# A STUDY OF THE SPATIAL CHARACTERISTICS OF A LASER PLASMA EXTREME UV CONTINUUM SOURCE FOR ABSORPTION SPECTROSCOPY 

A thesis for the degree of Master of Science

submitted to

## School of Physical Sciences

by

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## Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study to the award of Masters of Science, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed :


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#### Abstract

The spatial characteristics of a laser plasma Extreme UV continuum source used in absorption spectroscopy was examined. The spectroscopic system consists of a toroidal mirror in series with a 2.2 m grazing incidence spectrometer resulting in a quasi-stigmatic imaging system.

The experimental knife-edge response of the system was recorded in both the vertical and horizontal directions for energies of $50,75,100,125$ and 150 eV . Estimates of the spatial resolution of the system were obtained from the various experimental knife-edge responses of the system using the $25-75 \%$ and $10-90 \%$ criteria.

Using the ray tracing software SHADOW, the system was ray traced for each of the five energies above. The experimental knife-edge experiments in both the vertical and horizontal directions were then simulated in a series of virtual knife-edge experiments. The source size defined in SHADOW was varied until optimal correlation was achieved between the virtual and experimental knife-edge response of the system.

The source size that corresponded to best match between the experimental and virtual knife-edge experiments, for the different energies, was then defined in SHADOW and the system was ray traced using this source. The footprint of the rays on two screens, placed at the position of an absorbing plasma, provided estimates of the absorption volume. In any future work, the absorption volume estimates can therefore be used in the measurement of absolute photoabsorption cross-section data.


## Chapter 1

## Introduction

The requirement for cheap, compact and reliable sources of ionised species and XUV/VUV continuum is an important research goal. The laser produced plasma has proven to be one of the most cost effective and versatile sources of both ionised species and, since the discovery of continuum emission from a number of elements, XUV/VUV continuum. The beginning of this section will contain an overview of the laser produced plasma, its properties and the advantages it has over other methods of producing both ionised species and XUV/VUV continuum. Following this, a brief historical overview of the extension of the laser produced plasma into the dual laser plasma (DLP) technique will be presented.

When the output from a high power laser, greater than $10^{8}-10^{9} \mathrm{Wcm}^{-2}$, is focused onto a solid target in a vacuum, a short lived, high density, high temperature plasma is created. For example, a Q-switched Nd:YAG laser oscillator can produce energies of around 600 mJ with a pulse duration of about 15 ns . Such a pulse, focused onto a focal spot size of $\sim 100 \mu \mathrm{~m}$ in diameter, will produce an irradiance on the target of $\sim 5 \times 10^{11} \mathrm{Wcm}^{-2}$. With the inclusion of a single amplifier stage, energies in excess of 1 J delivered in a 15 ns pulse will result in an irradiance of $\sim 10^{12} \mathrm{Wcm}^{-2}$ resulting in electron densities up to $10^{22} \mathrm{~cm}^{-3}$ and electron temperatures up to 100 eV . In these conditions, matter is highly ionised and it is for this reason that the laser produced plasma has become one of the most widely used sources for the study of emission spectra from highly ionised species.

The laser plasma is also increasingly used as a source of XUV/VUV continuum radiation. The importance of the laser plasma continuum source has led it to be described as the "The Poor Man's Storage Ring" by Gullikson [1]. Early emission experiments frequently noted the presence of continuum emission from laser produced plasmas. In 1966, Ehler and Weissler [2] reported the predominance of VUV continuum emission from the higher Z elements of tungsten, tantalum and platinum in comparison to lower Z elements, such as
aluminium, whose VUV spectrum in the same conditions is dominated by line emission. A more complete study of the spectra from the rare-earth and neighbouring elements was completed by Carroll et al. [3]. The elements studied ranged from $\operatorname{tin}(Z=50)$ through to tungsten $(\mathrm{Z}=74)$, excluding xenon $(\mathrm{Z}=54)$ and promethium $(\mathrm{Z}=61)$. The higher Z elements platinum ( $\mathrm{Z}=78$ ), gold $(\mathrm{Z}=79)$, lead $(\mathrm{Z}=82)$, bismuth $(\mathrm{Z}=83)$ and uranium $(\mathrm{Z}=92)$ were also studied. The spectra of these elements were studied in the grazing incidence wavelength region up to $500 \AA$ with particular emphasis on the $40-200 \AA$ wavelength range. The rareearth elements from samarium ( $\mathrm{Z}=62$ ) to ytterbium $(\mathrm{Z}=70)$ were studied in the $500-2000 \AA$ wavelength region using a 3 m normal incidence spectrograph. The results of this study showed that in the $40-200 \AA$ wavelength range, the elements from $\operatorname{tin}(\mathrm{Z}=50)$ to neodymium $(\mathrm{Z}=60)$ contained strong line emission with varying amounts of continuum being present. The rare-earth elements from lanthanum ( $\mathrm{Z}=57$ ) to samarium ( $\mathrm{Z}=62$ ) had decreasing amounts of line emission, with increasing $Z$, until samarium which had essentially no line emission except for a few from impurities. The remaining rare-earth elements from samarium to ytterbium ( $\mathrm{Z}=70$ ) consisted of increasingly more uniform continuum which existed over a wider wavelength range. In the 200-2000 $\AA$ wavelength range, the rare-earth elements from samarium to ytterbium emitted good continuum up to $2000 \AA$ and beyond. The lower Z elements from tin to terbium $(\mathrm{Z}=65)$ contained some lines in this wavelength region but not enough to affect the overall character of the continuum. The results of this study showed that the rare-earth and some neighbouring elements provided a strong, essentially line free source of continuum which is tuneable over the wavelength range $40-2000 \AA$ with a suitable choice of target.

The origin of the continua emitted by the elements $62 \leq \mathrm{Z} \leq 74$, samarium to tungsten, was examined by O'Sullivan [4]. The emission of VUV continua, and the relative absence of lines, is a unique property of these elements. With the average degree of ionisation from the above elements being usually 10 or greater, for a typical irradiance of $10^{10} \mathrm{Wcm}^{-2}$, the origin of the continuum emission is mainly due to recombination (freebound) radiation, especially at shorter wavelengths. The contribution from bremsstrahlung (free-free) radiation varies between 10 and $20 \%$ with the greater contribution being at longer wavelengths [5,6]. The absence of spectral lines in the elements $62 \leq \mathrm{Z} \leq 74$ cannot be explained by plasma opacity alone and a detailed examination of the complex electronic
configurations is required. O'Sullivan discusses these complex configurations in detail and only a brief outline will be given here. In the case of the elements $62 \leq \mathrm{Z} \leq 74$, the low ion stages contain filled $5 s$ and $5 p$ subshells while the number of electrons in the $4 f$ subshell varies with the degree of ionisation and atomic number Z . As the degree of ionisation increases, the binding energy of the $4 f$ subshell increases more rapidly than the $5 s$ or $5 p$ subshells. At around the sixth ion stage, the $4 f$ level crosses the $5 p$ level and it passes the $5 s$ level at around the twelfth ion stage. The elements above therefore have $4 f$ electrons for each ion stage generated in the laser plasma. The proximity of these levels leads to an overlap of complex configurations that contain variable numbers of $5 s, 5 p$, and $4 f$ electrons. The possible transitions associated with these states for the various ion stages can result in hundreds of thousands of lines. As a result of this, the oscillator strengths are weakened enough so that these lines are smeared into a background continuum. The radiation emitted by the rare-earth and neighbouring elements is therefore not a true continuum as it contains a large number of unresolvable lines. High resolution studies in the XUV [7], and in the VUV [8] have shown that even at high dispersion these lines cannot be resolved and for all practical purposes can be considered as a true source of continuum.

Other studies on the properties of the continuum emission have revealed that the maximum emission from the laser plasma is observed when the angle of incidence of the laser and the observation angle are both at approximately $45^{\circ}$ [9-11]. Any angle greater than this leads to a reduction in observed emission intensity and some spectral changes. The dependence of the intensity emitted from a laser plasma with focus and laser energy was examined by Bridges et al. [12]. The laser plasmas were generated using a 0.5J low divergence Nd:YAG laser and a 4J ruby laser on a ytterbium target. In the case of changing the focus, Bridges et al. observed that while the intensity increased with tighter focusing, at the focus there was a dip in intensity. At the best focus, the power density on the target is a maximum and the irradiated area is a minimum. The maximum VUV intensity is observed when there is a slight defocusing, $\sim 2.4 \mathrm{~mm}$ out from the target, whereby the power density is reduced and the irradiated area is slightly larger. A similar effect was observed with the ruby laser and it appears therefore, that a saturation effect exists at the best focus. The variation in emission intensity with respect to the laser energy was investigated for both the tight focus ( 0 mm ) and the focus giving maximum emission intensity ( $\sim 2.4 \mathrm{~mm}$ defocus). The
results showed that there was a general linear dependence between the emission intensity and the incident laser energy. In the case of the ruby laser, with a $\sim 2.4 \mathrm{~mm}$ defocus, the emission output increased approximately linearly and then tailed off to a maximum value again indicating a saturation effect. There are many advantages associated with the laser plasma continua source and a summary of those is as follows [3,6,13-15],

- Ease of production and location : all that is required is a Q-switched laser and a suitable target
- Wide spectral coverage : with a suitable choice of target, a tuneable source of continuum over the $40-2000 \AA$ wavelength range is available
- Small spatial extent : the small spatial extent of the source is useful in experiments requiring spatial resolution
- High pealt intensity
- Purity of continuum
- Reproducibility
- Shortness of output pulses : suitable for time resolved studies
- Low sensitivity to ambient pressure variations : eliminates the need for differential pumping unlike synchrotrons which have many stages in order to protect the storage ring

As stated above, the existence of continuum emission was frequently observed in emission studies. One of the first reports on the absorption of laser plasma continuum by a second laser plasma was by Carillon et al. [16] in 1970. Up until this time, laser plasmas had only been used as a source of highly ionised species for emission studies. In this experiment, two aluminium laser plasmas were generated using a Nd:YAG laser. Aluminium spectra are dominated by line emission but there are a number of small wavelength intervals ( $\leq 10 \AA$ ) where continuum dominates. The small wavelength interval situated around $98 \AA$ was used to measure the time resolved continuum absorption of the radiation from the first aluminium plasma by the second aluminium plasma as a function of distance from the core. The results were recorded for inter plasma delays of 12 and 27 ns . At the short inter plasma delay, they observed a smooth decrease in absorption as they moved away from the plasma core
indicating that inverse bremsstrahlung is the main absorption process. At the longer inter plasma delay, a modulation on the absorption was observed at distances $\mathrm{d} \geq 3 \mathrm{~mm}$ with maximum absorption observed at $\sim 8 \mathrm{~mm}$. The increase in absorption was attributed to the fact that at the longer inter plasma delay and at greater distances from the plasma core, the ions and electrons have started to recombine. At a given distance from the plasma core, these lower stage ions exist in sufficient densities such that, the photoionisation of these ions adds a significant contribution to the overall absorption. With the narrow wavelength range of this particular experiment, its extension into photoabsorption experiments over a broader wavelength range was an important limitation. It did however show that the laser plasma could be used to provide both a source of continuum and a source of ionised species for absorption studies.

In 1977, Carroll and Kennedy [17] carried out the first controlled dual laser plasma experiment, using a broadband laser plasma continuum source, in a study of the photoabsorption spectrum of $\mathrm{Li}^{+}$. Carroll and Kennedy realised the potential of the dual laser plasma technique and utilised the broadband continuum emission from laser plasma created from the rare-earth and neighbouring elements. In their experiment, the output from a Qswitched ruby laser was used to produce both the backlighting continuum plasma, created on tungsten, and the absorbing plasma, created on lithium. The objective of this experiment was to observe the helium like doubly excited states of $\mathrm{Li}^{+}$. In conjunction with the point like backlighting plasma, the lithium target was positioned such that the expanding plasma plume was along the direction of the slit therefore yielding spatial information. In order to maximise the column density of the ground state $\mathrm{Li}^{+}$, the ruby laser beam was slightly defocused and the optimum conditions were found empirically. With this technique, the $1 s^{2} S_{0} \rightarrow 1 \operatorname{snp}{ }^{1} P_{1}$ principal series was recorded up to the $n=7$ member along with the doubly excited $1 s^{2} S_{0} \rightarrow 2 s n p{ }^{1} P_{1}$ series.

The measurement of photoionisation cross sections from photographic plates is difficult. Thus, in order to improve the speed and effectiveness of the dual laser plasma technique, Jannitti et al. [18] incorporated an important modification to their spectroscopic system in the form of a photoelectric detection system. The photoelectric detection system
consisted of a scintillator coated faceplate which is positioned on the Rowland circle. The light emitted by the scintillator is relayed to an image intensifier via a fibre optic bundle. The light from the image intensifier is incident on a photodiode array (PDA) of 512 pixels which is in turn read by an optical channel analyser (OMA). The introduction of this detection system along with the inclusion of a toroidal mirror, the benefits of which are discussed later in this chapter, provides the system with single shot sensitivity along with the ability to directly measure the relative absorption cross sections. This important modification therefore greatly reduces the time taken to produce an absorption spectrum along with the security of being able to observe the emission and absorption spectra in real-time.

Carroll and Costello [19] further enhanced the dual laser plasma technique by using two separate time-synchronised Q-switched lasers. The application of the dual laser plasma technique for obtaining absorption spectra of highly refractory and/or corrosive materials was also outlined in this work. Prior to this experiment, the backlighting and absorbing plasmas were created with a single pulse from the same laser. The single pulse would be divided and any delay between the pulses was introduced by the insertion of an optical delay line. With the introduction of a second laser and electronics to control the triggering of the laser pulses, variable time delays of between $250 \mathrm{~ns} \rightarrow 10 \mathrm{msec}$ could be introduced between the two output pulses of the lasers. A more complete overview of the dual laser plasma technique can be found in the article by Costello et al. [13] and references therein.

Many of the technical improvements outlined above have been incorporated into the multilaser multichannel 2.2 m instrument at Dublin City University (D.C.U.) [20]. A multichannel photoelectric detection system and a toroidal mirror, similar to the modifications implemented by Jannitti at al. above, have been included in the instrument. The instrument also uses two separate time-synchronised Q-switched lasers in order to enhance the dual laser plasma technique, as outlined by Carroll and Costello above. This system set-up allows greater control and flexibility in the spatial and temporal probing of laser plasmas and the single shot sensitivity allows the direct measurement of relative absorption cross sections. Using this spectroscopic instrument, Mosnier et al. [21] studied the inner-shell excitation spectra of $\mathrm{Mg}, \mathrm{Al}^{+}$, and $\mathrm{Si}^{2+}$ and Kiernan et al. [22] reported the first observation of a photon induced triply excited state in atomic Lithium. Other DLP
experiments on this system include the measurement of the photoabsorption of free $\mathrm{La}^{3+}$ ions in the $4 d$ excitation region [23] and a study the Xe and Mg isoelectronic sequences and the Ca isonuclear sequence [24].

The main objective of this work was to obtain an estimate for the volume $V$ of probed plasma that effectively contributes to an absorption spectrum obtained using the DLP technique on the 2.2 m instrument at D.C.U.. In any future work, the absorption volume estimates can therefore be used in the measurement of absolute photoabsorption cross-section data. The method used involved a combination of knife-edge experiments on the system and similar experiments using ray-tracing and this method will be outlined in Chapter 2. Using the results of the experiments and ray-tracing, it was also possible to estimate the spatial extent of the source that contributed to the final image at the detector. The ray-tracing of the system also allowed the footprints on the various system components to be observed.

In this chapter, Section 1.1 outlines how, with the introduction of a toroidal mirror, the speed and efficiency of the concave grating can be improved through the reduction of astigmatism. Section 1.2 describes the 2.2 m spectroscopic instrument at Dublin City University (D.C.U.) along with a brief description of its individual components. The final section details the objectives of this work along with the methods used to achieve them.

### 1.1 The Stigmatic Spectrometer

This section will begin with a brief description of the astigmatism associated with using a concave grating at grazing incidence. The remainder of the section details how, with the introduction of a toroidal mirror, the speed and efficiency of the concave grating can be improved through the reduction of astigmatism.
H. A. Rowland originally developed the idea of the concave diffraction grating in 1882. The function of the concave grating was not only to disperse the incident radiation but
to also collect and focus it. Since the development of the concave grating, considerable theoretical work has been completed by Beutler [25] and Namioka [26] using geometrical optics and by Mack, Stehn and Edlén [27] using physical optics. A more in-depth development of the theory of the concave grating is contained in Appendix 1. In the VUV region of the spectrum, the reflectance or transmittance of optics is very small. Thus, in order to obtain appreciable reflectivities only reflecting optics may be used at grazing angles of incidence.

At grazing incidence, the concave grating suffers from severe aberrations with astigmatism being the most significant (fig. 1.1). The effect of astigmatism is to spread the light, arriving at the Rowland circle, along long spectral lines. This spreading of the light along the spectral lines results in a loss of intensity.


Figure 1.1 Diagram illustrating the astigmatism of the concave grating when used at grazing incidence where $r_{1}^{\prime}$ is the location of the primary focus and $r_{2}^{\prime}$ is the location of the secondary focus measured from the centre of the grating. $S$ is the entrance slit.

In 1959, W. A. Rense and T. Violett [28] developed a method of increasing the speed of a grazing-incidence spectrograph used to photograph the ultraviolet spectrum of the sun. They did this by using a toroidal mirror in series with a concave grating. Using a toroidal mirror in series with a concave grating can produce, with a suitable choice of optical parameters, a stigmatic image at one wavelength. This method of creating a stigmatic grazing incidence spectrometer by using a toroidal mirror was further developed, for use with laser produced plasmas, by G. Tondello in 1979 [29]. The introduction of the toroidal mirror provides three main improvements to the grazing incidence spectrograph. The first improvement is that the toroidal mirror reduces the astigmatism of the concave grating. This reduction in the astigmatism of the concave grating means that the light, arriving at the Rowland circle, is concentrated into shorter and more intense spectral lines. The second improvement is that the toroidal mirror collects and focuses the light from the source onto the entrance slit. This results in a greater percentage of the light from the source getting through the entrance slit and onto the grating. The increase in the light arriving at the grating along with the reduction of the grating's astigmatism means that there is a large increase in the intensity at the detector. The third improvement is that the toroidal mirror, by way of its focusing, can be used to fill the optimum width of the grating and hence the optimum resolution can be attained.

In order to produce a stigmatic image on the Rowland circle, (Appendix 1) both the primary and secondary images of the grating must be formed on the Rowland circle. The position of the primary image, $r_{1}^{\prime}$ (fig. 1.1), is given by the equation,

$$
\begin{equation*}
r_{1}^{\prime}=R \cos \beta \tag{1.1}
\end{equation*}
$$

where $R$ is the radius of the concave grating and $\beta$ is the angle of diffraction. The secondary image, $r_{2}^{\prime}$, is formed after the Rowland circle (fig. 1.1) and the location of the secondary image is obtained from the equation given below,

$$
\begin{equation*}
\frac{1}{r}-\frac{\cos \alpha}{R}+\frac{1}{r_{2}{ }^{\prime}}-\frac{\cos \beta}{R}=0 \tag{1.2}
\end{equation*}
$$

where $r^{\prime}{ }_{2}$ gives the location of the secondary focus, $\alpha$ is the angle of incidence and $r$ is the distance from the grating to the entrance slit. An interesting point to note is that due to the wavelength dependence of diffraction, both the primary and secondary foci have unique positions for each wavelength with the primary focus always being focused on the Rowland circle. As the primary image is already located on the Rowland circle, the condition that the secondary focus of the grating be formed on the Rowland circle is obtained by substituting, $r_{1}^{\prime}=R \cos \beta$, into equation (1.2) for $r_{2}{ }_{2}$ giving,

$$
\begin{equation*}
r_{V}=\frac{R}{\cos \alpha-\sin \beta \tan \beta} \tag{1.3}
\end{equation*}
$$

where $r_{v}$ is the secondary effective source distance from the grating (fig. 1.2).


Figure 1.2 Schematic diagram of the $u w$ plane for the toroidal mirror and concave grating combination used to create a stigmatic spectrograph.

For grazing incidence, $r_{v}$ is negative and the source is therefore a virtual source. The term "secondary" is used to differentiate between the two effectively separate sources used to form the primary and secondary images. The primary image is formed by using the vertical slit, a horizontally divergent line source, as the object. The secondary image is formed by using the "secondary" effective virtual source formed a distance $r_{v}$ behind the
grating. In order to obtain a stigmatic image, the grating should be illuminated with converging light in the sagittal or $w l$ plane (fig. 1.3).


Figure 1.3 Schematic diagram of the $w l$ plane for the toroidal mirror and concave grating combination used to create a stigmatic spectrograph.

From the above description of the conditions required to produce a stigmatic image on the Rowland circle, it can be seen that the focusing element i.e. the toroidal mirror, is placed between the source and the entrance slit. The development of the theory of the toroidal mirror is similar to that of the concave grating. The concave grating can be considered as a special case of a toroidal surface whereby both the radii of curvature are the same. A development of the theory of the toroidal mirror is contained in Appendix 1 after the theory of the concave grating. The equation for the location of the primary focus of the toroidal mirror, $r_{1}{ }^{\prime}$, is given by,

$$
\begin{equation*}
\frac{1}{r}+\frac{1}{r_{1}{ }^{\prime}}=\frac{2}{R \cos \varphi} \tag{1.4}
\end{equation*}
$$

The location of the primary focus may be varied independently by changing the values of the major radius, $R$, the angle of incidence, $\varphi$, or the source to mirror distance, $r$. These values are chosen such that the primary focus of the toroidal mirror is located at the entrance slit in order to maximise the flux coupled into the system. The illuminated slit is then used as the object for the primary focus of the concave grating.

The location of the toroidal mirror's secondary focus, $r_{2}{ }^{\prime}$, is given by the equation,

$$
\begin{equation*}
\frac{1}{r}+\frac{1}{r_{2}{ }^{\prime}}=\frac{2 \cos \varphi}{\rho} \tag{1.5}
\end{equation*}
$$

The location of the secondary focus of the toroidal mirror may also be varied independently by changing the values of the minor radius, $\rho$, the angle of incidence, $\varphi$, or the source to mirror distance, $r$. These values are chosen such that the secondary image of the toroidal mirror is positioned at a distance $r_{v}$ behind the grating. The secondary image of the toroidal mirror then acts as a virtual source for the grating (fig. 1.2).

From the grating equation given below,

$$
\begin{equation*}
d(\sin \alpha+\sin \beta)=m \lambda \tag{1.6}
\end{equation*}
$$

it is clear that at a fixed angle of incidence on the grating, each wavelength will have a unique position on the Rowland circle. As $r_{v}$ depends on the angle of diffraction, $\beta$, it is therefore wavelength dependant. Due to this wavelength dependence, the stigmatic focusing conditions can only strictly be satisfied for a single wavelength. The optical parameters of the toroidal mirror may also be chosen to provide reduced astigmatism over a broad range of wavelengths, the spectrum then becomes quasi-stigmatic. This will be demonstrated in Chapter 4.

As stated above, one of the benefits of using a toroidal mirror is that it can be used to fill the optimum width of the grating and hence obtain the optimum resolution. The optimum width of the concave grating, $W_{\text {opt }}$, is given by [30],

$$
\begin{equation*}
W_{o p t}=2.51\left[\frac{R^{3} \lambda \cos \alpha \cos \beta}{\sin ^{2} \alpha \cos \beta+\sin ^{2} \beta \cos \alpha}\right]^{\frac{1}{4}} \tag{1.7}
\end{equation*}
$$

Equation (1.7) gives the optimum width of the grating that should be illuminated in order to optimise the resolving power of the grating. The width of the toroidal mirror that is required to produce optimum illumination of the grating is obtained by aperture matching and is given by [29],

$$
\begin{equation*}
w_{u w}=\frac{W_{o p t} r_{1}^{\prime}{ }^{\prime}}{R \cos \varphi} \tag{1.8}
\end{equation*}
$$

where $w_{u w}$ is the mirror width in the $u w$ plane. $W_{o p t}$ is the optimum width of the grating, $r_{1}{ }^{\prime}$ is the position of the primary focus of the toroidal mirror, R is the radius of curvature of the concave grating and $\varphi$ is the angle of incidence on the toroidal mirror.

### 1.2 The 2.2 m instrument at D.C.U.

A schematic layout of the Dual Laser Plasma (DLP) spectroscopic instrument at D.C.U. is shown in fig. 1.4 below. The basic system components will be described here and a more in depth discussion of the target and the knife-edge design will be presented in Chapter 3. The DLP system allows the measurement of both space and time resolved spectra of laser produced plasmas.

The system consists of a Spectron Laser Systems SL800 Nd:YAG laser to produce the laser plasma. The synchronisation of the laser(s) is controlled by a delay generator (DG). The delay generator used was a Stanford Research Systems Inc. Model DG535 Four Channel Digital Delay/Pulse Generator. In order to acquire the voltages necessary to trigger the laser(s), the TTL output from the delay generator is sent to an in-house voltage amplifier. There are four other lasers available, two dye lasers and a ruby laser and another Nd:YAG. These other lasers were not used throughout this work and they will therefore be omitted.

The output from the Nd:YAG laser is focused onto a tungsten (W) target using a 10 cm focal length spherical lens. A mounting plate for the knife-edge was designed to provide three degrees of freedom of movement. In between the target chamber and the toroidal mirror chamber is a Glass Capillary Array (GCA). This is used to prevent blow-off material from the plasma from coating the toroidal mirror. The toroidal mirror is positioned after the GCA and the specifications of the toroidal mirror will be given later in this section. Next is the entrance slit assembly, where there is a choice between a slit width of 10 and $20 \mu \mathrm{~m}$ respectively. Next is the concave grating and the specifications of this will also be given later in this section. The detector used to record the spectrum is a combination of a Micro-channel plate (MCP) and Photo Diode Array (PDA) detector. The output from the PDA is read by the Optical Multi-channel Analyser (OMA) and the output from the OMA is read by a Dell Dimension 486 P.C. The output from the OMA is also displayed on a Hewlett Packard digitising scope.


Figure 1.4 Schematic layout of the 2.2 m DLP spectrometer at D.C.U.

## The Nd:YAG laser

As stated previously, the laser used throughout this work was a Nd:YAG laser. The laser was a Spectron Laser Systems SL800 with an oscillator and an amplifier (fig. 1.5)


Figure 1.5 Schematic diagram of the Spectron Laser Systems SL800 Nd:YAG internal set-up.

The active material is a rod of yttrium-aluminium-garnate ( $\mathrm{YAG}, \mathrm{Y}_{2} \mathrm{Al}_{5} \mathrm{O}_{12}$ ) doped with neodymium $\left(\mathrm{Nd}^{3+}\right)$ ions. The Nd :YAG crystal is optically pumped by radiation from powerful discharge lamps to obtain a population inversion. The Pockel cell is used as an optical shutter in order to inhibit the normal lasing action. When the optimum population inversion is achieved, the Pockel cell is opened and all the stored optical energy in the Nd :YAG crystal is released in one single, giant optical pulse. This process is known as Qswitching. The optical pulse from the Nd:YAG SL800 is at the fundamental wavelength of 1064 nm (FWHM) and when operating in Q-switched mode, the optical pulse is typically 1 J in $15-20 \mathrm{~ns}$.

As stated at the beginning of this section, the output from the laser is focused onto the target using a 10 cm focal length lens. Using an optical microscope with a calibrated XYZ translation stage, the focal spot size of the lens on the target was measured to be between $75-80 \mu m$. The energy per pulse was measured using an energy monitor
manufactured by Radiant Dyes Chemie. The energy monitor converted the laser pulse into a voltage output with a conversion ration of 1.15 V per Joule. The energy per pulse was measured to be between $0.72-0.82 \mathrm{~J}$. Using these values, the irradiance on the surface of the target is calculated to be between $10^{11}-10^{12} \mathrm{Wcm}^{-2}$.

## Specifications of the toroidal mirror and concave grating

This section will contain the specifications of the Glass Capillary Array (GCA) and the optical parameters of the toroidal mirror. The optical parameters chosen were done so with the aid of the ray-tracing code SHADOW. SHADOW is public domain software written and maintained by Franco Cerrina et al. at The Centre of X-ray Lithography, Wisconsin University. SHADOW is unique in its ability to deal with strongly off-centre and asymmetric systems such as those encountered in the XUV. A more in depth discussion about SHADOW is contained in Chapter 2 and Appendix 2.

The GCA was supplied by the Galileo Electro-Optics Corporation. The GCA has a diameter of 25 mm and a thickness of 2 mm . The diameter of the pores are $50 \mu \mathrm{~m}$ and the ratio of length to diameter, $L / D$, is therefore equal to 40 . The GCA was inserted for two reasons. The first reason was that when a laser plasma was generated, the pressure in the target chamber fluctuated. To prevent pressure equalisation during operation, a differential pressure was maintained between the target and toroidal mirror chamber by the insertion of the GCA. The pressure differential during operation was from $\sim 10^{4} \mathrm{mbar}$ in the target chamber to $10^{-6} \mathrm{mbar}$ in the toroidal mirror chamber. The second reason was that it would help prevent the toroidal mirror from being contaminated by the blow-off from the laser plasma.

The instrument used throughout this work is a McPherson model 247M8 grazing incidence VUV monochromator. The entrance slit is positioned on the Rowland circle at an angle of incidence of $84^{\circ}$ to the grating normal. The concave grating has a radius of curvature, $R$, of 2.2176 m and has a gold coating. The grating contains 1200 lines per mm
and is blazed at an angle of $2^{\circ} 4^{\prime}$. The ruled area of the grating is 50 mm wide by 30 mm high. The spectral resolving power is of the order of 1500 at a photon energy of 150 eV . Using $r=R \cos \alpha$, the distance from the entrance slit to the grating is calculated to be 231.8 mm . Using equation (1.7), the optimum width of the grating, $W_{\text {opt }}$, for a wavelength of 18 nm is calculated to be approximately 25.43 mm .

A summary of the concave grating and toroidal mirror specifications is as follows:

| Concave grating | Toroidal mirror |
| :--- | :--- |
| Entrance slit to grating distance $=231.8 \mathrm{~mm}$ | Source to mirror distance $(r)=400 \mathrm{~mm}$ |
| Angle of incidence $(\varphi)=84^{0}$ | Angle of incidence $(\varphi)=82^{\circ}$ |
| Radius of curvature $(R)=2.2176 \mathrm{~m}$ | Primary focus position $\left(r_{1}{ }^{\prime}\right)=230.08 \mathrm{~mm}$ |
| Lines per $\mathrm{mm}=1200$ | Secondary focus position $\left(r_{2}{ }^{\prime}\right)=\infty$ |
| Blaze angle $=2^{\circ} 4^{\prime}$ | Major radius $(R)=2099 \mathrm{~mm}$ |
| Ruled area $(l \times b)=50 \times 30 \mathrm{~mm}$ | Minor radius $(\rho)=111.34 \mathrm{~mm}$ |
| Spectral resolving power $=\sim 1500 @ 150 \mathrm{eV}$ | Dimensions $(l \times b)=30 \times 30 \mathrm{~mm}$ |
|  |  |

## The MCP/PDA detector combination

The detector used in the 2.2 m spectrometer at D.C.U. is made up of a MicroChannel Plate (MCP), supplied by Galileo Electro-Optics Corporation, and an EG\&G® model 1453 Photo-Diode Array (PDA). Two high voltage power supplies were used to bias the MCP and they were a Bertan Associates Inc. Model 205B-05R for the voltage at $\mathrm{V}_{\mathrm{s}}$ and Model 205B-01R for the voltage at $\mathrm{V}_{\mathrm{i}}$. The analog video signal from the PDA is controlled and read by an EG\&G® Princeton Applied Research 1461 Detector Interface which forms the heart of the Optical Multi-channel Analyser (OMA). The output from the OMA is read by a Dell Dimension 486 P.C. and is also displayed on a Hewlett Packard HP54501A digitising scope.

An expanded view of the MCP/PDA combination can be seen in fig. 1.6 below. The MCP is surrounded by the purple dotted line. The MCP has channel diameters of $12 \mu \mathrm{~m}$ on $15 \mu \mathrm{~m}$ centres. The active area is $12.5 \mathrm{~cm}^{2}$ and presents approximately 40 mm of the surface at the Rowland circle. The channels are inclined at $8^{0}$ to the surface normal. At VUV wavelengths, the quantum efficiency of the MCP is $10-15 \%$. The MCP cannot be operated at pressures greater than $2 \times 10^{-6} \mathrm{mbar}$ and with the aid of an ion pump, a pressure of around $1.2 \times 10^{-7}$ mbar was maintained.


Figure 1.6 Schematic diagram of the MCP/PDA detector combination.

The operation of the MCP is described as follows. The input electrode, $\mathrm{V}_{\mathrm{i}}$, is biased up to about -1 kV with respect to the output electrode $\mathrm{V}_{0}$. When a photon, called the primary radiation, is incident on the input side of a channel, secondary electrons are generated. These secondary electrons are then accelerated along the channel and on impact with the channel walls, these secondary electrons themselves produce secondary electrons. The single photon on entering the channel therefore creates an avalanche effect, which typically yields up to $10^{4}$ electrons at the channel output. The electrons are then accelerated across a vacuum gap, of width 0.7 mm , by the positive potential difference provided by $\mathrm{V}_{\mathrm{s}}$
which can range from 3 to 4.2 kV depending on the intensity of the primary radiation arriving at the MCP's surface.

After the electrons are accelerated across the gap, they impinge on a phosphor coated fibre optic bundle. The phosphor converts the electrons into a visible photon signal which is then transmitted down the fibre optic bundle. The fibre optic bundle is tapered down from 40 mm to 25 mm which results in a demagnification factor of 1.6 . This visible photon signal is then read by the self-scanned 1024 pixel PDA. Each pixel is $25 \mu \mathrm{~m}$ by 2.5 mm high. The analog signal from the PDA is then read by the OMA and digitised. The output from the OMA can then be displayed on the scope or the data transferred to the P.C. The P.C. controls all aspects of the data acquisition by using software written to control the OMA.

### 1.3 Objectives of present work and associated methodology

In a gas phase photoabsorption experiment, the total atomic photoionisation crosssection at wavelength $\lambda\left(\sigma_{\lambda}\right)$ is obtained by measuring the intensity $\left(I_{\lambda}\right)$ transmitted by the absorbing medium, e.g. metallic vapour or rare gas, of a beam of initial intensity $\left(I_{0}^{\lambda}\right)$. These quantities are related by Beer-Lambert's law,

$$
\begin{equation*}
\frac{I_{\lambda}}{I_{0}^{\lambda}}=\exp \left[-\sigma_{\lambda} \frac{n}{V} l\right] \tag{1.9}
\end{equation*}
$$

where $\sigma_{\lambda}$ is the cross-section. The other quantities $n$ - the number of absorbing species, $V$ the absorption volume and $l$ - the sample length, have to be known. In a heat pipe or gas phase absorption cell, the number of absorbing species ( $n$ ) is obtained by measuring the pressure $(P)$ and the temperature ( $T$ ) and assuming ideal gas behaviour.

In a DLP experiment, the absorbing medium is a laser produced plasma, and thus the photoionisation cross-section of ions can be obtained. Due to the obvious difficulty in accessing the values of $n$ and $V, \sigma_{\lambda}$ is obtained on a relative rather than an absolute scale. The main objective of this work is to obtain an estimate for the volume $V$ of probed plasma that effectively contributes to an absorption spectrum obtained using the DLP technique (fig. 1.7). Thus, in principle, an absolute value for $\sigma_{\lambda}$ can still be obtained provided that $n$ is known. Currently, the precise measurement of ionic densities in laser produced plasmas poses severe experimental difficulties and thus the method described above is hardly applicable. However, if the atomic cross-section is precisely known in a different spectral region, accessible with the same experimental set-up, an estimate for the ionic densities can be obtained. Thus, the procedure described above becomes applicable in a region where the cross-section is unknown.

Also, from equation (1.9) above, in the case where the atomic cross-section is known, a quantitative analysis of the plasma can be carried out using the DLP technique. This is an important development for laser plasma diagnostics and the corresponding details can be found in Whitty et al. [31]


Figure 1.7 Diagram showing the volume that contributes to an absorption spectrum.

The methodology used to obtain $V$ is based on a comparison between the results of physical experiments on the DLP system and simulations of the same experiments using ray tracing. We now explain the essential stages of that procedure

First, a series of knife-edge traces are obtained by progressively obscuring the continuum beam used in the DLP absorption spectrometer and measuring the corresponding intensity variations as a function of knife-edge position. Second, each of these knife-edge traces is simulated on computer using ray tracing. From this simulation, the spatial characteristics of the continuum source are obtained. Finally, a detailed ray tracing study of the DLP system is carried for the spatial parameters of the source just obtained. Further analysis provides the sought absorption volume $V$.

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## Chapter 2

## Model of knife-edge traces and interpretation using Ray-tracing

In this chapter, a model of our knife-edge experiments will be presented along with a description of the ray tracing methods used to analyse them. Section 2.1 presents a basic mathematical description of the intensity variations at the detector position due to the progressive insertion of a knife-edge in the path of the backlighting continuum beam. Section 2.2 contains a description of the ray-tracing software SHADOW and its use in the generation of knife-edge traces. Section 2.3 outlines the method by which the experimental knife-edge traces were compared with the ray-traced ones and how this comparison provides some information about the spatial characteristics of the continuum source. Finally, in Section 2.4, the principle of the calculation of the absorption volumes is outlined.

### 2.1 Basic Model

In order to demonstrate the effect of the knife-edge, we assume that the laser plasma VUV continuum source is an extended 3-D source. The corresponding 2-D spatial distribution at wavelength $\lambda$ as seen by the system is $S_{0}^{\lambda}(x, y)$. The knife-edge, represented by $K(x, y)$, is positioned a distance $X$ along the $x$-axis (fig. 2.1). The introduction of the knife-edge modifies the spatial distribution of the source. The modified spatial distribution, $\theta_{x}^{\lambda}(x, y)$, is obtained from the convolution of $S_{0}^{\lambda}(x, y)$ and $K(x, y)$ and is given by,

$$
\begin{equation*}
\theta_{X}^{\lambda}(x, y)=\int_{-\infty}^{+\infty} \int_{-\infty}^{x} K(x, y) S_{0}^{\lambda}\left(x-x^{\prime}, y-y^{\prime}\right) d x^{\prime} d y^{\prime} \tag{2.2}
\end{equation*}
$$

where,

$$
\begin{align*}
K(x, y) & =1 \text { for all } x \leq X(\text { or } y \leq Y) \\
& =0 \text { for all } x>X(\text { or } y \leq Y) \tag{2.3}
\end{align*}
$$

From equation (2.2), it can be seen that $\theta_{X}^{\lambda}(x, y)$ depends parametrically on $X$.


Figure 2.1 Modelling overview of the system.

The resultant spatial distribution at the detector, $I_{X}^{\lambda}(x, y)$, is the convolution of $\theta_{X}^{\lambda}(x, y)$, and the instrument function, $\gamma^{\lambda}(\mathrm{x}, \mathrm{y})$. The instrument function of the system, $\gamma^{\lambda}(\mathrm{x}, \mathrm{y})$, is a convolution of all the distortion effects such as diffraction, magnification and aberrations, introduced by the system components (e.g. toroidal mirror). The spatial distribution of the image at the detector, $I_{X}^{\lambda}(x, y)$, is thus given by,

$$
\begin{equation*}
I_{X}^{\lambda}(x, y)=\int_{-\infty}^{+\infty+\infty} \int_{-\infty}^{\lambda} \gamma^{\lambda}(\mathrm{x}, \mathrm{y}) \theta_{X}^{\lambda}\left(\mathrm{x}-\mathrm{x}^{\prime}, \mathrm{y}-\mathrm{y}^{\prime}\right) \mathrm{d} \mathrm{x}^{\prime} \mathrm{d} \mathrm{y}^{\prime} \tag{2.4}
\end{equation*}
$$

Therefore, $I_{X}^{\lambda}(x, y)$ also depends parametrically on $X$.

Since we are dealing with a spectrally smooth continuum source, bandpass effects are neglected and a single pixel readout is the smallest monochromatic signal that can be measured by the system. The parameters of the system in Chapter 1 indicate that the
spectral line height can never be greater than the pixel height. Thus, the pixel output, $P_{X}^{\lambda}$, is the summation over the entire pixel area of the intensity distribution, $I_{X}^{\lambda}(x, y)$ and is given by,

$$
\begin{equation*}
P_{X}^{\lambda}=\iint_{\text {pixel arca }} I_{X}^{\lambda}\left(x^{\prime}, y^{\prime}\right) d x^{\prime} d y^{\prime} \tag{2.5}
\end{equation*}
$$

A typical knife-edge trace is then obtained when $P_{X}^{\lambda}$ is plotted as a function of $X$ (fig 2.2) with $X$ ranging from the fully retracted knife-edge position to the cut-off position. From the above analysis, it is seen that the profile of a knife-edge plot for a given system (fig 2.2) will be determined by the spatial distribution of the source $S_{0}^{\lambda}(x, y)$.


Figure 2.2 Typical knife-edge trace.

Several methods can be used to recover information on $S_{0}^{\lambda}(x, y)$ from a measured knife-edge trace as shown below.
(1) An estimation of the source dimension(s) in the $x(y)$ direction(s) can be obtained by taking the difference between $X_{10}\left(Y_{10}\right)$ and $X_{90}\left(Y_{90}\right)$, the $\mathrm{X}(\mathrm{Y})$ values corresponding the $10 \%$ and $90 \%$ values of the maximum intensity signal(s) $[1,2]$.
(2) Estimates based on a $25 \%-75 \%$ criterion can also be found in the literature [2-6].
(3) If $I_{X}^{\lambda}(x, y)$ were known, trial functions could be used for $S_{0}^{\lambda}(x, y)$ and the integrals (2.2) to (2.5) subsequently computed until the calculated knife-edge trace matches the measured one.

In the following (see Chapter 4), methods (1), (2) and (3) will be used. However, in our case, an analytical or numerical function describing the instrument response, $\gamma^{\lambda}(\mathrm{x}, \mathrm{y})$, is unknown. It is well known that, in the grazing incidence regime in particular, explicit calculation of $I_{X}^{\lambda}(x, y)$ from optical theory or its experimental characterisation constitute extremely arduous tasks. However, a complete characterisation of the system response can be obtained using ray-tracing methods and the analysis described in (3) remains applicable. A description of the characterisation of the system response using ray-tracing is described below.

### 2.2 System modelling using the XUV Ray-tracing Software SHADOW

This section will be subdivided in two subsections. The first subsection, section 2.2.1, will outline the general principles behind the ray-tracing software SHADOW. Section 2.2.2 provides an overview of how the 2.2 m DLP spectrometer at D.C.U. was defined in SHADOW. This section will also describe how the knife-edge traces were obtained using ray-tracing.

### 2.2.1 General principles of the ray-tracing software SHADOW

The ray-tracing program used throughout this work is called SHADOW (v2.2.0). SHADOW is public domain software written and maintained by Franco Cerrina et al. at The Centre for X-ray Lithography in Wisconsin University [7]. SHADOW studies the propagation of a photon beam through an optical system. The program is completely general but it is written specifically for use with XUV optics [8]. SHADOW is unique in its ability to deal with strongly off-centre and asymmetric reflective optics. It has been used
extensively to design XUV synchrotron radiation beamlines and monochromators around the world as well as in general applications ranging from flashlights to x -ray telescopes. A detailed overview of SHADOW and the SHADOW utilities that were used throughout this work is contained in Appendix 2.

The definition of any given optical system in SHADOW, must contain a source and at least one optical element (OE). The source has the ability to generate up to 20,000 rays with each ray being characterised as a point in a 15 dimensional space (starting point $P(x, y, z)$, direction $V\left(x^{\prime}, y^{\prime}, z^{\prime}\right)$, the electric field for the $s$ and $p$ component of the elliptical polarisation ( $E_{s x}, E_{s y}, E_{s z}$ ) and ( $E_{p x}, E_{p y}, E_{p z}$ ), the phases at the starting point ( $\phi_{s}, \phi_{p}$ ) and the wavelength $\lambda$ ) [8]. SHADOW can model different source distributions by generating $P(x, y, z)$ and $V\left(x^{\prime}, y^{\prime}, z^{\prime}\right)$ to a specified distribution function. Two such distributions under consideration for this work were the Uniform Distribution and the Gaussian Angle Distribution [9]. In the case of a Uniform Distribution, the incident intensity on a plane follows Lambert's law whereby the intensity varies as the cosine of the angle of the ray with respect to the optic axis. A Gaussian Angle Distribution is characterised by the standard deviation and the maximum divergence in $x^{\prime}$ and $z^{\prime}$. The source sampling used throughout this work was "random". In this case, the distribution of the rays in the source and the divergence of the rays are generated randomly within the dimensions and divergences defined in the source definition. The rays are produced using a Monte Carlo approach which results in the variables for each ray being generated anew [8,9].

SHADOW treats a given optical system as a series of individual optical elements, each one defined by a reflecting or refracting surface. The rays contained in the source output file (begin.dat) are intercepted by the first optical element (OE \#1). The intercepts of these rays with the optical element are stored in a data file (mirr.01, in the case of OE \#1). On intercepting with the optical element, the reflection angle for each ray is calculated using the parameters of the optical element and the original ray. The re-calculated rays then leave the optical element and arrive at the image or continuation plane (star.01, again in the case of $\mathrm{OE} \# 1$ ). If there is no other optical element then it is termed the image plane. If there is another optical element then it is termed the continuation plane and this
continuation plane becomes the source for the next optical element and so on until the rays reach the final image. The optical element geometry and Cartesian co-ordinate system used by SHADOW is illustrated in figure 2.3.


Figure 2.3 Optical element geometry in SHADOW (reproduced from SHADOW Primer 2.0)

### 2.2.2 Generation of knife-edge traces using ray-tracing

The first step in generating the knife-edge traces using ray-tracing was to model the 2.2 m DLP instrument at D.C.U. in SHADOW. The modelling of the instrument was successfully achieved by defining a source and three optical elements consisting of a fake transmission element, a toroidal mirror and a concave grating. A description of each of these components is contained in this section and their individual definitions are presented in Appendix 2.

In the physical experiments, the laser plasma source was viewed at an angle of $45^{\circ}$ by the 2.2 m instrument (fig. 1.4). In order to model this $45^{\circ}$ view of the source in SHADOW, the source must also be rotated by $45^{\circ}$. A source rotation can only be defined within the definition of an optical element and it therefore became necessary to define a
fake transmission element. In the definition of this fake transmission element, the source was rotated $315^{\circ} \mathrm{CCW}$ about the x -axis which results in a rotation of $-45^{\circ} \mathrm{CW}$. The fake transmission element is defined as a zero length "empty" element and it therefore rotates but does not translate the source. As the source is rotated by $-45^{\circ} \mathrm{CW}$, the vertical divergence in the source definition must be centred on $+45^{\circ}$ in order to bring the rays exiting the source back along the optic axis. The image file from the fake transmission element is then used as a virtual source for the rest of the system.

The second optical element to be defined was the toroidal mirror. Within the definition of an optical element is the option for an exit slit. In the case of the toroidal mirror, this option was used to define the entrance slit to the spectrometer. The third optical element to be defined was the concave grating which uses the entrance slit as the source of rays. A schematic of the optical system defined in SHADOW is shown in fig. 2.4. A fourth optical element, used as a 2-D imaging device, was also defined but is not shown in fig. 2.4.


Figure 2.4 A schematic of optical system defined in SHADOW for the 2.2 m DLP system.

Having defined the system in SHADOW, the next step is to generate the knife-edge traces using ray-tracing. For this purpose, the SHADOW utility "shadowit" was used [Appendix 2]. "shadowit" reads in a set of specified data files for a given system ray-traced
by SHADOW. Apertures can then be placed on any part of the system e.g. the dimensions of the toroidal mirror, entrance slit and concave grating. "shadowit" then labels the rays as either good or lost, depending on whether or not they go through the apertures defined. The apertures can be modified and the resulting effects studied without having to retrace the system using SHADOW. The knife-edge was simulated by inserting a screen 2 mm from the source (as was the case in the experiment). The system was then ray-traced with the screen in place and this provided a file containing the ray intercepts at the position of the knife-edge. This data file was then read in by "shadowit" along with the other data files associated with the source and the three optical elements. Apertures were then placed on the toroidal mirror, the entrance slit and the concave grating, corresponding to their physical dimensions. An aperture was also placed on the screen at the knife-edge position. In order to simulate the knife-edge, the aperture on this screen was reduced in size in a particular direction depending on whether it was a simulation of a horizontal or a vertical knife-edge scan. The aperture size was reduced until no rays got through the system. The ray-traced knife-edge trace was then obtained by plotting the number of rays that get through the system versus the value of the limit.

### 2.3 Comparison between measured and ray-traced knifeedge traces

The spatial characteristics of the source were obtained through a comparison between the measured and ray-traced knife-edge plots. This procedure is now detailed.

The experimental knife-edge response of the system was measured for five selected photon energies namely, $50,75,100,125$, and 150 eV and the method used is outlined in Chapter 3. Virtual knife-edge experiments were then generated in the same conditions as the experimental traces. The source parameters e.g. height, depth and distribution type, were varied in SHADOW until the best visual fit between the virtual and experimental knife-edge traces was obtained. An example of this fitting procedure is shown in figures 2.5 and 2.6.


Figure 2.5 Plot showing the experimental knife-edge response of the system in the vertical direction at 100 eV along with the results of the same virtual experiments carried out with three different source sizes.


Figure 2.6 Plot showing the experimental knife-edge response of the system in the horizontal direction at 100 eV along with the results of the same virtual experiments carried out with three different source sizes.

Then, the region of the source in which the rays that contribute to the final image originate, was extracted using "shadowif". The size and shape of the region defined by these rays provide a ray-traced estimation of the source dimensions. An example of this is shown in figure 2.7.


Figure 2.7 Footprint of the "good" and "lost" rays at the source. The "good" rays are rays that get through the system and contribute to the final image.

### 2.4 Principle of calculation of absorption volumes

Using the spatial information on the source obtained in section 2.3, it then becomes possible to calculate the absorption volume for a particular photon energy. As stated previously, using the SHADOW utility "shadowit", it is possible to extract the "good" rays that get through the system from any data file associated with a source, optical element or screen. Using the "good" rays from screens positioned at the location of the absorption plasma, it is therefore possible to obtain the size and shape of the plasma volume that is sampled by the continuum source.

In most DLP experiments carried out on the 2.2 m instrument, the absorption plasma is created $\sim 20 \mathrm{~mm}$ from the continuum source with a sample length $l$ of up to $\sim 10 \mathrm{~mm}$ (fig. 1.7). In order to obtain the size and shape of the absorption plasma that is sampled, a series of 10 screens with a 1 mm separation were inserted starting at 20 mm from the source (fig. 2.8).


Figure 2.8 Figure showing the 10 screens that were inserted at the position of the absorbing plasma in SHADOW.

The system was then ray-traced again with the source spatial characteristics obtained in section 2.3. "shadowit" then provided the "good" rays that get through the system on each of the screens 1-10. This procedure was repeated 50 times with a different random distribution of the rays in the source in order to more clearly define the boundaries
of the volume. In figure 2.9, the corresponding plot of the "good" rays on screen $1(20 \mathrm{~mm}$ from the source) for 100 eV is shown.


Figure 2.9 This figure shows the footprint of the "good" rays on screen 1, positioned 20 mm from the source, resulting from 50 separate ray-tracings. The figure also shows the rays that were extracted in order to calculate the absorption volume.

A computer program was written in order to extract the outermost rays from the ten screens (fig. 2.9). The surface area within the boundary line defined by the extracted rays was obtained by integration using plotting package Microcal Origin Version 3.5©. Assuming that the surface area is approximately constant over a 1 mm step, the volume of a slice of thickness $1 \mathrm{~mm}\left(v_{i}\right)$, (fig. 2.8), is simply the surface area $\times 1 \mathrm{~mm}$. This procedure is repeated 10 times for each of the 10 slices. The total volume, $V$, is obtained by summing the volumes of the individual slices $\left(v_{i}\right)$ i.e. $V=\sum_{1}^{10} v_{i}$.

## References

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## Chapter 3

## Experimental Measurements of Knife-edge traces

This chapter will outline the experimental designs and methods used in obtaining the knife-edge response of the system at the five selected energies. Section 3.1 will outline the experimental set-up used to obtain the knife-edge traces. In section 3.2, the initial and experimental alignment procedure for the target is described. Section 3.3 will examine the reproducibility of the knife-edge traces.

### 3.1 Experimental set-up

A schematic of the target chamber experimental set-up for obtaining the knife-edge response of the system is shown in figure 3.1.


Figure 3.1 Schematic of the target chamber with the positions of the continuum emitting plasma and the knife-edge indicated. The position where an absorption plasma would be generated is also shown.

The position of an absorption plasma is indicated but was not used in the coarse of this work. The target chamber was machined from a solid stainless steel cube of side 127 mm . Three mutually orthogonal holes of diameter 71 mm were machined through the cube to provide six entry ports. The top face contains an O-ring sealed, sliding flange and the backlighting target is mounted to this. This sliding flange can be positioned using a combination of micrometers and thumbscrews. This set-up enables good approximate positioning of the laser plasma with respect to the optic axis. The stainless steel knife-edges used throughout this experiment were obtained from an entrance slit assembly of a spectrometer. A separation of 2 mm between the laser plasma and the knife-edge was maintained in order to prevent the knife-edge being damaged by reflected Nd :YAG laser radiation from the plasma. In order to manipulate the knife-edge, an XYZ translation stage providing 3 degrees of freedom was designed. The designs of the various components manufactured for obtaining the knife-edge traces, such as the XYZ translation stage, are contained in Appendix 3.

One of the most important considerations in recording the knife-edge response of the system is the reproducibility of the experiments. The two main factors that will affect the reproducibility, assuming that the same experimental procedure is followed, are the source and the system. In this case, none of the system parameters such as the angles of incidence etc. are changed and therefore, the main factor affecting the reproducibility of the experiments will be the position of the laser plasma. Due to the spatial and temporal dependency of the population distribution in a laser plasma [1], it is essential that the same part of the laser plasma is used for each experiment. Previous estimates have shown that the system uses $\sim 0.5 \mathrm{~mm}$ of the laser plasma. With such a small dimension of the laser plasma being used, accurate positioning of the laser plasma is therefore critical.

In the case of the knife-edge experiments, accurate positioning of the laser plasma is critical both in terms of the optic axis and in its position along the optic axis. With this in mind, a target was designed with alignment holes which would utilise the HeNe alignment laser pointing along the optic axis (fig. 3.1) and the HeNe alignment laser pointing along the $\mathrm{Nd}:$ YAG beam (fig. 1.5, fig. 3.1). Utilising the alignment holes in conjunction with the HeNe alignment lasers allowed accurate positioning of the target with respect to the optic axis.

Even with accurate positioning of the target, there will exist slight variations in the position of the laser plasma itself. These variations are due to factors such as the shot to shot reproducibility of the laser, both in power and pointing stability, and the smoothness of the target's surface. The design of the target is now described.

## Target Design

The target was designed so that, within the constraints of the existing system, accurate alignment could be achieved. A source target of tungsten (W) was chosen for two main reasons, the first of which is the fact that it is a good source of continuum. The second reason for choosing tungsten was due to its inherent hardness and the fact that it is therefore more resistant to pitting. The disadvantage in using tungsten as the target material is that its inherent hardness means it is difficult to machine and it was not possible to drill the fine alignment holes necessary. In order to do this, a similar shaped piece of aluminium (Al) was bonded to the tungsten using a metal epoxy resin. The aluminium cylinder contained the two 1 mm diameter alignment holes, one for the HeNe alignment laser pointing along the $\mathrm{Nd}:$ YAG beam and the other for the HeNe alignment laser pointing along the optic axis. The advantage in using aluminium is that it is a good source of spectral lines which are used in the energy calibration of the continuum spectra. The design of the cylindrical target is shown in figure 3.2.


Figure 3.2 Schematic of tungsten and aluminium target design.

The angle of incidence of the laser and the angle of observation were chosen to be $45^{\circ}$. The alignment holes were therefore drilled at $45^{\circ}$ to a line of radius and perpendicular to each other (fig. 3.2). The position corresponding to $45^{\circ}$ down the length of the target was indicated by a scribe line slightly adjacent to the $45^{\circ}$ positions. Viewing the laser plasma at $45^{\circ}$ meant that the knife-edge could not be positioned close to the laser plasma. In order to allow the knife-edge to be positioned at the laser plasma, the lower part of the cylindrical tungsten target was cut away along the mill line indicated.

### 3.2 Experimental method

This section will outline the method used to obtain the vertical and horizontal knife-edge responses of the system for the selected energies of $50,75,100,125,150 \mathrm{eV}$. The initial alignment procedure outlined below was used to position the target so that the laser plasma was produced on the optic axis. In brief, it was a case of finding the height of the optic axis with the lens system. Then, the target was manipulated so that the alignment HeNe from the $\mathrm{Nd}:$ YAG laser (fig. 1.5, fig. 3.1) went through the alignment hole in the target (fig. 3.2). The target was then moved in the Z-direction (to/from the lens) and the position of maximum counts used to indicate that the laser plasma was on the optic axis. The alignment procedures used were a critical part of the knife-edge experiments as they ensured that the experiments were reproducible.

## Initial Alignment Procedure :-

(1) Due to the parameters chosen for the toroidal mirror, the laser plasma generated on the target has to be 400 mm from the centre of the toroidal mirror. The target was therefore positioned so that the line of $45^{\circ}$ on the target (fig. 3.2) was $\sim 400 \mathrm{~mm}$ from the centre of the toroidal mirror.
(2) The alignment HeNe from the Nd:YAG laser was positioned on the line of $45^{\circ}$ on the target. The focusing lens was moved up and down, and the focus was adjusted, until the
counts on the detector were a maximum. The focusing lens was not moved from this point on as it marked the correct height of the optic axis.
(3) When the height of the optic axis was found, the target was moved down so that the alignment HeNe from the Nd :YAG laser went through the alignment hole in the aluminium section of the target. If the alignment HeNe from the Nd :YAG laser did not go through the alignment hole correctly, the target was rotated and moved along the optic axis (Y-direction, to/from the toroidal mirror) until it did. This was indicated by a bright circular spot on the back wall of the target chamber.
(4) The target was then moved up until the alignment HeNe from the Nd:YAG laser was positioned at the same focal spot used in steps (1), (2), \& (3). It was important to use the same focal spot on the target each time, as the target is not completely symmetrical. The detector counts were then checked to see if the maximum counts, acquired in (2), were still obtained. If they were not, steps (2) \& (3) above were repeated.

At this stage in the alignment procedure the correct height of the optic axis was defined by the alignment HeNe from the Nd:YAG laser. The laser plasma generated was therefore at the correct height and the alignment procedure was then concerned with obtaining the correct position of the laser plasma in the Z-direction (to/from lens) (fig. 3.1).
(5) The target was moved in the Z-direction (to/from the lens) until the detector counts were a maximum. When the maximum counts were obtained, the target was moved up again to check if the alignment HeNe from the Nd:YAG laser still went through the alignment hole. If it did not go through the alignment hole, a combination of small rotations and small movements in the Y-direction were used to realign it. When this was completed, the target was moved back up so that the alignment HeNe from the Nd:YAG laser was positioned at the same focal spot and the maximum counts were rechecked.
(6) If the maximum counts were not obtained at this point, steps (2)-(5) were repeated until the maximum counts achieved in (5) were obtained again.
(7) The HeNe alignment laser pointing along the optic axis was then positioned so that its beam went through the optic axis alignment hole in the target. When the HeNe beam was going correctly through the alignment hole, a spotted pattern, due the GCA, was observed on the toroidal mirror.

The knife-edge experiments can begin once the above alignment procedure is completed. To do this, the knife-edge is positioned 2 mm from the point on which the laser plasma is generated. For a particular scan direction (horizontal or vertical) and energy, the knife-edge is moved in from the fully retracted position to the cut-off position (zero detector counts) in $20 \mu m$ steps. For each position of the knife-edge, 10 shots were taken and averaged. On completing the scan, an aluminium reference spectrum was taken using the aluminium section of the target. The aluminium reference spectrum was then used to convert pixel numbers to an energy scale. For a given energy, the detector counts from the pixel corresponding to $50,75,100,125$ or 150 eV , was extracted from each data file recorded for each position of the knife-edge. The knife-edge trace was then obtained by plotting the normalised detector counts versus the knife-edge position.

With an inch setting of 23.4 , the energy at the centre of the detector is $\sim 50 \mathrm{eV}$. At this setting, with the detector voltages set at +4 kV and -815 V , the maximum counts from the detector was $\sim 1000$. This was a set point for all the knife-edge experiments. If anything went wrong, the system was returned to these settings and the alignment procedure outlined above was repeated until $\sim 1000$ counts were obtained. Also, in order to ensure that the same part of the laser plasma was used when moving to a different energy and a fresh surface, the system was returned to the above settings and the target position was tweaked until the maximum counts of $\sim 1000$ were re-acquired. On completing this, the detector was then moved to the inch setting corresponding to the next energy.

The alignment procedure worked well and the maximum counts obtained above were very reproducible over long periods of time with approximately the same maximum counts being obtained during January 1997, October 1997 and February 1998 when knife-edge experiments were carried out.

### 3.3 Reproducibility of experimental design and set-up

In this section, the reproducibility of the knife-edge experiments will be examined along with the effect of pitting on the target. Ideally, the target should be moved so as to provide a fresh surface for each position of the knife-edge. With the current system design however, it was observed that any rotation or vertical movement of the target resulted in a large variation in the detector counts. The reason for this is that the target is not perfectly symmetric and as a result, the position of the laser plasma varies with respect to the optic axis. Within the constraints of the existing target chamber, the only other option was to use the same spot on the target for the duration of the individual knife-edge experiment.

In order to observe and quantify the effect that pitting has on the counts, a series of 600 shots were taken with the same focal spot for each of the five energies. Using an aluminium spectrum to calibrate the tungsten continuum spectra, the counts from the pixel corresponding to $50,75,100,125$ and 150 eV , was extracted from each of the 600 tungsten continuum data files. These detector counts were then plotted against the individual shot number. The results of this investigation are shown in figure 3.3. In order to quantify the level of decay in the detector counts as a result of pitting, a linear fit to the data was completed. The results of these linear fits are inset in the individual plots shown in figure 3.3.


Figure 3.3 Observation of the decay in detector counts as a result of pitting from 600 laser shots.

The mean counts for 50 eV was $\sim 903$ with a standard deviation of $\sim 31$ counts. The slope of the linear fit is $\sim-0.12$ counts/shot, which, after 600 shots, results in a decay of $\sim 72$ counts. Using the mean value of 903 counts, this represents a decay of approximately $8 \%$. The results of the pitting experiments are summarised in the table below,

| Photon <br> energy (eV) | Mean <br> (Counts) | Standard <br> deviation <br> (Counts) | Slope of linear fit <br> to pitting data <br> (Counts/shot) | Decay in <br> counts after <br> 600 shots | \% <br> decay |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{5 0}$ | 903 | 31 | -0.12 | 72 | $\sim 8 \%$ |
| $\mathbf{7 5}$ | 963 | 28 | -0.1 | 60 | $\sim 6 \%$ |
| $\mathbf{1 0 0}$ | 1047 | 40 | -0.06 | 36 | $\sim 3 \%$ |
| $\mathbf{1 2 5}$ | 965 | 50 | -0.15 | 90 | $\sim 9 \%$ |
| $\mathbf{1 5 0}$ | 860 | 44 | -0.11 | 66 | $\sim 8 \%$ |

The decay in the detector counts varies between $3 \%$ and $9 \%$ over the 600 shots. While these values are significant, they are better than the large variations observed when the target was rotated to provide a fresh surface. It was therefore decided that for each
individual knife-edge experiment, the target would not be moved. The resultant effect of this pitting into the target with respect to a knife-edge trace will be shown later in this section.

The reproducibility of the knife-edge traces can be seen in figures 3.4 and 3.5 on the following page. For each position of the knife-edge, 10 shots were taken and averaged. Using the aluminium reference spectrum to calibrate the tungsten continuum spectra, the counts on the pixel corresponding to 100 eV was extracted. These counts were then normalised and plotted against the knife-edge position. The results for three vertical scans at an energy of 100 eV are shown in figure 3.4. These vertical scans were taken on three separate occasions, January 1997, October 1997 and February 1998. It can seen from figure 3.4, that the results are quite reproducible. The set of results taken on October 1997 differ slightly from the other two sets taken on January 1997 and February 1998. This was due to the internal optics of the Nd:YAG laser being out of alignment which resulted in a slightly inconsistent energy per pulse. The set of results recorded during February 1998 were taken after the internal optics of the Nd:YAG had been realigned and they closely match the January 1997 set of results.

The results for four horizontal scans, again taken on three separate occasions, are shown in figure 3.5. The first three sets of results were taken in the normal fashion with a different focal spot for each of the individual knife-edge traces. The last knife-edge trace, recorded during February 1998, was completed using the same focal spot as the previous knife-edge trace taken during February 1998. This shows the effect of pitting is to effectively move the knife-edge trace to the right. The effect of this in terms of the source dimension estimates will be examined in Chapter 4.


Figure 3.4 Three vertical knife-edge traces at 100 eV .


Figure 3.5 Three horizontal knife-edge traces at 100 eV .

## References

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## Chapter 4

## Results and Analysis

### 4.1 Results from the physical and ray-traced knife-edge experiments

A detailed comparison between the experimental traces and the ray-tracing results, following the method outlined in Chapter 2, yielded the following common source features for all the selected photon energies.
(1) A satisfactory agreement could not be found with a 2-D source for any of the cases studied. The best fits were obtained for a 3-D source in the shape of a cylinder (fig. 4.1) with a uniform angle distribution and a constant depth distribution.


Figure 4.1 Schematic view of the source as defined in SHADOW.
(2) It was found that the fits in the vertical direction critically depended on the "width" of the source but had negligible dependence on its "height" or "depth". On the other hand, it was found that the fits in the horizontal direction critically depended on the "height" and "depth" of the source.
(3) Due to the axial symmetry of a laser produced plasma, the values of the source "width" and "height" were always kept equal. Thus, a vertical fit provides the values of "width/height" which are then used in the corresponding horizontal fit which itself provides the value of the source depth.
(4) Due to the rotation of the source by $45^{\circ}$ (see Chapter 2), the "effective height" along the z -axis that is viewed by the system is obtained by using from the following equation,

$$
\begin{equation*}
\text { "effective height" }=[(2 \times \text { height })+\text { depth }] \operatorname{Sin} 45^{\circ} . \tag{4.1}
\end{equation*}
$$

(5) The values of the source divergence in the $X$ and $Z$ direction were chosen so as to just fill the toroidal mirror (see Appendix 2) and therefore maximise the number of rays through the slit.

The vertical knife-edge response of the system completed at 50 eV is shown in figure 4.2 .


Figure 4.2 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 50 eV .

The estimate for the source dimension using the $25-75 \%$ criterion yields a value of approximately $205 \mu \mathrm{~m}$. Using the $10-90 \%$ criterion yields a value for the source dimension of approximately $373 \mu \mathrm{~m}$. In the case of the ray-traced knife-edge experiments completed using SHADOW, an overall source width of $550 \mu m$ (twice the "width" shown in fig. 4.1) provided the best fit between the physical and ray-traced knife-edge trace as shown in figure 4.2.

The horizontal knife-edge response of the system completed at 50 eV is shown in figure 4.3.


Figure 4.3 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 50 eV .

Using the $25-75 \%$ criterion yields a value of approximately $173 \mu \mathrm{~m}$ for the source dimension estimate in the horizontal direction. The source dimension estimate obtained using the $10-90 \%$ criterion is approximately $319 \mu \mathrm{~m}$. As the value of the source width and source height are kept the same, the value of $550 \mu m$, obtained from the fitting of the vertical knife-edge trace in figure 4.2, is used in the source definition for the ray-traced knife-edge experiments in the horizontal direction. The fitting of the horizontal knife-edge
trace is obtained by varying the source depth only. As shown in figure 4.3, a depth of $380 \mu \mathrm{~m}$ resulted in the best fit between the ray-traced and physical knife-edge traces. Using equation (4.1) above, the overall source height of $550 \mu \mathrm{~m}$, along with the source depth of $380 \mu \mathrm{~m}$, gives a value for the "effective height" of the source, as viewed by the spectrometer, as being approximately $658 \mu m$.

The system was then ray traced for a photon energy of 50 eV using the overall source height and width of $550 \mu \mathrm{~m}$ and the source depth of $380 \mu \mathrm{~m}$ that produced the best fit between the physical and ray-traced knife-edge traces in both the vertical and horizontal directions. The footprint of the good rays at the virtual source (star.0I) was then extracted and is shown in figure 4.4. This footprint provides the ray tracing source dimension estimates and they are approximately $536 \mu \mathrm{~m}$ in the $X$ or vertical direction and $440 \mu \mathrm{~m}$ in the $Z$ or horizontal direction.


Figure 4.4 Footprint of the "good" rays at the virtual source for a photon energy of 50 eV with the direction of the slit indicated.

The vertical knife-edge response of the system completed at 75 eV is shown in figure 4.5 .


Figure 4.5 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 75 eV .

Using the $25-75 \%$ criterion to estimate the source dimension yields a value of approximately $183 \mu \mathrm{~m}$ and the $10-90 \%$ criterion yields a value of approximately $299 \mu \mathrm{~m}$. For the ray-traced knife-edge experiments completed using SHADOW, an overall source width of $450 \mu \mathrm{~m}$ provided the best fit between the physical and ray-traced knife-edge trace as shown in figure 4.5.

The horizontal knife-edge response of the system completed at 75 eV is shown in figure 4.6 .


Figure 4.6 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 75 eV .

The $25-75 \%$ criterion yields a value of approximately $144 \mu \mathrm{~m}$ as an estimate for the source dimension in the horizontal direction. The $10-90 \%$ criterion yields a value of approximately $269 \mu \mathrm{~m}$. The overall source width of $450 \mu \mathrm{~m}$ obtained from the fitting of the vertical knife-edge trace is used in the source definition and only the source depth is varied for the ray-traced knife-edge experiments in the horizontal. As shown in figure 4.6, a depth of $325 \mu \mathrm{~m}$ resulted in the best fit between the ray-traced and physical knife-edge traces. Using equation (4.1) above, the overall source height of $450 \mu \mathrm{~m}$ along with the source depth of $325 \mu m$, gives a value for the "effective height" of the source as being approximately $548 \mu \mathrm{~m}$.

The system was then ray traced for a photon energy of 75 eV using the overall source height and width of $450 \mu \mathrm{~m}$ and the source depth of $325 \mu \mathrm{~m}$. The footprint of the good rays at the virtual source (star.01) is shown in figure 4.7. The source dimension estimates from this footprint are estimated to be $440 \mu m$ in the $X$ or vertical direction and $368 \mu \mathrm{~m}$ in the $Z$ or horizontal direction.


Figure 4.7 Footprint of the good rays at the virtual source for a photon energy of 75 eV .

The vertical knife-edge response of the system completed at 100 eV is shown in figure 4.8 .


Figure 4.8 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 100 eV .

The estimate of the source dimension using the $25-75 \%$ criterion yields a value of approximately $125 \mu \mathrm{~m}$. The $10-90 \%$ criterion yields a value of approximately $225 \mu \mathrm{~m}$. For the ray-traced knife-edge experiments completed using SHADOW, an overall source width of $400 \mu \mathrm{~m}$ provided the best fit between the physical and ray-traced knife-edge trace as shown in figure 4.8.

In figure 4.9, the horizontal knife-edge response of the system, for a photon energy of 100 eV , is shown along with a second horizontal knife-edge trace which was completed using the same focal spot.


Figure 4.9 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 100 eV .

The second knife-edge trace recorded using the same focal spot shows that the effect of pitting is to move the knife-edge trace to the right. Using the knife-edge trace recorded using a fresh surface on the target (black line), the horizontal source dimension estimate is approximately $99 \mu \mathrm{~m}$ using the $25-75 \%$ criterion and $206 \mu \mathrm{~m}$ using the $10-90 \%$ criterion. The source dimension estimates obtained from the knife-edge trace recorded using the same focal spot (green line) are approximately $118 \mu m$ and $234 \mu m$ using the $25-$ $75 \%$ and $10-90 \%$ criteria respectively. This shows that the effect of pitting is to increase the source dimension estimates. The value of $400 \mu m$, obtained from the fitting of the vertical knife-edge trace, is used in the source definition for the ray-traced knife-edge experiments in the horizontal. As shown in figure 4.9, a source depth of $350 \mu \mathrm{~m}$ resulted in the best fit between the ray-traced and physical knife-edge traces. Using equation (4.1) above, the
overall source height of $400 \mu \mathrm{~m}$, along with the source depth of $350 \mu \mathrm{~m}$, gives a value for the "effective height" of the source as being approximately $530 \mu \mathrm{~m}$.

The system was then ray traced for a photon energy of 100 eV using the overall source height and width of $400 \mu \mathrm{~m}$ and the source depth of $350 \mu \mathrm{~m}$. The footprint of the good rays at the virtual source (star.01) is shown in figure 4.10. The source dimension estimates from this footprint are estimated to be $392 \mu m$ in the $X$ or vertical direction and $368 \mu \mathrm{~m}$ in the $Z$ or horizontal direction.


Figure 4.10 Footprint of the good rays at the virtual source for a photon energy of 100 eV .

Figure 4.11 shows the vertical knife-edge response of the system completed at 125 eV .


Figure 4.11 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 125 eV .

Using the $25-75 \%$ criterion to estimate the source dimension yields a value of approximately $162 \mu \mathrm{~m}$. The $10-90 \%$ criterion yields a value of approximately $261 \mu \mathrm{~m}$ for the source dimension estimate. For the ray-traced knife-edge experiments completed using SHADOW, an overall source width of $425 \mu \mathrm{~m}$ provided the best fit between the physical and ray-traced knife-edge trace as shown in figure 4.11.

The horizontal knife-edge response of the system completed at 125 eV is shown in figure 4.12 .


Figure 4.12 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 125 eV .

The $25-75 \%$ criterion yields a value of approximately $117 \mu m$ for the source dimension estimate in the horizontal direction and the $10-90 \%$ criterion yileds a value of approximately $224 \mu m$. The value of $425 \mu \mathrm{~m}$ obtained from the fitting of the vertical knifeedge trace is used in the source definition for the ray-traced knife-edge experiments in the horizontal. As shown in figure 4.12, a depth of $325 \mu \mathrm{~m}$ resulted in the best fit between the ray-traced and physical knife-edge traces. Using equation (4.1) above, the overall source height of $425 \mu \mathrm{~m}$, along with the source depth of $325 \mu \mathrm{~m}$, gives a value for the "effective height" of the source, as viewed by the spectrometer, as being approximately $530 \mu \mathrm{~m}$.

The system was then ray traced for a photon energy of 125 eV using the overall source height and width of $425 \mu \mathrm{~m}$ and the source depth of $325 \mu \mathrm{~m}$. The footprint of the good rays at the virtual source (star.01) is shown in figure 4.13. The source dimension estimate from this footprint is estimated to be $416 \mu \mathrm{~m}$ in the $X$ or vertical direction and $360 \mu \mathrm{~m}$ in the $Z$ or horizontal direction.


Figure 4.13 Footprint of the good rays at the virtual source for a photon energy of 125 eV .

The vertical knife-edge response of the system completed at 150 eV is shown in figure 4.14.


Figure 4.14 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 150 eV .

Using the $25-75 \%$ criterion to estimate the source dimensions yields a value of approximately $145 \mu \mathrm{~m}$. Using the $10-90 \%$ criterion yields a value of approximately $274 \mu m$. For the ray-traced knife-edge experiments completed using SHADOW, an overall source width of $425 \mu m$ provided the best fit between the physical and ray-traced knifeedge trace as shown in figure 4.14.

The horizontal knife-edge response of the system, completed at 125 eV , is shown in figure 4.15.


Figure 4.15 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 150 eV .

The source dimension estimate in the horizontal direction is approximately $143 \mu \mathrm{~m}$ using the $25-75 \%$ criterion, and $294 \mu \mathrm{~m}$ using the $10-90 \%$ criterion. The value of $425 \mu \mathrm{~m}$ obtained from the fitting of the vertical knife-edge trace is used in the source definition for the ray-traced knife-edge experiments in the horizontal. As shown in figure 4.15, a depth of $380 \mu \mathrm{~m}$ resulted in the best fit between the ray-traced and physical knife-edge traces. Using equation (4.1) above, the overall source height of $425 \mu \mathrm{~m}$, along with the source depth of $380 \mu m$, gives a value for the "effective height" of the source, as viewed by the spectrometer, as being approximately $569 \mu m$.

The system was then ray traced for a photon energy of 150 eV using the overall source height and width of $425 \mu \mathrm{~m}$ and the source depth of $380 \mu \mathrm{~m}$. The footprint of the good rays at the virtual source (star.01) is shown in figure 4.16. The source dimension estimate from this footprint is estimated to be $416 \mu \mathrm{~m}$ in the $X$ or vertical direction and $392 \mu \mathrm{~m}$ in the $Z$ or horizontal direction.


Figure 4.16 Footprint of the good rays at the virtual source for a photon energy of 150 eV .

A summary of the source dimension estimates in the vertical (along the direction of the slit, X -axis) and horizontal directions obtained from the experimental knife-edge experiments using the $25-75 \%$ and $10-90 \%$ criteria is shown in Table 4.1. Table 4.1 also contains a summary of the source dimensions defined in SHADOW that gave the best fit between experimental and ray-traced knife-edge experiments along with the source dimension estimates obtained from the footprint of the good rays at the source

| Photon Energy (eV) | $\mathbf{5 0}$ | $\mathbf{7 5}$ | $\mathbf{1 0 0}$ | $\mathbf{1 2 5}$ | $\mathbf{1 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 5 - 7 5 \%}$ Vertical $(\mu \mathrm{m})$ | 205 | 183 | 125 | 162 | 145 |
| $\mathbf{2 5 - 7 5 \%}$ Horizontal $(\mu \mathrm{m})$ | 173 | 144 | 99 | 117 | 143 |


| Photon Energy (eV) | $\mathbf{5 0}$ | $\mathbf{7 5}$ | $\mathbf{1 0 0}$ | $\mathbf{1 2 5}$ | $\mathbf{1 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 0 - 9 0 \%}$ Vertical $(\mu \mathrm{m})$ | 373 | 299 | 225 | 261 | 274 |
| $\mathbf{1 0 - 9 0 \%}$ Horizontal $(\mu \mathrm{m})$ | 319 | 269 | 206 | 224 | 294 |


| Photon Energy (eV) | $\mathbf{5 0}$ | $\mathbf{7 5}$ | $\mathbf{1 0 0}$ | $\mathbf{1 2 5}$ | $\mathbf{1 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Overall source width/height <br> $(\mu \mathrm{m})$ | 550 | 450 | 400 | 425 | 425 |
| Source depth $(\boldsymbol{\mu m})$ | 380 | 325 | 350 | 325 | 380 |
| Effective source height $(\boldsymbol{\mu m})$ | 658 | 548 | 530 | 530 | 569 |
| Source dimension estimate in <br> the vertical (along $\mathbf{X})(\mu \mathrm{m})$ | 536 | 440 | 392 | 416 | 416 |
| Source dimension estimate in <br> the horizontal (along $\mathbf{Z})(\mu \mathrm{m})$ | 440 | 368 | 368 | 360 | 392 |

Table 4.1 This table shows the source dimension estimates obtained from the experimental knife-edge experiments using the $25-75 \%$ and $10-90 \%$ criteria along with the dimensions of the source used in the raytraced knife-edge experiments and the corresponding source dimension estimates obtained from the footprints at the source.

From Table 4.1 above, the overall trend is that the source dimension estimates decrease with increasing energy. This is consistent with the higher photon energies being emitted from a hot core and the lower energies being emitted from the cooler outer regions [1,2]. It can be seen that above 100 eV , the source dimension estimates do not decrease further in size which is inconsistent with the above description. This is due to scattered light in the system which results in the photons being recorded at the detector for 125 and 150 eV not being pure 125 and 150 eV photons [3].

In order to estimate the uncertainties in the ray-traced source dimension estimates, the fitting procedure between the experimental and ray-traced knife-edge traces was repeated using different source sizes until a good visual fit was no longer deemed to be attained. The uncertainties in the source dimension estimates were then obtained from the difference between the source size that provided a good visual fit and the source sizes that did not provide a good fit. This procedure was completed for both the vertical and
horizontal knife-edge traces recorded at a photon energy of 75 eV . The results of this procedure, carried out on the vertical knife-edge trace, is shown in figure 4.17. From figure 4.17, the uncertainty in the source width/height estimate of $450 \mu \mathrm{~m}$ is estimated to be between $-30 \mu \mathrm{~m}$ and $+50 \mu \mathrm{~m}$.


Figure 4.17 The results of the physical and ray-traced knife-edge experiments carried out in the vertical direction for a photon energy of 75 eV using different source sizes in order to estimate the uncertainty in the source dimension estimates.

The results of this procedure, carried out on the horizontal knife-edge trace, is shown in figure 4.18. From figure 4.18, the uncertainty in the source depth estimate of $325 \mu \mathrm{~m}$ is approximately $\pm 50 \mu \mathrm{~m}$. The uncertainties in the source dimension estimates could be reduced by increasing the number of rays that get through the system therefore improving the statistics. Increasing the numbers of rays can only be achieved by configuring and compiling the SHADOW source code to run with a larger number of rays e.g. 100,000. This however, could not be achieved on the current HP UNIX workstations due to a lack of storage space and CPU time.


Figure 4.18 The results of the physical and ray-traced knife-edge experiments carried out in the horizontal direction for a photon energy of 75 eV using different source sizes in order to estimate the uncertainty in the source dimension estimates.

### 4.2 Estimates of the absorbing volume using ray-tracing

This section will present the absorbing volume estimates obtained using ray-tracing. Due to the effect of scattered light above $\sim 100 \mathrm{eV}$, the volume estimates were only completed for the selected energies of 50,75 and 100 eV . The method used to obtain the absorbing volume estimates is outlined in Chapter2. In brief, for a particular energy, the system was ray-traced 50 times with a different random distribution of the rays in the source and the "good" rays that get through the system were extracted from 10 screens in each case. The outermost rays were then extracted from each of the ten screens and the surface area within these extracted rays was calculated. Assuming that the surface area is approximately constant over a 1 mm step, the volume of a slice of thickness $1 \mathrm{~mm}\left(v_{i}\right)$, (fig. 2.8 ), is the surface area $\times 1 \mathrm{~mm}$ and the total volume, $V$, is obtained by summing the individual slices. The source sizes used in the ray-tracing of the system at the selected energies are the shown in Table 4.1.

In figure 4.19 , the footprint of the good rays on screen 1 , positioned 20 mm from the source, resulting from 50 separate ray-tracings at a photon energy of 50 eV is shown. The rays that were extracted in order to calculate the absorption volume are also shown along with the area bounded by the extracted rays.


Figure 4.19 This figure shows the footprint of the "good" rays on screen 1, positioned 20mm from the source, resulting from 50 separate ray-tracings at a photon energy of 50 eV . The figure also shows the rays that were extracted in order to calculate the absorption volume.

In figure 4.20 , the footprint of the good rays on screen 10 , positioned 29 mm from the source, resulting from 50 separate ray-tracings at a photon energy of 50 eV is shown. The rays that were extracted in order to calculate the absorption volume are also shown along with the area bounded by the extracted rays.


Figure 4.20 This figure shows the footprint of the "good" rays on screen 10, positioned 29 mm from the source, resulting from 50 separate ray-tracings at a photon energy of 50 eV . The figure also shows the rays that were extracted in order to calculate the absorption volume.

The volume of absorption as defined by the extracted rays on screen 1 and screen 10 (figure 4.19 \& 4.20 respectively) is shown in figure 4.21 .


Figure 4.21 Figure showing the volume of absorption at a photon energy of 50 cV .

In Table 4.2, the results of the absorption volume estimates for the selected energies of 50,75 and 100 eV are shown. The volume of each of the individual slices defined by the 10 screens is also shown. For the case where the absorption volume is less than 10 mm in length, the absorption volume is obtained by summing the number of screens corresponding to its length.

| Photon Energy (eV) | $\mathbf{5 0}$ | $\mathbf{7 5}$ | $\mathbf{1 0 0}$ |
| :---: | :---: | :---: | :---: |
| Distance from source <br> (screen no.) | Volume of element <br> $\boldsymbol{i}\left(\boldsymbol{v}_{\boldsymbol{i}}\right)\left(\mathbf{m m}^{\mathbf{3}}\right)$ | Volume of element <br> $\boldsymbol{i}\left(\boldsymbol{v}_{\boldsymbol{i}}\right)\left(\mathbf{m m}^{\mathbf{3}}\right)$ | Volume of element <br> $\boldsymbol{i}\left(\boldsymbol{v}_{\boldsymbol{i}}\right)\left(\mathbf{m m}^{\mathbf{3}}\right)$ |
| $\mathbf{2 0 \mathrm { mm } ( \mathbf { 1 ) }}$ | 0.52 | 0.44 | 0.40 |
| $\mathbf{2 1 \mathbf { m m } ( \mathbf { 2 } )}$ | 0.53 | 0.44 | 0.43 |
| $\mathbf{2 2 m m ( 3 )}$ | 0.48 | 0.43 | 0.40 |
| $\mathbf{2 3 m m ~ ( 4 )}$ | 0.50 | 0.44 | 0.39 |
| $\mathbf{2 4 m m}(\mathbf{5})$ | 0.53 | 0.46 | 0.42 |


| $\mathbf{2 5 m m}(\mathbf{6 )}$ | 0.52 | 0.44 | 0.42 |
| :---: | :---: | :---: | :---: |
| $\mathbf{2 6 m m}(\mathbf{7})$ | 0.53 | 0.47 | 0.45 |
| $\mathbf{2 7 m m} \mathbf{( 8 )}$ | 0.55 | 0.55 | 0.46 |
| $\mathbf{2 8 m m} \mathbf{( 9 )}$ | 0.63 | 0.56 | 0.48 |
| $\mathbf{2 9 m m}(\mathbf{1 0})$ | 0.62 | 0.57 | 0.53 |
| Total volume (V) | 5.41 | 4.8 | 4.38 |

Table 4.2 Table showing the volume of the individual elements defined by each of the 10 screens along with the total absorption volume estimate for the selected energies of 50,75 and 100 eV .

### 4.3 Ray-traced images on the system components

In this section, the different footprints on the system's optical elements will be presented. The system was ray traced for a photon energy of 100 eV with a source "width" and "height" of $400 \mu \mathrm{~m}$ and a source depth of $350 \mu \mathrm{~m}$. Using "shadowit", the good rays on the toroidal mirror, grating and final image were extracted. In the case of the entrance slit, both the good and the lost rays were extracted. The footprints on the various optical elements below, are shown using the SHADOW's reference frame. In this reference frame, the toroidal mirror is lying flat and is facing up. In terms of the physical system, this is equivalent to looking along the optic axis towards the grating and rotating the system clockwise by $90^{\circ}$.

The footprint of the good rays on the toroidal mirror is shown in figure 4.22. The size of the toroidal mirror and the direction of the slit are also indicated.


Figure 4.22 Footprint on toroidal mirror for an effective source size of $400 \mu m \times 530 \mu m$ at 100 eV .

It can be seen that a 10 mm central portion of the toroidal mirror is used by the system which represent about $33 \%$ of the toroidal mirror's surface area. This is considerably more than the $10 \%$ previously thought to have been used by the system. This under utilisation of the toroidal mirror's height $(X)$ does not adversely affect the resolution of the grating as it is the width of the toroidal mirror $(Y)$ that determines the filling of the grating of the grating's width.

In figure 4.23, the footprint of both the good and the lost rays at the entrance slit are shown. The width of the entrance slit is also indicated.


Figure 4.23 Footprint at the entrance slit for an effective source size of $400 \mu m \times 530 \mu m$ at 100 eV .

The footprint of both the good and the lost rays on the entrance slit represent the primary image of the toroidal mirror. The primary image of the toroidal mirror is a curved line approximately 13 mm in height $(X)$ and approximately 0.48 mm in width $(Z)$. Of the 20,000 rays produced at the source, only 675 , or approximately $3.4 \%$, of them get through
the entrance slit toward the grating. The height $(X)$ of the slit utilised is approximately 10.25 mm .

The footprint of the good rays on the grating is shown in figure 4.24. The grating size and the direction of the rulings are also indicated.


Figure 4.24 Footprint on grating for an effective source size of $400 \mu m \times 530 \mu m$ at 100 eV .

In Chapter 1, the optimum width of the grating, at a wavelength of 18 nm , was calculated to be 25.43 mm . From figure 4.24 , it can be seen that approximately 42 mm of the grating's width ( $Y$ ) is illuminated and the rays are distributed quite evenly across its surface. The height $(X)$ of the footprint on the grating is approximately 10.5 mm . The final image from the grating, which falls on the detector, is shown in figure 4.25 .


Figure 4.25 Footprint at the final image for an effective source size of $400 \mu m \times 530 \mu m$ at 100 eV .

The final image at the detector is approximately 10.25 mm in height $(X)$ and 0.0315 mm in width.

When the instrument was designed, the parameters of the toroidal mirror were chosen such that, over the wavelength range of the system, the length of spectral lines at the detector would be approximately the same. In order to verify that the spectral lines are approximately the same height, the system was individually ray-traced for the five selected energies using the corresponding source sizes that were obtained from the fitting of the knife-edge experiments (Table 4.1). The cylindrical optical element defined on the Rowland circle provided a file with the intercepted rays on the Rowland circle. The energy of 100 eV was centred on the cylindrical optical element by setting the auto-tuning option to 100 ev in each case. The resulting final images for the five selected energies were then plotted on the same graph as shown in figure 4.26. As can be seen from figure 4.26, the height $(X)$ of the final images for the five different energies are of similar height.


Figure 4.26 Ray-traced images at the detector for five different energies with the grating tuned to 100 eV .
Each energy is traced with a separate source size obtained from the knife-edge traces.

## References

[1] Carroll, P.K., Kennedy, E.T., "Laser-Produced Plasmas", 1981 Taylor \& Francis Ltd.
[2] Tondello, G., Jannitti, E., Nicolosi, P., Santi, D., "Structure of the XUV emitting regions in a laser produced plasma", Optics Communications, Vol. 32, No. 2, February 1980.
[3] Whitty, W., Private communication.

## Conclusions

The experimental and target design, along with the initial and experimental alignment procedures, provided reproducible knife-edge experiments as shown in section 3.3. The knife-edge experiments were also proven to be reproducible over long periods of time. The problem of pitting into the target throughout the course of a single knife-edge experiment was also examined. The results of this study showed that while there was a reduction in detector counts due to pitting, the reduction was better that the large variations observed when moving the target to a fresh surface. The effect of pitting into the target was to move the knife-edge trace to the right resulting in a larger source dimension estimate being obtained. While the increase in the source dimension estimates for the horizontal knife-edge trace at a photon energy of 100 eV (figure 4.9) was significant, $\sim 19 \%$ (using the $25-75 \%$ criterion) and $\sim 13.5 \%$ (using the $10-90 \%$ criterion), this is an extreme case where more than 600 shots were taken on the target. A typical knife-edge experiment will have approximately 300 shots on the target with the expected result being that the increase in the source dimension estimates will be significantly less than the above increases.

The results of the experimental and ray-traced knife-edge traces are presented in section 4.1. Source dimension estimates were obtained using the $25-75 \%$ and $10-90 \%$ criteria and by ray-tracing. The source dimensions estimates obtained from these methods are presented in table 4.1. The estimates from the $25-75 \%$ and $10-90 \%$ criteria fell short of the estimates obtained from ray-tracing. The source dimension estimates obtained through ray-tracing are more reliable and accurate than the estimates obtained through the use of the $25-75 \%$ and $10-90 \%$ criteria. To obtain source dimension estimates via ray-tracing involves a slow and laborious procedure and the $25-75 \%$ criterion and, in particular the $10-90 \%$ criterion, should therefore only be used for comparison purposes or for quick and rough source dimension estimates when different lasers, focal spot sizes and target materials are used. In section 4.2, the absorption volume estimates for photon energies of 50, 75 and 100 eV are presented. The absorption volume estimates of $5.41 \mathrm{~mm}^{3}(50 \mathrm{eV}), 4.8 \mathrm{~mm}^{3}$ $(75 \mathrm{eV}), 4.38 \mathrm{~mm}^{3}(100 \mathrm{eV})$, will be used in any future experiments involving the measurement of absolute photoabsorption cross-section data.

In section 4.3, the footprints on the different optical elements obtained through raytracing are shown. The footprint on the toroidal mirror is shown in figure 4.22 and it can be seen that approximately $33 \%$ of the toroidal mirror's surface is used by the system. This is greater the $10 \%$ previously thought to have been used. In figure 4.23 , the footprint on the slit is shown and only $\sim 3.4 \%$ of the rays that are incident on the slit pass through to the concave grating. The footprint on the grating is shown in figure 4.24 and it can be seen that the optimum width of the grating is illuminated and thus, the optimum resolving power is achieved. The toroidal mirror parameters were chosen such that, over the wavelength range of the system, the length of the spectral lines at the detector would remain approximately the same. The system was ray-traced at five different photon energies $(50,75,100,125$, 150 eV ) using the source sizes obtained from the fitting of the knife-edge experiments. The resulting final images at the detector are shown in figure 4.25 and it can be seen that the final image heights are approximately the same for these five different energies. In any future work involving modifications/additions to the system, such as the insertion of apertures to improve the spatial resolution of the system, the implications to the imaging properties of the system can be examined using the source sizes obtained from the fitting of the knife-edge experiments

## Appendix 1

This appendix contains a brief introduction to the theory of the concave grating and the toroidal mirror. Much of the mathematical rigor will be omitted and only a summary of the important equations associated with the concave grating and the toroidal mirror will be presented.

## The Concave Diffraction Grating

The idea of a concave diffraction grating was originally developed by H. A. Rowland [1] in 1882. Since then, considerable work has been done by Beutler [2] and Namioka [3] using geometrical optics and by Mack, Stehn and Edlén [4] using physical optics. Prof. H. A. Rowland found that if a concave grating with a radius of curvature $R$, is placed at a tangent to a circle with a radius of curvature $0.5 R$, then the spectrum of an illuminated point lying on the circle will be focused onto this circle (fig. A1.1).


Figure A1.1 The Rowland circle. S is the illuminated point lying on the circle and the radiation is dispersed by the grating at $\lambda_{1}, \lambda_{2}$, etc. $\alpha$ and $\beta$ are the angles of incidence and diffraction respectively.

## General Theory

In this section, the general theory of the concave grating will be outlined using the geometrical approach developed by Beutler and Namioka. To develop the theory of the concave grating, a system of Cartesian co-ordinates must first be set up (fig. A1.2). Let the origin of the Cartesian co-ordinate system, $O$, be at the centre of the grating. The x-axis is the grating normal, the $y$-axis is perpendicular to the grating rulings and the $z$-axis is parallel to the grating rulings. Let the point $A$ represent the light source with its position described by the co-ordinates $x, y$ and $z$. Let the point $B$ represent the diffracted image of $A$ with its position described by the co-ordinates $x^{\prime}, y^{\prime}$ and $z^{\prime}$. Let any point on the grating, $P$, be represented by the co-ordinates $u, w$ and $l$ where only discrete values of $w$ are allowed.


Figure A1.2 Co-ordinate system used in the analysis of the concave gratings' imaging properties.

The condition for any two rays, which have been reflected by two adjacent rulings, to reinforce at $B$ is that the path difference be an integral number of wavelengths. The path difference between any two rays separated by $w$ is $m \lambda w / d$, where $m$ is the spectral order, $\lambda$ is the wavelength, $w$ is the separation of the two rulings and $d$ is the grating constant.

Therefore, any arbitrary light path $A P B$, may be represented by the characteristic or optical path function ${ }^{\dagger} F$, given by,

$$
\begin{equation*}
F=A P+B P+\frac{m \lambda w}{d} \tag{1.1}
\end{equation*}
$$

where

$$
\begin{equation*}
|A P|^{2}=(x-u)^{2}+(y-w)^{2}+(z-l)^{2} \tag{1.2a}
\end{equation*}
$$

and

$$
\begin{equation*}
|B P|^{2}=\left(x^{\prime}-u\right)^{2}+\left(y^{\prime}-w\right)^{2}+\left(z^{\prime}-l\right)^{2} \tag{1.2b}
\end{equation*}
$$

In the development of the concave grating theory it is convenient to express equations (1.2a) and (1.2b) in terms of the distances between the points $A$ and $B$ and the grating centre $O$, and the angles of incidence and diffraction, $\alpha$ and $\beta$ respectively. This requires the use of cylindrical co-ordinates and they are defined as follows,

$$
\begin{array}{ll}
x=r \cos \alpha & x^{\prime}=r^{\prime} \cos \beta  \tag{1.3}\\
y=r \sin \alpha & y^{\prime}=r^{\prime} \sin \beta
\end{array}
$$

where $\alpha$ and $\beta$ are the angles of incidence and diffraction respectively. The grating has a spherical surface of radius $R$ and any point $P$ on this surface can be represented by the equation,

$$
\begin{equation*}
(u-R)^{2}+w^{2}+l^{2}=R^{2} \tag{1.4a}
\end{equation*}
$$

which can be written as,

[^0]\[

$$
\begin{equation*}
u^{2}-2 R u+\left(w^{2}+l^{2}\right)=0 . \tag{1.4b}
\end{equation*}
$$

\]

The roots of the quadratic equation (1.4b) are given by,

$$
\begin{equation*}
u=R \pm\left[R^{2}-\left(w^{2}+l^{2}\right)\right]^{\frac{1}{2}} \tag{1.5}
\end{equation*}
$$

In equation (1.5), only the negative root is significant as the positive root gives the points on the other side of the diameter. Using a power series expansion, equation (1.5) can be written as,

$$
\begin{equation*}
u=\frac{w^{2}+l^{2}}{2 R}+\frac{\left(w^{2}+l^{2}\right)^{2}}{8 R^{3}}+\frac{\left(w^{2}+l^{2}\right)^{3}}{16 R^{5}}+\ldots . \tag{1.6}
\end{equation*}
$$

Now, using equation (1.4b), equation (1.2a) can be written as,

$$
\begin{equation*}
|A P|^{2}=x^{2}+y^{2}+z^{2}+2 R u-2 x u-2 y w-2 z l \tag{1.7}
\end{equation*}
$$

Introducing the cylindrical co-ordinates given by the equations in (1.3) and substituting equation (1.6) for $u$, equation (1.7) can be written as,

$$
\begin{align*}
&|A P|^{2}=(r-w \sin \alpha)^{2}+w^{2}\left(\cos ^{2} \alpha-\frac{r \cos \alpha}{R}\right)^{2}+l^{2}\left(1-\frac{r \cos \alpha}{R}\right) \\
&-2 z l+z^{2}+\frac{\left(w^{2}+l^{2}\right)^{2}}{4 R^{2}}\left(1-\frac{r \cos \alpha}{R}\right)\left(1+\frac{w^{2}+l^{2}}{2 R^{2}}+\ldots \ldots .\right) \tag{1.8}
\end{align*}
$$

The square root of $|A P|^{2}$ in the above equation (1.8) may be extracted using a further series expansion. This will lead to a final expression for $A P$ that is large and cumbersome with many higher order terms. A similar analogous result may be derived for $B P$. To simplify the treatment of the series expansion, $A P$ and $B P$ may be written as,

$$
\begin{align*}
& A P=F_{1}+F_{2}+F_{3}+\ldots \ldots .  \tag{1.9a}\\
& B P=F_{1}^{\prime}+F_{2}^{\prime}+F_{3}^{\prime}+\ldots \ldots . \tag{1.9b}
\end{align*}
$$

Each of the individual terms $F_{n}$, has a physical significance with regard to image formation and aberrations. For example, the terms $\left(F_{1}+F_{1}^{\prime}\right)$ and $\left(F_{2}+F_{2}^{\prime}\right)$ give the conditions for image formation for the plane and concave grating. The terms ( $F_{3}+F_{3}^{\prime}$ ) give the astigmatism and $\left(F_{4}+F_{4}^{\prime}\right)$ give the coma and curvature of the spectral lines. $\left(F_{5}+F_{3}^{\prime}\right)$ determines the spherical aberration which limits the useful size of the grating for any particular $\alpha, \beta, R$ and $d .\left(F_{6}+F_{6}^{\prime}\right)$ can be made equal to zero and $\left(F_{7}+F_{7}^{\prime}\right)$ represents a higher order aberration. In general, each successive term decreases in magnitude due higher inverse powers of $R, r$, or $r^{\prime}$ (excluding cases of very large angles of incidence $\alpha$, and diffraction $\beta$ ). For the scope of this work, it can be shown that the major over-riding aberration is astigmatism. The $4^{\text {th }}$ order terms and higher will therefore be omitted as their values are negligible in comparison to astigmatism. Using equations (1.9a) and (1.9b), the characteristic or optical path function $F$, may be written as,

$$
\begin{equation*}
F=F_{1}+F_{2}+F_{3}+F_{1}^{\prime}+F_{2}^{\prime}+F_{3}^{\prime}+\frac{m \lambda w}{d} \tag{1.10}
\end{equation*}
$$

where

$$
\begin{align*}
& F_{1}= r-w \sin \alpha  \tag{1.10a}\\
& \begin{aligned}
F_{2}= & \frac{1}{2} w^{2}\left(\frac{\cos ^{2} \alpha}{r}-\frac{\cos \alpha}{R}\right)+\frac{1}{2} w^{3} \frac{\sin \alpha}{r}\left(\frac{\cos ^{2} \alpha}{r}-\frac{\cos \alpha}{R}\right) \\
& +\frac{1}{2} w^{4} \frac{\sin ^{2} \alpha}{r}\left(\frac{\cos ^{2} \alpha}{r}-\frac{\cos \alpha}{R}\right)+\ldots . . \\
F_{3}= & \frac{1}{2} l^{2}\left(\frac{1}{r}-\frac{\cos \alpha}{R}\right)-l \frac{z}{r}+\frac{z^{2}}{2 r} \\
F_{1}^{\prime}= & r^{\prime}-w \sin \beta
\end{aligned} \tag{1.10b}
\end{align*}
$$

$$
\begin{align*}
& F_{2}^{\prime}= \frac{1}{2} w^{2}\left(\frac{\cos ^{2} \beta}{r^{\prime}}-\frac{\cos \beta}{R}\right)+  \tag{1.10e}\\
&+\frac{1}{2} w^{3} \frac{\sin \beta}{r^{\prime}}\left(\frac{\cos ^{2} \beta}{r^{\prime}}-\frac{\cos \beta}{R}\right) \\
&+\frac{1}{2} w^{4} \frac{\sin ^{2} \beta}{r^{\prime}}\left(\frac{\cos ^{2} \beta}{r^{\prime}}-\frac{\cos \beta}{R}\right)+\ldots . \cdot  \tag{1.10f}\\
& F_{3}^{\prime}= \frac{1}{2} l^{2}\left(\frac{1}{r^{\prime}}-\frac{\cos \beta}{R}\right)-l \frac{z}{r^{\prime}}+\frac{z^{2}}{2 r^{\prime}}
\end{align*}
$$

To further develop the theory of the concave diffraction grating and to obtain the conditions for image formation we need to invoke Fermat's Principle of Least Time. Fermat's Principle states that the optical path between two points taken by a beam of light is the one traversed in the least time [5]. Applying Fermat's principle to the concave grating, point $B$ is located such that the function $F$, will be an extreme for any point $P$. To focus $A$ at $B$, all these extremes for the various points $P$, must be equal. Since points $A$ and $B$ are fixed, and while $P$ may be any point on the grating's surface, the condition for $F$ to be an extreme for points along $w$ is,

$$
\begin{equation*}
\frac{\partial F}{\partial w}=0 \tag{1.11}
\end{equation*}
$$

and similarly, the condition for $F$ to be an extreme for points along $l$ is,

$$
\begin{equation*}
\frac{\partial F}{\partial l}=0 \tag{1.12}
\end{equation*}
$$

Equation (1.11) represents the condition for light, coming from points along the grating's horizontal, $w$, to be focused, and equation (1.12) represents the condition for light coming from points along the grating's vertical, $l$, to be focused. Equations (1.11) and (1.12) also have the significance of angles. If their values are zero then all the light coming from $A$ and reflected from $P$ will be directed strictly towards $B$. Therefore, if the partial derivatives in equations (1.11) and (1.12) were satisfied simultaneously, for any pair of $w$ and $l$ and for a fixed point $B$, a perfect focus would be obtained at $B$.

In the case of the concave grating, a perfect image cannot be obtained due to the partial derivatives given in equations (1.11) and (1.12) being non-zero. The deviation of these partial derivatives from zero indicates that some light falls outside $B$ and that the image formation is not perfect. The reason for this is that a ray, diffracted from a point $P(w, l)$, goes in the direction specified by $\beta$ and $z^{\prime} / r^{\prime}$ which are determined by equations (1.11) and (1.12). Since $\beta$ and $z^{\prime} / r^{\prime}$ are functions of $w$ and $l$, the direction of the diffracted ray changes slightly with each different point $P(w, l)$ on the grating's surface. Therefore, the rays diffracted from the different points $P(w, l)$ on the grating's surface arrive at slightly different points on the focal plane at $B$. In the case of a concave grating imaging a point source, the horizontal focusing will be achieved first, resulting in a vertical line. The vertical focusing will be achieved at a later point forming a horizontal line.

## The Rowland circle focusing conditions

It can be seen that the grating equation given by (1.17) has no information about the focusing conditions because $r$ and $r^{\prime}$ are not present. In the case of the concave grating, $R$, the radius of curvature of the grating, has finite values. The higher order terms contained in equation (1.10), therefore become more significant. The partial derivatives of these higher order terms must be evaluated and Fermat's principle applied to the result. On applying Fermat's principle to the partial derivatives of the second order terms, it is found that the result only goes to zero when the first term, shown below, goes to zero,

$$
\begin{equation*}
\frac{\cos ^{2} \alpha}{r}-\frac{\cos \alpha}{R}+\frac{\cos ^{2} \beta}{r^{\prime}}-\frac{\cos \beta}{R}=0 \tag{1.21a}
\end{equation*}
$$

This can be rewritten as,

$$
\begin{equation*}
\cos \alpha\left(\frac{\cos \alpha}{r}-\frac{1}{R}\right)+\cos \beta\left(\frac{\cos \beta}{r^{\prime}}-\frac{1}{R}\right)=0 \tag{1.21b}
\end{equation*}
$$

One of the solutions of this above equation (1.21b) is,

$$
\begin{equation*}
r=R \cos \alpha ; \quad r^{\prime}=R \cos \beta \tag{1.22}
\end{equation*}
$$

The solutions contained in (1.22) are the co-ordinates of a circle of diameter $R$, on which the points $r$ and $r^{\prime}$ lie. This circle is known as the Rowland circle. These solutions also make the second part of equation (1.20) equal to zero. It can be shown that all the higher terms in $F_{2}$ and $F_{2}^{\prime}$ cancel.

## Astigmatism

As stated previously, the major aberration associated with the concave grating is astigmatism. In the case of astigmatism, the horizontal or primary focus of a point source is completed in the Meridional plane (uw) first, i.e. the plane of dispersion, to produce a vertical line. The primary focus is sometimes referred to as the Meridional image. The vertical or secondary focus is achieved beyond the Rowland circle to produce a horizontal line in the Sagittal plane (wl), in which case the horizontal focus is lost (fig. A1.4). The secondary focus is sometimes referred to as the Sagittal image.


Figure A1.4 Diagram showing the astigmatism of the concave grating at grazing incidence.

As seen previously, the astigmatism of the concave grating is given by the third order terms in equation (1.10), namely $F_{3}$ and $F_{3}^{\prime}$. Applying Fermat's principle to the partial derivatives of the third order terms and introducing the Rowland focusing conditions given by the equations in (1.22) we have,

$$
\begin{array}{r}
\frac{\partial\left(F_{3}+F_{3}^{\prime}\right)}{\partial l}=l\left(\frac{1}{R \cos \alpha}-\frac{\cos \alpha}{R}+\frac{1}{R \cos \beta}-\frac{\cos \beta}{R}\right)  \tag{1.24}\\
-\frac{z}{R \cos \alpha}-\frac{z^{\prime}}{R \cos \beta}=0
\end{array}
$$

Using the following simplifications,

$$
\frac{1}{R \cos \alpha}-\frac{\cos \alpha}{R}=\frac{\left(1-\cos ^{2} \alpha\right)}{R \cos \alpha} \quad \text { and } \quad \frac{1}{R \cos \beta}-\frac{\cos \beta}{R}=\frac{\left(1-\cos ^{2} \beta\right)}{R \cos \beta}
$$

and knowing that $1 / R \neq 0$, gives,

$$
\begin{equation*}
\left(\frac{\sin ^{2} \alpha}{\cos \alpha}+\frac{\sin ^{2} \beta}{\cos \beta}\right)-\frac{z}{\cos \alpha}-\frac{z^{\prime}}{\cos \beta}=0 \tag{1.25}
\end{equation*}
$$

Thus, the length of the astigmatic image, $z^{\prime}$, is given by,

$$
\begin{equation*}
z^{\prime}=\left(z \frac{\cos \beta}{\cos \alpha}\right)+l\left(\sin ^{2} \beta+\sin ^{2} \alpha \frac{\cos \beta}{\cos \alpha}\right) \tag{1.26}
\end{equation*}
$$

where $\alpha$ is the angle of incidence, $\beta$ is the angle of diffraction and $l$ is the illuminated height of the grating's rulings. The first term gives the contribution due to the entrance slit of finite vertical length, $z$. The second term, which is independent of $z$, describes the astigmatism produced by a point source (i.e. $z=0$ ). As can be seen from the dependence of this equation on $\alpha$ and $\beta$, the angles of incidence and diffraction respectively, the effect of
astigmatism will be greater at larger angles of $\alpha$ and $\beta$ i.e. at grazing incidence. There is therefore little or no focusing in the sagittal plane, $w l$, of the grating [7].

If any point $z$ is to be focused at some point $z^{\prime}$, it is also expected that in the case of a point source in the plane of the Rowland circle $(z=0)$, the focus will also lie in the plane of the Rowland circle $\left(z^{\prime}=0\right)$. Using this relation, equation (1.23) may be written as,

$$
\begin{equation*}
l\left(\frac{1}{r}-\frac{\cos \alpha}{R}+\frac{1}{r^{\prime}}-\frac{\cos \beta}{R}\right)=0 \tag{1.27}
\end{equation*}
$$

Since $l \neq 0$, and for a focus to exist at all, the expression enclosed in the parenthesis must be zero. Therefore,

$$
\begin{equation*}
\frac{1}{r}-\frac{\cos \alpha}{R}+\frac{1}{r^{\prime}}-\frac{\cos \beta}{R}=0 \tag{1.28}
\end{equation*}
$$

where $r^{\prime}$ gives the location of the secondary focus. An interesting point to note is that due to the wavelength dependence of diffraction given by equation (1.17), both the primary and secondary foci have unique positions for each wavelength with the primary focus always being focused on the Rowland circle.

## Resolving power

It was shown by Namioka [3] and by Mack, Stehn and Edlén [4] that for a concave grating, the diffraction minima do not reach zero. To account for this they introduced the modified Rayleigh criterion which did not require that the maximum of one line should fall on the first minimum of the other line. However, the modified Rayleigh criterion still required that the ratio of the minimum intensity of the composite structure to that of either maximum is still $8 / \pi^{2}$ or 0.8106 (fig. A1.5).


Figure A1.5 Plot of relative intensity versus wavelength showing Rayleigh's modified criterion [1].

Using physical optics and the modified Rayleigh criterion, it can be shown that the resolving power of a concave grating is equal to $m N$ when the illuminated width of the grating, $W$, is $\leq W_{\text {opt }} / 1.18$. As the illuminated width of the grating increases beyond $W_{\text {opt }} / 1.18$, so too does the resolving power but not as rapidly as $m N$. The resolving power increases until it reaches a maximum when $W=W_{o p t}$. The optimum resolving power, $\mathscr{R}_{\text {opt, }}$ is then given by,

$$
\begin{equation*}
\mathscr{R}_{\mathrm{opt}}=0.92 \frac{W_{o p t} m}{d} \tag{1.34}
\end{equation*}
$$

An expression for the optimum width of the concave grating, $W_{\text {opt }}$, can be obtained from by analysing the $5^{\text {th }}$ order terms contained within equations (1.9a) and (1.9b) [2,3]. This is a rather lengthy and complex process and only the result is presented here. Therefore, the optimum width of the concave grating, $W_{\text {opt }}$, is given by [1],

$$
\begin{equation*}
W_{o p t}=2.51\left[\frac{R^{3} \lambda \cos \alpha \cos \beta}{\sin ^{2} \alpha \cos \beta+\sin ^{2} \beta \cos \alpha}\right]^{\frac{1}{4}} \tag{1.35}
\end{equation*}
$$

The above equation gives the optimum width of the grating that should be illuminated in order to optimise the resolving power of the grating.

## The Toroidal Mirror

A few years after Beutler published his theory on the concave grating, Haber, H. [8] applied the same analytical technique to develop the theory of the toroidal grating. The main difference between a toroidal and a spherical surface is that for a toroidal surface, the radius of curvature along its width differs from the radius of curvature along its height (fig. A1.6). A spherical or concave surface, where the radii of curvature along the width and height are the same, can therefore be considered a special case of a toroidal surface.


Figure A1.6 Generation of a torus by revolving a circle of radius $\rho$, lying in the $u^{\prime} l^{\prime}$ plane, about the axis $l^{\prime}$, where the centre of the circle lies a distance $R-\rho$ from the axis of revolution $l^{\prime}$.

In order to obtain the theory for the toroidal mirror, we must first develop the theory for the toroidal grating put forward by Haber. When this is completed, we can then extract the theory for the toroidal mirror by simply invoking the law of reflection, which states that the angle of incidence equals the angle of reflection. The main area of interest in the theory of the toroidal mirror is the location of the primary and secondary foci. Due to the fact that much of the theory for the toroidal grating follows closely that of the concave grating, much of the theory will be omitted and only the above areas of interest will be developed.

## General Theory

A torus is generated by revolving a circle of radius, $\rho$, lying in the $u^{\prime} l^{\prime}$ plane, about the axis $l^{\prime}$, where the centre of the circle lies a distance $R-\rho$ from the axis of revolution, $l^{\prime}$ (fig. A1.6). As can be seen from figure A1.6, a torus has two distinct radii of curvature. The radius of curvature in the $u w$ plane, $R$, is called the major radius. The radius of curvature in the $u l$ plane, $\rho$, is called the minor radius.

Using the same analytical technique as Beutler, Haber derived the optical path function, $F=A P+B P+m \lambda w / d$, in the same way except that the equation for the spherical surface, (1.4a), is replaced by the equation for the toroidal surface given below,

$$
\begin{equation*}
u^{2}+w^{2}+l^{2}=2 R u-2(R-\rho)\left\{R-\left[(R-u)^{2}+w^{2}\right]^{\frac{1}{2}}\right\} \tag{1.36}
\end{equation*}
$$

Note that if $\rho=R$, the above expression reduces to that of the spherical case developed by Beutler. The length of the light path, $|A P|^{2}$, is given by equation (1.2a).Expanding equation (1.36) in a power series and substituting the result into equation (1.37) yields the Cartesian expression for $|A P|^{2}$. In the spherical case, this substitution eliminated $u$ directly (see equation (1.8)). In the present case however, the elimination of $u$ requires that a quadratic equation in $u$ be formed from the power series expansion of equation (1.36). The negative radical of the solution of this quadratic equation is then substituted in for $u$. Converting to cylindrical co-ordinates and extracting the square root yields the final expression for $[A P]$. Similarly, an expression for the light path, $[B P]$, may be obtained by the same method.

Having obtained expressions for the optical path lengths $[A P]$ and $[B P]$, a final expression for the optical path function, $[A P]+[B P]$, may be obtained. The final expression for the optical path function may be separated into different terms in much the same way as

Beutler did for the concave grating. Each of the individual terms, $T_{n}$, has its own individual significance in relation to image formation and aberrations.

The terms, $T_{n}$, control the foci and aberrations in the image. The significance of the terms are as follows: the term $T_{1}$ describes the general grating equation; the term $T_{2}$ describes the primary (horizontal) focus condition on the Rowland circle; the term $T_{3}$ describes the secondary (vertical) focus which is produced as a result of astigmatism.

As stated previously, the two areas of interest for the toroidal mirror are the locations of the primary and secondary foci. These foci are obtained from the terms $T_{2}$ and $T_{3}$ respectively, and the higher order terms are therefore omitted. Also, as in the case of the concave grating, astigmatism (term $T_{3}$ ) is the main aberration and the magnitude of the higher order terms decreases with each successive term (again, as with the concave grating, excluding cases of very large angles of incidence $\alpha$, and diffraction $\beta$ ). The higher order terms are therefore less significant in the formation of the image and its aberrations.

The first two terms $T_{1}$ and $T_{2}$ for the toroidal grating are the same as the first two terms for the concave grating given by equations (1.10a), (1.10b), (1.10d) and (1.10e). The remaining higher order terms contain some differences that account for the differences between the concave and toroidal grating as far as image formations and aberrations are concerned.

## Primary focus

As stated previously, the conditions for the primary focus of the toroidal grating are contained within the term $T_{2}$. As in the case of the concave grating, Fermat's principle is applied to the partial derivative of the second term (taken w.r.t. $w$ ). Invoking the law of
reflection $(\alpha=\beta=\varphi)$ and taking into account that $\cos \varphi$ is small for large angles of incidence and that $w$ is small and $R$ is large, yields,

$$
\begin{equation*}
\frac{1}{r}+\frac{1}{r_{1}{ }^{\prime}}=\frac{2}{R \cos \varphi} \tag{1.40}
\end{equation*}
$$

The above equation defines the location of the primary (horizontal) focus, $r_{1}{ }^{\prime}$. As with the concave grating, the primary image is sometimes referred to as the Meridional focus. The other elements of this equation are the source to mirror distance, $r$, the major radius of the toroidal mirror, $R$, and the angle of incidence $\varphi$.

## Secondary focus

The secondary focus conditions can be obtained from the term $T_{3}$. Again, to convert from grating to mirror theory we invoke the law of reflection whereby we set $\alpha=\beta=\varphi$. To further simplify the analysis of the $T_{3}$ term, any squared terms are neglected, as their values are small. If any point $z$, is to be focused at some point $z^{\prime}$, it is also expected that in the case of a point source $(z=0)$ in the equatorial plane $(u w)$, the focus $\left(z^{\prime}=0\right)$ will also lie in the equatorial plane, $(u w)$. As before, applying Fermat's principle to the partial derivative of the third term (taken w.r.t. $l$ ), yields,

$$
\begin{equation*}
\frac{1}{r}+\frac{1}{r_{2}{ }^{\prime}}=\frac{2 \cos \varphi}{\rho} \tag{1.42}
\end{equation*}
$$

The above equation defines the location of the secondary (vertical) focus, $r_{2}{ }^{\prime}$. Again, as with the concave grating, the secondary focus is sometimes referred to as the Sagittal focus. The other elements of this equation are the source to mirror distance, $r$, the minor radius of the toroidal mirror, $\rho$, and the angle of incidence $\varphi$. The equations for the location of the primary, (1.40), and secondary, (1.42), foci are sometimes referred to as the Coddington equations [9].

## Summary of the focusing properties of the toroidal mirror

Using equations (1.40) and (1.42), it can be seen that for a given source distance, $r$, the position of the primary and secondary foci, $r_{1}{ }^{\prime}$ and $r_{2}{ }^{\prime}$ respectively, may be controlled independently by varying the various parameters. In the case of the primary focus, $r_{1}{ }^{\prime}$, the location of the focus may be controlled by varying the major radius, $R$, the angle of incidence, $\varphi$, or the source to mirror distance, $r$. The location of the secondary focus may be varied independently by changing the values for the minor radius, $\rho$, the angle of incidence, $\varphi$, or the source to mirror distance, $r$. A schematic for the positions of the foci, with respect to each other, is much the same as the one given for the concave grating in figure A1.4.

In order to gain a more complete understanding of the imaging aberrations associated with the toroidal mirror, a more detailed analysis of the optical path function must be completed. The method of analysis follows closely that of the concave grating but it is omitted here as it is beyond the scope of this work. While a more complete working is omitted, a brief outline of the important aberrations as follows.

The primary image, as in figure A1.4, is formed in the plane perpendicular to the principal ray. It has the form of a parabolic curve with its vertex positioned at the middle of the entrance slit. The curvature of the image depends directly on the values of $R, \rho, r$, and $\varphi$, where $R$ is the major radius of the toroidal mirror, $\rho$ is the minor radius of the toroidal mirror, $r$ is the source to mirror distance and $\varphi$ is the angle of incidence respectively. The primary image has a finite width due mainly to the spherical aberration of the mirror. The width of the image is minimised when the source to mirror distance, $r$, equals the mirror to entrance slit (location of primary focus), $r_{1}{ }^{\prime}$, which may be written as [7],

$$
\begin{equation*}
r=r_{1}^{\prime}=R \cos \varphi \tag{1.43}
\end{equation*}
$$

The secondary image, as in figure A1.4, is formed perpendicular to the primary image and is also curved. The secondary image, however, is not perpendicular to the principal ray, it is instead inclined at an angle. The angle of inclination is a function of the system aperture and the angle of incidence, $\varphi$. The inclination is therefore most pronounced at grazing incidence.

## References

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## Appendix 2

SHADOW [1] is released in the form of a FORTRAN and C library of subroutines. The core calculation engine, and many of the subroutines and utilities, are written in FORTRAN 77 with more recent additions for UNIX being written in C [2,3]. The graphic interpreter, Primvs, is built on top of the PLPLOT C-library. The user interface that controls SHADOW is written in the resident command language, in this case the Bourne shell. The program works in double precision with a typical accuracy of 1 part in $10^{13}$. No approximation is ever made throughout the code and the use of trig functions is kept to a minimum by vector algebra [2]. SHADOW can be obtained in either source code form or in precompiled form for a limited number of UNIX platforms and more recently for Windows 95©. Earlier versions of SHADOW (v2.0, v2.1.0) were distributed, in either source code or precompiled form, with the ability to generate only 5,000 rays. The latest versions of SHADOW (v2.2.0 onwards) have removed the restriction of having only 5,000 rays. The version used throughout this project was SHADOW v2.2.0 and it was compiled with 20,000 rays on a HP9000/720 with UNIX version hpux10.01. SHADOW is comprised of a core group of programs and a collection of utilities that are used to either process the output from the main programs or to provide data files required by SHADOW. The structure of SHADOW can be subdivided into 3 distinct groups and a brief description of each is as follows,

## Group 1 :- The Input/Output (I/O) session.

This group is used to define the source and optical elements contained within an optical system. There are three programs provided in this group namely "PROMPT", "MENU" and "BATCH". The first two can be used to define and ray-trace the optical system and the last is used to ray-trace the system with the existing parameter files. These programs are accessed via the startup command "GO" which is the outermost level of user interface that controls the flow of the programs. In "PROMPT" mode, a series of on screen questions prompt the user for the parameters required to define the source or an optical element. In "MENU" mode, the definition of the source or optical element is presented in a series of menus that are accessed from the command line. This mode is for the experienced
user and it offers improved flexibility and is less time consuming than the "PROMPT" mode. In "BATCH" mode, the user is requested to input filenames of previously generated parameter files (usually of the form "start. $X X$ " as described later) and these files are then used to ray-trace the system again. This mode does not allow the parameters of an optical system to be accessed and changed and is only used to regenerate the output data files.

## Group 2 :- Optics engine

This is where the actual calculations are performed i.e the system is ray-traced. This group consists of "SOURCE" and "TRACE" programs. The "SOURCE" program generates the rays in accordance with the specifications of source defined by the user. The "TRACE" program takes the rays developed by the "SOURCE" program and traces the rays through the optical system.

## Group 3 :- Analysis

This group can be further subdivided into pre-processors and post-processors. The pre-processors are a group of utilities that are used before the system is ray-traced. There are two types of pre-processors, one which generates an output file used by SHADOW when the system is ray-traced, e.g. "PREREFL" generates a file which contains the reflectivities for a particular optical coating such as gold, the other type provides general calculations such as the "torus" utility described later. The post-processors are used after the system is ray-traced and these utilities are required to interpret the results of the calculations performed by group 2 .

The programs contained in SHADOW are interactive and are run sequentially by the user. Communication between the various programs is via disk files. These disk files can also be categorised into 3 distinct groups as follows :-
(1) Data files :- These are usually unformatted binary files which increases accuracy and the speed of disk access. These files are the output from the optics engine, described in group 2 above, and they contain the rays in a given position of the optical system. Typical names are "begin.dat" for the source, "mirr.XX"
(2) for the beam at an optical element (where $X X$ is a number from 01 to 20), "star.XX" for the image of an optical element and "screen.XXYY" for the beam at a screen associated with a particular optical element.
(3) Parameter Files :- These files are usually of the form "start. $X X$ " and are of a NAMELIST format. NAMELIST format contains the values of each parameter used in the definition of a source or an optical element. They can be edited but it is not advisable to do so. The program "chval" is provided as a quick and safe way to change a single parameter. The other option is to change the parameter in "MENU" mode. All the data files can be regenerated from the "start. $X X$ " files.
(4) Analysis Files :- These are created by the analysis programs and they vary widely in their type and internal structure. The output from the analysis utilities can be in text, binary or ASCII format.

There are numerous utilities provided with SHADOW and a list of the utilities used throughout this work is as follows :-
h2kol :- This utility reads in a two column formatted file and creates a histogram of the frequency distribution of ray co-ordinates from a selected column.
plotxy :- This is a basic plotting utility for SHADOW. It will plot any combination of columns from an ASCII data file ("begin.dat", "mirr.XX", "star.XX", "screen.XXY"")
shadowit :- For a given optical system, "shadowit" reads in a set of specified data files computed by SHADOW (e.g. the source file, optical elements and their corresponding screens). Apertures can then be placed on any of these data files (e.g. the dimensions of the toroidal mirror, the entrance slit etc.). "shadowit" ignores the existing losses computed by SHADOW and it labels the rays as either good or bad depending on whether or not they have gone through all the apertures specified. The aperture sizes can be modified without having to retrace the system in SHADOW and the user can output the good (or bad) rays for any of the data files read in by "shadowit". All the information required by "shadowit" e.g. the list of data files to be read in, the aperture shape/size, the files on which the aperture
is placed etc. is contained in a file of NAMELIST format which is generated by "shadowit".
col_2 :- This utility reads in a formatted ASCII data file with a known number of columns. This utility allows the user to extract any two columns and write them to another file in any order specified by the user.
torus :- This utility computes the major and minor radii for a toroidal mirror given the setup parameters.
mirinfo :- This utility reads an exit record file e.g. of the type "end. $X X$ ", which is associated with an optical element (OE). It then creates an output file with information about that OE in a readable format.
sysinfo :- This utility reads all the exit record files and creates a file with a simplified description of the optical system.
sourcinfo:- This utility reads the source exit record file "end. 00 " and creates an output file with information about the source in a readable format.
grating_set :- This interactive utility computes the diffraction angles for blazed, constant incidence angle and constant included angle gratings. The user specifies the grating parameters, the incidence angle, the photon energy/wavelength and the utility calculates the diffraction angle for the given parameters.

An example of a source definition used in this work is shown below. This source definition describes a 3-D source in the shape of a cylinder. The vertical divergence shown represents a divergence of 0.0055 rads in the $+/-Z$ centred on $+45^{\circ}$. The values for the horizontal and vertical divergences were chosen so as maximise the numbers of rays through the slit without under-filling the toroidal mirror. These values were then used throughout all the ray-tracing work. In order to fit the experimental knife-edge traces, the
source size, under the Source Type option, and the source photon energy, under the Source Photon Energy option, were the only options that had their values altered ${ }^{\dagger}$.

In any of the following definitions, a ( + ) indicates that there are subdirectories which contain further options on category selected. Any indented text is a subdirectory of the $(+)$ directory preceding it.

## Source Definition

| File to store the rays | begin.dat |
| :--- | :--- |
| Source Type $(+)$ | RAN/RAN |
| Number of random rays $(+)$ | 20000 |
| Spatial Type $(+)$ | ELLIPSE |


| Source Spatial Characteristics:- <br> Depth |  |  |  |
| :--- | :--- | :--- | :--- |
|  | width | $[\mathrm{X}]$ | UNIFORM |
|  | height | $[\mathrm{Z}]$ | 0.2 |
|  | depth | $[\mathrm{Y}]$ | 0.2 |
|  |  |  | 0.38 |

Depth
Angle Distribution ( + )

UNIFORM
UNIFORM

| Source Angle Distribution:Angles in radians |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Horizontal divergence | [ $\mathrm{X}(+)$ ] | 0.023 |  |
|  | [ $\mathrm{X}(-)$ ] | 0.023 | This results in a divergence of 0.0055 rads in the $+/-\mathrm{Z}$ directions centred on $+45^{\circ}$ |
| Vertical divergence | [Z(+)] | 0.7798981 |  |
|  | [Z(-)] | -0.7908981 |  |
| Photon Energy distribution ( + ) | SLINE |  |  |
| Source Photon Energy:- |  |  |  |
| Energy distribution |  | SLINE |  |
| Units |  | EV |  |
| Line \# 1 |  | 50 (0 | $75,100,125,150 \mathrm{eV})$ |

The definition of the fake transmission element is shown below. It is positioned at the source and has zero length. The source is rotated in the definition of this optical element by $315^{\circ} \mathrm{CCW}\left(-45^{\circ}\right)$ about the x -axis in the source reference frame. This net $45^{\circ} \mathrm{CW}$ rotation is required in order to simulate the fact that the laser plasma source in the knifeedge experiments is viewed at $45^{\circ}$. As the source is rotated by $-45^{\circ}$, the vertical

[^1]divergence in the source definition must be centred on $+45^{\circ}$ in order to bring the rays exiting the source back along the optic axis.

## Fake Transmission Element

| MAIN MENU | Optical Element 1 |
| :---: | :---: |
| Source plane distance | 0.0 |
| Image plane distance | 0.0 |
| Incidence angle | 0.0 |
| Reflection angle | 180 |
| Mirror Orientation Angle | 0.0 |
| Source file | begin.dat |
| Type of element ( + ) | REFRACTOR |
| Figure (+) | PLANE |
| Limits check ( + ) | NO |
| Diffraction ( + ) | MIRROR |
| Source Movement (+) | YES |
| Source Movements:- <br> In SOURCE reference frame: <br> rotation [CCW ] around $X$ |  |
|  |  |
| Y | $Y \quad 0.0$ |
| Z | - 0.0 |

The definition of the screen at the knife-edge position ( 2 mm from the source) is shown below. This screen provides a file containing the intercepts of the rays at the position specified. In the case of the absorption volume calculations, the screens defined were the same as the one shown below except for the distance from the mirror.

## Definition of Screen 1 at knife-edge position

| View Screen/Slit Data | Optical Element 2 | Screen 1 |
| :--- | :--- | :--- |
| Position rel. to mirror | BEFORE |  |
| Distance (abs) from mirror | 398. |  |
| Aperturing | NO |  |
| Absorption | NO |  |

The definition for the toroidal mirror is shown below. When the source file is defined as "NONE SPECIFIED", the optical element automatically takes the output file from the previous optical element as the virtual source of rays. In this case, the virtual
source of rays are from the fake transmission element's output file, "star.01". The definition of the entrance slit is also contained in the definition of the toroidal mirror.

Toroidal Mirror


Mirror Shape:-
Shape selected
RECTANGLE
Mirror dimensions along axis (use ABSOLUTE values):

| $\mathrm{X}(+)$ Half Width | /Int Maj Ax | 15. |
| :--- | :--- | :--- |
| $\mathrm{X}(-)$ | / Ext Maj Ax | 15. |
| $\mathrm{Y}(+)$ | / Int Min Ax | 15. |
| $\mathrm{Y}(-)$ | / Ext Min Ax | 15. |
|  |  |  |
| MIRROR |  |  |
| YES |  |  |

Exit slit:-
Slit length (Sagittal)
15
width (Tangential) 0.01
tilt (CCW) 0.0

The grating is rotated by $180^{\circ}$ with respect to the toroidal mirror. The image distance and the reflection angle were set to zero so that the auto tuning option could be
used. With the auto tuning option turned on, these parameters were automatically computed by SHADOW. The only parameters changed in the definition of the grating was the energy to which the grating was auto tuned namely, $50,75,100,125,150 \mathrm{eV}$.

## Concave grating definition



Mirror Shape:-
Shape selected RECTANGLE
Mirror dimensions along axis (use ABSOLUTE values):rectangle/ellipse
X(+) Half Width / Int Maj Ax 15.
X ( - ) / Ext Maj Ax 15.
$\mathrm{Y}(+) \quad /$ Int Min Ax 25.
Y(-) /Ext Min Ax 25.
Diffraction (+)
GRATING
Define grating:-

| Ruling type (+) | CONSTANT |
| :--- | :--- |
| Lines/CM (at origin) | 12000. |
| Auto tuning | YES |
| Diffraction order | -1. |
| Energy/wavelength | EV |
| at eV. | $50 \quad$ (or $75,100,125,150 \mathrm{eV}$ ) |
| Mount type $(+)$ | ERG |

If the source is defined with more than one energy, only the energy to which the grating is auto tuned will be in focus. This is due the fact that SHADOW places a screen perpendicular to the position of the focus which is calculated using the diffraction angle associated with the auto tuned energy and the radius of curvature of the grating. In the case where there is more than one energy defined in the source, only the spectral line to which the grating is tuned will be in focus and the remaining spectral lines associated with the other energies will be out of focus. In order to counteract this, and to be able to view the resultant lines all at once, a fourth optical element was defined which essentially was a cylinder placed along the Rowland circle.

## Cylindrical optical element on the Rowland circle

| MAIN MENU | Optical Element 4 |
| :---: | :---: |
| Source plane distance | 0.0 |
| Image plane distance | 100 |
| Incidence angle | (Diffraction angle for the particular energy to be centred on the OE) |
| Reflection angle | (Diffraction angle for the particular energy to be centred on the OE ) |
| Mirror Orientation Angle | 0.0 |
| Source file | star. 03 (Output file from the grating.) |
| Type of element ( + ) | REFLECTOR |
| Figure ( + ) | SPHERICAL |
| OE specifications:Mirror parameters (+) | EXTERNAL |
|  | Mirror Parameters:-  <br> Type selected SPHERICAL <br> External parameters define $(+)$ n/a 0.0 |
|  | External paramcters:-  <br> Type selected SPHERICAL <br> Spherical radius 1108.8 |
|  | Focii and Continuation Planes COINCIDENT |
| Surface curvature | CONCAVE |
| Cylindrical | YES |
| Diffraction (+) | MIRROR |

The NAMELIST file used with "shadowil" throughout this work is shown below. The data files read in by "shadowit" are listed after "FILEIN" and the data files that will have apertures placed on them are selected in the "IFILE" line. In this case they are 4,6,7 and 8 which correspond to the files "screen.0201" (screen at the knife-edge position),
"mirr.02" (the toroidal mirror), "star.02" (the entrance slit) and "mirr.03" (the grating). The size of these apertures are defined by the limits in "XMAX", "YMAX", "XMIN" and "YMIN". An arbitrary limit of " 100 " is specified for the screen positioned at the knife-edge which ensures that all the rays pass this screen. In order to simulate the knife-edge experiments, the limits of the screen highlighted in red were varied. In order to simulate a horizontal knife-edge scan, the value of "YMAX" was varied and in order to simulate a vertical scan, the value of "XMAX" was varied.

## Shadowit NAMELIST file

| \$INPUT |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FLLEIN | $\begin{aligned} & =' \text { begin.dat', 'mirr.01', 'star.01', 'screen.0201', 'screen.0202', 'mirr.02', } \\ & \text { 'star.02', 'mirr.03', 'star.03', " } \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| XMAX | $=100$ | 15.0 | 7.5 | 15.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| YMAX | $=100$ | 15.0 | 0.005 | 25.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| XMIN | $=-100$ | -15.0 | -7.5 | -15.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| YMIN | $=-100$ | -15.0 | -0.005 | -25.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| KIND | = 1111000000 |  |  |  |  |  |  |  |  |  |
| NFILE | = 9 |  |  |  |  |  |  |  |  |  |
| NPOINT | $=20000$ |  |  |  |  |  |  |  |  |  |
| IFILE | $=4678000000$ |  |  |  |  |  |  |  |  |  |
| KOL | $=00$ | 00 | 00 | 13 | 00 | 12 | 13 | 12 | 00 | 00 |
| NCHECK | $=4$ |  |  |  |  |  |  |  |  |  |
| TRANS | $=0.0$ |  |  |  |  |  |  |  |  |  |
| \$END |  |  |  |  |  |  |  |  |  |  |

A brief explanation on the main parameters listed in the NAMELIST file required by "shadowit" are as follows,

FILEIN = List of data files to be read in by "shadowit".
XMAX $\quad=$ Limits in the +x direction on the files selected by IFILE below.
YMAX $\quad=$ Limits in the $+y$ direction on the files selected by IFILE below.
XMIN $\quad=$ Limits in the -x direction on the files selected by IFILE below.
YMIN $\quad=$ Limits in the $-y$ direction on the files selected by IFILE below.
KIND $\quad=$ Aperture type ( $1=$ Rectangular, $2=$ Elliptical, $3=$ Elliptical with hole $)$.
NFILE $\quad=$ Number of files to be read in (filenames listed in FILEIN above).
NPOINT $=$ Number of rays that are traced through the system.
IFILE $\quad=$ Lists the files that have apertures placed on them.
KOL $\quad=$ List the columns from each file that should be checked $(1=x, 2=y, 3=z)$.
NCHECK = Gives the number of files that have apertures placed on them.

## References

[1] SHADOW is a ray tracing software package developed and licensed by the University of Wisconsin, Madison; for further details see http://www.xravlith wisc.edu/
[2] Lai, B., Cerrina, F., "SHADOW: A SYNCHROTRON RADIATION RAY TRACING PROGRAM", Nuclear Instruments and Methods in Physics Research, A246, 337-341 (1986).
[3] Welnak, C., Chen, G.J., Cerrina, F., "SHADOW: a synchrotron radiation and X-ray optics simulation tool", Nuclear Instruments and Methods in Physics Research, A347, (1994), 344-347.

## Appendix 3

This appendix contains the designs of the various components manufactured for obtaining the knife-edge traces.

## Knife-edge and Knife-edge Mount

The dimensions of the stainless steel knife-edges from the entrance slit assembly of a spectrometer are shown in figure A3.1. The exact precision used in the manufacture of the knife-edge is unknown but it can be assumed that it is very high.


Figure A3.1 Schematic of knife-edge.

As the knife-edge experiments are carried out in both the vertical and horizontal directions, a mount that can hold two separate knife-edges was designed. This removed the necessity of having to bring the target chamber up to atmospheric pressure each time in order to change the knife-edge around for the different scan direction. The knife-edge mount was manufactured from aluminium and the design and dimensions are shown in figure A3.2.


Figure A3.2 Schematic of the knife-edge mount.

In order for the knife-edge to reach the centre of the chamber, an extension rod was manufactured with dimensions as shown in figure A3.3.


Figure A3.3 Schematic of the knife-edge extension rod.

This extension rod was in turn connected to a standard sliding type feed-through. In order to provide well-controlled movement of the knife-edge in the horizontal $(\mathrm{Z})$ direction, a mounting for the sliding feed-through, which incorporated a micrometer, was designed. The design of this feed-through is shown in figure A3.4.


Figure A3.4 Schematic of mounting for sliding feed-through.

This mounting consisted of three aluminium plates connected by four stainless steel rods. The end section of the shaft of the sliding feed-through was squared off and plate $B$ had a square hole machined in it which prevented the knife-edge mounted to the sliding feed-through from rotating throughout the experiment. A Mitutoyo micrometer, Model No. $150-802$, with a $0-25 \mathrm{~mm}$ range and a graduation of 0.01 mm with an overall accuracy of $\pm 5 \mu m$ was mounted to plate $C$.

## XYZ translation stage

The translation stage used to position the knife-edge was designed so as to provide 3 degrees of freedom. A fourth, rotational movement is also available by carefully loosening the clamping ring holding the sliding feed-through but this movement is not graduated and is only used to rotate the knife-edge into position for a vertical or horizontal scan. The translation stage consists of four separate pieces, an extension flange, a mounting plate and
two sliding plates kept in place by guide rails. An overall side view of the assembly, without the micrometers and guide rails, is shown in figure A3.5.


Figure A3.5 Schematic of XYZ translation mount assembly (without guide rails).

The design of the translation stage allowed an overall movement of 55 mm in both the vertical and horizontal directions. Before the translation stage was mounted to the target chamber, two modifications to the existing target chamber were required. The first modification was to mill the existing 71 mm circular hole in the target chamber's mounting face to a square of side 71 mm . This allowed the knife-edge to access the full range of movement provided by the translation stage. The second modification was to the inner sleeve of the target chamber. The purpose of this sleeve is to reduce the level of target debris that can possibly contaminate the toroidal mirror. The inner sleeve has a 15 mm diameter hole centred on the optic axis and a 50 mm hole perpendicular to this to allow the Nd:YAG laser beam through to the target. As with the target chamber's mounting face, a square hole of side 71 mm was cut out of the sleeve in order to allow the knife-edge access the full range of movement provided by the translation stage. For the sake on simplicity, each of the components are labelled in figure A3.5 and are described separately. The machining tolerances are $\pm 10 \mu \mathrm{~m}$ ( $\pm 0.01 \mathrm{~mm}$ ) unless otherwise stated.

The dimensions of the individual components are given in the figures. The first element, $A$, the extension flange is shown in figure A3.6.


Figure A3.6 Schematic of extension flange $A$.

This extension flange was required to allow sufficient room between the translation stage and the target positioning micrometers/thumbscrews situated on top of the target chamber. The extension flange was milled from a solid piece of aluminium. Aluminium is used in preference to stainless steel in most of these components due to its lightness and ease of machining. This extension flange has two O -ring grooves machined to a depth of 2.3 mm . The two Edwards O-rings used (Order No. H021-21-090) had an internal diameter of 104.5 mm and a section diameter or thickness of 3 mm . The internal diameter of 92 mm for the extension flange allowed the knife-edge to access the full 55 mm range of movement.

The design of mounting plate $B$ is shown in figure A3.7. This plate was also manufactured out of aluminium. The O-ring groove depth was again 2.3 mm and an Edwards O-ring (Order No. H021-21-092) with an internal diameter of 114.5 mm and a section diameter or thickness of 3 mm was used.


Figure A3.7 Schematic of mounting plate $B$.

This mounting plate was also used to support the whole translation stage by the addition of support legs made from aluminium. The design of these support legs is shown in figure A3.7a.


Figure A3.7a schematic of the support legs for the mounting plate $B$.

The mounting plate in figure A3.7 was also used to mount the micrometers and thumbscrews used to position the sliding plates. The design of the two thumbscrew mounts (1\&3) and the two micrometer mounts (2\&4) are shown in figure A3.7b. The number inset in the individual diagrams corresponds to the positions indicated in figure A3.7. All these
mounts were manufactured from aluminium and were bolted to the mounting plate using M8 bolts. The outer dimensions of the mounting plate in figure 3.7 were chosen so that when the thumbscrew/micrometer mounts were bolted in place, they acted as stops for the sliding plates. This prevented the sliding plates from being pushed passed the 0 -ring seals and therefore breaking vacuum.


Figure A3.7b Schematic of micrometer and thumbscrews holders for mounting plate $B$ in figure A3.7.

Two Mitutoyo micrometers, Model no. 151-255, were mounted to holders $2 \& 4$ in figure A3.7b. These micrometers were used to accurately position the two sliding plates. The measuring face of these micrometers is tipped with tungsten carbide. This ensures that the micrometer head in contact with the stainless steel sliding flanges is undamaged. The range of the micrometers is 50 mm with a graduation of $0.01 \mathrm{~mm}(10 \mu \mathrm{~m})$ and an overall accuracy of $\pm 5 \mu \mathrm{~m}$.

The first sliding plate $C$ is shown in figure A3.8.


Figure A3.8 Schematic of horizontal sliding plate $C$.

This plate is for the horizontal movement and is made from stainless steel. Stainless steel was used instead of aluminium so as so as to prevent the sliding plate from being damaged by the carbide tipped micrometer. The O-ring groove on the top face is 2.3 mm in depth and an Edwards (Order No. H021-21-087) O-ring with an internal diameter of 89.5 mm and a section diameter or thickness of 3 mm was used. The M4 tapped holes on the top face of this plate will be used for the guide rails for plate $D$. The design of the final sliding plate, $D$, is shown in figure A3.9.


Figure A3.9 Schematic of sliding plate $D$.

Again this sliding plate was made from stainless steel to prevent damage from the micrometer. A Caburn MDC 40KF nipple, made from anodised steel, was welded to this plate. The sliding feed-through was attached to this 40KF nipple using a hinged clamp. The design for the guide rails is shown in figure A3.10.


Figure A3.10 Schematic of the guide rails.

These rails are made from aluminium and they are secured to their respective plates using three M4 bolts. The bottom surface of all the sliding plates and the under side of the guide rails which are in contact with these sliding plates, were coated with vacuum grease. This ensured that the plates moved freely and that the O-rings would not be damaged over time.


[^0]:    ${ }^{\dagger}$ The characteristic or optical path function, equation (1.1), put forward by Beutler [2] is quite valid but some of the subsequent development of the theory was not. Namioka [3] pointed out and corrected some of these inaccuracies. However, not all of Beutler's results were invalid like in the case of the primary focusing conditions and the astigmatism of the concave grating. As these two results are the main areas of interest for the 2.2 m system at DCU, the corrections made by Namioka will be omitted.

[^1]:    ${ }^{\dagger}$ All text in red indicates that these values were varied for the ray-traced knife-edge traces.

