# Synthesis of 2 Dimensional Image Filters by Cellular Automata 

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9th. August 1991

## Table of Contents

Abstract ..... 3
1.0 Introduction ..... 4
1.1 What are Cellular Automata? ..... 5
1.2 Image Filtering ..... 9
1.3 Previous Work on Image filtering using Cellular Automata ..... 10
2.0 Filter Synthesis using a $3 \times 3$ Cellular Filter ..... 14
2.1 Assumptions ..... 14
2.2 The Filtering Operation ..... 15
2.3 Analytical Estimation of the Coefficients $\mathrm{K}_{0}, \mathrm{~K}_{1}$ and $\mathrm{K}_{2}$ ..... 21
2.3 Heuristic Estimation of the Coefficients $K 0, K_{1}$ and $K_{2}$ ..... 27
2.4.1 The Search Procedure ..... 27
2.4.2 Weighting Functions for Heuristic Search ..... 30
3.0 Example of the Synthesis of a Filter By a $3 x 3$ Kernel ..... 31
3.1 Test Pattern ..... 31
3.2 Synthesis of a 750 Hz . 5th. Order Low Pass Bessel Filter. ..... 32
3.2.1 Filter Coefficients Using Derived Equations ..... 40
3.2.2 Filter Coefficients Using Heuristic Search with $W_{1}(n)$ ..... 42
3.2.3 Filter Coefficients Using Heuristic Search with $W_{2}(n)$ ..... 45
3.2.4 Summary of Results ..... 49
3.3 Synthesis of a 2 kHz . 5th. Order Low Pass Bessel Filter ..... 49
4.0 Synthesis of Other Filter Types ..... 55
5.0 Parallel Implementation of Cellular Filters ..... 60
6.0 Conclusions ..... 65
7.0 References ..... 67


#### Abstract

The filtering action of some type of cellular automata has been known about since the mid 1960's. These cellular filters, based on a combination of reducing and augmenting kernels operating on a series of thresholded images, are derived empirically and have fixed characteristics. This thesis presents a method for synthesising 2 dimensional grey level image filters using cellular automata. This method takes as its starting point the characteristic equation of a filter expressed in the Laplace domain, and derives from this equation the kernel of the cellular filter which has the same characteristics as the filter defined by the Laplacian equation. The advantage of using cellular automata for image filtering is that they are highly parallel in nature, and as such, are a natural algorithm for image filtering on a parallel computational engine.


### 1.0 Introduction

The application of cellular automata to the task of image filtering has been known about since the mid 1960's. To date, this has been limited to empirical low pass filters with fixed amplitude response for use mainly on binary images. This thesis proposes a method based in the formal methods of linear system analysis, for the synthesis of any 2 dimensional grey level image filter which can be expressed by a transfer function in the Laplace domain.

The remainder of section 1 of this thesis introduces the concept of a cellular automaton, the process of image filtering and the previous work on the applications of cellular automata to the spatial filtering of images. In section 2 the method for synthesising 2 dimensional filters using cellular automata is developed using both an analytical approach and a heuristic approach. Several examples of filters are worked through in section 3 showing the validity of the method, and its limitations. In section 4 a method which extends this technique to the synthesis of other types of filters, such as high pass and band pass filters, is illustrated. The parallel nature, and the suitability of cellular filters for the parallel processing of images, is shown by the description, in section 5 , of one possible implementation of a cellular filter using a network of transputers. Finally, section 6 summarises the results of this technique, its limitations and possible areas for further research in the synthesis of filters using cellular automata.

### 1.1 What are Cellular Automata?

A Cellular Automaton is an array of processing elements (PEs). The functions computed by a PE is a function of its state and the states of its neighbouring elements. At its simplest, the role of a cellular operation is to transform an array of data s(i,j) as it, exists at a time $t$, given by $s(i, j, t)$, into an array of data $d(i, j, t+1)$. At time $(t+1)$, an element of the array of data $d(i, j)$ has a value determined by its original value s(i,j), along with the values of its nearest neighbours.

It is also assumed that the array of PEs is tightly coupled in that each $P E$ is connected to its immediate adjacent neighbouring PEs. A loosely coupled array of PEs would allow connections, through some form of switching network, to an arbitrary set of remote PEs.

A further restriction placed on the behaviour of the array of PEs is that all operations on a block of data in the array of PEs are independent of the position of the data in the array. If the data in the array is shifted to a new location in the array and operated upon, then the results are identical to the same operation conducted on the data in its original position in the array.

Operations on the array are assumed to be discrete and synchronous in time. By discrete in time, we mean that the operations are assumed to occur in discrete time with each step in time being a generation, iteration, or cycle. By synchronous in time, we mean that the state changes of the PEs (i.e. the change in the values of the PEs) occur simultaneously.

The neighbourhood of the PE $p(i, j)$ is defined as all the PEs coupled to $p(i, j)$. The state of $p(i, j)$ at a time (t+1) is determined by its present state and the states of the PEs in its neighbourhood. The kernel of the PE $p(i, j)$ includes both itself and the PEs in its neighbourhood. Consider the PE $p(i, j)$ and its neighbouring PEs arranged in a square tessellation, as follows.

$$
\begin{array}{ll}
p(i-2, j-2), p(i-1, j-2), p(i, j-2) & p(i+1, j-2), p(i+2, j-2) \\
p(i-2, j-1), p(i-1, j-1), p(i, j-1) & p(i+1, j-1), p(i+2, j-1) \\
p(i-2, j), p(i-1, j), p(i, j) & p(i+1, j), p(i+2, j) \\
p(i-2, j+1), p(i-1, j+1), p(i, j+1) & p(i+1, j+1), p(i+2, j+1) \\
p(i-2, j+2), p(i-1, j+2), p(i, j+2) & p(i+1, j+2), p(i+2, j+2)
\end{array}
$$

If we give $p(i, j)$ a $3 \times 3$ kernel, then the neighbourhood of $p(i, j)$ includes $p(i-1, j-1), p(i, j-1), p(i+1, j-1), p(i-1, j), p(i+1, j), p(i=$ $1, j+1) p(i, j+1)$ and $p(i+1, j+1)$. In general the neighbourhood of $p(i, j)$ is

$$
\{\mathrm{p}(\mathrm{i}+\mathrm{n}, \mathrm{j}+\mathrm{m}) \quad \text { i }-1<=\mathrm{n}<=+1,-1<=\mathrm{m}<=+1, \quad(\mathrm{n}, \mathrm{~m}) \ll(0,0)\}
$$

and the kernel of $p(i, j)$ is

$$
\{p(i+n, j+m) \quad \mid-1<=n<=+1, \quad-1<=m<=+1\} .
$$

And for an NxM kernel the neighbourhood of $p(i, j)$ is

$$
\left\{p(i+n, j+m) \quad \frac{-N}{2}<=n<=\frac{n}{2}, \quad \frac{-M}{2}<=m<=\frac{M}{2}, \quad(n, m) \ll \quad(0,0)\right\}
$$

and the kernel of $p(i, j)$ is
$\left\{p(i+n, j+m): \frac{-N}{2}<=n<=\frac{n}{2}, \frac{-M}{2}<=m<=\frac{M}{2}\right\}$.

The kernels, neighbourhoods and tessellations (the physical arrangement of the PEs) do not have to be rectangular. The tessellation can be any arrangement as long as operations on the array are isotropic. This tends to limit the tessellations to homogeneous regular geometric arrangements such as square, hexagonal and triangular which are capable of completely covering a plane surface. The kernels can be any combination the PE and its nearest neighbouring PEs. For example, figure 1 illustrates some of the possible neighbourhoods for a square tessellation.

The configuration of the array (i.e. the size and tessellation of the array and the state of each PE in the array) is called the signal, denoted by $s(r)$. Usually $r$ is a 2-dimensional space in which case the signal is s(i,j). The symbol used for the kernel is k(r).

The operation of the kernel $k(r)$ on an element of $s(r)$ (i.e. $s(i, j)$ a given $P E$ in the array defined by $s(r)$ ) is the weighted sum of the values held by the PEs in the neighbourhood of $s(i, j)$ and $s(i, j)$. The weights are defined by the kernel $k(r)$.

For example, if we define $k(r)$ to be a $3 x 3$ kernel as follows

$$
\begin{aligned}
& k(i-1, j-1)=1, k(i, j-1)=-1, k(i+1, j-1)=0 \\
& k(i-1, j)=-1, k(i, j)=1, k(i+1, j)=-1 \\
& k(i-1, j+1)=0, k(i, j+1)=0, k(i+1, j+1)=1
\end{aligned}
$$



Figure 1: Various neighbourhoods on a square tessalation
then the action of $k(r)$ on a specific element of $s(r)$, namely $s(i, j)$ at time t is

```
s(i,j,t+1)=s(i-1,j-1,t) - s(i,j-1,t) - s(i-1,j,t) + s(i,j,t)
```

    \(-s(i+1, j, t)+s(i+1, j+1, t)\)
    In general for 2-dimensional signal,

$$
\begin{equation*}
s(i, j, t+1)=\sum_{n, m}(k(i+n, j+m) \cdot s(i+n, j+m)) \tag{1}
\end{equation*}
$$

In the above example the kernel $k(r)$ was a binary kernel, that is a kernel which only contains the values 0 and $(+/-) 1$. Kernels are not limited to binary kernels and can contain any value. However binary kernels do have a particular interest since they are easily implemented in hardware.

This operation of the kernel $k(r)$ is applied to all the PEs in $s(r)$ simultaneously and after a single iteration the output of the 2dimensional array $s(r, t+1)$ is the discrete 2 -dimensional convolution of the kernel $k(r)$ with the signal $s(r)$ at a time $t$.

### 1.2 Image Filtering

Image filtering is the process by which certain spatial frequencies are removed from an image. If the low spatial frequencies are removed from an image, then those regions of the image in which the signal (intensity) is constant and/or changing slowly, are reduced
to a zero value. Only abrupt changes the image intensity remain. Such high pass filters, filters which attenuate low frequencies, are used for edge detection and enhancement because quick changes in image intensity is usually associated with the boundary, or edge, of a object and its background. High pass filters can also be used to remove shadows caused by diffuse or normal sunlight.

Low pass filters attenuate high spatial frequencies in an image and pass low spatial frequencies unaffected. These filters are very useful in removing speckling and snow effects from images. This type of high frequency noise in the image is generally caused by irregularities in the image capture system and changes in the environment. Such noise is expensive to completely eliminate since it would require high quality optics and a controlled environment.

A low pass filter can be combined with a high pass filter to produce a band pass filter which combines the operations of noise removal and edge enhancement. Band reject filters, that is filters which attenuate a specific band of frequencies, have applications in object recognition since certain textures can be characterised by a close band of frequencies localised in a region of the image.

### 1.3 Previous Work on Image filtering using Cellular Automata

To date image filters based on cellular automata have used the augmenting and reducing properties of binary multi-point kernels (kernels of 2 or more PEs).

For example consider the triple point kernel $n(x, y)$ shown in figure 2. When this kernel is applied to the signal data $s(x, y, t)$ the result is $s(x, y, t+1)$ which is no longer a binary signal. In order to transform the signal $s(x, y, t+1)$ back into a binary signal we need to threshold s(x,y,t+1). Figures 2(c), 2(d) and 2(e) are the results when $s(x, y, t+1)$ is thresholded using the values 0,1 and 2 . In figure $2(c)$ the signal data has been augmented, i.e. is larger than the original signal data, when the threshold value was 0 . When the threshold value was 1 , as in figure $2(d)$, the signal data has been reduced. When the threshold value was 2, as in figure 2(e), the result was a "residue" of the original signal, i.e. a single element in a background of elements of opposite value.

In general, these results hold true for dual-point, triple-point and multi-point kernels, since mutli-point kernels can be generated by a sequence of dual-point or triple-point kernels. Augmentation and reduction by means of an 8 step sequence using dual-point kernels, or by means of a 4 step sequence using triple-point kernels, can cause the removal of certain high spatial frequencies and change the duty cycle of lower spatial frequencies.

When used to filter grey level images these sequences of kernels must be applied to the set of binary signal images formed by thresholding the original image at each grey level. For example if the signal image has 8 grey levels, then the sequence of kernels would have to be applied to the 8 signal images formed by thresholding the original image at values $0,1, \ldots, 7$. Then the 8 resulting signals are recombined to produce the filtered grey level image.


Figure 2(a): Kernel


Figure 2(b): Signal data


Figure 2(c): Augmentation with threshold $=0$


Figure 2(d): Reduction with threshold $=1$


Figure 2(e): Residue with threshold $=2$

The work of Preston et al. has shown, by analysising the boundary propagation of the signal, that if $n_{i}$ is the number of iterations of reduction are followed by $n_{i}$ iterations of augmentation, with a sampling interval of $\Delta x$, then for a signal function of period $P$, then that sinusoid and all sinusoids of higher frequencies will be annihilated when

$$
n_{i}=\frac{P}{2\left(\Delta_{x}\right)}
$$

and therefore the cutoff frequency of the cellular filter is

$$
f_{\mathrm{co}}=\frac{1}{2 n_{\mathrm{i}}\left(\Delta_{\mathrm{x}}\right)}
$$

Lower frequencies will not be annihilated but will be clipped at their crests. The peak to peak amplitude $A_{p-p}$ is given by

$$
A_{p-p}(f)=\frac{1}{2\left[1+\operatorname{Cos}\left(\left((\pi) f / f_{\operatorname{co}}\right)\right)\right]}
$$

The advantage of this method of filtering a grey level image with $N$ grey levels using cellular automata is that it uses binary kernels, and is therefore it is easier to implement in hardware than multivalue kernel. However this method does have 4 major disadvantages:
(1) It requires the processing a $N$ thresholded images with $2 n_{i}$ iterations for each thresholded image.
(2) The characteristics of the filter produced by the cellular automata are fixed, i.e. it is not possible to alter the attenuation suffered by frequencies less than the cutoff frequency.
(3) It only produces low pass filters. This technique does not provide any means of synthesising high pass, band pass or any other type of filter.
(4) Non linear group delay causing a change in the duty cycle of certain low frequency components and image distortion.

This thesis proposes a new technique for the synthesis of 2 dimensional image filters using kernels with floating point values which overcomes the disadvantages of previous methods. Since the kernel does contain floating point values, each iteration does require more processing, but this technique is applied to the original image only and does not require $N$ thresholded images. Therefore it is more efficient when applied to high resolution grey level images.

### 2.0 Filter Synthesis using a $3 \times 3$ Cellular Filter

### 2.1 Assumptions

Throughout this thesis on filter synthesis using cellular automata, the following assumption are used.
(1) The image to be filtered is arranged in a square tessellation, and is represented by the 2 -dimensional signal $a(x, y)$.
(2) The image is a 8 bit gray level image which gives a range of -127 (black) to +127 (white).
(3) The filtered image is represented by the 2-dimensional signal $b(x, y)$.
(4) The behaviour of the filter is limited to isotropic behaviour. Therefore the results of the filtering process are independent of direction.
(5) The square root of $-1(\sqrt{-1})$ is represented by $j$.

### 2.2 The Filtering Operation

The action of $a$ filter $h(x, y)$ on $a$ signal $a(x, y)$ is defined by the equation

$$
\begin{equation*}
b(x, y)=\iint(a(x, y) \cdot h(x-X, y-Y)) d Y d X \tag{2}
\end{equation*}
$$

and hence is the convolution of the input signal $a(x, y)$ with $a$ filter characterised by $h(x, y)$.

In the 2-dimensional s'-domain, the Laplace transform of equation
(2) is written as

$$
\begin{equation*}
B\left(s^{\prime}\right)=H\left(s^{\prime}\right) \cdot A\left(s^{\prime}\right) \tag{3}
\end{equation*}
$$

where $H\left(s^{\prime}\right)$ is called the transfer function of the filter. $H\left(s^{\prime}\right)$ is the Laplace transform of $h(x, y)$, and so we can write,

$$
\begin{array}{ll}
H\left(s^{\prime}\right)=L\{h(x, y)\} & \text {... Laplace transform } \\
h(x, y)=L^{-1}\left\{H\left(s^{\prime}\right)\right\} & \text {... Inverse Laplace transform }
\end{array}
$$

An important property of the Laplace transform is the Laplace transform of the delta function $\delta(x, y)$.

$$
\operatorname{L}\{\delta(x, y)\}=1
$$

and hence, if we let the $a(x, y)$ be the delta function,
$B\left(s^{\prime}\right)=H\left(s^{\prime}\right) \cdot 1$
and

$$
\begin{equation*}
b(x, y)=h(x, y) \tag{5}
\end{equation*}
$$

This is why $h(x, y)$ is called the impulse response of the $f i l t e r$, since it is the output of a filter characterised by the transfer function $H\left(s^{\prime}\right)$ when a delta function, or impulse, is the input to the filter.

Since we have limited the behaviour of the filter $H\left(s^{\prime}\right)$ to isotropic behaviour, this also places limitations on the form of the filter's impulse response. Since the filter's response to any input signal should be independent of direction, we have

$$
\begin{equation*}
L^{-1}\left\{H\left(s^{\prime}\right)\right\}=h(x, y)=h(r, \theta)=h(r) \tag{6}
\end{equation*}
$$

where $(r, 8)$ are the polar coordinates of $(x, y)$.

We can therefore represent $H\left(s^{\prime}\right)$, the 2-dimensional filter transfer function, by a 1 -dimensional filter transfer function $H(s)$.

Now consider a $3 \times 3$ kernel $K$, which is to act as a filter. From equation (1) we know that the action of one iteration of the kernel $K$ on the input signal $a(x, y)$ is the discrete convolution of $K$ with $a(x, y)$. This is already very similar to the filtering action of a filter $h(x, y)$ as defined by equation (2). However, $h(x, y)$ is not Iimited to a $3 \times 3$ domain, and therefore multiple iterations of the kernel $K$ are necessary to approximate the action of $h(x, y)$.

Since the filter has to be isotropic, the action of the kernel $K$ must be isotropic, and $K$ must be symmetrical about its 4 major axes.

$$
\begin{array}{lll}
k(0,0) & k(1,0) & k(2,0) \\
k(0,1) & k(1,1) & k(2,1) \\
k(0,2) & k(1,2) & k(2,2)
\end{array}
$$

Therefore the 9 -element kernel $K$ can be simplified to a kernel with 3 distinct elements.

| $\mathrm{K}_{2}$ | $\mathrm{~K}_{1}$ | $\mathrm{~K}_{2}$ |
| :--- | :--- | :--- |
| $\mathrm{~K}_{1}$ | $\mathrm{~K}_{0}$ | $\mathrm{~K}_{1}$ |
| $\mathrm{~K}_{2}$ | $\mathrm{~K}_{1}$ | $\mathrm{~K}_{2}$ |

or formally as,

$$
\begin{align*}
k=\{k(i, j) \quad & 0<=i<=2,0<=j<=2, k(1,1)
\end{align*} \quad=K_{0}, \quad\left\{\begin{array}{l}
k(1,0)=k(0,1)=k(2,1)=k(1,2)=K_{1}, \\
\left.k(0,0)=k(2,0)=k(1,0)=k(2,2)=K_{2}\right\}
\end{array}\right.
$$

We will denote the action of one iteration of $K$ to a signal $a(x, y)$ as

$$
\mathrm{K} * \mathrm{a}(\mathrm{x}, \mathrm{y})
$$

If we let the signal array $a(x, y)$ be the impulse signal $\$(x, y)$ where

$$
\begin{aligned}
\delta(\mathrm{x}, \mathrm{y}) \quad & 1 ; \mathrm{x}=0, \mathrm{y}=0 \\
& 0 ; \mathrm{x}<>0, \mathrm{y}<>0
\end{aligned}
$$

Then,

$$
\begin{array}{llll}
\mathrm{K} * \delta(\mathrm{x}, \mathrm{y}) & \rightarrow & \left(\mathrm{K}_{2}\right) & \left(\mathrm{K}_{1}\right)
\end{array}\left(\mathrm{K}_{2}\right) .
$$

where $K_{0}$ is located at ( 0,0 ).

The application of $K$ to $K * \delta(x, y)$, denoted $K^{2} * \delta(x, y)$, is then

$$
\mathrm{K}^{2} * \hat{\delta}(\mathrm{x}, \mathrm{y}) \rightarrow
$$

| $\left(K_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{2}^{2}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}{ }^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{0} \mathrm{~K}_{1}+4 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ |
| $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{0} \mathrm{~K}_{1}+4 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{0}^{2}+4 \mathrm{~K}_{1}^{2}+4 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{0} \mathrm{~K}_{1}+4 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ |
| $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{0} \mathrm{~K}_{1}+4 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ |
| $\left(\mathrm{K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{1}^{2}+2 \mathrm{~K}_{2}^{2}\right)$ | $\left(2 \mathrm{~K}_{1} \mathrm{~K}_{2}\right)$ | $\left(\mathrm{K}_{2}^{2}\right)$ |

Since $K$ is isotropic, then $K^{2} * \delta(x, y)$ is symmetrical about its major axes (horizontal, vertical and diagonal). Repeated applications of $K$ to $\delta(x, y)$ will also be isotropic, therefore $K^{n} * \delta(x, y)$ is isotropic. $\mathrm{K}^{\mathrm{n}} * \delta(\mathrm{x}, \mathrm{y})$ is also non-zero for all ( $\left.\mathrm{x}, \mathrm{y}\right)$ where $\frac{-\mathrm{n}}{2}<=\mathrm{x}<=\frac{\mathrm{n}}{2}$ and $\frac{-\mathrm{n}}{2}<=\mathrm{y}<=\frac{\mathrm{n}}{2}$.

The impulse response of the filter $h(x, y)$ is an infinite continuous function, with 1 representing white and 0 representing black. However, since we have restricted the image intensity to 8 bits, we can consider $h(x, y)$ to be zero valued for all ( $x, y$ ) where $h(x, y)$ is less than the minimum discrete resolution, i.e.

$$
h(x, y)=h(1 R, m R)=0 \text { if } h(1 R, m R)<\frac{1}{2^{8}}
$$

where $R=$ resolution of the signal array.

Therefore $h(x, y)$ is also limited in the same manner as $K^{n} *(x, y)$. In order to synthesise the filter we need to choose $n, K_{0}, K_{1}$ and $K_{2}$ such that $K^{n} * \delta(x, y)$ approximates $h(x, y)$. Then $n$ iterations of the kernel $K$ to the delta function $\delta(x, y)$ will have the same effect as applying the filter to the delta function.

The value of $n$, the order of the cellular filter $K$, is easily determined. First, we derive the impulse response of the filter from the Laplacian transfer function of the filter. We then determine at which $\sqrt{2}$ multiple of the resolution $(\sqrt{2} m R)$ of the signal array does the impulse response remain less than the minimum resolution of the image intensity. The impulse response will always decay to a value less than the minimum image intensity resolution. This is because in order for $h(x, y)=h(r)$ to have any Laplace transform, $h(r)$ must meet the following condition.

$$
\int_{0}^{\operatorname{infinity}} i h(r) \mid \exp (-a r) d r<\operatorname{infinity}
$$

for some real, positive a. Therefore $\quad \mathrm{h}(\mathrm{r})$ ! $<M(\exp (a r))$ for all positive $r$, for some real positive $M$.

For this multiple (m) of the signal array resolution to exist at the furthest point of $K^{n}$, an $n x n$ response to an impulse function, from the origin, then the value of $n$ is given by

$$
\begin{equation*}
n=f l \operatorname{oor}\left(\frac{m}{\sqrt{2}}\right) \tag{8}
\end{equation*}
$$

```
where floor(x) = largest integer < x.
```

The values of $K_{0}, K_{1}$ and $K_{2}$ can be derived by two means, (a) analytically or (b) by a heuristic search.

### 2.3 Analytical Estimation of the Coefficients $K_{0}, K_{1}$ and $K_{2}$.

By induction, equations for the value of $K^{n} * \delta(x, y)$ at the different points, in terms of $K^{0}, K^{1}, K^{2}$ and $n$, will be derived. The actual value of $h(x, y)$ at these points can be set equal to the derived equations, and the values of $K^{0}, K^{1}$ and $K^{2}$ can be solved for.

Consider the following 3 elements of $K^{n} \boldsymbol{\delta}(x, y)$,

$$
\left.\begin{array}{rl}
K^{n} * \delta(x, y) \text { for } x=n, y=n & (\text { case } 1) \\
x & =n-1, y=n \\
& \text { (case } 2) \\
x & =n-1, y=n-1
\end{array} \quad \text { (case } 3\right)
$$

Case 1: $x=n \cdot y=n$

$$
\begin{aligned}
& \text { When } i=2, \text { for } x=i, y=i, \\
& K^{i} * \delta(x, y)=K^{2} * \delta(x, y)=K_{2}^{2}=K_{2}{ }^{i} \\
& \text { If } K \text { is applied to } K^{2} * \hat{\delta}(x, y) \text { then } \\
& K *\left(K^{2} * \delta(x, y)\right)=K_{2}^{2} \cdot K_{2} \\
& \Rightarrow K^{i+1} * \hat{\delta}(x, y) \quad \text { at } x=i+1, y=i+1
\end{aligned}
$$

Let $j=i+1$ and then

$$
K^{j} * \hat{\delta}(x, y)=K_{2}^{j} \quad \text { at } x=j, y=j
$$

This equation has the same form as $\mathrm{K}^{i} * \hat{o}(\mathrm{x}, \mathrm{y})$ and therefore , in general, $K^{n}{ }^{n}(x, y)$ at $x=n, y=n$ can be expressed as

$$
K^{n} * \delta(x, y)=K_{2}^{n} \quad \text { for } x=n, y=n
$$

Case 2: $x=n-1, y=n$

$$
\begin{aligned}
& \text { When i }=2 \text { for } x=i-1, y=i \text {, } \\
& K^{i} * \hat{0}(x, y)=K^{2} * \delta(x, y)=2 K_{1} K_{2}=i K_{1} K_{2} \\
& \text { If } K \text { is applied to } K^{2} * \delta(x, y) \text { then } \\
& \begin{aligned}
K^{*}\left(K^{2} * \delta(x, y)\right) & =2 K_{1} K_{2} \cdot K_{2}+K_{2}^{2} \cdot K_{1} \\
\Rightarrow K^{i+1} * \delta(x, y) & =2 K_{1} K_{2} 2+K_{1} K_{2}^{2}=3 K_{1} K_{2}^{2} \\
= & \text { at } x=i, y=i+1
\end{aligned}
\end{aligned}
$$

Let $j=i+1$ and then

$$
K j * \delta(x, y)=j K_{1} K_{2}^{j-1} \quad \text { at } x=j-1, y=j
$$

This not the same form as the expression for $K^{i} * \delta(x, y)$ so we apply another iteration of $K$ to $K^{\mathfrak{j}} * \hat{\delta}(x, y)$.

$$
\begin{aligned}
K *\left(K^{j} * \delta(x, y)\right) & =j K_{1} K_{2}^{j-1} \cdot K_{2}+K_{2}^{j} \cdot K_{1} \\
\Rightarrow K^{j+1} * \delta(x, y) & =(j+1) K_{1} K_{2}^{j}
\end{aligned}
$$

let $1=j+1$ and then

$$
\mathrm{K}^{\mathrm{l}} * \delta(\mathrm{x}, \mathrm{y})=1 \mathrm{~K}_{1} \mathrm{~K}_{2} \mathrm{l-1} \quad \text { at } \mathrm{x}=1-1, \mathrm{y}=1
$$

This equation has the same form as $\mathrm{K}^{\mathfrak{j}} * \hat{\delta}(\mathrm{x}, \mathrm{y})$ and therefore , in general, $K^{n} * \delta(x, y)$ at $x=n-1, y=n$ can be expressed as

$$
\mathrm{K}^{\mathrm{n} * \hat{\delta}(\mathrm{x}, \mathrm{y})=\mathrm{nK}_{1} \mathrm{~K}_{2} \mathrm{n}-1} \quad \text { for } \mathrm{x}=\mathrm{n}-1, \mathrm{y}=\mathrm{n}
$$

## Case 3: $x=n-1, y=n-1$

$$
\begin{aligned}
& \text { When } i=2 \text {, for } x=i-1, y=i-1, \\
& K^{i} * \delta(x, y)=K^{2} * \delta(x, y)=2 K_{0} K_{2}+2 K_{1}^{2}=i K_{0} K_{1}+i K_{1} i
\end{aligned}
$$

If $K$ is applied to $K^{2} * \delta(x, y)$ then

$$
\begin{aligned}
K *\left(K^{2} * \hat{\delta}(x, y)\right)=\left(2 K_{0} K_{2}\right. & \left.+2 \mathrm{~K}_{1}^{2}\right)\left(\mathrm{K}_{2}\right)+2 \mathrm{~K}_{1} \mathrm{~K}_{2} \cdot \mathrm{~K}_{1}+2 \mathrm{~K}_{1} \mathrm{~K}_{2} \cdot \mathrm{~K}_{1} \\
& +\mathrm{K}_{2}{ }^{2} \cdot \mathrm{~K}_{0}
\end{aligned}
$$

$$
\text { at } x=i, y=i
$$

$$
\begin{aligned}
\Rightarrow K^{i+1} * \delta(x, y) & =i K_{0} K_{2}{ }^{i}+i K_{1}{ }^{i} K_{2}+i K_{1}{ }^{i} K_{2}+i K_{1}{ }^{i} K_{2}+K_{0} K_{2}{ }^{i} \\
& =(i+1) K_{0} K_{2}{ }^{i}+3 i K_{1}{ }^{i} K_{2} \quad \text { at } x=i, y=i
\end{aligned}
$$

$$
\begin{aligned}
& \text { Let } j=i+1 \text { and then } \\
& K^{j} * \delta(x, y) \quad=j K_{0} K_{2}^{j-1}+X(j) K_{1}{ }^{2} K_{2} j-2 \text { at } x=j-1, y=j-1
\end{aligned}
$$

since the applications of $K$ to $K^{2} * \hat{\delta}(x, y)$ did not affect the power of $K_{0}$ in the first term and did not affect the power of $K_{1}$ in the second term. $X(j)$ denotes that the coefficient of the $K_{1}{ }^{2} K_{2}{ }^{j-2}$ term is some, as yet unknown, function of the variable $j$.

$$
\begin{aligned}
& \text { Applying } K \text { to } K^{j} * \delta(x, y) \text { gives, at } x=j, y=j \\
& K^{j+1 * \delta(x, y)}=\left(j K_{0} K_{2}^{j-1}+X(j) K_{1} 2_{K_{2}}{ }^{j-2}\right)\left(K_{2}\right) \\
& +j K_{1} K_{2}{ }^{j-1} \cdot K_{1} \\
& +j \mathrm{~K}_{1} \mathrm{~K}_{2}{ }^{\mathrm{j}-1} \cdot \mathrm{~K}_{1} \\
& +K_{2}{ }^{j} \cdot K_{0} \\
& \Rightarrow K^{j+1 * \delta(x, y)}=j K_{0} K_{2}^{j}+X(j) K_{1} 2_{K_{2}}{ }^{j-1}+2 j K_{1} 2^{2}{ }_{2}^{j-1}+K_{0} K_{2}{ }^{j} \\
& =(j+1) K_{0} K_{2}{ }^{j}+(2 j+X(j)) K_{1}{ }^{2} K_{2}{ }^{j-1}
\end{aligned}
$$

Let $1=j+1$ and then

$$
\begin{aligned}
& \mathrm{K}^{l} * \delta(\mathrm{x}, \mathrm{y})=1 \mathrm{~K}_{0} \mathrm{~K}_{2} \mathrm{l}-1+(2(1-1)+\mathrm{X}(1)) \mathrm{K}_{1}^{2} \mathrm{~K}_{2}^{1-2} \\
& \quad \text { at } \mathrm{x}=1-1, \mathrm{y}=1-1
\end{aligned}
$$

But $(2(1-1)+X(I))$ is a function equal to its last value plus twice the last iteration number, which can be written

$$
(2(1-1)+X(1))=2\left(\sum_{m=0}^{1-1}\right)
$$

And therefore,

$$
\mathrm{K}^{1} * \delta(\mathrm{x}, \mathrm{y})=1 \mathrm{~K}_{0} \mathrm{~K}_{2} 1-1+2\left(\sum_{\mathrm{m}=0}^{1-1}\right) \mathrm{K}_{1} 2_{\mathrm{K}_{2}}^{1-2}
$$

Since $K^{\perp} * \delta(x, y)$ has the same form as $K^{j} * \delta(x, y)$, in general, $K^{n} * \hat{\delta}(x, y)$ at $x=n-1, y=n-1$ can be expressed as

$$
K^{n} * \hat{O}(x, y)=n K_{0} K_{2}^{n-1}+2\left(\sum_{m=0}^{1-1}\right) K_{1} 2_{K_{2}}^{n-2}
$$

$$
\text { for } x=n-1, y=n-1
$$

In summary, the $n$ repeated applications of the kernel $K$ to the impulse function, yields:

$$
\begin{array}{ll}
K^{n} * \delta(x, y)=n_{0} K_{2}{ }^{n-1}+2\left(\sum_{m=0}^{1-1} m_{1}\right) K_{1} 2_{K_{2}}^{n-2} & \text { at } x=n-1, y=n-1 \\
K^{n} * \delta(x, y)=\mathrm{nK}_{1} K_{2}{ }^{n-1} & \text { at } x=n-1, y=n \\
K^{n} * \delta(x, y)=K_{2}^{n} & \text { at } x=n, y=n
\end{array}
$$

If $K^{n}$ is to have the same filtering effects as the filter defined by the Laplacian transfer function $H(s)$, then the response of $H(s)$ to
an impulse function should be the same at $(x, y)=\{(n-1, n-1),(n-1, n)$, $(\mathrm{n}, \mathrm{n})\}$ as $\mathrm{K}^{\mathrm{n}} *{ }^{8}(\mathrm{x}, \mathrm{y})$ at the same points.

The spatial response of $H(s)$ to an impulse function has already been shown to be (equation (6))

$$
h(r)=L^{-1}\{H(s)\}
$$

Hence, from the transfer function $H(s)$ of the filter,

$$
\begin{equation*}
\left.K^{n} * \delta((n-1) R,(n-1) R) \quad=h((n-1) R,(n-1) R)\right) \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
K^{n} * \delta((n-1) R, n R) \quad=h((n-1) R, n R) \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
K^{n} * \delta(n R, n R) \quad=h(n R, n R) \tag{11}
\end{equation*}
$$

And therefore,

$$
\begin{equation*}
n K_{0} K_{2}^{n-1}+2\left(\sum_{m=0}^{1-1}\right)_{1}^{1} K_{1} 2_{K_{2}}^{n-2}=h((n-1) R,(n-1) R) \tag{12}
\end{equation*}
$$

$\mathrm{nK}_{1} \mathrm{~K}_{2}{ }^{\mathrm{n}-1} \quad=h((\mathrm{n}-1) \mathrm{R}, \mathrm{nR})$

$$
\begin{equation*}
\mathrm{K}_{2}{ }^{\mathrm{n}} \quad=\mathrm{h}(\mathrm{nR}, \mathrm{nR}) \tag{14}
\end{equation*}
$$

Since $n$ has already been calculated, the values of $K_{0}, K_{1}$ and $K_{2}$ can be determined.

### 2.4 Heuristic Estimation of the Coefficients $K_{0}, K_{1}$ and $K_{2}$.

### 2.4.1 The Search Procedure

The coefficients of the cellular filter can be estimated by a heuristic search, as described in the flow chart in figure 3. For a given value of $n$ (the order of the cellular filter) a range of values for $K_{0}, K_{1}$ and $K_{2}$ is specified. Initially the filter coefficients are set to the beginning of the search ranges. In turn, starting with $\mathrm{K}_{2}$ and then progressing onto $K_{1}$ and $K_{0}$, each coefficient, $K_{i}$, is advanced in pre-defined steps, $d_{k}$, and at each triplet $\left\{K_{0}, K_{1}, K_{2}\right\}$, the response to the delta function is calculated. The value of the response at several points in the calculated impulse response for $\left\{K_{0}, K_{1}, K_{2}\right\}$ is compared with the values at the corresponding points in the actual impulse response. This error between the calculated impulse response and the actual impulse response at the different sample points is multiplied by a weighting function. The triplet $\left\{K_{0}, K_{1}, K_{2}\right\}$ with the minimum root mean square of the weighted error in the search range is recorded. The search range is then changed to $\left[K_{i}-d_{k}, K_{i}+d_{k}\right]$ and the step value is changed to $d_{k+1}$. The new interval $\left[K_{i}-d_{k}, K_{i}+d_{k}\right]$ is searched for a minimum root mean square weighted error. This process is repeated until $K_{i}$ reaches a desired accuracy. When $K_{i}$ reaches the desired accuracy this value of $K_{i}$ is recorded and is used for the searches involving the other coefficients.

When this procedure has been completed for $\mathrm{K}_{0}$, $\mathrm{K}_{1}$, and $\mathrm{K}_{2}$, the current pass is finished. Then the triplet $\left\{K_{0}, K_{1}, K_{2}\right\}$, with the minimum root mean square weighted error, and the error are recorded. Another pass is started, but this time only $K_{2}$ is set to the beginning

Figure 3: Heuristic search algorithim for Cellular Filter coefficients


of the specified search range. $K_{0}$ and $K_{1}$ remain at the values associated with the minimum root mean square weighted error during the last pass. When $\mathrm{K}_{2}$ reaches a minimum, $\mathrm{K}_{1}$, and then $\mathrm{K}_{0}$, are searched.

The search for the final coefficient values is completed if the minimum root mean square error for a pass is within $1 \%$ of the minimum root mean square error for the previous pass.

The program which implements this algorithm, find_ks.c (appendix A), initially performs a search for the filter coefficients with a 1 decimal point accuracy. This is done in order to quickly locate the region in which the minimum of the error function exists before performing the search to the desired degree of accuracy.

### 2.4.2 Weighting Functions for Heuristic Search

The weighting function is a function of the distance of the point from the origin and is used because the delta function, being a mathematical entity, can only be approximated in reality. The approximation used for the delta function in the heuristic search algorithm is the value of actual impulse response at the origin. The weighting function reduces the error introduced by this approximation of the delta function.

Two different weighting functions were used in the heuristic search algorithm, $W_{1}(i)$ and $W_{2}(i)$, defined as follows.

$$
\begin{aligned}
W_{1}(i) & =0.5 & & , 0<=i<1 \\
& =0.9 & & , 1<=i<2 \\
& =1.0 & & , i>=2 \\
W_{2}(i) & =0.0 & & , 0<=i<1 \\
& =0.01 & & , 1<=i<2 \\
& =0.1 & & , 2<=i<3 \\
& =0.5 & & , 3<=i<4 \\
& =0.9 & & , 4<=i<5 \\
& =1.0 & & , i>=5
\end{aligned}
$$

Both weighting functions reduce the error between the calculated and actual impulse responses, but $W_{2}(i)$ allows less contribution to the total error number than $W_{1}(i)$ at points close to the origin.

### 3.0 Example of the Synthesis of a Filter By a $3 \times 3$ Kernel

### 3.1 Test Pattern

In order to evaluate the cellular filter we will apply it to test image. This test image is defined by the following equation.

$$
\begin{equation*}
T(x, y)=\operatorname{Cos}\left(\frac{5(\pi)\left(x^{2}+y^{2}\right)}{R}\right) \tag{15}
\end{equation*}
$$

where $R$ is the grid resolution.

This generates a test image which consists of a sinusoid whose frequency varies linearly with distance.

$$
\begin{aligned}
& \omega=2(\pi) \mathrm{f}=\frac{10(\pi) \mathrm{r}}{\mathrm{R}}=10(\pi) \mathrm{m} \\
\Rightarrow & \mathrm{f}=5 \mathrm{~m}
\end{aligned}
$$

$$
\text { since } r=m R
$$

When this test image is passed through a threshold function, a function which maps the input to 1 if it is greater or equal to a given value or maps the input to 0 otherwise, then the test image appears as a series of concentric circles of different thicknesses. Figure 4 shows the test pattern with a threshold value of 0 . Note that the resolution of the images in figures $4-10$ is 72 dots per inch (dpi). Therefore when used to display 300 dpi images, a correction factor of 4.1667 should be used to calculate the actual frequencies from the images.

### 3.2 Synthesis of a 750 Hz . 5th. Order Low Pass Bessel Filter

The aim of this example is to illustrate how a $3 \times 3$ kernel for a cellular filter can be derived from a transfer function for a common filter. In this example a 5 th order low pass Bessel filter with 3 dB cutoff frequency of 750 Hz for use on a square tessellation of resolution $8.4667 \times 10^{-5} \mathrm{~m} .(300 \mathrm{dpi})$ will be synthesised. The Bessel filter was selected because it has a constant group delay, and therefore will not introduce distortion into the filtered image.


The normalised 5 th order low pass Bessel transfer function is

$\Rightarrow H\left(s_{n}\right)=$

945 \{

$$
\left(s_{n}+3.6467\right)\left[\left(s_{n}+3.352\right)^{2}+1.7427^{2}\right]\left[\left(s_{n}+2.3247\right)^{2}+3.571^{2}\right]
$$

$\Rightarrow H\left(s_{n}\right)=$

$$
945\left\{\frac{1}{\left(s_{n}+3.6467\right)\left(s_{n}^{2}+6.704 s_{n}+14.272\right)\left(s_{n}^{2}+4.649 s_{n}+18.156\right)}\right.
$$

$\Rightarrow H\left(s_{n}\right)=$

945 \{

$$
\frac{1}{\left(s_{n}+3.6467\right)\left(s_{n}-p_{1}\right)\left(s_{n}-p_{1} *\right)\left(s_{n}-p_{2}\right)\left(s_{n}-p_{2}^{*}\right)}
$$

where $\mathrm{p}^{*}{ }^{*}$ is the complex conjugate of p 1
and
$p_{1}=-3.352+j 1.742$
$p_{2}=-2.325+j 3.571$

Therefore,
$\Rightarrow H\left(s_{n}\right)=$
$945\{$ $\qquad$ $+\mathrm{k}_{2}+$ $-\mathrm{k}_{2}{ }^{\text {*}}+$ $+$ $\qquad$ $+$

$$
\left(s_{n}-p_{1}\right)
$$

$\qquad$ \}

$$
\left(\mathrm{s}_{\mathrm{n}}+3.6467\right)
$$ $\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{1}\right.$ *) $\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{1}\right)$ $\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{1}{ }^{*}\right)$

Calculating $\mathrm{k}_{1}$ :

$$
\begin{aligned}
& \mathrm{k}_{1}= \frac{1}{\left(\mathrm{~s}_{\mathrm{n}}^{2}+6.704 \mathrm{~s}_{\mathrm{n}}+14.272\right)\left(\mathrm{s}_{\mathrm{n}}^{2}+4.649 \mathrm{~s}_{\mathrm{n}}+18.156\right)} \\
& \text { at } \mathrm{s}_{\mathrm{n}}=-3.6467
\end{aligned} \quad \begin{aligned}
& \Rightarrow \mathrm{k}_{1}=\frac{1}{45.286}=0.0221
\end{aligned}
$$

## Calculating $\mathrm{k}_{2}$ :

$$
\begin{aligned}
& \mathrm{k}_{2}=\frac{1}{\left.\left(\mathrm{~s}_{\mathrm{n}}+3.6467\right)\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{1}\right)^{*}\right)\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{2}\right)\left(\mathrm{s}_{\mathrm{n}}-\mathrm{p}_{2}^{*}\right)} \\
& \text { at } \mathrm{s}_{\mathrm{n}}=-3.352+j 1.742 \\
& \Rightarrow \mathrm{k}_{2}=\frac{1}{(0.2947+j 1.742)(j 3.484)(-1.027-\mathrm{j} 1.829)(-1.027+j 5.313)} \\
&=\frac{1}{(-6.0691+j 1.0267)(-1.027-j 1.829)(-1.027+j 5.313)} \\
&=\frac{1}{(8.1108+j 10.046)(-1.027+j 5.313)} \frac{1}{-61.7042+j 32.7754} \\
&=\frac{1}{(69.8687) \exp (-j 0.4883)} \\
&=(0.0143) \exp (j 0.4883)
\end{aligned}
$$

Calculating ks:

$$
\begin{aligned}
\mathrm{k}_{3} & =\frac{1}{\left(s_{n}+3.6467\right)\left(s_{n}-p_{1}{ }^{*}\right)\left(s_{n}-p_{2}\right)\left(s_{n}-p_{2}^{*}\right)} \text { at } s_{n}=-2.325+j 3.571 \\
\Rightarrow k_{3} & =\frac{1}{(1.3217+j 3.571)(1.027+j 1.829)(1.027+j 5.313)(j 7.142)} \\
& =\frac{1}{(-5.174+j 6.085)(1.027+j 5.313)(j 7.142)} \\
& =\frac{1}{(-37.6433-j 21.2402)(j 7.142)} \\
& =\frac{151.6975-j 268.8484}{(308.6934) \exp (-j 1.0571)} \\
& =\left(3.2394 \times 10^{-3}\right) \exp (j 1.0571)
\end{aligned}
$$

Therefore the normalised transfer function of a 5 th order low pass Bessel filter is

$$
\begin{align*}
H\left(s_{n}\right)=945\{ & \frac{0.0221}{\left(s_{n}+3.6467\right)}+\frac{(0.0143) \exp (i 0.4883)}{\left(s_{n}+(3.352-j 1.742)\right)} \\
& +\frac{(0.0143) \exp (-. j 0.4883)}{\left(s_{n}+(3.352+j 1.742)\right)}+\frac{\left(3.2394 \times 10 \frac{-3) \exp (. j 1}{\left(s_{n}+(2.0571)\right.}\right.}{35-j 3.571))} \\
& +\frac{(3.2394 \times 10-3) \exp (-j 1.0571)}{\left(s_{n}+(2.325-j 3.571)\right)} \tag{16}
\end{align*}
$$

This transfer function is for a filter with a 3 dB cutoff frequency of 1 Hz . To scale the 3 dB cutoff frequency to $\omega_{\mathrm{c}}$ we transform $s_{n}$ to $\left(s / \omega_{c}\right)$.

Since the 3 dB cutoff frequency of the filter is 750 Hz . then

$$
\omega_{c}=2(\pi)\left(f_{c}\right)=4.7124 \times 10^{3} \mathrm{rads} / \mathrm{m}
$$

Hence the transfer function for a 5 th order low pass Bessel filter with a 750 Hz . 3 dB cutoff frequency is

$$
\begin{aligned}
& H(s)=945\left\{\frac{(1.0414 \times 10}{(s)} \frac{2}{4}\right) \\
& \left(s+1.7185 \times 10^{4}\right) \\
& +(6.7387 \times 101) \exp (.0 .4883) \\
& \left(s+\left(1.5796 \times 10^{4}-j 8.209 \times 10^{3}\right)\right) \\
& +\left(6.7387 \times 10^{1}\right) \exp (-. i 0.4883) \\
& \left(s+\left(1.5796 \times 10^{4}+j 8.209 \times 10^{3}\right)\right) \\
& +(1.5265 \times 101) \exp (i 1.0571) \\
& \left(s+\left(1.0956 \times 10^{\left.4-j 1.6828 \times 10^{4}\right)}\right.\right. \\
& +(1.5265 \times 101) \exp (-i 1.0571)\} \\
& \left(s+\left(1.0956 \times 10^{4}+j 1.6828 \times 10^{4}\right)\right. \\
& \Rightarrow H(s)=\frac{\left(9.8412 \times 10^{4}\right)}{\left(s+1.7185 \times 10^{4}\right)} \\
& +(6.3681 \times 104) \exp (. j 0.4883)- \\
& \left(s+\left(1.5796 \times 10^{4}-j 8.209 \times 10^{3}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\left(6.3681 \times 10^{4}\right) \exp (-i 0.4883)}{\left(s+\left(1.5796 \times 10^{4}+j 8.209 \times 10^{3}\right)\right)} \\
& +\frac{\left(1.4425 \times 10^{4}\right) \exp (. i 1.0571)}{\left(s+\left(1.0956 \times 10^{4}-j 1.6828 \times 10^{4}\right)\right.} \\
& +\frac{\left(1.4425 \times 10^{4} 4\right) \exp (-j 1.0571)}{\left(s+\left(1.0956 \times 10^{4}+j 1.6828 \times 10^{4}\right)\right.}
\end{aligned}
$$

Since $L^{-1}\left\{(s+a)^{-1}\right\}=\exp (-a r)$, then the impulse response is

$$
\begin{aligned}
h(r)= & \left(9.8412 \times 10^{4}\right) \exp -\left(1.7185 \times 10^{4}\right) r \\
& +\left(6.3681 \times 10^{4}\right) \exp (j 0.4883) \exp \left(-\left(1.5796 \times 10^{4}-j 8.209 \times 10^{3}\right) \mathrm{r}\right) \\
& +\left(6.3681 \times 10^{4}\right) \exp (-j 0.4883) \exp \left(-\left(1.5796 \times 10^{4}+j 8.209 \times 10^{3}\right) r\right) \\
& +\left(1.4425 \times 10^{4}\right) \exp (j 1.0571) \exp \left(-\left(1.0956 \times 10^{4}-j 1.6828 \times 10^{4}\right) r\right) \\
& +\left(1.4425 \times 10^{4}\right) \exp (-j 1.0571) \exp \left(-\left(1.0956 \times 10^{4}+j 1.6828 \times 10^{4}\right) \mathrm{r}\right)
\end{aligned}
$$

But since $\exp (j a)+\exp (-j a)=2 \operatorname{Cos}(a)$, the impulse response becomes

$$
h(r)=\left(9.8412 \times 10^{4}\right) \exp -\left(1.7185 \times 10^{4}\right) r
$$

$$
+\left(6.3681 \times 10^{4}\right) \exp \left(-\left(1.5796 \times 10^{4}\right) r\right) 2 \operatorname{Cos}\left(\left(8.209 \times 10^{4}\right) r+0.4883\right)
$$

$$
+\left(1.4425 \times 10^{4}\right) \exp \left(-\left(1.0956 \times 10^{4}\right) r\right) 2 \operatorname{Cos}\left(\left(1.6828 \times 10^{4}\right) r+1.0571\right)
$$

Therefore, the impulse response of a 5 th. order low pass Bessel filter with a 3 dB cutoff frequency of 750 Hz . is

$$
\begin{aligned}
h(r)= & \left(9.8412 \times 10^{4}\right) \exp -\left(1.7185 \times 10^{4}\right) r \\
& +\left(1.2736 \times 10^{5}\right) \exp \left(-\left(1.5796 \times 10^{4}\right) r\right) \operatorname{Cos}\left(\left(8.209 \times 10^{4}\right) \mathrm{r}+0.4883\right) \\
& +\left(2.885 \times 10^{4}\right) \exp \left(-\left(1.0956 \times 10^{4}\right) r\right) \operatorname{Cos}\left(\left(1.6828 \times 10^{4}\right) r+1.0571\right)
\end{aligned}
$$

and the envelope function is

$$
\begin{aligned}
\mathrm{h}_{\mathrm{e}}(\mathrm{r})= & \left(9.8412 \times 10^{4}\right) \exp -\left(1.7185 \times 10^{4}\right) \mathrm{r} \\
& +\left(1.2736 \times 10^{5}\right) \exp \left(-\left(1.5796 \times 10^{4}\right) \mathrm{r}\right) \\
& +\left(2.885 \times 10^{4}\right) \exp \left(-\left(1.0956 \times 10^{4}\right) \mathrm{r}\right)
\end{aligned}
$$

In order to calculate the order of the cellular filter which synthesises the 5 th. order Bessel filter, the impulse response and the impulse response envelope are calculated at multiples of $\sqrt{2}$ times the resolution of the grid being used. In this case the resolution $R=$ $8.4667 \times 10^{-5} \mathrm{~m}$.
$\underline{n R} \quad \sqrt{\underline{2} n R} \quad \underline{h}(\sqrt{\underline{2} n R}) \quad \underline{h}_{\mathrm{e}}(\sqrt{2} n R)$

| 8 R | 11.31370850 R | -0.09077836 | 0.83978480 |
| ---: | ---: | ---: | ---: |
| 9 R | 12.72792206 R | 0.19838985 | 0.22113703 |
| 10 R | 14.14213562 R | -0.04136556 | 0.05882040 |
| 11 R | 15.55634918 R | -0.00516089 | 0.01573345 |
| 12 R | 16.97056274 R | 0.00419819 | 0.00422143 |
| 13 R | 18.38477631 R | -0.00058706 | 0.00113457 |
| 14 R | 19.79898987 R | -0.00016676 | 0.00030522 |
| 15 R | 21.21320343 R | 0.00008129 | 0.00008215 |
| 16 R | 22.62741699 R | -0.00000674 | 0.00002212 |

A resolution of 8 bits in image intensity gives a minimum image intensity value of 0.003906 . The selection of the order of the cellular filter ( $n$ ), as described in section 2.2 , is to choose $n$ such that it is
the largest value for $n$ where $\mid h\left(\sqrt{2} n R ;>0.003906\right.$ and $h_{e}(\sqrt{2}(n+1) R)<$ 0.003906 . In this case n is 12.

### 3.2.1 Filter Coefficients Using Derived Equations

We have already determined that $n=12$. Therefore, from equation (14), we have

$$
\begin{aligned}
& K_{2}{ }^{n}=h(n R, n R) \\
\Rightarrow & K_{2}{ }^{12}=h(12 R, 12 R)=0.00419708 \\
\Rightarrow & K_{2}=0.63374
\end{aligned}
$$

Using this value for $\mathrm{K}_{2}$ in equation (13) gives

$$
n K_{1} K_{2} n-1=h((n-1) R, n R)
$$

$$
\Rightarrow 12 \mathrm{~K}_{1}(0.63374)^{11}=\mathrm{h}(11 \mathrm{R}, 12 \mathrm{R})=0.00510807
$$

$$
\Rightarrow \mathrm{K}_{1}=\frac{0.00510807}{12(0.63374)^{11}} \div 0.06428
$$

Substituting the values for $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ into equation (12) gives

$$
n K_{0} K_{2}^{n-1}+2\left(\sum_{m=0}^{1-1}\right) K_{1} 2_{K_{2}}^{n-2}=h((n-1) R,(n-1) R)
$$

$$
\begin{aligned}
& \Rightarrow 11 \mathrm{~K}_{0} \mathrm{~K}_{2} 11+2(66) \mathrm{K}_{1} 2 \mathrm{~K}_{2} 10=\mathrm{h}(11 \mathrm{R}, 11 \mathrm{R})=-0.00516913 \\
& \Rightarrow \mathrm{~K}_{0}(0.07947)+0.0056995=-0.00516913 \\
& \Rightarrow \mathrm{~K}_{0}=\frac{-0.0108686}{0.07947}=-0.13676
\end{aligned}
$$

## In summary,

$$
\begin{aligned}
& \mathrm{K}_{0}=-0.13676 \\
& \mathrm{~K}_{1}=0.06428 \\
& \mathrm{~K}_{2}=0.63374
\end{aligned}
$$

If we sum the elements of the kernel, we find that the sum is not unity.

$$
S_{k}=K_{0}+4 K_{1}+4 K_{2}=2.65532
$$

Because the sum of the filter kernel elements are not unity we will get a combination of amplification and inversion. This is avoided by normalising the kernel, such that the sum of the elements of the kernel is unity.

$$
\begin{aligned}
& \text { Therefore, } \\
& \mathrm{K}_{0 \mathrm{~N}}=\mathrm{K}_{0} / \mathrm{S}_{\mathrm{k}}=-0.0515 \\
& \mathrm{~K}_{1 \mathrm{~N}}=\mathrm{K}_{1} / \mathrm{S}_{\mathrm{k}}=0.02421 \\
& \mathrm{~K}_{2 \mathrm{~N}}=\mathrm{K}_{2} / \mathrm{S}_{\mathrm{k}}=0.23867
\end{aligned}
$$

When this cellular filter was applied to test image the results were a isotropic low pass filter with a 3 dB cutoff frequency of approximately 450 Hz . (figure 5). The amplitude response at 900 Hz . was approximately -13 dB which is consistent with the response of a 5 th. order low pass Bessel filter with a 3 dB cutoff frequency of 450 Hz . (figure 6).

### 3.2.2 Filter Coefficients Using Heuristic Search with $W_{1}(n)$

Using the heuristic search method, with weighting function $W_{1}(n)$, to determine the filter coefficients results in the following filter coefficients.

$$
\begin{aligned}
& \mathrm{K}_{0}=-0.40315 \\
& \mathrm{~K}_{1}=-0.15082 \\
& \mathrm{~K}_{2}=-0.00461
\end{aligned}
$$

which when normalised yields
$\mathrm{K}_{\mathrm{ON}}=0.39337$
$\mathrm{K}_{1 \mathrm{~N}}=0.14716$
$\mathrm{K}_{2 \mathrm{~N}}=0.0045$
and $n=12$.



When this cellular filter was applied to test image the results were an isotropic low pass filter with a 3 dB cutoff frequency of approximately 750 Hz . (figure 7). The amplitude response at 1500 Hz . was approximately -13 dB which is consistent with the response of a 5 th. order low pass Bessel filter (figure 8).

### 3.2.3 Filter Coefficients Using Heuristic Search with $W_{2}(n)$

Using the heuristic search method, with weighting function $W_{2}(n)$, to determine the filter coefficients results in the following filter coefficients.

$$
\begin{aligned}
& \mathrm{K}_{0}=-0.0487 \\
& \mathrm{~K}_{1}=0.15081 \\
& \mathrm{~K}_{2}=-0.03142
\end{aligned}
$$

which when normalised yields
$\mathrm{K}_{\text {ON }}=-0.11356$
$\mathrm{K}_{1 \mathrm{~N}}=0.35165$
$K_{2 N}=-0.07326$
and $n=12$.

When this cellular filter was applied to test image the results were an anisotropic filter (figure 9). At low frequencies, up to 1 kHz . the behaviour was low pass in nature, but still anisotropic. At higher


Figure 7: 750Hz W1 Heuristic Filter at - 3 dB



Fiqure 7: 750 Hz W2 Heuristic Filter et -SdB
frequencies there was no discernible behaviour other than anisotropic behaviour.

### 3.2.4 Summary of Results

Deriving the cellular filter coefficients from the equations 12 , 13 and 14 does not produce the expected results. This is due to the fact that we are trying to find a kernel which will match impulse response of the filter from only 3 points, far from the origin. If the procedure for deriving equations 12,13 and 14 was applied to other points, producing equations for the response at further points then the response of the cellular filter may be closer to the expected response.

Using the heuristic search algorithm with weighting function $W_{2}(n)$ does not produce good results since $W_{2}(n)$ overcompensates for errors near the origin during the search.

Using the heuristic search algorithm with weighting function $W_{1}(n)$ produced excellent results. The filter has the correct cutoff frequency and the correct amplitude response.
3.3 Synthesis of a 2 kHz . 5th. Order Low Pass Bessel Filter

From equation (16), section 3.2 , the normalised transfer function for a 5 th. order low pass Bessel filter is given by:

$$
\begin{aligned}
H\left(s_{n}\right)=945\{ & \frac{0.0221}{\left(s_{n}+3.6467\right)}+\frac{(0.0143) \exp (. j 0.4883)}{\left(s_{n}+(3.352-j 1.742)\right)} \\
& +\frac{(0.0143) \exp (-i 0.4883)}{\left(s_{n}+(3.352+j 1.742)\right)}+\frac{(3.2394 \times 10-3}{\left(s_{n}+(2.325-j 3.571)\right)} \\
& \left.+\frac{\left(3.2394 \times 10 \frac{-3}{}\right) \exp (-j 1.0571)}{\left(s_{n}+(2.32-j 3.571)\right)}\right\}
\end{aligned}
$$

If the cutoff frequency is $2 \mathrm{kHz} .$, then $2 \pi \omega_{c}=1.2566 \times 10^{4} \mathrm{rads} / \mathrm{m}$. Applying the transform $s_{n}$ to ( $s / \omega_{\mathrm{C}}$ ) to scale the 3 dB . cutoff frequency to $f_{c}$ gives the transfer function of a 5 th. order 500 Hz . low pass Bessel filter as

$$
\begin{aligned}
& \mathrm{H}(\mathrm{~s})=945\left\{\left(2.772 \times 10^{2}\right)\right. \\
& \left(s+4.5826 \times 10^{4}\right) \\
& +\left(1.797 \times 10^{2}\right) \exp (j 0.4883) \\
& \left(s+\left(4.2122 \times 10^{4}-\mathrm{j} 2.1891 \times 10^{4}\right)\right) \\
& +\quad(1.797 \times 10 \underline{2}) \exp (-\mathfrak{i} 0.4883) \\
& \left(s+\left(4.2122 \times 10^{4}+j 2.1891 \times 10^{4}\right)\right) \\
& +\left(4.0708 \times 10^{1}\right) \exp (i 1.0571) \\
& \left(s+\left(2.9217 \times 10^{4}-j 4.7136 \times 10^{4}\right)\right) \\
& +(4.0708 \times 101) \exp (-. j 1.0571)\} \\
& \left(s+\left(2.9217 \times 10^{4}+j 4.7136 \times 10^{4}\right)\right) \\
& \Rightarrow H(s)=\frac{\left(2.6245 \times 10^{5}\right)}{\left(s+4.5826 \times 10^{4}\right)} \\
& +\left(1.6982 \times 10^{5}\right) \exp (. j 0.4883) \\
& \left(s+\left(4.2122 \times 10^{4}-j 2.1891 \times 10^{4}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\left(1.6982 \times 10^{5}\right) \exp (-. j 0.4883)}{\left(s+\left(4.2122 \times 10^{4}+j 2.1891 \times 10^{4}\right)\right)} \\
& +\frac{\left(3.846 .9 \times 10^{4}\right) \exp (.11 .0571)}{\left(s+\left(2.9217 \times 10^{4}-j 4.7136 \times 10^{4}\right)\right)} \\
& +\frac{\left(3.8469 \times 10^{4}\right) \exp (-. j 1.0571)}{\left(s+\left(2.9217 \times 10^{4}+j 4.7136 \times 10^{4}\right)\right)}
\end{aligned}
$$

Using the inverse Laplace transform $L^{-1}\left\{(s+a)^{-1}\right\}=\exp (-a r)$, gives the impulse response as

$$
\begin{aligned}
h(r)= & \left(2.6245 \times 10^{5}\right) \exp \left(-\left(4.5826 \times 10^{4}\right) r\right) \\
& +\left(1.6982 \times 10^{5}\right) \exp (j 0.4883) \exp \left(-\left(4.2122 \times 10^{4}-j 2.1891 \times 10^{4}\right) r\right) \\
& +\left(1.6982 \times 10^{5}\right) \exp (-j 0.4883) \exp \left(-\left(4.2122 \times 10^{4}+j 2.1891 \times 10^{4}\right) r\right) \\
& +\left(3.8469 \times 10^{4}\right) \exp (j 1.0571) \exp \left(-\left(2.9217 \times 10^{4}-j 4.7136 \times 10^{4}\right) r\right) \\
& +\left(3.8469 \times 10^{4}\right) \exp (-j 1.0571) \exp \left(-\left(2.9217 \times 10^{4}+j 4.7136 \times 10^{4}\right) r\right)
\end{aligned}
$$

Since $\exp (j a)+\exp (-j a)=2 C o s(a)$, the impulse response function of a 5 th. order 2 kHz . low pass Bessel filter is

$$
\begin{aligned}
h(r)= & \left(2.6245 \times 10^{5}\right) \exp \left(-\left(4.5826 \times 10^{4}\right) r\right) \\
& +\left(3.3964 \times 10^{5}\right) \exp \left(-\left(4.2122 \times 10^{4}\right)\right) \cos \left(\left(2.1891 \times 10^{4}\right) r+0.4883\right) \\
& +\left(7.6938 \times 10^{4}\right) \exp \left(-\left(2.9217 \times 10^{4}\right)\right) \cos \left(\left(4.7136 \times 10^{4}\right) r+1.0571\right)
\end{aligned}
$$

and the impulse response envelope function is

$$
\begin{aligned}
h_{e}(r)= & \left(2.6245 \times 10^{5}\right) \exp \left(-\left(4.5826 \times 10^{4}\right) r\right) \\
& +\left(3.3964 \times 10^{5}\right) \exp \left(-\left(4.2122 \times 10^{4}\right)\right) \\
& +\left(7.6938 \times 10^{4}\right) \exp \left(-\left(2.9217 \times 10^{4}\right)\right)
\end{aligned}
$$

To calculate the order of the cellular filter which we will use to synthesise the 5 th. order 2 kHz . low pass Bessel filter, we calculate the impulse response and the impulse response envelope at multiples of $\sqrt{2}$ times the resolution of the grid we are using. Since the resolution $R=8.4667 \times 10^{-5}$ m., this gives:

| $\underline{n R}$ | $\underline{\underline{2} n R}$ | $\underline{h}(\sqrt{2} \underline{n})$ | $\underline{\underline{h}} \underline{\underline{e}}(\sqrt{2} \underline{n R})$ |
| :--- | :---: | ---: | ---: |
|  |  |  |  |
| $2 R$ | $2.82842712 R$ | 85.19818548 | 89.01947656 |
| $3 R$ | $4.24264069 R$ | 1.36309284 | 2.23882155 |
| $4 R$ | $5.65685425 R$ | 0.00462502 | 0.06506179 |
| $5 R$ | $7.07106781 R$ | -0.00104613 | 0.00195191 |
| $6 R$ | $8.48528137 R$ | -0.00005509 | 0.00005894 |
| $7 R$ | $9.89949493 R$ | -0.00000171 | 0.00000178 |

A resolution of 8 bits in image intensity gives a minimum image intensity value of 0.003906 . Therefore the selection of the order , $n$, of the cellular filter is such that it is the largest value of $n$ where $h(!h(\sqrt{2} n R))>0.003906$ and $h_{e}(\sqrt{2}(n+1) R)<0.003906$. From the data in the table above the correct value for the order of the cellular filter is 4.

Using the heuristic search technique, with wieghting function $W_{1}(n)$, to calculate the cellular filter coefficients, the following coefficients were obtained.
$K_{0}=-0.18406$
$\mathrm{K}_{1}=-0.03752$
$K_{2}=-0.00631$

Since the sum of the kernel, $\mathrm{S}_{\mathrm{k}}$, is not unity (-0.35938), the kernel must be normalised to avoid unwanted inversion and attenuation. This gives a normalised kernel for a cellular filter to implement a 5 th . order 2 kHz . low pass Bessel filter of:

$$
\begin{aligned}
& \mathrm{K}_{0 \mathrm{~N}}=\mathrm{K}_{0} / \mathrm{S}_{\mathrm{k}}=0.51216 \\
& \mathrm{~K}_{1 \mathrm{~N}}=\mathrm{K}_{1} / \mathrm{S}_{\mathrm{k}}=0.1044 \\
& \mathrm{~K}_{2 \mathrm{~N}}=\mathrm{K}_{2} / \mathrm{S}_{\mathrm{k}}=0.01756
\end{aligned}
$$

and the order of the cellular filter $n=4$.

When this cellular filter was applied to the test image it produced a low pass filter with an incorrect cutoff frequency which did not exhibit isotropic behaviour (figure 10).

This anisotropic behaviour and incorrect cutoff frequency highlight an important limitation of this method for synthesising 2dimensional image filters by cellular automata. The central idea of this technique is find a cellular automaton, which after $N$ iterations on the impulse signal, produces a result which is a sampled version of the desired filter's impulse response. The number of samples is equal to the number of iterations (order of the cellular filter) and the sampling rate of the impulse response is the resolution of the underlying grid. As in all sampling operations the Nyquist sampling criterion must be met for a set of samples to uniquely determine a


Fiqure 10: $2 k \mathrm{~Hz}_{\mathrm{W}} \mathrm{W} 1$ Heuristic Filter at -13dB
given function. The Nyquist sampling theorem states that in order for a set of samples to uniquely represent a given function, the sampling frequency must be greater than twice the highest frequency component in the function. In this technique the sampling rate is fixed by the resolution of the underlying grid and cannot be changed. Therefore, when the order of the cellular filter is low, not enough samples are obtained to uniquely represent the impulse response, and the cellular filter does not have the desired effects.

### 4.0 Synthesis of Other Filter Types

This technique for synthesising 2 -dimensional image filters by cellular automata can be extended to other types of filters, such as high pass, band pass and band reject filters, as well as low pass filters. Since the basic idea of the technique is to find a cellular automaton which after $N$ iterations on the impulse signal produces a sampled version of the impulse response of the filter, in order to synthesise other type of filters all that is needed is the transfer function of the filter to be synthesised. These transfer functions can be obtained by two methods:
(1) by consulting tables of transfer functions in references on filter design.
(2) by applying a frequency transform to a low pass filter transfer function

The use of frequency transformations on low pass filter transfer functions is a common way to derive the transfer functions of high pass, band pass, and other filters. Some of the common frequency transforms are:

## Low Pass to High Pass transform

For a given low pass transfer function, the high pass transfer function can be obtained by substituting $s^{-1}$ for $s$ in the transfer function.

## Low Pass to Band Pass transform

For a given low pass transfer function, the equivalent band pass transfer function can be obtained by the transform

$$
s->\frac{s^{2}+\omega_{0}^{2}}{\left(\omega_{3 d B}\right) s}=\frac{Q\left(s_{n}^{2}+1\right)}{s_{n}}
$$

$$
\begin{aligned}
& \text { where } \begin{aligned}
\omega_{3 \mathrm{~dB}} & =3 \mathrm{~dB} \text { pass band width } \\
\omega_{0} & =\text { the centre frequency defined by } \\
& \omega_{0}=\sqrt{\omega_{\mathrm{u}} \omega_{I}} \text { which approximates } \frac{\omega_{u+} \omega_{1}}{2} \text { for } \frac{\omega_{0}}{\omega_{3 \mathrm{~dB}}} \gg 1 \\
Q & =\text { the quality factor } \frac{\omega_{0}}{\omega_{3 \mathrm{~dB}}} \\
\text { and } s_{n} & =\frac{\mathrm{s}}{\omega_{0}}
\end{aligned}
\end{aligned}
$$

For a given low pass transfer function, the equivalent band pass transfer function can be obtained by the transform

$$
s \rightarrow \frac{\left(\omega_{3 d B}\right) s}{s^{2}+\omega_{0}^{2}}=\frac{s_{n}}{Q\left(s_{n}^{2}+1\right)}
$$

where $\omega_{0}, \omega_{3 \mathrm{~dB}}, Q$ and $s_{n}$ have the same definitions as for the low pass to band pass frequency transform.

As an example of how such a frequency transform can be used in this technique to synthesise a band pass filter consider the transfer function of a lst order low pass Bessel filter.

$$
H(s)=\frac{1}{s+1}
$$

In order to transform this into the transfer function of a lst. order band pass Bessel filter with lower and upper 3dB cutoff frequencies of 400 Hz and 800 Hz . respectively, the following transform is applied

$$
s \rightarrow \frac{s^{2}+\omega_{0}{ }^{2}}{\left(\omega_{3 \mathrm{~dB}}\right) s}
$$

```
where \(\omega_{0}=\sqrt{2 \pi(2800) 2 \pi(400)}=3.5543 \times 10^{3}\)
and \(W_{3 d B}=800-400=400\)
```

Hence the transfer function of such a band pass filter is

$$
\begin{equation*}
\mathrm{H}(\mathrm{~s})=\frac{\left(\psi_{3 \mathrm{~dB}}\right) \mathrm{s}}{\mathrm{~s}^{2}+\left(\psi_{3 \mathrm{~dB}}\right) \mathrm{s}+w_{0} 2} \tag{17}
\end{equation*}
$$

The inverse Laplace transform of $\frac{s+\alpha}{(s+\beta)^{2}+\gamma^{2}}$ is given by

$$
\begin{equation*}
L^{-1}\left[\frac{s+\alpha}{(s+\beta)^{2}+\gamma^{2}}\right)=\frac{1}{\gamma}\left[\sqrt{(\alpha-\beta)^{2}+\gamma^{2}}\right] \exp \left(-\alpha_{r}\right) \sin \left(\gamma_{r}+\phi\right) \tag{18}
\end{equation*}
$$

where $\phi=\operatorname{Tan}^{-1}\left(\frac{y}{\alpha-\beta}\right)$

But $\frac{s+\alpha}{(s+\beta)^{2}+\gamma^{2}}=\frac{s+\alpha}{s^{2}+2 \beta s+\left(\beta^{2}+\gamma^{2}\right)}$
and comparing this to equation (17) gives the following equalities

$$
\begin{aligned}
& \alpha=0 \\
& \beta=\frac{\omega_{3}}{2} \\
& y^{2}=\omega_{0} 2-\frac{\omega_{3 B}}{4} \mathrm{~dB}^{2}
\end{aligned}
$$

and therefore, by equation (18), the impulse response of the 1 st. order band pass Bessel filter is

$$
\begin{aligned}
& L^{-1}\{H(s)\}=h(r)= \\
y & \left.\frac{1}{y} \sqrt{(\phi-\beta)^{2}+\gamma^{2}}\right] \exp \left(-\omega_{r}\right) \sin \left(\psi_{r}+\phi\right) \\
\Rightarrow & \left.h(r)=\frac{\omega_{3} d B}{\sqrt{\omega_{0}{ }^{2}-\frac{\omega_{3}}{4 \mathrm{~dB}^{2}}}} \omega_{0} \sin \left(\sqrt{\omega_{0}^{2}-\frac{\omega_{3} \mathrm{~dB}^{2}}{4}} r+\phi\right)\right)
\end{aligned}
$$

> where $\phi=\operatorname{Tan}^{-1}\left(-\frac{\sqrt{\omega_{0}{ }^{2}-\frac{\omega_{3} \mathrm{~dB}^{2}}{4}}}{\frac{\omega_{3 \mathrm{~GB}}}{2}}\right)$
> $\Rightarrow h(r)=\left(4.0063 \times 10^{2}\right) \sin \left(\left(3.5487 \times 10^{3}\right) r-1.5145\right)$

Since this is a 1 st. order band pass transfer function, it has no negative exponential term. Therefore the method for selecting the order of the cellular filter is no longer valid since there is no value of $n=n_{0}$ such that $|h(\sqrt{2} n R)|$ < minimum image intensity for $n>n_{0}$. However the frequency of $h(r)$ is

$$
\begin{aligned}
& 3.5487 \times 10^{3} \mathrm{rads} / \mathrm{m}=2 \pi \mathrm{f} \\
\Rightarrow \quad & \mathrm{f}=564.79 \mathrm{~Hz} .
\end{aligned}
$$

On a grid of $300 \mathrm{dpi}\left(8.4667 \times 10^{-5} \mathrm{~m}\right)$ the number of samples needed to characterise a full cycle is

$$
\left(\left(8.4667 \times 10^{-5}\right)(564.79)\right)^{-1}=20.91
$$

Therefore the order of the cellular filter needed to synthesise the 1st. order band pass Bessel filter with lower and upper 3dB cutoff frequencies of 400 Hz , and 800 Hz . respectively is 21 . The coefficients of the cellular filter, i.e. the elements of the cellular automaton can now be found by heuristic search.

Cellular automata are an example of a highly symmetrical dataparallel computational system, in which the data decomposition between computational nodes is congruent. Within a given iteration, one cellular automaton does not depend on any other cellular automaton in order to produce its next state. All the cellular automata are independent of each other, and all of them can operate in parallel. This is the main advantage of using cellular filters for filtering 2dimensional images. Given a network of processors, a set of cellular automata can be assigned to each processor. Since the cellular automata have a congruent data decomposition, by adding more processors, the number of cellular automata assigned to each processor can be reduced linearly, and therefore the time taken to filter an image can be reduced approximately linearly. Since these processors need only be capable of floating-point addition and multiplication to a limited accuracy (8 bits in a system with 256 shades of grey), this allows for real time image filtering at relatively low costs.

The occam programs in Appendices $C$ and $D$ are just one possible parallel implementation of cellular filters. Appendix $C$ contains $a$ listing of netfilt.tsr, an occam program designed to run on a network of four transputers connected in a pipeline. Appendix $D$ contains a listing of host.tsr, an occam program which runs on the host transputer which can access the DOS file structure.

Each network transputer has two parallel processes running on it, a data.manager process and an image.filter process. The image.filter process receives a portion of the overall image, together with the
elements of the cellular automaton which will be used to synthesise the desired filter, from the host transputer via the network of data.manager processes on the different transputers. It then starts $M$ parallel processes, where $M$ is the number of pixels in the image portion. Each process is a cellular automaton. While there is a slight performance disadvantage in implementing the $M$ cellular automata on a single transputer in parallel rather then sequentially, it does emphasise the inherent parallel nature of cellular automata. After image.filter has performed one iteration of the cellular automata, the image portion is sent back the host transputer via the network of data.manager processes.

The data.manager process acts as a multiplexer for its associated image.filter process. The messages that the data.manager process receives from the image.filter process are reframed to include a node identification number. The messages are then sent to the host transputer via the data.manager process on the next transputer upstream on the pipeline of transputers. Messages which the data.manager process receives from the data.manager process on the next transputer downstream on the pipeline of transputers are passed on to the upstream data.manager process unaltered. When the data.manager process receives messages from the host transputer via the upstream data.manager processes, it checks the node identification number to determine if the message is destined for the image.filter process associated with it. If the message is destines for the image.filter process running on the same transputer as the data.manager process, the node identification number is stripped off and the message is passed on to the image.filter process. For all messages, except the quit message, if the data.manager process finds that the node identification number does not match this
node's identification number, the message is passed on to the next downstream data.manager process. Quit messages are handled differently. If the quit message is destined for this node, the data.manager process passes the quit message on to the image.filter process, which then terminates. The data.manager process then checks to see if all downstream nodes have received quit messages. If they have, the data.manager process then terminates itself. If all the downstream nodes have not received a quit message, it records itself as terminated, but does not actually terminate itself just then. The data.manager process stays active. If the message is destined for another node, prior to being passed on downstream, the node the quit message is destined for is recorded as terminating by this node. After the message has been passed on downstream the data.manager process then checks to see if all downstream nodes, including itself, have received quit messages. If all downstream nodes and this node have received quit messages the data.manager process will terminate itself.

The process running on the host transputer acts as a file manager, interfacing the network of image.filter processes to the user and to the DOS file system. After it has determined, from the user, the file containing the image to be processed, and the coefficients and order of the cellular filter, the host process initialises the network of data.manager processes and kick starts the network of image.filter processes by sending each image.filter process a request.something message. Each image.filter process will then request a portion of the image to which it will apply one iteration of the cellular filter. After an image.filter process has processed the image portion, the image portion is sent back to the host process for filing. The host process will send an acknowledgement to the appropriate image.filter
process to signal the image portion has been saved. The image.filter process will then request another image portion to process. When an image.filter process requests another image portion, and there is no more image portions left to be processed in this iteration of the cellular filter, the host process sends a hold message to the image.filter process. When the whole image has received an iteration of the cellular filter, another iteration is started by the host process sending request.something messages to the network of image.filter processes. This sequence of events continues until the required number of iterations has been carried out. The host process then shuts the transputer network down by sending quit messages to each data.manager process.

The following is the message structure between the host process and the data.manager process, and the data.manager process and the image.filter process.

Host process messages transmitted

| message name | parameters | final destination response |
| :--- | :--- | :--- |
| initialise | node id | data.manager |
| quit | node id | image.filter |
| hold | node id | image.filter |
| request.something | node id | image.filter |
| filter.param | node id, | image.filter |
|  | subimage id, | none |
|  | filter coefficients |  |


| image. row | node id, | image.filter none |
| :--- | :--- | :--- |
|  | subimage id, |  |
| row of image |  |  |
| image.saved | node id | image.filter none |

Host process messages received

| message name | parameters | finitial source | response |
| :--- | :--- | :--- | :--- |
| request. image | node id | image.filter | filter:param |
| image.filtered | node id | image.filter | none |
| filtered.row | subimage id |  |  |
|  | node id, | image.filter | image.saved |
|  | subimage id, |  |  |

The main disadvantage to this implementation of a cellular filter is the bottleneck introduced by the host process accessing the DOS file system. In a commercial implementation using a stand alone network of transputers, the host transputer would have enough memory to hold the entire image in memory, and the memory on each transputer in the network would only have enough memory to meet its requirements. Then, provided that the number of transputers in the network is an factor of the number of subimages in the total image, then the load balance is equal amoung all the transputers, and the performance of the filter increases linearly with the number of transputers.

The synthesis of a given 2 -dimensional grey level image filter by cellular automata can be successfully carried out using the technique described in this thesis. The basic goal of the technique is to find a cellular automaton which when replicated $M$ times (where $M$ is the size of the image, and each cellular automaton is assigned to each pixel in the image) will produce a sampled version of the impulse response of the image filter to be synthesised, after $N$ iterations on the impulse signal.

Two techniques for finding the kernel of such a cellular automaton were investigated, an analytical evaluation of the elements of the kernel and a heuristic search for the elements of the kernel.

The analytical technique did not produce good results, but this was due to the fact that only three points, at a distance from the origin of the impulse signal, were used to calculate the kernel. For this analytical technique to produce accurate results, further research is necessary to produce a general equation which describes the response, after $N$ iteration, of a set of cellular automata using a given kernel at any point in the resulting signal array.

The heuristic search for the elements of the kernel was investigated using two different wieghting functions for the error between the impulse response and the kernel after $N$ iterations. Wieghting function $W_{2}(n)$ was designed to mimic the analytical technique, by concentrating on matching the response of the kernel after $N$ iteration on the impulse signal, to the impulse response at
points distant from the origin of the impulse signal. Like the analytical technique, a heuristic search using wieghting function $W_{2}(n)$ does not produce good results. Wieghting function $W_{1}(n)$ was designed to give a closer fit to the impulse response at all points, while also allowing for misleading effects due to approximating the impulse signal. Heuristic searches using wieghting function $W_{1}(n)$ produced excellent results with the correct cutoff frequencies and accurate amplitude response characteristics.

Like all systems which seek to represent reality by a digitally sampled representation, this thesis has shown that cellular filters are also subject to the Nyquist sampling criterion. This limitation means that low order cellular filters are subject to aliasing effects and do not produce isotropic results.

In summary, the filtering behaviour of cellular automata on binary images has been known since the mid $1960^{\prime} \mathrm{s}$. In the late 1970's researchers, such as Preston and Duff, extended the use of cellular automata to the filtering of grey level images, by converting the grey level image into a series of binary images, applying cellular automata to these binary images and then reconstituting them back into a grey level image. These cellular filters were found empirically, and while the cutoff frequencies of these low pass filters could be altered, the response characteristics were fixed. In recent years, other researchers, such as Huang, Jenkins and Sawchuk, while working on digital cellular image processors and their associated algebra, have commented on the filtering characteristics on certain cellular automaton which have been found empirically. To date, no method has existed for deriving the kernel of a cellular filter which will have
characteristics defined by the user. The technique described in this thesis, based in the formal methods of linear systems analysis, enables the user to successfully derive the kernel of a cellular filter to match the behaviour of a known filter described by its Laplacian transfer function.

### 7.0 References

[1] "Modern Cellular Automata - Theory and Applications"; K. Preston and M.J.B. Duff; Plenum; 1984.
[2] "Binary Image Algebra and Optical Cellular Logic Processor Design"; K.S. Huang, B.K. Jenkins and A.A. Sawchuk; Computer Vision, Graphics and Image Processing vol. 45 no. 3 p295-345; 1989.
[3] "Introduction to Linear System Analysis"; Matrix Publishers; 1976.
[4] "Modern Control Systems"; R.C. Dorf; Addison-Wesley Publishing Co.: 1980.
[5] "Manual of Active Filter Design"; J.L. Hilburn and D.E. Johnson; McGraw-Hill; 1973.
[6] "Filter Design Tables and Graphs"; E. Christian and F. Eisenman; 2nd. Ed.; Wiley; 1966.
[7] "Introduction to the Theory and Design of Active Filters"; L.P. Huelsman and P.E. Allen; McGraw-Hill; 1980.

## Appendix A

This appendix contains a listing of the $C$ program, find_ks.c, which implements the heuristic search algorithm for the coefficients of the cellular filter which synthesises the filter whose impulse response is contained in the procedure impulse response.
／＊This program locates the coefficients of the kernal which has ＊／
/* the minimum mean square error from the impulse response for the $* /$
$\begin{array}{ll}\text { /* the minimum mean square error from the impulse response for the } \\ / * & \text { filter whose impulse response is defined by impulse_response (). */ }\end{array}$
/*
/*
Command lu */
Command line (Microsoft compiler): cl/W3 response,c */
$/ *$ */
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
/*
/*
/*
Version History
Version History */ */ */ */ */
/* Revision Notes */
/* ------- ----- */
$/ *$ */
/* 1.00 Initial Development Version */
$/ *$ */

$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ header files $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
我include "stdio.h"
全include "stdlib.h"
£include "conio.h"
£include "math.h"
£include "dos.h"
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ declaration files $* * * * * * * * * * * * * * * * * * * * * * * * * * * * /$

/* ASCII characters */
£define SPACE $0 \times 20$
/* ASCII graphics characters */
Sdefine LEFT_TOP COURNER 201
まdefine RIGHT_TOP_COURNER 187
£define LEFT_BOTTOM_COURNER 200
£define RIGHT_BOTTOM_COURNER 188
£define HORIZONTAL_LINE 205
sdefine VERTICAL_LINE 186
/* screen dimensions, counting from $0 * /$
£define SCREEN_WIDTH 79
fdefine SCREEN_DEPTH 23
$/ *$ miscellaneous */
£define EPSILON (double) 1.0e-08

```
&define TRUE
0
&define FALSE
-1
£define SWEEP_TABLE_SIZE 11
£define MAX_ERROR 1.0e+300
/* response size definitions */
£define ROOT2 (double) 1.414213562e+00
£define RESOLUTION
(double) 8.4667e-05
&define RESPONSE_WIDTH 31
£define RESPONSE_DEPTH 31
/* BIOS video services */
{define VIDEO 0x10
£define SET CURSOR_POS 0x02
£define READ_CURSOR_POS 0x03
£define DOS_SCROLL_WINDOW_UP 0x06
£define CURRENT_VIDEO__STATE 0x0f
/* DOS colours */
{define BLACK 0x00
/****************************** glubal variables ****************************/
/**************************** module variables ****************************/
int wieght_fn;
float filter_k0;
float filter_k1;
float filter_k2;
float delta_fn;
float response [RESPONSE_DEPTH+2][RESPONSE_WIDTH+2];
float resultant [RESPONSE_DEPTH+2][RESPONSE_WIDTH+2];
float step_table [SWEEP_TABLE_SIZE] = {1.0f, 5.0e-01f, 0.25f, 0.1f, 0.01f,
    0.001f, 0.0001f, 0.00001f, 0.000001f,
    0.0000001f, 0.00000001f};
int main ()
{
    /* declare local variables */
    int iterations, sweep, end_sweep, precision, pass;
    int quit_flag = FALSE;
```

int low resolution = TRUE;
float k0_at_min, k1_at_min, k2_at_min, step;
float best_k0, best_k1, best_k2;
float max_extent_k0, max_extent_k1, max_extent_k2;
float low_extent_k0, low_extent_k1, low_extent_k2;
float low_range, high_range;
double error, minimum_error, last_minimum_error;
/* function prototypes */
double root_mean_square_error (float, float, float, int);
double impulse_response (double);
float calculate_step (unsigned int);
void clear_screen (void);
void set_cursor_position (unsigned int, unsigned int);
void draw_border (void);
void filter_calc_data (char *, float, float);
void print_results (float, float, float, int, double);
void show_mean_square_error (double, double, float);
void get_filter_data (int *, int *, float *, float *, float *, float *, float *, float *, int *);
/* get cellular filter data boundaries */
get_filter_data (\&iterations, \&precision, \&low_extent_k0, \&max_extent_k0, \& low_extent_k1, \&max_extent_k $\overline{1}$, \&low_extent_k $\overline{2}$, \&max_extent_k2, \&wieght_fn);
/* initialise variables */
delta_fn $=(f l o a t)$ impulse_response ((double) 0.0);
best_k0 = low_extent_k0;
best_kl = low_extent_k1;
best k2 = low_extent_k2;
pass $=0$;
end_sweep $=4$;
filter_k0 $=$ low_extent_k0;
filter_k1 = low_extent_k1;
last_minimum_error = MAX_ERROR;
step $=1.0 \mathrm{f}$;
error $=$ MAX_ERROR;
do
\{
pass++;
minimum_error = MAX_ERROR;
filter_k2 = low_extent_k2;
k2_at_min = filter_k2;
/* find minimum root mean square error $K 2$ with $K 0$ and $K 1$ constant */
high_range $=$ max_extent_k2;
low_range $=$ low_extent_k2;
for (sweep $=1$; sweep < = end_sweep; sweep++)
\{
for (; filter_k2 <= high_range; filter_k2 += step)
/* calculate root mean square error between kernal */
/* and impulse response */
filter_calc data ("K2", low_range, high_range);
if (filter_k2 != low_range) show_meān_square_error (error, minimum_error, k2_at_min);
set_cursor_position $(20,15)$;
printf ("Pass \%s\%d", (end_sweep ==4? "pre-": ""), pass);
error $=$ root_mean_square_error (filter_k0, filter_k1, filter_k2, iterations);

```
    if (error < minimum_error)
        minimum_error = error;
        k2_at_min = filter_k2;
        }
    step = calculate_step (sweep);
    }
    low_range = k2_at_min - step;
    filter_k2 = low_range;
    high_range = k2_at_min + step;
    }
filter_k2 =k2_at_min;
minimum_error = MAX_ERROR;
filter_k1 = low_extent_k1;
k1_,at_min = filter_k1;
```

/* find minimum root mean square error $K 1$ with $K 0$ and $K 2$ constant */
high_range $=$ max_extent_k1;
low_range $=$ low_extent_k1;
for (sweep $=1$; sweep $<=$ end_sweep; sweept+)
\{
for (; filter_k1 <= high_range; filter_kl + = step)
\{
/* calculate root mean square error between kernal */
/* and impulse response */
filter_calc data ("K1", low_range, high_range);
if (filter_kl ! = low_range)
show_mean_square_error (error, minimum_error, k1_at_min);
set_cursor_position $(20,15)$;
printf ("Pass \%s\%d", (end_sweep = 4 ? "pre-": ""), pass);
error $=$ root_mean_square_error (filter_ko, filter_kl, filter_k2,
iterations);

```
    if (error < minimum_error)
        {
        minimum_error = error;
        k1_at_min = filter_kI;
        }
    step = calculate_step (sweep);
    }
    low_range = kl_at_min - step;
    filter_k1 = low_range;
    high_range = kl_at_min + step;
    }
filter_k1 =k1_at_min;
minimum_error = MAX_ERROR;
filter_k0 = low_extent_k0;
k0_at_min = filter_k0;
```

/* find minimum root mean square error $K 0$ with $K 1$ and $K 2$ constant */
high_range $=$ max_extent_k0;
low_range $=$ low_extent_k0;
for (sweep $=1$; sweep $<=$ end_sweep; sweep++)
for (; filter_k0<= high_range; filter k0 + = step)
\{
/* calculate root mean square error between kernal */
/* and impulse response */
filter_calc_data ("KO", low_range, high_range);
if (filter_k0 != low_range)
show_mean_square_error (error, minimum_error, k0_at_min);
set_cursor_position (20,15);
printf ("Pass \%s\%d", (end_sweep == 4 ? "pre-": ""), pass);
error $=$ root_mean_square_error (filter_k0, filter_kl, filter_k2,
iterations);
if (error < minimum_error)
\{
minimumerror = error;
kO_at_min = filter_k0;
\}
step $=$ calculate_step (sweep);
\}

```
low_range = k0_at_min - step;
filter_k0 = low_range;
high_range = k0_at_min + step;
}
```

filter_k0 =k0_at_min;

```
                    /* are we within 1% of last minumum error */
```

    if (minimum_error < (last_minimum_error * (double) 0.99))
    \{
    last_minimum_error = minimum_error;
    best_k0 \(=\) filter_k0;
    best_k1 = filter_k1;
    best_k2 \(=\) filter_k2;
        \}
    else
        if (low_resolution \(==\) TRUE)
            low_resolution = FALSE;
            end_sweep \(=\) precision +3 ;
            pass \(=0\);
            \}
        else
            quit_flag = TRUE;
    \}
    \}
    while (quit_flag == FALSE);
/* print out the results */
print_results (best_k0, best_k1, best_k2, pass, minimum_error);
return (0);
\}
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
/* This routine returns a measure of the root mean square error between */
/* the impulse response defined by impulse_response () and the cellular */
/* filter defined by K0, K1, K2 and N.
*/
double root_mean_square_error (float k0, float k1, float k2, int n)
\{
int i, l, pass, x, y;
float $* p$, *r;
double actual, calculated, error;
double error_squared_sum;
double impulse_response (double);
double weight (int);
/* initialise response with delta function */
for $(1=($ RESPONSE_WIDTH +2$) *($ RESPONSE_DEPTH +2$), p=($ float $*)$ response;

```
    1 != 0; 1--, p++)
    *p = 0.0f; /* clear out response buffer */
response [(RESPONSE_WIDTH+2)/2] [(RESPONSE_DEPTH+2)/2] = delta_fn;
for (l = (RESPONSE_WIDTH+2)*(RESPONSE_DEPTH+2), p = (float *) resultant;
    1 != 0; 1--, p++)
    *p = 0.0f; /* clear out resultant buffer */
```

/* apply filter to delta function */
pass $=1$;
while (pass+t $<=n$ )
for $(y=1 ; y<=((R E S P O N S E$ WIDTH +2$) / 2) ; y++)$
for $\left(x=y ; x<=\left(\left(R E S P O N S E \_W I D T H+2\right) / 2\right) ; x++\right)$
$p=\&(r e s p o n s e ~[x][y]) ;$
$r=$ \& (resultant [x][y]);
$* \mathrm{r}=(* \mathrm{p} * \mathrm{k} 0)$
$+(*(\mathrm{p}-(1)) * \mathrm{k} 1)$
$+(*(\mathrm{p}+(1))$ * k 1$)$
$+(*(\mathrm{p}-((($ RESPONSE_WIDTH + 2 $)+1)))$ * k 2$)$
$+(*(p-(($ RESPONSE WIDTH + 2 ) $)) * k 1)$
$+(*(\mathrm{p}-((($ RESPONSE WIDTH + 2) $)-1)))$ * k 2$)$
$+(*(\mathrm{p}+((($ RESPONSE_WIDTH + 2) $)-1)))$ * k2)
+ (*(p+((RESPONSE_WIDTH+2))) * k1)
$+(*(\mathrm{p}+((($ RESPONSE_WIDTH+2)+1))) * k2);
resultant $[y][x]=$ resultant $[x][y]$;
resultant [((RESPONSE_WIDTH+2)-1) - x][y] = resultant [x]!y];
resultant [y][((RESPONSE_WIDTH+2)-1) - x] = resultant [x][y]
resultant $\left.[x]\left[\left(\left(R E S P O N S E \_W I D T H+2\right)-1\right)-y\right]=r e s u l t a n t[x] \mid y\right] ;$
resultant $[((\operatorname{RESPONSE} W I \bar{D} T H+2)-1)-y][x]=$ resultant $[\mathrm{x}] \mid \mathrm{y}]$ i
resultant $[(($ RESPONSE_WIDTH +2$)-1)-x][((R E S P O N S E$ WIDTH +2$)-1)-$
$y]=$ resultant [x][y]:
resultant [((RESPONSE_WIDTH+2)-1) - y][((RESPONSE_WIDTH+2)-1) -
$x]=$ resultant $[x][y] ;$
\}
\}
/* copy resultant to response */
for $(p=$ (float $*)$ response, $r=$ (float *) resultant, $i=0$;
$\mathrm{i}<($ RESPONSE_WIDTH +2$)$ * (RESPONSE_DEPTH +2$)$; $i++$ )
$*(\mathrm{p}++)=*(\mathrm{r}++) ;$
\}
error_squared_sum $=($ double $) 0.0$;
for (i $=1 ; i<=n ; i++$ )
\{
actual $=$ impulse_response ((double) i * RESOLUTION);
calculated $=$ (double) response
[ (RESPONSE_DEPTH +2$) / 2][(($ RESPONSE_WIDTH +2$) / 2)+i]$;
error $=$ (actual - calculated) * weight (i);
error_squared_sum += error * error;
actual = impulse_response ((double) i * ROOT2 * RESOLUTION);
calculated $=$ (double) response
$[(($ RESPONSE_DEPTH + 2$) / 2)+i][(($ RESPONSE_WIDTH +2$) / 2)+i] ;$
error = (actual - calculated) * weight ((int) ((double) i * ROOT2));
error_squared_sum $+=$ error * error;
\}
error_squared_sum = error_squared_sum / (double) (2 * n) ;
return (sqrt (error_squared_sum));
\}
/* This routine returns the interval to the next filter coefficient to be */ /* checked.
float calculate_step (unsigned int sweep__number)
\{
return (step_table [sweep_number - 1]);
\}
/* This routine calculates the weighting factor for the error depending on */
/* its distance from the centre.
double weight (int distance)
\{
switch (distance)
\{
case 0:
if (wieght_fn == 1)
return ((double) 0.5); else
return ((double) 0.0); break;
case 1:
if (wieght_fn $==1$ )
return ((double) 0.9); else
return ((double) 0.01); break;
case 2
if (wieght_fn == 1)
return ((double) 1.0);
else
return ((double) 0.1); break;
case 3:
if (wieght_fn == 1)
return ((double) 1.0);
else
return ((double) 0.5);
break;

```
case 4:
    if (wieght_fn == 1)
        return ((double) 1.0);
    else
        return ((double) 0.9);
    break;
```

default:
return ((double) 1.0);
break;
\}
\}
/* This routine returns the value of the impulse response for a specified */
/* argument.
double impulse_response (double arg)
\{
double answer;
/* 500 Hz 5 th order low pass Bessel filter */
answer $=$ (double) 6.561e+04 * $\exp (($ double) $-1.1456 \mathrm{e}+04 * \arg )$
+ (double) $8.4908 \mathrm{e}+04 * \exp (($ double $)-1.0531 \mathrm{e}+04 * \arg ) *$
cos(((double) 5.4727e+03 * arg) + (double) 0.4883)
$+(d o u b l e) 1.9235 e+04 * \exp ((d o u b l e)-7.3042 e+03 * \arg ) *$
$\cos (((d o u b l e) 1.1219 \mathrm{e}+04$ * arg) + (double) 1.0571);
return (answer);
\}
/************************************************************************/
/* void draw_border (void)
/* This routine draws a border around the screen.
void draw_border (void)
\{
int i;
void set_cursor_position (unsigned int, unsigned int);
set_cursor_position (0,0);
putchar (LEFT_TOP_COURNER);
set_cursor_position (SCREEN_WIDTH,0);
putchar (RIGHT_TOP_COURNER);
set_cursor_position ( 0, SCREEN_DEPTH ) ;
putchar (LEFT_BOTTOM_COURNER);
set_cursor_position (SCREEN_WIDTH,SCREEN_DEPTH);
putchar (RIGH'_BOTTOM_COURNER);
set_cursor_position (1,0);
for ( $i=1 ; i<S C R E E N$ _WIDTH; $i++$ )
putchar (HORIZONTAL_LINE);
set_cursor_position (1,SCREEN_DEPTH) ;
for ( $i=1$; $i<S C R E E N \_W I D T H ; i++$ ) putchar (HORIZONTAL_LINE);
set_cursor_position $(0,1)$;
for ( $\left.i=1 ; i<S C R E E N \_D E P T H ; i++\right)$
\{
set_cursor_position (0,i);
putchar (VERTICAL_LINE); \}
set_cursor_position (SCREEN_WIDTH,1);
for ( $\mathrm{i}=1 ; \mathrm{i}$ < SCREEN_DEPTH; $i++$ ) \{ set_cursor_position (SCREEN_WIDTH,i); putchar (VERTICAL_LINE); \}
return;

```
/* void clear_entry (unsigned int, unsigned int, unsigned int) */
```

/*
/* This routine clears the screen with 31 spaces at the specified position */
/* and set the cursor position to the specified location.
void clear_entry (unsigned int $x$, unsigned int $y$, unsigned int repeat)
\{
unsigned int i;
void set_cursor_position (unsigned int, unsigned int);
set_cursor_position ( $x, y$ );
for ( $i=0 ; i<r e p e a t ; i++$ )
putchar (SPACE);
set_cursor_position ( $x, y$ );
return;
)
/* void clear_screen (void)
/*
/* This routine clears the entire screen.
*/
void clear_screen (void)
\{
void up_scroll_region (unsigned char, unsigned char, unsigned char,
unsigned char, unsigned char, unsigned char);
up_scroll_region $(0,0,79,23,0$, BLACK );
return;
\}
/* void up_scroll_region (unsigned char, unsigned char, unsigned char, /* unsigned char, unsigned char, unsigned char)
/* This routine uses DOS to clear a region of the screen.
void up_scroll_region (unsigned char x1, unsigned char y1, unsigned char $x 2$,
unsigned char y2, unsigned char up_scroll,
unsigned char attribute)
\{
union REGS reg86;
reg86.h.al = up_scroll;
reg86.h.cl = x1;
reg86.h.ch = y1;
reg86.h.dl $=x 2$;
reg86.h.dh = y2;
reg86.h.bh = attribute;
reg86.h.ah = DOS_SCROLL_WINDOW_UP;
int86 (VIDEO, \&reg86, \& $\bar{r} e g 86$ );
return;
1
/* void set_cursor_position (unsigned int, unsigned int)
/* This routine sets the cursor position to the specified co-ordinates.
void set_cursor_position (unsigned int $x$,unsigned int $y$ )
\{
union REGS in_reg, out_reg;
/* get page number */
in_reg.h.ah = CURRENT_VIDEO_STATE;
int86 (VIDEO, \&in_reg, \&out_reg);
/* set cursor position */
in_reg.h.ah = SET_CURSOR_POS;
in_reg.h.bh = out_reg.h.bh;
in_reg.h.dh $=$ (unsigned char) $y$;
in_reg.h.dl $=$ (unsigned char) $x$;
int86 (0x10, \&in_reg, \&out_reg);
return;
]
1 * This function displays the parameters of the cellular filter to be */
/* examined.
void filter_calc_data (char *p, float low, float high)
\{
void set_cursor_position (unsigned int, unsigned int);
void draw border (void);
void clear screen (void);
clear_screen ();
draw_border ();
set_cursor_position (20,3);
printf ("Calculating cellular filter coefficients");
set_cursor_position $(20,5)$;
printf ("Range for \%s is [\%9.8f, \%9.8f] $\mathrm{n}^{\prime \prime}$ ", p, low, high);
set_cursor_position $(20,7)$;
printf ("K0 = \%9.8f", filter_k0);
set_cursor_position $(20,8)$;
printf ("K1 = \%9.8f", filter_k1);
set_cursor position $(20,9)$;
printf ("K2 = \%9.8f\n", filter_k2);
return;
/* This function display the results for the programs search.
void print_results (float k0, float k1, float k2, int $n$, double error)
\{
int key;
void set_cursor_position (unsigned int, unsigned int);
void draw_border (void);
void clear_screen (void);
clear_screen ();
draw border ();
set_cursor_position (30,3);
printf ("Results:");
set_cursor_position $(25,7)$;
printf ("K0 = \%9.8f", k0);
set_cursor_position (25,9);
printf ("K1 = \%9.8f", k1);
set_cursor_position (25,11);
printf ("K2 = \%9.8f", k2);
set_cursor_position (25,15);
printf ("Number of passes $=\% \mathrm{~d}$ ", n );
set_cursor_position (10, 17);
printf ("Minimum root mean square error $=\% 20.8 f \backslash n "$, error);
set_cursor_position $(25,22)$;
printf ("Hit any key to return to DOS");
key =getch ();
clear_screen ();
return;
\}
/* This routine displays the mean square error and the minimum mean square */

```
void show_mean_square_error (double error, double minimum_error, float k_min)
```

\{
void set_cursor_position (unsigned int, unsigned int);
set_cursor_position (15, 11);
printf ("root mean square error $=\% 20.8 f \backslash n "$, error);
set_cursor_position (15,12);
printf ("Minimum error \%20.8f at \%9.8f", minimum_error, k_min);
return;
)
$/ *$ This routine asks the user for the parameters for the search for the $* /$ /* cellular filter coefficients and checks to see if they are reasonable. */ void get_filter_data (int $* n$, int *accuracy, float *low_k0, float *high_k0, float *low k1, float *high_k1, float *low_k2,

void clear_screen (void);
void draw_border (void);
void set_cursor_position (unsigned int, unsigned int);
void clear_entry (unsigned int, unsigned int, unsigned int);
clear screen ();
draw_border ();
set_cursor_position (35,3);
printf ("Enter Filter Data:");
set_cursor_position $(25,7)$;
printf ("Enter order of cellular filter: ");
scanf ("\%d", n);
while (*n (RESPONSE_WIDTH/2))
\{
set_cursor_position (22,20);
printf ("Error - maximum cellular filter order is \%d\n",
(RESPONSE_WIDTH/2));
clear_entry $(58,7,5)$;
scanf ("\%d", n);
\}
clear_entry $(22,20,45)$;
set_cursor_position $(25,9)$;
printf ("Enter decimal point precision : ") i
scanf ("\%d", accuracy);
while (*accuracy > (SWEEP_TABLE_SIZE-3))
\{
set_cursor_position $(22,20)$;
printf ("Error - maximum precision is \%d\n decimal places",
SWEEP_TABLE_SIZE-3);
clear_entry (57,9,5);
scanf ("\%d", accuracy);
\}
clear_entry (22,20,55);
da
\{
set_cursor_position $(25,11)$;
printf ("Initial range of KO");
clear_entry ( $20,12,50$ );
set cursor position (20,12);
printf ("Low : ");
scanf ("\%f", low_k0);
set_cursor_position $(40,12)$;
printf ("High : ");
scanf ("\%f", high_k0):
\}
while (*low_k0 >= *high_k0);
do
\{
set_cursor_position $(25,14)$;
priñtf ("Initial range of K1");
clear_entry $(20,15,50)$;
set_cursor position $(20,15)$;
printf ("Low : ");
scanf ("\%£", low_k1);
set_cursor position $(40,15)$;
printf ("High : ");
scanf ("\%f", high_k1);
\}
while (*low_k1 >= *high_k1);
do
\{
set_cursor position $(25,17)$;
printf ("Initial range of K2");
clear_entry ( $20,18,50$ );
set_cursor_position $(20,18)$;
printf ("Low : ");
scanf ("\%f", low_k2);
set_cursor_position $(40,18)$;
printf ("High : ") ;
scanf ("\%f", high_k2);
)
while (*low_k2 >= *high_k2);
do
[
clear_entry (25,20,50);
set_cursor_position (25,20);
printf ("Select wieghting function [1,2]: ");
scanf ("\%d", wf);
\}
while ((*wf !=1) \&\& (*wf != 2));
return;

## Appendix B

This appendix contains the listing of the $C$ program, cfilter, c, which implements a specified cellular filter on an image file. This program supports Hercules monochreome graphics and 9 pin Epson printers.

1/* \$Header: C:/msc/cfilter.c_v 1.1 25 Feb 1991 20:31:10 davids $\$ * /$
/***************************************************************************/
/*

* Filename: cfilter.c
/* Author: David Sinclair

/*
/*
/*
/*
Revision
This module is the main module for the program which implements
a specified $3 \times 3$ square tesselation cellular filter.
Revision: $1.2 \quad * /$
Author: David Sinclair
This program requires a Hercules display to shows images on */
screen and produces printer output for an Epson 9 pin printer. */
Last Edit: 16May91 */
*/
*/
*/
*/
Command line (Microsoft compiler): cl/W3/Ox/AL cfilter.c */
Version History */
*/
Revision Notes */
-------- ----~ */ *
1.00 Initial Development Version
*/
*/
*/

$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ File History $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
/* \$Log: C:/msc/cfilter.c_y \$
    * Rev 1.1 25 Feb 1991 20:31:10 davids
Rev 1.1 25 Feb 1991 20:31:10 davids
* Made show_image() data entry more compatible with the rest of the program.
*/
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ header files $* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
£include "stdio.h"
£include "stdlib.h"
£include "dos.h"
£include "conio.h"
$/ * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ declaration files $* * * * * * * * * * * * * * * * * * * * * * * * * * * * /$
/******************************* definitions *******************************/
/* ASCII characters */

| £define | BS | $0 \times 08$ |
| :--- | :--- | :--- |
| ddefine | LF | $0 \times 0 \mathrm{a}$ |
| £define | CR | $0 \times 0 \mathrm{~d}$ |
| ddefine | ESC | $0 \times 1 \mathrm{~b}$ |
| sdefine | SPACE | $0 \times 20$ |

/* ASCII graphics characters */
£define LEFT TOP COURNER

```
&define RIGHT_TOP_COURNER 187
&define LEFT_BOTTOM_COURNER 200
&define RIGHT_BOTTOM_COURNER 188
£define HORIZONTAL_LINE 205
fdefine VERTICAL_LINE 186
/* screen dimensions, counting from 0 */
&define SCREEN WIDTH 79
&define SCREEN DEPTH 23
£define SHOW_IMAGE_X_OFFSET 0
&define SHOW IMAGE_Y_OFFSET 0
/* miscellaneous */
{define TRUE 0
&define FALSE -1
&define ERROR -1
£define BEGINNING 0
£define STARTING 0
&define RUNNING 1
&define FINISHED 2
£define MAX_TITLE 45
/* BIOS video services */
£define VIDEO 0x10
£define SET_CURSOR_POS 0x02
£define READ__CURSOR_POS 0x03
{define DOS_SCROLL_WINDOW_UP 0x06
&define CURRENT VIDEO STATE 0x0f
/* DOS colours */
fdefine BLACK 0x00
/* image size definitions */
£define RESOLUTION 300
{define PRINTER_RESOLUTION 72
£define AN_INCH PRINTER RESOLUTION
{define IMAGE_WIDTH }71
£define IMAGE DEPTH 341
{define SLICE_DEPTH 10
£define PRINT_IMAGE_WIDTH (IMAGE_WIDTH - AN__TNCH)
{define PRINT_SLICE DEPTH 8
```

```
V*graphics definitions */
&define WHITE_ON_BLACK 0x00
£define BLACK_ON_WHITE Oxff
```

```
/**************************** global variables ****************************/
```

/**************************** global variables ****************************/
/**************************** module variables ****************************/
/**************************** module variables ****************************/
unsigned int iterations = 1;
char show threshold = 0;
char print_threshold = 0;
char input_file [16] = "input.img";
char output_file [16] = "output.img";
char show_file [16] = "input.img";
char image file [16] = "input.img";
char print_file [16] = "print.img";
char *temporary_file = "temp.img";
FILE *in_fp, *out_fp, *temp_fp; /* file discriptors */
float filter_k0 = (float) 1.0;
float filter_k1 = (float) 1.0;
float filter_k2 = (float) 1.0;
char image [SLICE_DEPTH+2][IMAGE_WIDTH+2];
char resultant [SLICE_DEPTH+2][IMAGE_WIDTH+2];
char print_buffer [PRINT_IMAGE_WIDTH];
char raw_buffer [PRINT_SLICE_DEPTH][IMAGE_WIDTH];
char init_file [10] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
/* graphics variables */
char ttable[] =
{0x61,0\times50,0x52,0x0f,0x19,0x06,0x19,0\times19,0x02,0x0d,0x0b,0x0c};
char gtable[] =
{0x35,0x2d,0x2e,0x07,0x5b,0x02,0x57,0x57,0x02,0x03,0x00,0x00};
unsigned char colorglb;
int xmax = 719,xmin = 0,ymax = 348,ymin = 0;
/********************************* modules **********************************/
int main ()
{
char quit flag = FALSE;
char ready_to_filter = FALSE;
int key;
void clear_screen (void), draw_screen (void), filter (void);
void set_cursor_position (unsigned int, unsigned int);
void show_image (void);
void print image (void);
char define_options (void);
int getche (void);

```
int getch (void);
```

whil.e (quit_flag == FALSE)
\{
clear_screen ();
draw_screen ();
switch (getche())
[
case '1':
ready to filter = define_options ();
break;

```
        case '2':
            if (ready_to_filter == TRUE)
            filter ();
                else
                    \{
                        clear_screen ();
                set_cursor_position (15, 12);
                printf ("Filter Options must be defined before filtering");
                key = getch ();
                \}
                break;
        case ' 3 ':
                show_image ();
                break;
        case '4':
            print_image ();
            break;
        case '0':
                quit_flag = TRUE;
                break;
        \}
    \}
clear_screen ();
return (0);
```

/* void draw_screen (void)
/* */
/* This routine draw the main program screen.
*/
void draw_screen (void)
{
void set_cursor_position (unsigned int, unsigned int), draw_border (void);

```
draw_border ();
set_cursor_position (25,2);
printf ("Cellular Automaton Filtering");
set_cursor_position \((20,8)\);
printf ("1 ... Define Cellular Filter Parameters");
set_cursor_position (20,10);
printf ("2 ... Filter Image");
set_cursor_position (20,12);
printf ("3 ... Show Image");
set_cursor_position \((20,14)\);
printf ("4 ... Print Image");
set_cursor_position (20,16);
printf ("0 ... Quit to DOS"):
set_cursor_position \((9,20)\);
printf ("Option .. [ ]");
set_cursor_position (20,20);
return;
/* void draw_border (void)
/* This routine draws a border around the screen.
void draw_border (void)
\{
    int i;
    void set_cursor_position (unsigned int, unsigned int);
set_cursor_position (0,0);
putchar (LEFT_TOP__COURNER);
set_cursor_position (SCREEN_WIDTH,0);
putchar (RIGHT_TOP_COURNER);
set_cursor_position (0, SCREEN_DEPTH);
putchar (LEFT_-BOTTOM_COURNER);
set_cursor_position (SCREEN_WIDTH,SCREEN_DEPTH);
putchar (RIGHT_BOTTOM_COURNER);
set_cursor_position (1,0);
for ( \(\mathrm{i}=1\); i < SCREEN_WIDTH; \(\mathrm{i}++\) )
    putchar (HORIZONTAL_LINE);
set_cursor_position (1, SCREEN_DEPTH);
for (i = 1; i < SCREEN_WIDTH; i++)
        putchar (HORIZONTAL_LINE);
set_cursor_position ( 0,1 );
for (i = 1; i < SCREEN_DEPTH; i++)
        \{
        set_cursor_position (0,i);
        putc̄har (VERTICAL_LINE);
    set_cursor_position (SCREEN_WIDTH,1);
    for ( \(\mathrm{i}=1\); i < SCREEN_DEPTH; \(\mathrm{i}++\) )
        \{
        set_cursor_position (SCREEN_WIDTH,i);
        putchar (VERTICAL_LINE);
        \}
    return;
\}

/* char define_options (void)
*/
/* */
/* This routine allows the user to define the filter coefficents, number */
/* of iterations, input file and output file.
char define options (void)
    char quit_flag \(=\) FALSE;
    void clear_entry (unsigned int, unsigned int, unsigned int);
    void clear_screen (void);
    void set_cursor_position (unsigned int, unsigned int);
    void draw_border (void);
    int getche (void);
    while (quit_flag == FALSE)
    \{
    clear_screen ();
    draw_border ();
    set_cursor_position (25,2);
    printf ("Define Cellular Filter Parameters");
    set_cursor_position \((20,5)\);
    printf ("1-.. Number of Iterations = \%d", iterations);
    set_cursor_position (20,7);
    printf ("2 ... Input File = \%s", input_file);
    set_cursor_position \((20,9)\);
    printf ("3... Output File = \%s", output_file);
    set_cursor_position \((20,11)\);
    printf ("4 ... Filter Coefficient \(K 0=\% 8,7 f "\) filter_k0);
    set_cursor_position \((20,13)\);
    printf ("5 ... Filter Coefficient K1 = \%8.7f", filter_kI);
    set_cursor_position (20,15);
    printf ("6...Filter Coefficient \(K 2=\% 8.7 f^{\prime \prime}\), filter_k2);
    set_cursor_position \((20,17)\);
    printf ("0 ... Return to Main Screen");
    set_cursor_position (9,20);
    printf ("Option .. [ ]");
    set_cursor_position \((20,20)\);
    switch (getche())
        \{
```

    case '1':
    clear_entry (49,5,5);
    scanf ("%d", &iterations);
    break;
        case '2':
    clear_entry (40,7,16);
    scanf ("%s", input_file);
    break;
        case '3':
    clear_entry (40,9,16);
    scanf ("%s", output_file);
    break;
        case '4':
    clear_entry (50,11,12);
    scanf ("%f", &filter_k0);
    break;
        case '5':
    clear_entry (50,13,12);
    scanf" ("%f", &filter_kl);
    break;
        case '6':
        clear_entry (50,15,12);
        scanf("%f", &filter_k2);
        break;
        case '0':
        quit_flag = TRUE;
        break;
        }
    }
    return (TRUE);
    }
/* void clear_entry (unsigned int, unsigned int, unsigned int)
/*
/* This routine clears the screen with (repeat) spaces at the specified */
*/
/* position and set the cursor position to the specified location. */
void clear_entry (x, y, repeat)
unsigned int x, y, repeat;
{
unsigned int i;
void set_cursor_position (unsigned int, unsigned int);
set_cursor_position ( }x,y\mathrm{ );
for (i = 0; i < repeat; i++)
putchar (SPACE);
set_cursor_position (x,y);
return;

```
\(/ *\) void filter (void) */
* */
/* This routine filters the input image with the cellular filter defined */
/* by the define_options () routine.
```

void filter (void)

```
\{
    unsigned char pass_state;
    char *p ,*r;
    unsigned int is slice number;
    unsigned long count, l, line_count;
    float centre_element;
    void cant_open (char *);
    unsigned long load_slice (unsigned int, unsigned int, unsigned char *);
    void write_slice (unsigned long, unsigned int);
    void set_cursor_position (unsigned int, unsigned int);
    void clear_screen (void);
    void draw_border (void);
    void fatal_error (char *, char *, char *);
    int getch (void);
    if ( (out_fp \(=\) fopen (output_file, "w+b")) == (FILE *) ERROR)
        cant_open (output_file);
        return;
        \}
else
        \{
        fwrite (init_file, sizeof (char), sizeof (init_file), out_fp);
        fclose (out_fp);
        \}
if \(((\) temp_fp \(=\) fopen (temporary_file, "w+b")) \(=(\) FILE *) ERROR)
    cant_open (temporary_file);
    return;
    \}
else
    \{
    fwrite (init_file, sizeof (char), sizeof (init_file), out_fp);
    fclose (temp_fp);
    \}
clear_screen ();
draw_border ();
set_cursor position \((30,2)\);
printf ("Filtering \%s", input_file);
for (i \(=0 ; i<i t e r a t i o n s ; ~ i++)\)
    \{
    if ((in_fp \(=\) fopen (input_file, "rtb")) \(==\) (FILE *) ERROR)
\{
cant_open (input_file);
return;
\}
```

if ((out_fp = fopen (output_file, "r+b")) == (FILE *) ERROR)
{
cant_open (output_file);
return;
}
else /* reset to start

```
    if (fseek (out fp, (long) 0, BEGINNING) == ERROR)
        fatal_error ("FATAL ERROR- fseek on", output_file, "in filter ()
failed - Hard Luck!");
```

if ((temp_fp = fopen (temporary_file, "r+b")) == (FILE *) ERROR)
cant_open (temporary_file);
return;
}
else /* reset to start

```
*/
if (fseek (temp_fp, (long) 0, BEGINNING) == ERROR) fatal_error ("FATAL ERROR- fseek on", temporary_file ,"in filter
() failed - Hard Luck!");
slice_number = 0;
pass_state = STARTING;
do
\{
set_cursor_position (25, 20);
printf ("Iteration \%2d of \%2d : Slice \%4d", (i+1), iterations, slice_number);
```

count = load_slice (i+1, slice_number, \&pass_state);
/* point p to image [1][1] */
p = (char *) image + (IMAGE_WIDTH+3);
r = (char *) resultant + (IMAGE_WIDTH+3);
for (l = count, line_count = 1; l != 0; l--, line_count++)
{
centre_element = ((float) *p * filter_k0)
+ ((float) *(p-(1)) * filter_k1)
+ ((float) *(p+(1)) * filter_k1)
+((float) *(p-(((IMAGE_WIDT\overline{H}+2)+1))) *
+((float) *(p-((IMAGE_WIDTH+2))) * filter_k1)
+ ((float) *(p-(((IMAGE_WIDTH+2)-1))) *
+ ((float) *(p+(((IMAGE_WIDTH+2)-1))) *
+((float) *(p+((IMAGE_WIDTH+2))) * filter_k1)
+((float) *(p+(((IMAGE_WIDTH+2)+1))) *

```
filter_k2)
filter_k2)
filter k2)
filter_k2);
```

    if (centre_element > (float) 127.0)
    *r = (char) 127;
    else
        if (centre_element < (float) -127.0)
        *r = (char) -127;
    else
        *r = (char) centre_element;
        if (line_count < IMAGE_WIDTH)
    {
        p++;
        r++;
        }
    else
    p++; p++; p++;
    r++; r++; r++;
    line_count = 0;
    }
    }
write_slice (count, i+1);
slice_number ++;
}
while (pass_state != FINISHED);
fclose (in_fp);
fclose (out_fp);
fclose (temp_fp);
}

```
return;
```

/* unsigned long load_slice (unsigned int, unsigned int, unsigned char *) */
/*
/* This routine loads a slice of the image into the image buffer and
/* returns the number of bytes in the buffer to be processed and whether
/* it has finished this iteration or not.
/* If the number of iterations is odd, then the slice comes from the */
/* output file on even passes, and from the temporary file on odd passes */
/* except for the first pass. If the number of iterations is even, then */
/* the slice comes from the output file on odd passes except for the first */
/* pass, and from the temporary file on even passes.
*/
*/
unsigned long load_slice (pass, slice_number, state)
unsigned int pass;
unsigned int slice_number;
unsigned char *state;
{
char *p;
long l;
FILE *file_fp;
long i, block_size, number_of_blocks, file_position;
void set_cursor_position (unsigned int, unsigned int);
void clear_screen (void);

```
void fatal_error (char *, char *, char *);
```

for (1 = (IMAGE_WIDTH+2)*(SLICE_DEPTH+2), p = (char *) image;
l != 0; l--, p++)
*p = 0; /* clear out image buffer */

```
for (1 = (IMAGE_WIDTH+2)*(SLICE_DEPTH+2), p = (char *) resultant;
    l ! = 0; 1--, p++)
    *p \(=0\); /* clear out resultant buffer
    if ((iterations \& 0x01) == 0x00 )
    /* first get the right source */
    /* even number of iterations */
    if ((pass \& \(0 \times 01)==0 \times 00)\)
/* even pass */
        file_fp = temp_fp;
    else
        if (pass ! = 1)
/* odd pass */
/* not first pass */
            file_fp = out_fp;
        else
/* first pass */
file_fp = in_fp;
else
    if \(((\) pass \& \(0 \times 01)==0 \times 00)\)
        file_fp = out_fp;
    else
        if (pass ! = 1)
            file_fp = temp_fp;
        else
            file_fp = in_fp;
*/
```

    i++, p += ((IMAGE_WIDTH+2)))
    {
    block size += fread (p, sizeof (char), IMAGE_WIDTH, file_fp);
    }
    if (block_size != (IMAGE_WIDTH * number_of_blocks))
    {
    *state = FINISHED;
    block_size = (block_size - IMAGE_WIDTH)>0?
        (long) (block_siz\overline{e}}\mathrm{ - IMAGE_WIDTH): (long) 0;
    }
    else
    block size = SLICE DEPTH * IMAGE_WIDTH;
    return (block_size);
    ```
\}
/* void write_slice (unsigned long, unsigned int)
/*
/* This routine write the processed slice of the processed image
/* (resultant) to the appropriate file. */
/* If the number of iterations is odd, then the slice is writen to */
/* the output file on odd passes, and to the temporary file on even */
/* passes. If the number of iterations is even, then the slice is writen */
/* to the output file on even passes and to the temporary file on odd */
/* passes.
void write_slice (length, pass)
unsigned long length; /* unreferenced at the moment */
unsigned int pass;
    char *p;
    int i;
    FILE *file_fp;
    if ((iterations \& 0x01) \(==0 \times 00\) )
    /* first get the right source */
    /* even number of iterations */
    if \(((\) pass \& \(0 \times 01)==0 \times 00)\)
    /* even pass */
        file_fp \(=\) out_fp;
    else /* odd pass */
        file_fp = temp_fp;
else
    if \(((\) pass \& \(0 \times 01)==0 \times 00)\)
        file_fp = temp_fp;
    else /* odd pass */
        file_fp \(=\) out_fp;
\(\mathrm{p}=(\) char \(*)\) resultant \(+((\) IMAGE_WIDTH +3\())\);
for (i = 0; \(\mathrm{i}<\) SLICE_DEPTH;
        i++, \(p+=((\) IMAGGE_WIDTH +2\()))\)
    fwrite (p, sizeof (char), IMAGE_WIDTH, file_fp);
    return;
\}
/* void cant_open (char *)
```

/*
/* This routine displays the error message that the file could not be
/* opened.
void cant_open (file)
char *file;
{
int debug_key;
void clear_screen (void);
void set_cursor_position (unsigned int, unsigned int);
int getche (void);
clear_screen ();
set_cursor_position (15,12);
printf ("ERROR - unable to open %s", file);
set_cursor_position (15,13);
printf ("Hit any key to continue");
debug_key = getche ();
return;
}
/* void fatal_error (char *, char *, char *) */
/* This routine prints the message passed to it and exits to DOS. */
void fatal_error (start_string, file, end_string)
char *start_string, *file, *end_string;
{
void clear_screen (void);
void set_cursor_position (unsigned int, unsigned int);
clear screen ();
set_cursor_position (15,12);
printf ("%s %s %s\n", start_.string, file, end_string);
fclose (in_fp);
fclose (temp_fp);
fclose (out_\overline{fp});
exit (1);
return;
}
/***************************************************************************/
/* */
/* void show_image (void)
/* This routine displays the image specified by the user. */
*/
void show_image (void)
{
unsigned char pass_state;
char *p;
char quit_flag = FALSE;
char show_image_flag = FALSE;
unsigned int key, slice_number, x_coord, y_coord;
unsigned long count, l;

```
void clear_screen (void);
void draw_border (void);
void clear_entry (unsigned int, unsigned int, unsigned int);
void set_cursor_position (unsigned int, unsigned int);
unsigned long load_slice (unsigned int, unsigned int, unsigned char *);
void cant_open (char *);
int getch (void);
int getche (void);
void init_graphics (void), leave_graphics (void);
void set_pixel (unsigned int, unsigned int);
void clear_graphics_screen (void);
```

while (quit_flag == FALSE)
{
clear screen ();
draw_border ();

```
    set_cursor_position (35, 3);
    printf ("Display Image\n");
    set cursor position (20, 9);
    printf ("1 ... File to display \(=\%\) ", show_file);
    set_cursor_position (20, 11);
    printf ("2 ... Set threshold \(=\% h{ }^{\prime}\) ", show_threshold);
    set_cursor_position (20, 13);
    printf ("3 ... Display \%s", show_file);
    set_cursor_position \((20,17)\);
    printf ("0 ... Return to Main Screen");
    set_cursor_position \((9,20)\);
    printf ("Option .. [ ]");
    set_cursor_position \((20,20)\);
    switch (getche())
        \{
        case '1':
        clear_entry \((44,9,16)\);
        scanf ("\%s", show_file);
        break;
        case '2':
            clear_entry \((42,11,5)\);
            scanf ("\%hd", \&show_threshold);
            break;
        case '3':
            show_image_flag = TRUE;
            quit_flag = TRUE;
            break;
        case '0':
            quit_flag = TRUE;
            break;
        \}
    \}
if (show_image_flag = FALSE \&\& quit_flag \(==\) TRUE) return;
```

    if ((in_fp = fopen (show_file, "r+b")) == (FILE *) ERROR)
    {
    cant_open (show_file);
    return;
    }
    ```
    init_graphics ();
    clear_graphics_screen ();
    x_coord \(=\) SHOW_IMAGE_X_OFFSET;
    Y_coord \(=\) SHOW_IMAGE_Y_OFFSET;
    slice_number \(=0\);
    pass_state \(=\) STARTING;
    do
    \{
    count \(=\) load_slice (1, slice_number, \&pass_state);
    \(\mathrm{p}=(\) char \(*)\) image + (MAGE_WIDTH + 3);
    for \(\left(\frac{1}{\{ }=\right.\) count \(\left.; 1 \quad!=0 ; 1--\right)\)
        if (*p >= show_threshold)
            set_pixel (x_coord, y__coord);
        if ( \(x\) _coord - SHOW_IMAGE_X_OFFSET) < (IMAGE_WIDTH-1))
            \{
            p++;
            x_coord++;
            \}
        else
            \{
                p++; p++; p++;
                \(\mathrm{x} \_\)coord \(=\)SHOW_IMAGE_X_OFFSET;
                y_coord++;
                \}
    slice_number ++;
    \}
while (pass_state ! = FINISHED);
    key = getch ();
    leave_graphics ();
    return;
/* void display_printing_options (char *, char, char *)
/* This rouitne displays the printing options. This was seperated from the */ /* print_image routine to allow global optimisation.
void display_printing_options (image_file, print_file, threshold, title)
char *image_file, *print_file, *title;
char threshold;
void clear_screen (void);
void draw_border (void);
void clear_entry (unsigned int, unsigned int, unsigned int);
void set_cursor_position (unsigned int, unsigned int);
clear screen ();
draw_border ();
set_cursor_position (35, 3); /* display user print options */
printf ("Print Image\n");
set_cursor_position (20, 7);
printf ("1...File to print \(=\% s\) ", image_file);
set_cursor_position (20, 9);
printf ("2 ... Set threshold \(=\% h{ }^{-1}\) ", threshold);
set_cursor_position (20, 11);
printf ("3 ... Print File \(=\% s^{\prime \prime}\), print_file);
set_cursor_position \((20,13)\);
printf ("4-.. Title \(=\% s^{\prime \prime}\), title);
set_cursor_position \((20,15)\);
printf ("5 ... Print \%s", image_file);
set_cursor_position (20,17);
printf ("0... Return to Main Screen");
set_cursor_position \((9,20)\);
printf ("Option .. [ ]");
set_cursor_position \((20,20)\);
return;
)
```

/* void print_image (void)
/* This routine prints the image specified by the user to a 9 pin EPSON
/* printer attached to LPT1:.
void print_image (void)
{
char *p, *rp;
char c, quit_flag = FALSE, print_image_flag = FALSE;
char title [MAX_TITLE];
unsigned char mask;
unsigned int i, j, slice_number, number_of__blocks, title_length;
unsigned long l, file_position;
FILE *file_fp, *print_fp;
char line_feeds [8] = {LF, LF, LFF, LF, LF, LF, LF,LF};
char printer_init_seq [2]= {ESC, '@'};
char line_step_and_unidirectional [6] = {ESC, 'A', 0x08, ESC, 'U', 0x01};
char graphics_seq [5] = {ESC, '氺', '5', 0x00, 0x00};
char bold_and_underline [5]={ESC, 'E', ESC, ',',0\times01};

```
```

char cancel_bold_and_underline $[5]={ }_{n}\left\{\operatorname{ESC}, \quad, F\right.$ ' ESC, $\left.{ }^{\prime}-{ }^{-1}, 0 \times 00\right\}$;
char *spaces = "
void clear_screen (void);
void draw_border (void);
void clear_entry (unsigned int, unsigned int, unsigned int);
void set_cursor_position (unsigned int, unsigned int);
void cant_open (char *);
void display_printing_options (char *, char *, char, char *);
void fatal error (char *, char *, char *);
int getch (void);
int getche (void);
title [0] = ' $\backslash 0$ ';
title_length $=0$;
while (quit_flag == FALSE)
\{
display_printing_options (image_file, print_file, print_threshold,

```
title);
```

switch (getche()) /* get user selection */
{
case '1':
clear_entry (42,7,16);
scanf ("%s", image_file);
break;
case '2':
clear_entry (42,9,5);
scanf ("%hd", \&print_threshold);
break;
case '3':
clear_entry (39,11,16);
scanf("%s", print_file);
break;
case '4':
clear_entry (34,13,(MAX_TITLE-1));
title_length = 0;
p = title;
while ( ((c = (char) getche ()) != '\r') \&\&
title_length < MAX_TITLE)
{
if (c != BS )
*p = c;
p++;
title_length++;
}
else
{
p--;
title_length--;

```
```

        }
            }
        *p}='\0'
        break;
        case '5':
        print_image flag = TRUE;
        quit_flag = TRUE;
        break;
        case '0':
        quit_flag = TRUE;
        break;
        }
    }
    if (print_image_flag == FALSE && quit_flag == TRUE)
        return;
    clear_screen ();
    draw_border ();
    set_cursor_position (15, 10); /* display user options */
    printf ("Printing %s thresolded at %hd to %s", image_file,
    print_threshold, print_file);
/* open file to print */
if ((file_fp = fopen (image_file, "r+b")) == (FILE *) ERROR)
cant_open (image_file);
return;
}
/* open output file */
if ((print_fp = fopen (print_file, "wb")) == (FILE *) ERROR)
cant_open (print_file);
return;
}
/* do a few line feeds */
fwrite (line_feeds, sizeof (char), 8, print_fp);
/* print escape sequence for 8 pin step line advance, unidirectional */
fwrite (printer_init_seq, sizeof (char), 2, print_fp);
fwrite (line_step_and_unidirectional, sizeof (char), 6, print_fp);
graphics_seq [3] = (char) (PRINT_IMAGE_WIDTH \% 256);
graphics_seq [4] = (char) (PRINT_IMAGE_WIDTH / 256);
slice_number = 0;
do
/* clear out unformatted print buffer */
for (l = (IMAGE_WIDTH)*(PRINT_SLICE_DEPTH), p = (char *) raw_buffer;

```
```

        1 != 0; 1--, p++)
        *p=0;
    /* clear out print buffer */
    for (1 = PRINT_IMAGE_WIDTH, p = (char *) print_buffer;
        l != 0; l--, p++)
        *p=0;
    ```
    /* position file pointer */
    file_position \(=\) (long) ( (long) IMAGE_WIDTH * (long) PRINT_SLICE_DEPTH *
slice_number)
    if (fseek (file_fp, file_position, BEGINNING) = = ERROR)
        \{
        fclose (file_fp);
        fatal_error ("FATAL ERROR- fseek in load_slice () failed - Hard
Luck!", "", "n");
    \}
    \(r_{p}=\) (char *) raw_buffer ;
    number_of_blocks = PRINT_SLICE_DEPTH;
    /* read section of image from file to unformatted print buffer */
    for ( \(i=0 ; i<n u m b e r \_o f \_b l o c k s ; i++, r p+=\) IMAGE_WIDTH)
    fread (rp, sizeof (char), IMAGE_WIDTH, file_fp);
\(r p=(c h a r *)\) raw_buffer;
\(p=\) (char *) print_buffer;
mask \(=0 \times 80\);
for ( \(j=0 ; j<\) PRINT_IMAGE_WIDTH; \(j++, r p++, p++\) )
    for ( \(i=0 ; i<8 ; i++\) )
        if \(\left(*\left(r p+\left(i * I M A G E \_W I D T H\right)+A N \_I N C H\right)>=\right.\) print_threshold)
            *p \(1=(\) mask \(\gg\) i) ;
    \}
    /* print graphics escape sequence */
    fwrite (graphics_seq, sizeof (char), 5, print_fp);
    /* print print_buffer */
    fwrite (print_buffer, sizeof (char), IMAGE_WIDTH, print_fp);
    /* print 〈cr〉<lf〉*/
    fwrite (cr_lf, sizeof (char), 2, print_fp);
    \}
while (++slice_number < (IMAGE_DEPTH/PRINT_SLICE_DEPTH))
/* do a few line feeds */
fwrite (line_feeds, sizeof (char), 8, print_fp);
```

    /* add title */
    fwrite (spaces, sizeof (char), 20, print_fp);
    fwrite (bold_and_underline, sizeof (char), 5, print_fp);
    fwrite (title, sizeof (char), title_length, print_fp);
    fwrite (cr_lf, sizeof (char), 2, print_fp);
    fwrite (cancel_bold_and_underline, sizeof (char), 5, print_fp);
    /* and reset printer */
    fwrite (printer_init_seq, sizeof (char), 2, print_fp);
    /* clean up on exit */
    fclose (file_fp);
    fclose (print_fp);
    return;
    }
/***************************************************************************/
/* void clear_screen (void)
/*
/* This routine clears the entire screen.
*/
void clear_screen (void)
{
void up_scroll_region (unsigned char, unsigned char, unsigned char,
unsigned char, unsigned char, unsigned char);
up_scrol1_region (0,0,79,23,0, BLACK);
return;
}
/* void up_scroll_region (unsigned char, unsigned char, unsigned char
/* unsigned char, unsigned char, unsigned char) */
/*
/* This routine uses DOS to clear a region of the screen.
void up_scroll_region (x1, y1, x2, y2, up_scroll, attribute)
unsigned char x1, y1, x2, y2, up_scroll, attribute;
{
union REGS reg86;
reg86.h.al = up_scroll;
reg86.h.cl = x1;
reg86.h.ch = y1;
reg86.h.dl = x2;
reg86.h.dh = y2;
reg86.h.bh = attribute;
reg86.h.ah = DOS_SCROLL_WINDOW_UP;
int86 (VIDEO, \&reg86, \&reg86);
return;

```
```

/* void set_cursor_position (unsigned int, unsigned int) */
/*
/* This routine sets the cursor position to the specified co-ordinates. */
void set_cursor_position (x,y)
unsigned int x,y;
{
union REGS in_reg, out_reg;
/* get page number */
in_reg.h.ah = CURRENT_VIDEO_STATE;
int86 (VIDEO, \&in_reg, \&out_reg);
/* set cursor position */
in_reg.h.ah = SET_CURSOR_POS;
in_reg.h.bh = out_reg.h.bh;
in_reg.h.dh = (unsigned char) y;
in_reg.h.dl = (unsigned char) x;
int86 (0x10, \&in_reg, \&out_reg);
return;
1
/* void not_yet_implemented (void) */
/* - - */
/* this routine informs the user that the requested function has not yet */
/* been implimented. */
void not_yet_implemented (void)
{
int debug_key;
void set_cursor_position (unsigned int, unsigned int);
void clear_screen (void);
int getche (void);
clear_screen ();
set_cursor_position (15,12):
printf ("Not Yet Implemented ... Hit any key to continue");
debug_key = getche ();
return;
}
/***************************************************************************/
/* void init_graphics (void) */
/* */
/* This routine initialises the display into grpahics mode. */

```
void init_graphics (void)
\{
void clear_graphics_screen (void);
int i;
colorglb \(=\) (unsigned char) WHITE_ON_BLACK;
outp (0x03bf,0x01);
outp (0x03b5,0x02);
for ( i=0; i<12; i++ )
\{
outp (0x03b4,i);
outp (0x03b5,gtable[i]):
\}
outp ( \(0 \times 03 \mathrm{~b} 8,0 \times 02+0 \times 08\) );
clear_graphics_screen ();
return:
/* void leave_graphics (void)
```

/* This routine leaves graphics mode and resets the display for text mode. */

```
void leave_graphics (void)
\{
    int i;
    void clear_graphics_screen (void);
    outp (0x03bf, 0x00) ;
    outp (0x03b8,0x20);
    for ( \(i=0 ; i<12 ; i++\) )
        \{
        outp (0x03b4,i);
        outp (0x03b5,ttable[i]);
        ]
    colorglb \(=0 ;\)
    outp ( \(0 \times 03 b 8,0 \times 20+0 \times 08\) );
    clear_graphics_screen ();
    return;
\}
/* void set_pixel (unsigned int, unsigned int)
/* void set_pixel (unsigned int, unsigned int)
/* This routine sets the pixel at the specified location. */
*/
void set pixel (x, y)
unsigned int \(x, y\);
unsigned char *p,bit;
    FP_SEG(p) \(=0 \times b 000\);
    FP_OFF \((\mathrm{p})=((\mathrm{y} \& 3) \ll 13)+90 *(\mathrm{y} \gg 2)+(\mathrm{x} \gg 3)\);
    bit \(=(\) unsigned char) \((0 \times 80 \gg(x \& 7))\);
    if (colorglb \(==\) (unsigned char) WHITE_ON_BLACK)
        *p \(=\) *p | bit;
    else
        *p \(=\) *p \& bit;
reiturn;
```

/* void clear_graphics_screen (void)

```
/* This routine clears a graphics screen.
```

void clear_graphics_screen (void)

```
unsigned char *p;
    for (FP_SEG(p) = 0xb000, FP_OFF (p) = 0x0000; FP_OFF(p) \(<0 \times 8000\);
        * \((\mathrm{p}++)=\) colorglb \()\);
    return;
\}

\section*{Appendix C}

This appendix contains a listing of the occam program, netfilt.tsr, which implements a cellular filter on a network of transputers connected in a pipeline.
```

{{{ EXE netfilt
{{{F netfilt
--:::T netfill.tsr

| File: netfilt.ocm | Revision: 1.0 |
| :--- | :--- |
| Author: David Sinclair | Last Edit: 21 Jul 91 |

```
\{ \{ \{ define constants
```

-- define constants

```

VAL image.width IS 714 :
VAL image.hieght IS 340 :
VAL block.width IS 42 :
VAL block.hieght IS 20 :
VAL number.of.processors IS 4 :
\} \} \}
\{\{\{ define channel protocols
```

                                    define channel protocols
    ```
```

PROTOCOL data.mgr.msgs
CASE
quit
hold
request.something
filter.params; REAL32; REAL32; REAL32; INT
image.row; INT::[]BYTE
image.saved

```
:
PROTOCOL image.filter.msgs
    CASE
        request.image
        image.filtered; INT
        filtered.row; INT: :[]BYTE
PROTOCOL from.file.mgr.msgs
    CASE
        initialise; INT
    quit; INT
    hold; INT
    request. something; INT
    filter.param; INT; REAL32; REAL32; REAL32; INT
image.row; INT; INT:: []BYTE
image.saved; INT
```

:

```
PROTOCOL to.file.mgr.msgs
    CASE
        request.image; INT
        image.filtered; INT; INT
        filtered.row; INT; INT: [ []BYTE
\} \} \}
\{\{\{ define channels
define channels
CHAN OF data.mgr.msgs from.data.mgr :
CHAN OF image.filter.msgs to.data.mgr :
CHAN OF to.file.mgr.msgs up.data.out :
CHAN OF to.file.mgr.msgs down.data.in :
CHAN OF from.file.mgr.msgs up.data.in :
CHAN OF from.data.mgr.msgs down.data.out :
\} \} \}
\{\{\{ define variables
define variables
\} \} \}
```

procedures

```
[number.of.processors] BOOL quit.table :
```

{{{ can.quit

```
Procedure: can.quit
\(-\quad\) This procedure updates the list of processors which have been told

PROC can.quit (INT id, INT node. number, BOOL active)
SEQ
quit.table [node.number] := TRUE
```

    active := TRUE
    SEQ i = id FOR (number.of.processors - id)
    IF
        quit.table [i] = FALSE
                active := FALSE
            quit.table [i] = TRUE
                SKIP
    {{{ data.manager

```
\} \} \(\}\)
Task: data manager
    This task adds messages from the image filter task to the stream of --
    messages being sent to the file manager. Messages which are sent
    down stream from the file manager are examined by the data manager
    to determine if the message should be sent to its assocated image
    filter task. Quit messages for down stream tasks are recorded to
    enable the data manager to terminate when it, and all down stream
    processors have been told to terminate.

PROC data.manager (VAL INT id,
                            CHAN OF from.file.mgr.msgs up.data.in, down.data,out
                        CHAN OF to.file.mgr.msgs up.data.out, down.data.in)
[(block.width+2)]BYTE block :
BOOL active :
INT index, node.number, block.size :
REAL k0, k1, k2 :
SEQ
    up.data.in ? CASE initialise; node.number
    PAR i \(=0\) FOR number.of.processors
        quit.table [i] := FALSE
    IF
        node. number < id
                down.data.out ! initialise; node.number
        node.number \(>=\) id
        SKIP
    WHILE active
        ALT
        to.data.mgr ? CASE
            request.image
                        up.data.out ! request.image; id
                image.filtered; index
                        up.data.out ! image.filtered; id; index
                filtered. row; block.size: :block
up.data.out ! filtered.row; id; block.size::block down, data.in? CASE
```

    request.image; node_number
    up.data.out ! request.image; node.number
    image.filtered; node.number; index
        SEQ
            up.data.out ! image.filtered; node.number; index
            SEQ i = 0 FOR (block.hieght+2)
                down.data.in ? CASE filtered.row; node.number;
                    block.size::block
            up.data.out ! filtered.row; node.number; block.size::block
    up.data.in ? CASE
quit; node.number
PAR
can.quit (id, node.number, active)
IF
node.number <> id
down.data.out ! quit; node.number
node.number = id
from.data.mgr ! quit
hold; node.number
IF
node.number <> id
down.data.out ! hold; node.number
node.number = id
from,data.mgr ! hold
request.something; node.number
IF
node.number <> id
down.data,out ! request.something; node.number
node.number = id
from.data.mgr ! request.something
image.saved; node.number
IF
node.number <> id
down.data.out ! image.saved; node.number
node.number = id
from.data.mgr ! image.saved
filter.param; node.number; k0; kl; k2; index
IF
node.number <> id
SEQ
down.data.out ! filter.param; node.number; k0; k1; k2;
index
SEQ i = 0 FOR (block.hieght+2)

```
up.data.in ? image.row; node. number; block.size::block down.data.out ! image. row; node. number; block.size::black
node. number \(=\) id
SEQ
from.data.mgr ! filter.param; k0; k1; k2; index SEQ i \(=0\) FOR (block.hieght+2)
up.data.in ? image. row; node. number; block.size::block from.data.mgr ! image.row; block.size::block
\{\{\{ image.filter
Task: image filter
-- This task applies the cellular automaton defined by k0, k1 and k2
to the image supplied to it.
-- filter
VAL block.size IS (block.width * block.hieght) :
INT size, block. number :
REAL32 k0, k1, k2 :
BOOL active :
[(block.hieght + 2)][(block.width + 2)]BYTE raw.image:
[(block.hieght + 2)][(block.width + 2)]BYTE filtered.image :
SEQ
active : = TRUE
WHILE active
SEQ
from.data.mgr ? CASE
quit -- shut down process neatly
active := FALSE
hold -- don't do anything just yet
SKIP
request. something \(\quad-\) ask for an image
to. data.mgr ! request.image -- request image to process
-- get cellular filter
-- parameters
filter. params; k0; k1; k2; block. number
\(\begin{aligned} & \text { SEQ } \\ & \text { SEQ } \\ & j\end{aligned}=0\) FOR (block. width +2 )
from.data.mgr ? CASE image.row; size::raw.image [j]
PAR i \(=0\) FOR block.size -- process each cell
VAL x.offset IS ((i / block.width) + 1) :
VAL y.offset IS ((i REM block.width) + 1) :

INT sum, \(\mathrm{b} 1, \mathrm{~b} 2, \mathrm{~b} 3, \mathrm{~b} 4, \mathrm{~b} 5, \mathrm{~b} 6, \mathrm{~b} 7, \mathrm{~b} 8, \mathrm{~b} 9\) :
SEQ
PAR
```

b5 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.offset][y.offset]))) -
128.0 (REAL32)) * k0))
IF
x.offset > 0
PAR
b4 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.offset - 1][y.offset]))) -
128.0 (REAL32)) * k1))
IF
y.offset > 0
b1 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.offset -1 ][y.offset -
1]))) - 128.0 (REAL32)) * k2))
y.offset <= 0
b1 := 0
IF
(y.offset + 1) < block.hieght
b7 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.offset - 1][y.offset +
1]))) - 128.0 (REAL32)) * k2))
(y.offset + 1) >= block.hieght
b7 := 0
x.offset <= 0
b4, b1, b7 := 0, 0, 0
IF
y.offset > 0
PAR
b2 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.offset][y.offset - 1]))) -
128.0 (REAL32)) * k1))
IF
(x.offset + 1) < block.width
b3 := (INT TRUNC (((REAL32 TRUNC
(INT (raw.image [x.affset + 1][y.offset -
1]))) - 128.0 (REAL32)) * k2))
(x.offset + 1) >= block.width
b3 := 0
y.offset <= 0
b2, b3 := 0, 0
IF
(x.offset + 1) < block.width
PAR
b6 := (INT TRUNC (((REAL32 TRUNC

```
```

                                    (INT (raw.image [x.offset + 1][y.offset])))
                                    128.0 (REAL32)) * k1))
            IF
            (y.offset + 1) < block.hieght
                        b9 := (INT TRUNC (((REAL32 TRUNC
                                    (INT (raw.image [x.offset + 1][y.offset +
                                    1]))) - 128.0 (REAL32)) * k2))
                    (y.offset + 1) >= block.hieght
                        b9 := 0
                    (x.offset + 1) >= block.width
                        b6, b9 := 0, 0
            IF
            (y.offset + 1) < block.hieght
                b8 := (INT TRUNC (((REAL32 TRUNC
                                    (INT (raw.image [x.offset][y.offset + 1]))) -
                                    128.0 (REAL32)) * k1))
            (y.offset + 1) >= block.hieght
            b8 := 0
            sum := (b1 + (b2 + (b3 + (b4 + (b5 + (b6 + (b7 + (b8 +
                b9)))))|) )
                    IF
        sum > 127
        sum := 127
        sum < (-127)
        sum := -127
        (sum >= (-127)) AND (sum <= 127)
            SKIP
            filtered.image [x.offset][y.offset] := BYTE (sum + 128)
    to.data.mgr ! image.filtered; block.number
SEQ j = 0 FOR (block.hieght + 2)
to.data.mgr ! filtered.row;
(block.width + 2)::filtered.image [j]
from.data.mgr ? CASE image.saved
to.data.mgr ! request.image -- request image to process

```

\section*{-- Logical Channels}

CHAN OF from.file.mgr.msgs app.in
CHAN OF to.file.mgr.msgs app.out
[number. of.transputers] CHAN OF from.file.mgr.msgs down.links :
[number. of.transputers] CHAN OF to.file.mgr.msgs up.links :

\section*{-- Configuration}
```

PLACED PAR
PROCESSOR 0 T4
PLACE app.in AT link1in :
PLACE app.out AT link1out :
PLACE down.links[0] AT link2out :
PLACE up.links[0] AT link2in :

```
        data.manger (0, app.in, down.links[0], app.out, up.links[0])
    image.filter ()
PLACED PAR i = 1 FOR (number.of.transputers - 1)
    PROCESSOR i T4
        PLACE down.links[i-1] AT link1in :
        PLACE up.links[i-1] AT linklout :
        PLACE down.links[i] AT link2out :
        PLACE up.links[i] AT link2in :
            data.manager (i, down.links[i-1], down.links[i],
                up.links[i-1], up.links[i])
            image.filter ()
\}\} \} F
...F code HT8
--::: : F netfilt.dcd
...E descriptor
--:: : ; F netfilt.dds
...F link
. .: : : F netfilt.dlk
. . . F debug
...-:::F netfilt.ddb

\section*{Appendix D}

This appendix contains a listing of the occam program, host.tsr, which interfaces between the cellular filter program, netfilt.tsr, running on a network of transputers, the user and the DOS file system.

File: host.ocm
Revision: 1.0
Author: David Sinclair
Last Edit: 19Jul91

This program is the host for the network version of the cellular filter.
\{\{\{ define constants
```

define constants

VAL image.width IS 71.4 :
VAL image.hieght IS 340
VAL block.width IS 42 :
VAL block.hieght IS 20 :
VAL number.of.blocks IS ((image.width / block.width) * (image.hieght / block.hieght)) :
\}\}\}
\{ \{\{ define channel protocols

```
define channel protocols
```

PROTOCOL from.file.ingr.msgs
CASE
initialise; INT
quit; INT
hold; INT
request. something; INT
filter.param; INT; REAL32; REAL32; REAL32; INT
image. row; INT; INT: :[]BYTE
image.saved; INT

PROTOCOL to.file.mgr.msgs
CASE
request.image; INT
image.filtered; INT; INT
filtered. row; INT; INT: : []BYTE
\} \} $\}$
\{\{\{ define channels
[number.of.blocks] BYTE image.map :
INT iterations:
INT in.file, out.file :
INT result, char, in.file.len, out.file.len :
[number, of.transputers] BOOL active.table :
[number.of.transputers] BOOL busy.processor :
[16] BYTE in.file.name :
[16] BYTE out.file.name :
[16] BYTE temp.file.name :
REAL k0, k1, k2 :
SEQ
temp.file.name $=$ "temp.img"
\{\{\{ get user data
goto. xy (screen, 0, 0)
clear.eos (screen)
goto.xy (screen, 0, 0)
write.full.string (screen, "Cellular Filter Parameters")
newline (screen)
newline (screen)
write.full.string (screen, "Number of iterations = ")
char : $=0$
read.echo.int (keyboard, screen, iterations, char)
newline (screen)
write.full.string (screen, "Filter Constant K0 = ")
char := 0
read.echo.real 32 (keyboard, screen, k0, char)
write.full.string (screen, "Filter Constant K1 = ")
char := 0
read.echo.real32 (keyboard, screen, k1, char)
write.full.string (screen, "Filter Constant K2 = ")
char := 0
read.echo.real32 (keyboard, screen, k2, char)
newline (screen)
write.full.string (screen, "File to be filtered = ")
char : $=0$
read.echo.text.line (keyboard, screen, in.file.len, in.file.name, char)
newline (screen)
write.full.string (screen, "Filtered filename = ")
char $:=0$
read.echo,text.line (keyboard, screen, out.file.len, out.file.name, char) newline (screen)
\} \} \}
\{\{\{ open image, temporary and filtered image files open.file (from.filer, to.filer, in.file.name, BinaryByteStream.Access, Update. Mode, Old.File, 0, image.file, out.result)
open.file (from.filer, to.filer, out.file.name, BinaryBytestrearn. Access, Update. Mode, New.File, 0, filtered.file, out.result)
seek (from.filer, to.filer, image.file, 0, out.result)
in.result : = Operationok
WHILE (in.result $=$ Operationok) $\quad--$ need filtered file to be same
SEQ -- size as image file
read.block (from.filer, to.filer, image.file, SIZE buffer, bytes.read, buffer, in,result)
write.block (from.filer, to.filer, filtered.file, buffer, bytes.read, out.result)
open.file (from.filer, to.filer, temp.file.name, BinaryByteStream. Access, Update. Mode, New.File, 0, temp.file, out.result)
seek (from.filer, to.filer, image.file, 0, out.result)
in.result : = Operationok
WHILE (in.result = Operationok) -- need temporary file to be same
SEQ -- size as image file as well read.block (from.filer, to.filer, image.file, SIZE buffer, bytes. read, buffer, in.result)
write.block (from.filer, to.filer, temp.file, buffer, bytes.read, out.result)
\} \})
to. workers ! initialise; number.of.transputers
SEQ i $=0$ FOR iterations
PAR $j=0$ FOR number, of.blocks -- initialise the map of the image.map [j] $:=$ FALSE filtered parts of the image

PAR $j=0$ FOR number. of.transputers
-- initialise table of active busy.processor [j] := FALSE
-- processar table
\{\{\{ assign input and output file for current pass IF

```
\((\) (iterations \(/ \backslash 1)=1\) )
                IF
            \(((\mathrm{i} / \backslash 1)=1) \quad-\) odd pass number
```

-- odd number of iterations
-- odd pass number
PAR
out.file : = filtered.file
IF
(i<>1)
in.file : = temp.file

PAR
out.file : = filtered.file
(i<>1) in.file : = temp.file

```
( \(\mathrm{i}=1\) )
        in.file := image.file
```

```
((i/\ 1) = 0)
                                    -- even pass number
        PAR
        out.file := temp.file
        in.file := image.file
    ((iterations /\1) = 0) -- even number of iterations
        IF
            ((i/\ 1) = 1)
                PAR
                out.file := temp.file
                IF
                    (i<> 1)
                        in.file := filtered.file
                    (i=1)
                        in.file := image.file
                            ((i)\ 1) = 0) -- even pass number
                out.file := filtered.file
                in.file := temp.file
```

\}\}\}
this is where the work is done

```
SEQ j = 0 FOR nomber.of,processors
    to.workers ! request.something; j
WHILE active
    from.workers ? CASE
        request.image; node.number
            SEQ
            index := 0
            WHILE (imagie.map [index] = FALSE) AND (index < number.of.blocks)
                index := index + 1
            IF
                index < number.of.blocks
                SEQ
                    to.workers ! filter.params; node.number; k0; k1; k2; index
                offset.x := (block.width * (index REM (INT(image.width /
                        block.width)))) - 1
                offset.y := ((index / (INT(image.width / block.width))) *
                                    block.hieght) - 2
                SEQ j = 0 FOR (block.hieght + 2)
                        SEQ
                        offset.y := offset.y + 1
                        start := 0
                        length := block.width + 2
                        IF
                        offset.x< <
                        PAR
                                length := block.width + 1
                                start := 1
```

```
    adjust := 1
offset.x > (image.width - (block.width + 2))
        PAR
                    block [block.width + 1]:= BYTE (0)
                    length := block.width + 1
                    (offset.x > = 0) AND (offset.x <= (image.width -
                                    (block.width + 2)))
```

                                    SKIP
    IF
(offset.y < O) OR (offset.y > $\quad$ image.hieght)
PAR $1=0$ FOR (block.width + 2)
block [l] := BYTE (0)
(offset.y $>=0$ ) AND (offset.y < image.hieght)
SEQ
offset $:=(($ offset.y * image.width $)+$ offset.x $)+$
adjust
seek (from.filer, to.filer, in.file, offset,
result)
read.block (from.filer, to.filer, in.file,
length, byte.count, block, result)
IF
start = 1
SEQ
SEQ $1=0$ FOR (block.width +1 )
block [(block.width + 2) - 1] :=
block [(block.width + 1) - 1]
block [0] := BYTE (0)
to. workers ! image. row; node. number; SIZE block::block
busy.processor [node. number] := TRUE
index $>=$ number.of.blocks
SEQ
to. workers ! hold; node. number
active := FLASE
SEQ $j=0$ FOR number.of.processors
IF
busy.processor [j] = TRUE
active : = TRUE
busy.processor [j] = FALSE
SKIP
image.filtered; node.number; index
SEQ
-- ignore the first row
from. workers ? CASE filtered.row; node. number; byte.count: block
offset. $x:=$ block.width * (index REM (INT(image.width /
block.width)) )
offset. $y:=((i n d e x /(I N T(i m a g e . w i d t h / b l o c k . w i d t h))) *$

```
SEQ j = 0 FOR block.hieght
    SEQ
                offset.y := offset.y + 1
                from,workers ? CASE filtered.row; node.number;
                    byte.count::block
                offset := (offset.y * image,width) + offset.x
                length := block.width
                seek (from.filer, to.filer, out.file, offset, result)
                short.block IS [block FROM 1 FOR block.width] :
                write.block (from.filer, to.filer, out.file, short.block,
                    length, result)
                from,workers ? CASE filtered, row; node.number; byte.count::block
                to.workers ! image.saved; node.number
                busy.processor [node.number] := FALSE
```

\{\{\{ shut down the processors tasks
SEQ $i=0$ FOR number.of.processors
to. workers ! quit; i
\} \} \}
[\{\{ close image, temporary and filtered image files
close.file (from.filer, to.filer, in.file, Close, Option, result)
close.file (from.filer, to.filer, out.file, close.Option, result)
close.file (from.filer, to.filer, temp.file, CloseDel.Option, result)
\} \} \}
\} \} \} F
...F code HT8
--::: F host.dcd
...F descriptor
--::: F host.dds
...F link
--::: F host.dlk
...F debug
--::: F host.ddb

