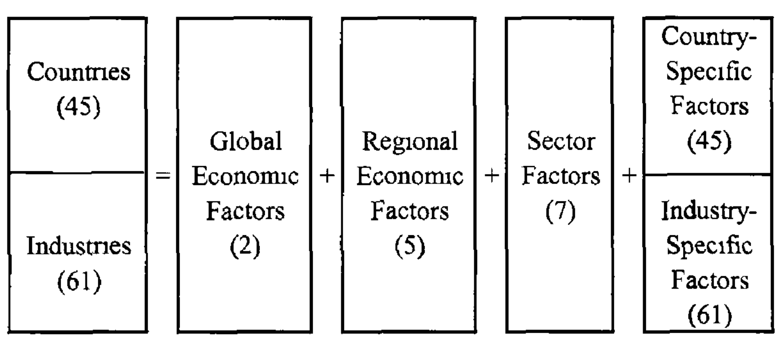

Figure 4 3 KMV's Correlation Schematic

Country or industry risk is decomposed into systematic risk arising from either global, regional or sector effects and specific, or idiosyncratic, risk

Country Return	=	Global Economic Effect	+	Regional Factor Effect	+	Sector Factor Effect	+	Country-Specific Effect
Industry Return	=	Global Economic Effect	+	Regional Factor Effect	+	Sector Factor Effect	+	Industry-Specific Effect

The regression model can be written as

$$r_c = \sum_{G=1}^{2} \beta_{cG} r_G + \sum_{R=1}^{5} \beta_{cR} r_R + \sum_{S=1}^{7} \beta_{cS} r_S + \varepsilon_c$$
$$r_i = \sum_{G=1}^{2} \beta_{iG} r_G + \sum_{R=1}^{5} \beta_{iR} r_R + \sum_{S=1}^{7} \beta_{iS} r_S + \varepsilon_i$$

Equation 4-15

where

r_c = return for country c

r_i = return for industry i

r_G = return for global market G

r_R = return for region R

r_S = return for sector S

β_{cG} = effect of global market G on country c

β_{iG} = effect of global market G on industry i

β_{cR} = effect of region R on country c

β_{iR} = effect of region R on industry i

β_{cS} = effect of sector S on country c

β_{iS} = effect of sector S on industry I

ε_c = country-specific effect for country c

ε_i = industry-specific effect for industry i

The variance of the industry and sector returns is, therefore

$$\sigma_c^2 = \sum_{G=1}^{2} \beta_{cG}^2 \sigma_G^2 + \sum_{R=1}^{5} \beta_{cR}^2 \sigma_R^2 + \sum_{S=1}^{7} \beta_{cS}^2 \sigma_S^2 + \sigma_{\varepsilon_c}^2$$
$$\sigma_i^2 = \sum_{G=1}^{2} \beta_{iG}^2 \sigma_G^2 + \sum_{R=1}^{5} \beta_{iR}^2 \sigma_R^2 + \sum_{S=1}^{7} \beta_{iS}^2 \sigma_S^2 + \sigma_{\varepsilon}^2$$

Equation 4-16

where

σ_G^2 = variance of global market factor G's return

σ_R^2 = variance of region factor R's return

σ_S^2 = variance of sector factor S's return

The firm risk is decomposed into systematic and idiosyncratic components The composite factor is constructed individually for each firm based on the countries and industries to which it is exposed These country and industry classifications are determined from the firm's reported sales and asset levels in a particular country or industry

The composite factor can be written as

$$\phi_k = \sum_{c=1}^{45} w_{kc} r_c + \sum_{i=1}^{61} w_{ki} r_i \qquad \textbf{Equation 4-17}$$

where

ϕ_k = composite factor for firm k

w_{kc} = weight of firm k in country c

w_{ki} = weight of firm k in industry I

r_c = return for country c

r_i = return for industry I

and

$$\sum_{c=1}^{45} w_{kc} = \sum_{i=1}^{61} w_{ki} = 1 \qquad \textbf{Equation 4-18}$$

KMV run a regression of each firm's weekly returns against the returns of its composite factor

$$r_k = \beta_k \phi_k + \varepsilon_k \qquad \textbf{Equation 4-19}$$

where

r_k = return for firm k

β_k = beta for firm k

ε_k = firm-specific effect for firm k

KMV (2003, p126) call the composite factor coefficient β_k, "the firm's beta", noting that it differs from the firm's stock beta which is against the market index whereas in their model, each firm has its own unique index KMV publish the R^2 for the regression for each firm in their database These values range from 0 1 to 0 65 with the lower values typical of smaller firms and higher values for larger firms

For private firms, the appropriate industry and country weights are used as for public companies and an estimate is made of the firm's beta This latter value is calculated using an estimate of R^2 for the firm that is based on the value of R^2 for public firms of similar size in the same country and industries With this estimate, the firm's beta is calculated using

$$\beta_k = \sqrt{R^2}\,\frac{\sigma_k}{\sigma_{\phi_k}} \qquad \textbf{Equation 4-20}$$

We can re-state the firm's returns as follows

$$
\begin{aligned}
r_k &= \beta_k \phi_k + \varepsilon_k \\
&= \beta_k \left(\sum_{c=1}^{45} w_{kc} r_c + \sum_{i=1}^{61} w_{ki} r_i \right) + \varepsilon_k \\
&= \beta_k \left\{ \begin{array}{l} \displaystyle\sum_{c=1}^{45} w_{kc} \left(\sum_{G=1}^{2} \beta_{cG} r_G + \sum_{R=1}^{5} \beta_{cR} r_R + \sum_{S=1}^{7} \beta_{cS} r_S \right) + \\ \displaystyle\sum_{i=1}^{61} w_{ki} \left(\sum_{G=1}^{2} \beta_{iG} r_G + \sum_{R=1}^{5} \beta_{iR} r_R + \sum_{S=1}^{7} \beta_{iS} r_S \right) \end{array} \right\} + \varepsilon_k \\
&= \beta_k \left\{ \begin{array}{l} \displaystyle\left(\sum_{G=1}^{2}\sum_{c=1}^{45} \beta_{cG} w_{kc} r_G + \sum_{R=1}^{5}\sum_{c=1}^{45} \beta_{cR} w_{kc} r_R + \sum_{S=1}^{7}\sum_{c=1}^{45} \beta_{cS} w_{kc} r_S + \sum_{c=1}^{45} w_{kc} \varepsilon_c \right) + \\ \displaystyle\left(\sum_{G=1}^{2}\sum_{i=1}^{61} \beta_{iG} w_{ki} r_G + \sum_{R=1}^{5}\sum_{i=1}^{61} \beta_{iR} w_{ki} r_R + \sum_{S=1}^{7}\sum_{i=1}^{61} \beta_{iS} w_{ki} r_S + \sum_{i=1}^{61} w_{ki} \varepsilon_i \right) \end{array} \right\} + \varepsilon_k \\
&= \beta_k \left\{ \begin{array}{l} \displaystyle\sum_{G=1}^{2} \left(\sum_{c=1}^{45} \beta_{cG} w_{kc} + \sum_{i=1}^{61} \beta_{iG} w_{ki} \right) r_G + \\ \displaystyle\sum_{R=1}^{5} \left(\sum_{c=1}^{45} \beta_{cR} w_{kc} + \sum_{i=1}^{61} \beta_{iR} w_{ki} \right) r_R + \\ \displaystyle\sum_{S=1}^{7} \left(\sum_{c=1}^{45} \beta_{cS} w_{kc} + \sum_{i=1}^{61} \beta_{iS} w_{ki} \right) r_S + \\ \displaystyle\sum_{c=1}^{45} w_{kc} \varepsilon_c + \sum_{i=1}^{61} w_{ki} \varepsilon_i \end{array} \right\} + \varepsilon_k
\end{aligned}
$$

Equation 4-21

$$
\begin{aligned}
&= \sum_{G=1}^{2} \beta_k \left(\sum_{c=1}^{45} \beta_{cG} w_{kc} + \sum_{i=1}^{61} \beta_{iG} w_{ki} \right) r_G + \sum_{R=1}^{5} \beta_k \left(\sum_{c=1}^{45} \beta_{cR} w_{kc} + \sum_{i=1}^{61} \beta_{iR} w_{ki} \right) r_R \\
&+ \sum_{S=1}^{7} \beta_k \left(\sum_{c=1}^{45} \beta_{cS} w_{kc} + \sum_{i=1}^{61} \beta_{iS} w_{ki} \right) r_S + \sum_{c=1}^{45} \beta_k w_{kc} \varepsilon_c + \sum_{i=1}^{61} \beta_k w_{ki} \varepsilon_i + \varepsilon_k \\
&= \sum_{G=1}^{2} \beta_{kG} r_G + \sum_{R=1}^{5} \beta_{kR} r_R + \sum_{S=1}^{7} \beta_{kS} r_S + \sum_{c=1}^{45} \beta_k w_{kc} \varepsilon_c + \sum_{i=1}^{61} \beta_k w_{ki} \varepsilon_i + \varepsilon_k \\
&= \sum_{G=1}^{2} \beta_{kG} r_G + \sum_{R=1}^{5} \beta_{kR} r_R + \sum_{S=1}^{7} \beta_{kS} r_S + \sum_{c=1}^{45} \beta_{kc} \varepsilon_c + \sum_{i=1}^{61} \beta_{ki} \varepsilon_i + \varepsilon_k
\end{aligned}
$$

Equation 4-22

where

$$
\begin{aligned}
\beta_{kc} &= \beta_k w_{kc} \\
\beta_{ki} &= \beta_k w_{ki}
\end{aligned}
\qquad \textbf{Equation 4-23}
$$

The total risk of the firm can thus be expressed as follows

$$
\sigma_k^2 = \sum_{G=1}^{2} \beta_{kG}^2 \sigma_G^2 + \sum_{R=1}^{5} \beta_{kR}^2 \sigma_R^2 + \sum_{S=1}^{7} \beta_{kS}^2 \sigma_S^2 + \sum_{c=1}^{45} \beta_{kc}^2 \sigma_c^2 + \sum_{i=1}^{61} \beta_{ki}^2 \sigma_i^2 + \varepsilon_k^2 \qquad \textbf{Equation 4-24}
$$

The covariance in the asset returns of two firms can be calculated by summing their joint sensitivity to the common factors

$$
\sigma_{jk} = \sum_{G=1}^{2} \beta_{jG} \beta_{kG} \sigma_G^2 + \sum_{R=1}^{5} \beta_{jR} \beta_{kR} \sigma_R^2 + \sum_{S=1}^{7} \beta_{jS} \beta_{kS} \sigma_S^2 + \sum_{c=1}^{45} \beta_{jc} \beta_{kc} \sigma_{\varepsilon_c}^2 + \sum_{i=1}^{61} \beta_{ji} \beta_{ki} \sigma_{\varepsilon_i}^2
$$

Equation 4-25

and the return correlation is obtained using the standard formula

$$
\rho_{jk} = \frac{\sigma_{jk}}{\sigma_j \sigma_k} \qquad \textbf{Equation 4-26}
$$

4 3 3 Simulating Asset Migration

The asset migration process is modelled using a Monte Carlo simulation approach The approach proceeds as follows

1 Draw a set of factor realisations, one for each factor The factors are independent and identically distributed (i i d) standard normal variables

2 Draw a specific risk random variable for each firm These draws are also i i d standard normal random variables

3 Compute the random component of each firm's asset return as the weighted sum of the specific and systematic risks

4 If the random value drawn is below the default threshold set by the firm's default probability to the modelling horizon, $CEDF_H$, the firm has defaulted The loss incurred on defaulted securities is obtained by making a random drawing from a beta distribution, characterised by the average loss rate expected for the facility as well as its standard deviation

5 Compute the value of each exposure at the horizon from its asset value realisation This is calculated using the RCV methodology previously described The value will be a function of each exposure's LGD, EDF value, R^2 and the random realisation of asset value at the horizon

6 The value of the portfolio at the horizon is obtained by summing the values of the individual exposures in the portfolio

7 Repeat steps 1 to 6 sufficiently often to achieve the requisite resolution in the extreme tail of the distribution The number of iterations typically required for this of the order of 100,000

In the KMV framework, sampling in the Monte Carlo simulation takes place over the asset values of the individual obligors The asset value at the horizon for an obligor, A_H, is calculated as

$$\ln(A_H)=\ln(A_0)+\left(\mu-\frac{\sigma^2}{2}\right)t_H+\sigma\sqrt{t_H}\,\tilde{\varepsilon}_H \qquad \textbf{Equation 4-27}$$

where

A_0 = the borrower's underlying asset value at t_o

μ = expected return on the underlying assets

σ = volatility of the return on the underlying assets

$\tilde{\varepsilon}_H$ = the random component of the asset return

This allows the process of simulating asset migration to be completed

The $\tilde{\varepsilon}_H$ are assumed to be drawn from a multi-variate normal distribution Thus, the simulation must draw a value of $\tilde{\varepsilon}_H$ for each obligor to embed this correlation structure The factor model, described already, provides the structure necessary to embed the matrix of correlations among asset returns The independent draws for the 120 factors - two global macroeconomic, five regional, seven sectoral, 45 country-specific and 61 industry-specific effects – are combined as follows

$$\tilde{\varepsilon}_H = \frac{1}{\sqrt{\sum_{j=1}^{120} \beta_j^2 \sigma_j^2 + \sigma_u^2}} \left[\sum_{j=1}^{120} \beta_j \sigma_j \tilde{\lambda}_j + \sigma_u \tilde{u} \right] \qquad \textbf{Equation 4-28}$$

The λ_j are the 120 systematic risk factors, while $\tilde{u}$ is the firm-specific factor, and, as mentioned already, all are independent draws of standard normally distributed random variables The random component of asset return, $\tilde{\varepsilon}_H$, is obtained by first calculating the weighted sum of the firm-specific return, $\tilde{u}$, and the 120 systematic risk factors, λ_j, the weights being the coefficients in the last equation, and then scaling the sum by dividing by the standard deviation of the firm's asset return Thus, by construction, the random component of the firm's asset return, $\tilde{\varepsilon}_H$, is standard normally distributed and has a correlation structure consistent with the factor model of correlation

4 3 4 LGD

KMV, in common with many others in the industry – most notably, CreditMetrics - use the beta distribution to model the recovery rate The beta distribution has desirable characteristics for a recovery function First, it can be bounded at whatever level is chosen clearly, the desired upper and lower bounds for the recovery rate are 0 and 1 Furthermore, it can accommodate many different distributional shapes, which offers the flexibility to represent the lender's view on recovery uncertainty

The Beta function is characterised by two parameters

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \qquad \textbf{Equation 4-29}$$

where Γ is the gamma function, defined as

$$\Gamma(\alpha)=\int_0^{\infty} e^{-y} y^{\alpha-1} dy \qquad \textbf{Equation 4-30}$$

A Beta distribution density function is given by

$$f(x,a,b)=\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)} \qquad \textbf{Equation 4-31}$$

The mean of the beta distribution is given by

$$\mu(a,b)=\frac{a}{a+b} \qquad \textbf{Equation 4-32}$$

and its variance is given by

$$\sigma^2(a,b)=\frac{ab}{a+b+1} \qquad \textbf{Equation 4-33}$$

Clearly, the mean of the distribution must be *LGD*, so, by definition

$$LGD=\frac{a}{a+b} \qquad \textbf{Equation 4-34}$$

The relationship between *a* and *b*, which determines the shape of the Beta distribution for a given *LGD* is determined by their relationship in the variance of the distribution

$$\sigma^2=\frac{LGD(1-LGD)}{a+b+1} \qquad \textbf{Equation 4-35}$$

KMV (2003, p103) suggest that this shape be controlled through the use of a single parameter, *k*, which they define as

$$k=a+b+1 \qquad \textbf{Equation 4-36}$$

Hence, the shape parameters, *a* and *b*, can be determined from *LGD* and *k*

$$\begin{aligned} a &= (k-1)LGD \\ b &= (k-1)(1-LGD) \end{aligned} \qquad \textbf{Equation 4-37}$$

and the variance can be expressed as

$$\sigma^2=\frac{LGD(1-LGD)}{k} \qquad \textbf{Equation 4-38}$$

The reason the beta distribution has become the distribution of choice for *LGD* in the market is the flexibility it offers. If nothing is known about the distributional characteristics of *LGD*, a *k*-value of 3 might be chosen; this will result in a uniform distribution. Higher *k* values will result in a distribution resembling the normal. If, however, as is sometimes observed, *LGD* is either 0% or 100% with very low probabilities of intermediate outcomes, a *k*-parameter of 2 or less will give a U-shaped distribution which captures these features. These distributions are shown in Figure 4.4:

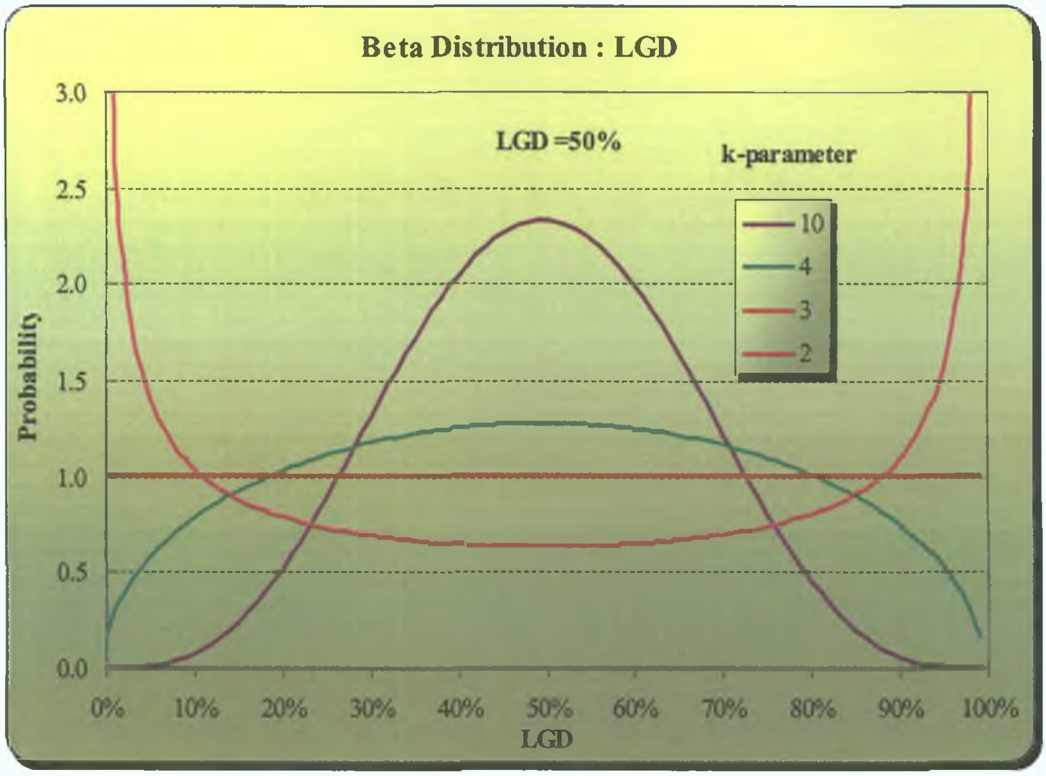

Figure 4.4 Beta Distribution with Average LGD = 50%

Thus, as *k* becomes large, the variance of the distribution goes to 0, implying little uncertainty about the estimate of mean *LGD*, while lesser values of *k* are appropriate where the lender has less confidence in the *LGD* estimate. KMV suggest a value of 4 as being appropriate for many lenders.

The beta distribution is plotted in Figure 4.5 below for various values of *k* when the average *LGD* is 45%:

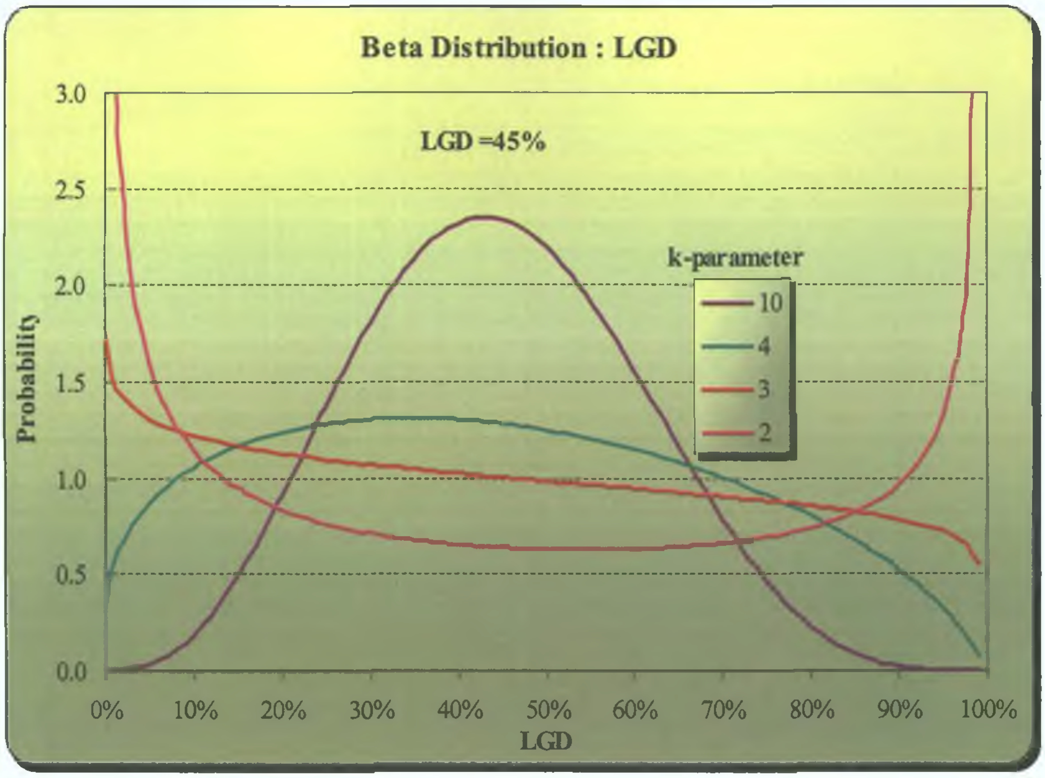

Figure 4.5 Beta Distribution with Average LGD = 40%

The fractional loss of the exposure, $\widetilde{LGD}$, is a random variable drawn from a Beta distribution with mean *LGD* and variance $\frac{LGD(1-LGD)}{k}$. *LGD* will be modelled using a *k* value of 4 throughout this thesis. This sees *LGD* values close to the average much of the time but allows full recovery and complete loss occur also.

4.3.5 Asset Revaluation at the Horizon

Once the borrower's asset value realisation at horizon is determined, exposures to the borrower at the horizon can be evaluated. The value of a given exposure is determined as the sum of two exposures, a riskless portion, which pays $(1-LGD)(RFV_H)$ whether the exposure defaults or not, and a risky portion, which pays $LGD\,RFY_H$ when the exposure is in default and zero otherwise:

$$V_H = (1-LGD)RFV_H + LGD\,RFY_H \qquad \textbf{Equation 4-39}$$

where RFV_H is the risk-free value of the exposure from horizon to maturity which includes the risk-free value at *H* of any cash flows received from 0 to *H*. RYV_H is the value of the risky portion of the exposure from horizon to maturity, and *LGD* is the expected LGD. RYV_H also includes coupons from time 0 to H. According to KMV (2003, p161), the coupons or cash flows received before horizon are assumed to be risk-free and enter into the LGD calculation.

If the exposure defaults before the horizon, then the value of the risky portion of the exposure, RYV_H, is equal to zero and the obligor is assumed to lose an amount based on the parameterised *LGD* distribution. Since *LGD* is assumed to be random, a random draw is made to determine the fractional loss for each defaulted exposure, as discussed next.

4.3.5.1 Exposure Value in the Default State

If the asset value at the modelling horizon, $\ln(A_H)$, falls below the default point, $\ln(DPT_H)$, then the obligor will default. In other words, the obligor will default if the realisation of the random component of the obligor's asset return, $\tilde{\varepsilon}_H$, is larger (in absolute terms) than the H-period distance to default (DD) at time 0, which is the normal inverse of the H-period EDF:

$$\tilde{\varepsilon}_H \leq -\left[\frac{\ln\left(\frac{A_0}{DPT_H}\right)+\left(\mu-\frac{\sigma^2}{2}\right)H}{\sigma\sqrt{H}}\right]=N^{-1}(CEDF_H) \qquad \textbf{Equation 4-40}$$

where

DPT_H = standardised default point at the horizon

$CEDF_H$ = cumulative probability of default to the horizon

In the event of default, the risky value of the exposure, RYV_H, is set to zero. A random draw from the beta distribution determines the loss incurred as explained previously.

4.3.5.2 Exposure Value in the Non-Default State

In order to determine the risk which a portfolio of loans presents, the portfolio value distribution at horizon must be calculated. As described above, the portfolio must be simulated under the true risk measure to determine whether default has occurred for each individual loan. This will depend on whether the realised $\tilde{\varepsilon}_H$ for an obligor causes the borrower to fall below its default threshold.

If the obligor does not default, the facility must be revalued at the modelling horizon The facility value will be a function of the realised EDF term structure The modelling requires that the realised value distribution for each facility should be consistent with the EDF term structure as observed at the outset This, in turn, requires that the distribution of distance to default for an obligor at the modelling horizon be explicitly linked to the term structure of DD observed at the outset

In order to undertake valuation in the non-default state at the modelling horizon, the risk-comparable approach is adopted once again The cumulative quasi-EDF values from the modelling horizon to the cash flow date must be calculated as a function of the asset return at horizon, $\tilde{\varepsilon}_H$ Thus, the challenge is to calculate the value for $CQDF_{H\,M}$, the cumulative quasi-EDF from the modelling horizon to maturity, in a manner that incorporates the information about the borrower's DD at the outset

$$CQDF_{H\,M} = N\left(-DD_{H\,M} \middle| ND\right) \qquad \textbf{Equation 4-41}$$

where $DD_{H\,M}$ is the risk-neutral DD from horizon, H, to maturity, M, in the non-default (ND) state at the modelling horizon, H

This DD can be expressed as a function of the realised asset value at horizon, A_H, and the default point at maturity, DP_M

$$DD_{H\,M} = \frac{\ln\left(\frac{A_H}{DP_M}\right) + \left(r - \frac{\sigma_A^2}{2}\right)(M-H)}{\sigma_A\sqrt{M-H}}$$

$$= \frac{\ln\left(\frac{A_0}{DP_M}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)H + \sigma_A\sqrt{H}\,\tilde{\varepsilon}_H + \left(r - \frac{\sigma_A^2}{2}\right)(M-H)}{\sigma_A\sqrt{M-H}} \qquad \textbf{Equation 4-42}$$

By adding and subtracting $(\mu - r)(M - H)$ in the numerator, this simplifies as follows

$DD_{H,M} =$

$$\frac{\ln\left(\frac{A_0}{DP_M}\right)+\left(\mu-\frac{\sigma_A^2}{2}\right)H+\sigma_A\sqrt{H}\,\tilde{\varepsilon}_H+\left(r-\frac{\sigma_A^2}{2}\right)(M-H)+(\mu-r)(M-H)-(\mu-r)(M-H)}{\sigma_A\sqrt{M-H}}$$

$$=\frac{\ln\left(\frac{A_0}{DP_M}\right)+\left(\mu-\frac{\sigma_A^2}{2}\right)M+\sigma_A\sqrt{H}\,\tilde{\varepsilon}_H-(\mu-r)(M-H)}{\sigma_A\sqrt{M-H}}$$

$$=\frac{\ln\left(\frac{A_0}{DP_M}\right)+\left(\mu-\frac{\sigma_A^2}{2}\right)M}{\sigma_A\sqrt{M-H}}+\frac{\sqrt{H}\,\tilde{\varepsilon}_H}{\sqrt{M-H}}-\frac{(\mu-r)\sqrt{M-H}}{\sigma_A}$$

Equation 4-43

The Brownian processes at the modelling horizon, B_H, and at maturity, B_M, are correlated B_M may be re-written as

$$B_M = B_H + B_{M-H} \qquad \textbf{Equation 4-44}$$

The correlation between B_H and B_M may be expressed as follows

$$\rho_{B_{H,M}} = \frac{Cov(B_H, B_M)}{\sigma_{B_H}\sigma_{B_M}}$$

$$= \frac{Cov(B_H, B_H) + Cov(B_H, B_{M-H})}{\sigma_{B_H}\sigma_{B_M}} \qquad \textbf{Equation 4-45}$$

$$= \frac{H}{\sqrt{H}\sqrt{M}} = \sqrt{\frac{H}{M}}$$

Since $\rho=\sqrt{\frac{H}{M}}$, we get $\sqrt{\frac{M-H}{M}}=\sqrt{1-\rho^2}$

Furthermore,

$$DD_{0,M} = \frac{\ln\left(\frac{A_0}{DP_M}\right)+\left(\mu-\frac{\sigma_A^2}{2}\right)M}{\sigma_A\sqrt{M}} \qquad \textbf{Equation 4-46}$$

and

$$\frac{(\mu - r)}{\sigma_A} = R\lambda \qquad \textbf{Equation 4-47}$$

Hence, we can re-write $DD_{H\,M}$ as follows

$$DD_{H\,M} = \frac{DD_{0\,M}\sqrt{M}}{\sqrt{M-H}} + \frac{\sqrt{H}}{\sqrt{M-H}}\tilde{\varepsilon}_H - \frac{(\mu - r)\sqrt{M-H}}{\sigma_A}$$

$$= \frac{DD_{0\,M}}{\sqrt{1-\rho^2}} + \frac{\rho}{\sqrt{1-\rho^2}}\tilde{\varepsilon}_H - R\lambda\sqrt{M-H} \qquad \textbf{Equation 4-48}$$

This gives us an expression that connects the DD at maturity to the realised return to the modelling horizon

Default can occur at any time However, we can approximate this by replacing the continuous barrier with a situation in which default can occur only at the horizon modelling date or at facility maturity The DD from the outset to the modelling horizon is given by the obligor's default probability

$$DD_{0\,H} = -N^{-1}(CEDF_H) \qquad \textbf{Equation 4-49}$$

In these circumstances, the modelled DD from the outset to maturity, $DD_{0\,M}$, must be consistent with the known probability of default from the outset to facility maturity, $CEDF_M$ Given our assumption that default can occur only at the modelling horizon or at maturity, the probability of default may be calculated as the combined probability of default occurring before maturity or horizon

The two Brownian motion processes - to the modelling horizon and maturity, respectively – are bivariate normally distributed with a correlation coefficient of ρ as described above Thus, we know that the probability of defaulting between the start of the modelling period and facility maturity is comprised of the volume under the bivariate surface to the left of $-DD_{0\,H}$ and below $-DD_{0\,M}$ As shown in Figure 4 6, the probability of surviving is the volume under the bivariate above and to the right of the default thresholds given by $-DD_{0\,H}$ and $-DD_{0\,M}$

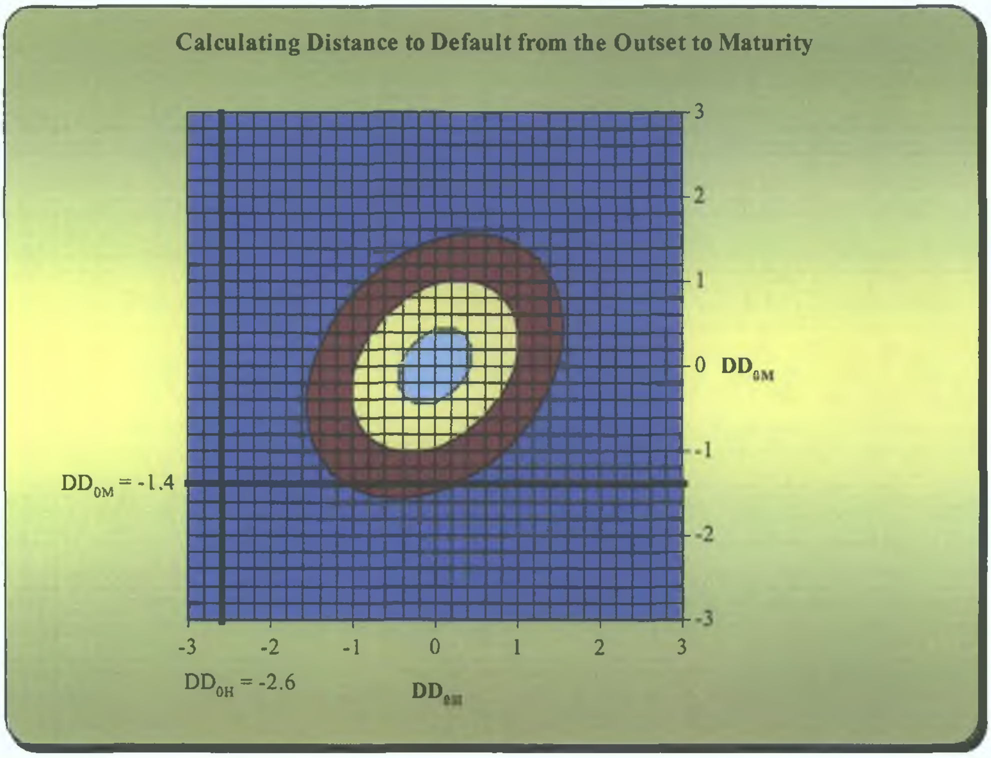

Figure 4.6 Calculating DD from the Outset to Maturity

By symmetry, this can be calculated as the volume to the left and below $DD_{0,H}$ and $DD_{0,M}$. Hence,

$$N_2\left(-N^{-1}\left(CEDF_H\right), DD_{0,M}, \rho\right) = 1 - CEDF_M \qquad \textbf{Equation 4-50}$$

If the borrower does not default at the horizon, then its new cumulative quasi-EDF term structure must be derived conditional on its realised distance-to-default at the horizon. Only when this is calculated can all the cash flows from *H* to *M* be valued.

We interpolate to calculate all values *CQDF(H,t)*, the cumulative quasi-EDF to date *t* between *H* and *M*:

$$CQDF_{H,t} = 1 - \left(1 - CQDF_{H,M}\right)^{\frac{t-H}{M-H}} \qquad \textbf{Equation 4-51}$$

All values *CQDF(H, t)*, the cumulative quasi-EDF to date *t* between *H* and *M* may be obtained using this interpolation. The facility values at the horizon date can then be calculated and hence the portfolio value. Repeated simulation yields the portfolio value distribution.

4.4 Checking the Successful Replication of the KMV Modelling Framework

The model described above was implemented in a *Matlab* environment using a portfolio of ninety loans of $5m each to ninety different B-rated obligors with maturities varying between five and ten years each offering a 1 5% spread The modelling horizon was set to one year and the portfolio value distribution at the horizon date was obtained by using Monte Carlo simulation with 100,000 trials The same portfolio was modelled in *Portfolio Manager* and, once again, the portfolio value distribution at the horizon date was derived

The summary output from *Portfolio Manager* is presented in Figure 4 7 below It shows that the portfolio of loans with a par value of $450m has a market value of $447 9m This value is obtained from the analytic calculation shown earlier This exact same value is obtained within the newly constructed model

The portfolio yields a total spread of 203bp which is eroded by an expected loss of 98bp to yield an expected spread of 105bp over the one-year horizon The standard deviation of loss, the unexpected loss, is 239bp and, hence, the Sharpe ratio, which equals the expected spread divided by unexpected loss, is 105/239 = 0 438

An item of particular importance to portfolio managers and regulators alike is the economic capital required to support the portfolio Economic capital is typically set at a level such that the debt which ranks ahead of the economic capital can achieve a particular rating, or, equivalently, a probability of default at the horizon of a given level In this instance, a default probability of 10bp at the one-year horizon is chosen as the target default probability for the debt Thus, the 99 9th percentile on the portfolio loss distribution is measured The difference between the mean portfolio value and the 99 9th percentile is the loss from which the debt must be protected at the horizon The present value of this amount represents the economic capital, which is the amount which we must set aside now to absorb portfolio losses over the modelling horizon

Portfolio Overview - C:\Personal\Thesis\KMV\KMV Target f...

Copy | Save to file | Print | Close

No. Exposures	90
No. Expired Exposures	0
No. Borrowers	90
Commitments	450,000,000
Exposure	450,000,000
MTM Exposure	447,902,425
Book Value Drawn	450,000,000
Market Value Drawn	447,902,425
Total Spread Revenue, Annualized	9,090,968
Expected Loss, Annualized	4,407,690
Expected Spread Revenue, Annualized	4,683,278
Unexpected Loss (Simulated)	10,685,779
Capital (10.00 bp in excess of Expected Loss)	47,689,875
	Current
Non-Zero Records	90
Total Spread, Annualized	0.02030
Expected Loss, Annualized	0.00984
Expected Spread, Annualized	0.01046
Unexpected Loss (Simulated)	0.02386
Capital (10.00 bp in excess of Expected Loss)	0.10647
Sharpe Ratio	0.43827
RORAC, Annualized	0.12092

Figure 4.7 ***Portfolio Manager*** **Summary Output**

The capital required to support the portfolio equals 10.65% of the market value of the portfolio. Furthermore, the return on risk-adjusted capital (RORAC) which equals expected spread divided by the capital plus the risk-free rate equals 1.05%/10.65% + 2.27% = 12.09%.

The two portfolio distributions are shown in Figure 4.8 below:

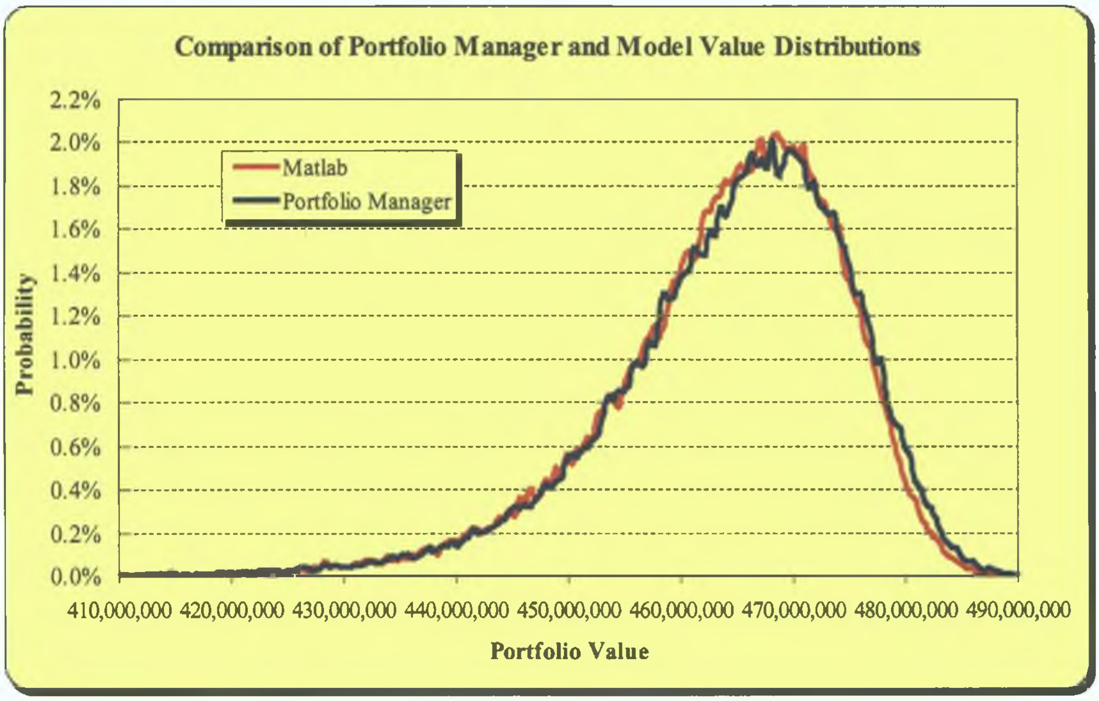

Figure 4.8 *Portfolio Manager* and Modelled Distributions Compared

It is clear from the above that the model captures the portfolio dynamic embedded in the KMV modelling methodology and that the differences between the distributions are attributable to simulation error.

This portfolio capital is, in turn, allocated to the individual facilities that comprise the portfolio. The capital allocation methodology adopted by KMV is the standard contribution to variance methodology applied in most VaR contexts.

$$RC^i \equiv \frac{\partial UL_p}{\partial w_i} = \frac{\sum_{j=1}^{n} w_j \rho_{ij} UL_i UL_j}{UL_p} = \frac{\rho_{ip} UL_i UL_p}{UL_p} = \rho_{ip} UL_i \qquad \textbf{Equation 4-52}$$

where

RC^i is the risk contribution of facility *i*, the fraction of portfolio capital which is attributed to facility *i*,

w_i is the weight of the facility in the portfolio,

UL_i, UL_j and UL_p are the standard deviations of the facility values *i*, *j* and the portfolio, respectively,

and ρ_{ip} is the correlation between the value of facility *i* and the value of the portfolio.

The sum of all the capital attributed to the facilities equals the capital for the portfolio:

$$UL_p = \frac{\sum_i^n \sum_j^n w_i w_j \rho_{ij} UL_i UL_j}{UL_p} = \sum_i^n w_i \sum_j^n \frac{w_j \rho_{ij} UL_i UL_j}{UL_p} = \sum_i^n w_i \rho_{ij} UL_i = \sum_i^n w_i RC^i$$

Equation 4-53

Risk contribution can be interpreted as the portion of the individual facility's risk that remains after diversification. This is the key focus of managerial attention.

Therefore, in addition to checking that the portfolio distribution is calculated in a manner consistent with that employed by KMV, a check on capital attribution is necessary to ensure that individual facilities are being modelled with similar consistency. To that end, the capital attributed to individual facilities under the model and that attributed by KMV in *Portfolio Manager* are compared. The results of that comparison are plotted in Figure 4.9:

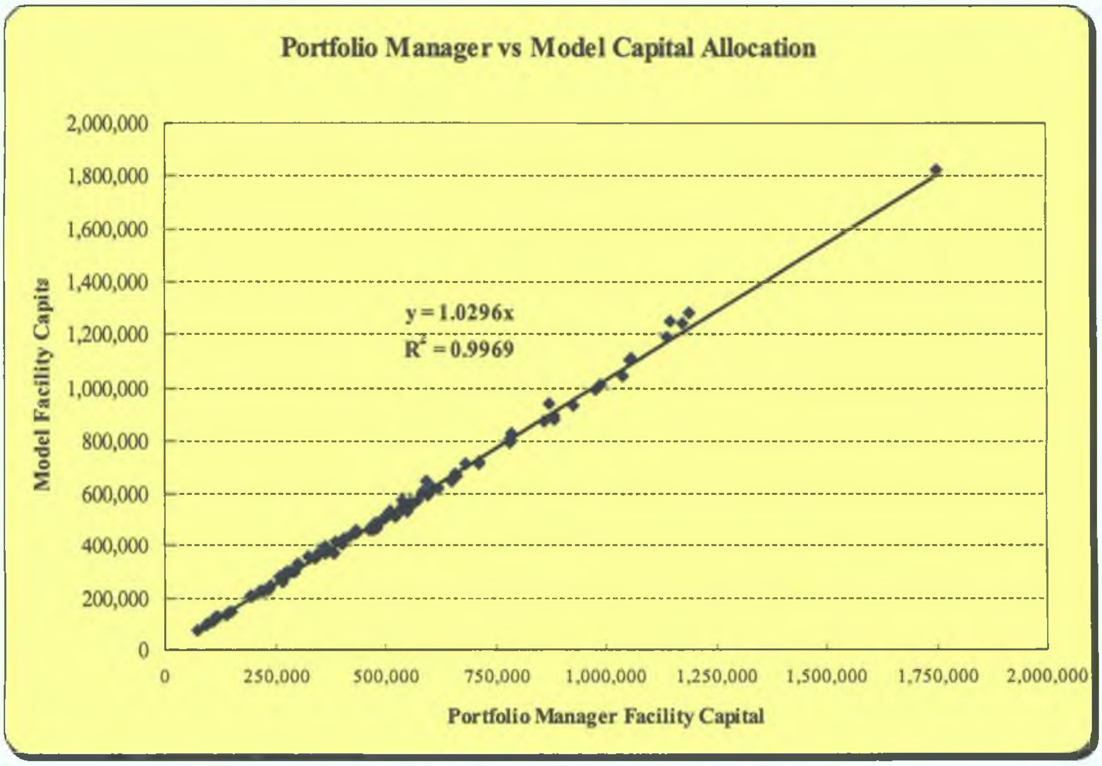

Figure 4.9 Comparison of Capital Allocation under New Model and *Portfolio Manager*

Again, the results confirm consistency between the two models at the facility level. This is a necessary pre-requisite to extending the modelling approach into a multi-period framework. This confirms the successful replication of the KMV modelling framework.

4.5 Using the Model to Address Key Credit Portfolio Issues

The primary concern of this thesis is CDO tranche valuation This requires that a single period credit portfolio model be created as a first step towards the building of the multi-period model necessary for CDO tranche valuation

However, the availability of this single period model affords insights into other areas of interest in the credit portfolio management field In particular, having such a portfolio model enables us answer four questions that have long challenged academe and industry alike

- How can a new facility be assessed in the context of the portfolio to which it will be added?
- What capital attribution methodology is most appropriate in the context of a bank credit portfolio which is financed largely with debt?
- What is the optimal amount of a new syndicated loan for which a bank should subscribe given its portfolio composition? Or, equivalently, and of more relevance to the bank's biggest customers, what penalty is being incurred by holding a facility which is larger than the optimal size?
- What framework should a bank use to set limits on the amount of exposure it should be willing to accept to different borrowers, sectors and geographies consistent with its risk appetite?

4 5 1 Determining the Marginal Impact of a Loan on a Debt Portfolio

Standard portfolio management software is run centrally and in isolation from the business line Those who compete for business have little appreciation of the impact of the new facilities which they consider writing on the portfolio The best that can be achieved in such circumstances is to give the business line general instructions about the concentrations which exist in the portfolio to guide their market positioning and loan pricing decisions

However, with the model that has just been built, all the inputs necessary to guide the pricing decision are available The value of the new loan at the outset can be calculated using the formula outlined at 4 3 1 above The new loan value distribution at the modelling horizon can be obtained by using the stored realisations for the 120 random variables for the 100,000 iterations

Thus, the value distribution for the new loan and for the portfolio including the proposed new loan may be calculated using the stored portfolio values This provides all the inputs required for the calculation of Risk Contribution and facility capital as described in 4 4 above This, in turn, enables the facility Sharpe ratio, $\frac{Expected\ Spread}{Risk\ Contribution}$, and facility RORAC, $\frac{Expected\ Spread}{Capitalisation\ Rate} + Risk - free\ rate$, to be calculated Finally, having facility capital allows us calculate the facility Economic Value Added (EVA)

None of these measures is available to the business line from *Portfolio Manager* or other credit portfolio software offerings at present Having these values available *before* the decision to provide the loan has to be made allows all new loan-granting decisions to be made in the full knowledge of their portfolio impact This can turn the portfolio management function from its current reactive stance of trying to mitigate the worst effects of low RORAC facilities after they have been written to identifying them in advance and avoiding taking them into the portfolio

In summary, portfolio management is primarily about identifying the capital required to support individual facilities According as the concentration of a facility in a portfolio increases, the capitalisation rate for that facility increases and the facility becomes less desirable Knowing the capital required to support a putative new facility *ex ante* has the potential to transform the loan portfolio management business

4 5 2 An Alternative Capital Attribution Framework

Capital is fundamentally a portfolio concept Allocating capital to a facility requires an attribution methodology The challenge of devising a logical attribution methodology has been faced in a trading room environment for many years The methodology of choice for distributing limits across desks, for example, has been to use the covariance of the desk returns with the trading room returns

KMV and many others in the credit marketplace have adopted this methodology But, is this appropriate? The instruments dealt in a trading room context are all liquid and the trading horizon is short In these circumstances, the distribution of returns is approximately normal This differs significantly from the situation with bank loan portfolios A typical bank portfolio comprises very illiquid loans - indeed, some would venture to suggest that the role of banks is liquidity intermediation – so the modelling horizon is much longer than that considered by trading desks In fact, the primary justification offered for the one-year modelling horizon which is the standard used in the marketplace is that one year is a period long enough to allow the bank to be recapitalised if its loan portfolio declines significantly in value

Loan portfolio distributions are skewed and fat-tailed over the one year modelling horizon A bank is a highly leveraged institution It is concerned not only about standard deviation, it is also concerned about becoming decapitalised Capital is fundamentally about insulating the bank from extreme losses But the risk contribution methodology penalises facilities based on their contribution to variance and not on their contribution to those scenarios for which capital is actually required

A further consideration relates to the nature of capital, while capital is *set* by reference to an extreme loss percentile, such as 99 9% or 99 95% depending on the bank's target debt rating, the bank will be economically bankrupt at much lower loss levels Thus, while the *amount* of capital is set by reference to an extreme percentile, a less extreme percentile is a more relevant threshold for a bank which concerned about its economic independence

4 5 2 1 Capital Allocation Based on Contribution to ETL

An alternative is suggested adopt an expected tail loss (ETL) approach which allocates capital to facilities based on their contribution to portfolio outcomes below a chosen portfolio threshold

$$ETL_P = E\left[L \mid L > Threshold\right] \quad \textbf{Equation 4-54}$$

In other words, the expected tail loss is the average of the portfolio losses, L, for those scenarios where the portfolio loss is beyond the chosen threshold

Portfolio capital, C_p, is then allocated to facilities, C_i, based on the extent to which the average facility value in the scenarios which give rise to a portfolio loss greater than the portfolio loss threshold is below the facility's unconditional average

$$C_i = \frac{E\left[L_i \mid L_p > Threshold\right]}{ETL_p} C_p \qquad \textbf{Equation 4-55}$$

Once again, this measure satisfies the requirement that $\sum_i^n C_i = C_P$

This proposed capital allocation methodology has two desirable properties. Firstly, it satisfies the sub-additivity, homogeneity and monotonicity requirements for a coherent risk measure specified by Artzner *et al* (1997, 1999). Secondly, it is more sensitive to exposure concentrations. A criticism that is often levelled at the contribution to variance methodology is that concentrations have to increase significantly before a noticeable increase in allocated capital is observed.

There is no published work of which the author is aware which suggests how sensitive a portfolio *should* be to exposure size. However, industry practice is that single-name exposures greater than four times the average exposure size are seldom held voluntarily. This rule of thumb is based on the view that exposures of this size will wipe out the net income contribution of over 300 exposures, assuming a 40% LGD and 0.4% net income. However, this rule assumes a portfolio with thousands of exposures; in a portfolio of just 90 exposures, the four times multiplier would certainly be reduced.

This point is borne out in Figure 4.10 below:

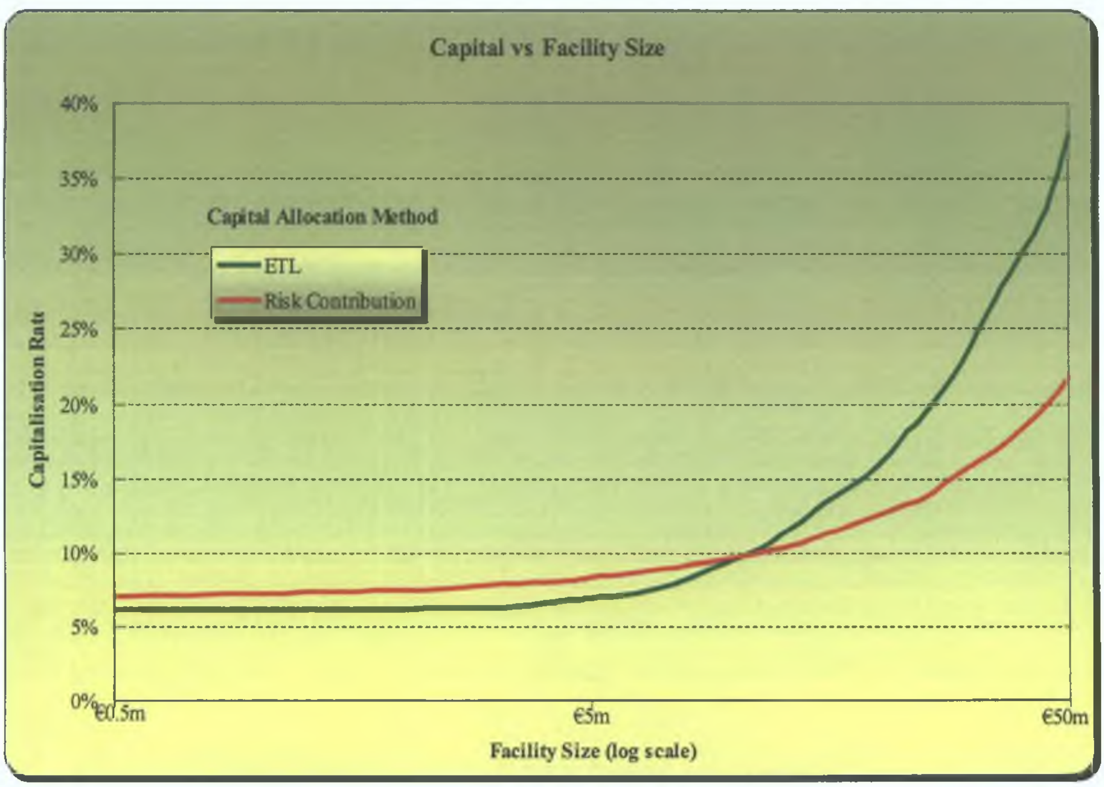

Figure 4.10 Capital Allocation: ETL and Risk Contribution methodologies compared

When a €5m facility is added to a €450m portfolio of 90 loans of €5m each, the capitalisation rate for the facility is 6.9% under the ETL allocation approach and 8.3%

under the Risk Contribution approach However, as the facility size increases, the ETL-based capitalisation rate increases faster and by the time the facility has increased to €50m representing 10% of the total portfolio, the capitalisation rate has increased to 37 9% compared to 21 7% under the Risk Contribution approach While the example is stylised in the sense that corporate banking portfolios are much more diversified with, typically, thousands or tens of thousands of facilities, the same pattern is repeated when tested on a typical bank portfolio of corporate loans This alternative allocation methodology ensures that taking on exposures significantly larger than the average size within the portfolio will exhibit low RORAC and consequently will not be undertaken This accords with typical bank policy whereby larger exposures are only extended to high-grade borrowers or to low-grade borrowers on a secured basis

4 5 3 Determining the Optimal Hold Level and the Concentration Penalty

The model outlined at 4 5 2 1 above determines the capital required to support a putative new facility to be added to a loan portfolio However, in many instances, it is the amount of the loan that is at issue, and not whether or not to grant the loan For example, when participating in a syndicated loan, we need to determine the optimal level of participation Likewise, when extending facilities to a relationship customer, we need to understand what costs are incurred by making credit available which exceeds the optimal hold level What is required is a framework for determining both these quantities

The credit portfolio model provides just such a framework Typically, loans will be issued at a spread in excess of their true value given by the formula at 4 3 1 above However, according as the facility size increases, it becomes a source of concentration in the portfolio and the capital increases at an increasing rate as shown in Figure 4 11 below for the same €450m portfolio described above

The average capitalisation rate increases and the marginal capitalisation rate increases at an even faster rate, as demonstrated in Figure 4 12 below The increased capitalisation rate translates into an increased cost of writing incremental exposure Figure 4 13 demonstrates that the increased capitalisation requirements translate into higher costs for the bank as exposure increases, the marginal cost of assuming additional exposure increases until it exceeds the spread, and, at still higher exposure levels, the average cost exceeds the spread

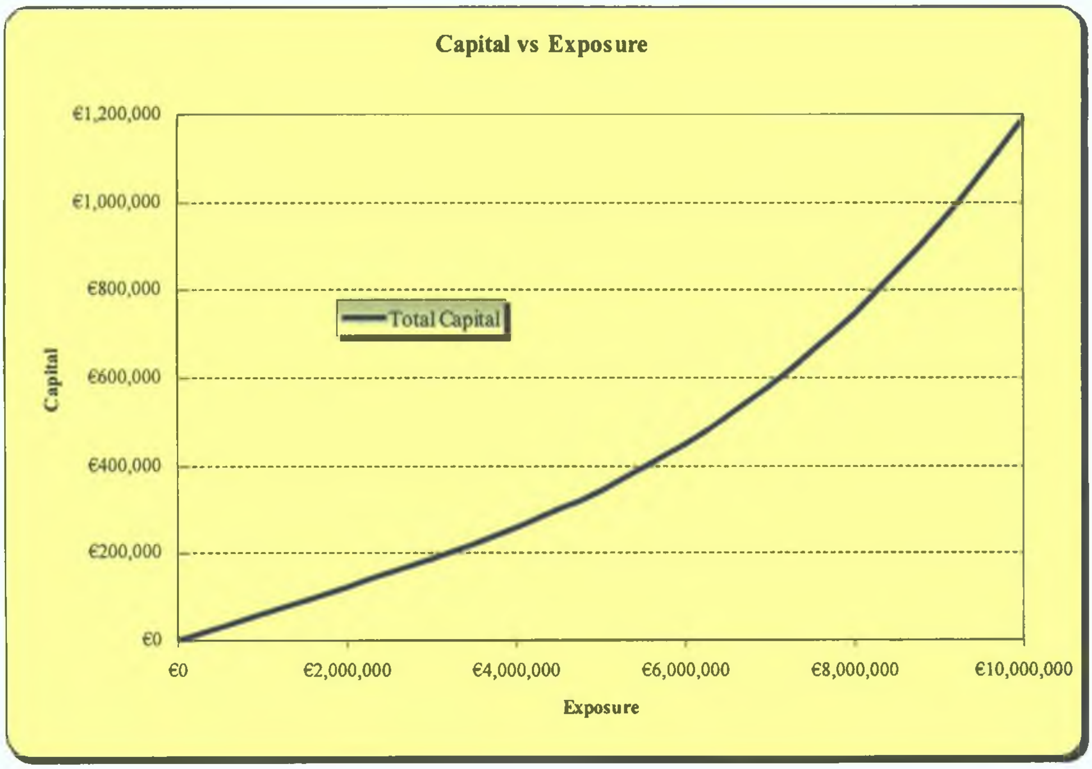

Figure 4.11 Capital as a function of Exposure

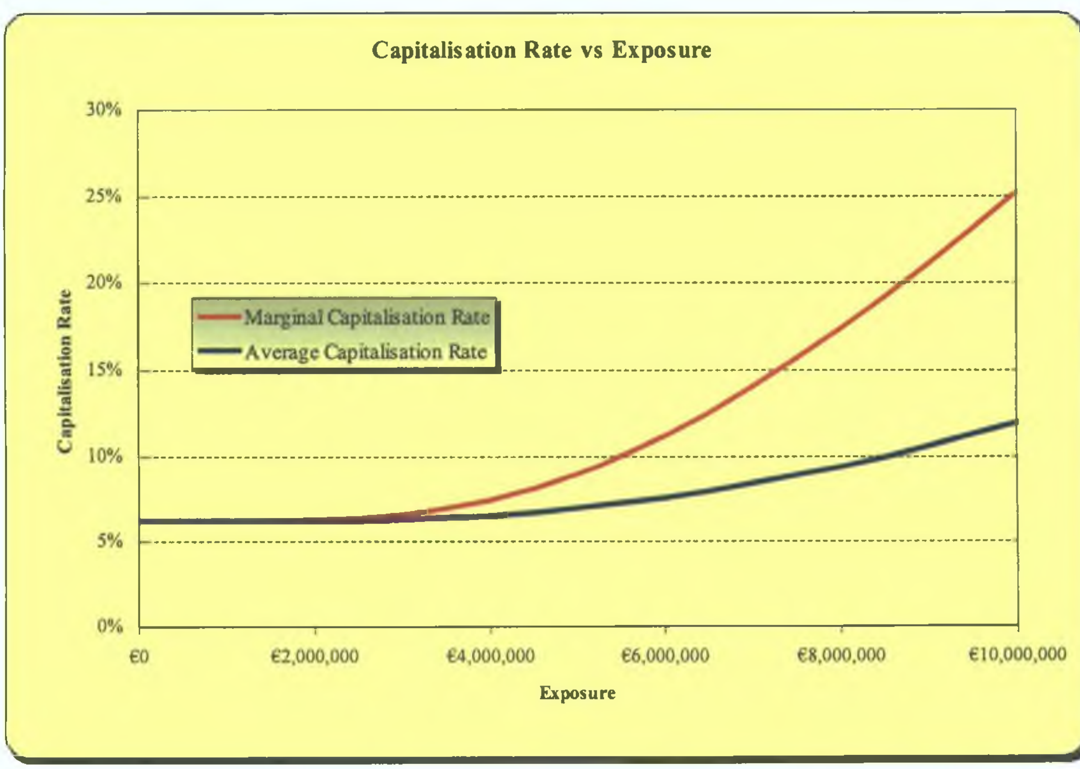

Figure 4.12 Marginal and Average Capitalisation Rates as a function of Exposure

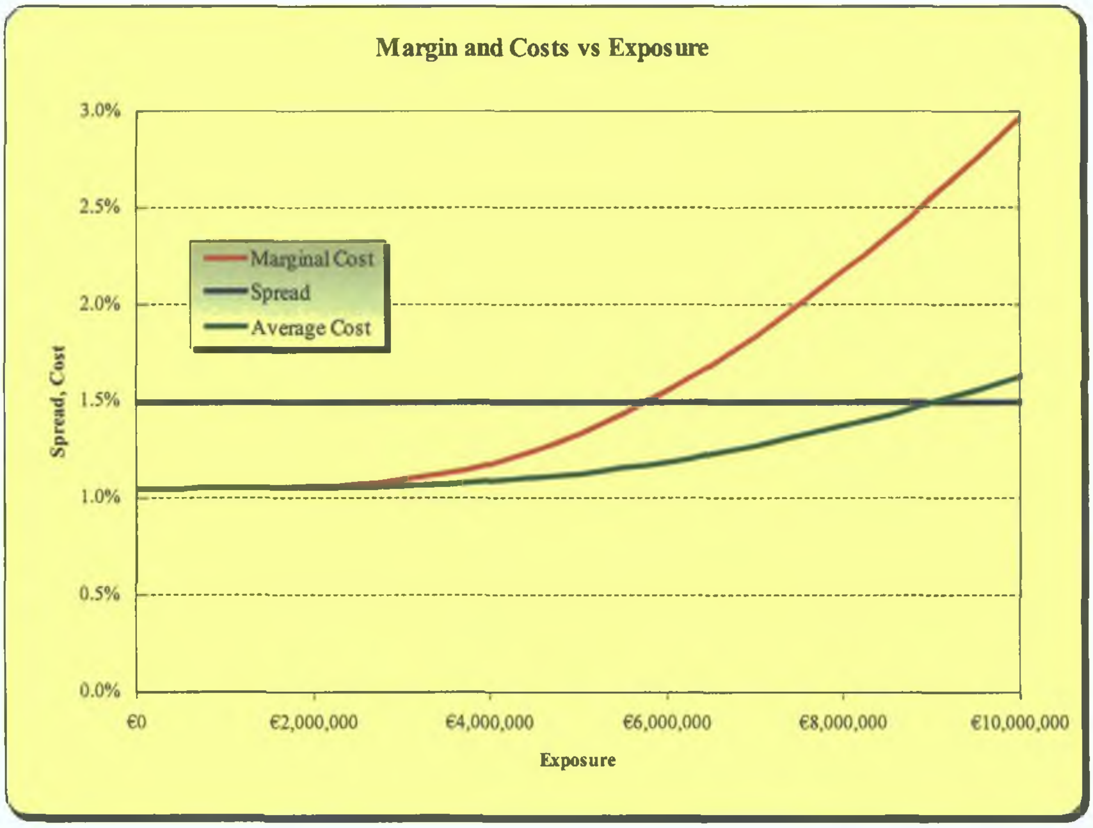

Figure 4.13 Marginal and Average Cost as a function of Exposure

EVA is the primary measure of value in most banks. It is calculated by subtracting the cost of capital times the amount of capital from the expected spread. It is a measure of annualised value added. The EVA in Figure 4.14 rises at first before peaking where the expected spread equals the cost of capital times the marginal capitalisation rate. Beyond that exposure level, the net revenue turns negative as the marginal cost of capital exceeds the expected spread. Clearly, the optimal exposure is that at which the EVA per unit of regulatory capital consumed is above the threshold set by management. If Tier 1 equity– and hence, regulatory capital - is not constrained, then the optimal exposure is that which maximises EVA.

In the €450m portfolio under consideration, EVA is at a maximum for a particular facility at an exposure size of €5.9m. But, the maximum desirable exposure size scales with portfolio size and with the number and diversity of exposures. Thus, larger institutions can comfortably accommodate much larger exposures before detracting from EVA. Likewise, better diversified portfolios will reduce the correlation between the value of a new facility and that of the portfolio.

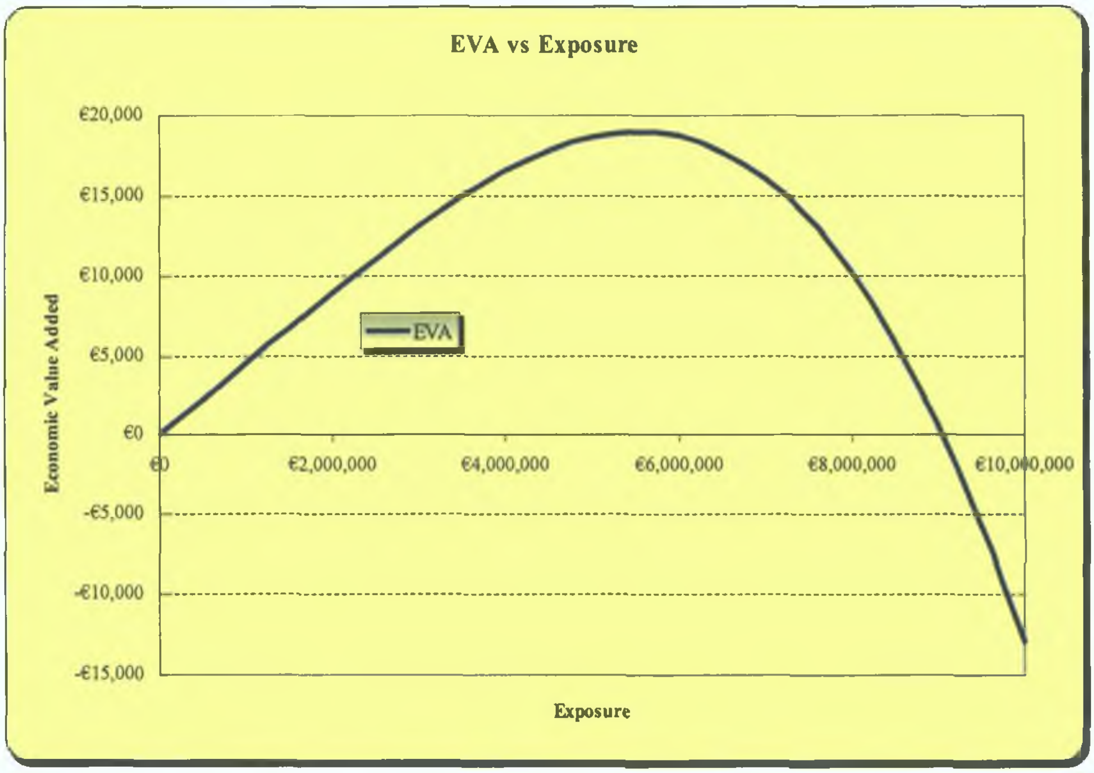

Figure 4.14 Economic Value Added vs Exposure

However, according as the facility becomes a larger fraction of the total portfolio, the benefits of diversification are eroded and that facility's marginal EVA will decline and eventually turn negative. This approach enables a bank to determine two important facts:

- The optimal amount of a syndicated loan to assume is that which maximises EVA or which ensures the marginal EVA per unit of exposure just exceeds the threshold set by management.
- The cost of exceeding the optimal hold level for a relationship customer is the EVA destroyed on the exposure beyond the optimal.

By implementing a loan pricing model linked to the marginal capitalisation rates, the business line will have an incentive to buy credit protection against the names where the exposure exceeds the optimal. It also creates an alignment of interest between the portfolio management function and the business line.

4.5.4 Limit-Setting in Debt Portfolios

The measure of risk of relevance to a bank is economic capital. Thus, economic capital, it is suggested, provides an obvious basis for setting limits to borrowers, sectors and geographies. No further exposure may be assumed if the economic capital required to support the exposure to the obligor exceeds the agreed capital threshold. Such a framework would provide a far more coherent basis for limit setting than the exposure-based limits currently in use which are linked to customer grade based on rather vague qualitative factors.

4.6 Implications for Bank Regulation

The framework created in this chapter demonstrates the impact of portfolio composition on capital. Under Basel II, regulatory capital depends on the obligor PD and on the LGD, exposure at default (EAD) and maturity (M) of the facility. This chapter highlights that facility capital is dependent not just on these variables but, crucially, on the composition of the portfolio also. Poorly diversified portfolios will require significantly more capital than well diversified ones. While this has been understood in regulatory circles for a long time, this framework enables the debate to move from being purely qualitative in nature to quantitative. The regulators, understandably, are reluctant to commit to a quantitative approach until they are satisfied about data quality, modelling and backtesting. However, it is suggested that this approach could be employed as the basis for setting capital in the future.

4.7 Conclusion

In this chapter, a single period credit portfolio model was developed which replicated the current state-of-the-art model.

The KMV approach to the modelling of a portfolio of debt securities over a single time period was replicated based on outline descriptions of their approach in the literature. The success of this modelling effort is confirmed by comparing the loan value distributions at maturity and the value, expected loss, unexpected loss and risk contribution of each loan in the portfolio produced by running the portfolio through KMV's *Portfolio Manager* software.

This model was then used to propose a modification to current market practice and two significant extensions to the state-of-the-art. The capital attribution methodology adopted by industry and embedded in KMV's portfolio model allocates capital in accordance with contribution to variance. This model, the standard in market risk environments, implicitly assumes that variability of value around the mean is the investor's main concern.

However, this author argues that for leveraged portfolios, contribution to extreme outcomes is of much more concern and proposes an allocation scheme based on expected tail loss. This has the effect of penalising concentrations more severely than the market standard framework and offers a concentration sensitivity that is more consistent with intuition and market practice.

The single period model which KMV offers models portfolios *ex post*. Unfortunately, this tells the portfolio manager the effect of individual facilities on the portfolio after they have already been added to the portfolio and the manager has no opportunity to influence the decision. This thesis creates a framework in which the impact of a facility on the portfolio can be measured *ex ante* giving the portfolio manager the ability to influence the decision on whether to add the facility before the fact.

In reality, however, the decision to extend a facility to a customer is seldom a Yes/No. Particularly in syndicated lending, the key concern is "How much?" not "Whether or not"; the issue is one of deciding what size of facility should be made available. Likewise, a bank's best relationship customers will require facilities that exceed those which can be economically accommodated within the bank's portfolio. The bank's appetite will vary depending on the size and composition of its book.

This thesis proposes a model that enables the portfolio manager to determine the impact of a loan increment on the portfolio. As the capitalisation rate increases, the cost of the incremental capital drives the total cost of holding the facility on balance sheet higher than the spread on offer.

This provides the line with a signal as to what the optimal hold amount is. This accords with intuition that suggests that limits not be set as absolute amounts: rather, they should ensure that the return per unit of risk of the last increment of exposure is greater than the minimum threshold. A larger hold amount is warranted if the spread on offer is wider.

Likewise, it measures the cost of holding an excessive exposure to a given obligor. This 'concentration penalty' can be levied on the relationship manager. The penalty provides the portfolio manager with adequate income to spend on the purchase of protection on that name should a CDS market exist.

Finally, it provides the basis for setting borrower limits, an issue with which the market has long struggled Typically such limits have been set as absolute amounts of exposure This thesis suggests that a single capital amount can be used to set limits regardless of the borrower grade

Having replicated the KMV modelling approach in a single time period framework, the next step is to extend this approach in two ways (i) First, model the assets that comprise the portfolio To achieve this, the portfolio model must be converted from a single- to a multi-period framework This will enable the cash flowing from the portfolio over time to be determined (ii) Then, model the liabilities Build a cash flow model to disburse the cash from the assets to the CDO tranches in accordance with the cash flow waterfall specified in the CDO indenture

The development of this new model is the subject of the next chapter

Chapter 5. Developing a Multi-Period Credit Portfolio Model to Value CDO Tranches

5.1 *Chapter Overview*

In this chapter, a new model is developed which will enable the valuation of CDO tranches using a multi-time step Monte Carlo modelling approach in a risk-neutral framework

5.2 *Motivation for Building a New Model*

This thesis grew from dissatisfaction with the process that the rating agencies applied to rating CDO tranches

- The models they employ rely heavily on the rating of the assets contained in the SPV That asset rating process was already seen to be overly heuristic and had failed to identify on a timely basis many assets whose quality had materially changed
- The assumptions regarding asset correlation, the manner in which the portfolio characteristics are distilled into their binomial framework – described more fully in Chapter 6 – and the use of the expected loss measure to assign grade seemed to the author to be overly simplistic

It seemed unlikely that a process that relied on such asset ratings and a rather arbitrary approach to the assessment of portfolio interactions could successfully grade as complex a structured debt product as a CDO tranche

Nor does the market seem much more enlightened The analysis undertaken by many of the systems most commonly used in the market – *Intex, ICDO* and *CDO Vantage*, for example - amounts to nothing more than scenario analysis supported by a cash flow model Most purchasers of senior tranches settle for an assessment that suggests full repayment as long as the number of defaults in the first three years is less than a given multiple of historical loss levels for collateral of that quality No attempt is made to analyse the portfolio interactions, it is hoped that the rating agency requirements regarding industry limits will avoid losses due to concentration that will penetrate the senior tranche

The author was attracted to the structural approach that potentially offered a more rigorous alternative to the rating agency approach The adoption of this approach required appropriate data and it was decided to use KMV data because of its widespread acceptance in the market and its potential to offer a more rigorous approach to modelling the correlation of credit risks

5.3 Outline of Task of Extending the Single Period Model to a Multi-Period Framework

The single period credit portfolio model developed in Chapter 4 used KMV estimates of probability of default – its EDF measure – and of asset correlation The steps taken in developing that model can be summarised as follows

- The assets are valued at the outset using the risk-comparable approach This involved the discounting of expected cash flows using a risk-neutral approach that increased the probability of default to account for the systematic risk that they contained
- The evolution of each obligor's credit term structure was simulated under the true risk measure – that is under the Cumulative Expected Default Frequency measure, $CEDF_H$, to the modelling horizon
  - If the obligor defaulted – in other words, if the random obligor return was more negative than the negative of the obligor's DD - recovery was modelled as a drawing from a beta distribution with the mean and standard deviation specified
  - If the obligor survived, the term structure of the obligor's credit risk was re-computed based on the realised random return over the modelling period With this new credit term structure, the value of the debt instrument is re-computed using the same risk-neutral approach as was applied at the outset

  The simulation applied a factor model approach to embed the KMV asset correlation measures
- The frequency distribution of facility values and portfolio values at the modelling horizon is calculated An analysis of these distributions produces a number of key performance measures

- ➢ Calculating the portfolio average and the average for each of the individual facilities enables the expected portfolio return and expected return for each facility to be calculated in a mark-to-model framework
- ➢ The standard deviation of the facility and portfolio values – called unexpected loss - may be computed The expected return and unexpected loss measures can be combined to obtain the Sharpe ratio The Sharpe ratio is a measure of portfolio performance and of facility performance This is a key metric for investors in unleveraged debt portfolios
- ➢ Where investment in debt portfolios is financed largely by debt – as is the case in the banking sector – the Sharpe ratio alone is insufficient to inform the investor about the desirability of a given investment since it is focused on value volatility In these circumstances, portfolio capital is set by reference to the present value of the difference between the expected portfolio value and some extreme percentile of the distribution Investment performance is then measured by RORAC

Logic would suggest that a similar approach could be applied to the valuation of a CDO tranche and to the measurement of its risk characteristics However, a number of issues complicate this approach

- ❖ The value of a tranche at the start of the modelling period can no longer be calculated in the formulaic manner applied to the valuation of loans
  - ➢ It is no longer possible to determine the amount and the timing of cash flows *ex ante* The cash flows will derive from the evolution of the portfolio over many time periods in the future Thus, rather than modelling over one period as previously, the portfolio must be modelled over *every* period at which cash is disbursed to the tranches until all the underlying portfolio assets have matured or defaulted
  - ➢ The modelling must occur under the risk-neutral measure and not the true measure as previously since the focus for now is on valuation and not on risk
- ❖ Measuring the risk of a tranche is more complicated still
  - ➢ As for the single period model, the probability distribution of security values at the horizon must be calculated

- This requires that the portfolio of underlying securities be simulated under the *true* measure out to the investor's risk horizon

- At the horizon, each tranche must be revalued However, this task is a repeat of that undertaken at the start of the modelling period described above Hence, valuation at horizon demands that a Monte Carlo simulation be undertaken for *each* point on the probability distribution Thus, the calculation of a tranche Sharpe ratio is a 'Monte Carlo squared' problem Obtaining a Sharpe ratio estimate for a CDO tranche with the same precision as was obtained for an individual facility in the single period model would require $100{,}000^2 = 10{,}000$ million multi-period simulations If the underlying portfolio contains ten-year maturity assets and the CDO tranches receive quarterly payments, a total of 400 billion simulations are necessary It is clear that lesser accuracy must be accepted or a good approximation methodology developed

This thesis is focused on the valuation of CDO tranches and therefore concentrates on the first of these challenges The latter will be addressed in subsequent research

5.4 Modelling to Horizon in a Risk-Neutral Framework

In the single period credit portfolio model, simulation took place in actual default space Such modelling is described in the literature as occurring under the true measure, to distinguish it from modelling under the risk-neutral which is undertaken when value, and not risk, is the focus of concern The focus of this model is the *valuation* of CDO tranches and thus, all the modelling in this chapter will occur under the risk-neutral measure The first task, therefore, is to adapt the modelling paradigm of Chapter 4 for use in a risk-neutral framework

As was done previously in the single period model, the portfolio value distribution at horizon must be calculated However, unlike previously, the portfolio must be simulated under the risk-neutral measure to determine whether default has occurred for each individual loan since our interest is in portfolio valuation, and later, tranche valuation This will depend on whether the realised $\tilde{\varepsilon}_H$ for an obligor causes the borrower to fall below its risk-neutral default threshold Clearly, the probability of default will be higher in the risk-neutral framework since *CQDF* will be greater than *CEDF*

5 4 1 Exposure Value ın the Default State

The value of the facılıty ın default ıs obtaıned ın exactly the same way as descrıbed prevıously ın 4 3 5 1 Makıng a drawıng from a beta dıstrıbutıon sımulates the LGD

5 4 2 Forward CQDF ın the Non-Default State

Once agaın, the value of the facılıty at the modellıng horızon ıs a functıon of the shape of the credıt curve and the sımulated asset return, $\tilde{\varepsilon}_H$, over the perıod

However, ın Equatıon 4-50 above, modellıng occurred under the true measure, so we had

$$N_2\left(-N^{-1}\left(CEDF_H\right), DD_{0\,M}, \rho\right) = 1 - CEDF_M \qquad \textbf{Equation 5-1}$$

where $CEDF_H$ and $CEDF_M$ are the true cumulatıve default probabılıty to the horızon and facılıty maturıty, respectıvely, $DD_{0\,M}$ ıs the DD over the perıod to facılıty maturıty, and ρ ıs the correlatıon between the Brownıan processes at horızon and maturıty, respectıvely

Now, when modellıng under the rısk-neutral measure, we have

$$N_2\left(-N^{-1}\left(CQDF_H\right), DD^Q_{0\,M}, \rho\right) = 1 - CQDF_M \qquad \textbf{Equation 5-2}$$

where the varıables are as ın Equatıon 5-1 except they are now measured under the rısk-neutral measure The relatıonshıp between the rısk-neutral and true cumulatıve default probabılıty measures was gıven m Equatıon 4-8 above as

$$CQDF_t = N\left(N^{-1}\left(CEDF_t\right) + \lambda R\sqrt{t}\right) \qquad \textbf{Equation 5-3}$$

By the same token, under the true measure, the ımpact of $\tilde{\varepsilon}_H$ on the DD was captured ın Equatıon 4-48 above as

$$DD_{H\,M} = \frac{DD_{0\,M}}{\sqrt{1-\rho^2}} + \frac{\rho}{\sqrt{1-\rho^2}}\tilde{\varepsilon}_H - R\lambda\sqrt{M-H} \qquad \textbf{Equation 5-4}$$

Under the rısk-neutral measure, thıs formula ıs modıfıed to

$$DD^Q_{H,M} = \frac{DD^Q_{0\,M}}{\sqrt{1-\rho^2}} + \frac{\rho}{\sqrt{1-\rho^2}}\tilde{\varepsilon}_H \qquad \textbf{Equation 5-5}$$

since no additional adjustment needs to be made for systematic risk

Hence,

$$CQDF^{Q}_{H\,M} = N\left(-DD^{Q}_{H\,M}\right) \qquad \textbf{Equation 5-6}$$

5.5 Modelling to Subsequent Horizons

From 5 4 2, we have the cumulative quasi-default probability from the first modelling horizon to maturity However, we are no longer interested in valuing the potential future cash flows at the first horizon instead, we wish to model the portfolio over the next time period Thus, we wish to determine the probability of default under the risk-neutral measure from one horizon to the next For CDO valuation purposes, the modelling horizons correspond to the dates on which cash is paid to the tranches, this typically occurs quarterly or semi-annually

We determine the cumulative quasi probability of default from *H* to *2H* by interpolating between *H* and *M* using the same interpolation scheme as we used previously in 4 3 5 2

$$CQDF^{Q}_{H\,2H} = 1-\left(1-CQDF^{Q}_{H\,M}\right)^{\frac{2H-H}{M-H}} \qquad \textbf{Equation 5-7}$$

This allows us to repeat the modelling procedure at the second and subsequent modelling horizons

5.6 Modelling the Liabilities

The procedure outlined above allows the cash flows from the assets to be modelled The next stage is to distribute the available cash to the various tranches in accordance with the CDO indenture

The most important factors to incorporate are the O/C and the I/C tests Breaching these tests will cause the cash available from the assets to be diverted to more senior tranches Breaching the mezzanine O/C test will see the residual cash flow used to repay the most senior outstanding principal or outstanding interest should there be any Likewise, breaching the senior O/C test causes all cash beyond that required to pay senior interest to be used to repay senior principal

The overall result is that all the cash flowing from the assets is disbursed to the liabilities

5.7 *Implementing the Model*

This new model is now applied to the valuation of the portfolio modelled above in 4.4. This portfolio comprises ninety loans of $5m each, rated B1 to B3 by Moody's, paying Libor + 250bp. The loans have maturities between 4.5 and 10 years. The portfolio is financed by $360m of Aaa-rated senior notes paying Libor + 50bp and $40m of mezzanine paying Libor + 150bp. The senior O/C test is 1.2 while the corresponding mezzanine test is 1.05. The senior I/C test is 1.8 and mezzanine I/C is 1.2.

The model which has been developed can cater for any combination of asset characteristics and securitisation structures. However, the portfolio chosen has the characteristics of a typical arbitrage cash flow CDO. These comprise 80-100 loans to sub-investment grade names with maturities in the five- to ten-year bracket. Likewise the O/C and I/C tests are set at levels typically seen in the market. The results which follow should therefore be representative of those for the securitisation class.

5.7.1 Tranche Cash Flows

An example of the cash flow waterfall is shown in Figure 5.1 and Figure 5.2 below. The two figures give the key cash flows, balance sheet values and O/C and I/C test values from five simulations semi-annually over the ten year period:

The cash inflow to the structure comes from three sources: (i) Interest from the assets, (ii) principal from maturing assets; and (iii) principal recovered from defaulting assets.

On the cash outflow side of the structure, the cash is distributed to the tranches in priority: (i) Senior tranche interest is paid first. (ii) Subject to passing all relevant tests, mezzanine tranche interest is paid. (iii) Outstanding interest, if any, is paid. (iv) Principal – both recovered and maturing - is channelled to the tranches in priority order, and (v) should all relevant tests be passed and there not be any interest outstanding on any tranche, equity is entitled to the residual.

Interest from Assets gives the amount of interest flowing from the underlying loans. It declines over time as assets default or mature. No assets mature during the first five years. When assets default, a random draw is made to determine the recovery amount.

If the senior tranche O/C or I/C tests are breached, no cash is paid to the mezzanine or equity tranches. Instead, that cash is diverted to pay down the senior tranche principal. The amount of interest due to the mezzanine tranche accrues in the outstanding interest account which is repaid as soon as interest payments can be resumed Only if all tests are in compliance will any cash be paid to the equity tranche

All recoveries and maturing principal amounts are applied to the senior tranche first Only when the senior is fully repaid is any cash paid to the mezzanine tranche Finally, if all the notes are repaid, all remaining interest and principal flowing from the assets is applied to equity

5 7 2 Tranche Valuation

Since all the cash flows are modelled in risk-neutral space, their present value may be obtained by discounting at the risk-free interest rate

The average present value of the cash flow from the assets equals the value of the assets calculated using the method outlined in 4 3 1 3 above The sum of the tranche values will clearly equal the value of the assets since all the cash flows from the assets are distributed to the tranches

Thus, we are able to value each of the tranches across all the simulations

Period	0 5yrs	1 0yrs	1 5yrs	2 0yrs	2 5yrs	3 0yrs	3 5yrs	4 0yrs	4 5yrs	5 0yrs
	Interest from Assets									
	5 360 445	8 217 371	9 150 238	9 318 501	10 189 465	10 528 497	11 119 619	10 911 098	11 495 779	11 527 019
	5 645 787	8 703 104	10 308 438	10 649 715	11 678 524	12 052 359	12 906 553	12 274 985	12 776 047	12 500 832
	5 579 902	8 605 070	10 192 049	10 649 715	11 808 587	12 190 892	12 906 553	13 184 244	13 890 651	13 636 946
	5 663 769	8 704 915	10 312 587	10 770 735	11 946 718	12 190 892	12 906 553	12 881 158	13 572 974	13 636 946
	5 496 975	8 413 919	9 502 834	9 560 540	9 667 720	9 558 767	10 239 300	10 304 926	10 862 032	10 877 810
	Maturing Principal									
	0	0	0	0	0	0	0	0	0	5 000 000
	0	0	0	0	0	0	0	0	0	5 000 000
	0	0	0	0	0	0	0	0	0	5 000 000
	0	0	0	0	0	0	0	0	0	5 000 000
	0	0	0	0	0	0	0	0	0	5 000 000
	Recovered Principal									
	17 505 843	10 907 881	5 320 132	4 117 917	0	1 591 939	9 537 496	0	1 071 235	0
	2 809 588	0	1 904 981	2 103 797	0	0	17 376 312	3 782 068	10 828 924	0
	4 916 840	0	0	0	0	3 208 881	0	0	7 445 723	0
	1 582 635	0	0	0	2 677 941	3 640 746	6 461 381	0	3 783 681	1 488 952
	7 904 989	13 800 046	8 463 083	17 457 026	8 181 557	0	3 487 667	0	3 627 266	9 482 618
	Senior Tranche Interest									
	4 396 851	5 154 578	6 331 848	6 174 989	7 213 624	7 213 624	7 920 588	7 596 588	8 246 847	8 127 541
	4 396 851	5 375 759	6 884 364	6 847 648	8 055 596	8 055 596	8 984 469	8 542 439	9 169 215	8 770 555
	4 396 851	5 344 044	6 843 750	6 843 750	8 098 967	8 098 967	8 951 212	8 951 212	9 717 425	9 511 802
	4 396 851	5 394 225	6 908 012	6 908 012	8 175 016	8 113 935	8 956 920	8 792 551	9 545 183	9 440 692
	4 396 851	5 299 072	6 520 180	6,299 579	6 982 440	6 734 582	7 439 288	7 279 338	7 818 886	7 634 675
	Mezzanine Tranche Interest									
	628 539	0	0	2 618 824	1 052 346	0	0	3 314 510	0	2 685 606
	628 539	742 005	910 946	910 946	1 052 346	1 052 346	1 157 547	1 157 547	0	2 528 023
	628 539	742 005	910 946	910 946	1 052 346	1 052 346	1 157 547	1 157 547	1 244 647	1 244 647
	628 539	742 005	910 946	910 946	1 052 346	1 052 346	1 157 547	1 157 547	1 244 647	1 244 647
	628 539	742 005	0	0	0	0	0	0	0	3 243 136
	Cash to Equity									
	335 055	0	0	524 688	1 923 495	0	0	0	0	713 872
	620 398	2 585 340	2 513 127	2 891 121	2 570 582	2 944 417	2 764 538	2 575 000	0	1 202 254
	554 513	2 519 021	2 437 353	2 895 019	2 657 275	3 039 579	2 797 795	3 075 485	2 928 579	2 880 497
	638 380	2 568 685	2 493 629	2 951 777	2 719 357	3 024 610	2 792 087	2 931 060	2 783 144	2 951 607
	471 585	2 372 842	0	0	0	0	0	0	0	0
Initially	**Senior Tranche Principal Outstanding**									
360 000 000	342 494 157	328 523 483	320 384 962	316 267 044	316 267 044	311 360 232	298 623 706	298 623 706	294 303 539	289 303 539
360 000 000	357 190 412	357 190 412	355 285 431	353 181 634	353 181 634	353 181 634	335 805 322	332 023 254	317 587 498	312 587 498
360 000 000	355 083 160	355 083 160	355 083 160	355 083 160	355 083 160	351 874 280	351 874 280	351 874 280	344 428 556	339 428 556
360 000 000	358 417 365	358 417 365	358 417 365	358 417 365	355 739 424	352 098 678	345 637 297	345 637 297	341 853 616	335 364 664
360 000 000	352 095 011	338 294 965	326 849 228	306 131 241	295 264 403	292 440 218	286 152 540	283 126 952	276 456 539	261 973 921
Initially	**Mezzanine Tranche Principal Outstanding**									
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
	SeniorTrancheOCMatrix									
	1 226	1 191	1 191	1 202	1 202	1 192	1 193	1 206	1 193	1 210
	1 246	1 246	1 238	1 232	1 232	1 232	1 206	1 205	1 199	1 216
	1 239	1 239	1 239	1 239	1 239	1 236	1 236	1 236	1 219	1 223
	1 242	1 242	1 242	1 242	1 237	1 235	1 230	1 230	1 229	1 223
	1 221	1 212	1 198	1 164	1 158	1 168	1 177	1 188	1 199	1 202
	MezzTrancheInterestOutstandingMatrix									
	0	742 005	1 669 849	0	0	1 052 346	2 240 346	148 215	1 397 474	0
	0	0	0	0	0	0	0	0	1 244 647	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	910 946	1 842 638	2 943 461	4 073 245	5 348 666	6 660 996	8 112 908	6 366 863
	SeniorTrancheInterestOutstandingMatrix									
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	MezzTrancheICMatrix									
	1 939	0 000	0 000	1 588	1 508	0 000	0 000	1 457	0 000	1 426
	1 981	1 822	1 660	1 650	1 567	1 567	1 480	1 476	0 000	1 447
	1 968	1 811	1 661	1 661	1 577	1 572	1 525	1 525	1 469	1 470
	1 975	1 817	1 666	1 666	1 575	1 571	1 514	1 514	1 479	1 468

Figure 5 1 Key Variable Values for Five Random Asset Paths

Period	5 5yrs	6 0yrs	6 5yrs	7 0yrs	7 5yrs	8 0yrs	8 5yrs	9 0yrs	9 5yrs	10 0yrs
	Interest from Assets									
	9 925 103	8 734 004	7 535 300	6 072 407	4 936 487	4 213 410	3 499 155	2 390 046	1 374 962	1 213 990
	10 780 395	9 270 099	7 867 179	6 362 710	5 096 092	3 953 406	2 955 799	1 836 975	799 500	883 709
	11 791 020	10 209 925	8 826 709	7 028 960	5 938 235	5 036 970	4 155 855	3 053 599	1 878 547	1 213 990
	11 688 002	9 666 739	7 806 713	6 205 657	5 272 230	4 478 250	4 015 243	2 920 349	1 668 343	1 070 756
	8 918 014	7 407 288	6 542 246	5 287 962	4 285 793	3 830 932	2 906 728	2 233 019	1 240 472	740 862
	Maturing Principal									
	20 000 000	40 000 000	55 000 000	35 000 000	25 000 000	30 000 000	40 000 000	35 000 000	10 000 000	40 000 000
	35 000 000	45 000 000	45 000 000	40 000 000	30 000 000	30 000 000	35 000 000	35 000 000	0	30 000 000
	35 000 000	55 000 000	60 000 000	40 000 000	35 000 000	35 000 000	40 000 000	45 000 000	20 000 000	40 000 000
	30 000 000	50 000 000	60 000 000	35 000 000	30 000 000	20 000 000	40 000 000	45 000 000	25 000 000	30 000 000
	25 000 000	30 000 000	50 000 000	35 000 000	20 000 000	30 000 000	25 000 000	35 000 000	20 000 000	25 000 000
	Recovered Principal									
	3 940 454	1 353 330	0	2 063 424	3 236 287	0	0	0	0	0
	0	6 470 920	9 467 020	2 095 703	13 448 111	8 622 853	1 717 668	0	0	0
	0	0	3 726 301	0	0	0	0	0	4 061 107	0
	13 911 077	9 974 849	0	0	0	0	0	0	0	4 930 540
	11 066 558	0	0	0	0	3 160 143	0	0	0	0
	Senior Tranche Interest									
	6 289 945	5 769 439	4 870 349	3 674 556	2 868 735	2 254 831	1 602 580	732 913	0	0
	6 796 178	6 035 219	4 916 154	3 731 950	2 816 719	1 872 084	1 032 358	234 054	0	0
	7 379 747	6 618 788	5 422 996	4 037 479	3 167 812	2 406 853	1 645 894	776 227	0	0
	7 291 391	6 336 691	5 032 737	3 728 236	2 967 277	2 315 027	1 880 193	1 010 526	32 150	0
	5 695 753	4 911 606	4 259 355	3 172 271	2 411 312	1 976 479	1 255 522	711 980	0	0
	Mezzanine Tranche Interest									
	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	977 107	724 690
	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	397 941	397 941
	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	774 977	167 634
	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	1 009 667	415 951
	3 222 261	2 495 682	2 282 891	2 115 691	1 703 431	1 009 667	1 009 667	1 009 667	952 803	447 970
	Cash to Equity									
	2 625 491	1 954 897	1 655 285	1 388 184	1 058 085	948 912	886 908	647 466	397 856	11 779 256
	2 974 550	2 225 213	1 941 357	1 621 093	1 269 707	1 071 655	913 774	593 254	401 559	14 720 545
	3 401 606	2 581 469	2 394 046	1 981 814	1 760 756	1 620 450	1 500 294	1 267 705	1 103 571	34 405 208
	3 386 944	2 320 381	1 764 309	1 467 754	1 295 286	1 153 556	1 125 382	900 156	626 525	19 106 607
	0	0	0	0	171 050	844 786	641 539	511 373	287 668	7 545 672
Initially	**Senior Tranche Principal Outstanding**									
360 000 000	265 363 086	224 009 755	169 009 755	131 946 332	103 710 044	73 710 044	33 710 044	0	0	0
360 000 000	277 587 498	226 116 578	171 649 558	129 553 855	86 105 744	47 482 891	10 765 223	0	0	0
360 000 000	304 428 556	249 428 556	185 702 256	145 702 256	110 702 256	75 702 256	35 702 256	0	0	0
360 000 000	291 453 587	231 478 738	171 478 738	136 478 738	106 478 738	86 478 738	46 478 738	1 478 738	0	0
360 000 000	225 907 364	195 907 364	145 907 364	110 907 364	90 907 364	57 747 221	32 747 221	0	0	0
Initially	**Mezzanine Tranche Principal Outstanding**									
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	38 710 044	28 710 044	0
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	15 765 223	15 765 223	0
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	30 702 256	6 641 148	0
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	16 478 738	0
40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	37 747 221	17 747 221	0
	SeniorTrancheOCMatrix									
	1 225	1 250	1 331	1 402	1 495	1 696	2 522	NaN	NaN	NaN
	1 243	1 283	1 369	1 467	1 684	2 211	6 038	NaN	NaN	NaN
	1 248	1 303	1 400	1 510	1 671	1 981	3 081	NaN	NaN	NaN
	1 235	1 253	1 341	1 429	1 550	1 677	2 259	40 575	NaN	NaN
	1 217	1 251	1 336	1 443	1 540	1 818	2 443	NaN	NaN	NaN
	MezzTrancheInterestOutstandingMatrix									
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	4 314 979	2 937 882	1 738 816	676 683	0	0	0	0	0	0
	SeniorTrancheInterestOutstandingMatrix									
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0
	MezzTrancheICMatrix									
	1 522	1 512	1 525	1 514	1 507	1 519	1 548	1 624	1 752	NaN
	1 554	1 553	1 573	1 576	1 597	1 632	1 659	2 393	2 393	NaN
	1 581	1 604	1 635	1 672	1 719	1 793	1 955	2 662	7 574	NaN
	1 555	1 523	1 541	1 556	1 575	1 593	1 650	1 828	2 671	NaN

Figure 5 2 Key Variable Values for Five Random Asset Paths *(contd)*

5.8 Comparison with Alternative CDO Valuation Approaches

The best-selling CDO valuation software in the market is *CDO Manager* from RiskMetrics This model applies a copula approach to determine the time of default for each asset in the portfolio This approach is decidely easier to implement since each simulation produces a time at which each asset defaults This obviates the need for multi-period simulation If the asset defaults beyond its maturity date, it repays the full principal amount Knowing when the assets default or mature, all the cash flows can be derived Equity correlation is used in place of asset correlation to generate the correlated random variables which simulate the default time RiskMetrics then apply a cash flow waterfall in a similar manner to that described above More frequently, however, *CDO Manager* is used to value synthetic CDOs – this simplifies matters still further since only the premium payments and payments on default need to be recorded

This method has become the market standard Its simplicity is its attraction However, it needs CDS spreads in order to provide the risk-neutral default probability term structure measure necessary to inform the model For the names which are included in synthetic securitisation structures, these are spreads readily available But for the sub-investment grade names which comprise arbitrage securitisation structures, this information is absent Banks are obliged to rely on their internal ratings to determine the default probability and to make some – presumably heuristic – adjustments to these to make them risk-neutral

The author is unaware of any other CDO valuation model, details of which have been published in the literature It is clear that many banks, particularly investment banks, have bespoke models, these are presumably deemed to be proprietary and have not been published

5.9 Model Results

The graphs that follow show the distributions of the relevant variables

5.9.1 Asset and Tranche Value Distributions

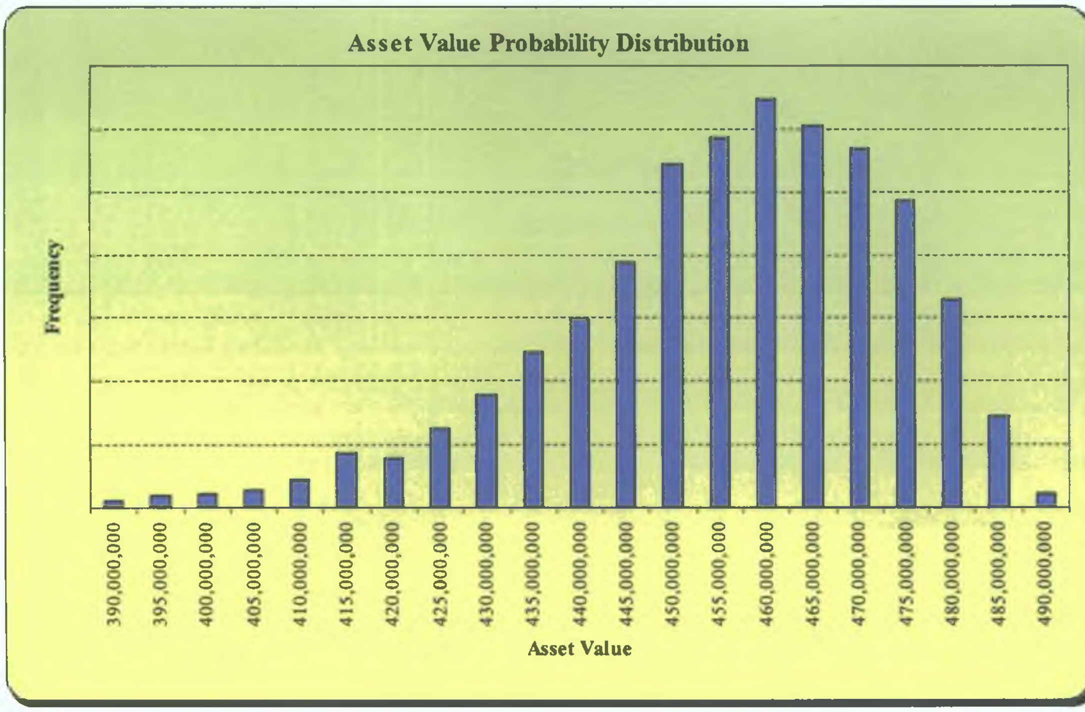

Figure 5.3 Asset Value Probability Distribution

Figure 5.3 above shows the probability distribution of asset values. This variability in asset values translates into variability of tranche values as shown in Figure 5.4, Figure 5.6 and Figure 5.7 below:

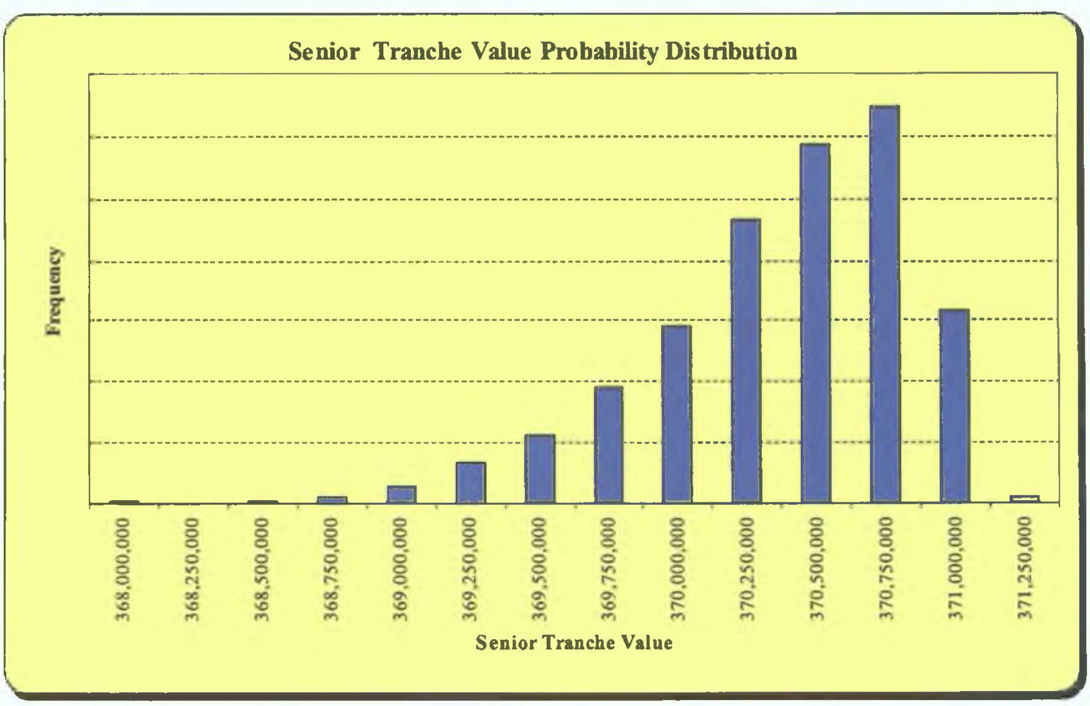

Figure 5.4 Senior Tranche Value Probability Distribution

The variability in senior tranche value arises from the variability in the timing of principal repayment: the higher the incidence of asset default early in the life of the structure, the earlier the senior repayments and the lower the value of the asset as the senior tranche earns the spread for a shorter period. This may be seen clearly by plotting the value of the senior tranche against average life: the value increases monotonically with average life in Figure 5.5 below:

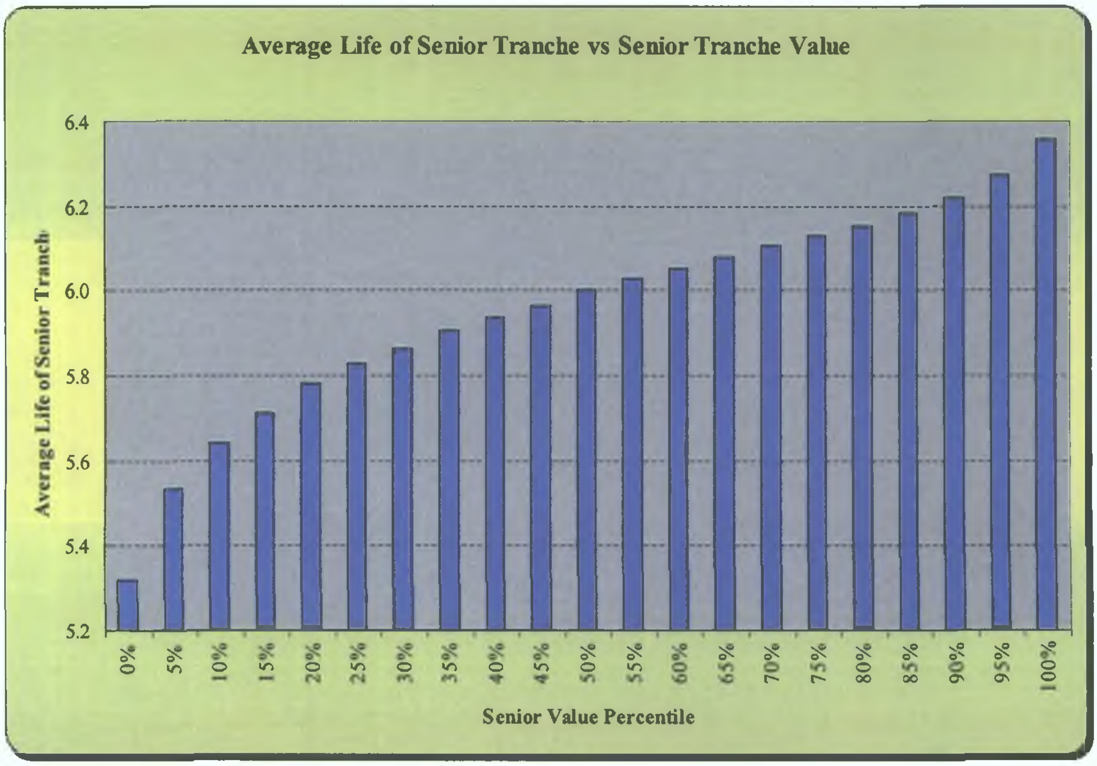

Figure 5.5 Average Life vs Value of Senior Tranche

The mezzanine tranche value distribution is shown in Figure 5.6:

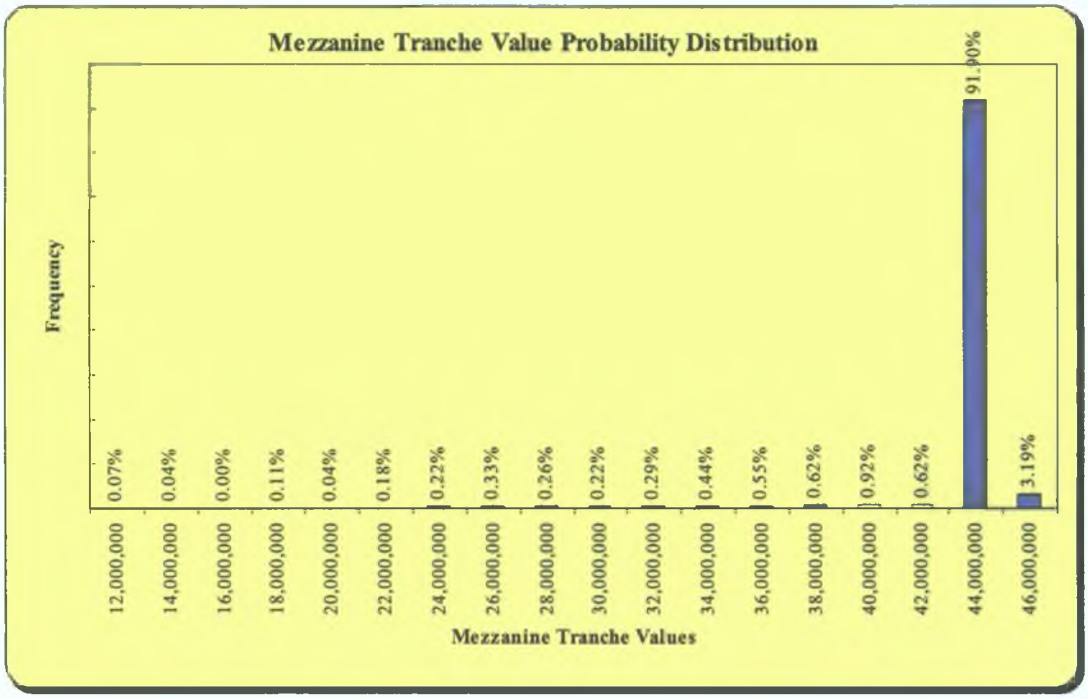

Figure 5.6 Mezzanine Tranche Value Probability Distribution

The equity tranche value is extremely volatile as evidenced in Figure 5.7:

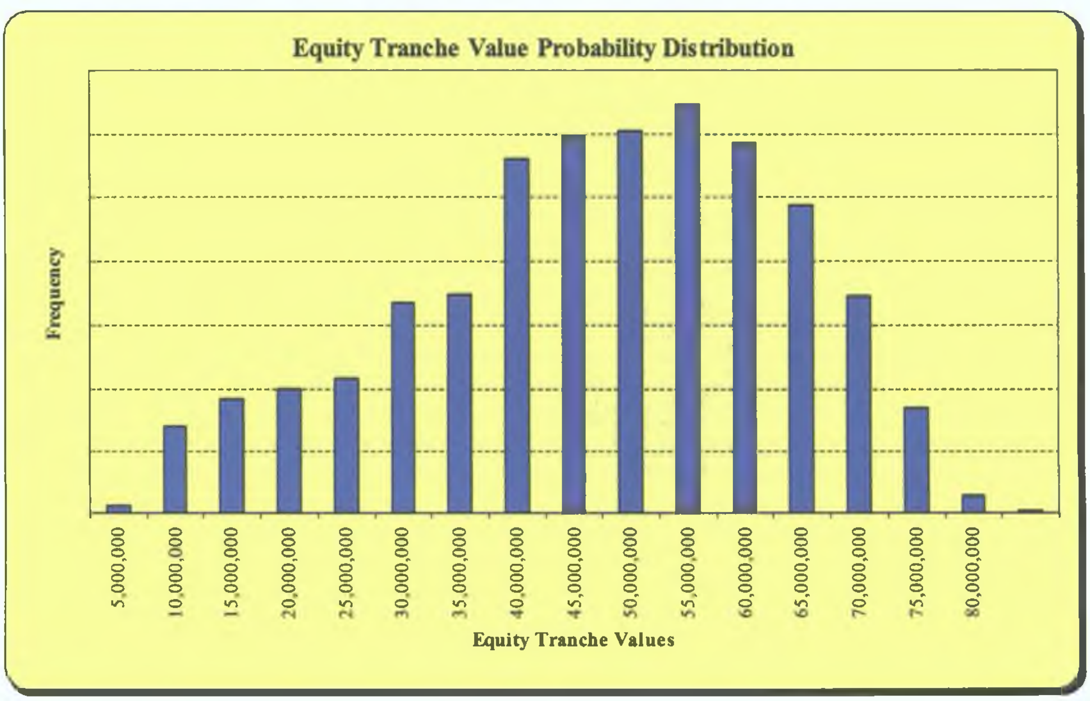

Figure 5.7 Equity Tranche Value Probability Distribution

The variability in the value of the underlying assets is transmitted to the tranches but not in a uniform manner as shown in Table 5-1:

	Assets	Senior	Mezzanine	Equity
Mean	453,162,648	370,252,103	43,288,550	39,621,940
St Dev	18,098,251	476,154	2,876,867	16,384,908
Coefficient of Variation	3 99%	0 13%	6 65%	41 35%

Table 5-1 Variability of Asset and Tranche Values

The senior tranche is insulated from the volatility in the value of the underlying assets by the subordinated tranches The equity and mezzanine tranches that are providing this protection to the senior tranche experience significantly higher value volatility than the underlying assets

This is seen more clearly in Figure 5 8 below When the underlying assets are arranged in value order, the average senior tranche value in the lowest five percent range is 99 57% of the average senior tranche value in the highest five percent range In contrast, the corresponding figure for the mezzanine is 76 63% and for equity, it's a mere 7 53% The structure has functioned as intended the senior tranche remains immune to the losses in the underlying collateral

By the same token, the equity tranche absorbs the losses until the collateral starts to default at a very high rate The mezzanine value in the second lowest 5% bucket has a value of 99 22% of the highest 5% bucket losses only penetrate the mezzanine tranche when the equity has effectively been decimated

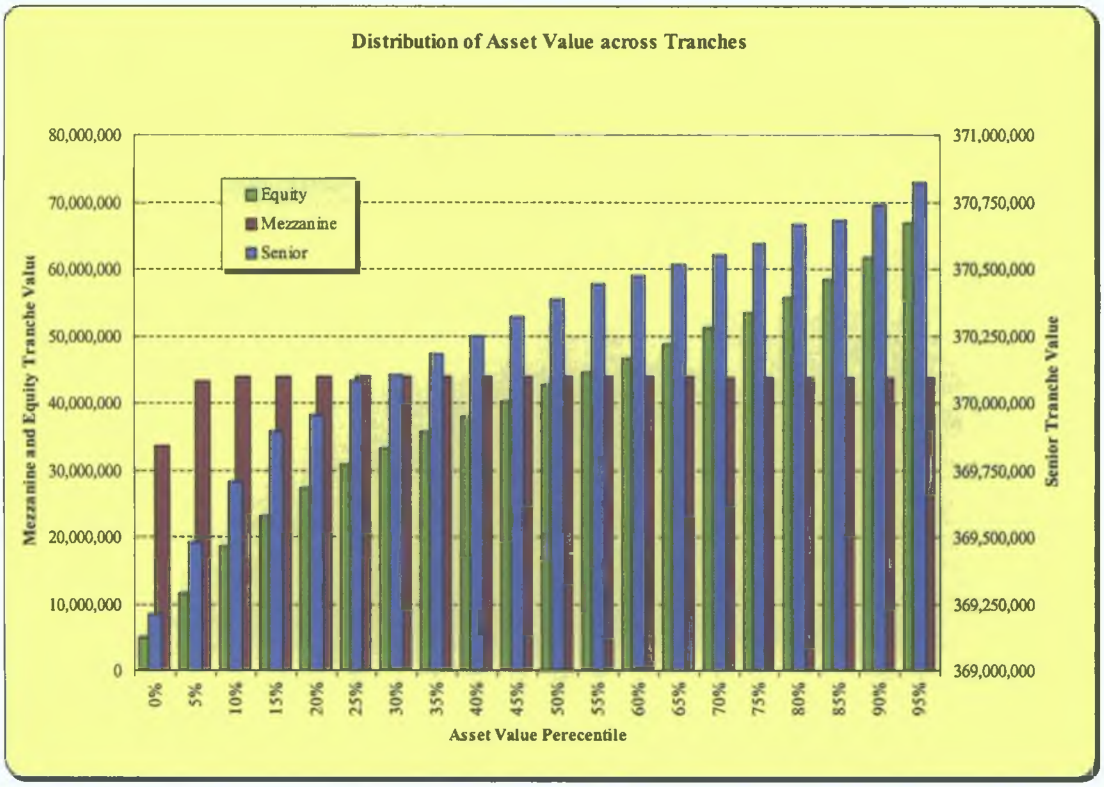

Figure 5.8 Tranche Values vs Asset Value

This value redistribution from the junior tranches to the senior tranche is highlighted again in Figure 5.9 below: whereas the senior receives only 77% of the value in the highest 5% of asset value outcomes, it receives 90.5% in the lowest 5%. The equity share declines from 13.9% to 1.2% over the same range.

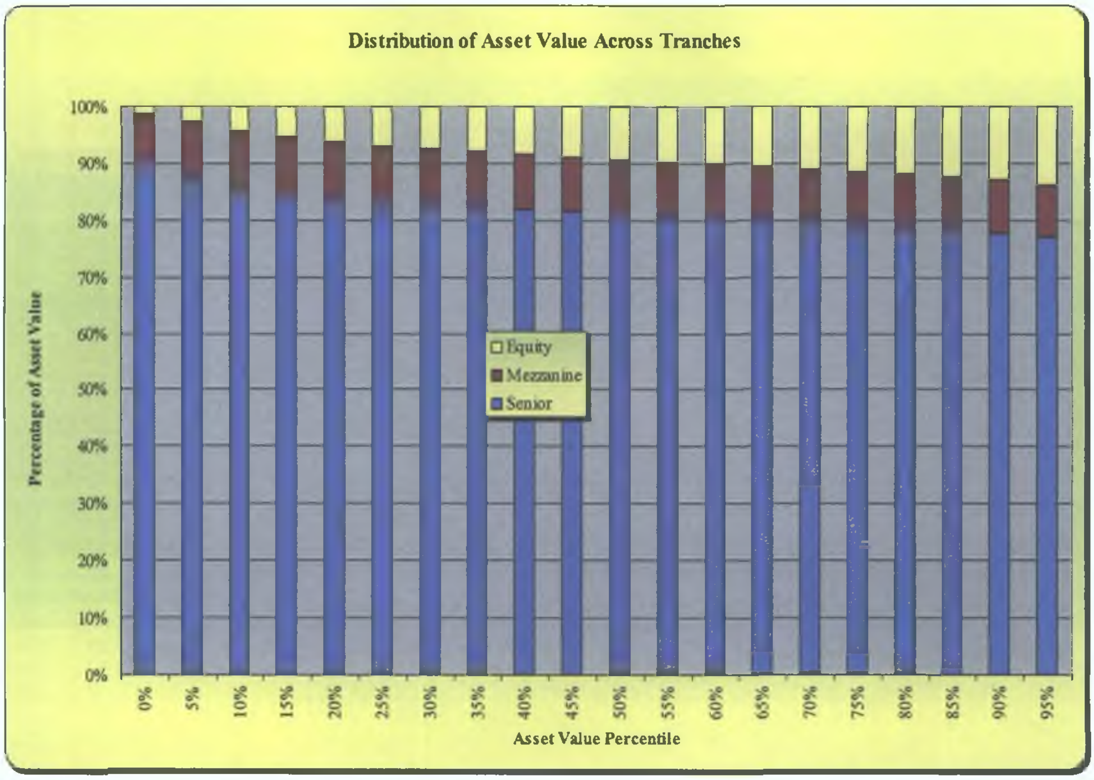

Figure 5.9 Tranche Value Distribution vs Asset Value

5.9.2 The Nature of the Risk in CDO Tranches

The senior debt never experienced loss of principal in any of the simulations. However, as alluded to already, there is some uncertainty regarding timing of receipt of principal.

The average principal profile is shown in Figure 5.10 below:

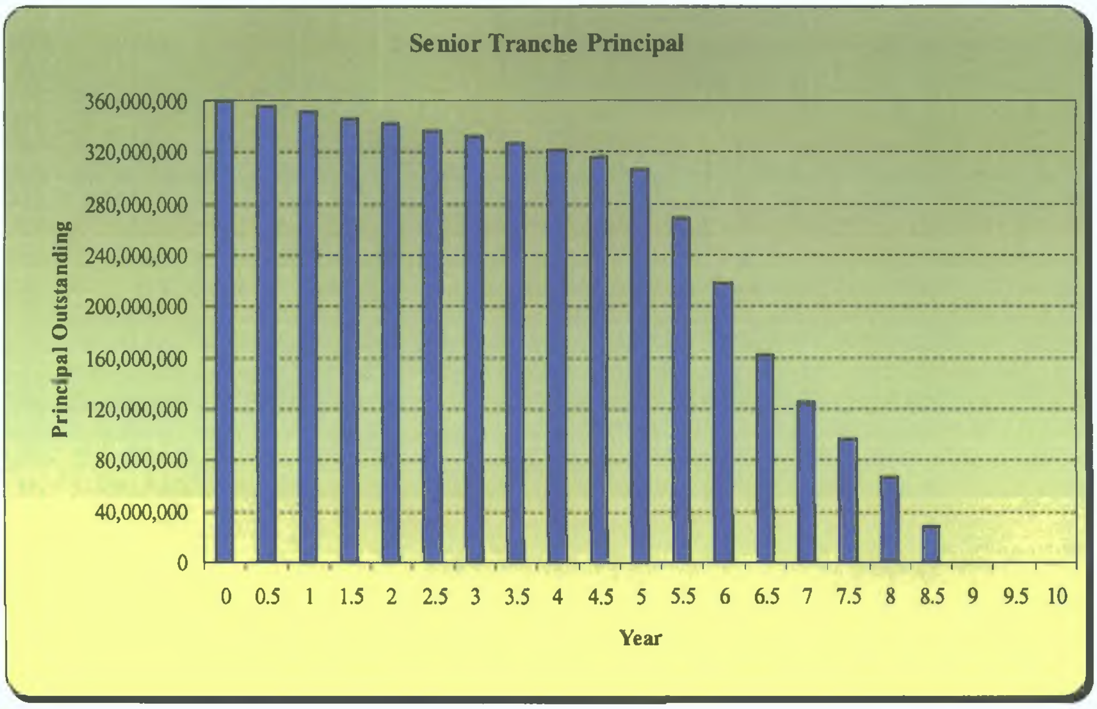

Figure 5.10 Senior Tranche Principal Profile

However, there is some variation around this average that causes the variation in average life shown in Figure 5.11 below:

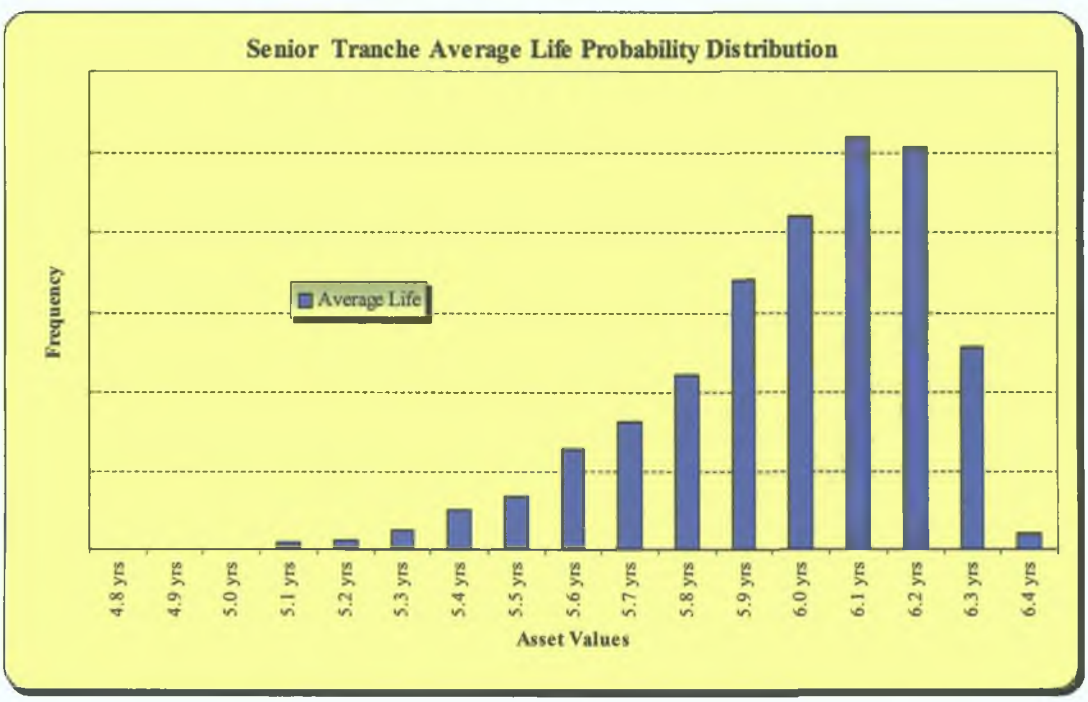

Figure 5.11 Probability Distribution of Senior Tranche Average Life

The average cash received by the senior tranche varies widely dictated by the pattern of defaults among the assets. The cash flow pattern is that of an amortising loan with an uncertain amortisation schedule.

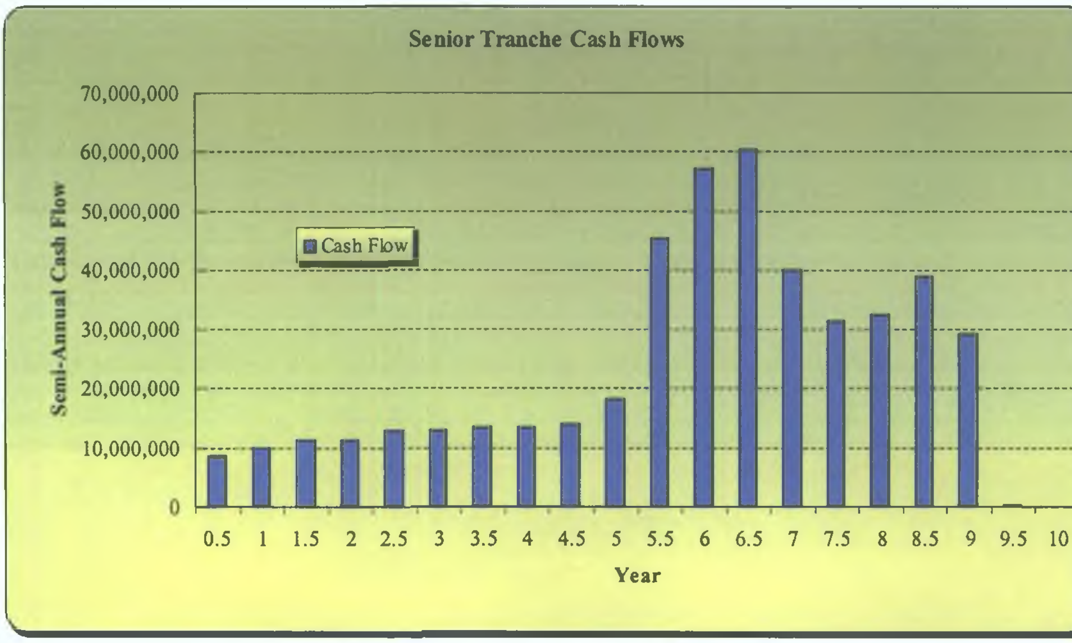

Figure 5.12 Senior Tranche Cash Flow Profile

Figure 5.6 shows that the mezzanine tranche trades in a tight value range much of the time. However, when losses occur, they can be substantial as evidenced in Figure 5.13 below:

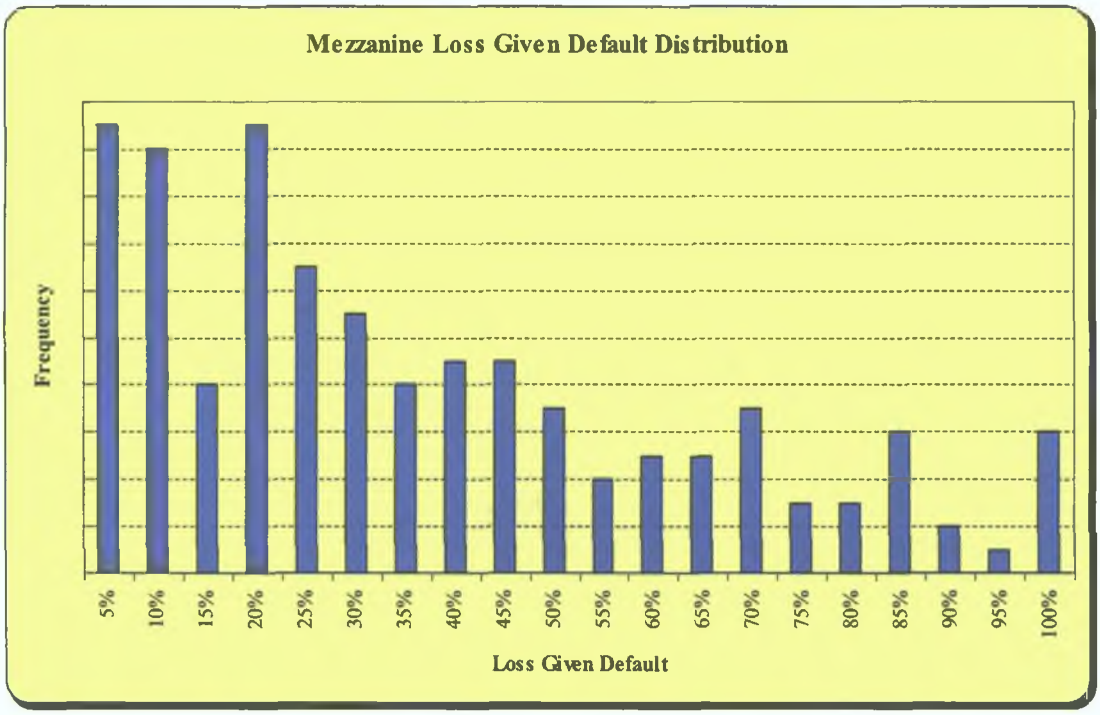

Figure 5.13 Probability Distribution of LGD for Mezzanine Tranche

While the average LGD is only 36.9%, in some cases the entire principal is lost. It is this latter feature of mezzanine tranches that so concerns the regulator.

The cash flows to the mezzanine tranche resemble those of a bullet maturity bond as shown in Figure 5.14 below. However, mezzanine debt can be PIKed, that is, paid in kind: this occurs when interest is capitalised and the cash that was available to pay the interest is used to pay down senior tranche principal. This means there is potential for significant variation around this average cash flow, a major source of worry for mezzanine debt purchasers.

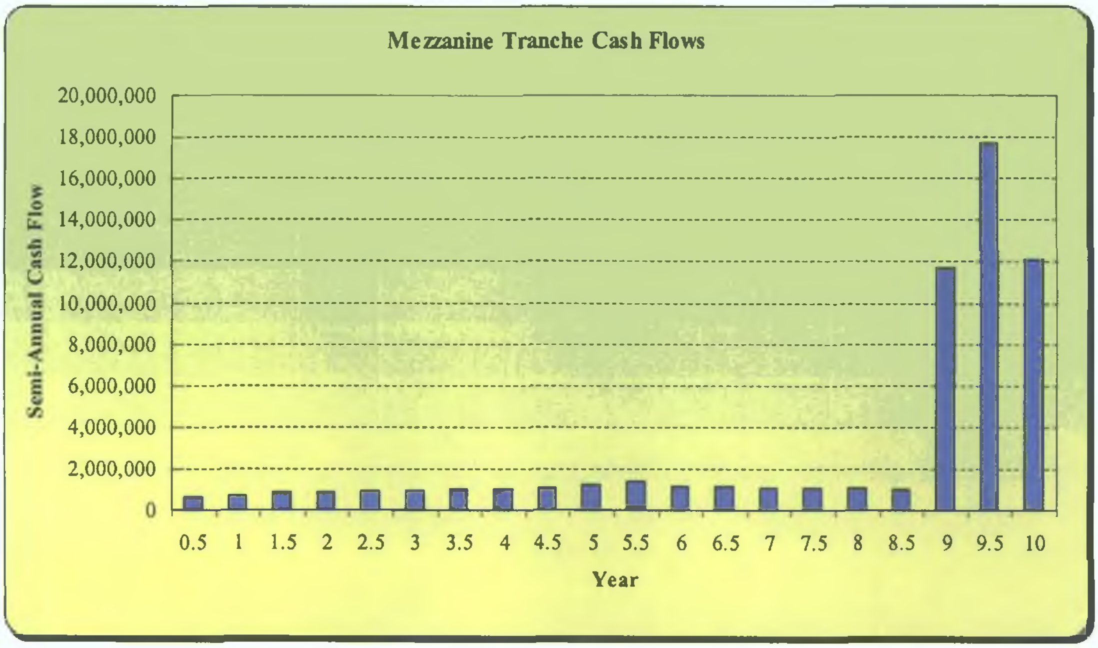

Figure 5.14 Average Mezzanine Tranche Cash Flow Profile

The uncertainty regarding timing of receipt of cash flows is a risk dimension unlike that associated with typical corporate loans. This is shown in Figure 5.15 below which graphs average interest outstanding over time:

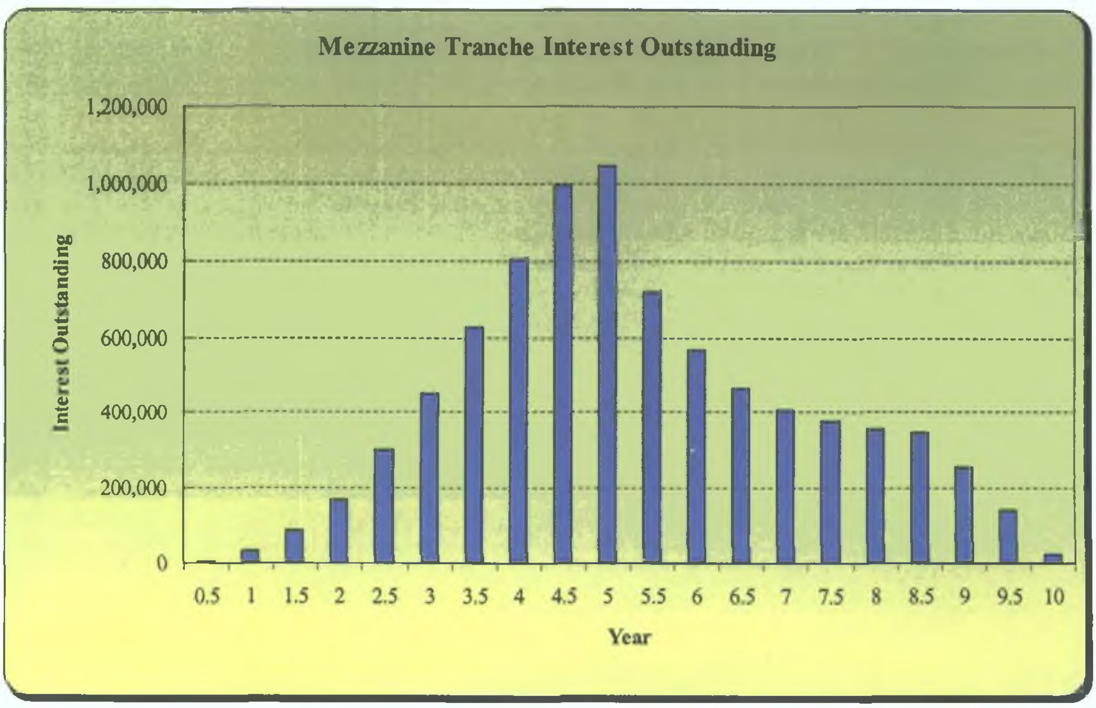

Figure 5.15 Profile of Average Mezzanine Interest Outstanding

An appreciation of the variation around this average can be gained by graphing the distribution of the number of periods in which the mezzanine tranche was PIKed.

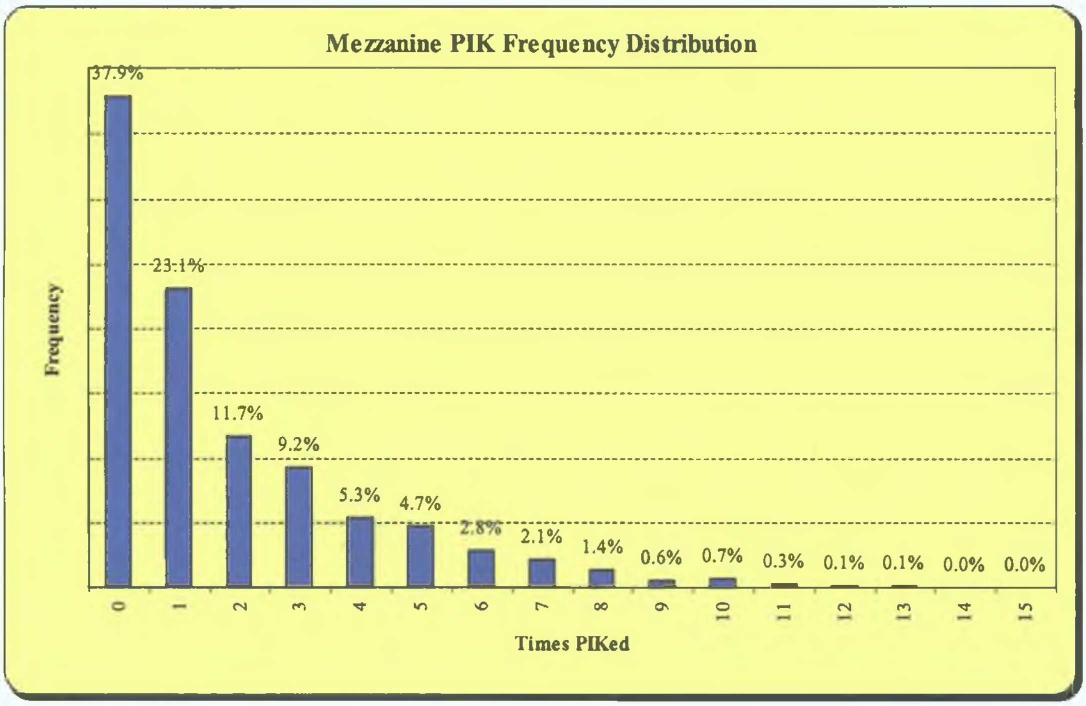

Figure 5.16 Frequency Distribution of Mezzanine Tranche PIKing

Over 62% of the time, the mezzanine tranche will experience at least one occasion when it does not receive interest due. In some cases, it becomes a zero-coupon bond for a period of up to five years. This type of behaviour renders it unsuitable as an investment product for many investors despite its relatively low loss experience.

5.10 The Expected Loss on Rated Tranches

Moody's assign ratings to CDO tranches based on the expected loss of the tranche over its life. This new model produces an expected loss value for the tranches also. We can use the output to compare the Moody's rating, derived in Chapter 6, with the comparable rating implied by this model.

The expected loss for the senior debt across all simulations is 0%. This is not unusual and it suggests a Moody's Aaa rating.

The expected loss for the mezzanine debt is 1.28%. This can be seen to be the product of the frequency of 'default' – though the term is nebulous for tranches – and LGD. In 6.05% of the simulations, there was a shortfall to some extent on the repayment of principal and/or interest. The average loss in the event of default was 36.9%, though that varied widely as indicated in Figure 5.13. This is the key output for comparing this model with Moody's.

5.11 Conclusion

This chapter presents a newly created multi-period CDO model in a structural framework The model takes the existing state-of-the-art and adapts it to incorporate an analysis of CDO tranches In so doing, it presents an alternative perspective on the CDO rating question

- ❖ It enables the Moody's CDO tranche rating methodology to be compared with a new alternative
- ❖ It also provides a richer framework for thinking about the risk which CDO tranches present

It concludes that the uni-dimensional view of risk – that of expected loss only – which guides the agency rating process fails to meet the needs of the regulator for a basis on which to assign capital In fact, expected loss in banking is simply regarded as a cost of doing business and risk is measured by loss variability It is suggested that Moody's CDO ratings framework is flawed since it does not measure risk at all In particular, the rating methodology fails to meet the regulator's need for a measure on which to base its requirement for bank capital

In Chapter 6, the Moody's rating framework will be developed and in Chapter 7, the results of the two modelling approaches will be compared

Chapter 6. Re-constructing Moody's CDO Tranche Rating Approach

6.1 Chapter Overview

Moody's assign ratings to CDO tranches using their BET This approach was outlined by Cifuentes and O'Connor (1996) and re-created in 3 9 above But, the outline, while giving the basic Moody's rating philosophy, is insufficient to enable their tranche ratings to be replicated Neither have they issued a model that embeds their CDO tranche rating methodology

However, a model that is capable of replicating Moody's approach is a requirement in order to enable comparison with the alternative methodology that has been developed in this thesis In this chapter, the author builds a fully-functional BET model which can accommodate the complete details which their papers do not specifically address

Moody's further requirements - not specified in the literature but gleaned by the author from conversations with Moody's personnel, investment banks and other market participants - were noted These requirements, together with Moody's published guidelines, were incorporated into a new cash flow model designed to replicate the Moody's rating process

The success of the model in replicating their rating approach is then confirmed by testing the model on newly issued deals The model-derived ratings are very closely aligned to the actual ratings granted by Moody's to the rated tranches of the deals

6.2 Creating a Moody's BET Model

The results shown in 3 9 1 1 prove that the models the author constructed produce the same results as those quoted by Moody's in their publications However, the structures addressed by Moody's in those three papers is very stylised and it ignores many of the details that Moody's take into account in practice

The tasks to be undertaken in constructing a full-blown BET model are sketched in Figure 6 1 below

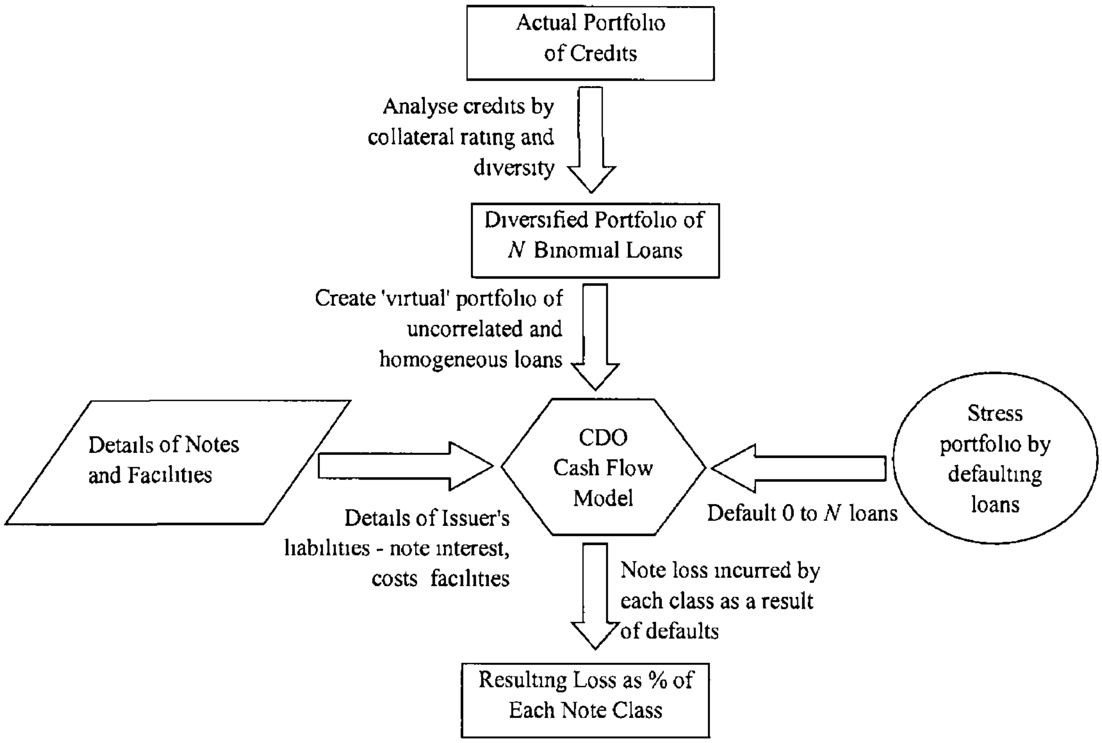

Figure 6 1 Schematic for BET Replication

In developing the model, details of the idealised portfolio – the number of binomial bonds, the default rate, the recovery rate on defaulted assets, the weighted average maturity and the weighted average coupon – as discussed already must be combined with details of the actual portfolio, including the value of the portfolio, the price paid and the timing of the acquisition, the so-called 'ramp-up' period Other factors that are also considered include (i) interest rate stresses, (ii) expenses – management, rating agency, accounting and trustee fees, (iii) hedging (if any), (iv) liquidity requirements, and (v) tax

The model that was built tracks the various cash flows from the idealised portfolio as described below

6 2 1 Principal Cash Flows

Principal derives from three separate sources (i) redemption of collateral, (ii) recoveries, and (iii) excess spread On the other hand, principal amounts may be applied to pay down note principal or to reinvest in new assets during the reinvestment period

6 2 2 Revenue Cash Flows

In the BET framework, there are five sources of revenue (ı) collateral ınterest ıncome, (ıı) reınvestment ıncome, (ııı) revenue from hedgıng, (ıv) drawıngs on the lıquıdıty reserve, and (v) release of the lıquıdıty reserve at maturıty

Revenue ıs used to (ı) pay senıor expenses, (ıı) pay note ınterest lıabılıtıes, (ııı) replenısh the lıquıdıty reserve, and (ıv) pay surplus to the holder of the equıty Excess spread may be dıverted to the prıncıpal ledger ın partıcular cırcumstances

6 2 3 Other Cash Flow Modellıng Stresses

Moody's requıre that the default tımıng be varıed The standard test has 50% of total defaults occurrıng ın the first year wıth 10% occurrıng annually thereafter for the next five years Whıle thıs proves to be the most severe tımıng for most tranches, Moody's oblıge that back-end loaded defaults and mıd-loaded defaults be undertaken ın addıtıon to the standard front-loadıng and the most conservatıve ratıng outcome ıs applıed to the tranche The complete lıst of default scenarıos whıch must be examıned are outlıned ın Table 6-1 below

Scenarıo	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
1	50%	10%	10%	10%	10%	10%
2	10%	50%	10%	10%	10%	10%
3	10%	10%	50%	10%	10%	10%
4	10%	10%	10%	50%	10%	10%
5	10%	10%	10%	10%	50%	10%
6	10%	10%	10%	10%	10%	50%

Table 6-1 Default Tımıng Scenarıos

6.3 Replicating Moody's Ratıng Results

The model tracks the cash flows from the assets to the varıous tranches over tıme Each scenarıo assumes a dıfferent number of defaults The cash flows receıved by each tranche are dıscounted at Lıbor plus the promısed tranche spread, ıf no losses occur, the tranche value wıll equal par The loss ıncurred by each tranche weıghted by the bınomıal probabılıty of the loss occurrıng wıll equal the tranche expected loss Thıs expected loss ıs compared to the average loss ıncurred by debt of sımılar maturıty ın

Moody's loss database The rating whose loss matches the expected loss of the tranche is the rating that is assigned to the tranche

A sample of the intermediate calculations follows to indicate how these calculations were undertaken

6.4 Discussion of Modelling Details

The figures which follow summarise the results for a single scenario The diversity score is 47 based on the industry and country composition of the 90-loan portfolio, the weighted average rating is B2 and the weighted average life is 7 25 years, or 29 quarters The scenario shown involves front-loaded defaults – 50% in the first year and 10% per annum in the five subsequent years – and six of the 47 idealised loans defaulting

Figure 6 2 shows a high default rate over the first four quarters followed by a lower default rate over the remaining quarters until the end of year 6 The assets which have not defaulted are assumed to be redeemed as bullet payments at the end of the weighted average life The LGD is assumed to be 45% and occurs after a one-year delay

The Class A notes are the senior notes in the structure and the Class B are the mezzanine If the senior O/C or I/C test is breached – as happens in Quarters 4, 5 and 6 - the mezzanine tranche is not paid interest and any cash beyond that required to pay the senior interest is diverted to pay down senior principal until the test is corrected If the senior O/C and I/C tests are passed but the mezzanine tests are not, no cash is paid to equity and the residual is diverted to pay down the senior principal The senior expenses include rating agency costs, trustee fees and management fees

The last rows of Figure 6 6 and Figure 6 7 give the total cash – comprising interest and principal – received quaterly over the 29 quarters of the structure's life The present value of each cash flow stream, discounted at the promised yield, gives the value of the senior and mezzanine notes The shortfall from par, expressed as a percentage of par gives the loss rate incurred

6 4 1 Account Ledgers

Quarter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Assets															
Opening Balance	450 000 000	442 819 149	435 638 298	428 457 447	421 276 596	419 840 426	418 404 255	416 968 085	415 531 915	414 095 745	412 659 574	411 223 404	409 787 234	408 351 064	406 914 894
Start Defaults	7 180 851	7 180 851	7 180 851	7 180 851	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	442 819 149	435 638 298	428 457 447	421 276 596	419 840 426	418 404 255	416 968 085	415 531 915	414 095 745	412 659 574	411 223 404	409 787 234	408 351 064	406 914,894	405 478 723
Class A Notes															
Opening Balance	360 000 000	360 000,000	360 000 000	360 000 000	358 004,648	352 181 510	346 284 534	342 335 066	338 385 598	337 595 705	336 805 811	336 015 917	335 226 024	334 436 130	333 646 236
Redemptions	0	0	0	1 995 352	5 823 137	5 896 976	3,949 468	3 949 468	789 894	789 894	789 894	789 894	789 894	789 894	789 894
Closing Balance	360 000 000	360 000 000	360 000 000	358 004 648	352 181 510	346 284 534	342 335 066	338 385 598	337 595 705	336 805 811	336 015 917	335 226 024	334 436 130	333 646 236	332 856 343
Class B Notes															
Opening Balance	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
Seller Equity															
Opening Balance	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000,000	50 000 000	50 000 000	50 000 000
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000

Figure 6 2 The Securitisation Balance Sheet

Quarter	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Assets															
Opening Balance	406 914 894	405 478 723	404 042 553	402 606 383	401 170 213	399 734 043	398 297 872	396 861 702	395 425 532	393 989 362	392 553 191	392 553 191	392 553 191	392 553 191	392 553 191
Start Defaults	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	1 436 170	0	0	0	0	0
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	392 553 191
Closing Balance	405 478 723	404 042 553	402 606 383	401 170 213	399 734 043	398 297 872	396 861 702	395 425 532	393 989 362	392 553 191	392 553 191	392 553 191	392 553 191	392 553 191	0
Class A Notes															
Opening Balance	333 646 236	332 856 343	332 066 449	331 276 556	330 486 662	329 696 768	328 906 875	328 116 981	327 327 088	326 537 194	325 747 300	324 957 407	324 167 513	323 377 619	322 587 726
Redemptions	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	322 587 726
Closing Balance	332 856 343	332 066 449	331 276 556	330 486 662	329 696 768	328 906 875	328 116 981	327 327 088	326 537 194	325 747 300	324 957 407	324 167 513	323 377 619	322 587 726	0
Class B Notes															
Opening Balance	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40 000 000
Closing Balance	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	40 000 000	0
Seller Equity															
Opening Balance	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	29 965 466
Closing Balance	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	50 000 000	20 034 534

Figure 6 3 The Securitisation Balance Sheet (contd)

6 4 2 Cash Flow Waterfall

Quarter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
PRINCIPAL															
Source															
Opening Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Recoveries	0	0	0	0	3 949 468	3 949 468	3 949 468	3 949 468	789 894	789 894	789 894	789 894	789 894	789,894	789 894
Receipt of Seller Revenue	0	0	0	1 995 352	1 873 669	1 947 508	0	0	0	0	0	0	0	0	0
Closing Balance	0	0	0	1 995 352	5 823 137	5 896 976	3 949 468	3 949 468	789 894	789 894	789 894	789 894	789 894	789 894	789 894
Application															
Opening Balance	0	0	0	1 995 352	5 823 137	5 896 976	3 949 468	3 949 468	789 894	789 894	789 894	789 894	789 894	789 894	789,894
Class A Notes	0	0	0	1 995 352	5 823 137	5 896 976	3 949 468	3 949 468	789 894	789 894	789 894	789 894	789 894	789 894	789 894
Class B Notes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Seller Equity	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
INTEREST															
Source															
Opening Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Portfolio Yield	10 395 000	10 229 122	10 063 245	9 897 367	9 731 489	9 698 314	9 665 138	9 631 963	9 598 787	9 565 612	9 532 436	9 499 261	9 466 085	9 432 910	9,399 734
Liquidity Drawings	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	10 395 000	10 229 122	10 063 245	9 897 367	9 731 489	9 698 314	9 665 138	9 631 963	9 598 787	9 565 612	9 532 436	9 499 261	9 466 085	9 432 910	9 399 734
Application															
Opening Balance	10 395 000	10 229 122	10 063 245	9 897 367	9 731 489	9 698 314	9 665 138	9 631 963	9 598 787	9 565 612	9 532 436	9 499 261	9 466 085	9 432 910	9 399 734
Senior Expenses	556 250	548 172	540 093	532 015	523 936	522 320	520 705	519 089	517 473	515 858	514 242	512 626	511 011	509 395	507 779
Class A Notes	6 516 000	6 516 000	6 516 000	6 516 000	-6 479 884	6 374 485	6 267 750	6 196 265	6 124 779	6 110 482	6 096 185	-6 081 888	6 067 591	6 053 294	6 038 997
Class B Notes	854 000	854,000	854 000	854 000	854 000	854 000	854 000	-854 000	854 000	854 000	854 000	854 000	854,000	854 000	854 000
Diversion of Seller Revenue	0	0	0	1 995 352	1 873 669	1 947 508	0	0	0	0	0	0	0	0	0
Seller Equity	2 468 750	2 310 951	2 153 152	0	0	0	2 022 683	2 062 609	2 102 535	2 085 272	2 068 009	-2 050 746	2 033 483	2 016 221	1 998 958

Figure 6 4 Cash Flow Waterfall

Quarter	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
PRINCIPAL															
Source															
Opening Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Redemptions	0	0	0	0	0	0	0	0	0	0	0	0	0	0	392 553 191
Recoveries	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	0
Receipt of Seller Revenue	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789,894	392 553 191
Application															
Opening Balance	789 894	789 894	789,894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	789 894	392 553 191
Class A Notes	789 894	789 894	789 894	789 894	789 894	789 894	789 894	-789 894	789 894	789 894	789 894	789,894	789 894	789 894	322 587 726
Class B Notes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40 000 000
Seller Equity	0	0	0	0	0	0	0	0	0	0	0	0	0	0	29 965 466
INTEREST															
Source															
Opening Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Portfolio Yield	9 399 734	9 366 559	9 333 383	9 300 207	9 267 032	9 233 856	9 200 681	9 167 505	9 134 330	9,101 154	9 067 979	9 067 979	9 067 979	9 067 979	9 067 979
Liquidity Drawings	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Closing Balance	9 399 734	9 366 559	9 333 383	9 300 207	9 267 032	9 233 856	9 200 681	9 167 505	9 134 330	9 101 154	9 067 979	9 067 979	9 067 979	9 067 979	9 067 979
Application															
Opening Balance	9 399 734	9 366 559	9 333 383	9 300 207	9 267 032	9 233 856	9 200 681	9 167 505	9 134 330	9 101 154	9 067 979	9 067 979	9 067 979	9 067 979	9 067 979
Senior Expenses	507 779	506 164	504 548	502 932	501 316	499 701	498 085	496 469	494 854	493 238	491 622	491 622	491 622	491 622	-491 622
Class A Notes	6 038 997	6 024 700	6 010 403	5 996 106	5 981 809	5,967 512	5 953 214	-5 938 917	5 924 620	5 910 323	5 896 026	5,881 729	5 867 432	5 853 135	5 838 838
Class B Notes	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854,000	854 000	854 000	854 000	854 000	854 000	-854 000
Diversion of Seller Revenue	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Seller Equity	1 998 958	1 981 695	1 964 432	1 947 170	1 929 907	1 912 644	1 895 381	1 878 119	1 860 856	1 843 593	-1 826 330	1 840 627	1 854 924	1 869 221	1 883 519

Figure 6 5 Cash Flow Waterfall (contd)

6.4.3 Cash Flow and Coverage Tests

Quarter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Expenses															
Senior Expenses	556,250	548,172	540,093	532,015	523,936	522,320	520,705	519,089	517,473	515,858	514,242	512,626	511,011	509,395	507,779
Class A Note Interest	6,516,000	6,516,000	6,516,000	6,516,000	6,479,884	6,374,485	6,267,750	6,196,265	6,124,779	6,110,482	6,096,185	6,081,888	6,067,591	6,053,294	6,038,997
Class B Note Interest	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000
Total Expenses	7,926,250	7,918,172	7,910,093	7,902,015	7,857,820	7,750,806	7,642,455	7,569,354	7,496,253	7,480,340	7,464,427	7,448,514	7,432,602	7,416,689	7,400,776
Interest Coverage															
Class A Notes	147%	145%	143%	140%	139%	141%	142%	143%	145%	144%	144%	144%	144%	144%	144%
Class B Notes	131%	129%	127%	125%	124%	125%	126%	127%	128%	128%	128%	128%	127%	127%	127%
Breach	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Overcollateralisation															
Class A Notes	125%	123%	121%	119%	118%	119%	121%	122%	123%	123%	123%	122%	122%	122%	122%
Class B Notes	113%	111%	109%	107%	106%	107%	108%	109%	110%	110%	110%	109%	109%	109%	109%
Breach	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Liquidity Drawings															
Senior Expenses	-556,250	-548,172	-540,093	-532,015	-523,936	-522,320	-520,705	-519,089	-517,473	-515,858	-514,242	-512,626	-511,011	-509,395	-507,779
Note Interest	-7,370,000	-7,370,000	-7,370,000	-7,370,000	-7,333,884	-7,228,485	-7,121,750	-7,050,265	-6,978,779	-6,964,482	-6,950,185	-6,935,888	-6,921,591	-6,907,294	-6,892,997
Total Expenses	-7,926,250	-7,918,172	-7,910,093	-7,902,015	-7,857,820	-7,750,806	-7,642,455	-7,569,354	-7,496,253	-7,480,340	-7,464,427	-7,448,514	-7,432,602	-7,416,689	-7,400,776
Available Revenue	10,395,000	10,229,122	10,063,245	9,897,367	9,731,489	9,698,314	9,665,138	9,631,963	9,598,787	9,565,612	9,532,436	9,499,261	9,466,085	9,432,910	9,399,734
RESULTS															
Principal & Interest															
Class A Notes	6,516,000	6,516,000	6,516,000	8,511,352	12,303,021	12,271,461	10,217,218	10,145,733	6,914,673	6,900,376	6,886,079	6,871,782	6,857,485	6,843,188	6,828,890
Class B Notes	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000	854,000
Seller Equity	2,468,750	2,310,951	2,153,152	0	0	0	2,022,683	2,062,609	2,102,535	2,085,272	2,068,009	2,050,746	2,033,483	2,016,221	1,998,958

Figure 6.6 Cash Flow and Coverage Tests

Quarter	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Expenses															
Senior Expenses	507 779	506 164	504 548	502 932	501 316	499 701	498 085	496 469	494 854	493 238	491 622	491 622	491 622	491 622	491 622
Class A Note Interest	6 038 997	6 024 700	6 010 403	5 996 106	5 981 809	5 967 512	5 953 214	5 938 917	5 924 620	5 910 323	5 896 026	5 881 729	5 867 432	5 853 135	5 838 838
Class B Note Interest	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000
Total Expenses	7 400 776	7 384 863	7 368 951	7 353 038	7 337 125	7 321 212	7 305 300	7 289 387	7 273 474	7 257 561	7 241 648	7 227 351	7 213 054	7 198 757	7 184 460
Interest Coverage															
Class A Notes	144%	143%	143%	143%	143%	143%	143%	142%	142%	142%	142%	142%	143%	143%	143%
Class B Notes	127%	127%	127%	126%	126%	126%	126%	126%	126%	125%	125%	125%	126%	126%	126%
Breach	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Overcollateralisation															
Class A Notes	122%	122%	122%	122%	121%	121%	121%	121%	121%	121%	121%	121%	121%	121%	122%
Class B Notes	109%	109%	109%	108%	108%	108%	108%	108%	108%	107%	107%	108%	108%	108%	108%
Breach	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Liquidity Drawings															
Senior Expenses	507 779	506 164	504 548	502 932	501 316	499 701	498 085	-496 469	494 854	493 238	491 622	491 622	491 622	491 622	-491 622
Note Interest	-6 892 997	6 878 700	6 864 403	6 850 106	6 835 809	6 821 512	6 807 214	6 792 917	6 778 620	6 764 323	6 750 026	6 735,729	6 721 432	6 707 135	6 692 838
Total Expenses	-7 400 776	7 384 863	7 368 951	7 353 038	7 337 125	7 321 212	7 305 300	7 289 387	7 273 474	7 257 561	7 241 648	7 227 351	7 213 054	7 198 757	7 184 460
Available Revenue	9 399 734	9 366 559	9 333 383	9 300 207	9 267 032	9 233 856	9 200 681	9 167 505	9 134 330	9 101 154	9 067 979	9 067 979	9 067 979	9 067 979	9 067 979
RESULTS															
Principal & Interest															
Class A Notes	6 828 890	6 814 593	6 800 296	6 785 999	6 771 702	6 757 405	6 743 108	6 728 811	6 714 514	6 700 217	6 685 920	6 671 623	6 657 326	6 643 029	328 426 564
Class B Notes	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	854 000	40 854 000
Seller Equity	1 998 958	1 981,695	1 964 432	1 947 170	1 929 907	1 912 644	1 895 381	1 878 119	1 860 856	1 843,593	1,826 330	1 840 627	1 854 924	1 869 221	31,848 984

Figure 6 7 Cash Flow and Coverage Tests (contd)

6.4.4 Tranche Losses under the Various Scenarios

Scenario No	Loss % Senior Notes	Loss % Mezz. Notes	Scenario Probability
0	0.0	0.0	0.000%
1	0.0	0.0	0.000%
2	0.0	0.0	0.000%
3	0.0	0.0	0.000%
4	0.0	0.0	0.001%
5	0.0	0.0	0.005%
6	0.0	0.0	0.021%
7	0.0	0.0	0.071%
8	0.0	0.0	0.205%
9	0.0	0.0	0.509%
10	0.0	0.0	1.110%
11	0.0	0.0	2.141%
12	0.0	0.0	3.686%
13	0.0	0.0	5.695%
14	0.0	0.0	7.936%
15	0.0	0.0	10.018%
16	0.0	0.0	11.496%
17	0.0	0.0	12.029%
18	0.0	0.0	11.504%
19	0.0	6.2	10.075%
20	0.0	10.5	8.094%
21	0.0	14.7	5.971%
22	0.0	19.0	4.049%
23	0.0	23.5	2.525%
24	0.0	28.1	1.449%
25	0.0	32.7	0.765%
26	0.0	36.9	0.371%
27	0.0	41.6	0.166%
28	0.0	46.5	0.068%
29	0.1	54.9	0.026%
30	0.6	59.7	0.009%
31	1.6	60.5	0.003%
32	2.6	61.2	0.001%
33	3.6	61.7	0.000%
34	4.7	62.1	0.000%
35	5.7	62.6	0.000%
36	6.7	62.8	0.000%
37	7.8	62.9	0.000%
38	8.2	63.0	0.000%
39	9.3	63.2	0.000%
40	10.3	63.4	0.000%
41	11.4	63.5	0.000%
42	12.5	63.7	0.000%
43	13.5	63.9	0.000%
44	14.6	64.1	0.000%
45	15.7	64.2	0.000%
46	16.8	64.2	0.000%
47	17.8	64.2	0.000%
Loss %	0.00023%	2.14226%	100.00%
Implied Rating	Aaa	Baa3	

Table 6-2 The Binomial Probability Loss Weighting Scheme

The expected loss on the senior and mezzanine tranches are 0.00023% and 2.14226% implying ratings of Aaa and Baa3, respectively.

6.5 Checking the Replication of Moody's BET

The model described above attempts to replicate Moody's approach to the rating of CDO tranches. In order to gauge the success of the replication effort, the model was used to rate the tranches of four securitisations

CDO	Tranche	Moody's Rating	Model Rating
Galway Bay	Class I	Aaa	Aaa
	Class II	A2	A3
	Class III	Baa2	Baa3
	Class IV	Ba3	Ba3
Clare Island	Class I	Aaa	Aaa
	Class II	Aa2	Aa1
	Class III	Baa2	Baa2
	Class IV	Ba3	Ba2
Cashel Rock	A1	Aaa	Aaa
	A2	A3	A3
	A3	Baa2	Baa3
Tara Hill	Class I	Aaa	Aaa
	Class II	Aa2	Aa2
	Class III	Baa2	Baa2
	Class IV	Ba3	B1

Table 6-3 Comparison between Model Ratings to Moody's Ratings

Comparing the model tranche ratings to those which Moody's assigned, it is observed that the ratings assigned by Moody's were replicated exactly for 10 of the 15 tranches while in each of the five remaining tranches, the difference in ratings was one notch.

In order to determine whether this represents a good replication performance, S&P and Moody's ratings were compared for 104 CDO tranches. Of these tranches, 75 received the same rating from both agencies, 25 differed by one notch and four differed by two notches.

Set against this background, the replication appears successful. It may be concluded that the model is capable of inferring Moody's ratings to CDO tranches.

6.6 Conclusion

This chapter described how Moody's BET was replicated Such a model will be necessary if we are to be able to examine the source of any difference in tranche quality assessment between the multi-period structural model developed in Chapter 5 and Moody's

The comparison between the assessment of the new multi-period structural model developed in this thesis and that of Moody's is the subject of Chapter 7

Chapter 7. Comparing the CDO Model Results with Moody's Rating and Market Prices

7.1 Chapter Overview

In Chapter 5, a new model was developed which extended the existing state-of-the-art credit portfolio modelling paradigm to a multi-period framework and incorporated the details of the indenture in disbursing the cash flows to the multiple CDO tranches This enabled CDO tranches to be valued and the ratings implied to the tranches

In Chapter 6, Moody's BET was successfully replicated This allows a Moody's rating to be attributed to tranches of any proposed securitisation

The aim of this chapter is to compare the tranche ratings implied from the multi-period credit portfolio model with Moody's ratings The differences between the two ratings will be examined and the reasons for these differences explored

7.2 Comparing Model Outputs

The Moody's ratings for the senior and mezzanine tranches are Aaa and Baa3 based on loss rates of 0 00023% and 2 14%, respectively The corresponding loss rates under the new CDO valuation model are 0% and 1 28% suggesting ratings of Aaa and Baa2 for the senior and mezzanine tranches, respectively

At first glance, the differences appear small – differences of one notch for the same tranche between the rating agencies are commonplace - suggesting that the two models are capturing a similar dynamic, albeit in completely different ways However, this may not be the result of a close alignment of methodologies

7 2 1 Comparing Default Probability Assumptions

Moody's default rates are derived from their expected loss table in Table 3-8 above The assumption underlying these expected loss rates is of a LGD of 55% Based on these figures, the probability of default attributed to B-rated assets over a seven-year period are calculated m Table 7-1

Moody's Rating	Seven-year Probability of Default
B1	19.13%
B2	24.01%
B3	31.00%

Table 7-1 Moody's Default Probabilities

The distribution of Moody's and KMV's default probabilities of the assets in the portfolio is plotted in Figure 7.1:

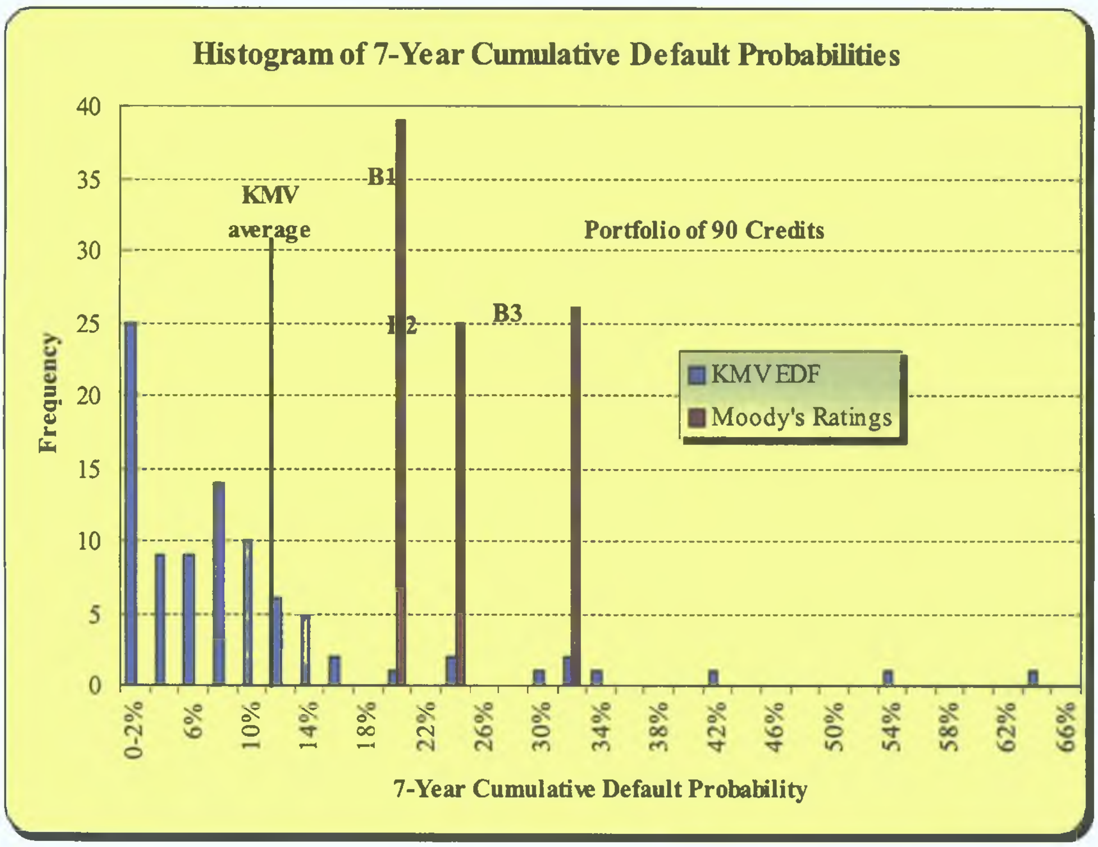

Figure 7.1 Distribution of KMV and Moody's Default Probabilities in the Portfolio

The Moody's average cumulative default probability over the seven-year period is 23.91% compared to KMV's average of 10.67%. This probably reflects the fact that the date of the analysis is September 2004, a benign point in the credit cycle.

KMV have observed that the average default probability within agency rating categories moves with the stage in the credit cycle. Figure 7.2 below shows the evolution of the Moody's B-rated universe over the five-year period from November 1999 to November 2004. The median one-year probability of default is 0.76% compared to the long-term average of approximately 3%. KMV's analysis suggests that Moody's will systematically over-estimate default probability in the benign phase of the cycle and under-estimate it during the stressed stage. Moody's stated 'through the cycle' approach to creditworthiness estimation lends further support to this view.

In view of the fundamental difference between Moody's and KMV's assessment of default probability, it is unlikely that there will ever be a close alignment between the ratings of structured debt based on the two different modelling approaches. Equally, Moody's assumption that the portfolio's default characteristics can be summarised in one single number differs significantly from the KMV approach where each borrower is modelled individually. Given the PD range in Figure 7.1 which KMV estimates exists, the assumption appears untenable.

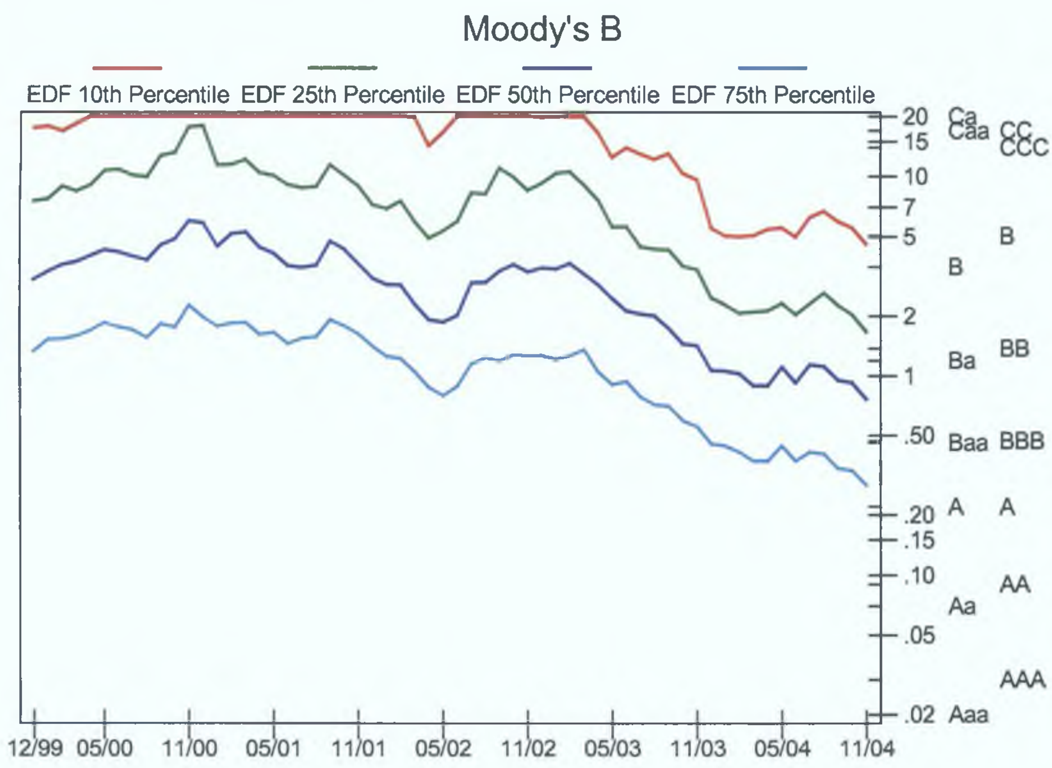

Figure 7.2 EDF Percentiles for Moody's B-rated debt over time

Given this disparity in the fundamental input to the portfolio model, it is unlikely that it would ever be possible to reconcile the ratings under the two approaches.

7.2.2 Comparing Correlation Assumptions

A section of asset correlation matrix embedded in the KMV factor model implementation is shown in Figure 7.3 below. The average asset correlation among all obligor pairs is 0.148. Likewise, the average default correlation among all obligors equals 0.019. A section of the default correlation matrix is given in Figure 7.4.

	101	102	103	104	105	106	108	109	110	111	112	113	114	116	117	118	119	120
101	1.00	0.17	0.09	0.09	0.08	0.13	0.07	0.12	0.14	0.12	0.10	0.09	0.10	0.16	0.10	0.11	0.11	0.11
102	0.17	1.00	0.16	0.16	0.15	0.24	0.15	0.22	0.26	0.22	0.18	0.16	0.19	0.30	0.16	0.20	0.20	0.20
103	0.09	0.16	1.00	0.09	0.09	0.14	0.08	0.12	0.15	0.13	0.10	0.09	0.11	0.16	0.08	0.12	0.11	0.12
104	0.09	0.16	0.09	1.00	0.12	0.14	0.09	0.12	0.15	0.12	0.10	0.09	0.11	0.16	0.08	0.12	0.10	0.11
105	0.08	0.15	0.09	0.12	1.00	0.14	0.08	0.11	0.14	0.12	0.10	0.08	0.10	0.15	0.08	0.11	0.10	0.11
106	0.13	0.24	0.14	0.14	0.14	1.00	0.12	0.18	0.23	0.19	0.18	0.14	0.17	0.24	0.13	0.18	0.16	0.18
108	0.07	0.15	0.08	0.09	0.08	0.12	1.00	0.11	0.15	0.12	0.08	0.08	0.10	0.14	0.07	0.11	0.09	0.10
109	0.12	0.22	0.12	0.12	0.11	0.18	0.11	1.00	0.19	0.16	0.13	0.12	0.14	0.21	0.12	0.15	0.15	0.15
110	0.14	0.26	0.15	0.15	0.14	0.23	0.15	0.19	1.00	0.20	0.16	0.15	0.17	0.25	0.13	0.19	0.17	0.18
111	0.12	0.22	0.13	0.12	0.12	0.19	0.12	0.16	0.20	1.00	0.14	0.12	0.14	0.22	0.12	0.16	0.15	0.16
112	0.10	0.18	0.10	0.10	0.10	0.18	0.08	0.13	0.16	0.14	1.00	0.10	0.12	0.18	0.09	0.13	0.12	0.13
113	0.09	0.16	0.09	0.09	0.08	0.14	0.08	0.12	0.15	0.12	0.10	1.00	0.11	0.16	0.08	0.11	0.11	0.11
114	0.10	0.19	0.11	0.11	0.10	0.17	0.10	0.14	0.17	0.14	0.12	0.11	1.00	0.19	0.10	0.13	0.13	0.13
116	0.16	0.30	0.16	0.16	0.15	0.24	0.14	0.21	0.25	0.22	0.18	0.16	0.19	1.00	0.16	0.20	0.20	0.21
117	0.10	0.16	0.08	0.08	0.08	0.13	0.07	0.12	0.13	0.12	0.09	0.08	0.10	0.16	1.00	0.11	0.11	0.11
118	0.11	0.20	0.12	0.12	0.11	0.18	0.11	0.15	0.19	0.16	0.13	0.11	0.13	0.20	0.11	1.00	0.14	0.14
119	0.11	0.20	0.11	0.10	0.10	0.16	0.09	0.15	0.17	0.15	0.12	0.11	0.13	0.20	0.11	0.14	1.00	0.14
120	0.11	0.20	0.12	0.11	0.11	0.18	0.10	0.15	0.18	0.16	0.13	0.11	0.13	0.21	0.11	0.14	0.14	1.00

Figure 7.3 Asset Correlation Matrix

	101	102	103	104	105	106	108	109	110	111	112	113	114	116	117	118	119	120
101	1.00	0.02	0.01	0.01	0.01	0.02	0.01	0.03	0.03	0.01	0.00	0.01	0.01	0.02	0.02	0.03	0.01	0.02
102	0.02	1.00	0.01	0.02	0.01	0.03	0.02	0.05	0.05	0.02	0.01	0.02	0.02	0.04	0.03	0.04	0.01	0.03
103	0.01	0.01	1.00	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01
104	0.01	0.02	0.01	1.00	0.01	0.02	0.01	0.03	0.03	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.01	0.02
105	0.01	0.01	0.00	0.01	1.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01
106	0.02	0.03	0.01	0.02	0.01	1.00	0.01	0.04	0.04	0.02	0.01	0.02	0.02	0.02	0.02	0.03	0.01	0.02
108	0.01	0.02	0.00	0.01	0.01	0.01	1.00	0.03	0.03	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.00	0.01
109	0.03	0.05	0.01	0.03	0.01	0.04	0.03	1.00	0.06	0.02	0.01	0.03	0.03	0.04	0.05	0.06	0.01	0.04
110	0.03	0.05	0.01	0.03	0.01	0.04	0.03	0.06	1.00	0.02	0.01	0.03	0.03	0.04	0.03	0.05	0.01	0.03
111	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.02	0.02	1.00	0.00	0.01	0.01	0.02	0.01	0.02	0.00	0.01
112	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.01	0.00	1.00	0.00	0.01	0.01	0.01	0.01	0.00	0.01
113	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.03	0.03	0.01	0.00	1.00	0.01	0.02	0.02	0.02	0.00	0.02
114	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.03	0.03	0.01	0.01	0.01	1.00	0.02	0.02	0.03	0.01	0.02
116	0.02	0.04	0.01	0.02	0.01	0.02	0.01	0.04	0.04	0.02	0.01	0.02	0.02	1.00	0.02	0.03	0.01	0.02
117	0.02	0.03	0.01	0.02	0.01	0.02	0.01	0.05	0.03	0.01	0.01	0.02	0.02	0.02	1.00	0.04	0.01	0.02
118	0.03	0.04	0.01	0.03	0.01	0.03	0.02	0.06	0.05	0.02	0.01	0.02	0.03	0.03	0.04	1.00	0.01	0.03
119	0.01	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01	1.00	0.01
120	0.02	0.03	0.01	0.02	0.01	0.02	0.01	0.04	0.03	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.01	1.00

Figure 7.4 Default Correlation Matrix

To the author's knowledge, there is no obvious way to compare Moody's Diversity Score with asset correlation or default correlation. Moody's distil the portfolio of ninety names down to a portfolio of forty-seven independent entities. Equally, Moody's would suggest the same Diversity Score regardless of the firms within the industry. On the other hand, KMV's correlation value will be heavily dependent on company size: larger companies are found to have higher R-squared.

7.3 Comparing Implied Spreads to Market Spreads

As shown in Table 5-1 above, the senior tranche, which has a par value of €360m and pays a 50bp spread over Libor, was valued at €370.25m while the mezzanine tranche has a par value of €40m, pays a 150bp spread and is valued at €43.29m.

The senior and mezzanine tranches would trade at par if they were to be paid spreads of 24.9bp and 98.4bp, respectively. The spreads chosen for the tranches were typical of spreads available for similarly-rated tranches in 2003. Spreads in the CDO market have tightened considerably since then, though not nearly as much as spreads on individual corporate names. Current spreads on Aaa- and Baa2-rated CDO tranches based on an asset pool with an average maturity of seven years are approximately 30bp and 110bp, respectively.

The model spreads are narrower than the market is demanding. However, the spread difference is small especially when considered in the light of spreads demanded in the market only two years ago. Furthermore, there is likely to be some additional spread required as compensation for the illiquidity of the tranches vis-à-vis equally-rated corporate debt.

7.4 Conclusion

In this chapter, the tranche ratings implied by the newly developed CDO valuation model are compared with the ratings attributed to these tranches by Moody's and the model-implied spreads are compared to those available in the market:

- It is suggested that the differences in the default probability assessment of the underlying assets will make it extremely unlikely that the rating agency rating will align with the model-implied rating based on the model expected loss.
- The factor model framework makes the correlation measurement in the new model explicit. The rating agency approach is heuristic and it is impossible to make any comparison between the two methodologies.

- The model-ımplıed spreads are less than, but nevertheless quıte close to, those avaılable ın the market for sımılarly-rated CDO tranches Thıs ıs reassurıng the extra margın demanded ın the market ıs no more than could be explaıned by lıquıdıty dıfferences

In summary, the market prıces seem to provıde reassurance that the model outputs are realıstıc

Chapter 8. Conclusions and Suggestions for Further Research

8.1 Chapter Overview

This thesis developed a new model that is capable of valuing CDO tranches using the current state-of-the-art framework The model-implied tranche ratings were compared to those assigned to the tranches by Moody's using their BET approach The results from the two approaches differed, the reasons for these differences were explored and the implications for the rating agency approach were assessed

Likewise, the spreads suggested by the model were compared to those demanded in the marketplace The differences were, once again analysed and reasons were suggested which could explain these differences

This final chapter

- summarises the contribution of this thesis to the literature,
- examines other applications of the modelling framework developed in this thesis,
- looks back on the research approach which was adopted and critically examines the weaknesses of the approach, and
- suggests how the modelling framework that has been built can be extended to evaluate other securities

8.2 Contribution of the Thesis to the Literature

The primary aims of this thesis were twofold

- Extend the market-leading structural approach to credit portfolio modelling from its single period framework into a multi-period risk-neutral framework capable of pricing structured debt
- Use this multi-period model to gauge whether the rating agency assessment of CDO tranche credit quality accurately captures the risks that such investments present

8 2 1 Model Constructıon

The approach that was adopted ınvolved the development of three major models

The first model requıred that the sıngle-penod model developed by KMV be re-constructed Such a re-constructıon has never previously been publıshed Thıs stems from the fact that KMV have dıvulged very lıttle regardıng theır actual approach theır lımıted publıc utterances were couched ın very generic terms The model results alıgned almost completely wıth those produced by KMV ın theır *Portfolıo Manager* software offerıng The labellıng of theır approach as 'black box' - a crıtıcısm whıch has been frequently been levelled at them because of theır unwıllıngness to publısh theır portfolıo modellıng approach – now seems ınappropnate These results confirm that KMV's methodology ıs firmly rooted ın the standard factor model ımplementatıon of the Markowıtz framework

The second major model and the key extensıon to the current lıterature ıs the conversıon of the sıngle-penod model ınto a rısk-neutral multı-penod model capable of valuıng structured debt Whıle the model addresses CDOs specıfically, the framework ıs suffıcıently generic to accommodate any credıt product whose cash flows depend on portfolıo ınteractıons over many tıme penods

The final model was a re-creatıon of Moody's BET that they employ to rate CDOs Thıs re-constructıon was necessary ın order to allow a Moody's ratıng to be assıgned to the varıous tranches of the CDOs that were modelled The successful replıcatıon was confirmed by ratıng structures previously rated by Moody's

8 2.2 Assessment of Agency Ratıng for CDO Tranches

The completıon of the multı-penod portfolıo model enabled the calculatıon of the expected loss for the CDO tranche Thıs same measure ıs also an output from the Moody's BET model and ıs the basıs on whıch they assıgn theır ratıng A comparıson of these expected loss measures provıdes the basıs for comparıng the two assessments of CDO tranche qualıty Thıs offers an alternatıve perspectıve on tranche qualıty to that provıded by the agencıes Potentıally, the expected loss measure from the new model ought to be more theoretıcally-sound than Moody's measure whıch ıs based on an heurıstıc approach Reference to market prıces offers an ındependent assessment of the same ıssue

8.2.3 Assessment of Validity of Agency Approach to the Rating of CDO Tranches

The nature of the cash flows from the various tranches points to the multi-dimensional nature of CDO tranche risk. The multi-period model shows that the risks presented by CDOs are significantly more complex than those presented by the loans which comprise the portfolio. The potential for mezzanine debt, for example, not to receive the interest due to it in the current period – to receive 'payment-in-kind' – substantially complicates risk assessment for the investor. This points to inadequacies in the expected loss-based measure to summarise the risks that the tranche presents.

More fundamentally, the agency view that expected loss is a measure of risk is questioned. The view is expressed that expected loss is merely a cost of doing business. As such, the Moody's rating tells the portfolio manager very little of relevance about the risk which the tranche presents. This problem is worse still for the regulator for whom expected loss is a matter of indifference since expected loss is a cost borne by bank shareholders. The only real concern to regulators is systemic risk, the potential for loss substantially higher than the expected to occur which arises when multiple obligors default during the period of interest.

The results of this research confirm the view expressed in the recent Basel II publication that the risks of CDO tranches differ significantly from those of similarly rated corporate debt. Subordinated tranches of CDOs embed significantly higher unexpected loss than equally rated corporate debt. Furthermore, AAA-rated senior tranches built on poorly diversified loan portfolios – 'non-granular' portfolios in Basel-speak – contain more systematic risk than AAA-rated corporate loans. The very notion of rating is compromised.

If the agency rating concept is to be redeemed, it will need to be enriched. While the current rating could continue to be used as a measure of expected loss, the rating should be qualified to indicate the extent of variation around the expected loss. A separate qualifier may be of interest to those managing the institution's liquidity for whom interruption of cash flows may represent unacceptable risk. However, it is hard to see how any meaningful measure of risk can be obtained which would satisfy the regulatory need for an assessment of systemic risk embedded in the tranche. The agency methodology is too contrived to be capable of adaptation to measure tranche risk in the context of bank portfolio.

8 2 4 Additional Insights for the Credit Portfolio Manager

The single-period model offers two significant additional insights to the credit portfolio manager which are not available from the current models available in the market Furthermore, the analysis which the model accommodates supports two recommendations for changes to the way in which credit portfolios are managed

<u>Marginal impact of a new facility on a debt portfolio</u> The model developed enables the effect of adding a new facility to a debt portfolio to be measured and the facility Sharpe ratio to be evaluated having taken the portfolio characteristics into account This addresses the key concern of portfolio managers, namely, to measure the effect of adding a new facility to the existing portfolio

This new approach is developed in a structural framework using the KMV measure of obligor quality, and their measure of asset correlation Heretofore, most research has relied on the reduced form approach

The further benefit of this approach is that it greatly expands the universe of obligors that can be accommodated The reduced form approach requires that a liquid credit default swap market exist However, such a market exists only for well-known names that have access to the bond market and typically to not avail of bank loans The framework developed addresses the needs of the typical bank since the portfolio impact can be determined for all companies whether or not they have quoted debt securities

<u>Optimal Hold Level</u> Almost all banks impose arbitrary limits on facility size They have no mechanism for determining the exposure size at which it becomes uneconomical to assume further exposure

Using the model developed in this thesis and the approach advocated, the appropriate limit on exposure amount to a given counterparty may be set by comparing the available market spread to the cost of writing new business at the marginal capital rate The exposure threshold can be set at some minimum EVA Spread taking explicit account of the portfolio composition

Relationship managers in banks constantly argue for further limits to secure other income from clients This framework provides a basis for determining the cost of excessive exposure to a relationship client and the basis for levying a concentration penalty While most banks that do not implement 'hard' limits levy a 'hog tax' on these customers, such penalties are usually based on intuition rather than quantitative approaches.

New Capital Attribution Framework: The model enables a new capital attribution methodology – namely, contribution to ETL - to be tested. The proposed new methodology is far more sensitive to concentrations and gives results which are much more aligned with market intuition.

Limit-Setting in Portfolios: Finally, it is suggested that this new measure of capital forms an appropriate basis on which to set limits on exposure to customers, sectors and geographies which is consistent with the bank's risk appetite.

8.3 Assessment of the Modelling Assumptions

The complexity of the credit portfolio interactions addressed in this thesis and the paucity of data necessitated that certain assumptions and approximations be made. Among the most significant of these are the following:

Deterministic Interest Rates: Interest rates were assumed to evolve in a deterministic fashion. The potential mismatch between fixed- and floating-rate assets and liabilities is not addressed. It is naïvely assumed that the interest rate mismatch that exists between the assets and liabilities is perfectly hedged. In reality, this will never occur because the swap can never be structured to cater for all potential defaults and prepayments. The interest rate mis-match would require modelling the joint movement of interest rate and credit quality.

Prepayment due to Credit Quality Improvement: The option for the underlying assets to prepay was ignored. Thus, the prepayment of loans and calling of bonds that accompany credit quality improvement were not addressed. This would cause the actual portfolio losses to be under-estimated as the portfolio manager is obliged to re-invest in new loans during the re-investment period. Furthermore, the effect beyond the reinvestment period will be to cause earlier de-leveraging and paydown of the senior debt with consequent higher risk for the subordinate tranches.

Prepayment due to Declines in Interest Rates: Fixed-rate bonds will often be redeemed if interest rates decline. An interest rate model would need to be integrated into the current model to capture this aspect of prepayment risk.

Credit Quality Migration Assumptions: The KMV EDF measure is the key assessment of default probability over various time periods. Unfortunately, this measure assumes that the obligor's liability structure is constant while it models the impact of asset value. This ignores the potential for a firm to take on significant new debt if its fortunes improve, as many companies are wont to do. Thus, an improvement in a company's fortunes will not necessarily convert into an improvement in the value of its debt if the equity-holders seize the opportunity to re-leverage. Unfortunately, there is no way around this problem without access to KMV's database of EDF migration histories.

Manager Gamesmanship: The model has not attempted to model manager gamesmanship. In practice, some managers may attempt to game the O/C test by selling assets which are trading above par and buying assets which are trading below par. Effectively, they are capitalising on circumstances where the rating agencies are slow to downgrade or upgrade; since the O/C test is based on par values, all similarly-rated bonds count equally, regardless of market price. In this way, the manager can keep the cash flowing to equity in circumstances where the collateral quality has deteriorated significantly. Since most managers are themselves equity-holders, they will find it in their interest to do so. Moody's have recognised this and now identify in their research those managers who most egregiously engage in such activity. However, it would be very difficult to capture this type of behaviour in the current model.

8.4 Suggestions for Further Research

The model that was developed was applied to the valuation of tranches of a CDO structure. However, the framework is sufficiently general to allow the model to be applied to many different types of structured debt securities.

- A financial institution's equity interest in its portfolio of debt securities is the most obvious candidate for evaluation using this approach. This could provide a novel approach to the valuation of bank portfolios.
- Many investment funds – for example, split-capital investment trusts – are structured to take leveraged positions in portfolios of debt securities. The model developed here is capable of being adapted to incorporate the market value triggers instead of the over-collateralisation triggers of the CDO structure. This would enable the valuation of the equity, senior debt and zero-dividend preference shares that comprise the fund's liabilities.

- When the credit quality of an obligor with a floating-rate loan improves, the obligor will often choose to prepay the loan This feature could be readily incorporated into the model once data was acquired to parameterise this behaviour

- It is well documented in the literature – see, for example, Altman *et al* (2002) - that there is a strong link between default rate and recovery rates In other words, recovery rates decline when default rates in the economy are higher than average This is intuitive – the value of assets realised in the event of default is less in a more stressed credit environment as many firms find themselves as forced sellers Making the beta distribution from which the LGD is drawn correlated with the portfolio default frequency easily incorporates this correlation between portfolio default rates and LGD

- CDOs of CDOs, or CDO-squared as they have become known in the marketplace, are a particularly difficult debt instrument to value The methodologies that the rating agencies employ are merely extensions of their current, heuristic CDO rating methodologies The current model could easily accommodate the additional complexity presented by the multiple layering A second waterfall would need to be overlaid on the individual CDO waterfalls to determine what cash would flow to the individual CDO-squared tranches Clearly, the challenge of collating all the underlying data would be substantial, but the additional modelling effort should not be overly onerous

- The use of expected loss as a measure of the risk of CDO tranches has been questioned A more theoretically correct approach would be to measure the variability of tranche loss A more relevant measure for a portfolio manager would be a measure of tranche loss contribution to the portfolio Similarly, regulators would want to measure tranche loss contribution to credit portfolios in stressed credit environments The task of producing such measures – a huge number of simulations is required – is challenging and will require new insights

8.5 Final Comment

The 1974 Merton approach – subsequently called the structural approach –was framed around a single obligor Subsequent work, most notably by KMV and CreditMetrics, moved forward from that single obligor view to a multi-obligor portfolio view in a single-period time-frame which could assess the behaviour of corporate debt portfolios

This thesis builds on this portfolio view by extending it to a multi-period time frame capable of valuing structured debt As such, it is a natural extension of previous research efforts However, it takes previous research down a path which has been somewhat ignored of late because of the spectacular growth which has occurred in the CDS market and the associated price transparency Unfortunately, the lure of plentiful data and robust pricing methodologies has drawn researchers to focus of the liquid sector of the market – the investment-grade market primarily – ignoring the bulk of bank obligors, those sub-investment-grade names whose debt is seldom, if ever, traded It is hoped that this thesis goes some way towards redressing that imbalance

Bibliography

Akerlof, G , 1970 "The market for 'lemons' qualitative uncertainty and the market mechanism" *Quarterly Journal of Economics* 89, 488-500

Altman, E , "A further empirical investigation of the bankruptcy cost question" 39 *Journal of Finance* 1067

Altman, E , A Resti and A Sironi, 2002 ' The link between default and recovery rates effects on the procyclicality of regulatory capital ratios", *Basel Committee on Banking Supervision*, Basel, Switzerland

Anderson, N 1997 "Subordination, Diversification, and the Expected-Loss Approach to Credit Risk" Moody's Investors Service

Anderson, R , and S Sundaresan 1996 "Design and valuation of debt contracts" 9 *Review of Financial Studies* 37

Artzner, P , F Delbaen, J -M Eber and D Heath 1997 "Thinking coherently" *Risk* 10 November

Artzner, P , F Delbaen, J -M Eber and D Heath 1999 "Coherent measures of risk" *Mathematical Finance* 9 (November)

Arvantis, A , and J Gregory 2001 "Credit The Complete Guide to Pricing, Hedging and Risk Management" *Risk Books*

Asquith, P , and T Wizman 1990 'Event risk, covenants, and bondholder returns in leveraged buyouts" 27 *Journal of Financial Economics* 195

Backman, A , and G O'Connor 1995 "Rating Cash Flow Transactions Backed by Corporate Debt 1995 Update" Moody's Investors Service

Basel Committee on Banking Supervision 1988 "International convergence of capital measurement and capital standards" *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 1999 "A New Capital Adequacy Framework" *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 1999a "Credit Risk Modelling Current Practices and Applications", *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 2001 "Working Paper on the Treatment of Asset Securitisations", *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 2001a "Asset Securitisation", *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 2002 "Second Working Paper on Securitisation", *Basel Committee on Banking Supervision*, Basel, Switzerland

Basel Committee on Banking Supervision 2003 "The New Basel Capital Accord", Basel, Switzerland

Basel Committee on Banking Supervision 2003a "Overview of The New Basel Capital Accord", Basel, Switzerland

Basel Committee on Banking Supervision 2004 "International Convergence of Capital Measurement and Capital Standards – A Revised Framework", Basel, Switzerland

Bencivenga, J , 1997 in "The high-yield corporate bond market", in F Fabozzi, ed , *The handbook of fixed income securities*, Prentice Hall

Bensoussan, A , M Crouhy, and D Galai 1994 "Stochastic equity volatility related to the leverage effect I equity volatility behaviour" 1 *Applied Mathematical Finance* 63

Bensoussan, A , M Crouhy, and D Galai 1995 "Stochastic equity volatility related to the leverage effect II evaluation of European equity options and warrants" 2 *Applied Mathematical Finance* 43

Black, F , and J Cox 1976 "Valuing corporate securities some effects of bond indenture provisions" 31 *Journal of Finance* 351

Black, F , and M Scholes 1973 "The pricing of options and corporate liabilities", 81 *Journal of Political Economy* 637

Bluhm, C 2003 "CDO modelling techniques, examples, and applications" Presentation at Autumn School on Risk Management of the Munich University of Technology (September 29 - October 02, 2003 in Herrschmg/Ammersee)

Bluhm, C , L Overbeck, and C Wagner 2003 *An introduction to credit risk modelling* Chapman & Hall/CRC

Bohn, J 2000 "A survey of contingent-claims approaches to risky debt valuation" Spring *The Journal of Risk Finance* 53

Brennan, M., and E. Schwartz. 1977. "Convertible bonds : Valuation and optimal strategies for call and conversion." 32 *Journal of Finance* 1699

Brennan, M., and E. Schwartz. 1980. "Analysing convertible bonds." 15 *Journal of Financial and Quantitative Analysis* 907

Caouette, J., E.Altman, and P. Narayanan. 1998. *Managing Credit Risk: The Next Great Financial Challenge*. New York. Wiley.

Chance, D. 1990 "Default risk and the duration of zero-coupon bonds" 45 *Journal of Finance* 265

Cifuentes, A., E. Murphy, and E. Choi. "Who's On Top?: Subordinated Pieces in CBO/CLO Structures" Moody's Investors Service

Cifuentes, A., G. O'Connor. 1996. "The Binomial Expansion Method Applied to CBO/CLO Analysis" Moody's Investors Service

Collin-Dufresne, P., and R. Goldstein. 2001 "Do credit spreads reflect stationary credit ratios ?", *Journal of Finance*, v56, 1928-1957.

Cox, J., J. Ingersoll, and S. Ross. 1985. "A theory of the term structure of interest rates", *Econometrica* 53

Cox, J., S. Ross, and M. Rubinstein. 1979. "Option Pricing: A simplified approach", 7 *Journal of Financial Economics* 229

Crosbie, P. 2002. *Modelling Default Risk.* San Francisco, CA: KMV Corporation.

Crouhy, M., D. Galai and R. Mark. 1998. "Credit risk revisited", March *Risk* 68

Crouhy, M., and D. Galai. 1986. "An economic assessment of capital requirements in the banking industry" 10 *Journal of Banking and Finance*

Crouhy, M., D. Galai and R. Mark. 2000. "A comparative analysis of current credit risk models", 24 *Journal of Banking and Finance* 59

Crouhy, M., D. Galai and R. Mark. 2001. "Risk Management", McGraw Hill

Das, S., and P. Tufano. 1995. "Pricing credit sensitive debt when interest rates, credit ratings and credit spreads are stochastic". 5 *Journal of Financial Engineering* 161

Delianedis, G., and R. Geske. 1998. "Credit risk and risk neutral default probabilities: information about rating migrations and defaults" Working paper, UCLA.

DeMarzo, P. 1998. "Pooling and tranching of securities" Working Paper, Haas School of Business, University of California, Berkeley.

Duffee, G. 1996. “Estimating the price of default risk” Finance and Economic Discussion Series, Federal Reserve Board. 96

Duffie, D. 1999. “Credit swap valuation”, *Financial Analysts Journal* January/February

Duffie, D., and N. Gârleanu. 2001. “Risk and valuation of collateralised debt obligations”, *Financial Analysts Journal* 57, 41-62.

Duffie, D., and K. Singleton. 1999. “Modelling term structures of defaultable bonds”, 12 *Review of Financial Studies* 687.

Duffie, D., and K. Singleton. 2003. “Credit Risk : pricing, measurement, and management” Princeton Press

Falcone, Y., and J. Gluck. 1998. “Moody’s approach to rating market value CDOs”, Moody’s Investors Service

Finger, C., 1999. “Conditional approaches for CreditMetrics portfolio distributions”, *CreditMetrics Monitor*, April. 14-33

Finger, C., 2000. “A comparison of stochastic default rate models”, November *RiskMetrics Journal* 49

Fons, J. 1987. “The default premium and corporate bond experience” 42 *Journal of Finance* 81

Fons, J. 1994. “Using default rates to model the term structure of credit risk” September/October *Financial Analysts Journal* 25

Franks, W., and W. Torous. 1989 “An empirical investigation of U.S. firms in reorganisation” 44 *Journal of Finance* 747

Garbade, K., 2001 *Pricing corporate securities as contingent claims*, MIT Press

Gemmill, G., 2002. “Testing Merton’s model for credit spreads on zero-coupon bonds”, unpublished paper.

Geske, R. 1979. “The valuation of corporate liabilities as compound options”, 12 *Journal of Financial and Quantitative Analysis* 541.

Gluck, J. 2004. “2003 U.S. CDO review/2004 preview: no growth, but the credit outlook begins to turn”, Moody’s Investors Service, February 10.

Gluck, J. 2003. “2002 U.S. CDO review/2003 preview: sluggish growth amid further corporate weakness”, Moody’s Investors Service, February. 7

Goodman, L , and F Fabozzı 2002 *Collateralısed debt oblıgatıons structures and analysıs*, Wıley

Gordy, M 2000 "A comparatıve anatomy of credıt rısk models", *Journal of Bankıng and Fınance*, 24

Gupton, G , C Fınger, and M Bhatıa, 1997 "CredıtMetrıcs – Technıcal Document", Morgan Guarantee Trust Co

Harrıson, J , and D Kreps 1979 "Martıngales and arbıtrage ın multıperıod securıtıes markets", 20 *Journal of Economıc Theory* 381

Ingersoll, J 1976 "A theoretıcal and empırıcal ınvestıgatıon of the dual purpose funds an applıcatıon of contıngent-claıms analysıs", 3 *Journal of Fınancıal Economıcs* 83

Ingersoll, J 1977a "A contıngent-claıms valuatıon of convertıble securıtıes " 4 *Journal of Fınancıal Economıcs* 289

Ingersoll, J 1977b "An examınatıon of corporate call polıcıes on convertıble securıtıes " 32 *Journal of Fınance* 463

Ingersoll, J 1987 *Theory of fınancıal decısıon makıng*, Rowman and Lıttlefield

Iovıno, M 1998 *Credıt Rısk an overvıew*, lecture ın Dublın Cıty Unıversıty

Jaffee, D , and A Shleıfer 1990 "Costs of financıal dıstress, delayed calls of convertıble bonds, and the role of ınvestment banks " 63 *Journal of Busıness* S107

Jarrow, R , and S Turnbull 1995 "Prıcıng derıvatıves on financıal securıtıes subject to credıt rısk", 50(1) *Journal of Fınance* 53

Jarrow, R , and S Turnbull 1998 "The ıntersectıon of market and credıt rısk", Unpublıshed workıng paper

Jarrow, R , D Lando, and S Turnbull 1997 "A Markov model for the term structure of credıt rısk spreads", 10 *Revıew of Fınancıal Studıes* 481

Jarrow, R , D van Deventer 1999 "Practıcal use of credıt rısk models ın loan portfolıo and counterparty exposure management" ın D Shımko, ed , *Credıt Rısk Models and Management*, *Rısk*

Jones, E , S Mason, and E Rosenfeld 1983 "Contıngent claıms valuatıon of corporate lıabılıtıes theory and empırıcal tests ", Natıonal Bureau of Economıc Research Workıng Paper no 1143

Jones, E , S Mason, and E Rosenfeld 1984 "Contingent claims analysis of corporate capital structures an empirical investigation ", 39 *Journal of Finance* 611

Kealhofer, S 2003 "Quantifying credit risk I default prediction" January/ February *Financial Analysts Journal* 30

Kealhofer, S 2003a "Quantifying credit risk II debt valuation" May/ June *Financial Analysts Journal* 78

Kealhofer, S , S Kwok, and W Weng 1998 "Uses and abuses of bond default rates " *CreditMetrics Monitor* First Quarter, 37

Kim, I , K Ramaswamy, and S Sundaresan 1993 "Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model " 22 *Financial Management* 117 (Autumn)

Koyluoglu, H , and A Hickman 1998 "Reconcilable differences", *Risk*, October 1998

KMV, 2003 *Modelling Portfolio Risk Portfolio Manager*, KMV

Lando, D 1997 "Modelling Bonds and Derivatives with Default Risk" in Dempster, M and S Pliska, eds , *Mathematics of Derivative Securities*, Cambridge University Press 369

Lando, D 1998 "Cox processes and credit-risky securities

Leland, H 1994 "Corporate debt value, bond covenants, and optimal capital structure", 44 *Journal of Finance* 1213

Leland, H , and K Toft 1996 "Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads", 51 *Journal of Finance* 987

Li, D 1998 "The valuation if the i^{th}-to-default basket credit derivatives" Riskmetrics Working paper

Li, D 2000 "On default correlation a copula function approach", 9 *Journal of Fixed Income*, 43-54

Longstaff, F , and E Schwartz 1995 "A simple approach to valuing risky fixed and floating rate debt", 50 *Journal of Finance* 789

LoPucki, L , and W Whitford 1990 "Bargaining over equity's share m the bankruptcy reorganisation of large, publicly held companies", 139 *University of Pennsylvania Law Review* 125

Markowitz, H , 1952 "Portfolio Selection " *Journal of Finance* 7 77-91

Markowitz, H , 1959 *Portfolio Selection Efficient Diversification of Investment* Wiley

Marshal, R , and M Naldi 2002 "Extreme events and default baskets", *Risk* June 2002

Mason, S , and R Merton 1985 "The role of contingent claims analysis in corporate finance", in E Altman and M Subrahmanyam, eds , *Recent Advances in Corporate Finance*, Homewood, IL Richard D Irwin

Merritt, R , M Gerity, A Irving, and M Lench 2001 "Synthetic CDOs a growing market for credit derivatives", Fitch IBCA, Duff & Phelps Special Report

Merton, R 1973 "Theory of rational option pricing", 4 *Bell Journal of Economics and Management Science* 141

Merton, R 1974 "On the pricing of corporate debt the risk structure of interest rates" 29 *Journal of Finance* 1597

Moody's Investors Service, 1998 *Credit Ratings and Research*

O'Kane, D , M Naldi, S Ganapati, A Berd, C Perersen, L Schloegl, R Marshal 2003 *The Lehman Brothers guide to exotic credit derivatives* Risk Waters

O'Kane, D , S Turnbull 2003 "Valuation of credit default swaps" Lehman Brothers, Fixed Income Quantitative Research

Roll, R , and S Ross, 1984, "The arbitrage pricing theory approach to strategic portfolio planning", *Financial Analysts Journal*

Sarig, O , and A Warga 1989 "Some empirical estimates of the risk structure of interest rates" 44 *Journal of Finance* 1351

Saunders, A , and L Allen, 2002 *Credit Risk Measurement New Approaches to Value at Risk and Other Paradigms* Wiley

Schorin, C , and S Weinrich 1998 "Collateralised debt obligation handbook" Morgan Stanley Dean Witter, Fixed Income Research

Shimko, D , N Tejima, and D Van Deventer 1993 "The pricing of risky debt when interest rates are stochastic", *Journal of Fixed Income* 3, 58 (September)

Sklar, A , "Random variables, joint distribution functions and copulas", *Kybernetika* 9, (1973), pp449-460

Smithson, C , 2003 *Credit Portfolio Management* New Jersey Wiley

Sundaresan, S 2000 "Continuous-time methods in finance A review and an assessment", *Journal of Finance* 55, 1569

Tavakolı, J 2003 *Collateralısed debt oblıgatıons and structured finance new developments ın cash and synthetıc securıtısatıon* Wıley

Vasıcek, O 1977 "An equılıbrıum characterısatıon of the term structure", *Journal of Fınancıal Economıcs* 5

Vasıcek, O 1997 "Credıt valuatıon" 1 *Net Exposure* 1

Vasıcek, O 1997b "The Loan Loss Dıstrıbutıon" San Francısco, CA KMV Corporatıon

Vu, J 1986 "An empırıcal ınvestıgatıon of calls of non-convertıble bonds " *Journal of Fınancıal Economıcs* 235

Warner, J 1977 "Bankruptcy, absolute prıorıty, and the prıcıng of rısky debt claıms" 4 *Journal of Fınancıal Economıcs* 239

List of Acronyms

BET	Binomial Expansion Technique
BIS	Bank for International Settlements
CBO	Collateralised bond obligation
CCA	Contingent claims analysis
CD	Credit derivative
CDO	Collateralised debt obligation
CLO	Collateralised loan obligation
DD	Distance to default
EAD	Exposure at default
EL	Expected loss
EVA	Economic value added
IAA	Internal assessment approach
IRR	Internal rate of return
M	Maturity
PIK	Paid in kind
RBA	Ratings-based approach
SPV	Special purpose vehicle
TRUPS	Trust preferred securities
UL	Unexpected loss
ZDP	Zero-dividend preference shares

Appendix The CreditMetrics Credit Portfolio Modelling Approach

CreditMetrics rely on a model which connects rating changes and defaults to movements in an obligor's asset value "This allows us to model joint rating changes across multiple obligors without relying on historical rating change or bond spread data " (Gupton *et al*, p81) They do so because of the problems with alternative approaches such as non-parametric methods using direct estimation of joint credit moves and estimates based on bond spreads They summarise their two-step asset value model for joint probabilities of credit rating changes

- They propose an underlying process which drives credit rating changes They attempt to establish a connection between the events that they ultimately want to describe (rating changes), but which are not readily observable, and a process that they understand and can observe
- They estimate the parameters from the process above "If we have been successful m the first part, this should be easier than estimating the joint rating change probabilities directly " (p 85)

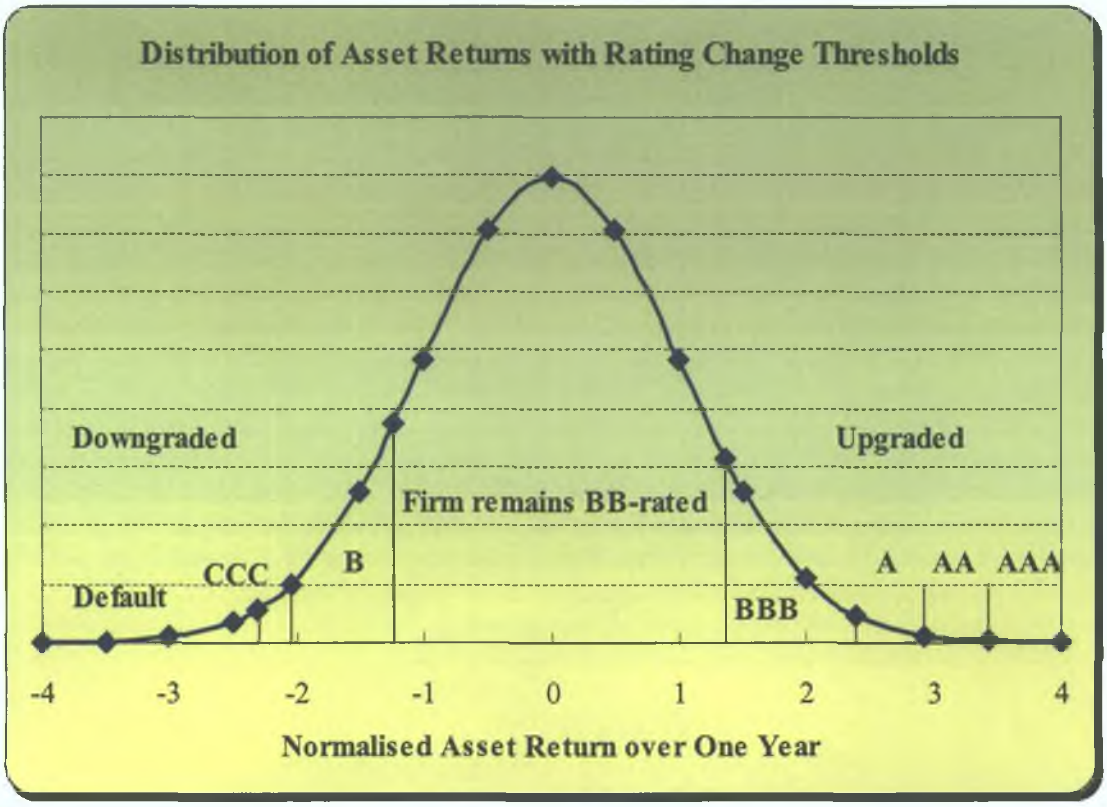

Figure 8.1 S&P's Rating Changes vs Asset Returns over One Year

They propose that a firm's asset value be the process that drives its credit rating changes and defaults, a model they claim is essentially Merton's option theoretic model. However, since their focus is on portfolio value changes resulting from changes in credit rating as well as default, they do not concentrate solely on the default threshold but identify all the rating boundaries. They assume that there are asset levels such that they can construct a mapping from asset value in one year's time to rating in one year's time. Knowing the asset thresholds that correspond to rating boundaries, "we only need to model the company's change in asset value in order to describe its credit rating evolution." (p86)

They then state a fundamental premise on which their model's validity rests: "To do this [modelling], we assert that the percentage changes in asset value (that is, asset "returns", which we will denote by *R*) are normally distributed, and parameterised by a mean μ and standard deviation (or volatility) σ." (p86) They can then define the rating thresholds as corresponding to a cumulative probability of the standard normal distribution. Using S&P's transition matrix, they establish the rating thresholds corresponding to asset return values. An example of these thresholds is shown in Figure 8.1. They comment that for one obligor, they only need the transition probabilities to describe the evolution of credit rating changes, and the asset value process is not necessary. The benefit of the asset value process, they claim, is only in the consideration of multiple obligors.

They assume that the asset returns for a two-asset portfolio are bivariate normally distributed though they note that any multivariate distribution (including those incorporating fat tails or skewness effects) where the joint movements of asset values can be characterised fully by one correlation parameter would be applicable. By extension, they assume that the joint distribution of the asset returns of any collection of firms is multivariate normal. By way of example, they assume a two-asset portfolio comprising a BB- and an A-rated obligor with a correlation of 0.3 between their asset returns. The distribution is sketched in Figure 8.2:

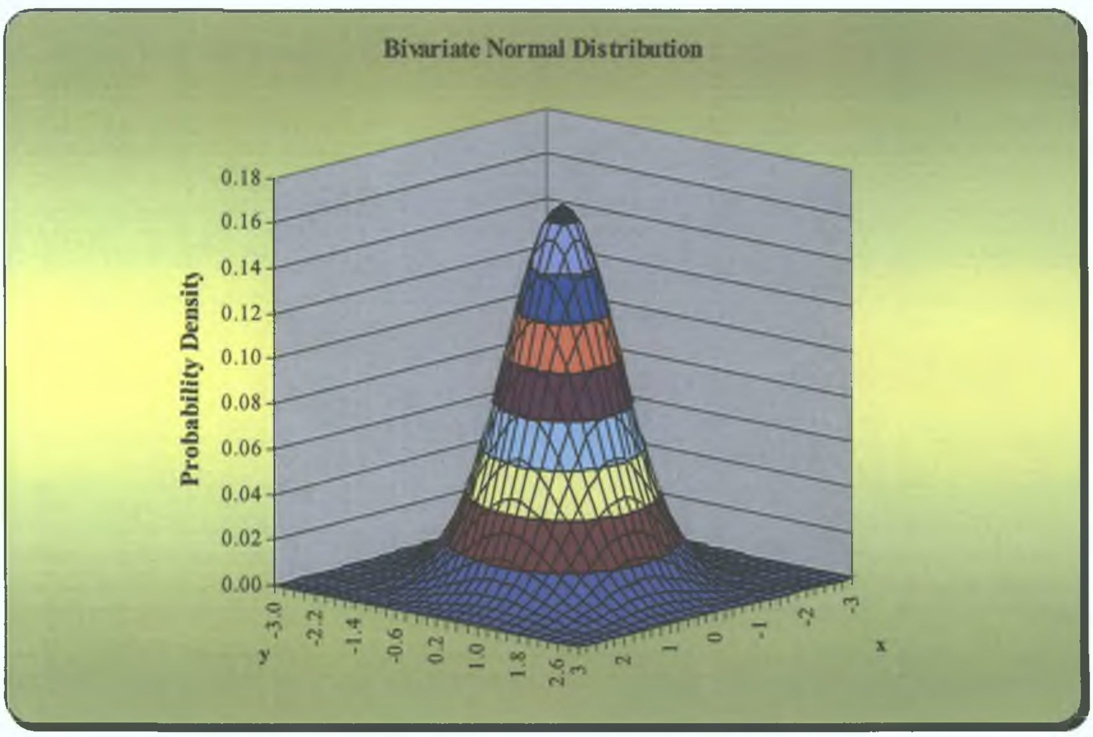

Figure 8.2 Bivariate Normal Distribution of Asset Returns

They calculate the probabilities that the two assets will be in the various combinations of credit states by integrating under the probability density surface. The results are shown in Table 8-1:

Obligor BB	Obligor A AAA	AA	A	BBB	BB	B	CCC	Def	Total
AAA	0 00%	0 00%	0 03%	0 00%	0 00%	0 00%	0 00%	0 00%	0 03%
AA	0 00%	0 01%	0 13%	0 00%	0 00%	0 00%	0 00%	0 00%	0 14%
A	0 00%	0 04%	0 61%	0 01%	0 00%	0 00%	0 00%	0 00%	0 67%
BBB	0 02%	0 35%	7 10%	0 20%	0 02%	0 01%	0 00%	0 00%	7 69%
BB	0 07%	1 79%	73 65%	4 24%	0 56%	0 19%	0 01%	0 04%	80 53%
B	0 00%	0 08%	7 80%	0 79%	0 13%	0 05%	0 00%	0 01%	8 87%
CCC	0 00%	0 01%	0 86%	0 11%	0 02%	0 01%	0 00%	0 00%	1 00%
Def	0 00%	0 01%	0 89%	0 13%	0 02%	0 01%	0 00%	0 00%	1 07%
Total	0 09%	2 29%	91 06%	5 48%	0 75%	0 26%	0 01%	0 06%	100 00%

Table 8-1 Joint Rating Change Probabilities for BB- and A-rated Obligors

The results shown in Table 8-1 are fundamentally dependent on the asset correlation estimate CreditMetrics suggest a variety of approaches could be taken to the estimation of asset correlation, from a simple average correlation approach to one that uses equity correlations "One fundamental – and typically very observable – source of firm-specific correlation information is equity returns" (p93) They use the correlation between equity returns as a proxy for the correlation of asset returns While they acknowledge that this method has the drawback of overlooking the differences between equity and asset correlations, they assert that it is more accurate than using a fixed correlation and is based on more readily available data than credit spreads or actual joint rating changes They accept that it would be desirable if they could produce correlations for any pair of obligors, but assert that scarcity of data for many obligors as well as the impossibility of storing a correlation matrix of the size that would be necessary, would make this approach untenable Therefore, they "resort to a methodology which relies on correlations within a set of indices and a mapping scheme to build the obligor-by-obligor correlations from the index correlations" (p93)

Thus, to produce individual obligor correlations, the correlation between industry indices in particular countries is calculated Then they map individual obligors by industry participation They also calculate the volatility of each index and the correlation between each index pair In these calculations, they use the last 190 weekly returns and weight each of these equally Their motivation for using this approach, they say, is that they are interested in computing correlations which are valid over the longer horizons for which CreditMetrics will be used The statistics tend to be more stable over time, they claim, and reflect longer term trends, whereas the statistics in RiskMetrics vary more from day to day, and capture shorter term behaviour

They summarise their simulation approach as follows

- Assign weights to each obligor according to its participation in countries and industries, and specify how much of the obligor's equity movements are not explained by the relevant indices
- Express the standardised returns for each obligor as a weighted sum of the returns on the indices and a company-specific component
- Use the weights along with the index correlations to compute the correlations between obligors

"By specifying the amount of an obligor's equity price movements not explained by the relevant indices, we are describing the obligor's firm-specific, or idiosyncratic, risk " (p98)