AN INVESTIGATION OF THE MATHEMATICAL LITERACY OF FIRST YEAR THIRD LEVEL STUDENTS IN THE REPUBLIC OF IRELAND

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ABSTRACT

This paper reports on a study carried out to measure the mathematical literacy of a selection of students entering third level education in Ireland. The study investigates how such students performed when confronted with mathematical tasks, which, though commensurate with their level of education, may not have been familiar to them, and to identify the factors influencing their performance. Moreover, the relationship between the skills measured by the test of mathematical literacy administered and those measured by state examinations was explored, as was the question of whether or not the concept of mathematical literacy is a useful one for third level educators.

Keywords: Mathematical literacy, confidence, gender, prior mathematical experience.

1. Introduction

It is recognised that the future economic success and social well-being of countries is closely linked to the knowledge and skills of their populations. The representative body of Irish business and employers, IBEC, recently emphasised that developing a knowledge economy depends on a strong supply of scientists and engineers, and for students to pursue these disciplines they must have a strong foundation in mathematics [1]. The United Kingdom has also identified the economic importance of producing both numerate citizens and top-class mathematicians, scientists and engineers [2], while Australia recognises that the decline in standards in school mathematics is impacting on enrollment in third level science and engineering courses and may, eventually, lead to an industry sector starved of skills [3]. The OECD's Programme for International Student Assessment (PISA) is designed to measure how prepared students are to meet the challenges of the future, rather than how well they have mastered particular curricula. To this end, the PISA studies measure the mathematical literacy of 15-year-olds by necessitating their formulation of, engagement with and solution of mathematical problems, highlighting the role mathematics plays in everyday life. An emphasis is placed on the understanding of concepts, the mastery of processes and the students' ability to function in a variety of real-life situations. PISA defines mathematical literacy as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen' ([4], p. 72).

There have been many studies looking at the factors influencing achievement in mathematics (for example [5]). These include gender, beliefs, attitudes and emotions such as self-confidence, anxiety, interest and desire to do or understand things, relevance to students' lives, prior mathematical experience, learning styles and strategies. In this paper we concentrate on three of these factors: gender, self-confidence, and prior mathematical experience. We examine how these affect students' performance in a test of mathematical literacy.

Gender

Much has been written about how gender affects achievement in mathematics. Since the seventies, studies have reported that males choose to study mathematics at more advanced levels, that they are more confident about learning mathematics, and believe that mathematics is and would be more useful to them than did females. ([6, 7, 8]).

In a review of studies concerning gender and achievement Stage, Kreinberg, Eccles and Becker [9] reported consistent results that suggested that at second level males perform better on reasoning tasks than females and some studies showed that females perform better than males on computational tasks. However in a later meta-analysis study, Hyde, Fennema and Lamon [10] concluded that gender differences in achievement are small. In Irish second level education in 2007, a higher percentage of males reached the highest grade in mathematics than females, but otherwise females outperformed their male counterparts at all grades and levels [11].

Tests measuring problem solving at the most complex cognitive level have found stronger gender differences in mathematics in favour of males (for example, [10, 12]). Sowder [13] found that children who can invent strategies are more likely to find sense in the mathematics they are learning and their understanding will lead to deeper confidence in their ability to do mathematics. It has been found that girls tend to use more standard algorithms than boys, who might invent or use more untaught strategies [12,14]. Gender differences have been found to vary depending on socioeconomic status and ethnicity, school and teacher.

Self-confidence

Confidence in one's ability to learn mathematics has been found to have a strong positive correlation with mathematical achievement, notably by Fennema and Sherman [6, 7]. It is not clear whether confidence has a causal effect on achievement, and the relationship between these variables is certainly complex. In a meta-analysis of the relationship between attitudes to mathematics (including confidence) and achievement in mathematics, Ma and Kishor [15] found some evidence for a causal effect. They recommended that more studies be carried out and that the mathematical ability of students be taken into account.

Bandura [16] introduced the notion of self-efficacy, which could be defined as the confidence in one's own ability to succeed at a specific task. Hackett and Betz [17] investigated the relationship between mathematics achievement and mathematics self-efficacy. They studied US university students and found that achievement and self-efficacy were correlated and furthermore that self-efficacy was a good predictor of whether a student would choose a mathematics-related major.

Carmichael and Taylor [18] investigated the relationship between students' beliefs, their self-confidence and their subsequent performance. They measured three levels of confidence: confidence to succeed in the course, in a mathematics topic and in a specific problem, and found that females and mature students reported lower levels of confidence on all three scales. They found that the specific measure of students' confidence (similar to Bandura's self-efficacy) was the most useful in predicting performance. In a study of first year students in the UK, Parsons [19] investigated self confidence and found that lower levels of previous achievement gave rise to lower confidence levels, and that confidence levels were decided well before reaching university.

Mathematical literacy was the major focus of the second cycle of PISA assessment in 2003. Students were asked about their self-efficacy regarding mathematical tasks. Significant gender differences in self-efficacy were found in almost all participating countries, with males showing higher levels of confidence. Irish students reporting a high level of self-efficacy also scored higher on the mathematical literacy test [20].

Prior Mathematical Experience

In a study of second level classroom practices in Ireland, Lyons, Lynch, Close, Sheerin and Boland [21] found that teaching was strongly didactic, and was especially so in mathematics classes. They found that teachers were under pressure to cover the syllabus and to prepare students for the state examinations and that this led to a reliance on drill and practice. They also found that teachers had profound influences on students' perceptions of mathematics. Negative attitudes and even 'mathematics anxiety' were associated with teachers going too fast, being too critical and not giving enough support. Students also reported that a fear of looking unintelligent prevented them from seeking help, and this appeared to be common across all students regardless of the level of mathematics studied or the type of school [21, p. 309]. English et al [22, p. 749] suggested that mathematics was associated with a strong sense of failure, of getting a wrong answer, and that “an over emphasis on procedures and techniques in the Irish context has led to a neglect of the ‘pleasure’ and aesthetic side of mathematics”.

The attitudes of mathematics teachers were studied as part of TIMSS [23] and PISA 2003 [24]. Over 70% of the Irish second year (second level) students in the TIMSS study were taught by teachers who believed that remembering formulae and procedures was very important for success in mathematics, and only 35% of these students were taught by teachers who thought it was very important for students to think creatively. Oldham [25, p. 272] comments that such beliefs could lead to lessons in which “lower-order skill is valued above higher-order, the procedural above the conceptual, and the decontextualised above the applicable”. The survey of Irish mathematics teachers carried out in conjunction with PISA 2003 found that teachers focus on skills and learning objectives assessed in examinations rather than those that are not [24].

The mathematical under-preparedness of students embarking on third-level courses in various disciplines has received much attention internationally [2, 3]. One factor identified as contributing to this situation is the emphasis in post-primary school on students’ examination performance (particularly in final examinations) and consequent examination-driven practice [26, 27]. In an effort to investigate such under-preparedness, many studies have assessed the basic mathematical skills of students entering third-level education using diagnostic tests [28, 29, 30]. However, even students with a good grasp of these basic skills may have problems transferring their knowledge to a different subject or situation [26, 31]. PISA’s measure of mathematical literacy thus becomes appropriate, as it aims to quantify how well students can use the mathematics they know. This idea of mathematical literacy is especially relevant for students studying Science or Engineering.

2. The Irish Educational System

The Irish Educational System has been credited as one of the main factors contributing to the recent success of the Irish Economy. One of the positive features of the system is that 85% of students complete second level education [32]. Irish students spend 5 or 6 years at second level, taking an examination called the Junior Certificate after 3 years and an examination called the Leaving Certificate (LC) at the end of their time in secondary school. These are state examinations and the results of the Leaving Certificate examination determine entry into third level education. Students typically take 7 subjects and because of third level matriculation requirements most students study English, Irish and Mathematics. In fact 96% of students who take the Leaving Certificate study Mathematics. This means that approximately 82% of Irish students study Mathematics until the age of (at least) 17. There are three different levels of Mathematics at Leaving Certificate – they are Foundation Level (FL), Ordinary Level (OL) and Higher Level (HL). There is some fluctuation in the numbers taking the various levels from year to year but typically 11% of students take FL, 71% take OL and 18% take HL. Students who study Mathematics at FL are not eligible for entry to third level so we will not consider these students here. The OL course is intended to provide “knowledge and techniques that will be needed in future study of scientific, economic, business and technical subjects” and that “emphasis can be given to aims concerned with problem-solving, abstracting, generalizing and proving” [33, p. 21]. The HL curriculum is a more advanced course than OL with more depth, particularly in the calculus sections which were designed originally for future mathematicians, physicists and engineers. Questions from the 2005 Leaving Certificate Examinations are included in the appendix for illustrative purposes.

Leaving Certificate Mathematics is assessed entirely by examination and it has been said that the predictable nature of these exams leads to teaching which emphasises procedure over conceptual understanding with teachers and students focussing on getting exam results to gain maximum points for entry to third level courses, to the detriment of learning knowledge and skills essential for the future [34]. Entwistle [35] found that when under pressure to cover material for assessment, a surface or sometimes strategic approach will dominate. This limited experience of mathematics education could ”potentially diminish one’s ability to make a successful transition to tertiary level mathematics-intensive courses” [27, p. 464]. One of the aims of this study was to examine such ideas.

In Ireland there are two main types of third level education: the University Sector (including colleges of education for future teachers) and the Institute of Technology Sector. About 64% of Irish school-leavers go on to study at third level with 59% of these at University and 41% at Institutes of Technology [32]. In this study we looked at four groups of students: a group of pre-service primary teachers who had chosen Mathematics as one of their academic subjects; a group of Humanities students at a University who were studying Mathematical Studies; a group of Science students at a University; a group of Engineering students at an Institute of Technology.

3. Test Instrument

In order to assess the mathematical literacy of first year students in Irish third level institutions and to identify their strengths and weaknesses, a PISA-style test was administered to the participants in this study. Test items released from PISA 2000 and PISA 2003 were used in order to better facilitate a comparison of the results obtained with the results of these studies. It was comparable to a single cluster used in PISA, containing 13 mathematical items to be completed in 30 minutes. The particular items chosen were spread across all six proficiency levels identified by PISA 2003, in such a way as to keep the average difficulty of the test (551.69) in line with those of the PISA clusters (ranging from 536.75 to 568.03) [36]. Moreover, the items were spread across the four content subdomains (Quantity, Space & Shape, Change & Relationships and Uncertainty), and the three broad clusters of competencies (Reproduction, Connections and Reflection) identified by PISA. Finally, all four PISA contexts (personal, public, educational/public, scientific) and three types of response format (multiple choice, closed constructed, open constructed response) were included.

Details of the test administered are given in **Table 1**. M136 Q03 Apples is included in the appendix as an example.

Item ID	Item Name	Subdomain	Competency	Level	Difficulty
M124Q01	Walking	Change & Relationships	Reproduction	5	611
M124Q03T	Walking	Change & Relationships	Connections	6	723
M438Q01	Exports	Uncertainty	Reproduction	2	427
M438Q02	Exports	Uncertainty	Connections	4	565
M509Q01	Earthquake	Uncertainty	Reflection	4	557
M555Q02T	Number Cubes	Space & Shape	Connections	3	503
M136Q03	Apples	Change & Relationships	Reflection	6	723
M037Q02	Farms	Space & Shape	Connections	3	524
M413Q01	Exchange Rate	Quantity	Reproduction	1	406
M413Q02	Exchange Rate	Quantity	Reproduction	2	439
M413Q03T	Exchange Rate	Quantity	Reflection	4	586
M266Q01T	Carpenter	Space and Shape	Connections	6	687
M547Q01T	Staircase	Space & Shape	Reproduction	2	421

Table 1: Description of the PISA items used.

In the PISA system, each question is assigned a difficulty level. Using Item Response theory and these difficulty levels we converted the raw scores (0-13) of the students to a score on the PISA scale [37]. In what follows, we will refer to this score as the PISA score. Furthermore, in PISA the Mathematics scale was divided into six mathematical literacy levels to represent degrees of proficiency. Level 1 students succeed only on the most basic tasks whereas level 6 students are able to handle complex problems and have advanced reasoning skills.

4. Questionnaire

After they had taken the PISA test, the students were asked to complete a questionnaire consisting of 28 questions. The questions could be grouped into five categories: personal, second level experience &

examination results, third level experience, study habits, attitude to Mathematics. The students had approximately 20 minutes to complete the questionnaire. Under the heading of personal information, each student's age and gender was recorded. The students were also asked to respond to a number of statements, such as 'I feel confident in approaching mathematics', using a 5-point Likert scale ranging from disagree strongly (1) to agree strongly (5). The survey was anonymous so that students would be more likely to give frank answers, and was administered at the beginning of the second semester, when the students had some, albeit limited, experience of third level.

5. Participants

The test was administered to 316 students in three third level institutions in Ireland: in 2006, 146 students were tested and 170 students took part in 2007. Preliminary results from the 2006 study were reported in [38]. The participants are students of the engineering department at the Institute of Technology, Tralee, the humanities and science faculties in the National University of Ireland, Maynooth, and the humanities and education faculties of St. Patrick's College, Drumcondra.
The numbers of students in each of the four subject groups is shown below.

		Frequency	Percent
Group	Engineering	66	20.9
	Arts	133	42.1
	Education	34	10.8
	Science	83	26.3
	Total	316	100.0

Table 2: Student Groups

There were 184 female students (58.2%) and 132 males (41.8%). There were only 29 (9.2%) mature students (in Ireland, students are classified as mature if they are 23 or over). The number of students who studied mathematics at HL at Leaving Certificate was 142 (45.7%), the number who studied at OL was 169 (53.5%), and 5 students (1.6%) did not take LC. Entry points for third level education are awarded to students based on their results in six LC subjects. Students passing an OL subject examination are awarded between 5 (for those achieving 40-44%) and 60 points (for those achieving 90-100%). Similarly, students passing a HL subject examination are awarded between 45 and 100 points. Students are awarded 0 points if they fail an OL or HL examination [39]. The LC points awarded to participants in this study based on their mathematics results were spread over the full range from 0 to 100. The median and mode were both 60 points.

	Higher Level	Ordinary Level	Total
Male	46 (36%)	82 (64%)	128
Female	96 (52%)	87 (48 %)	183
Total	142	169	311

Table 3: LEAVING CERTIFICATE Levels and Gender

6. Results & Analysis

6.1 *Overall Results*

PISA scores are scaled to give a mean of 500 and a standard deviation of 100. In 2003, mathematical literacy was the primary focus of PISA and Ireland achieved a mean score of 502.8 (with a standard deviation of 85.3) – a score not significantly different from the overall mean. Ireland was ranked 20th out of 40 participating countries (although Ireland's mean score was not significantly different to that of eight other countries). In PISA 2006, mathematics scores were benchmarked against those of PISA 2003 and Ireland's score was again not significantly different from the overall mean. This time Ireland was ranked 22 out of 57 participating countries.

In this study, the mean score for the group as a whole on the PISA test was 599.58 with a standard deviation of 91.2 Thus the mean is approximately one standard deviation higher than that for Irish 15-year-olds.

6.2 ***Literacy Levels***

In PISA, the Mathematics scale was divided into six mathematical literacy levels to represent degrees of proficiency. Table 4 shows the literacy levels reached by the students participating in this study. In PISA, 15-year-old students with a literacy level of 2 or below were said to be unsuitable for further education [36]. Thus, serious questions would have to be asked of the 6.9% of this sample remaining at level 2 or below.

Literacy Level	< 1	1	2	3	4	5	6
Frequency	3	3	16	52	122	48	72
Percentage	0.9	0.9	5.1	16.5	38.6	15.2	22.8

Table 4: Literacy Level

Only 22.8% of this sample has displayed evidence of the advanced mathematical thinking and ability to conceptualise or generalise information characterised by a level 6 proficiency. However, this is very favourable when compared with 2.2% of Irish students and 4.0% of participants from all OECD countries performing at this level in PISA 2003 (or 1.6% of Irish students and 3.3% of OECD students in 2006). Of more concern, perhaps, is the large number of students (23.4% of the entire sample) performing at level 3 or below.

6.3 ***PISA Scores and the Leaving Certificate***

In undertaking this study, one point of interest for us was whether PISA and the Irish Leaving Certificate examination measure similar skills. In PISA, students with good problem solving skills who can apply mathematics to real world problems score highly. Although the Leaving Certificate curriculum states that it aims "to provide them [the students] with the mathematical knowledge, skills and understanding needed for work" [40, p. 1], it has been said that the Irish mathematical education system rewards rote learning at the expense of understanding [34].

For the group studied here, the correlation between Leaving Certificate points in Mathematics and PISA scores was 0.422. This along with Figure 1 shows a weak to moderate positive relationship between Leaving Certificate points and the PISA score. A linear regression with the PISA score as the dependent variable and Leaving Certificate points as the independent variable gave an R-squared value of 0.174. The Leaving Certificate result is a statistically significant predictor of the PISA score ($p<0.0001$) but we can see that only 17.4% of the variation in PISA scores is explained by the linear relationship with Leaving Certificate points. The equation from this model is

$$\text{PISA Score} = 490.493 + 1.944(\text{LC points}).$$

However Figure 1 also shows that some students who scored highly in their Leaving Certificate exams have low scores on PISA and some students with low achievement at Leaving Certificate level perform very well on PISA-style questions. This may provide evidence that the Irish state examinations and PISA tests examine different skills.

In this study, 45% of the students (142) studied mathematics at HL at Leaving Certificate. The mean PISA score for these students was 638.24 (standard deviation 85.61) and for OL students was 570.44 (standard deviation 85.67). There is a significant difference (67.8 points, $p<0.0001$, t-test with equal variances not assumed) between these means, however the difference is not as great as that reported in the PISA 2003 (94 points) for students taking HL or OL at Junior Cycle.

It is clear from Table 5 that the HL students are more likely to attain a higher level of mathematical literacy than the OL students ($p<0.001$, chi-squared test). However LC level does not fully explain students' performance on the PISA-style test. For example, 13.4% of the HL students were classified as attaining no higher than literacy level 3.

		Literacy Levels			Total
		Level 3 or below	Level 4	Levels 5 and 6	
LCLevel	Higher	19 (13.4%)	49 (34.5%)	74 (52.1%)	142
	Ordinary	53 (31.4%)	70 (41.4%)	46 (27.2%)	169
Total		72 (23%)	119 (38%)	120 (38%)	311

Table 5: Literacy Levels and Leaving Certificate Levels

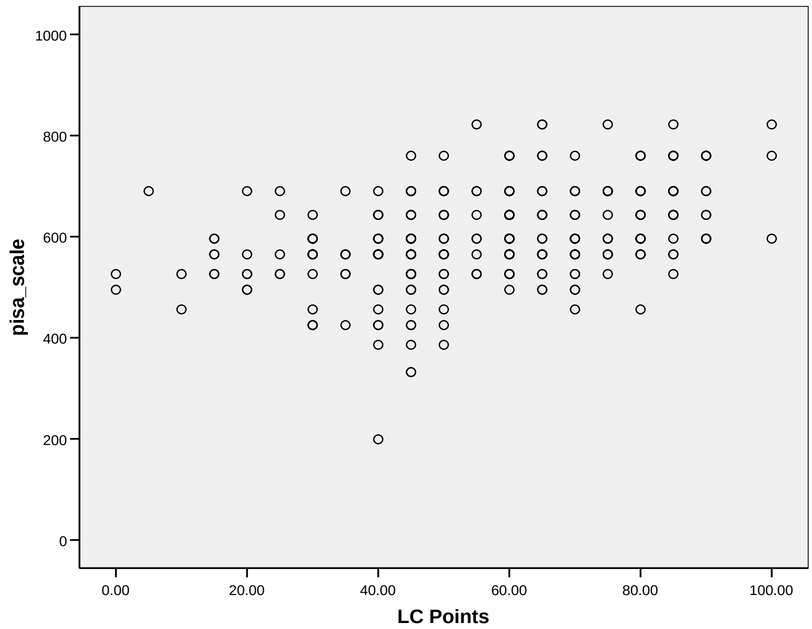

Figure 1: Scatterplot of PISA scores versus Leaving Certificate points.

6.4 ***Procedural Skills versus Conceptual Understanding***

In the questionnaire that formed part of this study, the students were asked to comment on the statement that an emphasis is placed on procedural skills rather than conceptual understanding of Mathematics in Irish schools. In this study, 64.9% (205) agreed with the statement, 9.5% (30) disagreed, 2.5% (8) were unsure, and 23.1% (73) did not respond. Looking more closely at the students who agreed or disagreed with the statement, there was no significant difference in the responses of male and female students ($p=0.294$, chi-squared test), or OL and HL students ($p=0.980$, chi-squared test) or confident and non-confident students ($p=0.605$, chi-squared test). The students were further separated into three groups according to their performance on the PISA test: those performing at levels 3 & below (23.4%), those performing at level 4 (38.6%) and those performing at levels 5 & 6 (38%). However, there was no significant difference in the responses of these three groups ($p=0.490$, chi-squared test).

6.5 ***PISA Scores and Gender***

In PISA 2006, Irish male students achieved a higher mean score (507.3) than female students (495.8) mirroring the pattern for all OECD participants (503.2 for males and 492.8 for females). PISA 2003 had similar findings.

The mean score for female students on the PISA-style test administered in this study was 590.51 (standard deviation 84.89), while the mean score for male students was 612.23 (standard deviation 98.29). There is a significant difference between these means ($p=0.041$, t-test equal variances not assumed) and there was also a significant difference in variances for the two groups ($p=0.049$). However the group of female students in this study had a significantly higher mean LC score than the male students ($p<0.0001$, t-test with equal variances not assumed). If we keep LC grade fixed, then gender and above average performance on PISA are not independent with males doing better. The Mantel-Haenzel common odds ratio confidence interval is (0.21, 0.664) ($p=0.001$). This means that the odds of a female achieving an above average score on PISA is no more than 66.4% of that of a male with the same grade, that is, males are more likely to achieve higher PISA scores than females who have the same LC grade.

6.6 ***PISA Scores and Confidence***

The students were asked to rate their level of confidence in approaching Mathematics on a five point scale with 5 being very confident. We were interested to see if confidence and high scores on PISA or on Leaving Certificate were related. Students were classified as confident if they awarded themselves a score of 4 or 5 on this scale and as non-confident otherwise. In this way, we identified 123 (39%) confident students and 191 (61%) non-confident students. A Chi-squared test carried out on the data as shown in Table 6 shows that confidence and LC level are not independent ($p<0.001$) with the HL students being more confident than the OL students.

		LC Level		Total
		Higher	Ordinary	
Confidence	Low	59 (32%)	128 (68%)	187
	High	82 (67%)	40 (33%)	122
Total		141(46%)	168 (54%)	309

Table 6: Confidence and LC Level

For this group, confidence and gender are not independent ($p=0.03$, chi-squared test) with male students being more confident in their mathematical abilities. If we split the group into HL and OL students, we still see the same pattern with male students displaying higher levels of confidence (chi-squared tests yielded $p=0.008$ for HL and $p=0.019$ for OL). In fact, if we keep LC grade fixed we get a Mantel-Haenzel common odds ratio confidence interval of (0.193, 0.598) ($p<0.001$). Thus the odds of a female being confident are no more than 60% of the odds of a male student with the same LC grade.

LC Level			Confidence		Total
			Low	High	
Higher Level	Gender	Male	12 (26%)	34 (74%)	46
		Female	47 (49%)	48 (51%)	95
	Total		59 (42%)	82 (58%)	141
Ordinary Level	Gender	Male	56 (68%)	26 (32%)	82

Female	72 (84%)	14 (16%)	86
Total	128 (76%)	40 (24%)	168

Table 7: Confidence and Gender

The mean score of the confident student group on PISA was 637.48 (standard deviation 86.08) and the mean score of the non-confident group was 574.23 (standard deviation 85.87). There is a significant difference between these means ($p<0.001$, equal variances not assumed). We can also see from Table 8 that literacy levels and confidence are not independent ($p<0.001$, chi-squared test).

		Literacy Levels			Total
		Level 3 or below	Level 4	Levels 5 and 6	
Confidence	Low	60 (31%)	78 (41%)	53 (28%)	191
	High	14 (11%)	44 (36%)	65 (53%)	123
Total		74 (23%)	122 (39%)	118 (38%)	314

Table 8: Literacy Levels and Confidence

In order to try to separate the effects of gender and confidence, we tested whether scoring above average on the PISA-style test and confidence are independent for the male and female students separately. We found that for both male and female students, confidence and above average scores are not independent ($p<0.001$ for males, $p=0.039$ for females, chi-squared tests), with confident students more likely to have high scores.

Gender			Pisa Score		Total
			Low	High	
Male	Confidence	Low	52 (73%)	19 (27%)	71
		High	23 (38%)	38 (62%)	61
	Total		75 (57%)	57 (43%)	132
Female	Confidence	Low	86 (72%)	34 (28%)	120
		High	35 (56%)	27 (44%)	62
	Total		121 (66%)	61 (34%)	182

Table 9: Confidence, Gender and PISA Scores

6.7 *Further Analysis*

It is clear that gender, confidence and LC level all have an influence on PISA-style test results. However, the data collected also draws attention to relationships existing between these variables themselves. In order to try to provide a clearer picture of the behaviour of the mathematical literacy of the participants and in an effort to clarify the relationships, a stepwise multiple regression model was used. The dependent variable was the PISA score and Leaving Certificate points, Leaving Certificate level, confidence (low confidence was coded as 0 and high confidence was coded as 1), and gender (coded as male=0 and female =1) were the independent variables. It was found that Leaving Certificate points ($p<0.001$), confidence ($p=0.005$) and gender ($p<0.001$) were all significant predictors of PISA scores. The adjusted R-squared value was 0.244, so this model explains about 25% of the variation in the PISA score. The equation from this model is

$$\text{PISA Score} = 511.934 + 1.889(\text{LC points}) + 29.665(\text{Confidence}) - 37.91(\text{Gender}).$$

Furthermore, a number of cluster analysis techniques (namely two-step clustering, single-link clustering and chi-squared automatic interaction detection) were implemented but the results were inconsistent and/or inconclusive.

7. Discussion

The mean score for the group of participants in this study, on the PISA test administered, was 599.58 with a standard deviation of 91.212. So with the benefit of 3 years extra schooling the mean has increased by about one standard deviation. This is in line with what was found in the TIMSS study, where 9- and 13- year olds were tested. There, the mean score for 13-year-olds was about one standard deviation higher than the mean for 9-year-olds [23]. However, it must be remembered that our sample is not representative of all 18-19 year-olds in Ireland. The participants in this study have chosen to study mathematics, science, or engineering at third level and 45% of them have studied HL mathematics at school (compared to 18% of the general population) and so one would expect them to be more mathematically literate than an average school-leaver.

Moreover, 23.4% of the sample involved attained literacy level 3 as defined in PISA 2003. A summary of level 3 proficiency skills is given below [37]:

> *At Level 3, students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.*

A further 38.6% of the sample has reached level 4. These students can work with explicit models for complex concrete situations that may involve constraints but cannot necessarily develop such models, identify the constraints or compare and evaluate problem-solving strategies for dealing with problems related to these. Students at this level can also construct and communicate explanations and arguments based on their interpretations and actions but are incapable of reflecting on these effectively and may lack the ability to formally characterise the situations they meet [37]. The gaps highlighted here in the skills sets of the students examined, may help to explain why mathematical knowledge and ability are often cited as critical factors in the retention of students on science and engineering courses [41], thereby sparking concerns for the effective functioning of a society more and more dependent on the knowledge economy.

Aside from the knowledge economy, Wilson and MacGillivray [30] are interested in the certain "at-homeness" with mathematics that enables any individual to cope with the mathematical demands of everyday life. They claim that studying higher level mathematics at second level consolidates the student's basic skills allowing him to develop this "at-homeness" and apply his skills. In their own study of 566 third level students enrolled on an introductory data analysis course in Queensland, Wilson and MacGillivray [30] found that students' carelessness increased with problem complexity, and that multi-step problems and abstraction caused obvious difficulties for students. The problems were more pronounced for those who hadn't taken higher level mathematics courses at second level. The London Mathematical Society [2] also bemoans the difficulties experienced by incoming third-level students in the United Kingdom in recent years when faced with simple multi-step problems and the related marked decline in their analytical skills.

Wilson and MacGillivray [30] appreciate that third level educators require an awareness of the level of students' basic mathematical skills in order to help them overcome their difficulties. Many third level educators expect their incoming mathematics students to display the advanced mathematical and reasoning skills that mark out those performing at PISA proficiency level 6 from other students. That is, the educators expect their incoming students to have the ability to conceptualise and generalise information based on investigations carried out and to be able to translate flexibly between different sources of information and representations, while developing new strategies and approaches for attacking novel situations [37]. However, in the study reported here, only 23.4% of the students who have selected to study mathematics and 22.8% of the overall group fell into this category. (Generally, there was no significant difference ($p=0.150$, chi-squared test) observed in the literacy levels attained by those choosing to study mathematics at third level and the science and engineering students who are obliged to take some mathematics modules.) This is a point of concern for those teaching mathematics at third level and may go some way to explain why students report difficulties in making the transition from second to third level mathematics. While the use of diagnostic testing of first year tertiary students and the decline in competence in their basic mathematical skills is well-documented [28, 29, 42], making an effort to routinely measure the mathematical literacy of incoming third level students as well as more routine algebraic skills, may then provide a means of familiarising educators with the reasoning, analytical and communication skills of such students. The benefits of the latter are threefold: not only would it help identify students at risk, and help to focus remedial measures on those aspects for which gaps are evident, but it would also remove unrealistic staff expectations.

Remembering that many of the participants in this study aim to become teachers (not only those students currently in the Education faculty but also many of those in Humanities or Science who may opt to study for a one-year postgraduate diploma in Education in order to qualify as a mathematics teacher) only serves to exacerbate the concerns of third level educators if no measures are taken to tackle the aforementioned problems. Hawks and Savage [29] claim that many mathematics teaching posts in the UK are filled by people who do not have a sound background in mathematics, while the LMS [2] ascertain that in order to improve what is taught and how it is taught, both the competence and the confidence of those who choose to become mathematics teachers must be increased. In Australia [3], it is feared that the quality of aspiring mathematics teachers is in decline, with new teachers having neither the enthusiasm nor the confidence to teach mathematics well. This lack of confidence has been blamed on their current educational system which fosters memorisation skills in place of deep learning approaches.

The mathematics syllabus for the Irish Junior Certificate cycle was recently revised and the new syllabus was first examined in 2003. As the revised curriculum aims to provide students with the mathematical knowledge, skills and understanding needed for life and work, it was perceived that it would better equip students for a mathematical literacy test such as PISA. The correlation between Junior Certificate scores in 2003 and PISA 2003 scores was 0.75 [20]. However, the correlation between Leaving Certificate points for mathematics and scores on the PISA-style test for the group studied here was only 0.427. This may point either to a change in emphasis in the syllabus and delivery of Leaving Certificate material or to a lack of understanding on the part of the students. These suggestions are corroborated by the teachers' responses to TIMSS and PISA questionnaires (as mentioned previously) and by national reports. The Chief Examiners Report on the performance of students in Leaving Certificate mathematics states that their weaknesses "relate to inadequate understanding of mathematical concepts and a consequent inability to apply familiar techniques in anything but the most familiar of contexts and presentations" [43, p. 49]. Some questions from the 2005 HL and OL examinations are given in the appendix. The National Council for Curriculum Assessment [44, p. 11] has stated that the Irish examination system "has contributed to increased emphasis on recall and the application of routine procedures". Moreover, the participants' responses to the questionnaire echo the NCCA's comment, providing evidence to support Entwistle's [35] comments on conditions leading to the adoption of surface approaches to learning. Perhaps this procedural approach to learning and teaching mathematics in the Irish system is a factor contributing to the disappointing performance of this study's participants on the PISA-style test.

In general, males performed better than females in the test administered in this study. The focus of the test items used would have been largely unfamiliar to the students and thus it is possible that the students could not rely on standard algorithms but needed to reason for themselves and try untaught strategies in answering the questions. This would lend support to the findings of previous studies (for instance, [9, 14]). If such a theory is true, we would then expect, following Sowder [13], males to be more confident than females: this was in fact borne out by this study.

Confidence was also shown to be a factor influencing the performance of participants on the PISA-style test, with those displaying higher levels of confidence performing better. In agreement with Parsons [19], it was found that lower levels of pre-university achievement gave rise to lower levels of confidence—Ordinary Level Leaving Certificate students being less confident than Higher Level students. However, gender was also a contributing factor and this study showed males to be more confident that females, as had PISA 2003.
It is worth mentioning that Wilson and MacGillivray's study [30] led to similar findings, with prior achievement and level of mathematics studied, gender, and self-efficacy all emerging as significant predictors of the score on the skills test administered. As in the study reported here, these predictor variables were found to be highly inter-dependent.

From our study we can conclude that the Irish education system does not seem to be achieving its aim of instilling the skills required for mathematical literacy, as defined by the OECD. The study also shows that there are complex interrelationships between gender, confidence, prior mathematical achievement and mathematical literacy, but further in-depth investigations are required to clarify the relationships.

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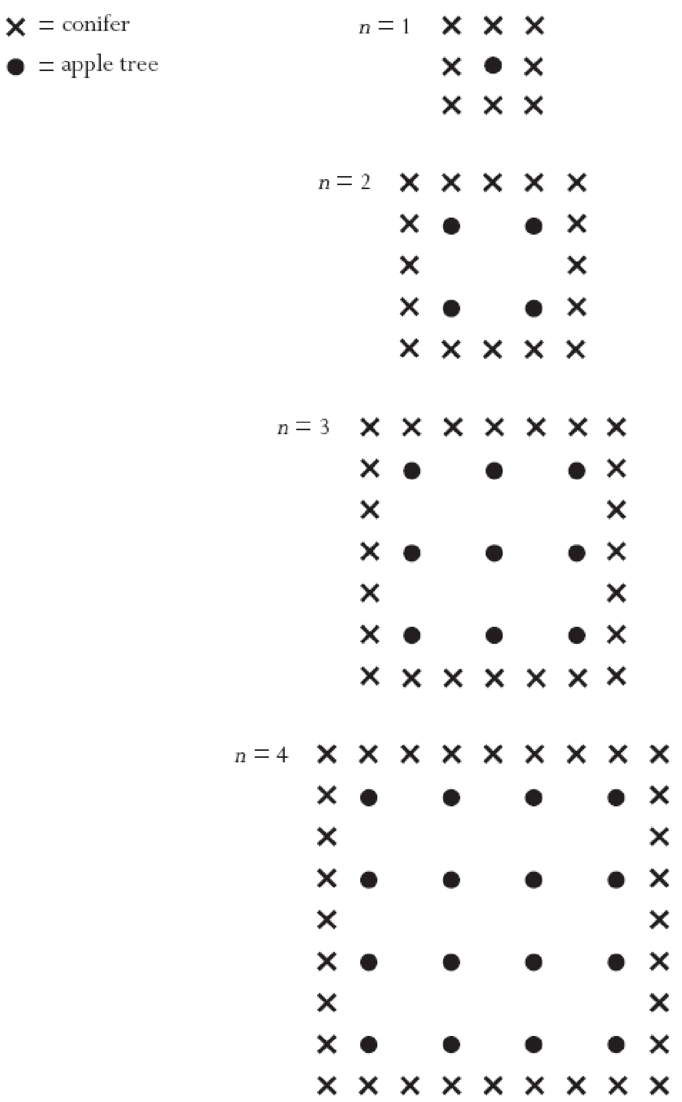
Appendix
PISA Item M136 Apples

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



Question 3: APPLES (M136Q03)

There are two formulae you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2

Number of conifers = $8n$

where n is the number of rows of apple trees.

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers? Explain how you found your answer.

Ordinary Level Leaving Certificate Mathematics Examination 2005 — Question 7 Paper 1

7. **(a)** Differentiate $9 + 3x - 5x^2$ with respect to x.

(b) **(i)** Differentiate $(3x^2 - 2)(x^2 + 4)$ with respect to x.

(ii) Given that $y = \frac{x^2}{x-1}$, find $\frac{dy}{dx}$ when $x = 3$.

(c) A car begins to slow down at p in order to stop at a red traffic light at q.

The distance of the car from p, after t seconds, is given by

$$s = 12t - \frac{3}{2}t^2$$

where s is in metres.

(i) Find the speed of the car as it passes p.

(ii) Find the time taken to stop.

(iii) The car stops exactly at q. Find the distance from p to q.

Higher Level Leaving Certificate Mathematics Examination 2005 - Question 7 Paper 1

7. **(a)** Find from first principles the derivative of x^2 with respect to x.

(b) **(i)** The parametric equations of a curve are:

$$x = 8 + \ln t^2$$
$$y = \ln(2 + t^2), \text{ where } t > 0.$$

Find $\frac{dy}{dx}$ in terms of t and calculate its value at $t = \sqrt{2}$.

(ii) Find the slope of the tangent to the curve $xy^2 + y = 6$ at the point (1, 2).

(c) **(i)** Write down a quadratic equation whose roots are $\pm\sqrt{k}$.

(ii) Hence use the Newton-Raphson method to show that the rule

$$u_{n+1} = \frac{(u_n)^2 + k}{2u_n}$$

can be used to find increasingly accurate approximations for $\sqrt{k}$.

(iii) Using the above rule and taking $\frac{3}{2}$ as the first approximation for $\sqrt{3}$, find the third approximation, as a fraction.