CERME7 Working Group 3: Algebraic thinking

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The Algebraic Thinking Working Group is an established CERME theme, and WG3 continued the work carried out in previous conferences. 13 papers and 4 posters were presented, representing 13 countries.

Four papers were focussed on the transition to algebraic symbolisation. Caspi and Sfard showed how 7th Grade Israeli students' discourse contains some algebra-like features, not normally found in everyday discourse. Dooley used epistemic actions to analyse and describe the development of algebraic reasoning amongst Irish pupils aged 9–11 years, and argued that the use of 'vague' language was central to such development. Gerhard exemplified the use of an analytic tool with secondary students in Germany, and highlighted the importance of focusing on the question of how algebraic knowledge interacts with arithmetic knowledge. Pytlak demonstrated how relatively sophisticated algebraic thinking can be achieved by children with geometric and numeric approaches, but without the use of symbols.

A second theme concerned equations and symbolisation. Alexandrou-Leonidou and Philippou found that primary children in Cyprus were capable of developing the dual meaning of the equals sign. Through a teaching intervention, children were enabled to solve equations in multiple representation formats. Didiş, Baş and Erbaş examined students understandings and errors when solving quadratic equations. Their findings added further weight to the literature, highlighting the ubiquity and problems of a purely instrumental, or procedural, understanding.

Other authors tackled technology. Chiappini demonstrated how AlNuSet software can enable students in Italy to overcome crucial epistemological obstacles with negative numbers and the equivalence of algebraic forms. Hewitt used the software Grid Algebra to analyse the activity of English students aged 9–10 years in order to examine the nature of algebraic activity. Maffei and Mariotti used Aplusix CAS in Italy to examine the interplay between different representations in algebra. They concluded that natural language has a dual role. Nobre, Amado, Carreira and da Ponte showed how Excel enabled three Grade 8 Portuguese students to engage with algebraic structure without the need for algebraic symbolisation. These students were able to model and solve a complex problem.

Generalisation was the fourth theme. Barbosa analysed the strategies used by 54 Portuguese students in 6th Grade working on generalisation. Students achieved better results with near generalisation than with far generalisation problems.

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Cañadas, Castro, and Castro tackled the different approaches to generalisation of 359 Spanish secondary students. Graphical approaches were used infrequently, and generally only when the problem was presented graphically. Chua and Hoyles discussed generalisation strategies used by 13 year-old students in Singapore. Express (higher attaining) students were more flexible, adopting a numerical approach for a linear problem, but using a constructive approach for a quadratic problem.

Algebraic thinking is a 'mature' domain within mathematics education research (e.g., Kieran 2006). The work on WG3 reflects this and, given this research history, there were many aspects of consensus, but also significant differences. The points of consensus concern the practice of algebraic thinking: (a) algebraic thinking provides insight into school mathematics, but translating these insights into general classroom practice is not straightforward; (b) classrooms tend to be dominated by procedures and manipulation; (c) technology is largely yet to be realised in most classrooms; (d) learners should acquire many ways to look at and work with algebra; and (e) it is important to consider multiple perspectives, talk and discourse, rich tasks and children's existing and naïve (mis)understandings. Some points of difference generated enriching debates related to: (a) the nature and importance of 'early algebra'; (b) the existence of a clear cognitive gap between (generalised) arithmetic and algebra; (c) the existence of one 'ideal' learning trajectory or several good-enough learning trajectories in which learning is inevitably somewhat idiosyncratic; and (d) the use and meaning of different terms, which was dependent on the theoretical framework used.

The group identified issues for future research. The early algebra debate in part reflects a current theme in the literature (Kaput, Carraher, and Blanton 2007), and it also reflects the policy context in which some countries are introducing algebra earlier. Re-contextualisation is a valid and important field of study, but we note that researchers need to demonstrate the contribution they make to the field as a whole through stronger literature reviews.

Translating research knowledge into practice was a concern for WG3 participants. For example, one debate was about how technology can help children do something that they would not otherwise do, and how teaching can then enable children to understanding ''independent of'' technology. Similarly, the group identified a need for further research into understanding group dynamics specific to algebraic thinking.

Note

The research papers and poster submissions published in the CERME7 proceedings related to this topic can be accessed from the hyperlink at http://www.erme.unito.it

References

Kaput, J.J., D. Carraher, and M.L. Blanton. 2007. Algebra in the early grades. Mahwah, NJ: Lawrence Erlbaum.

Kieran, C. 2006. Research on the learning and teaching of algebra. In Handbook of research on the psychology of mathematics education: Past, present and future, ed. A. Gutierrez and P. Boero, 11–49. Rotterdam: Sense.