Mathematical Skills and Attitudes of First Year Engineering Students

A thesis presented for the degree of Masters

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Table of Contents

Abstract

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This thesis reports on a study investigating the mathematical skills and attitudes of incoming engineering students in Dublin City University. The following questions are posed in this thesis: What are the students' strengths and weaknesses in basic areas of mathematics? What beliefs do the students express in relation to mathematics?

The research attempted to answer these questions using two approaches: Firstly, an attitudinal survey was undertaken to identify students' attitudes towards mathematics, under a selection of different headings. Secondly, mathematics tests were run, focusing on the core mathematical skills necessary for engineering students. Both the test and survey were run at the beginning and/or end of the academic years 2012/2013 and 2013/2014 to give comparative data.

The results of the survey reflected high levels of confidence in relation to mathematics and high goals in relation to mathematical achievement. The test results showed a number of strengths and weaknesses in understanding or dealing with basic mathematical problems and concepts. The test was graded using certainty-based marking, which allowed the identification of misconceptions as well as knowledge gaps in students' mathematical skills. Based on these findings, recommendations are made in relation to the mathematical education of engineers in DCU.

1 Introduction

1.1 Background to the study

Mathematics is increasingly the focus of educational studies, both nationally and internationally. This is due to the growing need of mathematical skills in today's technological and industrial world (Conway and Sloane 2005, European Commission 2011), along with the challenges and difficulties associated with its teaching and learning (Petocz et al. 2007, Hourigan and O'Donoghue 2007, Conway and Sloane 2005). An important element in the economic well-being of industrialised countries is the appropriate education of their engineers and scientists (Mustoe 2003). (Pyle 2001) considered the mastering of a "distinctive" knowledge of mathematics to be an essential skill for engineers. Although it is a "dense language", mathematics is the language of communication for scientists and engineers (Blockley and Woodman 2002). Moreover, the logical rigour defined by mathematics encapsulates the quality of knowledge required for engineers (Blockley and Woodman 2002). As a result, for engineering students, the mathematics education component of their studies has a major role to play (Mustoe 2003).

For that reason, undergraduate engineering courses include a substantial amount of mathematics, in order to provide students with the required level of mathematical knowledge to "create, manipulate, and interpret models relevant to the branches of engineering they study" (Pyle 2001). A wide range of mathematical areas are needed in engineering disciplines, although these may vary depending on the branch of engineering in question (Pyle 2001). Such areas include arithmetic, algebra, analysis, probability, calculus and discrete mathematics (Pyle 2001, Armstrong and Croft 1999). Blockley and Woodman (2002) emphasize the importance of mathematics for engineering students when they state that not only do students need to understand the subject, but they also need to feel at ease with it and be able to apply it to advanced applications found in engineering-related problems.

Despite the fact that mathematics is vital for engineering disciplines, studies show increasing concerns about students' mathematical level upon entry to higher education institutes (HEIs) (Carr et al. 2013, Mustoe 2003). McKay (2003) believes that students in engineering courses "traditionally struggle" with the mathematical elements of their modules. As a sizable number of students entering HEIs find themselves lacking both the core mathematical skills and the confidence needed to develop "university-level" concepts in engineering, many engineering departments internationally are concerned about this problem (Pyle 2001, The Engineering Council 2000, Williamson et al. 2003, Parsons 2004). For these reasons, efforts are underway to help engineering students grasp the necessary mathematics skills and master them. For example, five English universities undertook a project termed Helping Engineering Learn Mathematics (HELM 2005) which aimed to "enhance the mathematical education of engineering undergraduates by the provision of a range of flexible learning resources...(and)...drive student learning via a computer based assessment regime". Parsons (2004) also reported on a number of different forms of mathematical support provided to engineering students as a result of students' poor progression in engineering courses related to their failure in mathematics. Carr et al. (2013) described a project set up in Dublin Institute of Technology (DIT) attempting to improve engineering students' core mathematical skills, based on repeated online testing.

1.2 Motivation for the study

As mentioned above, research has shown that a large number of entrants into engineering programmes in higher education lack the basic mathematical knowledge and skills required to do well in engineering (MathsTEAM 2004, Golden 2002, Parsons 2004). Along with that, a number of those students have negative attitudes towards mathematics (Ernest 2002b). In recent years, the education system in Ireland has been through major developments and reform to curriculum and teaching approaches (NCCA 2012). As part of those changes, a reform of mathematics education has been implemented in post primary education, known as "Project Maths" (NCCA 2012). As it is believed that students' attitudes towards mathematics affect their performance in the subject (Ernest 2002a), aligned with the changes applied to the post-primary mathematics curriculum, a number of research questions arise:

- What are the attitudes and beliefs expressed by first-year engineering students about mathematics?
- What are these students' strengths and weaknesses in basic areas of mathematics such as number, algebra and calculus?

In order to address these research questions, this research aimed to:

- Investigate first-year engineering students' beliefs about and attitude towards mathematics;

- Compare attitudes professed at the start of their studies with those given after having studied mathematics as part of their engineering degree in first year;
- Test students' core mathematical skills;
- Attempt to highlight students' strengths and weaknesses in these core areas.

The basic mathematical skills and attitudes towards mathematics of incoming engineering students in Dublin City University have never been studied in-depth prior to this point. However, anecdotal evidence from examinations and attendance at the Mathematics Learning Centre (the DCU mathematics support centre) have suggested that many students' core skills may be weak upon entry. Given the changing post-primary mathematical background of these students, it is an ideal time to undertake such an investigation.

1.3 Brief overview of methodology

The study is comprised of two strands: a mathematics test and a survey. A paired-questions approach was used within the test, where each question was matched with another question testing a similar skill, and the test was then graded using a certainty-based marking (CBM) scheme. Both of these steps were taken in order to gain a deeper insight into the specific difficulties students face when confronted with basic mathematical questions. The attitudinal survey was designed to gain an overview of students' beliefs about mathematics and their confidence in their mathematical capabilities, as well as their prior experience of mathematics and theories of intelligence. Students were presented with a series of five-point Likert-scale questions to answer, in which there were a mixture of positive and negative statements, in order to ensure more accurate responses would be given. The study did not attempt to find a direct correlation between the test and the survey, but did link the results from certain survey questions to the confidence levels expressed by students in the test and the percentage of correct responses they gave.

1.4 Thesis structure

This thesis is structured into five chapters. Chapter one introduces the study, laying out the background, exploring the rationale for this work and giving an overview of the methodology. Chapter two reviews the relevant literature to the study, focusing on mathematics education in general and mathematics for engineering in particular, as well as recent work in diagnostic testing and the certainty-based marking scheme. Chapter three gives the methodology behind the study, focusing on the research methods used and justifications for these. Chapter four provides a comprehensive analysis of the results of the mathematical tests undertaken and discusses the subsequent findings. Chapter five then details the findings of the attitudinal surveys and discusses the students’ responses, as well as providing an analysis of the links between the results of the test and those of the survey. Finally, chapter six concludes this thesis, highlighting the key findings, and also provides recommendations for future work.

2 Literature review

In this chapter, the literature relevant to this study is reviewed. Firstly, an overview of the Irish education system is given, with some brief background about the mathematics curriculum taught at post-primary level. Diagnostic testing in mathematics in higher education is discussed at length, along with a particular technique for designing test questions known as a "paired-questions" approach. Marking schemes for such tests are also reviewed, focusing briefly on negative marking (NM) before looking in-depth at certainty-based marking (CBM). Finally, a short discussion of the affective domain in mathematics is given, focusing in particular on the elements relevant to this study.

2.1 The Irish Education System

Post primary education in the Irish system is named secondary level (Department of Education and Sciences. 2004). Students in secondary level in Ireland spend five or six years in school. Upon finishing secondary level they take a state examination called the Leaving Certificate (LC). The results of the LC examination are vital to students as their entry to higher education is determined by those results.

Moreover, students at LC are required to take a minimum of five subjects, although seven subjects is the norm. All subject examinations are offered at two levels: Ordinary Level (OL) and Higher Level (HL) apart from Mathematics and Irish which are also offered at Foundation Level (FL). The Higher Level curriculum, as its name suggests, is the most advanced level and includes more topics than the Ordinary Level curriculum, such as integration, for example (Faulkner, Hannigan and Gill 2010). 96% of students who take the LC study Mathematics

As part of the development of education in Ireland, the National Council for Curriculum and Assessment (NCCA) has led a reform of the mathematics curriculum in post-primary education under the name "Project Maths" (Project Maths Development Team). Project Maths is an approach to mathematics education, changing the syllabus, assessment, and teaching of mathematics, designed to equip students with substantial mathematical skills to achieve long term rewards demonstrated in improved understanding, thinking skills and examination results (Jeffes et al. 2012). It began as a result of educational concerns about mathematics education in Ireland (NCCA 2012). Conway and Sloane (2005), for example, addressed many concerns regarding mathematics education nationally and internationally. In particular, they emphasized the lack of students' capacity to apply mathematics in practical "real world" contexts, a skill that Blockley and Woodman (2002) observed as being vital to engineering students. A report by the NCCA (2011) further emphasized this point, declaring that a significant number of students in post-primary level lacked the skills needed in their academic and professional lives. In addition, Scanlan (2010) stated a number of other concerns including: students' performance levels in the Programme for International Student Assessment (PISA) tests; the small number of students taking mathematics at higher level in LC examinations; the difficulties with mathematics illustrated by higher education students; the lack of problem-solving skills of Irish students, as identified by employers; and the general need for qualified mathematical and scientific personnel for the knowledge economy.

2.2 Mathematics education for engineers

As stated in the introduction, mathematics education is increasingly becoming a concern for educators worldwide as a result of the reliance on economical, industrial, and technological careers in today's world (European Commission 2011, Conway and Sloane 2005). Thus, international studies have emphasized the importance of developing the mathematical literacy of students, with the aim of better preparing them for a number of disciplines, such as science and engineering, which rely heavily on mathematics and are in widespread demand (Petocz et al. 2007). One of the fundamental aims of these studies is to determine factors influencing students' achievements in mathematics (Goodykoontz 2008, Fadali, Velasquez-Bryant and Robinson 2004). It has been shown to be common across all disciplines in which mathematics is studied that mathematics is frequently disliked (Freeman et al. 2008) and studies have shown points of interest and correlations between students' beliefs and attitudes towards mathematics and their performance in the subject.

It is evident that mathematics is vital for engineering disciplines, as detailed in the introduction. Despite this, studies show increasing concerns about students' mathematical level upon entry to third level institutions (Carr et al. 2013). Efforts are being undertaken to help engineering students grasp the necessary mathematics skills and master them, for example: (Pidcock, Palipana and Green 2004, Parsons 2004, Carr et al. 2013).

2.3 Diagnostic testing

In recent years, as a result of widespread concerns about the prior mathematical knowledge of entrants into universities and other third level institutions, mathematics diagnostic tests are used in most third level institutions nationally and internationally (Sheridan 2013). Diagnostic testing is conducted mostly by science and engineering schools where mathematics is believed to be a core subject to such disciplines (Pinto et al. 2007, Lee et al. 2008). In order to gain more accurate information about students' prior mathematical knowledge and skills, studies are increasingly focusing on improving diagnostic tests and inventing new techniques for developing these tests and assessing them (Gillard, Levi and Wilson 2010, Carr et al. 2013, Lee and Robinson 2005, MathsTEAM 2004). The LTSN MathsTEAM, for example, supported by the Learning and Teaching Support Network in the UK carried out a comprehensive collection of case studies of diagnostic testing throughout the UK, as a follow-up study to the recommendation made by the British council to all mathematics-based disciplines to carry out diagnostic tests upon entry (MathsTEAM 2004). The study contained results to tests that had been undertaken, mentioned some barriers found in practice, and also provided some recommendations and supporting materials for future work. The recommendations included:

- Diagnostic testing is not of use if follow up support is not considered;
- Computer based tests are economically better and provide equivalent data to paper-based test;
- Paper-based tests could be used considering extra time;
- Attention should be given to which discipline the test is for, as that could change the focus of the test as well as the necessary difficulty levels to be considered;

In terms of aims and objectives when running mathematics diagnostic tests for mathematics-related disciplines, Lee and Robinson (2005); Sheridan (2013); Gillard, Levi and Wilson (2010); and The Engineering Council (2000) highlighted some of the important reasons for using such diagnostic tests. They illustrated the importance of using mathematics diagnostic tests as a tool for:

- gathering information about the cohort of university entrants;
- designing programmes and modules that take account of general levels of mathematical attainments;
- informing faculty and students about mathematical skills level;

- identifying students at risk of failing mathematics-based courses because of their mathematical deficiencies;
- focusing on preparation of support provision;
- providing realistic expectations of students' levels to staff members;
- targeting remedial help and support to those students most in need;
- identifying students who were likely to struggle with mathematical concepts in maths-related disciplines.

Diagnostic tests are in widespread use, and are given to students entering mathematics-based disciplines across most third-level institutions nationally and internationally, but addressing and assessing using differing methods. Whereas some institutions run computer-based tests, others rely on paper-based diagnostic tests (Carr, Bowe and Ní Fhloinn 2013, Sheridan 2013, Gillard, Levi and Wilson 2010).

2.3.1 Paired questions approach

Although diagnostic testing is widely used in a large number of universities, a variety of methods are applied in different institutions and universities as seen in the LTSN report (MathTEAM 2004). An innovative approach to diagnostic testing was introduced in Loughborough University in the UK in 2002, known as the "paired questions" approach. It was initially designed and applied to assessing a cohort of new engineering students in that year (Lee and Robinson 2005). The approach was designed "to allow easy identification and subsequent follow up of topics where the students needed extra help" (Lee and Robinson 2005). However, it was found that students participating in that test frequently got one question in the pair correct but not the other. By analyzing their data, and looking deeply in the structure of the test and the questions used, Lee and Robinson (2005) found that even though the two questions in a pair were meant to test the same skill, most of the pairs in the test (15 out of 20) tested different skills and involved a different number of steps. They suggested that using the paired question approach is very useful in determining students' knowledge in a subject and to detect whether they have partial knowledge in an area or made a slip if paired questions are chosen carefully (that is when both questions in a pair test the same skill and involve exactly the same number of steps). Thus, when a student responds correctly to both questions in such a pair, it indicates that they have the required knowledge in the topic, whereas responding incorrectly to both questions indicates a partial or lack of knowledge in that topic. Furthermore, responding correctly to one question in a pair and incorrectly to the other could indicate a genuine mistake. The paired questions approach was found to be useful and was used in a number of diagnostic tests applied by different institutions (Carr, Murphy and Ni Fhloinn 2011, Sheridan 2013).

2.4 Approaches to marking

Diagnostic tests are graded using a range of different marking schemes, as there are a wide variety of diagnostic tests in use. Whereas some apply negative marking, aiming to reduce the chances of guessing (Sheridan 2013), others use written tests and hand-mark them to highlight students' common mistakes and determine where the students lack knowledge or make a mistake (MathsTEAM 2004). However, some others do not assign any marks to the test and only provide qualitative feedback to students highlighting their weakness and strong points (The Engineering Council 2000). More advanced and complicated assessment schemes are also in practice, all with the aim of gaining more accurate information about students' background and mathematical knowledge levels, so that the lecturers have an accurate idea about students' mathematical abilities and in order to provide the appropriate support to those that need help in a certain topic. Two marking schemes will now be reviewed in greater detail: negative marking and certainty-based marking.

2.4.1 Negative Marking

Negative marking is a scheme undertaken by some educators in multiple-choice tests and examinations, aiming to discourage students' guessing when they are unsure of the correct answer (Holt 2006). Negative marking schemes are based on "rewarding" correct choices with a positive mark, "penalizing" incorrect answers with a negative mark, and unanswered questions are given a mark 0. The idea behind the marking scheme is that the expected value of a student guessing the answers at random should be zero (Lawson 2012).

2.4.2 Certainty-Based Marking

Certainty-Based Marking (CBM) or as it was formerly called Confidence-Based Marking (Issroff and Gardner-Medwin 1998) is a scheme for assessing students' knowledge in a multiple-choice question depending on how certain they are about their answer. When the CBM scheme was set up for computer-based assessment in University College London UCL, it was widely supported by Physiology departments in the university (Gardner-Medwin 1995). Since then, the scheme has been used by UCL medical students for voluntary study and self-assessment in physiology, anatomy, biomedical and medical science (Gardner-Medwin and Curtin 2007, Issroff and Gardner-Medwin 1998, Gardner-

Medwin 2014). It is also now available to use in the Virtual Learning Environment (VLE) Moodle, based on the same principle (Gardner-Medwin 2014).

In a CBM test scheme, after choosing an answer, students indicate their level of certainty about that answer as being low, medium or high. The marking scheme, illustrated in (Gardner-Medwin 1995), is set to encourage students to clarify their level of certainty (Gardner-Medwin and Curtin 2007) by choosing from low level when uncertain to high level when very certain, which indicates their level of knowledge.

Table 1 Certainty-Based Marking

Certainty Level	**Low**	**Medium**	**High**	**No Reply**
Mark if correct	1	2	3	0
Mark if incorrect	0	-2	-6	0

Gardner-Medwin and Curtin (2007) also explained the best certainty levels to be chosen by students when answering a question (Gardner-Medwin 2014). Indicating C=1 for low, C=2 for medium, and C=3 for high certainty levels, the best C level is the one that is highest at the point corresponding to the student's estimate of how likely they are to be correct. Each line of C levels shows how the expected mark depends on the student's estimation of the probability that they will be marked correct. According to Gardner-Medwin and Curtin (2007), "*The critical transition points, to merit using C=2 or C=3, are 67% and 80%*". Therefore, if a student is less than 67% certain about an answer being correct, it is best to leave it blank or choose C=1; and if they are more than 80% they are correct, it is best to choose C=3. This scheme is meant to estimate students' knowledge on a subject leaving minor chances for guessing. It was firstly designed for self-assessment but was eventually used for examinations as well. By asking students to state their confidence in their own answers, students are encouraged to distinguish reliable answers from uncertain ones.

According to Gardner-Medwin (2006), CBM has many advantages for students, such as encouraging better reflections upon knowledge, making them more realistic about their uncertainty and highlighting their misconceptions and in addition, they found the students to "like it". Furthermore, CBM could be more valid and reliable as a tool for knowledge measurement. Moreover, it produces further useful data on student assessments, given that selecting a high level of certainty with a correct answer indicates a good level of knowledge, whereas selecting the same level with an incorrect answer is an indicator of a misconception on the part of the student. On the other hand, when a large number of students select medium/low levels of certainty to a certain question, this indicates that students are unsure of their knowledge in the corresponding category, even if they have answered correctly.

In comparison to a "traditional" marking scheme, the mean CBM score is always lower than the "accuracy" (percentage of correct answers) as shown in the graphs below, taken from Gardner-Medwin (2014). Therefore, as an addition to the marking scheme, sometimes a CB bonus score, either positive or negative, is added to the accuracy to reflect how well or badly a student has distinguished their uncertainty from reliable answers. The exact approach employed depends on the nature of the assessment and feedback provided to students.

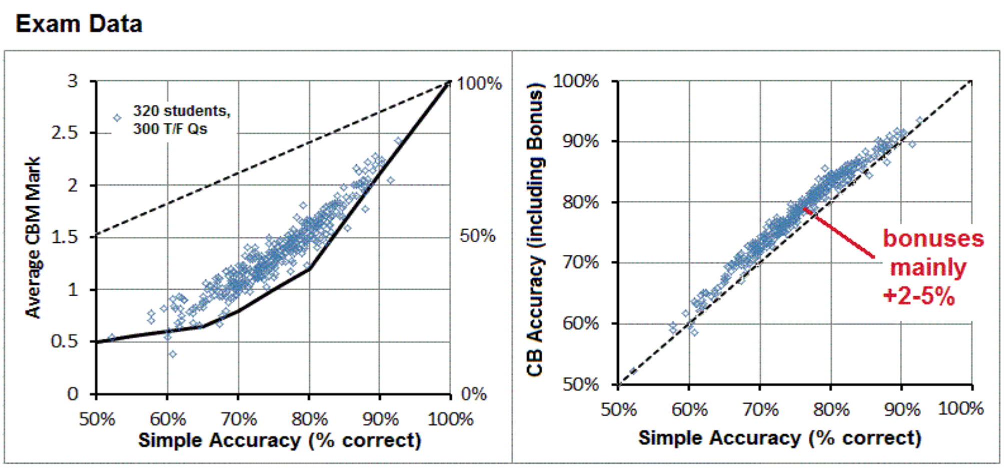

Figure 1 Example CBM data from a University College London medical exam (Gardner-Medwin 2013)

2.5 Affective domain in mathematics

"It has long been accepted that factors other than ability influence whether students use and develop their skills effectively and how they do so" (Breen, Cleary and O'Shea 2007). The affective domain in mathematics education is divided into attitudes, beliefs and emotions (McLeod 1992). However, it contains other areas of self-concept including confidence, self-efficacy, mathematical anxiety, and motivation. McLeod (1989) suggested that beliefs, attitudes, and emotions in particular are important factors in research on the affective domain in mathematics education.

A number of studies related students’ achievements to their perceived self-efficacy (Dweck 1986, Bandura 1993, Breen, Cleary and O'Shea 2007). Self-efficacy describes a person’s belief in his/her own ability to successfully participate in or cooperate with a specific activity (Bandura 1977). Bandura (1993) stated that students’ targets and ambitions, levels of motivation and academic successes are driven by their perceived self-efficacy to “regulate their own learning and to master academic activities”. Furthermore, Bandura (1993) stated that when people with low self-efficacy are presented with challenging tasks, they prevent themselves from trying to solve the task and become stressed and depressed easily and give up quickly. People with high or strong efficacy motivate themselves and when confronted with difficult tasks they perceive them as “challenges to be mastered rather than threats to be avoided”. More recently, Dogan (2012) observed that “students' concerns about mathematics can significantly affect their ability to learn and understand the subject.”

Based on the hypothesis that students’ beliefs about their capabilities to successfully undertake a task determine the way they approach it (Dweck 1986), many studies were undertaken to further investigate this theory in a mathematical context. As a result of the growing interest in this issue among mathematics and engineering educators in this area, a large number of studies have been conducted in many countries, such as Armstrong and Croft (1999), Petocz et al (2007), Fadali, Velasquez-Bryant and Robinson (2004) and Breen et al (2007, 2009). As part of these studies, a number of researchers and educators used surveys as tools to test, link and measure a variety of personal thoughts, beliefs, and attitudinal aspects that were believed to be factors influencing or linking to students’ performance in mathematics. Further specifics about a number of these studies will now be given.

In 1994-1995, a study conducted by Shaw and Shaw (1997) on the performance of first year engineering students carried out a survey that aimed to determine students’ attitudes toward mathematics and the difficulties they experienced with the subject. They developed a questionnaire looking at different aspects including: students background in terms of personal information such as gender, age, mathematics qualifications; mathematical experience pre and post entry to third level using Likert-scale questions; and a five-point scale rating difficulty in a number of mathematical topics (included numbers, algebra, calculus and probability).

Armstrong and Croft (1999) used confidence surveys based on three-point-scale items (40 items in total) exploring students' feelings about their confidence in regarding specific basic topics in mathematics. Petocz et al (2007) reported on international studies that were conducted in third-level institutions within five countries investigating students' conceptions and perceptions about mathematics. The studies undertook different approaches to assist their investigations. Qualitative surveys were used and included open-ended questions about students' conceptions about mathematics in general and about basic mathematics categories in particular.

Fadali, Velasquez-Bryant and Robinson (2004) carried out a study in Nevada University in the USA investigating the link between attitudes and capability in mathematics for students of first-year engineering. They used a survey that was adapted from one originally developed by Robinson and Maddux (1999) and aimed to investigate the hypothesis that incoming engineering students' attitudes towards mathematics were negatively affecting their capability in the subject (Fadali, Velasquez-Bryant and Robinson 2004). The survey used Likert-scale items investigating students' beliefs about engineering as well as their attitudes towards mathematics. Parsons, Croft, and Parsons, Croft and Harrison (2009) conducted a study of first-year engineering students in an English university, aiming to investigate factors affecting students' performance in mathematics. Items included open-ended questions as well as Likert-scale questions. The questionnaire gathered data about students' gender, background and mathematical qualifications; and more importantly gathered information about students' confidence, attitudes, and motivations with regard to mathematics in general, and to specific categories of mathematics in particular. Results from these studies showed a link between students' attitudes and their performance in the subject (Shaw and Shaw 1997). Over a three-year period, Parsons et al (2009) found consistent responses that showed high levels of motivation for engineering students and positive attitudes to learning mathematics. Furthermore, wherever their results showed some variation in students' confidence in relation to mathematics, they found that students' higher achievements in mathematics were associated with higher confidence in the subject. Petocz et al (2007) found that students' conceptions about mathematics differed and varied from the narrowest views to the broadest.

As mentioned, research is increasingly undertaken concerning students' beliefs and attitudes about mathematics, and how negative attitudes result in poorer achievements in the subject (Ernest 2002b). On the other hand, increasing students' self-confidence and persistence in mathematics has been found to enhance learning, and allow students to take advantage of the supports provided by their institutions, which in turn enhances their learning processes and improves their performance (Parsons 2004). Ernest (2002b) explained how negative attitudes lead to mathematics avoidance which in turn results in failure in the subject, whereas positive attitudes have the opposite effect. This is illustrated in the figure below.

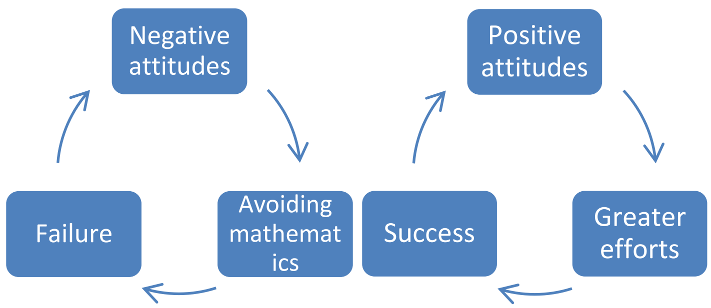

Figure 2 Failure and Success Cycles in Maths adapted (Ernest 2002)

In addition to undertaking an investigation into this area, it is clearly important that the correct research instrument is used. A study conducted by Breen, Cleary and O'Shea (2009a) attempted to evaluate the reliability and validity of a survey instrument that was used to measure a number of students' attitudes to and beliefs about mathematics. Their sample was of 186 first-year students enrolled in different disciplines in a number of higher education institutes (HEIs) in Ireland. The attitudinal survey used rating-scale items related to confidence, anxiety, theory of intelligence, goal orientation, and persistence. The questionnaire items used in their study were modified or used directly from a range of previous studies. Items included in goal orientation and persistence scales were mostly used from the study of Stipek and Gralinski (1996), along with some of the questions related to theory of intelligence. The remaining theory of intelligence questions were modified from Schoenfeld (1985). Confidence scale items were collected and adapted from three studies: Mulhern and Rae (1998), OECD (2003) and Chapman (2003). The rest of the items used were constructed by the researchers for the purpose of that study (Breen Cleary and O'Shea 2009b). The survey instrument used in their study was found to be valid (the survey measures what it is claimed to measure) and reliable (the survey results are consistent in different setting which should not affect the result) as a result of their analysis

3 Methodology

In this chapter, the aims of the research project are laid out, along with the research questions considered. A description of the research instruments is given, along with the rationale for this choice. The sample of students involved in the study is identified and the approach to data collection is described. Methods of data analysis are briefly discussed, along with any ethical implications or limitations of the study.

3.1 Research Aims

While it is widely known that mathematics is essential for a number of disciplines such as science and technology, it is also known that mathematics is essential and fundamental for engineering (LTSN 2003). Nonetheless, it is consistently seen that students entering higher education have poor core mathematics skills which cause difficulties for them in a number of engineering areas (Carr et al. 2013). The overarching aim of this study was to investigate the mathematical skills of first-year engineering students in Dublin City University (DCU) and to consider their attitudes towards mathematics. In order to address this, this research aimed to:

1. Investigate first-year engineering students' beliefs about and attitude towards mathematics;
2. Compare attitudes professed at the start of their studies with those given after having studied mathematics as part of their engineering degree in first year;
3. Test students' core mathematical skills;
4. Attempt to highlight students' strengths and weaknesses in these core areas.

3.2 Research Questions

The research conducted attempted to answer the following questions:

1. What are the attitudes and beliefs expressed by first-year engineering students about mathematics?
2. What are these students' strengths and weaknesses in basic areas of mathematics such as number, algebra and calculus?

3.3 Research Instruments

The research instruments employed in this study were a mathematics test and a survey. The selection and design of each instrument will now be described in detail.

3.3.1 Selection and design of mathematics test

The aim of this mathematics test was to look at some of the core mathematical skills required for engineering students, meaning that a series of short questions was the most appropriate design, allowing certain skills to be isolated in each question. To allow for easier and more consistent marking of the test, it was decided to adopt a multiple-choice format, as is usually done in diagnostic testing (MathsTEAM 2004, p. 2). However, in this study, a "paired-questions" approach was taken to the mathematics test. This approach was introduced in Loughborough University in the U.K. in 2002 (Lee and Robinson 2005). It was designed for mathematics diagnostic testing and was first introduced to a cohort of first year engineering students (Lee and Robinson 2005), which is a similar group to the samples used in this study. Each pair of questions were designed with the aim of testing the same skill, to gain a better overall idea of students' understanding of a topic.

The test, which is to be found in full in Appendix A1, consisted of 40 questions (20 pairs) testing three main areas: numbers, algebra and calculus. In each area, a pair of questions tested a particular mathematical skill. These skills were chosen based on areas of weakness previously identified in the same university (Carr, Murphy and Ní Fhloinn, 2009) and other areas of concern noted for similar cohorts in another Irish Higher Education Institute (HEI) (Carr, Murphy and Ní Fhloinn, 2009). Table 2 details the categories involved.

Table 2: Categories of mathematical topics examined in the mathematics test, with each topic covered by a pair of questions. Adapted from Lee and Robinson (2005) and Carr, Murphy and Ní Fhloinn (2009).

		Ques.	Category	Pair
Numbers	Number	1,2	Indices	1
		3,4	Addition/Multiplication of Fractions	2
Algebra	Algebra	5,6	Removing Brackets	3
		7,8	Evaluating expressions for given values of	4
		9,10	Indices	5
		11,12	Factorising	6
		13,14	Addition/Subtraction of Fractions	7
		15,16	Equivalent Fractions	8
		17,18	Transposition of Formula	9
		19,20	Quadratic Equations	10
		21,22	Equations of a straight line	11
		23,24	Simultaneous Equations	12
		25,26	Laws of Logarithms	13
		27,28	Laws of Exponentials	14
		29,30	Partial Fractions	15
Calculus	Differentiation	31,32	Basic	16
		33,34	Chain Rule	17
		35,36	Product Rule	18
	Integration	37,38	Basic	19
		39,40	Substitution	20

As can be seen in Table 2, the test consisted of four Number questions, 26 Algebraic questions, and six Calculus questions. The Number and Algebra questions were chosen from the work of Lee and Robinson (2005). Questions for the calculus section were chosen from Carr, Murphy and Ní Fhloinn (2009).

Certainty-based marking (CBM) responses were introduced into the test for the purposes of the marking scheme. These required students to indicate their level of certainty (low, medium or high) for each question for which they had chosen an answer. This was done in order to provide an additional layer of information regarding the students' knowledge of a topic: a student who answers correctly but chooses a low level of confidence in their answer may not be sure about the mathematical steps involved; one who answers incorrectly but chooses a high level of confidence is unaware of their misconceptions in this area.

3.3.2 Selection and design of survey

The aim of the survey used in this work was to learn more about incoming engineering students' attitudes towards and beliefs about mathematics, and to look at these again at the end of one year in higher education. Due to time constraints in the delivery of the survey (which was to be done during the same class period as the test), it was decided to use Likert-scale questions rather than open-ended questions, to allow students to answer more questions in the short time available. A five-point Likert scale was used where (1) represented 'Strongly agree, (2) 'Agree', (3) 'Not sure', (4) 'Disagree' and (5) 'Strongly disagree'. However, all questions chosen had been designed in both positive and negative formats, to counteract the problem of students marking the same response for all items (Fadali, Velasquez-Bryant and Robinson 2004).

Questions used in this survey were drawn from a number of sources, in order to cover all areas of interest in this project. The majority of the questions were based on the work of Breen, Cleary and O'Shea (2009a), who undertook their study with a sample of first year students from HEIs in Ireland. The reliability and validity of the questions used in their study were evaluated using Rasch analysis, which is "*a means of constructing an objective fundamental measurement scale from a set of observations of* *ordered categorical responses*" (Breen, Cleary and O'Shea 2009a, p.334). In addition, a number of questions were adapted from an attitudinal survey designed for new engineering students in Nevada University to assess their beliefs about engineering and mathematics (Fadali, Velasquez-Bryant and Robinson 2004). Finally, two questions were taken from the work of Jeffes et al (2013) which looked at the use of mathematics in real life situations.

The questionnaire collected personal information (including gender, year of birth, level of mathematics achievement at Leaving Certificate) from the participants initially. Then it was divided into questions on eight different scales: Confidence, Anxiety, Theory of Intelligence, Learning Goals, Persistence, Approach, Prior Experience, and General. The first five of these scales are taken from Breen, Clearly and O'Shea (2009a), while the two questions from Jeffes et al (2012) feature in the General scale, and all other questions are from Fadali, Velasquez-Bryant and Robinson (2004).

3.4 Sample

There were three samples of students involved in this study, involving two different class-groups. All students were taking a mathematics module as part of an engineering undergraduate degree in Dublin City University and were in their first year. The first sample was from April 2013; the second from September 2014 and the final one from May 2014. The last two samples were from the same class-group, but with some variation, based on which students attended on each of the two test days. The sample included 14% females in the first and third samples, and 11% females in the second sample.

3.5 Data Collection

In an attempt to also determine whether students' beliefs and attitudes towards mathematics and their basic mathematical skills were affected by their first year of higher education, the tests and surveys were to be administered at the beginning and the end of the academic year within the period of study. The first test and survey took place in December 2012 and was a pilot, after which a number of changes were made. Three subsequent tests and surveys were issued, in April 2013, September 2014 and May 2014, as mentioned above.

As the test and survey were to be distributed during a single class period, which is 50 minutes long, it was decided to allow ten minutes for the survey and 40 minutes for the test, as that allowed the students an average of one minute per test question to select the answer.

Students were asked to attend at the normal class time. For the first two samples, they were informed in advance by their lecturer that they should attend the class session but this did not occur with the third sample, where there was a notable decline in student numbers (96 and 117 students for the first and second sample respectively, but only 43 students in the third sample). They were free to leave at any point during the survey or test. Students were informed that both the test and survey were for the purposes of research and would not impact upon their results in any way, nor would their lecturers be in a position to look at their test or survey responses.

3.6 Data Analysis

The analysis of the test responses was done using Excel. The responses from each student for each question were recorded and the CBM score calculated, using the marking scheme described in the previous chapter (where the marks given vary based on the level of confidence selected combined with whether or not the answer was correct). Although CBM was the main marking scheme used in the test, the results for a number of other, more traditional, marking approaches were also calculated for each question, in order to provide a comparison. Therefore, for each question, the percentage of correct responses was calculated, along with the score that would have been received if negative marking was used. In the negative marking scheme used in this analysis, 4 marks were given for a correct answer, -1 for an incorrect answer and 0 for a blank, giving an expected score of zero if all questions were guessed. Questions were studied in pairs to provide greater insight into the likelihood of slips or misconceptions having occurred.

The responses to the survey questions were also recorded in Excel, using a five-point Likert scale. A count was done for each question to establish the number of responses falling into each of the five possible response categories. In addition, a mean Likert score was calculated for each of the eight scales identified. In order to do this, given that some of the questions were phrased in a positive way and others in a negative way, the results for the negative questions were reversed to provide a more accurate overview of responses in that area. Standard deviations were also provided for these scores.

Finally, the responses given for relevant questions on the survey were compared with overall results from the test. The intention had been to undertake an extensive analysis, linking the responses from all questions with the scores students received on the test and their chosen levels of confidence in their answers. However, as will be seen in the next chapter, the results of the survey were far more homogeneous than anticipated, rendering such an analysis redundant for a large number of questions. Therefore, a reduced analysis was done on relevant questions instead.

3.7 Ethical Issues

This study falls into the category of low-risk research involving human participants and presents no particular challenges in relation to ethical issues. The DCU "Guidelines on Best Practice in Research Ethics" (https://www4.dcu.ie/system/files/research/pdfs/guidelines_research_ethics.pdf) were followed where applicable.

3.8 Limitations of the Study

The principal limitation of the study in question is the small number of samples involved. Ideally, such a study would be undertaken over a longer time period, allowing for a greater number of comparisons, both between data from different years and data from different time periods (e.g. start of each academic year, end of each academic year). In addition, the samples involved were taken exclusively from DCU and did not involve engineering students from any other HEI in Ireland. The lecturer was present during the administration of the test and survey, which may have had an impact upon the responses given by students, regardless of anonymity guaranteed. Only 20 areas were examined in the mathematics test and it is possible that other core skills were overlooked that might have provided valuable information. Due to time constraints, students were allocated only ten minutes for the survey, which may not have provided them with sufficient time.

4 Mathematics test analysis

The questions used in the test are part of a diagnostic test built on paired questions introduced in Loughborough University. Lee and Robinson (2005) have suggested not using their exact questions, as some pairs do not test the same mathematical skills or depend on a different number of steps to get the correct answers. In this test, however, we have used some of the pairs from that test in order to investigate the use of certainty-based marking. In fact, all the questions in the Numbers and Algebra sections are adopted from the work of Lee and Robinson (2005). In addition, the sample of students used in the Lee and Robinson (2005) study was also a first-year engineering group, but based in the U.K., allowing for an international comparison between the answers given by first-year engineering students in the Irish system and those in the U.K. study. For that reason, in the following detailed test analysis, student responses to this study were compared with their equivalent in the Lee and Robinson (2005) study. Wherever differences were seen in students' performance or where the power of using CBM scheme provided greater clarity regarding unclear aspects of students' work, this was discussed for the relevant question. Furthermore, the questions for which the same percentage of correct responses were received from students in this study as in the Lee and Robinsons (2005) study were highlighted to provide an overall idea of how students in this study compared with their counterparts in the U.K..

4.1 Overview of the analysis

The test was run over three academic years. The first group was a pilot and many changes were made subsequently to the test, so in the test analysis we will not include the pilot group but will only mention interesting points of comparison. The three sample groups that were tested consisted of two different groups: one group who were tested at the end of their first academic year in April 2013 (sample 1), and a second group who were tested twice: once at the beginning of their first year (September 2013, known as sample 2) and once at the end of first year (May 2014, known as sample 3).

In order to compare students' performances across the three samples, an overall mark was calculated using three different correction schemes: 1. Number of correct answers, referred to as CA hereafter; 2. Certainty-Based Marking (CBM) as it is the marking scheme this study adopted in analyzing data; and 3. Negative marking (NM) to provide comparative data, as many multiple-choice tests are corrected using this marking scheme. As mentioned in the previous chapter, the mean negative marking score is designed to be zero if all questions are guessed with no knowledge. Therefore, 4 marks were awarded for a correct answer, -1 for an incorrect answer and 0 for a blank answer in the NM scheme. The CBM marking scheme is presented in the table below:

Table 3 CBM marking scheme

Certainty Level	**Low**	**Medium**	**High**	**No Reply**
Mark if correct	1	2	3	0
Mark if incorrect	0	-2	-6	0

As can be seen in Table 4, the weakest student performance was in sample 2, where the test took place in September 2013. That is not surprising, as this sample is the only one to undertake the test at the beginning of the academic year. The other two samples took the test at the end of their first academic year in engineering, having taking a mathematics module for the full academic year. This difference was seen clearly when comparing the groups of September 2013 and May 2014 which are the same cohort of students, tested at the beginning and end of the same academic year.

Table 4 Comparison between students' performance from the three samples

	Correct Answers Out of 40	**Correct Answers %**	**Mean CBM Out of 120**	**CBM %**	**Mean NM Out of 160**	**NM%**
Apr-13	25.63	64%	38.77	32%	94.64	57%
Sep-13	22.81	57%	24.92	21%	78.85	50%
May-14	26.60	66%	39.86	33%	95.88	60%

To further investigate any improvement which may have occurred during the course of the year, we compared the performance of the 39 students who attended both tests on September 2013 and on May 2014 and the results are shown on Table 5.

Table 5 Comparison between students who completed the test in both September 2013 and May 2014 (n=39)

	CA	**CA%**	**CBM**	**NM%**
Sep13	23.95	60%	29.74	52%
May14	26.28	66%	41.18	60%

Furthermore, in order to highlight the areas of strength and weakness of each group, questions were rated on a poorest to best performance basis, and then separated on different charts based on the percentage of correct responses. Figures 3, 4, and 5 show questions that got less than 40% correct responses for the three groups. For the September group, eight questions were included that fell below that threshold whereas the April and May groups included only five and four questions respectively. Furthermore, questions 8, 20, 29 and 40, which are all from the Algebra section apart from number 40, were always included amongst the most poorly answered questions, although ranked differently. These questions are highlighted later under each subcategory analysis.

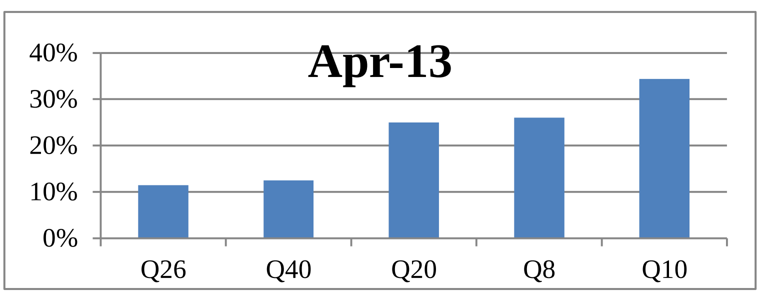

Figure 3 Questions that received less than 40% correct responses in April 2013

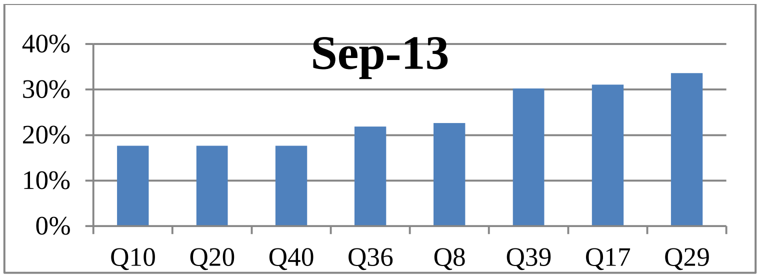

Figure 4 Questions that received less than 40% correct responses in September 2013

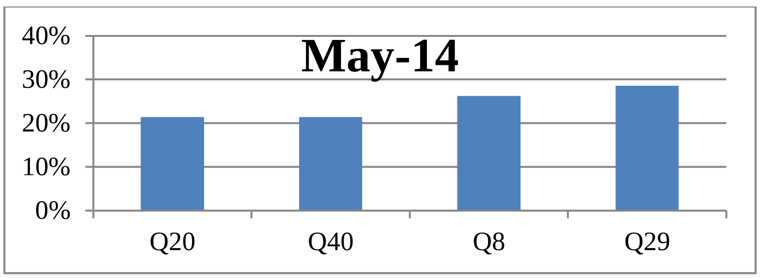

Figure 5 Questions that received less than 40% correct responses in May 2013

To further investigate each student group's points of strength and weakness, questions were sorted again from poorest to best performance but this time based on certainty-based marking (CBM). Under the CBM marking scheme, a negative mean mark is a sign of student misconceptions in an area and so, the figures below were produced to include any question that scored a negative mean mark using CBM.

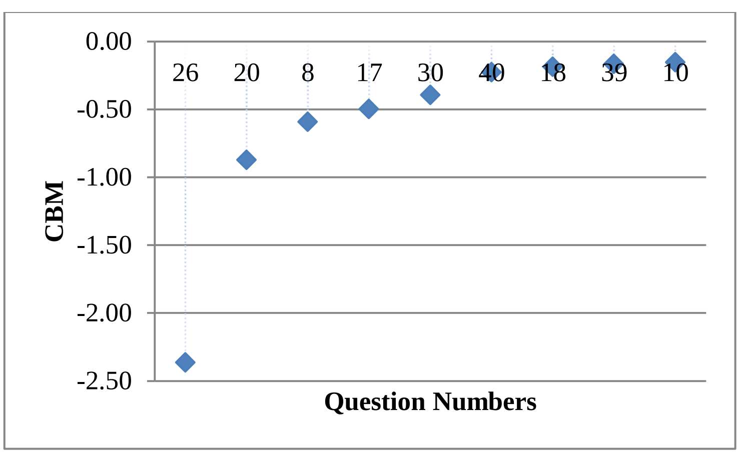

Figure 6 Questions that received a negative mean CBM score, April 2013

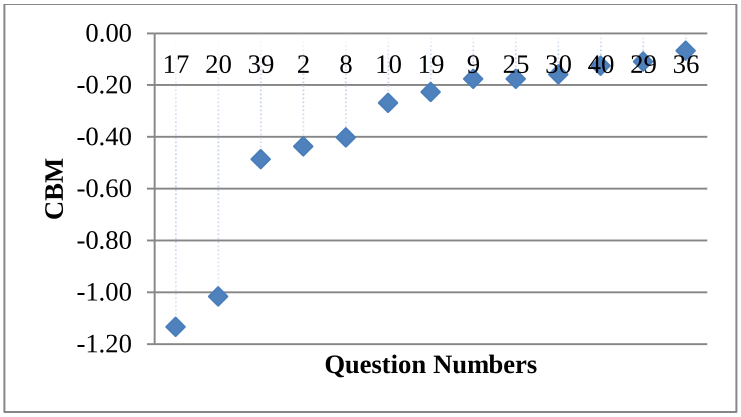

Figure 7 Questions that received a negative mean CBM score, September 2013

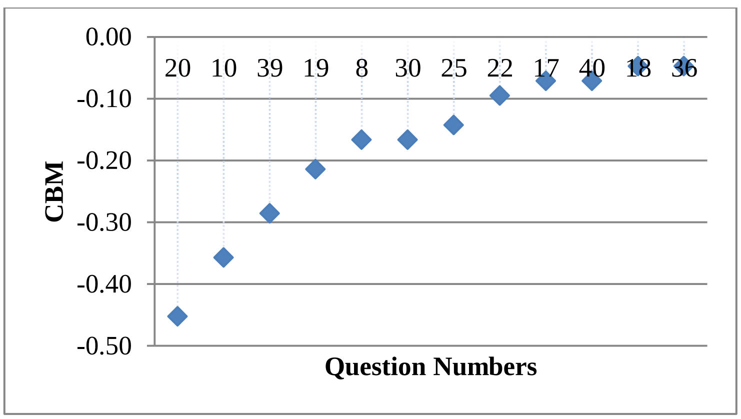

Figure 8 Questions that received a negative mean CBM score, May 2014

The figures above show that the September group recorded the highest number of questions with a negative CBM score, 13 in total. However, the other two group samples included 9 questions for the April sample and 12 questions for the May sample. Although at first it seems that the May group got a high number of questions with a negative CBM score, when a closer look is taken at the marks, it is seen that this group got a minimum of -0.5 compared to a minimum of -2.5 for Apri13 and -1.2 for Sep13 group. Thus, although they had a high number of questions receiving a negative CBM score, the score itself suggests that most students who answered incorrectly expressed only moderate confidence in their answer, indicating some awareness of the fact that they might not be fully competent in this area.

When comparing the worst answered five questions in terms of the percentage of correct responses with the questions which resulted in the lowest mean CBM score for the April 2013 sample, it was found that questions 8, 20 and 26 appeared in both groups, as shown in

Figure 3 and Figure 6. These questions test evaluating an expression for a given value of x, quadratic equations using the quadratic formula and laws of logarithms respectively. For the September 2013 sample, seven questions appeared in the lowest rankings with respect to the percentage of correct responses and the mean CBM score. These questions were 8, 10, 17, 20, 29, 36 and 39, and deal with the following areas: evaluating an expression of a given value of x, indices, transposition of formula, quadratic equations using the quadratic formula, partial fractions, product rule of differentiation, and integration by substitution. Three out of the four questions that received less than 40% correct responses by the May 2014 group were included in the worst answered with respect to the mean CBM score. These were questions 8, 20 and 40, and it should be noted that the first two of these were also among the poorest answered in the April 2013 cohort, the other test administered at the end of the academic year. Question 40 also dealt with integration by substitution.

Another interesting point shown in the figures above is that when questions were rated based on CBM, they changed to a different order in terms of worst answered, which shows the importance of CBM and that will be discussed in more detail when analyzing each category. What is worth mentioning here is that question 29, which featured in the most poorly answered questions in terms of correct answers alone, was not included in the questions that received a negative CBM score, which indicates that students are aware of their lack of knowledge in the area of partial fractions. A detailed description of each question and data analysis is found in the sections that follow.

4.2 Number

Students of all groups displayed a good performance in the four questions in the Numbers section as they only test basic numerical skills.

4.2.1 Indices

1. The value of 3^{-2} is

A) $\frac{1}{6}$ B) -6 C) $\frac{1}{9}$ D) -9 E) None of the above

2. The value of $4^{\frac{1}{2}} + 9^{\frac{1}{2}}$ is

A) 29 B) 43 C) 5 D) 19 E) None of the above

Figure 9 Pair 1: Indices

The first pair in the Numbers section tested indices. Where the first question tested negative indices, the second one tested fractional indices. Even though it was found in the Lee and Robinson study that both questions do not test the same skill, it was decided to keep both questions in order to examine students' reflections on their certainty levels. When looking at the percentages of students who answered the questions correctly, the first question got more correct responses in respect to all the three samples. However, for all groups, question number one got more than 77% correct responses whereas the second one got only 55% correct responses for Sep13 group and more than 72% correct responses from the other two groups. Details are described in Table 6.

On the other hand, when taking into account the certainty levels selected by the students to each question, the second question in the pair which tested fractional indices got much lower mean of students' marks based on their certainty level.

Table 6 Percentages of correct responses, mean Confidence-Based Marking score and negative marking score for indices questions

	Q1			**Q2**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**88**	**1.95**	**3.38**	**72**	**0.80**	**2.60**
Sep 2013	**77**	**1.27**	**2.88**	**55**	**-0.44**	**1.80**
May 2014	**83**	**1.38**	**3.07**	**79**	**1.40**	**2.93**

The most significant difference in CBM between the two questions is shown by the group of September 2013. Where students got a mean of 1.2 to the first question, they got -0.4 to the second one which indicates a considerable misconception of fractional indices. In comparison with students' performance in the study led by Lee and Robinson (2005), there were similar differences in correct responses to the same pair (81% correct responses to Q1and 51% to Q2). In that study the researchers found that because the questions do not test the same skill, they were not able to predict whether the mistakes were the result of slips made by students. In contrast, students' reflections of their certainty levels in this study suggest that most of the students were confident about their wrongly selected answer which indicates a misconception in understanding fractional indices. On the other hand, as can be seen in the table, in the third sample, May 2014, approximately the same mean CBM score was achieved in both questions. This would seem to indicate that students have increased their certainty in this topic during the course of their first year studies in engineering.

4.2.2 Addition/Multiplication of Fractions

3. The value of $\frac{4}{5} + \frac{1}{4}$ is

A) $\frac{21}{20}$ B) $\frac{5}{9}$ C) $\frac{24}{20}$ D) $\frac{19}{20}$ E) None of the above

4. The value of $\frac{5}{7} \times \frac{3}{4}$ is

A) $\frac{8}{28}$ B) $\frac{53}{74}$ C) $\frac{15}{28}$ D) $\frac{20}{21}$ E) None of the above

Figure 10 Pair 2: Addition/Multiplication of fractions

The second pair in the Number section tests addition of fractions in the first question and multiplication of fractions in the second one. Although the first question required an additional step in solving it, it was found that students performed better in the first one. Students' correct responses to both questions for the three samples are shown in the table below.

Table 7 Percentages of correct responses, Confidence-Based Marking and negative marking to addition and multiplication of fractions questions

	Q3			Q4		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**93**	**2.31**	**3.65**	**77**	**1.47**	**2.88**
Sep 2013	**84**	**1.65**	**3.23**	**74**	**1.19**	**2.74**
May 2014	**88**	**2.21**	**3.40**	**69**	**0.40**	**2.45**

In order to understand students' misconceptions about multiplying fractions, a closer look was taken at the incorrect answers and the certainty levels selected. As the highest differences between the two questions were found in the May group, it was decided to further investigate these responses. What was found was that, amongst the 30% of students who answered question four incorrectly, 62% selected answer "D" (meaning that they "cross-multiplied" the numerator from the first fraction with the denominator from the second fraction to give a new numerator and vice versa to give a new denominator). Furthermore, 23% of those students were highly certain about their answer and 32% selected medium certainty, which resulted in an overall CBM score of 1.19 for the whole group compared to 1.65 for question three. Even though the other two groups did not perform as differently to each question as the May group, the most common incorrect answer was still found to be answer "D".

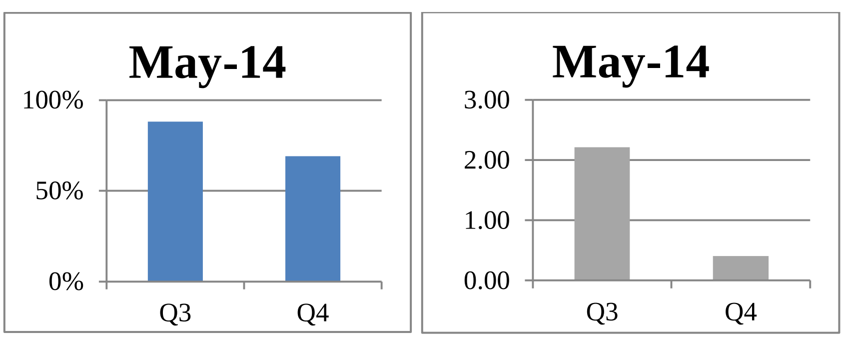

Figure 11 Left: percentages of correct responses; Right: Mean CBM score for addition/multiplication of fractions

4.3 Algebra

The Algebra section of the test consists of 26 questions, which differ in their difficulty levels, all of which are adopted from the work of Lee and Robinson (2005).

4.3.1 Removing Brackets

5. The expression $-2x^3 + 5x^2 + 3 - (x^2 - 6x - 4)$ can be simplified to

A) $-2x^3 + 4x^2 - 6x - 1$ B) $-3x^3 + 5x^2 + 6x + 7$ C) $-2x^3 + 4x^2 + 6x + 7$

D) $-2x^3 + 6x^2 - 6x - 1$ E) None of the above

6. Removing the brackets in the expression $(s + 2)(s^2 - 3)$ gives

A) $s^3 + 2s^2 - 3s - 6$ B) $s^3 - s^2 - 6$ C) $s^3 + 2s^2 - 3s + 6$

D) $s^3 + s^2 - 6$ E) None of the above

Figure 12 Pair 3: Removing brackets

The two questions involving removing brackets test different skills. Whereas the first question tests multiplying a bracket by a negative sign, the second question tests multiplying two brackets, which got more correct responses among the three groups. This can be seen in the table below, which shows the percentage of correct responses given by students from each of the three samples to the two questions in this pair.

Table 8 Percentages of correct responses, Confidence-Based Marking and negative marking to removing brackets questions

	Q5			Q6		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**70**	**0.7**	2.65	**81**	**1.9**	3.24
Sep 2013	**76**	**0.9**	2.64	**90**	**1.8**	3.37
May 2014	**66**	**0.0**	**2.33**	**88**	**2.0**	**3.40**

If we look at the CBM score for these two questions, again we see that the second question has a consistently higher than the first, showing that students both answered this question better and were more confident in their knowledge of this area. There are two possible reasons for this difference that seem natural: one is that students are less competent and confident dealing with a negative sign outside a bracket; the other is that students were unsure or made errors while merging the two quantities of x^2 or the two integers.

For Q5, as can be seen in Table 10, there is a drop between the percentage of correct answers achieved in Sept 2013 and May 2014, by the same cohort of students. This drop is mirrored in a lower mean CBM score also. A closer examination of the data reveals that every student in the May 2014 cohort expressed the highest level of confidence in their answers to this question, indicating that a portion of students in this group are not aware of their misconceptions in this topic.

4.3.2 Evaluating expressions for given values of x

7. If $x = -1$, the expression $\frac{2x^2-x+1}{3-x}$ has the value

A) 0 B) $\frac{1}{2}$ C) 1 D) $-\frac{1}{2}$ E) None of the above

8. The value of $3x^{-\frac{1}{2}}$ when $x = \frac{1}{9}$ is

A) 1 B) 9 C) -1 D) 3 E) None of the above

Figure 13 Pair 4: Evaluating expressions for given values of x

The two questions under this category tested different skills and required a different number of steps to solve them. For the first question, Q7, *x* was given a negative value whereas for the second question, Q8, *x* was given a fractional value and the question involved a negative rational power of the variable. Students got more than 81% correct responses to the first question among the three groups, but they only got a maximum of 26% correct answers to the second question. When comparing these results to students' responses in Loughborough university, it was found that they got a similar range of marks for the first question but there was a noticeable difference in the second one as about 50% of the Loughborough students' responses were correct. Taking a closer look at Q8, it can be seen that the formula included both negative and fractional indices which might have made it more complicated for the students. On the other hand, when looking at students' responses to questions on indices in the Number section of the test (in 4.2.1), we saw that students got more than 70% correct responses to both questions among the three groups. To further investigate the drop evident in student performance on indices in Question 8, a closer look was taken at the levels of certainty chosen by students for that particular question.

Table 9 Percentages of correct responses, Confidence-Based Marking and negative marking to "Evaluating expressions for given values of x" questions

	Q7			**Q8**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**81**	**1.72**	3.22	**25**	**-0.59**	0.52
Sep 2013	**82**	**1.17**	2.87	**24**	**-0.40**	0.34
May 2014	**86**	**1.83**	3.29	**26**	**-0.17**	0.57

Figure 14 shows that the students who responded to correctly to Q8 were confident with their answers as more than 80% of them selected medium to high certainty levels. (note that in this figure and similar figures below, H, M and L refer to high, medium and low levels of confidence respectively).

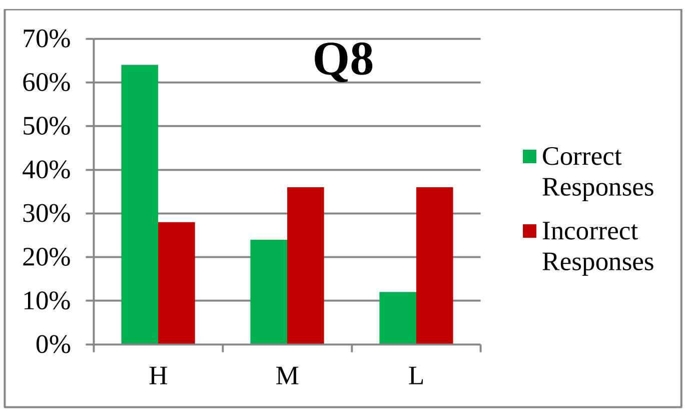

Figure 14 Percentages of different certainty levels selected by students to Q8 on April 2013

On the other hand, about 40% of the students who answered incorrectly selected medium and high certainty levels which affected the overall mean CBM score shown in Table 9. That indicates that about 40% of the students in both groups have misconceptions when evaluating the value of *x* in different equations.

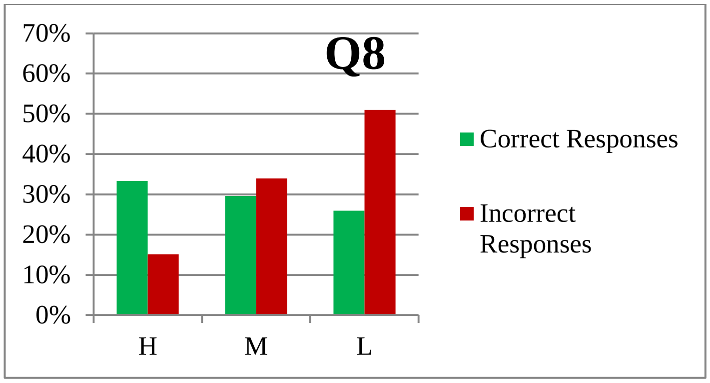

Figure 15 Percentages of different certainty levels selected by students to Q8 in September 2014

The difference for the September group, shown in Figure15, is that about 60% of the students who answered correctly selected medium and high certainty levels, and less than 30% of the students who did not select the correct answer selected the same levels of certainty. This suggests that a higher number of students in this cohort were aware of their lack of knowledge in this topic.

4.3.3 Indices

9. The expression $(y^x)^3$ is equal to

A) $3y^x$ B) y^{3x} C) y^{x+3} D) y^{x^3} E) None of the above

10. The expression $\frac{4x^{-2}y^2}{x^3y^{-4}}$ can be written as

A) $4\left(\frac{x^5}{y^6}\right)$ B) $\frac{4}{xy^{-2}}$ C) $\frac{y^6}{4x^5}$ D) $4\left(\frac{y^6}{x^5}\right)$ E) None of the above

Figure 16 Pair 5: Indices

In this pair, it is seen that the second question is more complicated than the first one and so a substantial difference in students' responses is seen for those questions. Table 10 shows that a high number of students responded correctly but the mean CBM score shows that they either showed low levels of certainty or incorrect responses were selected with high levels of certainty.

Table 10 Percentages of correct responses, Confidence-Based Marking and negative marking to indices questions

	Q9			Q10		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**68**	**0.70**	1.73	**33**	**-0.16**	0.2
Sep 2013	**56**	**-0.18**	2.55	**16**	**-0.27**	0.94
May 2014	**83**	**1.64**	3.17	**43**	**-0.36**	1.33

Figure 17 and Figure 18 show that students showed high levels of certainty when either choosing the correct or incorrect answers.

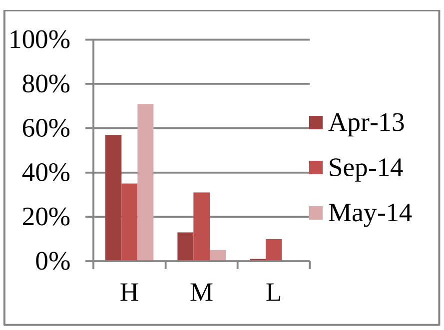

Figure 17 Certainty levels associated with incorrect answers to Q9

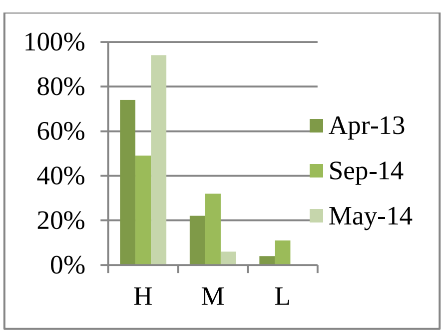

Figure 18 Certainty levels associated with correct answers to Q9

Most of the students who answered this question incorrectly among the three groups chose answer “C” which clearly indicates a misconception when raising indices to another power. It would appear that students are confusing this procedure with that of multiplying indices together as they have chosen to add the powers. Not surprisingly, given that students performed poorly in other questions containing negative indices (e.g. Q1 and Q8), the results for Q10 were low, with a negative mean CBM score for every cohort.

4.3.4 Factorising

11. The quadratic expression $x^2 - 7x - 18$ can be factorised as
A) $(x-9)(x+2)$ B) $(x+9)(x-2)$ C) $(x-6)(x+3)$
D) $(x+6)(x-3)$ E) None of the above

12. The quadratic expression $25a^2 - 9b^2$ can be factorised as
A) $(25a^2+9b^2)(a-b)$ B) $(25a^2-9b^2)(a+b)$ C) $(2a-3b)^2$
D) $(5a-3b)(5a+3b)$ E) None of the above

Figure 19 Pair 6: Factorising

Table 11shows that all groups performed very well on both factorising questions with more than 85% correct responses to both questions along with high certainty levels shown in CBM. It is worth mentioning that Loughborough’s student group got only 71% correct responses to the second question in this pair, considerably lower than any of our groups.

Table 11 Percentages of correct responses, Confidence-Based Marking and negative marking for factorising questions

	Q11			**Q12**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**90**	**2.41**	3.69	**85**	**1.98**	3.46
Sep 2013	**89**	**2.05**	3.5	**89**	**2.14**	3.36
May 2014	**90**	**2.10**	3.52	**100**	**2.71**	4

4.3.5 Addition/Subtraction of Fractions

13. Expressing $\frac{1}{x} - \frac{3}{y}$ as a single fraction, gives

A) $\frac{-2}{x-y}$ B) $\frac{-2}{xy}$ C) $\frac{y-3x}{xy}$ D) $\frac{y-3x}{x-y}$ E) None of the above

14. Expressing $\frac{2}{2x+3} - \frac{1}{x+1}$ as a single fraction, gives

A) $\frac{1}{(2x+3)(x+1)}$ B) $\frac{-1}{(2x+3)(x+1)}$ C) $\frac{1}{x+2}$ D) $\frac{-x-2}{(2x+3)(x+1)}$ E) None of the above

Figure 20 Pair 7: Addition/Subtraction of fractions

Question 13 and 14 test addition and subtraction of fractions respectively in algebraic formulae. As students performed very well in addition and subtraction of fractions in the Number section (1.2), similarly they performed well in this category in the Algebra section, showing an ability to transfer their skills in adding and subtracting fractions from numeric formulae to Algebraic ones. Their results are shown in Table 12.

Table 12 Percentages of correct responses, Confidence-Based Marking and negative marking for addition and subtraction of fractions questions

	Q13			**Q14**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**83**	**2.01**	**3.38**	**67**	**1.07**	**2.55**
Sep 2013	**86**	**1.92**	**3.33**	**70**	**0.66**	**2.31**
May 2014	**86**	**2.05**	**3.29**	**64**	**0.62**	**2.24**

By looking at the table, it can be seen that Q14 got less correct responses and lower CBM among the three groups. This is not surprising, as the second question was more complicated and required more steps to solve it.

4.3.6 Equivalent Fractions

15. The expression $\frac{6x-12y}{-3}$ is equivalent to

A) $2x-4y$ B) $4y-2x$ C) $\frac{4y-2x}{3}$ D) $12y-6x$ E) None of the above

16. The expression $\frac{2+4x-x^3}{2x^2}$ can be written as

A) $\frac{1}{x^2}+\frac{2}{x}-\frac{x}{2}$ B) $1+\frac{2}{x}-\frac{x}{2}$ C) $\frac{2}{x^2}+\frac{2}{x}-\frac{x}{2}$ D) $\frac{1}{x^2}+\frac{2}{x}-\frac{x^2}{2}$ E) None of the above

Figure 21 Pair 8: Equivalent fractions

Although this pair of questions tests the same skill, the only group of students who scored the same percentage of correct answers in both questions was the May group, as seen in Table 13.

Table 13 Percentages of correct responses, Confidence-Based Marking and negative marking for equivalent fractions questions

	Q15			**Q16**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**72**	**1.08**	2.75	**61**	**1.43**	2.33
Sep 2013	**68**	**0.19**	2.21	**49**	**0.61**	1.97
May 2014	**71**	**0.60**	2.57	**71**	**1.12**	2.62

As the mean CBM score gives us further insight into the level of knowledge that students assume they have, it is clear that even though students got fewer correct responses to the second question, they had a higher mean CBM score for that question, suggesting that the percentages of correct responses to question 16 are more likely to reflect the students' knowledge in equivalent fractions.

4.3.7 Transposition of Formula

17. If $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$, then z is given by

A) $z = \frac{x+y}{xy}$ B) $z = \frac{xy}{x+y}$ C) $z = x + y$ D) $z = -\frac{1}{x} - \frac{1}{y}$ E) None of the above

18. If $x = \frac{a}{b-c}$, then b is given by

A) $b = \frac{a}{x-c}$ B) $b = \frac{a}{x} - c$ C) $b = \frac{a+c}{x}$ D) $b = \frac{a}{x} + c$ E) None of the above

Figure 22 Pair 9: Transposition of formula

Both questions in this category were poorly answered, as seen in the very poor CBM scores, along with the low numbers of correct responses shown in Table 17. When comparing these results with Loughborough's students' results, it was found that Loughborough showed comparable performance in the first question with about 40% correct responses, but performed better in the second question with more than 70% correct responses to same question.

Table 14 Percentages of correct responses, Confidence-Based Marking and negative marking for transposition of formula questions

	Q17			**Q18**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**42**	**-0.50**	1.25	**54**	**-0.19**	1.83
Sep 2013	**29.6**	**-1.13**	0.63	**55**	**0.00**	1.87
May 2014	**50**	**-0.07**	1.52	**52**	**-0.05**	1.64

When looking closely at students' selections of incorrect answers, in order to indicate whether students were inclined to select a particular answer, it was found that their incorrect answers were spread between the incorrect choices, combined with different levels of certainty from low to high, indicating misconceptions and a lack of knowledge when transposing the subject of a given formula. Percentages of correct responses, Confidence-Based Marking and negative marking to transposition of formula questions are shown in Table 14.

4.3.8 Quadratic Equations

19. The solutions of the quadratic equation $x^2 + 4x - 77 = 0$ are
A) $x = -77, x = 1$ B) $x = 77, x = -1$ C) $x = 11, x = -7$
D) $x = -11, x = 7$ E) None of the above

20. The solutions of the quadratic equation $x^2 + 6x + 2 = 0$ can be written as
A) $x = -3 + \sqrt{28}, x = -3 - \sqrt{28}$ B) $x = -6 + \sqrt{28}, x = -6 - \sqrt{28}$
C) $x = -3 + \sqrt{7}, x = -3 - \sqrt{7}$ D) $x = 3 + \sqrt{7}, x = 3 - \sqrt{7}$ E) None of the above

Figure 23 Pair 10: Quadratic equations

The two quadratic equations questions rely on different methods to solve them. Whereas the first question can be solved by factorising, the other question requires the use of the quadratic formula.

Table 15 Percentages of correct responses, Confidence-Based Marking and negative marking for quadratic equations questions

	Q19			**Q20**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**66**	**0.54**	2.48	**24**	**-0.88**	0.4
Sep 2013	**58**	**-0.23**	1.97	**14**	**-1.02**	0.2
May 2014	**62**	**-0.21**	2.12	**21**	**-0.45**	0.48

Although the first question in the pair got between 58% and 66% correct responses, the mean CBM score was negative for the two last groups, which indicates a high number of students selecting high certainty levels with incorrect answers. Most of the incorrect responses were answer ‘C’, showing that students understood the concept of direct factorising but dealt poorly with the negative sign, as we have seen previously in other questions.

The second question in the pair, which required the use of the quadratic formula, showed students’ poor knowledge in this category with less than 25% correct responses in April 2013 and May 2014, and only 14% correct responses in September 2013. Moreover, the mean CBM score for that question also indicated that high certainty levels were chosen with incorrect responses. However, in the case of this particular question, it should be stated that students would have had access to a "log book" [1] while taking state examinations in second-level, and the exact format of the quadratic formula is available in that book. Therefore, it is possible that some of the poor scores in Question 20 are attributable to the fact that students did not know the precise formula by heart and used a slightly incorrect version to answer the question.

4.3.9 Equations of a straight line

21. The equation of the line through the point $(2, -2)$ and with slope -3 is
A) $y + 3x - 4 = 0$ B) $y - 3x + 8 = 0$
C) $3y + x + 4 = 0$ D) $-3y + x - 8 = 0$ E) None of the above

22. The equation of the straight line through the points $(3, 0)$ and $(1, -1)$ is
A) $y - 2x + 6 = 0$ B) $y + 2x - 6 = 0$
C) $2y - x + 3 = 0$ D) $2y + x - 3 = 0$ E) None of the above

Figure 24 Pair 11: Equations of a straight line

The two questions in this pair test different skills and required different numbers of steps. Whereas Q21 required 4 steps, Q22 required 7 steps, as found in Lee & Robinson (2005).

Table 16 Percentages of correct responses, Confidence-Based Marking and negative marking for equations of a straight line questions

	Q21			**Q22**		
	CA	**CBM**	**NM**	**CA**	**CBM**	**NM**
Apr 2013	**54**	**0.95**	1.95	**45**	**0.59**	1.57
Sep 2013	**59**	**0.50**	2.1	**39**	**0.04**	1.24
May 2014	**64**	**0.12**	2.26	**40**	**0.10**	1.24

Although the second question got less correct responses, the mean CBM scores do not show a considerable difference between both questions. It is also seen amongst the three groups

[1] A log book is a booklet provided to students in examinations which provide a variety of formulas and tables.

for both questions that a high number of students left it blank and this indicates students' awareness of their lack of knowledge in this area. Students would have been used to being given the formula for the equation of the line in their log book and so might not have known how to answer the question without this information to hand.

4.3.10 Simultaneous Equations

23. If $4x + 3y = 15$ and $x - 3y = -10$, then x is equal to
A) 1 B) 5 C) -1 D) -5 E) None of the above

24. The solution of the simultaneous equations $5x + y = 8, -3x + 2y = -10$ is
A) $x = \frac{6}{5}, y = 2$ B) $x = 1, y = 3$ C) $x = -2, y = 18$
D) $x = 2, y = -2$ E) None of the above

Figure 25 Pair 12: Simultaneous Equations

The number of correct responses along with the mean CBM score for both questions in this pair indicates a good knowledge of simultaneous equations among the three groups of students, as shown in Table 17.

Table 17 Percentages of correct responses, Confidence-Based Marking and negative marking for simultaneous equations questions

	Q23			**Q24**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**85**	**2.40**	3.47	**66**	**1.16**	2.51
Sep 2013	**82**	**1.20**	2.76	**75**	**1.46**	2.8
May 2014	**81**	**1.74**	3.1	**74**	**1.26**	2.74

However, the second question still got fewer correct responses as it requires more steps to solve it. It is worth mentioning that this was not the same case for Loughborough's students, who performed better in the second question compared with their performance on the first one.

4.3.11 Laws of Logarithms

25. Expressing $2\log x + 3\log y$ as a single log term gives

A) $\log(2x+3y)$ B) $\log(6xy)$ C) $\log(x^2+y^3)$ D) $\log(x^2y^3)$ E) None of the above

26. Expressing $x\log 3 - 2\log x$ as a single log term gives

A) $\log(\frac{3^x}{x^2})$ B) $\log(3^x - 2x)$ C) $\log(3^x - x^2)$ D) $\log(\frac{3}{2x})$ E) None of the above

Figure 26 Pair 13: Laws of Logarithms

The two questions testing laws of Logarithms were found to be testing the same skill (Lee and Robinson 2004).

Table 18 Percentages of correct responses, Confidence-Based Marking and negative marking for laws of logarithms questions

	Q25			**Q26**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	56	**0.15**	1.6	**76**	**1.29**	**2.33**
Sep 2013	41	**-0.18**	1.18	**49**	**0.71**	1.95
May 2014	**55**	**-0.14**	1.79	**67**	**1.43**	2.43

Nonetheless, students performed better in the second question than the first one, as seen in Table 18. This is surprising, as students were inclined to perform more poorly on questions involving negative signs. However, the low mean CBM score for both questions indicates students' poor levels of knowledge in relation to logarithms.

4.3.12 Laws of Exponentials

27. Simplifying the expression $3e^{4t}e^{-2t}$ gives

A) $3e^{2t}$ B) $3e^{-8t}$ C) $(e^{2t})^3$ D) $(e^{-8t})^3$ E) None of the above

28. Simplifying the expression $\frac{(e^x)^2}{e^{2x}}$ gives

A) $e^{(x^2-2x)}$ B) $2e^{-x}$ C) 1 D) $e^{x^2} - e^{2x}$ E) None of the above

Figure 27 Pair 14: Lows of Exponentials

The study of Lee and Robinson (2005) found that the above questions do not test the same skill and involve a different number of steps. However, students' performance in both questions was comparable to some extent.

Table 19 Percentages of correct responses, Confidence-Based Marking and negative marking for lows of exponentials questions

	Q27			**Q28**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**65**	**1.17**	2.51	**51**	**0.73**	1.78
Sep 2013	**50**	**0.82**	2.08	**38**	**0.34**	1.42
May 2014	**64**	**1.12**	2.29	**62**	**0.67**	2.17

Continuing the trend for most pairs, the second question on the pair received lower numbers of correct responses and a poorer CBM mean score in respect to each group. This is not surprising, as it was more complicated and involved more steps. Percentages of correct responses, Confidence-Based Marking and negative marking to lows of exponentials questions are shown in Table 19.

4.3.13 Partial Fractions

29. The expression $\frac{1}{(x+2)(x+3)}$ when split into partial fractions is

A) $\frac{3}{x+2}+\frac{2}{x+3}$ B) $\frac{3}{x+2}-\frac{2}{x+3}$ C) $\frac{1}{x+2}-\frac{1}{x+3}$

D) $\frac{1}{x+2}+\frac{1}{x+3}$ E) None of the above

30. When expressing $\frac{2x}{(x-2)(x+1)^2(x^2+1)}$, in partial fractions, the appropriate form, with a, b, c, d, e and f constants, is

A) $\frac{a}{x-2}+\frac{b}{x+1}+cx+\frac{d}{x^2+1}$ B) $\frac{a}{x-2}+\frac{b}{(x+1)^2}+cx+\frac{d}{x^2+1}$

C) $\frac{a}{x-2}+\frac{b}{x+1}+\frac{cx+d}{(x+1)^2}+\frac{ex+f}{x^2+1}$ D) $\frac{a}{x-2}+\frac{b}{x+1}+\frac{c}{(x+1)^2}+\frac{dx+e}{x^2+1}$ E) None of the above

Figure 28 Pair 15: Partial fractions

The two questions on this pair test different skills and require different numbers of steps. The table below shows the low performance of the three groups, especially in the second question.

Table 20 Percentages of correct responses, Confidence-Based Marking and negative marking for partial fractions questions

	Q29			**Q30**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**40**	**0.42**	1.19	**8**	**-0.40**	-0.1
Sep 2013	**34**	**-0.12**	0.87	**8**	**-0.16**	-0.1
May 2014	**29**	**0.02**	0.69	**12**	**-0.17**	0.14

It also shows that the majority of the students were aware of their lack in knowledge as indicated by the mean CBM score and the NM mark. The second question got the lowest performance for two of the groups and within the five poorest for the third one.

4.4 Calculus: Differentiation

All the questions in the calculus section were adopted from the study of Carr, Murphy and Ní Fhloinn (2011). They were part of an advanced diagnostic test assigned to engineering students. The test used the paired-questions approach and attempted to test the same skills for each pair. Yet, it was found that some of the second questions of a pair were more complicated than the first one.

4.4.1 Basic Differentiation

31. Find $\frac{dy}{dx}$ where $y = x^4$

A) $4x^3$ B) $4x^4$ C) $5x^5$ D) x^3 E) None of the above

32. Find $\frac{ds}{dt}$ where $s = t^7$

A) $7t^6$ B) t^6 C) $7t^8$ D) $\frac{x^8}{7}$ E) None of the above

Figure 29 Pair 16: Basic differentiation

Questions 31 and 32 test the same skill of basic differentiation. Students in all groups showed a very high knowledge of this topic, as indicated by the high percentages of correct responses followed by high levels of certainty to both questions in the pair, as shown in Table 21.

Table 21 Percentages of correct responses, Confidence-Based Marking and negative marking for basic differentiation questions

	Q31			**Q32**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**94**	**2.85**	3.91	**93**	**2.78**	3.85
Sep 2013	**93**	**2.50**	3.78	**89**	**2.41**	3.66
May 2014	**98**	**2.93**	3.9	**95**	**2.69**	3.79

In fact, both questions in the pair are to be found within the top 5 questions in terms of students' performance amongst the three groups as shown in Figure 30 and Figure 31.

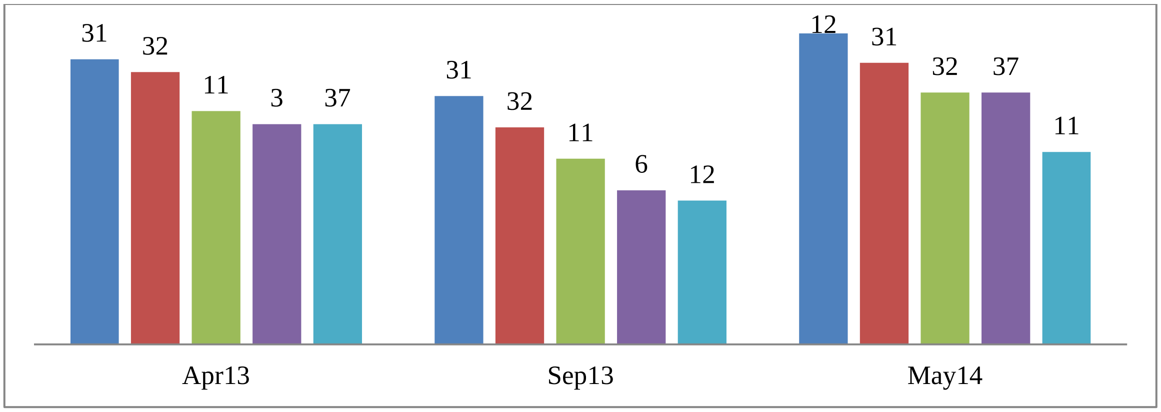

Figure 30 Top 5 questions in terms of students' performances with regard to correct responses

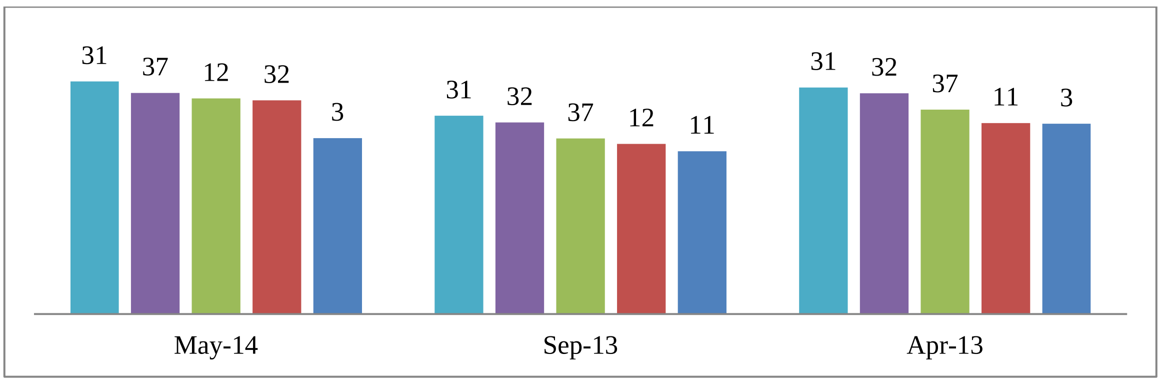

Figure 31 Top 5 questions in terms of students' performance with regard to mean CBM score

One point to note is that the second question is slightly more poorly answered than the first; this could be as a result of the fact that the variables in question were *s* and *t* instead of *x* and *y*, given that, anecdotally, this can cause problems to some of the weaker students.

4.4.2 Chain Rule

33. Solve $\frac{d}{dx}\sin(2x+1) =$

A) $2\cos(2x+1)$ B) $2\sin(2x+1)$

C) $\cos(2x+1)$ D) $-\cos(2x+1)$ E) None of the above

34. Find $\frac{ds}{dt}$ where $s = \cos(5t-4)$

A) $\sin(5t-4)$ B) $-5\sin(5t-4)$

C) $-5\cos(5t-4)$ D) $-\sin(5t-4)+5$ E) None of the above

Figure 32 Pair 17: Chain rule

Both questions in this pair test the chain rule using trigonometric functions. Despite this, no group scored exactly the same mean CBM or NM score for both questions.

Table 22 Percentages of correct responses, Confidence-Based Marking and negative marking for chain rule questions

	Q33			**Q34**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**80**	**1.90**	3.22	**73**	**1.31**	2.88
Sep 2013	**41**	**0.39**	1.54	**41**	**0.13**	1.53
May 2014	**83**	**1.89**	3.19	**76**	**1.69**	2.86

However, even though there were no major differences between the questions as shown in
Both questions in this pair test the chain rule using trigonometric functions. Despite this, no group scored exactly the same mean CBM or NM score for both questions.

Table 22 and Figure 33 , it is notable that for the three groups, the second question got lower marks. This raises a number of questions, such as were the students better dealing with ‘sin’ rather than ‘cos’? Or was it the negative sign within the ‘cos’ function that led to confusion? Was it as a result of the choice of variables? Or was it simply a result of slips made by students?

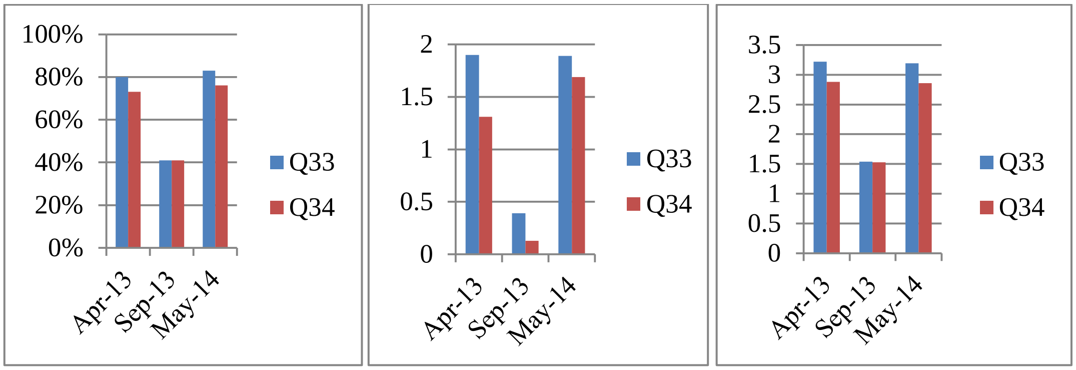

Figure 33 Percentages of correct responses, mean CBM scores, and mean NM scores respectively to Questions 33&34: Chain rule from Calculus section of the test

To investigate this, the author had a look at the incorrect responses and the certainty levels indicated with each answer. By highlighting those who answered the second question incorrectly but the first one correctly, we found that 50% of them selected answer "C" and 21% selected answer "D" with the majority of them indicating medium to high certainty levels. This indicates that, even though these students responded correctly to the first question, they either made the mistake of differentiating one function and leaving the other, or else were unsure of how to deal with the integer once there was a negative sign in front of the trigonometric function. There was a clear improvement in both questions for the cohort who took the test at the start of first year and again at the end, and their May 2014 score is more in line with the April 2013 score, suggesting that students gained greater clarity and accuracy with this skill during their first year in engineering.

4.4.3 Product Rule

35. Solve $\frac{d}{dx}(x^3 \cos x) =$

A) $3x^2 \cos x - x^3 \sin x$ B) $3x^2 + \cos x$

C) $3x^2 + x^3 \cos x$ D) $-x^3 \sin x$ E) None of the above

36. Find $\frac{dy}{dx}$ where $y = x^2 \sin 3x$

A) $2x \sin 3x + 6 \cos 3x$ B) $2x \sin 3x - 3x^2 \cos 3x$

C) $2 \sin 3x + x^2 \cos 3x$ D) $2x \sin 3x + 3x^2 \cos 3x$ E) None of the above

Figure 34 Pair 18: Product Rule

Even though the two questions in this pair at first glance seem to be testing the same skill of solving a product rule problem including a trigonometric function, the second question requires the use of the chain rule as well as the product rule. This extra requirement in the second question resulted in a notable difference in students' results as shown in Table 23.

Table 23 Percentages of correct responses, Confidence-Based Marking and negative marking for product rule questions

	Q35			Q36		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**81**	**1.82**	3.19	**44**	**0.59**	1.39
Sep 2013	**43**	**0.87**	2.08	**22**	**-0.07**	0.44
May 2014	**83**	**2.12**	3.26	**48**	**-0.05**	1.45

When comparing the performances of the three groups, as in the previous pair, a significant difference is seen between the groups who had the test at the end of the year (Apr13& May14), and the group who had the test at the beginning of the year (Sep13). The first and third group showed similar performance in both questions with a slightly higher score for the May group. They both got more than 81% correct responses and a mean CBM score of more than 1.8 for Q35, compared with only 43% correct responses followed by a mean CBM score of 0.87 for the same question in the September group. Again, both their competency and their confidence in this question seem to have improved dramatically during the course of their first year.

When comparing students' performance in questions related to the chain rule with their performance in Q35, the product rule, it can be seen that, even though the students' performance indicated good knowledge of both rules, a high number of them failed to merge both skills together in one problem for Question 36.

4.5 Calculus: Integration

4.5.1 Basic integration

37. Find $\int x^3\,dx$

A) $\frac{x^4}{4}+c$ B) $4x^4+c$ C) x^3+c D) $3x^3+c$ E) None of the above

38. Find $\int t^{-4}\,dt$

A) $-4t^{-3}+c$ B) $\frac{-t^{-3}}{3}+c$ C) $t^{-5}+c$ D) $\frac{t^{-5}}{5}+c$ E) None of the above

Figure 35 Pair 19: Basic integration

The two questions in this pair test the same skill of basic integration but with different signs for the indices and different variables used. Again, when students were exposed to negative indices in the second question, they performed more poorly.

Table 24 Percentages of correct responses, Confidence-Based Marking and negative marking for basic integration questions

	Q37			**Q38**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**89**	**2.57**	3.67	**63**	**0.82**	2.34
Sep 2013	**80**	**2.21**	3.42	**58**	**0.56**	2.18
May 2014	**95**	**2.79**	3.79	**79**	**1.31**	2.95

Although there is a noticeable difference between the percentages of correct responses for Q37 compared with Q38 across the three samples, the difference in the mean CBM scores for the two questions is starker, as can be seen in the table. Upon closer analysis of the data, the difference in the mean CBM scores was observed to be as a result of low levels of confidence expressed with correct responses, as well as high levels of confidence expressed with incorrect responses. This is an indication that not only do the students make more mistakes when dealing with the negative sign, but also they show lower confidence when dealing with it. However, bearing in mind that many students fail to deal correctly with the negative sign in a wide range of questions, supported by the fact that the highest number of incorrect responses indicated answer ‘D’ (meaning that they got an answer of -5 when adding 1 to -4) , in general, students have a good basic knowledge of integration.

4.5.2 Substitution

39. Evaluate the following integral $\int \cos(x+2)\,dx$

A) $-\sin(x+2)+c$ B) $\sin(x+2)+c$

C) $2\cos(x+2)+c$ D) $-2\cos x+c$ E) None of the above

40. Evaluate the following $\int x(4x^2-7)^3\,dx$

A) $\frac{(4x^2-7)^2}{8}+c$ B) $\frac{(4x^2-7)^2}{32}+c$ C) $\frac{(4x^2-7)^4}{8}+c$

D) $\frac{(4x^2-7)^4}{32}+c$ E) None of the above

Figure 36 Pair 20: Integration by Substitution

From the percentage of correct responses shown in Table 25, we see that students who did not respond correctly to both questions mostly did not respond at all, which indicates students’ awareness of their lack of knowledge in the corresponding questions.

Table 25 Percentages of correct responses, mean Confidence-Based Marking score and negative marking score for integration by substitution questions

	Q39			**Q40**		
	CA%	**CBM**	**NM**	**CA%**	**CBM**	**NM**
Apr 2013	**46**	**-0.17**	1.63	**15**	**-0.23**	0.07
Sep 2013	**23**	**-0.49**	0.86	**08**	**-0.13**	0.39
May 2014	**55**	**-0.29**	1.9	**12**	**-0.71**	0.48

Further analysis was undertaken to determine the number of responses to each of the possible answer choices and the results are shown on Figure 37 and Figure 38 below. Answer “X” on the scale indicates no response to the question. Figure 38 details responses to the first question and it shows that most of the incorrect answers were to “A” which indicates lack of knowledge of the sign of the “cos” function when integrated. This is not surprising, as students are used to being able to double-check this result with a log book in their examinations but were not supplied with one for this test.

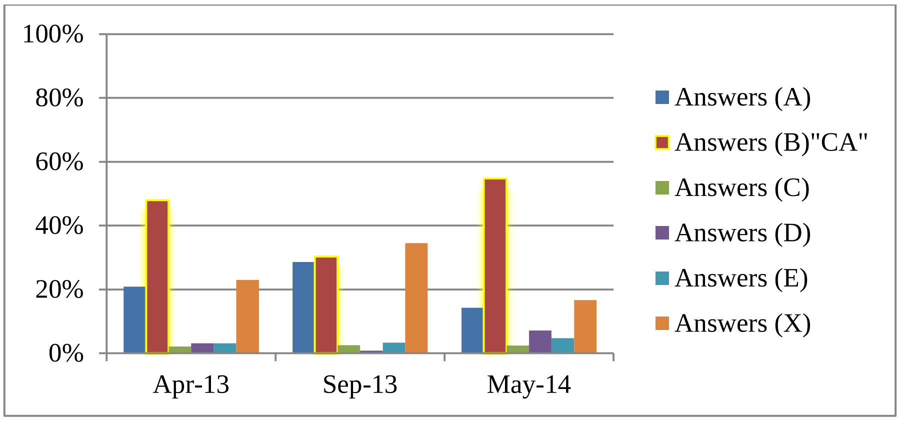

Figure 37 Percentages of different answers selected by students to Question 39 (Integration by Substitution)

Unlike the first question, student responses were widely spread among all the answers provided for the second question, along with large number of students (more than 40% of each group) who avoided answering the question as shown in Figure 38. This indicates students’ awareness of their deficiencies in that particular topic.

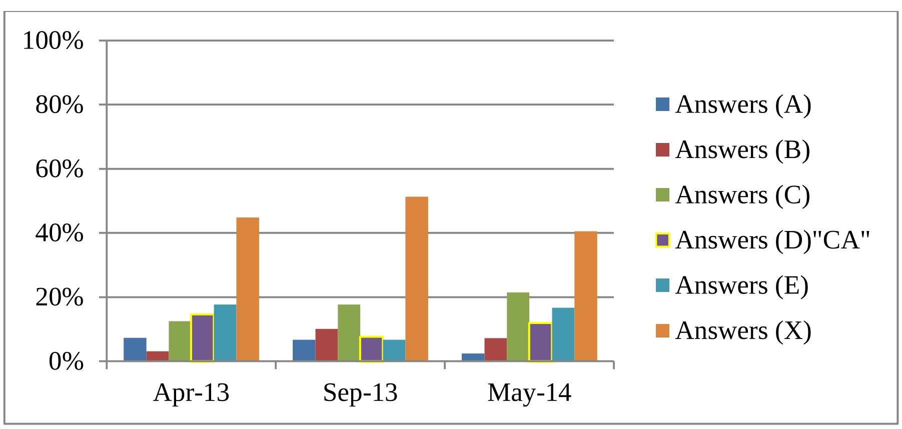

Figure 38 Percentages of different answers selected by students to Question 40 (Integration by Substitution)

4.6 Additional Discussion

Paired questions in the test showed a number of interesting points. Firstly, as was suggested in the work of Lee and Robinson (2004), when the two questions test different skills or include a different number of steps, it is not initially easy to determine the lack of knowledge and slips/mistakes. However, as the paired questions approach was supported with the CBM scheme, in many cases students' lack of knowledge and their misconceptions in a certain category were clearly shown. We believe that using properly paired questions is still to be recommended, but time constraints should be borne in mind. Covering the range of areas of basic mathematical skills and concepts which would be considered core for a number of disciplines, along with using a pair of questions to test each skill, will result in a large number of test items. Running more than one test to cover all the areas targeted would be a possible solution to this.

When questions were sorted from worst to best answered, a number of questions were repeatedly seen to feature, whether with respect to percentages of correct responses or the mean CBM score. Question 8 (evaluating an expression for a given value of x, involving negative indices) and question 20 (a quadratic equation that required the use of the quadratic formula) were both included among the questions with the least number of correct responses and the lowest mean CBM score across all three samples. On the other hand, questions 11 and 12 (factorisation), 31 and 32 (basic differentiation) and 37 (basic integration) were included in the top ten answered questions in relation to both of these measures.

The use of CBM in the data analysis in this study raised a number of interesting points. Whereas the paired questions used in the test can show the possibility of incorrect answers as a result of minor slips or correct answers by lucky guessing, the mean CBM score for each question gives us a more reliable estimate of the students' level of knowledge in a category or a subject. For example, if we look at Section 4.1, when students were confronted with positive indices in an integration problem, 80-95% responded correctly with high levels of confidence. But when comparing with the other question in the pair which tests the same skill with negative indices, students only gave 58-80% correct responses. Without the benefit of CBM, it may be assumed that 20% of the students made a slip in the second question; but by focusing on the levels of certainty indicated by students to both questions, it is clear that, despite answering correctly, students indicated much lower levels of certainty for the second question. This resulted in a notable difference between the mean CBM score for both questions, confirming the suspicion that students find greater difficulties when confronted with negative indices in integration

However, our data analysis also showed that 82% of high certainty levels were selected with correct answers, which shows that students selected high levels fairly wisely, as shown in the figures below. On the other hand, more than 45% of medium certainty levels were selected with incorrect answers which raises concerns about students' awareness of indicating the correct level of certainty when assuming being correct with a percentage of about 50%. i.e. the scheme suggests that students should have selected a low level of certainty if they were less than 67% certain about an answer whereas student responses suggest that when students thought they were about 50% certain about an answer, they tended to choose a medium level of certainty. This raises the question of justifying the marking scheme and in future work, it would be suggest the exploration of a different marking scheme to reflect students' choices.

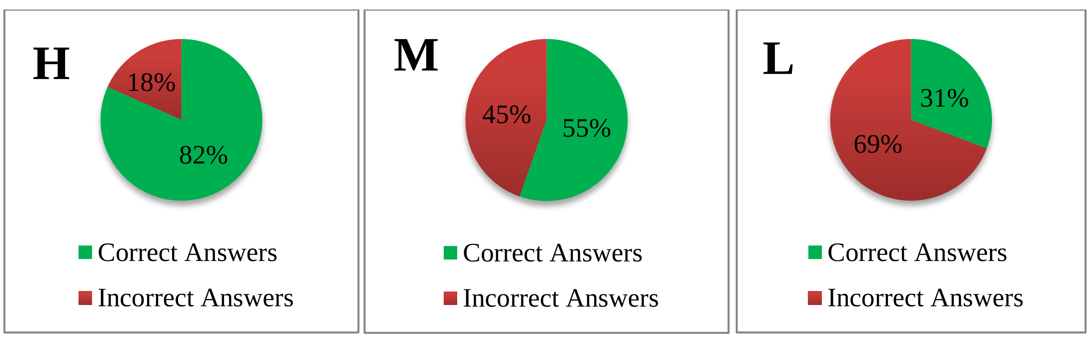

Figure 39 Percentages of correct and incorrect answers to the correspondent certainty levels (H = high, M = medium, L = low) indicated in mathematics' tests for all groups

What is more regarding the selected low levels of certainty, about 31% of the selected low levels were for correct answers, and this percentage highlights the usage of CBM. That is to say that even though a large number of answers were indicated correctly they do not necessarily reflect a good level of students' knowledge upon the corresponding question.

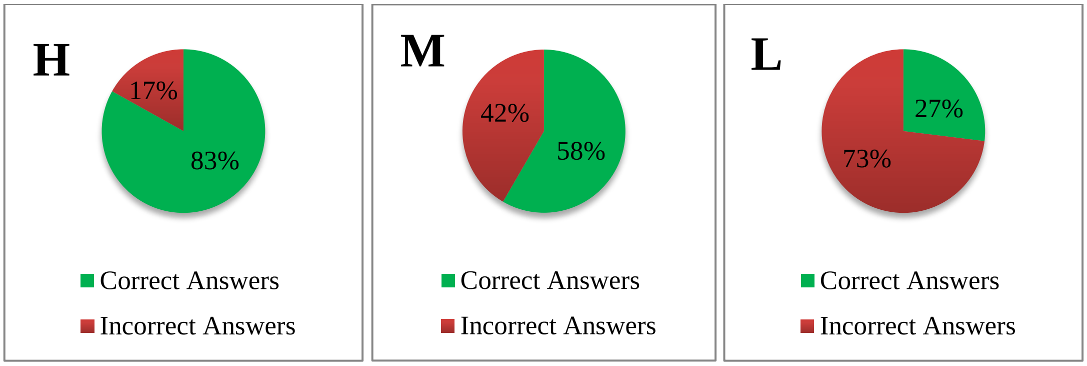

Figure 40 Percentages of correct and incorrect answers to the correspondent certainty levels indicated in mathematics' tests for April 13 group

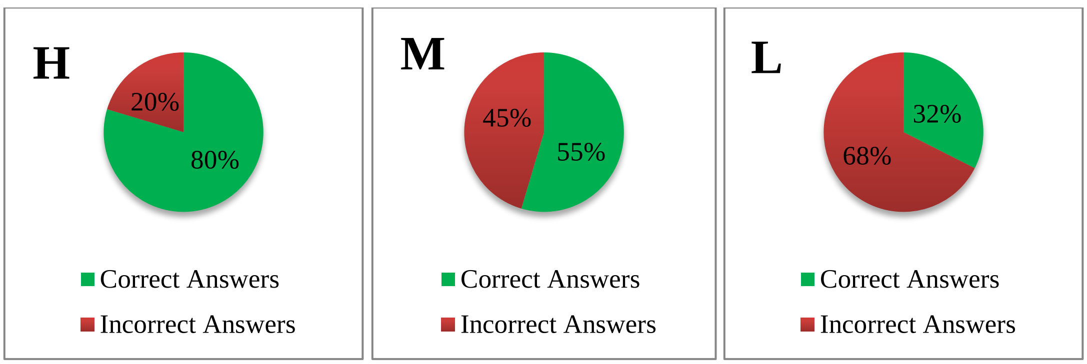

Figure 41 Percentages of correct and incorrect answrs to the correspondent certainty levels indicated in mathematics' tests for September 13 group

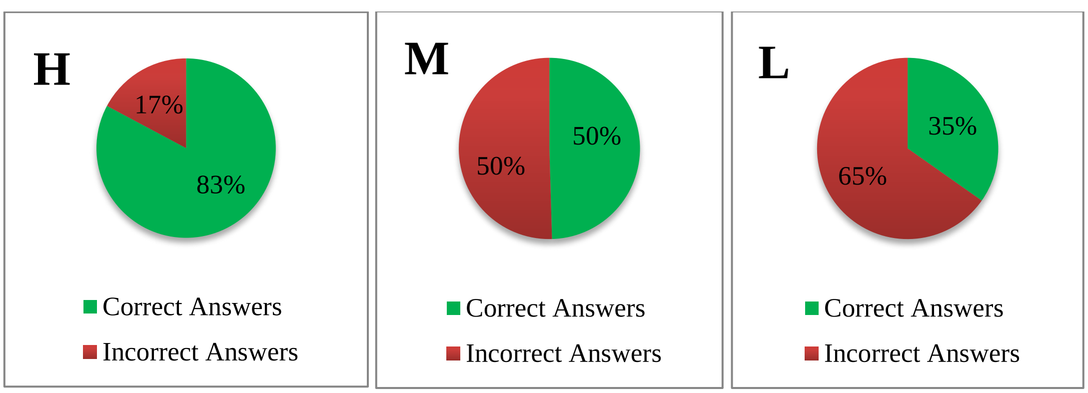

Figure 42 Percentages of correct and incorrect answers to the correspondent certainty levels indicated in mathematics' tests for May 14 group

It can be seen from Figures 41, 42 and 43 that there was little variation between the overall levels for all groups involved, in terms of correct and incorrect answers at each confidence level.

5 Survey Analysis

In December 2012, 34 students were included in a pilot study. The pilot survey included 44 Likert-scale questions. On the basis of this pilot study, some of the questions were removed from the main study questionnaire and other questions were amended.

In the main study, the test and survey were presented at the beginning and/or end of the students' first academic year in engineering. Thus, we had three samples, two of which were examined at the end of the academic year, April 2013 and May 2014, and the third one was examined in September 2013, at the beginning of the academic year 2013-2014. Therefore, as the pilot sample was not comparative to any of the main study's samples, and as it included only a small number of students which, data collected from the pilot study are not presented in this work.

In this chapter, a detailed analysis of the survey is provided, highlighting points of interest in each part of the survey.

5.1 Confidence Scale

Confidence Scale	
Q1.	I learn mathematics quickly.
Q2.	I feel confident in approaching mathematics.
Q3.	I can get good marks in mathematics.
Q4.	I have trouble understanding anything with mathematics in it.
Q5.	Mathematics is one of my worst subjects.
Q6.	I am just not good at mathematics.

Figure 43 Questions from the Confidence scale of the survey

The survey started with six questions examining students' confidence regarding mathematics, all of which are adopted from the study of Breen, Cleary and O'Shea (Breen, Cleary and O'Shea 2009a). While the first three questions (Q1-Q3) in the confidence scale address positive statements regarding confidence in mathematics, the following three questions (Q4-Q6) address negative confidence statements about mathematics. Figures 45, 46 and 47 show that in students' responses across all three samples (April 2013, September 2013 and May 2014), their confidence in mathematics was expressed mainly in a positive way. This is evident in the overall agreement with the positive statements and the disagreement with the negative ones.

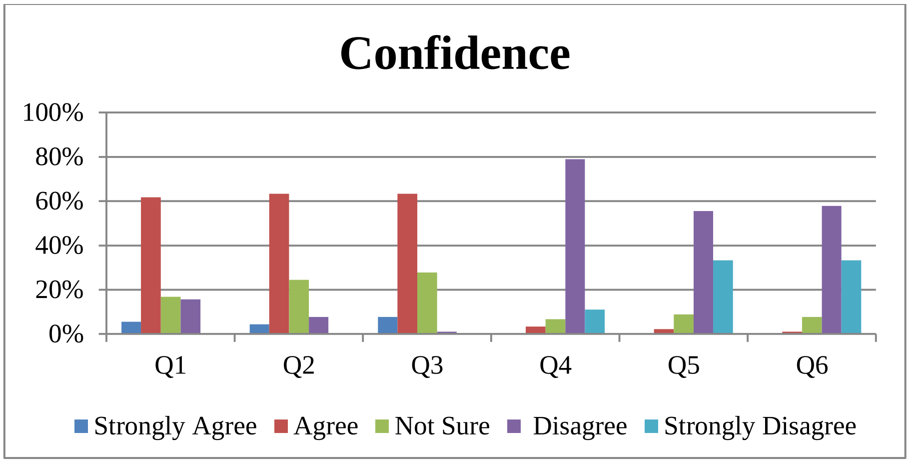

Figure 44 Percentages of students' responses to confidence questions on the survey from April 2013

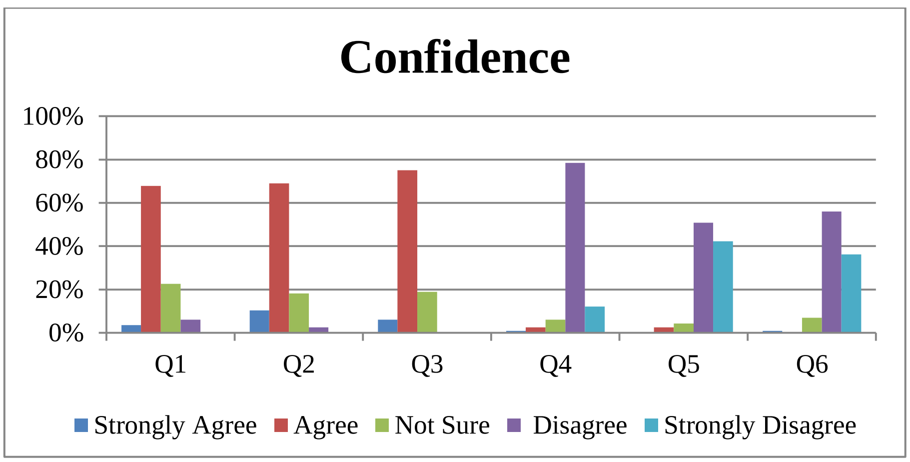

Figure 45 Percentages of students' responses to confidence questions on the survey from September 2013

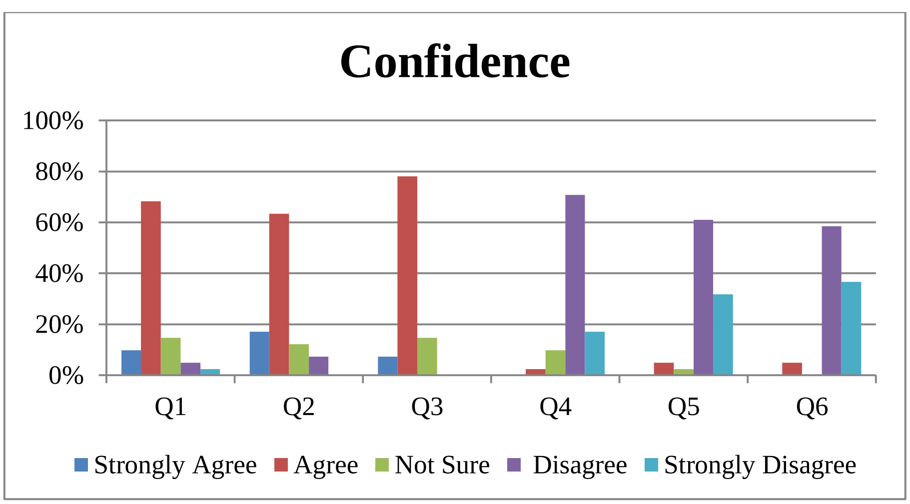

Figure 46 Percentages of students' responses to confidence questions on the survey from May 2014

For instance, most of the students, between 62% and 68% of students across the three samples, “agree” that they can learn mathematics quickly (Q1) and they feel confident in approaching mathematics (Q2). Furthermore, more than 60% of the April sample, and more than 75% of the two other samples agreed that they could get “*good marks*” in mathematics (Q3). Less than 5% of students among the three samples reported having problems with mathematics or understanding it, with the majority students disagreeing with that statement. What is more, when students were confronted with negative statements about mathematics being one of their “*worst subjects*” (Q5) or about being “*just not good at mathematics*” (Q6), between 51% and 61% disagreed and between 32% and 42% strongly disagreed with both statements among the three samples.

Therefore, responses to confidence scale items indicate that the cohort included in this study showed high levels of confidence in relation to mathematical abilities, across the three samples. It could be seen from the graphs above that the cohort of students that were examined at the beginning and the end of the same year showed consistency in their responses to the scale.

5.2 Anxiety Scale

Anxiety Scale	
Q7.	I get very nervous during maths lectures.
Q8.	I often worry that it will be difficult for me in maths lectures.
Q9.	I often feel helpless when doing a maths problem.
Q10.	Mathematics makes me feel uneasy and confused.
Q11.	I usually feel at ease doing mathematics problems.

Figure 47 Questions from the Anxiety scale of the survey

The anxiety scale consisted of five questions, all of which were taken from the Breen, Cleary and O’Shea (2009a) study. Unlike the confidence scale, the three samples here showed more variable responses, although the overall tendency was towards low anxiety expressed towards mathematics, as shown in the figures below.

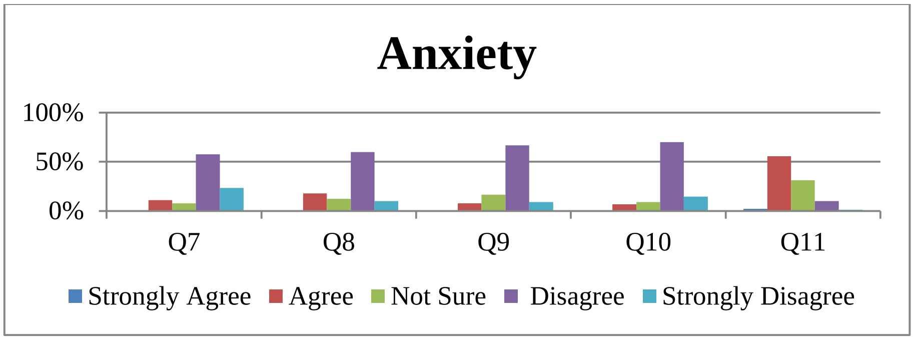

Figure 49 Percentages of students' responses to anxiety questions on the survey from April 2013

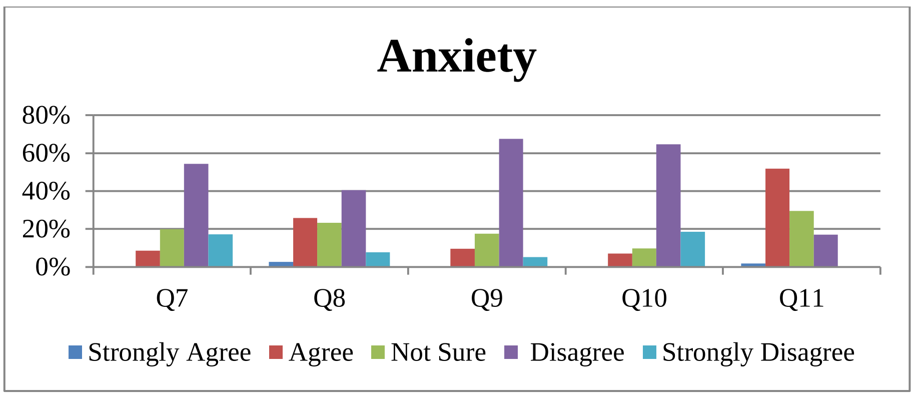

Figure 48 Percentages of students' responses to anxiety questions on the survey from September 2013

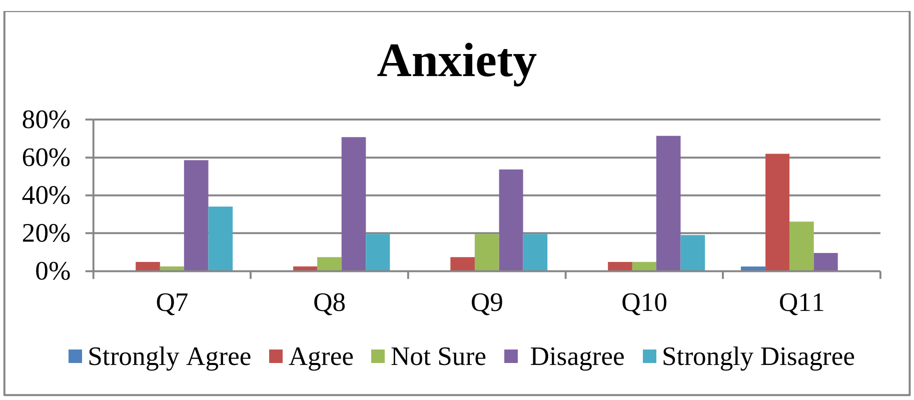

Figure 49 Percentages of students' responses to anxiety questions on the survey from May 2014

Between 50% and 80% of students either disagreed or strongly disagreed with the negative opinions reflected in the statements. Nonetheless, there are about 30% of the students among the three samples that agreed or strongly agreed or at least were "not sure" as to whether they get nervous or feel it will be difficult or feel helpless during mathematics lectures, which may be a concern for mathematics lecturers and educators.

5.3 Theory of Intelligence

Theory of Intelligence Scale	
Q12.	You have to be smart to do well in maths.
Q13.	People are either good at maths or they are not.
Q14.	Some people will never do well in maths no matter how hard they try.
Q15.	You can succeed at anything if you put your mind to it.
Q16.	You can succeed at maths if you put your mind to it.
Q17.	It is possible to improve your mathematical skills.
Q18.	Everyone can do well in maths if they work at it.

Figure 50 Questions from the Theory of Intelligence scale of the survey

There are seven items in the theory of intelligence scale, all of which are adopted from the study of Breen, Cleary and O'Shea (2009a). Students' responses to these questions are shown in the figures below.

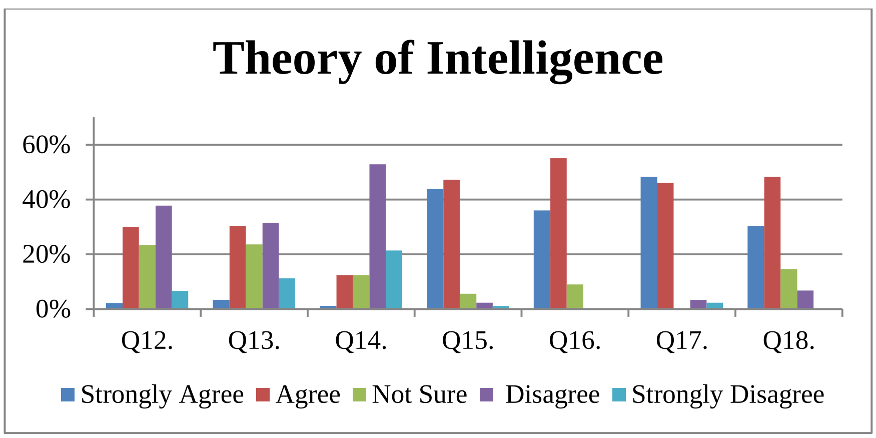

Figure 51 Percentages of students' responses to Theory of Intelligence scale on the survey in April 2013

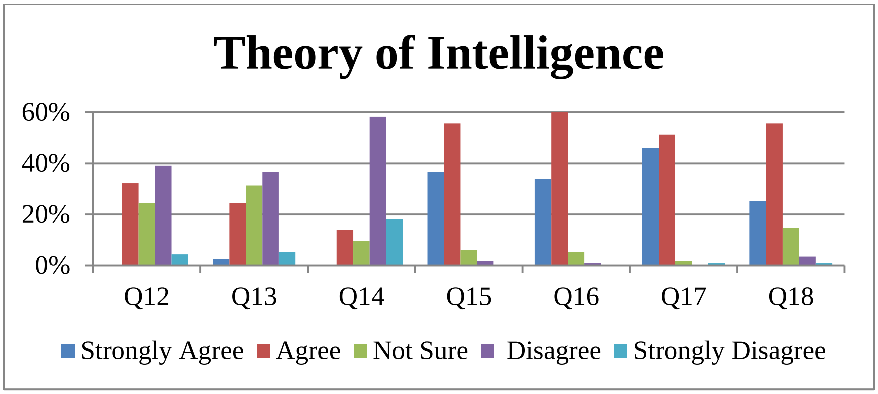

Figure 52 Percentages of students' responses to Theory of Intelligence scale on the survey in September 2013

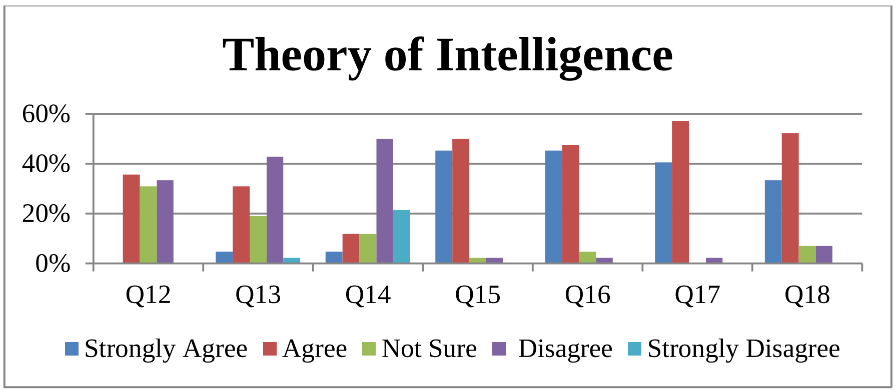

Figure 53 Percentages of students' responses to Theory of Intelligence scale on the survey in May 2014

The theory of intelligence scale showed a variety of responses to students' beliefs of intelligence in general, and in terms of mathematics in particular. In particular, the first two questions on this scale showed a range of different responses across the three samples, from agreement through to disagreement with the statements: *"You have to be smart to do well in maths"* and "*People are either good at maths or they are not"*. Despite this, responses to the other questions on the scale showed students' belief about the possibility of improvement or success at *"anything"* as well as at mathematics, provided sufficient work was done. This was particularly seen in the majority of the responses to Q14, where more than 90% of the three samples agreed or strongly agreed that mathematical skills could be improved or success in mathematics could be gained if one put one's mind to it or worked at it.

5.4 Persistence Scale

Persistence Scale	
Q19.	I will risk showing that I don't know something in order to acquire new mathematical knowledge.
Q20.	I am most proud of my mathematical performance when I feel I have done my best.
Q21.	When presented with a choice of mathematical tasks, my preference is for a challenging task.
Q22.	When presented with a mathematical task I cannot immediately complete, I increase my efforts.
Q23.	When presented with a mathematical task I cannot immediately complete, I persist by changing strategy.
Q24.	When presented with a mathematical task I cannot immediately complete, I give up.
Q25.	When presented with a choice of tasks, my preference is for one I know I can complete.

Figure 54 Questions from the Persistence scale of the survey

There are seven persistence questions in the attitude survey. All are adopted from the study of Breen, Cleary and O’Shea (Breen, Cleary and O’Shea 2009a). The overall outcome to these questions was that students reported strong tendencies towards persistence in mathematics. Students among the three groups also displayed high levels of persistence in their receptivity to seek help, increase their efforts or change their strategy when confronted with a challenging mathematical task, as shown in the figures below.

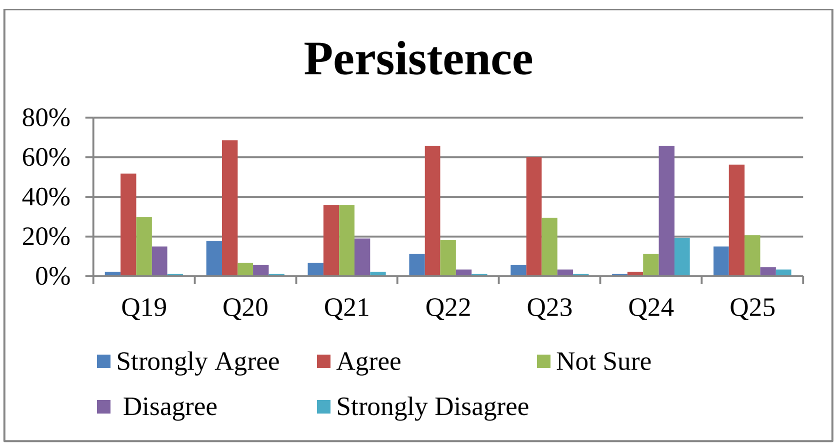

Figure 55 Percentages of students’ responses to Persistence scale on the survey in April 2013

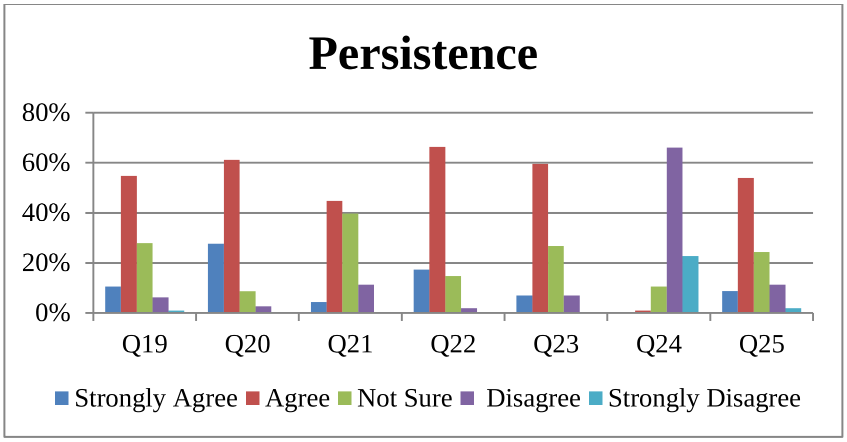

Figure 56 Percentages of students’ responses to Persistence scale on the survey in September 2013

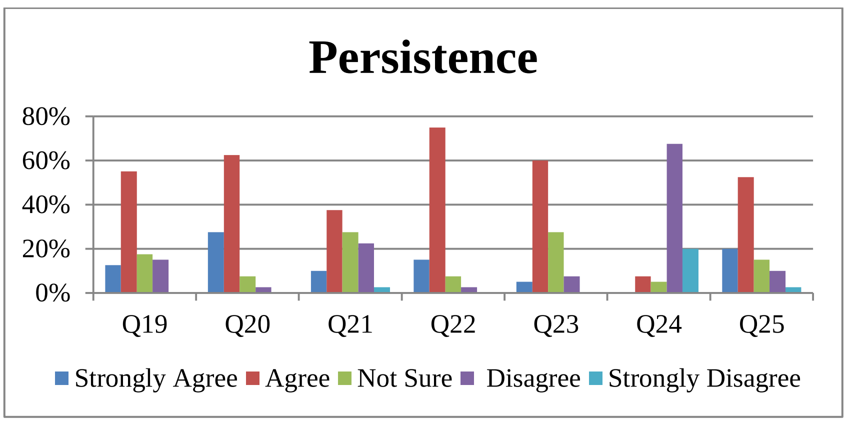

Figure 57 Percentages of students’ responses to Persistence scale on the survey in May 2014

Indeed, Q24, which asked about students’ tendency to give up when confronted with a mathematical task they cannot immediately complete, got the highest level of disagreement, with percentages of approximately 64% disagreeing and an extra 23% who strongly disagreed. However, even though students might persist to achieve their targets in mathematics (as their responses to the other questions suggest), their responses to Q21 suggest that it might not be their preference to select the more challenging mathematical task. That is seen with percentages of more than 50% of the students disagreed or were not sure about the statement: “*When presented with a choice of mathematical tasks, my preference is for a challenging task*”.

5.5 Learning Goals Scale

Learning Goals scale	
Q26.	I work at maths because I like finding new ways of doing things.
Q27.	I work at maths because I like learning new things.
Q28.	I work at maths because I like figuring things out.
Q29.	I work at maths because I want to learn as much as possible.
Q30.	I work at maths because it is important for me that I understand the ideas.

Figure 58 Questions from the Learning Goals scale of the survey

The learning goals scale consists of the five questions shown in Figure 58, again all of which were used by Breen, Cleary and O'Shea (2009a). The majority of students who responded to these questions reflected a positive point-of-view regarding their mathematical learning goals, as shown in the figures below.

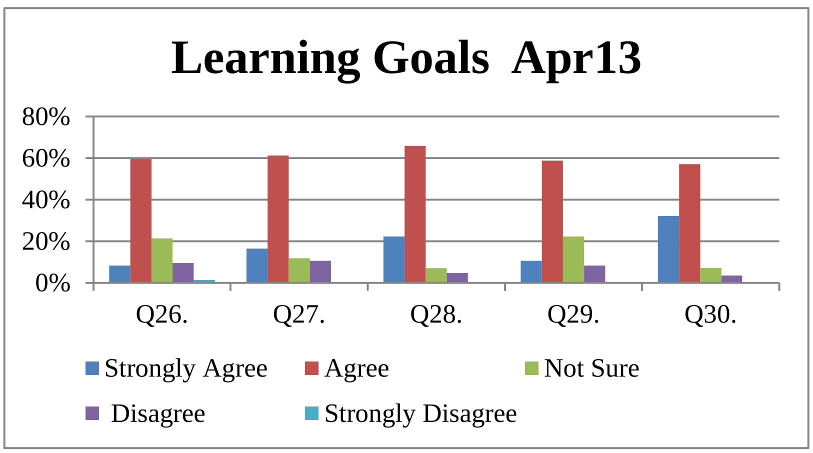

Figure 59 Percentages of students' responses to Learning Goals scale on the survey in April 2013

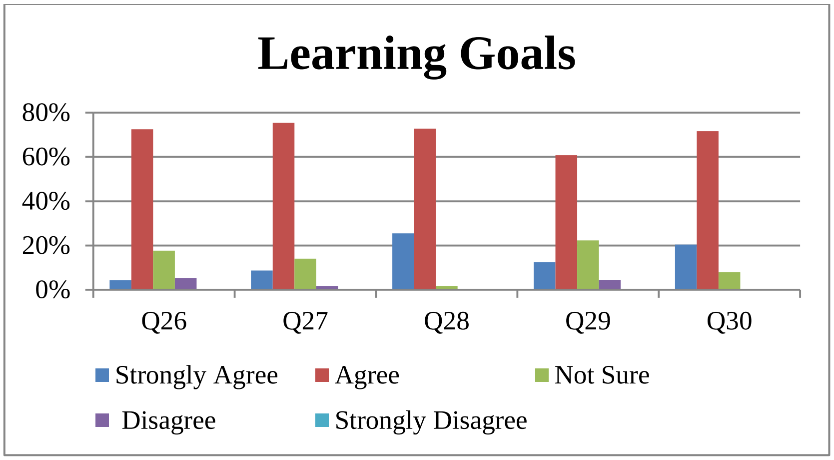

Figure 60 Percentages of students' responses to Learning Goals scale on the survey in September 2013

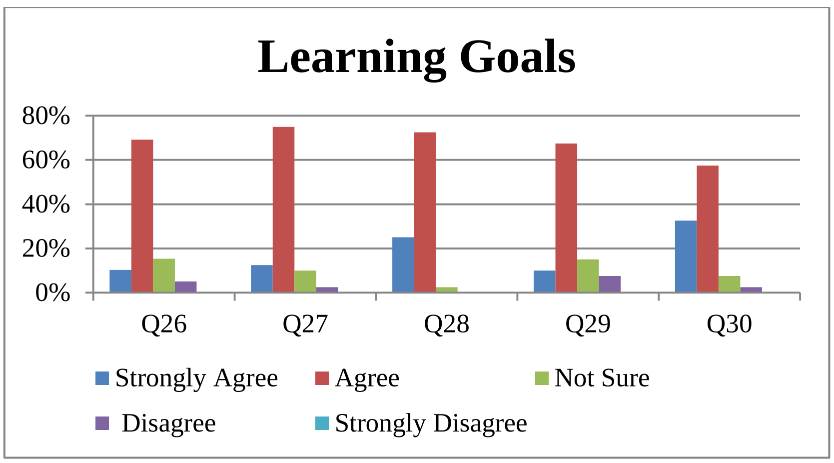

Figure 61 Percentages of students' responses to Learning Goals scale on the survey in May 2014

Most students agreed that the goal of working at mathematics is for the pleasure of learning, figuring things out, and finding new ways or ideas. Responses to the learning goals scale not only reflected students' awareness of the importance of studying mathematics, but also their motivations in working at the subject.

5.6 Approach Scale

Approach Scale	
Q31.	I learn mathematics by understanding the underlying logical principles, not by memorizing the rules.
Q32.	If I cannot solve a mathematical problem, at least I know a general method of attacking it.

Figure 62 Questions from the Approach scale of the survey

The approach scale consisted of two questions. Both questions were adapted from an attitudinal survey designed for new engineering students in Nevada University to assess their beliefs about engineering and mathematics (Fadali, Velasquez-Bryant and Robinson 2004). The selected questions on this scale are an attempt to investigate students' approaches to learning mathematics and determine whether it is by memorizing mathematics rules or understanding the principles of mathematics.

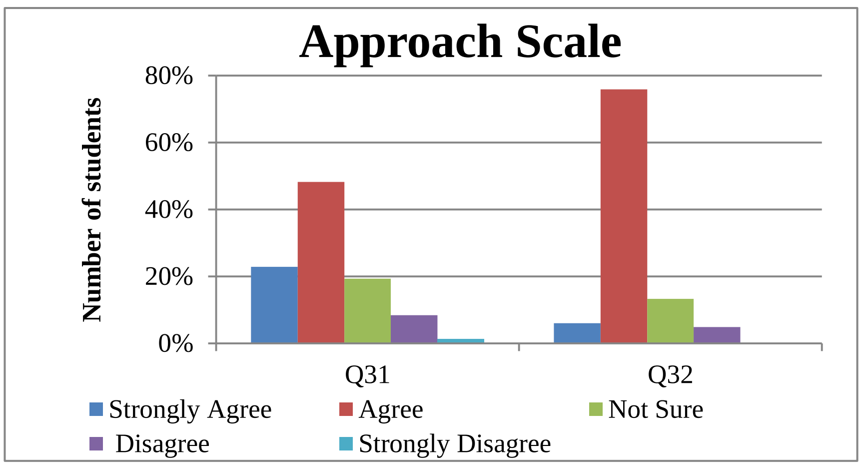

Figure 63 Percentages of students' responses to Approach scale on the survey in April 2013

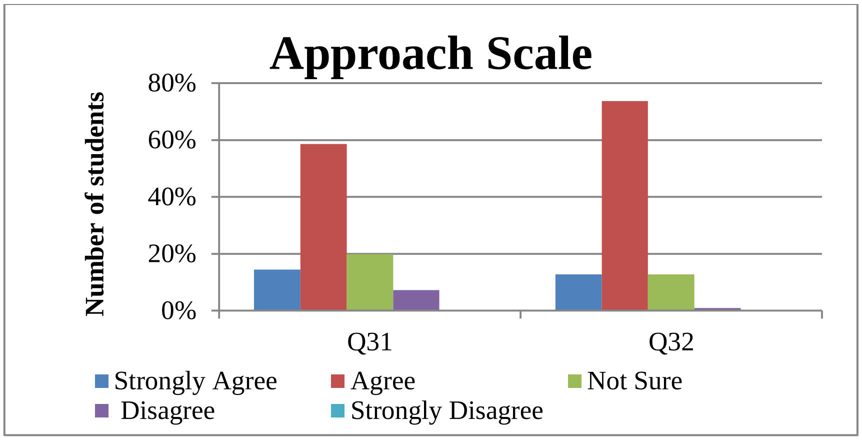

Figure 64 Percentages of students' responses to Approach scale on the survey in September 2013

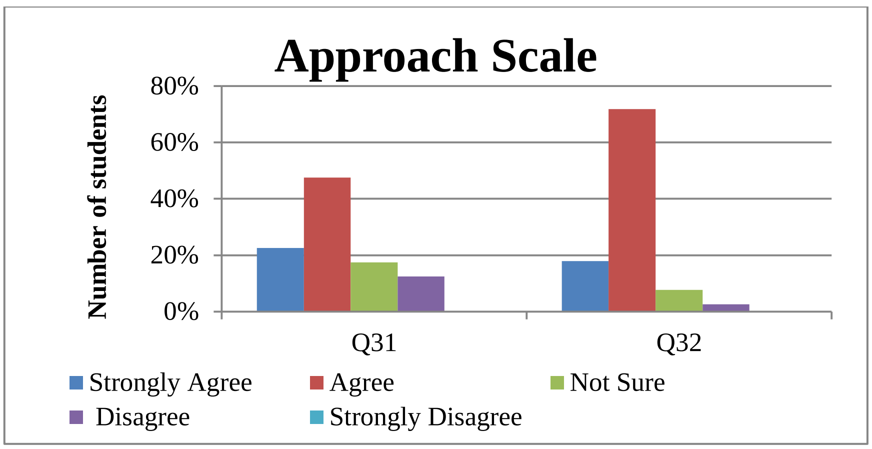

Figure 65 Percentages of students' responses to Approach scale on the survey in May 2014

Looking at Q31, which states the appropriate approach to learning mathematics is to understand its principles instead of memorizing rules, responses reflected a variety of opinions. However, more than 60% of the students overall agreed with the statement. Q32 illustrates an overall positive approach to mathematics, with more than 70% of students overall who agreed and between 6% and 18% who strongly agreed that they knew a general method of attacking a question, even if they could not solve it.

5.7 Prior Experience Scale

Prior Experience Scale	
Q33.	Mathematics is a course in school which I have always enjoyed studying.
Q34.	I have forgotten many of the mathematical concepts that I have learned in secondary school.
Q35.	Maths was enjoyable in secondary school.
Q36.	I have a good background in mathematics.

Figure 66 Questions from the Prior Experience scale of the survey

There are four items questioning prior mathematical experience, as shown in Figure 66. They are specially designed to investigate students' experiences with mathematics in school and specifically in secondary level. They attempt to determine if students' experiences and feelings in relation to post-primary level mathematics are positive overall, bearing in mind the phased implementation of Project Maths that is currently underway. The results of students' responses to these questions are shown in the figures below.

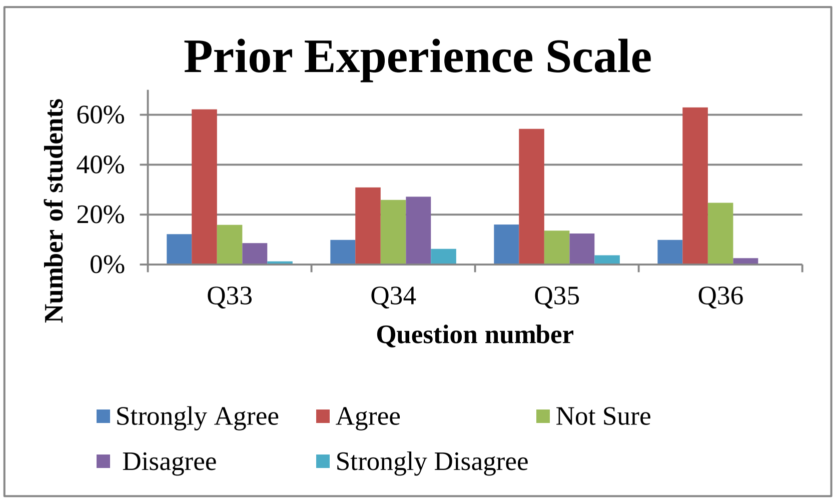

Figure 67 Percentages of students' responses to Prior Experience scale in April 2013

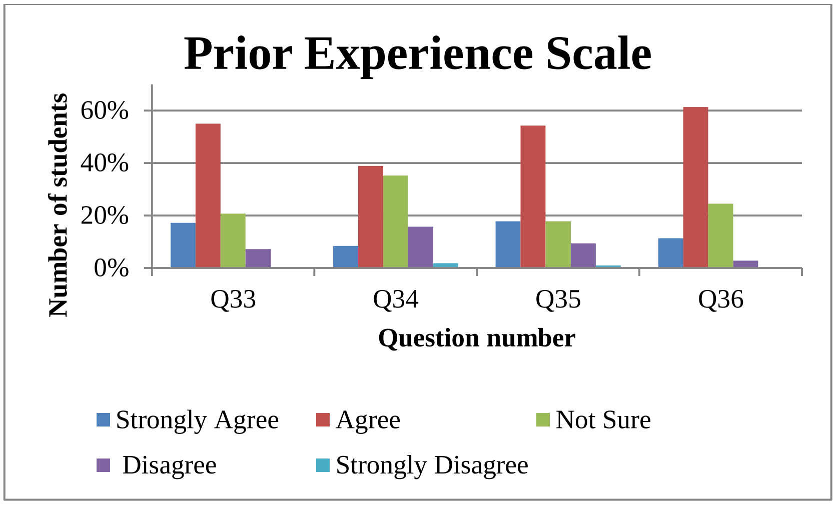

Figure 68 Percentages of students' responses to Prior Experience scale in September 2013

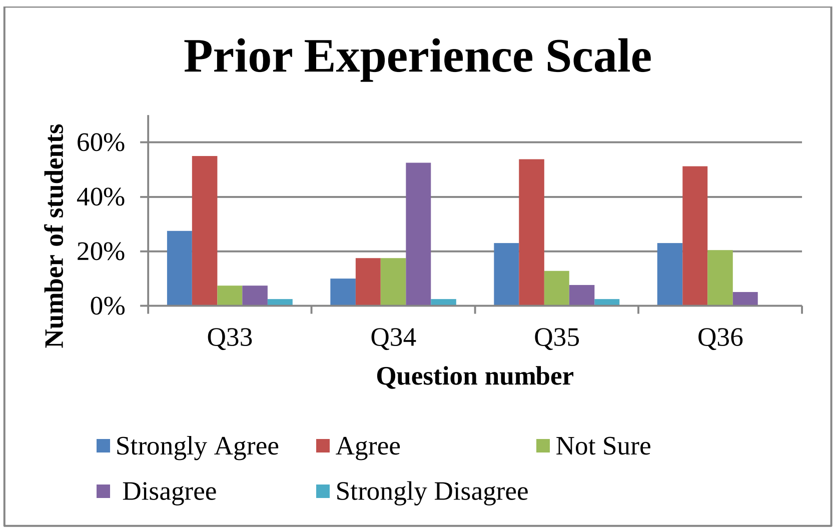

Figure 69 Percentages of students' responses to Prior Experience scale in May 2014

Q33 showed a tendency to agree that mathematics was always "enjoyable" in school, with around 70% who either agree or strongly agree with that statement. When focusing upon emphasizing mathematical enjoyment in secondary school in Q35, comparable results were found. Unlike the rest of questions on the questionnaire, Q34 varied from one sample to another; between 40-50% of the responses of April 2013 and September 2013 groups either agreed or strongly agreed about forgetting mathematical concepts learnt in secondary level, while less than 30% of student responses in May 2014 showed any agreement with that statement. Furthermore, more than 50% of the May 2014 group responses disagreed with that statement. While it might have been predicted that there would be some variation in the responses to this question, it would have been expected that the April 2013 and May 2014 groups would show greater agreement, given that these students had just completed several months of mathematics without a break, whereas the September 2013 cohort had done no mathematics for the previous three months.

5.8 General Scale

General Scale	
Q37.	I apply what I learn in mathematics to real-life situations.
Q38.	I think about maths problems and plan how to solve them.
Q39.	I regularly use computer in mathematics to help me solve maths problems.
Q40.	I set goals and targets about my mathematics learning.
Q41.	I copy what the lecturer writes on the board then practice using examples.

Figure 70 Questions from the General scale of the survey

Lastly on the survey were five questions that explored students' opinions about a number of statements related to mathematics. Students in the three groups again gave similar responses to all of the five questions, as shown in the figures below.

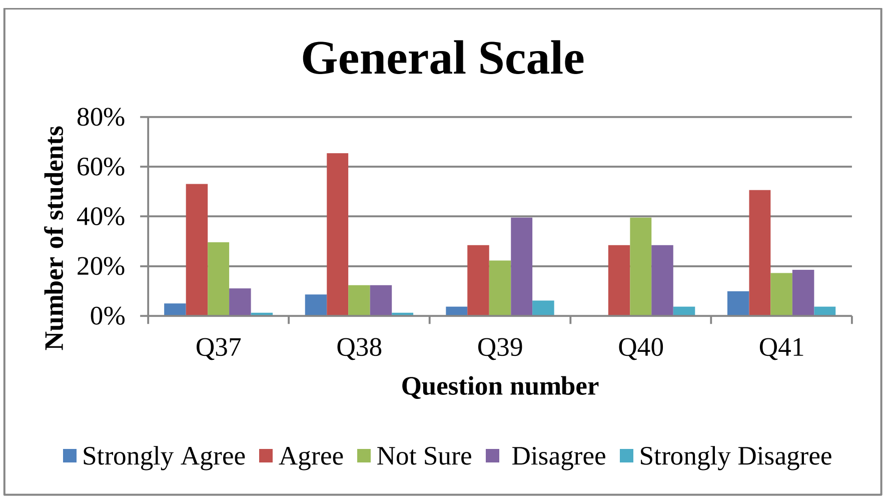

Figure 71 Percentages of students' responses to General scale in April 2013

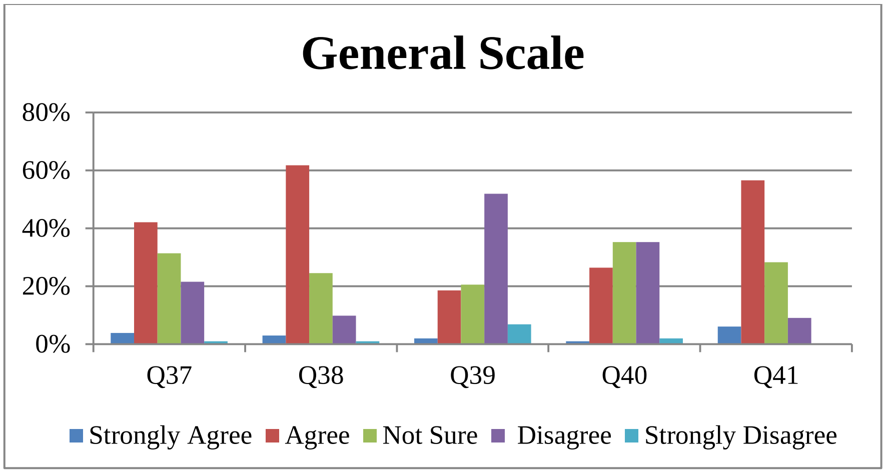

Figure 72 Percentages of students' responses to General scale in September 2013

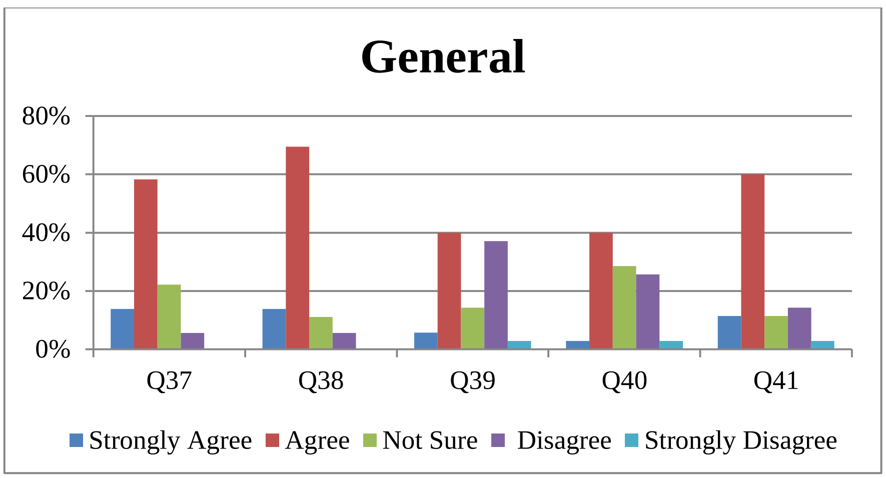

Figure 73 Percentages of students' responses to General scale in May 2014

Students agreed strongly with Q38, which spoke about thinking about mathematics problems and planning how to solve them. Furthermore, students showed strong agreement with Q41 which spoke about copying what lecturers write on the board and then practicing examples, with more than 50% agreeing with the statement and between 6% and 10% strongly agreeing with it.

5.9 Further discussion

Further analysis was carried out on the survey results of all the groups. Firstly, it was decided to separate the responses based on the students' Leaving Certificates mathematics grades, to ascertain whether or not the percentage of students in each sample with high prior achievements in mathematics had impacted upon the responses in the survey. Therefore, responses were categorized into two groupings, Higher Level B3 or higher in mathematics in the first group, and lower than a Higher Level B3 in the second group. The breakdown of the percentage of students in each category is shown in Figure 74.

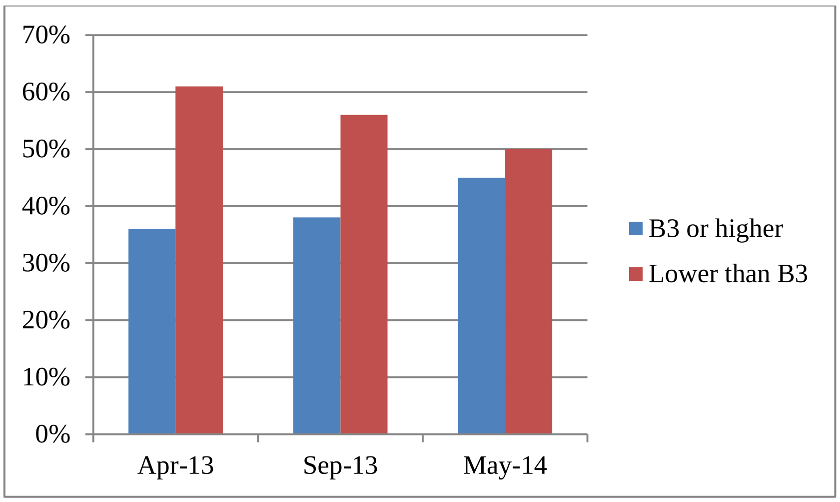

Figure 74 Comparison of students' LC mathematics grades among all samples

It is seen in the figure above that there are differences are apparent between students' grades in the September 13 and May 14 samples, although these students are from the same class. There are two reasons for this difference: firstly, this figure shows the breakdown as a percentage of the sample, and a large number of students who took the test in September 2013 did not do so in May 2014. Secondly, it was observed that a significant proportion of students wrote down a different grade when asked for their LC mathematics grade in May than they had in September.

In order to determine whether students' LC grades are correlated with their attitudes, we excluded the students who had LC grades of B3 or higher from the three main samples. Generally speaking, no noticeable differences were shown compared to the original samples. For instance, by looking at Q3 which stated: *"I can get good marks in mathematics"*, the original samples and those who had lower than B3 gave similar responses, which may be related to students' definitions of a "good mark" or also to their beliefs about their ability to gain good marks despite their previous experiences. Students' responses to that question for all groups are contrasted in Figure 75.

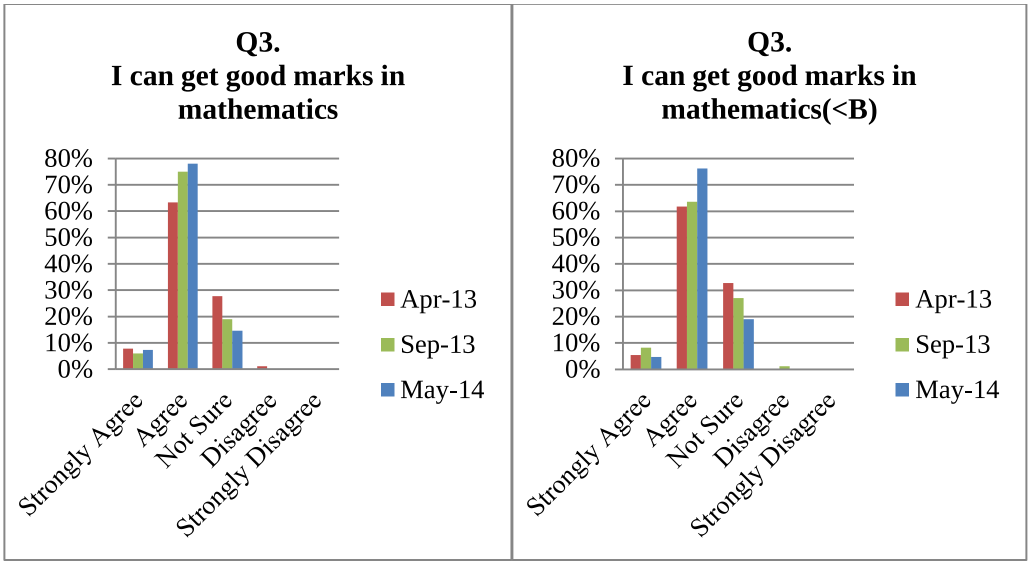

Figure 75 Students' responses to Q3 of the confidence scale of the survey Left: Responses of students of all samples; Right: Responses of students who got lower than HL B3 at LC

Again, when comparing all the questions from the Prior Experience scale between the two cohorts, no significant differences were observed in the students' responses, with both groups responding similarly when asked about their enjoyment of mathematics at school.

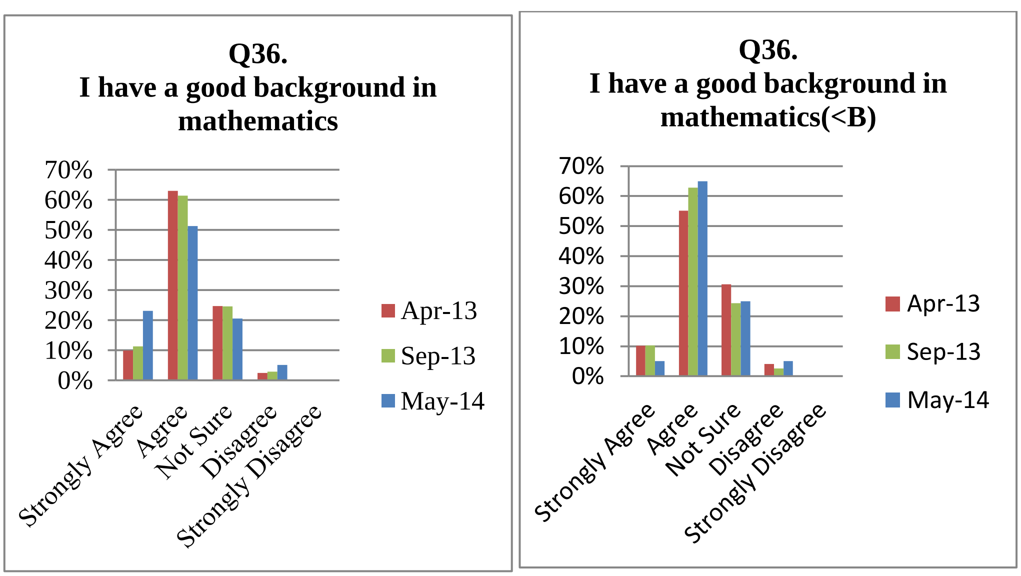

Figure 76 Students' responses to Q36 of the Prior Experience scale of the survey Left: Responses of students of all samples; Right: Responses of students who got lower than HL B3 at LC

For Q36, which related directly to students' experiences in mathematics in post-primary level, the same cohort of students who took the test at the beginning and the end of the same year (samples from Sep13 and May14) showed a change on their responses from strongly disagreeing to only agreeing with the statement: *"I have a good back ground in mathematics"*.

5.10 Links between test and survey results

Although there is no attempt to establish a direct correlation between the results of the test and the survey responses in this study, a comparison between the relevant outcomes is warranted. As mentioned in the methodology, originally an extensive analysis had been planned investigating individual student responses with the outcome of their test, but the homogeneity of the responses in the survey rendered this unnecessary in many cases. However, there remain a number of questions of interest and these are explored now.

Each individual student was given an overall mark in respect of the number of correct responses they obtained and their mean CBM mark. In respect of the number of correct responses, the results of three cohorts of students were isolated from the main sample: those who obtained less than 40% correct answers (as that would be considered a failure in the DCU grading system); those who obtained less than 50% correct answers (indicating over half their answers were incorrect) and those who achieved more than 80% correct answers. With respect to CBM, two cohorts were isolated: this time, those who received less than mean CBM grade of 1 were in one category and those who received more than a mean CBM grade of 2 were in another. It can be seen in the table below that between 17% and 34% of the students across the three samples answered half or more of the test questions incorrectly, whereas less than 20% of the students of the sample one and three, and only 3% of the students of sample two responded correctly to 80% of the questions.

Table 26 showing the percentage of students in each category (less than 40% correct answers; less than 50% correct answers and more than 80% correct answers; mean CBM score less than 1 and mean CBM score greater than 2)

	CA <40%	**CA <50%**	**CA >80%**	**Mean CBM<=1**	**Mean CBM>=2**
Apr-13	3%	21%	19%	46%	14%
Sep-13	8%	34%	3%	71%	3%
May-14	5%	17%	14%	18%	7%

In contrast, between 78% and 95% of the students across the three samples agreed or strongly agreed that they can get *"good marks"* in mathematics. It should be noted at this point that it may be possible that students can achieve "good marks" in mathematics with some weak core skills, depending on how the grading for the examination is carried out. O'Sullivan, Breen and O'Shea (2012) note that "*there is a growing tension between criticism of the predictability of the state examinations and the public's demand for familiarity in order to be fair to candidates"*. Students were not specifically prepared for the mathematics test they took as part of this study and so it tested the core skills they knew without revising.

Moreover, when a closer look was taken at the September 2013 group, tested at the beginning of the year, it was found that more than 70% of the students in that sample agreed or strongly agreed that they have a good background in mathematics, whereas only 3% of that group got more than 80% of the test questions correct, and more than 34% of the students of that group answered incorrectly to over half the test questions.

A closer look was also taken at Q40 in the survey which stated: "I set goals and targets about my mathematics learning", as this question received a variety of responses. It was decided to investigate the test performance of students who did not agree with the statement and the mean number of correct answers for that group was found to be 62%.

Overall, many of the survey questions showed little difference between students with varying mathematical prior achievements and different test scores, as the responses to the survey were homogeneous across almost all scales explored.

Overall, this gives rise to a picture of students who have what could be considered to be a false confidence in their mathematical abilities. This is discussed further in the Conclusions.

6 Conclusions and recommendations

This study investigates first year engineering students' attitudes and beliefs towards mathematics, and tests some basic mathematical skills that are vital to their studies in the future. It was conducted on Dublin City University's (DCU) first year engineering students during the academic years of 2012-2013 and 2013-2014 at the beginning and/or the end of their first year.

The test used a paired questions approach and involved a certainty based marking (CBM) scheme to provide a more advanced assessment of the students' knowledge in each category of the test. The test consisted of 40 questions which were categorized into three main areas of mathematics: number, algebra and calculus.

The survey was structured using a five-point Likert scale and consisted of 41 questions. The questions were divided into eight different scales: Confidence, Anxiety, Theory of Intelligence, Learning Goals, Persistence, Approach, Prior Experience, and General.

While both the test and survey were adapted from existing material, they have not been used in combination on the same sample of students before this work. In addition, part of the significance of this work is that the introduction of the CBM grading system to the diagnostic test provided a new approach to marking such a mathematics test and allowed links to be made between the confidence expressed by students in their answers to basic mathematical questions and the confidence they expressed in their mathematical ability in general.

6.1 Findings

The results of this study have been reported in detail in the previous two chapters. The main findings are now highlighted below.

1. When negative numbers (including negative indices) featured in questions, these were routinely more poorly answered than questions involving only positive numbers. In addition, students expressed lower certainty levels when dealing with such questions. Such findings are not unique to this cohort of students, as recent studies (Cangelosi et al. 2013, Gullick and Wolford 2014, Altiparmak and Özdoğan 2010) have reported similar issues in relation to students ranging from elementary school right up to university students.

2. Questions involving basic differentiation showed consistently good student performance, both in terms of the percentage of correct responses and the mean CBM score. Such questions were constantly in the top five best-answered questions on the test.
3. Students also performed consistently well in questions testing basic factorisation, with high percentages of correct answers and high mean CBM scores.
4. When dealing with a quadratic equation which required the use of the quadratic formula, students performed poorly. While this may have been as a result of not having access to a log-book to check the formula, the CBM results showed high levels of confidence associated with incorrect answers, indicating that students were unaware of their lack of knowledge of the precise formula. This means they may be inclined to try to solve such an equation without referencing the log-book, or they may be using the formula incorrectly.
5. Integration by substitution was the most poorly answered in terms of the percentage of correct answers given to the question. However, it was not in the bottom five questions in terms of mean CBM score, which indicates that students are aware of their knowledge gap in this area.
6. Transposition of formulae was another area that caused difficulties for students. While the percentage of correct answers for such questions were not among the poorest, the mean CBM score was the lowest of all questions, indicating high levels of confidence paired with incorrect responses and low levels of confidence paired with correct responses.
7. In the paired-questions approach, even if two questions in a pair accurately examined the same skill and involved the same number of steps, it was found that the second question of that pair was more likely to get incorrect responses.
8. The sample who performed most poorly on the mathematics test was those students who took the test at the start of first-year engineering. The other two samples, tested at the end of the year, scored better. However, no such difference was observed between responses to the survey.
9. Students reported high levels of confidence in relation to their mathematical abilities when responding to the survey. This agrees with the findings of Parsons, Croft and Harrison (2009) who found that the majority of first-year engineering students were fairly confident in relation to mathematics. This confidence was reflected in the mathematics test, where students marked high levels of confidence for 51%-68% of

the questions, with medium levels for 13%-22%. As mentioned in other findings above, there were several notable instances where these high levels of confidence were mostly accompanying incorrect answers.

10. Responses to survey questions reported low levels of mathematical anxiety among the engineering students, although up to 30% of students agreed or were unsure as to whether they felt nervous during mathematics lectures or worried about the difficulty of the lectures.
11. Students self-reported high levels of positive attitudes towards mathematics and motivation towards achieving in the subject. Again, this agrees with the findings of Parsons, Croft and Harrison (2009) for a similar cohort of students.

6.2 Implications

The results of this study have implications for a number of different stakeholders, both within DCU and beyond, including students, lecturers and mathematics support personnel. Each of these will now briefly be addressed in turn.

- This study allowed the students involved to gain a greater awareness of the extent of their knowledge in various core areas of basic mathematics. This was particularly the case for questions where students indicated a high level of confidence in incorrect answers, as this was indicative of misconceptions of which the students were unaware. Once this was brought to their attention, they were in a position to work on these areas and improve their overall mathematical skillset.
- Lecturers within DCU, particularly those directly involved with the cohort in this study, were provided with additional knowledge regarding areas of weakness of the students, e.g. laws of logarithms, integration, negative indices. The results of the survey also indicated that some students who may be confident in their mathematical ability have weak core skills, meaning that lecturers in general need to be aware that such confident students may be less likely to seek additional help to rectify these skills, due to a lack of awareness of their part.
- For those working in mathematics support centres, or those directly involved in diagnostic testing, there is a clear added advantage to introducing a paired-questions approach in addition to a CBM marking scheme into such tests, based on the research undertaken in this project. This provides much richer data than traditional marking schemes as well as highlighting knowledge gaps and misconceptions. This

in turn will make it easier to provided targeted support at an early stage on topics which are poorly answered in such a test.

6.3 Future work

The research conducted in this study could be enhanced and extended in a number of ways, based on the findings of this study and the limitations inherent in its design and scope. Some suggestions for future work are now given.

1. This study is limited in that it was only undertaken over the course of two academic years with a relatively small group of students from a single HEI. As a result, only three samples of the test and survey were available for analysis. In order to draw any strong conclusions from this work, it would be necessary to continue to administer both over the course of a number of years, possibly including a larger sample of students, or those from several different HEIs. Alternatively, students other than engineering students (e.g. science students, preservice mathematics teachers) could be included in a larger study. However, this study provides the necessary groundwork for this to be undertaken, as well as having highlighted important findings to date.
2. In the paired-questions approach, it had been assumed that when students responded correctly to only one question in a pair, that might indicate either a lucky guess or a minor slip. But, as mentioned in the findings above, even if two questions in a pair examined the same skill in the same number of steps, the second question was more poorly answered. Possible reasons for this may include that students may be more likely to make a slip in the second question as their concentration is lower when repeating the same operations for the second time. Alternatively, when producing paired questions on test, there may be a tendency on the part of the examiner to subconsciously increase the difficulty level in the second question, even where it is intended that the questions be well matched. To examine this issue to greater detail, a series of tests could be distributed in which the order of the questions in each pair are swapped to ascertain whether this has an impact upon student responses or not. However, levels of certainty indicated with both questions should be also considered.
3. The survey in this study was distributed at the start and/or end of first year. However, the pilot was distributed at the end of the first semester and showed

strikingly different results. Undertaking such a study at three points during the year instead of two would allow for a comparison between students' attitudes at various points during their studies, particularly as the period towards the end of the first semester has been identified as a critical one in terms of student retention (Yorke and Longden 2008).

4. The CBM scoring used in the test in this study may not be the optimal one to reflect students' declared confidence levels in their answers. Investigating the use of different weightings within the scoring system would be of use in future mathematics tests.
5. This study was undertaken against the backdrop of a major change in the teaching and learning of mathematics at post-primary level, in the form of Project Maths. Investigating students' continued performance on such mathematical skills as well as their attitudes towards mathematics as Project Maths is phased in fully would be of interest.

6.4 Recommendations

As a result of the findings of this study, a number of recommendations can be made in terms of improving the teaching and learning of mathematics on a large scale.

1. Particular attention should be given in post-primary mathematics education to dealing with negative numbers. This might be done by ensuring, where appropriate, that students are exposed to more problems involving negative numbers than positive ones, providing opportunities for them to practice and refine the techniques needed.
2. Intensive interventions should be put in place during the course of first-year engineering programmes to ensure that students' core mathematical skills are robust. Although improvements were made in the course of the year for most of the questions on the test, significant deficiencies were still identifiable in basic skills.
3. Online mathematics self-assessment opportunities using CBM-based marking schemes should be introduced for engineering students in HEIs. These assessments should be available to students beyond first year, as this research shows that some serious knowledge gaps and misconceptions remain at the end of first year. The CBM approach will inform students about their areas of weakness and highlight any misconceptions they have. Such ongoing assessments are in place in several HEIs

already (Carr, Bowe and Ní Fhloinn 2013) but without the benefit of including CBM.

Data collected from this study and analysis of the data has provided some evidence of engineering students' levels of mathematics upon entry to a HEI in Ireland, as well as their beliefs and attitudes about mathematics.

7 Appendix

Appendix A: Mathematics Test

Name (BLOCK CAPITALS):

Course:

Student Number:

INSTRUCTIONS FOR TEST

1. This test consists of 40 multiple-choice questions.

2. Each question has five possible answers (A, B, C, D or E), one of which is correct. Circle the letter beside the appropriate answer.

3. If you do not know how to do a particular question, leave it blank - DO NOT GUESS.

4. Answer as many questions as you can. You should work entirely on your own, and not consult friends or textbooks.

5. **No calculators are to be used**.

6. After each question, indicate how certain you are (Low, Medium or High) about your answer by circling this option. YOU MUST CIRCLE ONE OF THESE OPTIONS OR YOU WILL RECEIVE NO MARKS FOR THAT PARTICULAR QUESTION.

7. The test is **negatively marked**. You get the following marks for each question:

Certainty Level	Low	Medium	High	No Reply
Mark if correct	1	2	3	0
Mark if incorrect	0	-2	-6	0

8. NOTE: THE WAY TO DO BEST IN THIS TEST IS TO BE **HONEST** ABOUT YOUR LEVEL OF CERTAINTY IN EACH QUESTION.

9. As a guide, this test should take about 40 minutes to complete.

1. The value of 3^{-2} is

A) $\frac{1}{6}$ B) -6 C) $\frac{1}{9}$ D) -9 E) None of the above

Certainty Level: Low Medium High

2. The value of $4^{\frac{1}{2}} + 9^{\frac{1}{2}}$ is

A) 29 B) 43 C) 5 D) 19 E) None of the above

Certainty Level: Low Medium High

3. The value of $\frac{4}{5} + \frac{1}{4}$ is

A) $\frac{21}{20}$ B) $\frac{5}{9}$ C) $\frac{24}{20}$ D) $\frac{19}{20}$ E) None of the above

Certainty Level: Low Medium High

4. The value of $\frac{5}{7} \times \frac{3}{4}$ is

A) $\frac{8}{28}$ B) $\frac{53}{74}$ C) $\frac{15}{28}$ D) $\frac{20}{21}$ E) None of the above

Certainty Level: Low Medium High

5. The expression $-2x^3 + 5x^2 + 3 - (x^2 - 6x - 4)$ can be simplified to

A) $-2x^3 + 4x^2 - 6x - 1$ B) $-3x^3 + 5x^2 + 6x + 7$ C) $-2x^3 + 4x^2 + 6x + 7$

D) $-2x^3 + 6x^2 - 6x - 1$ E) None of the above

Certainty Level: Low Medium High

6. Removing the brackets in the expression $(s + 2)(s^2 - 3)$ gives

A) $s^3 + 2s^2 - 3s - 6$ B) $s^3 - s^2 - 6$ C) $s^3 + 2s^2 - 3s + 6$

D) $s^3 + s^2 - 6$ E) None of the above

Certainty Level: Low Medium High

7. If $x = -1$, the expression $\frac{2x^2-x+1}{3-x}$ has the value

A) 0 B) $\frac{1}{2}$ C) 1 D) $-\frac{1}{2}$ E) None of the above

Certainty Level: Low Medium High

8. The value of $3x^{-\frac{1}{2}}$ when $x = \frac{1}{9}$ is

A) 1 B) 9 C) -1 D) 3 E) None of the above

Certainty Level: Low Medium High

9. The expression $(y^x)^3$ is equal to

A) $3y^x$ B) y^{3x} C) y^{x+3} D) y^{x^3} E) None of the above

Certainty Level: Low Medium High

10. The expression $\frac{4x^{-2}y^2}{x^3y^{-4}}$ can be written as

A) $4\left(\frac{x^5}{y^6}\right)$ B) $\frac{4}{xy^{-2}}$ C) $\frac{y^6}{4x^5}$ D) $4\left(\frac{y^6}{x^5}\right)$ E) None of the above

Certainty Level: Low Medium High

11. The quadratic expression $x^2 - 7x - 18$ can be factorised as

A) $(x-9)(x+2)$ B) $(x+9)(x-2)$ C) $(x-6)(x+3)$

D) $(x+6)(x-3)$ E) None of the above

Certainty Level: Low Medium High

12. The quadratic expression $25a^2 - 9b^2$ can be factorised as

A) $(25a^2+9b^2)(a-b)$ B) $(25a^2-9b^2)(a+b)$ C) $(2a-3b)^2$

D) $(5a-3b)(5a+3b)$ E) None of the above

Certainty Level: Low Medium High

13. Expressing $\frac{1}{x} - \frac{3}{y}$ as a single fraction, gives

A) $\frac{-2}{x-y}$ B) $\frac{-2}{xy}$ C) $\frac{y-3x}{xy}$ D) $\frac{y-3x}{x-y}$ E) None of the above

Certainty Level: Low Medium High

14. Expressing $\frac{2}{2x+3} - \frac{1}{x+1}$ as a single fraction, gives

A) $\frac{1}{(2x+3)(x+1)}$ B) $\frac{-1}{(2x+3)(x+1)}$ C) $\frac{1}{x+2}$ D) $\frac{-x-2}{(2x+3)(x+1)}$ E) None of the above

Certainty Level: Low Medium High

15. The expression $\frac{6x-12y}{-3}$is equivalent to

A) $2x-4y$ B) $4y-2x$ C) $\frac{4y-2x}{3}$ D) $12y-6x$ E) None of the above

Certainty Level: Low Medium High

16. The expression $\frac{2+4x-x^3}{2x^2}$can be written as

A) $\frac{1}{x^2}+\frac{2}{x}-\frac{x}{2}$ B) $1+\frac{2}{x}-\frac{x}{2}$ C) $\frac{2}{x^2}+\frac{2}{x}-\frac{x}{2}$ D)$\frac{1}{x^2}+\frac{2}{x}-\frac{x^2}{2}$ E) None of the above

Certainty Level: Low Medium High

17. If $\frac{1}{z}=\frac{1}{x}+\frac{1}{y}$,then z is given by

A) $z=\frac{x+y}{xy}$ B) $z=\frac{xy}{x+y}$ C) $z=x+y$ D) $z=-\frac{1}{x}-\frac{1}{y}$ E) None of the above

Certainty Level: Low Medium High

18. If $x=\frac{a}{b-c}$, then b is given by

A) $b=\frac{a}{x-c}$ B) $b=\frac{a}{x}-c$ C) $b=\frac{a+c}{x}$ D) $b=\frac{a}{x}+c$ E) None of the above

Certainty Level: Low Medium High

19. The solutions of the quadratic equation $x^2 + 4x - 77 = 0$ are

A) $x = -77, x = 1$ B) $x = 77, x = -1$ C) $x = 11, x = -7$

D) $x = -11, x = 7$ E) None of the above

Certainty Level: Low Medium High

20. The solutions of the quadratic equation $x^2 + 6x + 2 = 0$ can be written as

A) $x = -3 + \sqrt{28}, x = -3 - \sqrt{28}$ B) $x = -6 + \sqrt{28}, x = -6 - \sqrt{28}$

C) $x = -3 + \sqrt{7}, x = -3 - \sqrt{7}$ D) $x = 3 + \sqrt{7}, x = 3 - \sqrt{7}$ E) None of the above

Certainty Level: Low Medium High

21. The equation of the line through the point $(2, -2)$ and with slope -3 is

A) $y + 3x - 4 = 0$ B) $y - 3x + 8 = 0$

C) $3y + x + 4 = 0$ D) $-3y + x - 8 = 0$ E) None of the above

Certainty Level: Low Medium High

22. The equation of the straight line through the points $(3, 0)$ and $(1, -1)$ is

A) $y - 2x + 6 = 0$ B) $y + 2x - 6 = 0$

C) $2y - x + 3 = 0$ D) $2y + x - 3 = 0$ E) None of the above

Certainty Level: Low Medium High

23. If $4x + 3y = 15$ and $x - 3y = -10$, then x is equal to

A) 1 B) 5 C) -1 D) -5 E) None of the above

Certainty Level: Low Medium High

24. The solution of the simultaneous equations $5x + y = 8, -3x + 2y = -10$ is

A) $x = \frac{6}{5}, y = 2$ B) $x = 1, y = 3$ C) $x = -2, y = 18$

D) $x = 2, y = -2$ E) None of the above

Certainty Level: Low Medium High

25. Expressing $2\log x + 3\log y$ as a single log term gives

A) $\log(2x+3y)$ B) $\log(6xy)$ C) $\log(x^2+y^3)$ D) $\log(x^2y^3)$ E) None of the above

Certainty Level: Low Medium High

26. Expressing $x\log 3 - 2\log x$ as a single log term gives

A) $\log(\frac{3^x}{x^2})$ B) $\log(3^x-2x)$ C) $\log(3^x-x^2)$ D) $\log(\frac{3}{2x})$ E) None of the above

Certainty Level: Low Medium High

27. Simplifying the expression $3e^{4t}e^{-2t}$ gives

A) $3e^{2t}$ B) $3e^{-8t}$ C) $(e^{2t})^3$ D) $(e^{-8t})^3$ E) None of the above

Certainty Level: Low Medium High

28. Simplifying the expression $\frac{(e^x)^2}{e^{2x}}$ gives

A) $e^{(x^2-2x)}$ B) $2e^{-x}$ C) 1 D) $e^{x^2}-e^{2x}$ E) None of the above

Certainty Level: Low Medium High

29. The expression $\frac{1}{(x+2)(x+3)}$ when split into partial fractions is

A) $\frac{3}{x+2}+\frac{2}{x+3}$ B) $\frac{3}{x+2}-\frac{2}{x+3}$ C) $\frac{1}{x+2}-\frac{1}{x+3}$

D) $\frac{1}{x+2}+\frac{1}{x+3}$ E) None of the above

Certainty Level: Low Medium High

30. When expressing $\frac{2x}{(x-2)(x+1)^2(x^2+1)}$, in partial fractions, the appropriate form, with a,b,c,d,e and f constants, is

A) $\frac{a}{x-2}+\frac{b}{x+1}+cx+\frac{d}{x^2+1}$ B) $\frac{a}{x-2}+\frac{b}{(x+1)^2}+cx+\frac{d}{x^2+1}$

C) $\frac{a}{x-2}+\frac{b}{x+1}+\frac{cx+d}{(x+1)^2}+\frac{ex+f}{x^2+1}$ D) $\frac{a}{x-2}+\frac{b}{x+1}+\frac{c}{(x+1)^2}+\frac{dx+e}{x^2+1}$ E) None of the above

Certainty Level: Low Medium High

31. Find $\frac{dy}{dx}$ where $y = x^4$

A) $4x^3$ B) $4x^4$ C) $5x^5$ D) x^3 E) None of the above

Certainty Level: Low Medium High

32. Find $\frac{ds}{dt}$ where $s = t^7$

A) $7t^6$ B) t^6 C) $7t^8$ D) $\frac{x^8}{7}$ E) None of the above

Certainty Level: Low Medium High

33. Solve $\frac{d}{dx} \sin(2x+1) =$

A) $2\cos(2x+1)$ B) $2\sin(2x+1)$

C) $\cos(2x+1)$ D) $-\cos(2x+1)$ E) None of the above

Certainty Level: Low Medium High

34. Find $\frac{ds}{dt}$ where $s = \cos(5t-4)$

A) $\sin(5t-4)$ B) $-5\sin(5t-4)$

C) $-5\cos(5t-4)$ D) $-\sin(5t-4)+5$ E) None of the above

Certainty Level: Low Medium High

35. Solve $\frac{d}{dx}(x^3 \cos x) =$

A) $3x^2 \cos x - x^3 \sin x$ B) $3x^2 + \cos x$

C) $3x^2 + x^3 \cos x$ D) $-x^3 \sin x$ E) None of the above

Certainty Level: Low Medium High

36. Find $\frac{dy}{dx}$ where $y = x^2 \sin 3x$

A) $2x \sin 3x + 6\cos 3x$ B) $2x \sin 3x - 3x^2 \cos 3x$

C) $2\sin 3x + x^2 \cos 3x$ D) $2x \sin 3x + 3x^2 \cos 3x$ E) None of the above

Certainty Level: Low Medium High

37. Find $\int x^3\,dx$

A) $\frac{x^4}{4}+c$ B) $4x^4+c$ C) x^3+c D) $3x^3+c$ E) None of the above

Certainty Level: Low Medium High

38. Find $\int t^{-4}\,dt$

A) $-4t^{-3}+c$ B) $\frac{-t^{-3}}{3}+c$ C) $t^{-5}+c$ D) $\frac{t^{-5}}{5}+c$ E) None of the above

Certainty Level: Low Medium High

39. Evaluate the following integral $\int \cos(x+2)\,dx$

A) $-\sin(x+2)+c$ B) $\sin(x+2)+c$

C) $2\cos(x+2)+c$ D) $-2\cos x+c$ E) None of the above

Certainty Level: Low Medium High

40. Evaluate the following $\int x(4x^2-7)^3\,dx$

A) $\frac{(4x^2-7)^2}{8}+c$ B) $\frac{(4x^2-7)^2}{32}+c$ C) $\frac{(4x^2-7)^4}{8}+c$

D) $\frac{(4x^2-7)^4}{32}+c$ E) None of the above

Certainty Level: Low Medium High

Appendix B: The attitudinal survey

Name ____

Class____

Student Number _______________________________

Gender □M □F

Mature student □Yes □No

Year of leaving cert. ______

Leaving Cert Maths Level:

□HL □OL □None

Leaving Cert Maths grade:

□A1 □A2 □B1 □B2 □B3 □C1 □C2 □C3 □D1 □D2 □D3 □Other

Thank you for cooperation in participating in this survey.

Confidentiality Clause: Your name is being used only to match data for statistical analysis. It will not be used for any other purpose. Data will be collected throughout the semester, and once the data is matched, all names will be removed. Your lecturer(s) will not have access to your opinions.

Each of the statements on this survey expresses a feeling that a particular person has toward mathematics. There are no right or wrong answers. You are to express, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are:

- Strongly agree
- Agree
- Not sure
- Disagree
- Strongly disagree

You are to circle that best indicates how closely you agree or disagree with the feeling expressed in each statement.

1. I learn mathematics quickly.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

2. I feel confident in approaching mathematics.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

3. I can get good marks in mathematics.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

4. I have trouble understanding anything with mathematics in it.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

5. Mathematics is one of my worst subjects.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

6. I am just not good at mathematics.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

➢ **How confident you are that you can do well in maths?**

..
..
..
..

7. I get very nervous during maths lectures.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

8. I often worry that it will be difficult for me in maths lectures.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

9. I often feel helpless when doing a maths problem.
   ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

10. Mathematics makes me feel uneasy and confused.
    ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

11. I usually feel at ease doing mathematics problems.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **Compared to other subjects, how much do you worry about how well you are doing in maths? And why?**
...

12. You have to be smart to do well in maths.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

13. People are either good at maths or they are not.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

14. Some people will never do well in maths no matter how hard they try.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

15. You can succeed at anything if you put your mind to it.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

16. You can succeed at maths if you put your mind to it.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

17. It is possible to improve your mathematical skills.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

18. Everyone can do well in maths if they work at it.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **What level of intelligence do you think is necessary to do well in maths? Please explain**.
...

19. I will risk showing that I don't know something in order to acquire new mathematical knowledge.

●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

20. I am most proud of my mathematical performance when I feel I have done my best.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

21. When presented with a choice of mathematical tasks, my preference is for a challenging task.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

22. When presented with a mathematical task I cannot immediately complete, I increase my efforts.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

23. When presented with a mathematical task I cannot immediately complete, I persist by changing strategy.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

24. When presented with a mathematical task I cannot immediately complete, I give up.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

25. When presented with a choice of tasks, my preference is for one I know I can complete.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **What do you do when presented with mathematical tasks that you can't immediately do?**
...
...
...
..

26. I work at maths because I like finding new ways of doing things.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

27. I work at maths because I like learning new things.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

28. I work at maths because I like figuring things out.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

29. I work at maths because I want to learn as much as possible.

●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

30. I work at maths because it is important for me that I understand the ideas.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **What are your goals in learning mathematics? ....**

31. I learn mathematics by understanding the underlying logical principles, not by memorising the rules.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

32. If I cannot solve a mathematical problem, at least I know a general method of attacking it.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

33. Mathematics is a course in school which I have always enjoyed studying.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

34. I have forgotten many of the mathematical concepts that I have learned in secondary school.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

35. Maths was enjoyable in secondary school.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

36. I have a good background in mathematics.
●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **How do you describe your math experience in secondary school? ....**

37. I apply what I learn in mathematics to real-life situations.

●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

38. I think about maths problems and plan how to solve them.
    ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

39. I regularly use computer in mathematics to help me solve maths problems.
    ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

40. I set goals and targets about my mathematics learning.
    ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

41. I copy what the lecturer writes on the board then practice using examples.
    ●Strongly agree ●Agree ●Not sure ●Disagree ●Strongly disagree

- **In one or two sentences, please describe what most influenced your decision to study Engineering**
  ..
  ..
  ..
  ..

If you did your Leaving Cert last year in Ireland, please answer the following two questions:

- **What do you think about Project Maths?**
  ..
  ..
  ..

- **What effects did Project Maths have upon your maths abilities \ experience?**
  ..
  ..
  ..
  ..

- **Any other comments?**
  ..
  ..
  ..

End of Survey
Thank you

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