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An Investigation into the Characteristics of Equity Volatility and its Implications for Derivative Strategies

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ABSTRACT

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The development of an effective mechanism for pricing options has inspired a large volume of academic research and has ultimately changed the landscape of the financial markets. Since the publication of Black and Scholes' (1973) seminal paper on option pricing, the finance literature has explored and at least partially resolved many of the limitations associated with the original model. The reality of stochastic volatility contradicts a key assumption of the Black-Scholes model and addressing this has motivated the development of more appropriate volatility models. The improved specification and forecasting of asset price volatility has been influenced by the demands of risk management and portfolio functions. The increased use of quantitative methods in portfolio management is due, in part at least, to successful academic research into asset volatility.

Existing research is extended in this thesis by first examining the forecasting power of implied volatilities from traded UK equity options. Composite implied volatilities are created using weighting techniques that efficiently capture the predictive information in traded options. These implied volatilities are benchmarked against subsequently realized stock price volatility estimated from high-frequency stock price data. The predictive information provided by the options market is compared against that available from sophisticated statistical models such as the generalized autoregressive conditional heteroskedastic (GARCH) model and the exponential-GARCH (E-GARCH) model. Comparison of implied and statistical forecasts is carried out over a number of forecasting horizons using regression analysis as well as robust pairwise tests.

The second part of this thesis uses semi-parametric techniques to examine the long-run dynamics of UK equity volatility. The nature of volatility persistence found in both the implied and realized volatility series of a number of companies is carefully examined. Testing the time-domain properties of the volatility series identifies the extent to which structural breaks in volatility contribute to observed levels of persistence in our sample of companies. The nature of the long-run relationship between implied and realized volatility is also examined. The relevance of these empirically observed volatility characteristics is examined in the final part of this thesis.

Using dynamic programming techniques together with Monte Carlo simulation, optimal portfolio weights are determined for a derivative strategy implemented in discrete time. The derivative strategy is activated across a six-month investment horizon and rebalancing occurs at the beginning of each month. The creation of a series of variance grid points at each time step makes the dynamic programming approach computationally feasible. Progressing backwards from the end of the investment horizon, optimal portfolio weights are found for each of the variance grid points. The optimisation procedure assumes that volatility is driven by a short-memory affine process. The economic cost associated with omitting long-memory effects is isolated by simulating a fractionally integrated process across the same investment horizon and applying the previously assigned weights at each time step. The relevance of omitting possible regime shifts in the volatility process are evaluated in the same manner. Portfolio outcomes are derived for the optimal case, that is, when actual volatility follows a short memory process. Outcomes are also derived for the alternative conditions, that is, a 'true' long memory, fractionally integrated process as well as the 'spurious' long memory or regime-switching case. The impact of volatility mis-specification is captured in the characteristics of the portfolio's terminal wealth distribution.

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Contents

CHAPTER 1

Introduction

1.1. Introduction

The fund management industry has been the venue for significant research and innovation over the past fifty years. The wider application of risk-adjusted performance measures has meant that stock selection and asset allocation decisions are increasingly sensitive to volatility effects over short- and medium-term horizons. In a survey carried out by The Intertek Group, the management of equity portfolios has shown a greater reliance on quantitative modeling in recent years[1]. This trend in the fund management industry is a product of the development of a number of volatility forecasting models in the academic literature. Increasingly sophisticated time series forecasting models have been produced by Engle (1982), Bollerslev (1986), Nelson (1991) and Glosten, Jagannathan and Runkle (1993) among others. Although these models have become progressively more attuned to empirically observed volatility characteristics, their relative complexity has limited their appeal for market participants. The academic literature has

[1]The 2006 Survey by the Intertek Group and Frank J. Fabozzi notes that 84% of respondents reported that the percentage of equity assets under quantitative management had increased over the previous two years.

also extensively examined the capacity of the options market to provide forward-looking information on asset volatility [Bates (1991), Christensen and Prabhala (1998), Poteshman (2000)]. This thesis compares implied volatility forecasts against popular time series methods and the relationship between the equity options market and the underlying assets is explored within the context of the implied-realized volatility relationship. The thesis also investigates how long-run volatility dynamics impact on the construction of optimal investment strategies.

1.2. Research Motivation

The initial motivation for this thesis emerged from an interest in the behaviour of financial market participants around the time of the collapse in telecom, media and technology stocks at the beginning of this decade. The predominance of investor sentiment as an influencing factor on asset values led to some preliminary investigations into how measures of sentiment could be used to inform decision making in the financial markets. The function fulfilled by financial options, that of hedging against and speculating on future risk presented a potentially valuable line of enquiry. The early part of this research focused on the volatility forecasting literature and in particular on the advances made in the class of statistical forecasting methodologies such as ARCH and GARCH. The ability of these models to predict volatility over short forecast horizons has been well documented in the existing literature [Poon and Granger (2003)]. The forecasting performance of implied volatilities backed out of the market prices of traded options provides an alternative forecasting method and a possible indicator of investor sentiment. This thesis focuses on individual equity options which have been relatively neglected in the existing literature, in principle due to limitations with data availability. Generating a composite implied volatility estimate using close-to-the-money options efficiently utilises the forward-looking information contained in option prices [Mayhew (1995), Ederington and Guan (2002)]. A comparison of this forecast against statistical GARCH forecasts generated from historical patterns in the volatility of the underlying asset is also carried out. The comparison between both forecasting methodologies provides an insight into the relationship between the market for individual equity options and the underlying equities.

Empirical research into the volatility dynamics of foreign-exchange markets and equity indices has identified levels of persistence that can be modeled as fractional integration in the autocorrelation structure of volatility. These findings have been challenged by more recent research showing that structural breaks in volatility can induce similar levels of persistence [Granger and Hyung (2004)]. Budek, Schotman and Tschering (2006) note that correctly modeling long-run volatility dynamics is critical for the development of effective criteria for risk management procedures.

In the financial markets there has been an increased flow of capital towards investment strategies other than the traditional 'long-only' strategy. Evaluating the optimal portfolio outcomes of investment strategies that allow for positions in derivatives has been facilitated by developments in dynamic programming techniques. Improvements in computing speeds has also made dynamic programming within a realistic framework possible. This thesis uses dynamic programming techniques to examine the relevance of long-memory effects for market practitioners engaged in discrete time derivative strategies.

1.3. Objectives of the Research Study

This thesis achieves a number of objectives that enhance the current understanding of volatility dynamics and the relevance of these dynamics in portfolio applications. The following specific research objectives are proposed.

- To evaluate the ability of implied volatilities to forecast subsequently realized volatility for a set of individual companies traded on the FTSE-100 index.
- To test for the presence of fractional integration in the realized volatility of individual equities.
- To test for the presence of structural breaks in the volatility process (spurious long memory).

- To model the long-run relationship between implied and realised volatility, testing whether the relationship is a fractionally cointegrating one.
- To employ dynamic programming techniques and Monte Carlo simulation in the investigation of long memory effects, produced as a result of fractional integration or induced through structural breaks in the mean level of volatility can impact on the performance of a derivative strategy.

1.4. Structure of the Thesis

Chapter Two - The Forecasting Performance of Implied Volatilities on Individual Equity Options compares the forecasts produced by implied volatilities from individual equity options to statistical forecasts produced by GARCH models. There are three datasets employed in the study. Daily end-of-day data on individual equity options for the period 1997-2003 is provided by LIFFE and daily stock price data for the same period is obtained from Datastream. Forecasts produced by the class of GARCH models are generated using the daily stock price data. An accurate estimate of realized volatility is calculated using a third dataset of high-frequency tick-price data provided by the London Stock Exchange (LSE). The raw dataset obtained from the LSE contained some incorrect and non-unique prices, which had to be removed before the dataset could be used in the construction of a daily realized volatility time series. The results of this research show that traded prices on individual equity options contain information on future idiosyncratic or stock-specific risk that is not available in GARCH or E-GARCH models. The results also show that individual UK equity options are the optimal forecasting method particularly over 10-day forecasting horizons. These findings provide some insight into the short-run relationship between the options and stock market and the results confirm the important signal function of individual equity options. Therefore, this research has potential relevance for risk managers who actively manage equity portfolios and are engaged in an ad-hoc rebalancing strategy.

Chapter Three - Modeling the Implied and Realized Volatility Relationship in Individual Equities extends the analysis in Chapter 2 by looking at the long-run dynamics of realized volatility for a sample of FTSE-100 stocks. Semiparametric techniques used in this case are the Geweke Porter-Hudak (1983) estimator and the feasible exact local Whittle estimator. Both estimates provide evidence that stock specific shocks observed in realized volatility decay at a very slow, hyperbolic rate. Recent research has shown that similar levels of persistence can be induced by occasional breaks in the volatility process. It is therefore important that the contribution of structural breaks to observed levels of persistence are measured. This chapter includes a series of time-domain tests that identify structural breaks in both realized and implied volatility series. Fractional integration is shown to be present in a number of volatility series. The similar levels of persistence observed in the implied and realized volatility series of a number of companies indicates that both volatility series may be fractionally cointegrated. Using recently developed techniques we formally tests for the presence of fractional cointegration between implied and realized volatility. The more complete understanding of the long-run dynamics provided by this analysis leads to a re-assessment of the relationship between implied and realized volatility.

Chapter Four - Long Memory Effects in Portfolio Planning applies the insights of previous chapters to a realistic portfolio planning problem. The research examines a derivative strategy under specific volatility conditions. Simulations are used to show how the distribution properties of terminal wealth are influenced by long-memory effects. This chapter also examines whether modelling for long memory effects in asset volatility has an economic value when constructing an optimal derivative strategy in discrete time. The impact of long memory effects on the terminal wealth distribution is specifically identified. A clear distinction is made between long memory effects that arise from a fractionally integrated process and those induced by regime shifts in the volatility structure. The optimal investment strategy in discrete time is found using backward induction combined with a numerical optimization and Monte Carlo simulation. The optimization procedure assumes that asset volatility follows a short memory process and using this portfolio policy the outcomes from actual volatility following a fractionally integrated or regime-shifting model are examined. In the context of a covered call strategy, portfolio performance is found to deteriorate if the underlying data generating process is fractionally integrated and these effects are not included in the optimisation process. If fractional integration is not considered in a discrete-time rebalancing decision, the investor is more likely to include non-optimal derivatives within the portfolio.

Chapter Five - Summary Discussion and Conclusions involves a summary discussion of the research carried out. The limitations of the research are also flagged as are suggestions for further research.

CHAPTER 2

The Forecasting Performance of Implied Volatilities on Individual Equity Options

2.1. Introduction

The volatility forecasting literature includes a large number of studies that examine the volatility implied in the market prices of index options[1]. The increased use of index options by equity fund managers as part of a hedging strategy has meant that implied volatilities backed out of the traded market prices of index options are now a generally accepted indicator of investor sentiment. These instruments are useful for those funds whose performance is closely correlated with a specific equity index. However in recent years, greater amounts of capital have been devoted to active investment strategies that rely less on index-wide diversification. At the beginning of this decade actively managed equity funds incurred additional expenditure of $20 billion per year measured as the expense ratio between active and passive funds [Wermers (2000)]. Cai and Zhang (2004) show that over the sample period, 1981 to 1996, the average institutional traders trades in approximately 75%

[1] A partial list of these studies includes Bates (2000), Canina and Figlewski (1993), Christensen and Prabhala (1998), Jackwerth and Rubinstein (1996).

of stocks in each quarter. Fund performance is dependent on the fund manager's stock-picking ability and as part of this process the expected volatility pattern of individual stock returns over a number of horizons are an important input. This chapter explores whether implied volatility from individual equity options can contribute to effective decisions, in this regard, by providing appropriate estimates of expected volatility.

Individual equity options are driven by a set of factors that are in many cases distinct from those that influence index option prices. New 'firm-specific' information, such as quarterly reports or product releases lead to a temporary increase in trading and influence short-term volatility patterns. The period immediately prior to these announcements is often characterised by a decline in trading volume and volatility as traders await the arrival of new information [Ederington and Lee (1996), Malz (2003)]. These short-term variations have been attributed to feedback trading and it has been observed in stock index returns by Koutmos (1998) and Venetis and Peel (2003). If the market in individual equity options is efficient, the timing of expected company announcements and the associated short-run volatility patterns should be reflected in the pricing of individual equity options. Dubinsky and Johannes (2005) show how equity options capture stock specific uncertainty around earnings announcements dates in a sample of 20 U.S. stocks. This feature distinguishes these instruments from index options where stock specific announcements impact market wide volatility only in exceptional cases. Further evidence of the differences between index and individual equity options is the existence of commonly used analytical strategies such as dispersion trading[2].

This chapter examines the ability of implied volatility estimates from individual equity options to forecast subsequently realized volatility over the life of the option. Estimates of stock volatility over extended horizons are important considerations for risk managers seeking to control and measure their risk exposure and minimise rebalancing costs. The increased use of risk-adjusted portfolio performance coupled with the practice of ad-hoc rebalancing decisions at the end of each trading month increases the importance of accurate volatility estimates between rebalancing dates. Research into the predictive ability of equity options has been hampered by the relatively low levels of liquidity in these markets compared to index options. Testing procedures were also constrained by the reliance on daily returns data when benchmarking implied volatility forecasts. In this research the availability of extensive options data and high frequency price data on the underlying stocks contribute to the robust results. Options data was obtained on all constituent companies making up the FTSE-100 as at December 2003.

[2]Dispersion trading exploits the fact that index volatility has traded at a premium (due to hedging demand from fund managers), while individual stock volatility has been fairly priced. The strategy can be operationlised by combining a short position on Index options with a long position on options on its constituent stocks.

Those companies with short trading histories and low option trading volume were not considered, so that, the research presented here uses options data on fifteen FTSE-100 companies. The options considered are those contracts for which data is continuously available over a seven year period. Estimation of the composite implied volatility forecast uses a weighted average of implied volatilities drawn from a few close-to-the-money equity options. Benchmarking of the predictive ability of these estimates is then carried out against realized volatility estimated from intraday tick-by-tick price data on individual stocks. This research also ranks implied volatility forecasts against multi-step ahead forecasts produced by popular time-series models such as GARCH [Bollerslev(1986)] and E-GARCH [Nelson(1991)].

The following section discusses the relevant literature in the area. The subsequent section details the process for estimating realized volatility and the methods used to generate each of the forecasting models. This section also describes the testing methodology. Empirical results and conclusion then follow.

2.2. Volatility Forecasting

The development of an effective option pricing framework by Black and Scholes (1973) and Merton (1973) contributed to the rapid expansion of exchange traded options on an increasingly broad range of assets. Traditional option pricing models such as the standard Black-Scholes model for pricing

European options as well as the binomial approach developed by Cox, Ross and Rubinstein (1979) for American-style options rely on the assumption of constant volatility over the life of the option contract. Subsequent models have incorporated greater realism by modeling the dynamics of stochastic volatility observed in empirical studies, however despite the obvious limitations of traditional option models they remain widely used among market participants. The volatility estimate implied by the market price of the option is generally obtained using this traditional framework and is considered an efficient forward-looking estimate of the expected volatility of the underlying asset.

Unsurprisingly a significant amount of research has focused on the predictive ability of implied volatility forecasts from equity index options which are traded in relatively large volumes. Canina and Figlewski (1993) examined implied volatility from S&P index options and found that it is a weak predictor of subsequently realised volatility. Option market inefficiency and the assumptions underlying the Black and Scholes (1973) option-pricing model are identified as key factors in this result. Figlewski (1997) shows that non-continuous trading in the options market and the inability to observe the 'true' equilibrium option price due to bid-ask spreads are key reasons for the weak results found in those early studies. Inaccuracies that are caused by market frictions are exacerbated when using deep in-the-money (ITM) and deep out–of-the-money (OTM) options so that even a narrow bid-ask spread has a significant impact on the implied volatility estimate [3].

Despite these findings, commercially available implied volatility indices such as the Chicago Board of Trade Volatility Index (VIX) have been available since 1993. The VIX is a synthetic implied volatility measure produced as a weighted average of 8 OEX (S&P 100 Index) put and call options. Academic research has more recently provided greater support for the use of these indices as forward-looking indicators of market sentiment. An important contribution to the literature was made by Christensen and Prabhala (1998) who looked at the relationship between implied and realized volatility on S&P 500 Index options over a sample period of $11\frac{1}{2}$ years. Christensen and Prabhala's (1998) use of a non-overlapping dataset allows for more robust regression results and implied volatility is found to have a much higher explanatory power than past volatility. In addition, this approach reduces the positive implied volatility bias that is suggested in Jorion's (1995) study. This bias is found consistently in the literature and is generally attributed to the unrealistic assumption of constant volatility in standard option pricing models. Penttinen (2001) notes that studies testing implied volatility forecasts have been entirely based on a period of historically low volatility.

[3] Bates' (1991) study of S&P 500 options for the period 1985-1987 looked at relative prices of OTM put options and OTM call options with a view that "unusually" expensive OTM puts could indicate the market's assessment of an imminent downturn. In the case of the October 1987 stock market crash expectations of a market downturn were reflected in options prices in the year before the crash, however in the period immediately preceding the crash downside risk was not very pronounced.

Penttinen (2001) argues that the bias between ex-ante implied and ex-post realized volatility should be attributed to the rational but unrealised expectations of infrequently occurring jumps in volatility.

Although notably less research has examined the market in individual equity options, an extensive review of the volatility forecasting literature by Poon and Granger (2003) identifies some relevant studies. Chiras and Manaster (1978), Beckers (1981) and Gemmill (1986) do not show consistent support for forecasts provided by implied volatility backed out of individual equity options. The use of narrow sample periods and limited options data in these studies is likely to have an impact on the findings produced. Lamoureux and Lastrapes (1993) and Vasilellis and Meade (1996) backed implied volatility estimates out of a single equity option price and tested its predictive ability over a limited number of forecast horizons. Lamoureux and Lastrapes (1993) tested the forecast performance of implied volatilities drawn from ten European-style individual equity options traded on the Chicago Board of Exchange over a two-year period (April 1982 - March 1984). Due to limitations of the options data, one-step-ahead implied volatility forecasts were obtained from options with maturities of 129 trading days in some cases. These forecasts were found to contain information above that available in historical prices. Vasilellis and Meade (1996) examined implied volatilities on the options of twelve FTSE-100 companies using weekly data. The implied volatilities were shown to contain information not available from historical data. Implied volatility forecasts, generated both from single option prices as well as using weighting schemes provide forecasts that are superior to forecasts produced by the GARCH model. In this chapter the use of high frequency data for both stock and option prices, facilitates a robust analysis that updates and extends the study carried out by Vasilellis and Meade (1996).

2.2.1. Data Description

The task of evaluating the accuracy of volatility forecasts is difficult since 'true' volatility cannot be observed. The use of high-frequency data (HFD) confers a number of benefits on this research and distinguishes the analysis carried out here to similar studies by Day and Lewis (1992) and Lamoureux and Lastrapes (1993) that use weekly and daily data respectively in their estimation of 'true' volatility. Andersen and Bollerslev (1998) studied the conditional variance estimates provided by forecasting models against squared returns (r_t^2) , calculated from end of day stock prices. Returns are calculated as, $r_t = \ln(S_t) - \ln(S_{t-1})$, where the return at time t, r_t is the natural log difference of the stock price (S_t) at time t. Daily squared returns are shown to be a noisy estimate of volatility and in all cases a regression of the volatility forecast against squared returns would indicate weak explanatory power. Awartani and Corradi (2004) show that even if 'true' unobservable volatility, $\sigma_t^{\dagger 2}$, is replaced with squared returns, r_t^2, then the correct ranking of models based on any quadratic loss function will be maintained. A detailed analysis of the predictive ability of volatility forecasts benefits from the use of HFD which contains significantly more information than daily price data.

For each of the companies included in the study, tick-by-tick data was obtained from the London Stock Exchange (LSE) for the period 1997 to 2003. In order to calculate an accurate measure of realized volatility it is important that market microstructure effects are mitigated where possible. Microstructure effects such as bid-ask spreads are shown by Meddahi (2002) and Oomen (2006) to contribute to less robust estimates of realized volatility. Volatility estimates become increasingly biase over finer sampling intervals. The problems associated with bid-ask spreads are overcome in this research by using the series of best prices available at the times each of the trades occurred. The best price series provided by the LSE is a raw dataset that presents a number of exchange-related issues that require preliminary manipulation of the dataset. The best price dataset is obtained for each of the sample companies and it includes all trades published on a given day, irrespective of the time of trade execution or reporting. Furthermore, cancellation trades and late trade corrections are included on the day the correction is published. Late and overnight trades are also included on the day these trades are published, that is, within the trading hours of the following trading day. The reporting of cancellation trades as well as late and overnight trades present a number of issues in relation to the recording and sequencing of prices that must be addressed before the series could be used. The timing and type of trading that is carried out means that the large tick datasets is are very often characterised by incorrect, simultaneous and consecutive non-unique price reports that if left unadjusted severely affect the estimation of realized volatility.

In a study of microstructure effects in the US stock market, Brownlees and Gallo (2006) address the not insignificant matter of how best to manage unwieldy financial data. This research produced a set of useful algorithms that were amended and used here to filter and manage the LSE data[4]. The initial step was to remove incorrect price reports including zero prices and suspiciously large price observations. Consecutive non-unique prices that are the result of late or overnight trades also affect the correct calculation of realized volatility and are excluded. This process resulted in a time series of intra-day prices with irregular intervals between each reported price. Recently a number of studies have addressed how HFD can be optimally used in the estimation of realized volatility. Although the estimation of realized volatility should approximate 'true' volatility as the sampling frequency increases, Jacod and Shiryaev (2003) show that realized volatility estimates become biased and inconsistent as the sampling frequency increases. To overcome these effects a simple nonparametric measure of realized volatility can be produced that first converts the irregular series of tick prices to a lower frequency, equally-spaced time series. In this instance, a time series of 30-minute prices is created using an aggregation function that finds the last price before the end of each interval. If $\ln S_t, t = 1, ..., T$ is a series of daily stock prices and let $\ln S_{t+k\xi}, k = 1, ..., m$ and $\xi = 1/m$ denote a series of 30-minute observations, then a daily estimate of realized variance can be constructed as $RV_{t,m} = \sum_{k=0}^{m-1} (\ln S_{t+(k+1)\xi} - \ln S_{t+(k)\xi})^2$. This approach is

[4]Brownlees and Gallo's (2006) Matlab computer code is publicly available.

evidenced in studies by Andersen, Bollerslev, Diebold and Labys (2001) and Barndorff-Nielsen and Shephard (2002).

More recently, studies have observed that neglecting more finely sampled data in this manner excludes a large proportion of available information. Zhang, Myland and Ait-Sahalia (2006) consider the method of ad-hoc sparse sampling along with a number of alternative realized volatility estimators. The approach is ranked as the fourth best estimator of the five considered and is shown to be less effective than sampling sparsely at an optimally determined frequency. Sparse sampling, either by ad-hoc or optimal methods, excludes data and is thus inferior to approaches that use subsampling and averaging. The use of subsampling incorporates a greater amount of the available information and avoids the process of discarding data that is necessarily part of sparse sampling. Zhang, Mykland and Ait-Sahalia (2006) identify the optimal estimator as one that uses subsampling and averaging and also corrects for bias. The two-scales realized volatility (TSRV) estimator combines two estimators, one that utilises all tick prices and a second that averages estimates of realized volatility across regular intervals. In this instance realized volatility is estimated in a number of stages over slow and fast time scales that correct for biases produced by microstructure effects. An examination into multi-scale realized volatility (MSRV) by Zhang (2006) shows that the estimator converges to 'true' volatility at a faster rate than alternative methods. In this research, the estimation of daily realized volatility using either the TSRV or the MSRV estimator imposes a considerable computational burden when applied to the large number of tick observations for FTSE-100 stocks over the sample period 1997 to 2003. In light of more recent evidence, sparse sampling produces a relatively crude estimator of realized volatility. The relative simplicity of this approach means it is computationally feasible when applied to sizeable datasets examined in this research. The properties of the daily realized volatility series for each company, summarised in Table 2.1, show a wide variation in the descriptive statistics among the cross-section of companies.

Deriving Implied Volatilities from Individual Equity Options

American-style equity options trade on the London International Financial Futures Exchange (LIFFE) and prices are quoted for all companies on the FTSE-100 exchange. The dataset procured from LIFFE contains details such as the trade and expiry dates, the volume traded on each option, as well as an annualized implied volatility backed out of the market price of the option. Implied volatility estimates for each option are produced by LIFFE Euronext using the Cox, Ross and Rubinstein (1979) binomial option pricing method that incorporates dividends and early exercise. The exact procedure for constructing the binomial tree is proprietary and the details are retained by LIFFE Euronext. The LIFFE Euronext data includes options contracts that were priced but were not traded. A preliminary filtering process carefully removed options data on quoted contracts that were not traded. For each company considered in this study a number of daily options contracts were traded on each day with a range of maturities and strike prices. The moneyness of an option refers to the distance between the price at which the option can be exercised, that is, the strike price and the current price on the underlying asset. A number of studies have examined how best to use implied volatilities on traded options. The majority of studies specifically examined the sensitivity of implied volatility from index options to contract specifications such as maturity and moneyness.

Over the past decade there has been greater agreement among academics and practitioners on how to optimally use implied volatility data. Ederington and Guan (1999) show that at-the-money options demonstrate greater sensitivity to the volatility of the underlying asset than far-from-the-money options. Furthermore, the positive bias found empirically in implied volatility forecasts is minimized by using options trading close to the money. These findings are confirmed by Bodie and Merton (1995) who show that the bias associated with implied volatility forecasts can be mitigated by using short-dated options that are close-to or at-the-money. Ederington and Guan (2002) estimate a composite implied volatility forecasts using four or eight option contracts trading close-to-the-money. This method produces estimates that are superior to estimates generated using all available options, including those that are far-from-the-money. Although simple averaging of implied volatilities has been applied by Jorion (1995) and Weber (1996) in studies on forecasting, Mayhew (1995) and Ederington and Guan (2002) show that weighting implied volatilities according to the moneyness of the contract minimises the variance of the implied volatility estimate and is thus more efficient than simple averages. These findings are reflected in the investment industry where commercially available implied volatility indices are estimated using only a few at-the-money or near-the-money options and are weighted according to contract moneyness. One commercial example is the implied volatility index (VIX) produced by the CBOE that is constructed as a weighted average of the four call options and four put options that are trading nearest the money[5]. In this paper, composite implied volatilities are created using a moneyness weighted scheme similar to that applied by commercial vendors such as the CBOE.

Forecasts are examined across short-, medium- and long-term horizons of 10-days, 30-days and 60-days respectively. A prerequisite to estimating an implied volatility forecast is that sufficient contracts are available at each forecast date to enable calculation of the synthetic implied volatility estimate. To satisfy this condition options are grouped according to the length

[5]Details on the construction of the The VIX Index are contained in the CBOE technical document "The New CBOE Volatility Index - VIX", available on www.cboe.com.

of contract maturity. Three subgroups of options are created and each group aligned to a specific forecast horizon. Short-term implied volatility forecasts are generated from contracts with maturities between 10 and 30 days. Medium-term forecasts are generated from option contracts with maturities between 30-60 days and contracts with 60 to 90 days maturity are used in long-term forecasts. Once the contracts are bundled, the annualized implied volatility estimate provided by LIFFE Euronext is adjusted to the appropriate forecast horizon using the square root of time rule. Assuming 252 working days in the year, the h-day-ahead implied volatility forecasts are estimated as, $\sqrt{\frac{h}{252}}IV$, as per Giot and Laurent (2007). For example, an implied volatility produced by an option contract in the short-maturity (10-day) bundle may have 25 days to expiry. The implied volatility from this contract is adjusted to a ten-day horizon as follows, $\sqrt{\frac{10}{252}}IV_{25-day}$. The optimal method for constructing a composite implied volatility estimate has been examined in a number of studies [Mayhew (1995), Ederington and Guan (2002)]. The weighting scheme used in this chapter is guided by the results produced by Ederington and Guan (2002) that recommend methodologies used by commercial vendors such as the CBOE-VIX. The scheme approximates the commercial approach as closely as possible. It differs only in the use of four rather than eight options (a reflection of the greater trade volume in Index options compared to equity options). The composite implied volatility forecast incorporates the implied volatilities of the four nearest-to-the-money options traded on each day in a manner similar to that applied by commercial vendors. This procedure uses the time-adjusted implied volatilities from the two call options (IV_{c1} and IV_{c2}) and the two put options (IV_{p1} and IV_{p2}) that are trading nearest the money. The weighting method used to estimate the composite implied volatility takes account of the respective option's moneyness as follows,

$$
\begin{aligned}
IV_{t,T} &= 0.50\left(\frac{X_{c2}-F}{X_{c2}-X_{c1}}\right).IV_{c1} + 0.50\left(1-\frac{X_{c2}-F}{X_{c2}-X_{c1}}\right).IV_{c2} \\
&\quad +0.50\left(\frac{X_{p2}-F}{X_{p2}-X_{p1}}\right).IV_{p1} + 0.50\left(1-\frac{X_{p2}-F}{X_{p2}-X_{p1}}\right).IV_{p2} \qquad (2.1)
\end{aligned}
$$

where F is the underlying stock price (face value), X_{c1} and X_{c2} ($X_{p1}$and X_{p2}) are the strike prices of the nearest-the-money call (put) options and IV_{c1} and IV_{c2} (IV_{p1} and IV_{p2}) are the corresponding implied volatilities. This approach focuses the analysis on those options trading near the money which are most sensitive to volatility in the underlying asset, therefore minimizing measurement errors and clientele effects that affect out-of-the-money options. A daily composite implied volatility times series is produced for each forecast horizon, that is, the 10-day, 30-day and 60-day horizon. These forecasts are benchmarked against realized volatility estimated as the sum of daily realized volatility across the forecast horizon from the day after the trade date, as follows, $RV_{t,h} = \sum(RV_{t+1}, ..., RV_{t+h})$, where RV is realized volatility estimated from 30-minute intraday returns, t is the option trade date and h is the forecast horizon. The procedure results in paired implied and realized volatility series at 10-day, 30-day and 60-day overlapping forecast horizons. The use of overlapping horizons in regression analysis has been shown by Christensen and Prabhala (1998) to produce results that are not robust since residuals based on regressions across overlapping horizons are likely to be correlated.

Christensen and Prabhala (1998) show that by constructing a series of non-overlapping forecasts, correlation in the residual series is mitigated and more robust results are produced. This procedure has a disadvantage in that it diminshes the sample size used in the analysis. The creation of a series of 10-day forecasts across non-overlapping horizons allows us to compare our results with those produced by Christensen and Prabhala (1998). The non-overlapping series selects option contracts trading at 10-day intervals. The problem of shallow trading in 10-day contracts is overcome by including all options with a contract maturity between 10 and 60 days. Where the contract maturity extends beyond ten days, annualized implied volatility from these contracts is adjusted to the 10-day horizon using the square root of time rule. The composite implied volatility forecast is then estimated from the four closest-to-the-money options using the moneyness weighing scheme. Realized volatility is estimated for the ten days from the option trade date using HFD. Implied volatility forecasts are also compared to forecasts produced by statistical methods, namely, the generalized autoregressive conditional heteroskedastic (GARCH) model as well as exponential GARCH (E-GARCH) model.

Statistical Approaches to Modeling Volatility

The performance of implied volatility estimates are ranked against statistical forecasting models. We apply the generalised ARCH or GARCH model proposed by Bollerslev (1986) to daily stock price returns,

$$\sigma_{t+h}^2 = \varpi + \alpha\varepsilon_t^2 + \beta\sigma_t^2 \tag{2.2}$$

where σ_t^2 is the variance at time t, ϖ is the mean variance, α and β are constant parameters and ε_{t-1}^2 is squared innovation from the previous time period. The daily stock price return is estimated as the log difference between the end-of-day closing stock price S_t and the previous day's closing price S_{t-1}, as follows, $r_t = \ln(S_t) - \ln(S_{t-1})$. The parsimonious use of parameters in the GARCH (1,1) model have contributed to its relative popularity among practitioners in the financial markets. Within the academic literature, the GARCH (1,1) model represents only one of a myriad of GARCH-type models. Despite its relative simplicity, Hansen and Lunde (2005) show that GARCH (1,1) produces robust volatility forecasts. Hansen and Lunde

(2005) implement the test for superior predictive ability (SPA) in order to compare one-day forecasts from 330 GARCH-type models. The SPA test allows comparison of multiple models and is based on the pairwise methodology developed by Diebold and Mariano (1995). The Hansen and Lunde (2005) study show that GARCH (1,1) predicts out-of-sample forecast of conditional variance on the DM-$ exchange rate that is equivalent to more sophisticated models. A similar analysis of GARCH models is carried out on IBM returns. Hansen and Lunde (2005) show that GARCH models that capture volatility asymmetry outperform GARCH (1,1) in this context.

The GARCH (1,1) model uses the absolute values of the innovations not their sign, therefore, both postive and negative price movements are modeled identically. The asymmetric nature of asset volatility is captured by alternative specifications that take account of the fact that negative and positive price shocks have different effects on future volatility. Nelson's (1991) exponential GARCH or EGARCH model introduces a leverage term into the conditional variance equation, and is expressed as:

$$\log \sigma_t^2 = \varpi + \beta_1 \log \sigma_{t-1}^2 + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}, \tag{2.3}$$

where α, β and γ are constant parameters. The use of logged conditional variance, $\log \sigma_{t-1}^2$, relaxes the positiveness constraints. Furthermore, asymmetry in volatility effects is included through $\varepsilon_{t-1}/\sigma_{t-1}$ as long as $\gamma \neq 0$.

When $\gamma < 0$, positive shocks generate less volatility than negative shocks ('bad news') reflecting conditions observed empirically.

The GARCH and E-GARCH models are applied to daily returns on each FTSE-100 company included in the sample. A rolling estimation procedure is used to generate parameter estimates, so that, the first rolling prediction,, $g_{i,n+1}(\widehat{\beta}_{i,R})$, uses model parameter estimates $\widehat{\beta}_{i,n}$ estimated using data observations 1 to n, the second prediction $g_{i,n+2}(\widehat{\beta}_{i,n+1})$, is created using model parameter estimates $\widehat{\beta}_{i,n+1}$ estimated from observation 2 to $n+1$ and so on. Overlapping GARCH and E-GARCH forecasts are generated for 10-day, 30-day and 60-day horizons and ranked against implied volatility forecasts. Furthermore, non-overlapping GARCH and E-GARCH forecasts are co-ordinated with 10–day non-overlapping implied volatility forecasts. The three forecast methodologies are examined directly using regression analysis and are also ranked using pairwise test statistics.

2.2.2. Testing Methodology

Two approaches are used to test volatility forecast accuracy: in-sample tests based on Mincer-Zarnowitz regressions and out-of-sample predictive tests. A standard Mincer-Zarnowitz (1969) regression of forecasts is set up as follows,

$$RV_k^h = \beta_0 + \beta_1 f_{k,m}^h + e_k, \tag{2.4}$$

where RV_k^h is the volatility experienced from day following the option trade date to the end of the forecast horizon, $f_{k,m}^h$ is the forecast is provided by one of the selected models, m. The encompassing regression includes intraday realized volatility observed on the day prior to the forecast date, RV_{k-1}, as an additional explanatory variable. The efficiency of forecast models with respect to observed realized volatility is thus examined,

$$RV_k^h = \beta_0 + \beta_1 f_k^h + \beta_2 RV_{k-1} + e_k, \tag{2.5}$$

An examination of regression output from an unbiased forecast model will produce $\beta_0 = 0$ and $\beta_1 = 1$ and $\beta_2 = 0$, if the forecast captures all the information contained in the observed volatility. The downward bias observed in the slope coefficient when implied volatility is regressed on subsequently realized volatility has been attributed in part to errors in estimating the implied volatility forecast. Christensen and Prabhala (1998) address the errors-in-variables (EIV) problem in the context of examining the implied and realized volatility relationship. The EIV problem is caused by a number of market and model related issues in the estimation of implied volatility forecasts. Christensen and Prabhala (1998) point to the use of the Black-Scholes model as one of a number of contributory factors since it does not allow for dividends and early exercise. This problem is overcome in this research by the use of the Cox-Ross-Rubinstein (CRR) model that allows for dividends and early exercise. The procedure used to estimate the composite implied volatility excludes deep out-of-the-money options as well as contracts with less than ten days to maturity, thus reducing the EIV problem associated with bid-ask spreads in option prices. Christensen and Prabhala's (1998) study into index implied volatilities observes that non-synchronous measurement of option prices and index levels contributed to the EIV problem. This is not a concern in the examination of the implied and realized volatility relationship at the level of individual assets. In addition to carrying out a regression-based analysis, a pairwise comparison of the performance of two forecast models can be examined using the test statistic developed by Diebold and Mariano (1995). The Diebold-Mariano (DM) test examines the differential loss from using model 1 $g\left(e_{1i}^2\right)$ versus model 2 $g\left(e_{2i}^2\right)$ in any period i, where the loss is typically given by the mean square error $\left(e_i^2\right)$. The differential loss in period i from using model 1 versus model 2 is then $d_i = g\left(e_{1i}^2\right) - g\left(e_{2i}^2\right)$ and the mean loss is,

$$\overline{d} = \frac{1}{H}\sum_{i=1}^{H}\left[g\left(e_{1i}^2\right) - g\left(e_{2i}^2\right)\right] \tag{2.6}$$

If the predictive ability of both forecast models is equal then $\overline{d} = 0$. The DM test is robust over one-step ahead forecasts but is shown to be inconsistent over longer forecast horizons. An encompassing test developed by Harvey, Leybourne and Newbold (HLN) (1998) adjusts the DM test statistic for use over forecast horizons longer than 1-step ahead. If γ_i is the i-th autocovariance of the d_t sequence and if the first q values of γ_i are non-zero, then the variance of $\overline{d}$ can be approximated by $var(\overline{d}) = \left[\gamma_0 + 2\gamma_1 + ... + 2\gamma_q\right](H-1)^{-1}$. The Harvey, Leybourne and Newbold (HLN) (1998) test statistic is given as,

$$HLN = \overline{d}/\sqrt{\left[\gamma_0 + 2\gamma_1 + ... + 2\gamma_q\right]/(H-1)}. \tag{2.7}$$

The sample HLN statistic is compared to a $t-$statistic with $H-1$ degrees of freedom. In this research, the HLN statistic is an appropriate measure since forecasts are evaluated across multi-step horizons. Results for the DM test statistic are included for comparison.

2.3. Empirical Results

The sample period runs from 1^{st} October 1997 to 31^{st} December 2003 and analysis was carried out on fifteen companies listed on the FTSE-100. The companies used in this chapter are those for which options were continuously traded over the entire sample period and daily contract volumes were sufficient to enable estimation of the composite implied volatility forecast. Analysing the regression coefficients allows us to make inferences about the quality of implied volatility forecasts as well as forecasts produced by GARCH and E-GARCH models. Table 2.2 provides a summary of the key regression results for the series of overlapping forecasts. The complete set of regression results are contained in Tables 2.2.1 to 2.2.15. The dependent variable in each case is realized volatility observed across the 10-day, 30-day and 60-day forecast horizon. The cross-section of results show wide variability in the proportion of realized volatility explained by implied volatility. The shortest maturity options (10-30 days) produce implied volatility forecasts with relatively strong predictive ability as measured by R^2 in the case of GlaxoSmithKline (21.32%), Hilton (21.92%), Hanson (42.11%), HSBC (45.51%), Kingfisher (61.97%), Prudential (31.25%) and Reuters (27.16%). Across the same forecast horizon, implied volatility has weak predictive ability in the case of Aviva, British Aerospace, British Airways, Cadburys, Diageo, Dixons, Marks and Spencers and Royal Sun Alliance. This variation in R^2 is consistent with the findings reported in early studies by Chiras and Manaster (1978) and Beckers (1981) on US equity stock options. The explanatory power of equity options is not consistent across the companies sampled. Furthermore, the proportion of realized volatility predicted using equity options is in many cases lower than the predicted volatility associated with index options [Blair, Poon and Taylor (2001), Poteshman (2001)]. GARCH and E-GARCH forecasts demonstrate good explanatory power over short horizons for a number of companies and this deteriorates markedly as the forecast horizon is extended.

The implied volatility bias observed in index options is not as prominent in the results for equity options. The relatively neutral intercept term, β_0 in the majority of cases suggests that individual equity options are generally fairly priced. Although implied volatility forecasts from short maturity options over-estimate realized volatility in the case of Kingfisher, Prudential and Reuters, this bias becomes negligble over longer forecast horizon. The low forecast bias supports the idea that implied volatility is appropriately priced for infrequently occurring jumps in volatility [Penttinen (2001)]. The use of a composite implied volatility forecast based on options that are approximately ATM is likely to be a factor that minimizes the forecast bias. This finding contrasts with studies on index options, such as those by Christensen and Prabhala (1998) on the S&P 100 and Ederington and Guan (2002) on the S&P 500 that show a consistent positive bias for equity index options. The results on UK equity options contained here are also inconsistent with Gemmill's (1986) study of UK equity options. The use of high frequency intra-day returns to calculate actual volatility has been shown by Poteshman (2000) to significantly reduce the bias and this is a feature that distinguishes the research here and the earlier study by Gemmill (1986).

The encompassing regression includes the realized volatility observed on the day prior to the option trade date as an additional explanatory variable and an examination of its coefficient, β_2, indicates the extent to which information provided by observed intra-day volatility is incorporated into forecasts.

Forecast efficiency is supported by $\beta_2 = 0$ and implied volatility forecasts from short-maturity options produce estimates of β_2 approximately close to zero in the case of Cadburys (0.02), GlaxoSmithKline (-0.09), Hilton (0.033), HSBC (0.051), Hanson (-0.099), Prudential (0.002) and Reuters (-0.003). This finding appears consistent across forecast horizons and relatively low β_2 coefficients are also observable for GARCH and E-GARCH forecasts. Although there has been no recent research into the information content of equity options, some comparison can be made with encompassing regressions carried out by Giot and Laurent (2007) in their study of S&P 100 and S&P 500 index options. Rather than use lagged realized volatility in the encompassing regression Giot and Laurent (2007) instead use the continuous/jump decomposition of historical realized volatility. Index options exhibit a high information content with little additional information produced by the switch from using solely implied volatility to using a model that contains implied volatility and the full decomposition of realized volatility. The results reported in this chapter show that the precision of implied volatility forecasts produced by equity indices is similarly produced in the market for individual equity options.

Test statistics developed by Diebold and Mariano (DM) (1995) and Harvey, Leybourne and Newbold (HLN) (1998) facilitate a direct comparison of implied volatility forecasts against the statistical techniques of GARCH and E-GARCH. A summary of results for the HLN statistic across the long forecast horizon using overlapping samples is provided in Table 2.3. The entire set of results for both the DM and HLN statistic are included in Tables 2.3.1-2.3.15. A comparison of implied volatility forecasts against GARCH and E-GARCH forecasts across overlapping 10-day horizons supports the use of GARCH and E-GARCH approaches in the majority of companies sampled. Over the longest forecast horizon (60 days) implied volatility is superior to statistical forecast methods in all but six companies, as measured by the HLN statistic. The Durbin-Watson statistic indicates correlations in the residual series resulting from the use of overlapping forecast horizons.

Christensen and Prabhala (1998) demonstrate that more robust results are available by testing predictive ability across non-overlapping forecast horizons. Forecasts are tested against the sum of daily squared returns experienced over the ten-day period following the option trade date so that results can be compared to the analysis of index options produced by Christensen and Prabhala (1998). Results are also produced for a second series of regressions that calculate the dependent variable from realized volatility over the ten-day period following the option trade date, estimated using intraday data. Figures 1 to 5 illustrate the respective times series used in these regressions. The upper panel plots implied volatility derived from options with maturities between 10 and 60 days, scaled to a 10-step-ahead forecast horizon. This panel also includes 10-step-ahead GARCH and E-GARCH forecasts as well as realized volatility calculated using daily squared returns over that horizon. The centre panel compares realized volatility calculated from intra-day returns against implied volatility over similar horizons. The pattern of the two time series suggests that the positive bias of implied volatility forecasts is justified by occasional jumps in volatility in keeping with Penttinen's (2001) findings. From the lower panel, a wide variation in the volume of option contracts traded on each day is observed. Table 2.4 reports the mean squared error (MSE) and mean absolute error (MAE) produced by each of the forecast models across the non-overlapping ten-day forecast horizon. The error measures show wide-variation in error terms for the cross section of companies and evaluation across multi-step horizons produces large forecast errors in some cases, particularly Diageo, Hansen and Reuters.

Tables 2.5 provides a summary of the results for the Mincer-Zarnowitz and encompassing regressions across non-overlapping horizons. In the more detailed results produced in Tables 2.5.1-2.5.14, it is clear from the Durbin-Watson (DW) statistic reported for the series of MZ and encompassing regressions demonstrates that the use of non-overlapping forecast horizons significantly reduces correlation in the error terms. In almost all cases implied volatility explains a significant proportion of daily squared returns experienced over the following 10-day period. The results from the MZ regression show that, with the exception of British Airways and Dixons, implied volatility explains between 16% and 45% of daily squared returns. GARCH and

E-GARCH forecasts also exhibit strong explanatory power for a number of the companies sampled. Of the two statistical models considered a dominant forecasting approach is not indicated by the MZ regression results, although, E-GARCH forecasts provide greater explanatory power than GARCH in nine of the fifteen companies sampled. Both statistical methods produce low R^2 statistics for British Airways, Cadburys and Marks and Spencers. When realized volatility is calculated from intraday data, the explanatory power of all of the forecasting models considered shows a marked decline. Composite implied volatility has an explanatory power greater than the time series models in nine of the fifteen companies studied. Overall, implied volatility forecasts explain more than 15% of subsequently realized volatility in only four stocks, Aviva (15.84%), British Aerospace (16.14%), British Airways (15.64%) and Dixons (18.60%). Furthermore, implied volatility forecasts produce a negative intercept term for the majority of companies, indicating that implied volatility overestimates subsequently realized volatility. Interestingly the signficant increase in explanatory power produced by the encompassing regression suggests that implied volatility doesn't fully incorporate information available in intraday historical data in all cases. The use of implied volatility and historical volatility as explanatory variables explains a significant proportion of realized volatility in the case of Aviva (20.40%), British Aerospace (37.11%), British Airways (16.69%), Dixons (48.78%), Marks and Spencers (30.89%), Prudential (47.30%) and Reuters (19.30%). When examined across 10–day non-overlapping forecast horizons, the predictive ability of GARCH and E-GARCH forecasts is similarly inconsistent across the sample of companies. The superiority of implied volatility forecasts over GARCH-type forecasting methods supports the findings of Vasilellis and Meade (1996).

Table 2.6 reports the results from pairwise tests of forecast accuracy across the sample of non-overlapping horizons. The interested reader is directed to Tables 2.6.1-2.6.14 for the entire set of results for both the HLN and DM tests. Forecast accuracy is benchmarked against both daily squared returns and realized volatility. Implied volatility is shown to provide superior forecasts when ranked against either GARCH or E-GARCH models. The HLN statistic for GlaxoSmithKline (-0.325) comparing implied volatility to E-GARCH is the only instance where a time series model significantly outperforms implied volatility. Using the HLN test, the superiority of implied volatility forecasts when ranked against GARCH forecasts is statistically significant for Aviva, Cadburys, GlaxoSmithKline, HSBC, Kingfisher, Marks and Spencers and Prudential when 'true' volatility is approximated using daily squared returns. Implied volatility significantly outperforms GARCH in the case of British Aerospace, British Airways, Diageo, Dixons and HSBC when forecasts are benchmarked against realized volatility. The generally positive HLN statistic for implied volatility compared to E-GARCH suggests a marginal difference in favour of implied volatility but the results are not found to be statistically significant in any of the companies sampled.

2.4. Conclusion

UK equity options contain predictive information on stock volatility across a range of forecast horizons. The results show that a composite implied volatility produces forecasts that are significantly better than sophisticated time series methods in many cases. This finding is consistent irrespective of whether forecasts are benchmarked against daily squared returns or realized volatility. The information content yielded by traded equity options has applications in practical investment and risk management functions.

Risk-adjusted performance measures such as the Sharpe ratio and Jensen's alpha are now widely used to measure and rank fund performance. The results produced here suggest that the implied volatility is the optimal metric of volatility across medium-term horizons and should be incorporated in the allocation of asset weights. Relying solely on statistical forecasting methods even over relatively short forecast horizons may omit important information on volatility effects caused by expected company announcements. Furthermore, equity options contain predictive information unavailable in historical price patterns in the underlying asset. In the context of portfolios benchmarked against an equity index, commercially available implied volatility indices provide a useful signal. The construction of a composite implied volatility estimate is shown here to provide information that is potentially useful for the management of active equity funds that are partially diversified. In this context managers who engage in ad hoc rebalancing strategies over short and medium intervals are likely to benefit significantly from the predictive information contained in equity options data.

2.5. Appendix A: Supporting Material for Chapter 2

Table 2.1: Descriptive statistics for daily realized volatility.

Stock Name	Mean	Std Dev	Kurt.	Skew.
Diageo (dge)	1.668	2.846	18.683	3.289
Hanson (hns)	1.368	2.955	44.265	5.316
RoyalSunAlliance (rsa)	1.111	2.369	34.654	4.917
Hilton (hg)	1.073	2.136	34.797	4.883
Aviva (av)	0.966	3.067	50.571	6.549
BritishAirways (ba)	0.901	1.577	26.440	4.170
Kingfisher (kgf)	0.850	1.671	29.371	4.376
Prudential (pru)	0.592	1.642	148.230	10.245
BritishAerospace (bae)	0.559	1.217	214.590	11.170
Cadburys (cbry)	0.472	1.397	250.451	13.480
Reuters (rtr)	0.422	0.873	105.57	8.397
Marks&Spencers (mks)	0.325	1.060	558.429	20.298
Dixons (dxn)	0.300	0.830	261.545	13.837
GlaxoSmithKline (gsk)	0.244	1.401	188.171	12.917
HSBC (hsbc)	0.238	0.941	150.089	11.008
Average	**0.739**	**1.732**	**141.057**	**8.990**

Mean, standard deviation, skewness and kurtosis for daily realized volatility for FTSE-100 firms from October 1997 to December 2003 (1233 observations). The skewness and kurtosis are computed as $\frac{1}{n-1}\sum t_\theta^3$ and $\frac{1}{n-1}\sum t_\theta^4$, respectively, after studentizing the relevant quantity, θ, say, as $t_\theta = \frac{(\theta_t - \bar{\theta})}{\sigma(\theta)}$, where $\sigma(\theta)$ is the standard deviation of θ.

Table 2.2: Summary statistics from Mincer-Zarnowitz regression. Statistics indicate the information content produced by implied volatility (IV), GARCH and E-GARCH forecasts across overlapping forecast horizons (60-90 Days). Results are ranked according to R^2produced by IV forecasts.

	IV			GARCH			E-GARCH		
Stock Name	R^2	β_0	β_1	R^2	β_0	β_1	R^2	β_0	β_1
Marks&Spencers (mks)	42.46%	-2.889	0.432	4.85%	3.111	0.686	8.67%	1.850	0.863
RoyalSunAlliance (rsa)	29.29%	-5.994	0.664	26.73%	-10.249	4.272	37.75%	-18.768	5.466
Diageo (dge)	22.90%	-3.906	0.769	3.20%	5.073	0.744	12.86%	-0.151	1.485
GlaxoSmithKline (gsk)	21.08%	-0.213	0.224	1.02%	1.157	0.680	2.85%	1.182	0.622
HSBC (hsbc)	21.02%	0.106	0.183	11.24%	-0.656	0.958	15.51%	-2.855	1.428
Cadburys (cbry)	16.42%	0.407	0.188	12.65%	-1.095	1.431	2.92%	1.016	0.735
Prudential (pru)	13.34%	-0.164	0.384	9.37%	1.283	1.621	6.64%	-1.607	2.265
Aviva (av)	11.81%	1.453	0.273	21.14%	-0.686	1.692	16.55%	-6.606	3.013
Kingfisher (kgf)	6.04%	1.724	0.219	8.02%	2.012	0.939	12.22%	0.211	1.278
Hilton (hg)	3.82%	4.326	0.176	12.77%	2.893	0.888	13.40%	0.348	1.239
Reuters (rtr)	1.60%	7.049	0.201	7.88%	5.767	0.953	3.31%	8.402	0.561
Dixons (dxn)	1.58%	1.886	0.070	19.73%	-2.967	1.703	13.86%	-0.368	0.898
Hanson (hns)	1.35%	4.709	0.078	7.57%	3.012	0.705	13.14%	2.554	0.692
BritishAerospace (bae)	0.68%	13.665	-0.125	8.40%	2.844	1.455	11.00%	-3.283	2.752
BritishAirways (ba)	0.08%	7.799	-0.017	2.86%	3.368	0.523	1.85%	4.355	0.388

Table 2.3: Summary results for the Harvey-Leybourne Newbold (HLN) statistic comparing implied volatility (IV) forecasts against GARCH and E-GARCH forecasts. Statistic reported is for overlapping long-horizon (60-90 days) forecasts, dependent variable is daily squared returns.

Stock Name	IV-GARCH	IV-E-GARCH
Diageo (dge)	0.673	0.259
Marks&Spencers (mks)	0.639	0.297
HSBC (hsbc)	0.463	0.393
GlaxoSmithKline (gsk)	0.432	0.181
Cadburys (cbry)	0.283	-0.109
RoyalSunAlliance (rsa)	0.248	0.361
Prudential (pru)	0.210	-0.100
Kingfisher (kgf)	0.074	0.377
Hilton (hg)	0.033	0.114
BritishAerospace (bae)	0.026	0.147
BritishAirways (ba)	0.010	-0.004
Dixons (dxn)	-0.008	-0.047
Aviva (av)	-0.032	–0.124
Reuters (rtr)	-0.039	-0.068
Hanson	-0.069	0.278

Table 2.2.1: Aviva. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	11.34%	39.79%	42.28%	9.82%	6.86%	13.48%	11.81%	21.14%	16.55%
β_0	-0.046	-9.200	-13.386	0.783	0.205	-8.042	1.453	-0.686	-6.606
	(2.634)	(2.492)	(2.913)	(2.258)	(1.563)	(4.010)	(1.728)	(1.656)	(3.225)
β_1	0.644	2.492	4.076	0.428	1.563	3.246	0.273	1.692	3.013
	(0.206)	(0.439)	(0.546)	(0.124)	(0.554)	(0.791)	(0.075)	(0.330)	(0.683)
DW	0.705	0.555	0.543	0.353	0.307	0.325	0.266	0.295	0.266
Enc. Reg.									
R-Square	13.85%	39.95%	42.63%	10.44%	7.03%	13.56%	12.85%	22.21%	17.46%
β_0	-0.527	-9.184	-13.340	0.898	0.667	-8.634	1.873	-0.326	-6.015
	(2.634)	(2.506)	(2.924)	(2.264)	(3.102)	(4.429)	(1.771)	(1.682)	(3.274)
β_1	0.586	3.053	3.981	0.3955	1.421	3.401	0.262	1.657	2.924
	(0.208)	(0.460)	(0.565)	(0.130)	(0.641)	(0.930)	(0.075)	(0.331)	(0.688)
β_2	2.182	0.567	0.829	0.838	0.483	-0.343	-0.917	-0.092	-0.085
	(1.477)	(1.264)	(1.222)	(0.973)	(1.094)	(1.071)	(0.085)	(0.080)	(0.082)
DW	0.744	0.562	0.549	0.331	0.3301	0.3321	0.274	0.303	0.273

Mincer-Zarnowitz (MZ) Regression estimates of specification (2.4), $RV_k^h = \beta_0 + \beta_1 f_{k,m}^h + e_k$.
Here $f_{k,m}^h$ denotes the composite implied volatility (IV), GARCH or E-GARCH forecast across horizon, h, RV_k^h denotes the ex-post realized volatility over the horizon.
Encompassing Regression estimates of specification (2.5), $RV_k^h = \beta_0 + \beta_1 f_{k,m}^h + \beta_2 RV_{k-1} + e_k$.
The data consists of overlapping forecast horizons covering the period October 1997 to December 2003.
Numbers in parentheses denote standard errors.
Durbin-Watson (DW) statistic indicates that errors are positively autocorrelated.

Table 2.3.1: Aviva. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.195	0.090	0.103	0.280	-0.235	-0.187
	(0.188)	(0.188)	(0.158)	(0.158)	(0.165)	(0.165)
HLN	-0.069	0.163	0.273	0.345	-0.032	-0.124
	(0.188)	(0.188)	(0.158)	(0.158)	(0.165)	(0.165)

Diebold-Mariano (DM) estimates of specification (2.6), $\overline{d} = \frac{1}{H}\sum_{i=1}^{H}[g(e_{1t}^2) - g(e_{2t}^2)]$,

$g(e_{1t}^2)$ is the loss from model 1 (IV), $g(e_{2t}^2)$ is the loss from model 2 (GARCH/E-GARCH).
Harvey-Leybourne-Newbold (HLN) estimates of specification (2.7),

$HLN = \overline{d}/\sqrt{[\gamma_0 + 2\gamma_1 + ... + 2\gamma_q]/(H-1)}$, γ_i is the $i-th$ autocovariance of the d_t sequence.

The data consists of overlapping forecast horizons covering the period October 1997 to December 2003. Standard errors are in parentheses.

Table 2.2.2: British Aerospace. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	2.36%	64.54%	61.80%	7.28%	72.36%	73.14%	0.68%	8.40%	11.00%
β_0	4.579	-12.24	-15.227	2.628	-4.782	-10.964	13.665	2.844	-3.283
	(4.002)	(1.996)	(2.381)	(2.508)	(0.872)	(1.161)	(2.685)	(2.090)	(3.111)
β_1	0.415	3.830	4.502	0.418	2.600	3.914	-0.125	1.455	2.752
	(0.314)	(0.334)	(0.417)	(0.128)	(0.137)	(0.203)	(0.115)	(0.366)	(0.596)
DW	0.724	0.989	1.027	0.311	0.516	0.534	0.148	0.168	0.172
Enc. Reg.									
R-Square	3.09%	64.69%	61.80%	9.82%	73.08%	73.58%	2.54%	8.67%	11.16%
β_0	5.653	-11.938	-15.247	3.670	-4.798	-10.852	14.336	2.835	-3.154
	(4.274)	(2.084)	(2.511)	(2.540)	(0.863)	(1.158)	(2.693)	(2.093)	(3.126)
β_1	0.366	3.812	4.504	0.333	2.552	3.851	-0.188	1.404	2.683
	(0.322)	(0.337)	(0.424)	(0.134)	(0.138)	(0.206)	(0.119)	(0.373)	(0.610)
β_2	-0.743	-0.324	0.017	1.412	0.587	0.460	1.107	0.411	0.317
	(1.014)	(0.601)	(0.630)	(0.585)	(0.307)	(0.306)	(0.613)	(0.578)	(0.517)
DW	0.682	0.979	1.029	0.323	0.534	0.526	0.165	0.167	0.170

Details on Table 2.2.1

Table 2.3.2: British Aerospace. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.247	-0.073	-0.331	0.0218	-0.761	0.084
	(0.193)	(0.193)	(0.141)	(0.141)	(0.125)	(0.125)
HLN	0.0034	0.1210	0.012	0.307	0.026	0.147
	(0.193)	(0.193)	(0.141)	(0.141)	(0.125)	(0.125)

Details on Table 2.3.1

Table 2.2.3: British Airways. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	0.01%	5.18%	7.89%	1.32%	0.99%	2.46%	0.08%	2.86%	1.85%
β_0	7.993	2.200	-0.253	6.392	6.008	3.222	7.799	3.368	4.355
	(2.837)	(2.843)	(3.222)	(1.702)	(2.205)	(2.655)	(1.233)	(1.750)	(1.646)
β_1	-0.017	0.858	1.213	0.125	0.427	0.812	-0.017	0.523	0.388
	(0.191)	(0.413)	(0.466)	(0.077)	(0.304)	(0.364)	(0.044)	(0.225)	(0.209)
DW	0.832	0.907	0.958	0.213	0.221	0.230	0.160	0.187	0.168
Enc. Reg.									
R-Square	0.32%	5.21%	7.95%	1.99%	1.86%	3.83%	2.17%	4.98%	3.75%
β_0	8.142	2.194	-0.264	6.297	5.649	2.144	7.641	2.748	3.914
	(2.867)	(2.861)	(3.242)	(1.704)	(2.217)	(2.721)	(1.226)	(1.763)	(1.651)
β_1	-0.057	0.878	1.238	0.115	0.426	0.898	-0.030	0.540	0.385
	(0.208)	(0.433)	(0.484)	(0.077)	(0.303)	(0.367)	(0.044)	(0.224)	(0.207)
β_2	0.628	-0.187	-0.242	0.356	0.404	0.511	0.468	0.467	0.442
	(1.264)	(1.186)	(1.157)	(0.309)	(0.307)	(0.307)	(0.238)	(0.232)	(0.233)
DW	0.826	0.912	0.965	0.223	0.234	0.256	0.180	0.207	0.188

Details on Table 2.2.1

Table 2.3.3: British Airways. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.150	0.119	0.010	0.077	-0.075	-0.039
	(0.184)	(0.184)	(0.117)	(0.117)	(0.122)	(0.122)
HLN	0.008	0.144	0.075	0.130	0.010	-0.004
	(0.184)	(0.184)	(0.117)	(0.117)	(0.122)	(0.122)

Details on Table 2.3.1

Table 2.2.4: Cadburys. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	0%	1.20%	4.63%	11.10%	8.00%	1.81%	16.42%	12.65%	2.92%
β_0	4.276	2.588	-0.114	0.635	0.515	1.710	0.407	-1.095	1.016
	(1.631)	(2.094)	(2.690)	(0.933)	(1.158)	(1.481)	(0.739)	(1.241)	(1.510)
β_1	0.002	0.486	1.226	0.215	0.882	0.488	0.188	1.431	0.735
	(0.166)	(0.589)	(0.743)	(0.070)	(0.343)	(0.412)	(0.041)	(0.370)	(0.417)
DW	0.722	0.724	0.748	0.629	0.472	0.467	0.376	0.278	0.268
Enc. Reg.									
R-Square	0.09%	1.33%	4.84%	25.05%	19.24%	16.73%	20.81%	17.07%	8.41%
β_0	4.358	2.529	-0.211	0.636	1.355	2.655	0.408	-0.926	1.257
	(1.684)	(2.122)	(2.727)	(0.863)	(1.122)	(1.397)	(0.722)	(1.217)	(1.477)
β_1	0.008	0.496	1.245	0.189	0.531	0.119	0.177	1.323	0.609
	(0.174)	(0.595)	(0.751)	(0.065)	(0.341)	(0.395)	(0.041)	(0.365)	(0.410)
β_2	0.020	0.023	0.029	0.909	0.855	0.966	0.274	0.276	0.307
	(0.089)	(0.085)	(0.084)	(0.243)	(0.264)	(0.263)	(0.115)	(0.118)	(0.124)
DW	0.714	0.718	0.741	0.736	0.569	0.591	0.421	0.311	0.308

Details on Table 2.2.1

Table 2.3.4: Cadburys. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.082	0.240	0.050	-0.187	0.034	-0.154
	(0.218)	(0.218)	(0.188)	(0.188)	(0.161)	(0.161)
HLN	-0.00	0.348	0.245	-0.141	0.283	-0.109
	(0.218)	(0.218)	(0.188)	(0.188)	(0.161)	(0.161)

Details on Table 2.3.1

Table 2.2.5: Diageo. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	9.13%	39.67%	49.84%	20.61%	10.27%	21.98%	22.90%	3.20%	12.86%
β_0	3.305	-1.852	-4.743	-3.925	1.513	-3.469	-3.906	5.073	-0.151
	(1.459)	(1.218)	(1.283)	(2.211)	(2.044)	(2.061)	(1.882)	(1.827)	(1.840)
β_1	0.475	1.583	1.979	1.026	1.353	2.002	0.769	0.744	1.485
	(0.147)	(0.191)	(0.196)	(0.158)	(0.315)	(0.297)	(0.106)	(0.309)	(0.279)
DW	0.669	0.987	1.047	0.239	0.290	0.319	0.168	0.114	0.137
Enc. Reg.									
R-Square	10.43%	42.40%	51.30%	20.64%	10.58%	22.34%	23.42%	4.99%	14.19%
β_0	3.442	-2.255	-4.963	-3.843	1.658	-3.330	-3.768	5.395	0.099
	(1.459)	(1.209)	(1.270)	(2.249)	(2.056)	(2.069)	(1.886)	(1.824)	(1.867)
β_1	0.421	1.562	1.945	1.023	1.352	2.004	0.742	0.579	1.404
	(0.153)	(0.188)	(0.194)	(0.159)	(0.315)	(0.297)	(0.109)	(0.320)	(0.297)
β_2	1.436	1.995	1.634	-0.129	-0.461	-0.4944	1.415	2.674	1.163
	(1.176)	(0.903)	(0.832)	(0.581)	(0.614)	(0.573)	(1.305)	(1.476)	(1.428)
DW	0.694	1.101	1.125	0.239	0.291	0.323	0.165	0.133	0.142

Details on Table 2.2.1

Table 2.3.5: Diageo. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.245	0.254	0.140	0.248	0.265	0.197
	(0.161)	(0.161)	(0.129)	(0.129)	(0.124)	(0.124)
HLN	0.115	0.336	0.539	0.298	0.673	0.259
	(0.161)	(0.161)	(0.129)	(0.129)	(0.124)	(0.124)

Details on Table 2.3.1

Table 2.2.6: Dixons. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	7.40%	18.85%	8.46%	6.23%	39.56%	50.26%	1.58%	19.73%	13.86%
β_0	11.025	-4.038	0.085	1.540	-1.419	-0.655	1.886	-2.967	-0.368
	(3.181)	(3.028)	(2.707)	(0.896)	(0.721)	(0.296)	(1.623)	(1.564)	(1.183)
β_1	-0.467	2.272	1.183	0.099	1.248	0.942	0.070	1.703	0.898
	(0.252)	(0.719)	(0.593)	(0.045)	(0.180)	(0.109)	(0.064)	(0.399)	(0.260)
DW	0.579	0.634	0.661	0.648	0.880	0.999	0.420	0.527	0.478
Enc. Reg.									
R-Square	15.32%	19.05%	9.21%	18.58%	47.35%	54.67%	2.03%	20.46%	14.68%
β_0	11.18	-3.175	0.792	1.272	-1.369	-0.555	1.79	-2.957	-0.349
	(3.079)	(3.222)	(2.980)	(0.844)	(0.678)	(0.478)	(1.63)	(1.568)	(1.186)
β_1	-0.550	2.158	0.952	0.099	1.183	0.884	0.078	1.738	0.929
	(0.247)	(0.810)	(0.715)	(0.042)	(0.170)	(0.107)	(0.066)	(0.402)	(0.263)
β_2	3.129	0.543	1.135	0.859	0.687	0.524	-0.428	-0.533	-0.564
	(1.579)	(1.697)	(1.927)	(0.260)	(0.210)	(0.198)	(0.734)	(0.648)	(0.674)
DW	0.896	0.677	0.743	0.792	1.142	1.182	0.422	0.547	0.494

Details on Table 2.2.1

Table 2.3.6: Dixons. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.151	-0.166	-0.200	0.283	-0.214	-0.111
	(0.248)	(0.248)	(0.191)	(0.191)	(0.190)	(0.190)
HLN	0.298	-0.104	0.0063	0.317	-0.008	-0.047
	(0.248)	(0.248)	(0.191)	(0.191)	(0.190)	(0.190)

Details on Table 2.3.1

Table 2.2.7: GlaxoSmithKline. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	21.32%	15.62%	27.29%	25.43%	16.52%	23.46%	21.08%	1.02%	2.85%
β_0	-0.781	-3.193	-6.192	-0.918	-5.340	-5.931	-0.213	1.157	1.182
	(0.727)	(1.268)	(1.249)	(0.514)	(1.215)	(1.038)	(0.473)	(1.570)	(0.927)
β_1	0.535	1.865	2.560	0.347	2.424	2.443	0.224	0.680	0.622
	(0.074)	(0.314)	(0.303)	(0.035)	(0.302)	(0.245)	(0.025)	(0.388)	(0.210)
DW	0.331	0.186	0.230	0.264	0.198	0.213	0.231	0.115	0.123
Enc. Reg.									
R-Square	21.47%	15.96%	27.58%	25.43%	16.62%	23.56%	21.72%	1.70%	3.46%
β_0	-0.746	-3.178	-6.165	-0.922	-5.346	-5.940	-0.175	1.257	1.272
	(0.730)	(1.269)	(1.250)	(0.514)	(1.217)	(1.039)	(0.473)	(1.569)	(0.928)
β_1	0.534	1.871	2.561	0.374	2.422	2.442	0.223	0.662	0.607
	(0.074)	(0.314)	(0.303)	(0.035)	(0.303)	(0.245)	(0.025)	(0.388)	(0.210)
β_2	-0.090	-0.135	-0.124	0.068	0.100	0.101	-0.083	-0.086	-0.081
	(0.150)	(0.155)	(0.144)	(0.159)	(0.168)	(0.161)	(0.053)	(0.060)	(0.059)
DW	0.331	0.186	0.230	0.265	0.199	0.215	0.221	0.114	0.120

Details on Table 2.2.1

Table 2.3.7: GlaxoSmithKline. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.097	0.444	0.101	0.119	0.240	0.105
	(0.119)	(0.119)	(0.091)	(0.091)	(0.095)	(0.095)
HLN	0.324	0.553	0.310	0.213	0.432	0.181
	(0.119)	(0.119)	(0.091)	(0.091)	(0.095)	(0.095)

Details on Table 2.3.1

Table 2.2.8: Hilton. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	21.92%	20.45%	35.47%	16.34%	13.30%	23.47%	3.82%	12.77%	13.40%
β_0	-3.781	-6.051	-13.198	0.805	0.908	-2.781	4.326	2.893	0.348
	(3.562)	(4.283)	(4.211)	(2.035)	(2.250)	(2.385)	(2.199)	(1.597)	(2.234)
β_1	1.063	3.017	4.110	0.406	1.253	1.752	0.176	0.888	1.239
	(0.263)	(0.781)	(0.727)	(0.108)	(0.377)	(0.372)	(0.095)	(0.251)	(0.341)
DW	0.964	0.732	0.939	0.635	0.588	0.684	0.269	0.345	0.344
Enc. Reg.									
R-Square	21.93%	20.62%	35.50%	16.89%	13.72%	24.32%	3.85%	13.43%	13.74%
β_0	-3.829	-5.885	-13.102	1.017	1.103	-2.600	4.315	2.566	0.047
	(3.690)	(4.343)	(4.290)	(2.065)	(2.285)	(2.397)	(2.278)	(1.652)	(2.304)
β_1	1.065	3.020	4.106	0.412	1.269	1.789	0.176	0.974	1.306
	(0.267)	(0.787)	(0.734)	(0.109)	(0.379)	(0.375)	(0.102)	(0.274)	(0.363)
β_2	0.033	-0.206	-0.085	-0.339	-0.297	-0.424	-0.002	-0.099	-0.069
	(0.596)	(0.598)	(0.540)	(0.495)	(0.504)	(0.473)	(0.129)	(0.125)	(0.121)
DW	0.961	0.748	0.945	0.633	0.587	0.693	0.269	0.322	0.326

Details on Table 2.2.1

Table 2.3.8: Hilton. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.0286	0.287	0.092	0.427	-0.191	0.028
	(0.214)	(0.214)	(0.193)	(0.193)	(0.177)	(0.177)
HLN	0.379	0.438	0.223	0.520	0.033	0.114
	(0.214)	(0.214)	(0.193)	(0.193)	(0.177)	(0.177)

Details on Table 2.3.1

Table 2.2.9: Hanson. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	42.11%	11.64%	16.67%	1.06%	6.36%	16.97%	1.35%	7.57%	13.14%
β_0	-1.094	2.668	1.791	5.165	3.242	0.822	4.709	3.012	2.554
	(2.021)	(2.327)	(2.345)	(3.895)	(3.291)	(3.155)	(2.578)	(2.109)	(1.778)
β_1	0.680	0.724	0.853	0.107	0.704	1.094	0.078	0.705	0.692
	(0.183)	(0.457)	(0.437)	(0.239)	(0.619)	(0.555)	(0.115)	(0.422)	(0.305)
DW	1.794	1.577	1.584	0.847	0.924	1.032	0.611	0.682	0.689
Enc. Reg.									
R-Square	43.04%	13.63%	19.71%	3.82%	8.39%	18.85%	5.61%	9.64%	14.61%
β_0	-1.081	2.389	1.368	5.358	3.745	1.317	4.733	3.350	2.763
	(2.060)	(2.403)	(2.420)	(3.948)	(3.438)	(3.295)	(2.560)	(2.151)	(1.810)
β_1	0.690	0.816	0.977	0.136	0.696	1.085	0.065	0.595	0.636
	(0.187)	(0.486)	(0.466)	(0.245)	(0.630)	(0.564)	(0.115)	(0.442)	(0.319)
β_2	-0.099	-0.151	-0.188	-0.592	-0.501	-0.482	0.161	0.116	0.098
	(0.183)	(0.235)	(0.228)	(0.824)	(0.794)	(0.747)	(0.132)	(0.134)	(0.130)
DW	1.794	1.577	1.584	0.848	0.891	0.988	0.600	0.649	0.656

Details on Table 2.2.1

Table 2.3.9: Hanson. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.277	0.205	-0.277	0.397	-0.171	0.242
	(0.369)	(0.369)	(0.369)	(0.369)	(0.278)	(0.278)
HLN	0.434	0.238	-0.070	0.511	-0.069	0.278
	(0.369)	(0.369)	(0.369)	(0.369)	(0.278)	(0.278)

Details on Table 2.3.1

Table 2.2.10: HSBC. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	45.51%	45.77%	49.23%	27.19%	32.80%	32.94%	21.02%	11.24%	15.51%
β_0	-2.493	-2.986	-4.473	-1.953	-4.104	-5.658	0.106	-0.656	-2.855
	(0.434)	(0.492)	(0.558)	(0.470)	(0.567)	(0.677)	(0.336)	(0.590)	(0.763)
β_1	0.683	1.641	1.998	0.427	1.917	2.242	0.183	0.958	1.428
	(0.047)	(0.120)	(0.136)	(0.035)	(0.141)	(0.164)	(0.019)	(0.143)	(0.177)
DW	0.782	0.375	0.415	0.202	0.157	0.154	0.174	0.143	0.151
Enc. Reg.									
R-Square	48.56%	45.82%	49.28%	27.27%	33.14%	33.30%	21.06%	11.38%	15.65%
β_0	-2.520	-3.019	-4.504	-1.921	-4.076	-5.635	0.110	-0.680	-2.886
	(0.440)	(0.498)	(0.563)	(0.473)	(0.567)	(0.676)	(0.336)	(0.591)	(0.765)
β_1	0.685	1.644	2.002	0.423	1.921	2.248	0.183	0.068	1.440
	(0.047)	(0.120)	(0.137)	(0.035)	(0.140)	(0.164)	(0.019)	(0.144)	(0.178)
β_2	0.051	0.056	0.052	-0.074	-0.159	-0.161	-0.042	-0.072	-0.073
	(0.121)	(0.124)	(0.120)	(0.118)	(0.113)	(0.113)	(0.094)	(0.100)	(0.097)
DW	0.782	0.374	0.413	0.203	0.164	0.161	0.177	0.146	0.154

Details on Table 2.2.1

Table 2.3.10: HSBC. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.047	0.108	-0.066	0.005	0.268	0.298
	(0.110)	(0.110)	(0.084)	(0.084)	(0.087)	(0.087)
HLN	0.396	0.166	0.159	0.088	0.463	0.393
	(0.110)	(0.110)	(0.084)	(0.084)	(0.087)	(0.087)

Details on Table 2.3.1

Table 2.2.11: Kingfisher. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	61.97%	58.95%	59.57%	8.18%	23.92%	21.86%	6.04%	8.02%	12.22%
β_0	-5.466	-6.235	-7.562	0.443	-1.736	-3.093	1.724	2.012	0.211
	(1.073)	(1.215)	(1.326)	(2.154)	(1.546)	(1.872)	(1.726)	(1.406)	(1.554)
β_1	0.906	2.257	2.475	0.396	1.759	2.016	0.219	0.939	1.278
	(0.084)	(0.223)	(0.242)	(0.126)	(0.299)	(0.363)	(0.078)	(0.289)	(0.311)
DW	1.311	1.118	1.140	0.333	0.308	0.288	0.290	0.284	0.306
Enc. Reg.									
R-Square	62.04%	58.96%	59.58%	8.20%	24.89%	22.77%	7.50%	8.84%	12.26%
β_0	-5.446	-6.227	-7.555	0.407	-2.287	-3.689	1.508	2.104	0.267
	(1.081)	(1.226)	(1.336)	(2.175)	(1.612)	(1.943)	(1.727)	(1.408)	(1.552)
β_1	0.909	2.259	2.478	0.396	1.819	2.086	0.244	0.967	1.318
	(0.085)	(0.225)	(0.244)	(0.127)	(0.302)	(0.368)	(0.080)	(0.290)	(0.312)
β_2	-0.097	-0.318	-0.038	0.046	0.317	0.307	-0.308	-0.226	-0.255
	(0.274)	(0.284)	(0.282)	(0.292)	(0.268)	(0.272)	(0.224)	(0.218)	(0.213)
DW	1.310	1.120	1.142	0.334	0.332	0.305	0.312	0.307	0.323

Details on Table 2.2.1

Table 2.3.11: Kingfisher. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.035	0.039	-0.184	-0.109	-0.066	0.287
	(0.194)	(0.194)	(0.156)	(0.156	(0.149)	(0.149)
HLN	0.285	0.174	0.031	-0.022	0.074	0.377
	(0.194)	(0.194)	(0.156)	(0.156)	(0.149)	(0.149)

Details on Table 2.3.1

Table 2.2.12: MKS. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	8.55%	3.64%	0.79%	29.87%	6.20%	7.45%	42.46%	4.85%	8.67%
β_0	2.781	3.711	4.993	0.218	4.113	3.427	-2.889	3.111	1.850
	(1.178)	(1.363)	(1.540)	(0.808)	(0.879)	(0.976)	(0.837)	(1.158)	(1.158)
β_1	0.306	0.541	0.279	0.393	0.571	0.688	0.432	0.686	0.863
	(0.095)	(0.264)	(0.296)	(0.044)	(0.165)	(0.180)	(0.037)	(0.224)	(0.207)
DW	0.651	0.597	0.592	0.276	0.194	0.197	0.225	0.110	0.113
Enc. Reg.									
R-Square	10.71%	6.92%	4.88%	30.04%	6.89%	8.25%	42.94%	5.64%	9.30%
β_0	3.096	4.005	5.191	0.203	4.137	3.429	-2.789	3.093	1.905
	(1.185)	(1.354)	(1.518)	(0.810)	(0.879)	(0.975)	(0.840)	(1.157)	(1.158)
β_1	0.270	0.452	0.207	0.387	0.521	0.641	0.433	0.718	0.879
	(0.097)	(0.264)	(0.293)	(0.045)	(0.170)	(0.184)	(0.037)	(0.225)	(0.207)
β_2	0.204	0.249	0.276	0.488	0.981	1.041	-0.391	-0.506	-0.451
	(0.125)	(0.126)	(0.127)	(0.726)	(0.850)	(0.834)	(0.327)	(0.410)	(0.400)
DW	0.687	0.660	0.669	0.276	0.199	0.202	0.238	0.107	0.116

Details on Table 2.2.1

Table 2.3.12: MKS. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.137	-0.133	0.287	0.094	0.528	0.234
	(0.155)	(0.155)	(0.122)	(0.122)	(0.121)	(0.121)
HLN	0.230	-0.081	0.408	0.134	0.639	0.297
	(0.155)	(0.155)	(0.122)	(0.122)	(0.121)	(0.121)

Details on Table 2.3.1

Table 2.2.13: Prudential. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	31.25%	29.38%	32.37%	25.08%	15.59%	13.23%	13.34%	9.37%	6.64%
β_0	-4.008	-3.218	-7.771	-2.729	-1.208	-4.444	-0.164	1.283	-1.607
	(1.499)	(1.467)	(1.893)	(1.266)	(1.437)	(2.148)	(1.675)	(1.704)	(2.827)
β_1	0.916	2.173	3.207	0.540	1.748	2.524	0.384	1.621	2.265
	(0.111)	(0.276)	(0.381)	(0.066)	(0.287)	(0.457)	(0.072)	(0.371)	(0.626)
DW	0.546	0.465	0.451	0.255	0.184	0.149	0.158	0.104	0.096
Enc. Reg.									
R-Square	31.28%	29.56%	32.68%	25.28%	15.78%	13.52%	13.73%	9.98%	7.33%
β_0	-4.010	-3.386	-8.047	-2.648	-1.124	-4.381	-0.046	1.339	-1.667
	(0.1512)	(1.495)	(1.926)	(1.273)	(1.445)	(2.151)	(1.681)	(1.704)	(2.824)
β_1	0.916	2.187	3.240	0.540	1.746	2.530	0.382	1.635	2.306
	(0.112)	(0.278)	(0.383)	(0.066)	(0.288)	(0.457)	(0.072)	(0.371)	(0.626)
β_2	0.002	0.111	0.144	-0.107	-0.101	-0.127	-0.148	-01.87	-0.198
	(0.177)	(0.180)	(0.176)	(0.1449)	(0.153)	(0.155)	(0.163)	(0.167)	(0.169)
DW	0.546	0.469	0.460	0.255	0.188	0.154	0.162	0.115	0.107

Details on Table 2.2.1

Table 2.3.13: Prudential. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.024	0.087	0.142	-0.080	0.074	-0.192
	(0.135)	(0.135)	(0.116)	(0.116)	(0.121)	(0.121)
HLN	0.197	0.134	0.300	0.004	0.210	-0.100
	(0.135)	(0.135)	(0.116)	(0.116)	(0.121)	(0.121)

Details on Table 2.3.1

Table 2.2.14: RSA. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	11.03%	12.45%	26.37%	34.24%	41.11%	46.72%	29.29%	26.73%	37.75%
β_0	2.606	-0.963	-8.555	-4.335	-10.032	-12.321	-5.994	-10.249	-18.768
	(3.060)	(4.116)	(4.358)	(2.078)	(2.413)	(2.379)	(2.875)	(3.726)	(3.963)
β_1	0.523	1.899	3.063	0.734	3.852	4.092	0.664	4.272	5.466
	(0.204)	(0.691)	(0.703)	(0.093)	(0.424)	(0.402)	(0.101)	(0.693)	(0.688)
DW	0.511	0.411	0.429	0.460	0.536	0.510	0.265	0.319	0.375
Enc. Reg.									
R-Square	26.19%	22.93%	35.71%	32.25%	41.22%	47.43%	30.82%	26.73%	37.78%
β_0	1.266	0.208	-7.949	-4.502	-9.960	-12.372	-6.050	-10.222	-18.618
	(2.843)	(3.924)	(4.125)	(2.075)	(2.425)	(2.374)	(0.285)	(3.878)	(4.061)
β_1	0.497	1.442	2.689	0.723	3.813	4.042	0.644	4.264	5.427
	(0.187)	(0.677)	(0.677)	(0.093)	(0.433)	(0.403)	(0.101)	(0.748)	(0.721)
β_2	1.331	1.143	1.065	0.228	0.078	0.192	0.499	0.009	0.061
	(0.407)	(0.429)	(0.387)	(0.169)	(0.163)	(0.153)	(0.331)	(0.363)	(0.325)
DW	0.736	0.648	0.626	0.503	0.551	0.563	0.286	0.319	0.376

Details on Table 2.2.1

Table 2.3.14: RSA. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	-0.036	0.283	-0.140	0.144	0.047	0.251
	(0.224)	(0.224)	(0.151)	(0.151)	(0.161)	(0.161)
HLN	0.154	0.337	0.109	0.305	0.248	0.361
	(0.224)	(0.224)	(0.151)	(0.151)	(0.161)	(0.161)

Details on Table 2.3.1

Table 2.2.15: Reuters. Information content of IV (implied volatility), GARCH and E-GARCH forecasts.

	Short-Horizon (10-30 Days)			Medium-Horizon (30-60 Days)			Long-Horizon (60-90 Days)		
MZ Reg.	IV	GCH	E-GCH	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	27.16%	21.66%	22.56%	9.58%	15.16%	10.57%	1.60%	7.88%	3.31%
β_0	-4.926	-1.757	-1.789	1.680	4.551	6.189	7.049	5.767	8.402
	(4.343)	(4.325)	(4.226)	(3.228)	(1.980)	(2.024)	(3.617)	(1.986)	(1.970)
β_1	1.462	2.612	2.517	0.569	1.364	1.061	0.201	0.953	0.561
	(0.280)	(0.581)	(0.545)	(0.143)	(0.264)	(0.252)	(0.124)	(0.256)	(0.239)
DW	0.749	0.688	0.653	0.356	0.353	0.321	0.146	0.189	0.160
Enc. Reg.									
R-Square	27.16%	21.69%	22.57%	22.60%	25.05%	22.86%	9.13%	13.00%	10.64%
β_0	-4.926	-1.722	-1.766	3.180	5.477	6.622	7.631	6.512	8.265
	(4.389)	(4.359)	(4.267)	(3.011)	(1.879)	(1.888)	(3.490)	(1.951)	(1.900)
β_1	1.462	2.627	2.522	0.417	1.000	0.764	0.154	0.765	0.483
	(0.283)	(0.592)	(0.554)	(0.136)	(0.262)	(0.243)	(0.120)	(0.257)	(0.231)
β_2	-0.003	-0.293	-0.124	3.972	3.555	3.885	1.610	1.358	1.585
	(1.677)	(1.750)	(1.734)	(0.796)	(0.804)	(0.800)	(0.442)	(0.442)	(0.437)
DW	0.749	0.693	0.654	0.680	0.634	0.637	0.263	0.282	0.281

Details on Table 2.2.1

Table 2.3.15: Reuters. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts.

	Short-Horizon		Medium-Horizon		Long-Horizon	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
DM	0.063	0.089	-0.107	-0.146	-0.143	-0.124
	(0.191)	(0.191)	(0.134)	(0.134)	(0.129)	(0.129)
HLN	0.249	0.175	0.025	-0.092	-0.039	-0.068
	(0.191)	(0.191)	(0.134)	(0.134)	(0.129)	(0.129)

Details on Table 2.3.1

Table 2.4: Forecast errors measures for composite implied volatility (IV), GARCH and E-GARCH forecasts across 10-day non-overlapping horizons. Forecast errors are given by mean square error (MSE) and mean absolute error (MAE).

	MSE			MAE		
Stock Name	IV	GARCH	E-GARCH	IV	GARCH	E-GARCH
Reuters (rtr)	156.00	157.52	160.98	7.57	7.57	7.81
Hanson (hns)	135.66	138.99	136.63	8.04	7.63	7.49
Diageo (dge)	130.06	149.33	153.31	7.90	8.09	7.91
RoyalSunAlliance (rsa)	84.91	85.25	85.81	5.43	5.45	5.40
Cadburys (cbry)	56.50	57.62	57.26	3.56	3.58	3.58
Prudential (pru)	27.98	28.55	28.34	3.40	3.45	3.48
Kingfisher (kgf)	24.87	24.15	24.31	3.65	3.59	3.62
Hilton (hg)	16.72	15.30	16.06	2.70	2.79	2.83
BritishAirways (ba)	15.52	16.96	18.37	2.51	2.78	2.92
Aviva (av)	15.46	15.22	14.93	2.22	2.22	2.24
BritishAerospace (bae)	12.10	13.75	13.86	2.43	2.55	2.56
GlaxoSmithKline (gsk)	11.74	11.74	11.72	1.47	1.47	1.48
Marks&Spencers (mks)	8.48	8.70	8.68	1.60	1.59	1.56
HSBC (hsbc)	3.91	4.00	4.01	1.17	1.27	1.26
Dixons (dxn)	3.54	3.90	3.85	1.15	1.28	1.25

The mean square error (MSE) is $\frac{1}{H}\sum_{i=1}^{H} e_i^2$ and the mean absolute error (MAE) is $\frac{1}{H}\sum_{i=1}^{H} |e_i|$.

Table 2.5: Summary statistics from Mincer-Zarnowitz regression. Statistics indicate the information content produced by implied volatility (IV), GARCH and E-GARCH forecasts across a 10-Day non-overlapping forecast horizon. The dependent variable is realized volatility estimated from intraday tick data. Results are ranked according to R^2produced byIV forecasts.

	IV			GARCH			E-GARCH		
Stock Name	R^2	β_0	β_1	R^2	β_0	β_1	R^2	β_0	β_1
Dixons (dxn)	18.60%	-20.618	3.859	6.54%	-5.007	3.557	4.05%	-0.229	2.179
BritishAerospace (bae)	16.14%	-3.221	0.903	4.70%	1.003	0.411	3.92%	0.518	0.526
Aviva (av)	15.84%	-1.715	0.775	17.12%	-2.601	1.290	18.71%	-4.965	1.805
BritishAirways (ba)	15.14%	-0.871	0.620	7.83%	0.431	0.588	0.19%	3.623	1.388
Marks&Spencers (mks)	10.19%	-1.458	0.544	7.90%	-0.794	0.579	8.14%	-1.367	0.677
Reuters (rtr)	9.85%	-6.274	2.502	8.97%	2.448	1.837	6.97%	4.093	1.507
Diageo (dge)	9.72%	-0.395	0.398	0.35%	1.407	0.060	1.63%	0.904	0.137
Prudential (pru)	5.79%	-0.551	0.617	3.86%	0.356	0.749	4.58%	-1.417	1.160
Hilton (hg)	5.32%	1.168	0.546	13.36%	-1.660	1.233	9.02%	-0.334	0.910
RoyalSunAlliance (rsa)	2.57%	2.518	0.604	2.18%	2.222	0943	1.54%	2.772	0.799
HSBC (hsbc)	2.57%	0.384	0.189	0.13%	1.705	-0.068	0%	1.479	-0.011
Cadburys (cbry)	1.99%	-1.887	0.981	0.045%	4.221	-0.262	0.66%	7.442	-1.173
GlaxoSmithKline (gsk)	0%	1.359	0.012	0.02%	1.025	0.103	0.19%	2.521	-0.263
Kingfisher (kgf)	0%	4.701	-0.006	2.89%	7.675	-0.614	2.25%	7.680	-0.607

Table 2.6: Summary results for the Harvey-Leybourne Newbold (HLN) statistic comparing implied volatility (IV) forecasts against GARCH and E-GARCH forecasts. Statistic reported is for non-overlapping 10-Day horizon. Dependent variable is realized volatility.

Stock Name	IV-GARCH	IV-E-GARCH
BritishAirways (ba)	0.355	-0.032
Dixons (dxn)	0.303	-0.044
HSBC (hsbc)	0.253	-0.007
BritishAerospace (bae)	0.246	-0.016
Diageo (dge)	0.217	0.135
Cadburys (cbry)	0.145	0.109
Marks&Spencers (mks)	0.137	0.097
Prudential (pru)	0.127	0.093
Reuters (rtr)	0.127	-0.046
Aviva (av)	0.123	0.144
RoyalSunAlliance (rsa)	0.108	-0.031
GlaxoSmithKline (gsk)	-0.001	-0.325
Kingfisher (kgf)	-0.001	-0.071
Hilton (hg)	-0.010	-0.061

Table 2.5.1: Aviva. Information content of implied volatility, GARCH and E-GARCH over 10-day-ahead non-overlapping horizons. Realized volatility is the dependent variable estimated from intraday tick data.

Number of Observations = 68	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	21.09%	16.46%	21.54%	15.84%	17.12%	18.71%
β_0	-3.712	-3.591	-9.156	-1.715	-2.601	-4.964
	(2.777)	(3.170)	(3.981)	(1.606)	(1.768)	(2.268)
β_1	1.598	2.260	3.460	0.775	1.290	1.805
	(0.380)	(0.626)	(0.812)	(0.220)	(0.349)	(0.463)
DW	0.841	0.927	0.860	2.130	2.290	2.326
Encompassing Regression						
R-Square	22.39%	17.24%	22.62%	20.40%	20.28%	22.72%
β_0	-3.671	-3.304	-8.857	-1.672	-2.276	-4.642
	(2.776)	(3.201)	(3.996)	(1.574)	(1.759)	(2.236)
β_1	1.538	2.139	3.324	0.712	1.154	1.660
	(0.384)	(0.647)	(0.825)	(0.218)	(0.355)	(0.462)
β_2	0.495	0.388	0.452	0.319	0.439	0.487
	(0.474)	(0.498)	(0.475)	(0.268)	(0.273)	(0.265)
DW	0.936	0.978	0.931	2.176	2.287	2.336

Table 2.6.1: Aviva. Pairwise comparison of implied volatility (IV) to GARCH / E-GARCH forecasts, across 10–day-ahead non-overlapping horizons.

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.104	0.129	-0.026	0.073
	(0.201)	(0.201)	(0.201)	(0.201)
Harvey-Leybourne Newbold	0.269	0.175	0.123	0.144
	(0.201)	(0.201)	(0.201)	(0.201)

Table 2.5.2
British Aerospace (Details on Table 2.5.1)

Number of observations = 56	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	16.04%	63.86%	66.59%	16.14%	4.70%	3.92%
β_0	-2.156	-5.977	-11.100	-3.221	1.003	0.518
	(3.622)	(1.606)	(2.021)	(2.096)	(1.531)	(1.984)
β_1	1.558	2.641	3.744	0.903	0.411	0.526
	(0.484)	(0.264)	(0.360)	(0.280)	(0.252)	(0.354)
DW	1.024	1.671	1.327	0.940	0.915	0.881
Encompassing Regression						
R-Square	16.37%	64.85%	67.02%	37.11%	34.20%	34.55%
β_0	-1.713	-5.969	-11.671	-1.124	1.222	0.662
	(3.782)	(1.622)	(2.027)	(1.898)	(1.285)	(1.653)
β_1	1.461	2.632	3.689	0.456	0.146	0.257
	(0.531)	(0.275)	(0.367)	(0.266)	(0.218)	(0.300)
β_2	0.638	0.113	0.696	3.018	3.391	3.402
	(1.430)	(0.878)	(0.837)	(0.718)	(0.695)	(0.683)
DW	1.034	1.664	1.291	1.478	1.545	1.533

See Table 2.2.1. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.2
British Aerospace (Details on Table 2.6.1)

	IV-GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	-0.239	0.036	0.130	-0.040
	(0.222)	(0.222)	(0.222)	(0.222)
Harvey-Leybourne Newbold	0.039	0.150	0.246	-0.016
	(0.222)	(0.222)	(0.222)	(0.222)

See Table 2.3.1. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.3
British Airways (Details on Table 2.5.1)

Number of observations = 58	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	0.05%	0.95%	2.35%	15.64%	7.83%	0.19%
β_0	8.219	6.063	3.213	-0.871	0.431	3.623
	(3.725)	(3.934)	(4.965)	(1.728)	(1.915)	(2.534)
β_1	0.072	0.407	0.807	0.620	0.588	0.117
	(0.414)	(0.554)	(0.694)	(0.192)	(0.269)	(0.254)
DW	0.866	0.963	1.024	1.775	1.608	1.388
Encompassing Regression						
R-Square	0.23%	1.13%	2.65%	16.69%	13.72%	6.96%
β_0	8.475	5.892	2.819	-0.557	-0.060	2.681
	(3.843)	(4.002)	(5.092)	(1.773)	(1.887)	(2.513)
β_1	0.012	0.397	0.819	0.546	0.560	0.147
	(0.459)	(0.559)	(0.700)	(0.212)	(0.263)	(0.345)
β_2	0.424	0.390	0.500	0.519	1.117	1.196
	(1.348)	(1.222)	(1.212)	(0.622)	(0.576)	(0.598)
DW	0.853	0.947	1.012	1.849	1.812	1.642

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.3
British Airways (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV-GCH	IV-E-GCH
Diebold Mariano	-0.062	0.075	0.130	-0.157
	(0.218)	(0.218)	(0.218)	(0.218)
Harvey-Leybourne Newbold	-0.002	0.114	0.355	-0.032
	(0.218)	(0.218)	(0.218)	(0.218)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.4
Cadburys (Details on Table 2.5.1)

Number of observations = 51	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	21.37%	8.38%	5.06%	1.99%	0.045%	0.66%
β_0	–0.960	0.448	0.628	-1.887	4.221	7.442
	(1.318)	(1.577)	(1.944)	(5.452)	(5.983)	(7.225)
β_1	0.882	0.983	0.889	0.981	-0.262	-1.173
	(0.242)	(0.464)	(0.550)	(0.983)	(1.760)	(2.045)
DW	0.955	0.828	0.805	2.101	2.078	2.093
Encompassing Regression						
R-Square	24.82%	11.64%	9.50%	4.47%	3.88%	5.00%
β_0	-0.809	0.964	1.236	-1.429	6.255	9.623
	(1.305)	(1.612)	(1.959)	(5.344)	(6.107)	(7.290)
β_1	0.821	0.777	0.659	0.796	-1.074	-2.000
	(0.242)	(0.485)	(0.563)	(0.994)	(1.839)	(2.096)
β_2	0.403	0.399	0.459	1.219	1.576	1.648
	(0.266)	(0.300)	(0.299)	(1.091)	(1.138)	(1.113)
DW	1.009	0.834	0.805	2.131	2.136	2.145

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.4
Cadburys (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV -GCH	IV - E-GCH
Diebold Mariano	0.225	-0.218	0.066	0.067
	(0.233)	(0.233)	(0.233)	(0.233)
Harvey-Leybourne Newbold	0.392	-0.135	0.145	0.109
	(0.233)	(0.233)	(0.233)	(0.233)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.5
Diageo (Details on Table 2.5.1)

Number of observations =62	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	17.52%	40.39%	48.32%	9.72%	0.35%	1.63%
β_0	-0.692	-3.781	-6.402	-0.395	1.407	0.904
	(2.686)	(2.029)	(2.077)	(0.885)	(0.826)	(0.902)
β_1	1.698	2.060	2.370	0.398	0.060	0.137
	(0.475)	(0.323)	(0.316)	(0.156)	(0.131)	(0.137)
DW	0.981	1.264	1.250	1.931	1.714	1.73-
Encompassing Regression						
R-Square	20.97%	40.86%	48.33%	10.25%	2.79%	3.12%
β_0	-0.167	-3.547	-6.471	-0.331	1.576	1.143
	(2.670)	(2.067)	(2.173)	(0.896)	(0.835)	(0.937)
β_1	1.448	1.968	2.390	0.367	0.050	0.070
	(0.494)	(0.351)	(0.358)	(0.166)	(0.142)	(0.154)
β_2	3.5664	1.352	-0.228	0.439	0.970	0.787
	(2.221)	(1.976)	(1.916)	(0.745)	(0.798)	(0.827)
DW	1.312	1.372	1.331	1.940	1.790	1.777

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.5
Diageo (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	-0.160	0.116	0.114	0.098
	(0.211)	(0.211)	(0.211)	(0.211)
Harvey-Leybourne Newbold	0.202	0.198	0.217	0.135
	(0.211)	(0.211)	(0.211)	(0.211)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.6
Dixons (Details on Table 2.5.1)

Number of observations = 50	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	1.88%	29.76%	14.24%	18.60%	6.54%	4.05%
β_0	1.703	-4.963	-0.961	-20.618	-5.007	-0.229
	(3.189)	(2.198)	(2.078)	(9.191)	(8.022)	(6.955)
β_1	0.388	2.379	1.291	3.859	3.557	2.179
	(0.404)	(0.531)	(0.457)	(1.165)	(1.940)	(1.530)
DW	0.643	1.161	1.016	1.045	0.974	0.955
Encompassing Regression						
R-Square	4.93%	32.87%	16.77%	48.78%	43.92%	42.20%
β_0	1.247	-4.814	-0.758	-16.077	-6.646	-2.720
	(3.194)	(2.174)	(2.076)	(7.418)	(6.286)	(5.473)
β_1	0.490	2.444	1.314	2.836	3.042	1.892
	(0.410)	(0.526)	(0.455)	(0.095)	(1.521)	(1.201)
β_2	-0.233	-0.231	-0.208	2.325	2.538	2.561
	(0.190)	(0.156)	(0.174)	(0.441)	(0.453)	(0.459)
DW	0.698	1.208	1.061	1.591	1.547	1.479

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.6
Dixons (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV-GCH	IV-E-GCH
Diebold Mariano	-0.283	-0.214	0.145	-0.086
	(0.235)	(0.235)	(0.235)	(0.235)
Harvey-Leybourne Newbold	-0.029	-0.142	0.303	-0.044
	(0.235)	(0.235)	(0.235)	(0.235)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.7
GlaxoSmithKline (Details on Table 2.5.1)

Number of observations: 52	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	31.54%	21.31%	31.06%	0%	0.02%	0.19%
β_0	-2.017	-5.088	-6.314	1.359	1.025	2.521
	(1.107)	(2.108)	(1.862)	(1.752)	(3.110)	(2.933)
β_1	1.071	2.338	2.549	0.012	0.103	-0.263
	(0.186)	(0.529)	(0.447)	(0.294)	(0.781)	(0.704)
DW	1.006	0.834	0.928	2.119	2.117	2.125
Encompassing Regression						
R-Square	32.78%	22.10%	31.85%	2.49%	1.89%	2.41%
β_0	-2.589	-4.703	-6.091	2.419	1.700	3.013
	(1.212)	(2.148)	(1.881)	(1.911)	(3.156)	(2.946)
β_1	1.206	2.216	2.460	-0.237	-0.142	-0.460
	(0.219)	(0.549)	(0.458)	(0.346)	(0.807)	(0.718)
β_2	-1.574	1.098	1.083	2.917	2.211	2.381
	(1.373)	(1.294)	(1.196)	(2.165)	(1.900)	(1.873)
DW	1.021	0.873	0.966	2.132	2.124	2.130

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.7
GlaxoSmithKline (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.129	0.167	-0.007	0.010
	(0.193)	(0.193)	(0.193)	(0.193)
Harvey-Leybourne Newbold	0.366	0.294	-0.001	-0.325
	(0.193)	(0.193)	(0.193)	(0.193)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.8
Hilton Group (Details on Table 2.5.1)

Number of observations = 46	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	23.53%	19.59%	32.32%	5.32%	13.36%	9.02%
β_0	-5.009	-5.063	-8.381	1.168	-1.660	-0.334
	(3.694)	(4.148)	(3.709)	(2.493)	(2.611)	(2.608)
β_1	1.894	2.461	2.839	0.546	1.233	0.910
	(0.514)	(0.751)	(0.619)	(0.347)	(0.473)	(0.435)
DW	0.587	0.875	0.887	2.050	1.994	2.029
Encompassing Regression						
R-Square	23.77%	21.40%	33.70%	14.03%	17.68%	15.84%
β_0	-4.990	-6.057	-8.697	1.099	-0.798	0.091
	(3.731)	(4.242)	(3.729)	(2.405)	(2.637)	(2.547)
β_1	1.924	2.781	2.989	0.435	0.956	0.707
	(0.526)	(0.807)	(0.640)	(0.339)	(0.501)	(0.437)
β_2	-0.239	-0.755	-0.586	0.875	0.654	0.789
	(0.651)	(0.701)	(0.619)	(0.419)	(0.436)	(0.423)
DW	0.574	0.925	0.903	2.103	2.014	2.058

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.8
Hilton Group (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.059	0.401	-0.080	-0.114
	(0.246)	(0.246)	(0.246)	(0.246)
Harvey-Leybourne Newbold	0.237	0.537	-0.010	-0.061
	(0.246)	(0.246)	(0.246)	(0.246)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.9
HSBC (Details on Table 2.5.1)

Number of observations = 73	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	45.27%	40.71%	42.45%	2.57%	0.13%	0%
β_0	-2.736	-3.417	-4.973	0.384	1.705	1.479
	(0.898)	(1.075)	(1.248)	(0.798)	(0.930)	(1.096)
β_1	1.192	1.815	2.153	0.189	-0.068	-0.011
	(0.155)	(0.260)	(0.297)	(0.138)	(0.224)	(0.261)
DW	1.083	1.180	1.147	1.200	1.227	1.230
Encompassing Regression						
R-Square	45.61%	42.46%	44.14%	2.71%	0.90%	0.68)
β_0	-2.653	3.328	-4.835	0.419	1.744	1.537
	(0.910)	(1.069)	(1.242)	(0.811)	(0.935)	(1.104)
β_1	1.162	1.747	2.076	0.176	-0.098	-0.044
	(0.162)	(0.262)	(0.299)	(0.144)	(0.229)	(0.266)
β_2	0.466	1.032	1.012	0.197	0.456	0.428
	(0.704)	(0.706)	(0.696)	(0.627)	(0.618)	(0.619)
DW	1.061	1.190	1.012	1.207	1.246	1.247

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.9
HSBC (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.077	0.076	0.105	-0.030
	(0.194)	(0.194)	(0.194)	(0.194)
Harvey-Leybourne Newbold	0.405	0.156	0.253	-0.007
	(0.194)	(0.194)	(0.194)	(0.194)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.10
Kingfisher (Details on Table 2.5.1)

Number of observations = 71	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	17.54%	15.39%	15.98%	0%	2.89%	2.25%
β_0	-0.130	-0.618	-1.553	4.701	7.675	7.680
	(1.725)	(1.987)	(2.193)	(3.024)	(2.510)	(2.838)
β_1	1.618	1.488	1.662	-0.006	-0.614	-0.607
	(0.298)	(0.414)	(0.452)	(0.437)	(0.492)	(0.554)
DW	0.905	1.112	1.094	1.440	1.429	1.426
Encompassing Regression						
R-Square	17.85%	15.74%	16.15%	14.26%	17.18%	16.19%
β_0	-0.171	-0.643	-1.527	5.655	7.107	7.032
	(1.736)	(1.998)	(2.207)	(2.846)	(2.348)	(2.663)
β_1	1.416	1.458	1.631	-0.299	-0.701	-0.674
	(0.303)	(0.419)	(0.461)	(0.421)	(0.460)	(0.518)
β_2	0.183	0.193	0.138	1.087	1.058	1.044
	(0.357)	(0.361)	(0.363)	(0.019)	(0.012)	(0.014)
DW	0.926	1.122	1.100	1.448	1.402	1.419

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.10
Kingfisher (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.040	0.030	-0.092	-0.097
	(0.194)	(0.194)	(0.226)	(0.226)
Harvey-Leybourne Newbold	0.379	0.088	-0.001	-0.071
	(0.194)	(0.194)	(0.226)	(0.226)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.11
Marks and Spencers (Details on Table 2.5.1)

Number of observations = 59	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	25.51%	6.46%	5.32%	10.19%	7.90%	8.14%
β_0	-0.132	3.120	2.898	-1.458	-0.794	-1.367
	(1.419)	(1.467)	(1.732)	(1.403)	(1.310)	(1.536)
β_1	0.956	0.582	0.609	0.544	0.579	0.677
	(0.216)	(0.293)	(0339)	(0.213)	(0.262)	(0.301)
DW	1.206	1.189	1.158	1.816	1.653	1.662
Encompassing Regression						
R-Square	28.58%	12.00%	11.10%	30.89%	31.85%	32.64%
β_0	-0.123	2.798	2.511	-1.437	-1.397	-2.084
	(1.402)	(1.445)	(1.705)	(1.241)	(1.145)	(1.336)
β_1	0.914	0.578	0.616	0.445	0.570	0.690
	(0.215)	(0.287)	(0.332)	(0.190)	(0.227)	(0.260)
β_2	0.745	0.994	1.014	1.744	1.861	1.882
	(0.481)	(0.529)	(0.532)	(0.425)	(0.419)	(0.417)
DW	1.176	1.134	1.093	2.095	1.853	1.857

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.11
Marks and Spencers (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.299	-0.084	0.049	0.034
	(0.216)	(0.216)	(0.216)	(0.216)
Harvey-Leybourne Newbold	0.342	-0.053	0.137	0.097
	(0.216)	(0.216)	(0.216)	(0.216)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.12
Prudential (Details on Table 2.5.1)

Number of observations = 68	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	36.53%	26.53%	23.54%	5.79%	3.86%	4.58%
β_0	-4.339	-2.448	-5.212	-0.551	0.356	-1.417
	(1.879)	(1.973)	(2.722)	(2.296)	(2.263)	(3.049)
β_1	1.545	1.956	2.621	0.617	0.749	1.160
	(0.250)	(0.400)	(0.581)	(0.306)	(0.459)	(0.651)
DW	1.034	0.993	0.866	1.390	1.426	1.294
Encompassing Regression						
R-Square	41.42%	30.62%	27.30%	47.30%	44.91%	44.92%
β_0	-4.536	-2.448	-4.916	-1.128	0.357	-0.447
	(1.821)	(1.932)	(2.679)	(1.732)	(1.726)	(2.339)
β_1	1.503	1.858	2.460	0.494	0.439	0.630
	(0.243)	(0.395)	(0.578)	(0.231)	(0.353)	(0.504)
β_2	0.997	0.916	0.881	2.912	2.911	2.896
	(0.427)	(0.468)	(0.480)	(0.407)	(0.418)	(0.419)
DW	1.100	0.981	0.847	1.540	1.540	1.503

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.12
Prudential (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	0.132	-0.092	0.064	0.058
	(0.201)	(0.201)	(0.201)	(0.201)
Harvey-Leybourne Newbold	0.270	-0.008	0.127	0.093
	(0.201)	(0.201)	(0.201)	(0.201)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.13
Royal Sun Alliance (Details on Table 2.5.1)

Number of observations = 58	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	22.54%	22.64%	31.44%	2.57%	2.18%	1.54%
β_0	-2.966	-5.053	-8.944	2.518	2.222	2.772
	(3.474)	(3.961)	(3.943)	(4.195)	(4.796)	(5.088)
β_1	1.661	2.823	3.349	0.604	0.943	0.799
	(0.411)	(0.697)	(0.660)	(0.496)	(0.844)	(0.852)
DW	0.727	0.918	0.961	1.703	1.697	1.689
Encompassing Regression						
R-Square	24.06%	23.88%	32.58%	7.89%	7.49%	7.13%
β_0	-3.081	-4.984	-8.917	2.287	2.376	2.838
	(3.472)	(3.966)	(3.946)	(4.117)	(4.707)	(4.986)
β_1	1.589	2.695	3.239	0.459	0.658	0.539
	(0.416)	(0.711)	(0.670)	(0.494)	(0.844)	(0.847)
β_2	0.600	0.545	0.520	1.210	1.213	1.241
	(0.572)	(0.576)	(0.540)	(0.679)	(0.683)	(0.682)
DW	0.751	0.929	0.975	1.841	1.830	1.827

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.13
Royal Sun Alliance (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV - GCH	IV - E-GCH
Diebold Mariano	-0.002	0.265	0.041	-0.063
	(0.218)	(0.218)	(0.218)	(0.218)
Harvey-Leybourne Newbold	0.151	0.334	0.108	-0.031
	(0.218)	(0.218)	(0.218)	(0.218)

See Table 2.2b. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.5.14
Reuters (Details on Table 2.5.1)

Number of observations = 60	Daily Squared Returns			Realized Volatility		
MZ Regression	IV	GCH	E-GCH	IV	GCH	E-GCH
R-Square	17.76%	18.41%	12.99%	9.85%	8.97%	6.97%
β_0	-6.024	1.303	3.454	-6.274	2.448	4.093
	(5.735)	(3.646)	(3.722)	(8.664)	(5.557)	(5.554)
β_1	2.328	1.823	1.425	2.502	1.837	1.507
	(0.657)	(0.504)	(0.484)	(0.993)	(0.768)	(0.722)
DW	0.977	1.092	0.965	1.217	1.214	1.228
Encompassing Regression						
R-Square	18.07%	18.44%	13.02%	19.30%	17.07%	15.00%
β_0	-6.306	1.241	3.405	-8.553	1.036	2.971
	(5.807)	(3.700)	(3.769)	(8.316)	(5.384)	(5.377)
β_1	2.315	1.813	1.414	2.395	1.610	1.255
	(0.663)	(0.512)	(0.494)	(0.949)	(0.745)	(0.705)
β_2	0.974	0.324	0.318	7.841	7.341	7.338
	(2.127)	(2.138)	(2.216)	(3.046)	(3.111)	(3.161)
DW	0.980	1.092	0.968	1.296	1.279	1.281

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

Table 2.6.14
Reuters (Details on Table 2.6.1)

	Daily Squared Returns		Realized Volatility	
	IV - GCH	IV - E-GCH	IV- GCH	IV - E-GCH
Diebold Mariano	-0.015	-0.185	0.027	-0.083
	(0.214)	(0.214)	(0.214)	(0.214)
Harvey-Leybourne Newbold	0.134	-0.119	0.127	-0.046
	(0.214)	(0.214)	(0.214)	(0.214)

See Table 2.2a. Note: Results contained here are for non-overlapping forecast horizons.

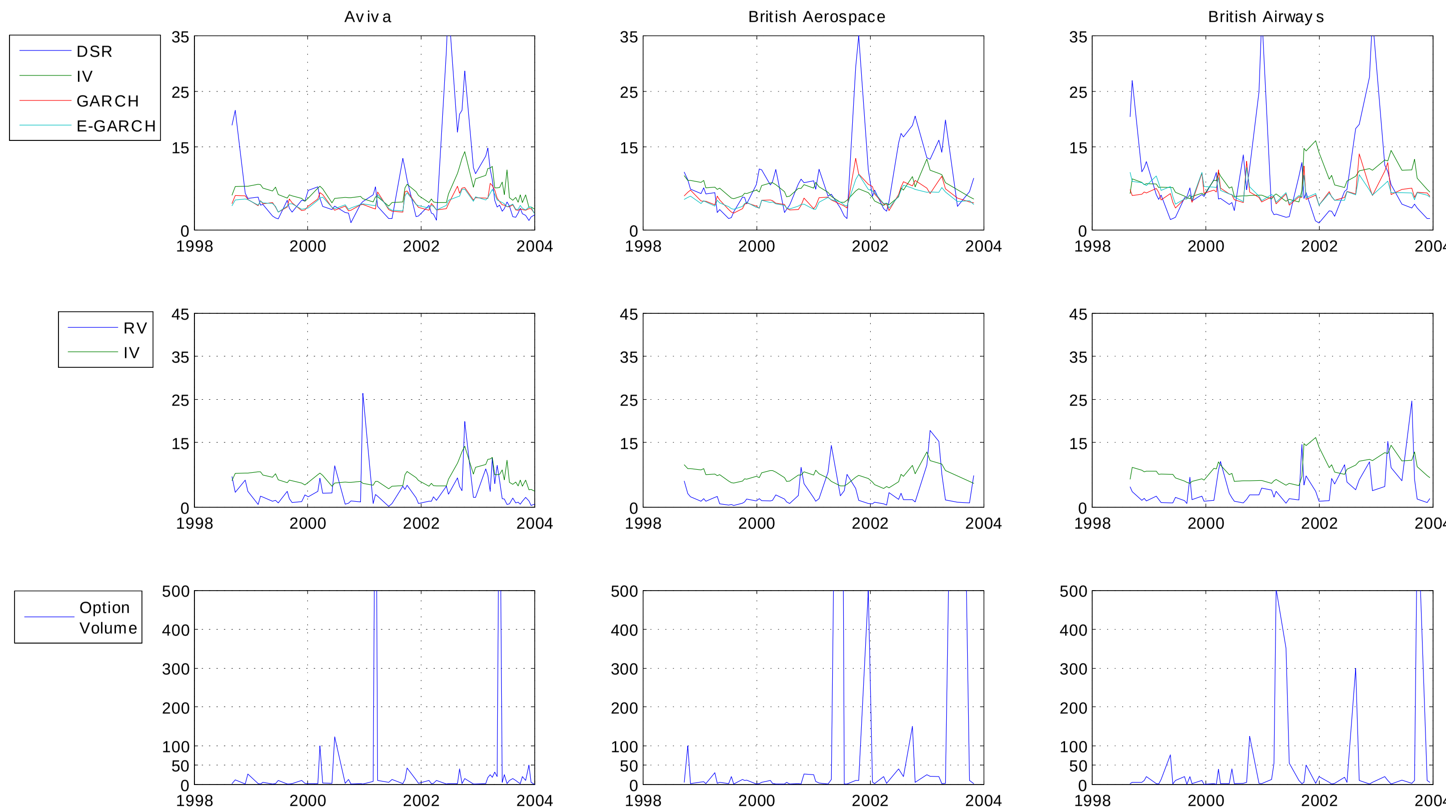


Figure 2.1: Upper panel plots 10-day ahead composite implied volatility (IV), GARCH and E-GARCH forecasts and daily squared returns (DSR). The centre panel plots the composite implied volatility and realized volatility series across 10-day horizons. The volume of option contracts traded on the relevant forecast days are illustrated in the lower panel.

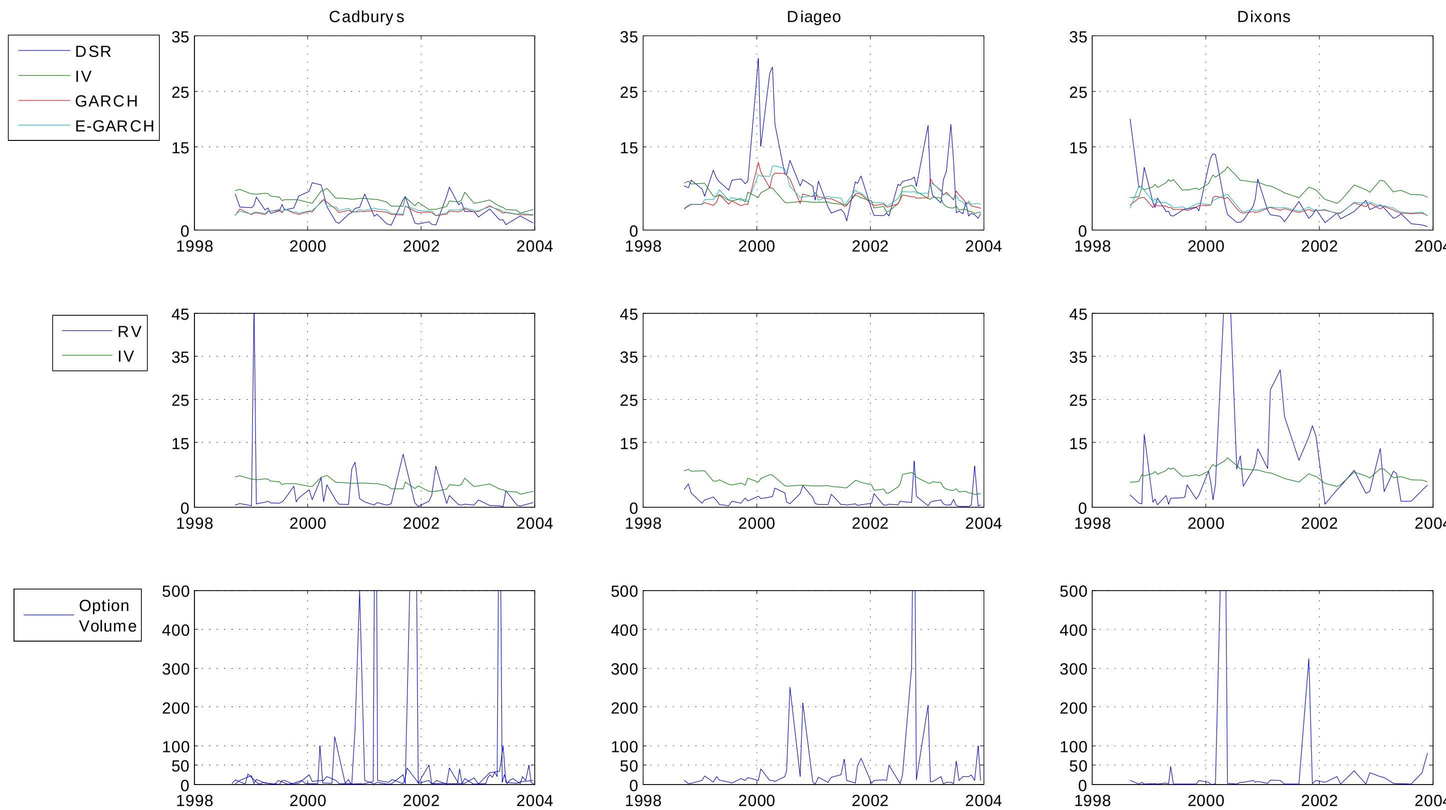


Figure 2.2: Upper panel plots 10-day ahead composite implied volatility (IV), GARCH and E-GARCH forecasts and daily squared returns (DSR). The centre panel plots the composite implied volatility and realized volatility series across 10-day horizons. The volume of option contracts traded on the relevant forecast days are illustrated in the lower panel.

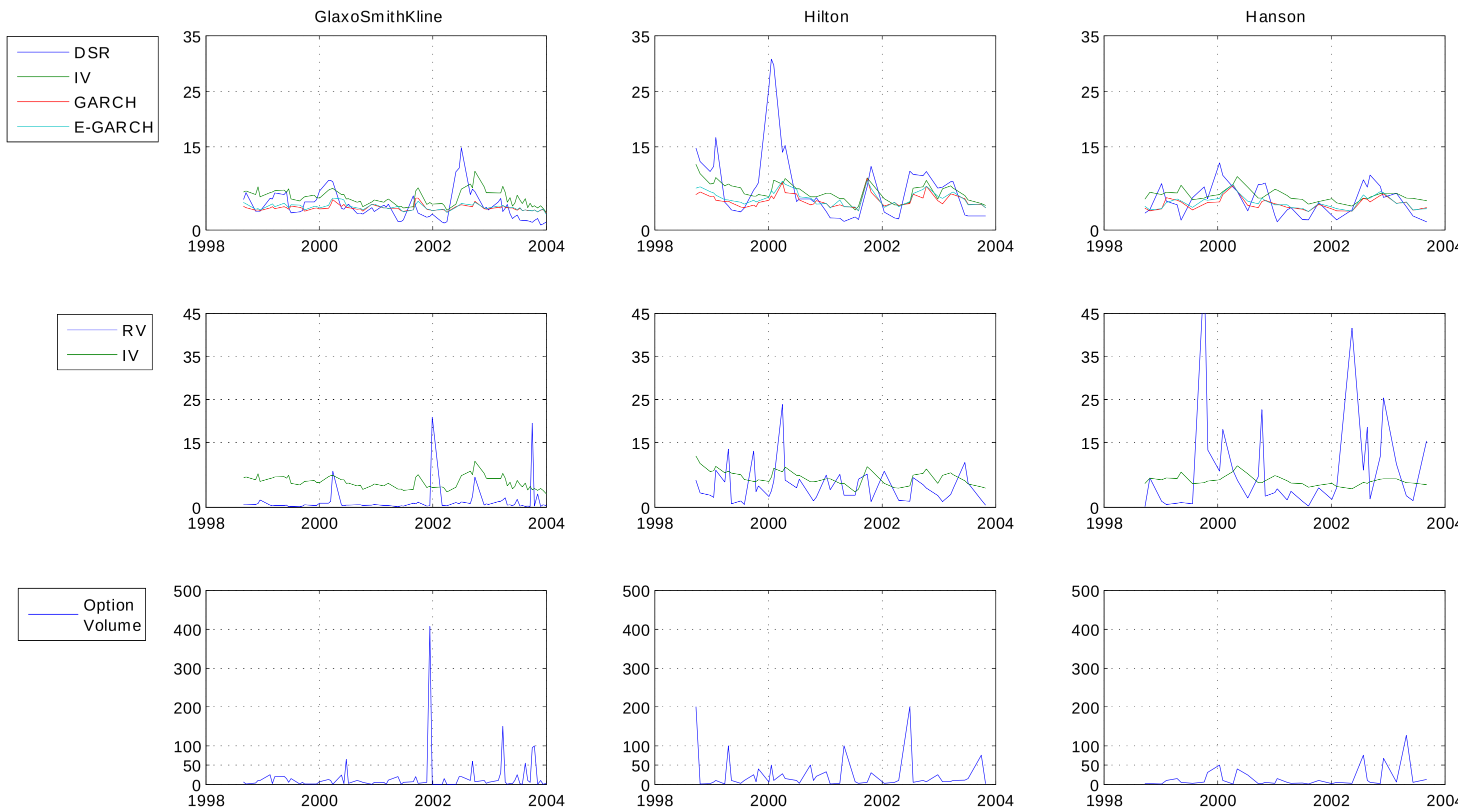


Figure 2.3: Upper panel plots 10-day ahead composite implied volatility (IV), GARCH and E-GARCH forecasts and daily squared returns (DSR). The centre panel plots the composite implied volatility and realized volatility series across 10-day horizons. The volume of option contracts traded on the relevant forecast days are illustrated in the lower panel.

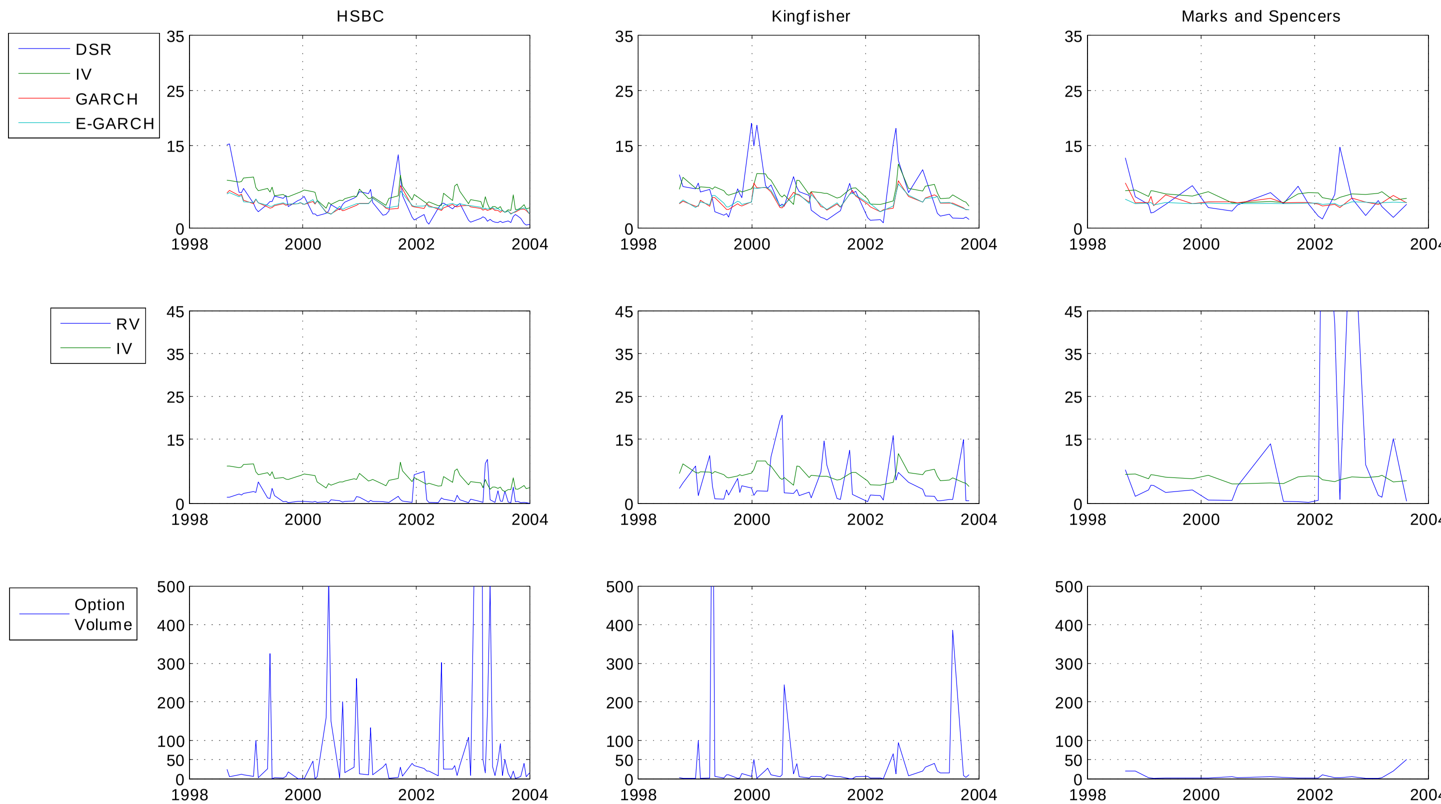


Figure 2.4: Upper panel plots 10-day ahead composite implied volatility (IV), GARCH and E-GARCH forecasts and daily squared returns (DSR). The centre panel plots the composite implied volatility and realized volatility series across 10-day horizons. The volume of option contracts traded on the relevant forecast days are illustrated in the lower panel.

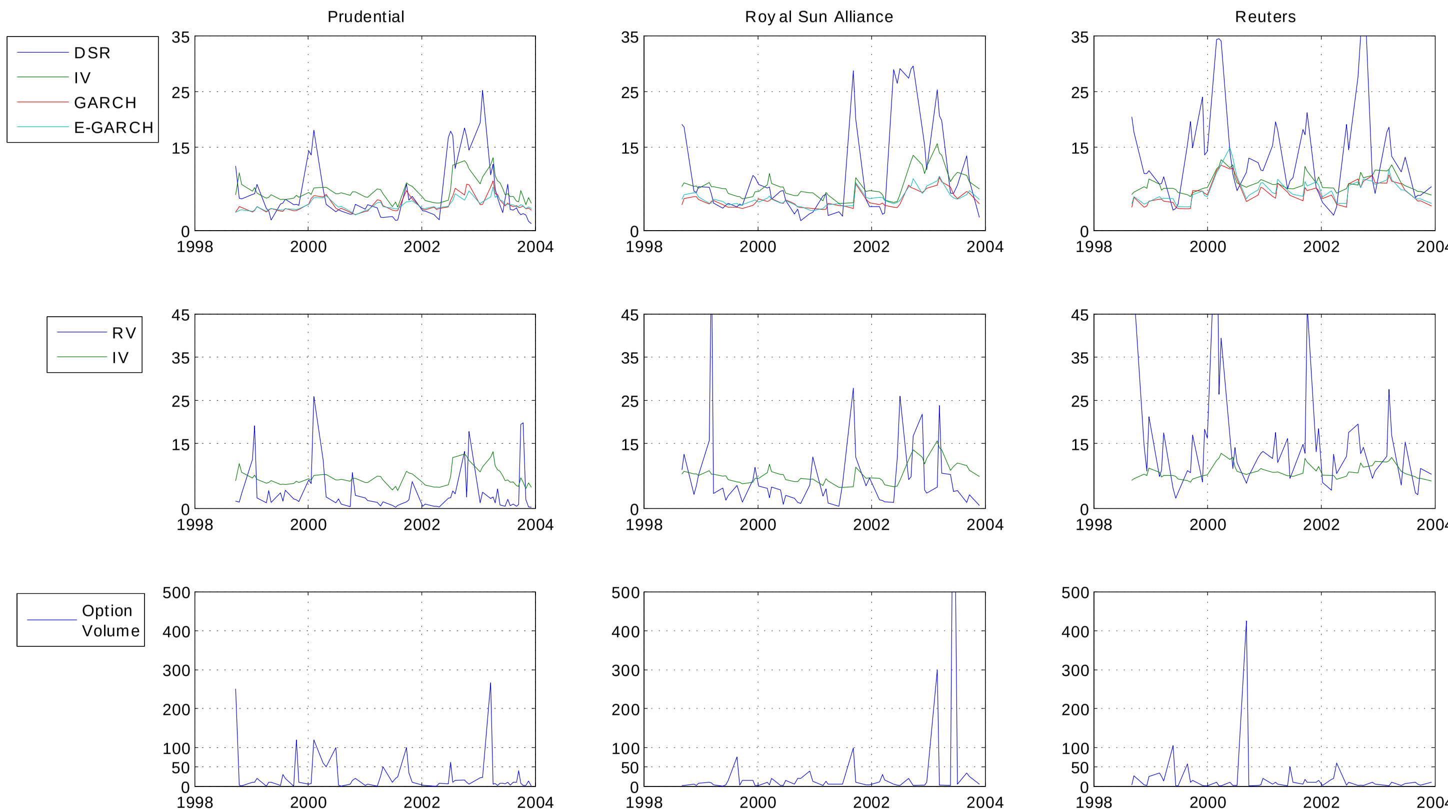


Figure 2.5: Upper panel plots 10-day ahead composite implied volatility (IV), GARCH and E-GARCH forecasts and daily squared returns (DSR). The centre panel plots the composite implied volatility and realized volatility series across 10-day horizons. The volume of option contracts traded on the relevant forecast days are illustrated in the lower panel.

CHAPTER 3

An Investigation into Fractional Dynamics in UK Equity Volatility

3.1. Introduction

Successful modeling and forecasting of financial asset volatility relies on accurate measures of how shocks persist in the autocorrelation function. In an $I(0)$ process, shocks decay at an exponential rate while no mean reversion occurs in an $I(1)$ process. A fractionally integrated long memory process is defined as $I(d)$, that is, fractionally integrated of order d if its d'th difference is $I(0)$, where d can be any real number. This is based on McLeod and Hipel's (1978) definition of a fractionally integrated long memory process y_t, that is integrated of order d, or $I(d)$ if

$$(1-L)^d y_t = u_t \tag{3.1}$$

where L is the lag operator, $-0.5 < d < 0.5$ and u_t is stationary. In a fractionally integrated long memory process shocks decay at a slow rate and cannot be modelled using a finite number of autoregressive and moving average terms. Baillie and King (1996) originate the concept of fractionally integrated processes and long memory processes with studies in hydrology by Hurst (1951, 1957) and Mandelbrot and Wallis (1968). The introduction of I(d) models into economics and finance however didn't occur until the 1980s with the development of the autoregressive fractionally integrated moving average (ARFIMA) process by Granger (1980), Granger and Joyeux (1980) and Hosking (1981) and Geweke and Porter-Hudak's (1983) development of an effective technique for measuring the d parameter.

Over the past twenty years, studies in economics and finance have mapped long memory effects in a range of data sets. The nonlinear Exponential Smooth Autoregressive model (ESTAR) allows for long-range persistence in low level shocks and it has been used by Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003) to describe the dynamics of purchasing power parity (PPP) deviations. Paya and Peel (2006) use simulation and bootstrap methods to show that estimates of the speed of adjustment to shocks are upwardly biased in the ESTAR model and must be adjusted for. In a review, Baillie (1996) notes that evidence of long memory has been found in forward premums, interest rate differentials and rates of inflation as well as in the volatility of financial asset returns. Empirical analysis of spot exchange rates by Baillie and Bollerslev (1994) and Diebold, Husted and Rush (1991) and S&P 500 volatility by Bollerslev and Mikkelsen (1996) have identified a level of persistence in the autocorrelation function that is consistent with an $I(d)$ process with $0 < d < 1$.

Granger (1998) challenges the evidence supporting long memory and makes the distinction between 'true' long memory arising out of fractional integration and 'spurious' long memory produced by structural breaks in volatility. Granger and Hyung's (2004) analysis of S&P 500 data shows that structural breaks in volatility can reproduce a hyperbolic rate of decay in autocorrelations. Motivated by the renewed debate on long memory effects in asset volatility, this chapter tests for the presence of fractional integration in the realized volatility series of sixteen FTSE-100 companies, where realized volatility is constructed from intraday tick data. Implied volatility backed out of the market prices of traded equity options are similarly tested for the presence of long memory and the results for fractional integration are checked for false positives that could be induced by structural breaks. A second objective is to examine whether fractional integration in realized volatility informs our understanding of the long-run relationship between realized volatility and option implied volatility. Using testing procedures developed by Robinson and Yajima (2002) and Nielsen and Shimotsu (2007), the implied and realized volatility series of individual equities are jointly tested for fractional cointegration and the cointegrating rank of these series is determined.

This chapter shows that the realized and implied volatility series for a number of FTSE-100 companies are fractionally integrated. The results also reveal a significant degree of variation in the levels of long-run dependence among the cross-section of companies sampled. The validity of this finding is tested by examining the time domain properties of each series using tests developed by Shimotsu (2006). These tests consider the presence of 'spurious' long memory induced by structural breaks and the results show that although there is evidence of structural breaks it is not sufficient to explain the level of persistence found in our sample. This robust examination and confirmation of fractional integration in both realized volatility and implied volatility extends similar findings on equity indices produced by Bandi and Perron (2006). The results highlight the weakness of traditional volatility modelling approaches to fully capture the dynamics of equity volatility. This chapter also shows that the realized and implied volatility of individual equities can be fractionally cointegrated. An examination of the cointegrating rank of the two series shows that a maximum of one cointegrating relationship exists between the implied and realised volatility series of a number of FTSE-100 companies. The evidence of a long-run equilibrium relationship has potential implications for both the signal function of options markets and the optimal implementation of derivative strategies that use both options and the underlying asset. In this chapter, implied and realized volatility series on FTSE-100 companies are robustly tested for the presence of long memory in this manner. Los (2005) shows that long memory effects are relevant in the construction of appropriateValue-at-Risk (VAR) estimates, while Taylor (2001) and others demonstrate its non-negligible effects on options pricing. Thus, the results found here have important consequences for the development of effective risk management and asset allocation strategies involving UK equities.

The remainder of this chapter proceeds as follows. Section 3.2 comprises a review of the current research on long memory effects in financial data. Section 3.3 describes the construction of the realized and implied volatility time series used in this paper. It also describes the methods used to measure long memory and how the long-run relationship between implied and realised volatility is modeled. Section 3.4 discusses our empirical findings and Section 3.5 concludes.

3.2. Implied Volatility and Long Memory

Lo (1991) points out that long-term forecasting, optimal consumption /savings decisions, portfolio optimisation and the pricing of derivatives are all sensitive to the investment horizon. Given the evidence of long memory effects in financial and economic time series, both Lo (1991) and Sowell (1992) note that long term forecasts should allow for greater flexibility in the order of integration. Similarly, Engle and Patton (2001) recommend the inclusion of volatility characteristics such as mean-reversion, asymmetric effects and persistence if a volatility forecast is to be used effectively in portfolio and risk management applications. The requirement to allow for long-run persistence in volatility encouraged the development of fractionally integrated variants of the autoregressive integrated moving average (ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH) class of models. The ARFIMA (p,d,q) model introduced by Granger and Joyeux (1980) and Granger (1980, 1981) and the fractionally integrated GARCH (p,d,q) (FIGARCH) model of Baillie, Bollerslev and Mikkelsen (1996) allow d to assume values between *0* and *1*, thus producing forecasts that allow the effect of shocks to dissipate hyperbolically over time. The application of the latter model by Bollerslev and Mikkelsen (1996) shows that it efficiently captures the mean-reversion properties of S&P 500 volatility. This evidence supporting fractional integration in financial data was reinforced by Breidt, deCrato and Lima (1998) who used semiparametric techniques to identify long memory effects in both the squared returns and the logarithms of squared returns estimated from daily data on several market indices. An important observation was made by both Robinson (1978) and Granger (1980) who found that long memory effects can emerge in equity index volatility as a result of aggregation even though the individual series do not exhibit this characteristic.

There are however a number of studies that have looked at individual financial time series data. Barkoulas and Baum (1997) examine daily returns data on thirty companies quoted on the Dow Jones Industrial Average Index

(DJIA). Evidence supporting long memory effects is sparse, with no fractal structure identified in the returns series of twenty-two of the companies sampled. Ray and Tsay (2000) randomly select 100 S&P 500 companies and find strong evidence of long range dependence in the volatility structure of the majority of companies sampled. Levels of persistence were similar among firms from the same industry implying that, if firms are categorised by industry, firm volatilities are driven by common components and are thus tied together in the long run. The greater availability of high-frequency data on financial assets has encouraged research into the relationship between intraday periodicity and long memory effects. Andersen and Bollerslev (1997) attempt to resolve the short-run decay associated with news arrivals in intraday data and long memory effects observed in daily returns. In a sample of 5-minute returns on the DM-$ exhange rate, they found that this conflict could be resolved by adjusting for the U-shape observed in the intra-day periodic structure, i.e. volatility is high at the open and close of trading. An examination of 5-minute return volatility in US Treasury bond futures by Bollerslev, Cai and Song (2000) also shows that long memory effects become more prominent once adjustments were made to account for repetitive intra-day trading patterns.

Evidence of long-run dependence in asset volatility has been used to explain a number of empirical anomalies in option pricing. Using simulations to generate prices for hypothetical long-term equity anticipation securities

(LEAPS) on the S&P 500 index, Bollerslev and Mikkelsen (1996) show the importance of modeling long memory effects in option pricing. The results produced by Bollerslev and Mikkelsen (1996) indicate that the S&P 500 Index is efficiently modeled by a fractionally integrated process. Sundaresan (2000) suggests that long memory can explain some of the empirical anomalies that arise using standard option pricing models. The more pronounced smile effect observed in short-maturity options compared to long-term options is identified as a possible consequence of long memory effects being omitted from the modeling procedure.

The evidence supporting fractional integration in financial data has encouraged research into the nature of the long-run relationship between fractional series. While cointegration traditionally examines nonstationary $I(1)$ series for the presence of stationary $I(0)$ linear relations, fractional cointegration facilitates greater flexibility when modelling the relationship between series. The concept of fractional cointegration has applied in a number of contexts within econometrics. For example, Davidson, Peel and Byers (2006) use a fractionally cointegrating error correction model (FVECM) to describe patterns in UK political poll results and Cheung and Lai (1993) as well as Robinson and Marinucci (2001) have examined the presence of fractional cointegration in financial data. Given the documented existence of fractional integration in financial asset volatility, Robinson and Yajima (2002) and Nielsen and Shimotsu (2007) have developed more rigorous techniques that facilitate investigation into potential long-run equilibrium relationships between series that are not strictly $I(1)$. An examination of the long-run relationship between realized volatility and implied volatility on the S&P 100 by Bandi and Perron (2006) has shown some evidence of fractional cointegration. Their findings are closely related to the forecasting literature which shows that although implied volatility contains predictive information it consistently overestimates subsequently realized volatility. This has been demonstrated in the case of equity indices by Christensen and Prabhala (1998) and for individual equities by Garvey and Gallagher (2007). The positive bias has been partially attributed to the use of overlapping data, errors-in-variables and missing variables which can compromise the results produced by the classical regression approach used in previous studies [Christensen and Prabhala (1998)]. Pan (2002) has shown that a jump-risk premium can be observed in option prices and that this premium has a positive relationship with volatility levels in the underlying market. This finding is clearly an important factor when attempting to understand the dynamics of the implied-realized volatility relationship and it has motivated the addition of a volatility risk premium to the implied-realised volatility regression described in (3.14) below [Poteshman (2000), Chernov (2007)]. Bandi and Perron (2006) point out that explicitly accounting for a time-varying risk premium in this manner can be misleading. Their study shows that an examination of the long run implied-realised relationship using semi-parametric techniques avoids the issues associated with a classical regression approach by focusing on very low (harmonic) frequencies and ignoring short-run dynamics.

The primary objective of this chapter is to establish the presence of fractional integration in the implied and realized volatility series of individual equities. Establishing a robust estimate of the long memory parameter is complicated by the existence of structural breaks which have been shown to induce persistence in asset volatility. Hamilton and Susmel (1994), Granger and Marmol (1997), Mikosch and Starica (1999) and Granger and Hyung (2004) all show that regime-switching or occasional break models can produce the long memory property in a finite sample. Using the semi-parametric GPH estimator described later in this paper, Granger and Hyung (2004) show that extracting the long memory properties in the autocorrelation of a break model becomes more difficult as the number of breaks increases. Results from an investigation into the macroeconomic determinants of stock market volatility by Morana and Beltratti (2004) suggests that volatility of the S&P 500 is characterised by both structural changes and long memory. They also show that breaks in the volatility of macroeconomic factors such as interest rates and money growth, produces breaks in stock market volatility. Shimotsu (2006) shows how sample splitting and d'th differencing can be used to distinguish between true long memory and a spurious long memory process that is produced by structural breaks. Empirical analysis of realized volatility on the S&P 500 Index by Shimotsu (2006) as well as

Morana and Beltratti (2004) shows that persistence is likely to be explained both by regime shifts and long memory effects.

Ohanissian, Russell and Tsay (2008) examine the relevance of distinguishing between 'true' long memory or fractional integration and 'spurious' long memory that is produced by regime-shifts in the volatility process. Under the assumptions that the true volatility model is known and can be estimated, the omission or mispecification of the long memory parameter leads to significant option mispricing. If actual volatility follows a 'spurious' long memory process and it is modelled as a short memory or 'true' long memory process then the resulting call options will be underpriced. Furthermore, when volatility follows a 'true' fractionally integrated process, then the use of either a short-memory model or a 'spurious' long memory model leads to an overpricing of call options. These findings by Ohanissian, Russell and Tsay (2008) clearly illustrate the importance of correct specification of the persistence parameter for market participants.

3.3. Modeling Volatility

Data on both individual FTSE-100 stocks and their related options for the period 1^{st} October 1997 to 31^{st} December 2003 are obtained from the London Stock Exchange (LSE) and the London International Financial Futures Exchange (LIFFE) respectively. I select the maximum number of FTSE-100 stocks (sixteen) for which options data is continuously available over that time period. Implied volatility estimates are backed out of traded prices of American-style individual equity options traded on the London International Financial Futures Exchange. Tick-by-tick price data obtained from the London Stock Exchange is used to construct a time series of realised volatility for each FTSE-100 stock included in the study.

Volatility is estimated nonparametrically using this high-frequency data, however the raw price series must be adjusted for the presence of noise due to imperfections of the trading and recording process. Each price series used in our sample first undergoes a filtering process that removes non-unique and incorrect observations. Once these so-called market microstructure effects are corrected for, the irregular tick-by-tick price series is then converted to regular series of thirty-minute intervals. Research by Oomen (2006) distinguishes between sampling schemes based on transaction time, business time, and calendar time. Based on IBM transaction data over the period 2000-2004, Oomen (2006) shows that the mean square error (MSE) of realised volatility can be reduced by sampling returns on a transaction time scale rather than the more common sampling approach using calendar time. However, if market microstructure noise is dominant then the simplicity of calendar time sampling may produce superior estimates. Oomen (2006) results show that the optimal sampling frequency of about 3 minutes was strongly dependent on variations related to market liquidity.

In this chapter, calendar time sampling is used by selecting the mid-price recorded closest to the end of each thirty-minute interval between 8.30am and 4.30pm each day. Given the range of stocks included in our sample the interval length is arbitrarily selected as a tradeoff between market microstructure effects and the accuracy associated with approximations at higher frequencies. The approximation of 'true' volatility from high frequency data has become increasingly sophisticated. Empirical evidence from foreign exchange and equity markets has pointed to a daily U-shaped pattern in return volatility [Andersen and Bollerslev (2003)]. Calendar time sampling does not take account of this repetitive intraday trading pattern. The creation of a regularly spaced time series using calendar time sampling also excludes a significant amount of intraday data [Zhang and Mykland (2006)]. Despite this valid criticism, Andersen, Bollerslev, Diebold and Labys (2003), Barndorff-Nielsen and Shephard (2004) and Meddahi (2002) show that sampling intraday tick prices in this manner produces accurate estimates of integrated variance. Shimotsu's (2006) study on identifying long memory effects also uses this approach to estimate daily realized volatility of the S&P 500 index. The use of calendar time sampling as a preliminary step in estimating realized volatility on UK equities facilitates later comparison against Shimotsu's (2006) results. Daily realized volatility is calculated as the sum of thirty-minute squared returns. Annualized realized volatility is calculated as follows

$$\sigma_t^R = \sqrt{\frac{1}{n}\sum_{j=1}^{n_t} r_j^2 \times 252}, \tag{3.2}$$

where r_j is the sum of the 30-minute squared returns calculated on day j, and n is the number of days from option trade date to option expiry, assuming 252 working days in the year.

The implied volatility series is calculated as a weighted average of implied volatility from the four closest-to-the-money options traded on each day. This follows the approach used by commercial providers of implied volatility indices and has been shown by Ederington and Guan (2002) to provide stronger forecasts than approaches that include away-from-the-money options data. To mitigate correlation in the residuals a nonoverlapping series is created by selecting options traded on the first business day of each month with a maturity between 15-22 days. The weighted implied volatility backed out of those options are used to contruct our implied volatility series. The matching realized volatility series is calculated as the average realized volatility of the underlying asset experienced over the life of the option.

Preliminary statistics for the implied and realized volatility series are contained in Table 3.1. For the cross-section of companies considered in this paper, the sample average implied volatility (38.6%) exceeds subsequently realized volatility (33.9%). This somewhat supports the empirical evidence of a positive bias in option implied volatilities. Average levels of kurtosis are also similar for both implied and realized volatility while both series display similar levels of positive skewness. This suggests that stock price volatility and option prices are driven by broadly similar dynamics.

Testing for Long Memory

There are a number of methods used to test for long memory in time series, with some of the earlier approaches such as the rescaled range (R/S) statistic developed by Hurst (1951, 1957) discussed in Baillie (1996). However, Lo (1991) shows that the R/S statistic is not robust to short memory and heteroskedasticity and proposes a modified version,

$$Q_T = R_T/\sigma_T(q), \tag{3.3}$$

where R is the range,

$$R_T = \max_{0\leqslant j\leqslant T}\left\{\sum_{j=1}^{T}(y_j - j\overline{y})\right\} - \min_{0\leqslant j\leqslant T}\left\{\sum_{j=1}^{T}(y_j - j\overline{y})\right\}, \tag{3.4}$$

and the sample standard deviation, $\sigma_T(q)$, is available from

$$\sigma_T^2(q) = c_0 + 2\sum_{j=1}^{q} w_j(q)c_j, \tag{3.5}$$

where y is the series under consideration, T is the number of observations available and $\overline{y}$ is the sample mean. In (3.5) above, c_j is the jth-order sample autocovariance of y_t and $w_j(q)$ are the Bartlett window weights of,

$$w_j(q) = 1 - [j/(q+1)] \text{ for } q < T. \tag{3.6}$$

Baillie (1996) notes that no criteria exists for the optimal choice of q and its use is not supported by simulation tests.

The use of semi-parametric methods does not require us to make any assumptions about the short-run dynamics of the series under consideration. Two semi-parametric estimators that provide relatively consistent estimates of the persistence parameter d are applied. Specifically, the memory parameter d is estimated using the log-periodogram approach proposed by Geweke and Porter-Hudak (1983) and the feasible exact local Whittle estimator developed by Shimotsu and Phillips (2005). Both approaches assume the spectral density, $f(\lambda)$ of the process X_t satisfies

$$f(\lambda) \sim G\lambda^{-2d}, \text{as } \lambda \to 0+,$$

where $d \in (-\frac{1}{2}, \frac{1}{2})$ and $G \in (0, \infty)$. Robinson (1995) defines a long memory, fractionally integrated process as follows

$$(1-L)^d X_t = u_t, \tag{3.7}$$

where L is the lag operator and u_t is a covariance stationary process whose spectral density is bounded and bounded away from zero at the zero frequency $\lambda = 0$. The approach developed by Geweke and Porter-Hudak (GPH) (1983) is based on the following log-periodogram (LP) regression,

$$\ln P(\lambda_s) = c - d\ln(4(\sin^2(\frac{\lambda_s}{2}))) + \varepsilon(\lambda_s), \tag{3.8}$$

where the periodogram of the data is $P_x(\lambda_s) = |w_x(\lambda_s)|^2$ and is computed at the following Fourier frequencies close to zero, $\lambda_s = \frac{2\pi s}{n}$ $(s = 1, ..., m < \frac{n}{2})$, c is a constant and ε denotes the regression residual. The discrete Fourier transform is defined as

$$w_x(\lambda_s) = (2\pi n)^{-1/2}\sum_{t=1}^{n} X_t e^{it\lambda_s} \tag{3.9}$$

and the estimator is semi parametric in that it only employs local assumptions (near the zero frequency) and treats the spectral density away from the origin nonparametrically. The distribution of the long memory parameter, d will be asymptotically normal with the variance $\pi^2/6n$. The GPH approach is relatively simple to apply, although Geweke and Porter-Hudak (1983) show that it is biased and inefficient when the regression residuals are substantially autocorrelated.

An alternative estimation procedure is the local Whittle (LW) estimator originally developed by Kunsch (1987) and Robinson (1995). LW estimation uses the following Gaussian objective function,

$$Q_m(G,d) = \frac{1}{m}\sum_{s=1}^{m}\left[\log(G\lambda_s^{-2d}) + \frac{\lambda_s^{2d}}{G}P_x(\lambda_j).\right] \tag{3.10}$$

$\lambda_s = \frac{2\pi s}{n}$, $s = 1, ..., n$ and estimation takes place over the bandwidth, m, which is some integer less than n. The LW estimator minimizes $Q_m(G,d)$ and Robinson (1995) shows that the asymptotic standard errors for this estimator are, $\sqrt{m}(\widehat{d}_{n,m} - d) \Longrightarrow N(0, \frac{1}{4})$. It is more efficient than the GPH estimator in the stationary case ($|d| < \frac{1}{2}$), however both the LW and LP estimators are inconsistent in the non-stationary case, when $|d| > \frac{1}{2}$ [Kim and Phillips (1999)]. The exact local Whittle (ELW) estimator proposed by Phillips and Shimotsu (2004) is given as

$$\widetilde{d} = \underset{d\in[\Delta_1,\Delta_2]}{\arg\min} R(d), \tag{3.11}$$

where d is bounded by Δ_1 and Δ_2 and

$$R(d) - \log\widehat{G}(d) - 2d\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j, \tag{3.12}$$

$$\widehat{G}(d) = \frac{1}{m}\sum_{j=1}^{m} I_{\Delta d_x}(\lambda_j). \tag{3.13}$$

Assuming that the mean of the time series X_t in (3.7) is known, Phillips and Shimotsu (2004) show that the ELW is consistent and asymptotically normally distributed when the true value of $d \in (\Delta_1, \Delta_2)$ if $\Delta_2 - \Delta_1 \leq \frac{9}{2}$.

The feasible exact local Whittle (FELW) estimator set out by Shimotsu and Phillips (2005) allows estimation of the d parameter when the mean is unknown. Its development was motivated by simulation results which indicated that in finite data sets the estimated ELW estimator is inconsistent if the error in estimating the mean is not controlled. The FELW estimator provides consistent estimates of d for $d > -\frac{1}{2}$ by assigning an appropriate weighting scheme for ELW estimates of d using the unadjusted data series for estimates of μ_0 when d is small and the demeaned data series when d is large. The FELW estimator is appropriate in this context as a general purpose estimator is required that can allow for a substantial range of stationary and nonstationary regions of d.

The implied $\left(\sigma_t^{IV}\right)$and realized volatility $\left(\sigma^{RV}\right)$ series for each of the companies included in our sample are plotted in Figures 3.1a and 3.1b. Both series consist of 75 non-overlapping monthly observations for the sample period from the 1st October 1997 to 31st December 2003 and a visual examination reveals an expected similarity in the behaviour of both series across time. The long memory parameter, d for the implied and realized volatility series is estimated using both the GPH and FELW approach across a range of seven bandwidths, m between $[n^{0.4}]$ and $[n^{0.7}]$. The GPH and FELW estimates of d are also found for the residual series produced by the OLS regression

$$\sigma^{RV} = \alpha + \beta\sigma_t^{IV} + \varepsilon_t \tag{3.14}$$

GPH and FELW estimates are also found for the differences $\sigma^{RV} - \sigma_t^{IV}$ which is essentially constraining the d estimate of implied and realized volatility to be the same. The results of our estimation procedure as well as the 95% confidence bands are plotted in Figures 3.2a-3.2g. The plots show that both estimation procedures demonstrate an upward trend in the d estimate as the size of the bandwidth increases. It is also seen that the FELW provides lower estimates of d than the GPH method. Long-memory estimates are shown to be reasonably consistent across bandwidths (with the exception of the FELW estimate for Dixons (realised volatility) and Hilton (implied volatility)). For each company included in the sample, Tables 3.2a-3.2d provide a summary of parameter estimates for the entire sample period over three bandwidths $m = n^{0.50}, m = n^{0.60}$ and $m = n^{0.70}$ as well as their associate standard errors. At the lower frequencies the GPH estimation of d for both the implied and realized series is in the stationary region ($\widehat{d} < \frac{1}{2}$) for Aviva, GlaxoSmithKline, Kingfisher, Lonmin, Prudential and Reuters. The parameter estimates produced by the FELW estimate confirm these results for the implied and realized series of Prudential and Reuters and suggest it may additionally be the case for Cadburys and Hanson. For the majority of companies in the sample the estimate of d moves into the nonstationary region ($\widehat{d} > \frac{1}{2}$) once the bandwidth increases to $n^{0.60}$ prompting attention to the results for the FELW estimator since the GPH estimator has been shown to be inconsistent in this region. The cross-sectional variation in the long memory parameter is similar to previous findings by Barkoulas and Baum

(1997). Using the local Whittle estimator proposed by Robinson (1995), Barkoulas and Baum (1997) reported wide variability in the estimates for the persistence parameter estimated from the daily prices of thirty DJIA companies. These results can also be compared to those of Ray and Tsay (2000) who test the fractional properties of daily volatilities of individual companies quoted on the S&P 500 using the GPH estimator. Ray and Tsay's (2000) estimates are markedly lower than those produced by the FTSE-100 companies sampled here. Daily volatilities for the 100 randomly sampled S&P 500 companies produce mean estimates estimates for the persistence parameter of 0.3768 with a standard deviation of 0.0875. Ray and Tsay (2000) note that theoretical results produced by Deo and Hurvich (1998) as well further simulation tests suggest that the level of persistence is likely to be higher and much closer to our results for FTSE-100 companies.

The results are tested for spurious effects induced by structural breaks or regime shifts in the volatility process. Shimotsu (2006) shows that estimates of the long memory parameter from subsamples should be similar to those produced by the full sample if the series is a 'true' long memory fractionally integrated process. Differences in the long memory parameter among subsamples suggests that the long memory effect is a product of structural breaks in volatility. Under the sample-splitting approach, if a series X_t is an $I(d)$ process then the average of $\widehat{d}^{(1)}, ..., \widehat{d}^{(b)}$estimates from the b subsamples should approximate the $\widehat{d}$ estimate of the full sample. A visual examination of average $\overline{d}$ estimates when we split the sample into two subsamples ($b = 2$) should demonstrate some consistency while a more formal test for parameter constancy is the Wald statistic (W). The number of subsamples used in the testing procedure is constrained by the size of the data set which consists of 75 nonoverlapping observations. Shimotsu (2006) defines the Wald statistic for testing the null hypothesis of parameter constancy among subsamples, as follows;

$$W = 4mA\widehat{d}_b(A\Omega A')^{+}(A\widehat{d}_b)', \tag{3.15}$$

where b is the number of subsamples used and $\widehat{d}_b$ and A are given as

$$\widehat{d}_b = \begin{pmatrix} \widehat{d} - d_0 \\ \widehat{d}^{(1)} - d_0 \\ \\ \widehat{d}^{(b)} - d_0 \end{pmatrix} \text{ and } A = \begin{pmatrix} 1 & -1 & ... & 0 \\ .. & . & & . \\ .. & . & & . \\ 1 & 0 & ... & -1 \end{pmatrix},$$

and $(A\Omega A')^{+}$ is the generalized inverse of $A\Omega A'$. Hurvich and Chen (2000) found that the Wald test overrejects long memory. This is overcome by using the modified version (W_c) that selects the number of periodogram ordinates equal to m/b so that the subsample estimation uses the same width of frequency band as used to estimate the full sample, as follows,

$$W_c = 4m(c_{m/b}/(m/b))A\widehat{d}_b(A\Omega A')^{+}(A\widehat{d}_b)'. \tag{3.16}$$

Simulation results provided by Shimotsu (2006) show that the modified Wald statistic performs well despite the bias in semiparametric estimates that arise from the short-run dynamics in the data. The Wald statistic is compared to a Chi-squared distribution at the 95% level. Shimotsu (2006) proposes a second test that identifies the accuracy of the long memory parameter estimate (d). The process involves d'th differencing of the series under consideration and the resulting series should be $I(0)$. Despite the relative simplicity of both approaches, Shimotsu (2006) demonstrates that they can be effectively used to identify spurious long memory in both the stationary and nonstationary case. The differenced series is tested for stationarity using the Z_t unit root test [Phillips-Perron (1988)] and the KPSS test [Kwiatkowski, Phillips, Schmidt and Shin (1992)][1].

Table 3.3a and 3.3b summarise the test results for true long memory in implied and realized volatility respectively. Long memory is validated using both the split sample and d'th differencing approaches. An examination of the realized volatility series for each company rejects the null hypothesis of true long memory using the modified Wald statistic in the case of Aviva and Royal Sun Alliance. True long memory is rejected in the implied volatility series of Cadburys. The modified Wald statistic does not reject parameter

[1]The KPSS test was designed to test the null hypothesis of *I(0)* against the alternative hypothesis of *I(1)*. First, it involves taking the residuals e_t from the regression of a process y_t on an intercept and time trend and forming the partial sum S_T of the residuals as follows, $S_T = \sum_{t=T}^{T} y_t$. The KPSS test for stationarity is then, $\eta_t = T^{-2} \sum S_T^2 / \sigma_T^2(q)$ where $\sigma_T^2(q)$ is defined in (5) above.

constancy among subsamples for all other companies included in the sample. Our estimation of the long memory parameter is based on a bandwidth of $m = n^{0.60}$ and because of the size of the options dataset under consideration the number of subsamples (b) used are limited to 2. Some variation in estimates of $\overline{d}$ among subperiods means that we cannot exclude structural breaks within the sample period. The level of variation in $\overline{d}$ is unlikely to explain the observed level of the persistence in the log realized volatility. The results provided by d'th differencing tests in particular suggest that true long memory effects are present in the realized volatility of individual stocks. The values for the Phillips-Perron (Z_t) and the KPSS (η_μ) statistic applied to the d'th differenced series are shown in tables 3.3a and 3.3b respectively. Comparing both test statistics against their critical values at the 5% level supports the null hypothesis that the differenced series is $I(0)$ cannot be rejected in any instance for either the realized or implied volatility series. The generation of non-overlapping observations reduces the size of the sample and the persistence parameters that are estimated are characterised by large standard errors. These limitations are mitigated by the application of rigourous tests for 'true' long memory. Both the split sample and d'th differencing tests provide support for parameter constancy in both the implied and realized series. Estimates of the long memory parameter are shown to be consistent and these estimates support long memory fractional integration in the volatility of individual stocks, particularly when estimation is carried out at lower frequencies.

The findings in this research are similar to the results found in other financial time series. Choi and Zivot (2007) show that stationary long memory is found in the forward discount series even after allowing for structural breaks. Shimotsu (2006) tests for 'true' long memory in the S&P 500 realized volatility series by splitting the sample of 5,000 observations into subperiods of 1,000 observations. 'True' long memory is supported by the Phillips-Perron and KPSS test although variation in subsample estimates of d is partly attributed to sampling error. Although the presence of jumps and/or structural breaks is accepted they cannot explain the level of perisistence found in the S&P 500. The studies by Shimotsu (2006) and Choi and Zivot (2007) and the results presented in this chapter indicate that 'true' long memory is a condition of a number of financial time series and that it can be successfully distinguished from spurious long memory induced by structural breaks. The following section extends the investigation into implied and realized volatility by examining the existence and rank of fractional cointegration between these series.

Specifying the rank of fractional cointegration in the implied-realized volatility relationship

Engle and Granger (1987) identify the $p \times 1$ vector variate X_t as cointegrated $CI(d,b)$ if it has $I(d)$ elements, and for some, $b > 0$, there exists β such that $\beta' X_t$ is $I(d-b)$. Although the original definition restricts $d = b = 1$, the evidence supporting fractional integration in financial data requires some flexibility to be built in to this definition. Fractional cointegration between two series is suggested if they exhibit similar levels of long-run dependence and long-memory estimates produced by the residual series are lower. Based on this criteria a comparison the FELW estimates indicates some level of fractional cointegration between implied and realized volatility in a number of companies sampled. Fractional cointegration is suggested between the implied and realized volatility series of the following companies; Aviva ($FELW_{IV}^{av} = 0.509$, $FELW_{RV}^{av} = 0.543$), British Aerospace $\left(FELW_{IV}^{bae} = 0.940,\ FELW_{RV}^{bae} = 0.941\right)$, British Airways ($FELW_{IV}^{ba} = 0.620$, $FELW_{RV}^{ba} = 0.428$), Cadburys ($FELW_{IV}^{cbry} = 0.721$, $FELW_{RV}^{cbry} = 0.762$), Hilton ($FELW_{IV}^{hg} = 0.583$, $FELW_{RV}^{hg} = 0.570$), (Prudential $FELW_{IV}^{pru} = 0.637$, $FELW_{RV}^{pru} = 0.523$) and Reuters ($FELW_{IV}^{rtr} = 0.939$, $FELW_{RV}^{rtr} = 1.063$). The relevant GPH and FELW estimates of d and their associated standard errors are summarised in Tables 3.4a and 3.4b respectively.

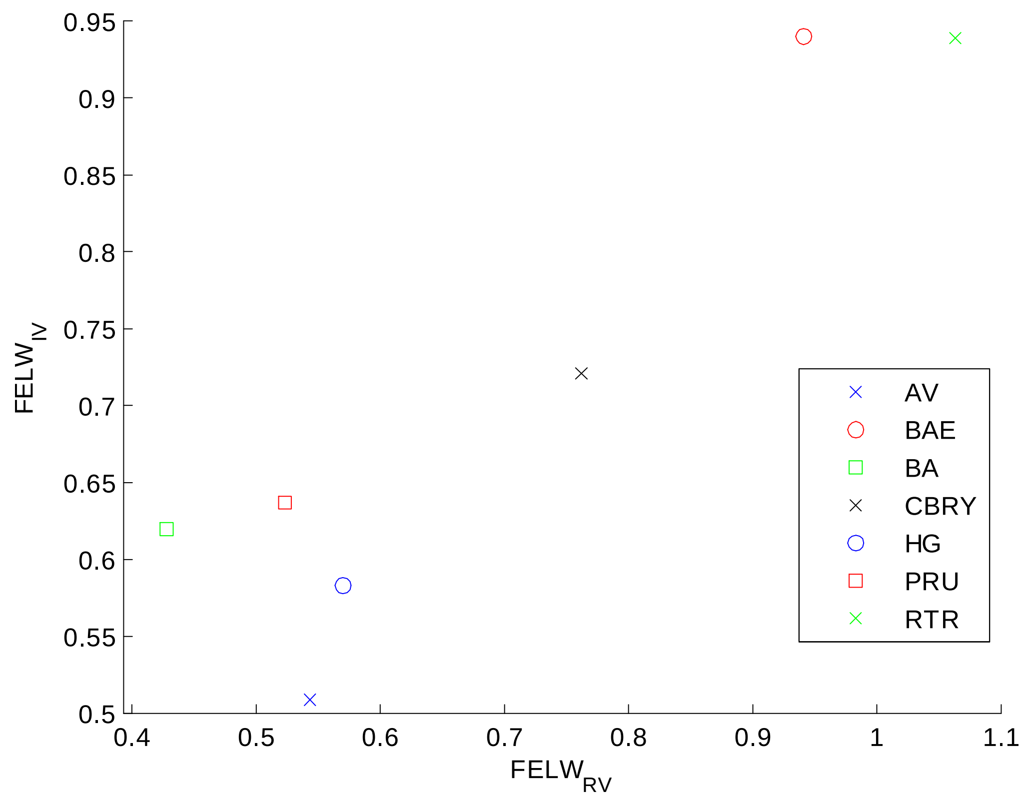


Figure 3.1: Scatterplot of feasible exact local Whittle (FELW) estimates for implied and realised volatility series of FTSE-100 companies. The proximity of the FELW estimate for both the IV and RV series is suggestive of some level of fractional cointegration.

The summary of GPH and FELW estimates contained in Tables 3.6a and 3.6b shows that the implied and realized volatility series for FTSE-100 companies exhibit properties associated with fractional cointegration. It can be seen from the full set of results (Tables 3.2a-3.2d) that parameter estimates are sensitive to the estimation procedure employed as well as the choice of bandwidth. Although the analysis uses the longest continuously available dataset for individual UK equities (1997 - 2003), cointegration analysis benefits from the use of even longer sample periods where possible. The findings justify a more rigorous investigation into fractional cointegration. Fractional cointegration is more formally tested using the methodology developed by Nielsen and Shimotsu (2007). The approach is an extension of the cointegration rank determination procedure proposed by Robinson and Yajima (2002). The use of FELW estimation means that the approach is sufficiently flexible to model both (asymptotically) stationary and nonstationary processes and it assumes that the mean value of the series, X_t is unknown. The consistency of FELW estimation over both the stationary and nonstationary regions of d is important as our earlier results show that both implied and realized volatility of many FTSE-100 companies lie in the nonstationary region. The analysis in this section is focused on the implied and realized volatility series of the seven companies identified in Table 3.4b, namely, Aviva, British Aerospace, British Airways, Cadburys, Hilton, Prudential and Reuters. The volatility series of these companies produce close FELW d estimates that indicate cointegration and warrant a more formal investigation.

The presence of fractional cointegration is examined by first testing the hypothesis of pairwise equality of the persistence parameters, where a and b are the two series under consideration. Following Nielsen and Shimotsu (2007), a t-type test proposed by Robinson and Yajima (2002) is used to tests for parameter equivalence in the implied and realized volatility series. Robinson and Yajima (2002) note that although fractional cointegration requires that the persistence parameters are equivalent for both series, the sensitivity of d estimation suggests greater variability in d should be facilitated. Robinson and Yajima (2002) proceed by redefining d as the $p \times 1$ vector $d = (d_1, ..., d_p)'$ where d_p is the long memory parameter estimated for series p using FELW estimation. Also $\widehat{d} = (\widehat{d}_1, ..., \widehat{d}_p)'$ and $D = diag\{G_{11}, ..., G_{pp}\}$ where G is defined previously in equation (3.14) and G_{ab} is the *(a,b)*th element of G. They propose a consistent multivariate version of the FELW estimator

$$\widehat{G} = \frac{1}{m_1}\sum_{j=1}^{m_1} \mathrm{Re}\left\{\lambda(\lambda_j)^{-1} I_j \lambda(\lambda_j)^{-1*}\right\} \tag{3.17}$$

where m is the bandwidth used, $I_j = I(\lambda_j)$ and $I(\lambda) = w(\lambda)w(\lambda)^*$, $w(\lambda) = (w_1(\lambda), ..., w_p(\lambda)))'$ where $w_p(\lambda)$ is the discrete fourier transform of series p defined in (3.13) above. Robinson and Yajima (2002) show that X_{at} and X_{bt} are cointegrated if $G_{ab}^2 = G_{aa}G_{bb}$, while there is no cointegrating relationship between the series when $G_{ab}^2 < G_{aa}G_{bb}$. This can be expressed in the form of a test statistic as follows,

$$\widehat{T}_{ab} = \frac{m^{1/2}(\widehat{d}_a - \widehat{d}_b)}{\{\frac{1}{2}(1 - \widehat{G}_{ab}/(\widehat{G}_{ab}\widehat{G}_{bb}))\}^{1/2} + h(n)}, \tag{3.18}$$

where $h(n) > 0$ and

$$h(n) + \frac{(\log\, m)m^{1/2+\xi}/n^{\xi} + (\log\, m)^2 m^{-1/6}}{h(n)} \longrightarrow 0 \text{ as } n \longrightarrow \infty. \tag{3.19}$$

If X_{at} and X_{bt} are fractionally cointegrated $\widehat{T}_{ab} \longrightarrow_d 0$ while if no cointegration exists $\widehat{T}_{ab} \longrightarrow_d N(0,1)$. We see that parameter equivalence is supported at bandwidth $m = 13$ and from Table 6, the proximity of the $\widehat{T}_{IV-RV}$ statistic to zero indicates cointegration between the implied and realised volatility series of the companies under consideration.

Determination of the cointegration rank consists of finding estimates of G and its eigenvalues and then determining their limit distribution. From (3.17) above,

$$G(d_*) = \frac{1}{m}\sum_{j=1}^{m}\lambda_j^{2d_*}\,\mathrm{Re}(I_j) \tag{3.20}$$

where d_* is the common value of $d_1, ..., d_p$, and in this paper $p = 2$. In both the multivariate log periodogram method [Robinson (1995)] and the multivariate local Whittle method [Lobato (1995)], G is given as $\widehat{G}(\widehat{d}_*)$. Robinson and Yajima (2002) note that these approaches are only consistent under narrow conditions and the assumption of full rank for G means they are not valid if X_t is cointegrated. The solution proposed by Robinson and Yajima (2002) and applied in this chapter is to pool estimates of d_* based on the individual elements of X_t, using a bandwidth m_1 that increases faster than m. This provides an estimate of d_a, $\widetilde{d}_a$ that uses the bandwidth m_1 instead of m, that gives,

$$\overline{d}_* = \frac{1}{p}\sum_{a=1}^{p}\widetilde{d}_a$$

so that as $n \longrightarrow \infty$ the limit distribution is,

$$m^{1/2} vec(\widehat{G}(\overline{d}_*) - G) \longrightarrow_d N(0, \frac{1}{2}(G \otimes G + (G \otimes G_1, ..., G \otimes G_p))). \tag{3.21}$$

Results are provided for $\widehat{G}(\overline{d}_*)$ as well as the correlation matrix $\widehat{P}(\overline{d}_*) = \widehat{D}(\overline{d}_*)^{-1/2}\widehat{G}(\overline{d}_*)\widehat{D}(\overline{d}_*)^{-1/2}$, where $\widehat{D}(\overline{d}_*)$ is a scale-invariant version of the $\widehat{G}(\overline{d}_*)$ and has the same rank as $\widehat{G}(\overline{d}_*)$. Tables 3.6a to 3.12a present the eigenvalues of $\widehat{G}(\overline{d}_*)$ and $\widehat{P}(\overline{d}_*)$ estimated for bandwidth parameters at $m_1 = [n^{0.55}] = 24$, $m = 32$ and $m_1 = [n^{0.45}] = 13, m = 18$. The proximity of at least some of the eigenvalues of G to zero suggests cointegration. The results of the rank determination analysis for the seven companies under consideration are contained in Tables 3.6b to 3.12b. The results support fractional cointegration at $m_1 = [n^{0.45}] = 13$, $m = 18$ with weaker support evident across bandwidths, $m_1 = [n^{0.55}] = 24$, $m = 32$. Overall there is evidence of only at most only one cointegrating relationship between implied and realized volatility. The existence of a fractionally cointegrating relationship between implied and realized volatility is consistent with similar results for stock indices [Bandi and Perron (2006)]. Kellard, Dunis and Sarantis (2007) tested for fractional cointegration between implied and realized volatilities on a number of foreign exchange rate over the period January 1991 to September 2005. Traded implied volatility obtained from brokers and foreign exchange realized volatility over 172 non-overlapping observations are shown to be fractionally cointegrated. Los (2005) and Budek, Schotman and Tschering (2006) have examined the implications of fractional integration for risk management and long-term portfolio choice. Los (2005) observes that the Value-at-Risk (VAR) measure omits long memory components that are evident in dynamic market pricing. The fractal dynamics observed in the return series of many financial assets means that these assets thus possess an undefined or 'infinite' variance. Los (2005) argues that VaR incorrectly and from the perspective of risk managers, dangerously, relies on a computed standard deviation that has been shown not to converge over time. Budek, Schotman and Tschering (2006) examine the practical implications of persistence in asset returns for long-term choice in a portfolio composed of US equities, Treasury bonds and cash. Using a multi-variate fractionally integrated process, Budek, Schotman and Tschering (2006) show that long-term portfolio weights and as a consequence long-term portfolio risk are highly dependent on the estimation of the long memory parameter. The results produced in this chapter identify the presence of fractionally integrated long memory in UK equity volatility. Long-memory effects in asset volatility as well as the identification of the implied-realized volatility relationship in UK equities as a fractionally cointegrated one also has implications for the practical implementation of derivative strategies not yet considered in the existing literature.

3.4. Summary and Conclusion

This chapter presents a careful examination of the fractional properties of volatility on individual FTSE-100 equities. The results from a univariate analysis of the implied and realized volatility series of sixteen FTSE-100 companies over a number of bandwidths show little evidence of a consistent long memory parameter among the stocks selected. The apparent absence of a strong common factor driving volatility indicated by this finding is in contrast to the results of Ray and Tsay (2000) who found similar levels of volatility persistence among related stocks. Fractional integration in the implied and realized volatility series was tested for possible 'spurious' results induced by structural breaks in volatility. Variation in the persistence parameter was evident among subsamples suggesting that breaks did occur in volatility across the sample period. The Wald statistic which checked for parameter consistency showed that fractional integration could only be rejected in the realized volatility series of Aviva, Hanson and Royal Sun Alliance. Tests for incorrect estimation of the persistence parameter (d) were also carried out using d'th differencing. Application of the Phillips-Perron test (Z_t) also suggested spurious long memory effects in the realized volatility series of British Aerospace, Diageo and Hilton. Although the results indicate the presence of structural breaks in implied and realized equity volatility, the presence of fractional integration could not be rejected.

The final part of this chapter formally examines whether implied and realized volatility on individual equities are fractionally cointegrated processes. Univariate estimates of d on implied and realized volatility as well as the residual series produced by an OLS regression suggest the presence of fractional cointegration in a number of companies. A closer examination is facilitated by applying Robinsons and Yajima's (2002) test statistic for parameter equivalence. The multivariate FELW estimation described by Nielsen and Shimotsu (2007) shows that at least one cointegrating vector can be identified even though the results are sensitive to the choice of bandwidth.

The results presented in this chapter are relevant for forecasting and risk management since levels of persistence are shown to affect long-run predictions of stock returns. The inclusion of long-run dependence can be achieved through the application of appropriate fractionally integrated volatility specifications such as the process developed by Comte, Coutin and Renault (2003). The long-run equilibrium relationship between individual equities and their associate options is worthwhile exploring in greater detail with more extensive datasets. Recent research has pointed to the profitability of derivative strategies [Santa-Clara and Saretto (2006), Doran and Fodor (2006), Branger, Breuer, Schlag (2006)]. Budek, Schotman and Tschering (2006) have shown the importance of long-memory effects in selecting the optimal long-term portfolio. The evidence in this chapter supporting fractional integration is equity volatility compliments the recent research in portfolio optimisation and derivative strategies.

3.5. Appendix B: Supporting Material for Chapter 3

Table 3.1. Descriptive Statistics for Implied and Realized Volatility Series.

Stock	Implied Volatility				Realised Volatility			
Name	Mean	Std Dev	Kurt.	Skew.	Mean	Std Dev	Kurt.	Skew
Aviva (av)	39.3	10.17	4.51	1.32	33.6	11.14	4.08	1.27
BritishAerospace (bae)	40.5	8.72	5.94	1.30	35.9	8.77	4.58	1.29
BritishAirways (ba)	47.5	14.28	2.92	0.86	41.8	15.48	3.40	1.00
Cadburys (cbry)	30.6	5.82	2.80	0.09	27.0	5.58	3.34	0.79
Dixons (dxn)	42.8	8.41	2.55	0.36	25.1	6.86	2.57	0.50
Diageo (dge)	32.4	7.95	6.21	1.24	40.4	11.97	2.56	0.46
GlaxoSmithKline (gsk)	34.5	7.14	2.66	0.46	23.9	6.11	3.06	0.79
Hilton (hg)	39.0	9.45	2.85	0.49	41.4	8.50	2.24	0.29
Hanson (hns)	35.4	6.71	3.66	0.73	41.2	14.27	3.80	1.03
HSBC (hsbc)	35.3	9.57	2.61	0.29	21.2	5.50	4.45	0.93
Kingfisher (kgf)	37.4	7.04	2.56	0.25	34.1	7.77	2.85	0.87
Lonmin (lnr)	33.7	4.25	2.84	0.60	33.4	5.52	2.62	-0.00
Marks&Spencers (mks)	35.8	8.91	3.87	0.88	31.2	7.42	2.25	0.32
Prudential (pru)	40.1	9.52	5.67	1.51	32.7	11.07	3.23	1.21
RoyalSunAlliance (rsa)	44.9	13.64	6.07	1.68	41.6	16.44	3.52	1.23
Reuters (rtr)	49.3	9.36	2.65	0.39	38.5	12.07	2.91	0.63
Average	**38.6**	**8.81**	**3.77**	**0.78**	**33.9**	**9.65**	**3.22**	**0.79**

Mean, standard deviation, skewness and kurtosis for implied and realized volatility series for FTSE-100 firms from October 1997 to December 2003. The data consists of 75 monthly nonoverlapping observations. The skewness and kurtosis are computed as $\frac{1}{n-1}\sum t_\theta^3$ and $\frac{1}{n-1}\sum t_\theta^4$ respectively, after studentizing the relevant quantity, θ, say, as $t_\theta = \frac{(\theta_t - \bar{\theta})}{\sigma(\theta)}$, where $\sigma(\theta)$ is the standard deviation of θ.

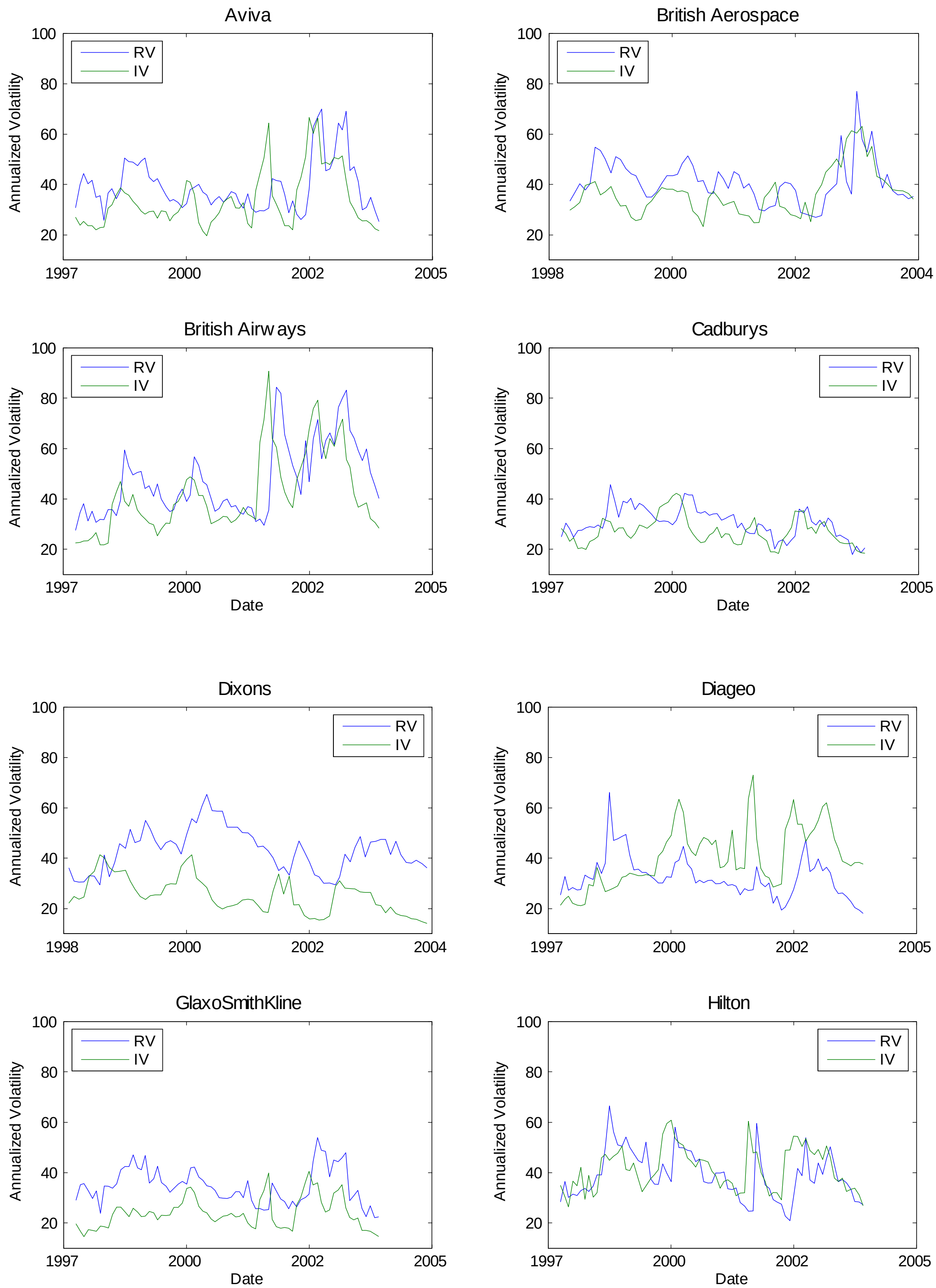


Figure 3.1a Implied (IV) and realized volatility (RV). Each time series consists of 75 non-overlapping observations generated using end-of-day equity options data (implied) and intraday tick data (realized).

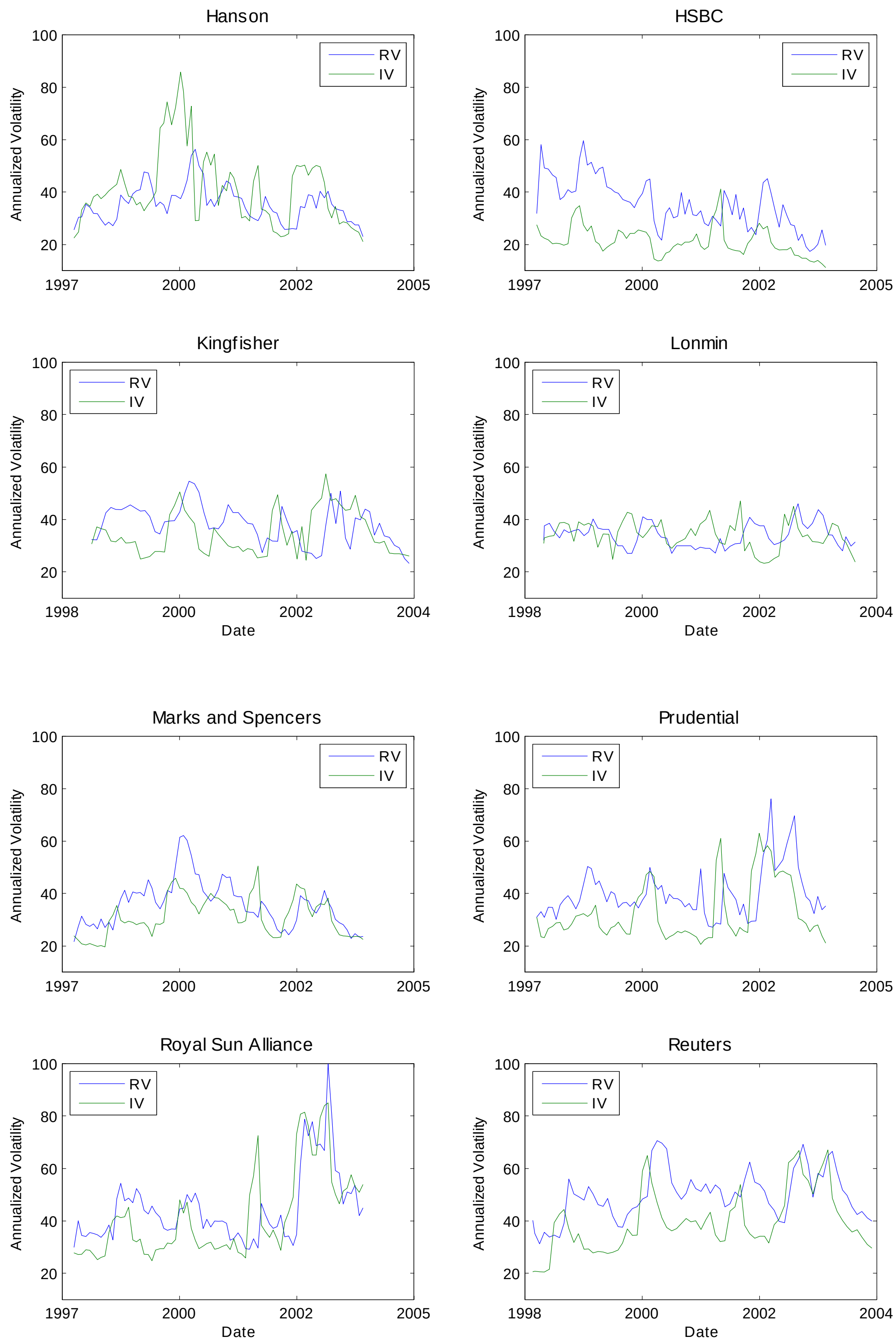
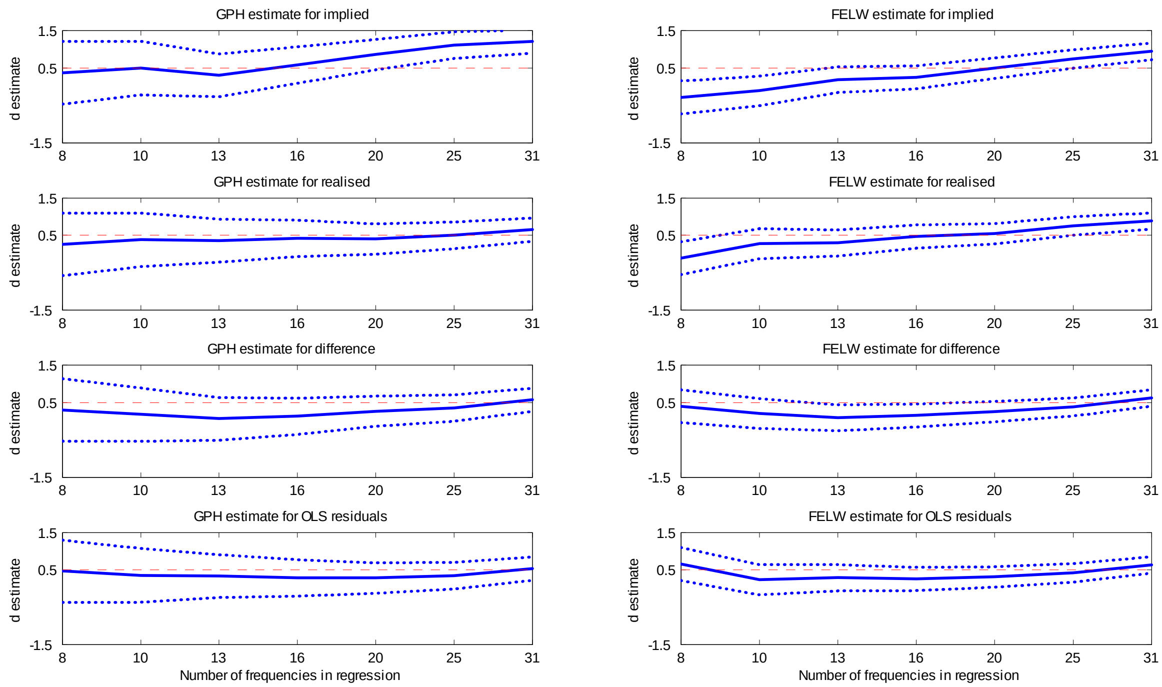
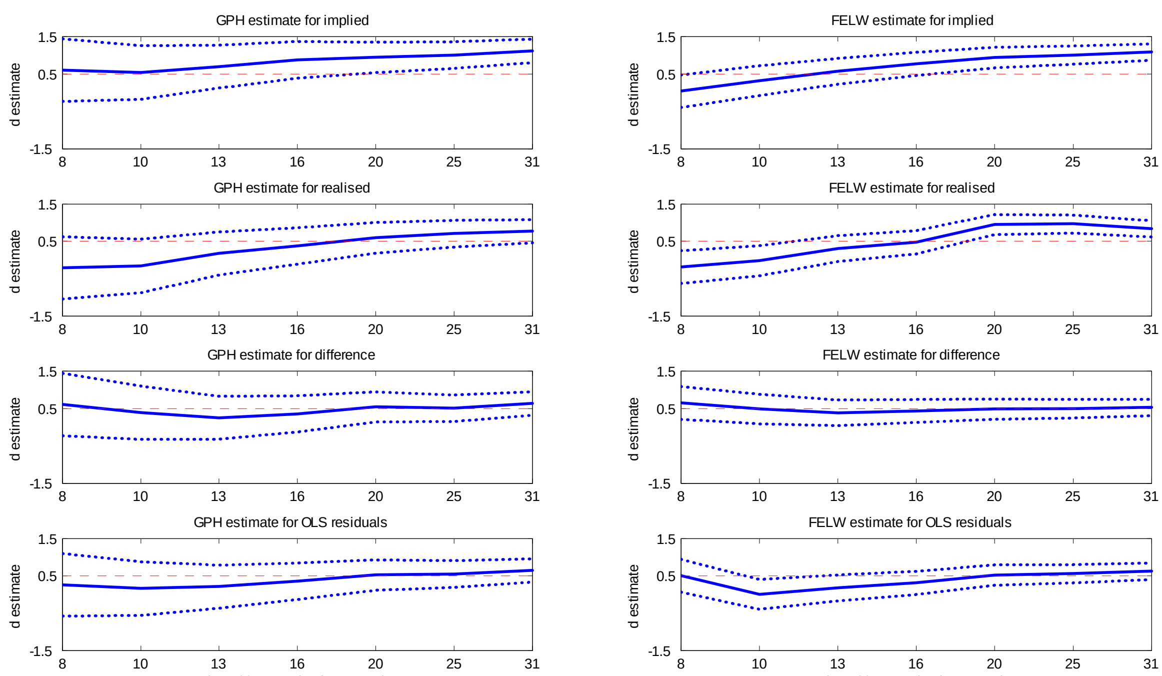


Figure 3.1b Implied (IV) and realized volatility (RV). Each time series consists of 75 non-overlapping observations generated using end-of-day equity options data (implied) and intraday tick data (realized).

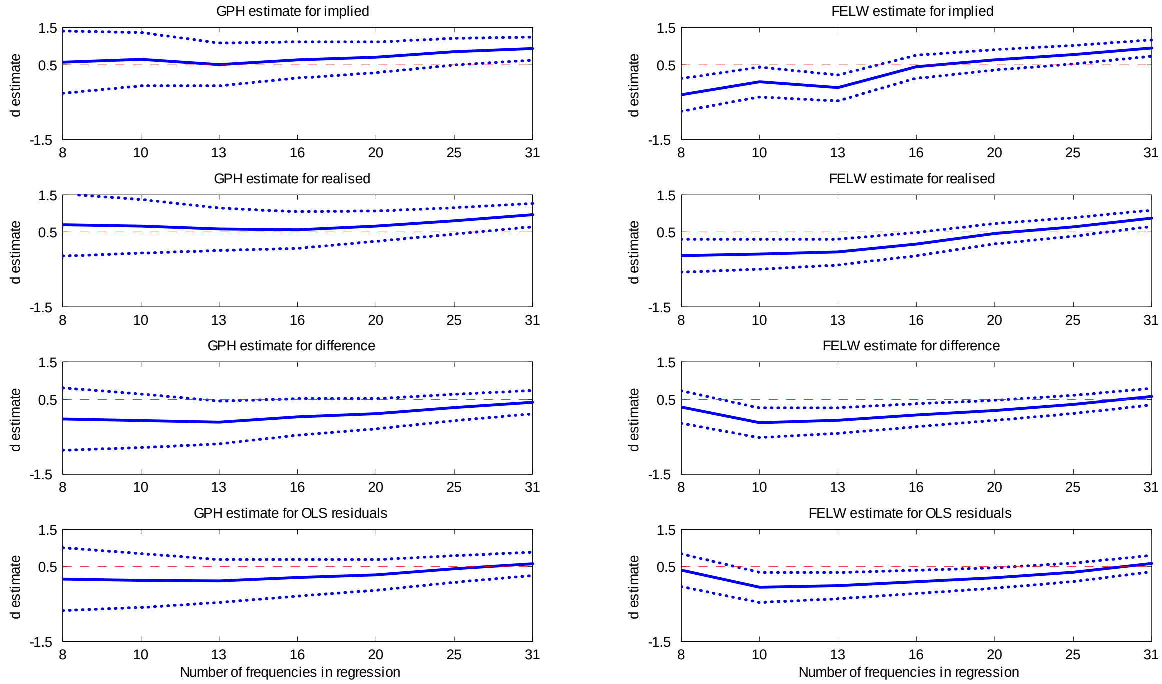


Aviva

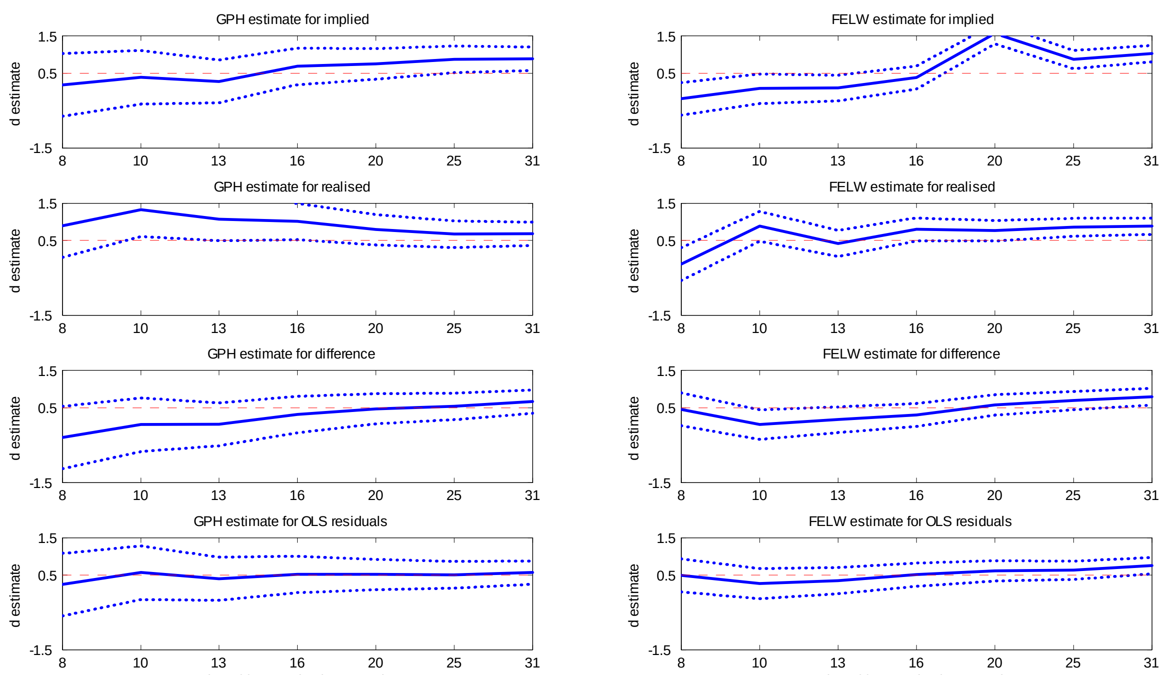


British Aerospace

Figure 3.2a Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

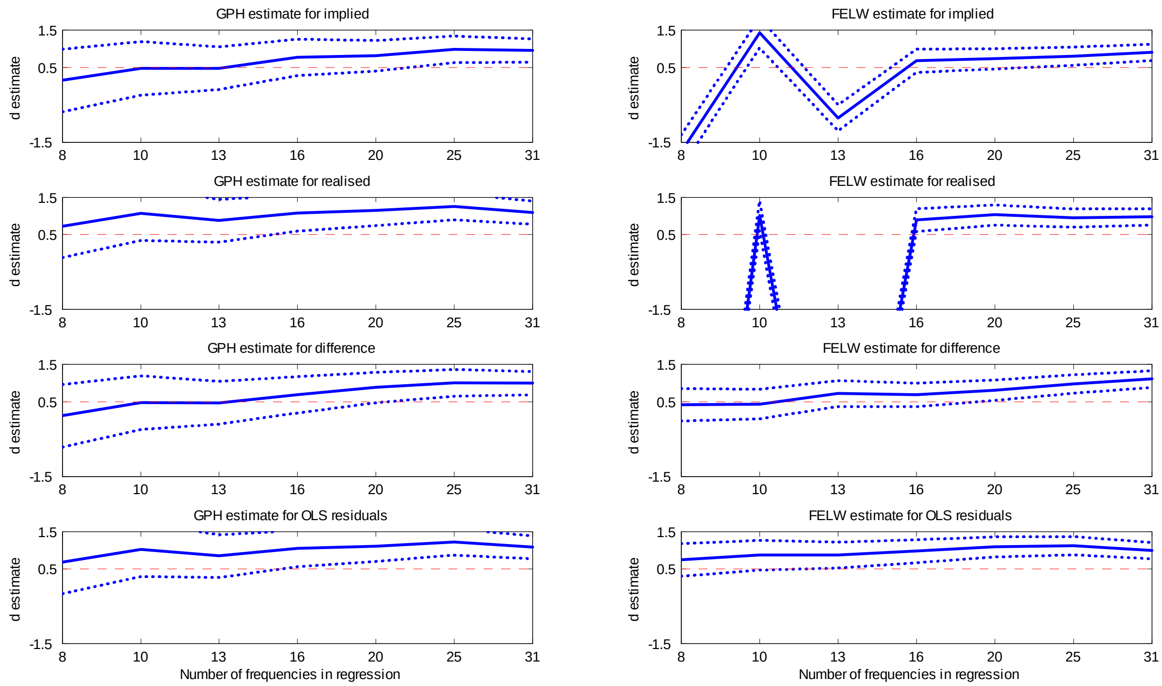


British Airways

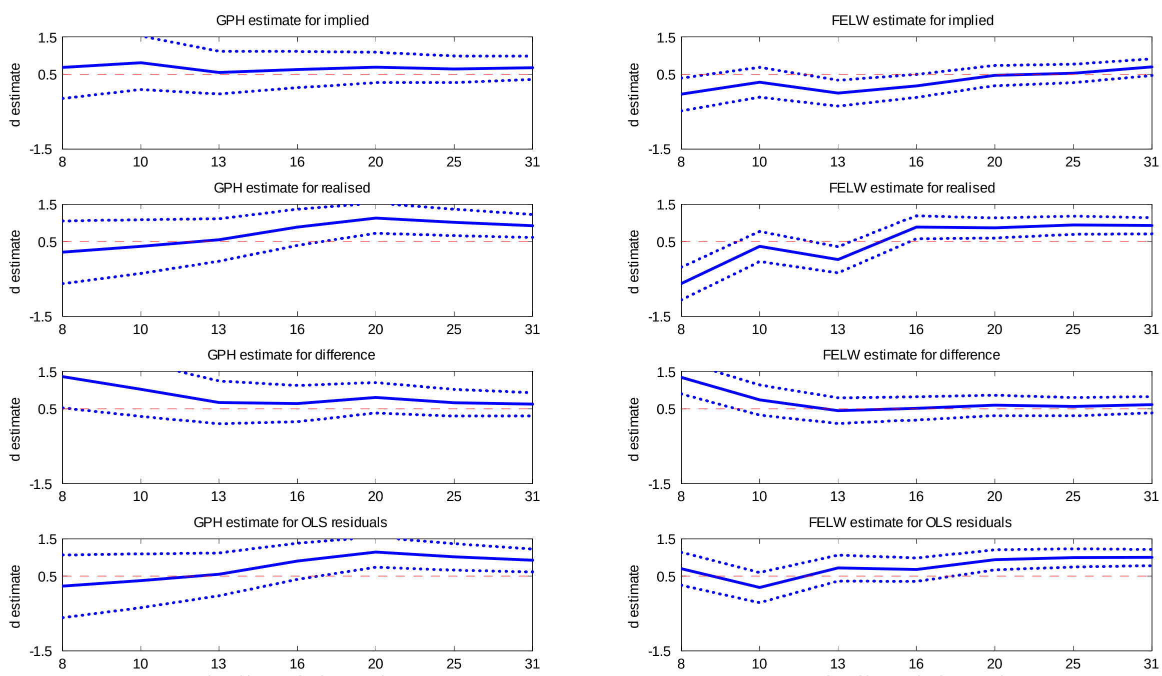


Cadburys

Figure 3.2b Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

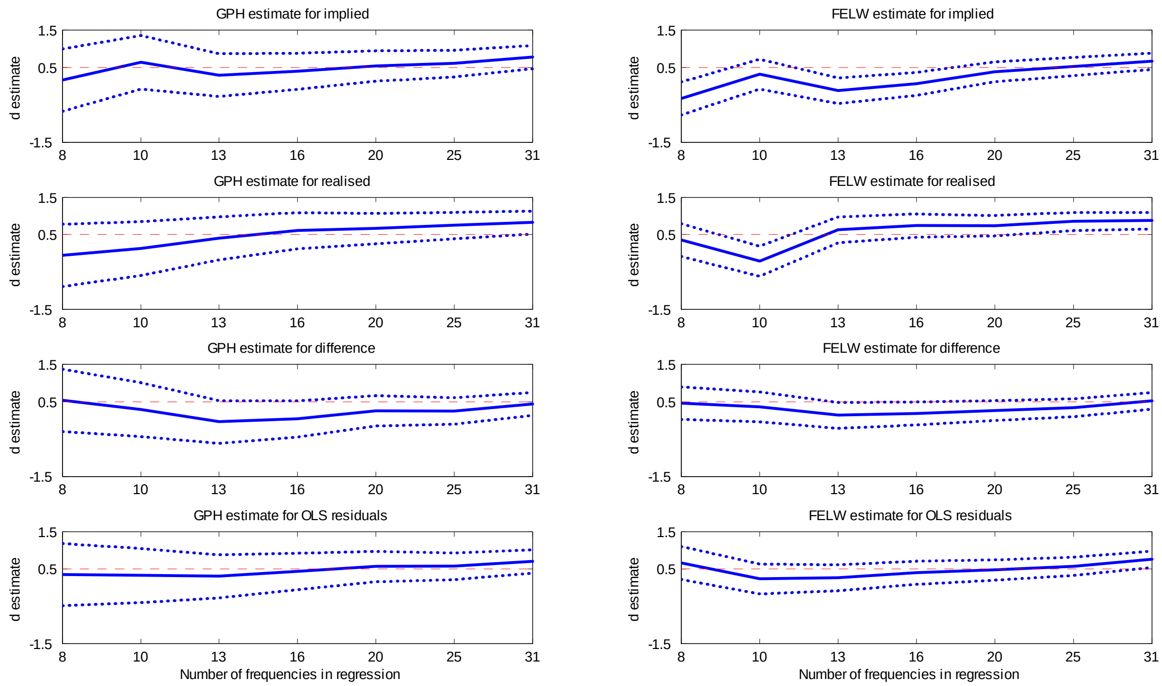


Diageo

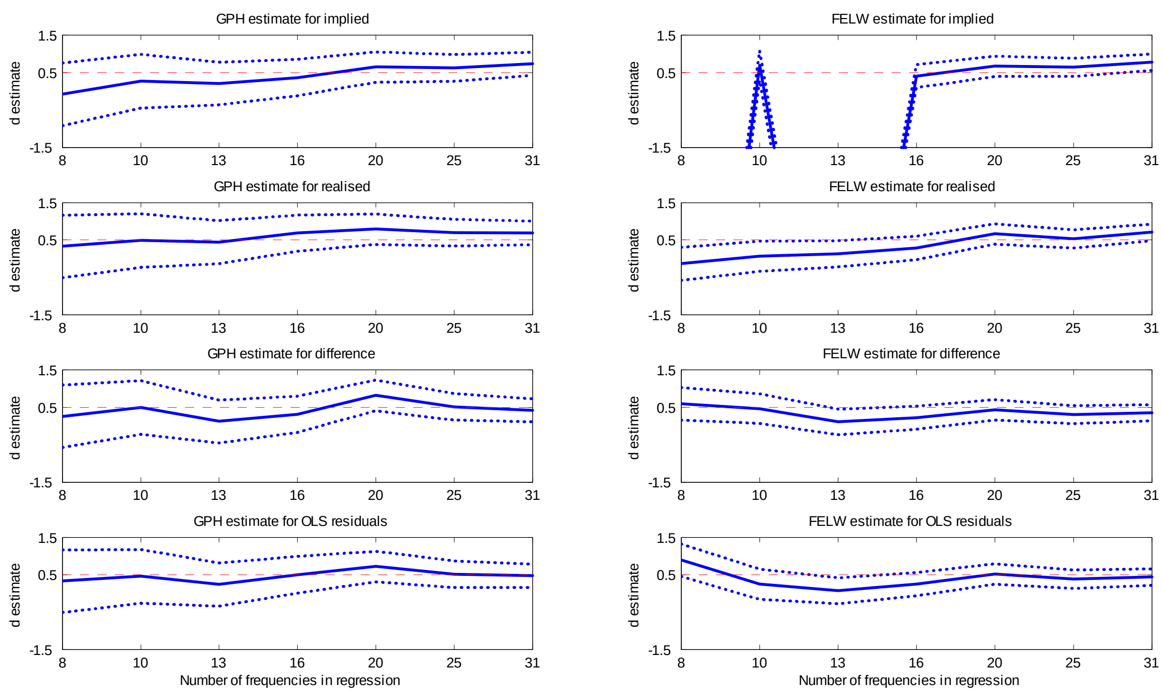


Dixons

Figure 3.2c Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

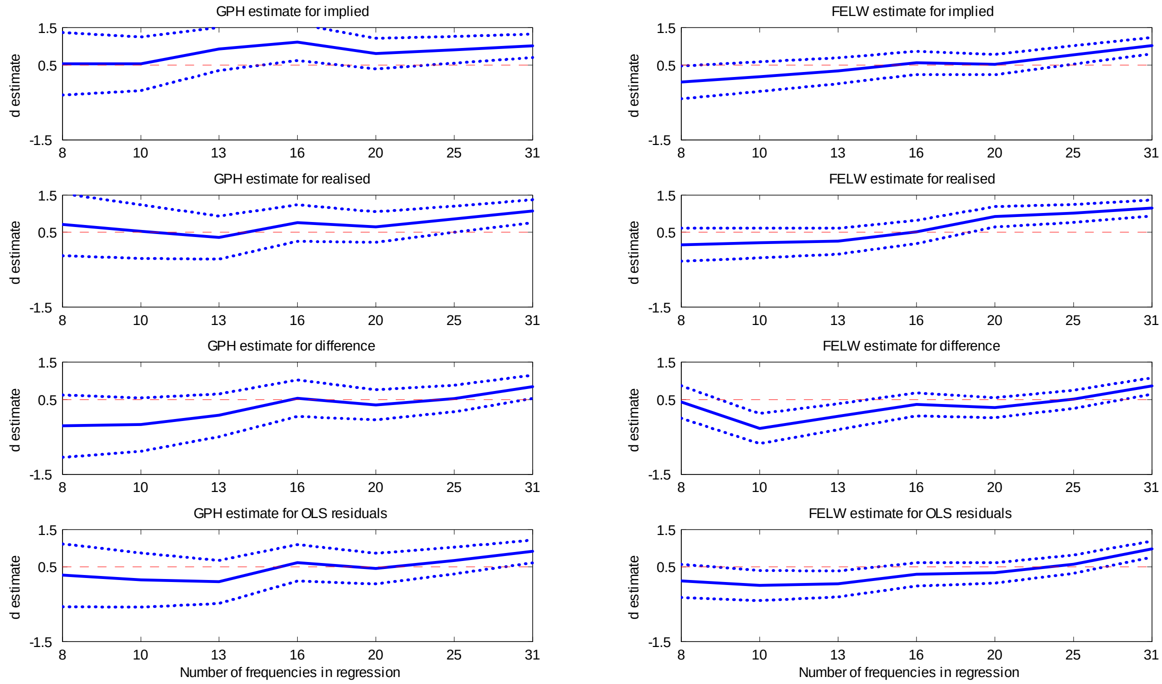


GlaxoSmithKline

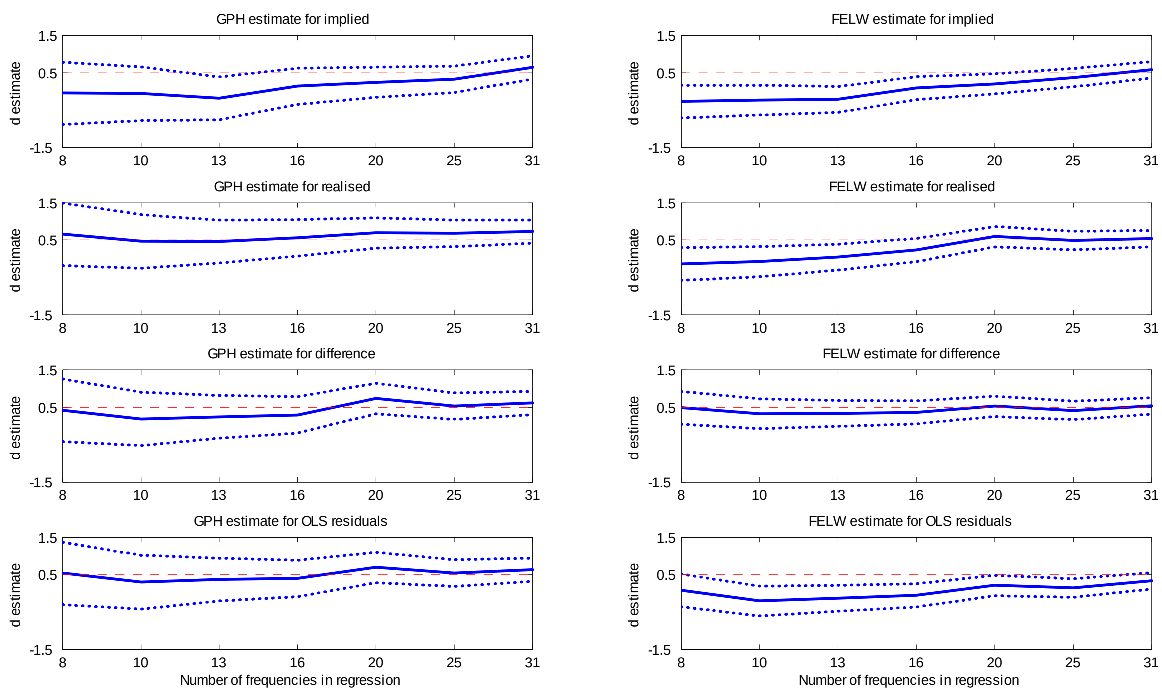


Hilton

Figure 3.2d Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

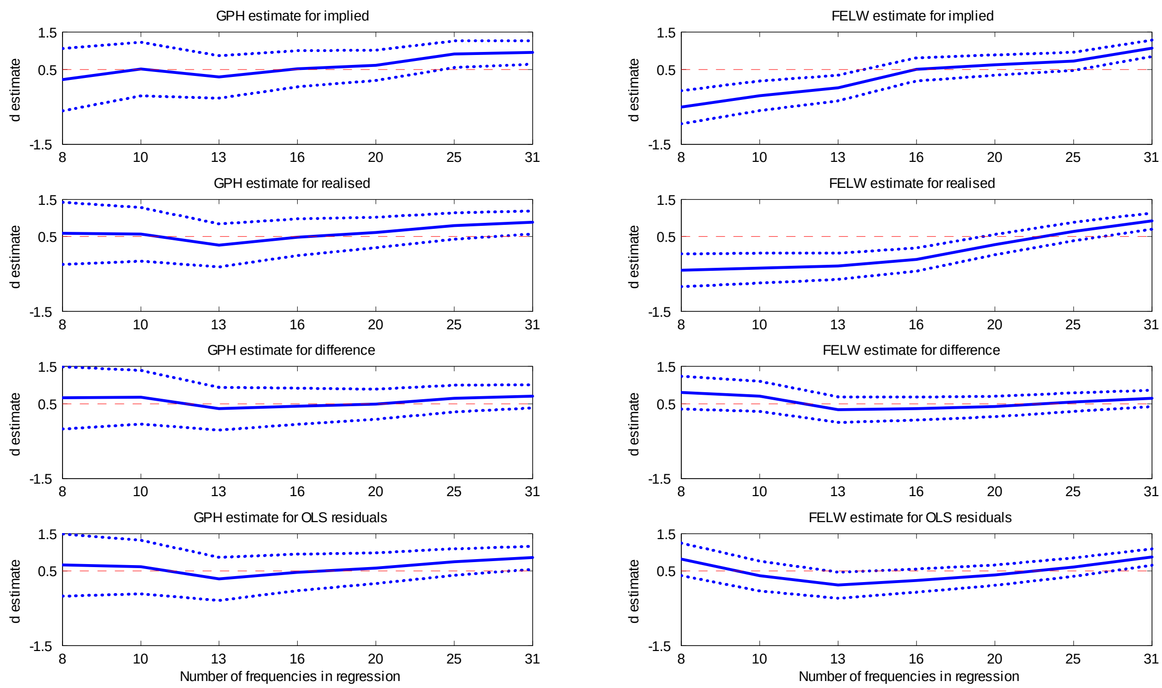


Hanson

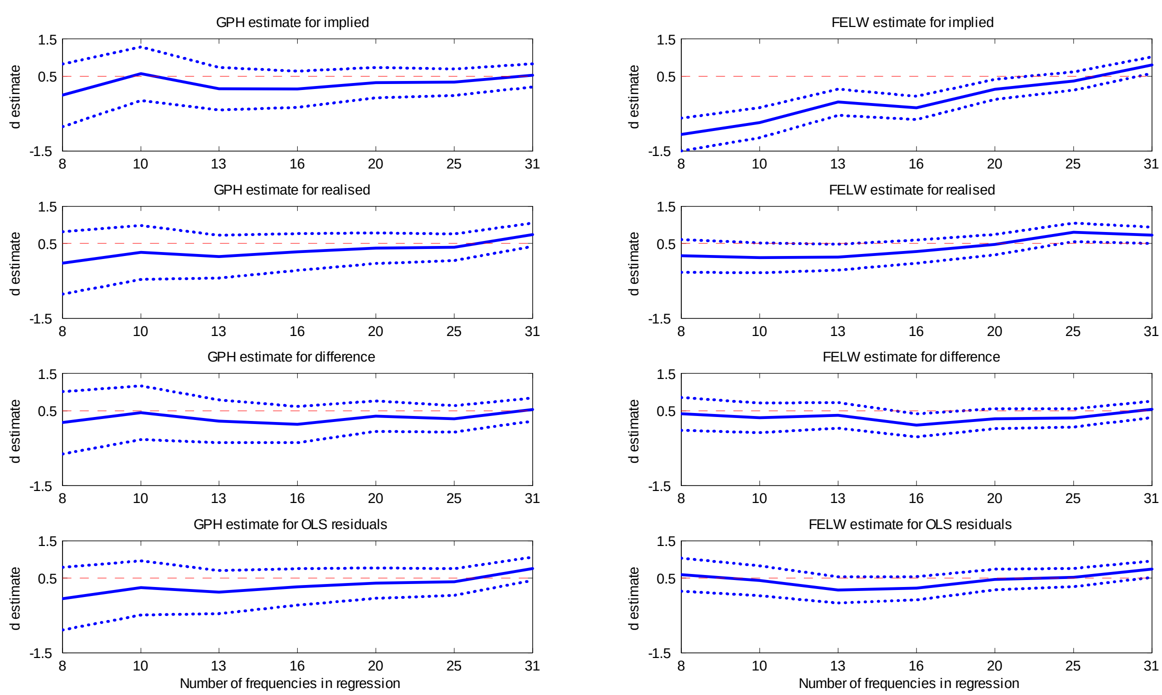


HSBC

Figure 3.2e Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

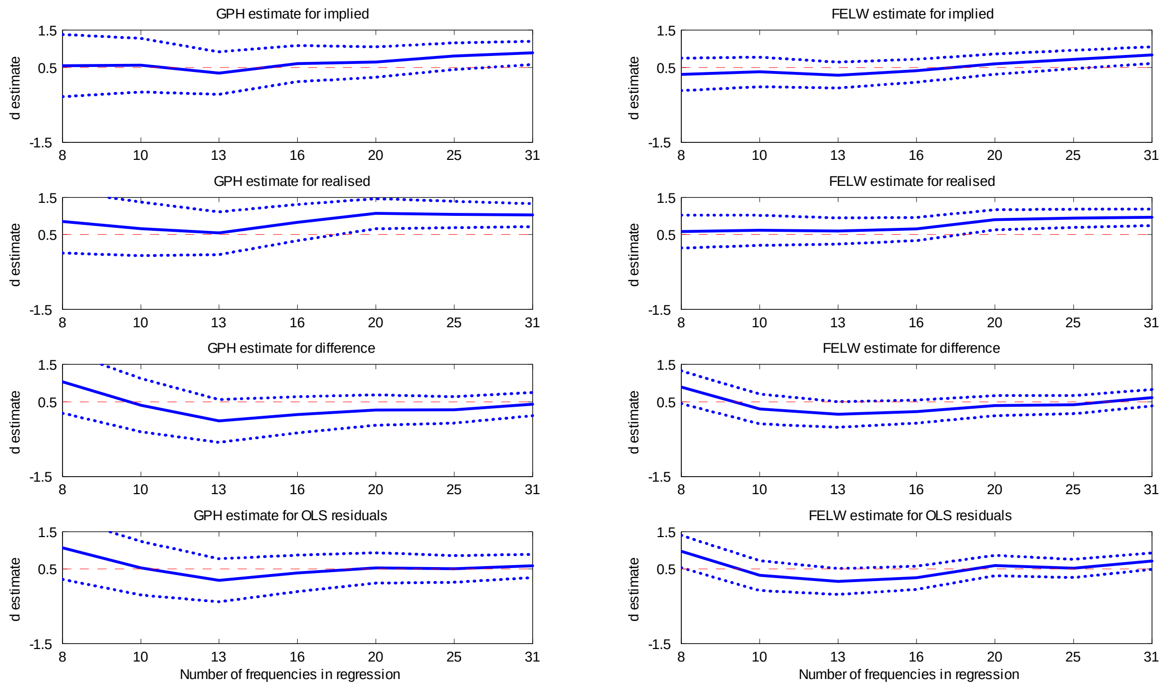


Kingfisher

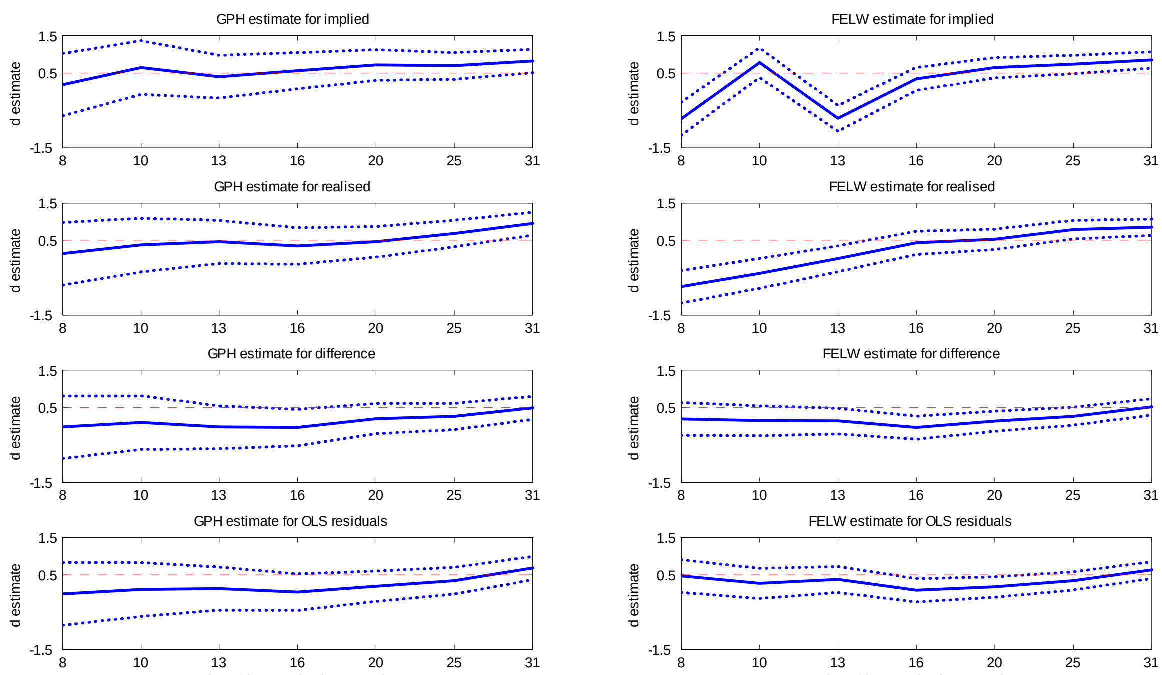


Lonmin

Figure 3.2f Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

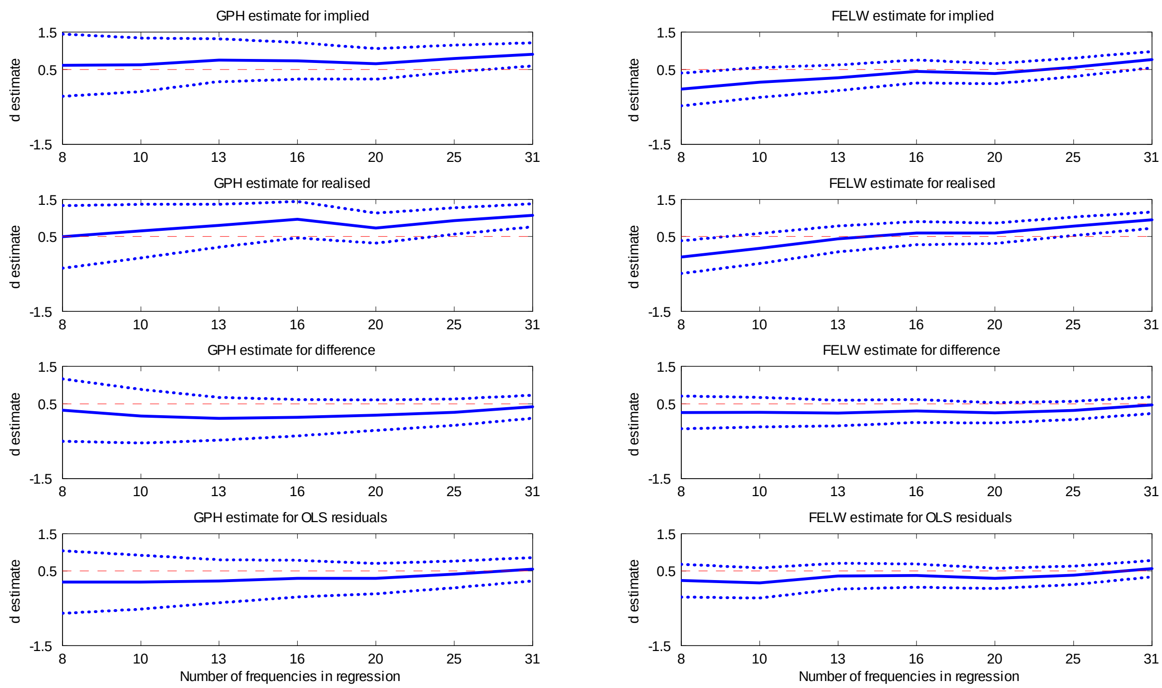


Marks and Spencers

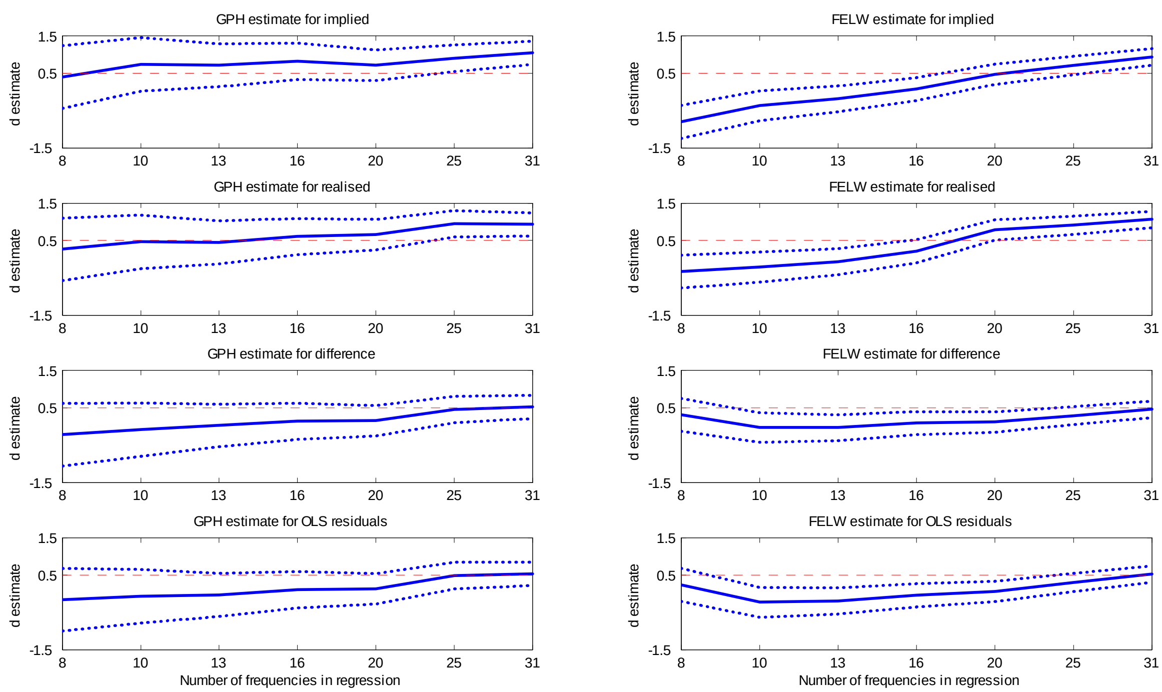


Prudential

Figure 3.2g Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.



Royal Sun Alliance



Reuters

Figure 3.2h Long memory parameter estimates and standard error bands as a function of the number of frequencies. Parameter estimates produced by log-periodogram (LP) and feasible exact local Whittle (FELW) approach.

m	$\widehat{d}$ (implied)		$\widehat{d}$ (realised)		$\widehat{d}$ (residuals)		$\widehat{d}$ (differences)	
	LP	FELW	LP	FELW	LP	FELW	LP	FELW
				Aviva				
$[n^{0.50}] = 9$	0.308	-0.082	0.357	0.442	0.336	0.180	0.069	0.096
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.862	0.509	0.401	0.543	0.284	0.314	0.272	0.260
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	1.209	0.943	0.651	0.884	0.535	0.639	0.575	0.634
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
				British Aerospace				
$[n^{0.50}] = 9$	0.699	0.719	0.178	0.396	0.212	0.259	0.255	0.387
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.947	0.940	0.595	0.941	0.525	0.544	0.548	0.489
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	1.121	1.093	0.776	0.836	0.647	0.633	0.642	0.540
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
				British Airways				
$[n^{0.50}] = 9$	0.507	-0.1363	0.580	-0.032	0.117	0.202	-0.113	-0.066
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.703	0.620	0.661	0.428	0.278	0.324	0.116	0.207
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.937	0.942	0.958	0.864	0.579	0.606	0.426	0.578
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
				Cadburys				
$[n^{0.50}] = 9$	0.282	0.106	1.068	0.499	0.404	0.462	0.060	0.033
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.754	0.721	0.792	0.762	0.518	0.715	0.475	0.538
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.889	1.030	0.683	0.884	0.570	0.788	0.671	0.797
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)

Table 3.2a. Long memory parameter estimates for implied and realized volatility of Aviva, British Aerospace, British Airways and Cadburys. Estimates are produced using the LP and FELW Estimator across a number of frequencies.We also show estimates for the residual series produced by the residual series from an OLS regression and for the differences RV-IV. Standard errors are in parenthesis.

m	$\widehat{d}$ (implied)		$\widehat{d}$ (realised)		$\widehat{d}$ (residuals)		$\widehat{d}$ (differences)	
	LP	FELW	LP	FELW	LP	FELW	LP	FELW
Dixons								
$[n^{0.50}] = 9$	0.482	-0.447	0.875	-5.322	0.850	0.785	0.474	0.494
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.814	0.736	1.150	1.027	1.112	1.094	0.887	0.814
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.954	0.907	1.093	0.973	1.081	0.993	1.003	1.113
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Diageo								
$[n^{0.50}] = 9$	0.546	-0.007	0.544	0.189	0.550	0.646	0.673	0.452
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.687	0.465	1.129	0.864	1.144	0.963	0.802	0.601
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.676	0.693	0.920	0.924	0.924	1.006	0.628	0.612
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
GlaxoSmithKline								
$[n^{0.50}] = 9$	0.294	-0.162	0.403	0.554	0.306	0.314	-0.333	0.139
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.542	0.380	0.667	0.738	0.566	0.516	0.259	0.269
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.777	0.664	0.827	0.874	0.705	0.777	0.444	0.531
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Hilton								
$[n^{0.50}] = 9$	0.210	-6.661	0.441	0.126	0.242	0.232	0.123	0.116
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.653	0.583	0.794	0.570	0.722	0.585	0.825	0.434
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.742	0.783	0.691	0.703	0.473	0.475	0.424	0.360
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)

Table 3.2b. Long memory parameter estimates for implied and realized volatility of Dixons, Diageo, GlaxoSmithKline and Hilton. Estimates are produced using the LP and FELW Estimator across a number of frequencies.We also show estimates for the residual series produced by the residual series from an OLS regression and for the differences RV-IV. Standard errors are in parenthesis.

m	$\widehat{d}$ (implied)		$\widehat{d}$ (realised)		$\widehat{d}$ (residuals)		$\widehat{d}$ (differences)	
	LP	FELW	LP	FELW	LP	FELW	LP	FELW
Hanson								
$[n^{0.50}] = 9$	0.931	0.500	0.359	0.261	0.101	0.079	0.081	-0.035
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.810	0.519	0.643	0.836	0.458	0.363	0.363	0.287
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	1.017	0.998	1.065	1.164	0.917	0.989	0.844	0.870
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
HSBC								
$[n^{0.50}] = 9$	-0.181	–0.210	0.463	0.066	0.371	0.428	0.250	0.341
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.247	0.199	0.694	0.511	0.694	0.664	0.741	0.534
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.651	0.570	0.730	0.538	0.632	0.619	0.617	0.544
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Kingfisher								
$[n^{0.50}] = 9$	0.303	0.185	0.269	-0.233	0.288	0.216	0.372	0.346
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.614	0.611	0.611	0.176	0.579	0.436	0.489	0.431
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.960	1.022	0.883	0.863	0.856	0.882	0.702	0.650
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Lonmin								
$[n^{0.50}] = 9$	0.169	–0.628	0.152	0.134	0.129	0.079	0.223	0.084
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.330	0.013	0.376	0.396	0.367	0.449	0.360	0.291
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.528	0.716	0.737	0.726	0.755	0.735	0.537	0.542
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)

Table 3.2c. Long memory parameter estimates for implied and realized volatility of Hanson, HSBC, Kingfisher and Lonmin. Estimates are produced using the LP and FELW Estimator across a number of frequencies.We also show estimates for the residual series produced by the residual series from an OLS regression and for the differences RV-IV. Standard errors are in parenthesis.

m	$\widehat{d}$ (implied)		$\widehat{d}$ (realised)		$\widehat{d}$ (residuals)		$\widehat{d}$ (differences)	
	LP	FELW	LP	FELW	LP	FELW	LP	FELW
Marks and Spencers								
$[n^{0.50}] = 9$	0.351	0.306	0.543	0.597	0.196	0.308	-0.012	0.163
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.651	0.593	1.065	0.941	0.531	0.561	0.283	0.378
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.892	0.835	1.024	0.964	0.583	0.757	0.439	0.611
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Prudential								
$[n^{0.50}] = 9$	0.402	0.104	0.457	0.226	0.135	0.229	-0.218	-0.039
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.718	0.637	0.461	0.523	0.198	0.217	0.207	0.136
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.820	0.849	0.950	0.845	0.679	0.626	0.493	0.522
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Royal Sun Alliance								
$[n^{0.50}] = 9$	0.750	0.287	0.798	0.546	0.228	0.252	0.107	0.256
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.653	0.391	0.731	0.590	0.296	0.305	0.197	0.260
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	0.908	0.766	1.076	0.944	0.550	0.559	0.423	0.219
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)
Reuters								
$[n^{0.50}] = 9$	0.402	0.104	0.457	0.226	0.135	0.229	-0.218	-0.039
	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)	(0.573)	(0.346)
$[n^{0.60}] = 14$	0.718	0.637	0.461	0.523	0.198	0.217	0.207	0.136
	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)	(0.406)	(0.271)
$[n^{0.70}] = 23$	1.052	0.939	0.934	1.063	0.536	0.527	0.527	0.463
	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)	(0.310)	(0.219)

Table 3.2d. Long memory parameter estimates for implied and realized volatility of Marks and Spencers, Prudential, RoyalSunAlliance and Reuters. Estimates are produced using the LP and FELW Estimator across a number of frequencies.We also show estimates for the residual series produced by the residual series from an OLS regression and for the differences RV-IV. Standard errors are in parenthesis.

Ticker	$\widehat{d}$	$\overline{d}(b=2)$	W_c $(b=2)$	Z_t	$\widehat{\eta}_t$
av	0.483 (0.271)	1.257 (0.271)	**6.558**	-1.932 (-2.841)	0.071 (0.435)
bae	0.870 (0.271)	1.170 (0.271)	2.799	**-3.177 (-2.850)**	0.075 (0.462)
ba	0.569 (0.271)	0.860 (0.271)	0.001	-0.958 (-2.792)	0.143 (0.423)
cbry	0.846 (0.271)	0.850 (0.271)	0.652	-2.087 (-2.852)	0.259 (0.462)
dxn	1.130 (0.271)	1.546 (0.271)	0.882	-2.808 (-2.850)	0.075 (0.460)
dge	0.998 (0.271)	1.478 (0.271)	0.649	**-3.021 (-2.849)**	0.109 (0.460)
gsk	0.830 (0.271)	1.133 (0.271)	0.290	-2.575 (-2.853)	0.140 (0.462)
hg	0.765 (0.271)	1.044 (0.271)	3.769	**-3.009 (-2.849)**	0.151 (0.458)
hns	0.627 (0.271)	1.296 (0.271)	**7.938**	-2.186 (-2.788)	0.203 (0.428)
hsbc	0.643 (0.271)	0.845 (0.271)	0.820	-1.338 (-2.797)	0.254 (0.432)
kgf	0.411 (0.271)	0.758 (0.271)	2.048	-1.530 (-2.888)	0.161 (0.439)
lnr	0.459 (0.271)	1.081 (0.271)	1.489	-2.184 (-2.872)	0.055 (0.437)
mks	0.961 (0.271)	1.132 (0.271)	0.059	-2.326 (-2.849)	0.206 (0.460)
pru	0.417 (0.271)	0.715 (0.271)	0.325	-1.696 (-2.885)	0.108 (0.439)
rsa	0.640 (0.271)	1.525 (0.271)	**10.094**	-1.339 (-2.795)	0.076 (0.432)
rtr	0.509 (0.271)	0.703 (0.271)	1.125	-1.600 (-2.841)	0.103 (0.433)

Table 3.3a. Implied Volatility (UK Equity). Test results for fractional dynamics. Full sample estimates of d are denoted as $\widehat{d}$. Estimates of d when the number of subsample estimates, $b = 2$ are given by $\overline{d}$. Estimation in both cases used a bandwidth $m = n^{0.60}$.Results are also provided for the Phillips-Perron test (Z_t) and the KPSS test ($\widehat{\eta}_\mu$) with their associated critical values at the 5% level are given in parenthesis. The Wald statistic is evaluated against the Chi distribution at 5% level ($\chi^2_{0.95}(1) = 3.84$).

Ticker	$\widehat{d}$	$\overline{d}(b=2)$	W_c $(b=2)$	Z_t	$\widehat{\eta}_t$
av	0.586 (0.271)	0.860 (0.271)	0.477	-1.751 (-2.777)	0.075 (0.421)
bae	0.913 (0.271)	1.060 (0.271)	2.207	-2.097 (-2.849)	0.053 (0.461)
ba	0.634 (0.271)	1.056 (0.271)	0.763	-1.339 (-2.791)	0.080 (0.430)
cbry	0.848 (0.271)	1.757 (0.271)	12.049	-2.219 (-2.853)	0.072 (0.462)
dxn	0.829 (0.271)	1.184(0.271)	0.667	-2.037 (-2.853)	0.129 (0.462)
dge	0.564 (0.271)	1.082 (0.271)	1.764	-1.241 (-2.795)	0.095 (0.424)
gsk	0.423 (0.271)	0.973 (0.271)	1.930	-1.970 (-2.881)	0.079 (0.438)
hg	0.708 (0.271)	1.487 (0.271)	0.002	-2.356 (-2.819)	0.094 (0.442)
hns	0.665 (0.271)	1.050 (0.271)	0.000	-2.335 (-2.813)	0.242 (0.440)
hsbc	0.400 (0.271)	0.786 (0.271)	0.819	-1.437 (-2.892)	0.208 (0.440)
kgf	0.618 (0.271)	1.375 (0.271)	0.066	-2.069 (-2.779)	0.066 (0.424)
lnr	0.283 (0.271)	0.696 (0.271)	2.960	-1.873 (-2.933)	0.169 (0.446)
mks	0.597 (0.271)	0.799 (0.271)	0.749	-1.687 (-2.769)	0.119 (0.419)
pru	0.631 (0.271)	1.039 (0.271)	0.801	-2.013 (-2.790)	0.061 (0.429)
rsa	0.573 (0.271)	0.796 (0.271)	0.721	-0.637 (-2.788)	0.223 (0.423)
rtr	0.577 (0.271)	0.695 (0.271)	0.023	-1.264 (-2.785)	0.089 (0.422)

Table 3.3b. Realized Volatility (UK Equity). Test results for fractional dynamics. Full sample estimates of d are denoted as $\widehat{d}$. Estimates of d when the number of subsample estimates, $b = 2$ are given by $\overline{d}$. Estimation in both cases used a bandwidth $m = n^{0.60}$.Results are also provided for the Phillips-Perron test (Z_t) and the KPSS test ($\widehat{\eta}_{\mu}$) with their associated critical values at the 5% level are given in parenthesis. The Wald statistic is evaluated against the Chi distribution at 5% level ($\chi^2_{0.95}(1) = 3.84$).

	$\widehat{d}$ *(implied)*	$\widehat{d}$ *(realised)*	$\widehat{d}$ *(residuals)*
Lonmin (lnr)	0.169	0.152	0.129
	(0.573)	*(0.573)*	*(0.573)*
Prudential (mks)	0.402	0.457	0.135
	(0.573)	*(0.573)*	*(0.573)*
Reuters (rtr)	0.402	0.457	0.135
	(0.573)	*(0.573)*	*(0.573)*

Table 3.4a Summary of GPH parameter estimates that suggest fractional cointegration between implied and realized volatility. Parameter values and standard errors are estimated across a bandwidth $m = [n^{0.50}]$.

	$\widehat{d}$ *(implied)*	$\widehat{d}$ *(realised)*	$\widehat{d}$ *(residuals)*
Aviva (av)	0.509	0.543	0.314
	(0.271)	*(0.271)*	*(0.271)*
British Aerospace (bae)	0.940	0.941	0.544
	(0.271)	*(0.271)*	*(0.271)*
British Airways (ba)	0.620	0.428	0.324
	(0.573)	*(0.573)*	*(0.573)*
Cadburys (cbry)	0.721	0.762	0.715
	(0.271)	(0.271)	(0.271)
Hilton (hg)	0.583	0.570	0.585
	(0.271)	(0.271)	(0.271)
Prudential (pru)	0.637	0.523	0.217
	(0.271)	(0.271)	(0.271)
Reuters (rtr)	0.637	0.523	0.217
	(0.271)	(0.271)	(0.271)

Table 3.4b Summary of FELW parameter estimates that suggest fractional cointegration between implied and realized volatility. Parameter values and standard errors are estimated across a bandwidth $m = [n^{0.50}]$.

	$\widehat{T}_{IV-RV}$
Aviva (av)	0.2011
British Aerospace (bae)	0.0843
British Airways (ba)	0.1583
Cadburys (cbry)	0.0131
Hilton (hg)	0.1545
Prudential (pru)	0.8519
Reuters (rtr)	0.0987

Table 3.5. $\widehat{T}_{IV-RV}(\widehat{T}_{ab})$ Estimates of parameter equivalence.

Aviva: Estimated eigenvalues of $10,000 \times \widehat{G}(\overline{d}_*)$ and $\widehat{P}(\overline{d}_*)$ for implied and realized volatility. Based on $\overline{d}_* = (\widehat{d}_{IV} + \widehat{d}_{RV})/2$.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\overline{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	2.2034	1.2713
$m_1 = [n^{0.45}] = 13, m = 18$	5.4181	0.8459
Eigenvalues of $\widehat{P}(\overline{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.2622	0.7378
$m_1 = [n^{0.45}] = 13, m = 18$	1.6968	0.3032

Table 3.6a

Aviva: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

$L(u)$	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-0.9074	-0.8156	-0.6999	-0.5543	-0.3710
$\widehat{r}$	0	0	0	0	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.2502	-1.1626	-1.0578	-0.9324	-0.7824
$\widehat{r}$	0	1	1	1	1

Table 3.6b

British Aerospace: Estimated eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$ and $\widehat{P}(\bar{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	6.3894	2.5225
$m_1 = [n^{0.45}] = 13, m = 18$	2.2737	0.5492
Eigenvalues of $\widehat{P}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.4073	0.5927
$m_1 = [n^{0.45}] = 13, m = 18$	1.5416	0.4584

Table 3.7a

British Aerospace: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

L(u)	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-1.0525	-0.9606	-0.8449	-0.6993	-0.5160
$\widehat{r}$	0	0	0	1	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.0951	-1.0075	-0.9026	-0.7773	-0.6273
$\widehat{r}$	0	1	1	1	1

Table 3.7b

British Airways: Estimated eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$ and $\widehat{P}(\bar{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	2.8782	1.8637
$m_1 = [n^{0.45}] = 13, m = 18$	5.6707	1.5695
Eigenvalues of $\widehat{P}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.2134	0.7866
$m_1 = [n^{0.45}] = 13, m = 18$	1.5524	0.4476

Table 3.8a

British Airways: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

L(u)	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-0.8585	-0.7667	-0.6510	-0.5054	-0.3221
$\widehat{r}$	0	0	0	0	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.1058	-1.0182	-0.9134	-0.7880	-0.6380
$\widehat{r}$	0	1	1	1	1

Table 3.8b

Cadburys: Estimated eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$ and $\widehat{P}(\bar{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	3.0631	1.9098
$m_1 = [n^{0.45}] = 13, m = 18$	1.1471	0.1750
Eigenvalues of $\widehat{P}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.2134	0.7866
$m_1 = [n^{0.45}] = 13, m = 18$	1.4697	0.5303

Table 3.9a

Cadburys: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

$L(u)$	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-0.7495	-0.6576	-0.5420	-0.3963	-0.2130
$\widehat{r}$	0	0	0	0	0
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.0232	-0.9355	-0.8307	-0.7053	-0.5554
$\widehat{r}$	0	1	1	1	1

Table 3.9b

Hilton: Estimated eigenvalues of $10,000 \times \widehat{G}(\overline{d}_*)$ and $\widehat{P}(\overline{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\overline{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	2.8782	1.8637
$m_1 = [n^{0.45}] = 13, m = 18$	4.9081	0.7105
Eigenvalues of $\widehat{P}(\overline{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.2134	0.7866
$m_1 = [n^{0.45}] = 13, m = 18$	1.7257	0.2743

Table 3.10a

Hilton: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\overline{d}_*)$.

L(u)	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-1.0642	-0.9724	-0.8567	-0.7111	-0.5278
$\widehat{r}$	0	0	0	1	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9371	-0.7221	-0.4714	-0.1714
L(1)	-1.2792	-1.1916	-1.0868	-0.9614	-0.8114
$\widehat{r}$	1	1	1	1	1

Table 3.10b

Prudential: Estimated eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$ and $\widehat{P}(\bar{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	3.0631	1.9098
$m_1 = [n^{0.45}] = 13, m = 18$	4.2093	0.3381
Eigenvalues of $\widehat{P}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.1043	0.8957
$m_1 = [n^{0.45}] = 13, m = 18$	1.7958	0.2042

Table 3.11a

Prudential: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

L(u)	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-0.7697	-0.6779	-0.5622	-0.4166	-0.2333
$\widehat{r}$	0	0	0	0	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.3493	-1.2617	-1.1569	-1.0315	-0.8815
$\widehat{r}$	0	1	1	1	1

Table 3.11b

Reuters: Estimated eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$ and $\widehat{P}(\bar{d}_*)$ for implied and realized volatility.

Bandwidth	$\widehat{\delta}_1$	$\widehat{\delta}_2$
Eigenvalues of $10,000 \times \widehat{G}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	3.0631	1.9098
$m_1 = [n^{0.45}] = 13, m = 18$	3.8051	1.2106
Eigenvalues of $\widehat{P}(\bar{d}_*)$		
$m_1 = [n^{0.55}] = 24, m = 32$	1.1043	0.8957
$m_1 = [n^{0.45}] = 13, m = 18$	1.5172	0.4828

Table 3.12a

Reuters: Rank estimates for implied and realized volatility using the model selection procedure with $\widehat{P}(\bar{d}_*)$.

L(u)	$v(n) = m_1^{-0.45}$	$v(n) = m_1^{-0.35}$	$v(n) = m_1^{-0.25}$	$v(n) = m_1^{-0.15}$	$v(n) = m_1^{-0.05}$
$m_1 = [n^{0.55}] = 24, m = 32$					
L(0)	-1.2904	-1.1066	-0.8753	-0.5841	-0.2175
L(1)	-1.0320	-0.9402	-0.8245	-0.6789	-0.4956
$\widehat{r}$	0	0	0	1	1
$m_1 = [n^{0.45}] = 13, m = 18$					
L(0)	-1.1070	-0.9317	-0.7221	-0.4714	-0.1714
L(1)	-1.1070	-0.9831	-0.8783	-0.7529	-0.6029
$\widehat{r}$	0	1	1	1	1

Table 3.12b

CHAPTER 4

Long Memory Effects in Portfolio Planning

4.1. Introduction

Results from empirical studies on a range of financial assets have demonstrated the presence of long memory effects in asset volatility [Bollerslev and Mikkelsen (1996), Baillie (1996)]. This pattern of persistence or long memory was originally identified as a hyperbolic rate of decay in an asset's autocorrelation structure. More recently, research has demonstrated that these properties can be reproduced by a short-memory model with breaks. This observation has led to a distinction between true long memory produced by fractional integration and spurious long memory that is induced by structural breaks in volatility. Granger and Hyung (2004) show that empirically distinguishing between a true and a spurious long memory process is a difficult task.

The complexity associated with modeling fractional dynamics has meant that long memory is rarely considered in the practical implementation of risk models or portfolio optimisation. Los (2005) and Budek, Schotman and Tschering (2006) are recent studies on long memory effects in risk management and long term asset allocation decisions. This chapter examines portfolio allocation under discrete time rebalancing, focusing on the effect of the underlying volatility process. It looks at portfolio performance when a long memory process is incorrectly assumed to be short memory. A number of recent studies have demonstrated that many financial assets exhibit levels of fractional integration that is not induced by structural breaks [Granger and Hyung (2004), Bandi and Perron (2006), Garvey and Gallagher (2007)]. The support for fractional dynamics found in the academic literature is not as yet reflected in practical implementation of mainstream decision-making in the financial markets. The research in this chapter is motivated by the requirement to bridge the gap between the academic observation of long memory effects and the practical requirements of market participants. This research is part of a renewed interest in dynamic portfolio choice driven by simulation-based methods that allow for more realistic conditions to be considered when finding portfolio solutions.

The theoretical underpinnings of portfolio optimization present a utility maximizing investor who is required to rebalance his portfolio, either continuously or periodically within an investment period. Merton (1969) was the first to note that time-varying investment opportunities produce a hedging demand for the multiperiod investor. The investor's portfolio selection should account for predictable changes in future investment opportunites. Merton's (1969, 1971) work is generally considered to be the starting point for the literature on dynamic portfolio choice while Fama (1970) and later

Dumas and Luciano (1991) and Pliska (1997) attempt to extend this framework to solving multiperiod optimisation problems. The portfolio optimisation literature has developed by addressing the challenges associated with more realistic conditions such as transaction costs and constraining the allocation decision to discrete intervals. Estimating the optimal portfolio where the selection of the current portfolio is dependent on asset returns beyond the initial rebalancing period has proved particularly challenging[1]. Kim and Omberg (1996) provide an analytic solution for a nonmyopic investor in continuous time under a set of restrictive assumptions that include, limiting the investor to investing in one risk-free asset and a risky asset that follows a simple mean-reverting diffusion process. The investor is also restricted to consuming wealth at the end of the investment horizon. Wachter (2002) also generates a closed-form solution to the multiperiod portfolio problem, however the solution provided is similarly restricted to tight parametrizations of the asset return dynamics. The introduction of greater realism into the portfolio choice problem has limited the availability of closed-form solutions and increased the application of methods that provide numerical and approximate solutions.

The numerical approach used by Balduzzi and Lynch (1999) facilitates the inclusion of transaction costs and asset return dynamics similar to those

[1]Nonmyopic portfolio behaviour is observed when the current selection of assets weights takes account of the distribution of asset returns over revision periods beyond the current one.

observed for the U.S. stock market. By discretizing the state space it is possible to maximise the investor's utility backwards from the end of the investment horizon using the Bellman equation, thus providing an optimal solution for a nonmyopic investor. The insights providing by this technique have been similarly used in studies by Brandt (1999), Barberis (2000) and Dammon, Spatt and Zhang (2001).

Studies by Merton, Scholes and Gladstein (1978, 1982) represent early attempts to model the risk and return characteristics of portfolios that include call and put options. More recently, a number of studies have explored dynamic asset allocation for portfolios that include derivative securities. Liu and Pan (2003) directly include derivatives within the dynamic portfolio choice framework set out by Merton (1981). The resulting portfolio provides the investor with exposure to specific characteristics of volatility, such as diffusion risk and jump risk. Asset allocation in a derivative portfolio requires the investor to think in terms of volatility exposures and select options with specific characteristics, such as the moneyness. This can be demonstrated by the exposures available from using at-the-money call options which are sensitive to market volatility and out-of-the-money put options which are sensitive primarily to jump risk. Liu and Pan (2003) provide a closed-form solution for an optimal derivative portfolio in continuous time under an assumption that jumps can only occur in the stock price and their analysis crucially demonstrates the superiority of derivative portfolios compared to portfolios excluding derivatives in terms of certainty equivalent wealth outcomes. Branger, Schlag and Schneider (2008) use numerical techniques combined with simulations to find the continuous-time portfolio solution when jumps occur in both the stock price and in levels of volatility.

The profitability of derivative strategies in the presence of transaction costs and margin calls has been established by Santa-Clara and Saretto (2005), while Doran and Fodor (2006) has shown that combination strategies such as writing covered calls or protective puts using S&P 100 and S&P 500 options can provide positive abnormal performance. Using Jensen's alpha, Doran and Fodor (2006) shows that derivative strategies outperform the underlying index in both bull (1996-2000) and bear (2000-2003) market conditions. The limitation of implementing strategies in discrete time is addressed by Branger, Breuer and Schlag (2006). Deriving optimal strategies in discrete time is shown to provide superior outcomes when ranked against outcomes from so-called naive strategies that are derived in continouous time and implemented at discrete intervals. The numerical and approximation techniques used by Branger, Breuer and Schlag (2006) provide an insight into discrete time implementation of a derivative strategy and allow the examination of borrowing constraints and margin requirements that affect practitioners. The study shows that derivative strategies remain profitable even when rebalancing frequency is limited to monthly intervals.

The numerical and simulation approaches used to provide insights into the characteristics of derivative strategies can be usefully extended to include an examination of the role of volatility persistence. A long memory, fractionally integrated time series $I(d)$ is defined as one where shocks decay hyperbolically over time. It can be distinguished from an $I(0)$ process where shocks die out at an exponential rate and also from an $I(1)$ process where no mean reversion occurs. Using 5-minute returns on both the DM-$ exchange rate and S&P 500 futures, Andersen and Bollerslev (1997) show that long memory is strongly present in realized volatility once repetitive intraday trading patterns are adjusted for. Bollerslev and Mikkelsen (1999) also find evidence of long memory in S&P 500 Index options with maturities between nine months and three years (LEAPS). Evidence of long memory has been challenged by studies which show how regime-shifts in volatility can produce levels of persistence in the autocorrelation structure similar to those produced by true long memory processes. Analysing returns on the S&P 500 Index, Granger and Ding (1996) show that the sample level of d is likely to be induced by structural breaks. Spurious long memory is suggested by significant variations in parameter estimates observed in subperiods. Using similar data, Granger and Hyung (2004) show that a model incorporating structural breaks provides forecasts that are only marginally less competitive than those provided by an $I(d)$ model. Monte Carlo methods employed by Diebold and Inoue (2001) demonstrate that true long memory $I(d)$ is difficult to distinguish from spurious long memory induced by regime shifts.

More recently, a number of pre-existing tests have been adapted to distinguish between true and spurious long memory. Shimotsu (2006) shows that a clear distinction between varieties of long memory can be made using two tests: (i) parameter constancy among subperiods and (ii) stationarity after $d'th$ differencing. Evidence supporting long memory even after accounting for structural breaks has been subsequently found in returns produced by the S&P 500 [Beltratti and Morana (2006)] and in the log realized volatility of FTSE-100 firms [Garvey and Gallagher (2008)].

This chapter explores the relevance of these findings in the context of finding a solution to the dynamic portfolio choice problem. The presence of long memory effects highlights the weakness of conventional modelling approaches such as ARMA or GARCH, which fail to capture fractional orders of integration where $0 < d < 1$. This has motivated the development of the class of fractionally integrated GARCH (FIGARCH) processes [Baillie, Bollerslev and Mikkelsen (1996)]. FIGARCH allows d to assume a range of values, thus allowing for greater flexibility when modelling temporal dependencies in the volatility structure. Using an extension of the FIGARCH model, Taylor (2001) examines whether there is an economic value associated with modelling long memory in the context of option pricing. Using options data on the S&P 100 from 1984 to 1998 the study compared implied volatilities obtained from short and long memory specifications. Importantly, it was found that prices on long maturity options continue to reflect long memory effects in the volatility structure and the application of a long memory specification for the data generating process caused implied volatilities to differ by more than 1% when compared to short memory implied volatilities. In a related study Ohanissian, Russell and Tsay (2004) used simulation methods to show that serious option mispricing can result from mis-specifying the long memory property in volatility. Black-Scholes implied volatilities were generated using a short-memory, spurious long memory and a true long memory specification. Option prices produced by each alternative specification were compared thus demonstrating ignoring long memory results in an underpricing of call options with maturities between one month and two years. Importantly, Ohanissian, Russell and Tsay (2008) showed that options are underpriced if the data generating process follows a 'true' long memory but is mis-specified using the 'spurious' long memory process. Mis-pricing is most acute for short maturity options and decreases as option maturity becomes longer. This finding supports similar results produced by Taylor (2001).

Empirical studies on option prices have shown that volatility smile effects are stronger in short maturity options than in long maturity options. This observation suggests that the randomness of volatility persists even over very long horizons. Sundaresan (2000) notes that this pattern observed in the term structure of volatility smiles presents a challenge for correctly modelling volatility. The inclusion of jump components in the return process should be modelled if smile effects for short maturity options are to be mitigated, however model specifications that include jumps lead to unrealistic volatility behaviour for long maturity options. Comte, Coutin and Renault (2003) attempt to resolve this by developing a continuous time stochastic volatility model with long memory and an associate option pricing model. Their approach allows for persistence in the long run while persistence is negligble in the short run.

The relevance of long memory effects in a realistic discrete time setting is examined here. In practice, asset allocation decisions are made with relative frequency and in many cases do not account for fractional behaviour in the underlying volatility process. This chapter measures the impact of omitting fractional dynamics in the portfolio optimization process and examines the effects of this omission both in terms of portfolio allocation and economic cost over the entire investment horizon. The methodology applied here benefits from recent developments in portfolio optimisation and dynamic programming as well as the findings from a number of recent studies on the performance of derivative strategies.

The analysis focuses on implementing a covered call strategy where the investor has access to a risky asset, a call option on that asset and the money market account. The investment period T, covers six months and the portfolio is rebalanced at the beginning of each month, with the final rebalancing decision made a the beginning of the final period, $T-1$. Optimisation at each interval assumes that the underlying process follows a short-memory specification. Actual volatility is then simulated across the entire investment horizon when volatility follows a short-memory, spurious long memory and a true long memory specification, respectively. This methodology identifies whether portfolio policy and performance are influenced by the omission of long memory effects and it also examines the relative economic consequences when long memory is a true fractionally integrated process or is induced by shifts in the volatility regime.

The results show that the covered call strategy does provide positive wealth outcomes when volatility is correctly specified. Long memory effects, produced either by fractional integration or structural breaks in volatility must be included in the rebalancing criteria. If the underlying volatility is fractionally integrated and the investor incorrectly assumes short memory then the covered call strategy is likely to produce negative returns. Similarly, if the investor ignores occasional structural breaks in volatility then portfolio returns are negatively affected.

The remainder of the chapter is structured as follows. Section 4.2 describes the portfolio choice problem and presents the three model specifications used in this paper. Implementation and associated issues in Section 4.3, Section 4.4 presents the results and Section 4.5 concludes.

4.2. Model Specification and Portfolio Choice

This section first describes the three stochastic volatility specifications used to model the short memory, spurious long memory and true long memory process, respectively. Details of the portfolio choice problem are presented and the dynamic programming techniques used to solve this problem is described. Each of the three model specifications used here are variations on the affine stochastic process used by Heston (1993). The use of this class of volatility model in simulations allows one to isolate portfolio results that arise from long memory effects specifically. The standard affine model is itself a short-memory process and Ohanissian, Russell and Tsay (2008) show how it can be adjusted to produce a volatility process that switches between a high state and low state (spurious long memory). Additionally, Comte, Coutin and Renault (2003) adapt the standard model to create a fractionally integrated volatility structure. Each of the volatility specifications allows for correlation between the price and volatility processes, where the correlation in each case is represented by ρ.

Specifications

A short memory (or no long memory) model is simulated using the specification provided by Heston (1993). It is represented in its objective form as,

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^{(1)}, \tag{4.1}$$

$$dV_t = \kappa(\overline{v} - V_t) - \sigma\sqrt{V_t}\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right), \tag{4.2}$$

where S_t is the price process, V_t is a square root variance process and $W_t^{(1)}$ and $W_t^{(2)}$ are two independent standard Wiener processes with correlation ρ. The mean of the variance process is given by $\overline{v}$, while κ is the speed of adjustment. An occasional break model is created by introducing a switching mean in volatility to the standard Heston specification as follows,

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^{(1)},$$
$$dV_t = \kappa(v_t - V_t) - \sigma\sqrt{V_t}\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right),$$

$$dv_t = [v^h + v^l - 2v_t]dq_t^*$$

In this model, the mean level of the variance process v_t, switches between a high $\left(v^h\right)$ and a low state $\left(v^l\right)$ where the switching behaviour is determined by a Poisson jump process q_t^*. Granger and Hyung (2004) show that breaks in the volatility process can induce levels of persistence in the volatility structure similar to those observed for a fractionally integrated process. The occasional break model provides an appropriate proxy for spurious long memory process in this chapter.

The third model considered here is a true long memory process that is simulated through an adaption to the standard square root model. This specification was developed by Comte, Coutin and Renault (2003) and used by them to propose a long memory extension of the Heston (1993) option pricing model. The square root process, $\widetilde{V}_t$ is adjusted using a fractional integration operator $\left(I^{(\alpha)}\right)$ as follows:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^{(1)}$$

$$V_t = \overline{v} + I^{(\alpha)}(\widetilde{V}_t - \overline{v})$$

$$d\widetilde{V}_t = \kappa(\overline{v} - V_t) - \sigma\sqrt{V_t}\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right),$$

The fractional integration operator in a finite sample is approximated as $I_0^{(\alpha)}$ and results in the process,

$$I_0^{(a)}(X)(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} X(s) ds.$$

The discrete-time long memory process is then found by applying a recursive discretization method to fractional integrals. Carmona, Coutin and Montseny (2000) rewrite the fractional integrals using the Laplace inverse transform to show that a function f, continuous on $[0, T]$ satisfies

$$I_0^{(\alpha)}(f)(t) = \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \int_0^\infty x^{-\alpha} \Psi(x, t, f) dx,$$

where,

$$\Psi(x,t,f) = \int_0^t e^{-x(t-s)} f(s)ds$$

Comte, Coutin and Renault (2003) then apply a geometric subdivision of R^+, $x_i = r^i$, $i = -n, -n+1, ..., 0, 1, ..., n-1$, for some $r \in [1,2]$ and n going to infinity. In this manner, a discretised volatility process with long memory characteristics can be produced from the volatility process $\widetilde{V}$ using the following scheme

$$V^{r,n,\Delta}(t_i) = \overline{v} + \frac{1}{\Gamma(\alpha)\Gamma(1-\alpha)} \sum_{j=-n}^{n-1} c_j \Psi^{\Delta}(\xi_j, t_i, \widetilde{V} - \widetilde{v})$$

c_j and ξ_j are given by

$$c_j = \left(\frac{r^{1-\alpha} - 1}{1-\alpha} \right) r^{(1-\alpha)j}$$

$$\xi_j = \left(\frac{1-\alpha}{2-\alpha} \right) \left(\frac{r^{2-\alpha} - 1}{r^{1-\alpha} - 1} \right) r^j$$

and Ψ^{Δ} is

$$\Psi^{\Delta}(x, t_{i+1}, f) = \Psi^{\Delta}(x, t_i, f)e^{-x\Delta} + f(t_i)\frac{1 - e^{-x\Delta}}{x}$$

$$\Psi^{\Delta}(x, t_0, f) = 0$$

The fractional integration operator is applied to the short memory volatility process centered on its empirical mean rather than its theoretical mean which cannot be observed and Comte, Coutin and Renault (2003) show that when the long memory parameter α is zero the standard square root model is recovered exactly

Parameter values for each of the model specifications described have been estimated by Ohanissian, Russell and Tsay (2008) from actual data on the S&P 500 Index. The estimation procedure is an implied state-generalised method of moments (IS-GMM) approach similar to that used by Pan (2002). Ohanissian, Russell and Tsay (2008) consider a time series of realised volatility calculated from a 9-year sample of intra-day spot data (1990-1998) gathered on the index as well as 1-year of option data (September 2, 1993 - August 31, 1994) to generate the parameter estimates. The long memory parameter, α is estimated using the log-periodogram regression framework developed by Geweke and Porter-Hudak (1983). Ohanissian, Russell and Tsay (2008) show that the long memory scheme is effective for values $r = 1.3$, $n = 1000$, and $\Delta = 0.10$. In the case of the regime-switching model, the mean of the process is found by calculating the filtered probability of being in each state at each time using Hamilton's (1990) EM algorithm. The parameter values produced by the estimation procedure are listed in Table 4.1.

Dynamic Portfolio Choice

This research considers an investor who engages in a covered call (also called a 'buy-write') strategy. The investor writes a call contract while simultaneously owning an equivalent number of shares in the underlying stock. This popular strategy is effective in a sideways or slightly bullish market and is employed by buy-and-hold type investors. McIntyre and Jackson (2007) apply a covered call strategy to a number of FTSE-100 stocks and find that in many cases call writing strategies produce better returns than buy-and-hold strategies. The principle of the strategy is that income received from writing the call provides some protection against a decline in the stock value while the upside potential of the strategy is limited because the payout on the call will become non-zero as the stock price increases above the strike price at maturity. The strategy is employed if the investor wishes to hedge against any short-term pull back in the value of an asset while expecting its long-term prospects to be good. Therefore, it can be viewed as a sort of mean reversion play and provides a good platform from which to analyse our three model specifications since they describe alternative behaviours of volatility with respect to some long-run mean level.

The investor is faced with a six-month investment horizon and is limited to rebalancing his portfolio at the beginning of each month. The investor has access to 3-month, 6-month and 12-month options respectively. In each case, the investor takes action at the rebalancing date by closing out the written call contract and avoiding the obligation to sell his stock at the call's strike price. At the rebalancing date, the investor then selects a weight in another option with full maturity for the following investment interval. The investor is thus never required to make a decision on an option at expiry and instead relies on the price dynamics of the call option over the first month of its life to earn a return from his position in the contract.

Starting with a wealth level, W_0, for each rebalancing date t, $0 \leqslant t \leqslant T$, the investor places a fraction ϕ_t of wealth in an asset S_t and a fraction ψ_t in a derivative security C_t. He can also access a money market account which returns the risk-free rate, r. We assume that the investor behaves according to a constant relative risk aversion (CRRA) utility function, $U(W) = W^{1-\gamma}/(1-\gamma)$ with a coefficient of risk aversion γ. Following from Liu and Pan (2003), the portfolio choice problem requires us to maximise the investor's expected utility of terminal wealth W_T

$$\max_{\{\phi_t, \psi_t, 0 \leqslant t \leqslant T\}} E\left(\frac{W^{1-\gamma}}{1-\gamma}\right) \tag{4.3}$$

where the dynamics of wealth in discrete time are as follows,

$$W_{t_i} = W_{t_{i-1}}\left[\phi_{t_{i-1}}\frac{S_{t_i}}{S_{t_{i-1}}} + \psi_{t_{i-1}}\frac{C_{t_i}}{C_{t_{i-1}}} + \left(1 - \phi_{t_{i-1}} - \psi_{t_{i-1}}\right)e^{r(t_i - t_{i-1})}\right]. \tag{4.4}$$

The investor in this instance uses one call option, however this approach can be easily extended to include more than one option. Merton (1971) defined the indirect utility function as,

$$J_{t,w,v} = \max_{\{\phi_t,\psi_t,0\leqslant t\leqslant T\}} E\left(\frac{W^{1-\gamma}}{1-\gamma}|W_t = w, V_t = v\right),$$

and using the stochastic control principle we find the optimal portfolio by backward induction from time $T-1$ to t_0. This can be rewritten as,

$$J(t,w,v) = \max_{\left\{\phi_{t_i},\psi_{t_i}\right\}} E_{t_i}\left(J\left(W_{t_{i+1}}, V_{t_{i+1}}, t_{i+1}\right)|W_{t_i} = w, V_{t_i} = v\right) \tag{4.5}$$

where wealth is given by Equation (4.4) and volatility (V) follows the standard square root process described previously. This procedure thus finds the approximate optimal weights for a covered call strategy when the true data generating process underlying stock volatility is short memory. It is worth noting that the indirect utility function described above is separable in that the current state of W_{t+1} reached from W_t by applying J_t depends only on W_t and J_t and not on the past history $V_0, ..., V_{t-1}$. This Markovian state property is a central requirement for implementing this procedure. This requirement implies that the optimal policy cannot be similarly found for a fractionally integrated process which is by definition non-Markovian and depends on the history of volatility observations.

4.3. Implementation

Since an analytical solution does not exist for this problem we are required to solve the optimal portfolio policy numerically. The use of CRRA means that the utility function is homothetic in wealth and we can thus without loss of generality, normalize $W_t = 1$ so that the value function is a backward functional equation that depends only on the horizon and state variables as follows,

$$J(w, v, t) = w^{1-\gamma} J(1, v, t). \tag{4.6}$$

Discretizing the state space has emerged as a popular method of solving the dynamic portfolio choice problem [Balduzzi and Lynch (1999), Barberis (2000) and Damon, Spatt and Zhang (2001), Branger, Breuer and Schlag (2006)]. By discretizing the state space for each time period we are reducing an infinite set of optimisation problems in continuous state space to a fixed number of possible states. The first step is to discretize the state variable variance V into a grid of 10 equally spaced intervals between 0.002 and 0.10 and 10 equally spaced intervals between 0.10 and 0.60. The number of grid points is limited since the speed of the solution is roughly proportionate to the number of points used in approximating volatility. Another trick in constructing the grid relates to the optimal interval size which is not necessarily constant across volatility levels. The choice of intervals between grid points is motivated in part by Carroll's (2006) applications of this technique to solve microeconomic dynamic stochastic optimization problems. Carroll (2006) demonstrates that by increasing the density of the variance grid at lower variance levels the accuracy of the subsequent solution is improved. The state variable time, t is also discretized, t_i $(i = n-1, n-2, ..., 1, 0)$, so that in our case of an investor with a six-month investment horizon we have a 5×19 time-variance grid.

The optimal portfolio policy and indirect utility function at each variance gridpoint are then found beginning at time *T-1.* This is done by simulating a large number of paths, 100,000 in this case, from each variance grid point. These simulations are carried out using the short memory model specification described earlier. The starting stock price is assumed to be one and the starting price for the call option is valued using the Heston option pricing model. For each path an end-of-period stock value and volatility level is obtained and used to find the end-of-period call option price, thus a 3-month option will have a 2-month maturity at the end of the investment period. For such a large number of simulations, pricing each path using the Heston approach is unfeasible. The procedure is made more efficient by creating a variance-moneyness grid, finding the option price for each variance grid point, and approximating the call price for each path by interpolation. Branger, Breuer and Schlag (2008) examine derivative optimisation in a similar manner.

Once an expected stock price and call price are obtained, sequential quadratic programming is used to find the optimal weight in each asset. The algorithm used here is a generalised version of Newton's method for unconstrained optimisation that finds a step away from the starting point by minimizing a quadratic model of the problem. The optimisation is carried out with a starting portfolio position of 1 in the stock and 0 in the call option and the procedure is constrained in this instance by margin requirements on the stock and call option that are detailed below. Once the optimal portfolio weights are selected at time *T-1* we proceed backwards, repeating the optimisation procedure across all variance grid points for each time step to time 0. Most dynamic programming methods involve recursions on the approximated value function. vanBinsbergen and Brandt (2007) show that iterating on portfolio weights can be superior under some conditions, such as short sales constraints. The dynamic portfolio choice problem presented in this chapter is solved by recursing on the optimised portfolio weights rather than on the utility function thus implementing the method put forward by vanBinsbergen and Brandt (2007). This method of iteration provides an insight into the portfolio choice problem and allows an extended comparison of portfolio policy under different volatility model specifications.

Retaining the grid of optimal portfolio weights derived under the assumption of short memory we can examine the distributional properties of terminal wealth when actual volatility is either fractionally integrated or contains occasional breaks. A large number of paths (100,000) is simulated under the two alternative volatility specifications and at each rebalancing period, for every path portfolio weights are assigned based on the existing grid estimated under the short memory condition. Portfolio weights are approximated by interpolation using the pre-determined grid of optimal weights. As in the short-memory case, this procedure is carried our recursively, beginning at time T-1 and proceeding backwards to time 0.

Measuring Performance. The distributional characteristics of terminal wealth are first observed when an investor in a derivatives portfolio ignores long memory effects during the rebalancing procedure. At each time step, optimal portfolio weights for each variance grid point are derived under the assumption that volatility follows a short memory specification. This grid is then used to approximate the optimal portfolio when volatility is either a regime-switching or a fractionally interated process respectively. The difference in performance is a measure of the economic costs of not accounting for long-run effects when carrying out short run rebalancing. The portfolios are compared using the annualized percentage difference in certainty equivalent wealth, an approach used previously in the context of derivative strategies by Liu and Pan (2003).

The maximum utility achievable is when volatility follows a short memory (no long memory) process and the associate certainty equivalent wealth is defined using the indirect utility function as

$$J_{NLM}(W_0, V_0, 0) = \frac{W_{NLM,w}^{1-\gamma}}{1-\gamma} \tag{4.7}$$

and the certainty wealth equivalent wealth for the spurious long memory (SLM) and true long memory (TLM) are likewise defined as,

$$J_{SLM}(W_0, V_0, 0) = \frac{W_{SLM,w}^{1-\gamma}}{1-\gamma} \tag{4.8}$$

and,

$$J_{TLM}(W_0, V_0, 0) = \frac{W_{TLM,w}^{1-\gamma}}{1-\gamma} \tag{4.9}$$

respectively.

Margin Constraints. The inclusion of margin requirements introduces greater realism into our portfolio problem. These margin requirements constrain the positions taken by the investor in stocks and derivatives. In common with Branger, Breuer and Schlag (2008), we use the margin requirements set out by Interactivebrokers.com and it is assumed that the margin requirements are imposed only at rebalancing times. The margin requirements for a single call option are explained below. The calculation are easily extended when there are two or more call options used. The margin is determined by the respective weight selected, which is denoted by ϕ in the case of the stock and ψ for the call option. It is useful to note that the number of options per unit of wealth is $\frac{-\psi}{C}$, where C is the call option price.

For long and short stock positions the required margin M_{ST}, is 50% of the stock price, so that $M_{st} = 0.5\phi$ per unit of wealth. The margin requirement for a long call position, M_{LC} is equivalent to the price of the call, $M_{LC} = \psi$. For covered short positions, the margin requirement depends on the option's moneyness, as follows,

$$M_{SC_c} = \min\{\frac{-\psi}{C}, \phi\}.C$$

where the number of covered calls is $\min\{\frac{-\psi}{C}, \phi\}$ and they are trading in-the-money (ITM), while the margin requirement is 0 if they are trading at-the-money (ATM) or out-the-money (OTM). For short naked calls the margin requirement is the market value of the option plus the maximum of 15% of the underlying market value minus the out-of-the-money amount and 10% of the underlying market value. The number of naked calls is,

$$\frac{-\psi}{C} - \min\{\frac{-\psi}{C}, \phi\} = \min\{\frac{-\psi}{C} - \phi, 0\}$$

and the margin requirement is:

$$M_{SC_N} = \min\{\frac{-\psi}{C} - \phi, 0\}[C + \max\{0.15 - \max(Strike - 1.0, 0), 0.1\}].$$

These margin requirements constrain the positions taken in each of the risky assets as follows,

$$M_{ST} + M_{LC} + M_{SC_C} + M_{SC_N} \leqslant 1.$$

4.4. Results

The results are interpreted in three stages. First, the performance of the optimal portfolio devised under the assumption that volatility follows a short memory process is examined. This represents our benchmark portfolio and is used to illustrate the return characteristics of a covered call strategy. The short memory process is generated using the model parameters contained in Table 4.1. A series of optimal portfolio weights are found for every variance gridpoint at each rebalancing interval using this optimisation procedure. The second part of the analysis starts by using this predetermined grid of portfolio weights, which are only optimal under the short memory condition. The optimality of these weights are examined under two alternative volatility specifications, namely a fractionally integrated process and a regime switching process. These processes are simulated across the investment horizon, with portfolio weights at each time step allocated according to the predetermined grid. In this way, the economic consequences of omitting long memory effects are identified. The results also distinguish between outcomes produced by a fractionally integrated process ('true' long memory) and those produced by a regime shifting process ('spurious' long memory).

Finally, the robustness of the results to variations in the persistence parameter used in the true long memory model are explored. The model parameters contained in Table 4.1 are adopted from Ohanissian, Russell and Tsay (2008). The persistence parameter is estimated using the semi-parametric technique developed by Geweke and Porter-Hudak (1983). This produces estimates that are characterised by wide standard errors. To account for estimation error, the portfolio wealth distribution is examined under a long memory process with a higher level of persistence ($\alpha = 0.80$).

The results are presented using a common vertical scale that allows comparison across model specification and option maturity. The first column of panels in Figure 4.1 illustrates the expected distributional properties of terminal wealth for a portfolio using 3-month call options ranging from 10% in-the-money (ITM) to 20% out-of-the-money (OTM). The upper plot shows the portfolio results when underlying volatility is assumed to follow a short memory model, called here a no long memory (NLM) process. It can be observed that at-the-money (ATM) options provide the best mean wealth across simulations. The sensitivity of ATM options to volatility results in a greater number of negatively skewed outcomes and higher associated risk as proxied by standard deviations. Although mean portfolio returns are slightly lower using out-of-the-money (OTM) options the upswing in positive skewness indicates a higher proportion of more positive returns coupled with markedly lower standard deviation in portfolio returns. The results are in keeping with previous findings on covered call strategies by McIntyre and Jackson (2007) where OTM options provide good performance in practice, while overall the strategy would appear to be attractive for an investor who assumes the S&P 500 follows a short-memory process.

The middle and lower plots describe the distribution of terminal portfolio wealth when the actual data generating process follows a spurious long memory (SLM) and true long memory (TLM) process respectively. The second plot suggests that the investor's decision to engage in the strategy would not be influenced if the underlying asset followed an SLM model as simulated portfolio wealth is almost identical to the NLM case across strikes. The lower plot which illustrates portfolio outcomes when underlying volatility follows a fractionally integrated (true long memory) process is noticeably compressed by the strong variation in skewness for 15% and 20% OTM options. The increased number of both positive (15% OTM strike) and negative (20% OTM strike) outliers can be explained by the hyperbolic decay towards a long-run mean that characterises a long memory process. It illustrates that a higher proportion extreme positive or negative returns are possible on the options side of the portfolio when OTM option positions are closed out at each rebalancing period.

The implications of using longer maturity call options vary according to the assumed underlying volatility process. The upper plot in the second and third columns of plots in Figure 4.1 show that contract maturity is not a signficant factor if volatility contains no long memory conditions. Portfolio wealth distribution exhibits broadly similar characteristics for both a 3-month and a 6-month call option. If volatility is a SLM process the differences in wealth distribution become more accentuated for OTM options. An examination of the wealth distribution of portfolios that use 20% OTM options shows that the use of 6-month rather than 3-month options leads to a higher proportion of positively skewed portfolio returns with lower standard deviations. A higher proportion of negatively skewed returns a produced by a fractionally integrated process. Portfolio outcomes for a covered call strategy using 12-month options are illustrated in the third column of panels in Figure 4.1. Greater risk is induced by trading close-to-the-money contracts, while trading OTM contracts leads to lower risk outcomes. This pattern is common to all three volatility specifications. In general, the patterns of terminal wealth illustrated in Figure 4.1 suggests that long-memory effects become less dominant if the investor's strategy uses 12-month options compared to 3-month or 6-month options. The use of OTM options is consistently the best strategy, providing stable, positively skewed returns irrespective of the process driving volatility.

The covered-call strategy that includes OTM call options appears to be consistently superior from the simulations carried out here. This finding supports the empirical study carried out by McIntyre and Jackson (2007) and in simulation results in this chapter. Table 4.2 summarises the performance of the covered call strategy in terms of certainty equivalent wealth focusing on 20% OTM options. Assuming that true volatility is known and that it follows a short memory process the optimal strategy that emerges is for the investor to select 3-month options. The contained in Table 4.2 show that positive portfolio performance is also achieved if underlying volatility follows a regime-switching process. The same strategy produces negative returns when volatility is fractionally integrated.

Table 4.2 shows that portfolio performance is significantly affected by the choice of volatility model. The covered call strategy is clearly a worthwhile strategy in certain market conditions but appears vulnerable in particular to long memory effects in underlying volatility. This is explored further by focusing on outcomes from implementing a covered call strategy using 20% OTM options. Kernel density estimates for portfolio wealth are generated both when volatility contains no long memory, that is, alpha = 0, and when the true long memory model produces an alpha = 0.30 and 0.80, respectively. The kernel density estimates in Figure 4.2, illustrate a tight distribution of returns when volatility follows a short memory process and demonstrate a clustering of positive portfolio returns when 3-month call options are used in the covered call strategy. The returns from the covered call strategy decliness as the level of persistence in the volatility of the underlying asset increases. This distinction can be observed by comparing the distribution of returns when alpha is 0.30, illustrated in the centre row of panels and when alpha is 0.80, shown in the lower row of panels. This finding has clear implications for the correct inclusion of long memory dynamics in the application of investment strategies. The wide standard errors the accompany estimates of long memory parameters are an important consideration in the decision to include long memory dynamics if modelling volatility with a view to implementing investment strategies in discrete time.

4.5. Conclusion

This chapter examines the relevance of long memory effects in a realistic portfolio setting. Long memory effects in volatility have been attributed to both fractional integration and regime switching in the data generating process. We first estimate the numerical performance of a derivative strategy implemented in discrete time under the assumption of no long memory and we subsequently compare this against the performance of the derivative strategy when volatility follows a true long memory or spurious long memory process. Using a dynamic programming approach we find that covered call strategies provide positive, stable returns if it is assumed that the investor knows the model specification driving volatility.

The results can be related to the discussion in Liu and Pan (2003) that describes derivative positions in terms of the exposure they offer to respective aspects of volatility. The selection of an optimal weight in a derivative security depends on the sensitivity of that derivative to volatility in the underlying asset. They note, for example, that at-the-money options are more sensitive to market volatility, thus providing exposure to volatility risk. The results here show that the optimal covered call strategy requires the selection of short maturity options that are trading 20% OTM. This result is clearly sensitive to volatility parameters estimated under varying market conditions. If the data generating process switches between a state of high volatility or low volatility then this strategy may not be optimal. However based on our simulation results, the economic loss from omitting the regime-switching component is not significant. If actual underlying volatility follows a fractionally integrated process (true long memory) and the optimisation procedure ignores this, then a higher proportion of negative returns are observed. The results also show that ignoring fractional integration is less consequential when longer maturity options are used in the covered call strategy. Portfolio performance and the distribution properties of terminal wealth begin to converge when 6-month and 12-month options are selected in preference to 3-month options as part of the strategy.

Long memory effects are clearly important for investors rebalancing derivative portfolios in discrete time. Performance of the simple covered call strategy is shown to be highly sensitive on the maturity and moneyness characteristics of the selected options. The optimal strategy derived under the assumption of short memory may result in signficant losses if actual volatility is fractionally integrated.

4.6. Appendix C: Supporting Material for Chapter 4

Table 4.1: This table shows the model parameters for each model used in our simulations. Estimates are produced by Ohanissian, Russell and Tsay (2008) from Standard and Poors 500 data using an implied state generalised method of moments (IS-GMM). approach.

	μ	κ	σ	$\overline{\nu}$	ν^l	ν^h	ρ	λ^l	λ^h	η^V	α
No Long Memory	0.12	3.6	0.34	0.015			-0.55			-2.1	
	(0.02)	(1.0)	(0.03)	(0.003)			(0.05)			(1.2)	
Spurious Long Memory	0.13	5.9	0.34		0.014	0.022	-0.59	2.9	1.6	-2.8	
	(0.02)	(1.3)	(0.03)		(0.004)	(0.004)	(0.07)	(1.0)	(1.0)	(1.2)	
True Long Memory	0.12	4.3	0.32	0.014			-0.65			-2.9	0.30
	(0.02)	(1.1)	(0.03)	(0.004)			(0.07)			(1.3)	(0.10)

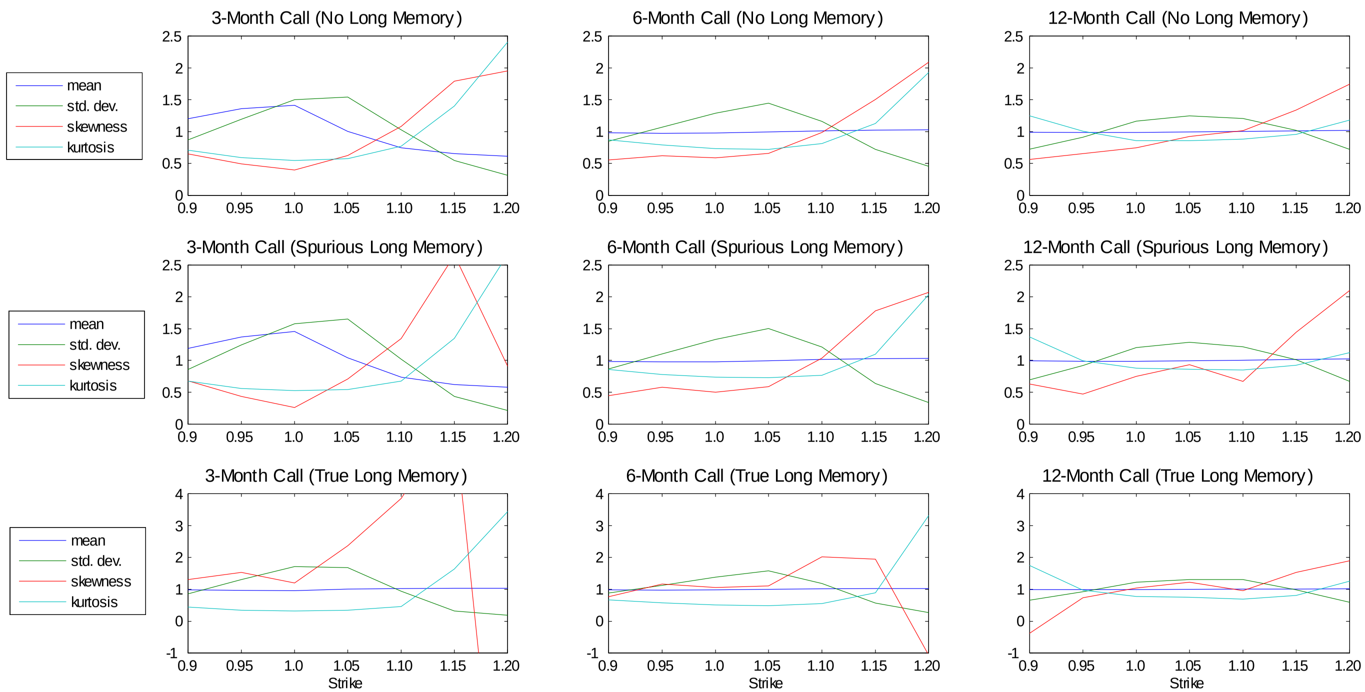


Figure 4.1: The distributional properties of terminal wealth for a covered call strategy implemented using 3-Month, 6-Month and 12-Month call options. In each case results are provided for options with strikes ranging from 10% ITM (0.90) to 20% OTM (1.20). The mean, standard deviation, skewness and kurtosis are standardised by their mean for illustrative purposes. Rebalancing takes place at the beginnning of each month and optimal positions in the stock and call option are selected under the assumption that volatility follows a short memory model. Portfolio performance when volatility follows a spurios long memory process and a true long memory process are illustrated in the centre and lower row of panels respectively.

Table 4.2: Covered Call Strategy Using 20 Percent OTM Options. Certainty Equivalent Wealth and Portfolio Weights.

Call Option Time to maturity: 3-Months			
	CE_w	ϕ_0	ψ_0
NLM	0.0078	0.5136	-0.4826
SLM	0.0077	0.5146	-0.4835
TLM	-0.1706	0.5149	-0.4838
Call Option Time to maturity: 6-Months			
NLM	-0.1703	0.5136	-0.4826
SLM	-0.1705	0.5146	-0.4835
TLM	-0.1707	0.5149	-0.4838
Call Option Time to maturity: 12-Months			
NLM	-0.1694	0.5137	-0.4826
SLM	-0.1694	0.5147	-0.4834
TLM	-0.1700	0.5149	-0.4838

Summary of portfolio performance measured using certainty equivalent wealth. The optimal strategy is derived assuming volatility follows a short memory process. Using simulations we examine the performance of this strategy when actual volatility follows a short memory model (no long memory), a regime switching model (spurious long memory) and a fractionally integrated model (true long memory).

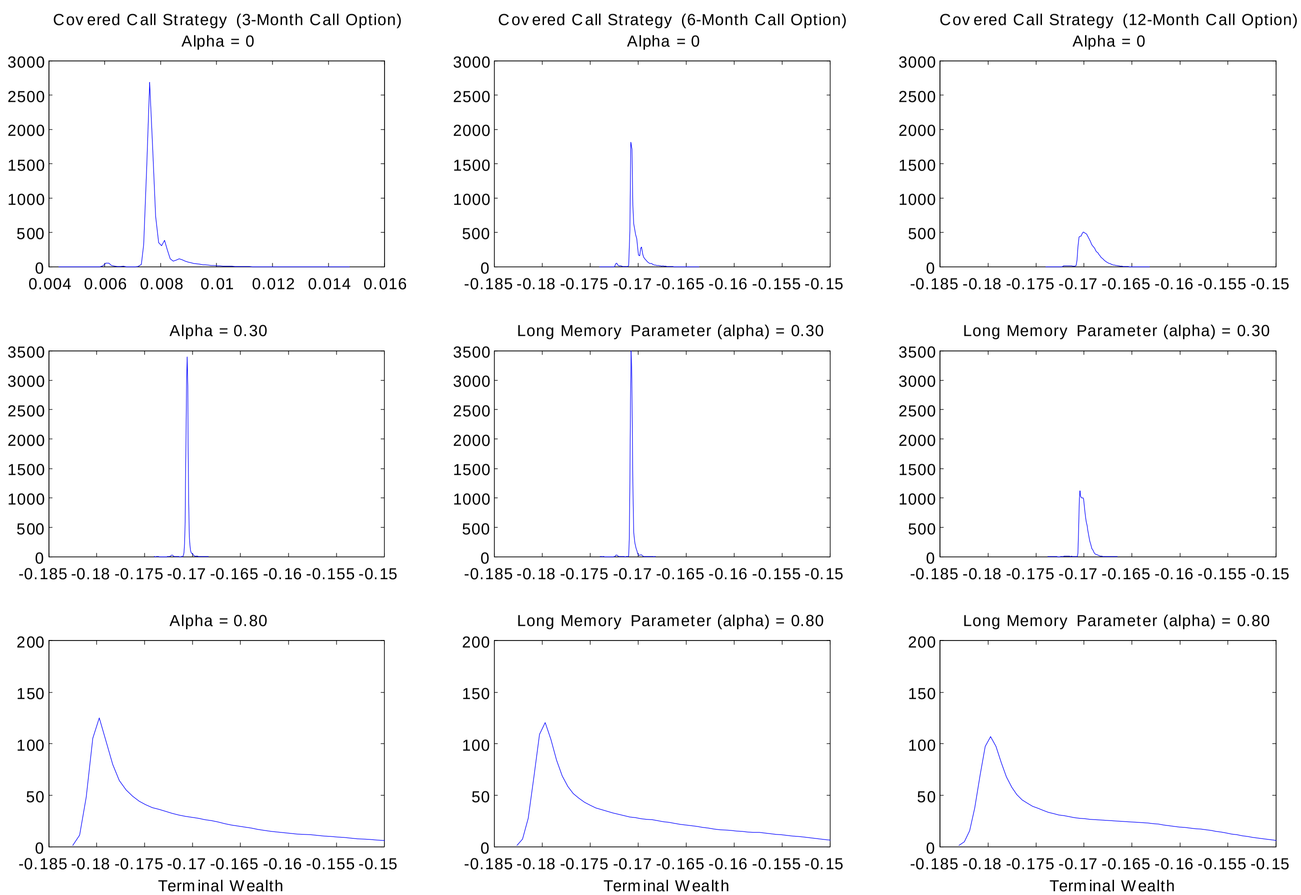


Figure 4.2: Kernel Density Estimate of the Distribution of Terminal Wealth. Portfolio uses 20% OTM call options at 3-Month (left column), 6-Month (centre columns) and 12-Month (right column) maturities. Volatility in each case is simulated (100,000 paths) using no long memory (alpha =0) and long memory models (alpha = 0.30 and 0.80). [Note: Alternative horizontal scale used for outcomes for Covered Call Strategy (3-Month Call Option), Alpha=0.]

CHAPTER 5

Summary Discussion and Conclusions

5.1. Introduction

Modeling and forecasting asset volatility is an important and challenging task both for academic researchers and professional risk managers. The past twenty years has seen the development and expansion of a range of sophisticated forecasting models within the academic literature as well as an increasing reliance on these models within the financial markets for a number of risk management functions. Statistical forecasting methods that rely on historical price return patterns produced by economic and financial assets have become increasingly complex. Relatively simple, easily implemented moving average approaches such as the exponentially weighted moving average (EWMA) models have been superceded in the academic literature by the class of ARCH and GARCH models. These statistical methods are effective over very short forecast horizons and all exhibit a declining predictive ability as the forecast horizon extends beyond one-day. Research into the forward-looking information provided by the options market has shown that an effective alternative is available for risk managers. Implied volatilities are easily obtained and provide forecasts over multi-day horizons that are in many cases more robust that those provided by statistical methods. The first part of this thesis belongs to the body of work testing and ranking alternative volatility forecasting methods.

Improvements in computing power and the adoption of techniques originally developed in the physical sciences have facilitated analysis of long-run persistence in asset volatility. These subtle long-run effects can be identified using semi-parametric techniques and more recently, academic research in finance has shown the importance of long-memory effects in risk management and option pricing. This thesis contributes to the finance literature by developing a greater understanding of both UK equity volatility and the equity options market. The characteristics of stock specific volatility are carefully defined and the distinct role of volatility effects in discrete time derivative strategies are observed. The research specifically contributes to the existing finance literature as follows.

- Analysis of composite implied volatility forecasts in the context of individual UK equities.
- A comparative analysis of forecasts produced composite implied volatilities obtained from individual equity options and GARCH and E-GARCH forecasts.
- An examination of the long-run dynamics of realized and implied volatility of UK equities using semi-parametric techniques.

- An investigation into the presence of structural breaks in UK equity volatility.
- The application of dynamic programming techniques to establish the relevance of long-memory effects in constructing optimal derivative strategies in discrete time.

5.2. Research Findings

The research findings can be separated into a number of distinct strands. The results presented in Chapter 2 show that implied volatility contains predictive information on subsequently realised asset volatility and that this forward-looking information is superior to that available in GARCH-type forecasts. This is an extension of the existing research into index options carried out elsewhere and it demonstrates that equity options can provide a useful signal function for stock specific risk. A composite implied volatility estimate is constructed as a weighted average of implied volatilities drawn from close-to-the-money options. Tests were carried out across ten-day forecast horizons using non-overlapping UK stock data. Furthermore, predictive information available from implied volatilities is shown to be superior to that available from both GARCH and E-GARCH models. The superior performance of implied volatilities is shown by both Mincer-Zarnowitz and encompassing regression results. Direct comparison of implied volatilities against statistical methods using Diebold-Mariano and Harvey Leybourne Newbold pairwise tests confirm the superiority of implied volatility forecasts.

The existence of long memory effects in stock price volatility is examined in Chapter 3. Semi-parametric techniques are used to identify volatility persistence in the volatility series of a number of FTSE-100 companies. Spurious long memory, induced by structural breaks in volatility is tested for by examining the time-domain properties of stock volatility using subsampling and differencing techniques. Variations in persistence parameter estimates are observed among subsamples indicating the presence of structural breaks in a number of volatility series. Fractional integration in the autocorrelation structure is shown to be the primary cause of long memory effects for both the implied and realized volatility. Further examination shows that structural breaks do not explain the levels of persistence observed in many volatility series. For the first time, this research also models the long-run relationship between implied and realized volatility for individual stocks using recently developed semi-parametric techniques. For a number of companies, both volatility series are shown to be linked by a fractionally cointegrating relationship.

Chapter 4 examines the implications of long memory effects in a portfolio selection problem. The analysis is carried out within a realistic framework, where an investor with a six-month investment horizon, engages in a covered-call strategy and is constrained to rebalancing his portfolio at monthly intervals. The asset allocation decision is also constrained by margin requirements. An optimal derivative strategy is constructed under the assumption that asset volatility follows a short memory process. Dynamic programming techniques are used to estimate optimal portfolio weights for each time step beginning at the last investment period. Optimal portfolio weights are then determined for a series of predetermined variance grid points at each time step. The performance of these optimal portfolio weights is examined by simulating long memory volatility processes across the investment horizon. The results show that the omission of long-memory effects in asset volatility in the context of this derivative strategy substantially alters the distribution of terminal wealth. Portfolio performance is negative if the assets under consideration follow a long memory, fractionally integrated process and this is not included in the optimisation procedure. This result implies that fractional integration or true long memory must be considered in the construction of optimal derivative strategies.

5.3. Research Issues and Limitations

There are a number of issues and limitations associated with the research undertaken that merit consideration. The LIFFE Euronext options dataset obtained for this research provided data for every company quoted on the FTSE-100. The relatively short trading history of many options series limited the analysis to sixteen FTSE-100 companies that traded continuously from 1997 to 2003. Composite implied volatility estimates were constructed from the four closest-to-the-money options traded on a given day amd contract maturities were matched as closely as possible to the forecast horizon being considered. The creation of this estimate was however subject to liquidity constraints since traded options on some days were limited to one or two option contracts. To overcome this limitation and optimally use the available data, the implied volatility estimate was constructed from the closest to the money options which meant backing implied volatility out of a single option in some cases. Similar limitations have applied to previous studies into individual equity options carried out in the US market [Gemmill (1986), Lamoureux and Lastrapes (1993)].

The analysis of long-memory effects in Chapter 3 relies on the application of semi-parametric techniques, namely the Geweke and Porter-Hudak estimate and the exact local Whittle estimate. These techniques are widely used in the literature to examine time series behaviour at very low, harmonic frequencies. They are however, characterised by wide error bands which weaken the inference that can be drawn from the results. The limitation of this methodology is for the most part overcome by the additional time domain tests used to show consistency in the persistence parameter among subsamples. This analysis as well as the examination of fractional cointegration carried out in Chapter 3 would benefit from a more extended sample period. US data would possibly provide a longer sample period with which to examine the long-run behaviour of asset volatility.

5.4. Areas for Further Research

The analysis of the implied-realized volatility relationship in Chapter 2 should be extended to include a wider sample of companies. This could be achieved using LIFFE Euronext options data for equities quoted on other European indices. A more liquid dataset may yield option contracts appropriate for the construction of shorter non-overlapping horizons, for example, bi-weekly. The analysis of a long time series would provide stronger regression results. The univariate analysis carried out in this thesis could be usefully applied in a multivariate, portfolio context. The findings suggest that the increased use of equity options data would provide a simple but effective tool in active equity fund management. Portfolio construction requires a measure of correlation between the securities included in the portfolio. Accurate measures of the dynamic correlation between assets are important inputs for portfolio optimisation and risk management. Lopez and Walter (2000) and Skintzi and Refenes (2006) examine the forward looking information contained in index options on the Dow Jones Industrial Average (DJIA). Skintzi and Refenes (2006) show that implied correlation has a high explanatory power and is a good forecast of realized correlation. Existing research should be extended to consider the forward-looking information contained in equity options in the context of partially diversified portfolios. Implied correlations reflected in options data could be compared against the performance of multivariate statistical methods, such as the dynamic conditional correlation multivariate GARCH (DCC- MV GARCH) model [Engle and Sheppard (2001), Kearney and Poti (2006)]. Composite implied volatilities from individual equity options are also an appropriate mechanism for the analysis of strategic (long-term) and tactical (short-term) asset allocation decisions. According to Rey (2004), tactical asset allocation is ostensibly used to "realign the return and risk profile of a long-term strategic benchmark portfolio". Arnott and Fabozzi (1988) define it more specifically as, "shifting the asset mix of a portfolio in response to the changing patterns of reward available in the capital markets". Tactical asset allocation is implemented across different investment horizons and decisions are evaluated according to a range of criteria. Philips, Rogers and Capaldi (1996) note that the objective of the fund manager is to outperform benchmark returns on a risk-adjusted basis. Implementing tactical asset allocation using the information content of individual equity options across various investment horizons has not been examined to date in the academic literature. Future research should address the economic benefits of opportunistic asset allocation decisions that exploit forward-looking, stock-specific information contained in equity options.

The calculation of realized volatility using high-frequency data is likely to benefit from the use of a longer sample period as well as the development of more nuanced approaches such as those suggested by Andersen and Bollerslev (1997) and Oomen (2006) that take account of the intra-day trading patterns. The multi-scale realized volatility (MSRV) estimate proposed in Zhang and Mykland (2005) and evaluated in Zhang's (2006) study are likely to produce more efficient estimates of realized volatility. The research findings from Chapter 4 which show the creation of optimal derivative portfolio strategies using dynamic programming techniques should be further validated using empirical market data. The analysis could also be extended beyond the simple covered call strategy to a range of derivative strategies. One implication of this research is the potential development of trading strategies that exploit derivative mispricing that arises from fractionally integrated volatility. The relevance of volatility persistence should be examined through alternative derivative strategies that optimally allocate weights to option contracts expiring at different maturities. Using market data presents challenges such as potential liquidity constraints and microstructure effects which must be overcome when estimating and implementing optimal strategies in practice. Future research extending from the findings in Chapter 4 should address the role of margin constraints. A comparison of the solution both with and without margin constraints should provide an insight into the economic cost of such constraints.

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Appendix D: Sample Matlab Code

5.5. Matlab Programme 1: Importing and Formatting Data

% STEP 1: Import daily stock price data, used to generate GARCH/EGARCH forecasts

[date, cua, ba,aws, cad, dix, gns, gxo, ldb, hsn, hsb, kgf, lnr, ms, pru, ryl, rut] =

textread('C:\Data\price_data.txt','%s %f%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f');

for i = 1:size(date,1)

dstr =date{i};

date_1(i) = datenum(dstr);

end

dates = date_1';

dates(1)=[];

% Co-ordinate ticker symbols!!

av = cua; ba = ba; bay = aws; cbry = cad; dxn = dix; dge = gns; gsk = gxo; hg = ldb; hns = hsn;

hsba = hsb; kgf = kgf; lmi=lnr; mks = ms; pru = pru; rsa = ryl; rtr = rut;

% returns: underlying asset returns (N x 16)

prices = [av, ba, bay, cbry, dge, dxn, gsk, hg, hns, hsba, kgf, lnr, mks, pru, rsa, rtr];

returns = price2ret(prices);

% Previously, STEP1_OPTDATA_LOAD.m

% STEP 2: Import data.

fname1 = ['C:\Program Files\MATLAB\R2007a\work\screenoptdata\databank\hg.mat'];

load (fname1);

fname2 = ['C:\Program Files\MATLAB\R2007a\work\screentickdata\data\30min_ts.mat'];

load (fname2);

RV = rv1day_8; % Realized Volatility

DR = [dates, returns(:,8)]; % Daily Returns

optdata = optdata_all;

% STEP 3: Create expiry dates and convert to numerical form - expiries are third Friday of every month

expiry = optdata(:,3);

for i=1:size(expiry,1)

dstr = num2str(expiry(i));

if length(dstr) == 5

mm = ['0',dstr(1)];

yy = dstr(2:5);

else

mm = dstr(1:2);

yy = dstr(3:6);

end

mth = str2num(mm);

yr = str2num(yy);

calend = calendar(yr,mth);

if calend(1,1:6) == zeros(1,6) % if month begins on Sat, third Fri is in fourth row of calendar month matrix

dy = calend(4,6);

else

dy = calend(3,6); % if month begins on Fri, or before this, third Fri is in third row of calendar month matrix

end

expiry(i) = datenum(yr,mth,dy);

end

optdata(:,3) = expiry;

clear dstr mm yy mth yr calend dy i;

%%%% moneyness %%%%

ctm = abs(optdata(:,9)-optdata(:,4));

%%%% time-to-maturity %%%%

ttm=optdata(:,3)-optdata(:,1);

optdata = [optdata, ctm, ttm];

% exclude options with volume traded < 30

[B] = find (optdata(:,6)>= 30);

optdata = optdata(B,:);

clear B

% select options with a maturity between X1 and X2 days

[B] = find (10 < optdata(:,13)& optdata(:,13) < 30);

```
optdata = optdata(B,:);
% % TEST: select only options traded after September 11th 2001 (731105).
% [B] = find (optdata(:,1)>731217);
% optdata = optdata(B,:);
```

% Calculate Realized Volatility from Trade Date to Expiry Date.

```
option_trade_dates = unique(optdata(:,1));
stock_trade_dates = RV(:,1);
common=zeros(1,1);
for j=1:length(stock_trade_dates)
[A]=find(option_trade_dates==stock_trade_dates(j));
common=[common;A];
end
common(1)=[];
option_trade_dates = option_trade_dates(common);
clear j
submat=zeros(1,9);
for j=1:length(option_trade_dates)
% 1) Find trade date.
[A]=find(optdata(:,1)==option_trade_dates(j));
```

% 2) Break option matrix into blocks (based on trade dates).

```
eval(['sub_optdata_',num2str(j),'= optdata(A,:);']);
% 3) Calculate realised volatility from trade date to expiry.
```

```
% Function: RlzV.

eval(['FCAST_',num2str(j),'= RlzV(sub_optdata_',num2str(j),',RV,DR);']);

eval(['submat=[submat;FCAST_',num2str(j),'];']);

end

submat(1,:)=[];

clear F*

FCastMatrix=submat;

% FcastMatrix: C1=Dates, C2=Volume, C3=TTM, C4=Realised Vol., C5=EncompRV,
C6=DSR, C7=Implied Vol, C8/9=GARCH/EGARCH forecasts.

FTS1 = fints([FCastMatrix(:,1), FCastMatrix(:,6),FCastMatrix(:,7),FCastMatrix(:,8),FCastMatri

FTS2= fints([FCastMatrix(:,1),FCastMatrix(:,4),FCastMatrix(:,7)]);

FTS3=fints([FCastMatrix(:,1),FCastMatrix(:,2)]);

subplot(3,1,1); plot(FTS1); legend('Sqd Returns','IV','GARCH','E-GARCH');legend('location',
'NorthWest');

subplot(3,1,2); plot(FTS2); legend('Realised Volatility','Implied Volatility');
legend('location', 'NorthWest');

subplot(3,1,3); plot(FTS3); legend ('Option Volume Traded');legend('location',
'NorthWest');

% Run MZ Regression.

[RSQ,BETA,DW,RESID] =MZReg3(FCastMatrix);

% Run MZ Regression.

[RSQrv,BETArv,DWrv,RESIDrv] =MZReg4(FCastMatrix);

% Run Encompassing Regression.

[EncRSQ,EncBETA,EncDW] =ENCReg3(FCastMatrix);
```

% Run Encompassing Regression.

[EncRSQrv,EncBETArv,EncDWrv] =ENCReg4(FCastMatrix);

% Use RESID from MZReg3 to calculate DM statistic.

[DM,HLN]=DM_HLN_TEST(RESID);

% FcastMatrix: C1=Dates, C2=Volume, C3=TTM, C4=Realised Vol., C5=EncompRV, C6=DSR, C7=Implied Vol, C8/9=GARCH/EGARCH forecasts.

save ('C:\Program Files\MATLAB\R2007a\work\screenoptdata\databank\fixedh2008_8.mat')

5.6. Matlab Programme 2: Measuring and Testing Volatility Persistence (Long Memory).

clear;

file = ['C:\Program Files\MATLAB\R2007a\work\screenoptdata\databank2\all_tsa.mat'];

load (file);

n = 16;

s=75;

dv=0.4:0.05:0.70;

m = fix(s.^dv);   % m is the number of frequencies used in estimation.

l=length(m);

LPxall=zeros(1,3);

LPyall=zeros(1,3);

LPeall=zeros(1,3);

LPdiffall=zeros(1,3);

% Estimates variations on the Exact Local Whittle Estimator over different

% bandwidths.

```
whichstats = {'r'}; model = {'linear'};
for k = 1:n
eval(['x_',num2str(k),' = novlp_tsa_',num2str(k),'(:,2);']);
eval(['y_',num2str(k),' = novlp_tsa_',num2str(k),'(:,3);']);
eval(['diff_',num2str(k),'=x_',num2str(k),'- y_',num2str(k),';']);
% Regression
eval(['r_',num2str(k),' = regstats(x_',num2str(k),',y_',num2str(k),');']);
eval(['e_',num2str(k),' = r_',num2str(k),'.r;']);
for f=1:l
eval(['LPx',num2str(k),' = LP_calc(x_',num2str(k),',dv(',num2str(f),'));']);
eval(['LPy',num2str(k),' = LP_calc(y_',num2str(k),',dv(',num2str(f),'));']);
eval(['LPe',num2str(k),' = LP_calc(e_',num2str(k),',dv(',num2str(f),'));']);
eval(['LPdiff',num2str(k),' = LP_calc(diff_',num2str(k),',dv(',num2str(f),'));']);
eval(['LPxall = [LPxall; LPx',num2str(k),',];']);
eval(['LPyall = [LPyall; LPy',num2str(k),',];']);
eval(['LPeall = [LPeall; LPe',num2str(k),',];']);
eval(['LPdiffall = [LPdiffall; LPdiff',num2str(k),',];']);
end
eval(['LPxall',num2str(k),'= LPxall(2:end,:);']);
eval(['LPyall',num2str(k),'= LPyall(2:end,:);']);
eval(['LPeall',num2str(k),'= LPeall(2:end,:);']);
eval(['LPdiffall',num2str(k),'= LPdiffall(2:end,:);']);
```

LPxall=zeros(1,3);

LPyall=zeros(1,3);

LPeall=zeros(1,3);

LPdiffall=zeros(1,3);

end

status = 'done'

save ('C:\Program Files\MATLAB\R2007a\work\P_2\code\LongMemory_Mar08\Data\LPresu

function [d, nobs, tasy, sigasy, tols, sigols] = gph (series, incl, excl)

% Available from: http://fmwww.bc.edu/repec/bocode/g/gph.m

%Copyright (c) 10 March 1998 by Ludwig Kanzler

% Department of Economics, University of Oxford

% Postal: Christ Church, Oxford OX1 1DP, U.K.

% E-mail: ludwig.kanzler@economics.oxford.ac.uk

% Homepage: http://users.ox.ac.uk/~econlrk

% $ Revision: 1.31 $$ Date: 15 September 1998 $

function [d] = felw2st(x,m,p,rep)

% function[d] = felw2st(x,m,p,rep)

% FELW2ST.M computes the 2-step feasible exact Whittle likelihood

% estimator.

%Reference: Shimotsu and Phillips (2004), Shimotsu (2004)

% Code written by, Katsumi Shimotsu, July 2004

%Depenency: this code requires fracdiff.m and veltaper.m

%INPUT x: data (n*1 vector)

%m: truncation number

% p: the order of the taper, p = 2 or 3, 3 is preferable

%rep: number of steps

5.7. Matlab Programme 3: Testing for Long Memory versus Structural Breaks.

% Test 1: Visual examination of d against avg(d1,...d4);

```
n =16;

stack2 = zeros(1,3); stack4 = zeros(1,3);

for j = 1:n

eval(['[dfs2_',num2str(j),',dhat2_',num2str(j),', W2_',num2str(j),'] = lm_teststats(2, LWfs_',num2str(j),', LW2_',num2str(j),');']);

eval(['d2_',num2str(j),' = [dfs2_',num2str(j),',dhat2_',num2str(j),',W2_',num2str(j),'];']);

eval(['stack2 = [stack2;d2_',num2str(j),'];']);

eval(['[dfs4_',num2str(j),',dhat4_',num2str(j),', W4_',num2str(j),'] = lm_teststats(4, LWfs_',num2str(j),', LW4_',num2str(j),');']);

eval(['d4_',num2str(j),' = [dfs4_',num2str(j),',dhat4_',num2str(j),', W4_',num2str(j),'];']);

eval(['stack4 = [stack4;d4_',num2str(j),'];']);

end

subsample_fs_2_4 = [stack2,stack4(:,2)];

subsample_fs_2_4(1,:)=[];
```

```
Wstat = [stack2(:,3),stack4(:,3)];

Wstat(1,:)=[];

clear dfs* dhat* d2* d4* stack*
```

function [dfs, dhat,W] = lm_teststats(b,d_fs, d_sub)

```
% INPUT

% d_fs = LW d estimate for full sample

% d_sub = LW d estimates for subsamples

% b = number of subsamples

% n = length of full sample

% OUTPUT

% Wald statistic for testing H0

m = floor (1343./b); % subsample size

d = zeros(1,1);

for g = 1:b

eval(['d',num2str(g),'= d_sub.LW',num2str(g),'(2).lw;']);

eval(['d = [d;d',num2str(g),'];']);

end

d(1,:)=[];

dhat = mean (d);

dfs = d_fs(3).lw;

dhat_ts = [d;dfs];

dhat_ts = mean(dhat_ts);
```

% Create b+1 vector, c Shimotsu (2006).

dhat_b =[dfs;d]-dhat_ts;

A = ones(b,1);

I_1 = -1.*eye(b);

A = [A,I_1];

iotab = ones(b,1);

bIb = b.*eye(b);

% A.*Omega.*Atranspose is as follows

AOAt = bIb-(iotab*iotab');

invAOAt =pinv(AOAt); % generalised inverse

Adb = A*dhat_b;

% Wald Statistic

W = m*(Adb'*invAOAt*Adb);

% modified Wald statistic

% Test for long-memory v structural breaks in volatility ts.

% Application of the Phillips-Perron (1988) test.

status = 'start'

fname1 = ['C:\Program Files\MATLAB71\work\P_2\results\30minrv_test1.mat'];

load (fname1);

df = zeros(1,1);

n = 16;

for j = 1:n

```
% Shimotsu,2006 (p18)

eval(['[adf',num2str(j),',adfresid',num2str(j),',df',num2str(j),',dfresid',num2str(j),',d',num2str(j),']
= unitroot_fxn(log(ts1day',num2str(j),'.^0.5),LWfs_',num2str(j),');']);

% Vector contains the PP stats from stocks 1-n.

eval(['df = [df;df',num2str(j),'(2,1)];']);

end

df(1)=[];

status = 'done'

% reject a unit root if t_1 <= 0.10
```

5.8. Matlab Programme 4: Optimal Discrete-Time Covered Call Strategy.

```
tic

a_setup_params;

global mcs

vol=zeros(TimeSteps,1);

mcs=zeros(NoRebal,NoSims);

d=linspace(200,1000,NoRebal);

for i = 1:NoSims %No. of simulations

st=s0;

vt=mu;

for j=1:TimeSteps % No of timesteps

[eigvecs,eigvals] = eig(corr_matrix);
```

```
eigvals = diag(eigvals)';
eps_vec = randn(2,1)';
z1= sum(eigvecs(1,:).*sqrt(eigvals).*eps_vec);
z2= sum(eigvecs(2,:).*sqrt(eigvals).*eps_vec);
st=st+R*st*dt+sqrt(vt)*st*z1*sqrt(dt);
vt=vt+KappaQ*(v-vt)*dt+Sigv*sqrt(vt)*z2*sqrt(dt);
vt=abs(vt);
vol(j)=vt;
end
mcs(:,i)=vol(d);
end
toc
```

5.8.1.

```
a_setup_params;
global mcs
vol=zeros(NoTSteps,1);
mcs=zeros(NoRebal,NoSims);
d=linspace(200,1000,NoRebal);
for i = 1:NoSims %No. of simulations
st=s0;
vt=mu;
```

```
for j=1:NoTSteps % No of timesteps
[eigvecs,eigvals] = eig(corr_matrix);
eigvals = diag(eigvals)';
eps_vec = randn(2,1)';
for k = 1:2
eval(['z',num2str(k),' = sum(eigvecs(',num2str(k),',:).*sqrt(eigvals).*eps_vec);']);
end
JprobL=ldl*vt*dt;
JumpL = binornd(1,JprobL);
JprobH=ldh*vt*dt;
JumpH=binornd(1,JprobH);
st=st+R*st*dt+sqrt(vt)*st*z1*sqrt(dt);
regimeSigvP=SigvP+(vh-vl)*JumpL+(vl-vh)*JumpH;
vt=vt+KappaP*(regimeSigvP-vt)*dt+Sigv*sqrt(vt)*z2*sqrt(dt);
vt=abs(vt);
vol(j)=vt;
stk(j)=st;
end
mcs(:,i)=vol(d);
end
a_setup_params;
global mcs
```

```
TimeSteps=6;
vol=zeros(TimeSteps,1);
stk=zeros(TimeSteps,1);
vmat_nlm=zeros(TimeSteps,NoSims);
smat_nlm=zeros(TimeSteps,NoSims);
n=8; % length of sample
d = 3; % 3-dimensional CIR process
r = 1.04; %
tau =1;
for h = 1:NoSims %No. of simulations
N = n + (n-1);
m = 1./(gamma(alpha)*gamma(1-alpha));
% eta & c vectors from -N...N-1
eta = zeros(1,N);
c = zeros(1,N);
i = -n:n-1;
lg = length(i);
for t = 1:lg
eta(t)= ((1-alpha)./(2-alpha))*((r^(2-alpha)-1)./(r^(1-alpha)-1)).*(r^i(t));
c(t)= ((r^(1-alpha)-1)./(1-alpha)).*(r^((1-alpha)*i(t)));
end
e1 = exp(-eta*dt);
```

```
e2 = (1-exp(-eta*dt))./eta;

psi = zeros(n+1,lg);

lmsr = zeros(n,1);

% Create short memory volatility process,

% Apply fractional integration operator to get lm version.

rho = 0.5; rho1 = 0.25; rho2 = -0.5; % Correlation coefficient

lag = 1;

%sigv = 0.48;

corr_matrix = [1,rho,rho1;rho,1,rho2;rho1,rho2,1];

theta = d.*g^2./4;

vt(1,1:3)=mu; vt = [vt;zeros(n-lag,3)];

s(1) = 0; s =[s;zeros(n-lag,1)];

cir(1) = 0; cir = [cir;zeros(n-lag,1)];

std_sq(1) = 0; std_sq = [std_sq;zeros(n-lag,1)];

rtsm(1) = 0; rtsm = [rtsm;zeros(n-lag,1)];

rtlm(1) = 0; rtlm = [rtlm;zeros(n-lag,1)];

for j = 1:n-1

% generate correlated normal variates z1, z2 and z3

[eigvecs,eigvals] = eig(corr_matrix);

eigvals = diag(eigvals)';

eps_vec = randn(3,1)';

for k = 1:3
```

```
eval(['z',num2str(k),' = sum(eigvecs(',num2str(k),',:).*sqrt(eigvals).*eps_vec);']);

end

% d-dimensional Ornstein-Uhlenbeck process (d=3)

vt(j-lag+2,:) = vt(j-lag+1,:) + (-(kappav./2).*vt(j-lag+1,:).*dt) + (1/2)*sqrt(dt)*g.*randn(1,3);

s(j-lag+1) = sum(vt(j-lag+1,:).^2);

cir(j-lag+2) = s(j-lag+1) + kappav*(theta-s(j-lag+1))*dt + g*sqrt(dt)*sqrt(s(j-
lag+1))*z2;

% psi: rows = 0,1,...,N , columns = -N...0...N-1

psi(j-lag+2,:) = psi(j-lag+1,:).*e1 + (cir(j-lag+1)-mean(cir(:))).*e2;

lmsr(j-lag+1) = theta + m.*sum(c.*psi(j-lag+2,:));

% construct sample paths of the log returns

rtsm(j-lag+2) = rtsm(j-lag+1) + sqrt(abs(cir(j-lag+1))).*z1.*sqrt(dt);

rtlm(j-lag+2) = rtlm(j-lag+1) + sqrt(lmsr(j-lag+1)).*z1.*sqrt(dt);

end

rtsm(1:2)=[];

rtlm(1:2)=[];

vmat_nlm(:,h)=rtsm;

smat_nlm(:,h)=rtlm;

clear rtsm rtlm

end

vmat=vmat_nlm;

smat=smat_nlm;

mcs=vmat;
```

```
% setup_parameters.m
global mu KappaQ v Sigv Rho
global SigvP KappaP LambdaP
global s0 T dt NoSims NoTSteps corr_matrix
global d Gamma Beta R
% Volatility Parameters: OHT (2003, P10).
mu=0.12; % mu
KappaQ=2.10; % kappa
v=0.016; % vhat
Sigv=0.33; % sigma
Rho = -0.51; % rho
% Heston requires additional parameters.
SigvP=0.021721;
KappaP=5.3;
LambdaP=1.84156;
s0=1; % Initial Stock Price
T=1;
dt=0.10;
NoSims=500000; %Number of Simulations
TimeSteps=1000;
NoRebal=6;
corr_matrix = [1,Rho;Rho,1];
```

```
d=0.34;

Gamma = 3; %(* Coefficient of Relative Risk Aversion *)

Beta = 0.96; %(* Discount factor *)

R = 0.03; %(* Gross interest rate *)

%(* Construct the grid of possible values of MuVec (Variance Grid) and
TVec (Time Grid) *)

global MuVec

lowvariance = linspace(0.002,0.10,12);

highvariance=linspace(0.10,0.60,8);

variance=[lowvariance,highvariance];

MuVec=unique(variance);

% d_setup_options

global strikes mat_1 mat_2

% Moneyness vector

strikes=linspace(0.9,1.2,7);

mat_1 = (T/12)*3; % 3-month

mat_2 = (T/12)*2; % 2-month

a_setup_params;

b_setup_gridpts;

c_setup_options;

global KappaQ v Sigv Rho

global SigvP KappaP LambdaP
```

```
global s0 T dt NoSims NoTSteps corr_matrix
global d Gamma Beta R
global mcs MuVec strikes
% Output:
% series of option prices associated with variance grid,
% atm1 (start) of period: 19 x 7: 19 variance grid points, 7 strike prices,
% atm2 (end) of period : same size matrix
grdpt_call1=zeros(length(MuVec),length(strikes));
grdpt_call2=zeros(length(MuVec),length(strikes));
for j=1:length(MuVec)
for s=1:length(strikes)
grdpt_call1(j,s)=hestonnlm(mat_1,s0,strikes(s),MuVec(j),SigvP,KappaP,Sigv,Rho,LambdaP,R);
grdpt_call2(j,s)=hestonnlm(mat_2,s0,strikes(s),MuVec(j),SigvP,KappaP,Sigv,Rho,LambdaP,R);
end
end

% Employ backwards recursion - HJB.
% Interpolation used to find optimal Wgts, Util. and Ret for each path.
Beta=1/(1+R);
DistETW=zeros(1,4); AvgHoldings=zeros(1,2);
KDE_Matrix=zeros(NoSims,1);
for s=1:length(strikes)
ETW=zeros(NoSims,1); % Expected Terminal Wealth
```

```
EPW=zeros(NoSims,2); % Expected Portfolio Weights

eval(['Weights= WeightsX',num2str(s),';']);

eval(['Returns=ReturnX',num2str(s),';']);

% Based on optimal policy at variance grid points, find interpolated

% portfolio return (ir) and wealth.

% Proceed backwards from time, T-2 to start of intvestment period.

for j=NoTSteps-1:-1:1

[ir, iw]=intpol(mcs(j,:),MuVec,Weights,Returns);

TF = isnan(ir);

ir(TF)=0;

TF2 = isnan(iw);

iw(TF2)=0;

ETW=ETW+Beta.*(ir');

EPW=EPW+iw(:,1:2);

end

KDE_Matrix=[KDE_Matrix,ETW];

% Statistics for Distribution of Terminal Wealth

meanETW=mean(ETW); stdETW=std(ETW); skewETW=skewness(ETW);
kurtETW=kurtosis(ETW);

DistETWX=[meanETW,stdETW,skewETW,kurtETW];

DistETW=[DistETW;DistETWX];

% Average Portfolio Holdings (by Strike)

meanEPW=mean(EPW);
```

```
AvgHoldings=[AvgHoldings;meanEPW];
end
DistETW(1,:)=[];
AvgHoldings(1,:)=[];
meanDistETW=mean(DistETW);
for s=1:length(strikes)
DistETW(s,:)=DistETW(s,:)./meanDistETW;
end

tic
% Step 1: Import Parameters.
clear all;
setup_parameters;
setup_nlm;
setup_gridpts;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initialisation %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
X0=[1,0,0];
% Store Option Prices at Variance Grid Point.
for k =1:length(MuVec)
[call10(k)]=hestonnlm(((T/12)*6),s0,K,MuVec(k),SigvP,KappaP,Sigv,Rho,LambdaP,R);
[call20(k)]=hestonnlm(((T/12)*6),s0,K2,MuVec(k),SigvP,KappaP,Sigv,Rho,LambdaP,R);
```

```
[path_returns]=NLMgenerate(MuVec(k),NoSims,call10(k),call20(k),K,K2,...
T,R,X0,s0,KappaP,KappaQ,MuX,MuY,v,Sigv,SigvP,Rho,EtaB2,EtaB1,...
LambdaP,LambdaQ,dt);
VGPT(k).pathreturns=path_returns;
VGPT(k).price=[s0,call10(k),call20(k)];
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Backwards recursion %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
EUUPA=zeros(1,1);
EPRET=zeros(1,1);
EWGT=zeros(NoTSteps-1,3);
VGPTpathweights=zeros(NoSims,3);
VGPTpathutility=zeros(NoSims,1);
VGPTpathreturn=zeros(NoSims,1);
VGPTweights=zeros(length(MuVec),3);
VGPTutility=zeros(length(MuVec),1);
VGPTportreturn=zeros(length(MuVec),1);
for t = NoTSteps-1:-1:1
for k =1:length(MuVec)
for j=1:NoSims
[VGPTpathweights(j,:),VGPTpathutility(j),VGPTpathreturn(j)]=sqpII(VGPT(k).pathreturns(j,:)
```

```
end
VGPTweights(k,:)=mean(VGPTpathweights);
VGPTutility(k)=mean(VGPTpathutility);
VGPTportreturn(k)=mean(VGPTpathreturn);
end
% Interpolation applied to cross section of simulated volatility
% at time T-1,T-2,..,1,0
[Uint,Rint,Wint]=interpolII(vmat(t,:),MuVec,VGPTutility,VGPTweights,VGPTport

Utility_at_t=mean(Uint);
Return_at_t=mean(Rint);
Weights_at_t=mean(Wint);
% Proceed recursively, iterating on the optimal portfolio weights from
% the previous time period
X0=Weights_at_t;
EUUPA=EUUPA+Utility_at_t;
EPRET=EPRET+Return_at_t;
EWGT(t,:)=Weights_at_t;
end
save ('C:\Program Files\MATLAB71\work\DSOP_code\data\der2_fiveper_itm.mat')
toc

function [x,fval,Portfolio_Rt]=sqp(ret,price,X0,R,K,K2)
```

```
options = optimset('Display', 'off', 'LargeScale','off');
% Optimisation
[x,fval]=fmincon(@objfun,X0,[],[],[],[],[],[],@constrfun,options);
Portfolio_Rt= x(1)*ret(1) + x(2)*ret(2) + x(3)*ret(3) + (1-x(1)-x(2)-x(3))*(1+R);
function f = objfun(x)
w_ti_1=1;%(1+R)^t;
w_ti= w_ti_1*(x(1)*ret(1)+x(2)*ret(2)+x(3)*ret(3)+(1-x(1)-x(2)-x(3))*exp(R));
obval=marg_ut(w_ti)*ut(1);
f=-obval;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Constraints %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [c,ceq] = constrfun(x)
% Equality Constraint
ceq=[];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Margin Requirements (MR) %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% MR_S (50% x Stock Price x Weight Held)
if x(1)>= 0
MR_S=0.5*x(1);
```

```
elseif x(1) < 0

MR_S=1.5*x(1); % investopedia.com: 150% of the value of the short sale

end

% Long Call (LC), Covered Call (CC) and Naked Call (NC).

% In case of a short stock position -

% Equates to zero for the purposes of

% MR calculation.

if x(1)>=0

NumStocks=x(1)/price(1);

elseif x(1)<= 0

NumStocks=0;

end

MR_LC1=[]; MR_CC1=[]; MR_NC1=[];

if x(2)>=0;

MR_LC1=price(2);% Long Position: Weight x Der1Price

elseif x(2) < 0 & price(1) > 1; % Short, ITM-Call

NumShortCalls1=-x(2)/price(2);

Covered1=min(NumShortCalls1, NumStocks);

Naked1=max(abs(NumShortCalls1-NumStocks),0);

MR_CC1=Covered1*price(2);

MR_NC1=Naked1*(price(2)+ max(0.15*price(1)-(max(K-price(1),0)),0.1*price(1)));

elseif x(2) < 0 & price(1) <= 1; % Covered OTM/ATM-Call
```

```
MR_CC1==0;

NumShortCalls1=-x(2)/price(2);

Naked1=max(abs(NumShortCalls1-NumStocks),0);

MR_NC1=Naked1*(price(2)+ max(0.15*price(1)-(max(K-price(1),0)),0.1*price(1)));

end

MR_LC2=[]; MR_CC2=[]; MR_NC2=[];

if x(3)>=0;

MR_LC2=price(3);% Long Position: Weight x Der2Price

elseif x(3) < 0 & price(1) > 0.95; % Short Position: Covered ITM-Call

NumShortCalls2=-x(3)/price(3);

Covered2=min(NumShortCalls2, NumStocks);

Naked2=max(abs(NumShortCalls2-NumStocks),0);

MR_CC2=Covered2*price(3);

MR_NC2=Naked2*(price(3)+ max(0.15*price(1)-(max(K2-price(1),0)),0.1*price(1)));

elseif x(3) < 0 & price(1) <= 0.95; % Covered OTM/ATM-Call

MR_CC2=0;

NumShortCalls2=-x(3)/price(3);

Naked2=max(abs(NumShortCalls2-NumStocks),0);

MR_NC2=Naked2*(price(3)+max(0.15*price(1)-(max(K2-price(1),0)),0.1*price(1)));

end

c=[-1+(MR_S + (MR_LC1 + MR_CC1 + MR_NC1) + (MR_LC2 +
MR_CC2 + MR_NC2))];

end
```

end