Assessment, Development and Consolidation of Advanced Skills in Engineering Mathematics

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ABSTRACT

In the Dublin Institute of Technology there are two routes to obtaining an Honours Degree in Engineering. The first is direct entry onto a four year Honours degree. To qualify for this route students must have achieved a grade of 55% or higher in Higher Level Mathematics at second level. For many students who do not have this level of mathematics they may enter a three year ordinary degree and then have the option of transferring into year 3 of the Honours degree. Many of our students struggle with this transition, in particular with the level of Maths.

In a survey of almost 300 Engineering students, it was shown that a large percentage struggled with key skills in maths and tended to avoid questions on differentiation and integration. Here, we have developed an Advanced Mathematics Diagnostic exercise for students entering into 3rd year of the Honours degree. In addition, we have set up an online test that encourages students to revise (or perhaps learn for the first time) many of the key skills required for later years of an Honours Engineering degree.

KEYWORDS

Mathematical Competencies;Engineering Mathematics

1. INTRODUCTION

Many students who enter third level engineering have problems with core mathematical skills. This has been borne out in the results of diagnostic tests carried out in many third level institutions, both in Ireland ([7], [10]) and in the U.K. ([12]). These problems with core concepts can lead to comprehension difficulties in numerous modules, both in mathematics itself and in related subjects.

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In addition some students enter the later years of Engineering courses in Ireland having come through several different routes, both within a particular college and from other colleges. It can be difficult to quantify exactly what knowledge of Engineering maths they have from their earlier years. In an attempt to quantify this problem we are devising an Advanced Maths Diagnostic exercise similar to the Diagnostic exercises already given to many students on entry to third level. Students who fail to reach a satisfactory result in the Advanced Maths Diagnostic exercise will be required to sit online tests until such time as they reach the required standard in Engineering Maths to complete the 3rd and 4th year of an Honours degree in Engineering.

1.1 Entry to Honours Engineering

Mathematics is compulsory for all second level students in Ireland up to and including the final exam (Leaving Certificate), where mathematics can be sat at Higher, Ordinary and Foundation levels. About 16-17% of students sit mathematics at Higher Level.

There are two distinct routes to achieving an Honours degree (Level 8) in Engineering in the Dublin Institute of Technology(DIT). Students who have achieved a C (55%) or better in higher level Mathematics in the Irish Leaving Certificate (final secondary school exam in Ireland) are eligible to enter directly onto a 4 year Honours degree in Engineering. Students who do not have this level of mathematics but have a pass in Ordinary level Maths may enter onto a 3 year Ordinary Degree (Level 7) in Engineering. Upon successful completion of this award students may progress to the 3rd year of the Honours degree. These students tend to struggle with the Mathematical level of the Honours degree [3]. Traditionally an Ordinary Degree in Engineering was a preparation for work as a Technician. Increasingly students from this back ground proceed to get an Honours degree (over 50% of students now continue to get an Honours degree). In theory students cover the same mathematics syllabus over the three years of the ordinary degree as a student covers over the two years of the Honours degree. Given that Ordinary level students are weaker to begin with they are invariably weaker at mathematics at the end of their Ordinary degree than their counterparts in the Honours degree.

1.2 Choice on Papers

Brown et. al observed that "Assessment defines what students regard as important, how they spend their time and how they come to see themselves as students and then as graduates. If you want to change student learning then change the methods of assessment.' [1]. Despite this, many mathematics papers in Ireland give students a choice between questions on different topics. This allows students the opportunity to omit or avoid more difficult topics. This was strongly borne out by a survey carried out in the DIT [5,6]. From the 276 responses received, 64% had avoided integration at some point, and 47% felt they had struggled as a result. A quarter of respondents felt that choice should be removed from at least some mathematics papers, while 60% felt there should be compulsory questions on certain topics.

2. CORE SKILLS ASSESMENT

Many engineering undergraduates begin third-level education with deficiencies in core mathematical skills. In the DIT, a diagnostic test is given to first-year students, consistently revealing problems in basic mathematics. Many students do not seek help to address these problems and struggle through several years of engineering, carrying a handicap of poor core mathematical skills, as confirmed by exploratory testing of final year students [5].

In order to improve these skills in engineering students, a "module" in core mathematics was developed. The course material was basic, but a grade of 70% or higher was required to pass the module. Students were allowed to repeat the module as often as they liked until they passed. An automated examination for this module was developed.

There has been a systematic improvement in the core mathematical abilities of the students participating in the core skills initiative. As an illustration of this we show the results for 36 students studying preliminary engineering, in table 1 below. The results show a systematic improvement in the results of the students. On the first test only 1 out of 36 students achieved a mark of over 90%, by the end that had increased to 7. More importantly 25 out of 36 failed to achieve a mark of 70% (pass mark) in their first attempt. This number was reduced to 11 by the end of semester. Similar improvement was observed in other classes.

Table 1: Grades of 36 preliminary engineering students in the core skills assessment [3]

	Mean	**More than 90%**	**More than 70%**	**Less than 70%**
1st Attempt	54%	1	10	25
Dec 2009	65%	6	14	16
Apr 2010	73%	7	18	11

There is further evidence in the feedback we are getting from focus groups, in particular

- the students supported the high pass mark and some felt it should be even higher.
- The students appreciated the chance to take the test multiple times and could clearly articulate the formative effect this had on their learning .
- The importance in engineering of the mathematics examined by the test was evident to all students but particularly to the students in the latter stages of their engineering programmes.

2.1 Advanced Core Skills Assessment

It was decided to set up an advanced core skills assessment in mathematics, similar to the core skills initiative already developed for first year students in the DIT described above [3,4]. This consists of a multiple-choice test based on a randomised question bank. The material covered by the test consists of the more important aspects of undergraduate engineering mathematics covered in the first two years of the Honours degree programme and/or the three years of the Ordinary degree programme . The pass mark will be set at 90 %. Students will be allowed to re-sit the assessment as frequently as required until they pass.

2.2 Pilot Project Overview

Beginning September 2011 we will pilot the advanced core skills assessment. It will be worth 10% of the continuous assessment of the mathematics module. In the first instance, the students will sit the Advanced Mathematics Diagnostic. Those who score 90% will receive nine marks out of ten for this component, whilst those who scored less than 90% will receive no marks and will have to take the advanced core skills assessment at a later date. These students will continue to sit the assessment on a monthly basis until they achieved the required pass mark. Those who don't pass will receive 0 for this component of the module.

Students will be given access to a website with resources tailored for each question and will be encouraged to attend the Student Maths Learning Centre (SMLC). After their second and subsequent attempts, special classes on problem topics will be provided.

2.3 Advanced Diagnostic test

The test consists of 10 paired questions on each of the subtopics shown below. For logistical reasons the test is restricted to 1 hour. In this time it is not possible to cover the wide range of topics that we would like our students to know from the previous years. After several iterations we settled on the following 10 subtopics.

2.3.1 Differentiation

It was decided to include four sets of paired questions on differentiation in the test. In the survey of DIT Students[5,6] mentioned above 39 % of our students consistently avoided questions on differentiation and

the majority of these students struggled chronically because of it.

2.3.2 Integration

Two sets of paired questions are included on integration. This area is avoided by 64% of our students [6], and after differentiation is the second most fundamental area of engineering mathematics.

2.3.3 Matrices

This area is important, as we test the ability to manipulate numbers, basic operations conducted in the correct order and multiplication by zero.

2.3.4 Complex Numbers

The question on complex numbers involves the manipulation of complex numbers at their most basic.

2.3.5 Differential equations

We only consider one type of 1st order differential equations, separation of variables. For 2nd order differential equation we only consider the most basic type of homogeneous equations.

Topic	**Sub-Topic**
Differentiation	Basic
	Product Rule
	Quotient Rule
	Chain Rule
Integration	Basic
	Substitution
Differential Equations	1st Order ODE's
	2nd Order ODE's
Matrices	Matrix Multiplication
Complex Numbers	Multiplication

2.4 Evaluation of the Advanced Mathematic Diagnostic Test

An evaluation strategy has been devised in order to enhance and develop the Advanced Diagnostic test and the way in which it is implemented in, and integrated into, the module. The evaluation is essentially a comparison between aims and objectives of the development and implementing the test and the reality of the students' learning and development. It is the authors' intention to obtain data that will inform the subsequent changes and refinements. The methods of data collection will be questionnaires, focus groups, test results, the amount of attempts made by the students, and attendance at tutorials. In addition the list of topics on the test will be circulated to lecturers for discussion with a view to improvement of the test.

3. FUTURE WORK

This work will begin in September 2011 with a cohort of 180 students. The pilot year will allow us to develop and improve our diagnostic test. We hope to consolidate the test in the first two years. We have identified several key stages to this:

1. Diagnostically test third year students
2. Allow students to repeat the core skills initiative until they pass.
3. Diagnostically test all 4th year students
4. Create a series of practice tests
5. Anonymous online survey of all students
6. Focus groups and interviews of selected students
7. Analysis of paired questions
8. Feedback from staff in the Faculty

This work will allow us to identify areas in the earlier years of the undergraduate mathematics that need improvement. This test will also be useful in assessing the suitability of graduates from other colleges who apply to enter the 3rd year of our Honours degree and in the placement of Erasmus students within our courses.

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APPENDIX:

ADVANCED DIAGNOSTIC TEST

1. Find $\frac{dy}{dx}$ *where* $y = x^4$

a) $4x^3$ b) $4x^4$ c) $5x^5$ d) x^3

2. Find $\frac{dy}{dx}$ *where* $y = x^7$

a) $7x^6$ b) x^6 c) $7x^8$ d) $\frac{x^8}{7}$

3. Find the solution of the following first order Differential Equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$$

a)$Ae^{-3x} + Be^{x}$ b) $Ae^{-3x} + Be^{-x}$

c) $Ae^{3x} + Be^{x}$ d) $Ae^{x} + Be^{3x}$

4. Find the solution of the following second order Differential Equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 0$$

a)$e^{-3x}Acosx + Bsin$ b)$e^{1x}Acos3x + Bsin3x$

c) $Acos3x + Bsinx$ d) $e^{1x}Acos3x + Bsin3x$

5. Find $\int x^3 dx$

a)$\frac{x^4}{4}$ b)$4x^4$ c)$3x^3$ d) x^3

6.Find $\int x^{-4} dx$

a)$-4x^{-3}$ b)$-\frac{x^{-3}}{3}$ c)x^{-5} d) $\frac{x^{-5}}{5}$

7. $\begin{pmatrix} -1 & -2 & 1 \\ 2 & -3 & 2 \\ 1 & 2 & -3 \end{pmatrix}\begin{pmatrix} -2 & 7 \\ 0 & 3 \\ 5 & -1 \end{pmatrix} =$

a)$\begin{pmatrix} 7 & -14 & 6 \\ 3 & 3 & 4 \\ 2 & 1 & 7 \end{pmatrix}$ **b**) $\begin{pmatrix} 7 & -14 \\ 6 & 3 \\ -17 & 16 \end{pmatrix}$

c) $\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 16 \end{pmatrix}$ d) $\begin{pmatrix} 3 & -14 \\ 6 & 7 \\ -13 & 10 \end{pmatrix}$

8. $\begin{pmatrix} 6 & 3 & 0 \\ 2 & 5 & 1 \\ 9 & 8 & 6 \end{pmatrix}\begin{pmatrix} 7 & 4 \\ 6 & 7 \\ 5 & 0 \end{pmatrix} =$

a) $\begin{pmatrix} 65 & 45 \\ 49 & 42 \\ 141 & 92 \end{pmatrix}$ **b**) $\begin{pmatrix} 60 & 45 \\ 49 & 43 \\ 141 & 92 \end{pmatrix}$

c) $\begin{pmatrix} 65 & 45 \\ 49 & 43 \\ 141 & 98 \end{pmatrix}$ d) $\begin{pmatrix} 65 & 42 \\ 49 & 43 \\ 140 & 98 \end{pmatrix}$

9. Solve $\frac{dy}{dx} = \frac{2x}{y+1}$

a) $\frac{y^2}{2} + y = x^2 + c$ b) $\frac{y^2}{2} + y = 2x^2 + c$

c) $\frac{y^2}{2} + y = \frac{x^2}{2} + c$ d) $y^2 + y = x^2 + c$

10 . Solve $\frac{dy}{dx} = (1 + x)(1 + y)$

a)$1 + y = x + \frac{x^2}{2} + c$ b)$1 + y^2 = x + \frac{x^2}{2} + c$

c)$\ln(1 + y) = x + x^2 + c$ d) $\ln(1 + y) = x + \frac{x^2}{2} + c$

11. $\frac{d}{dx}\sin(2x + 1) =$

a) $2\cos(2x + 1)$ b) $2\sin(2x + 1)$

c) $\cos(2x + 1)$ d) $-\cos(2x + 1)$

12. Solve $\frac{d}{dx}\cos(5x - 4) =$

a) sin(5x-4) **b**) -5sin(5x-4)

c)-5cos(5x-4) d)-sin(5x-4) +5

13. Solve $\frac{d}{dx}x^3 cosx =$

a) $3x^2 cosx - x^3 sinx$ b) $3x^2 + cosx$

c) $3x^2 + x^3 cosx$ d)$-x^3 sinx$

14. Find $\frac{dy}{dx}$ where $y = x^2 sinx$

a)$2xsinx + 2cosx$ b) $2xsinx - x^2 cosx$

c) $2sinx + x^2 cosx$ **d**) $2xsinx + x^2 cosx$

15.Find $\frac{dy}{dx}$ where $y = \frac{cosx}{x^2}$

a) $\frac{(xsinx+2cosx)}{x^2}$ **b)**$-\frac{(xsinx+2cosx)}{x^2}$

c) $\frac{-(sinx+2cosx)}{x^4}$ d) $\frac{-(xsinx+cosx)}{x^2}$

16. Find $\frac{dy}{dx}$ where $y = \frac{x^2+5}{2x-4}$

a) $\frac{(20-8x)}{(2x-4)^2}$ **b)** $\frac{(2x^2-8x-10)}{(2x-4)^2}$

c) $\frac{(6x^2-8x-10)}{(2x-4)^2}$ d) $\frac{(2x^2-8x-5)}{(2x-4)^2}$

17. If $z_1 = 9 - 2j$ and $z_2 = 2 - 4j$ find $z_1 z_2$

a) 18-32 j b) 18+40j c)18-40j **d**) 10-40j

18. If $z_1 = 4 + 2j$ and $z_2 = 1 - 8j$ find $z_1 z_2$

a) 4+46j b) 4-46j c)20-34j **d)** 20-30j

19. Evaluate the following integral $\int \cos(x + 2)\, dx$

a) –sin(x+2) b)sin(x+2) c) 2cos(x+2) d)-2cos(x)

20. Evaluate the following $\int x(4x^2 - 7)^3 dx$

a) $\frac{(4x^2-7)^3}{8}$ b) $\frac{(4x^2-7)^3}{32}$ c) $\frac{(4x^2-7)^4}{8}$ **d)** $\frac{(4x^2-7)^4}{32}$