Co – Movements among Financial Stocks and Covariance Matrix Analysis

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Declaration

I hereby certify that this material, which I now submit for assessment on the program of study leading to the award of M.Sc. in Computing is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Abstract

The major theories of finance leading into the main body of this research are discussed and our experiments on studying the risk and co-movements among stocks are presented

This study leads to the application of Random Matrix Theory (RMT) The idea of this theory refers to the importance of the empirically measured correlation (or covariance) matrix, C, in finance and particularly in the theory of optimal portfolios However, this matrix has recently come into question, as a large part of it does not contain useful information but rather noise Therefore, recent work has indicated that the theory of optimal portfolios, which depends on C, is not adequate We use RMT in order to measure the noise component of C, and then we examine the methods of differentiating noise from information We go on to develop a novel technique of stability analysis for the eigenvectors of C after noise removal

Further, changes in the portfolio associated with the riskiest position, (as given by the largest eigenvalue and associated eigenvector), are investigated using the results of the previous chapters From the results, we observe periods of co-movements of stocks, which change regularly because of some key events in the market These periods are characterised by a linear relationship between price and eigenvalue change However, the residuals in this model are strongly dependent on granularity (i e sampling rate) with fit breaking down at rates smaller than five days Possible reasons for this breakdown are presented in detail

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Chapter 1
Introduction

A financial market is a market where *financial assets* are traded Although the existence of a financial market is not a necessary condition for the creation and exchange of an asset, in most economies, assets are created and consequently traded in some type of financial market

The role of financial market is to indicate how the funds should be allocated among assets as well as providing an environment, which forces or motivates an investor to sell an asset [Fabozzi et al , 1998] Because of these properties, it is said that a financial market offers *liquidity* [Fabozzi et al , 1998], which is the ability of an asset to be converted into cash quickly [Investor, 2003]

1 1 Stock Markets around the world

In terms of *market value*, the stock markets of the United States and Japan are the largest in the world The third largest market, but far behind the United States and Japan, is the UK market

Trading of common stock occurs in a number of trading locations such as "national stock exchanges" or "over-the-counter" (OTC) markets [Teweles et al , 1998] Stock exchanges are made up of members who use the facilities to exchange certain common stocks To be *listed*, a company must apply and satisfy requirements established by the exchange [Fabozzi et al , 1998] To have the right to trade stocks on the floor of the exchanges, firms or individuals must buy a seat on the exchange, i e they must become a member of the exchange A member firm may trade for its own account or on behalf of a customer In the latter case it is acting as *broker*

The top national stock exchange in the United States is the New York Stock Exchange (NYSE), popularly referred to as the Big Board It is the largest exchange with over 3,000 companies' shares listed

Unlisted stocks are also traded electronically in an over-the-counter market There are about 5,000 common stocks included in the NASDAQ, the electronic quotation system with a total market value of over $2 trillion (reported in NASDAQ web site 2003 [NASDAQ, 2003])

1 2 Stock Market Indicators

A stock market *indicator* (or *index*) is a statistical construct that measures price changes and/ or *returns* in stock market The purpose of the index calculation is usually to provide a single number whose behaviour is representative of the movement of prices of all listed stocks and indicative of behaviour of the market as a whole

The most commonly quoted stock market indicator is the Dow Jones Industrial Average (DJIA) Other stock market indicators cited in the financial press are the Standard & Poor's 500 Composite (S&P 500), the New York Stock Exchange Composite Index (NYSE Composite), the American Stock Exchange Market Value Index (AMEX), the NASDAQ Composite Index, and FTSE 100

In general, market indices represent only stocks listed on one exchange Examples are DJIA and the NYSE Composite, which represent only stocks listed on the Big Board By contrast, the NASDAQ includes only stocks traded over the counter But the most popular index is the S&P 500 as it contains both NYSE - listed and OTC - traded shares The DJIA uses only 30 of the NYSE-traded shares, while the NYSE Composite includes every one of the listed shares [Teweles et al , 1998] The NASDAQ also includes all shares in its universe, while the S&P 500 has a sample that contains only 500 of the more than 8000 shares [Teweles et al , 1998] FTSE 100 includes 100 most highly capitalised blue chip companies, representing approximately 80% of the UK market [FTSE, 2003]

1 3 Derivatives

Derivative (or derivative security) is a financial instrument whose value depends on the values of another asset Derivative markets have an important role in 'risk management' and 'price discovery' [Hull, 2000, Chance, 1995] as they enable investors with a lower level of risk preference to transfer the risk to the investors with higher level of risk preference "Future" and "forward" contracts are agreements whereby two parties agree to transact some financial assets at a predetermined price at a specified future date One party agrees to buy the financial asset, the other agrees to sell the financial asset Both are obligated to perform, and neither party charges a fee unless they do not want to perform the contract

Option contract, the other substantial derivative, gives the owner of the contract the right, but not obligation, to buy (or sell) a financial asset at a specified price from (or to) another party The buyer of the contract must pay the seller a fee, which is called the option price

1 4 Market declines

When a crash occurs on the market all the people around the world suffer directly or indirectly and millions of Dollars are lost Investors lose their money overnight and the entire economy is affected The largest single day decline in the history of most of the world's stock markets, occurred on Monday, October 18, 1987 On that day, popularly referred to as Black Monday the DJIA lost 22 6% and S&P500, 20 5% of their total values and other market indexes declined to a similar extent [Roll, 1988] On Black Monday Wall Street lost 15,000 jobs in the financial industry [Facts, 2002] and all outstanding US stocks declined to approximately one trillion dollars [Sopris, 2002] Also a record loss of £50 6 billion on the London Stock Exchange was proceeded by the fall of Wall Street [Guardian, 2003]

Market situation at present is represented by a prolonged decline plus a high volatility that analysts describe it as a period of low returns and high risk, which will be continued far into the future [Business, 2003c] This critical decline is exposed in the S&P500, which has lost 45% of its value from March 2000 to March 2003 together with considerable volatility, six declines and five rises over the three years The rallies and declines show how fast the market is changing investors' portfolio values In contrast to the last decade, the average length of the rallies over the last year was only 74 days before the market slumped 10% whereas for 7 years when the market climbed from October 1990 to October 1997 there was no decline of this size [Business, 2003a] Stocks are swinging up and down now, more often than any time since 1938 [Business, 2003b] As recently as 1995, the S&P 500 traded all year without once changing 2% in a day but in 2002, it swung that much or more on 52 days [Business, 2003b]

Clearly, war with Iraq, and threat of terrorism made for a nervous market during last year Millions of buy-and-hold investors, who have been a major stabilising force in the stock market, are bailing out [Business, 2003b]

Figure 1 1 represents evolution of the four major indexes around the world over the last decade The volatility and decline of the market are clearly seen over recent years, in comparison with previously The other general message, here, is that the market is reverting to pre-mid 90's levels

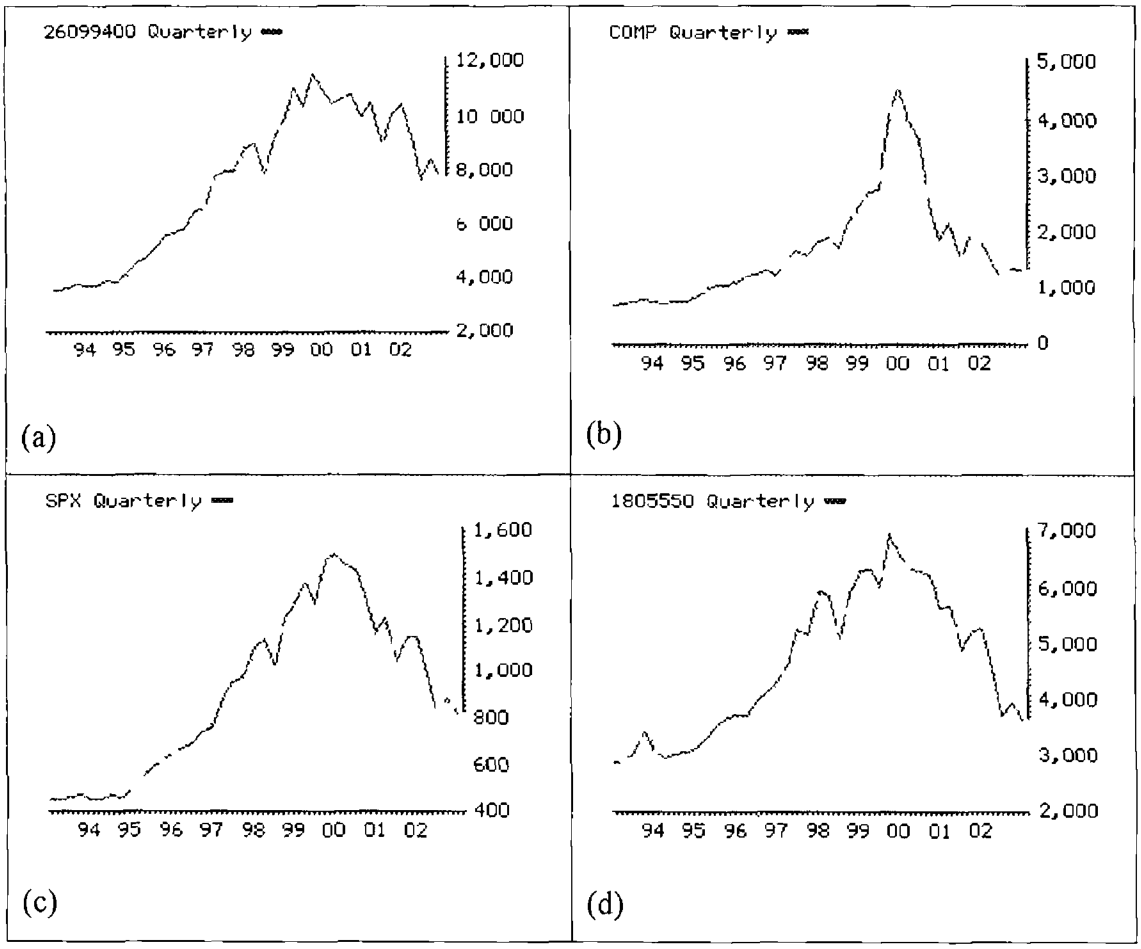

Figure 1 1 A decade market evolution of (a) DJIA, (b) NASDAQ Composite, (c) S&P500 and (d) FTSE 100

Also it is useful to look at the currency situation, where Euro has been hardly appreciating versus US Dollar since last year Figure 1 2 shows Euro-US-Dollar movement over the last three years The US Dollar is in a worsening condition against the Euro such that as of time of writing[1] it has hit a 4-year low against Euro

[1] To the date of 10 March 2003

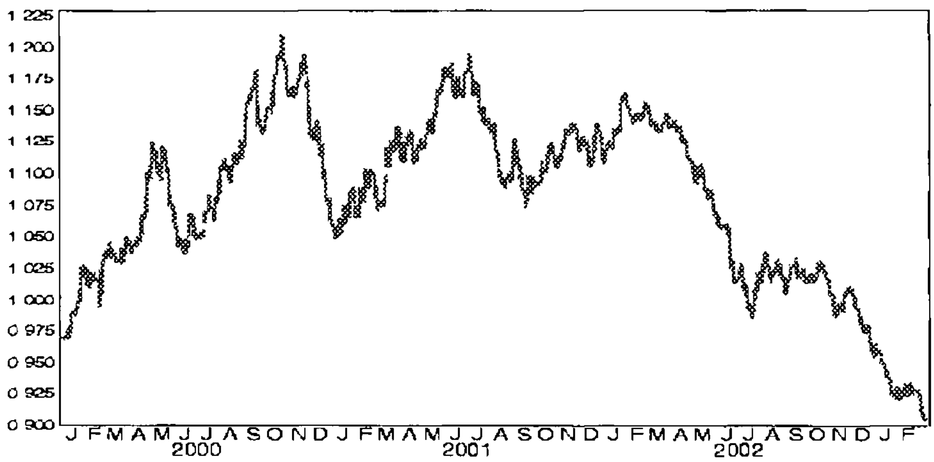

Figure 1 2 US Dollar dropping versus Euro over last three years The lowest is the last day shown i e 10 March 2003

However, market volatility is the investors' intensive concern as it determines whether profit or loss on their portfolios is going to take place Today, a financial analysts' job is to determine a reasonable estimation of what the future market behaviour will be and how the customers' portfolio value is going to change He or she will design the best portfolio for their clients to optimise the risk-return relationship on the combined assets in the portfolio That is the importance of analysing and predicting the market future, which has reached a significant place in the field of finance Estimating the future value of a portfolio and the probability of how likely it is to be realised requires some features and instruments that have introduced a new field in finance, known as portfolio management or investment strategy [Fuller et al , 1987] The first element of investment strategy is how to combine assets to gain the most return with respect to the level of risk associated with each investment Indeed, different investors have different levels of risk preferences [Fuller et al , 1987] Some people prefer to deal with a low-risk investment even though it has a lower level of return Others prefer the chance of gaining high return from their investment despite increased risk An optimisation process, however, can imply sets of assets that result in either most return for a fixed level of risk (dependent on the investor's risk preference) or the lowest amount of risk for a fixed level of return [Elton et al , 1981] In any such process, the way that stocks move together is an essential element to be studied The problem of how stocks inter-relate is determined by the correlation coefficient, which measures co-movements between different stocks [Elton et al , 1981] For a set of assets combined in a portfolio, the correlation coefficients between each pair of stocks can be displayed in a correlation matrix

1 5 Scope of the thesis

In Chapter 2, we talk about theories of portfolio management citing key literature on the area More details about the important role of the correlation matrix of stocks in the portfolio management are explained, and the difficulties in calculating this matrix are demonstrated

In Chapter 3, of this thesis, we discuss error problems inherent in the structure of such correlation matrices This error problem occurs for several reasons, such as the finite number of records in the stock's prices To distinguish the error in the correlation matrix, C, a technique, known to random matrix theory (RMT), is applied [Laloux et al , 1999] This allows us to differentiate the error from true information In this chapter, also, we demonstrate a simulation approach to study the result of RMT, when scattering (or noise) in stock's prices is not in the same

In Chapter 4, methods of removing errors in C are investigated However, the stability of the matrix is the important matter, which should be preserved in all cases Accordingly, we apply a statistical model introduced by Krzanowski (1984) to examine the stability of C after removing errors from that Our results indicate that the stability of the matrix notably reduces and therefore, the analysis on that cannot be, simply, reliable The method deals with the stability of eigenvalues and eigenvectors of C, in which for a small change in an eigenvalue, it measures the changes of eigenvectors Concerning this method, we extract a new method of removing errors in the way that stability of the matrix is mostly kept

In the last chapter we take the most reliable part of the correlation matrix, which is the maximum eigenvalue, to study the movement of the market The largest eigenvalue of the correlation matrix is always an effective measure of risk and the corresponding eigenvector represents the most-risky combination of stocks Therefore we focus on the daily changes of the maximum eigenvalue to investigate the risk on the set of stocks This study is based on a theorem in linear algebra called the Spectral Theorem, and suggests a method to measure the day-to-day changes of the correlation matrices Our results demonstrate a linear relationship between the changes in eigenvalues' ratio and the changes in the prices of the stocks We model this relationship and discuss possible causes and breakdowns of the model

Finally, an overall conclusion plus some explanation about the future work is presented

Chapter 2

Literature Review

In order to gain the most return from the set of assets of which it consist a portfolio we need to estimate the risk and return of the portfolio In this chapter, we review the literature in "portfolio theory" [Elton et al , 1981] that leads to the problem of risk prediction Different parts of this process include the study of correlations among stocks and the risks associated with individual assets In this study, also, we review other risk measurements used in "Capital Asset Pricing Model" [Fuller et al , 1987], which is one of the most important models in finance This chapter leads up to the point of my project and a history about that We start by considering "Markowitz portfolio theory" [Markowitz, 1991]

2 1 A review of Markowitz portfolio theory

The modern portfolio theory developed by **Markowitz** in the early 50s [Markowitz, 1952] shows how to measure the risk associated with various securities and how to combine these securities in a portfolio to get the maximum return for a level of risk that investors are willing to accept

The cornerstone of the theory of Markowitz is the concept of "diversification" Diversification is a portfolio strategy to reduce the risk by combining a variety of investments It implies that as the number of various stocks gets higher, the overall risk of the combined portfolio gets lower Strongin et al (2000) indicate that the risk of the portfolio is always a function of $1/\sqrt{N}$, where N is the number of distinct assets

One of the results of the theory of Markowitz is the clarification of the definition of risk Before his theory of modern portfolio, there were other definitions for risk like the one suggested by Graham et al (1962) Graham et al defined risk as a "margin of safety", based on the idea that the analyst should independently estimate the value for the security in respect of its earning power and financial characteristics and despite the market price

Also another definition by Sharpe (1981) considers the below-the-mean variability, since, for most investors, risk is related to the chance that future portfolio values will be "less" than expected However, empirical studies have shown [Blume, 1970] that it makes little difference whether one measures variability of returns on one side or both sides of the expected return Since working with the below-the-mean variability is not easy, total variability of returns has been used widely as a proxy for risk and the most commonly used measures have been the variance and standard deviation of return [Fuller et al , 1987] In general standard deviation is preferable because the standard deviation of a portfolio's return can be determined from the standard deviations of the returns of its component securities [Sharpe, 1981] No other variability measures are as simple to use as standard deviation Therefore the risk or variability of a portfolio consist of N assets is measured as

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{\substack{k=1 \\ k\neq i}}^{N} X_i X_k \sigma_{ik} , \qquad (2\ 1)$$

(where σ_i^2 is the variance of component security i and X_i are the fractions of the investor's funds invested in the security i and σ_{ik} is the covariance between security i and security k expressed as $\sigma_{ik} = \frac{1}{T}\sum_{j=1}^{T}(r_{ij} - \mu_i)(r_{kj} - \mu_k)$, in which T is the number of records for each asset and μ_i is the average of assets i records and r_{ij} is the return on the i[th] asset)

2 2 A review of Capital Market Theories and predicting models

After the theory of Markowitz developed, there was a need to implement it in the real world Sharpe in 1961 in his PhD dissertation created the basis for a practical model called the "single-index model" or "one-factor model" The major assumption of this simplified model is that all the co-movements of stocks can be explained by a single factor One version of the model called the "market model", uses a market index such as the S&P 500 as the factor affecting stock movements [Elton et al , 1981] According to the market model it is assumed that when the market goes up most stocks tend to increase in price, and when the market goes down, most stocks tend to decrease in price

As well as the single-index model, King (1966) presented evidences on the existence of other factors beyond the market factor He measured effects of common movements among stocks beyond market effects and found an additional covariation of stocks associated with industry This model is one of the family of "multi-index model" [Elton et al , 1981] Although multi-index models attempt to capture some of the non-market factors and, hence to incorporate additional information, they fail to provide a better description of the stocks behaviour That is because the cost of introducing additional indices increases the chance to pick up random noise rather than real information Elton et al (1973), and (1978) study "averaging techniques" to predict the co-movements among stocks Such techniques smooth the entries in the historical correlation matrix to reduce random noise and so provide a better forecast However, the disadvantage of averaging models is that real information may be lost All the models described above are statistical and are used to explain the covariance[1] between asset returns There is also an economic model of equilibrium returns which includes Capital Asset Pricing Model (or CAPM), one of the most famous of all financial models [Fuller, 1987] This model has been developed by Sharpe (1964), Lintner (1965), and Mossin (1966) in 1960s and it is based on the idea that investors demand additional expected return (called risk premium) if asked to accept additional risk The expected return of a stock equals the rate on a risk-free asset plus a risk premium According to CAPM, risk is defined in terms of volatility, and it is measured by the investment's β[2] coefficient, which is the ratio of covariance between stock's return and market's return to the variance (or volatility) of the market This type of risk was identified by Sharpe (1964) to be "systematic risk", and also known as "market risk" Since systematic risk is only associated with the market, it cannot be diversified away Beside the systematic risk, there is "unsystematic risk", which is the risk specific to a company's fortunes and can be reduced by increasing the number of various assets

An alternative asset-pricing model to the CAPM was created by Ross (1976) and it is called the Arbitrage Pricing Model (or APT) Unlike CAPM, APT may specify return on an asset as a (linear) function of more than one single factor The strength of this model lies in the fact that it is based on the no-arbitrage condition, which means that two identical or similar items cannot be sold at different prices on different markets

2 3 Historical covariance matrix, A problem

Implicitly, in all the theories and models we have mentioned so far, the calculation of the covariance matrix is required as a practical stage For example in the first step of calculating the historical portfolio risk one needs to construct the covariance matrix

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[1] A measure of the co-movements of two assets

[2] $\beta_i = \dfrac{Cov_{im}}{\sigma_m^2}$, in which Cov_{im} is the covariance between ith stock and market and σ_m^2 is the variance of the market

on a past period Also for estimating β in the models mentioned, the need to calculate the historical covariances between a huge number of stocks can be seen Studying the correlation[3] (or covariance) matrix, C, is also of great interest in data analysis in order to extract information from experimental signals or observations e g in pattern recognition [Theodoridis et al , 1999], weather forecasting [Jolliffe et al , 2003], and economic data analyses [Laloux et al , 1999] Moreover, many statistical tools such as principal component analysis and factor analysis [Kim et al , 1979] try to obtain the meaningful part of the signals in a correlation matrix in order to reduce the dimensionality of the data set

However, use of this matrix, C, which has been widespread over decades, came into question as two different research groups realised that a large part of it does not include useful information but rather significant errors [Laloux et al , 1999, Plerou et al , 1999] Laloux et al (1999) and Plerou et al (1999) simultaneously worked on the application of some physical theories, (discussed below), in the field of finance and concluded that C does not include pure information In particular, the difficulties associated with determining the true correlations between financial assets arise primarily due to

- Non-stationary[4] nature of the correlations between stocks
- A finite number of observations of asset price movements

The issue then becomes how to identify the true correlated assets when there is error in the measured correlations

2 3 1 Physical history

In their book, Mantegna and Stanley (2000) stated that, since 1990, a growing number of physicists have attempted to analyse and model financial markets The interest of these people in financial systems was initially stimulated by work such as that of Majorana (1942) on the essential analogy between statistical laws in physics and in the social sciences In 1999 similarities between correlations in financial and physical time series were noted by two different groups of physicists [Bouchaud et

[3] which differ from covariance matrices in having the variance normalised out

[4] It means the correlation between any two pairs of stocks changes with time

al , 2000, Plerou et al , 1999] Plerou et al (2000a) explained that financial markets are examples of complex systems in which a huge amount of data exist The correlations between the price changes of stocks can be compared to those of movements of molecules in a box containing many gas molecules, in which there are some random pair-wise bonds between some of the gas molecules Alternatively, in a more complicated system, not just random pair-wise bonds are involved, but rather bonds connecting clusters of molecules, which evolve over time Thus, new molecules are connected to the existent clusters and others, which are part of one cluster, connect to different clusters

In all these cases one can calculate the empirical measured correlation matrices but in Finance, unlike for most physical systems [Plerou et al , 2000b], there is no algorithm to calculate the "interaction strength" between two companies and there are difficulties in quantifying stock correlations

A further difficulty is that of strictly limited number of observations, which causes random errors in correlation matrices (as mentioned above)

In nuclear physics, the problem of understanding the properties of matrices with random entries has a rich history beginning with the work of Wigner (1951a, 1951b, and 1956) and subsequently Dyson (1962, 1963) and Mehta (1963, 1991) In the fifties, physicists faced the problem of understanding (and measuring) the energy levels of complex nuclei[5], which existing sub-atomic particle models failed to explain Large amounts of data on the energy levels were becoming available but were too complex to be explained by model calculations because the exact nature of the interactions was unknown To solve this problem, Wigner (1951a, 1951b) made the assumption that the interactions between the constituents comprising the nucleus are so complex that they can be modelled as random Then, he assumed that the Hamiltonian[6] describing a heavy nucleus[7] could be described by a matrix H with independent random elements Based on this assumption, Wigner (1956) derived properties for the statistics of eigenvalues of the **random matrix** H whose elements

[5] The nucleus, like the atom, has discrete energy levels [Ernest Orlando Lawrence Berkeley National Laboratory, http //www lbl gov/abc/wallchart/teachersguide/pdf/Ch06-EnergyLevels%20doc pdf]

[6] Hamiltonian is a series of operators associated with the system energy

[7] Heavy nuclei are composed of many interacting constituents

are mutually independent random variables Eventually he found these properties in remarkable agreement with experimental data [Wigner, 1956]

A long time of about three decades after this result in nuclear physics, Laloux et al (1999) applied Random Matrix Theory (RMT) to finance and showed that C can be partitioned into noisy and non-noisy components From a more formal mathematical point of view, this phenomenon was studied also by a group at the University of Boston by Plerou et al (1999) In their own independent research they used the method of RMT to study the correlations of stock price changes

Based on this earlier work on RMT in Finance, Bouchaud and Potters (2000) argued that Markowitz's theory of *optimal portfolio* is not adequate on its own Therefore, in [Laloux et al , 2000] the authors introduced a technique to remove noise from the matrix by cleaning the noisy band and they suggest that the risk of the optimised portfolio obtained using a *cleaned correlation matrix* is more reliable They, therefore, claim that the cleaned correlation matrix is more stable Plerou et al (2000b, 2001b) and Mounfield et al (2001) discuss the stability of C by examining the overlap (i e measured by scalar product) of its eigenvectors over two consecutive sub-periods For those showing higher overlap (values near unit) over two sub-periods, the stability is assumed to be higher and for those showing lower overlap values, stability is lower The evidence suggests that the part of C known as noisy (in RMT), tends to show lower level of stability (in overlap)

In research on techniques of Principal Component Analysis [Jolliffe, 1986] the stability of the eigenvectors is also studied Green (1977) and Bibby (1980) discuss the rounding effect of Principal Components (PCs) on the variance (eigenvalues) of the matrix They show that rounding PCs to a few decimal places does not make a great change though the Principal Components are no longer exactly orthogonal

Krzanowski (1984), however, considers the opposite problem Instead of looking at the effect of small changes in the eigenvectors on the eigenvalues, he examines the effect of small changes of the eigenvalues on the eigenvectors He then argues that this gives important information on the stability of Principal Components [Jolliffe, 1986] It is thus possible, using the Krzanowski technique, to examine the stability of C before removing noise and afterwards This demonstrates how noise-removing methods can effect the stability of the modified correlation matrix and how a cleaning method, in general, can preserve the stability behaviour of the correlation matrix This is discussed in detail in the following chapters with experimental results used to demonstrate these points for our data

The focus of this review has been

(a) topics in our project and literature on the area,

(b) noise in the correlation matrix,

(c) stability behaviour of the correlation matrix

Practical implementations are demonstrated in the following chapters Further, we bring together a number of these features in the study of *co-operative behaviour* in market, [Crane et al , 2002, Keogh et al , 2003] in chapter 5

Chapter 3
Random Matrix Theory

We study the basis for applying Random Matrix Theory (RMT) on an empirically-measured correlation matrix, C, of financial data type and demonstrate that this matrix contains a large amount of noise

Firstly, we simulate a set of data and add different volumes of random noise to see the results of the theory on each data set

Secondly, we apply RMT on empirical data and estimate the percentage of noise in C using eigenvalue and eigenvector analysis The experimental results for each of them are presented

3 1 Random Matrix prediction

Wigner (1956) and Dyson et al (1963) describe Random Matrix predictions as an average of all possible interactions in a nucleus Additionally, they explain that deviations from the universal predictions of "Random Matrix Theory" identify non-random properties that are specific to the considered system

Agreement between the distribution of the eigenvalues of a matrix M, with those from a matrix made up of random entries implies that M has entries that contain a considerable degree of randomness, as has been shown in the literature [Plerou et al , 2000b, Laloux et al , 1999] This matrix made up of random entries with unit variance and zero mean is called a *random matrix* [Mehta, 1991] In the case of a correlation matrix, agreement between eigenvalues' distribution of C and those from a random matrix, represents randomness (or noise) and therefore deviations from RMT represent genuine correlation [Plerou et al , 2000a] This is exactly our problem, to identify the true information (i e correlated assets) among noise (or randomness) in the financial correlation matrix The method is to compare the distribution of eigenvalues of correlation matrix against the "null hypothesis" of a random matrix Since the correlation matrix is symmetric the random matrix, which it is compared to, should also be symmetric [Plerou et al , 2001b] Any agreements between them should pinpoint noise and any deviations should reflect genuine correlated assets In other words, the part of the correlation matrix that has the same behaviour as the random matrix is considered to be the noisy part while that which contains information, is considered to be non-noisy

3 2 Empirically-measured correlation matrix

Normally, the *price changes* (or *return*) of stocks are employed to quantify the empirical correlation matrix [Plerou et al , 2001a] Therefore, we need to calculate the price changes of assets $i=1, \ldots, N$ over a time scale Δt For a price $S_i(t)$ of the i^{th} asset at time t, one can define its price change/ return $G_i(t)$ as

$$G_i(t) = \ln S_i(t+\Delta t) - \ln S_i(t) \tag{3 1}$$

It should be noticed that the terms "return" and "price changes" are sometimes used interchangeably (as in [Plerou et al , 2001a] for instance) but strictly speaking, they are different

Also, it needs to be mentioned that different stocks have varying levels of volatilities In the literature review chapter (chapter 2), we explained that volatility or risk associated with the stock's price changes can be measured by its variance (or alternatively standard deviation) With respect to various standard deviations for different time series, one usually defines a normalised return to standardise the different stock volatilities Therefore, we normalise G_i with respect to variance σ_i as follows

$$g_i(t) \equiv \frac{G_i(t) - \overline{G_i(t)}}{\sigma_i}, \qquad (3\ 2)$$

where σ_i is the standard deviation of G_i for assets $i=1, \ldots ,N$ and $\overline{G_i}$ is the time average of G_i over the period studied

There is a standard definition [Plerou et al , (2000b), (2001a)] for the correlation matrix C, with elements C_{ij},

$$C_{ij} = \overline{g_i(t) g_j(t)} \qquad (3\ 3)$$

Here the bar denotes a time average over the period studied In matrix notation, the correlation matrix can be expressed as

$$C = \frac{1}{T} G G^T, \qquad (3\ 4)$$

where G is an $N \times T$ matrix with elements $\{g_i(m),\ i = 1, \ldots ,N,\ m = 0, \ldots ,T-1\}$, T is the number of records and G^T denotes the transpose of G

3 3 Eigenvalue distribution of the correlation matrix

As stated above, our aim is to extract real information about correlations from C So, the properties of C are compared with those of a random correlation matrix as has been done by Laloux et al (1999) and Plerou et al (1999) According to Equation 3 5 a "random" correlation matrix R is considered [Plerou et al , 2001b] as,

$$R = \frac{1}{T} A A^T \tag{3 5}$$

where A is an $N \times T$ matrix containing N time series of T random elements with zero mean and unit variance, which are mutually uncorrelated Statistical properties of random matrices such as R have been known for many years in physical literature [Dyson, 1971, Edelman, 1988, Sengupta et al , 1999] Particularly, under the condition of $T \to \infty$, $N \to \infty$ and providing that $q = \frac{T}{N} \geq 1$ is fixed, it was shown [Sengupta et al , 1999] that the distribution of eigenvalues λ of the random correlation matrix R is given by

$$P_r(\lambda) = \begin{cases} \dfrac{q}{2\pi\sigma^2} \dfrac{\sqrt{(\lambda_{\max} - \lambda)(\lambda - \lambda_{\min})}}{\lambda} & \lambda_{\min} \leq \lambda \leq \lambda_{\max} \\ 0 & elsewhere \end{cases}, \tag{3 6}$$

where σ^2 is the variance of the elements of G, (in the case of a normalised matrix G, it is therefore equal to unity), and $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues of R respectively, given by

$$\lambda_{\substack{\max \\ \min}} = \sigma^2 \left(1 + \frac{1}{q} \pm 2\sqrt{\frac{1}{q}}\right) \tag{3 7}$$

These are the theoretical maximum and minimum eigenvalues that determine the bounds of the theoretical distribution of eigenvalues All the eigenvalues of the random matrix are located between these two values. If eigenvalues of C are beyond λ_{max} it is said that they deviate from the random (or theoretical) bound.

To see the results of RMT in practice we apply it first on generated data (a novel approach) and second on a set of real data. With generating data and having the advantage of controlling the volume of noise, we aim to examine the effect of different volume of added noise on the result of RMT. While using real data is the main purpose to approximate noise in a real historical correlation matrix.

3.4 Simulation

In order to see the results of RMT on a correlation matrix made up of a simulated data we generate a set of sinusoidal random time series. A set of 450 sinusoidal time series with 1500 observations (i.e. the same size as our real data set) is generated with random amplitude and random phase. Figure 3.1 shows some of these time series.

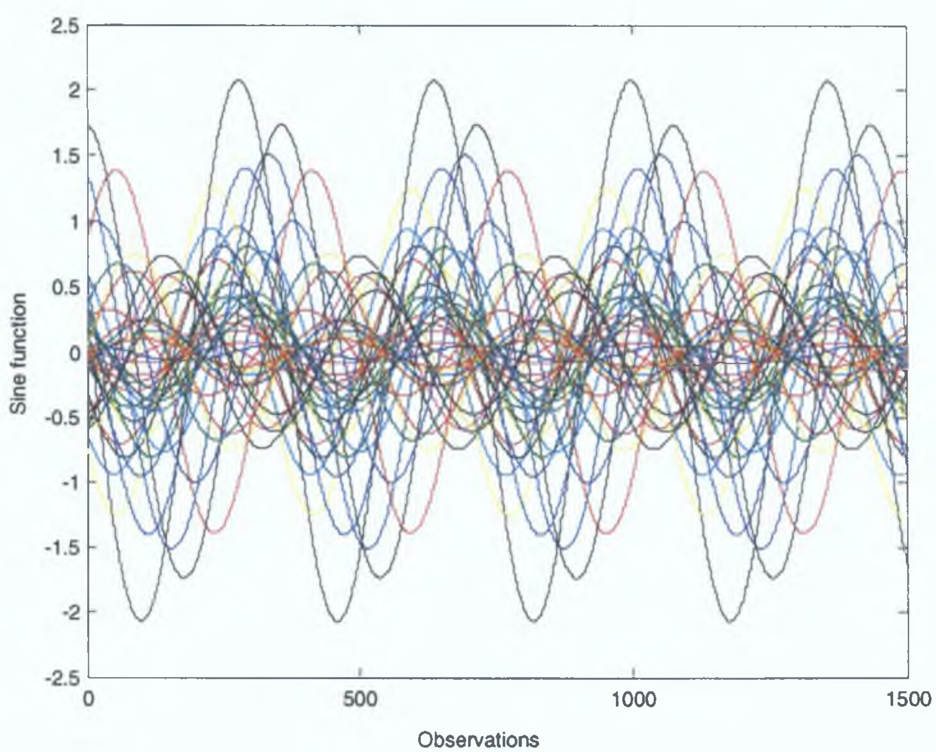
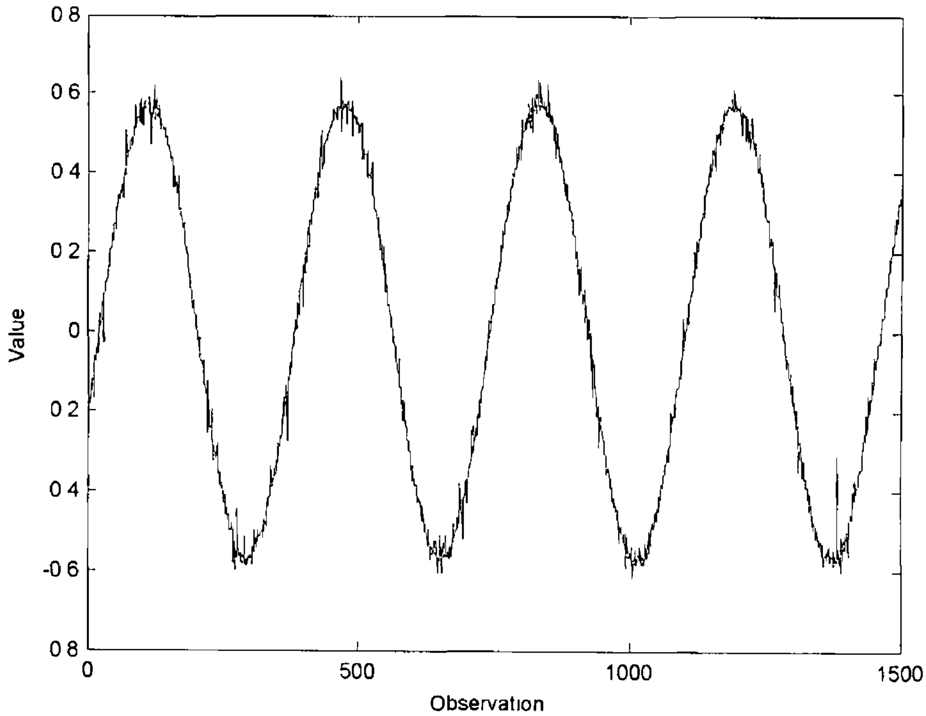
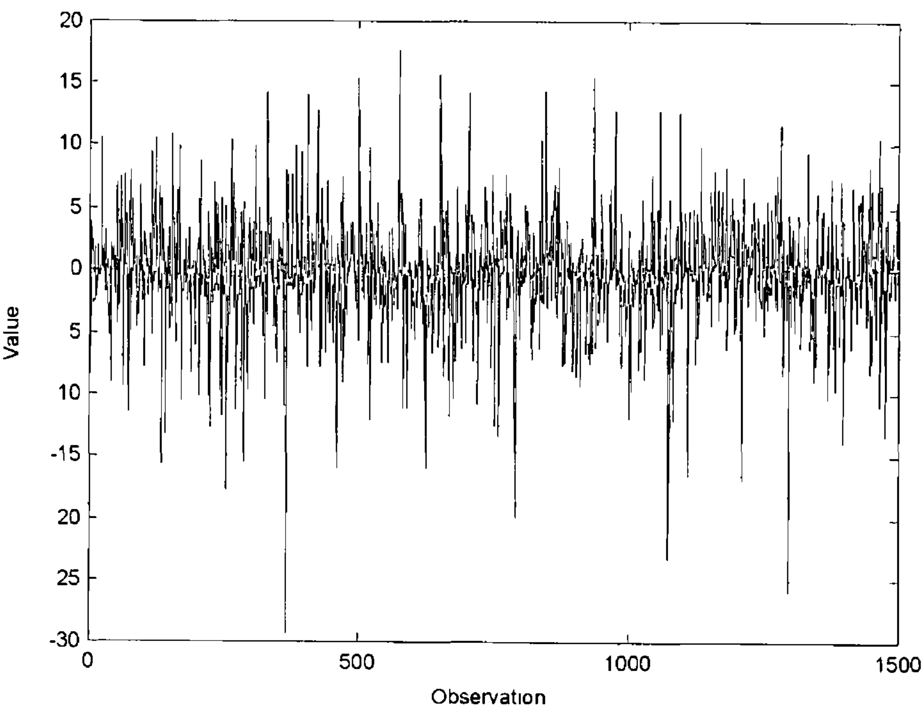


Figure 3.1 Randomly generated sinusoidal time series



(a)



(b)

Figure 3 2 A sinusoidal time series with added random noise (a) with noise standard deviation equal to 0 02, (b) with noise standard deviation equal to 4

Next, we add some random noise normally distributed with zero mean and a particular standard deviation to the time series We control the volume of noise added to the generated data by changing its standard deviation Figure 3 2 shows one of the time series with two different volumes of added noise In Figure 3 2(a) the time series is quite clear, while, in Figure 3 2(b), it is mainly dominated by noise We study the behaviour of the correlation matrix constructed from those noisy time series by applying RMT in order to see the effect of different volumes of noise in time series on noise in C

First by using Equation 3 3 the correlation matrix C is constructed Since the number of observations is $T=1500$ and the number of time series is $N=450$ the inequality $q = \frac{T}{N} \geq 1$ is satisfied Therefore, we can apply the RMT to our generated data and plot the distribution of the eigenvalues of C

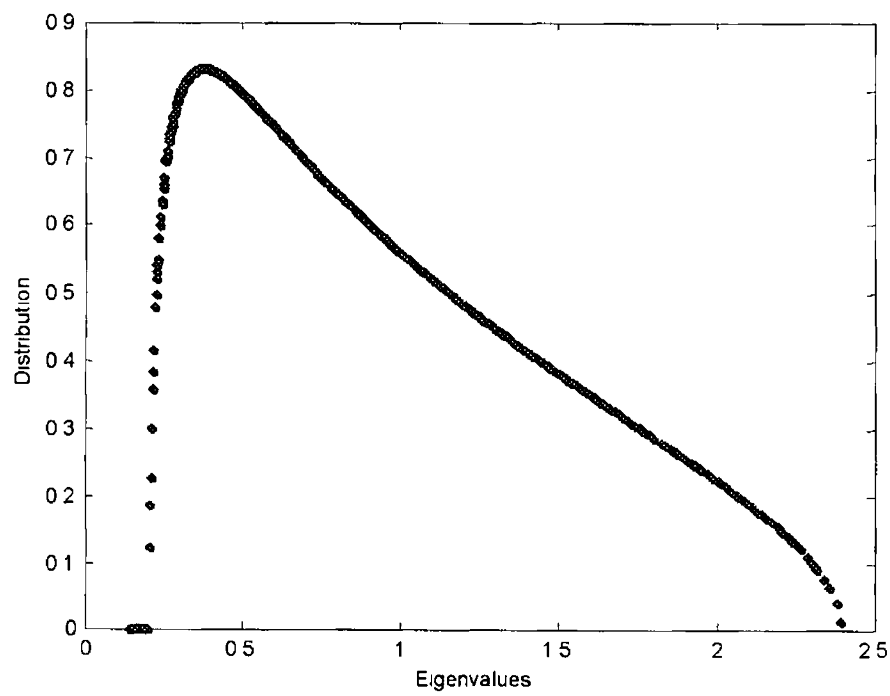


Figure 3 3 The theoretical distribution of the eigenvalues of the generated random matrix

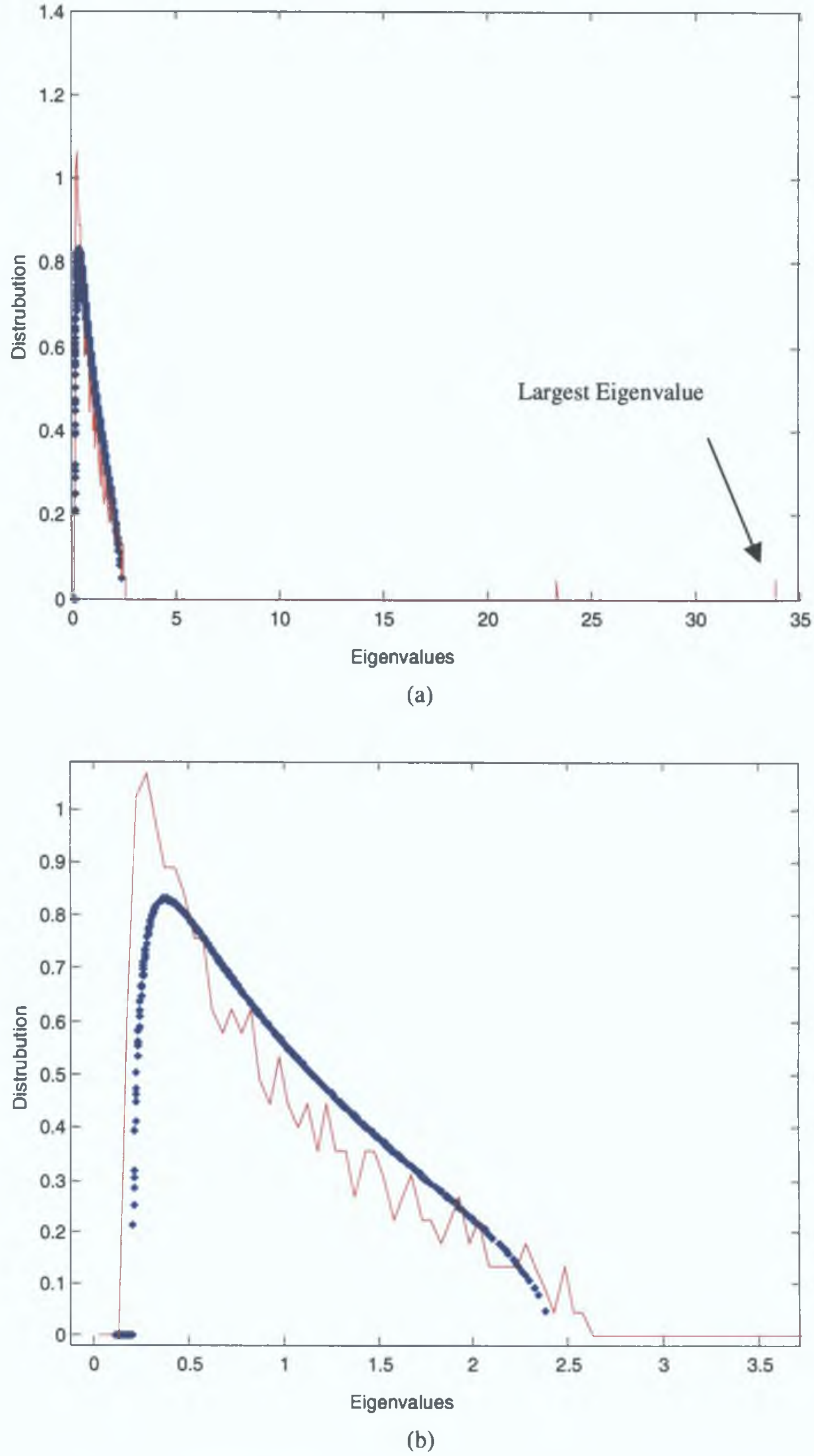


Figure 3.4 (a) The empirical distribution (solid line) and the theoretical distribution (diamonds) of eigenvalues, (b) a closer look at the beginning part of the graphs.

Using Equation 3 6 the theoretical distribution of the eigenvalues of the correlation matrix is calculated This distribution is shown in Figure 3 3 The actual (empirical) distribution of the eigenvalues of C is also calculated and together with the theoretical one is shown in Figure 3 4 In this experiment some random noise with standard deviation equal to 0 02, (Figure 3 2(a)), is added to the generated sinusoidal time series It is observable, from Figure 3 4, that a large part of the empirical graph is similar to the theoretical one This part (Figure 3 4(a)) that is carrying noise corresponds to the noisy band of the correlation matrix

However, there are some eigenvalues that deviate from the theoretical graph, which are called non-noisy eigenvalues corresponding to that part of the correlation matrix that contains real information

In the case that the standard deviation of the noise, σ_N, is 0 02, the number of deviated eigenvalues is 7 out of 450 (i e about 1 6%)

3 4 1 Experimental analysis

In the next stage, we increase the volume of the added noise by increasing its standard deviation The largest amount of noise added to the time series has a standard deviation equal to 4 The same time series of Figure 3 2(a) with this new (increased) volume of added noise is shown in Figure 3 2(b) The distribution of the eigenvalues of the correlation matrix made up of this noisy data for both the theoretical and empirical cases are shown in Figure 3 5 The number of deviated eigenvalues is 18, which is 4% of the total number of eigenvalues

However, in order to estimate the exact effect of the added noise we increase the volume of noise gradually Starting from 0 02, standard deviations of 0 05, 0 08, 0 5, 1, 1 5, 2, 2 5, 3, 3 5 and 4 are examined and the number of deviated eigenvalues is estimated The results are presented in Table 1 and plotted in Figure 3 6(a)

At the beginning, by increasing σ_N an increase in the number of deviated eigenvalues is observed This is shown in figure 3 6(a) for the standard deviations from 0 02 to 0 08 However, for σ_N varying from 0 08 to 4 no dramatic increase is observed From this we deduce that except for small values of added noise, the volume of the added noise has no effect on the number of deviated eigenvalues

Consequently, we conclude that the number of non-noisy eigenvalues is independent of added noise. On the other hand, in Figure 3.5 it is shown that the maximum eigenvalue of C, say *max(e)*, for the case when $\sigma_N = 4$, is closer to the $\lambda_{\max}$, theoretical maximum value from Equation 3.6, than *max(e)* when $\sigma_N = 0.02$, Figure 3.4(a). This motivates us to investigate the effect of the volume of added noise on the distance between $\lambda_{\max}$ and *max(e)*. For the same experiments as Table 3.1 this distance is examined and the result is plotted in Figure 3.6(b). Again it is seen that after the first few points, which show the maximal differences, the rest of them fall on an almost horizontal line. Again showing that $(\lambda_{\max} - max(e))$ is almost independent of added noise.

We also computed the eigenvalues of an ideal C with absolutely no noise. Only two of the largest eigenvalues are non-zero and the rest are all zeroes. Additionally, those non-zero eigenvalues are much larger than $\lambda_{\max}$. This means that as C approaches the ideal correlation matrix, more eigenvalues approach zero and the difference between the smallest and largest eigenvales (that has a value of about 260) is significant. That is the reason why we see less deviated eigenvalues for a very low volume of noise in our experiments, i.e. $\sigma_N = 0.02$.

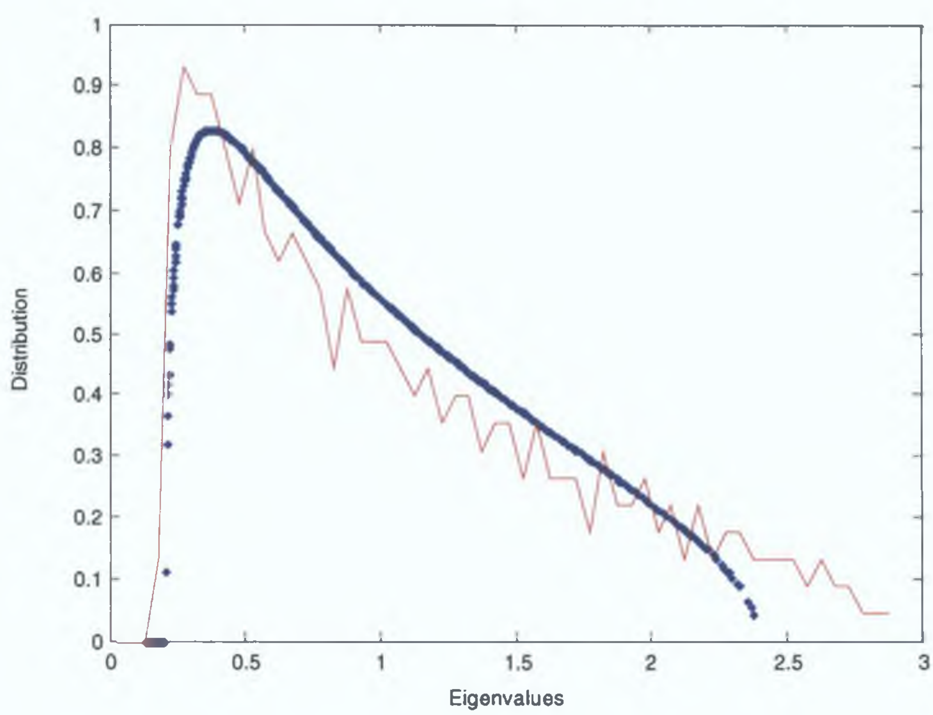


Figure 3.5 The empirical distribution of eigenvalues (solid line) and the theoretical distribution (diamonds). Added noise with standard deviation equal to 4

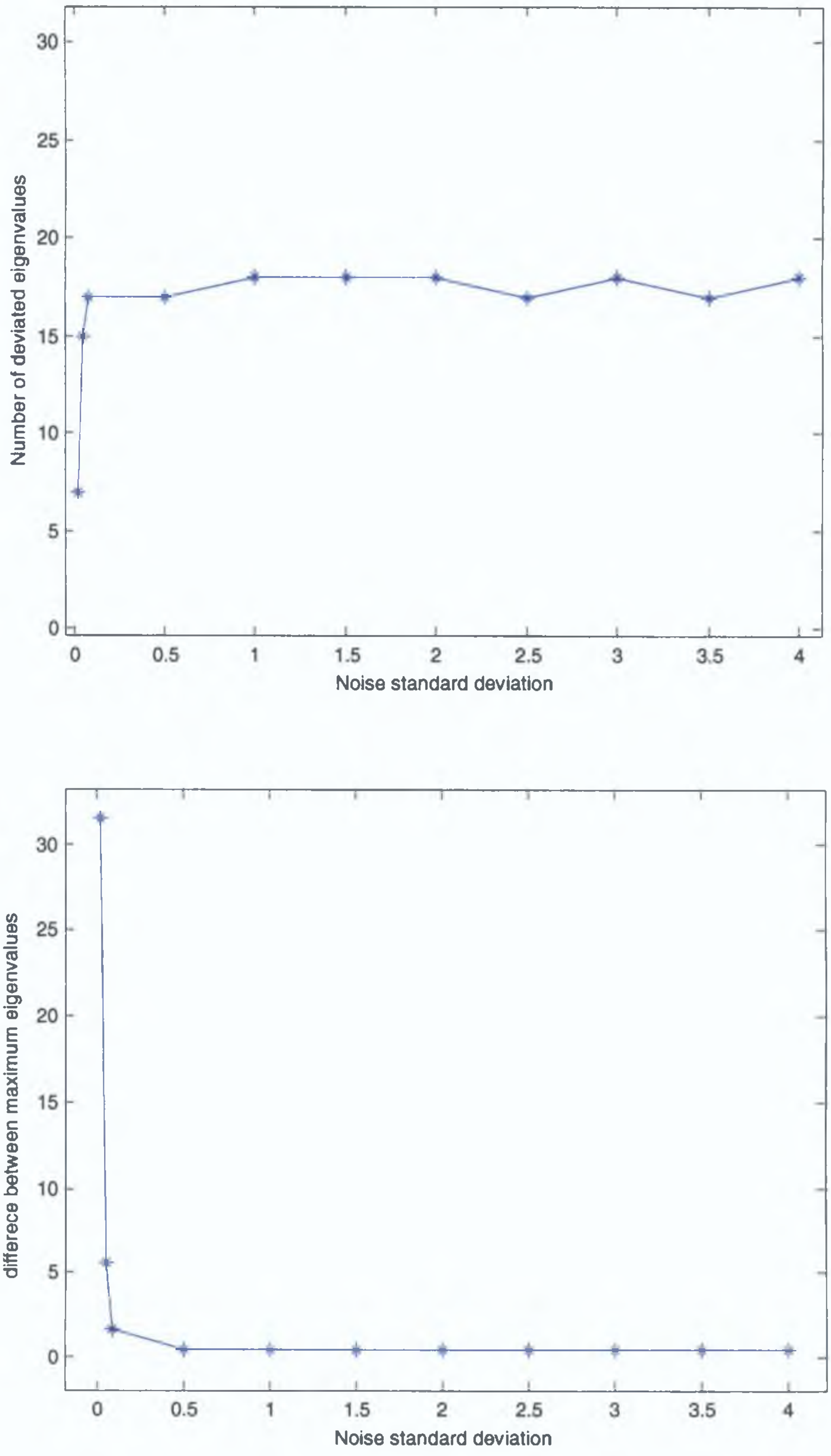


Figure 3.6 Relationship between the number of deviated eigenvalues from the noise band and the volume of noise

Noise STDV	0 02	0 05	0 08	0 5	1	1 5	2	2 5	3	3 5	4
# deviated eigenvalues	7	15	17	17	18	18	18	17	18	17	18

Table 3 1 The volume of noise versus the number of deviated eigenvalues

The implications really are that the strong signal core is shown by this method and is contained in a few relatively unaffected values

3 5 Application to empirical data

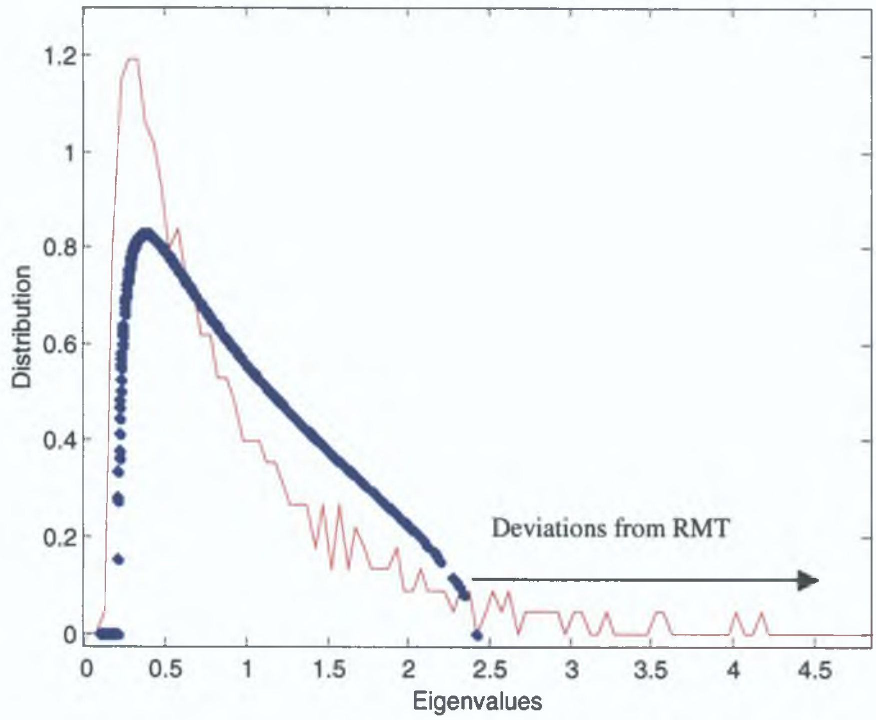
The set of data we have for our experiments consists of 30-minute intra-day prices from S&P500 over the period started from the beginning of April 1997 to the beginning of April 1999 Since the number of observations (records) is important for using RMT, the intra-day data can provide a large number of records on an even small period of time This set of data with about $N=450$ companies, and over $T=1500$ observations is appropriate for our purpose since according to the constraint on using Equation 3 5, $q = \frac{T}{N} \geq 1$ is satisfied

3 6 Numerical results

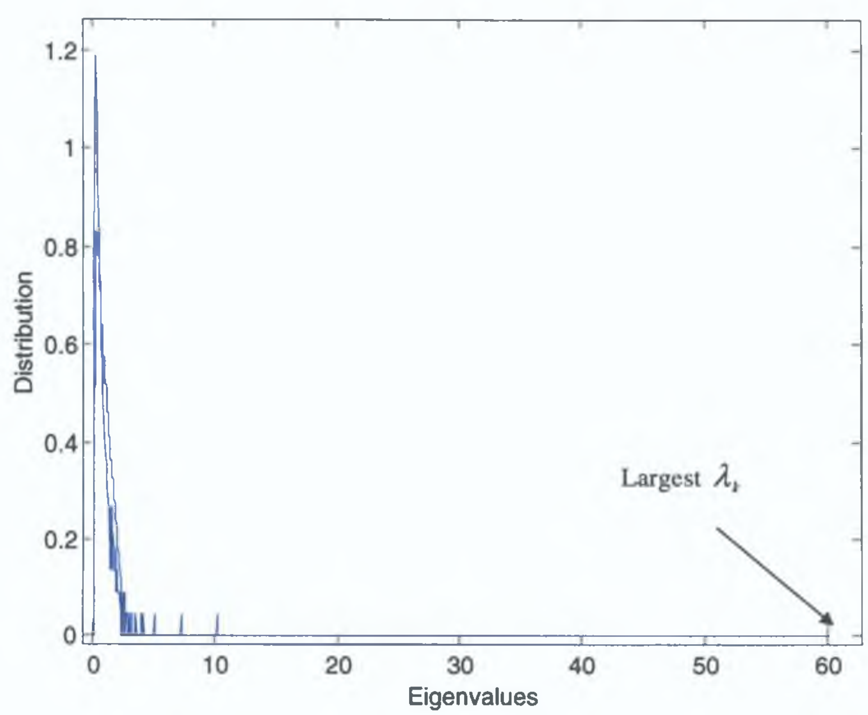
Firstly, we construct the empirically-measured correlation matrix C by using the Equation 3 6 as in the previous section, and then compute the eigenvalues λ_k where $k=1, \ldots, N$ is in ascending order

The distribution of the eigenvalues of the corresponding random correlation matrix is also calculated using Equation 3 6 Figure 3 7 shows the results of our experiments on the 30-minute data for 452 stocks and 1500 records The same as the results of RMT on the simulated data we can observe two things from Figure 3 7

- The bulk of the eigenvalues of C conform to those of a random matrix with graphs consistent with the latter This consistency means that there is a measure of randomness in the bulk of the eigenvalues Therefore, as stated in [Laloux et al , 1999] we conclude that the corresponding part of eigenvalues is random and we consider this part as the noisy band



(a)



(b)

Figure 3.7 (a) Eigenvalue distribution for C constructed from the 30-minute prices for 452 stocks of S&P 500 for 1500 records started from April 1997. The diamond curve shows the RMT result for $P_r(\lambda)$ in Equation 3.6. Several eigenvalues outside the RMT upper bound λ_{max} can be seen. (b) a wider view of the graph (a) including the highest eigenvalue

- In Figure 3 7(a) and (b), which represent the same quantity on different scales, deviations from RMT for a small number of the largest eigenvalues can be clearly seen Our experiments indicate that 22 eigenvalues are beyond the noise band and the rest are consistent with RMT results In other words, just 4 7 % of the eigenvalues deviate from the RMT prediction

This is in agreement with literature, e g [Laloux et al , 1999] which argues that at most 6% of the eigenvalues are non-noisy

In the case of our data, approximately 95 3% of the total number of eigenvalues fall in the region where the theoretical formula (3 6) applies Thus, less than 5% of the eigenvalues appear to carry most of the information These are similar to the results of Laloux et al (1999) and Plerou et al (2000b)

In addition, the noise and information content of the correlation matrix can be examined by eigenvector analysis This looks at the structure of the eigenvectors and compares the eigenvector component distribution with those of the random matrix

3 6 1 Eigenvector analysis

The eigenvector components of the random matrix are normally distributed with zero mean[1] as stated by Laloux et al (1999) Thus, it is expected that the eigenvectors corresponding to the noise band of the correlation matrix follow a similar distribution to the random ones Figure 3 8 represents the distribution of the eigenvector components corresponding to our actual correlation matrix The eigenvectors associated with the largest eigenvalue, and some of the smaller ones are shown It is seen that the distribution of the market eigenvector[2], in black, does not follow the same structure as the others The components of the market eigenvector are distributed around a mean of 0 045 and a variance of 0 05, whereas the other eigenvector components are distributed with zero mean and a much wider variance In fact the dispersion of the components around the mean gets broader as one examines eigenvectors associated with progressively smaller eigenvalues

[1] This distribution is independent of the distribution of the random matrix elements

[2] The eigenvector associated with the largest eigenvalue which has approximately equal component values

Although the eigenvector analysis we have done is not as precise as the eigenvalue analysis above, it does suggest that the market eigenvector behaves differently to the eigenvectors of the random matrix and, therefore, represents the most reliable part of the correlation matrix.

So far, it has been shown that the results from the theory of random matrices are of great interest in understanding the statistical structure of the empirical correlation matrices. The central result of this study is in recognising that there exists a large amount of noise in the eigenvalues and corresponding eigenvectors of C.

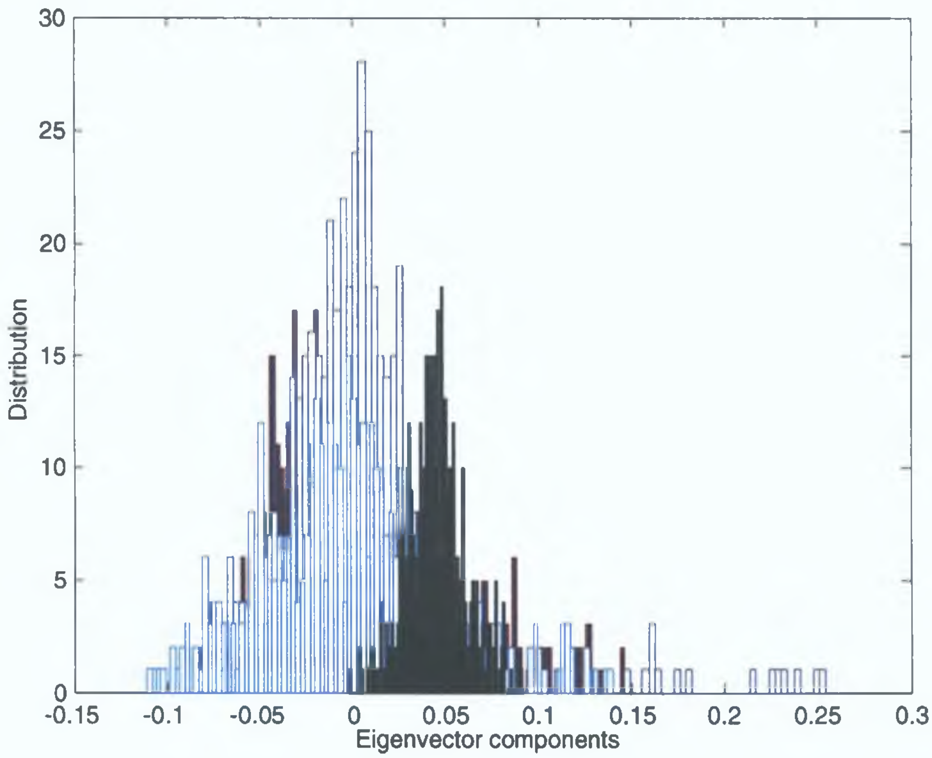


Figure 3.8

Distribution of eigenvector components

3.7 RMT prediction and portfolio theory

We have noted specifically that the risk associated with a particular portfolio consisting of N assets, which can be expressed (in some sense) as the total variance σ_p^2, in which the risk is directly associated with the correlations between stocks. In portfolio optimisation, σ_p^2 should be minimised for a given value of the return of the portfolio, R_p The result of the optimisation analysis indicates that the smallest eigenvalue is associated with the "least risky portfolio" and the corresponding eigenvector determines the weights (or fraction) of stocks in the portfolio (as stated in [Chan et al , 1999, Alexander, 2001] for instance) Therefore, the composition of the "least risky portfolio" has a large weight on the eigenvectors of C corresponding to the smallest eigenvalues

However, from the RMT results, we saw that the smallest eigenvalues of C and corresponding eigenvectors are not trustworthy, as they contain a large amount of noise Therefore, Markowitz's portfolio theory, which depends on a purely historical correlation matrix, comes into question as the smallest eigenvalues of C (determining the smallest risk-portfolio) are dominated by noise

This shows the importance of differentiating noise from information in C In the next chapter we use the suggested noise removal method by Bouchaud et al (2000) and discuss on it in detail

3 8 Conclusion

We have applied Random Matrix Theory to determine the noise in an empirically-measured correlation matrix, C For a set of actual data from S&P500 we found that approximately 95% of eigenvalues of C do not hold useful information and can considered as noisy and less than 5% of them carry useful information This is supported by the evidence from literature [Bouchaud et al , 2000], which demonstrates that at most 6% of C carries useful information Our results are based on an eigenvalue analysis of C The corresponding eigenvector analysis specifies that the market eigenvector (the eigenvector corresponding to the largest eigenvalue) has a different construction to other eigenvectors and this implies that the market eigenvector represents most of the information of C

In addition, we examined RMT results in simulated data with various volumes of noise Interestingly, we observed that the number of deviated eigenvalues from the random bound does not depend on the volume of the added noise In general, all noise volumes considered gave almost the same number of deviated eigenvalues This means that when actual data sets, which naturally contain various amount of noise, are used to construct C, the ratio of noisy part of eigenvalues of C to the non-noisy part is identical The correlation matrix always holds a constant amount of noise

Chapter 4

Matrix Stability

In the previous chapter we observed that a large number of the eigenvalues and eigenvectors of an empirically-measured correlation matrix, C, for financial data, contain noise We explained how the noisy eigenvalues and eigenvectors of C can make the results of the optimisation process[1] inaccurate We now concentrate on the separation of the noisy part from the non-noisy part in C Differentiating noise from information, in the first instance and then removing the noise, makes the optimisation process more reliable and this leaves the analyst in a better position to estimate the risk of the constructed portfolio However, the suggested technique by Bouchaud et al (2000) for cleaning (removing noise in) C needs to be studied We apply this technique of cleaning C on the previous data set from S&P500 and then discuss its associated problems A statistical model suggested by Krzanowski (1984) is used then to study the stability of the cleaned C

[1] To optimise risk-return relationship in a portfolio

4 1 Noise removal from correlation matrix

We wish to differentiate and separate C into two parts

- The part of C that conforms to the properties of a random correlation matrix ("noise")
- The part of C that deviates from that predicted by RMT ("information")

In the first approximation, as stated by Laloux et al (1999), the location of the theoretical (or random) edge, determined by the theoretical maximum and minimum eigenvalues, allows us to distinguish "information" from "noise" Indeed, the edge of theoretical eigenvalues differentiates the eigenvalues consistent with the random bound from those that deviate

After separating the noisy and non-noisy parts, we go on to remove the noisy part of C For this purpose we use the method that Bouchaud et al have applied in [Bouchaud et al , 2000] The idea is to replace the restriction of the empirical correlation matrix to the noise band subspace by the identity matrix with a coefficient such that the trace of the matrix is conserved The idea behind this technique is that the eigenvalues corresponding to the noise band are not expected to contain real information, so one should not distinguish between the different eigenvalues in this sector In effect, they suggest flattening, (see Figure 4 1), the noise part by replacing it with a multiple of identity matrix, while keeping the trace the same Maintaining the same trace is important since the trace or the sum of all eigenvalues is always equal to the trace of the correlation matrix, because

$$C = V^T D V \,, \qquad (4\ 1)$$

(where V is the matrix of eigenvectors on the columns, D the diagonal matrix of the eigenvalues, and V^T is the transpose of V)

Then taking the trace of both sides of (3 8), we get

$$trace(C) = trace(V^T D V) \,, \qquad (4\ 2)$$

By the rules of matrix tracing, we know that for square matrices *A*, *B*, and *C* *trace(ABC)= trace(BCA)= trace(CAB)*. Since $VV^T = 1$, for normalised eigenvalues we have,

$$\begin{aligned} trace(C) &= trace(VV^T D) \\ &= trace(D). \end{aligned} \tag{4.3}$$

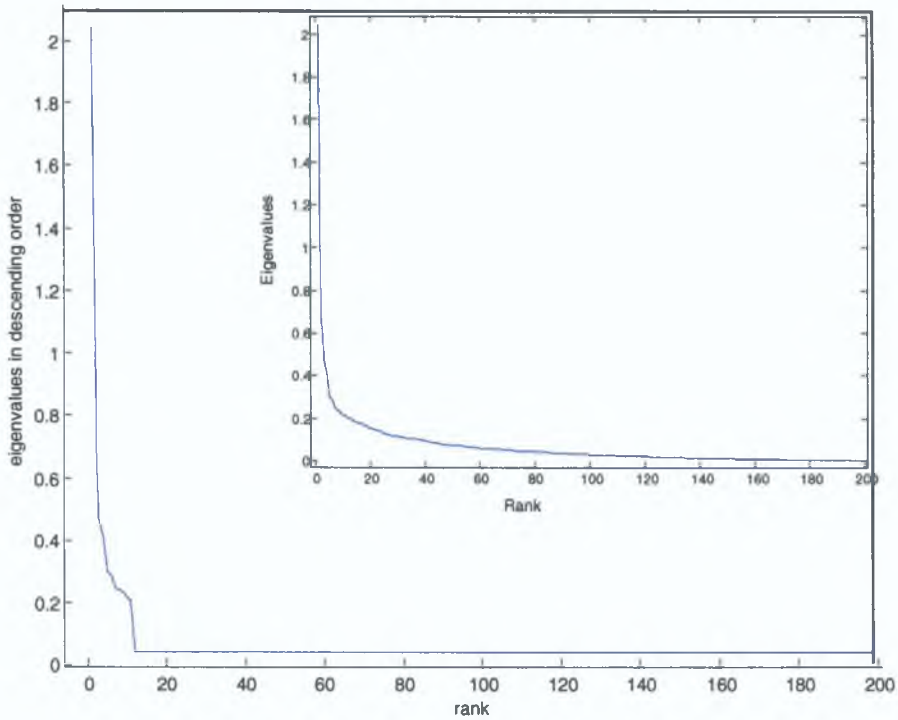


Figure 4.1 Flattening of eigenvalues for noisy part. The non-noisy largest values are untouched, but other parts have been replaced by their average. Inset: The eigenvalues of the original *C* in descending order.

Equation (4.3) indicates that the sum of eigenvalues should always be fixed. So if *trace (D)* equals η, therefore we can write

$$\sum_{i=1}^{N} \lambda_i = \eta \qquad \text{or}$$

$$\underbrace{\sum_{i=1}^{m}\lambda_i}_{m\ \text{non-noisy items}} + \underbrace{\sum_{i=m}^{N}\lambda_i}_{(N-m)\ \text{noisy items}} = \eta \tag{4 4}$$

$$\Rightarrow \quad \sum_{i=1}^{m}\lambda_i + \underbrace{\mu+\mu+\ \ +\mu}_{N-m} = \eta, \text{ where } \mu = \frac{\sum_{i=m}^{N}\lambda_i}{N-m} \tag{4 5}$$

It is evident from the analysis above how the method is applied and that the noisy part of the eigenvalues is replaced by the mean of those items (Figure 4 1)

After replacing noisy eigenvalues by their mean, we need to compute the cleaned-C, which is calculated by substituting cleaned-D in the equation (4 1)

Finally this cleaned-C, where the noise has been removed, will be used to construct an optimal portfolio

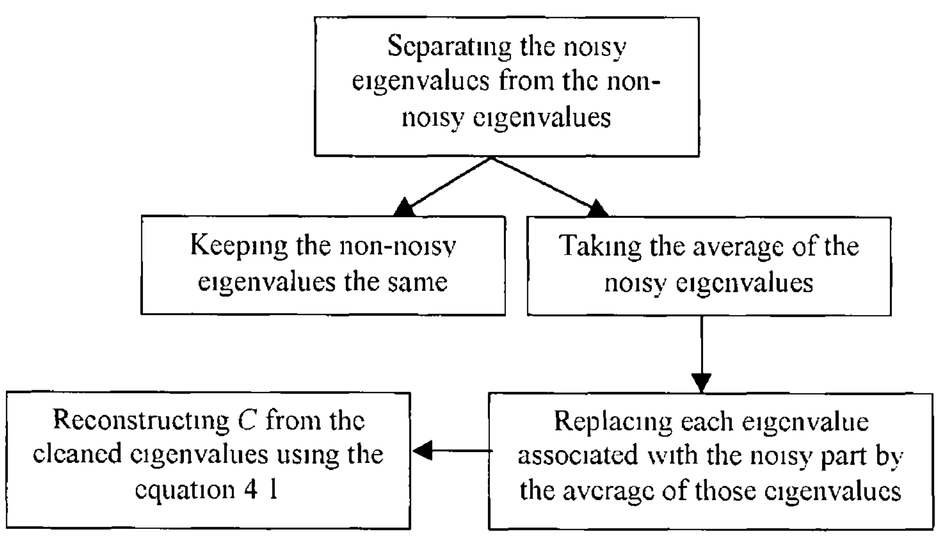


Figure 4 2 The procedure of cleaning C and removing the noise according to the Bouchaud et al [Bouchaud et al 2000] technique

To implement this idea in practice we use Laloux et al 's suggestion in [Laloux et al , 2000] where the prediction of risk obtained using noisy-C is compared with that of cleaned-C (Figure 4.3). In this way, we divide the total available time period into two equal sub-periods. So we can take the return in the second sub-period as an estimate of the future return i.e. bootstrapping effectively when we are using the first set of data from the first sub-period. By taking the return in the second sub-period, therefore, we have assumed that the investor has "perfect" predictions on the future average returns.

4.1.1 Experimental analysis

The data set we used for RMT prediction tests is also used for the rest of experiments, but with the restriction that a smaller window of the set is considered. It is just to avoid a heavy time consuming computation.

First of all, we construct the correlation matrix using the first 600 data points for 200 stocks.

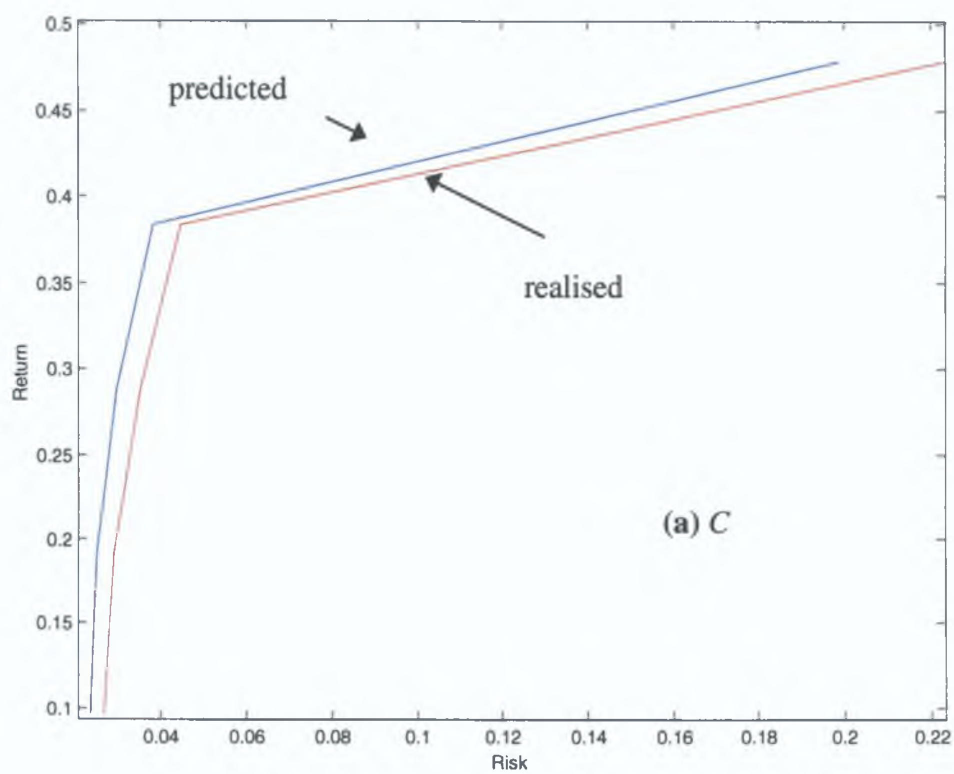
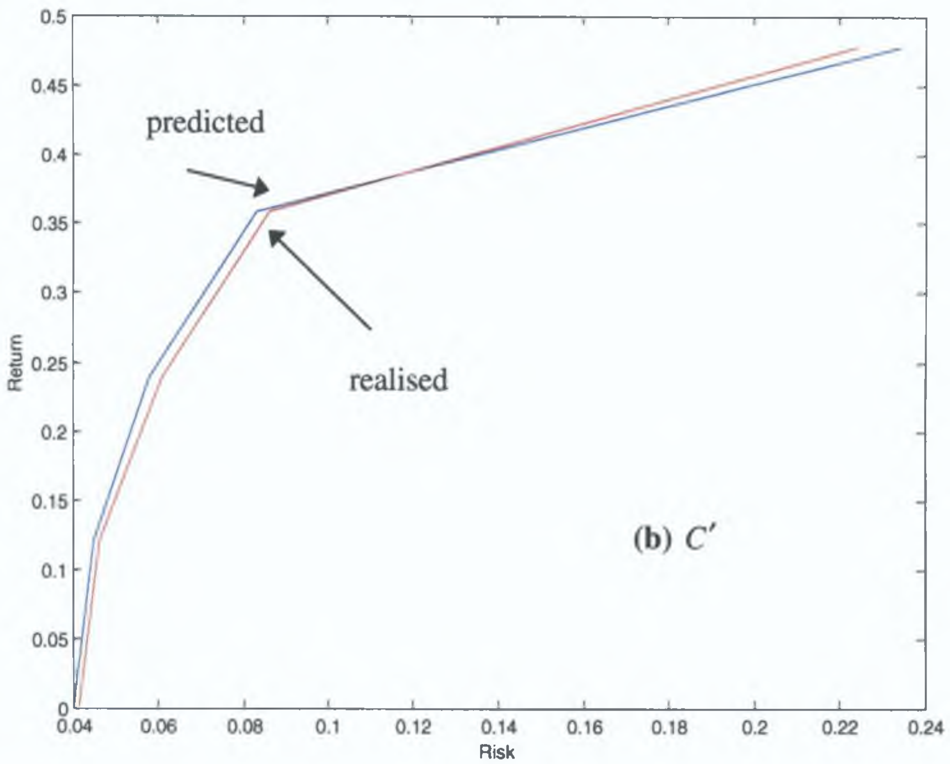


Figure 4.3 (a) Portfolio return versus risk for the family of optimal portfolios constructed from the original matrix C. The top curve shows the predicted risk of the family of optimal portfolios calculated using 30-min returns started from 01/04/1997. The bottom curve shows the realised risk.



(Continued from last page) **(b)** Risk-return relationship for the optimal portfolios constructed using cleaned correlation matrix. The top curve shows the predicted risk and the bottom curve shows the realised risk.

Next, we clean the matrix by following the procedure above (Figure 4.2). Subsequently, we need to extract the *optimal portfolios* and *efficient frontiers* of both noisy (original) and cleaned-C to compare their prediction of risks. The so-called efficient frontier refers to the set of portfolios that will be preferred by all investors who exhibit risk avoidance and who prefer more return to less and it is given by

$$\frac{\partial}{\partial p_i}(D_p - \xi G_p)\bigg|_{p_i = p_i^*} = 0 \qquad (4.6)$$

(where p_i and p_i^* denote the asset weights and G_p is the mean return and $D_p = \sum_{i,j=1}^{N} p_i p_j C_{ij}$, and ξ is some parameter).

Here, we define the expressions "predicted risk" and "realised risk", which will be used frequently in the reminder of this chapter. The efficient frontier calculated using the return on the second sub-period and the correlation matrix for the first sub-period is called *prediction* of the portfolio (Figure 4 4(a)) and the associated risk is called the *predicted risk* Using the return and correlation matrix calculated using the second sub-period combined with the weights of the same family of portfolios as the predicted ones, we design another set of portfolios, which is called in the literature [Laloux et al , 2000] the *realisation* of the portfolio, Figure 4 4(b) The associated risk is also known as *realised risk*

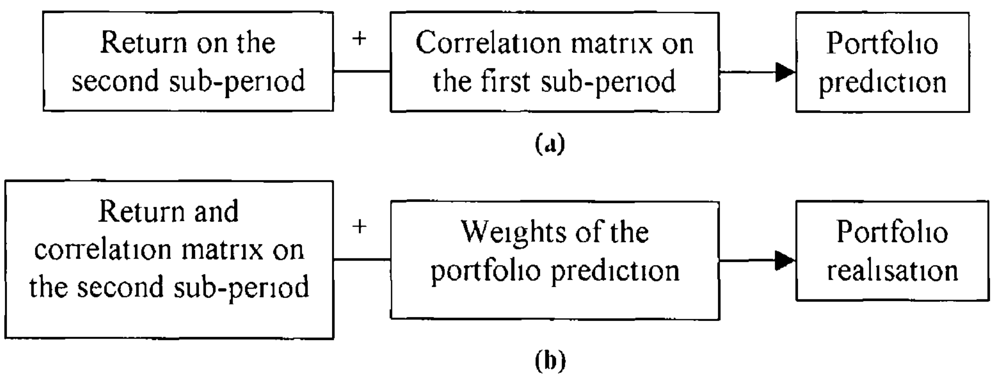


Figure 4 4 definition of (a) portfolio prediction and (b) portfolio realisation

Laloux et al (2000) state that the predicted and realised risks get closer (Figure 4 3(b)) when the cleaned matrix is used in delineating the efficient frontier as would be desired They attribute the closeness of the mentioned curves to the power of cleaned_C in predicting the future risk and they conclude that the stability of the cleaned_C is higher than the stability of the original C

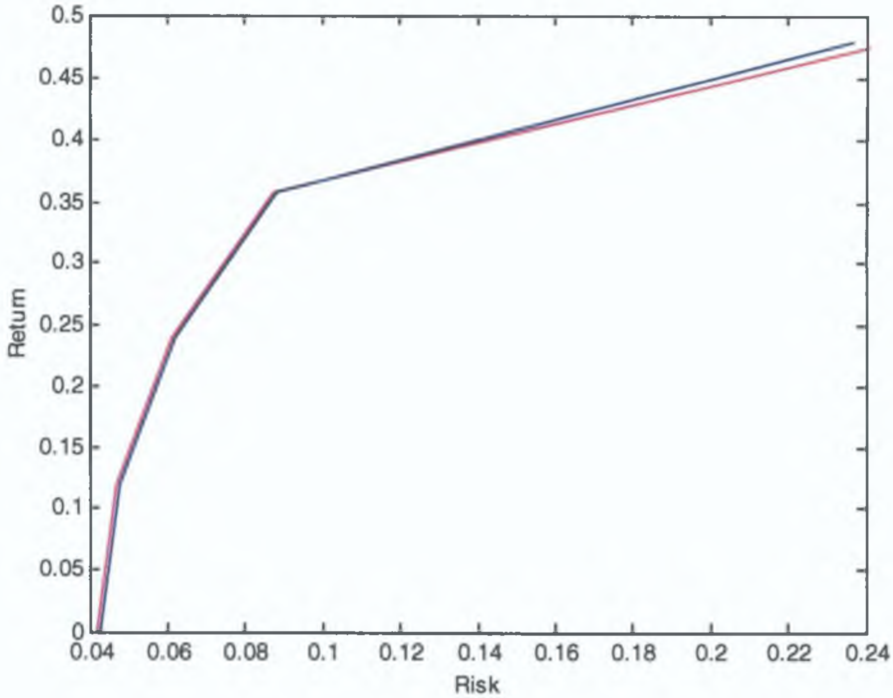


Figure 4.5 The same quantity as the figure 4.3b, but with the assumption that only three eigenvalues are non-noisy and therefore, untouched in the cleaning process. The curves are very close.

We have replaced more eigenvalues (i.e. 197) with their average and it is interesting to observe, in Figure 4.5, the increasing closeness between the two curves. Therefore, the conclusion about the higher stability of the cleaned-C seems unlikely to be true because it seems that the reason for the closeness of the curves is related to the similarities of the replaced eigenvalues.

Hence (as we go on to show) we believe that not only does the suggested technique by Bouchaud et al (2000) not improve the stability of C but it could actually reduce it.

4.2 Optimal portfolio: A stability approach

In this section we study the correlation matrix from a stability point of view. The stability of the correlation matrix is in fact an important aspect that should always be considered. In the statistical literatures e.g. [Jolliffe, 1986] it is stated that eigenvectors and principal components can only be confidently interpreted if they are stable. Now the question is: what would happen to the stability of C after cleaning it?

Is the stability of cleaned-C higher or lower? If it is lower how can we remove noisy elements from C such that the most stability is conserved?

In a number of published articles, (such Lee, 2001), on this subject the overlap of the eigenvectors of two consecutive time sub-periods is considered to study the consistency (or convergence) of the eigenvectors Overlap of two vectors means the amount of rotation of the second vector with respect to the first one (Figure 4 6) In the case where the vectors are normalised, the dot product of the vectors represents the cosine of the angle between them and gives a measure of the overlap If the eigenvectors' directions remain similar over the two sub-periods, then the cosine value should be significant (very much closer to unity) Otherwise, it will be small (close to zero)

We compare the overlap of eigenvectors over two consecutive sub-periods for our real data set, section 4 1 1 The first sub-period is the first 600 records of our data for 200 stocks and the second is the second 600 records Figure 4 7 shows that after first few eigenvectors (corresponding to the first largest eigenvalues), the overlap falls below a line, which is called in the literature [Strongin et al , 2000] the *noise level* and it is measured by $\frac{1}{\sqrt{N}}$, where N is the number of eigenvectors

The first eigenvectors, as we argued earlier in this chapter, are the ones that deviate from the random bound and they are considered as providing information in contrast to the next eigenvectors, which are considered noisy As expected, the eigenvectors associated with the largest eigenvalues show more stability and the degree of overlap is significant

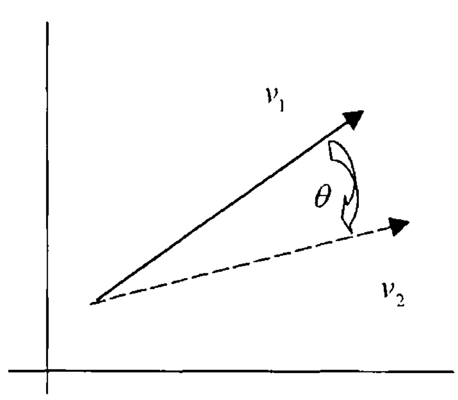


Figure 4 6 the angle between two vectors v_1 and v_2 indicates their overlap

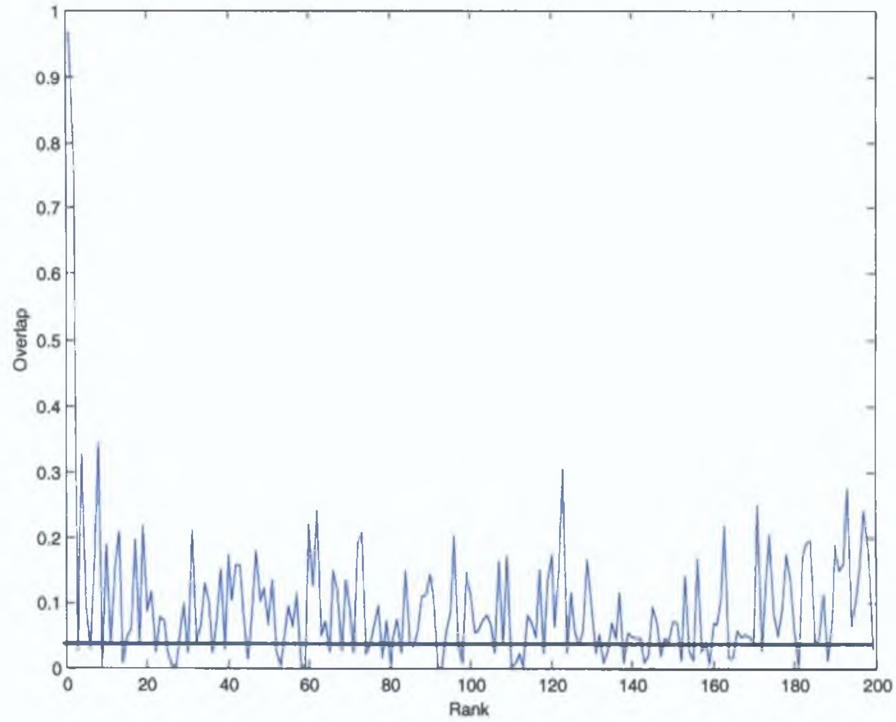


Figure 4.7 Eigenvector overlap between the two sub-periods. The horizontal line is the noise level

However, to measure more formally the stability of a matrix and its eigenvectors, we employ another approach in the literature on principal components suggested by Krzanowski (1984). Krzanowski (1984) examines the effect on v_k (kth eigenvector) of small changes in the value of λ_k (kth eigenvalue) and he argues that this is important because it gives information on the stability of the principal components. The principal components can only be securely interpreted if they are stable with respect to small changes in the values of the λ_k's. Specifically, he investigates the perturbation of an eigenvector derived for a small reduce/ increase, ε, in the corresponding eigenvalues. He determines the component, *v(i)*, which diverges as much as possible from the ith eigenvector, v_i, but whose eigenvalue is at most ε greater/ less than that of v_i, such that the angle θ between *v(i)* and v_i can be calculated by equation (4.7).

$$\cos\theta = \begin{cases} (1+\dfrac{\varepsilon}{\lambda_i - \lambda_{i+1}})^{-1/2} & \textit{if } \varepsilon \textit{ is decreased from } \lambda_i \\ (1+\dfrac{\varepsilon}{\lambda_{i-1} - \lambda_i})^{-1/2} & \textit{if } \varepsilon \textit{ is increased to } \lambda_i \end{cases} \qquad (4\ 7)$$

where $\lambda_1 \geq \lambda_2 \geq \quad \geq \lambda_n$ This equation demonstrates that the effect on v_i of an ε change in λ_i is an inverse function of $\lambda_i - \lambda_{i+1}$ Thus it is not the **absolute size** of the eigenvalue which determines whether that component is stable or not but rather its **separation** in terms of eigenvalue from the next component Relatively isolated (early) components with large eigenvalues should therefore be fairly stable, but later components all of which have similar non-zero variances will not be stable So the largest non-zero eigenvalue and corresponding eigenvector can be used to find the smallest perturbation in v_i which leads to a change ε in λ_i

4 2 1 Stability of the correlation matrix

In this section we study the stability of C for our real data using the Krzanowski's model [Krzanowski, 1984] in equation 4 7 The angle between eigenvector i of original C and $v(i)$ is calculated, where $v(i)$ is the perturbation of i^{th} eigenvector derived for a small change in λ_i (Figure 4 8) Also ε is determined by the empirical changes in the average of λ_i from the first sub period to the second sub period It approximates to 0 2% in our experiments So, effectively a perturbation method is implied

Cosine θ

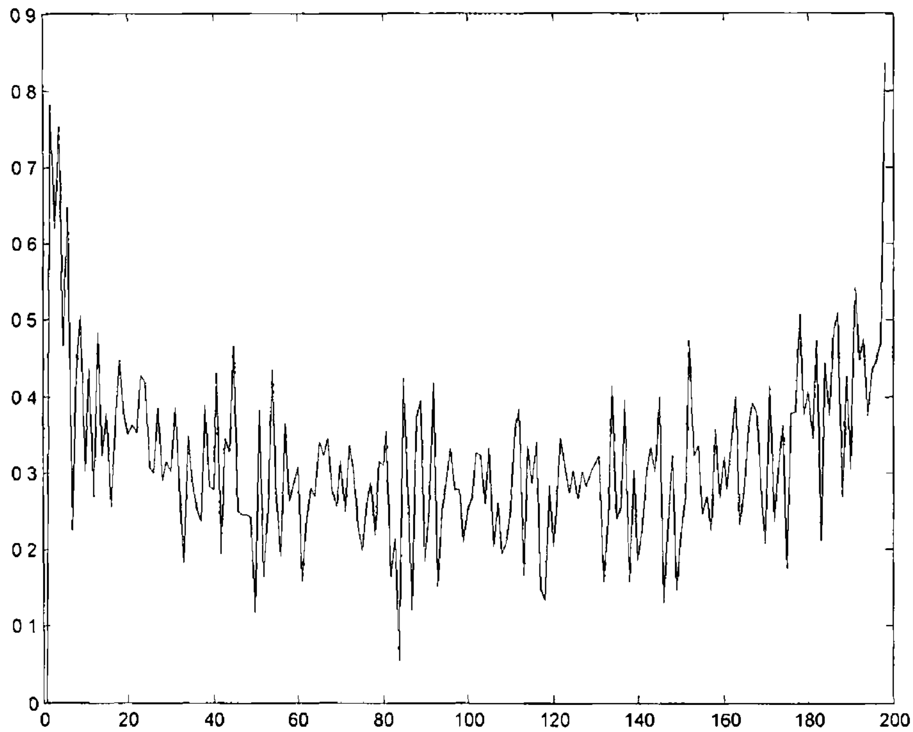


Figure 4 8 Cosine θ for eigenvectors of original correlation matrix where θ represents the perturbation of v_i derived for the change of 0 2 % in λ_i

As expected, Figure 4 8 shows that the first (largest) eigenvectors are the most stable ones ($cos\,\theta$ large) Also the last (smallest) eigenvectors show higher stability than the middle ones, which is because the smallest eigenvalues approach zero and therefore in Equation 4 7, $cos\,\theta$ represents greater values

Examining figure 4 3(a) again one can see that the top end of the predicted and realised curves are further apart whereas the bottom and the middle area are closer (about 19% of the top distance) Since the area of the efficient frontier associated with the highest risk, is corresponding to the largest eigenvalues, and the largest ones are the most stable ones, then we can conclude that as stability gets progressively lower the curves gets progressively closer This contradicts the conclusion of Bouchaud et al (2000) They attribute the closeness of the mentioned curves to the higher stability whereas we conclude that it is due to reduction in stability We now discuss this in greater detail

4.2.2 Stability of the cleaned correlation matrix

According to Equation 4.7 we examine the stability of cleaned_C where the method of Bouchaud et al (2000) is used for cleaning (Figure 4.9).

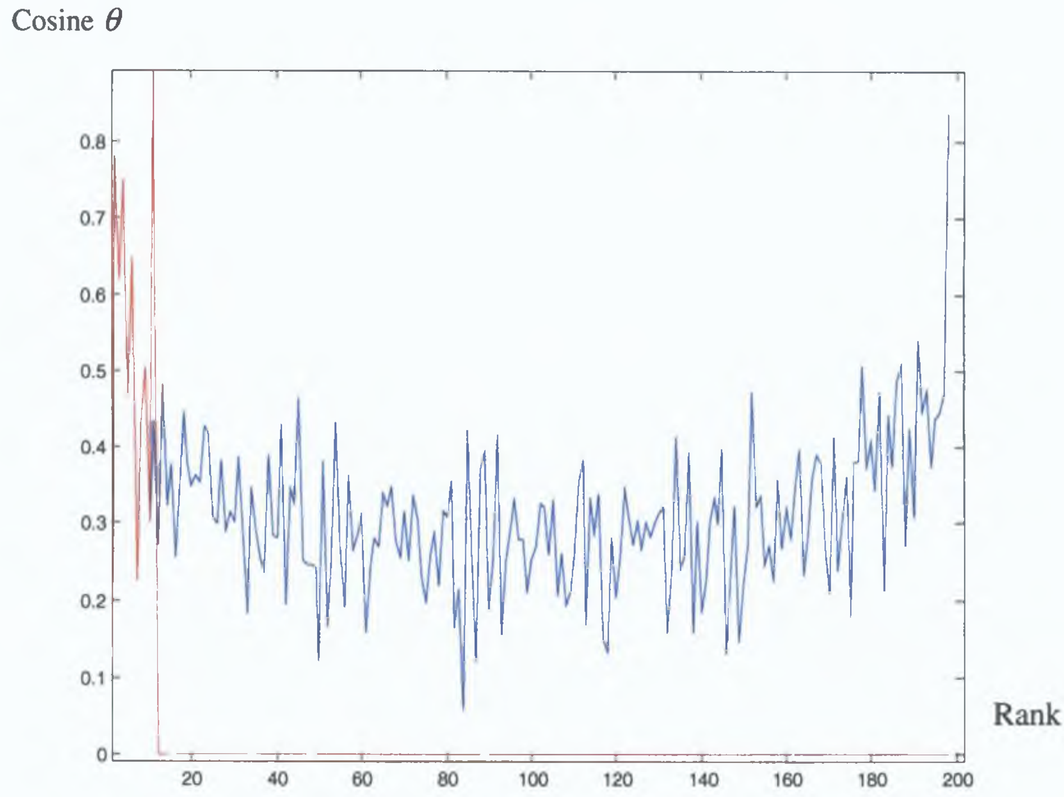


Figure 4.9 cosine θ for eigenvectors of original C (solid blue curve) and cleaned_C (dashed red curve) where θ represents the perturbation of v_i derived for the change of 0.2 % in λ_i.

It can be seen from the Figure 4.9 that the stability of the eigenvectors of cleaned_C has declined noticeably to a low point after the 11^{th} eigenvector (the edge of the noisy/ non-noisy determined of RMT prediction).

That happens because the noisy band of eigenvalues is replaced by their average, which means no separations between the eigenvalues at all.

Also Figure 4.9 (in contrast to Figure 4.3b) again illustrates the negative relationship between stability of eigenvectors (and therefore C) and the distance between two predicted and realised curves. As the stability decreases, the two curves get closer and conversely when stability increases the distance between curves gets larger.

4.3 A new approach to matrix filtering

We propose a new method of filtering C to preserve the stability of the matrix as much as possible. The principle is to replace the noisy eigenvalues with components that have most separation from each other, while maintaining a fixed sum (according to Equation 4.3, the sum of eigenvalues should be constant). In Figure 4.10 the noisy part of the graph is changed to an oblique line. The slope is determined so that on one hand the most separation between components is attained and on the other hand none of the eigenvalues is replaced by negative values (as all the eigenvalues of the correlation matrix are positive).

To have an idea of how the method works, Figure 4.11 shows the eigenvalues of filtered_C (or cleaned_C).

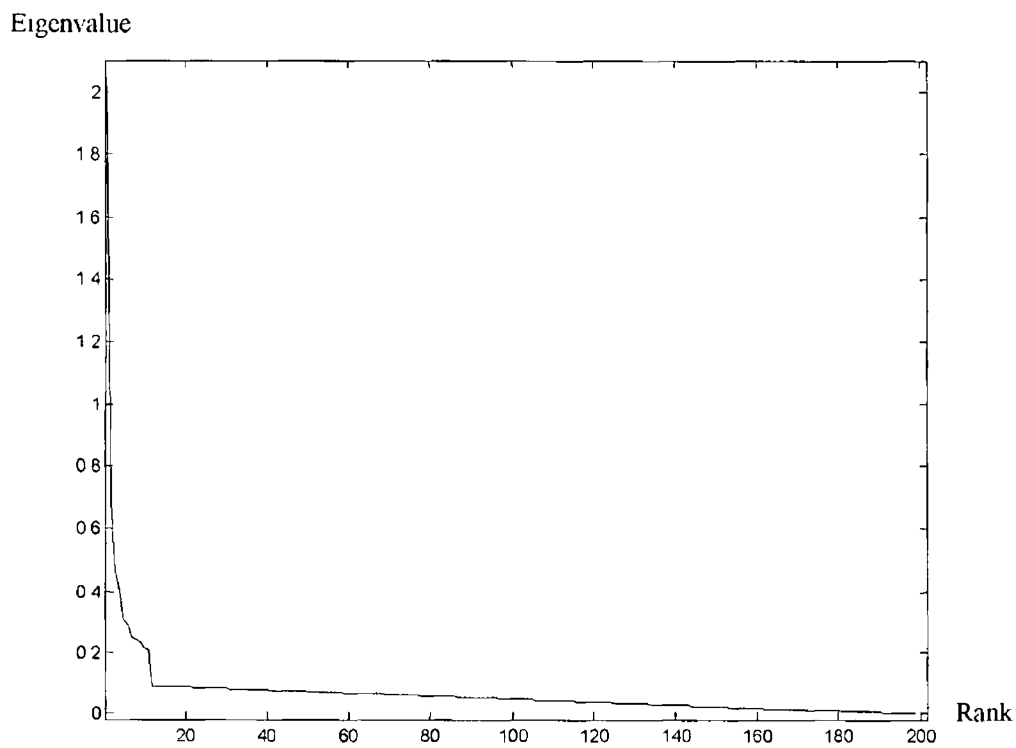


Figure 4.10 Replacement of eigenvalues. The non-noisy largest values are untouched, but other parts have been replaced such that each two ones are kept in the most distance.

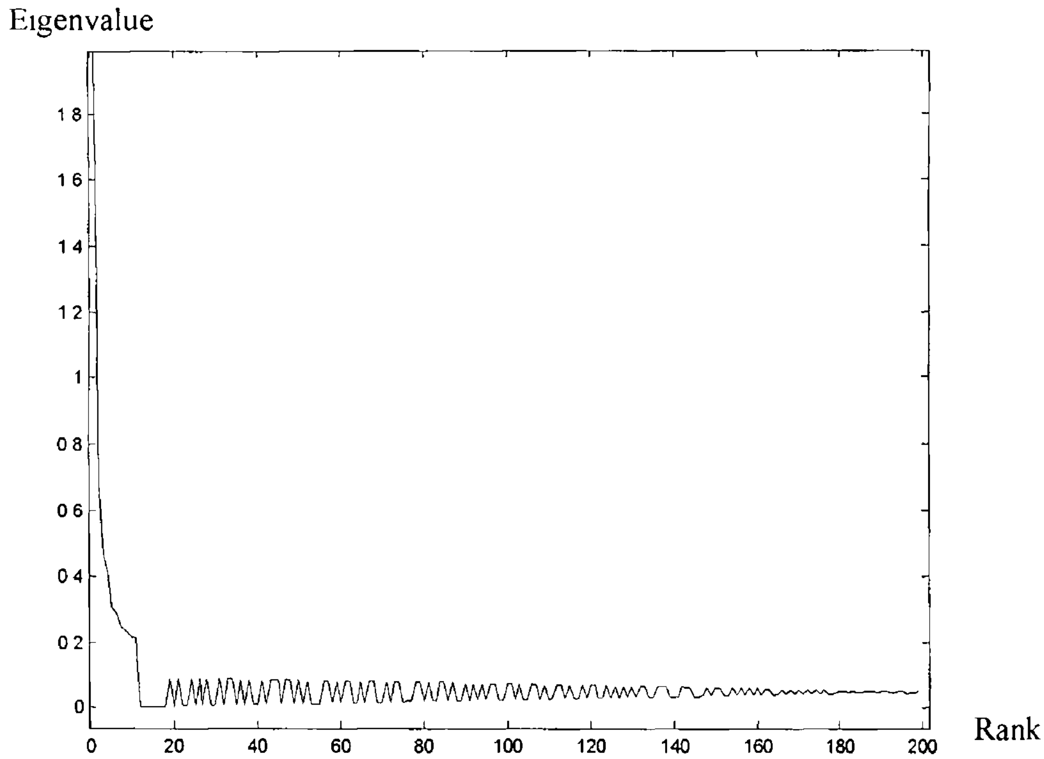


Figure 4 11 Eigenvalues of filtered_C Eigenvalue after 11th oscillate since each two ones are kept in the most distance as possible

To observe the stability of this new filtered_C we compute its $\cos\theta$ (in Equation 4 7) This is shown in Figure 4 12 As can be seen the stability of noisy eigenvectors of the original matrix is higher than those of the filtered matrix up to approximately 150th eigenvector and it is lower after 150th Thus, this method keeps stability much higher in comparison with the flattening of the eigenvalues, (the method of Bouchaud et al (2000)) Concerning the previous results from section 4 2 1, we expect that the predicted and realised risk-return curves are closer on graph in the interval between 11th to 150th and farther apart after 150th in comparison to those of original C

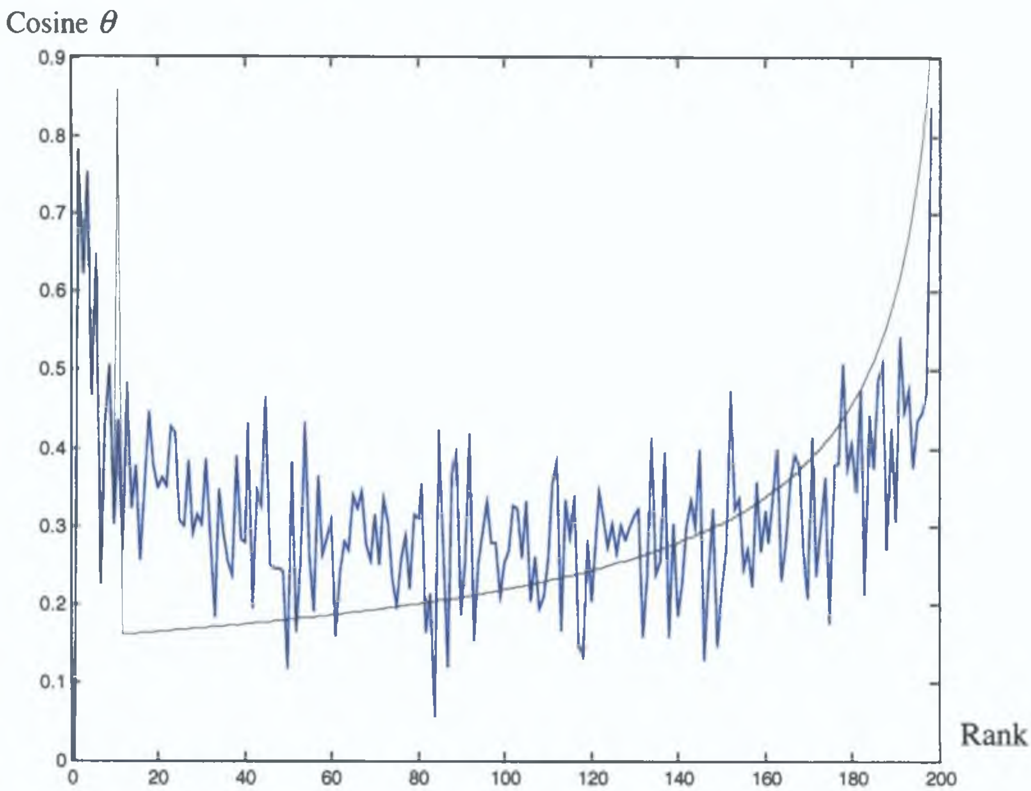


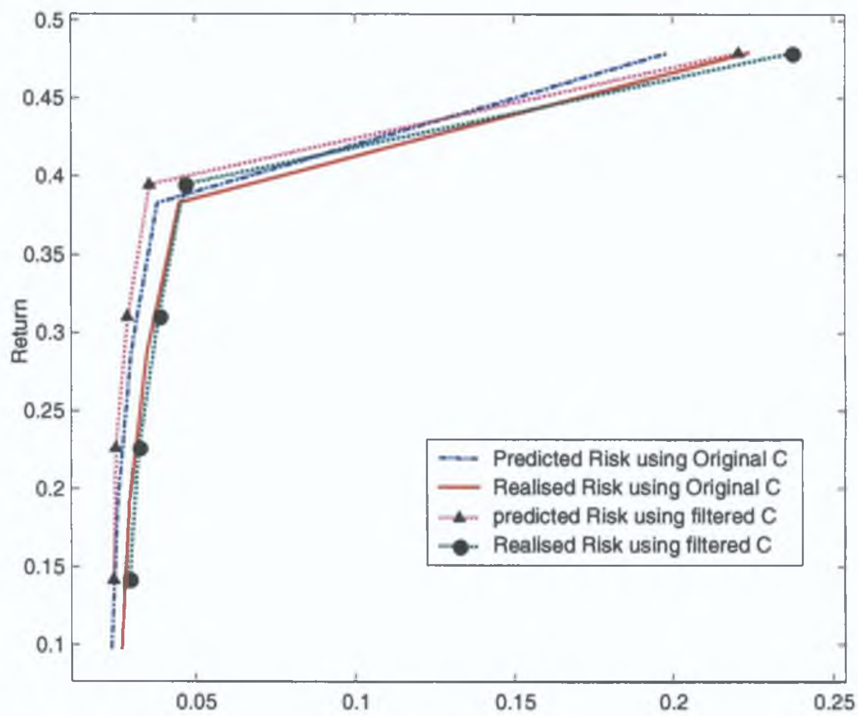
Figure 4.12 Cosine θ for the original matrix C (solid line) and filtered-C by the method based on Krzanowski results (dashed line)

Again the largest eigenvalues correspond to the riskiest portfolios exposed in the top area of the efficient frontier in Figure 4.13. Equally the smallest eigenvalues correspond to the least risky portfolios, exposed in the lower area. As can be seen, the upper end, d_U^{filt}, of the curves in Figure 4.13 converge whereas the middle parts, d_M^{filt}, are less close than those for original_*C*. This is another indication of the validity of our assertion that stability has an inverse relation to the distance between the predicted and realised risks. As the stability gets higher the closeness between curves decreases and vice versa. If d_U^{orig} stands for the distance between the upper end of the curves for original_*C*, and d_M^{orig} for those of middle parts, then we have found that:

Table 4.1 % Closeness of curves

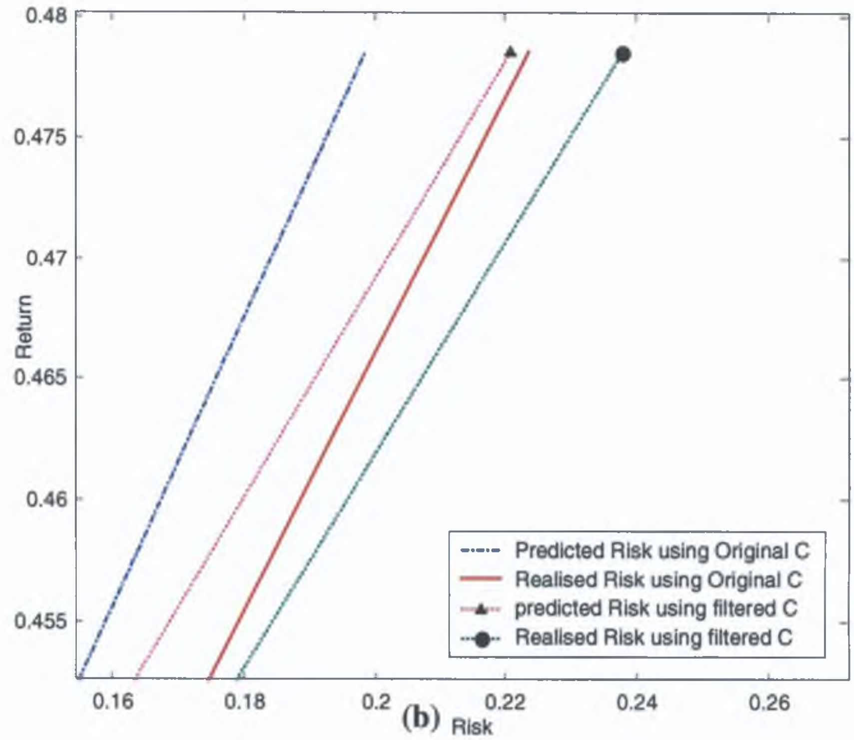
$d_M^{orig} = 63\%\ d_M^{filt}$
$d_U^{filt} = 68\%\ d_U^{orig}$

We conclude from these findings is that the method of cleaning C suggested by Bouchaud et al (2000) has a detrimental impact on the stability of C. Furthermore, the closeness of the predicted and realised curves does not necessarily represent the power of prediction of risk in future. Indeed, when the correlation matrix is less stable, the predicted and realised curves are closer than the case with more stability.

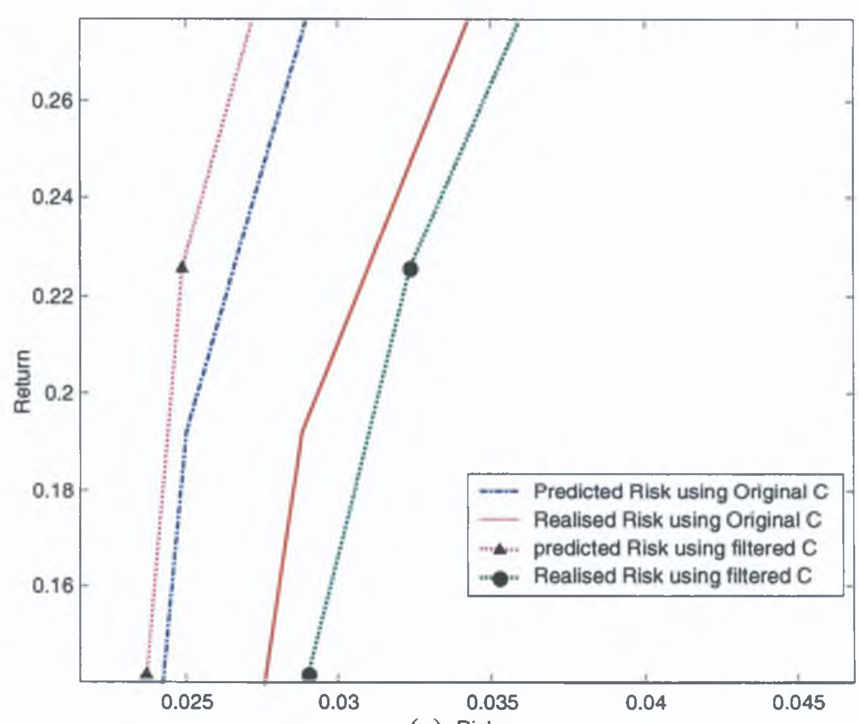


(a)

Figure 4.13 (a) Portfolio return versus risk for the family of optimal portfolios constructed from the original matrix C (red and blue curves) and filtered_C (triangle-green and circle-magenta curves).



(b)



(c)

(Continued from last page.) **(b)** A closer look at the top of the graph of (a); d_U^{filt} is smaller than d_U^{orig} **(c)** The closer look at the bottom area of the graph (a); d_M^{orig} is smaller than d_M^{filt} .

4 4 Conclusion

In this chapter we have examined the principally-used technique of noise removal for the correlation matrix, C This technique, which proposes flattening the noisy part of the eigenvalues, largely decreases the level of stability of C We have applied Krzanowski's stability model to study the stability of the financial correlation matrix after removing the noise According to this model, we have discovered that the advocated technique for noise removal destroys the stability of C

Based on the Krzanowski's model we proposed a novel technique to filter C such that the stability of the matrix is preserved This model keeps the noisy eigenvalues at maximum separation from each other while the trace of C is kept the same

To see the effect of noise removal, Bouchaud et al (2000) have suggested comparing the realised and the predicted optimal portfolios They have found a shorter distance between the realised risk and the predicted risk for the cleaned_C than that of the original C They have attributed this as a higher stability of the cleaned_C

In our study we have shown on a set of itra-day data from S&P500 that this is not the case and in fact there is a negative relationship between the stability of C and the closeness of the predicted and realised risks This assertion is also demonstrated through experiments of filtering C based on the Krzanowski's model Therefore the common technique of noise removal not only does not promote the stability and hence power of prediction, but actually leads to a noticeable deterioration and should be avoided

Chapter 5
Epochs

This chapter is about co-movement among stocks as influenced by market price changes We propose a new approach to study market reaction to high volatility and use the Spectral Theorem [Strang, 1988] to measure the day-to-day co-movement of stocks A new concept of *epochs* is introduced where these represent patterns defined in terms of daily change in the largest eigenvalue and daily change in market sector prices The evidence suggests a strong linear relationship between price and the largest eigenvalue in the epochs but the error terms in the linear models show correlated behaviour Therefore, a modified model is introduced Further, some break points are observed in the modified model, for which we discuss possible reasons

5 1 Volatılıty

After the crash of October 1987, few people would argue that stock market volatılıty changes randomly over tıme Researchers (begınnıng wıth Black (1976)) have found evıdence that stock prıces are negatıvely correlated wıth volatılıty A prıce drop ıncreases the rısk of a company goıng bankrupt, and ıts stock therefore becomes more volatıle It ıs also saıd, [Nelson, 1991], that volatılıty tends to rıse ın response to bad news and to fall ın response to good news The economıc reason for thıs ıs unclear Although ıt has been long understood [Chrıstıe, 1982] that fınancıal leverage plays a role, ıt has not been possıble to explaın the extent of the asymmetrıc response of volatılıty to large sudden posıtıve and negatıve returns

What ıs known ıs that large negatıve stock returns are more common than large posıtıve ones, so stock returns are negatıvely skewed (For example, accordıng to an ınvestıgatıon by [Cutler et al , 1989], 8 out of 10 largest movements ın the S&P500 have ınvolved declınes sınce World War 2, and only two have ıncreased)

However, we have found work [Black, 1976][Chrıstıe, 1982][Campbell et al , 1992][Bekaert et al , 2000] on volatılıty and crashes to be retıcent on the effect of prıce changes on "market movements" What we mean by "market movement" ıs combıned upward or downward behavıour of stocks Thıs ıs the quantıty measured by the largest eıgenvalue of the covarıance matrıx[1] made up of the stocks prıces/ prıce changes [Alexander, 2001]

Regardıng chapter 3 and 4, we know that the largest eıgenvalue of the covarıance matrıx ıs the most "relıable" eıgenvalue, ı e the most devıated one from the noıse band Therefore, although the covarıance matrıx contaıns much noıse, the largest eıgenvalue appears to contaın true ınformatıon about correlated movements of stocks and, unlıke other eıgenvalues, all the analyses dependıng on that are lıkely to be adequate In thıs study we propose a novel approach to rısk recognıtıon (or rısk response) The suggested method ıs based on measurıng the day-to-day changes of the rıskıest posıtıon on stock combınatıons, whıch ıs determıned by the largest eıgenvalue of the covarıance matrıx

[1] It should be mentıoned that the covarıance and correlatıon matrıces are most of the tıme used ınterchangeably They have the same propertıes except that the correlatıon matrıx removes the varıances by normalısatıon

5 1 1 Risk

Investors should not and indeed do not hold a single asset (or even asset class) Holding a multiple-asset portfolio, on the other hand, makes returns on assets 'uncertain', since two assets yielding different 'certain' returns cannot both be available (as everyone will want to invest in the higher yielding asset and no one will purchase the lower yielding one) If everyone knew with certainty the returns on all assets, then a general framework for a rational portfolio selection could be introduced In the case of common stocks it is impossible to predict the value of portfolio at any future date The best an investor can do is guess the most likely estimates In this case, it is said that the investor bears risk In order not to incur risk, an investor might hold a portfolio of Treasury Bills, and of course, face no uncertainty, because the value of the portfolio at the maturity of the securities will be identical with the predicted value In the literature review earlier, we explained that variance (or standard deviation) of return is the most common feature to measure the risk From Markowitz diversification [Markowitz, 1991], the variance (and hence overall risk) of a combination of two assets can be even less than the variance of either of the assets, themselves This is because the risk of a combination of assets is calculated from equation 2 1 For the case when the portfolio of assets is equally weighted, the equation 2 1 can reduce to

$$\sigma_p^2 = \frac{\overline{\sigma_i^2} + \overline{\sigma_{ij}}}{N} + \overline{\sigma_{ij}}, \qquad (5\ 1)$$

(where bar notation denotes a time average and N is the number of assets) Hence, when the number of assets gets large enough, the variance of a selection of assets depends only on the average of the covariances between stocks To calculate the covariance between assets price/ price change, similar to the previous chapters we have

$$\sigma_{ij} = \overline{G_i(t) G_j(t)}, \qquad (5\ 2)$$

(where G_i represents assets price or price change (dependent on what is used in practice) and the bar denotes a time average over the period studied) In matrix notation the covariance matrix can be calculated as following,

$$C^{\bullet} = \frac{1}{T} GG^{T}, \tag{5 3}$$

(where G is an $N \times T$ matrix with elements $G_i(t)$, $i=1, \quad ,N, \ t=1, \quad ,T$)

It should be mentioned that the matrix of the assets time series, G, is not normalised with respect to the variance as in Equation 3 1 The normalisation process includes removing the volatility information on co-movements of stocks, which is not particularly useful for our experiments here since we aim to study the volatility behaviour itself Therefore, the covariance matrix that we construct is not a normalised covariance matrix

5 2 Methodology

From the spectral theorem we know that any $N \times N$ symmetric matrix can be decomposed into

$$\begin{aligned} H_{N \times N} &= \sum_{i=1}^{N} \lambda_i v_i v_i^{T} \\ &= \sum_{i=1}^{N} \lambda_i p_i \end{aligned} \tag{5 4}$$

(where $p_i = v_i v_i^{T}$, the outer product of the eigenvectors) Since the covariance matrix is symmetric, therefore spectral decomposition can be used, i e

$$C^{\bullet}{}_{N \times N} = \sum_{i=1}^{N} \lambda_i p_i \tag{5 5}$$

Hence, the $N \times N$ covariance matrix is decomposed into a product of its eigenvalues and eigenvectors

Our suggested method to measure the day-to-day variation of co-movements of stocks is to take the difference between the two covariance matrices corresponding to two consecutive days/ observations and examine the effect on the maximum eigenvalue It means that if $C^{\bullet}{}_{m}$ represents the covariance matrix corresponding to *m* recent days (i e an average based on a window of *m* days) and $C^{\bullet}{}_{m+1}$ represents the covariance matrix corresponding to *m+1* days, then the difference between $C^{\bullet}{}_{m+1}$ and $C^{\bullet}{}_{m}$ will be the covariance matrix on day *m+1* With respect to the spectral theorem we can write

$$C^{\bullet}{}_{m+1} - C^{\bullet}{}_{m} = \sum_{i=1}^{N} \lambda_i^{m+1} p_i^{m+1} - \sum_{i=1}^{N} \lambda_i^{m} p_i^{m} \tag{5 6}$$

$$= \sum_{i=1}^{N} \lambda_i^{m+1} p_i^{m+1} - \sum_{i=1}^{N} \lambda_i^{m} p_i^{m+1} + \sum_{i=1}^{N} \lambda_i^{m} p_i^{m+1} - \sum_{i=1}^{N} \lambda_i^{m} p_i^{m} \tag{5 7}$$

$$= \sum_{i=1}^{N} (\lambda_i^{m+1} - \lambda_i^{m}) p_i^{m+1} + \sum_{i=1}^{N} \lambda_i^{m} (p_i^{m+1} - p_i^{m}) \tag{5 8}$$

Equation 5 8 clearly shows how the daily variation of prices can be reflected in the daily variation of the eigenvalues of the covariance matrix As we have mentioned earlier in this chapter, our analysis focuses on the most reliable eigenvalue, which contains information about the co-operation of the stocks Also, for more convenience, the study of $\lambda_i^{m+1} - \lambda_i^{m}$, where i gets values from 1 to *N*, can be replaced by the study of $\frac{\lambda_i^{m+1}}{\lambda_i^{m}}$ (shown in appendix A)

To our belief use of the daily changes of the largest eigenvalue as a barometer of the 'risk perception' by the market is a novel approach We define these changes of the maximum eigenvalues as the change in *cohesion* As the largest eigenvalue reflects the high level of coherent trading activity in the market, we consider the degree of uniformity in response to the *coherence* Therefore, $\frac{\lambda_{\max}^{T+1}}{\lambda_{\max}^{T}}$ in our study is recognised as the change in cohesion with time and is denoted ΔCoh_T

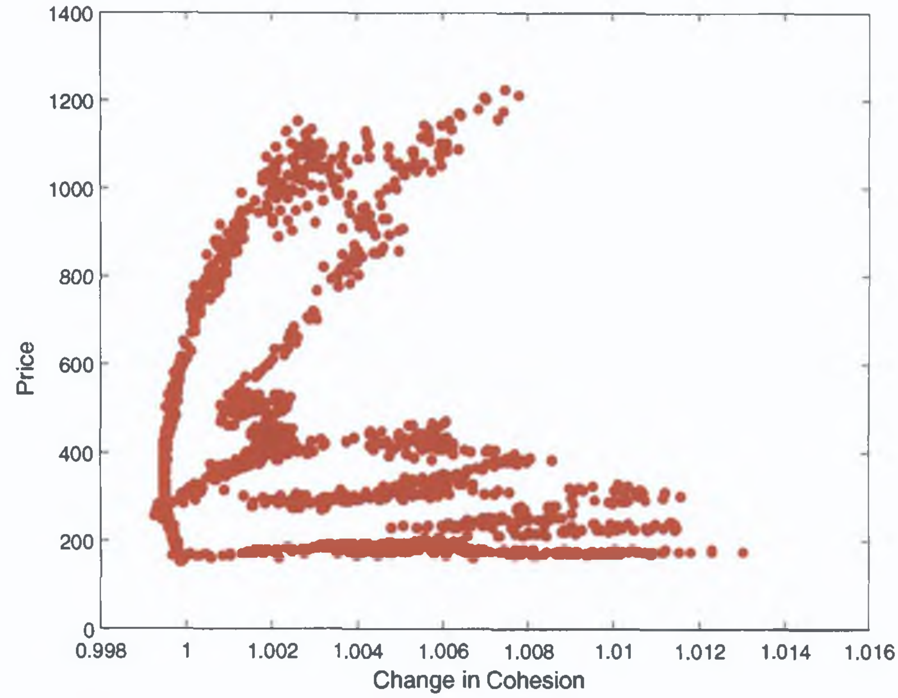
5 3 Experimental results

5 3 1 Epochs Definition

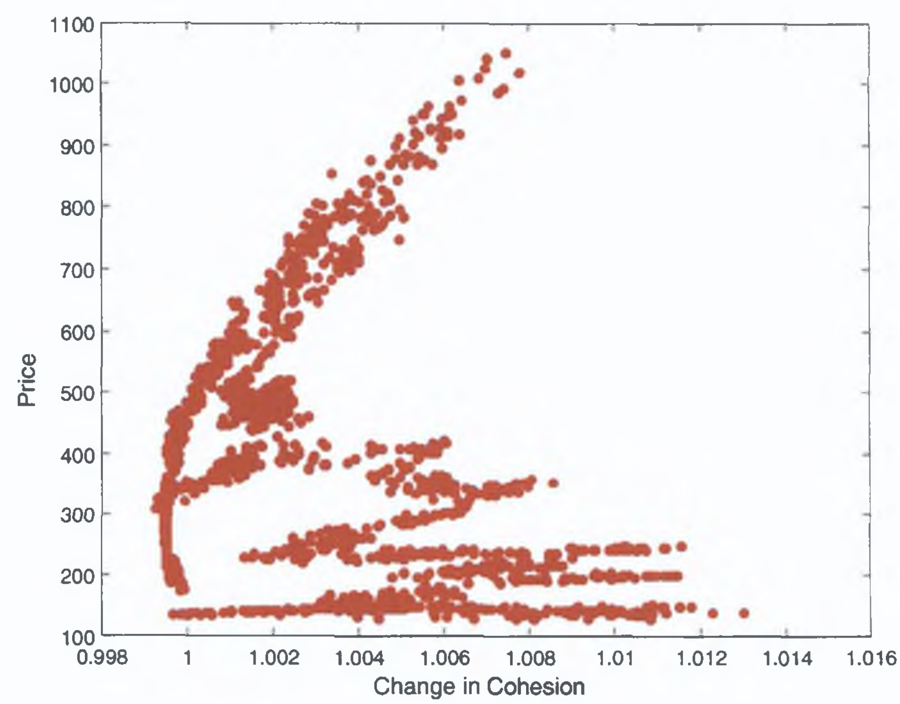
The data set we use for this part, is daily market sector prices from the Dow Jones EURO STOXX[2] over a time period started from the first of January 1996 to the first of October 2002 This data set is divided into different market sectors allowing us to examine change in cohesion for each

First, we take an initial time window of one year (i e 260 days) and step-by-step move on to construct the covariance matrices on each day using Equation 5 3 Next, calculating the change in cohesion (as mentioned by the ratio of consecutive maximum eigenvalues) over consecutive time periods, we plot it versus the price for each market sector index Figure 5 1 shows the daily change in cohesion against price for Technology, Telecom, and Industry indices An interesting result is the linear relationships between the price index and the change in cohesions, which may be clearly observed Also additional patterns are seen in plots, which seem to show how the market is organising itself We call these growth patterns "epochs" Epochs are the focus of our discussion here, and we will characterise them and seek to explain their occurrence

[2] http //www stoxx com

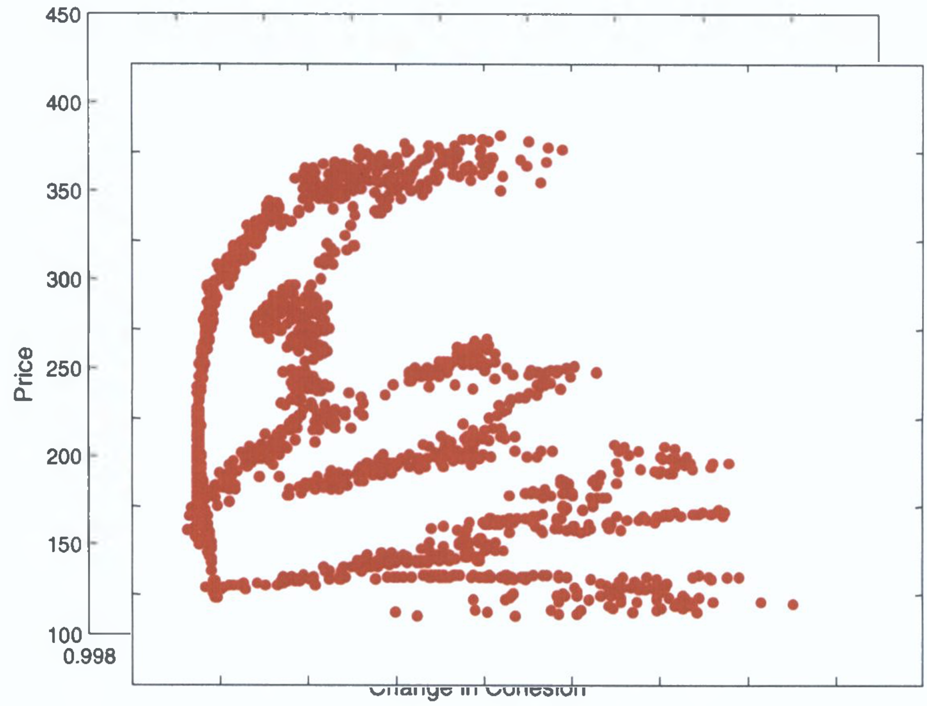


(a)



(b)

Figure 5.1 epochs skeleton for (a) Technology, (b) Telecom, and (c) Industry



(c)

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5.3.2 Epochs: Discussion

The linear relationship between price changes and the cohesion degree changes of particular points, where the coherence appears to return to a lower value at the end of each epoch. In addition, it seems that epochs are characterised by progressively higher slopes until this expansion or inflation can no longer be maintained in the prices. This also supports the indecisive parts of the end of the epochs, which tend to happen after a period of maximum prices and for which no higher degree of cohesion can be observed.

For a better understanding of this phenomenon, it is instructive to pinpoint the turning point/ end of each epoch. For this purpose, we measure the statistics of the progressive regression fits. If linear fits start from a couple of points at the beginning of data and move toward the next point every time, there should appear a sharp drop in the value of R-Squared[3] at a particular point. We consider this sudden fall as the turning point of each epoch. In fact the progressive fits continue to maintain a stable

[3] A measure of how good a fit is.

or constant value until the point where the relationship between price and cohesion changes and the model fits start to diminish. As a measure of how good a model fit is, we use the R-Squared statistic. Therefore, by looking at the progressive R-Squared values we determine where the model fits tend to fall. Before getting to the turning point, the values of R-Squared tend to improve and in some stages they show stabilising behaviour (i.e. getting to a consistent value of R-Squared). As they get to a turning point, a sudden drop in the R-Squared value is observed. Figure 5.2 shows the change of R-Squared for one of the epochs observed in Figure 5.1(a). Before the 40th point (Figure 5.2), an improvement in the model fits is observed but at 40th point a sudden drop occurred. This point is associated with the date of 14/09/1998, which we believe should be one of the key turning points. Some of these identified points are presented in Table 5.1. In order to determine what happened on these days we searched the archive of newspapers or weekly news such as 'Guardian', 'Irish Times', and 'Business Week'. The results of that are shown in Table 5.1. For example the date of 14/09/1998 is corresponding to the 'global crisis' when markets all around the world were in decline due to the Asian Crisis.

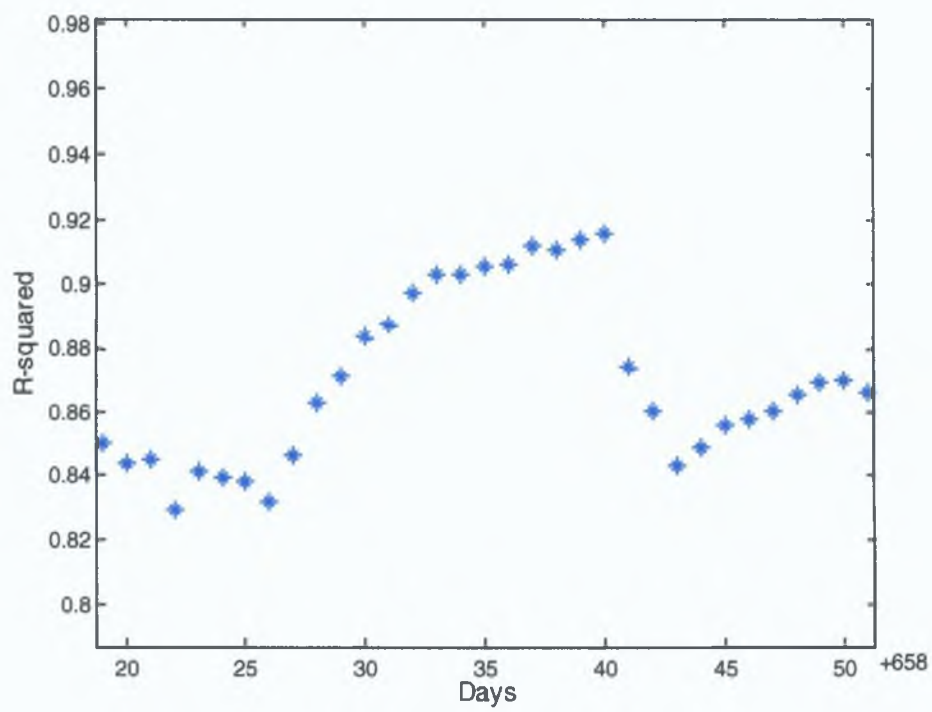


Figure 5.2 Progressive R-Squared values corresponding to the Technology- Cohesion graph. A sudden drop on the point of 40th day is observed.

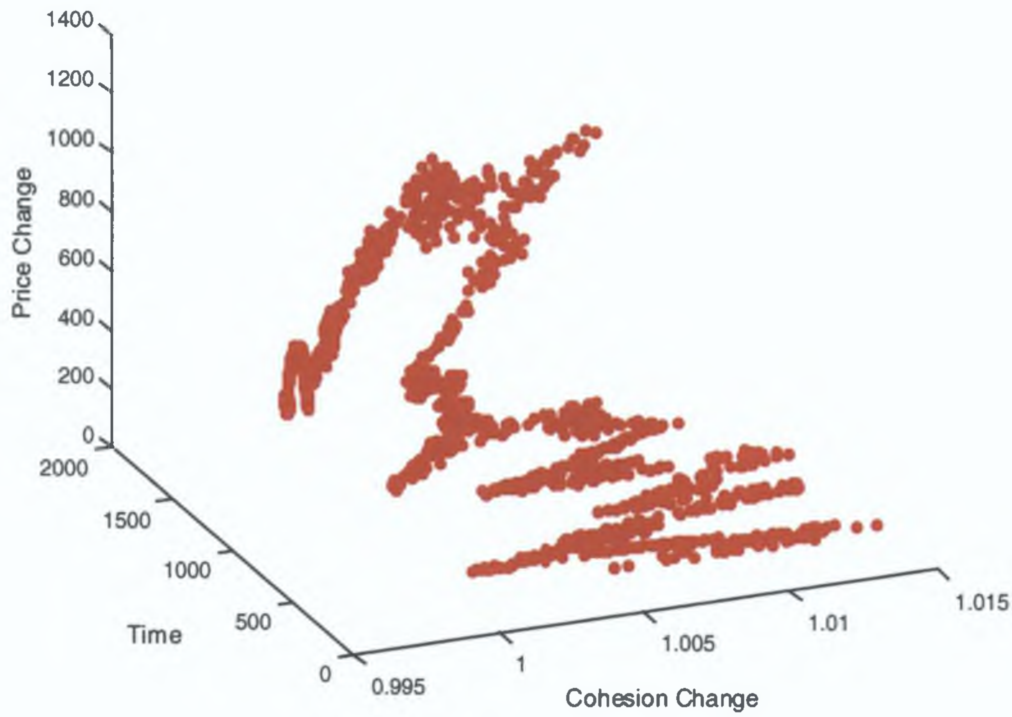


Figure 5.3 The time dependence of epochs for Technology plot

It should be mentioned, however, that some of the epochs are very volatile and identifying an exact turning point is sometimes difficult. In addition, the identified points in a few cases are not consistent with what one sees. This appears to be the case mostly in the epoch with the largest slope. What one sees is different from what theory describes (Table 5.1). But if we plot a 3-dimension graph of epochs, where a time dimension is added to the two other dimensions, it could be easier to distinguish the turning points of epochs. Figure 5.3 shows the 3-dimensional plot of figure 5.1(a), when the time axis is the third dimension.

However, the model fits should be specified and its parameters estimated, while assumptions and conditions underlying the model should be explored. As each epoch is identified earlier data should be dropped to identify the next one.

Table 5 1 The identified turning points of epochs corresponding to key events in the market

Date	Event
06/02/1996	Apple Computer in decline
17/03/1997	Fall of Dow Jones after the Warren Buffet's speech, UK pre-election period
31/07/1997	European markets fell sharply, worrying about the Greenspan report to Congress
29/08/1997	Fears of higher German interest rates
20/04/1998	Microsoft fear of splitting
20/07/1998	European markets fall Japanese and Russian economic crisis
14/09/1998	Global crisis due to Asian crisis
25/05/1999	China-US relationship crisis
23/07/1999	Market slump after Greenspan address
14/10/1999	Inflation, interest rate rise fear and a warning from Greenspan
24/04/2000	Backlash behind the anxiety over Globalisation
01/05/2000	Fall of Dot-com
12/12/2000	US Supreme Court Judgment Ratifies President Bush
18/12/2000	Tech slump
28/03/2001	Cut in Federal Reserve rate
11/09/2001	September 11th
02/07/2002	Scandal in Worldcom unfolding
12/07/2002	European markets tumbled after the SAP-Deutschland warning

5 3 3 Epochs Models

Cohesion graphs in Figure 5 1 appear to be characterised by periods of linear regression fits between price and change in cohesion The linear regression model is of the form

$$Y_t = a_t X_t + b_t + \varepsilon_t \tag{5 9}$$

(where Y_t is the daily price, X_t is the daily change in cohesion, ε_t is a random error term with zero mean and a_t and b_t are regression coefficients, both functions of time) In this model, the random errors should be independently distributed i e $cov\{\varepsilon_t , \varepsilon_t\}=0$ for all i, j, $i \neq j$ [Neter et al , 1990] To test for independence of error in the regression model, it is usually suggested [Neter et al , 1990] to prepare a time plot of residuals The purpose of plotting residuals versus time is to see if there is any auto-correlation between the error terms over time If a trend can be seen in the residuals, it is concluded that the errors are non-independent, otherwise they are presumed to be independent Therefore, we need to calculate and plot the residuals of our model over time Figure 5 4 shows two samples of these plots calculated from 06/02/1996 to 17/03/1997 and the other from 29/08/1997 to 20/04/1998 It should be mentioned that these four dates are obtained from Table 5 1

From Figure 5 4 it is evident that there is a correlation between the error terms Negative residuals are associated mainly with the early trails, and positive residuals with the later trails[4] It clearly can be observed that there is a trend in Figure 5 4 (a) and (b) This means, therefore, that ε_t are not independent Although we have got good fits (R_Squared values between 0 45- 0 66) for most market sectors, the dependency of error terms is against the model's assumptions and, therefore, should be eliminated In this case, a transformation that is often helpful to remedy such dependence is to work with first differences of the model Hence, we take the first differences of the Equation 5 9

$$Y_t' = \gamma X_t' + \beta + \delta(tX_t)' \tag{5 10}$$

(where β is found to be typically small and positive and γ, δ are large and negative, positive respectively)

The Durbin-Watson[5] (DW) test is then applied to test whether the error terms for the transformed model are uncorrelated We re-examine the transformed model and find that the DW statistic is satisfied with the values very much close to 2 Therefore, there are no more patterns in the residuals left (white noise) However, the regression

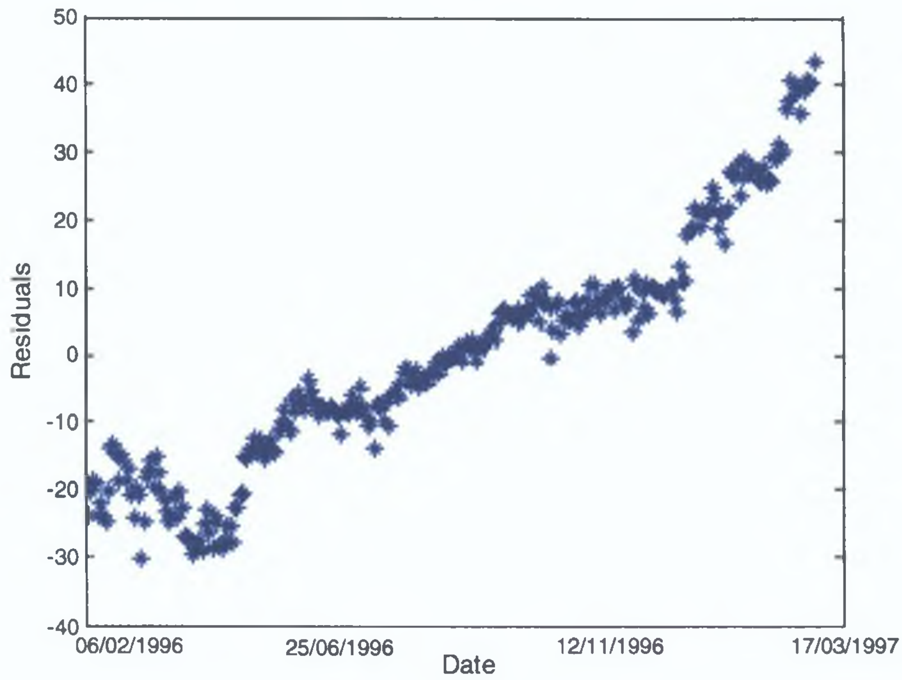
[4] When the error term are independent it is expected that the residuals fluctuate in a more random pattern around the base line zero

[5] It is a test for serially correlated (or autocorrelated) residuals If the residuals are uncorrelated, the expected DW statistic is 2

model shows poorer fit with the R-Squared values smaller after modifying the regression model.

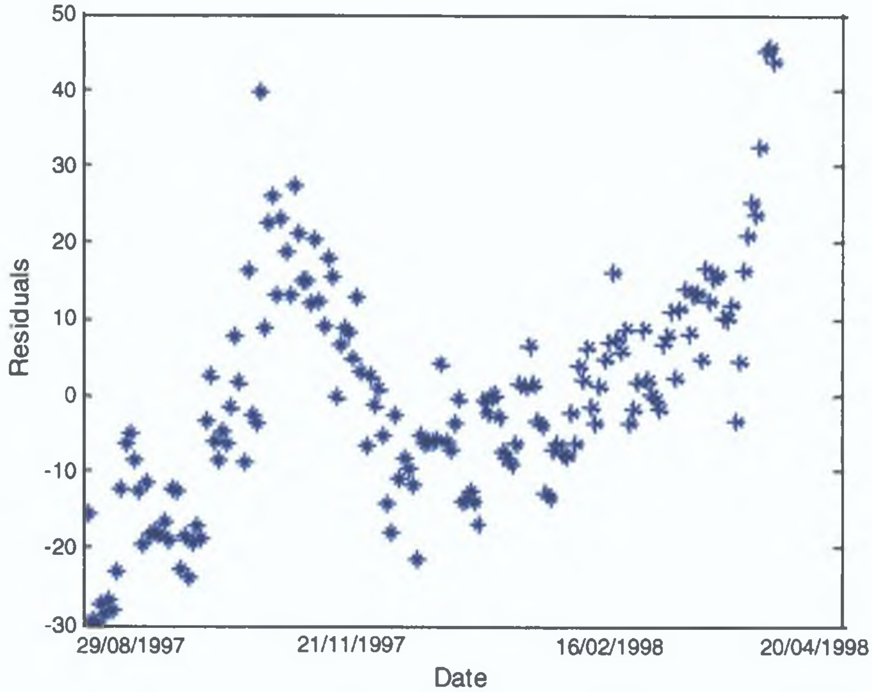
5.3.4 Sampling rate analysis

We tried to examine different sampling rates other than daily rates to see the effect of differencing on the lower sampling rates. Two days, 5 days, 10 days and monthly sampling rates are tested. Table 5.2 represents the reduction of the R-Squared for the Technology market sector index. As can be seen, DW statistics were obtained for a variety of granularities.



(a)

Figure 5.4 Residual- time plots illustrating nonindependence of error terms. (a) The residuals associated with the model fitted over the period from 06/02/1996 to 17/03/1997 and (b) from 29/08/1997 to 20/04/1998.



(b)

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Table 5.2 Model fit statistic for technology at different sampling rates

Sampling Rate	R-Squared	DW Statistic
Monthly	0.46	1.97
Every 10 Days	0.40	2.05
Every 5 Days	0.40	2.20
Every 2 Days	0.20	2.05

What can be seen in Table 5.2 is that the goodness of fit is reduced for the higher resolution data. This might suggest that there exists a high volatility in data, which induces the poor fit for higher sampling rates. In other words, although at daily rates, modifying the model results satisfactorily in white noise, the prediction power of the modified model is poorer.

The model failure for fine granularity is discussed in the following section.

5.4 Possible reasons for model failure

Possible reasons for regression model breakdown can be:

1 Day of the week, which means that market behavior on particular days of the week, is reflecting the different level of activities affected by closing, starting or mid-week time

2 Outliers, which means that large price changes, are only reflected in the high resolution data

The first case can be rejected because the eigenvalue changes show high variability on all days of the week, and there is distinctive pattern for variability over particular days The second one is refuted too, since the model fits are satisfactory for lower resolution though not for higher ones Also the epoch skeleton can be observed for all the sampling rates including the monthly case (Figure 5 5)

Furthermore, outlying points seem to have little effect on the R-Squared and DW statistic, as their number is very small compared to the overall number of observations

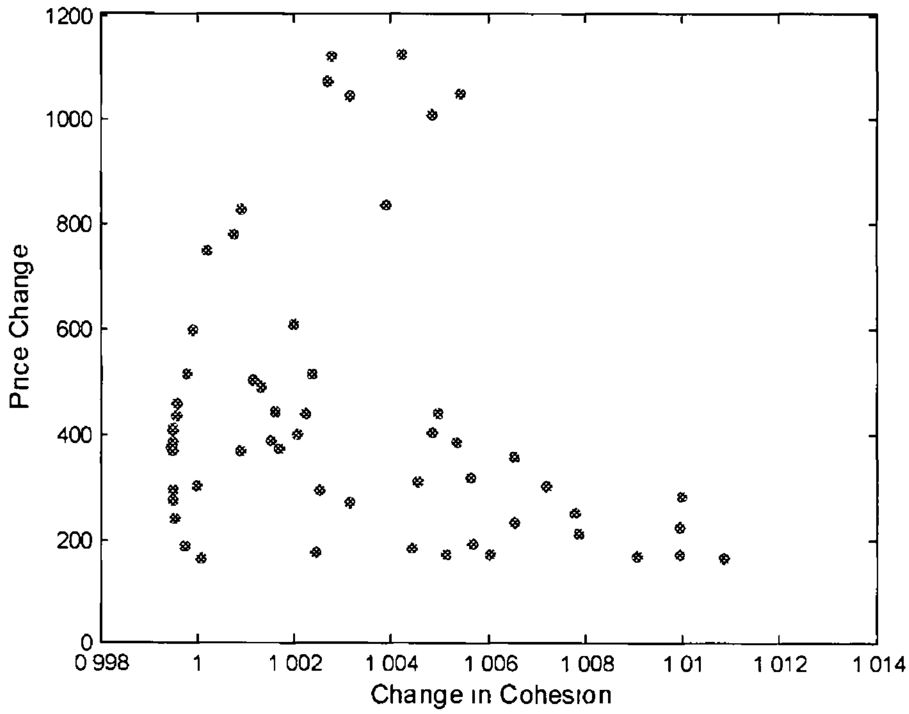

Figure 5 5 Epoch skeleton for monthly sampling rate is retained

Finally, there exist some periods (of a few-days-length) in the market data, which reflect the failure of the general model fits Clearly, these periods, where the short-term volatility increases, are associated with changes in cohesion As we explained in the previous sections $\lambda_{\max}^{T}$ could be considered as a measure of volatility and therefore, the change in cohesion ΔCoh_T is interpreted as the proportional change in volatility This high volatility naturally represents 'short-term' lack of co-operative behaviour Since coherence describes the change in common perception of risk, lack of common perception provides fluctuations over the short-term and this reduces the goodness of fit In the epoch plots, the coherence (or co-operative behaviour in market trading) is clearly lower at the end of epochs, which is due to the lack of common perception of risk However, it is suggested that the breakdown in coherence can be used as a measure of critical market uncertainty

5 5 Conclusion

In this chapter we introduced the concept of cohesion, which is co-operative behaviour in market trading and also demonstrated the existence of epochs or growth patterns in the Dow Jones Euro STOXX market sector Epochs represent the linear relationship between the sector price change and the daily change in cohesion They are defined in all market sectors but can be observed more strongly in Technology, Telecom and Industry (less so in "defensive" sectors such as Food or Automobile) The end of each epoch, where the coherence or cohesion has returned to a lower value, represents a relatively critical situation in the market However, the existence of high volatility, which is naturally dominant over short-term periods, diminishes the power of predictability of sustained changes

Chapter 6
Conclusions and Future work

The overall goal of this thesis was to study the risk and co-movements of financial stocks We have studied the role of the correlation matrix in order to determine the risk on a set of assets The major theories in finance are based on a true measure of correlations among assets We have shown that the so-called historical correlation matrix, C, contains a large amount noise and we applied the theory of random matrices to determine the noise in C For a set of actual data from S&P500 we realised that approximately 95% of eigenvalues of C do not contain true information This is supported to some extent by other work [Plerou et al , 1999][Laloux et al , 1999], which considers that at most 6% of C carries useful information The noise involved in C is due to the finite number of records and the non-stationary correlations between stock prices We also performed an eigenvector analysis in complement to the correlation matrix analysis, where we expect that the eigenvector corresponding to the maximum eigenvalue has a different construction from those representing true information

In order to see the result of RMT on different volumes of noise involved in financial data, we presented simulations creating a sinusoidal time series By adding different volumes of random noise controlled by the amount of standard deviation, different noise volumes in time series were studied and RMT applied to each The evidence suggests that the number of deviated eigenvalues from the random bound does not in general depend on the volume of the added noise All volumes of noise examined gave approximately the same number of deviated eigenvalues These were associated with the non-noisy part of C and the value of RMT here would appear to be in distinguishing the non-noisy or information-rich values

After applying RMT and identifying the amount of noise in C, some techniques are needed to separate and remove the information from the noise We used the principal technique of noise removal [Laloux et al , 2000], which proposed flattening the noisy part of the eigenvalues However, in examination of the stability of the matrix after noise was removed, we concluded that the standard method may actually destroy stability On the basis of results for stability obtained from the Krzanowski's stability model, we therefore proposed a technique to filter C such that the stability of the matrix is preserved This model keeps the noisy eigenvalues at maximum separation from each other while the trace of C is kept the same

To test the effects of noise removal, Bouchaud et al (2000) have suggested comparing the realised and the predicted optimal portfolios They have found a shorter distance between the realised risk and the predicted risk for the cleaned_C than that of the original C They have attributed this to higher stability of the cleaned_C

In our study we have demonstrated that this is not the case and, in fact, there is a negative relationship between the stability of C and the closeness of the predicted and realised risks This assertion is also showen by experiments in filtering C based on Krzanowski's model We conclude therefore that the common technique of noise removal not only does not promote the stability and therefore the power of prediction, but actually leads to some deterioration

- A practical application for the result of RMT

With respect to principal component analysis, we know that the minimum eigenvalue of the correlation matrix is associated with the least risky position on a stocks combination and the maximum eigenvalue is associated with the most risky position The result from RMT tells us that the combination of assets corresponding to the most risky position is the most reliable one Therefore, unlike for minimum eigenvalues, analyses dependent on the maximum eigenvalue are likely to be adequate

By studying the daily changes of the riskiest position on stocks combinations, we have aimed to study the market risk behaviour New concepts of '*cohesion*', and '*epochs*' have been introduced We have demonstrated the existence of epochs in EURO-STOXX market sector data, for which linear relationships between change in cohesion and market sector prices are observed The end of each epoch, sees a fall in coherence representing a relatively critical situation in the market The evidence supports a relationship between volatility and the change in the cohesion, but volatility is naturally dependent on short-time periods and therefore the existence of high volatility, diminishes the predictability of the model fits

As a final note, it is suggested that coherence provides a measure that can be directly linked to critical market uncertainty

- Future work

Some analysts would claim that noise is a reflection of the various trading strategies of the investors in the market [Long et al , 1990] In order to estimate the causes of noise, we have considered the use of fractional calculus, (which deals with applications of integral/ derivatives of arbitrary order) The recent application of fractional calculus model in finance has provided the modelling of non-Gaussian probability distributions, as are found in financial data where large fluctuations are to be expected In the case of the correlation matrix, application of the fractional calculus model will draw on more detailed analysis of the statistical feature of risk-return on combination of assets in relation to time-dependent behaviour of the eigenvalues, which reflect stock movements

Glossary

β the ratio of covariance between stock's return and market's return to the variance (or volatility) of the market

Broker an individual or firm, which facilitates a deal between a buyer and seller

Change in cohesion the change of the maximum eigenvalue

Cleaned correlation matrix the correlation matrix re-constructed after removing noise also expressed as cleaned_C

Co-operative behaviour the common understanding of selling/ buying stocks in a market

Derivatives Derivative (or derivative security) is a financial instrument whose value depends on the values of another asset

Dividend taxable payments declared by a company's board of managers given to its shareholders out of the earning of the company at the present Usually quarterly and given as cash

Efficient frontier the line on a risk-return graph comprised of all optimal portfolios

Epochs progressive patterns defined in terms of daily change in the largest eigenvalue and daily change in market sector prices

Financial assets financial assets are instruments to claim amount of cash at some future time The price of a financial asset equals the present value of all cash dividends that the asset provides its owners

Indicator a statistical construct that measures price changes and/ or returns in stock market

Liquidity the ability of an asset to be converted into cash quickly

Listed stock Stocks that are traded on an exchange are said to be listed stocks

Market value the value of a company's outstanding shares, as measured by shares times the current price Also known as market capitalisation

Optimal portfolio a portfolio that provides the greatest expected return for a given level of risk, or equivalently, the lowest risk for a given expected return, also called efficient portfolio

Price change the difference between asset's values on two consecutive days

Rate of return the annual rate of return on an asset expressed as a percentage of the total amount invested, also called return

Return return for a particular time period is equal to the sum of the price change divided by the price at the beginning of the time period

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Appendix A

Suppose

$$\frac{\lambda_i^{m+1}}{\lambda_i^m} = 1 + \varepsilon \qquad (*)$$

It means that if $\lambda_i^m = 1$, then $\lambda_i^{m+1} = 1 + \varepsilon$, and ε is a negligible part With multiplying λ_i^m in both hands we have $\lambda_i^{m+1} = \lambda_i^m + \varepsilon\lambda_i^m$, and therefore $\lambda_i^{m+1} - \lambda_m^m = \varepsilon\lambda_i^m$ Since ε vanishes, any finite multiplication of that vanishes, thus $\lambda_i^{m+1} - \lambda_i^m = \varepsilon^m$ Finally from equation * we can conclude that $\frac{\lambda_i^{m+1}}{\lambda_i^m} \approx 1 + (\lambda_i^{m+1} - \lambda_i^m)$, which implies the study of $\frac{\lambda_i^{m+1}}{\lambda_i^m}$ is the same as the study of $\lambda_i^{m+1} - \lambda_i^m$

Appendix B

```
%%%
%%% This function calls for other functions
%%%  to construct the portfolio prediction and portfolio realisation
%%%  and plot the graphs
%%%

load SPnonzero  %getting the data set from S&P500 intraday
data=dataSP',

%selectmg the first 200 time series over the first sub-period of 600 days
d1=data(1 200, 1 600),
% the same 200 time series over the next sub-period of 600 days
d2=data(1 200, 601 1200),
clear dataSP data
remove3  % removing those zero entries
clear d1 d2
cov1=covar(dd1),  %constructing the covariance matrix on the first sub-period
cov2=covar(dd2),  %constructing the covariance matrix on the second sub-
period
ret2=findreturn(dd2),  %calculating daily returns on the second sub-period
clear dd1 dd2
[V1, D1]=eig(cov1),
[V2, D2]=eig(cov2),
[V1, D1]=sort_correl(V1, D1),  %sorting the eigenvalues
[V2, D2]=sort_correl(V2, D2),

figure
plot(diag(D1)),  %plotting the eigenvalues
%cleaning C using Bouchaud technique
[C_b1, V_b1, D_b1]=get_clean1(V1, D1, 3),
[C_b2, V_b2, D_b2]=get_clean1(V2, D2, 3),

%constructing portfolio prediction using original C
[PortRisk1, PortReturn1, PortWts1]=portopt(ret2, cov1, 5),

%constructing portfolio realisation using original C
[PortRisk2, PortReturn2]=portstats(ret2, cov2, PortWts1),

%constructing portfolio prediction using cleaned-C
[PortRisk_b1, PortReturn_b1, PortWts_b1]=portopt(ret2, C_b1, 5),

%constructing portfolio realisation using cleaned-C
[PortRisk_b2, PortReturn_b2]=portstats(ret2, C_b2, PortWts_b1),

C_k1=clean_matrix10(V1, D1, 11), %filtering C using Krzanowski technique
C_k2=clean_matrix10(V2, D2, 11),
```

```
%constructing portfolio prediction using filtered-C
[PortRisk_k1, PortReturn_k1, PortWts_k1]=portopt(ret2, C_k1, 5),

%constructing portfolio realisation using filtered-C
[PortRisk_k2, PortReturn_k2]=portstats(ret2, C_k2, PortWts_k1),
figure
hold on
plot(PortRisk1, PortReturn1, 'b- '),
plot(PortRisk2, PortReturn2, 'r--'),
hold off

figure
hold on
plot(PortRisk_b1, PortReturn_b1, 'm^ '),
plot(PortRisk_b2, PortReturn_b2, 'g* '),
hold off

figure
hold on
plot(PortRisk1, PortReturn1, 'b- '),
plot(PortRisk2, PortReturn2, 'r--'),
plot(PortRisk_k1, PortReturn_k1, 'm^-'),
plot(PortRisk_k2, PortReturn_k2, 'go-'),
hold off
```

```
function covr=covar(data)
% This function makes the covariance matrix for chapter4

size_data=size(data),
obsr=size_data(2)-1, %the number of observations
n=size_data(1), % the number of time series (or stocks)
for j=1 obsr
   for i=1 n
      data(i,j)=(data(i,j+1))-(data(i,j)), %calculating the price changes
   end
end
data=data( , 1 obsr),
covr=cov(data'),
```

```
function ret=findreturn(data)
%To calculate daily return on data
data_size=size(data, 1),
obs_size=size(data, 2),
ret=[],
ret_sum=0,
for i=1 data_size
   ret_sum=0,
   for j=1 obs_size-1
      ret_sum=ret_sum+((data(i, j+1)-data(i, j))/data(i, j)),
   end
   ret(i)=ret_sum,
end
```

```
function [C_cl, Vc, Dc, sorted_cov]=get_clean1(V, D, n)
% This function calculates the cleaned-C
% V is the matrix of eigenvectors
% D is the matrix of eigenvalues
% n is the number of nonnoisy eigenvalues

[Vs, Ds]=sort_correl(V, D), %To sort the eigenvalues
sorted_cov=Vs*Ds*Vs',
clear V D
C_cl=clean_matrix(Vs, Ds, n), %C_cl is the cleaned-C
clear Vs Ds
%Vc and Dc are eigenvectors and eigenvalues of the cleaned-C
[Vc, Dc]=eig(C_cl),
```

```
function C_cl=clean_matrix(V, D, n)
% This function gets the matrices of eigenvalues (D) and eigenvectors (V)
%  and returns the cleaned-C
% n is the number of non-noisy eigenvalues
% C_cl is the cleaned-C


D_size=size(D, 1);

% D_mean is the mean of the noisy eigenvalues
D_mean=mean(diag(D(n+1:D_size, n+1:D_size)));

% D_cl is the cleaned eigenvalue matrix
D_cl=zeros(D_size);

% Non-noisy part
for j=1:n
   for i=1:n
      D_cl(i, j)=D(i, j);
   end
end

% Noisy part is replaced by the identity matrix * mean of the noisy eigenvalues
for j=n+1:D_size
   for i=1:D_size
      if (j==i) & (i>n)
         D_cl(i, j)=D_mean;
      end
   end
end
figure
hold on
plot(diag(D_cl))
title('Cleaned "eigenvalues"')
hold off
% Cleaning C
C_cl=V*D_cl*V';
```

```
function C_cl=clean_matrix10(V, D, n)
% using Krzanowski technique with respect to the constraint that eigenvalues
%shouldn't become zero
% This function gets the matrices of eigenvalues (D) and eigenvectors (V)
%  and returns the cleaned-C
% n is the number of non-noisy eigenvalues
% C_cl is the cleaned-C


[V, D]=sort_correl(V, D),
D_size=size(D, 1),
small_n=D_size-n,

D_sum=sum(diag(D(n+1 D_size, n+1 D_size))),
big_noisy_ev=1e-8,

% D_cl is to be the cleaned eigenvalue matrix
D_cl=zeros(D_size),
increase_step=(2/(small_n*(small_n-1)))*(D_sum-small_n*big_noisy_ev)

% Non-noisy part
for i=1 n
      D_cl(i, i)=D(i, i),
end


% Biggest noisy eigenvalue
D_cl(D_size, D_size)=big_noisy_ev,

% Noisy part
for j=D_size-1 -1 n+1
   D_cl(j,j)=D_cl(j+1, j+1)+increase_step,
end

figure,
plot(real(diag(D_cl)))
sum(diag(D))
sum(diag(D_cl))

C_cl=V*D_cl*V',
[Vc, Dc]=eig(C_cl),
hold on
plot(real(diag(Dc))),
```

```
function eign =epoch(filename, start_date, ending_date)
%%%
%%% This function calculate day-to-day C and day-to-day eigenvalues
%%% and eigenvectors
%%% for the purpose of plotting "epochs"
%%%
% filename is the file of data set, which is a text file with 3 headerlines
% eign is the day-to-day eigenvalues of C
% start_date/ ending_date determines the time-period to study

[Date, Bas, Consum1, Energy ,Indust, Consum2, Tech, Util, Autom, Bank ,
Chemic , Telecom, Constr, Health , Financ, Food ,Insur, Media
,Retail]=textread(filename,'%s%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f
%f','headerlines',3,'delimiter',','),

market=[Bas, Consum1, Energy ,Indust, Consum2, Tech, Util, Autom, Bank ,
Chemic , Telecom, Constr, Health , Financ, Food ,Insur, Media  ,Retail],

size_m=size(market),
size_row=size_m(1),
n=size_m(2)


%%%  To construct covariance matrices on the progressive windows

start_date=strcat(start_date),
ending_date=strcat(ending_date),

%% Attempt to find the given dates (firt_point and ending_date) on the data set

win_end=1,
while sum(start_date==char(aa(win_end,1)))~=10
   win_end=win_end+1,
end

last_point=win_end,
while sum(ending_date==char(aa(last_point,1)))~=10
   last_point=last_pomt+1,
end

%%

% To go further on the time-windows

increased_step=1,
while win_end+increased_step<last_point+1
   sub=market(1 win_end+increased_step-1, ),
   covr=cov(sub),
% % %     If a normalised correlation matrix is needed
% % %     for count_col=1 n
```

```
%%%      for count_row=1 n
%%%       corr(count_row,
%%%count_col)=covr(count_row,count_col)/sqrt(covr(count_row,
%%%count_row)*covr(count_col, count_col)),
%%%      end
%%%    end
   [ev, d]=eig(covr),
   eign(increased_step,n)=d(n,n),
   increased_step=increased_step+1,
end
```

```
function plot_epoch(eigen_file,sec_name)
% This function plot epochs
% eigen_file is the file of saved day-to-day eigenvalues
% sec_name is the name of the sector to study

for i=1 size(eigen_file, 1)-1
   ratio(i)=eigen_file(i+1)/eigen_file(i),
end

[Bas, Consum1, Energy ,Indust, Consum2, Tech, Util, Autom, Bank , Chemic ,
Telecom, Constr, Health , Financ, Food ,Insur, Media  ,Retail]
=textread('UpTo10_2002 txt',
'%*s%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f','headerlines',3,'delimit
er',','),

ratio=ratio',
switch sec_name
case 1
   sec=Bas(263 1991, ),
case 2
   sec=Consum1(263 1991, ),
case 3
   sec=Energy(263 1991, ),
case 4
   sec=Indust(263 1991, ),
case 5
   sec=Consum2(263 1991, ),
case 6
   sec=Tech(263 1991, ),
case 7
   sec=Util(263 1991, ),
case 8
   sec=Autom(263 1991, ),
case 9
   sec=Bank(263 1991, ),
case 10
   sec=Chemic(263 1991, ),
case 11
   sec=Telecom(263 1991, ),
case 12
   sec=Constr(263 1991, ),
case 13
   sec=Health(263 1991, ),
case 14
   sec=Finance(263 1991, ),
case 15
   sec=Food(263 1991, ),
case 16
   sec=Insur(263 1991, ),
case 17
```

```
   sec=Media(263 1991, ),
case 18
   sec=Retail(263 1991, ),
end
 figure
 plot(ratio,sec, 'r '),
```

```
function [b,stats]= findfit(x, y)
% This function is to find a linear fit to a set of data

x_matrix=[ones(size(x)),x],
[b,bint,r,rint,stats] = regress (y,x_matrix),
```

```
function stats = pinpoint(E,sector, c1, c2)
%To pinpoint the turning point of epochs
% sector is the name of sector e g Tech
% E is the file of maximum eigenvalue
%% c1 may start from 1 and c2 usually takes the value from 100 or 200

for i=1 size(E, 1)-1
   ratio(i)=E(i+1)/E(i),
end
ratio=ratio',

[Bas, Consum1, Energy ,Indust, Consum2, Tech, Util, Autom, Bank , Chemic ,
Telecom, Constr, Health , Financ, Food ,Insur, Media  ,Retail]
=textread('UpTo10_2002 txt',
'%*s%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f','headerlines',3,'delimit
er',','),
switch sector
case 1
   sec=Bas(263 1991, ),
case 2
   sec=Consum1(263 1991, ),
case 3
   sec=Energy(263 1991, ),
case 4
   sec=Indust(263 1991, ),
case 5
   sec=Consum2(263 1991, ),
case 6
   sec=Tech(263 1991, ),
case 7
   sec=Util(263 1991, ),
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case 10
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case 12
   sec=Constr(263 1991, ),
case 13
   sec=Health(263 1991, ),
case 14
   sec=Finance(263 1991, ),
case 15
   sec=Food(263 1991, ),
case 16
   sec=Insur(263 1991, ),
case 17
```

```
   sec=Media(263 1991, ),
case 18
   sec=Retail(263 1991, ),
end

j=1,

%% contr_point2 should be greater than contr_point1 and c2 should be less than
%%1728, the number of records
contr_point1 = c1,
for contr_point2 = c1+1 c2
   sector=sec(contr_point1 contr_point2),
   ratio1=ratio(contr_point1 contr_point2),
   [b( ,j),stats(j,  )]=findfit(ratio1,sector),
        %% b(1, ) is the tangent slope of the fitted line and b(2, ) is the y-
        %%intercept
        %% stats(j,1) gives R_2 at each stage(j=1,   )

   j=j+1,
end
figure
plot(stats( , 1),'c*'),  %stats1 shows the R_2 of the fitting line
```

```
function m_point=densityrmt(covr, l)
% To plot the density of eigenvalues in RMT and empirical

sizcov=size(covr),
n=sizcov(1)
q=l/n,
[v,e1]=eig(covr),
disp 'eig finished'
for i=1 n
   e(i)=e1(i, i),
end
maxee=max(e),
minee=min(e),
sigma=1,

% To calculate the theoretical max and min eigenvalues
emax=sigma*(1+1/q+2*sqrt(1/q))
emin=sigma*(1+1/q-2*sqrt(1/q)),

%To plot the density of the theoretical ones
for i=1 n
   pd(i)=q*sqrt((emax-e(i))*(e(i)-emin))/(2*pi*e(i)*sigma),
end
Figure
for k=1 size(e, 2)
   if (e(k)<=emax)
    plot(e(k), pd(k),
'bd','LineWidth',1,'MarkerEdgeColor','b','MarkerFaceColor','b','MarkerSize',3),
   end
end
%

% To count the number of deviated eigenvalues from noise bound
e_deviate=0,
for i=1 size(e, 2)
  if e(i)>emax
    e_deviate=e_deviate+1,
  end
end
e_deviate
%

% To plot the empirical density of eigenvalues
w=0 05,
k=0,
i=1,
sum(1 8000)=0,

while (k+1)*w<=max(e)+w
```

```
  i=i+1,
  sum(i)=sum(i-1),
  for j=1 n
    if (e(j)>k*w) & (e(j)<=(k+1)*w)
      sum(i)=sum(i)+1,
    end
  end
  k=k+1,
end
num_intval=i,
sum=sum/n,
m_point(1, 1)=w/2,
m_point(1, 2)=sum(1),

for j=2 num_intval
  m_point(j, 1)=m_point(j-1, 1)+w,
  m_point(j, 2)=(sum(j)-sum(j-1))/w,
end

hold on,
plot(m_point( , 1), m_point( , 2), 'r-', 'Linewidth',
1,'MarkerEdgeColor','r','MarkerFaceColor','r','MarkerSize',1),
hold off,
%
```

```
function a=generate_sinus(n, T)
% This function generates n time series of sinusoidal waves
% with T observation

for i=1 n
   r=randn(1),
   omega=randn(1),
   for theta=1 T
      th=theta*pi/180,
      a(i, theta)=r*sin(th+omega),
   end
end
```

```
function a=add_noise(a, zarib)
% This function adds noise to a time series a

for i=1 size(a, 1)
      %randn('state',sum(100*clock)),
   for j=1 size(a, 2)
      p=zarib*randn(1),
      r=randn(1),
      %r=1,
      a(i, j)=a(i, j)+p*r,
   end
end
```

```
function covr=correlation(data)
% This function makes the correlation matrix

siz=size(data),
t=siz(2)-1,
n=siz(1),
for j=1 t
   for i=1 n
      data(i,j)=(data(i,j+1))-(data(i,j)), %/data(i,j)),
  end
end

data=data( , 1 t),
STD=diag((1/t)*(data*data')),
covr=cov(data'),
for i=1 n
   dmean=mean(data(i,  )),
   varn=var(data(i,  )),
%   if varn==0
%       i
%   end
   data(i,  )=(data(i,  )-dmean)/sqrt(varn),
end
disp ('normalized'),

covr=cov(data'),
```

Epochs in Market Sector Index Data - Empirical or Optimistic?

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Dublin City University,
Dublin, Ireland

Abstract We introduce here the concept of "epochs" in market movements (i e periods of co-movements of stocks) These periods in EURO-STOXX market sector data are characterised by linear relationships between price and eigenvalue change The evidence suggests strong time dependence in the linear model coefficients but residuals are strongly dependent on granularity (i e sampling rate) with fit breaking down at rates smaller than five days Possible reasons for this breakdown are presented together with additional arguments on the relative merits of correlation and variance-covariance matrix eigenanalyses in measuring co-movements of stocks

1 INTRODUCTION

The leverage effect (i e the fact that at-the-money volatilities tend to increase for asset price drops) in financial markets has been much studied over recent years This increase has been shown to apply for different forecasting horizons, dependent on whether studies focus on the volatility in auto-correlations of actual stock prices or on those of index data The literature is reticent, however, about the effect of price changes on the combined upward or downward movements of shares and it is this aspect which we address Specifically, we have developed, for different market sector data from Dow Jones EURO STOXX, a novel approach to the way in which the market recognises and responds to risk This, implicitly, reflects the volatility, where risk reaction clearly changes as prices rise or fall or crashes occur The method is based on examining change in the riskiest position, as determined by the maximum eigenvalue from an eigenanalysis of the variance-covariance matrix from day-to-day, and relating this to effects on prices for the various sectors The change in the largest eigenvalue acts, in some sense, as a barometer of market risk

Results to date indicate that there are periods in the sector data, for which change in the largest eigenvalue varies linearly with price As each period appears to end, the relationship changes and a smaller change in the eigenvalue is required to bring about the same change in price In the limit and during a crash, the slope is still positive (and quite large) This is evidence, we believe, that crash patterns are well-defined, due to common perception of risk change, while upward trends are less predictable Evidence tends to support an implicit relationship between the instantaneous volatility and the daily changes in the eigenvalues, not least because alternative analysis of the correlation matrix, (for the data normalised with respect to the variance), results in a disappearance of these periods The above effects vary from sector to sector, being seen to be most pronounced in the technology and telecoms sectors and less so in more defensive sectors

In summary, evidence of periods in eigenvalue-price correlations is presented together with additional arguments on the relative merits of correlation and variance-covariance matrix analyses in measuring co-movements of stocks

2 THE BASICS

In order to make money (or more euphemistically "maximise their utility") on stock markets, investors buy and sell assets As the overall risk associated with a portfolio of stocks can be shown [Elton et al (2002)] to decrease with the number of assets, more is better when it comes to assets However, by having more assets, the investor will potentially take on more risk in order to generate higher expected returns A balance needs to be struck, therefore, between the risk a new asset will add to the portfolio and the expected return This problem of balance requires a knowledge of the volatility of and correlation between the assets, quantities that only become available (if then!) with time

In order to measure the correlation between assets, we use the variance-covariance matrix **C** based on a dataset from EURO-STOXX market sector indices The covariance matrix is updated daily and the individual covariances thus calculated at time T and over a time horizon M are given by (see for example [Litterman & Winkelmann(1998)])

$$\sigma_{ij}^{T}(M) = \frac{\Sigma_{s=0}^{T} w_{T-s} r_{i,T-s} r_{j,T-s}}{\Sigma_{s=0}^{T} w_{T-s}} \tag{1}$$

where $r_{i,T}$ is the daily return on the ith asset at date T and w_T is the weight applied at date T

For our purposes, we use asset (i e index) price instead of return, $r_{i,T}$ and take unit weights for previous days' data (i e $w_{T-s} = 1$ for all s,T) in the calculation of covariances

As regards volatility measurement, it has been known for some time (see e g [Bouchaud & Potters(2000)]) that the eigenstates of the correlation matrix of assets are useful in the estimation of risk in a portfolio made up of those assets In this paper, we suggest using the *covariance* matrix **C** because, as we will show, this seems to contain more long term information on co-movements of stocks as the covariances retain volatility information which is lost in the normalisation process Furthermore, the need to use correlation does not apply for indices For volatility, we suggest the use of day-to-day (or period-to-period) change in the largest eigenvalue of the covariance matrix as a measure of the change in the riskiest position as perceived by the majority of market participants This, we believe to be novel, up to now, while it has been recognised that the largest eigenvalue (λ^T_{max}) and corresponding eigenvector (z^T_{max}) represented "the market" (with the eigenvector representing in some way the riskiest portfolio), the change in λ^T_{max} over time has not (to our knowledge) been used before as a barometer of the perception of risk in the market

Specifically, we define the *change in cohesion*, ΔC_{T+1} as being

$$\Delta C_{T+1} = \frac{\lambda^{T+1}_{max}}{\lambda^{T}_{max}} \tag{2}$$

3 RESULTS AND DISCUSSION

3 1 Epochs Definition and Possible Causes

We have taken as our data, daily market sector index prices from EURO-STOXX covering 18 different market sectors[1] and, starting with a time horizon of 200 days, have calculated the covariance matrices for successive days as per Equation 1 By calculating the ratio of the change in largest eigenvalue λ_{max} over successive time periods, and plotting this against the price of the particular market sector index, we obtain a view of how the market is tending to organise itself Referring to Figure 1, which shows the daily change in λ_{max} for the telecom and technology indices, we see evidence for the epochs mentioned above Central to the idea of the epochs is the notion that "there are many ways for a market to go up but just one to go down" Various authors ([Zumbach et al (1999)] and others) have identified key events which can have an (occasionally disproportionate) impact on financial markets Similarly we have identified events which have acted as market breakpoints and can be clearly identified as marking the end of epochs Some of these are given in Table 1

Table 1 Key Events as Breakers in Market Cohesion

Date	Event
07/97 - 11/97	Asian Market Crisis
14/09/98	Bad News from South America
23/07/99	Plunge after Market Highs & Greenspan Address
12/12/00	US Supreme Court Judgement Ratifies President Bush
28/03/01	Cut in Federal Reserve Rate
18/09/01	Post September 11th Reaction

It should be emphasised, however, that not all these events appear to be seen by the market as bad *per se* Often, it seems to be just a pause for breath or a lack of any particular cohesion Successive epochs seem to be characterised by progressively higher slopes in price change for an increased degree of cohesion, each epoch being brought to an end by a particular event Finally, it would

[1] c f www stoxx com

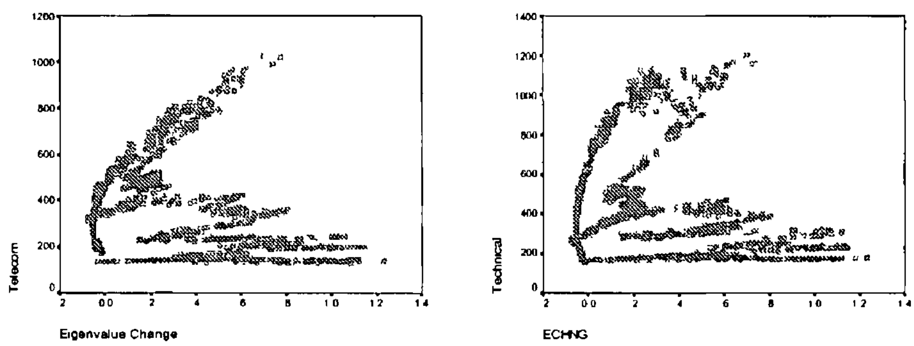
appear that, as extreme prices are reached, further increases in cohesion become impossible and an avalanche will occur with cohesion and price both falling together

3 2 Epochs Models and Breakdown

From Figure 1, it will be apparant that cohesion graphs such as those shown, are characterised by periods of simple linear correlation between price and changes in cohesion Fitting a simple linear model of the form

$$Y_T = \alpha_T + \beta_T X_I + \epsilon_t \tag{3}$$

(where α_I, β_I are functions of time, Y_T are the daily prices, X_T, the daily change in cohesion and ϵ_t are the errors) we get a good fit ($R^2 \approx 0.45 - 0.66$) for most market sectors However, ϵ_t, which should be independently distributed, actually follows a random walk There is clearly, therefore, time-dependence of order one in ϵ_t with partial auto-correlation function (PACF) at lag one very close to one and a Durbin-Watson[2] (DW) statistic d very much less than 2



(a) Telecoms Sector

(b) Technology Sector

Fig 1 Daily Change in Cohesion ($\lambda_{max}^{T+1}/\lambda_{max}^{T}$) vs Price for Different Market Sectors

Hence, this implies we should take first differences of Equation 3

$$\Delta Y_I = \beta + \gamma \Delta X_T + \delta \Delta (T X_T) \tag{4}$$

(where β is small and positive and γ, δ are large and (negative, positive) respectively) Re-examining the modified model we find that the DW statistic is satisfied $d \approx 2$ so this is now consistent with the null hypothesis of no positive autocorrelation and with no remaining systematic pattern in the residuals (white noise) However, at the *daily* sampling rate, the model fit becomes very poor after taking first differences Examining different sampling rates (see Table 2), the degradation in R^2 can be clearly seen for the technology market sector index

[2] testing for positive autocorrelation in the residuals

Table 2 Model Fit Statistics for Technology at Different Sampling Rates

Sampling Rate	R^2 Statistic	DW Statistic
Monthly	0 46	1 97
Every 10 Days	0 40	2 05
Every 5 Days	0 40	2 20
Every 2 Days	0 20	2 05

This data suggests that the variability (or volatility) increases markedly as the resolution increases and conversely that there is a settling period after which short term effects do not have an impact on subsequent data

3 3 Possible Reasons for Model Breakdown

As we have seen above, the simple linear regression models break down for fine granularity of sampling There are several possible reasons for this

- *Day of the week* It seems possible that market behaviour on certain days of the week, is subject to increased/reduced activity, reflecting start-up, closure and similarly We rejected this explanation, however, as eigenvalue changes and hence changes in cohesion were highly variable for all same-day data without exception
- *Statistical leverage or outliers* This presumes that extreme values occurring at certain time points or reflecting abnormally large localised price fluctuations are peculiar to a high sampling rate This does not seem to be the case, since even very coarse-grained sampling, (i e monthly), retains the epoch skeleton Furthermore, while epoch turning points are clearly significant, the fit is satisfactory for lower sampling rates, but not for high ones Similarly, outlying points appear to have little influence on the R^2 or Durbin-Watson statistics for all rates and, in any case, are relatively few in number It does seem likely, however, that there exists a settling period in the market, a few days in length, during which high variability diminishes the fit
- *Other systematic features* Even at the daily rate, (fine-grained sampling), first-order filtering satisfactorily results in white noise Unfortunately, the relatively poor level of fit achieved for these higher sampling rates indicate that the model is significantly less reliable in terms of predictability value

What does seem clear is that the increased variability over a few days, reflects high volatility associated with changes in cohesive or co-operative behaviour, with frequent changes of the sign of the largest eigenvalue, (the linear trend is in any case strictly non-monotonic) It is, therefore, instructive to consider the nature of the volatility measure in more detail

Clearly, implied/price volatilities will change *no matter what* so that the historical basis should be relatively bias free Equally, λ^T_{max} is an effective measure of volatility, so that the change ΔC_{T+1} in the coherence is equivalent to looking at proportional change in volatility Typically, these high-frequency fluctuations are self-correcting, representing a short-term and not particularly far-reaching lack of cohesive market behaviour, i e $\Delta C_{T+1} \approx 1$ In other words, if coherence represents the change in commonly-perceived risk, deviation from the common perception is sufficient to ensure fluctuating variability over the short-term and corresponding reduction in fit, as observed for our data

While this position is usually resolved over a slightly longer period, it is the lack of a sustained common perception of risk, (i e lack of coherence), which leads to a drop in price The quantity ΔC_{T+1} is clearly large and negative at the turning point of an epoch, corresponding to large change in the maximum eigenvalue, as well as a change of sign, since no change of sign for λ^T_{max} between consecutive time points implies that C is incremented Unfortunately, since high variability is naturally associated with short-term effects, as mentioned previously, it is non-trivial to determine whether a sustained change can be determined in advance A number of points are worth noting, however Firstly, use of the covariance measure exposes the detail of these changes The epochs, while present to some extent in the correlation matrix information, are far less distinct and thus less useful for our purposes, while dimensionality considerations do not arise for indices Secondly, we note that outliers are few, so that tolerances can more readily be established on the size of high-frequency fluctuations (It seems clear that the price change distributions involved do not scale simply, since volatility is dependent on the time interval as usual) It is suggested that coherence provides an intermediate measure which can be directly linked to critical market uncertainty

4 Conclusions

We demonstrate the existence of epochs in EURO-STOXX market sector data, where the change in the largest eigenvalue of the covariance matrix of daily prices, varies linearly with time The evidence supports an implicit relationship between instantaneous volatility and the change in the maximum eigenvalue, but volatility patterns are, typically, dependent on the time intervals observed At the end of an epoch, the relationship changes, with smaller eigenvalue changes leading to the same price change Epochs are present in all market sectors, but are most strongly defined in the less-defensive sectors, such as technology, telecoms The epoch end is followed by a return to low coherence in market trading, due to disparate perception of risk Co-operative behaviour (or strong coherence) in market trading suggests that up-turn patterns are more varied and hence less easy to predict, whereas down-turn patterns are well-defined, due to a common perception of risk change As a final note, it should be mentioned that we attempted to replicate the results for exchange rate data but as data were limited, partial support only was obtained for results reported here

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Noise in the correlation matrix: A simulation approach

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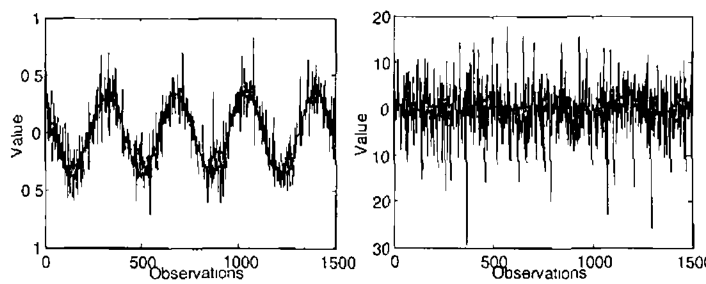
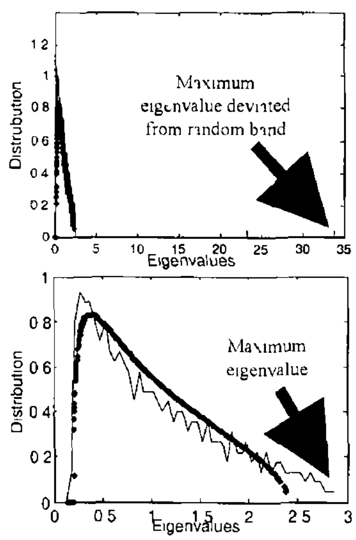
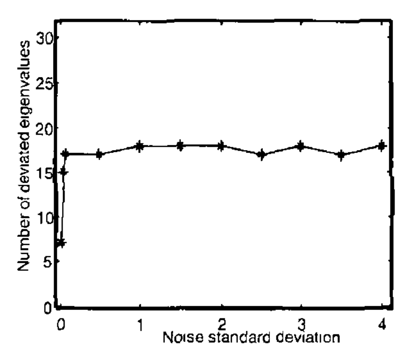
The results of applying Random Matrix Theory (RMT) in finance suggest that the historical correlation matrix, , carries a large amount of noise The purpose of RMT is to compare the statistical properties of with those of a random matrix By applying RMT we study how responds to the <u>different volumes of noise in data</u>

We generate a set of 450 random sinusoidal time series and add some random noise normally distributed with zero mean and a particular standard deviation The volume of noise is controlled by its standard deviation





The distributions of the eigenvalues in historical and random cases are plotted A large part of the historical graph is similar to the random one This part that is carrying noise corresponds to the noisy band of There are some eigenvalues that deviate from the random graph These eigenvalues and the corresponding part of have information and are known as the non-noisy band To estimate the exact effect of the added noise we increase the volume of noise gradually Starting from 0 02 11 different values of standard deviation are examined and the number of deviated eigenvalues is estimated



At the beginning by increasing the standard deviation an increase in the number of deviated eigenvalues is observed But for the standard deviation varying from 0 08 to 4 no dramatic change is observed Therefore it indicates that from a particular point onward there is no relationship between the volume of noise and the number of non noisy eigenvalues or in the other words, amount of noise has no effect on the number of noisy eigenvalues

Consequently we conclude that the volume of noise in the data has almost no effect on the genuine information part of the eigenvalues of the correlation matrix This represents that except for very small volumes of noise involved in data RMT result is the same for different amounts of noise In the case of stock markets we conclude that RMT result does not depend on the different volumes of noise involved in stocks prices Whatever noisy the stocks prices are RMT estimates the same percentage of the deviated eigenvalues of from the noisy bound

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