Markov Decision Processes in the Optimisation of Culling Decisions for Irish Dairy Herds.

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Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Master of Science in Computer Applications, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed: ____________ Date: 30/9/1997.

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Abstract

This thesis presents the development of a decision support system to optimise replacement and insemination decisions in Irish dairy herds under market conditions. The technique used was developed specifically for animal replacement problems, and is known as a 'Hierarchic Markov Process'. The model optimises culling decisions on the basis of lactation, production level, calving date, fertility and calving interval. Production, involuntary culling rates, feed costs, carcass values and other economic factors are allowed to vary according to the traits of a particular animal. The output from the model is a series of retention payoffs (RPO) upon which culling decisions are based. RPO is the expected future return in keeping a cow for an additional stage rather than replacing her with a heifer.

A study of culling rates in commercial dairy herds was also carried out and the effect of culling strategies on the genetic level of dairy herds was investigated using Monte Carlo simulation.

Chapter 1

Introduction

1.1 The Importance of the Dairy Industry to the Irish Economy

Ireland is a predominately rural economy and society. About 43% of the population live in rural areas, including towns of fewer than 1,500 people. Population density is 50 persons per square kilometre, the lowest density level in the European Union (Anon (1994a)). The agricultural sector in Ireland accounts for around 12.6% of employment, compared with an average European Union figure of 6.2%. When the food sector is included, this accounts for some 17% of the total employment in Ireland (Anon (1994b)).

The agriculture and food industries are vital elements not only in the Irish rural economy but in the wider national economy. The Gross Domestic Product for Ireland in 1995 was estimated by the Central Statistics Office to be £34,199 million, with the agriculture, forestry and fishing sector accounting for £2,880 million of this, equivalent to 7.5% of the total. This represents an increase from £2,687 million and £2,575.7 million for 1994 and 1993 respectively. However, expressed as a percentage of the total gross domestic product, the gross product from this sector has decreased: 7.7% for 1994 and 7.9% for 1993. The proportion of GDP accounted for by agriculture in Ireland is, however, high in comparison with other EU countries, where the average is 2.7% (Anon (1994a)).

Exports from the agricultural sector account for some 22% of total exports and the low import content of agri-food exports means that approximately 40% of Ireland's net foreign exchange earnings come from this sector. Preliminary estimates of the value of exports from agricultural produce in Ireland for 1996 (January to November) was estimated at £2,287.3 million. The value of exported agricultural produce for the years 1993- 1995 are shown in Table 1.1.

Year	Agricultural Produce IR£ million
1993	2,540.3
1994	2613.4
1995	2940.8

Table 1.1 The value of exported agricultural produce for the years 1993- 1995.

In 1996, the overall value of Gross Agricultural Output was estimated at £3,494 million, a decrease of 2.2% from 1995 (Anon (1997)). £1,838.6 million of this figure was attributed to livestock, of which 62.5 % (£1,149.9 million) was accounted for by cattle. Income from livestock products was dominated by milk production (£1,210.2 million; 97.4% of £1,242.4 million). These and other relevant figures are presented in Table 1.2

	Year		
	1994	**1995**	**1996**
Livestock	**1,825.5**	**1,885.6**	**1,838.6**
• Cattle	1,282.4	1,323.7	1,149.9
• Pigs	200.2	233.2	292.5
• Sheep / lambs	168.7	155.3	198.2
Livestock Products	**1,173.3**	**1,237.8**	**1,242.4**
• Milk	1,140.9	1,204.2	1,210.2
Crops	**401.9**	**449.6**	**413.1**
• Cereals	100.2	137.5	130.3
• Root crops	136.1	136.1	103.6
Gross Agricultural Output	**3,400.6**	**3,573.0**	**3,494.1**

Table 1.2 Estimates of Outputs in Agriculture, 1996.

At the end of December 1996, the Irish cattle herd consisted of 6,756,600 animals of which 2,334,900 were cows, 1,272,400 (54%) being dairy cows (Anon, 1997). The number of dairy heifers in calf was estimated at 240,300 at this time. The national dairy herd is increasing (0.4% from 1995 to 1996), and the number of other cows is also increasing (7.5% from 1995 to 1996). This results in an overall increase in the national cow herd of 3.5% in the period December 1995 – December 1996.

1.2 The Importance of Culling Strategies for Profitability

The profitability of a dairy herd is directly influenced by the actions of the decision-maker, i.e. the dairy farmer. The decisions that the farmer makes are based mainly on economic considerations, rather than biological considerations (Van Arendonk(1985b)). The management by the dairy farmer is directed towards the maximisation of total profit on the farm. A decision type that greatly influences the herd profitability is the culling strategy (Renkema and Stelwagen(1979); Kuipers(1982)). The culling strategy is the process of making decisions on whether to keep or replace animals in the herd. A decision to replace a cow in the dairy herd will be taken by a farmer because he/she expects higher profits by replacing that cow than by keeping it in the herd.

In some instances, replacement decisions may be outside the control of the dairy farmer, e.g. if the cow contracts a serious illness. Such instances, when the replacement of a cow in the herd is not a management decision, are classed as '*involuntary culling*'. Culling on the basis of a management decision is referred to as '*voluntary culling*', and is the subject of research in this thesis. The advantages of a culling strategy can be improved by maximising the proportion of cows that are culled for voluntary as distinct from involuntary reasons. In a study of culling some years ago (Crosse and Donovan(1989)), the proportion of cows culled for involuntary reasons was found to be 13.68% compared with 3.88% on average for voluntary reasons.

1.3 Summary of Research Objectives

1.3.1 Study of Culling in Irish Dairy Herds

A study of culling rates in large intensive commercial dairy herds in Ireland is described in Chapter 2 of this thesis. Data from the DairyMIS database (Crosse(1991); Cliffe(1994)), which were recorded over a five year period, were analysed, together with cow disposal rates from the National Farm Survey over the period 1990- 1993 (Anon(1993a)).

While cow replacement strategies exert a considerable effect on farm profitability, other studies have found that they have a negligible effect on the genetic improvement of the herd (Korver and Renkema(1979); Allaire(1981)). A Monte Carlo simulation model (Jørgnsen(1996)) was used to simulate the effects of culling strategies based on the lactation adjusted production on physical and financial parameters in a model dairy herd (Moorepark Blueprint for summer production). This model was also used to simulate the effect of different breeding goals in the herd (i.e. sires of varying genetic merit).

1.3.2 Optimisation of Culling Decisions

Mathematical models have been developed to determine the optimal replacement (and insemination) policy for dairy cows under different price and production circumstances. These optimum decisions need to be based on the expected future performance of the cows already in the herd and of the future replacement heifers. Optimisation techniques that have been applied to dairy cattle replacement were described and evaluated by Kristensen(1993).

The optimisation technique that has been applied in most studies of dairy replacement is dynamic programming. This technique, first introduced by Bellman(1957) , can be used to determine the replacement decisions which result in maximum expected income (where income is defined by an objective function) over time. The application of this optimisation technique to the dairy replacement problem is outlined in Chapter 3 of this thesis. Two techniques, the value iteration method and the policy iteration method, are examined and their advantages and disadvantages outlined. Relevant objective functions for dairy replacements are studied, including an objective function for a situation where milk quota acts as a constraint on production.

Kristensen (1988; 1991) developed the notion of Hierarchic Markov Processes for application to animal replacement problems. This method is a hybrid of the value and policy iteration methods, and is designed to overcome the so-called 'curse of dimensionality', associated with these traditional dynamic programming methods. This approach, has been applied to dairy replacement problems (Houben et al(1994); Kristensen(1987), and its benefits are explained in Chapter 4.

The Irish dairy and beef industries are highly seasonal (Ryan (1997)). This pattern of seasonal production essentially reflects farmers' efforts to maximise returns from their resources and farming systems. 'Seasonality of production arises from the fact that milk and beef can be produced at lower farm cost in the summer months because of Irish grass growth rates' (Anon (1993b)). The optimisation technique of Hierarchic Markov Processes described by Kristensen(1989; 1991) has to be modified to include seasonality (Kristensen(1997)), which is essential to the modelling of an Irish dairy system. The necessary changes to the iteration cycle are explained in Chapter 5 of this thesis.

A model for dairy replacement in the Irish dairy industry was developed and the physical traits describing dairy cows and the economic factors used are described in Chapter 6. The analysis of economic inputs and effect of season was carried out using the 'Moorepark Dairy Planner' (Walsh(1995)), and this computer programme was modified to take cognisance of recent developments in milk production technologies. The results obtained from the model for optimal replacement are presented in Chapter 7. Finally the conclusions from these results and future research in this area are discussed in Chapter 8.

Chapter 2

Culling in Irish Dairy Herds

2.1 Dairy Cow Disposal Rates from Commercial Dairy Farms in Ireland

2.1.1 Introduction

Cow disposal rates from dairy herds participating in DairyMIS (Crosse(1991); Cliffe(1994)) were analyzed over a five-year period (1990-1994). In total over this period, there were 22,000 animal records from 53 herds. The number of farms recorded varied over the years due to farms entering and leaving the system, though the turnover rate of farms was relatively small. The primary reasons for dairy cow disposal together with the effect of parity of animal, seasonality of disposal and farm effect were retrieved from the computer records. In addition, cow disposal rates from the National Farm Survey (NFS) database were also analyzed over the period 1990-1993.

The long-term profitability of the dairy herd will be affected by whether dairy cows leave the herd for 'voluntary' reasons such as low milk production or for 'involuntary' reasons such as animal disease, infertility or mortality. Many studies have reported the reasons (and their relative frequencies) for removal of dairy cows (Allaire et al(1977); O'Connor and Hodges(1963); Gartner(1983); Young et al(1983); Walsh(1983) and Crosse and O'Donovan(1989)). The actual herd culling rates can vary widely between herds and between years and are largely determined by herd management practices (Gartner(1983); Walsh(1983); and Crosse and O'Donovan(1989)). In a study of experimental herds in Ireland, Walsh(1983) reported an average culling rate of 21.6 per cent per annum, varying between herds from 16.4 to 27.2 per cent. A more recent survey on dairy cow culling in Ireland by Crosse and O'Donovan(1989) reported an average dairy cow disposal rate of 17.6 per cent, this figure being made up of 13.7 per cent for involuntary reasons and 3.9 per cent for voluntary reasons. Reproductive problems were the most common reason given for involuntary culling and mastitis was the second most common reason. The culling rate for older animals was higher than for the other age classes, and the seasonal distribution of cow disposals was relatively constant with the peak of culling in January.

Dairy farmers in Ireland have now been operating with a quota constraint on milk production for over a decade. The objective of this study was to update the information available on dairy cow disposal rates on commercial dairy farms under this new economic reality. The development and implementation of the DairyMIS computer system is described in detail in Crosse(1991) and Cliffe(1994). The farms included in the DairyMIS system are representative of intensive dairy farms in Ireland and are mainly located in the South of Ireland. The NFS data base is more representative of dairy farms nationally; further details in relation to the National Farm Survey are presented in detail in Anon(1993a).

2.1.2 The DairyMis System

2.1.2.1 Data capture - DairyMIS

Each animal entering the computer system had to be identified with a unique number. Initially, the following data were assembled on each animal: animal number, lactation number, sire, dam, breed, date of birth, calving dates and status (i.e. in milk or non-lactating). Stock events such as calving date, sales, and deaths were then recorded in diaries on the farm and were collected monthly by a recorder, who coded the data for computer input. In the case of culling data, up to 31 primary reasons for animal sales were assigned codes and these codes were then used for computer input. The data were validated both at entry to computer and when the main system files were updated.

2.1.2.2 Data presentation

The results are presented as percentages, the denominator for each of the percentage values calculated being equal to the number of cows at risk (available at any time during year) for the DairyMIS data. The average number of cows in the herd was used as the denominator for the NFS data. The data were analyzed using chi-square analysis where relevant.

2.1.3 Culling Rates for the DairyMIS Data

The following results refer to the DairyMIS database only. The disposal rates of cows by primary reason for disposal are shown in Table 2.1. Up to 31 primary reasons for culling were recorded initially but a number of the less important reasons have been grouped together under 'other reasons' for this analysis.

Primary reason for culling	Year					
	1990	1991	1992	1993	1994	Average
Tuberculosis	0.19	0.48	0.08	0.18	0.11	0.21
Abortion	0.13	0.34	0.23	0.20	0.18	0.22
Brucellosis	0.00	0.00	0.02	0.04	0.00	0.01
Calving problems	0.06	0.09	0.10	0.04	0.12	0.08
Infertility	3.10	4.12	5.02	2.53	3.18	3.59
Limb and foot disorders	0.34	0.50	0.63	0.42	0.25	0.43
Late calving	0.19	0.11	0.02	0.55	0.41	0.26
Low production	1.10	1.35	0.81	1.02	1.72	1.20
Mastitis	1.56	1.78	2.81	1.20	1.83	1.84
Old age	0.42	0.78	0.33	0.67	1.85	0.81
Pining	0.13	0.05	0.04	0.09	0.11	0.08
Teat and udder injuries	0.23	0.21	0.15	0.09	0.18	0.17
Surplus	2.59	2.86	1.44	2.05	1.99	2.19
Other reasons	6.82	3.04	3.04	3.36	4.16	4.08
Total	16.87	15.71	14.70	12.44	16.09	15.16

Table 2.1 Cow disposals - by primary reason for disposal (%)

The average cow disposal rate was 15.2%, averaged over the five years ranging from 12.4% in 1993 to 16.9% in 1990. The most significant primary reasons for culling on average over the five years were infertility (3.6%), surplus of stock (2.2%), mastitis (1.8%), and low production (1.2%). While there were differences in these figures between years, the relative importance of these reasons for cow disposal remained over the years. The disposal rates of cows by parity (lactation number) are shown in Table 2.2.

Parity of Dam	Year						
	1990	1991	1992	1993	1994	Average	Significance
Lactation 1	16.49	10.85	9.09	8.37	12.15	11.39	***
Lactation 2	11.64	11.09	9.41	7.41	10.76	10.06	*
Lactation 3	13.09	11.36	10.88	10.46	10.93	11.34	NS
Lactation 4	14.24	14.45	12.96	9.46	13.08	12.84	NS
Lactation >4	20.84	20.70	20.91	18.05	23.59	20.82	***
Significance	***	***	***	***	***		

Table 2.2 Cow Disposals: by parity (lactation number) of animal (%)

The disposal rate of cows was significantly influenced by the parity of the animal ($P<0.001$). The highest incidence of culling (20.8 per cent) within each parity category was recorded for cows of greater than 4 lactations. The culling rates in the other parity classes were quite similar. The reasons for culling by the parity groups are shown in Table 2.3 and it can be seen that infertility is cited as the overall most important main reason for disposal, though for first lactation animals this reason is ranked second. A surplus of cows is the second most important reason accounting for 14% of all cullings, though it assumes a somewhat higher importance in first and second lactation disposals. Mastitis, which accounts for 12% of cullings overall, becomes particularly important in later lactations. Low production, on the other hand, is given as a relatively important reason in early lactations and it is quite insignificant in later lactations. As would be expected, cow disposals due to old age in parities 1, 2 and 3 are very low.

Primary reason for culling	Parity of dam					
	1	2	3	4	> 4	All
Infertility	19.50	27.29	28.66	30.73	21.66	23.48
Surplus	24.82	16.86	13.61	13.02	11.46	14.29
Mastitis	6.94	5.17	9.69	13.75	14.77	12.10
Low production	10.17	12.87	13.40	5.00	6.03	7.94
Old age	0.21	0.97	0.52	11.46	9.20	5.45
Limb and foot disorders	1.26	1.95	2.58	3.23	3.31	2.80
Late calving	1.05	1.27	1.34	2.92	1.90	1.76
Abortion	1.75	3.22	1.86	1.56	0.85	1.40
Tuberculosis	1.61	1.75	2.37	1.56	0.91	1.30
Teat and udder injuries	1.40	0.78	0.82	1.04	1.17	1.12
Pining	0.70	0.49	0.52	0.83	0.50	0.56
Calving problems	1.40	0.49	0.52	0.00	0.41	0.53
Brucellosis	0.21	0.00	0.31	0.00	0.05	0.08
Other reasons	28.96	26.80	23.92	25.21	27.73	27.20
Total	100.0	100.0	100.0	100.0	100.0	100.0

Table 2.3 Cow disposal rate - reason for disposal by parity of cow

The remainder of cullings are attributable to a very large number of reasons each of which is cited as the reason for the culling in only a very small percentage of cases. Indeed it was necessary to group eighteen such reasons into one grouping "other reasons" in Table 2.3, and while each reason was of very minor importance, the grouping accounted for approximately one quarter of cullings.

The seasonal distribution of cow disposals for the years 1990-1994 is shown in Table 2.4. It can be seen that there have been year-to-year fluctuations in this pattern. While May accounts for the highest percentage of annual disposals averaged over the five years, it does not represent the peak in every year. In general, however, it can be seen that the period December to May accounts for a high proportion of cullings, typically 65%.

	Year					
Month	1990	1991	1992	1993	1994	Average
January	10.25	15.15	10.07	13.02	7.71	11.15
February	10.02	10.50	10.34	7.32	7.09	9.10
March	16.36	7.57	10.88	13.02	16.16	12.86
April	11.86	9.61	14.15	10.93	6.28	10.49
May	8.71	14.39	11.50	21.46	26.17	16.22
June	8.71	5.22	8.64	6.59	7.40	7.32
July	2.85	4.77	2.59	3.05	4.29	3.56
August	8.89	8.02	2.72	2.81	1.86	5.01
September	4.27	7.26	12.18	7.15	2.42	6.53
October	5.99	3.50	1.70	2.65	3.11	3.50
November	4.86	5.54	5.99	5.55	9.07	6.20
December	7.23	8.47	9.25	6.43	8.39	7.98
Total	100.0	100.0	100.0	100.0	100.0	100.0

Table 2.4 Cow disposals - by month of disposal (%)

During the other six months, June to November, disposals are typically low, with occasional peaks occurring in individual months.

Twenty-three farms that were recorded over a five-year period were selected (matched sample) to quantify the culling rate on individual farms. The data are shown in Table 2.5. It can be seen that the average disposal rate ranged from 10 per cent to 20 per cent. The difference in the disposal rates between farms was significant for four out of the five years ($P<0.001$). The difference in cow disposal rates for many of the individual farms across years was also significant ($P<0.05$). The culling rates fluctuated from year to year for many of the farms, as can be seen in Table 2.5.

Farm Code	Year						
	1990	1991	1992	1993	1994	Average	Significance
2	21.9	13.5	12.1	11.4	19.4	15.7	**
3	28.0	18.3	17.3	8.5	7.0	15.8	***
4	25.6	13.8	19.6	5.0	9.6	14.7	***
5	22.8	16.6	13.5	12.3	18.7	16.8	NS
6	27.7	12.3	17.2	0.9	10.9	13.8	***
7	19.1	13.6	8.4	7.5	10.6	11.8	**
9	18.6	13.3	13.1	9.7	14.7	13.9	NS
11	25.7	13.9	18.1	2.0	13.9	14.7	***
12	17.1	16.5	17.2	23.6	25.6	20.0	NS
13	17.1	16.5	12.5	15.6	10.2	14.4	NS
15	0.0	10.1	22.3	6.9	12.8	10.4	***
16	17.5	13.3	4.9	12.5	9.9	11.6	**
19	0.0	17.1	8.2	13.2	16.9	11.1	**
22	7.5	24.4	19.1	12.1	13.4	15.3	***
23	26.2	25.4	11.9	8.5	8.2	16.0	*
25	11.4	15.6	19.0	12.1	25.1	17.5	***
27	15.6	15.6	19.0	12.1	25.1	17.5	**
29	19.4	14.4	10.1	20.1	22.3	17.3	*
30	0.0	13.9	10.3	16.2	9.7	10.0	NS
32	13.8	11.0	19.4	18.6	16.1	15.8	*
33	44.2	10.9	6.1	9.7	12.0	16.6	***
34	18.8	16.3	16.7	16.8	9.5	15.6	NS
35	10.0	20.0	14.3	3.0	15.8	12.6	***
Significance	***	NS	***	***	***		

Table 2.5 Cow disposals by farm

2.1.4 Discussion of Results

The disposal rates of cows in the National Farm Survey database for the years 1990-93 are shown in Table 2.6, with categories for dairy herd size.

Herd Size	Year				
	1990	1991	1992	1993	Average
3 - 30 cows	12.31	14.42	19.58	18.60	16.23
31 - 60 cows	13.78	12.83	15.21	15.55	14.34
60+ cows	12.95	16.63	17.63	15.05	15.57
Total	12.91	14.36	17.92	17.01	15.55

Table 2.6 Cow disposal rate by herd size category (NFS data-base)

The average disposal rate was 15.6%, ranging from 12.9% in 1990 to 17.9% in 1992. The range in cow disposal rates for different herd size categories was also relatively small (14.3% - 16.2%).

The dairy herds participating in the DairyMIS system are representative of the larger intensive dairy farmers in Ireland. The larger herds are however very significant in terms of total milk supply. Fingleton(1995) estimated the number of herds of 3-30 cows, 30-60 cows, and greater than 60 cows to be 26,500, 12,500 and 4,200, respectively. The data from the NFS was included to see if the culling rate in the National dairy herd was similar to that found in DairyMIS herds.

The average culling rate from the DairyMIS data set was 15.2 per cent compared to an average culling rate of 15.6 per cent from the NFS database. The cow disposal rates for farms in the different herd size categories was relatively similar. This culling rate is lower than that found in a similar previous study which was carried out in the early 1980's (Crosse and O'Donovan(1989)), when the average culling rate of 17.6 per cent was recorded. The culling rate found in this study is considerably lower than the culling rate of 21.6 per cent reported by Walsh(1983) for experimental herds in Ireland. The influence of disease eradication programs, and in particular the Brucellosis Eradication Program was very significant in the study by Walsh.

The overall average culling rate found in this study also appears low when compared with rates reported in similar studies in the UK. The Milk Marketing Board reported an average rate of 17.7 % (Anon(1972)) and a similar rate of 17.4 was reported by Beynon and Howe(1974). Gartner(1983), on the other hand, reported an average culling rate of 21.0 %.

The major advantages of culling strategies for herd improvement are achieved by maximizing the proportion of cows that are culled for voluntary as distinct from involuntary reasons (Walsh(1983)). Generally, culling for low production is considered to be voluntary culling. Since the advent of milk quotas, culling of surplus cows has assumed much more importance, and it is likely that dairy farmers would cull their poorer animals if they had surplus stock. Culling of surplus stock together with culling for low production is therefore considered to be voluntary culling. In this study, it was found that about three quarters of all cullings were involuntary and only one quarter were culled for voluntary reasons. These results are very similar to those reported in the previous study by Crosse and O'Donovan(1989). Infertility was the most important single reason given for involuntary culling and while culling for infertility was particularly associated with older cows, a significant proportion of younger animals were also culled for this reason. Mastitis was the next single most important reason given for involuntary culling and this was also associated more with older cows. These results are consistent with those reported elsewhere; several surveys have highlighted infertility and mastitis as the predominant reasons for involuntary culling in dairy herds (Crosse and O'Donovan(1989); Gartner(1983); Anon(1971-72); Walsh(1983) and Beynon and Howe(1974)).

The highest incidence of culling (20.8 per cent) was recorded for animals with greater than four lactations. This was associated with infertility, mastitis, old age, low production as well as a collection of other reasons. There was little difference in the percentage of cows culled within the other age categories. Other studies have shown that, in general, culling rates increase with age (Crosse and O'Donovan(1989); O'Connor and Hodges(1963); Gartner(1963); Beynon and Howe(1974)). The relative ranking of animals in different parities in terms of disposal is important. The potential genetic gain from a selection policy is maximized by the disposal of older cows of inferior genetic quality.

The disposal of dairy cows was well distributed throughout the year, with higher than average being recorded in the period December to June. While most of the herds recorded were Spring-calving, some of the distribution of cullings throughout the year may be due to the presence of some Autumn-calving herds in the survey. It is likely that many dairy cows culled for involuntary reasons are sold in poor body condition. Walsh(1985) demonstrated that both carcass weight and carcass grades of cull cows can be improved considerably by allowing a fattening period prior to disposal. The introduction of milk quotas has meant that land availability is no longer a constraint on many dairy farms. It is therefore possible to keep cull cows for longer periods on farms and thus improve their value prior to sale (though some animals will have to be salvaged because of serious illness); this could significantly improve farm income.

The large farm-to-farm variation in culling rates recorded in this study as in the previous study by Crosse and O'Donovan(1989) would indicate that management is important. A reduction of involuntary culling should allow for more culling for low production and other reasons, which may enhance the genetic and general health of the dairy herd.

2.2 A Model for Culling and Breeding Goals in Irish Dairy Herds

2.2.1 Development of a Simulation Model

The breeding goal on many commercial dairy farms is to maximise profitability, and in this regard, the genetic quality of the dairy herd is an important determinant of farm profitability. There has been a marked increase in the rate of genetic improvement in dairy herds in Ireland on recent years. These developments present new opportunities for increased profit on dairy farms as well as major challenges to milk production systems that are mainly based on grass.

The simulation techniques known as Monte Carlo methods were used in this study; these techniques are outlined in Appendix A. A model using Monte Carlo methods was developed to simulate the physical and financial consequences of alternate breeding goals and alternative levels of voluntary culling for a hypothetical Irish spring-calving herd. The characteristics for this herd are based on the Moorepark Blueprint for summer milk production. This model farm represents a typical intensive dairy farming system in Ireland where the EU milk quota is the most limiting constraint on production and where there has to be a combination of enterprises to use the land available. This allowed the opportunity cost of capital and land to be evaluated

2.2.2 State Variables Included in the Model.

A cow's Relative Breeding Index (RBI95) describes, in a single figure, a cow's genetic merit for the production of fat and protein (Diskin(1995)). RBI95 values are calculated from PD95 (Predicted Difference) values for kg of fat, kg of protein and protein % and yield using the following formula:

RBI95=100+0.36(PD Fat Kg)+1.64(PD Prt Kg) +74(PD Prt%)-0.014(PD milk yield)

All available pedigree information is used to identify links between relatives so that they can contribute to the proof of that animal. Information on ancestors can be important in the case of animals with few performance records. As information from the animals' own performance increases, the importance given to ancestry information declines.

A cow's first 5 lactations are eligible for inclusion in the analysis. A reliability figure, which is a measure of the reliability of milk ratings, is also published along with a cow's Breeding Index. The reliability varies according to the amount of information coming from an animal's own lactation records and from all its identified relatives. Reliability will generally be only 35-40% for cows with 1-2 lactations and up to 65% for cows with 5 lactations.

RBI95 was included as a state variable for each animal in the herd, as were lactation number and production level. Production level was described by one of 15 levels ($\leq$74%, 74 to 78%,...... , 122 to 126% and $\geq$126%), and defined relative to the mature equivalent production. In this model, the lactation of an animal was measured as 1^{st}, 2^{nd} and 3+.

2.2.3 Other Model Parameters

Current input and output prices were used in the analysis, and are presented in Table 2.7. An involuntary culling rate of 18% was assumed. It was further assumed that 50% of calves born were male, and that an overall calf mortality rate of 6% prevailed. The RBI of a calf was calculated as the average of the sire and dam's RBI95. At the end of each year, new heifers entered the herd until the quota could be expected to be met in the following year (although production levels for the following year were unknown, expected yields on the basis of current production levels could be calculated). Replacement heifers are 2 year-olds, which were bred and reared within the herd. The RBI95 of these heifers was assumed to be equal to the average RBI95 of female calves surviving from 2 years previously.

Prices (£ IR)	
Land value /Ha /year	150
Feed costs /animal /year	60
Milk Price /litre	0.22
Calf value (female)	130
Calve value (male)	80
Replacement cost	840
Carcass value	350
Interest rate (%)	8

Table 2.7 Current input and output prices used for the simulation model.

The RBI95 of an animal in the herd had an effect on physical performance and on financial factors. The following were dependent on the RBI95 of animals in the herd:

- milk yield
- feed costs
- calf value
- carcass value
- land usage

2.2.4 Culling and Breeding Goals Tested by the Model.

The physical and financial performance factors were simulated for two breeding goals: (1) where sires with an average RBI95 of 130 were used; (2) where the average RBI of the sires used was 150. For each of these breeding goals, the RBI95 of the average sire was allowed to increase (by 2 units per time stage) over time to reflect the improvements in sires used over time (i.e. RBI Sire(t) = RBI Sire(0) + (2*t)).

Three contrasting levels of voluntary culling 0, 10 and 20 % were also evaluated. At the end of each stage of the simulation, a number of animals (based on the current herd size and on the voluntary culling rate) were culled according to the lowest lactation adjusted milk yield.

At each stage of the simulation, the total yield, total revenue, land in use, and average RBI of the herd were calculated. The farm parameters used are presented in Table 2.8.

Calf mortality (%)	6
Involuntary culling (%)	18
Quota (Litres)	520,000
Capital available	100,000
Land available	87
Housing space available (cows)	110

Table 2.8 Other parameters used for the model farm.

2.2.5 Stochastic Elements Included in the Model.

A planning horizon of 20 years (20 stages) was used. At each stage, each animal in the herd could be culled involuntarily with a probability of 18%. Survival was determined stochastically, as were the transitions from one production level to another, where the transition probabilities were calculated as described by Van Arendonk (1985b). Transitions in production levels were assumed to take place at the end of lactation (stage).

For each calf in the herd, sex and survival were also determined stochastically. Transitions in 'lactation number' could be determined deterministically since only one transition was possible for an animal at any stage (i.e. lact(n+1) = lact(n) +1). An animal's RBI95 remained constant for its lifetime, so again the transitions for this state variable were deterministic.

2.2.6 Results of Physical and Financial Factors.

As the simulation model was run, certain physical and financial factors were measured at each stage. These were:

- Average RBI95 of the herd.
- Total milk yield of the herd.
- Average milk yield per cow.
- Current herd size.
- Land in use for dairy enterprise.
- Capital investment.
- Total revenues.

Breeding the herd to high index sires was found to have large effect on the average RBI of the herd (Figure 2.1).

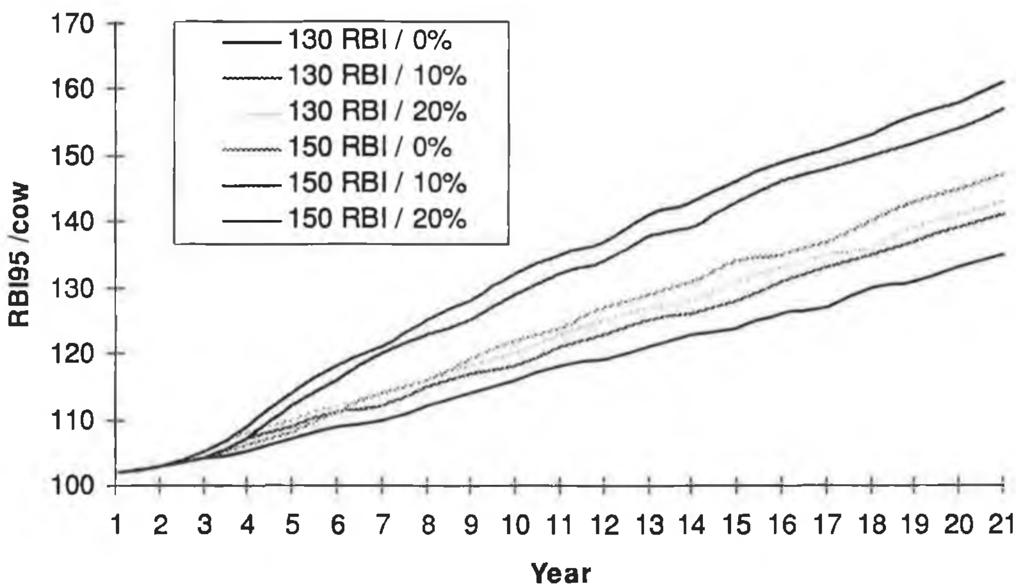

Figure 2.1 The average RBI95 of the herd under differing breeding goals and voluntary culling rates.

For the latter half of the planning horizon, a higher breeding goal resulted in higher herd RBI95's, no matter what culling rate was chosen. Where the lower breeding goal was combined with high culling rates (20%), the average RBI95 of the herd rose quickly, but within 6-7 years, the average RBI95 where the higher breeding goal was used (regardless of the culling rate) was always higher. When there was no voluntary culling, increases of 18 and 35 units of RBI95 for years 10 and 20 respectively were recorded for the lower breeding goal (Sires with RBI95 130). This compares with an increase of 24 and 47 units of RBI95 respectively, for the higher breeding goal (Sires with RBI95 of 150). The increases in RBI95 associated with higher culling rates are due to the faster introduction of genetically superior animals to the herd. The increases due to culling policy are greater for the higher breeding goal because, with the lower breeding goal, the genetic level of the herd soon approached that of the replacement heifers.

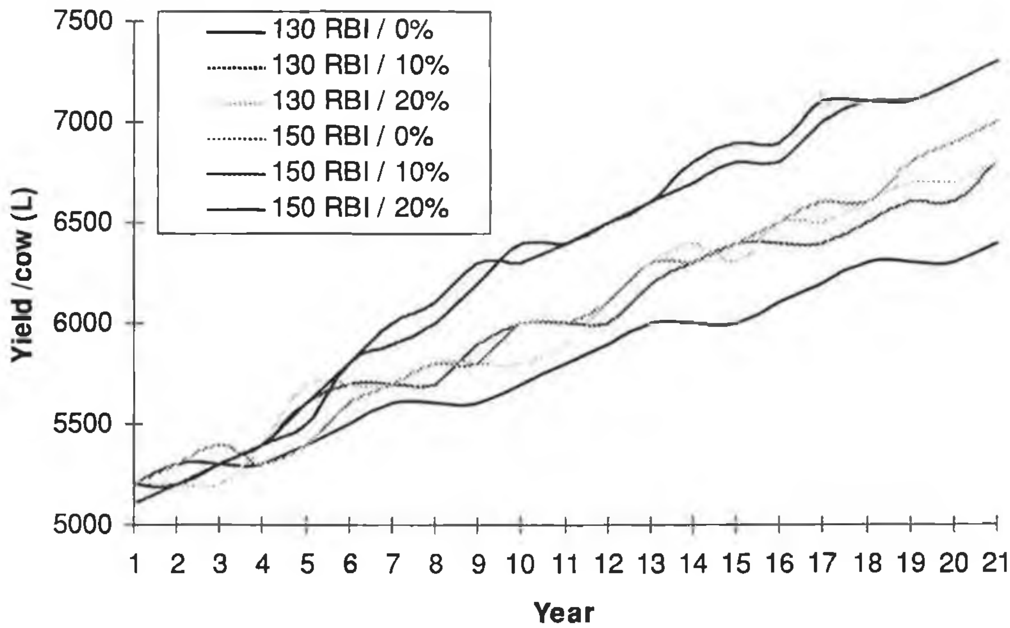

Figure 2.2 The average milk yield of cows in model herd under different culling gaols and voluntary culling rates.

In the model, a herd member's milk yield was adjusted for RBI95 level. So, for a certain age and production level, an animal with higher RBI95 would have a higher yield than one with a lower RBI95 level. Therefore, it is not unexpected that the use of the higher breeding goal (which results in higher average RBI95 per animal (Figure 2.1)) results in a higher milk yield per animal as shown in Figure 2.2. Because of the inclusion of a milk quota (Table 2.8), higher milk yields per cow (associated with higher breeding goals) meant that herd sizes had to be reduced, as illustrated in Figure 2.3. When a voluntary culling rate of 10% was applied, herd sizes of 85 and 76 were recorded for years 10 and 20 respectively, when the lower breeding goal was applied. While for the higher breeding goal, the respective figures were 79 and 70. So the application of the higher breeding goal rather than the lower, results in a 11% reduction in herd size after 10 years and a 30% reduction in herd size after 20 years.

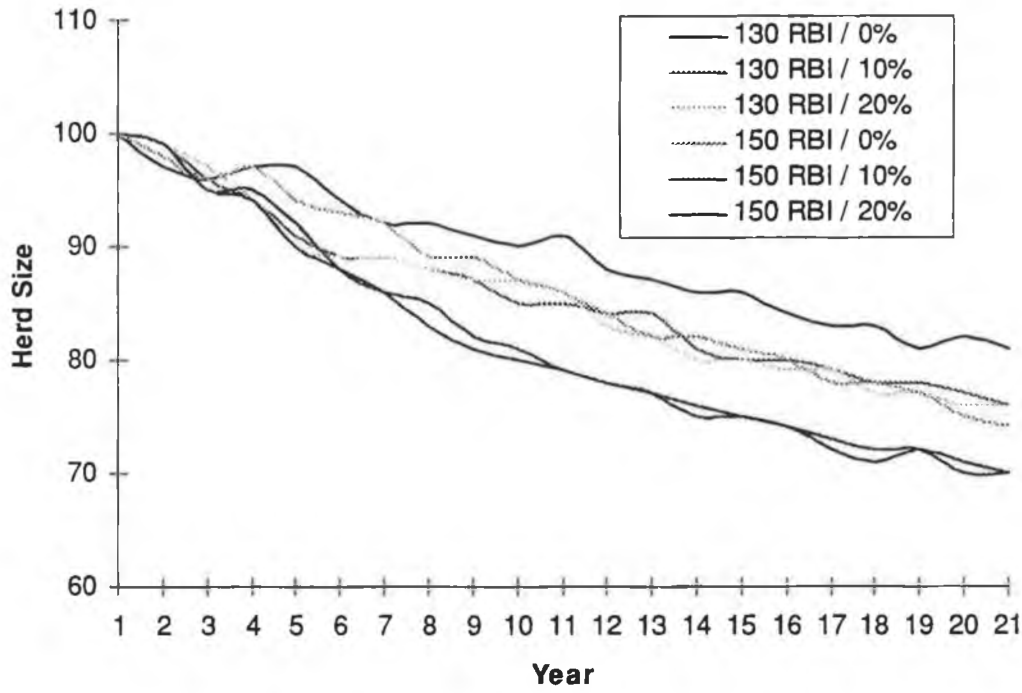

Figure 2.3 The size of the herd under the different breeding and culling strategies.

The land used for the dairy enterprise was based on the genetic makeup of the herd. For each animal in the herd, land usage was calculated as follows:

Land Usage = 0.85+((RBI-100)*0.003) hectares

For the higher breeding goal, the average RBI was greatest (Figure 2.1). However, the fewer animals required in the herd to reach quota (Figure 2.3) had the effect of reducing the land area required for the dairy enterprise (Figure 2.4).

A measure of farm profit was taken at each stage under the various breeding and culling strategies. These measures are shown in Figure 2.5. The calculation of these figures was based on milk revenue, calf sales, feed costs, replacement costs, land usage and capital expenditure.

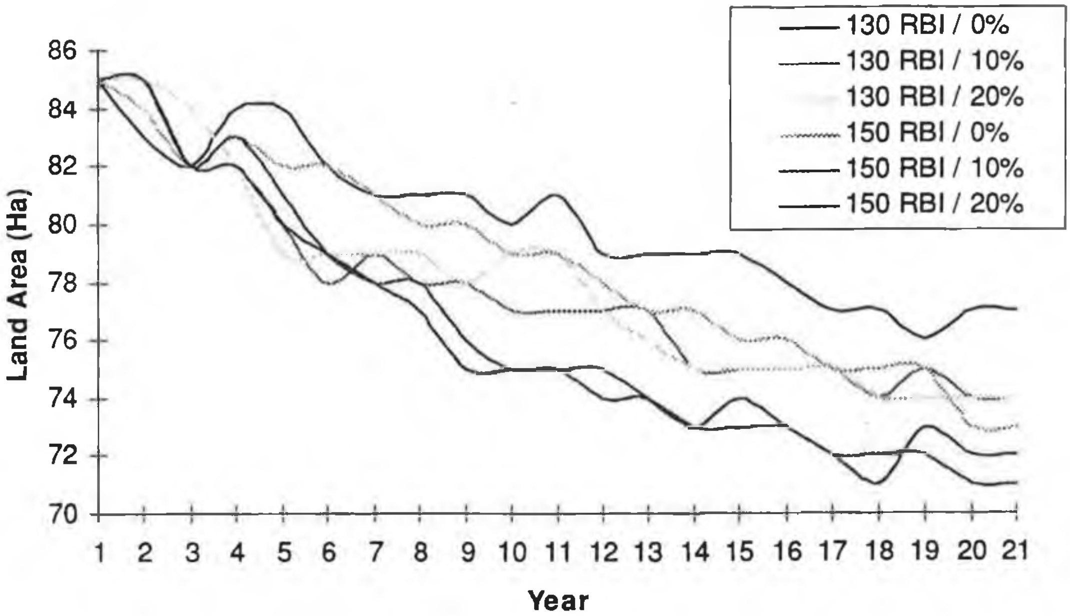

Figure 2.4 Land area required for the dairy enterprise under differing breeding goals and voluntary culling rates.

A culling policy with no voluntary culling resulted in higher profits than either a 10% or 20% voluntary culling rate (and the 10% rate resulted in higher profits than the 20% rate). The use of higher merit sires (150 RBI95) resulted in higher profit rates over the 20 year planning horizon.

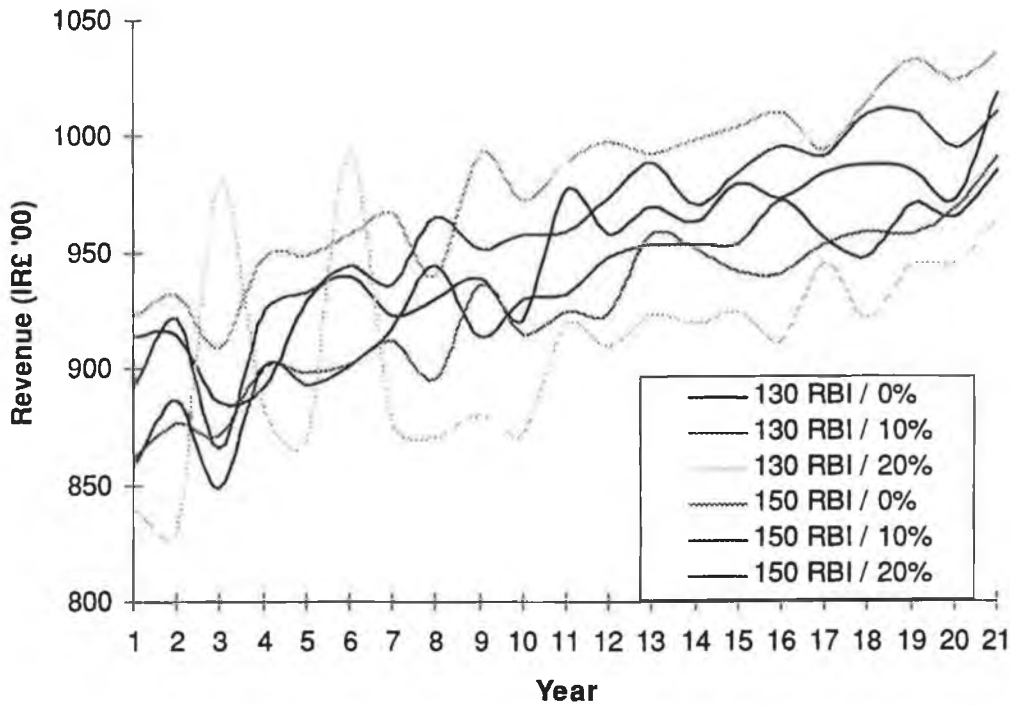

Figure 2.5 Revenue for model farm under different breeding and culling strategies.

2.2.7 Conclusions from the Simulation Model

The results of the simulation suggest that the use of high merit sires had a large effect on the genetic merit of the herd and consequently on milk yield per cow. This resulted in increased farm profit even though cow numbers had to be reduced to stay within the quota. Increasing the rate of voluntary culling also increased the genetic merit of the herd, especially in the early years. This had positive effect on milk yield per cow as well as on other farm parameters. However, the cost of achieving this higher milk yield per cow was outweighed by the corresponding higher cost of providing replacement animals. Therefore, culling on the basis of production alone resulted in a reduction in total farm profit.

Chapter 3
Traditional Dynamic Programming Techniques

3.1 Replacement Problems

3.1.1 Representing Operational Systems

In the analysis of operational systems, it is often possible and convenient to consider the system as having a finite number of possible states (properties). The system may then consist of a sequence of transitions between these states over time. For example, if the problem were that of machine reliability, the state of the system may be described by the age and state of repair of the machine. Similarly, in the case where the operational system being described is a dairy cow, the system may be described by the age and production level of the animal.

3.1.2 Replacement Problems

The determination of an optimal replacement strategy is an example of a sequential decision problem. If an asset is used in a production process, it is relevant to consider at regular time intervals (stages) whether the present asset (described by its state) should be replaced or kept for an additional period. The system can therefore be influenced by a decision maker, who at each stage, uses one of a feasible set of actions (e.g. Keep or Replace). The replacement problem is to determine the set of decisions (one for each state at every stage) that will optimise some objective function. This set of decisions is referred to as the optimal policy.

3.1.3 Dynamic Programming

If the traits of the asset are well defined and their precise behaviour over time is known in advance, there are deterministic methods that might be applied to determine analytically the optimal replacement time. If however, the traits of the asset in question are affected by random variation over time and among assets (as is the case where the asset is a dairy cow), the replacement decision will depend on the present observation of the traits. In such cases, a technique known as dynamic programming is a relevant tool in the determination of an optimal replacement policy.

In the late 1950's Bellman(1957) published a book entitled 'Dynamic programming'. Bellman was the first to appreciate the wide range of applicability of a computational procedure, which is referred to here and in most literature as the *value iteration method.* The value iteration method was first applied to the dairy replacement problem as early as 1963, when a model consisting of one state variable, lactation, described by 12 levels, was introduced by Jenkins and Halter(1963).

3.2 Dynamic Programming Techniques under a Finite Planning Horizon

3.2.1 Criteria of Optimality

Before considering dynamic programming as an optimisation technique for a sequential replacement problem, we must first consider our objective function (what it is that we wish to maximise/minimise). The following are two traditional objective functions under a finite planning horizon.

A finite planning horizon is applied in the situation where a farmer knows that farm production will cease after *N* stages, where *N* is fixed and finite. A stage may be of any length, such as a year or a month. A relevant criterion under a finite planning horizon may be to find the policy (a set of decisions at each (stage, state)) that maximises the total expected rewards over the planning horizon (i.e. over the *N* stages). Under this criterion the objective function would be:

$$h(s^1,\ldots,s^N) = E\left(\sum_{n=1}^{N} r_{i(n)}^{s^n}\right) \qquad \textit{(Equation 3.1)}$$

where s^n is the policy at stage *n*, and $r_{i(n)}$ is the reward for the (unknown) state *i* at stage *n*.

If the decision maker has a time preference, such that he/she prefers an immediate reward to an identical reward at a later stage, then the maximisation of total *discounted* rewards would be a more relevant criterion. The objective function would then be :

$$h(s^1,\ldots,s^N) = E\left(\sum_{n=1}^{N} \beta^{n-1} r_{i(n)}^{s^n}\right) \quad , 0 < \beta < 1 \qquad \textit{(Equation 3.2)}$$

where β is the discount rate defined by the interest rate and the stage length.

3.2.2 The Value Iteration Method

The value iteration method is often referred to in literature as *dynamic programming*, *successive iteration* or *successive approximation*. The value iteration method has its basis in what Bellman(1957) described as the principle of optimality.

> 'An optimal policy has the property that what ever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision'

This statement implies that if the optimal policy from stage n onwards $(\xi(n) = (\sigma(n), \sigma(n+1), \ldots, \sigma(N)))$ is known, then the optimal policy from stage *n-1* onwards$(\xi(n-1))$ will include$\xi(n)$. Thus, as we proceed, more and more non-optimal sub-policies can be rejected and disregarded for the remainder of the calculation.

The value iteration method uses functional equations of the following type , which embody the principle of optimality, to sequentially determine optimal policies:

$$f_i(n) = \max{}^d \{r_i^d + b\sum_{j=1}^{u} p_{ij}^d f_j(n+1)\}$$

$$i = 1,2,\ldots,u \qquad , n = N, N-1, \ldots, 1.$$

(***Equation 3.3***)

where
- r_i^d is the immediate reward when a decision d is made for state i.
- p_{ij}^d is probability of transition from state i to state j when the decision d is taken.
- The discount factor b is included if the decision maker has a time preference so that he prefers an immediate reward to an identical reward at a later stage. If discounting is not to be performed, then b=1.

- The function $f_i(n)$ is the total expected rewards from the process when it starts from state i and will operate for n stages before termination. Thus $f_i(N)$ is the salvage value of the system when it is in state i.

The action d maximising the right hand side of Equation 3.3 is optimal for state i. Equation 3.3 illustrates backward recurrence : starting at stage $n=N$ and working backward to the first stage ($n = N, \ldots, 1$) an optimal policy is chosen using the Equation 3.3.

2.2.3 The Advantages of Dynamic Programming

The advantage of the dynamic programming method over a method of successive enumeration is in its use of the criteria of optimality. Working backward from the final stage N, once the optimal policy has been found at a particular stage, for a particular state, no other decision need be considered at that stage, for that state.

Consider a very small model of two states, as shown in Figures 3.1 and 3.2. At each stage in the model, transitions between the two states are possible, depending on the decision made. The use of the backward recurrence relation, Equation 3.3, is shown in Figure 3.1. At the first step of the value iteration algorithm (N-1), two comparisons are required. Again, at the second step (N-2), two comparisons are required, since the optimal sub-policy from N-1 onwards is already known, and, as the principle of optimality implies, must be on the optimal path.

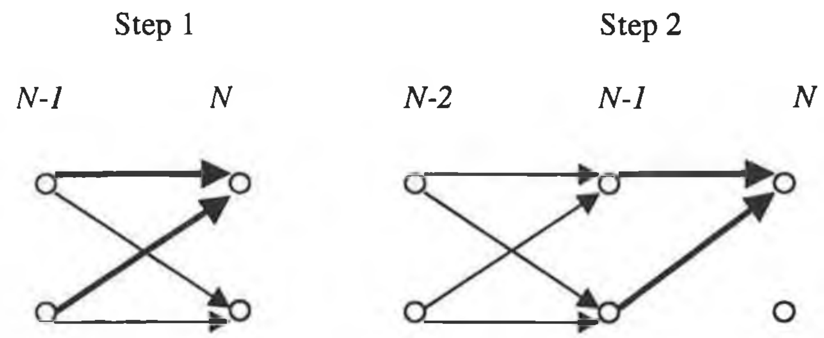

Figure 3.1 Illustration of the principle of optimality.

Using a method of complete enumeration, as shown in Figure 3.2, where the number of stages in the model is 2, results in eight comparisons. These eight comparisons, correspond to the eight possible complete paths through the model.

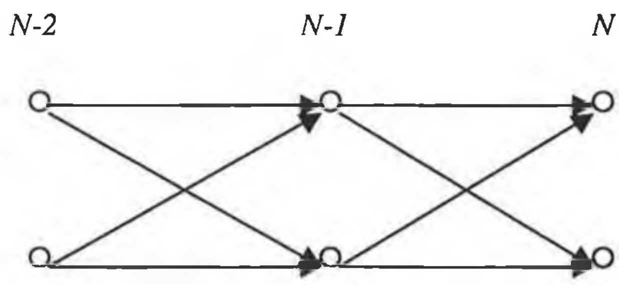

Figure 3.2 Illustration of Complete Enumeration.

In general, with a model of N stages, where d decisions are possible, the value iteration method requires $N \times d$ comparisons. A method of complete enumeration would require d^N comparisons. Thus, even for quite small models, a method of complete enumeration could be intractable.

3.2.4 The Value Iteration Method over an Infinite Planning Horizon

The value iteration method described uses a finite planning horizon. In other words, the number of stages in the model is known and finite. In such cases, some salvage value (the value of an asset in state i) can be placed on the system at stage N.

Of course, often the decision maker may not know in advance when production will cease. In such cases, we have an infinite number of stages (i.e. $N=\infty$). The value iteration method requires a finite planning horizon and cannot solve the problem exactly over an infinite planning horizon. However, it can be used to approximate the optimal policy over an infinite planning horizon. It can be shown (Howard(1960)) that

$$\lim_{n \to \infty} f_i(n) = f_i \ , \quad i = 1,2,\ldots,u \qquad \textbf{(Equation 3.4)}$$

where f_i for fixed i is constant. So by using Equation 3.3 over a large number of stages, we will eventually observe that $f_i(n+1)$ is equal to $f_i(n)$ for all i, and that the same policy is repeated over a number of stages. While we cannot be sure that this is in fact the optimal policy over an infinite planning horizon, we can assume that it is close to this optimal policy.

3.2.5 Application to Animal Replacement

McArthur(1973) designed a stochastic dynamic programming model for a New Zealand herd-tested factory supply herd, breeding A.B. Jersey replacements. He then evaluated the culling decisions derived from this model by comparing the resulting profits with those of a herd culled on genetic value alone. For this model the state variables were age and production level. McArthur(1973) concluded that optimal culling rules derived from his model did not appreciably increase the gains from culling.

Stewart et al(1977) developed a dynamic programming model in which Holstein cows in a Canadian dairy herd were described by lactation number, body weight, average milk fat % and milk yield. To minimise the number of state variables needed in the model, cows were described by estimated 305-day milk yield; this resulted in a state space of size 2695. A 10-year planning horizon was used and milk returns, beef sales, feed costs and cow depreciation costs were all included in the calculation of rewards at each stage.

When a cow was removed at any stage, it was assumed that she would be replaced by an 'average' heifer in her first lactation. To determine the milk production and body weight production of an 'average' heifer, first lactation records of 5,049 Holstein cows were studied. Linear regression estimates of the probability of involuntary removal, death and survival to the next stage (i.e. next lactation) were derived. A study of 41,896 lactation records was carried out to predict body weight transitions and it was concluded that no change was large enough to cause transitions between body weight categories, with the result that in the model a cow was assumed to remain in the same body weight category for its lifetime. The study looked at the sensitivity of this dynamic programming model to changes in prices of relevant factors. It was found that optimal policies for 2,557 of the 2,695 states did not change regardless of the milk price, feed price, or interest rate.

Killen and Kearney(1978) developed a dynamic programming model for Irish spring calvers. A value iteration model was used with lactation as the only state variable. The model developed found the optimal culling policy in terms of expected future returns from milk and butterfat production. The model was used to compare retrospectively the actual culling rate nationally with the optimum culling rate over the 20 year period 1957-1976. The model found that the optimal culling rate under the market conditions for those years was between 17% and 20%, which was slightly higher than the actual culling rate.

The model developed by Harris(1988) described cows by lactation number, future milkfat production and calving date, resulting in 2600 states; this model was then used to rank cows based on future profitability. Incorporation of calving date into the model allowed the variation in individual cow lactation length and the likelihood of induced calving to be accounted for in the coming and future seasons. Future milkfat production was defined in such a way that it remained constant for the lifetime of a cow. The model calculated, using the value iteration method, an annualised present value for all states at the present stage and used these values to rank cows in a herd. The study, which was carried out in New Zealand, was applied to a herd of 168 Jersey cows. Sensitivity analysis, using Spearman Rank correlation, was carried out on the parameters of the model. The author concluded by noting the advantages of the dynamic model over the use of production indices, those advantages being that it incorporates economic variables and looks further into the future.

In Harris(1990) this model was developed further to include milk production, milk protein production and breed as state variables. In this study, as with Van Arendonk(1985), milk production state variables were expressed as percentages of the mature equivalent. The optimal policies were found both for replacement rates and the insemination dates for cows. This was applied to 10 dairy herds and gave an increase of NZ$1.80, per 1% culled within herd in the net present value per cow from culling, which outperformed the culling on the payment production index by NZ$1.10.

Van Arendonk(1985b) in his model described cows by lactation number, stage of lactation and the level of milk production during the previous and present lactations, which resulted in a state space of size 29880. A 20-year planning horizon was used and a large number of factors, such as calf revenues, carcass value, feed costs, cost of replacement heifers and the probability of and financial loss associated with involuntary replacement, were considered.

It was assumed that a cow would be automatically replaced if it reached its 12th lactation. There were 15 alternatives for production level, which were defined relative to the mature equivalent production in the absence of genetic improvement and voluntary replacement. The production level of a cow remained constant during the lactation period and transition to other production levels took place only at the end of the lactation period. Transition to different production levels depended on production in the present and previous lactation for cows of second lactation or higher. For heifers, these probabilities depended only on production in the present lactation.

The probabilities of realisation were used to calculate the marginal probability of voluntary and involuntary replacement during each lactation, and the total proportion of cows which were disposed of voluntarily. In addition, the average herd life of the cows was derived. The average herd life in the optimum situation was 42.9 months, which corresponded to an annual replacement rate of 28%. In total 26% of the replacements were voluntary, resulting in a voluntary replacement rate of 7.3%.

Changes in the price of a replacement heifer or the carcass price for culled cows were found by Van Arendonk(1985b) to have a considerable effect on the optimum replacement policy. A reduction in the difference between the carcass value of culled cows and the replacement costs resulted in a higher rate of voluntary replacement. Changes in the price of milk, calves or feed, the production level of the herd or the rate of genetic improvement did not greatly affect the optimum policy.

The model used in the previous study was extended in Van Arendonk(1985c) to allow for variation in time of conception. Up to two months after calving, voluntary replacement or insemination was not considered. Three alternatives were considered from 2 to 7 months after calving for open cows namely a) inseminating the cow with a given probability of success, b) leaving her open, and c) replacing her immediately. For the remaining months of the current lactation of open and pregnant cows, the alternatives to keep or replace immediately were considered. In the study, a 15-year planning horizon was used to save computation time.

The marginal probability of conception was calculated from the probability of first and/or later inseminations occurring and the probability that conception took place after insemination. The effect of the length of the previous calving interval on the net revenues during the current lactation was accounted for during the transition to the present lactation. It was found that, as expected, insemination was generally continued longer for high producing cows than for low producers. The optimum replacement policy was greatly affected by the size of the difference between the replacement heifer price and the carcass value of culled cows, while price changes for milk, feed or calves had a negligible effect.

Van Arendonk (1986) then extended the model further to take into account seasonal variation in biological factors and prices; a new variable, month of calving was therefore added. The month of calving influenced the production of milk, fat and protein, the probability of conception, feed costs and the prices of milk, calves and culled cows. Due to the large number of possible states of a cow, it was necessary to omit the previous lactation's production level as a variable in the model. Seasonal differences in milk production, feed costs and calf price contributed considerably to the differences in expected income from heifers freshening at different months. After exclusion of the seasonal variation in the production of milk, fat and protein, only minor differences in income remained.

The model of DeLorenzo(1992) had 151,200 states, the variables being lactation, production, month of calving, month of lactation and days open. The model used was an adaptation of the model used by Van Arendonk(1986) and was solved over a 20-year planning horizon. This model considered three decisions at each stage : keep, keep and inseminate or replace. To test optimal strategies computed from the model, a simulation was written that could implement the optimal strategies or use alternative insemination and culling policies. Expected monthly net revenues per cow from the model agreed closely with the monthly net revenues from simulation. Some slightly higher net revenues resulted from the simulation, but this would be expected. As a stochastic problem, dynamic programming seeks the optimised expected, or long run average, net return.

McCullough and DeLorenzo(1996) devised an approach to evaluating the results from this model using sensitivity and behavioural analysis. The state vector described by DeLorenzo(1992) was revised to include three additional days open classes. Sensitivity analysis was defined as the quantification of the various outputs resulting from uncertain price and production inputs. Behavioural analysis determined how outputs changed when model specifications changed. Twelve outputs were considered in these analyses, including, percentage of states with insemination decisions, percentage of states with replacement decisions, voluntary culling rates and involuntary culling rates. The model specifications that were varied were decision horizon, number of milk production classes, and number of days open classes.

After the consideration of the changes in the 12 outputs compared with the large savings in computation time, a 5 year decision horizon was determined to be justifiable. This is an important result as many other models use planning horizons of 15-20 years. In models which have an infinite planning horizon (policy iteration, Hierarchic Markov Processes), the planning horizon need not be considered.

It was found that for the model defined by DeLorenzo et al(1992), the number of production levels could be reduced from 15 to 5 without changing results significantly, but the number of days open should not be reduced from 10 and the variable for month of calving should be retained.

3.3 Infinite Planning Horizons

3.3.1 The Policy Iteration Method

The value iteration method is exact over a finite planning horizon and can be used to approximate an infinite planning horizon (Equation 3.4). Howard(1960) introduced the policy iteration method, which could solve sequential problems exactly over an infinite planning horizon. In infinite stage problems we assume that the system is to be operated under the same policy at every stage. Our aim is to find a policy, which, if repeated indefinitely, will have better limiting properties than other policies. To do this, the policy iteration algorithm generates a sequence of stationary policies, each with improved utility over the preceding one.

Since the collection of all stationary policies is finite and an improved policy is generated at every iteration, it follows that the algorithm will find an optimal stationary policy in a finite number of iterations. Howard(1960) proved this under the assumption that the discount factor was fixed. However, this proof can be extended so that the discount factor is dependent on the policy and state of the system (Kristensen(1988)).

3.3.2 Optimisation using the Policy Iteration Method

If an infinite planning horizon is assumed, the vector of present values $g^s = (g_1^s, g_2^s, \ldots g_u^s)'$ under the policy s is calculated as :

$$g^s = (I - B^s P^s)^{-1} r^s$$

Where I is the $u*u$ identity matrix, P^s is the matrix of transition probabilities under policy s and B^s is the diagonal matrix whose non-zero elements are the discount factors $\beta_1^s, \beta_2^s, \ldots, \beta_u^s$. An optimal policy can then be defined as that policy that maximises the elements of the vector g^s (i.e. a policy s' is optimal iff it satisfies the condition $g^{s'} = max\{g^s\}$). The policy iteration algorithm can now be stated as follows :

Step 1 : (Initialisation) Choose an arbitrary policy s^0. Go to Step 2.

Step 2 : (Policy Evaluation) Given the stationary policy s^k, compute the corresponding cost function g^{s^k} from the linear system of equations :

$$g^{s^k} = (I - B^s P^s)^{-1} r^s$$

Go to Step 3.

Step 3 : (Policy Improvement) For each state i, determine the action $d^{k+1} \in D$ that maximises

$$r_i^d + \beta_i^d \sum_{j=1}^{u} p_{ij}^d g_j^{s^k}$$

And put $s^{k+1}(i)=d^{k+1}$. If $s^{k+1}=s^k$ then stop since an optimal policy has now been found. Otherwise $k=k+1$ and go to Step 2.

3.3.3 Criteria of Optimality

Referring to the traditional criteria of optimality discussed in Chapter 3.2.1, the objective function (Equation 3.1) (the maximisation of total expected rewards) cannot be applied under an infinite planning horizon, as the function will not converge where $N = \infty$.

Since, by definition, the discount factor $\beta<1$, the function (Equation 3.2) (the maximisation of total expected discounted rewards) will converge towards a fixed value for a large value of N. Thus, under an infinite planning horizon, the objective function for the maximisation of total expected discounted rewards is defined and given by :

$$h(s^1, \ldots, s^N) = E(\sum_{n=1}^{N} \beta^{n-1} r_{i(n)}^{s^n}) \qquad \textbf{\textit{(Equation 3.5)}}$$

As with a finite planning horizon, each animal and its successors are represented by a separate Markov decision process. Therefore criterion 3.5 is equal to the maximisation of total discounted net revenues *per animal.*

Another relevant criterion of optimality under an infinite planning horizon is the maximisation of expected average reward per unit of time. If all stages in the system are of equal length, this equates to the maximisation of expected average rewards per stage. This criterion along with (3.5) was developed by Howard(1960). The objective function can be stated as:

$$h(s) = g^{s} = \sum_{i=1}^{u} \pi_i^s r_i^s \qquad \textbf{\textit{(Equation 3.6)}}$$

Where π_i^s is the limiting state probability under the policy s (i.e. when the policy is kept constant over an infinite number of stages)[1].

The maximisation of total expected discounted rewards, and the maximisation of expected average rewards per stage over an infinite planning horizon were developed by Howard(1960). Later Jewel(1963) presented a criterion for maximisation of average rewards over time (per stage), where the stage length could vary according to the state i and the action d. The objective function for this criteria is given by :

$$h(s) = g^{s} = g_r^s \Big/ g_t^s = \sum_{i=1}^{u} \pi_i^s r_i^s \Big/ \sum_{i=1}^{u} \pi_i^s t_i^s \qquad \textbf{\textit{(Equation 3.7)}}$$

where t_i^s is the expected stage length at state i under decision d.

The appropriate equations for this objective function, as well as (3.5) and (3.6), under the policy iteration method, are presented in Table 3.1.

---

[1] In practice, the optimal policies determined by criteria (3.5) and (3.6) are virtually identical (Kristensen(1993)).

Objective Function.	Linear equations of step 2.			Expression Step 3.
	Equation (i=1,..u)	Unknowns	Addition equation	
(3.5)	$f_i^s = r_i^s + \beta \sum_{j=1}^{u} p_{ij}^s f_j^s$	$f_1^s, \dots, f_u^s$	——	$r_i^d + \beta_i^d \sum_{j=1}^{u} p_{ij}^d g_j^{s^k}$
(3.6)	$g^s + f_i^s = r_i^s + \sum_{j=1}^{u} p_{ij}^s f_j^s$	$g^s, f_1^s, \dots, f_u^s$	$f_u^s=0$	$r_i^d + \sum_{j=1}^{u} p_{ij}^d f_j^s$
(3.7)	$g^s t_i^s + f_i^s = r_i^s + \sum_{j=1}^{u} p_{ij}^s f_j^s$	$g^s, f_1^s, \dots, f_u^s$	$f_u^s=0$	$r_i^d - t_i^s g^s + \sum_{j=1}^{u} p_{ij}^d f_j^s$

Table 3.1 Equations to be used for the policy iteration method for different objective functions

We can see clearly from Table 3.1 that the objective function (3.6) is just a special case of objective function (3.7) where $t_i^d = 1$ for all i and all d. Both Howard(1960) and Jewell(1963) interpreted t_i^d as the expected stage length.

3.3.4 Criteria of Optimality under Quota Constraints

In the situation where no production quota is imposed upon a farmer, the most limiting criterion is the number of cows he can have on his farm (grazing or housing capacity). In this situation, the most appropriate criterion of optimality would appear to be the maximisation of expected net rewards per cow in the long term.

An alternative criterion is needed, however, if there is some other restriction imposed on the system. This restriction may be in relation to inputs to the system (e.g.

a limited supply of heifers), or on the outputs of the system (e.g. a quota on production). Since milk production quotas are in place in all EU countries, including Ireland, such alternative criteria must be considered. In such a situation, where the amount of milk produced is the most limiting restriction, an obvious criterion would appear to be the maximisation of average rewards per kg of milk produced. This criterion of optimality was first applied by Kristensen(1989). The theoretic development of this criterion was later presented by Kristensen(1990). Under the traditional average rewards criterion (Chapter 3.3.3) the optimal policy is that policy that maximises

$$g_r^s = \sum_{i=1}^{u} \pi_i^s r_i^s$$

Further we define that if a state i is observed and a decision d is made, a certain physical output (milk yield) m_i^d is involved. In the same way as the average rewards can be calculated under policy s, so the average physical output under s can be calculated as

$$g_m^s = \sum_{i=1}^{u} \pi_i^s m_i^s$$

The criterion of optimality that is appropriate for the quota situation is the maximisation of expected rewards per unit of physical output. Therefore, the optimal policy would be that policy that maximises

$$g^s = g_r^s / g_m^s = \left(\sum_{i=1}^{u} \pi_i^s r_i^s\right) \Big/ \left(\sum_{i=1}^{u} \pi_i^s m_i^s\right) \qquad \textit{(Equation 3.8)}$$

The value of g^s can be interpreted as the average rewards per unit of physical output when the policy s is kept constant over an infinite number of stages. To ensure that g^s is always defined (and that its sign is always determined by g_r^s) we assume, that for all s,

$$g_m^s = \sum_{i=1}^{u} \pi_i^s m_i^s > 0$$

To implement this criterion of optimality, the maximisation of average net rewards over time, presented by Jewell(1963) can be considered. Jewell considered the optimisation of this criterion where the stage length was considered as being a stochastic variable whose distribution depended on the state i and the decision made, d (Chapter 3.3.3).

Denoting the expected stage length, given i and d, as t_i^s, Jewell presented an algorithm that maximised

$$h(s) = g^s = g_r^s / g_t^s = \sum_{i=1}^{u} \pi_i^s r_i^s \Big/ \sum_{i=1}^{u} \pi_i^s t_i^s \qquad \textbf{\textit{(Equation 3.9)}}$$

It is quite clear that equations (3.9) and (3.8) are analogous: In (3.8) m_i^d is interpreted as the expected physical output, dependent on i and d. In (3.9) t_i^s is the expected stage length dependent on i and d. Therefore, the maximisation of average expected rewards per kg of milk can be applied using the equations in Table 3.1 for criterion (3.7) where we substitute m_i^d for t_i^s.

2.3.5 Policy Iteration - Discussion

The policy iteration has the advantage over the value iteration algorithm in that it provides an exact solution to sequential problems (e.g. replacement problems) over an infinite planning horizon. The policy iteration method also converges quickly. It has further advantages, in that the equations in Table 3.1 are general. It is quite possible to calculate the economic consequences of following any policy s. This allows us to compare non-optimum policies with the optimal policy by carrying out one iteration of the non-optimal policy. This can be done by comparisons of the relative values under each policy considered.

The criteria of optimality developed by Jewell(1963) and modified by Kristensen(1990) are also useful in calculating other technical results under a given policy by redefining r_i^d and t_i in Table 3.1 (3.7). Examples of such interpretations were given by Kristensen(1990).

1) If r_i^d is defined as the milk yield of a cow in state i under policy d, and t_i^d is defined as the stage length when state i is observed under policy s. Then g^s is the average milk yield per cow per year under policy s.

2) Let $r_i^d = 1$ if state i represents the purchase of a heifer and zero otherwise and let t_i^d be defined as the stage length when state i is observed under policy s. Then g^s is the annual replacement rate under policy s.

3) Let $r_i^d = 1$ if calving takes place and zero otherwise and let t_i^d be defined as in 2) and 1) as the stage length when state i is observed under policy s. Then g^s is the average number of calvings per cow per year under policy s.

4) Let $r_i^d = n$ and $t_i^d = 1$ if a calving takes place after a calving interval of n weeks, and both are zero otherwise. Then g^s is the average length of the calving interval under s.

The value iteration algorithm finds optimal policies at each stage (i.e. policy is dependent on stage number), so it is possible to associate different rewards with each stage (i.e. one could model an expected improvement in milk price at n=3). Because the policy iteration method maximises the objective function over an infinite number of stages ($N = \infty$) and finds a stationary policy that is a general solution (i.e. policy is the same at each stage), it is not possible to allow rewards, outputs etc.. to depend on stage. It is however possible to include a rate of inflation/deflation in the model by use of the discount factor β.

The more complicated formulation of the policy iteration method involves finding the exact solution of g^s in Step 2. This involves solving the system of simultaneous linear equations :

$$g^s = (I - B^s P^s)^{-1} r^s$$

The dimension of this system of equations is equal to the number of states in the model u. The solution of this set of equations requires the inversion of a matrix of dimension $u*u$, which is rather complicated. Thus, even for quite small models, the policy iteration method involves solving large sets of simultaneous equations. Because of this, when the model is large, the policy iteration method quickly becomes infeasible. Due to this constraint, the policy iteration method has not been widely applied to animal replacement problems.

Ben-ari et al(1983) examined the effect of a finite planning horizon on the results of a dairy replacement problem. It was argued that the use of a finite planning horizon ignored data beyond the planning horizon and caused severe distortion of the results. These problems were illustrated by comparing the results of a model formulated using the value iteration method with 5, 7 and 10 year planning horizons. The model itself described a cow by 3 state variables : milk yield, age and bodyweight. A stage length of one year was used and the decisions considered at each stage were to keep the animal for an additional year or to replace it with a heifer. This model was then formulated using the policy iteration method and designed as an interactive tool.

The policy iteration method was also applied to the dairy replacement problem by Reenberg(1979) and by Kristensen and Ostergaard(1982). In both of these models the state space was small, containing 9 and 177 states respectively.

Chapter 4

Hierarchic Markov Processes

4.1 The Curse of Dimensionality

4.1.1 Systems with Large State Spaces

The variation in traits in dairy cows is considerable and thus, models with large state spaces are often needed in the modelling of the dairy cow as a production unit. The difficulty with the policy iteration method, and to a lesser extent the value iteration method, is that the optimisation of models with large state spaces is infeasible. This problem is often referred to in literature as 'the curse of dimensionality' (Kristensen(1993)).

The notion of Hierarchic Markov Processes was introduced by Kristensen(1988;1991) to help overcome this problem and make possible the exact solutions of models with very large state spaces over infinite planning horizons. In order to illustrate how Hierarchic Markov Processes help solve systems with large state spaces, we will need to first look at how the 'curse of dimensionality' arises.

4.1.2 Including Age as a State Variable

Consider a simple dairy model where a cow is described by age and yield (Kristensen 1996). In this instance, we will say that the variable, age, can only take on the values *1…4,* so that here, an animal must be replaced after at most 4 lactations. Yields can take on one of 3 values: high, average or low. We will assume that the probability that an animal will remain at the same level of yield to be 0.6. The transition to another level is assumed to be 0.3 from low (or high) to average and 0.1 from low to high or vice versa, if the cow is kept. On the other hand, if the decision is to replace, we will assume that there is an equal chance of the replacement heifer being in each of the three levels. The probabilities for transition between the different yield levels are presented in Table 4.1.

Table 4.1 is, in effect, our transition matrix P_{ij}, if the only state variable in our model were yield.

	Decision = 1 (Keep)			Decision = 2 (Replace)		
	j=1 (L)	j=2 (A)	j=3 (H)	j=1 (L)	j=2 (A)	j=3 (H)
i=1 (L)	0.6	0.3	0.1	1/3	1/3	1/3
i=2 (A)	0.2	0.6	0.2	1/3	1/3	1/3
i=3 (H)	0.1	0.3	0.6	1/3	1/3	1/3

Table 4.1 Probabilities between different yield levels.

The inclusion of lactation gives us the transition matrix for a model with both yield and age as state variables. The inclusion of age as a state variable has increased the dimensions of P_{ij} by a factor of 4 (corresponding to the number of levels permissible for the variable 'age'). The new transition matrix P_{ij} is presented in Table 4.2 and Table 4.3. Table 4.2 shows the transition matrix where the decision = 'keep', and Table 4.3 gives the transition matrix where the decision = 'replace'. Empty spaces in these matrices correspond to zeros.

P_{ij}^{keep}		j	1	2	3	4	5	6	7	8	9	10	11	12
		L	1	1	1	2	2	2	3	3	3	4	4	4
i	L	y	L	A	H	L	A	H	L	A	H	L	A	H
1	1	L				0.6	0.3	0.1						
2	1	A				0.2	0.6	0.2						
3	1	H				0.1	0.3	0.6						
4	2	L							0.6	0.3	0.1			
5	2	A							0.2	0.6	0.2			
6	2	H							0.1	0.3	0.6			
7	3	L										0.6	0.3	0.1
8	3	A										0.2	0.6	0.2
9	3	H										0.1	0.3	0.6
10	4	L	1/3	1/3	1/3									
11	4	A	1/3	1/3	1/3									
12	4	H	1/3	1/3	1/3									

Table 4.2 Transition matrix for decision = 'keep'.

$P_{ij}^{replace}$		j	1	2	3	4	5	6	7	8	9	10	11	12
		L	1	1	1	2	2	2	3	3	3	4	4	4
i	L	Y	L	A	H	L	A	H	L	A	H	L	A	H
1	1	L	1/3	1/3	1/3									
2	1	A	1/3	1/3	1/3									
3	1	H	1/3	1/3	1/3									
4	2	L	1/3	1/3	1/3									
5	2	A	1/3	1/3	1/3									
6	2	H	1/3	1/3	1/3									
7	3	L	1/3	1/3	1/3									
8	3	A	1/3	1/3	1/3									
9	3	H	1/3	1/3	1/3									
10	4	L	1/3	1/3	1/3									
11	4	A	1/3	1/3	1/3									
12	4	H	1/3	1/3	1/3									

Table 4.3 Transition matrix for decision = 'replace'.

It is important to note that the transition matrices in these tables are sparse because transitions in age are deterministic. Considering a cow in lactation i, if the decision to keep for a further stage is taken, only a transition to age $i+1$ is possible (assuming that the stage length is one year). If the decision is to replace the animal, then only a transition to lactation 1 (replacement heifer) is possible. All other transitions will be equal to zero.

4.1.3 Including Permanent Traits in the Model

In addition to the state variables for age and production, we will now define a state variable for the genetic merit of the cow. This variable can take on the values: bad (1), average (2) or good (3). Once a heifer enters the herd, a measure of this genetic merit is taken. The animal will retain the same level of genetic merit for its lifetime in the herd. When a heifer first enters the herd, we will further assume, that the probability of its genetic level being good, average or bad is as follows:

Good = 0.3

Average = 0.4

Bad = 0.3

The size of the state space is now 36 = 3 * 4 * 3. There are 36 different possible states (combinations of state variables) that an animal can occupy. The transition matrix for this model has now increased in size from a 12*12 matrix to a 36*36 matrix. This new transition matrix *Pij* is shown in Appendix B. For the decisions 'keep' and 'replace'. This new matrix shows even greater sparcity than the matrices in Tables 4.2 and 4.3. This is because genetic merit is defined as a permanent trait of the animal. So, if the decision 'keep' is taken, transitions to states with a different genetic level to the present state are not possible, and as such, these probabilities are equal to zero.

4.1.4 Dimensionality in Dynamic Programming

It is quite clear that if a new state variable of *n* levels is added to the model, it results in an increase in the state space by a factor of *n*. In the small model discussed here, the size of the state space does not pose significant problems to optimisation. However, in more realistic models, where several traits are represented by state variables, with a realistic number of levels, the size of the state space can soon become prohibitive. In order to help circumvent this 'curse of dimensionality', the notion of 'Hierarchical Markov Processes' was introduced by Kristensen(1988;1991).

4.2 Hierarchic Markov Processes

4.2.1 The Structure of Hierarchic Markov Processes

In the traditional Markov decision model, a decision to 'replace' is simply considered as a transition from one state to another. Hierarchic Markov Processes take into account the fact that when a decision to replace is taken, there is a fundamental change in the process (replacement of the current animal with a replacement heifer). These processes omit age as a state variable and, moreover, take advantage of the fact that when a replacement occurs the process (life cycle of the replacement animal) is restarted (Houben et al(1994)).

In a Hierarchic Markov Process, the model is split into one main process and a set of subprocesses. In the main process, the size of the state space is equal to the number of subprocesses and each state in the main process corresponds to a subprocess. In each subprocess there is a finite number of stages N, where N is the maximum life-span of an animal in the herd. The structure of these processes is shown in Figure 4.1.

A subprocess begins when an animal enters the herd. When a replacement occurs, transition to an absorption state occurs. The subprocess then remains in this state until stage N, when a new subprocess (representing the replacement heifer) enters the herd. When a subprocess is in an absorption state, the stage length is always zero, and all rewards and outputs are equal to zero.

4.2.2 Notation and Terminology

In the following sections, the notation and terminology to be used in this and following chapters in regard to Hierarchic Markov Processes are defined. To avoid ambiguity, Greek letters (ι,κ etc...) are used to denote states in the main process (Kristensen(1993)).

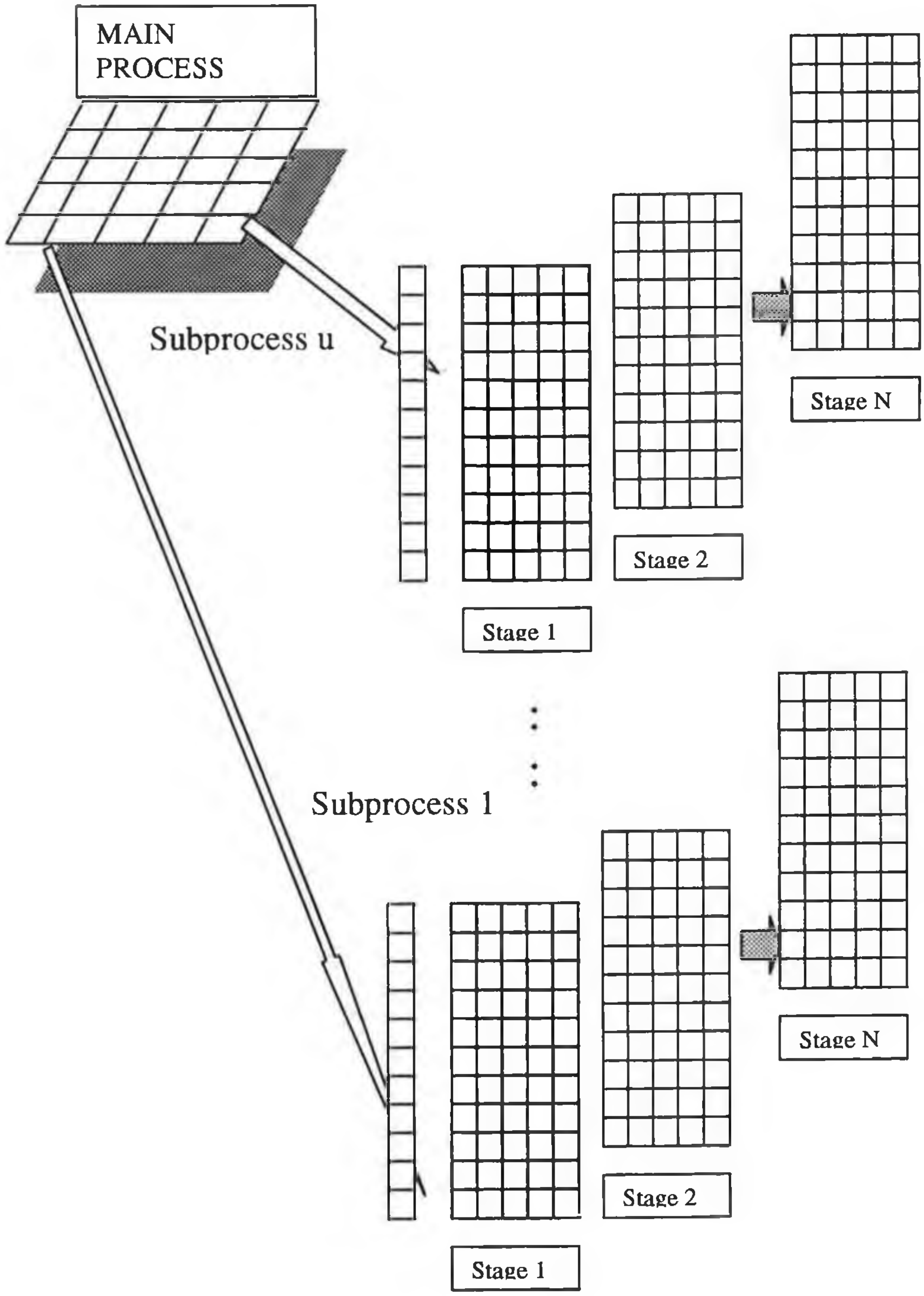

Figure 4.1 The transition probability structure of a Hierarchic Markov Process

4.2.2.1 Notation for Subprocesses

A Hierarchic Markov Process is a series of Markov decision processes called subprocesses built into one Markov decision process called the main process. A subprocess is a finite time Markov decision process with N stages and a finite state space $\Omega_n = \{1,\ldots,u_n\}$ for stage n, $1 \leq n \leq N$. The action set D_n of the nth stage is assumed to be finite, too. A policy s of a subprocess is a map assigned to each stage n and state $i \in \Omega_n$ an action $s(n,i) \in D_n$. The set of all possible policies of a subprocess is denoted Γ. When the state i is observed and the action d is taken, a reward $r_i^d(n)$ is gained. The corresponding physical output is denoted as $m_i^d(n)$. Let $p_{ij}^d(n)$ be the transition probability from state i to state j where i is the state in the nth stage, j is the state in the $n+1$th stage and d is the action taken at stage n. We further define $p_i(0)$ as the initial probability of being in state i at stage $n=0$.

Under criterion (3.5), we must also define the discount factor in state i under the action d as $\beta_i^d(n)$. We assume that the stage length is given by stage, state and action.

4.2.2.2 Notation for the Main Process

Assume that we have a set of v subprocesses each having its own set of parameters. The main process is then a Markov decision process running over an infinite number of stages and having a finite state space $\{1,\ldots,v\}$. Each stage in this process represents a particular subprocess. The action sets of the main process are the sets Γ_ι, $\iota = 1,\ldots,v$, of all possible policies of the individual subprocesses. A policy σ is a map assigning to each state ι of the main process an action $\sigma(\iota) \in \Gamma_\iota$. The transition matrix of the main process has the dimension $v*v$, and is denoted $\Phi = \{\phi_{\iota\kappa}\}$. The transition probabilities are assumed to be independent of the action taken. The reward f_ι^σ and the physical output h_ι^σ in state ι of the main process are determined from the total rewards and output functions of the corresponding subprocess of the value iteration method (Shown for f_ι^σ in Figure 4.2). This is possible as the subprocesses run over a finite number of stages N.

h_ι^σ and the discount factor for subprocess ι, B_ι^σ can be calculated analogously. This is shown in Appendix C.

$$f_i^s(n) = r_i^s(n) \quad , \quad n = N.$$

$$f_i^s(n) = r_i^s(n) + \beta_i^s(n) \sum_{j=1}^{U_{n+1}} p_{ij}^s(n) f_j^s(n+1) \quad , \quad n = N-1,......,1.$$

and

$$f_\iota^\sigma = \sum_{i=1}^{U_1} p_i(0) f_i^s(1) \quad , \quad s = \sigma(\iota).$$

Figure 4.2 Calculation of the reward for a subprocess

4.2.3 Formulation and Optimisation

The main advantage of the Hierarchic Markov Process is directly related to its structure. A subprocess has a well defined and finite planning horizon (equal to the life-span of a cow). This and its large state space make value iteration the ideal optimisation method to use. The main process has a small state space and an infinite planning horizon, and so, the policy iteration method is the ideal optimisation tool.

The hierarchic approach combines these two optimisation techniques to allow the solution of replacement problems with large state spaces over an infinite planning horizon. It can be shown mathematically that results of a model formulated as a Hierarchic Markov Process are equal to that of a general Markov process optimised using the policy iteration method.

The form of the optimisation cycle for Hierarchic Markov Processes is that of the policy iteration method. Step 3 of the policy iteration method would be prohibitive in this case as the number of alternatives action that would need to be compared is Γ_ι, the number of alternative policies in the subprocess ι. However, because of the structure of the Hierarchic Markov Process, the value iteration method can be used in the subprocess and the results used in Step 3 of the policy iteration method of the subprocess.

As we are optimising over an infinite planning horizon, the objective functions considered for the policy iteration method (Chapters 3.3.3 and 3.3.4) are relevant. The iteration cycle of the Hierarchic Markov Process in its general form is as follows:

Step 1: Choose an arbitrary policy σ. Go to Step 2.

Step 2: Solve the following set of linear simultaneous equations for $F_1^\sigma, F_2^\sigma, ...F_v^\sigma$ and in the case of the criteria 3.7 and 3.8 g^σ:

$$g^\sigma h_\iota^\sigma + F_\iota^\sigma = f_\iota^\sigma + B_\iota^\sigma \sum_{k=1}^{v} \phi_{\iota k} F_k^\sigma$$

In the cases of criteria 3.7 and 3.8 an additional equation $F_v^\sigma = 0$ is needed in order to determine a unique solution (since in this case we have *v+1* unknowns). Go to Step 3.

Step 3: Define

$$T_\iota = \sum_{k=1}^{v} \phi_{\iota k} F_k^\sigma$$

Under criterion (3.5) and $T_\iota = 0$ under criteria 3.6, 3.7, 3.8.

For each subprocess ι, find by means of the recurrence relations

$$\tau_{\iota,i}(n) = \max^d \{r_i^d(n) - m_i^d(n) g^\sigma + \beta_i^d(N) T_\iota\},$$
$$n = N$$

$$\tau_{\iota,i}(n) = \max^d \{r_i^d(n) - m_i^d(n) g^\sigma + \beta_i^d(n) \sum_{j=1}^{U_{n+1}} p_{ij}^d(n) \tau_{\iota,j}(n+1)\},$$
$$n = 1, ..., N-1.$$

a policy s' of the subprocess. The action $s'(n,i)$ is equal to the d' that maximises the right hand side of the recurrence equation of state i at stage n. Put $\sigma'(\iota) = s'$ for $\iota = 1,...v$. If $\sigma' = \sigma$, then stop since an optimal policy is found. Otherwise, redefine σ according to the new policy (put $\sigma = \sigma'$). Go to Step 2.

This general formulation can be altered for each of our criteria:

- Under criterion (3.5) all physical outputs ($m_i^d(n)$ and accordingly h_ι^σ) are put equal to zero.
- Under criterion (3.6) all physical outputs ($m_i^d(n)$ and h_ι^σ) and all discount factors ($\beta_i^d(n)$ and B_ι^σ) are put equal to 1.
- Under criterion (3.7) all discount factors are put equal to 1.

4.2.4 Discussion

This iteration cycle makes it possible to solve for large state spaces over an *infinite* planning horizon. State variables of the main process are those traits that remain constant over the lifetime of an asset (dairy cow). Examples of such traits may be:

- Genetic merit of sire.
- Genetic merit of dam.
- Month of first calving.
- Breed.
- Age of first calving.

Once an animal occupies one of these states (subprocesses) it remains there for the duration of its time in the herd. Therefore, these permanent traits need not be considered as state variables in the processes.

The size of the state space in the main process determines the number of linear simultaneous equations to be solved in Step 2 of the iteration cycle. It is because of this step that it is not feasible to solve large models using the policy iteration method. Under criterion 3.6, the number of simultaneous equations is v (the number of states in the main process), while under criteria 3.7 and 3.8 the number of equations to be solved is $v+1$. The size of the main process' state space (determined by the permanent traits included in the model) is usually small (Kristensen(1987) and Kristensen(1989) had a main state space of size 5; Houben et al(1992) had a main state space of size 1), which makes it feasible to solve even large models exactly over an infinite planning horizon.

All other traits to be included in the model would be included as state variables in the subprocesses. Examples of traits (variables) that could be included in the model are:

- Month of calving.
- Calving interval.
- Production in current lactation.
- Production in previous lactation.
- Body weight.
- Fertility status.

In a traditional Markov decision process, the age of the dairy cow (lactation, stage of lactation) might also be included as a state variable (Chapter 4.1).

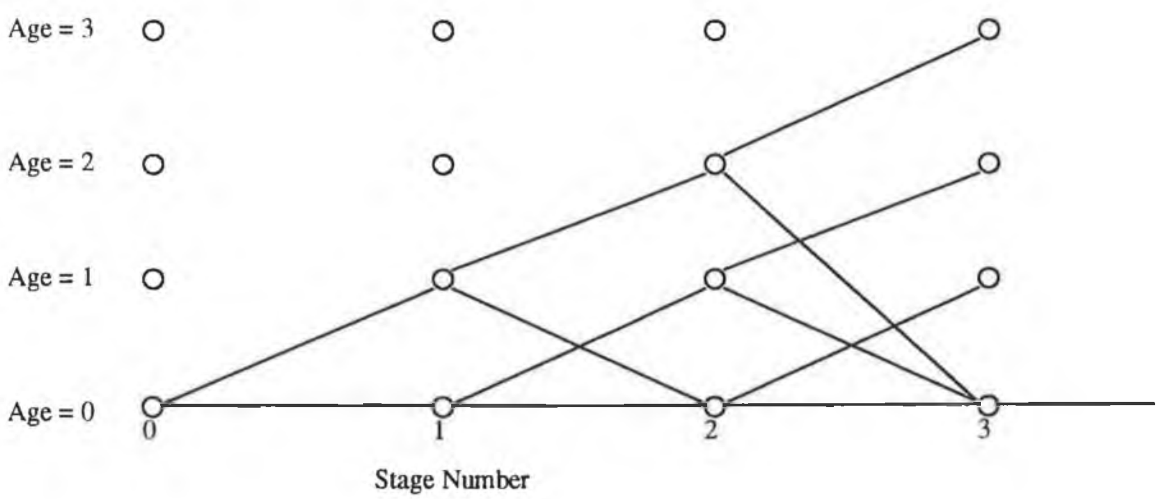

Figure 4.3 A traditional Markov process with age as the only state variable (0-3). Under this structure, the process represents an animal and its future replacements (a transition to 'age = 0' represents a replacement by a heifer).

However, under a hierarchic structure, a new subprocess begins when an animal enters the herd. Because of this, age need not be included as a state variable, since it will correspond directly to current stage of the subprocess (Figure 4.4).

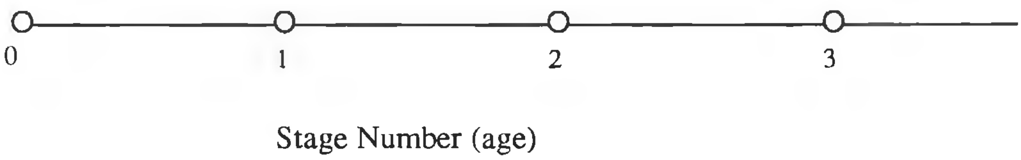

Figure 4.4 A subprocess in a hierarchical structure. Age is not required as a state variable as it equals the stage number. In this structure, the stage number is not a stage in a planning horizon, but rather a stage in the life cycle of the asset (dairy cow). This is because when an animal enters the herd it enters the start of a subprocess (which subprocess depends on the permanent traits of the asset.

4.2.5 The Advantages of Hierarchic Markov Processes

The Hierarchic Markov Process is designed especially to fit the structure of animal decision problems where the successive stages of the subprocesses correspond to the age of the animal in question (Kristensen(1988)). If the model is designed in such a way that the main process (which is solved using the policy iteration method) has very few states, and the subprocesses (which are solved using the value iteration method) have large state spaces, it is possible to find optimal solutions to even very large sequential decision problems. The optimal solution given is exact (optimal over an infinite planning horizon), fast and can handle very large models.

Models formulated as Hierarchic Markov Processes can be formulated as an ordinary Markov decision process. This could then be solved over an infinite planning horizon using the policy iteration method. In this situation, each combination of main state (ι), subprocess state (i) and subprocess stage (n) should be interpreted as a state (denoted $\iota n i$). Parameters for this ordinary Markov decision process are easily found from the hierarchic model:

$$r^d_{\iota n i} = r^d_i(n),$$

$$m^d_{\iota n i} = m^d_i(n),$$

$$\beta^d_{\iota n i} = \beta^d_i(n),$$

$$p^d_{(\iota n i)(\kappa m j)} = \begin{cases} p^d_{ij}(n), & \iota = \kappa \ \ and \ \ m = n-1 \\ \Phi_{\iota\kappa}\, p_i(0), & n = N \ \ and \ \ m = 1 \\ 0, & otherwise \end{cases}$$

Note: The parameters on the right hand side of these equations belong to the ιth subprocess except $p_i(0)$ which belongs to subprocess κ.

This formulation would result in the same optimal policy as if the hierarchic formulation were applied. The hierarchic method however, has considerable computational advantage over the ordinary Markov decision process. The policy iteration method is only relevant when solving small models, but the value iteration method has been used to solve very large models. The time spent on optimisation using the hierarchic formulation is much lower than even the value iteration method (and it has the advantage of solving exactly over an infinite planning horizon).

4.2.6 Computational Advantages

Step 2 of the Hierarchic Markov formulation involves the solving a set of v equations. With large models which have small state spaces in the main process, the time spent on this step is small. For this step, we also require the calculation of the rewards (f_ι^σ) and either the physical output or the expected discount factor according to equations of the type shown in Figure 4.2. Since these parameters are calculated for a known policy (σ), their calculation (whichever two are relevant under our criterion) involves approximately the same number of operations as one iteration of the value iteration method (if the number of alternative actions to be compared for each state using the value iteration method is two). If more than two actions were considered, the number of operations required would be lower than that of one step of the value iteration method.

Step 3 of the hierarchic formulation involves exactly the same number of operations as one iteration of the value iteration method (the choice of decision for all i, for some stage n). So, in conclusion, each iteration of the Hierarchic Markov Process involves approximately the same number of operations as two iterations of the value iteration method. For example, a model reported by Kristensen(1991)was optimised using Hierarchic Markov Processes under a range of different price conditions. The number of iterations that were required was between 3 and 6. This corresponds to between 6 and 12 iterations of the value iteration method. The age of the animal was measured every 4 weeks. If a finite planning horizon of 20 years were assumed and the value iteration method used for optimisation, this planning horizon would represent 260 iterations (stages) of the value iteration method (as compared with 6-12 using the hierarchic approach).

Kristensen(1988) compared the performance of the Hierarchic Markov Process with the value iteration method and the policy iteration method for criteria 3.5 - 3.8. A small model, with 48 states when formulated as an ordinary Markov decision process, was used. When formulated as a Hierarchic Markov Process, the main process of the model has three states and the subprocesses have 4 states. The results of these comparisons are shown in Figure 4.5 and 4.6.

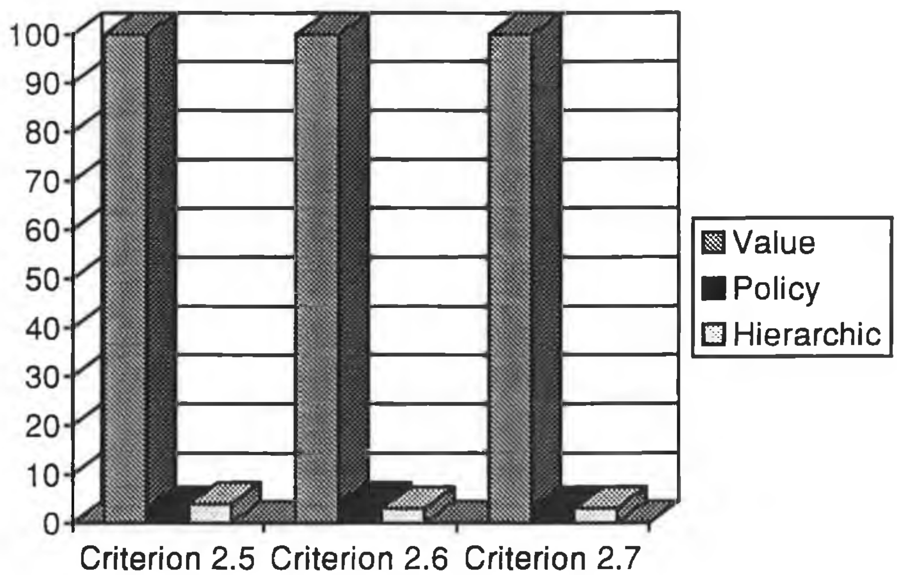

Figure 4.5 Number of iterations required for 3 different methods under 3 different criteria of optimality.

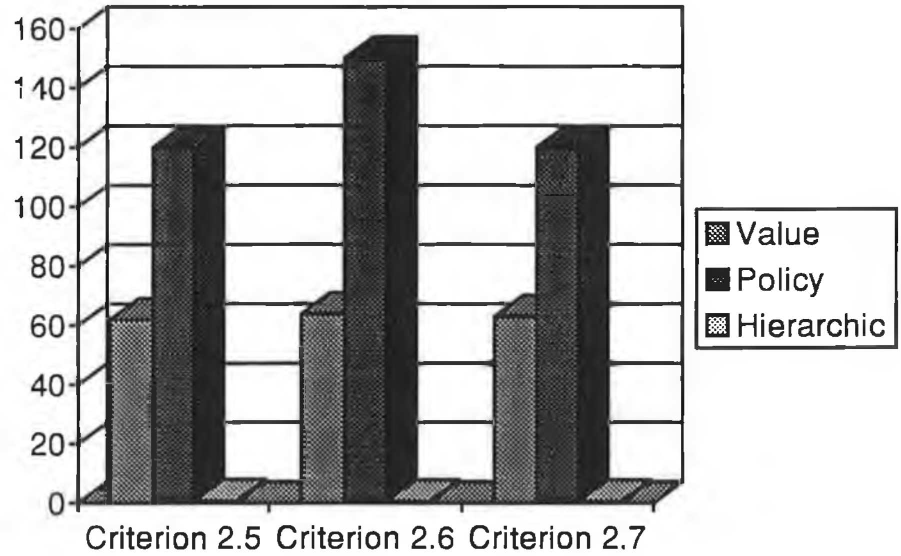

Figure 4.6 Relative computer time for the three optimisation methods under 3 different criteria of optimality.

In Figure 4.5, it can be seen that the number of iterations required by the value iteration method is far greater than for either of the other two methods considered. The relative computer time for the 3 optimisation techniques is shown in Figure 4.6. As we would expect, the hierarchic model's performance was superior to that of the policy iteration method due to the smaller number of linear equations solved during optimisation. In this example, an iteration of the hierarchic model is performed even faster than one of the value iteration method applied to the same (transformed) model. The reason for this is that the value iteration method had not been programmed in the most efficient way (Kristensen(1988)).

4.2.7 Application to Animal Replacement.

In the Kristensen(1987) model, a cow was described in terms of lactation number, stage in lactation, level of milk yield in the current and previous lactation, length of calving interval, and genetic class, defined by the sire. It was concluded that milk yield of previous lactation was not needed as a state variable when the other variables were present in the model.

The future profitability, calculated from the optimal solution, was used for ranking of cows in the herd. Replacement decisions were allowed to depend on the genetic class of the heifers. While the weight of the cow was not included as a state variable, a standard weight curve describing weight as a function of lactation number and stage of lactation was used. Beyond the states defined by the state variables, additional states were: 1) a replacement state, to which the system immediately transfers when a replacement takes place, 2) a disease state, the probability of which is equal to the probability of involuntary disposal, and 3) an infertility state, which the process occupies if the cow is not known to be pregnant 40 weeks after calving.

This is the only study containing genetic class, defined by the breeding value of the sire, as a state variable. It was found that there was a significant difference in the average herd life of different genetic classes. Genetic class also made it possible to compare the heifers available in the herd with present cows, and to rank them. Unlike other studies where transitions to different classes of milk yield could only take place at the end of a lactation, no such simplification was made in this paper.

Houben(1994) looked at the effects of mastitis occurrence on optimal replacement policies. He found that although mastitis had a considerable effect on expected income, in most cases the optimal decision was to keep and treat rather than to replace the cow. The dairy replacement problem was modelled as a Hierarchic Markov Process. The state variables used in the model were lactation, production levels in the current and previous lactation, calving interval, clinical mastitis in current month(binary) and accumulated number of clinical quarters in the current and previous lactation. Exclusion of infeasible states resulted in a Hierarchic Markov Process of 6,821,724 states with the result that optimisation of this model took over 6 hours of computer time. This model optimised three decisions : 1) keep the cow for at least one more month and do not inseminate, 2) keep the cow for at least one more month and inseminate her, 3) replace the cow immediately with a replacement heifer.

In the model, production level was defined relative to cows of the same age and month of lactation and production level transitions were allowed on a monthly basis. When transitions were only allowed at the start of a new lactation, it resulted in an overestimation of high production animals and an underestimation of low producing animals. The gross margin model of Van Arendonk(1985a) was extended to include effects of clinical mastitis. Production losses due to mastitis and transition probabilities were taken from Houben(1993). In Houben(1993) a stepwise least squares method was used to obtain unbiased estimates of milk, fat and protein losses, and logistic regression was used to estimate the probability that a cow would have clinical mastitis in the next month. Analysis of the effect of the variables related to mastitis was carried out and it was found that the state variable that accounted for clinical quarters in the previous lactation had little influence on the optimal policy. Clinical mastitis in the current lactation, especially in the current month, however, had a significant effect on expected income.

Chapter 5

Seasonality, and its Inclusion in Markov Models

5.1 Seasonality in the Irish Dairy Industry

5.1.1 Milk Supply Patterns

Ireland, in contrast with other EU countries, has a highly seasonal milk supply pattern. This can be seen in Figure 5.1, where the milk supply patterns for Ireland, France, the Netherlands, and Denmark are shown. While the supply curves for these other EU countries are relatively flat, the milk supply in Ireland is at its highest in May and at its lowest in the winter months and the difference in supply during these times is large.

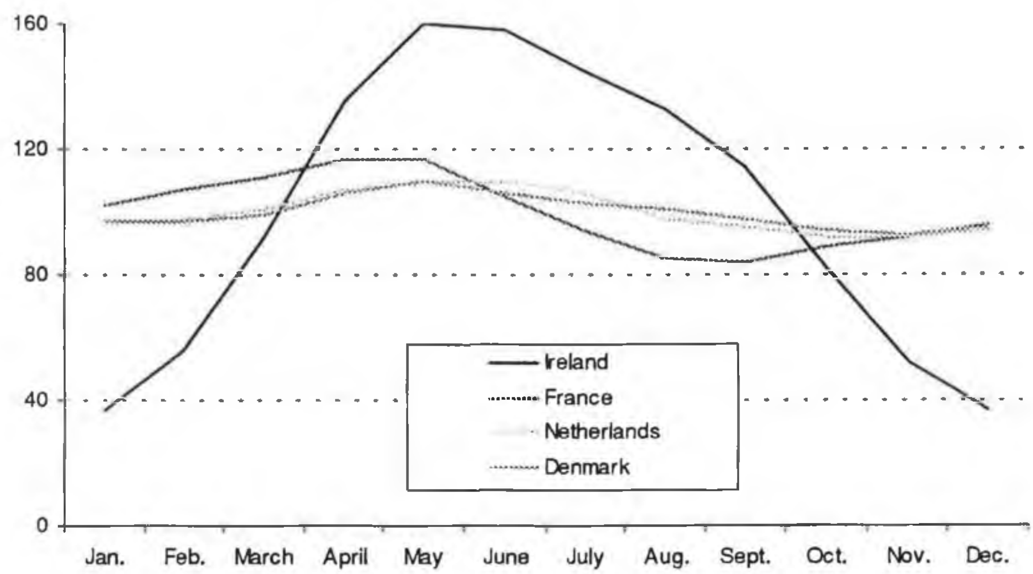

Figure 5.1 A comparison of milk supply curves for four EU countries.

This results from the fact that the lactation period in Ireland is associated with the availability of grazed grass in the production system (Anon(1993b)). In Ireland, this grazed grass, which is an inexpensive source of feed (relative to silage and concentrates) is available during the summer. A study by Ryan(1997) was carried out to contrast three calving patterns for Irish dairy herds. The resultant production systems are shown in Figure 5.2 and Figure 5.3 for spring and winter calving herds respectively.

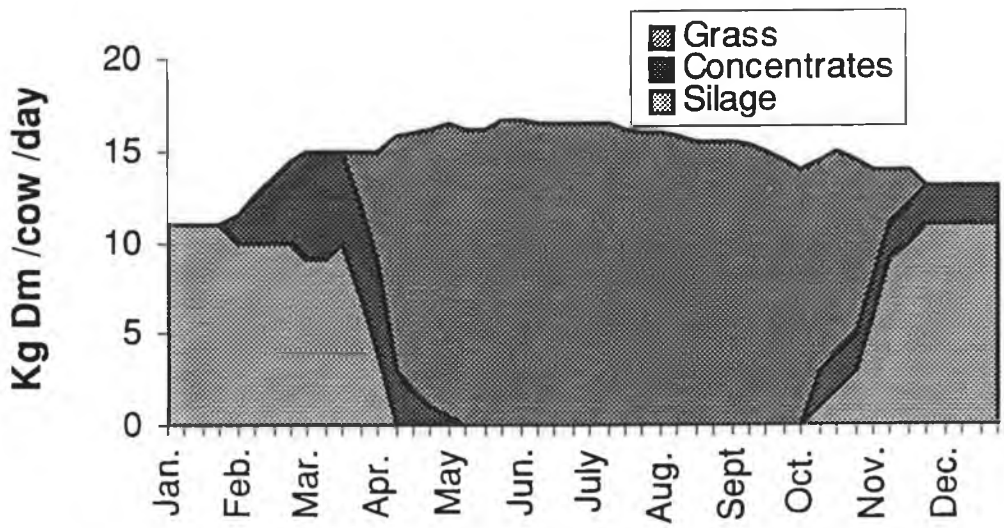

Figure 5.2 Spring milk production system.

Under the spring production system 1.4 tons of Silage dry matter, 580 Kg of concentrate dry matter and 3.5 tons of grass of dry matter were required per cow.

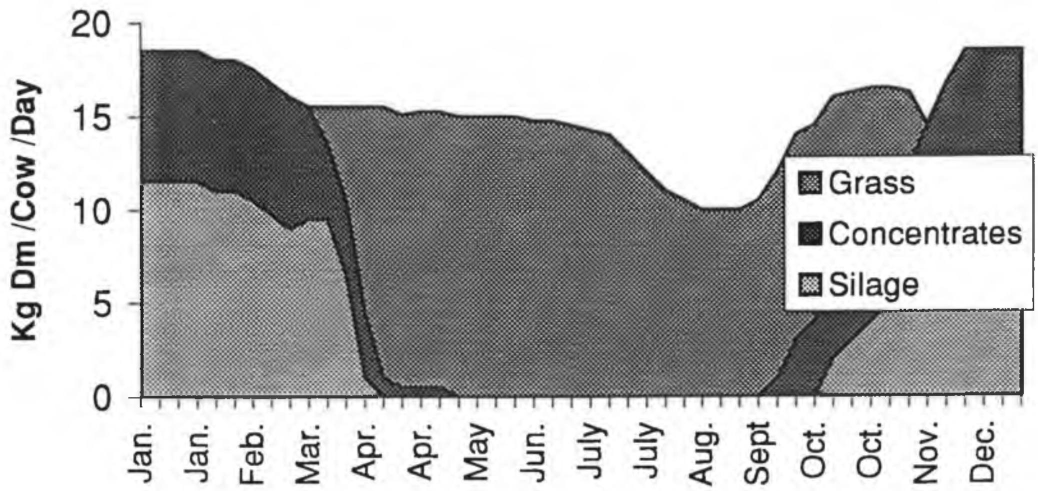

Figure 5.3 Winter milk production system.

Under the winter milk production system each cow required, on average, 1.5 tons of silage dry matter, 1.3 tons of concentrates dry matter and 2.8 tons of grass dry matter. This illustrates clearly the impact of season of calving on feed costs.

5.1.2 Seasonality in Markov Processes

Seasonality was included in the model of Van Arendonk(1986) which was formulated as a traditional Markov process. The hierarchic models of Kristensen(1987) and Houben et al(1994) which modelled Danish and Dutch production systems respectively did not include seasonality. Due to the impact of season of calving on the costs and revenues in Irish dairy herds, seasonality should be included in any attempt to model Irish dairy herds.

5.2 Including Seasonality in Hierarchic Models

5.2.1 Problems with Seasonality in Hierarchic Models

To include seasonality in a Hierarchic model, it would appear that we should include an additional state variable in our subprocesses. This variable could be called 'month' and have 12 possible values, where each value corresponds to a particular month of the year (Jan, Feb,...,Dec). However, the inclusion of this state variable leads to certain problems and requires reformulation of the Hierarchic Markov Processes discussed in Chapter 4.2. To illustrate the problems associated with the inclusion of seasonality, consider a model with month as the only state variable (i.e. $\Omega = \{1,\ldots,12\}$).

In this model we further assume that if a decision to replace is taken, a replacement heifer enters that herd the following month (e.g. if a replacement decision is taken in June, the replacement heifer enters the herd in July (Figure 5.4)).

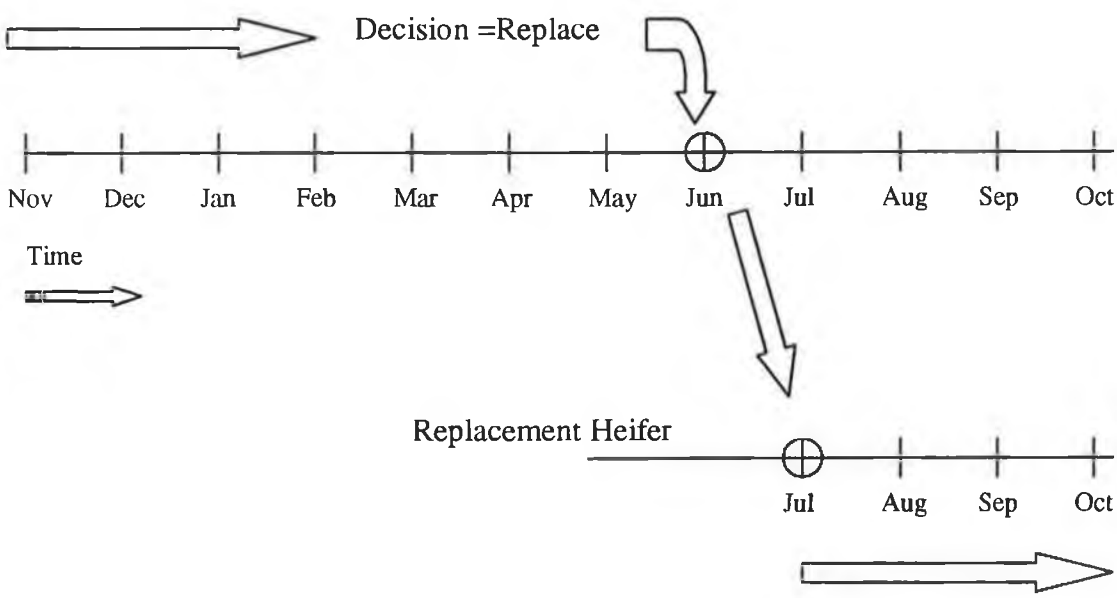

Figure 5.4 Illustration of how the month of entry of a replacement heifer is dependent on the animal it is replacing.

So when the replacement heifer (Figure 5.4) enters the herd, it enters into the state corresponding to July with probability equal to one, and to any other state with a probability of zero.

i.e. $p_i(0) = 1$ iff $i = 7$ (July)
$= 0$ otherwise

Of course, if the heifer entered the herd in another month, the initial probabilities $p_i(0)$ would change accordingly. So, depending on the month in which a cow enters the herd, we have 12 distinct initial probability distributions. To account for this we would need 12 subprocesses, corresponding to a main state variable 'month of entry to the herd', where each of these subprocesses have distinct initial probabilities $p_i(0)$ (Figure 5.5).

	Subprocess	$p_i(0)$
Month of entry = Jan	(1)	[1 0 0 0 ... 0]
Month of entry = Feb	(2)	[0 1 0 0 ... 0]
Month of entry = Mar	(3)	[0 0 1 0 ... 0]
.	.	.
.	.	.
Month of entry = Dec	(12)	[0 0 0 0 ... 1]

Figure 5.5 The initial probabilities $p_i(0)$ for the 12 subprocesses corresponding to month of first entry into the herd.

The model now has a main process with a state space of size 12, and each subprocess in the main process has a state space of size 12. A further implication becomes clear when one considers the transitional probabilities in this main process (required for Step 2 of our Hierarchic Markov Process iteration cycle). The iteration cycle of Chapter 4.2.3 was based on the assumption that the transition probabilities of the main process Φ were independent of policy σ (i.e. independent of decisions made in the subprocesses). Under this formulation, this is no longer true. The probability of an animal which enters the herd in month i (subprocess ι) being replaced by an animal which will enter the herd in month j (subprocess κ) is dependent on the decisions made in the subprocess ι. For example, if a cow's 'month of entry' to the herd is June, the probability that its replacement heifer's 'month of entry' will be 'January' is dependent on the time of replacement (which is a result of policy σ). To account for these problems, some reformulation of the Hierarchic model is needed.

5.2.2 An Extension to the Hierarchic Model

The problem of how to extend Hierarchic Markov Processes to relax the assumption of Φ being independent of policy was studied by Kristensen(1988), although not in relation to seasonality. Kristensen(1988)'s model ,described as an 'extended model' was devised for a situation where h different qualities ($\iota = 1,\ldots,h$) of an asset existed, and where these qualities could be ranked from the least preferred to the most preferred.

i.e. ι_i is preferred to ι_j iff $\iota_i > \iota_j \quad \forall \quad i, j = 1,2,\ldots,h$.

For this situation, it was assumed that one could order a certain quality of asset, but that there was a limited supply for each quality. So, if an asset of quality ι was ordered there was a probability π_ι ($\pi_\iota < 0 \ \ \forall\iota$) that it could be delivered. In this model, the action set Ω ={keep, replace} was extended. In each subprocess, h actions of the type ' replace if an asset of quality ι is available ' were defined. If the action 'keep' is referred to as action $h+1$, this resulted in $h+1$ possible actions.

Because of the additional actions (actions 1,...,h) which result in the replacement of the asset (although with different heifers), one replacement state was no longer sufficient in a subprocess. For every action of the type 'replace if an asset of quality ι is available' an absorption state was defined, this resulted in h absorption states $\lambda_1,\ldots,\lambda_h$. These states were defined in all state spaces. These replacement states retain all the properties of replacement states, defined in Chapter 4.2 and additionally:

$$P_{ij} = 0 \quad \text{if} \quad i \neq j$$
$$= 1 \quad \text{if} \quad i = j \quad \forall\, i, j = \lambda_1, \ldots, \lambda_h$$

(i.e. transitions between replacement states are not possible).

The inclusion of these additional absorption states, had no effect on transition probabilities (P_{ij}) if the action 'keep' ($h+1$) was chosen. If one of the replacement decisions (i.e. 1,...,h) was taken, transition probabilities to the different absorption states could be calculated (Equation 5.1) and the probability that an asset was kept was $1 - p^{\iota}_{i\lambda_\kappa}(n)$.

$$p^{\iota}_{i\lambda_\kappa}(n) = \begin{cases} 0, & \kappa < \iota, \\ \pi_\kappa \prod_{j=\kappa+1}^{h} (1 - \pi_j), & h > \kappa \geq \iota, \\ \pi_\kappa, & \kappa = h \end{cases} \qquad \textbf{\textit{(Equation 5.1)}}$$

Since the only valid decision at stage N-1 is to replace (a necessary condition for Hierarchic Markov Processes), at the end of a subprocess the process will always be in one of the replacement states ($\lambda_1, \ldots, \lambda_h$). Depending on the replacement state occupied at stage N, the old asset would be replaced by a new asset of the corresponding quality if it were available.

This 'extended model' also required a reformulation of the iteration cycle described in Chapter 4.2. There are analogies between this extended model and a model which includes seasonality. The necessary changes to the iteration cycle of Hierarchic Markov Processes are dealt with in the next section, with particular reference to seasonality.

5.2.3 Adapting the 'Extended Model' for Seasonal Effect

An adaptation of the 'extended model' described by Kristensen(1988) can be used to include seasonal effect in a Hierarchic Markov Process. We must include season (month) as a state variable in the main process as well as the subprocesses (Chapter 5.2.1). A value of e.g. 4 in the main process means that the current subprocess *started* in April. On the other hand, a value of 4 in a subprocess means that the *current* month at that stage in the subprocess is April. The state of the main process directly determines the probability distribution at the beginning of the subprocess (Chapter 5.2.1).

As with the 'extended model', new absorption states must be defined. In this case, where season is described by 'month', 12 absorption states must be defined (λ_1,... ,λ_{12}), each corresponding to a month in which the replacement heifer could enter the herd.

- Replace with a heifer which enters the herd in January.
- Replace with a heifer which enters the herd in Febuary.

:

:

- Replace with heifer which enters the herd in December.

These absorption states are defined in all state spaces (Figure 5.6).

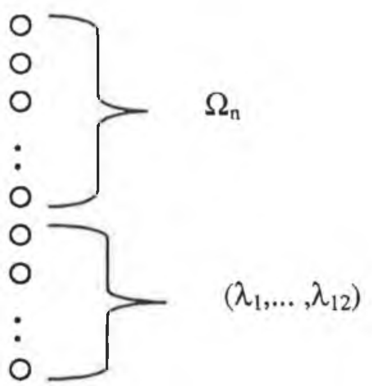

Figure 5.6 At any stage *n* in all subprocesses, 12 additional (absorption) states (λ_1,...,λ_{12}) are defined.

Analogously to the 'extended model', new actions of the type 'replace with heifer in month ι' must be defined (were ι = 1,…,12 correspond to the months Jan,…,Dec). As before, we also have a further action 'keep', which results in 13 possible actions. The inclusion of these additional absorption states has no effect on transitions where the decision made is to 'keep'. However, when a decision to replace an animal is taken, transition probabilities to these replacement states are as follows :

$$p^{\iota}_{i\lambda_{\kappa}}(n) = \begin{cases} 1, & \kappa = i \\ 0, & otherwise \end{cases},$$

Note: The assumption is made here that if a decision to replace an animal is made in month *i*, the animal is culled at the end of month *i* and the heifer enters the herd the same month.

These absorption states have the same properties as before; once a process enters an absorption state it remains in that state for the remainder of the process (Figure 5.4).

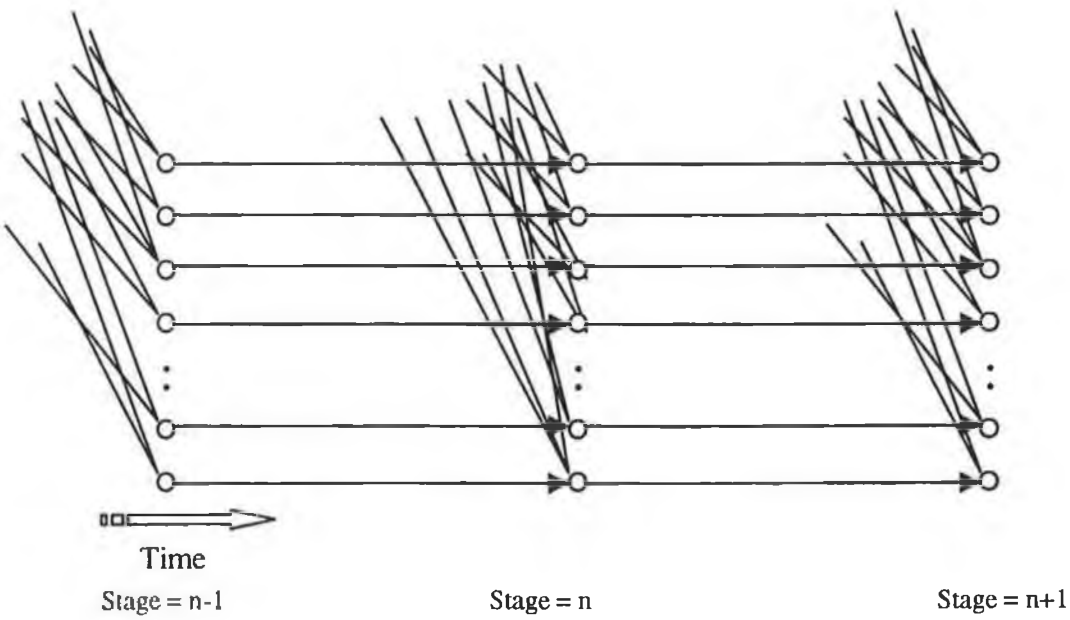

Figure 5.4 Once a process enters an absorption state, it remains in that state for the remainder of the process.

5.2.4 Calculating Main State Transitions

Transition probabilities for the main process, Φ, which are required for Step 2 of the iteration cycle for Hierarchic Markov Processes (Chapter 4.2), now depend on the policies of the subprocesses. For each state in the main process (month of entry), one must calculate the probability that it ends in the ith absorption state λ_i ($i = 1,\ldots,12$). These main process transition probabilities can be calculated using the following set of recurrent equations for a given κ and for $\iota=1,\ldots,12$.

$$q_i^s(N) = \begin{cases} \beta_i^s(N), & \iota = \kappa \\ 0, & \iota \neq \kappa \end{cases}, \qquad i = 1,\ldots,12$$

$$q_i^s(n) = \beta_i^s(n)\sum_{j=1}^{w_n} p_{ij}^s(n)q_j^s(n+1), \qquad i = 1,\ldots,w_n, \quad n = N-1,\ldots,1$$

$$\Phi_{\iota\kappa}^s = \sum_{i=1}^{w_1} p_i^\iota(0)q_i^s, \qquad \iota = 1,\ldots,12.$$

The main transition probabilities are denoted here as $\Phi_{\iota\kappa}^\sigma$, since these transitions now depend on the current policy σ. These transition probabilities must be calculated for each iteration of our cycle, as the policy σ changes. These formulae are identical for all 12 subprocesses, except for the third line (which differs since $p_i(0)$ is dependent on subprocess) and therefore all $\Phi_{1\kappa}^\sigma, \ldots, \Phi_{12\kappa}^\sigma$ can be calculated simultaneously for κ.

5.2.5 Seasonal Effect under the Average Reward / Output Criterion

Under the average reward/output criterion described in Chapter 4.2.3, the recurrent equations described (Chapter 5.2.4) can be used to calculate the main state transitions, but no discounting is applied.

i.e. $\beta_i^s(n) = 1 \quad \forall \; i, s, n.$

A further change to the iteration cycle described in Chapter 4.2.3, is required in the first equation of Step 3 of the cycle (Kristensen(1996)). This change is required as the terminal value T_ι now depends on the final state of the subprocess (i.e. which absorption state is occupied). The new equation is as follows:

$$\tau_{\alpha,i}(n) = \max_d \{r_i^d(n) - m_i^d g^\sigma + F_i^\sigma\} \qquad ,n = N$$

Where F_i^σ is the ith of the relative values for the main process calculated in Step 2 of the iteration cycle. The full iteration cycle for a Hierarchic Markov Process where the criterion of optimality is average reward/output and where seasonality is included in the model is shown in Appendix C.

5.2.6 The Impact of Seasonality on the Complexity of a Model

Under the formulation described, the model consists of 12 subprocesses. Usually, under a Hierarchic Markov Process design, each subprocess would have different rewards ($r_i^d(n)$), physical outputs ($m_i^d(n)$) and transition probabilities $P_{ij}^d(n)$. This, however is not the case for this formulation, since 'month' is included as a state variable in the subprocesses *as well as* the main process. All parameters in the model are in fact the same, with the exception of the initial probability distribution ($p_1(0),\ldots,p_\omega(0)$) which is dependent on the subprocess ι. Because of this, the initial probabilities should be indexed by subprocess (i.e.$p_1^\iota(0),\ldots,p_\omega^\iota(0)$ for subprocess ι).

Since month is included as a state variable in the subprocesses, the policy for each subprocess will also be the same. Computationally, the consequences of this are that the new policy determined in Step 3 of the Hierarchic Markov Process iteration cycle need only be calculated for one process. Since only $P_i(0)$ differs between subprocesses, many of the calculations in Step 2 of the iteration cycle can be carried out simultaneously. The calculation of h_ι^σ and f_ι^σ for the 12 subprocesses are identical with the exception of the 3rd line. So in each case only this 3rd line need be applied to each subprocess individually.

The inversion of a 12*12 matrix is now needed where it would not in the absence of seasonality in the model. While the inversion of such a small matrix would not be a cause for concern, if other variables were included as main state variables, difficulties may arise. If a new main state variable with q possible values was defined, this would result in the inversion of a $12q*12q$ matrix. So, although the inclusion of seasonality results in an increase in the state space by a factor of 12, the consequences are very small, since almost all calculations can be done simultaneously for all subprocesses.

Chapter 6

A Hierarchic Model for Irish Dairy Herds

6.1 State Variables, Stages and Decisions

6.1.1 State Space in the Main Process

A model was designed and applied to the Irish dairy replacement problem, which included in the decision making process low production, fertility, calving interval, seasonality, month of calving and various economic factors. This hierarchic model consisted of twelve subprocesses built into one main process. Each state in the main process (corresponding to a subprocess) corresponded to a 'month of entry into the herd' (as described in Chapter 5.2).

6.1.2 Stages in the Model

With a Hierarchic Markov Process design, stage length in the main process will depend on the decisions made in the subprocesses. In this model, the stage length was not equal for all stages in the subprocesses but was dependent on the stage number. A stage in any of the subprocesses ended and a new one began 0, 2, 3, 4, 5 and 6 months after calving (Figure 5.1) and immediately when an animal was replaced (i.e. stage length = 0 replacement occurs).

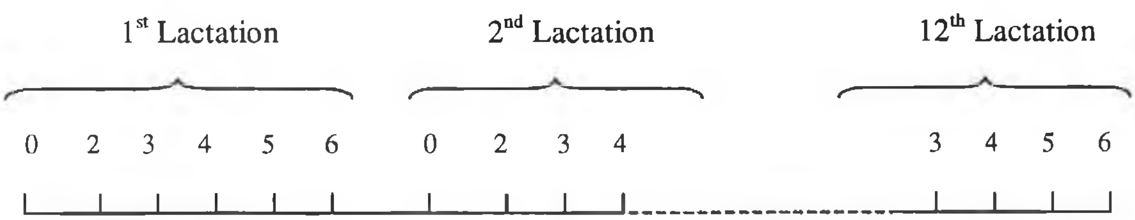

Figure 6.1 Structure of the stages in each subprocess.

The length of a stage beginning 6 months after calving depended on the calving interval of the animal for that particular lactation.

In total a subprocess consisted of 73 stages (6 substages * 12 lactations = 72). The 73^{rd} stage was required to allow transition to an absorption state 6 months after the 12^{th} calving (the maximum lifespan on a cow allowed in the model), and its length is therefore zero (Figure 6.2).

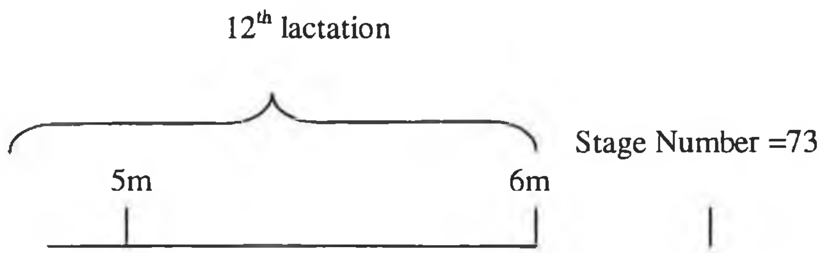

Figure 6.2 Only transitions to an absorption state are possible at the 72^{nd} stage (6 months after 12^{th} lactation).

6.1.3 Replacement and Insemination Decisions

Both replacement decisions and insemination decisions were considered in this model. At the time of calving (0m), the only decisions to be considered were 'keep' and 'replace'. Then, from 2 months after calving, at monthly intervals, up until the 5^{th} month after calving, the following three decisions were possible:

- 'keep' — To keep the animal for a further stage, but not to attempt fertilization.
- 'inseminate' — To keep the animal for a further stage and to attempt to fertilize with a certain probability of success.
- 'replace' — To replace the animal at the end of the current month.

Six months after calving, the decision to inseminate the animal was not considered (Figure 6.3). Additionally, if an animal was found to be 'open' (not pregnant) 6 months after calving, then the decision to 'replace' was taken immediately.

	0m	2m	3m	4m	5m	6m
Keep	x	x	x	x	x	x[1]
Insem		x	x	x	x	
Replace	x	x	x	x	x	x

Figure 6.3 Valid decisions considered at each substage.

6.1.4 State Variables in the Model

Production levels, expressed as a percentage of the mature equivalent were used as a state variable in the model. The model allowed for 15 alternative production levels. These production levels were:

- ≤ 74% of mature equivalent.
- 74 to 78% of mature equivalent.
- 78 to 82% of mature equivalent.

:

:

- 122 to 126% of mature equivalent.
- ≥ 126% of mature equivalent.

[1] Only valid under the condition that the animal is not 'open' at this stage.

The limits and mean values for each of these alternatives were calculated using formulae for 'intensity of selection' described by Van Arendonk(1985b). The mean values, denoted av_m (m = 1,..,15) are calculated using Equation 6.1.

$$av_m = 100 - vc(z(x_m^u) - z(x_m^l))/(p(x_m^u) - p(x_m^l))$$ ***Equation 6.1***

- z(x) = height of distribution ordinate at point x.

  $= \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

- p(x) = proportion with production lower than x.

  $= p(x_m) = \int_{-\infty}^{x} z(t)dt \quad ; \qquad t \approx N(0,1)$

- vc = variation co-efficient of lactation production.

  = 12% (Van Arendonk(1985b)).

- x_m^u = standardized upper limit of level m (x ~ N(0,1)).

  = (y_m – 100)/vc

- x_m^l = standardized lower limit of level m.

  = x_{m-1}^u for m>1 and $-\infty$ for m = 1.

- y_m = upper limit of production level m.

The limits and mean values (calculated using Equation 6.1) for the 15 production levels are shown in Table 6.1.

Production level (m)	Limits (%)	Average (%)
1	74	69.74
2	74 - 78	76.22
3	78 - 82	80.18
4	82 - 86	84.15
5	86 – 90	88.11
6	90 – 94	92.07
7	94 - 98	96.04
8	98 - 102	100.00
9	102 – 106	103.96
10	106 – 110	107.93
11	110 – 114	111.89
12	114 – 118	115.85
13	118 – 122	119.82
14	122 – 126	123.78
15	126	130.26

Table 6.1 The limits and average production levels for the 15 production levels.

The inclusion of seasonality in the model meant that the state variable 'current month' had also to be included in the subprocesses. This state variable had 12 valid alternatives corresponding to the months of the year. This subprocess state variable is distinct from the variable 'month of entry' which is the only state variable in the main process. The value of the main process variable does, however, directly determine the initial probability of the variable 'current month' in a subprocess (Chapter 5.2).

The fertility status of an animal was also considered in culling and insemination decisions. A state variable, which will be referred to here and elsewhere as 'status', was included in the model to allow this. This variable could take on one of five values. The first 4 of these possible values: 2m, 3m, 4m and 5m indicated the stage in the cows current lactation in which the animal had been successfully fertilized. If successful insemination took place, it is assumed that it would be observed the following month. So, if insemination were attempted 2 months after calving, the success or otherwise of the insemination attempt would be observed at the following stage (3 months after calving). The fifth valid value for the variable 'status' was 'open', used to indicate that the animal had not as yet been successfully inseminated.

6.1.5 Invalid States

At some stages of a subprocess, not all the possible state values were valid. For example, 3 months after calving a 'status' of 5m (indicating that the animal in question had been successfully fertilized 5 months after calving) would not be valid. Admissible values for the state variable 'status' are shown in Figure 6.4.

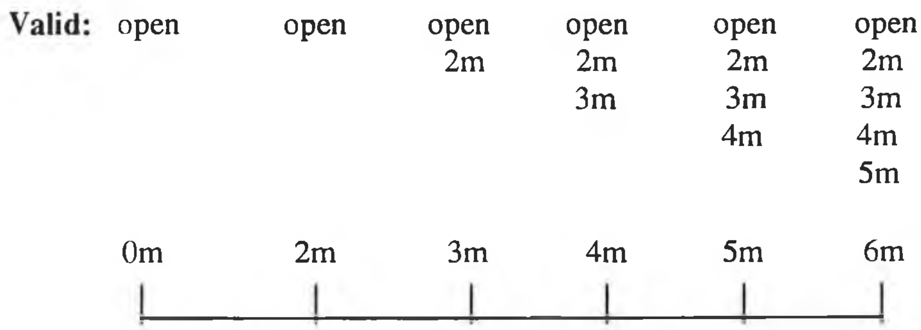

Figure 6.4 Valid values for 'status' for each substage.

6.1.6 Absorption states

In every subprocess and at every stage, 12 additional states (absorption states) were defined. These 12 states were required since the transition probabilities in the main process depended on the policy in the subprocesses (Chapter 5.2). The absorption states were:

- Replace with a heifer entering the herd in January.
- Replace with heifer entering the herd in February.

:

:

- Replace with heifer entering the herd in December.

At any stage, if a process was in any of these absorption states, the stage length at that stage was equal to zero. In each subprocess, the state space was of size 912 (Figure 6.5).

$$\begin{bmatrix}\text{15} \\ \text{production} \\ \text{levels}\end{bmatrix} * \begin{bmatrix}\text{12} \\ \text{`current month'} \\ \text{levels}\end{bmatrix} * \begin{bmatrix}\text{5} \\ \text{`status'} \\ \text{levels}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{12} \\ \text{absorption} \\ \text{states}\end{bmatrix} = 912$$

Figure 6.5 The size of the state space in each subprocess.

6.2 Transition Probabilities

6.2.1 Transitions in Production Level

Transitions in production level were assumed, in the model, to take place at the end of lactation. A cow would remain in the production level occupied at the start of its current lactation until the time of next calving. The initial distribution ($p_i(0)$) and transition probabilities for the production level for a cow in production level *m* at the end of a lactation was defined as in Van Arendonk(1985b). The initial probabilities $p_i(0)$ are calculated using Equation 6.2 (where the notation is as used for Equation 6.1).

$$p_m(0) = \int_{x'_m}^{x''_m} z(t)dt \qquad \textbf{\textit{Equation 6.2}}$$

$$= p(x''_m) - p(x'_m)$$

For a cow in production level *m* in the current lactation, the probability of transition to production level *m'* in the next lactation can be calculated for the following equations (Equation 6.3).

$$P_{mm'} = p(x''_{m'}) - p(x'_m) \qquad \textbf{\textit{Equation 6.3}}$$

- $x''_m = ((y_{m'} - 100) - \hat{a}_m) / vc'$
- $\hat{a}_m = (av_m - 100)b$
- $vc' = vc\sqrt{(1-b^2)}$

- $x^{l}_{m'} = x^{u'}_{m'-1}$ for m'>1 and $-\infty$ for m = 1

6.2.2 Transitions in the Variable 'Current Month'

At any stage *n*, under a decision *d*, only one transition in 'current month' is possible. These deterministic transitions were based on the stage length at *n* under decision *d*. For the month of calving (0 months), the transition probabilities for the state variable 'current month' where the decision to 'keep ' was taken were:

$$P_{mn}(0m) = 1 \quad \text{iff} \quad n = (m+2) \bmod 12$$
$$= 0 \quad \text{otherwise}$$

Where the stage number corresponds to 2, 3, 4 or 5 months after calving, a stage length of one month is observed and transitions in 'current month' where the decision was is to 'keep' or 'inseminate' were:

$$P_{mn}(2m, 3m, 4m, 5m) = 1 \quad \text{iff} \quad n = (m+1) \bmod 12$$
$$= 0 \quad \text{otherwise}$$

Six months after calving, if the animal was still found to be open, the decision taken had to be to replace the animal. If, however, the animal was found to be pregnant at this time the decisions 'keep' and 'replace' were valid. If the decision was taken to 'keep' (i.e. until the next calving), transition probabilities for the variable 'current month' were dependent on the calving interval (which determines the stage length 6 months after calving). The calving interval and thus, the stage length 6 months after calving could be calculated from the 'status' of the cow. From this variable we know how many months after the last calving conception occurred.

The earliest time when conception could occur in the model was 2 months after calving, which gives a minimum calving interval of 11 months (a gestation period of 9 months) and therefore, a minimum stage length (for a stage corresponding to 6 months after calving) of 5 months (Figure 6.6)

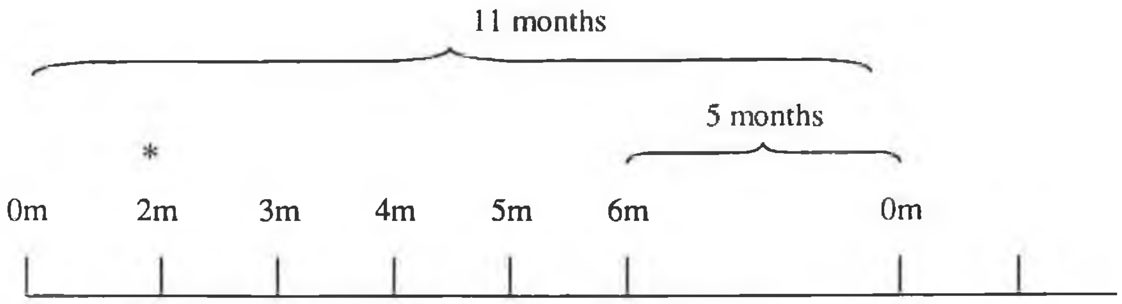

Figure 6.6 A calving interval of 11 months.

Similarly, if conception occurred 2 months after the earliest possible date (i.e. occurred 4 months after calving), the resulting calving interval would be 13 months, resulting in a stage length 6 months after calving of 7 months (Figure 6.7).

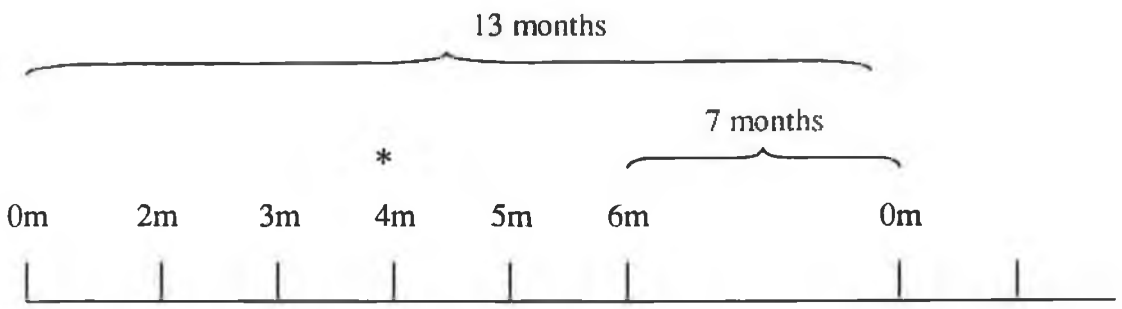

Figure 6.7 A calving interval of 13 months.

The transition probabilities (for decision = 'keep') for the variable 'current month' 6 months after calving follow directly from these stage lengths. i.e.

P_{mn} (6m)	=	1	iff	n = (m+5+'status') mod 12
	=	0	otherwise	

6.2.3 Transitions in Fertility Status

The probability of successful conception, if the decision to inseminate is taken, has been found to improve the later after calving that insemination is attempted (Figure 6.8)

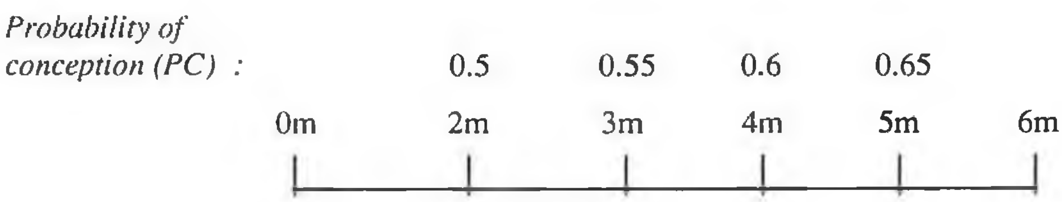

Figure 6.8 Probabilities of conception.

If the decision to inseminate an animal between 2 and 5 months after calving was taken, transition probabilities for the state variable 'status' were calculated from these probabilities of successful fertilization:

P_{sr} (n = 2m, 3m, 4m, 5m)	=	PS (n) iff	*r* = 'status' level corresponding to the current substage
	=	1 – PS (n) iff	*r* = 'open'

The decision to inseminate was only valid at stage *n* if the animal was 'open' at that stage (i.e. s = open). If the decision taken was to 'keep' the animal for an additional stage, but not to attempt insemination, the transition probabilities for the variable 'status' were:

$$P_{sr}\ (n = 0m, 2m, 3m, 4m, 5m) \quad = \quad 1 \quad \text{iff} \quad s = r$$
$$= \quad 0 \quad \text{otherwise}$$

For stages corresponding to 6 months after calving, if the animal is open, it is said to be infertile and 'replace' is the only valid decision. Otherwise, the actions 'keep' and 'replace' were valid at these stages.

6.2.4 Transition Probabilities for the Decision = 'Replace'

If the decision was to 'replace' the animal at the end of the current month, transitions to the absorption states were dependent on the current month (i.e. if replacement occurs in May, transition to the absorption state corresponding to June occurs). These probabilities have already been shown in Chapter 5.2 (Equation 5.2).

6.3 Physical Outputs and Rewards

6.3.1 Materials and Methods

A computer program called the Moorepark Dairy Planner, originally described by Walsh(1995), was used for this analysis. The program was modified for this project taking cognisance of recent developments in milk production technology. The developments in milk production technology in recent years are described in detail by Crosse(1996); Dillon(1996); Dillon and Crosse(1997); Gordon(1996); Mayne(1996); O'Farrell et al(1997); Stakelum(1997). The Moorepark Dairy planner is a computer programme by which the dairy farm manager can calculate the effect of a range of decisions and management practices on factors such as milk production, milk composition, feed inputs, seasonality of milk output, seasonality of feed input, inputs of variable and fixed costs and margin over feed costs and margin over all costs.

6.3.2 Physical Outputs

Under the average reward / output criterion, which was the criterion of optimality applied in this model, $m_i^d(n)$ is the milk yield for a cow in state i at stage n when a decision d is taken. In the model, the mature equivalent lactation production was set at 5500 litres. In the calculation of $m_i^d(n)$ this figure had first to be adjusted for the average of the production level associated with the state i (Table 6.1). This figure was then adjusted for the lactation of the animal, which could be calculated from the stage number n. The lactation adjustment multipliers are shown in Table 6.2

Lactation	1	2	3	4	5	6	7	8	9	10	11	12
Adjustment	.77	.83	1.0	1.0	.99	.95	.94	.93	.92	.91	.90	.89

Table 6.2 Adjustment factors for lactation production .

The month of calving at any stage *n* and for any state *i* could easily be calculated by backstepping the appropriate number of months (depending on the stage in lactation) from the month associated with state *i* (Figure 6.9).

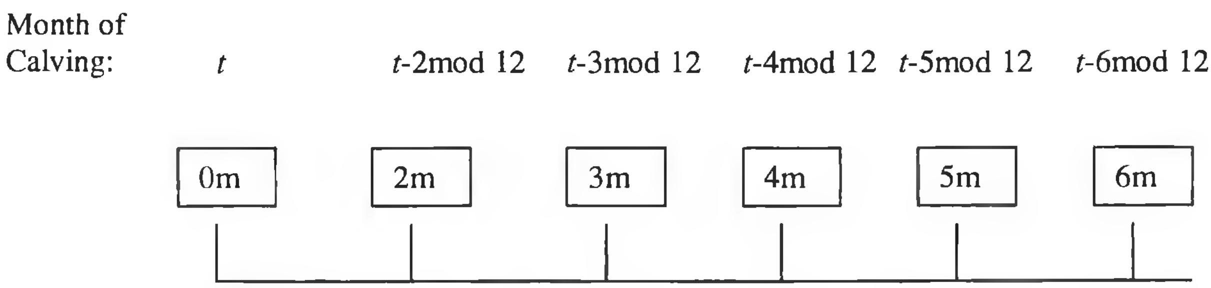

Figure 6.9 Calculation of month of calving, where *t* is the value of the variable month for state *i*.

The production for the lactation associated with stage *n* was then adjusted for month of calving (multiplication factors shown in Table 6.3).

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adjustment	1.0	.98	.96	.95	.95	.97	.98	1.0	1.01	1.01	1.006	1.004

Table 6.3 Adjustment factors for lactation production.

This lactation production was then divided between the stages of the lactation. The distribution of production over a lactation is dependent on the month of the year in which calving takes place (For example, Figure 6.10 shows the cumulative production when calving takes place in January and in August)

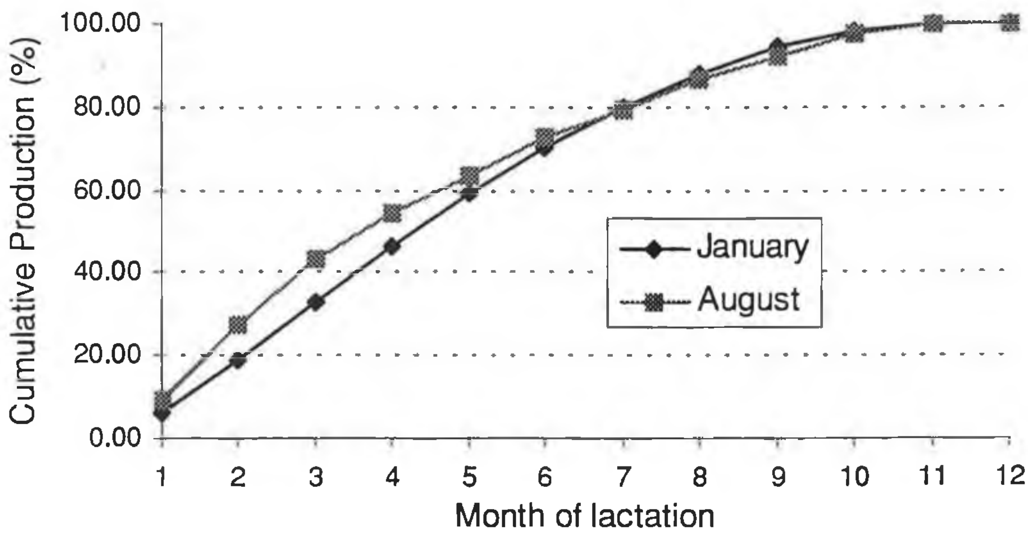

Figure 6.10 Cumulative production (%) for two different months of calving.

The lactation curves for month of calving are given in full in Appendix D. For stages of length greater than one, cumulation of these proportions was necessary in the calculation of the milk yield of that stage.

6.3.3 Calculation of Gross Margin

The immediate expected rewards $r_i^d(n)$ for all states i (a combination of production level, fertility status and month) at all stages n (a combination of lactation and stage in lactation) and for all decisions d (keep, inseminate or replace) had to be calculated. A gross margin was first calculated for all (i,n,d). The gross margin model included

- Income from milk production
- Calf Sales
- Feed Costs
- Sundry Costs.

With the information from the gross margin model the immediate expected rewards for all (i,n,d) could be calculated. The gross margin ($Gm_i(n)$) was calculated as:

$Gm_i(n)$ = Milk Revenue – Feed Costs – Sundry Costs + Calf Revenues

6.3.4 Income from Physical Output

The income from milk yield for state i at stage n under decision d were based on a milk price, adjusted for fat and protein content. Standard fat and protein curves were used (Appendix E). For every month, an associated fat and protein yield was calculated using these curves. The milk price could then be adjusted for these figures.

Adjusted Milk Price = Standard Milk Price + Fat Correction + Protein Correction.

Where Fat Correction $= \left(\left(\frac{\text{Total Fat Content}}{\text{Total Yield}}\right) - 0.036\right) * 0.24$

Protein Correction $= \left(\left(\frac{\text{Total Protein Content}}{\text{Total Yield}}\right) - 0.033\right) * 0.43$

6.3.5 Calf Sales

At the time of calving, calf sales were included in the gross margin. Calf value was adjusted for month using the multiplication factors of Table 6.4.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adjustment	1.2	1.2	.90	.90	.90	.90	.80	.75	.95	1.0	1.1	1.2

Table 6.4 Adjustment factors for calf sale revenues.

6.3.6 Feed Costs

Feed costs were calculated on the basis of

- Grass usage
- Silage usage
- Concentrate usage

Grass, the cheapest form of feed, is in supply during the summer but not in the winter months (Figure 6.11).

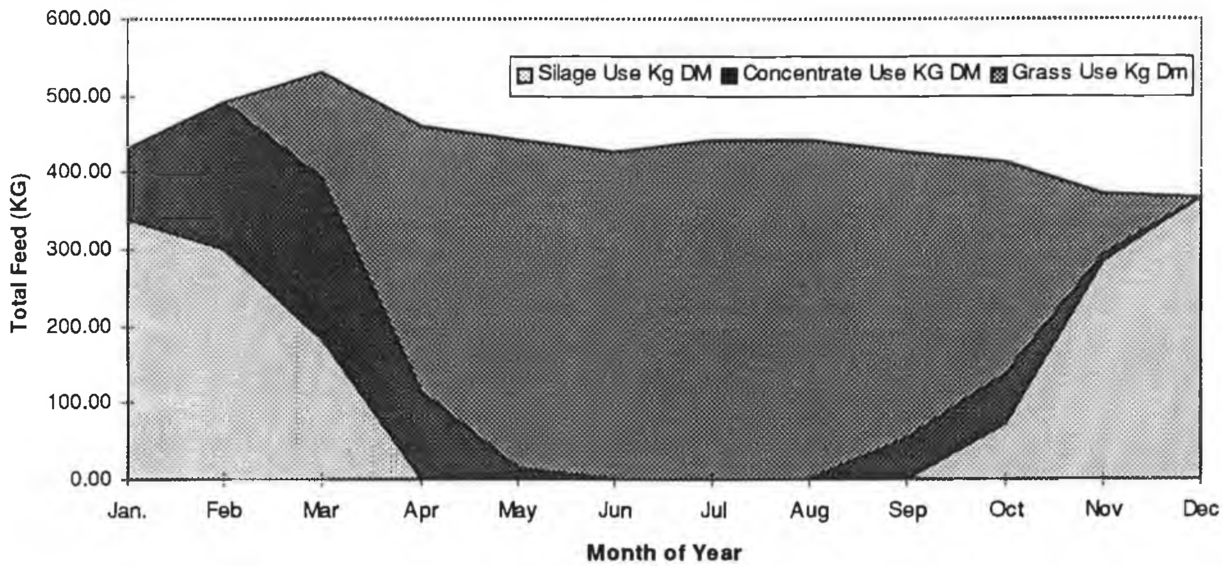

Figure 6.11 The seasonality of the different feed factors.

The seasonality of these feed supply patterns were handled in the model as follows. Depending on the month of calving (calculated as in Figure 6.9), the grass, silage and concentrate needed for the current lactation of the cow was calculated (Table 6.5). Depending on the stage of lactation and the stage length at *n*, the proportion of necessary grass, silage and concentrate for the lactation required at stage *n* were calculated (the proportions used are presented in Appendix F).

Month	Grass needed /cow /kg DM	Silage needed /cow /kg DM	Concentrate needed /cow /kg DM
January	2950	1550	750
February	3300	1425	570
March	3400	1400	470
April	3300	1400	500
May	3000	1620	550
June	2970	1589	650
July	2835	1634	750
August	2600	1600	1100
September	2500	1600	1200
October	2525	1650	1075
November	2600	1750	880
December	2700	1750	780

Table 6.5 Grass, silage and concentrate requirements for each month of calving.

Grass and silage used was also adjusted for lactation (Table 6.6).

Lactation	1	2	3	4	5	6	7	8	9	10	11	12
Adjustment	.83	.97	1.0	1.0	1.0	.99	.98	.97	.96	.95	.94	.93

Table 6.6 Adjustments in Grass and silage requirements for lactation.

6.3.7 Sundry Costs

Sundry costs were also allowed for in the model. In the basic model, additional costs of IR£ 320 for 12 months were allowed for each cow in the herd. This results in monthly additional costs of IR£ 26.66. At stage *n*, the sundry costs were then based solely on the stage length of *n*, under decision *d*.

6.3.8 The Calculation of Immediate Expected Rewards

With the information from the gross margin model, the immediate expected rewards for state *i*, at stage *n*, when decision *d* is taken, were calculated as follows:

$$r_i^{keep}(n) = ((1 - PIV(n)) * GM_i(n)) + (PIV(n)*((CV(n) - LIV) - HC))$$

$$r_i^{insem}(n) = ((1 - PIV(n)) * GM_i(n)) + (PIV(n)*((CV(n) - LIV) - HC)) - IC$$

$$r_i^{replace}(n) = GM_i(n) + CV(n)$$

where $PIV(n)$ is the marginal probability of involuntary disposal at stage *n*.
$GM_i(n)$ is the gross margin at stage *n* for state *i*.
$CV(n)$ is the carcass value at stage *n*.
LIV is the loss in carcass value due to involuntary culling.
HC is the cost of a replacement heifer.
IC is the cost of insemination.

6.3.9 Marginal Probability of Involuntary Disposal

The marginal probability of involuntary disposal at stage *n* was calculated on the basis of lactation number and stage length. These probabilities were estimated from the DairyMis records described in Chapter 2.1. Infertility, late calving, low production, old age and surplus were not considered as involuntary culling as these management decisions were included in the model. The probability of involuntary disposal at lactation *l* is give in Table 6.7 for *l* = 1,..,12.

l	1	2	3	4	5	6	7	8	9	10	11	12
	0.0580	0.0437	0.0542	0.0669	0.0785	0.1072	0.0936	0.1294	0.1500	0.1621	0.1624	0.2008

Table 6.7 The probability of involuntary disposal for lactation *l*.

The proportion of involuntary culling in each month of lactation was then calculated on the basis of culling date and date of disposal. These proportions (up to a maximum 15 months allowed in the model) are shown in Table 6.8.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0.1542	0.0825	0.0590	0.0654	0.0609	0.0584	0.0527	0.0761	0.0685	0.0577	0.0438	0.0774	0.0615	0.0438	0.0381

Table 6.8 The proportion of involuntary culling which takes place in month k of lactation.

Chapter 7

Optimal Replacement Policies for Irish Dairy Herds

7.1 Interpretation of Results

7.1.1 The Optimal Ranking of Dairy Cows

The output from the Hierarchic model is a series of rankings. The dynamic programming approach can enable one to inform a farmer which cows in the herd should be replaced, on the basis that a replacement heifer (and its future successors) are expected to be more profitable than the current cow. However, in many situations it is more relevant to the dairy farmer to know which cow in the herd is the least profitable, rather than which animals in particular should be replaced.

Often, replacements are determined by the calvings of new heifers (such situations often arise when only home reared heifers are used in the herd). In such a situation, the availability of a ranking of the animals in the herd is more important than an optimal policy. If a replacement is then to take place, the least profitable cow in the dairy herd should be replaced. Rankings for all dairy cows defined in the model can be calculated from Step 3 of the Hierarchic Markov Process iteration cycle (Chapter 4.2.3). This ranking (retention payoff (Houben et al, 1994)) can be calculated as:

$$RPO_i(t) \quad = \quad \max(\, \tau_i(\text{n, keep}) \,,\, \tau_i(\text{n, insem}) \,) \quad - \quad \tau_i(\text{n, replace})$$

So, the retention payoff for a cow in state i at time t ($RPO_i(t)$) is the expected future profit from keeping (or inseminating and keeping) the cow for an additional stage, rather than replacing it at the end of the current month. An example of these rankings for stage n=17 (six months after the third calving) is shown in Figure 7.1[1]. At this stage, the option to inseminate is not available; if an animal is found to be open at this stage then the only available decision is to replace that animal. In terms of the optimal

[1] These rankings result from the implementation of the model for Irish Dairy herds described in the previous chapter. These and other results are analysed and discussed in the following Section (7.2).

Calving Interval (months)	Class of Milk Yield	Current Month											
		0	1	2	3	4	5	6	7	8	9	10	11
13	0	-158	-37	81	215	222	214	193	131	69	39	-26	-142
13	1	-134	-8	119	252	259	259	240	177	111	75	4	-117
13	2	-118	12	145	278	284	287	270	206	138	99	24	-101
13	3	-100	34	172	305	311	317	300	236	165	123	44	-83
13	4	-81	57	200	333	339	347	330	266	193	147	65	-66
13	5	-61	82	229	362	368	377	361	295	222	173	87	-47
13	6	-39	108	259	393	398	408	391	325	250	198	109	-27
13	7	-16	136	289	423	429	439	422	356	279	224	132	-6
13	8	8	164	321	455	460	470	453	386	308	251	156	16
13	9	34	193	352	487	491	501	484	416	337	278	180	39
13	10	61	223	384	519	523	532	515	446	366	305	205	63
13	11	89	253	416	551	555	563	546	477	396	333	230	88
13	12	117	284	448	583	587	594	577	507	425	360	255	113
13	13	146	315	480	615	619	625	608	537	454	388	281	140
13	14	193	364	531	667	670	675	657	585	502	433	324	183

Figure 7.1 The expected gains if the cow is kept 6 months after 3rd calving, where the animal has a calving interval of 13 months.

policy, a negative *RPO* means that the cow in question should be replaced at the end of the current month, whereas a positive *RPO* would mean that the animal should be kept until the time of next calving (since the next stage is zero months after fourth calving).

7.1.2 Including Herd Level Effects

The difficulty with the direct application of an optimal policy from a dynamic programming model to a dairy herd is that these models are single-component, whereas a dairy herd is in fact a multi-component system (Kristensen(1992)) . By this it is meant that, in practice, the culling decision for a cow in a herd does not only depend on the state of that cow but on the state of other cows in the herd. Examples of such 'herd-level effects' could be a limited supply of heifers or a quota constraint. A quota acts as such a constraint since, although a cow may have a positive retention payoff, at the herd level, keeping this cow may result in over-production for which penalties may accrue.

While a multi-component system (dairy herd) may be formulated as an ordinary Markov process (this is demonstrated by Kristensen(1992)), the resulting model would be far too large to be solved by any known methods. An approximate method to include herd-level effects called 'Parameter iteration' was introduced by Ben-Ari and Gal(1986). The model of Ben-Ari and Gal(1986), which included only 180 states ,was then improved and analysed by Kristensen(1992). Both of these models attempted to include the possibility of replacement heifer shortage.

An effort to include quota as a herd level constraint was introduced by Houben et al(1995). A genetic algorithm (Davis, 1991) was used to attempt to calculate optimal herd composition. This genetic algorithm used the results of the dynamic programming model of Houben et al(1994). The object of the model was to optimise the herd value (HV), HV being defined as the sum of expected future economic profitability of all cows, determined by a dynamic programming model. The chromosome used in the genetic algorithm had a binary alphabet, and was split into two parts, the first holding the decision on whether individual animals should be kept, and the second holding the decision on herd size. The final gene in the chromosome, a marked gene, could not be mutated while the rest could. A check was needed to ensure that a chromosome could not reflect a situation where the advised reduction of herd size was larger than the number of cows to be culled immediately. Key parameters for the model, crossover probability and mutation probability, were found by measuring the performance of the operator over a recent interval. Using a dataset of 16 cows, experiments showed that good results were found when the crossover probability was about 0.6 and the mutation probability was 0.01.

The model, when implemented, was found to be robust and quick, but only included the herd level effect of quota for the current year and did not account for the effect herd composition in the current year might have on herd composition (and herd profitability) in future years. This meant that, generally, the lowest ranking cows in the herd were culled until the quota constraint was satisfied.

7.2 Optimal Replacement and Insemination Decisions

7.2.1 Basic Results

The parameters used in the Hierarchic model for Irish dairy herds are shown in Table 7.1. The model calculated optimal culling and insemination policies under these parameters.

	£ IR
Carcass value	400
Price of replacement heifer	800
Base price of milk /Litre	0.22
Cost of grass /Kg DM	0.028
Cost of silage /Kg DM	0.085
Cost of concentrates /Kg DM	0.17
Sundry costs /cow /year	320
Insemination cost	10
Loss in carcass value due to involuntary disposal	50
Other	
Mature Equivalent Production (L)	5500
Age at first calving (months)	24

Table 7.1 The basic economic parameters applied in the dairy replacement model

For each run of the model, certain results could be calculated, as described in Chapter 2.3.5, describing the optimal solution. In Chapter 2.3.5 the calculation of these technical results were described for the policy iteration method. However, they can be applied analogously to Hierarchic Markov Processes, the method used in this study. The technical results calculated for the optimal policy were:

- Average milk yield, per cow, per year
- Average replacement rate per year

- Average number of calves born, per cow, per year
- Average return from milk, per cow, per year
- Average feed costs, per cow, per year
- Average gross margin, per cow, per year
- Average calving interval, per cow, per year

These technical results, calculated under the optimal policy (and where the economic inputs of Table 7.1 were applied) are presented in Table 7.2

Basic Results		
Average milk yield /cow /year	(L)	5257.89
Average replacement rate /year	(%)	17.8
Average calves born /year /cow	(number)	1.112
Average return from milk /cow /year	(£IR)	1157.7
Average feed costs /cow /year	(£IR)	320.63
Average gross margin /cow /year	(£IR)	774.76
Average calving interval /cow /year	(months)	11.7

Table 7.2 Results from the basic model

Examples of the rankings resulting from the optimal replacement and insemination policy found by the model are shown in Figure 7.2. In Figure 7.2, only the cows which were 'open' 3 months after their 4^{th} calving are shown. Because these animals are 'open' at this stage, the option to 'inseminate' the animal was considered valid by the model. As before, a negative ranking at stage n, for a cow in state i ($RPO_i(n)$) means that the optimal decision is to 'replace'. A positive $RPO_i(n)$ means that the optimal decision is to keep the animal for at least a further stage. Where the optimal decision is to keep the animal for a further stage, but not to attempt insemination (i.e. the decision = 'keep'), the ranking is marked with the symbol ⊗. It can be seen that, at this stage, in almost all cases where it is optimal to keep the cow for a further stage, it is also optimal to attempt insemination. Only for those animals

Class of Milk Yield	Current Month											
	J	F	M	A	M	J	J	A	S	O	N	D
0	-122	11	30	78	60	104	144⊗	127⊗	155⊗	86⊗	57⊗	-18
1	-94	52	69	127	112	155	183	153	168⊗	94⊗	61⊗	-13
2	-68	80	96	160	145	187	210	172	175⊗	99⊗	63⊗	-11
3	-40	110	126	194	179	220	237	192	183⊗	104⊗	66⊗	-8
4	-9	141	158	229	213	253	267	217	191⊗	111⊗	69⊗	-5
5	22	173	191	265	248	286	297	242	207	118⊗	71⊗	-2
6	55	206	226	301	283	320	328	267	226	125⊗	74⊗	1
7	89	241	260	337	319	354	359	293	246	132⊗	77⊗	24
8	124	276	296	374	355	389	391	319	265	139⊗	86	56
9	159	312	332	410	391	423	423	348	285	157	105	90
10	195	349	368	447	427	458	455	377	308	174	131	124
11	231	386	405	484	463	493	487	407	334	198	159	159
12	268	423	442	521	499	528	519	437	362	227	190	194
13	304	460	478	558	535	563	552	467	392	256	222	230
14	363	519	537	617	592	619	604	517	441	305	274	287

Figure 7.2 Rankings for 'open' cows, 3 months after 4th calving, under the optimal policy. All cows, where the optimal decision was to keep but not to inseminate are marked by the symbol ⊗.

with low production levels, and which last calved in the summer months, was it optimal to keep the cow for a further stage without attempting insemination.

7.2.2 The Effect of Production Level

The effect of production level in the model is apparent in Figure 7.2. All animals for which the optimal decision was to 'replace' (negative $RPO_i(n)$) had low production levels. The production level of the dairy cow has been included in almost all dynamic programming models for dairy replacement strategies (Houben et al(1994); Kristensen(1985); Kristensen(1987); Van Arendonk(1986)). In this model, as in others, the inclusion of production level had a considerable effect on the optimal replacement strategy. Table 7.3 gives the average retention payoff associated with each of the 15 production levels considered in this model. These average figures are only used to illustrate the positive effect of high production on ranking, and do not include the probability of realising any of these states.

Production Level *	Average *RPO*
≤ 74%	49.1876
74 – 78%	70.4887
78 – 82%	84.9217
82 – 86%	100.5150
86 – 90%	117.1652
90 – 94%	134.9231
94 – 98%	153.8353
98 – 102%	173.7557
102 – 106%	194.6619
106 – 110%	316.5100
110 – 114%	239.1527
114 – 118%	262.4493
118 – 122%	286.3136
122 – 126%	310.4110
≥ 126%	350.0683

* Relative to the mature equivalent.

Table 7.3 Average retention payoff for each of the 15 production classes under the optimal policy.

It can be seen in Table 7.3 that the average retention payoff increases with production level. This would be expected, as the higher relative production of the higher classes is not offset by additional expenses. Therefore, the higher production levels should perform better (higher RPO's) under the average reward/output criterion.

The effect of production level in the model was then studied by reformulating the model so that production level was not included as a state variable. This reduced the size of the state space by a factor of 15. Removing production level as a variable was equivalent to assuming all cows were in the production class associated with 100% of the mature equivalent (5500 L). The technical parameters calculated in this situation are presented in Table 7.4 [2].

Technical Result	Basic Situation [1]	No Production Levels [2]	No Production Transitions [3]
Av. Milk yield /cow /year	5257.89	5178.44	5615.558
Av. Replacement rate %	17.8	15.76	24.57
Av. No. calves /cow /year	1.112	1.09233	1.1756
Av. Return from milk /cow /year	1157.7	1140.20	1236.64
Av. Feed costs /cow /year	320.63	319.17	321.34
Av. Gross margin /cow /year	774.76	726.80	957.22
Av. Calving Int. /cow /year	11.7	11.7	11.69

[1] 15 Production levels, with transitions in these levels.
[2] No production levels included in the model.
[3] 15 production levels, but with no transitions allowed.

Table 7.4 Technical results for three different production level types in the model.

In the absence of a state variable for production level, there was a drop in the replacement rate under the optimal policy. This would indicate that production level should be included in the dairy replacement model, as many of the replacement decisions are made on the basis of the production level of the cow (i.e. different decisions are optimal for cows with all the same traits, except for production level). When production level is included as a state variable, the optimal policy involves culling of low producing cows and keeping higher producing cows, which results in higher average milk yield (5257.89 l) than the situation where the production level of the cow is not included in the model (5178.44 l). This lower average milk production when production level is not considered, results in lower returns from milk production, and in lower average gross margins (Table 7.4).

The inclusion of transitions in production levels in the model was also studied. The model was reformulated so that transitions in production levels could not occur. Once a heifer entered the herd at a certain production level, it remained at that level for the remainder of its time in the herd. Technical results calculated in this situation are also presented in Table 7.4. Since low producing animals had to remain in their initial production levels, culling in these lower production levels increased. This resulted in a much higher replacement rate (24.57%) and a large increase in the average milk yield of cows under this optimal policy (again, reflected in higher returns from milk production and gross margins). The higher culling rate, meant that the number of calves born also increased. When transitions in production levels were removed, 11.5% of the optimal decisions (keep, inseminate or replace) differed from the situation where they were included. This indicates that the inclusion of these transitions has a considerable effect on optimal policy and should be included in the model.

7.2.3 The Effect of Seasonality on Optimal Policy

The seasonality of milk production in Ireland, which was included in the model as explained in earlier chapters, is inevitably reflected in the optimal culling and replacement policies found. Considering Figure 7.2 again, only for animals which were in the 3rd month of there 4th lactation in January or December it was optimal to 'replace'. Similarly the optimal insemination policy was highly seasonal; the option to 'keep' was only optimal in Figure 7.2 during the months July – November. Under the optimal policy, these cows would be kept in the herd, without attempting insemination until a heifer could replace a cow in the herd at a more profitable time. The effect of month of calving is illustrated in Figure 7.3, again using average retention payoffs. As can be seen in Figure 7.3, cows that calved in the summer months had the lowest average retention payoffs, whereas those that calved in the early months of the year had the highest.

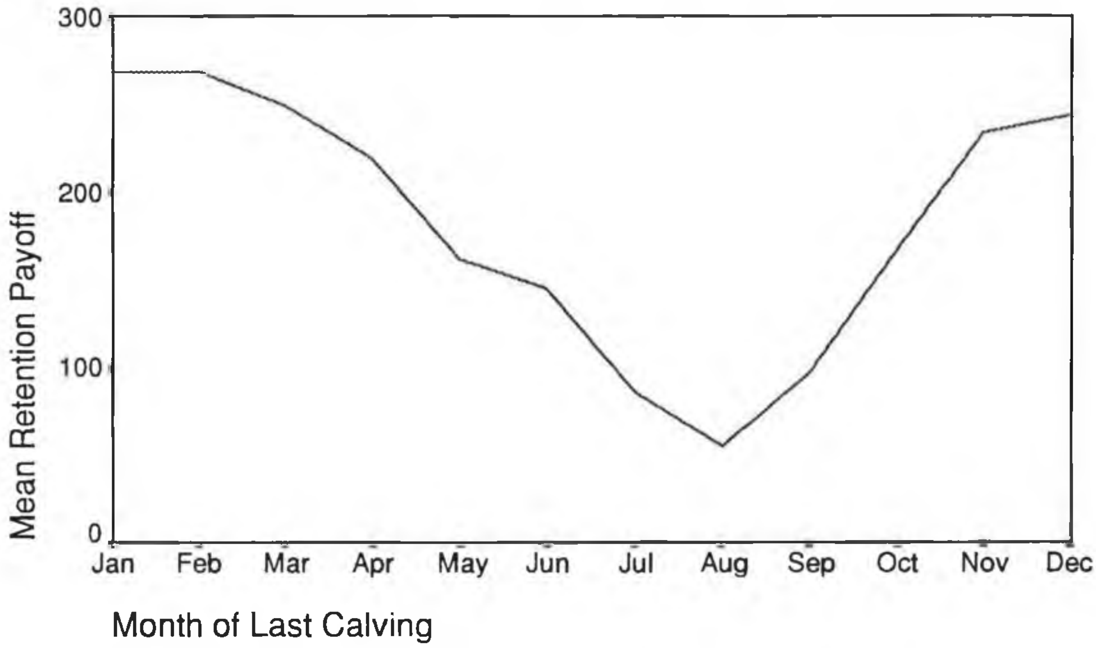

Figure 7.3 Average RPO by month of last calving.

The effect of including seasonality in the model is also illustrated in Figure 7.4, where a simple count of the number of 'open' cows for which the optimal decision was to 'inseminate' rather than to 'keep' or 'replace' is shown. Stages where the option to 'inseminate' was not valid were not considered. It can be seen in Figure 7.4, that the majority of decisions to inseminate took place in the summer months. This is be expected as insemination at this time results in Spring calving, which in turn takes advantage of the Irish grass growth pattern. The influence of the seasonality on insemination policy (Figure 7.4) and the large variance in the average RPO's for each month of calving (Figure 7.3) show clearly that seasonality should be included in any culling model for the Irish dairy Industry.

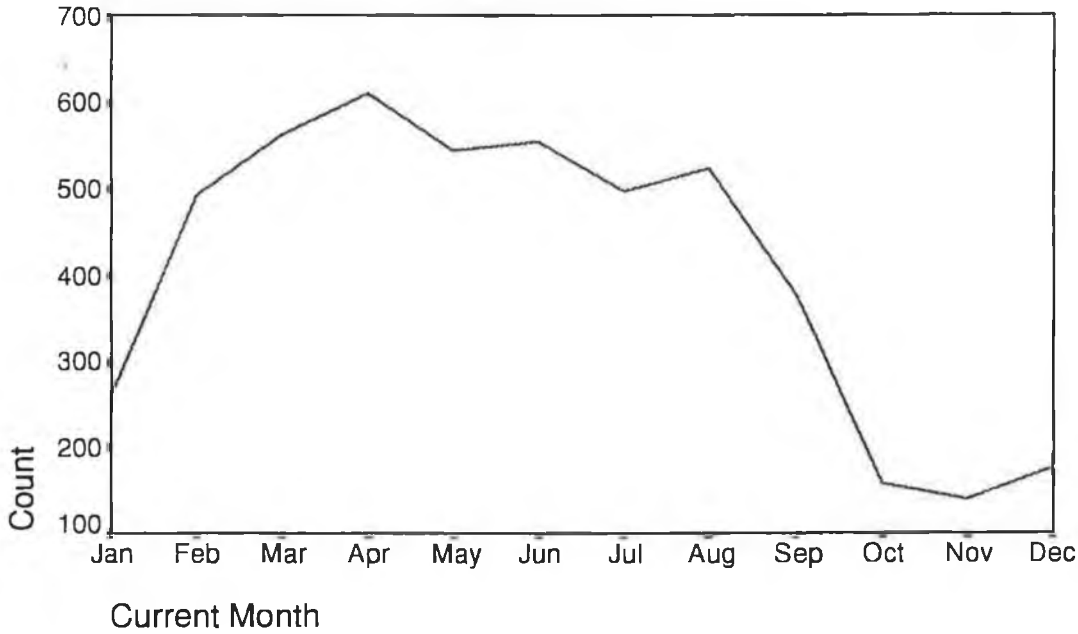

Figure 7.4 A count of the number of 'open' cows where the optimal decision was to inseminate.

7.2.4 Including Conception Rates

In the model, the probability of a successful conception was said to be dependant on the stage in lactation. The later in lactation that insemination was attempted, the higher the probability of that insemination being successful. Where the option to inseminate was valid, and where the decision to keep the animal for a further stage (i.e. positive *RPO*) was optimal, it was found that under the optimal policy insemination was attempted in 68.24% of cases.

The effect that the uncertainty of insemination had on the optimal policy was studied by changing the model so that if insemination was attempted at any (valid) stage, the probability that it would result in fertilisation was equal to 1. Because of this change, where insemination decisions were not based on uncertainty, the decision to inseminate could by delayed until the most profitable time. Under these circumstances, insemination was only carried out in 48.78% of cases where the optimal policy was to keep the cow for a further stage. The removal of the uncertainty of successful insemination also had a large affect on the technical results of the model.

These parameters are shown in Table 7.5, and it can clearly be seen that if conception is certain when attempted, the herd parameters improve greatly. The average milk yield per cow per year has increased to 5914.2 litres, an increase of 12.5% and gross margin, per cow, per year has increased by 23.27%. The calving interval also substantially decreased in length and more selective culling was carried out (replacement rate of 19.07%).

	Basic Situation [1)]	Conception Rate =1 [2)]
Av. Milk yield /cow /year	5257.89	5914.19
Av. Replacement rate %	17.8	19.07
Av. No. calves /cow /year	1.112	1.22
Av. Milk return /cow /year	1157.7	1301.65
Av. Feed costs /cow /year	320.63	310.567
Av. Gross margin /cow /year	774.76	955.11
Av. Calving Int. /cow /year	11.7	11.117

Table 7.5 Technical results; [1)] where uncertainty of conception is included in the model and [2)] where conception always occurs when attempted.

7.2.5 Lactation and Stage of Lactation

The stage in the lifetime of the cow had a considerable affect on its *RPO*. In Figure 7.5, the average retention payoff of animals in the 12 possible lactations in the model are shown. The highest average *RPO*'s were found in the second and third lactations. The average *RPO* then decreased, as the cows got older.

Replacement and insemination decisions within lactation were also allowed in the model, and had a large effect on optimal policy. If within-lactation decisions had not been allowed in the model, then decisions would have to be made only at the end of each lactation. If this were the case, optimisation of insemination decisions would not be possible in the model.

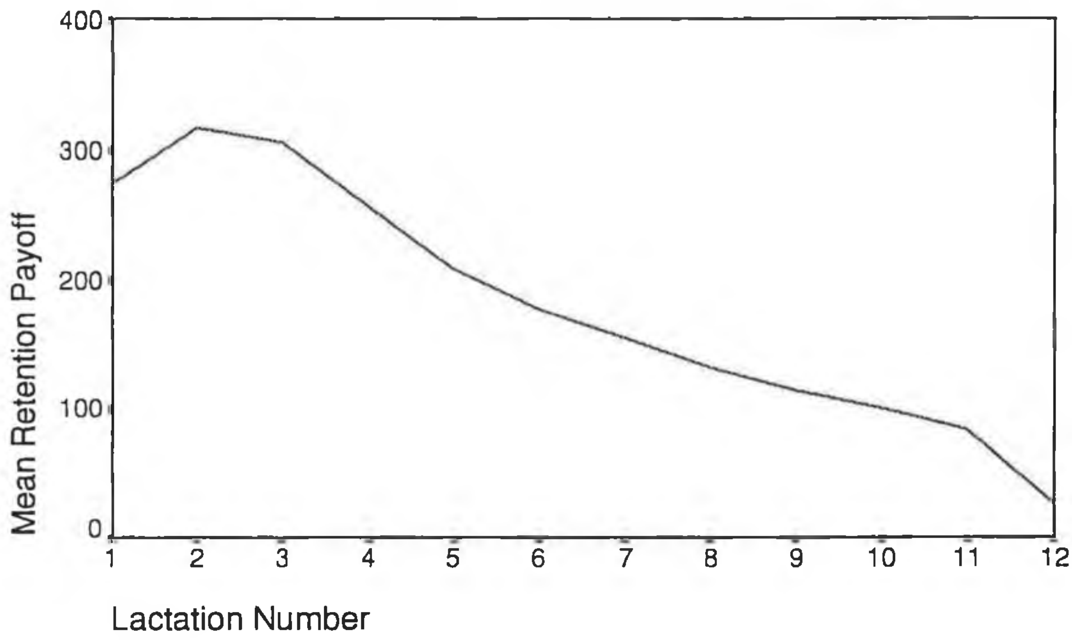

Figure 7.5 Average RPO by lactation number.

In Table 7.6, the percentage of cases for each substage where the three permissible decisions were optimal under the optimal policy are presented (only 'open' cows were considered, as the decision to inseminate was not valid for any other fertility status).

Decision	Substage (months after calving)					
	0	2	3	4	5	6
Keep	88.9	28.2	25.8	25.5	26.2	-
Inseminate	-	63.2	60.9	55.7	47.3	-
Replace	11.1	8.5	13.3	18.8	26.6	100[1]
	100 %	100 %	100 %	100 %	100 %	100 %

[1] Six months after calving, if the cow was found to be 'open', the only valid decision was to replace

Table 7.6 The percentage of decisions to keep, inseminate and replace for open cows at different stages of lactation.

It can be seen in Table 7.6 that the majority of replacement decisions were taken in the later stages of lactation. Because of this, returns from the most productive stage of a cow's lactation (in terms of milk yield) could be accrued.

7.2.6 Replacement Costs

In other dynamic programming models designed for dairy replacement, optimal policies were found to be sensitive to changes in the costs and revenues associated with culling. These costs and revenues are the cost of a replacement heifer and the carcass value of the cow being replaced respectively. Sensitivity analysis was carried out using the hierarchic model for these two parameters. In each case, the optimal policy where the parameter was set at 90% and 110% of its base value (Table 7.1) were studied. The technical results from the optimal policy resulting from three replacement heifer costs are shown in Table 7.7

Technical Parameter	Cost of replacement heifer as a % of basic cost		
	90%	100%	110%
Av. Milk yield /cow /year	5281.49	5257.89	5240.64
Av. Replacement rate %	20.66	17.8	16.12
Av. No. calves /cow /year	1.134	1.112	1.100
Av. Milk return /cow /year	1162.82	1157.7	1153.88
Av. Feed costs /cow /year	319.11	320.63	322.13
Av. Gross margin /cow /year	825.92	774.76	743.15
Av. Calving Int. /cow /year	11.7	11.7	11.7

Table 7.7 Technical results from optimal policy under 3 price conditions for replacement heifers.

When the cost of the replacement heifer was 90% of the basic value, it was optimal to have more strategic culling in the herd (20.66%). This increased the average milk yield per cow to 5281.49 litres and the gross margin to IR£ 825.92 (an increase of 6.6% from the basic replacement cost). An increase in the replacement cost (110% of the basic replacement cost) had the opposite affect on the technical parameters calculated from the optimal policy. The replacement rate was now 16.12% and the gross margin was IR£ 743.15 (a decrease of 4.08%). The replacement cost was found to have no effect on the average calving interval per cow, which remained at 11.7 months, and to have only a marginal effect on the number of calves born per year.

The technical results from the optimal policies for carcass value at differing levels is shown in Table 7.8. As with changes in replacement cost, changes in the value of the cull cow were found to have a considerable affect on the optimal policy and its results. The effects of these changes were not as large as changes in replacement costs. When the value of the cull cow's carcass was 90% of the basic situation, the associated replacement rate was 16.71% and the average gross margin IR£ 755.18 (a decrease of 2.5%).

Technical Parameter	Carcass value as a % of basic		
	90%	100%	110%
Av. Milk yield /cow /year	5251.36	5257.89	5267.80
Av. Replacement rate %	16.71	17.8	19.61
Av. No. calves /cow /year	1.10	1.112	1.13
Av. Milk return /cow /year	1156.16	1157.7	1159.96
Av. Feed costs /cow /year	321.89	320.63	319.27
Av. Gross margin /cow /year	755.18	774.76	806.17
Av. Calving Int. /cow /year	11.7	11.7	11.7

Table 7.8 Technical results from optimal policy under 3 price conditions for the carcass value of a culled animal.

When the value of the cull cow's carcass was 110% of the basic situation, the associated replacement rate was 19.61% and the average gross margin IR£ 806.17 (an increase of 4%).

Chapter 8
Conclusions

8.1 Conclusions

The purpose of this research has been to investigate methodologies for determining optimal culling decisions on dairy farms, with particular reference to farms operating in the Irish dairy industry. The typical technique applied for replacement problems, where the traits of the asset in question are affected by random variation over time and between assets (e.g. animal replacement problems), is dynamic programming. Traditional dynamic programming methods, the value iteration method (Bellman(1957)) and the policy iteration method (Howard(1960)) were considered as possible optimisation methods to be developed for this model. However, due to the size of the state space required when using these techniques for animal replacement problems, the hierarchic approach was taken. Hierarchic Markov Processes help solve the 'curse of dimensionality' associated with the policy iteration method (and to a lesser extent the value iteration method), without having to make any assumptions on the length of the planning horizon.

The model developed allowed decisions to be made at certain stages of a cow's lactation. Replacement decisions were considered at the time of calving and 2, 3, 4, 5 and 6 months after calving. Insemination decisions were made 2, 3, 4 and 5 months after calving. These replacement and insemination decisions were made on the basis of level of production, fertility, calving interval, season, month of calving and various economic factors. The seasonality of grass production in Ireland meant that it was necessary to include seasonality in the model. This required some reformulation of the Hierarchic Markov Process iteration cycle described in Chapter 4 (Kristensen(1997)) and the necessary enhancements to the iteration cycle were outlined in Chapter 5.

The output from the model, described in Chapter 7, was a series of rankings, based on retention payoffs (*RPO*). *RPO* was calculated as the expected future profit from keeping (or keeping and inseminating) a cow for an additional stage, rather than replacing her at the end of the current month. These *RPO*'s allowed animals in a herd to be ranked, and for culling decisions in a herd to be made on the basis of these rankings (i.e. the lowest ranked cows in the herd would be culled first).

Several variations in the model were tested (e.g. with and without transitions in production level), and for each resulting optimal policy (and ranking), certain technical parameters were calculated. These technical parameters were then used to analyse the effect of changes in the model on optimal policy. It was found that, if transitions in the state variable 'production level' were not allowed, the optimal policy changed greatly (the optimal policy was found to be different in 11.5% of cases). The model was also reformulated with the variable 'production level' not included, and this resulted in the replacement rate increasing from 17.8% to 24.57%. The optimal policy was also found to be highly seasonal, which was to be expected as lactation curves, based on month of calving, for milk yield, protein yield, fat yield and feed costs, were included in the model.

When insemination was attempted in the model, conception occurred with a certain probability of success. The effect of this uncertainty of conception was studied by subsequently removing it from the model. This showed that in fact the inclusion of the uncertainty of conception had a considerable impact on the model results. Higher milk yields per cow, shorter average calving intervals and more calves per cow could be achieved when insemination could be assumed to be always successful. The effect of the costs and revenues associated with culling were also studied. When the cost of a replacement heifer was 10% lower, the optimal policy resulted in a replacement rate of 20.66%, as compared with 17.8% in the basic model.

8.2 Future Research

The dynamic programming approach does not allow inclusion of herd-level effects and further research into including these herd-level effects in decision support for culling decisions would be useful. The model of Houben et al(1995), which used a search algorithm (genetic algorithm) to maximise herd value (HV), was a contribution to this, but in that work, the impact of quota (the herd level constraint) was only considered in the current year. The model of Houben et al(1995) drew on results from an earlier dynamic programming model (Houben et al(1994)) which included culling decisions based on mastitis incidence. In the study of culling rates on DairyMIS farms described in Chapter 2.1, decisions to cull due to mastitis accounted for 12.14% of all culling decisions. It would be possible to extend the model described in this thesis to consider culling due to mastitis as a 'voluntary culling' decision. However, considerable research would be needed into the calculation of transition probabilities for variables associated with mastitis, and in the economic effects of mastitis incidence.

The method of Hierarchic Markov Processes allows replacement models with large state spaces to be solved exactly. A further extension of this technique is described by Kristensen and Jørgensen(1996), and is called 'Multi -level Hierarchic Markov Processes'. This new technique takes advantage of the fact that transitions in some variables are not possible at all stages (e.g. the variable for production level in our study). Kristensen and Jørgensen(1996) used a sow replacement example to illustrate how this technique might be applied, though the model was not actually implemented. At this time, the algorithm for Multi-level Hierarchic Markov Processes has not been included in an animal replacement model, but is another extension of this thesis which would be worthy of investigation.

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Appendices

Appendix A
Monte Carlo Simulation

A simulation model is simply a model of a system, which is used for the study of the real system's behaviour under different conditions. The inputs for a simulation model consist of a set of parameters Φ, and a set of decision rules Θ.

The parameter set $\Phi = (\Phi_0, \Phi_s)$, where Φ_0 are the initial values of the parameters at the start of calculation (state of nature), and Φ_s are parameter values that change during simulations (state variables). As state variables should be considered at each stage of the simulation, $\Phi_s = (\Phi_{s1}, \Phi_{s2}, \ldots, \Phi_{sT})$ where T is the number of stages in the planning horizon.

The decision rules Θ specify the setting of input factors as well as other decisions within the system. A decision rule can, for example, be simply a straightforward rule of thumb to make culling decisions or it could implement the results of a Dynamic Programming model for culling decisions.

The purpose of a simulation model is to calculate the expectation of a response function, e.g. the expected utility

$$\overline{U}_M(\Theta) = \int_{-\infty}^{\infty} U_M(\Theta, \Phi = \varphi)\, p(\Phi = \varphi)\, d\varphi$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_M(\Theta, \varphi_s \backslash \Phi_0 = \varphi_0)\, p(\Phi_s = \varphi_s \backslash \Phi_0 = \varphi_0)\, p(\Phi_0 = \varphi_0)\, d\varphi_s\, d\varphi_0$$

(***Equation A.1***)

where $\overline{U}_M(\Theta)$ is the utility function under the model M. This can refer to any response function of the output variables.

There have been two categories of simulation models implemented within animal production in literature, stochastic models and deterministic models. The stochastic nature of the system is ignored in deterministic models. Stochastic models can be either probabilistic models (e.g. Markov Chains, Bayesian networks) or Monte Carlo models, the simulation technique used to model Irish dairy herds here.

Monte Carlo techniques rely on the drawing of random numbers. Every time the model encounters a stochastic variable, a (pseudo-) random variable is drawn from the appropriate distribution and this value is used in the subsequent calculations. Each completed calculation (simulation run) with the model represents a random drawing from the simultaneous distribution of input and output variables. By increasing the number of calculations, the distribution of the output variables can be specified to any degree of precision. The expected utility is found from :

$$\overline{U}_M(\Theta) \approx \frac{1}{k}\sum_{i=1}^{k} U_M(\Theta, \varphi_i)$$

where φ_i is a random drawing from the multidimensional distribution of the parameters, and k is the number of random drawings (Figure A.1).

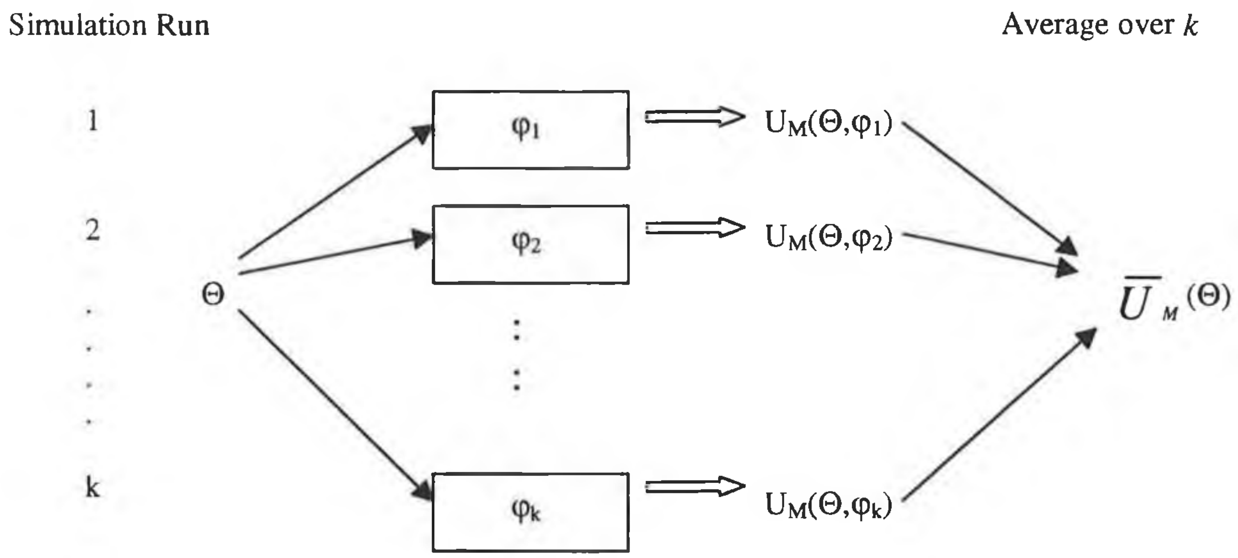

Figure A.1 Monte Carlo simulation.

How a simulation model is formulated will depend on the purpose for which it is intended. The simulation model may be intended to help improve the understanding of a complex system, or for decision support.

If, as in this case, the purpose of the model is to improve the understanding of a complex system, then for Φ_0 (the initial state of nature), we have a fixed and known set of parameters. For this scenario, the expected value of the utility function can be calculated as:

$$\overline{U}_M(\Theta \backslash \Phi_0 = \varphi_{0j}) \approx \frac{1}{k}\sum_{i=1}^{k} U_M(\Theta, \varphi_{si} \backslash \Phi_0 = \varphi_{0j})$$

i.e. Only the inner part of the integrand in Equation A.1 must be calculated.

Appendix B

Transition Probabilities for a 36- State Model

p_{ij}^{keep}				1	2	3	4	5	6	7	8	9	10	11	12
				Bad Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1	B	1	L				0.6	0.3	0.1						
2			A				0.2	0.6	0.2						
3			H				0.1	0.3	0.6						
4		2	L							0.6	0.3	0.1			
5			A							0.2	0.6	0.2			
6			H							0.1	0.3	0.6			
7		3	L										0.6	0.3	0.1
8			A										0.2	0.6	0.2
9			H										0.1	0.3	0.6
10		4	L	1/9	1/9	1/9									
11			A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13	A	1	L												
14			A												
15			H												
16		2	L												
17			A												
18			H												
19		3	L												
20			A												
21			H												
22		4	L	1/9	1/9	1/9									
23			A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25	H	1	L												
26			A												
27			H												
28		2	L												
29			A												
30			H												
31		3	L												
32			A												
33			H												
34		4	L	1/9	1/9	1/9									
35			A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B1 Transition probabilities from state *i* to state *j* under the action 'keep' (*j*=1,…,12)

p^{keep}_{ij}				13	14	15	16	17	18	19	20	21	22	23	24
				Average Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1			L												
2		1	A												
3			H												
4			L												
5		2	A												
6	B		H												
7			L												
8		3	A												
9			H												
10			L	1/9	1/9	1/9									
11		4	A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13			L				0.6	0.3	0.1						
14		1	A				0.2	0.6	0.2						
15			H				0.1	0.3	0.6						
16			L							0.6	0.3	0.1			
17		2	A							0.2	0.6	0.2			
18			H							0.1	0.3	0.6			
19	A		L										0.6	0.3	0.1
20		3	A										0.2	0.6	0.2
21			H										0.1	0.3	0.6
22			L	1/9	1/9	1/9									
23		4	A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25			L												
26		1	A												
27			H												
28			L												
29		2	A												
30			H												
31	H		L												
32		3	A												
33			H												
34			L	1/9	1/9	1/9									
35		4	A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B2 Transition probabilities from state *i* to state *j* under the action 'keep' (*j*=13,…,24)

P^{keep}_{ij}				25	26	27	28	29	30	31	32	33	34	35	36
				Good Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1	B	1	L												
2			A												
3			H												
4		2	L												
5			A												
6			H												
7		3	L												
8			A												
9			H												
10		4	L	1/9	1/9	1/9									
11			A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13	A	1	L												
14			A												
15			H												
16		2	L												
17			A												
18			H												
19		3	L												
20			A												
21			H												
22		4	L	1/9	1/9	1/9									
23			A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25	H	1	L				0.6	0.3	0.1						
26			A				0.2	0.6	0.2						
27			H				0.1	0.3	0.6						
28		2	L							0.6	0.3	0.1			
29			A							0.2	0.6	0.2			
30			H							0.1	0.3	0.6			
31		3	L										0.6	0.3	0.1
32			A										0.2	0.6	0.2
33			H										0.1	0.3	0.6
34		4	L	1/9	1/9	1/9									
35			A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B3 Transition probabilities from state *i* to state *j* under the action 'keep' (*j*=25,…,36)

$P_{ij}^{replace}$				1	2	3	4	5	6	7	8	9	10	11	12
				Bad Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1			L	1/9	1/9	1/9									
2		1	A	1/9	1/9	1/9									
3			H	1/9	1/9	1/9									
4			L	1/9	1/9	1/9									
5		2	A	1/9	1/9	1/9									
6	B		H	1/9	1/9	1/9									
7			L	1/9	1/9	1/9									
8		3	A	1/9	1/9	1/9									
9			H	1/9	1/9	1/9									
10			L	1/9	1/9	1/9									
11		4	A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13			L	1/9	1/9	1/9									
14		1	A	1/9	1/9	1/9									
15			H	1/9	1/9	1/9									
16			L	1/9	1/9	1/9									
17		2	A	1/9	1/9	1/9									
18			H	1/9	1/9	1/9									
19	A		L	1/9	1/9	1/9									
20		3	A	1/9	1/9	1/9									
21			H	1/9	1/9	1/9									
22			L	1/9	1/9	1/9									
23		4	A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25			L	1/9	1/9	1/9									
26		1	A	1/9	1/9	1/9									
27			H	1/9	1/9	1/9									
28			L	1/9	1/9	1/9									
29		2	A	1/9	1/9	1/9									
30			H	1/9	1/9	1/9									
31	H		L	1/9	1/9	1/9									
32		3	A	1/9	1/9	1/9									
33			H	1/9	1/9	1/9									
34			L	1/9	1/9	1/9									
35		4	A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B4 Transition probabilities from state *i* to state *j* under the action 'replace' (*j*=1,...,12)

$P^{replace}_{ij}$				13	14	15	16	17	18	19	20	21	22	23	24
				Average Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1			L	1/9	1/9	1/9									
2		1	A	1/9	1/9	1/9									
3			H	1/9	1/9	1/9									
4			L	1/9	1/9	1/9									
5		2	A	1/9	1/9	1/9									
6	B		H	1/9	1/9	1/9									
7			L	1/9	1/9	1/9									
8		3	A	1/9	1/9	1/9									
9			H	1/9	1/9	1/9									
10			L	1/9	1/9	1/9									
11		4	A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13			L	1/9	1/9	1/9									
14		1	A	1/9	1/9	1/9									
15			H	1/9	1/9	1/9									
16			L	1/9	1/9	1/9									
17		2	A	1/9	1/9	1/9									
18			H	1/9	1/9	1/9									
19	A		L	1/9	1/9	1/9									
20		3	A	1/9	1/9	1/9									
21			H	1/9	1/9	1/9									
22			L	1/9	1/9	1/9									
23		4	A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25			L	1/9	1/9	1/9									
26		1	A	1/9	1/9	1/9									
27			H	1/9	1/9	1/9									
28			L	1/9	1/9	1/9									
29		2	A	1/9	1/9	1/9									
30			H	1/9	1/9	1/9									
31	H		L	1/9	1/9	1/9									
32		3	A	1/9	1/9	1/9									
33			H	1/9	1/9	1/9									
34			L	1/9	1/9	1/9									
35		4	A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B5 Transition probabilities from state *i* to state *j* under the action 'replace' (j=13,…,24)

$P^{replace}_{ij}$				25	26	27	28	29	30	31	32	33	34	35	36
				Good Genetic Merit											
				1st lactation			2nd lactation			3rd lactation			4th lactation		
				L	A	H	L	A	H	L	A	H	L	A	H
1			L	1/9	1/9	1/9									
2		1	A	1/9	1/9	1/9									
3			H	1/9	1/9	1/9									
4			L	1/9	1/9	1/9									
5		2	A	1/9	1/9	1/9									
6	B		H	1/9	1/9	1/9									
7			L	1/9	1/9	1/9									
8		3	A	1/9	1/9	1/9									
9			H	1/9	1/9	1/9									
10			L	1/9	1/9	1/9									
11		4	A	1/9	1/9	1/9									
12			H	1/9	1/9	1/9									
13			L	1/9	1/9	1/9									
14		1	A	1/9	1/9	1/9									
15			H	1/9	1/9	1/9									
16			L	1/9	1/9	1/9									
17		2	A	1/9	1/9	1/9									
18			H	1/9	1/9	1/9									
19	A		L	1/9	1/9	1/9									
20		3	A	1/9	1/9	1/9									
21			H	1/9	1/9	1/9									
22			L	1/9	1/9	1/9									
23		4	A	1/9	1/9	1/9									
24			H	1/9	1/9	1/9									
25			L	1/9	1/9	1/9									
26		1	A	1/9	1/9	1/9									
27			H	1/9	1/9	1/9									
28			L	1/9	1/9	1/9									
29		2	A	1/9	1/9	1/9									
30			H	1/9	1/9	1/9									
31	H		L	1/9	1/9	1/9									
32		3	A	1/9	1/9	1/9									
33			H	1/9	1/9	1/9									
34			L	1/9	1/9	1/9									
35		4	A	1/9	1/9	1/9									
36			H	1/9	1/9	1/9									

Table B6 Transition probabilities from state *i* to state *j* under the action 'replace' (*j*=25,…,36)

Appendix C

The Iteration Cycle for Hierarchic Markov Processes [1]

[1] where the average reward / output criterion is applied and seasonality is included in the model

Step 1 Choose an arbitrary policy p.

Step 2a For $\kappa = 1,..12$, find for $\iota = 1,..12$ the main transition probability $\Phi_{\iota\kappa}^{p}$ under policy p.

$$q_i^p(N) = \begin{cases} 1, & \iota = \kappa \\ 0, & \iota \neq \kappa \end{cases}, \qquad i = 1,\dots,12$$

$$q_i^p(n) = \sum_{j=1}^{W_n} p_{ij}^p(n) q_j^p(n+1), \qquad i = 1,\dots,W_n, \quad n = N-1,\dots,1$$

$$\Phi_{\iota\kappa}^p = \sum_{i=1}^{W_1} p_i^p(0) q_i^p, \qquad \iota = 1,\dots,12.$$

Step 2b Solve the following set of linear simultaneous equations for F_1^p, $F_2^p, \dots, F_v^p$ and g^p:

$$g^p h_\alpha^p + F_\alpha^p = f_\alpha^p + \sum_{k=1}^{v} \phi_{\alpha k} F_k^p \qquad \alpha = 1, \dots, v$$

Step 3 For each subprocess α, find by means of the recurrence equations a policy s' of the subprocess. Put $p'(\alpha)=s'$ for $\alpha = 1,\dots,v$.

$$\tau_{\alpha,i}(n) = \max^d \{reward_i^d(n) - output_i^d(n) g^p + F_i^p\},$$
$$n = N$$

$$\tau_{\alpha,i}(n) = \max^d \{reward_i^d(n) - output_i^d(n) g^p + \sum_{j=1}^{U_{n+1}} prob_{ij}^d(n) \tau_{\alpha,j}(n+1)\},$$
$$n = 1,\dots,N-1.$$

If $p' = p$, then stop since an optimal policy is found. Otherwise, redefine p according to the new policy (set $p = p'$). Go to Step 2.

Appendix D

Lactation curves for % milk supply

	Month of Calving											
	JAN	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEPT**	**OCT**	**NOV**	**DEC**
JAN	6.40	0.00	4.00	6.00	6.40	8.00	8.00	8.80	10.90	13.10	13.00	13.70
FEB	12.80	6.50	0.00	4.00	5.60	6.00	7.00	7.10	10.50	10.80	12.00	13.00
MAR	13.90	13.00	7.00	0.00	4.00	5.50	6.00	6.90	9.00	10.80	12.00	13.00
APR	13.50	13.80	13.00	7.50	0.00	3.50	5.00	5.90	8.00	9.30	11.00	12.50
MAY	12.40	13.00	13.00	14.50	7.00	0.00	3.00	5.20	6.70	8.20	10.00	12.00
JUN	10.90	12.00	12.00	13.50	15.00	8.00	0.00	2.50	5.20	6.60	9.00	10.00
JUL	9.90	10.50	11.40	11.50	14.00	14.00	9.00	0.00	2.30	5.00	6.00	8.50
AUG	8.30	9.50	10.50	10.50	12.00	13.00	16.00	9.20	0.00	2.60	4.00	5.00
SEPT	6.40	8.00	9.10	9.00	11.00	12.00	14.00	18.00	8.40	0.00	2.00	2.30
OCT	3.80	6.50	8.00	8.50	10.00	11.00	13.00	16.20	14.00	8.00	0.00	2.00
NOV	1.70	4.00	6.50	8.00	8.00	10.00	10.00	10.60	13.00	13.00	8.00	0.00
DEC	0.00	3.20	5.50	7.00	7.00	9.00	9.00	9.60	12.00	12.60	13.00	8.00
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).

Appendix E

Lactation Curves for Fat and Protein

Lactation curves for Fat %.

	Month of Calving											
	JAN	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEPT**	**OCT**	**NOV**	**DEC**
JAN	3.80	3.60	4.10	4.50	4.20	4.08	3.93	3.73	3.56	3.42	3.40	3.52
FEB	3.60	3.85	3.60	4.50	4.50	4.20	4.02	3.81	3.39	3.46	3.35	3.35
MAR	3.40	3.60	4.00	3.60	4.50	4.50	4.16	3.91	3.79	3.52	3.35	3.35
APR	3.40	3.50	3.60	3.81	3.60	4.50	4.40	4.13	3.94	3.69	3.40	3.36
MAY	3.30	3.45	3.50	3.35	3.81	3.60	4.65	4.42	4.22	4.04	3.60	3.40
JUN	3.30	3.45	3.30	3.14	3.25	3.73	3.60	4.52	4.36	4.02	3.70	3.55
JUL	3.50	3.55	3.58	3.25	3.20	3.32	3.78	3.60	4.65	4.20	4.00	3.70
AUG	3.64	3.65	3.70	3.43	3.29	3.28	3.39	3.87	3.60	4.00	4.40	4.00
SEPT	3.75	3.80	3.86	3.66	3.47	3.39	3.35	3.46	3.98	3.60	4.00	4.73
OCT	4.30	4.00	4.40	4.08	3.82	3.71	3.59	3.54	3.67	4.00	3.60	4.80
NOV	4.30	4.25	4.30	4.52	4.11	4.07	3.90	3.77	3.67	3.40	4.00	3.60
DEC	3.60	4.22	4.00	4.50	4.07	4.14	3.94	3.72	3.65	3.50	3.70	4.00
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).

Lactation curves for Protein %

	Month of Calving											
	JAN	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEPT**	**OCT**	**NOV**	**DEC**
JAN	3.30	3.27	3.60	3.80	3.80	3.61	3.48	3.31	3.18	3.05	2.90	
FEB	2.90	3.30	3.27	3.80	3.80	3.80	3.57	3.38	3.03	3.09	3.00	3.00
MAR	2.80	3.20	3.40	3.27	3.80	3.80	3.64	3.44	3.30	3.11	3.00	2.90
APR	3.15	3.10	3.30	3.47	3.27	3.80	3.98	3.75	3.60	3.37	3.21	2.89
MAY	3.18	3.20	3.20	3.13	3.54	3.27	4.28	4.08	3.91	3.60	3.45	3.08
JUN	3.20	3.20	3.20	3.00	3.10	3.53	3.27	4.24	4.10	3.75	3.61	3.25
JUL	3.20	3.30	3.27	2.95	2.91	3.01	3.41	3.27	4.11	3.80	3.70	3.30
AUG	3.30	3.40	3.40	3.11	2.99	2.98	3.07	3.48	3.27	3.80	4.01	3.40
SEPT	3.50	3.50	3.50	3.42	3.29	3.18	3.13	3.24	3.68	3.27	4.45	3.60
OCT	3.75	3.60	3.85	3.74	3.51	3.41	3.31	3.27	3.38	3.40	3.27	4.00
NOV	3.70	3.60	3.65	4.01	3.65	3.63	3.48	3.48	3.33	3.30	3.40	4.00
DEC	3.27	3.62	3.60	3.87	3.52	3.59	3.42	3.28	3.18	3.00	3.00	3.27
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).

Appendix F
Feed Intake Curves

Lactation Curves for Grass Usage

	Month of Calving											
	JAN	**FEB**	**MAR**	**APR**	**MAY**	**JUN**	**JUL**	**AUG**	**SEPT**	**OCT**	**NOV**	**DEC**
JAN	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FEB	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MAR	4.7	4.7	4.0	3.5	4.1	4.0	5.3	6.2	6.1	6.1	6.0	5.1
APR	11.8	11.3	11.7	11.9	10.1	10.1	12.0	13.2	17.6	15.8	16.0	14.3
MAY	14.5	15.0	14.9	13.4	11.9	12.0	11.0	14.4	16.5	16.3	16.0	16.4
JUN	14.5	14.5	14.5	15.5	15.2	13.0	10.0	10.0	15.9	15.3	15.0	16.4
JUL	15.0	15.0	14.9	14.6	15.7	15.7	16.0	10.0	10.2	14.8	15.0	17.0
AUG	15.0	15.0	15.0	15.5	16.2	16.0	16.7	16.0	9.3	9.2	13.5	14.0
SEPT	12.7	12.7	12.6	12.8	13.7	14.6	15.0	16.0	12.7	9.5	9.0	9.6
OCT	9.3	9.3	9.7	10.0	10.1	11.2	11.0	11.0	8.3	10.2	8.0	7.2
NOV	2.7	2.7	2.8	2.8	3.0	3.4	3.0	3.2	3.4	2.9	1.5	0.0
DEC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).

Lactation Curves for Silage Usage

	Month of Calving											
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEPT	OCT	NOV	DEC
JAN	21.9	22.9	21.0	21.4	23.0	22.5	21.0	22.2	22.3	23.2	21.0	18.0
FEB	19.3	20.3	20.0	20.0	20.4	20.1	21.0	19.0	22.3	23.9	19.4	18.6
MAR	11.9	11.5	12.0	12.0	11.5	12.1	12.9	12.4	11.9	11.6	11.6	10.8
APR	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
MAY	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
JUN	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
JUL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AUG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SEPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
OCT	4.8	4.6	6.0	6.0	6.0	6.1	6.5	6.2	6.5	2.3	10.0	18.6
NOV	18.4	17.8	19.0	19.0	17.0	17.3	17.7	19.2	15.6	15.1	17.0	16.8
DEC	23.8	22.9	22.0	21.6	22.2	21.8	21.0	21.0	21.6	23.9	21.0	17.1
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).

Lactation Curves for Concentrate Usage

	Month of Calving											
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEPT	OCT	NOV	DEC
JAN	12.6	0.0	1.9	13.2	11.3	13.8	8.5	12.4	16.9	18.6	23.0	25.0
FEB	25.7	16.8	0.0	0.8	10.0	8.9	8.4	12.3	16.7	18.8	23.4	25.4
MAR	27.9	32.6	17.6	0.0	0.0	4.5	8.1	8.9	11.5	11.2	15.9	20.7
APR	15.2	20.6	27.7	15.5	0.0	0.0	8.3	0.0	0.0	2.7	3.2	10.5
MAY	1.9	2.6	3.5	12.2	9.1	0.0	0.0	0.0	1.1	1.3	0.0	1.7
JUN	0.0	0.0	0.0	0.0	12.3	7.4	0.0	0.0	0.0	0.0	0.0	0.0
JUL	0.0	0.0	0.0	0.0	0.0	9.7	6.7	0.0	0.0	0.0	0.0	0.0
AUG	0.0	0.0	0.0	0.0	0.0	0.0	8.3	6.5	0.0	0.0	0.0	0.0
SEPT	7.2	9.7	12.9	17.1	12.0	14.0	13.5	13.2	5.0	0.0	0.0	3.4
OCT	8.5	10.0	13.2	17.6	17.2	14.2	12.8	15.7	14.8	10.0	0.0	1.0
NOV	1.1	6.2	14.6	11.6	17.2	14.0	13.0	16.0	17.5	17.9	10.2	0.9
DEC	0.0	1.5	8.5	12.0	11.0	13.5	12.4	15.1	16.7	19.5	24.2	11.5
Total	100	100	100	100	100	100	100	100	100	100	100	100

Source: MoorePark Dairy Planner (Walsh(1995)).