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An Exploration of Young Children's Number Sense on Entry to Primary School in Ireland

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Abstract

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The importance of number sense lies in its relationship with mathematical proficiency. Consequently, number sense is increasingly emphasised in curriculum documentation related to mathematics. The *Primary School Curriculum: Mathematics* (Government of Ireland, 1999b) is no exception. It implicitly stresses the importance of number sense and draws teachers' attention to a range of key aspects. My study is premised on the belief that young children's number sense is developed as a result of their everyday experiences with numbers, and these experiences form the basis on which further development of number sense, both in and out of school, takes place. Two questions are investigated: What number sense do young children demonstrate on entry to school?; What is the congruence between young children's number sense and the curriculum for the first year of primary school?

The first question is explored through individual experience-based flexible, focused interviews with fourteen children (eight boys and six girls) early in their first term at primary school. The age-range of the children was 4 years, 1 month to 5 years, 1 month. The second question is addressed by means of an analysis of relevant curriculum documentation and then linking that analysis to the data analysis.

A framework for exploring number sense in young children was developed. This included four aspects: Pleasure and interest in number; Understandings of the purposes of number; Quantitative thinking; and Awareness/Understanding of numerals.

Affective issues emerged as centrally important in the development of children's number sense. Counting purposes were most transparent for children, with generic, communicative and label purposes also identified by them. There were striking differences in the ways in which girls and boys constructed their metacognitive framework in relation to number. Children's

responses to quantification tasks provided interesting insights into how children perceived such tasks. The process of estimation appears to be challenging for many four-year-old children.

A number of areas of omission were identified in the teacher guidelines that accompany the curriculum documents. Implications were drawn with respect to how aspects of children's number sense, for example estimation, are developed at school. Implications for teacher education are also discussed.

Statement

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Education, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Chapter 1: Introduction

Background

There has been a change in emphasis in many countries in recent decades in relation to mathematics curricula for young children (Campbell, 1999; Anghileri, 2000). This change is characterised by a move from viewing mathematics as a set of skills and procedures, to seeing problem solving as a central focus. Mathematics, Campbell suggests, is now being viewed as a way of thinking about quantity, relationships and patterns. Anghileri describes reforms of the mathematics curriculum in Britain as shifting from an emphasis on standard procedures for calculating, to each child being able to observe patterns and relationships and to make connections so that they develop a 'feel' for number. She observes that the term 'number sense' is widely used in reform documents in many countries including the United States and Australia. Number sense generally refers to a type of 'flexibility' with number and with strategies for quantifying. But it goes beyond skills and understandings and includes '... *the nurturing of a positive attitude and confidence* ...' (Anghileri, 2000: 2) Thus it appears that 'number sense' is a relatively new and important concept in relation to learning and teaching mathematics in recent years.

Rationale

In Ireland a revised curriculum for primary schools was introduced in 1999. Prior to that, the last major review of the curriculum was in 1971. In the *Primary School Curriculum: Introduction* (Government of Ireland, 1999a) it is claimed that the revised curriculum incorporates current educational thinking and the most innovative and effective pedagogical practice. However, unlike the documents cited above, the curriculum documentation related to mathematics in schools in Ireland makes only one direct reference to number sense. It is stated that '*Quick recall of number facts and a strong number sense are important for efficient estimation.*' (Government of Ireland, 1999b: 32) Thus number sense is not emphasised, nor is it explicitly highlighted for discussion in the curriculum documentation pertaining to schools in Ireland. However, an initial analysis of that documentation reveals that the development of number sense is implicitly addressed in

several areas of the documentation. Thus it appears that 'number sense' is a topic that needs explication and investigation in the context of teaching and learning mathematics in schools in Ireland.

My interest in the topic

My personal interest in issues related to young children's number sense arose as a result of my appreciation, from the time that I was very young, of mathematics as an interesting and absorbing area of study. When my own son was about four- or five-years-of age, he began to show interest in large numbers and asked on several occasions what the biggest number was. I observed how he looked at me when I explained that whatever big number we could think of we could always add to it, and so it was an impossible question to answer. For several months afterwards he loved to play a game that consisted of him suggesting a particular number as the biggest in the world and me replying that I knew a bigger one, i.e., his number *and one more*. At that time I was teaching junior infant children (i.e., those in the first year of school) and I had observed a similar interest in number and appreciation of number amongst many of them. These experiences suggested to me that number is of interest to many young children. Consequently, a question that has always puzzled me was why so many children grow into adults that profess to hate mathematics or not to be able to understand how it works.

My professional interest in the issue of young children's number sense arises from my work in teacher education. As a lecturer in Early Childhood Education, I am responsible for planning and teaching a course on mathematics in the early years at school. When I began lecturing in 1998, one of my first tasks was to plan and prepare course work for student teachers. As I researched the course in early years mathematics, I frequently came across the term 'number sense' but it was rarely explained or developed. In 1999, the revised curriculum for primary schools in Ireland was published. It included supporting documentation that detailed the curriculum content and recommended teaching approaches for the various subject areas, including mathematics. Number sense had already emerged as an issue in my reading of the international literature, and I sought to see how

it was reflected in the documentation pertaining to teaching and learning of number and mathematics in Ireland.

The research questions

In this study, I set out to explore the number sense of a group of young children through ascertaining their number knowledge, their interest in/disposition towards number and their views of the usefulness and purposes of such knowledge. I also explore how the number sense of a specific group of four-year-old children 'fits' with the mathematics curriculum that those children will experience at school. Thus I state my research questions as follows:

- What number sense do young children demonstrate as they start formal schooling?
- What is the congruence between the findings of this study in relation to children's number sense and the statutory curriculum in mathematics for the first year of school in Ireland?

Bridging the gap between informal and formal learning

Ginsburg and Seo (1999) have observed that it has long been asserted that mathematics education should in some way build on children's informal knowledge, i.e., the knowledge which they have developed as a result of everyday interactions with adults and other children. School mathematics has been characterised as de-personalised, self-contained activity that is divorced from other aspects of children's lives (Confrey, cited in Cobb and Yackel, 1998). Furthermore, there is considerable research evidence that shows a gap between formal, i.e., school mathematics, and the mathematics used to solve tasks in everyday life, generally characterised as informal mathematics (e.g., Resnick, 1987; Nunes *et al.*, 1993; Lave and Wenger, 1999a). Indeed I observed such a gap on many occasions over the years I worked with young children. The insights gained from this study offer an opportunity to consider how, from an integrated (affective and cognitive) perspective, the perceived gap between formal and informal learning can be minimised in respect of number sense. As a result of my findings, I explore how teachers can find out about young children's prior knowledge in relation to number sense. I also consider a number of areas of pedagogy that

should be specifically addressed in relation to the development of children's number sense during the first year at school (See Chapter 6).

Learning frameworks

I have explored a number of theoretical perspectives in my search for a theory of learning that would provide a framework within which to explore my research questions.

Phenemonography

My journey began with Pramling's (1983) unique and innovative study of children's views of learning. I found her study inspiring in that it demonstrated what was to me, a really interesting and novel approach to thinking about learning in young children. Pramling (1996) worked from the premise that learning is about understanding and making sense of the world, and that in order to understand young children's learning it is necessary to view it from their perspectives. She used a phenomenographic perspective in her study of children's views of learning, i.e., one that focuses on children's experiences of a particular phenomenon as the object of research. In the pilot study, I adopted a phenomenographic approach and I explored in some depth the possibility of using it in the main study to investigate children's views of the purposes of number. Also, I followed a phenomenographic procedure for the analysis of data in the pilot study (as I understood it then) but I found that when I had presented the categories of description, i.e., the findings, I was left with a sense of unfinished business. Saljo (1996) also describes sometimes experiencing a similar '*Is that all?*' reaction to phenomenographic research and he suggested that, in his opinion, this reaction may well be typical also of those who read this type of research. Certainly, in the pilot study, I found that it enhanced the presentation of findings to look to other theoretical frameworks to help me interpret the data. As I understand it, in pure phenomenography, there is little point in dwelling on aspects such as talk, meaning of words or context, since the focus is children's experiences and the differences in these. This raised a dilemma for me in terms of a theoretical framework because I now realised that the social and cultural aspects of children's self-reported experiences were the ones that I found most interesting and these also

appeared to shed most light on my research questions. As a result of my reading, I learned that one of the strengths of sociocultural approaches is that they do take account of these aspects of learning situations and this was what initially drew me to explore these frameworks further.

A sociocultural perspective on learning

I explored the similarities and differences between phenomenography and other perspectives on learning (Marton and Booth, 1997). Then, having also investigated sociocultural approaches to learning, I decided to adopt a broadly sociocultural framework since related theories appear to share some similarities with the features of phenomenography that attracted me in the first instance. For example, phenomenographers, with their focus on learners' experiences, claim to focus on the child's world rather than on the child (Marton, 1981). The context of learning is key for socioculturalists also, their emphasis being on '... *the conditions for the possibilities for learning.*' (Cobb and Yackel, 1998: 184) Sociocultural theories build on earlier theories such as those of Piaget and Vygotsky, extending some aspects of them in particular ways and rejecting others (e.g., Rogoff, 1999). While individual activity is recognised as important in learning by both socioculturalists and by constructivists, in sociocultural approaches priority is given to sociocultural processes while psychological processes are prioritised by constructivists (Cobb and Yackel, 1998).

In particular, the sociocultural approach to understanding learning as outlined by Rogoff (1990; 1998) is important to me. I found that her perspective fitted comfortably with the ways in which I observe the learning and development of individuals. My model of mind is one that sees reality as constructed, not made. I consider that this construction takes place as a result of participation in social activity and from collaboration with other people. My position is that learning is both an individual and a collaborative process. It arises from both individual activity and from participation in social activity. I share Rogoff's (1998: 715) view that

> ... *children's participation in sociocultural activities is complexly and multidimensionally structured with important contributions from individuals, their social*

> *partners of varying status and expertise, and the structure of the cultural/historical activities in which they participate and which they contribute to shaping further.*

Using Rogoff's perspective, learning is viewed as a collaborative process and occurs in shared involvement in community/institutional endeavours. Learning is participation, and development is seen as a transformation of participation. From this perspective, the terms learning and development can be used interchangeably. This accords with the phenomenographic view as expressed by Pramling (1995: 136) that since children '... *are in an interaction with the surrounding world at all times, learning cannot be separated from development.*' I too consider that it is difficult to differentiate between development and learning and I see them as aspects of an overall process. The collaborations that lead to learning can take many forms: '*Intended or accidental, mutual or one-sided, face-to-face, shoulder-to-shoulder, or distant, congenial or contested ...*' (Rogoff, 1998: 728)

I very much agree with Rogoff's view that we need to devote greater attention to the process of collaboration in other sociocultural activities beyond social interaction with experts or with peers in situations that are largely intended as instructional. Specifically, from a sociocultural perspective, there is a need to focus on collaborations between adults and preschool children in informal everyday activity.

Rogoff (1995) places emphasis on the individual learner, but also on what she terms *interpersonal* and *community* aspects. Her view is that individual learning cannot be understood outside of an activity or of the people participating in it. Thus she conceptualises learning as the development of mind in sociocultural context. She adopts a non-dualist perspective on person and world (i.e., they are not separate but mutually constitutive) that is similar to the phenomenographic perspective (Marton and Booth, 1997). Rogoff's (1990: 7) framework is one that conceives of children as '*... apprentices in thinking, active in their efforts to learn from observing and participating with peers and more skilled members of their society ...*'

Children's active participation is seen as the process through which they gain facility in an activity (Rogoff, 1995). However, she argues that children

need to be guided in that participation. She explains the term 'guided participation' as :

> ... *the processes and systems of involvement between people as they communicate and co-ordinate efforts while participating in culturally valued activities. This includes not only the face-to-face interaction, which has been the subject of much research, but also the side-by-side joint participation that is frequent in everyday life and the more distal arrangements of people's activities that do not require co-presence ... The 'guidance' referred to in guided participation refers to observation, as well as hands on involvement in an activity.* (Rogoff, 1995: 700)

Phenomenographic and sociocultural approaches differ on the question of what characterises learning, i.e., they differ in their responses to the important question of 'What is it to learn? In phenomenographically-oriented studies '... *people's experiences and their awareness are seen as the main objects of research, experience is viewed as a particular relation between object and subject which encompasses them both.*' (Pramling, 1996: 565) From a sociocultural perspective the main object of research is the development in participation in sociocultural activity (Rogoff, 1990; 1998). Learning is seen to be a consequence of collaboration in social activity for socioculturalists, whereas for phenomenographers it derives from the child's experience, but the interrelationships that constitute the activity are not seen as central. Because of their different understandings of learning, the approaches to the analysis of data within the phenomenographic and sociocultural frameworks are quite different. In a sociocultural framework, the unit of analysis is the sociocultural activity. In contrast, phenomenography focuses on the individual's experience as the unit of analysis.

Rogoff (1998: 688) expressed her sociocultural approach to analysis thus:

> ... *an examination of individual, interpersonal, and community/institutional developmental processes involves*

> *differing planes of analysis, with any one plane being the focus, but with the others necessarily observed in the background ... In an analysis focusing on individual contributions to sociocultural activities, the individual's contributions are in focus while those of the other people are blurred, but one cannot interpret what the individual is doing without understanding how it fits with ongoing events.*

Analysis in the *personal plane* focuses on the personal processes by which, through engagement in an activity, individuals change and become prepared to engage in subsequent similar activities. In relation to the *community plane*, the focus is on children participating with others in culturally organised activities. In that case, the nature of the activity, how it relates to the processes and institutions of the community and the tools and artefacts used are important. When dealing with the *interpersonal plane,* the focus is on how people communicate and co-ordinate efforts in a variety of interactions. In Chapters 4 and 5, I use these different planes of analysis to construct a holistic picture of young children's number sense.

Other influences

Analysis of Piagetian and neo-Vygotskian theories (e.g., Rogoff, 1998) show that while they emphasise different phenomena, they also have some ideas in common. My study of number sense is situated in the domain of mathematics where, for many years, constructivist frameworks dominated ideas about learning. Such frameworks pervade much of the writing in relation to various aspects of number sense. Consequently, in this study, some constructivist influences (e.g., von Glasersfeld) emerge in the review of the literature (See Chapter 2) and in the discussions of the findings (See Chapter 4). Increasingly, research in mathematics presents a situative perspective and that (marginal) influence is also discernible in my study. In particular, Greeno's (1991) theoretical analysis of number sense from a situative perspective (See Chapter 2: Section B) resonated strongly with me, even though my perspective on learning falls short of subscribing to the view that learning is located entirely in lived experiences. Since neither the views expressed by Greeno (1991) nor those of von Glasersfeld (1987) are

incompatible with those expressed by Rogoff above, I included their perspectives (and other related ones) in my study where they were useful (Sfard, 1998).

In summary, this study draws heavily on Rogoff's (1990; 1995; 1998) sociocultural theory of learning with the influences of other theoretical perspectives discernible at various points. For example, the phenomenograpic influence is particularly obvious in relation to those aspect of the study concerned with children's views and experiences related to purpose of number (See Chapter 2; Chapter 4: Section B). Where other approaches had something to offer as I explored young children's number sense, I made use of them.

Description, aims and scope of the study

This study explores, from young children's perspectives, aspects of the sociocultural context of early number learning. I argue that young children develop a number sense through their participation in sociocultural events that involve number. As they participate and collaborate in everyday activity they develop a 'number sense'. This number sense is not just related to what children know about number; it is also related to the conditions of their learning, the processes by which they learn, and the affective impact of such learning on their dispositions as learners (See Chapter 2). These concerns have been referred to as the *who, what* and *why* of learning (Anning and Edwards, 1999). In my study, children's views and understandings of their early experiences with number are derived through the process of informal individual interviews with fourteen children, eight boys and six girls. It is generally acknowledged that interviewing is not a simple method to adopt with young children, but one that requires considerable care and attention (e.g., Doverberg and Pramling, 1993; Ginsburg, 1997). It is especially important to ensure that children are comfortable with the process and willing to communicate their views, experiences and understandings in an open way. Issues related to consent are discussed in Chapter 3.

The interviews that provided the data for my study were conducted at a very specific point in time, i.e., during weeks two, three and four of term one. I interviewed children as they started school for a number of reasons. Firstly I

wanted to explore children's informally acquired number sense, as they revealed this to me, and I felt that I needed to do this as soon as possible after entry, and before they gained any significant experience of school. I felt that if I didn't explore children's viewpoints then, they might respond later in ways that they perceived as ones that were expected at school. Their responses might be somewhat different than those that were not yet influenced by the process of becoming a pupil (Brooker, 2002).

In my experience there is an eagerness about the new experience of school and about learning in those early weeks that is not quite as discernible later. Indeed, Pramling (1983) found that there was an important qualitative change in children's conception of learning at around the age of entry to school. She speculates that when children begin school, they direct a lot of their attention and interest to differences between the school and pre-school experiences. Children's responses concerning their expectations of school then reflect their attention to such differences. Munn (1997) also suggested that, on the basis of her findings, the transition to school seemed to trigger a change in some children's beliefs about the purpose of counting. From these arguments, I concluded that there is a limited window within which to explore children's informally acquired number sense. I judged this to be the point of entry to school, since, at this point, children are likely to meaningfully engage in discussions about learning and about number but, as yet, are relatively unaffected by the experiences of school.

The study

This is a study about the number sense of young children at the point of entry to formal schooling in Ireland. It focuses on learning about number and developing a number sense in early childhood. As such, it is multi-disciplinary in nature, drawing from a number of different disciplines, including developmental psychology, cognitive psychology and mathematics education.

The chapters that follow tell how:

- I developed a framework for exploring number sense with young children (Chapter 2);

- I used a variant of the clinical interview method to explore number sense in young children (Chapter 3);
- I analysed the curriculum documentation in order to assess the attention therein to the central aspects of number sense in young children (Chapter 3);
- I arrived at the conclusion that number sense in young children is different in different contexts and is linked to specific tasks (Chapter 3, 4);
- I surmised, on the basis of the data, that boys and girls differ in relation to their participation in number-related experiences (Chapter 4);
- I discerned, on the basis of the data analysis, aspects of young children's quantitative thinking as they responded to various tasks (Chapter 4);
- I concluded that estimating cannot make sense to many four- and five-year-old children (Chapter 4);
- Children's accounts of their participation in collaborative activity contribute to our understanding and appreciation of the factors that influence their number sense (Chapter 5);
- My practice in teacher-education is being influenced by this research (Chapter 6);
- Findings of this study will potentially affect the thinking of curriculum planners and reviewers, and ultimately the pedagogy of infant teachers (Chapter 6).

A clear and comprehensive statement of what is meant by number sense (as it relates to four- and five-year-old children) was essential to this study. Thus, I began my enquiry with the search to clarify the term 'number sense' and to ensure that the meaning that I attributed to it adequately reflects the experiences and interests of young children.

Chapter 2: Reviewing the Literature

From a sociocultural perspective, for most four-year-old children in Western European culture, early experiences in the home are likely to provide the best opportunities for informal learning about number, and thus for developing number sense. Therefore, I surveyed the extensive literature related to informal learning in the domain of number and from these I selected a number of key studies that focused on children's home experiences and early interactions about number. These studies were important in underpinning my argument that young children's number sense is developed as a result of their everyday interactions in their families and communities. I then explored the meanings/usages of the term number sense. As a result of the review, I established that there was no suitable framework for considering number sense in young children. Thus I developed a suitable framework arising from the review and using this framework, I restated the research questions in greater detail than previously (See Chapter 1) and in line with the findings of this review.

Section A: Evidence of home use of mathematical language

For the purposes of this study, I define 'informal learning' about number as that which children experience as a result of everyday interactions with other people. This may or may not include explicit teaching, and it may occur in a variety of settings including school. An important area in which research shows children have clearly established informal learning is in relation to number words. From a sociocultural perspective, Durkin *et al.* (1986) studied the interpersonal contexts in which numbers are first encountered and used. Children were seen to encounter number and counting in the course of interactions with their parents/care givers, and children's use of number words in conversation and in count sequences increased over time. Mothers used many number words incidentally, but they also used a variety of discernible strategies to familiarise children with numbers. However, much of the numeric input from mothers was heavily laden with situational and linguistic ambiguity, leading to the conclusion that teaching and learning in the acquisition of number words and counting

is not straightforward. Furthermore, the authors suggested that the conflict introduced as a result of the ambiguity actually promoted development. This suggests to me that the nature of the guidance of children's participation in numeric activity is central for the development of their understanding of the various uses of number. Indeed, Saxe *et al.* (1987), as a result of an extensive socioculturally focused quantitative research study, suggested that adult guidance was instrumental in providing children with a framework for understanding quantitative tasks. Their study revealed a reciprocal dependence between children's everyday activities and their numerical understanding. Children regularly participated in mathematical activities that they had generated themselves as well as in activities that were socially generated. Also, children adjusted their goals to their mother's efforts at guidance by responding to their mothers' directions. These findings demonstrate that the numerical environment of the preschool child is constituted through a process of active participation and negotiation on the part of ***both*** children and adults.

Further evidence of the nature of children's mathematical experiences in the home was provided by Walkerdine (1988: 81) who described two types of tasks or strategies involving number that were a feature of practices in the home. She termed these *instrumental* and *pedagogic.* Instrumental referred to tasks in which the main focus and goal was a practical accomplishment and in which numbers were an incidental feature of the task as in, for example, cake making. In the pedagogic tasks, numbers were the specific focus of the task, with the teaching and practise of counting central. From her situative perspective, Walkerdine argues, convincingly, that what children learn and the mathematical/numerical meanings they create within these practices relate, '... *to the power and relationships inscribed within them.*' (p. 31) She uses the example of the production of terms such as *more* which develop meaning in terms of '... *their relation with a practice of regulation of consumption.*' (p. 31) These are specific lived practices and the meanings produced reflect that.

Aubrey *et al.* (2000) carried out an analysis, from a mathematical perspective, of a sample of the Wells transcripts dating from the 1970's. These related to a longitudinal study of the linguistic development of a

group of 128 three- and four-year-old children. The analysis focused on the content area of the mathematical discourse in relation to practical activities, in order to consider the sorts of tasks taking place and what was being said about them. They found that two-thirds of the mathematical references made were related to number and counting, with the remainder mainly related to measure. This strongly suggests to me that aspects of young children's counting will provide some important insights about their number sense.

Variations in children's experiences about number

Fuson (1988), in a seminal study, argued that children's numeric thinking was very dependent on specific experiences provided in their daily lives. The validity of this claim was borne out when variations in relation to number experience in the home were seen to affect the level of concept development of children starting school in New Zealand (Young-Loveridge, 1989). Here the focus was on five-year old children's numerical understandings and the extent and nature of their mathematical experiences in the preschool period. The findings are important since they give a detailed picture of the unique 'mathematical' experiences of the (six) children studied. The range of potentially rich mathematical experiences which were engaged in by the children to varying degrees included activities such as setting the table, shopping, playing dice games, telling the time and reading the calendar. Experience with numbers in the home varied and was related to children's scores on a range of number tasks. The high scorers had been involved in many mathematically related experiences with practical purposes and, most importantly, were guided in these by parents/adults who encouraged the child's interest in number or related skills such as counting. These adults seemed to hold high expectations of the children in relation to number and they included children in activities such as card playing, sometimes explicitly teaching them related skills. They also ensured that children had a wide range of experiences related to number. The nature of the interactions between parents and children, whose number understanding was high at entry to school, was different to those where children didn't score highly. Thus, this study indicates to me that the nature and extent of experiences, but *especially* the guidance that the children received, was seen to be critical in relation to the development of their mathematical

understandings. In particular, this study suggests that explicit and focused attention to the role of numbers in a variety of activities is crucial.

More recently, Aubrey *et al.* (2003) compared the interactional patterns of two mother-child dyads and found them to be strikingly different in terms of quantity and increase in rate of mathematical dialogue over time. A qualitative analysis distinguished different parental pedagogical styles of supporting children's early mathematical development. It follows that young children's number sense is differently supported/guided in their interactions with parents/adults, with some forms of guidance more effective than others. However much of the research that concerned itself with interactions and relationships as well as activities (i.e., Walkerdine 1988; Aubrey *et al.* 2003) has focused on what occurs between young girls and their mothers, but there is very little data about how parents support young boys in this area of learning.

To summarise, research shows that:

- Children's interactions about number in the home vary in quantity and quality and this impacts on children's number sense;
- Many of children's informal experiences about mathematics revolve around number and counting;
- Children's understandings about number/number-related situations vary depending on the nature and extent of their informal experiences with number;
- The nature of the guidance that children receive in developing their number sense is crucial;
- While there is some data on how parents support young girls in the development of their number sense, there is comparatively less known about how they support young boys.

Section B: Number sense and young children

The questions that drove this section of the review were 'What is number sense?', 'How does the term *number sense* relate to terms such as *numeracy*, number and mathematics?' and 'What does number sense in young children look like?'

Characterising number sense

Being 'numerate' has been described as:

> ... *an 'at-homeness' with numbers and an ability to cope with the practical mathematical demands of everyday life ... an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease.* (Cockcroft, 1982: para 34).

Although young children (and some adults, too) could not yet be said to be numerate, they are, nevertheless, in the process of developing an understanding of some of the ways in which number can be used in everyday activity, and in that sense, can be said to be developing number sense. The term *number sense* came into usage because of a desire amongst reformers to replace the word 'numeracy' by one that did not have the same abstract ring (McIntosh *et al.*, 1992). It seems that what was involved in coining this new phrase was an attempt to embrace a variety of applications of number in real-world situations and to balance the skills based approach, often prevalent in past curricula, with other aspects of understanding number. A satisfactory definition of number sense has proven to be very elusive (Sowder and Schappelle, 1989). Interestingly, Resnick (1989) suggests that, rather than look at definitions, we might be better to describe the characteristics of number sense. Sowder's (1992) interpretation of number sense as an holistic concept of quantitative intuition, or a feel for numbers and their interrelationships, suggests something that is wide-ranging and so difficult to pin down. While I agree that number sense is a holistic concept, I think it is important to try and break it down to constituent aspects if it is to be addressed in the curriculum.

Demonstrating number sense

As Chapter 1 suggests, a number of countries have included reference to number sense in their curriculum documentation. Number sense is often equated in such documents with '*flexibility*' and '*inventiveness*' for calculation (Anghileri, 2000: 2). It is suggested that, as children make

connections between numbers, they develop a flexibility in their thinking in relation to numbers. Indeed, Anghileri describes this development in flexibility as something that is happening from the earliest stages of learning onwards. The idea that number sense is something that begins to develop long before formal schooling is also emphasised by McIntosh *et al.* (1992). However, their framework for examining number sense which includes knowledge of, and facility, with numbers; knowledge of, and facility with, operations; and application of this knowledge to computational situations, is not sufficiently consistent with their emphasis on number sense as something that begins in the preschool period. In particular, it does not explicitly recognise the nature of early learning.

At this point, number sense could be said to be a holistic concept related to everyday use of number and to encompass skills, understandings, disposition and flexibility. However, Bereiter and Scardamalia (1996: 488) who characterise number sense as '*knowing one's way around numerical domains*' express a contrastive view. They consider that number sense manifests itself in the ability to make flexible, efficient and almost effortless use of mathematical facts. They further describe it as a basis for shortcuts, error checking, approximation and taking advantage of situation characteristics. However, as I argued above, when we talk about number sense in relation to young children, we must consider how they are experiencing number in the course of their everyday lives because everyday life is the source of all of their learning, including their number sense. Young children's number sense has to differ from that displayed by more experienced number users since the experience of four-year-old children is relatively limited. From a sociocultural perspective, it appears that Bereiter and Scardamalia's characterisation is too narrow for the purposes of considering number sense in young children. However, that characterisation does show that, for some, the nature and extent of the learner's everyday experiences of number is not considered as an important aspect of number sense. Such a characterisation also illustrates the need to identify characteristics of number sense that might apply to four–year-old children, and which take into account key aspects of early learning such as experience and affect.

Number sense and everyday experiences

Greeno's (1991) in-depth theoretical analysis of number sense looks beyond definition, beyond relational understandings and beyond constituent skills. He asks us to imagine number sense as a form of '*cognitive expertise*' in the domain of number and quantity. He describes such expertise as '*... knowledge that derives from extensive activity in a domain ...*' (p.170) He describes the domain of number as an environment which is large and complex and which is populated with numerical objects that a person can get to know and use. He also describes how it is necessary to develop understanding of the domain, and the objects within it, at two different levels. Firstly, as he explains, understanding must be developed at the level of knowing the objects, where they are and the relations among them. Secondly, he argues, it is also necessary to know how to use these objects and the different uses to which they can be put. In this way, according to him, the structure of the environment of number and quantity become familiar. Thus Greeno's analysis extends the notion of number sense to encompasses dynamic everyday applications, much as was done by both Fuson (1988) and Howden (1989). He also suggests that as the quantitative objects, i.e., numbers, gradually develop in meaning, these concepts come to have properties that make them particularly useful for mathematical reasoning. Greeno (1991: 197) also explicitly acknowledges the role of adults in relation to the development of number sense, recognising that '*... someone who already lives in the environment is an important resource for a newcomer ...*' I feel that Greeno's characterisation is very useful in relation to young children since it appears to take account of the fact that the development of their number sense needs to be guided by more experienced others and is intrinsically bound up in everyday experiences. It acknowledges that it is through participation in numerical experiences that young children come to appreciate a range of possibilities for the use of numbers. With Greeno's characterisation, we get more emphasis on the conditions of learning rather than on the *what* aspect that appears to dominate, for example, Bereiter and Scardamalia's characterisation as discussed earlier.

The relational aspect of number sense

Howden's (1989: 6) description of number sense is rooted in children's experiences. She describes children who are developing number sense as those who have *'... a special feel for numbers, an intuition about how they are related to each other and the world around them.'* Howden provides us with an example of number sense: in response to her question asking them what they thought of when they heard the number *'twenty-four'*, first-graders (aged seven years) gave a range of responses that demonstrated their number sense. These included *'Two dozen eggs'*, *'Two dimes and four pennies'*, *'My mother was twenty- four last year'*, *'The day before Christmas'* and *'My brother got a cut this long and it took seventeen stitches to sew it up. That's almost twenty-four.'* (p. 6) Howden's approach is very useful, I feel, since it brings everyday experiences most clearly into focus. A number of other researchers also consider relational understanding to be important in number sense, but they appear to be using the word *'relational'* in a narrower sense than Howden. For example, Hiebert (1989) suggests that what is involved is an understanding of the relationships between what he refers to as *'the two functions of numerical symbols.'* (p. 82) He identifies those as *'records of something already known'* (p. 82) and as *'tools for thinking'* which he equates with *'objects of thought.'* (p. 83) Alternatively, they can be seen as relationships between quantity and counting (Van deWalle, 1990) or as networks of mathematical connections (National Council of Teachers of Mathematics [NCTM], 1998). In summary, awareness of the relations between numbers certainly emerges as central to number sense but some authors (e.g., Fuson, 1988; Howden, 1989; Greeno, 1991) understand *'relational'* in a wider sense than others. Either way, it is generally regarded as a defining feature of number sense (Anghileri, 2000).

Numerals (i.e., written numbers) and number sense

An understanding of the everyday use of numbers implies, in my view, that young children are also aware of numerals in communication between people. Indeed, a sense of numerals has been identified above as an essential aspect of number sense (Hiebert, 1989). I suggest that there is a need to consider also children's understandings of the everyday use of these. For Greeno (1991) and Gray (1997), the importance of knowing the symbols

resides in knowing about symbol ambiguity; the same symbol can have different meanings in different situations. For example, the numeral 50 on the front of a bus means something quite different to 50 on a road speed sign. Greeno (1991; 196) highlighted '*knowing notations*' which he equated with '*fluency in the notations of arithmetic*' as an important aspect. He considers such knowledge as essential since it incorporates recognition of the ambiguities of symbols and of the necessity to pay attention to the context or situation in which the symbol is being used. This is important to me since, from a sociocultural perspective, the context is part of the meaning.

Displaying a friendliness with numbers

Silver (1989: 92) argued that number sense may be as much about dispositional considerations as it is about cognitive matters, and he describes '... *an important, more subtle aspect of number sense* [than specific cognitive competence]*: namely the disposition toward numerical activity, or more generally toward mathematics, that it make sense.*'

In Chapter 1 (p. 1) the nurturing of positive attitudes and confidence was seen to be important in relation to number sense. My own view is that descriptions of number sense, (e.g., Howden, 1989), that focus around '*friendliness*' and '*a special feel for numbers*', do imply a dispositional aspect. Indeed disposition has been identified by DeCorte *et al.* (1996) as an important aspect of learning in the domain of mathematics. These authors adopt the characterisation of disposition of Perkins and colleagues (Perkins *et al.*, 1993: 4), which has three components: *inclination, sensitivity* and *ability*. DeCorte and colleagues use this three-dimensional characterisation to argue that dispositions cannot be taught directly but that they flourish over an extended period of time. Children play an active role in the development of their disposition towards number by participating and collaborating in number related activity. Indeed, Rogoff (1990: 171) draws our attention to what she refers to as '... *the essential nature of children's own eagerness to partake in ongoing activity.*'

Some authors have sought to include the dispositional issue in their characterisations of number sense. For instance, the notions of *inclination*

and *ability* featured in McIntosh *et al.*'s (1992: 3) description of number sense (as it pertains to older children and adults): '*Number sense ... reflects an inclination and ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information.*'

To summarise, research indicates that:

- Number sense is a holistic construct that is difficult to define;
- Number sense is concerned with the development of a wide range of understandings, skills and attitudes about number that extend beyond those generally associated with numeracy and encompass everyday uses;
- Flexibility with number is a key aspect of number sense;
- Characterisations of number sense often appear to concern themselves with aspects pertaining mainly to older children and adults. Consequently, there is a need to develop a framework that relates specifically to younger children;
- Number sense, particularly as it pertains to young children, must take account of everyday experiences;
- Disposition must be acknowledged as an aspect of young children's number sense.

In the next section, I consider how number sense can be explored with young children as they start school.

Section C: Exploring number sense in young children

It appears from the review above that number sense is best described as a multi-faceted concept that, for the purposes of discussion and analysis, can be described as having a number of different aspects. These include:

- A dispositional aspect;
- An aspect that recognises everyday experiences of number;
- A flexibility and inventiveness aspect;
- An aspect that relates numbers to each other;
- An aspect that pertains to understanding numerals;
- An aspect that recognises different uses of number;
- A quantification aspect.

Indeed, Silver (1989: 28) suggested that there is a need to understand better the separate 'pieces' of number sense in order to see how they might all tie together. He suggested this 'pieces' approach as a way of getting a handle on what he called '*a paralyzingly large phenomenon.*'

I am very much in agreement with Carpenter (1989) when he suggests that number sense in very young children may well look different from that of older learners. We have seen earlier (See Section A of this Chapter) that preschool children's numerically related experiences vary considerably from child to child. From the review above (See Section B of this Chapter), and my observations at the pilot study stage of this research, I conclude that a framework for examining number sense in very young children will need to encompass a number of aspects. It will need to differ considerably in emphases to the framework presented by McIntosh *et al.* (1992) which appears to emphasise aspects more in keeping with older, more experiences learners who have been exposed to formal teaching of number in a school setting.

I propose to follow Silver's advice above and adopt a 'pieces' approach in constructing a framework related to number sense as it pertains to young children. However, the pieces will reflect the holistic understandings related to number that four-year-old children might reasonably be expected to have developed from the accumulation of their experiences and interactions about number.

A framework for considering number sense in young children

Arising from the considerations above, I present the following framework reflecting key aspects of number sense as it relates to four-year-old children:

- Pleasure and interest in number (Silver, 1989; Howden, 1989; DeCorte *et al.*,1996; Anghileri, 2000);
- Understandings of some of the purposes of number (Fuson, 1988; Howden, 1989; Greeno, 1991);
- Quantitative thinking, for example counting; relating numbers to other numbers; subitizing; estimating (Greeno, 1991; McIntosh *et*

al., 1992; Bereiter and Scardamelia, 1996; NCTM, 1998; Anghileri, 2000);

- Awareness/understanding of numerals (Hiebert, 1989; Greeno, 1991; Gray, 1997).

In the following section, I explore the literature related to each of these aspects of the framework.

Pleasure and interest in number

McLeod (1992) argues strongly that the affective and cognitive domains are intimately linked and that any research in mathematical learning really needs to pay attention to both aspects because you cannot attend to one without taking account of the other. This converges with the views of Silver (1989) and others cited earlier (See Section B of this Chapter), and with my own view of the holistic nature of early learning. In the pilot study, I observed that children's accounts of their experiences had great potential in determining the types of affective factors that were operating in relation to number sense. As they related their experiences and talked about their views, it was possible to begin to understand how they engaged with number, how they viewed number and the contexts in which number was visible for them. I was able to observe how individual children positioned themselves in relation to number and when they appeared to 'call up' number as a salient feature of their everyday interactions in social and physical settings. Some children explicitly expressed a liking for number and number-related activity as they talked about their experiences. Similarly, the disinterest of some children was evident in the ways that they responded to the tasks and questions. Essentially, my contention is that children's disposition towards number was discernible in the data. Research does indicate that dispositions can be detected in very young children, as early as three- and four-years of age (Dweck, 2002). Bertram and Pascal (2002: 94) describe dispositions in early childhood as '... *environmentally sensitive. They are acquired from and affected by interactive experiences with the environment, significant adults and peers ... positive dispositions are learnt but they are rarely acquired didactically.*' Young learners' dispositions are increasingly seen as a central aspect of learning in the early years (e.g., Katz, 1988; Carr and Claxton, 2002). In the pilot study, I

observed that some children displayed more awareness of number in the environment than others; were more inclined to have a sustained conversation about numbers; and showed more enthusiasm for number-related activity, all of which seem to me to demonstrate a disposition towards number.

Looking once more at the components of disposition as delineated by Perkins *et al.* (1993), I suggest that the word *sensitivity* is synonymous with awareness because in respect of number sense they both imply to me, the idea of being 'tuned into' number-related aspects of the environment. Carr and colleagues (1994: 265) have referred to a heightened awareness as '*an orientation to numeracy*' and their findings suggested that this was fostered in families where attention was drawn to numbers and their usefulness for solving everyday problems. They considered such family practices as essential '... *for helping young children to develop a good number sense early on.*' (p. 265)

Carruther's account of her daughter's self-initiated numeracy-related actions on the world included an account of Savoy's high level of interest in, enthusiasm for, and playfulness with number over a two-year period from the time she was about twenty months old. She described the acquisition by Savoy of what she terms a '*mathematical set*' (Worthington and Carruthers, 2003: 203), which she defined (after Holdaway, 1979) as '*the ability to tune in with appropriate action* [in mathematical contexts].' This ability was seen to be closely related to family culture and was characterised by the development of an increasing awareness of number and of the ability to use number in dealing with the world. I think that what is described here in relation to Savoy is an example of an orientation to numeracy as described above by Carr and her colleagues.

In summary, in the context of my study it seems possible to detect some young children's orientation/disposition to number sense on the basis of their responses in discussions about number, and in their interactions in relation to number-related tasks.

Evidence of understandings related to the purposes of number

Exploring children's experiences

Observational studies of young children in the home (e.g., Durkin *et al.*, 1986; Saxe *et al.*, 1987; Bottle, 1999; Aubrey *et al.*, 2003) and in organised preschool settings (e.g., Munn, 1995), have given us some information about the interactions children have with others regarding the use and purpose of numbers in everyday activity. One mother's diary related to her young children's uses of number words in early childhood gave a fascinating insight into the effects of specific experiences on the development of concepts of number. It showed the importance of the availability of resources in the environment, in particular people, with whom to discuss number and number-related experiences (Fuson, 1988).

The literature suggests that one key way in which children may convey an ease with number is through their understandings of the ways in which numbers relate to the world around them (Howden, 1989; Greeno, 1991). Children's own accounts of their experiences relating to number can also offer insights into their number sense, since it is through these that children convey, amongst other things, their implicit understandings of purpose.

Seeking children's views

There are few studies that address the question of young children's views on issues related to learning either generally or in domain-related topics, such as number. Pramling's (2004: 4) analysis is that researchers seem to think that '... *children must be 'close to becoming adults' in order to be allowed to be heard and to express their perspectives.*'

Pramling's (1983) early work in which she sought children's views on learning is a useful benchmark in an otherwise sparsely populated field of studies that seek to ascertain children's views about metacognitive issues. Similarly, Munn's (1994) study related to children's beliefs about counting is one important exception to the lack of research into young children's views in the domain of number. Her findings provide compelling evidence that children's ideas about the purposes for counting are very different to the ideas of adults. Children in that study (and perhaps the adults they interacted

with) saw counting as a linguistic and affective activity rather than a functional one, thus pointing to the necessity for making quantitative purposes explicit for children. More recently, young children (aged between 3 and 7 years) were asked to articulate preferences and choices and to reflect upon their own strengths, weaknesses and development in the context of child interviews related to independent learning (Hendy and Whitebread, 2000). The findings suggested that children's abilities to think and act independently are underestimated by teachers, particularly the older children in the study.

Ascertaining understandings of the purposes of written numerals

Sinclair and Sinclair (1984) focused on four-, five- and six-year-old children's understandings of written numbers in everyday contexts for the purposes of ascertaining the extent to which they understood the purpose for which these written numbers were being used. The numerals in the pictures they showed the children represented some of the common uses of numerals in the environment. They asked them what was missing from pictures (of birthday cakes, buses, bus stops, lifts, speed limit signs, car licence plates) from which the numerals had been removed. Children were asked what information the numerals conveyed on these objects but the study didn't report children's understanding of purposes of number. In their discussion of their findings, Sinclair and Sinclair (1984) pointed out how little we knew then about the actual interactions children have with others regarding the use and purpose of written numerals. We do know however that children are curious about written numerals and that for them written numerals appear to be interesting objects to explore prior to, or in the absence of, formal instruction (Atkinson, 1992; Tolchinsky, 2003). However, there has been almost no research that explores children's interactions about number/experiences with number, as reported by children themselves. A few studies have sought to examine how children choose to represent numerical information (e.g., Hughes, 1986; Munn, 1994) and from these, some of the challenges children face in learning how to communicate using an abstract symbolic system become clear.

In an attempt to explore young children's understanding of purpose Ewers-Rogers and Cowen (1996) asked them about the meaning of numbers in the wider environment, i.e., on buses, phones, coins and birthday cards. They found that most three-year-olds and a number of four-year-olds didn't notice when the numerals were absent from pictures of these objects. Whether or not the children would have noticed if writing or other elements of the pictures were missing was not an issue that was considered by the authors. Based on their findings, Ewers-Rogers and Cowen surmised that their children's understanding of the purpose of the numerals on these objects typically preceded spotting that they were missing. In their study the authors were concerned to present children with what they considered to be tasks focused on *'ordinary human activity'* (p. 152) as opposed to decontextualised number tasks. They sought to ensure that they maximised the possibility that children saw a purpose for the activity and thus would display what they knew about the use of numerals. The authors observed that children rarely fabricated explanations for numerals on objects, thus indicating, and rightly so I think, that when children were aware of purpose it was because they had discussed a particular issue with others who understood conventional purpose.

A study which drew on parental diary entries (Worthington and Carruthers, 2003) indicated that few children got opportunities to see their parents write numbers for their own interest or work. As the authors point out, and I agree, it is such situations that help establish the sociocultural context of number for children. We know that children and adults do come to share an understanding of purpose in relation to conventional numerals. But Munn (1994) argues that this is a result of a slow dawning of comprehension, on the part of children, of the function of these symbols. Before this realisation kicks in children are very likely working on a combination of no beliefs or their own subjective beliefs or working theories about these symbols. Gradually these are replaced by a more universal theory of purpose.

To summarise, research indicates that:

- Some purposes of number are generally more visible to young children than others;

- Many four-year-old children are engaged in constructing meanings for written numbers in various contexts and they often capture essential aspects of them, using context to interpret meaning;
- In certain situations, for example in relation to telephones and birthday cards, most four year-olds can explain the purpose of numerals;
- There is still relatively little known about the conditions in which purpose is conveyed to young children and about the role of adults and of children themselves in situations that convey purpose. My study seeks to explore with children both how purpose is conveyed to them and also how they understand it.

Quantitative thinking

For most people, quantitative thinking is an essential element of mathematics. Young children construct their knowledge of quantification in the course of their everyday experiences, leading them to quantify sets of objects. The elements that I deem important in relation to quantification in the four- year-old age group are subitizing, counting and estimating. Below I take a detailed look at the literature on the relationship between subitizing, counting and estimation since this issue emerged as central in relation to the analysis (See Chapter 4: 93-106).

Perceptual quantity

Nunes and Bryant's (1996) summarised research pertaining to the genesis of mathematical experiences of young children and they speculate that it is located in the ability of infants to distinguish between sets of one to three objects on the basis of numerosity. They also suggest that infants realise when the set size changes when one object is added or taken away. Steffe and Cobb (1988) however take the position that such discriminatory ability is unrelated to number words, counting or any kind of numeral system. Fuson (1988: 18) concluded that one- and two-year-old children possess the ability to label small numerosities with distinctive 'words'. For instance, a diary entry related to her daughter (aged 1 year 8 months) read '*You know two items and use it correctly in new situations: two means one in each hand.*' This she describes as a perceptual process. Subitizing implies '[T]*he immediate correct assignment of number words to small collections of*

perceptual items ...' (von Glasersfeld 1987: 303) Clements (1999: 400) uses the term *'perceptual subitizing'* to describe this early ability and to differentiate it from *'conceptual subitizing'*. The former, according to him, is a process of recognising a quantity without using mathematical processes. The latter he describes as *'an ability to group and quantify sets quickly'* (p. 401) and he relates it to a situation where people [his term] 'just know' how many dots are on, for example, an eight-dot domino. Clements suggests that where an array is organised as opposed to randomly displayed, they just know the domino's number because they recognise the number pattern as a composite of parts, i.e., two groups of four, and as a whole, i.e., eight.

Where quantities are not easily or accurately differentiated by perceptual means, children who are able to count can use their cultures counting system to determine how-many. Fuson's (1988) study demonstrated the ways in which children first relate the counting and cardinality meanings of number words. Her research showed that by the age of 4 and a half, most children can quantify using an incremental counting procedure starting from one. They count large numbers of objects in rows (to 20) with considerable accuracy. This confirmed earlier studies which concluded that most five-year-old children in the United States had considerable experience and competence in counting sets of up to thirty objects on occasions (e.g., Gelman and Gallistel, 1978; Saxe *et al.*, 1987). However, research also consistently demonstrates a wide variation in counting related activity amongst children at the age of entry to school (Fuson, 1988; Wright, 1991; Aubrey, 1997). We can conclude from this that in relation to counting, experience and practise matter greatly.

It appears that overt counting behaviour, such as pointing and saying number words aloud, undergoes progressive internalisation across the age range 3 to 6 years (Fuson, 1988). Most three-year-old children in the Fuson study counted aloud, while most five-year-old children counted silently, thus suggesting that for most children overt counting is a precursor of silent counting. Fuson also observed that the first internalisations of counting may result in less accurate counting. It follows that when children offer an 'inaccurate' response to a quantification problem, then the possibility that they are counting silently must be considered. Fuson also provided an

overview of the ways in which counting enables children between the ages of 2 and 8 to become more proficient in handling numeric relations. It appears that once children appreciate the ordering of number words in the number word sequence, this permits equivalence and order relations to be established on the sequence of number words and after/before and just after/just before can then be derived. Thus children's use of counting, as suggested earlier (See this Chapter, p. 15), appears to be a critical issue to consider in relation to number sense.

Relationship between subitizing and counting

Von Glasersfeld (1987) has argued that the realisation that the last number word used when counting an array is the same as the subitized word for that set, is what enables children to learn and generalise that the last word used answers the question of 'how-many'. Similarly, Steffe and Cobb's (1988) review of the literature led them to conclude that children count perceptual patterns at a very young age (c. three years) in order to form the semantic links between patterns and number. The position of others (e.g., Gelman and Gallistel, 1978) was that learning to quantify does not depend on subitizing but rather on accurate counting. Fuson (1988) reviewed various positions on the relationship between (perceptual) subitizing and counting. She sought to clarify the developmental relationship between labelling a subitized situation with the correct number word, and counting in order to label that situation.

Fuson (1988) found many of the two- and three-year-old children in her study did not seem able to, or did not spontaneously use subitizing in a counting situation. This suggests that for these children subitizing and counting are not yet related. She interpreted this as undermining the subitize and count position above. She claimed that Gelman and Gallistel's (1978) proposition that children's ability to count does not depend on their ability to subitize was supported by her data. She concluded that children follow different routes to early relations between counting and cardinality. In her study, the subitize and count account applied for some children, but not for others. Thus she argued that that the connection between counting and the 'how-many' question may be made in many ways. This suggests that in a study such as mine, quantification problems should elicit a range of

approaches including subitizing, overt counting, silent counting, estimating and perhaps guessing.

The perception of composite figural patterns, i.e., spatial or temporal patterns, is seen by some as fundamental to children's understanding of number (von Glasersfeld, 1987) and indeed perceptual patterns (e.g., such as those seen on dominoes) have been shown to play a key role in children's construction of arithmetic meanings and strategies (Steffe and Cobb, 1988). It is claimed that four-year-old children begin to recognise and visualise sets of quantities from five to ten as special patterns that they can see when looking inside numbers (Fuson *et al.*, 2001). Such patterns appear to be the key to *'conceptual'* subitizing as described above by Clements (1999). Indeed he argues that children use counting and patterning abilities to develop conceptual subitizing. This raises the question of whether there is a developmental sequence whereby perceptual subitizing emerges before counting, which in turn emerges before conceptual subitizing. Fuson's work dispels the notion of a developmental sequence between subitizing and counting, since she found that children follow different routes to quantification.

Wood (1998) points out that theoretical debate about how developmental changes should be interpreted are very complex. He reviewed a number of longitudinal studies that investigated aspects of young children's mathematical development. The findings revealed that children of any particular age generally used a variety of strategies to solve problems, rather than just one. Some children used different strategies on the same problem on different occasions. They also applied newly adopted strategies to problems they could already solve by other means. Conceptually more advanced strategies were then applied to more challenging problems and when successful, soon became a dominant strategy. All this suggests that a more loosely defined relationship may exist between the processes of counting and subitizing, but not a developmental sequence. In other words, there is no suggestion that learning of one must precede the learning of the other. It also suggests that while a count and subitize strategy may account for some children's responses to certain quantification tasks, it is most likely

that this is just one of a number of possible strategies that children may draw on to solve certain quantification problems.

Estimated quantity

Two decades ago, Fuson and Hall (1983) reviewed the research evidence available in relation to estimation. This included a study, which claimed that where time permits, adults will count (mentally) sets of up to six objects even when asked to estimate. Above this size they tended to estimate. They found very little research on estimation in young children and indeed they speculated that because of the likely lack of experience amongst young children, and thus of stored mental representations of numerosities, the estimating abilities of young children would be quite weak. Sowder (1992) also expressed the view that estimation may be too difficult a process for young children since they lack experiences that permit the establishment of mental pictures of quantities or a feel for size. Both of these, in her view, are prerequisites for being able to estimate. MacNamara (1996) cites evidence that a seven-year old child in her study could quantify sets of up to ten items without overt counting. When asked how he did this he described a strategy that involved subitizing some of the items and then counting on the remainder. This compression of the counting process (Gray, 1997) in order to efficiently generate a solution to the quantification problem would appear then to be one important base that children can use to estimate quantity. Fuson (1988: 349) also identified what she termed a '*subitize and add*' strategy amongst the older children in the age-range three- to six-years on sets of four to six objects. Clements (1999) suggests that conceptual subitizing, which involves creating and using various number patterns together with counting, is essential for estimating.

Estimating and guessing

In her review of research related to the question of whether young children estimate, Sowder (1992) cites research suggesting that children (aged five years) offered estimates that appeared to make no sense when related to the numbers in the problem presented to them. Aubrey (1997: 86) offers further evidence that estimation tasks make little sense to children of four-or-five-years of age. She found that in general the children in her study, except those without counting skills, immediately started to count the items when

asked to 'guess' and furthermore '... *were confused when prompted again to guess when the items were removed.*' Practitioners too have observed similar reluctance on the part of some young children to make a guess. Andrews (1995) reported that children in her kindergarten class often attempted to count the number of items in response to an estimation task, and furthermore, she noted that they frequently became frustrated when they were precluded from doing so because of the arrangement of the objects in a jar. So while the word 'guess' is sometimes suggested as a way of clarifying what is involved in estimation (See Government of Ireland, 1999c), Andrews speculated that the actual effect for some children may be to trivialise the mathematical process of estimation. It does so, she argues, by suggesting that there is no information available against which to draw a logical conclusion. Sowder (1992) suggests that confusion between guessing and estimation may be a characteristic of poor estimators. If she is correct in this, then encouraging children to guess may actually be working against developing estimation abilities. Sowder's (1992) review indicated that even older children (aged nine to fourteen years) did not understand estimation. Indeed Andrews (1995) suggests that children need structured experiences that enable them to understand the differences between a guess, an estimate and an exact answer. It also appears that they need to be able to use counting in a flexible way and in different kinds of situations.

Affective issues

Affective factors are seen as important in relation to estimation and Sowder (1992) argues that children are more likely to use estimation if they see the value of it. Indeed both she and Silver (1989) agree that what is being asked of older children when estimation is requested contradicts what they have generally learned at school about the primacy of one right answer. It is possible that younger children may face a similar dilemma when they are asked to say 'how-many' without counting, i.e., to estimate. Such tasks may not yet make any sense to them in the light of the enormous importance and power that counting assumes for them at this stage of their numerical development (Fuson, 1988). Much of children's home experience also emphasises 'correctness', for example, enough spoons for everyone to have

ice-cream is a practical requirement, and *too many* or *not enough* often causes comment.

The how-many question

The question of 'how-many' seems to function for some young children as a request for the counting act rather than as a request for information gained from that (Fuson and Hall, 1983). The use of the how-many question appears to predispose some children to overt counting and so to ignore or fail to see the utility of other related strategies. As we have seen, many children do not seem to be able to use subitizing in a counting situation (Fuson 1988), or at least do not do so spontaneously. Also, subtle differences between the ways in which questions related to 'how-many' are posed can affect whether or not children re-count a set in response, and consequently to different inferences about the children's understanding of the task. Nunes and Bryant (1996), in their review of how children learn to count, concluded that four -year-old children often fail to connect their use of counting with other aspects of the logic of number, such as transitivity. This led them to argue that knowing how to count is one skill, but knowing when counting is a good problem-solving strategy and presumably the converse, i.e., when it is not, is an entirely different and more advanced skill. On the basis of the literature review, it seems that what children have to choose between is whether to count aloud, or whether to use some other (counting) strategy such as silent counting or very rapid counting, i.e., to use counting in a more flexible way. It seems that we know a lot about children's counting but relatively little about how they become flexible and efficient users of counting as a quantification process. We have very little information about how they integrate counting with other skills and processes in order to solve quantification tasks.

Fuson (1988) suggests that the importance of the ability to subitize lies in the role it may play in providing an early base for addition and also in facilitating the addition and subtraction of very small numbers. She argues that it is facilitative because in perceptual quantification, children 'see' the addend and the sum and thus perform the addition. Similarly, von Glasersfeld (1987: 280) argues that perceptual recognition and subsequent representation of groups of items (to 4 or 5) plays an '*indispensable*' role in

the development of arithmetic operations. Prior to this, he observed, subitizing had been regarded as marginal in the acquisition of numerical skills. Marton and Neuman (1990: 69) provided empirical evidence to support the importance of perceptual processes and they argued that it was these '*seeing*' skills or '*analytic*' skills that enabled seven-year-old children in their study to solve quantification problems. However, the weight of the evidence seems to suggest that for many children counting is the key skill in relation to solving quantification problems.

In summary, young children use a number of enumerative strategies to quantify sets. These include subitizing, counting and estimation, and perhaps guessing. Research suggests that counting is not dependent on subitizing. Different children use different strategies at different times. However, when children can count, then they may combine this skill with their ability to perceive numerosity in small sets and their ability to create and use patterns related to quantity. Thus they develop a more powerful way of quantifying sets containing more than three or four items. In this way, subitizing may be important for estimating for some children in some contexts. Thus it seems that conceptual change in the form of developing relational understanding between subitizing, counting and cardinality is important for children's developing number sense in the preschool period. At four-years-of-age counting is very important to children (Fuson, 1988), and for pedagogic reasons, extensively practised in the home (Aubrey *et al.*, 2000). However, adult purposes for counting are generally not well understood by children of this age (Munn, 1994). During the early years, quantitative thinking becomes increasingly more complex. Counting is used not just to quantify, but children can learn to make inferences based on this counting in order to reason quantitatively (Fuson, 1988; Nunes and Bryant, 1996). The extent to which young children have mastered counting to quantify, and can use it in a flexible manner is an important aspect of their number sense. Such flexibility of use enables the development of various strategies that they can then use to solve a variety of quantification problems, including estimation.

Awareness/understanding of numerals

In my opinion, exploring young children's number sense implies an exploration of children's understandings of numerals in various situations. A study of children's number knowledge at school entry in England found that half of the forty-eight children could read between five and nine or more numbers. A further third of the children recognised between two and four numerals and only about one fifth of children were unable to read only one of the numerals, or none (Aubrey, 1997). Many children, then, may have had the requisite experiences for reading numerals during the preschool period. During the pilot-study, I was struck both by the extent to which the children were interested in written numbers and their efforts to communicate using these. When we experience number in everyday life, numerals often feature as an essential aspect of those experiences. The ability to understand and use the language of number, both spoken and written, is fundamental to children's learning of number in our culture, and to the development of their number sense. Indeed, Greeno (1991: 177) sees numeric symbols as ' ... *helpful-even essential in the activities of learning.*' However, he rates experiences in conceptual environments as more important. I would qualify this view, since, in my experience, some children at least see writing and reading numerals as very important. I think that for such children, learning to recognise, name and write the numerals may well be their way into number, and then to the conceptual environments. From my perspective, meaningful activities that involve young children writing numbers are central to the development of their understanding in the domain of number. In relation to how children learn to communicate using number, we can look to the research in relation to children's understanding and use of conventional number symbols (e.g., Atkinson, 1992; Tolchinsky, 2003).

In summary, in this section of the review, I have considered each of the four key aspects of my framework for exploring number sense in young children. Some of the aspects, i.e., children's understanding of the purposes of number, and children's quantitative thinking, have been reviewed in more depth than the other aspects. These two aspects were ones that the pilot study suggested were potentially rich data sources, and the review of the curriculum documentation concurred with this.

Section D: Contribution of the study and clarification of the research questions

As a result of the review above, I have established that my study differs from previous studies that looked at children's views or dispositions in relation to number, or their number sense in the following ways:

- Children's pleasure and interest in number

While dispositions in early childhood have received considerable research attention little has been written about how we might discern children's disposition towards number by means of considering their pleasure and interest in number;

- Children's views about the purposes of number

This is an area almost entirely unexplored. I explore it through discussion with children of their number-related experiences and by questioning them in relation to their interpretations of these experiences. My focus then is children's experiences and the knowledge of uses that these suggest. [Ewers-Rogers and Cowen (1996) questioned children about the meaning/purpose of the numeral in specific contexts, seeking to ascertain how much children noticed numerals in the environment.];

- Children's quantitative thinking

While there has been a great deal of research related to counting and other aspects of quantification, there have been few studies that specifically focus on children's responses to estimation tasks. This aspect of quantification is a particular focus of my study [Task 9 was specifically designed to get at estimation.] (See Appendix 1);

- Children's understandings of the purposes of numerals, i.e., written numbers

While there has been some research activity related to this aspect of number sense (e.g., Ewers-Rogers and Cowen, 1996), my approach which seeks to elicit children's experiences in order to ascertain their understandings of purposes of written numbers has not been used previously. [Sinclair and Sinclair (1984) presented children with particular scenarios where it was assumed that they would notice numbers. These situations were then discussed with children. My approach differs in that I seek to determine

which of children's experiences, from their own accounts and perspectives are about the functions of written numerals].

The research questions

Question One

What number sense do young children demonstrate as they start formal schooling?

This question will be investigated through the exploration, by means of individual discussion with children at the point of entry to school, of each of the following related sub-questions:

- To what extent do children convey pleasure and interest in numbers?
- To what extent do children demonstrate understanding of the different purposes of number?
- To what extent do children demonstrate quantitative thinking?
- To what extent do children demonstrate awareness/understandings of numerals?

Question Two

What is the congruence between the findings of this study in relation to children's number sense and the references to number sense in the statutory curriculum documents related to mathematics for the first year of school in Ireland?

In Chapter 3, which follows, I discuss the methodology that I used to explore number sense with young children as they commenced primary school. I also outline the steps I took in analysing the curriculum documentation in order to assess the congruence between that documentation and the number sense demonstrated by the children who participated in this research study.

Chapter 3: Methodology

I investigated the number sense of young children starting school using a variant of the clinical interviewing method that I term 'experience-based flexible focused' (EBFF) interviewing. In the first section of this chapter, I discuss the issues surrounding the use of interviews with young children in pedagogical contexts and I describe my use of the interview methodology in this study. I elaborate on what I mean by EBFF interviewing. I also discuss what I consider to be some of the consequences of this methodology. In Section B, I describe how I analysed the curriculum documentation for references to number sense, both explicit and implicit.

Section A: Interviewing the children

Child interviewing in pedagogical contexts

Almost two decades ago Tammivara and Enright (1986: 226) commented that '*The difficulties inherent in communicating with young children have, in effect, kept ethnographers and many other researchers from examining the world of the child from the vantage point of the child for many years.*'

Things have moved on somewhat and there is now a good deal of literature regarding both the procedural and ethical issues related to interviewing young children (e.g., Smith *et al.*, 2000). In the past, there was some resistance to the idea that interviewing children was an acceptable research tool (David, 1992) but this situation has also changed considerably. Brooker (2001) identified two complementary principles that have evolved over recent years and which underlie recent change in attitudes: a belief in children's rights and a belief in children's competence.

In researching my questions related to number sense and young children, I used a variant of the clinical interview method. In relation to this methodology, Piaget (1929: 8) wrote

> *Since the clinical method has rendered such important service in a domain where formerly all was disorder and confusion, child psychology would make a great mistake to neglect it.*

Below I outline the arguments that Piaget, and others since, have put forward in support of clinical interviewing.

Clinical interviewing

Piaget was very much a pioneer in the area of interviewing children. In *The Child's Conception of the World* (1929), he discussed the challenge of exploring cognition and thinking in young children and of the necessity for developing a sufficiently sensitive way of carrying out such explorations. Piaget developed what is now known as the *clinical interview method* in order to investigate underlying patterns in children's thinking. In fact, his methodology was greatly influenced by the psychoanalytic movement and by Freud's approach to cognition. This approach was one that distinguished between what is observed on the surface and what underlies it (Ginsburg, 1997). The interpretative stance to behaviour assumed by Freud, and his description of the work of the analyst as that of interpreting behaviour to illuminate the underlying process, is one that had a pervading influence on Piaget as he began to think about ways to investigate children's thinking.

Characterising 'clinical' child interviews

The clinical interview, as a class of methods, has been developed over the years since first used by Piaget. This development has taken place across a number of areas of enquiry. It has been especially of interest to those researchers in the area of cognitive research who found traditional methods of enquiry, for example, testing or observation, inadequate for the kinds of explorations needed to uncover children's thinking.

According to Ginsburg (1997: 39), the clinical method is an approach to interviewing children that is characterised by '... *a particular kind of flexibility involving the interviewer as measuring instrument.*' There is considerable flexibility in the interview design and an anticipation that the questioning will emerge and develop as the interview progresses. A number of researchers have alluded to the challenge inherent in using this method (e.g., Doverberg and Pramling, 1993; Ginsburg, 1997) and Piaget (1929) himself suggested that it took at least a year of daily practice to become skilled at this sort of enquiry. In discussing the clinical interview in

psychological research and practice, Ginsburg (1997: ix) makes the point that '*... this phrase refers to a class of flexible interview methods the nature of which is very difficult to capture in a single phrase.*' The approach is '*deliberately non-standardised*' (p. 29) and so directly challenges the traditional point of view concerning scientific method in research and practice. The word *clinical* is used to describe an aspect of the methodology. As Ginsburg (1979: 109) explains, it is used '*... not in the sense of focusing on pathology but in the sense of great sensitivity to and understanding of the* ***individual.***' [original emphasis] The interviewer must act as a clinician in judging how to respond to different children by '*... being sensitive to the nuances of individual needs.*' (p. 140) Pramling (1983: 48) argued that such sensitivity is a key element in the success of the interviewer in getting individual children to express themselves freely and '*expose*' their thoughts.

Clinical interviews: The centrality of purpose

One of the ways in which the clinical interview methodology has been developed and refined over the decades is the purpose for which it is used. Initially Piaget concerned himself with similarities and differences in children's perceptions that he could detect empirically as in, for example, *The Child's Conceptions of Number* (Piaget, 1952). Later his main interest seemed to be in characterising cognitive structures (Marton, 1981).

It is useful to compare differences in the nature of the questions/tasks used by Piaget and that of others who later used the clinical interviewing method. While Piaget was interested in the child's responses to questions mostly about natural science and derived his theories about cognitive structures from these, Pramling (1983) used the interview situation to elicit children's conceptions about different phenomena. She describes her purpose as 'exploratory' rather than 'evaluative'. She was not seeking 'correct' answers as such. Her focus was on the child's subjective experiences related to the phenomena in question, i.e., learning. Piaget's investigations, on the other hand, sought to examine children's responses and reactions and evaluate and classify them with reference to what he referred to as '*the right answer.*' (1997: 21)

Pramling used the same method as Piaget, but for a different purpose. In her phenomenographic study of children's learning, the content being talked about had a central place in the analysis and in the presentation of the findings. In contrast, for Piaget, the content of the discussion was secondary, the focus of the research was on underlying structures of intelligence. Pramling's interviews with children were dialogical in nature. This implied reciprocity and mutual turn taking in communicating and both the interviewer and the child became significantly involved in the development of the conversation (Doverberg and Pramling, 1993). In Pramling's (1983) study, interviews were carried out and analysed within a phenomenographic framework that focused on children's perceptions as formed by their experiences. In Piaget's works (e.g., 1997), interviews were carried out and analysed using an epistemological framework which focused on the formation of knowledge and of the human mind (Bringuier, 1980).

In summary, in Pramling's (1983) study of learning from a phenomenographic perspective, the purpose of the questioning was to come as close as possible to the child's world by enabling children to describe their experiences. This is the elicitation of what phenomenographers (e.g., Marton, 1981; Pramling, 1983) call the second-order perspective. Marton (1981) described how Piaget became increasingly concerned in his research with the first-order perspective and interpreted this as a shift in which the child rather than the child's world became the focus for attention. Pramling (1983) distinguished her work from that of Piaget by reference to her (phenomenographic) interest in children's experiences and she argued that it is these, rather than the underlying structure of the mind, that form the basis for children's views about particular phenomena such as learning. In Chapter 2, I argued that children's views related to the role of number in everyday life were an important aspect of their number sense. I strongly agree with Pramling's argument that children's views are based on their experiences. Consequently, exploring experiences that young children consider to be related to number is imperative in eliciting implicit understandings that may otherwise remain unarticulated.

A contemporary rationale for using a clinical interview methodology

Ginsburg (1997) presented a contemporary rationale for using the clinical interview. He argued that the acceptance of the constructivist position (and in respect of this study, a sociocultural one as defined in Chapter 1) in relation to knowledge makes it imperative to use a method that recognises and seeks to capture the distinctive nature of the child's thoughts. He also argued that the flexibility inherent in the methodology enables the interviewer to seek to establish *'subjective equivalence'*, by which he means to *'... help the child to interpret the problem in the same way as other children ... in the same way I do.'* (1997 [1929]: 61) This is possible since the interviewer has the opportunity to vary the questions and tasks for each child depending on the child's reactions during the interview. Of course, as Ginsburg also points out, this in no way guarantees subjective equivalence, but it does offer a possibility not offered by standardized procedures. Furthermore, he suggests that this methodology helps reveal the fluidity of children's thinking: children can appear to possess certain knowledge but can only exhibit it in certain situations, or in relation to particular tasks. Also the dynamic aspects of children's thinking may be revealed as they interact with the ideas and tasks raised in conversation during the interview. This methodology, Ginsburg (1997: 65) further claims, may be extremely useful for filling in the *'mental context'*, i.e., *'... the context of motion, interest, motivation and the like.'* The example he offered relates to reading: *'We want to know as much about the child's fear of reading and the conditions that elicit it as about the reading strategies the child employs.'* (1997: 65) On this point Ginsburg's view of the potential use of the methodology differs from Piaget's view. According to Ginsburg, Piaget really had no interest in context, seeing it only as something to transcend as he investigated his real interest, i.e., the basic and universal structures of mind. Ginsburg's own position is that *'... mental context is not something to be shunted aside in order to study thinking; rather, the two* [thinking and affect]*, being inseparable, must be examined together.'* (p.65) Ginsburg's position aligns very well with my own on the holistic nature of early learning and is implicit in the sociocultural perspective outlined earlier and adopted in this study (See Chapter 1: 5-8).

Finally, Ginsburg (1997: 68) also argued, convincingly in my view, that because of sensitivity to the individual and the ensuing flexibility in relation to carrying out the interview, this methodology is *fair*. This type of *fairness* '*... treats people differently-as individuals-in an effort to identify their abilities and problems, strengths and weaknesses.*' In using a variant of this methodology, this study serves to test a number of Ginsburg's claims. For example, the above claim that clinical interviewing is useful for exploring affective dimensions of children's thinking is seen, as a result of both the pilot study and my main study, to be a valid claim.

The study

In Chapter 1 (p. 9), I described my objective as one of exploring young children's number sense through ascertaining their number knowledge, their views of the usefulness and purposes of such knowledge and their ideas related to how people acquire this type of knowledge. This objective was reflected in the four key aspects of number sense identified in the framework, presented in Chapter 2 (p. 22). With these aspects in mind, I developed a number of tasks and questions designed to explore young children's number sense. These served to guide the interviews. I was also guided by the need for flexibility, and the principle that what is involved is '*... much more than discovering whether a particular task the researcher has devised works with certain children. Rather it is discovering how the children 'work' and then devising tasks.*' (Ginsburg, 1997: 163) In relation to the latter, see the discussion below (p. 46) regarding themes of interest to the children.

The research instruments

Two interviews were designed to explore children's pleasure and interest in number; their understandings of the purposes of number; their abilities to think quantitatively; and their awareness/understandings of number

Interview 1 (Focused on questions, See Appendix 1a)

The purpose of this interview was to ascertain children's number sense in relation to their views and experiences of number. I sought to engage the children and challenge them to think deeply about their views and experiences. To this end, I developed a number of questions to guide the

interview but I found that, in most cases, the discussion took on a particular and individual character once the child became engaged with the topic. This individuality was critical in relation to the ethos of the interview, as described below (pp. 48-50).

I reviewed the questions used in previous studies where children's views/perspectives of aspects of learning/mathematical learning were sought. These included studies concerned with the topic of counting (e.g., Doverberg and Pramling, 1993; Munn 1994; 1997), number (e.g., Burton, 2002), formative assessment (e.g., Turnstall and Gipps, 1998) and mathematics learning (e.g., Kloosterman *et al.*, 1996). These studies, and others, offered interesting and productive strategies for interviewing young children and for focusing them on the issues. For instance, in relation to the challenge of establishing rapport and getting children talking, Burton (2002) reported a study wherein children were invited to respond to statements that young children like them didn't know much about large numbers. She felt that such statements opened up discussion. This alerted me to the fact that I needed to plan how I might open the discussion with the children.

Interview 2 (Focused on tasks, See Appendix 1b)

The purpose of this interview was to ascertain children's number sense in relation to specific tasks. I decided on the tasks after research and review of the concept of number sense and a preliminary analysis of the curriculum content for this stage of schooling (Government of Ireland, 1999b). The tasks I used were based on a number of successful ones found in the literature (e.g., Hughes, 1986; Saxe *et al.*, 1987; Munn, 1994; Aubrey, 1997; Cook *et al.*, 1997; Clements, 1999; Van de Walle, 2001). I observed that one of the ways in which recent studies often built on previous studies was by refining the approaches/tasks that earlier researchers had used. For instance, Hughes (1986) reported considerable success in engaging young children in tasks using magnetic numerals and he illustrated how these numerals could assist children in conveying informal understandings of number-related concepts. This work was helpful to me in designing ways in which children could communicate their understandings about number. I also drew on my extensive professional experience as a teacher of young

children in making intuitive judgements about the suitability of the tasks for eliciting children's number sense.

The pilot study that I carried out with nine children provided an excellent opportunity to gauge children's responses to various questions and tasks. My analysis of the responses enabled me to refine those questions and tasks that were most useful for exploring my research questions. The discussions that I had with children revealed some central themes and these themes were important because they made visible some of the sociocultural contexts within which the children were developing their number sense. Age emerged as a central theme of interest for most children. The theme of family and friends was central also. My experience, together with the research I did in preparation for the interviews, led me to the conclusion that 'birthdays' are really powerful triggers for number and other early mathematical 'talk' (See also Carr, 2001).

In relation to Interview 1, I had learned that an effective way to engage children was to focus on the their families and friends. In relation to Interview 2, I built almost all the tasks around a birthday theme (i.e., that of the fourth birthday of a toy character, Coco, from a popular breakfast cereal). Some tasks revolved around 'games' with Coco, though I was aware that this strategy was not without its effects in that the discourse associated with such games might not do justice to children's understandings and skills (Walkerdine, 1988). However, I felt from my teaching experience with young children that overall these themes/strategies offered potential for exploring the four key aspects of number sense identified for this study.

Carrying out the study

A total of fourteen children, eight boys and six girls, were interviewed for the study. This was carried out with careful consideration to issues such as gaining access, establishing relationships with the participants, and ethics. A detailed account of how I arranged access, and collected the data can be found in Appendix 2a.

The interviews in the boys' school were carried out over the first two weeks of the new school year in September. The interviews in the girls' school

were carried out during the third and fourth weeks of September. Each child was interviewed twice. The age-range of the children was 4 years 1 month to 5 years 1 month. On average, the first interview lasted about twenty minutes and the second about thirty minutes. Interviews were fitted around the events of the day, and I took care that children were not interviewed at recess time, playtime or snack-time or during other activities where it was deemed desirable or necessary for children to be with the class group. Earlier I stated the main reasons why I chose to carry out the interviews during the first few weeks of school (See Chapter 1: 9-10). I was also influenced by research which had shown that children just starting school were as yet unfamiliar with the rules of discourse pertaining in classrooms (Willes, 1984). For this reason, I felt that children would be more likely to answer more freely and spontaneously, than might be the case later. Indeed Ginsburg (1997: 185) refers to what he terms '... *traditional school games, amongst them self-imposed suggestibility*', i.e., where children try to 'read' what's in the interviewer's head and so provide what they perceive to be the desired response. He suggests that such 'games' can apply in the interview situation where children have experienced the discourse of schooling. In summary, I contend that at the point of entry to school, children's responses may be less constrained than later.

I was conscious that school was a new environment for the children participating in the study and that this was something that I needed to consider. Tizard and Hughes' (1984) study of children's learning at home and at school demonstrated that it is not always easy to get children talking at school since many children are unwilling to engage in conversations in particular types of unfamiliar physical and social contexts. These findings had implications for the way in which I designed the interview situation and presented it to the children. I was aware in planning the interviews that the relationship that I created with the children, their interpretation of my intentions, and the degree of mutual understanding or inter-subjectivity achieved would determine the conversation that unfolded. Carr (2000) observed that young children often have their own ideas about the intent and the topic in relation to learning at school and I had no doubt but that the same would hold true of the interview situation that I was about to create.

Indeed, I was very conscious of Gollop's (2000) assertion that the entire context and process of an interview can have a huge impact on the outcome and I kept this in mind as I planned each step of the process of interviewing the children.

Accuracy of children's reports

Hughes and Baker (1990) argue that it is difficult to give a definitive answer to the question of the age at which children can be expected to report accurately on past events. But does it matter, in a study of young children's number sense, whether children's accounts of past events are accurate or not? I think not. What is important in this study is the fact children have linked particular events/situations with number.

My experience as a teacher, together with my theoretical understanding of the role of language in conveying meaning in discussion, heightened my awareness of the necessity to guard against making suggestions that children might, for one reason or another, just agree with. However, it appears that children are not as susceptible to adult suggestions as we might think. Young children have been seen to resist suggestions concerning events that are central to them (Ginsburg, 1997).

Issues related to ethics: Power and ethos

Diminishing the imbalance in power relations

Carr (2000) has argued that in the interview situation the data comes from the children's interpretation of the researcher's intent. In this study, I sought to ensure that my intent, i.e., to find out about their learning in relation to number, was known to the children and that as far as possible, they understood this intent.

Throughout the process of collecting the data, I also sought to ensure that the discussion was respectful of the children's views and opinions, and that this respect was conveyed to them. I did so in a number of ways. Firstly, as part of the invitation to be interviewed, I suggested to children that they had some important ideas that I would like to hear. I explained this, usually, in the following way '*I am really interested in hearing about how children learn all sorts of things and your Mam/Dad agreed that you might help me*

because you know lots of things.' In the course of the discussions, I used enabling and supportive phrases such as *'That's really interesting'* to encourage the child to talk on. At the conclusion of the discussions with the children, I always sought to assure the children that the information they had given me was very useful and important for my work.

Where, as it occasionally happened, a child indicated that she/he wished to return to their classroom, we did so immediately. I also gave children the choice of concluding the interview wherever they showed signs of tiredness or where they seemed to become anxious, for example in relation to who was collecting them from school or other such concerns.

The ethos of the interview

Ginsburg (1997) describes the interviewer's goal as one of uncovering children's thinking and involving them in the process to the extent that they introspect about their own thinking and express it so that it can be understood by another. In clinical interviews, the ethos of the interview is critical in enabling this to take place. An ethic of respect for their responses is established even before the interview itself when the interviewer suggests to the children that what they have to say is interesting, useful and helpful to others as they try to understand learning. Ginsburg contrasts this ethic with the ethic of control and evaluation that he associates with standardised procedures. The fact that an adult and child are the players in this type of research immediately introduces an asymmetric relationship and we cannot deny that these power asymmetries exist.

My interview plan was drawn from my research project and was based on my professional knowledge of young children. Furthermore, I had indeed developed a set of questions and tasks to guide the interviews. It could be argued that in such situations the power remains with the researcher. However, the interaction that takes place within the clinical interview may also be described as one in which the interviewer is responsive to the child's comments and the path of the interview is determined by this developing interplay. In this way, the interviewer influences the child and the child influences the interviewer. Scheurich (1995: 247) has argued that in relation to interviews in general *'... interviewees are not passive objects; they are*

active participants in the interaction.' He viewed this activity, from a critical theorists perspective, as including the notion of '*resistance*' by those being interviewed. For instance, I found that that the young children that I interviewed sometimes demonstrated their agency by responses that suggested resistance, for example, by answering the question that they wished to answer rather than the one put, or perhaps by changing the topic to one that they wished to discuss. The clinical interview encourages and indeed depends for its success on such agency. Success, then, can be said to depend on both the child and the interviewer. The ethos of the interview can be described as democratic. It assumes a relocation of some of the power in the interview from interviewer to child. This can be achieved in the first instance by adopting a methodology, such as the clinical interview, that is non-invasive, non-confrontational and participatory (Morrow and Richards, 1996).

In conclusion, I think it is important to recognise that as adults we do not 'give' children power in an interview. Rather we create the conditions in which they themselves can manage the power play within the interview, just as they do in other social engagements.

Experience-based flexible and focused interviewing

My interest was in the social, affective, sense-making, awareness and identity aspects of children's number sense as well as their demonstrated ability with number. I used a variant of clinical interviewing, EBFF interviewing, in order to explore different dimensions of number sense, thereby placing children's thinking '... *within its general psychological context.'* (Ginsburg, 1997: 64)

The purposes of the interviews then, from my perspective, were:

- to try to understand how particular children viewed number;
- the significance, if any, they attached to number;
- the contexts that stimulated their thinking about number;
- the associations they made with number.

While the EBFF interviewing methodology is modelled on the clinical interview as described by Piaget and discussed above, there were a number of aspects that differed.

For instance, I found that asking young children questions such as *'Do you know any numbers?'* or *'Do you know anything about numbers?'* rarely elicited any response other that silence. Using the flexibility of the interview situation, I was able to experiment with a number of different openings. I found that questions such as *'Can you count?'* and *' Will you count as high as you can?'* were relatively successful in enabling children to understand what the intended focus of the interview was, i.e., number, and appeared to clarify my goal for them. Doverberg and Pramling (1993) have argued that the starting point should be a situation or experience with which the child is familiar. Counting related questions appeared to constitute such a situation.

Also, where I judged it necessary to check for robustness or strength of conviction of a child's answer, I preferred to do so by revisiting the same question or idea later in the interview, perhaps worded differently, a strategy also used extensively by Pramling (1983). I frequently used the question *'Why do you say that?'* or *'How did you figure that out?'* or some related question to try and elicit children's explanations and reasoning. My perspective is that sociocultural contexts, and individual experiences within such contexts, largely determine the ways in which children respond to questions and tasks within the interview.

To summarise, EBFF interviewing is differentiated from clinical interviews as used by Piaget on three counts. The sociocultural context is regarded as centrally important in respect of what children convey about their number sense; children's views and experiences are explored and clarified, not through counter-suggestion, but through 'intensive' questioning; and the focus of the interview is clearly conveyed to children at the outset.

A flexible approach to questioning

I have described above the process involved in the design of the interview. The tasks and questions that I had prepared were the starting points for the discussions with the children. The flexibility inherent in the method permits the interviewer, from there on, to vary the questions and tasks for each

child. It also enables the interviewer to probe the child's responses. This flexibility also means that the professional knowledge and skills of the interviewer are central. The establishment of joint meaning and shared purposes with the child being interviewed is imperative for the success of the interview. I sought to achieve this by using the flexibility of the interview to interpret children's responses and then improvising in terms of contingent questions and follow-up responses.

As the interview progressed, it was often necessary for me to make an immediate interpretation of the child's response and to make a judgement based on that. Sometimes I repeated or echoed the child's response, sometimes hoping for further comment from the child, or sometimes just checking that the response remained stable. Sometimes I might suspect that the child was taking a specific perspective on the task and I would then seek to investigate this hypothesis further.

For example, Síle was asked to suggest a number bigger than 10 (See Appendix 1b: Task 3):

Síle: *I say seven*

Liz: *Are you sure about that?*

Síle: *Yes*

Liz: *Seven is bigger than ten?*

Síle: *Yes one is small and ten is all ... zero*

Her response didn't tally with her earlier responses to (single-digit) items in the Task. Thus I pursued her reasoning in relation to this particular pair of numbers. What emerged was a fascinating glimpse of the logic that she applies to judging the numerical value of ten. Her number sense in relation to ten derives from her current knowledge of the digits 1 and 0: 1 is small and 0 is nothing. Síle has developed her own theory in relation to how number size is judged.

Sometimes I judged that it was appropriate to tease out the reasoning behind a child's response. I sought to explore further any unexpected or unusual responses to specific tasks. On a number of occasions, I found it useful to improvise and perhaps to change the presentation of the task or the order of the task, or indeed the focus of the task. This freedom within the interview

allowed me to be truly responsive to individual children, sometimes testing hypotheses or sometimes exploring.

The following piece of dialogue illustrates how I used this flexibility to introduce a change of wording where I hypothesised that the language related to the task *(See Appendix 1b: Task 4)* was presenting a problem for Tom:

Liz: *This is going to be a game that you're going to do in your head. Let's pretend that Coco had three presents and he got one more ... how many had he then?*

Tom: *Hm?*

Liz: *He had three presents and he got one more how many has he now?*

Tom: *None*

Liz: *We'll just say it again ... he had three presents ... and then his Mammy gave him one more*

Tom: *Yes*

Liz: *How many has he now?*

Tom: *One*

Liz: *Are you sure? Altogether?* [He nodded]

Had the use of 'had' given Tom the impression that the objects were no longer there and only the additional one was now available for consideration?

A few moments later:

Liz: *Now the next day when he counted his presents he had nine altogether and Granny came and gave him another one ... how many has he now?*

Tom: *Ten*

Liz: *Yes. And then another day he looked at his presents and he had only three!*

Tom: *Where are the other ones gone?*

Liz: *He must have put them away somewhere ... but he has only three presents on the table and then Grandad gave him one more ... how many has he now?*

Tom: *Four*

Here I changed the context of the task by being more specific with the details, and this may have clarified the situation for Tom. Changing from the use of *one more* to *another one* may also have helped Tom deal with the question I was asking.

As I interviewed the children, I was very careful about the wording of the questions. I was conscious of the argument that metacognitive experiences are most likely to occur in situations that stimulate in children a lot of careful, highly conscious thinking (Flavell *et al.,* 1995). For this reason I incorporated some prompts to create just such an effect, for example *'I want you to think hard now'* and *'Are you sure about that?'* These proved to be very effective. I observed in the study that children hesitated when they heard this and they appeared to stop and think before responding. See, for example, Tom and Síle above.

Seeking to expose a child's thinking

Researchers such as Doverberg and Pramling (1993), who have used a flexible interview methodology extensively, have suggested that one of its important features is that it can serve to stimulate children to think about things in a way that they had not done previously. Similarly, Ginsburg (1997) claimed that the occasion of the clinical interview, carefully conducted, could become for the child a place for thinking rather than an occasion for getting the right answer.

Sometimes I created small amounts of structuring based on individual responses, thus eliciting something more from the children than would have been the case had I not followed up an initial response with more probing questions.

For example, consider the following dialogue with Lara:

Liz: *So now you're in big school ... will you learn about numbers here?* [She nodded in agreement] *What will you learn?*

Lara: *I will learn 4,5,6,7,8,9,10*

Liz: *But you already know that*

Lara: *No, I don't*

Liz: *Yes you do*

Lara: *No*

Liz: *You're good at that*

Lara: *But I can't do the other ones and I can't think about it in my head*

Liz: *You can't think about it in your head? Do you have to think about it in your head?* [She nodded in agreement] *Which ones can you not do ... after what?*

Lara: *After twenty-five*

Liz: *After twenty-five it's tricky is it ... and you can't think about them in your head ... do you ever think about the other ones in your head?*

Lara: *No*

Liz: *I wonder does Mammy ever think about numbers in her head?*

Lara: *Well she does but I don't know them*

Liz: *How do you know she thinks them in her head?*

Lara: *Because she told me*

Liz: *What did she say to you?*

Lara: *She said 'Do you want to learn numbers with me?'*

As a result of my structuring and probing, Lara's ability to assess her learning needs emerges, as does her theory of mind and her idea that her thinking is different to that of her mother. This supports previous research which demonstrated that young children can describe their own abilities and difficulties at an impressive level (Hendy and Whitebread, 2000). As a result of intensive questioning, it was possible to elicit from Lara a more qualitative assessment of her own strengths and weaknesses in relation to counting. It has been observed (Coltman, 2005) that awareness of personal capabilities, at this level of detail, was only infrequently expressed when children were independently engaged in mathematical tasks.

Based on my experience interviewing these children, I very much concur with Doverberg and Pramling's (1993: 40) argument that in order to do justice to young children's thinking it is essential to conduct the interview in

a probing way. They advise that '*We must keep asking questions until we are sure the child has no more to say.*'

The dynamic nature of the interview

It has been argued that the interview gives young children an opportunity to develop their thinking (Doverberg and Pramling, 1993; Brooker, 2001). Their thinking is developed as they struggle to articulate ideas in response to the questions put to them. Ginsburg (1997: 37) suggests that the structuring of questions is an inherent aspect of sensitive interviewing, and provides a means by which the child may access pre-existing knowledge. But he also suggests that as a result of the interview the child may acquire some new abilities '*In the clinical interview the child sharpens, or even acquires the ability to introspect and express thinking.*' (p. 113)

Acknowledging the intensity of the interview experience

The EBFF interview, properly conducted, is an intense experience for both child and interviewer. The differences in roles and responsibilities between the interviewer and the child demonstrate the source of the intensity for each and are discussed further below.

The interviewer

Interviewing young children is not like interviewing adults. Hughes and Baker (1990) conclude that the main difference resides in the fact that young children possess different communicative abilities to adults. Lewis and Lindsay (2000) relate the differences to the wider issues of developmental status and the psychological and legal implications arising from child interviews. Children are younger and much less powerful than the adult interviewer. There is, then, a need for adults to recognise the challenge inherent in enabling children to communicate and to share their experiences and ideas. But there is also an onus on adults to assist children in expressing themselves. A high level of involvement is needed from the outset to establish and maintain rapport with the children being interviewed. Indeed, Ginsburg (1997: 130) argued that what is needed is something deeper than rapport and he described what is established between interviewer and child as '... *an intimate relationship.*' The responsibility for establishing this is very much the responsibility of the adult and will

ultimately determine the success, or otherwise, of the interview. I discussed above how constant decision making on the part of the interviewer is also something that is very much to the fore with this method. The interviewer has to decide whether or not to pursue a response further. It is this pursuit of children's meanings, and the efforts that this entails for the interviewer that led Doverberg and Pramling (1993: 37) to describe this type of activity as '*... deep (intensive) interviewing.*'

The child

For the child, the intensity of the experience arises from a number of features of the method. Firstly, the one-to-one nature of the sustained interaction with an adult will create an intense situation for some children. Ginsburg (1997) describes the interview as a complex form of social interaction. He does so on the basis that it is usually sustained over a considerable length of time (perhaps a half-hour, or sometimes longer), it involves considerable give-and-take between the interviewer and child, and it is quite unlike much of what usually happens between adult and child. The desire to co-operate and 'help' the interviewer is demanding on the child. Also, the nature of the questions and tasks may involve the child in introspection. This introspection may emerge in the interview context as a result of the probing of the interviewer.

Garbarino *et al.* (1992: 59) argued that young children are largely unaware of their own thought processes and have considerable difficulty reflecting on them. They also suggested that '*... they are often unaware of the boundaries between what they know and what they don't know.*' However, I found extensive evidence that the young children in this study were metacognitively aware. In the context of the interview situation, the children in this study displayed considerable awareness of what they knew and what they did not know. This awareness was expressed by them through their use of metalanguage. (See the framework developed by Coltman (2005), where a typology for categorising children's metalanguage is presented).

Síle's description below of her own abilities and difficulties is impressive, and again confirms previous research that demonstrated that children of this age are very capable in this regard (Hendy and Whitebread, 2000). Síle is

very articulate about the aspect of reading numbers that she finds puzzling, and in the process, she gives us a lead as to how her understandings in this respect might be developed. As she does so, we can sense the intensity of her thinking and how demanding it is for her to express her thoughts and to articulate her complex ideas:

Liz: *And do you read the calendar with Mam?*

Síle: *No. I can't read*

Liz: *But does she read it for you?*

Síle: *Do you know ... em ... there's no words*

Liz: *What's on the calendar?*

Síle: *Just numbers and pictures ... three pictures*

Liz: *Just lots of numbers and three pictures?*

Síle: *Yes*

Liz: *And can you read the numbers on the calendar?*

Síle: *Yes*

Liz: *Can you? Good girl*

Síle: *Well if there's ... em ... 2 and 3, I can't do that*

Liz: *2 and 3 together*

Síle: *No .*

Liz: *Right...how far can you go... can you read 1?*

Síle: *Well if... two 1's is easy ... makes 2*

Liz: *Two 1's makes two?*

Síle: *Yes*

Liz: *When you have two 1's together, that's two is it?*

Síle: *Yes*

Liz: *Right ... on the calendar*

Síle: *And ... well there's not two 1's on the calendar ... there's ... em ... two, number two*

Liz: *Right ... after number one it's number two*

Síle: *Yes... and two ones can ... is eleven as well*

Trustworthiness and credibility

Merriam (1998) argues, and I agree, that the applied nature of educational inquiry makes it imperative that researchers and others have confidence in the way the investigation was conducted and in the results derived from it.

However, as Robson (2002) points out, the evaluative criteria applied in the quantitative tradition are not feasible or appropriate with qualitative research. I adopt the stance advocated by Robson and focus on the credibility and trustworthiness of my study. I follow his advice that while we certainly need to pay attention to the standard evaluative criteria generally applied to scientific research, in qualitative research we need to operationalise these concepts so that they are appropriate to the conditions and circumstances of qualitative enquiry.

Trustworthiness

I used a number of strategies to enhance the trustworthiness of my study. Firstly, data were derived from two separate interviews with each child participant. While each of the two interviews had the same ethos (democratic) and focus (number sense), the first engaged the child in intensive discussion while the second engaged the child in number tasks (this format did not, however, preclude discussion). Internal validity was also addressed by checking my interpretations (where possible) of the child's meaning as the interview proceeded. I did this by intensive questioning and pursuit of an idea, once that idea been introduced by the child. Over the course of the interview, I sometimes revisited particular questions, usually with slightly different wording, searching for consistency and clarification. Langstead (1994) suggests that the problems of reliability and validity differ in evaluating research with children. He argues that consistency of response is important and that the criterion for validity must be derived from the child and the consistency and the logic that can be discerned by looking at the interview as a whole. In Chapter 4 (p. 75), I describe how consideration of the whole interview was an aspect of the way I analysed the data.

In carrying out analyses, I drew on audio-recordings of the interviews. Even though there was a considerable amount of data involved (fourteen children with approximately one hour of audiotape for each child), I decided after some consideration, to transcribe the interviews myself. I did so in order to maximise the accuracy of the transcripts, and also to become more familiar with the data. The first interview with each child (Interview 1) was transcribed in full. In relation to the second interview (Interview 2), I used a

recording sheet to record the children's responses to the various tasks during the course of the interview. After the interviews I transcribed and inserted into the record of the interview any additional discussion pertaining to the various tasks and not recorded on the sheet at the time of the interview. (See, for example, Appendix 4: 189-190).

As a further strategy to enhance the validity of my study, I sought comments from colleagues on drafts of chapters and on conference papers in order to clarify my own position, biases and assumptions. I also submitted, and had accepted, papers for publication and I found the peer review process useful for challenging assumptions that I may have made.

Credibility

The credibility of my study was enhanced by explaining the assumptions and the theory underlying my study and by describing in great detail the various steps and decision-making in designing and conducting the research. I also describe in detail how I derived findings from the data and how I reached my conclusions. In this way, readers can judge for themselves whether or not the conclusions drawn from the data make sense, whether they are consistent and dependable (Merriam, 1998). In my reporting, I strove for accuracy and honesty. I sought to provide as non-biased an account as possible in mapping the route through which I reached my various decisions and conclusions. In short, I sought to use a reflexive approach throughout.

In writing the account of my study, I have sought to provide the *web of evidence* that Ginsburg (1997: 182) suggests is necessary for making judgements about validity. He argues, quoting Messick's (1989) proposal of an integrated evaluative judgement in relation to assessing validity, that the body of evidence as a whole must be coherent and convincing. The web of evidence that I supply includes the observations I made of children's reactions to the interview situation; the ways in which the children responded to my probes; and their spontaneous verbalisations about number and about learning in that domain. Since judgements about credibility are based on evidence, I provide readers with as much evidence as possible so

that the dependability of the study and the conclusions drawn may be assessed.

I drew extensively on detailed records that I had kept of my activities while carrying out the study. These records pertained to all aspects of the study and include data, my research notes and diary, and details of the various stages of analysis. The appendices contain key information in relation to a number of issues related to trustworthiness and credibility since they focus on issues related to procedures (See Appendix 1, 3, 4, 5, 6), access, relationships, and ethics (See Appendix 2). The information I provide should also facilitate readers to determine how closely their situations match my research situation, and hence, whether findings from my study might also be applicable in their situations. It is important also that the information I provide is sufficient to permit the study to be replicated. However, replication of my study will not yield the same results, since different children have different experiences to relate and so different views, knowledge and attitudes in relation to numerically related issues, i.e., a different number sense. However, it is essential that readers concur that, given the data collected, the results make sense. As Merriam (1998: 206) argues '*The question then is not whether findings will be found again but* ***whether the results are consistent with the data collected.***' [original emphasis]

I found Ginsburg's (1997) suggested criteria for evaluating the clinical interview very useful. He suggests that evaluation should consider five issues:

- Was the interview conducted in an adequate manner?;
- Was the child engaged with the content and did s/he respond seriously?;
- Was the interpretation of the interview plausible and rationally derived from the internal evidence?;
- Can the study be replicated?;
- Did the interview have beneficial effects on the child and on the interviewer?

While I intend to revisit some of these questions later (See Chapter 4, 5 and 6), I strove to ensure that the audit trail provided is detailed enough to allow readers to make such judgements for themselves, and also to permit a similar study to be conducted by another researcher.

Section B: Analysing the curriculum documents

I carried out a content analysis on the mathematics curriculum for children in the first year of school. Three separate but related documents were involved. The first of these was the *Primary School Curriculum: Introduction* (Government of Ireland, 1999a). I will refer to this as Document A. It is a general introduction to the curriculum and the curriculum areas it embraces. It discusses the aims, principles and the defining features that apply across all curriculum areas; the principles of learning on which the curriculum is based; and the vision in relation to its implementation. This introductory text serves to introduce the seven curricular areas, namely, Language, Mathematics, Social environmental and scientific education, Arts education, Physical education, Social, personal and health education and Religious education. The *Primary School Curriculum: Mathematics* (Government of Ireland, 1999b) is a detailed statement of content in the form of skills and concepts to be acquired and learning objectives to be achieved. I will refer to this as Document B. The *Primary School Curriculum: Mathematics: Teacher Guidelines* (Government of Ireland, 1999c) accompanies the curriculum statement and is a book of teacher guidelines described as '... *an aid and resource for teachers and schools as they encounter the curriculum and begin to implement its recommendations.*' (p. 66) I will refer to this as Document C. The guidelines sought to explore a wide range of approaches and methodologies that develop the new emphases and give expression to new thinking on teaching and learning. They also explicate the content of the curriculum. In addition, they include what are described as detailed exemplars and sample lessons that demonstrate the newer approaches.

The curriculum objectives are presented in concise form in Document B. These objectives served as the units of analysis in analysing that part of Document B. The other two documents, i.e., the *Teacher Guidelines* (Document C) and the *Introduction* (Document A) and the section of

Document B that considers assessment, are less structured, and in those cases I used the sentences as the units of analysis. After an initial reading of the documents it was relatively straightforward to select those sections of the documents that pertained to learning and teaching about number, and thus number sense, in the early years at school. These provided the sample for analysis. A complete list of sections sampled is provided in Appendix 3.

Next, I examined the selected sections of the three documents for references to the four aspects of number sense that I identified in Chapter 2:

- Pleasure and interest in number;
- Understandings of the purposes of number;
- Quantitative thinking;
- Awareness/understanding of symbols.

These provided the lens with which I examined the identified sections of the three documents referred to above.

In the next chapter, I outline the way in which I analysed the data generated using the above data collection techniques and I present the findings that I derived.

Chapter 4: Findings 1

Section A: Findings from the analysis of curriculum documents

The nature of the curriculum documents

The introduction into schools in 1999 of the revised primary curriculum was generally seen as a significant landmark in the history of primary education in Ireland. Prior to that, the last major review of the curriculum was in 1971. In Document A, it is claimed that the curriculum incorporates current educational thinking and the most innovative and effective pedagogical practice. It was heralded as a curriculum that set out clearly not only *what* the child should learn but *how* the child should learn most effectively (National Council for Curriculum and Assessment [NCCA], 2000). It was envisaged that the implementation of the revised curriculum was a process that would take a number of years to complete. It was intended that teachers would be involved in planning for implementation at school level and in professional development programmes on each of the areas of learning, including mathematics. To date, six years after the introduction of the curriculum to schools, teachers have received just two days in-service development in relation to the revised mathematics programme. Schools have also been involved in some whole school planning, sometimes with the assistance of an advisor from the Primary Curriculum Support Programme. However, these advisors were not selected for their particular expertise in mathematics or for their expertise in early years, but were generally teachers who had obtained some additional qualifications and who had some experience in giving voluntary in-service courses of short duration to teachers, for example during the summer holidays.

Content analysis of the *Primary School Curriculum* in relation to aspects of number sense

As outlined in Chapter 3, relevant sections of the three separate but related documents were analysed. Table 4.1 shows the analysis of text across the three relevant documents.

Table 4.1: Content analysis of selected documents of Primary School Curriculum in relation to aspects of number sense

Categories	Introduction (Document A)	Mathematics Curriculum * (Document B)	Teacher Guidelines (Document C)
Children's pleasure and interest in number	'Within the framework of the curriculum schools are afforded flexibility to plan a programme that is appropriate to the individual school's circumstances and to the needs, aptitudes and interests of the children.' (p.11) '... many children begin formal schooling from the age of four. This requires a curriculum that is appropriate to the developmental and learning needs of young children...There is a need for a continuing process whereby the child's experiences in the infant classes interacts with the developmental experience of home and family.' (p. 30)	Assessment: 'The cognitive and affective areas should be assessed ... attitudes towards mathematics, including confidence, interest, willingness to take risks, and perception of the usefulness of mathematics, are assessed by observing the enthusiasm with which the child approaches a task. Attitudes also encompasses the interest the child shows in completing tasks and in using mathematics confidently in other curricular areas and in real-life situations. Teacher observations of such attitudes contribute to an overall picture of the child's mathematical development and are continuing and informal.' (pp. 115-6)	'It is hoped that the new emphases will lead to an enhancement of the child's mathematical education and to a heightened pleasure and interest in the subject.' (p. 30)
Children's understandings of purposes of number	'The curriculum gives particular consideration to the social importance and relevance of mathematics. It emphasises the value of real contexts for mathematical activity in school and gives prominence to the constructivist approach. The child is encouraged to be active in learning and to engage in discussion with the teacher and other children.' (p. 47)		'Society needs people who can think and communicate quantitatively and who can recognise situations where mathematics can be applied to solve problems.' (p. 2) 'The child learns from the people and materials around him/her. It is experience of the social and physical world that is the source of concepts, ideas, facts and skills. Integration of those experiences is the vital ingredient.' (p. 3)

Categories	Introduction (Document A)	Mathematics Curriculum * (Document B)	Teacher Guidelines (Document C)
Children's quantitative thinking Counting; relating number; subitizing; estimation.	'Prediction and estimation skills are taught from junior Infants onwards. This will enable children to develop the ability to gauge the accuracy and validity of possible solutions' (p. 47) 'An important goal of the curriculum is the development of the children's numeracy skills. The understanding and memorisation of number facts and the ability to apply them appropriately are fundamental to children's wider mathematical development and to their ability to use mathematics to attain an understanding of the world.' (p. 48)	The child should be enabled to • count the number of objects in a set, 1-10 (p. 22) • recognise solutions to problems (p. 18) • explore the components of number, 1-5 (p.18) • combine sets of objects, totals to 5 (p. 18) • partition sets of objects, 1-5 (p. 18) • devise and use mental strategies and procedures for carrying out mathematical tasks (p. 19) • use appropriate manipulatives to carry out mathematical tasks and procedures (p. 19) • compare equivalent and non-equivalent sets 1-5 by matching without using symbols...more than/less than/same as (p. 23) • subitize (tell at a glance) the number of objects in a set, 1-5 (p. 24) • tell at a glance how many objects are in a set; estimate using a known set; without counting classify the other sets as less than/about the same as/more than the given set (p. 24) • solve simple oral problems , 0-5 (p.25) • solve practical tasks and problems using number (p.34)	'Throughout all the strands of the curriculum emphasis has been placed on the development of estimation strategies...' 'The teacher will have to lead the work by encouraging them to make a sensible 'guess', to test their guess and revise it where needed...' 'Children must also be taught to investigate the reasonableness of their results. They can be encouraged to develop their own ways of deciding when an answer is reasonable.' (p. 32)

Categories	Introduction (Document A)	Mathematics Curriculum * (Document B)	Teacher Guidelines (Document C)
Children's awareness / understanding of numerals	'As well as learning to manipulate numbers, symbols and shapes it is important that the child becomes familiar with the language of mathematics and develops the ability to use it to communicate mathematical ideas accurately.' (p. 48)	The child should be enabled to • discuss and explain mathematical activities (p.18) • record the results of mathematical activities concretely and using diagrams, pictures and numbers (p. 18) • discuss problems presented concretely, pictorially or orally. (p.18) • recognise the relationship between verbal, concrete, pictorial and symbolic modes of representing numbers (p. 18) • recall and understand terminology (p. 19) • read, write and order numbers 1-5 (p. 24) • identify the empty set and the numeral zero (p. 24)	'The same care and attention should be given to the formation of numbers as is given to the formation of letters. Children should practise forming and writing numerals...'(p. 31) 'Discussing and interpreting symbols in the environment is a good starting point for introducing mathematical symbols...' (p. 30) 'Introducing mathematical symbols and numerals is the last step in the learning process...'(p. 31)

* Only selected content/skill objects are cited here. Full a full list of objectives related to the various skills and strands of the curriculum see Primary School Curriculum: Mathematics (Government of Ireland, 1999b).

Below I discuss the curriculum documents with reference to each of the four aspects of number sense under consideration.

Children's pleasure and interest in number

Document A highlights the importance of providing conditions in schools within which children's pleasure and interest in the various curriculum areas, including number, can be promoted. Document B expresses the hope that the new emphases will contribute to the heightening of children's pleasure and interest in the subject (See Table 4.1). The importance of children's informal learning, and the need to ensure that subsequent learning

recognises that which has gone before, is also emphasised in that document. Consequently, the absence in Document C, of any discussion of how informal experiences may effect children's engagement with number-related experiences in school is a serious omission. For instance, we know that particular kinds of early experiences play a critical role in children's later attainment in mathematics (See Chapter 2).

Recognition of the importance of the affective, as well as the cognitive areas, is emphasised in Document B in relation to ongoing informal assessment. This is in keeping with research that places 'affect' as a centrally important element in learning and recognises that the question of how to assess affective dimensions is an important issue in relation to mathematics education (e.g., Silver, 1989; McLeod, 1992). However, there is no corresponding discussion in Document C in relation to why it is important and how it can be developed or assessed, beyond the advice offered in Document B regarding *'... observing the enthusiasm with which a child approaches a task.'* (pp. 115-6) This is a surprising omission in light of the fact that the importance of doing so is particularly emphasised in Document A. In the absence of such advice, I think that the statement in relation to assessment of young children's mathematical understandings could be interpreted narrowly by teachers. In that event, they may then assess children with tasks rather than assess children while they are engaged in tasks, or through discussions regarding children's ideas and understandings.

I suggest that guidance for teachers needs to emphasise that, for young children, the issue of their interest in number or their *'at-homeness'* (Cockcroft, 1982: par. 38) is central when thinking about number sense. Indeed my findings will suggest that young children starting school may already have developed a liking or enthusiasm for number, or otherwise, based on their experiences during the preschool period, i.e., their disposition towards number is already developing. Hence, guidelines should also emphasise that issues related to disposition need to be investigated by teachers in order to work effectively with children in relation to the further development of their number sense.

Children's understandings of purposes of number

The importance of understanding purpose in the domain of mathematics received some attention in the curriculum documents analysed. For instance, Document C appears to suggest that such understanding is strengthened by integrating informal and formal learning and by enabling children to recognise mathematics in the environment. Document B encourages teachers to focus on activities and discussion that explicitly enable children to understand the different purposes of number in everyday life. It is suggested that mathematics education will help to provide the child with the means to deal effectively with everyday life, with their physical world and with their social experiences.

The social nature of children's learning is explicitly recognised in all three documents and the integration of the different experiences that children have is seen as important for development in relation to mathematics. These experiences are recognised in Document C as the source of children's concepts, ideas, facts and skills. My findings will show that children's experiences are very individualistic in nature and the learning derived from them is dependent on a number of variables, including the extent of children's participation in events and the level of expertise of those who participate with them in their learning. Document B appears to foreground mathematics education as the impetus and motivator for children's understandings of mathematical purpose, while at the same time acknowledging the centrality for learning mathematics of other people and materials. I believe, based on my data and analysis, that for young children, it is the drive to make sense of the world that is the primary motivator for their learning. The number-based experiences that takes place either in/out of school helps shape their learning, and consequently, their number sense.

If children's real-life experiences are to be considered when planning learning experiences, as argued in Document A, then teachers will need to investigate these experiences and consider how they can be used to develop children's number sense. I think that there is an important issue here, since teachers of young children otherwise can only depend on their knowledge of the universal child as derived from developmental psychology to guide pedagogy. Walkerdine (1988: 5-6) argued that there was no such child as a

'universal child.' She further argued that the *'scientific truth'* about children's reasoning has become a very powerful tool in pedagogic practice and that *'... current psychological and pedagogical 'truths' about children's learning'* need to be prised apart. I suggest that teachers of young children can temper such 'truths' if they have information related to children's individual experiences and understandings.

Children's quantitative thinking

In Document B, it is proposed that children's understanding of how numbers relate to each other should be developed through a range of content objectives and these are comprehensively outlined for teachers. These are generally uncontroversial and similar to those found in early primary school curricula in a number of countries. In the Irish context, a major new emphasis at infant level is the introduction of activities related to *subitizing*, and what is referred to in Document C as sensible 'guessing' or *estimation.* Sowder (1992) observed that until recently, few primary school curricula included any mention of estimation for very young children. According to her, its perceived importance now appears to lie in its relationship with number sense as research has concluded that children must develop a quantitative intuition or number sense in order to be able to estimate. When or how estimation should be introduced to young children are issues for debate among researchers, cognitive psychologists and practitioners (See Chapter 2: 32-34). The curriculum documents take a strong pro-stance on the introduction of estimation activities to very young children at school. For example, Document A advises that estimation is something that should be emphasised right from the beginning of school mathematics. Others such as Sowder (1992) take a rationalistic standpoint and suggest that educators should not be in too much of a rush to teach estimation skills' since children might be too young to master them in anything but a superficial manner. One aspect of my study sought to explore how young children handled estimation tasks by engaging them in estimation activities and I discuss the findings related to this, and other aspects of quantification, later in the chapter.

Children's awareness/understanding of written numerals

Document B emphasises the need to develop a number of skills in relation to children's awareness of different aspects of number (knowledge of number names, understanding of symbols) but the comments found in Document C relate mainly to the formation of numerals (See Table 4.1). My observation is that teaching children how to form the numerals is a comfortable area of number-related activity for many teachers. It is relatively easy to teach and assess and it provides tangible evidence of 'work'. The advice in Document C is that children should practise forming and writing the numbers. My experience suggests that one undesirable outcome of this advice may be that teachers will continue with the traditional emphasis on practise of numeral formation. I have observed that this sometimes extends well beyond the stage within which it is beneficial for children. This is not to suggest that teaching related to the writing of numbers is not an important aspect of young children's number sense. Indeed, my data will show that writing numbers is of considerable interest to many children. I suggest that guidelines need to emphasise the importance of assessing children's ability in this area and of teaching this skill *selectively* where it needs to be taught. An important task for the teacher is to ensure meaningful contexts where children can practise such skills. I suggest also that the guidelines need to advise teachers that responsiveness to children's interests in this area is an important consideration during the first year at school. Where children are interested and actively seeking to write numerals, they should be encouraged and assisted to do so. To assist teachers who encounter children who appear not interested in writing conventional numerals, the guidance needs to emphasise the importance and role of children's invented symbols, of which there is no mention at all in the curriculum documents. They also should suggest ways in which such symbols may be used as a stepping stone to the introduction of conventional written numbers.

Conclusion

The curriculum documentation, taken as a whole, makes reference to each of the key elements of number sense that I identified in my framework. Document A establishes a good balance between the emphasis placed on the

unique ways in which individual children construct mathematics, the role of sociocultural processes and the mathematical content. The documentation also clearly establishes the affective and social dimensions of learning as relevant, alongside the cognitive aspect. However, my analysis shows that Document C does not demonstrate *how* teachers can develop a number pedagogy that builds on children's informal knowledge. This is a serious situation in a climate where there has been little provision for continuing professional development and consequently for teachers to develop an understanding of the new emphases. My analysis also leads me to conclude that the affective aspect of the curriculum is not at all explicit in Document B. It is given very brief and general consideration in the discussion about assessment and almost no consideration in the discussion about content. This is especially obvious when compared with the detailed and extensive listings related to the development of skills and the attainment of objectives in the same document, i.e., the cognitive aspect. As a result, the skills and knowledge aspects of number appear to be foregrounded in the documentation. Consequently, these are the aspects that are likely to be interpreted as priorities, especially since teachers remain generally unaware of the rationale behind the new emphases related to affect and to building on informal learning. In my experience, as a teacher-educator and as a researcher, teachers generally do not appreciate the role and influence of either affect or informal learning.

The imbalance in relation to how different aspects of number sense are treated in Document B is not redressed in any way in the discussion in Document C. The values conveyed in Document A regarding the relevance of curriculum in terms of children's interests, the provision of appropriate contexts for learning and the importance of localising the curriculum to suit particular circumstances/children, appears to be somewhat diminished in the tone of Document B. Also, the emphasis on objectives and skill development evident in Document B may give teachers the impression of a 'programme' to be worked through from start to finish, rather than the flexible framework described in Document A (See Table 4.1). Teachers are told that:

> ... *a significant proportion of the teacher guidelines is directed to a detailed explanation of approaches and methodologies in teaching and learning. A particular emphasis is placed on the newer approaches envisaged in the curriculum and detailed exemplars and sample lessons that demonstrate these are included.* (Government of Ireland, 1999a: 67)

My analysis demonstrates that Document C falls considerably short of that promise. Indeed, there appears to be a tension between the documents in relation to the position of the areas related to affect *vis-à-vis* those related to skills and content.

In conclusion, my analysis of the curriculum documentation indicates that specific guidance is needed in Document C in relation to the following areas:

- Engaging young children in discussions about number;
- Developing and assessing the affective areas of learning in relation to number;
- Identifying and building on children's informal number-related learning;
- Providing young children with opportunities to construct and apply understandings and skills in contexts drawn from their own experiences, interests and environments;
- Developing newly emphasised number skills in young children (e.g., estimation).

Section B: Findings from the interview data

Considerations related to analysis of interview data

In Chapter 3 (p. 60), I discussed the importance of the *web of evidence* (Ginsburg, 1997: 182) for assessing the trustworthiness of a study. This applies at the analysis stage as much as at any other stage of the work. Kvale (1995) suggested that the monitoring of the validity of qualitative research as it progresses involves a process of checking, questioning and theorising. In assessing validity by relativist standards, he argues that the quality of the craftsmanship of the research is a critical issue. An essential

aspect for consideration is the interpretation and verification of data through the process of analysis. Ginsburg (1997) similarly advises that the question of the interpretation of data derived from the interviews, and the findings based on such interpretations, must be capable of being checked. Ensuring that such checking is possible constitutes an essential step in establishing the validity or trustworthiness of the study.

In this chapter, and in Chapter 5 that follows, the analysis of children's responses and the findings derived from this was undertaken using the three planes of analysis, as outlined by Rogoff (1998) and as discussed earlier (See Chapter 1: 6-8). However, since it is impossible to discuss all three planes at once, I will focus mostly on the personal plane in this chapter, thereby allowing the interpersonal and community/institutional planes to recede into the background. In Chapter 5, I will focus mainly on the interpersonal plane and allow the personal and community/institutional planes to recede. I discuss this approach to analysis in more detail later in this Chapter (pp. 111-113). Presenting the analysis and findings in two complementary chapters permits me to offer a holistic account of children's participation in number-related activity and a comprehensive account of the development of their number sense.

Data Reduction

My data consisted of 14 sets of interviews. There were two separate interviews for each of the fourteen children who participated in the study. In Chapter 3, I described how the first interview with each child was focused around questions and the second interview was focused around tasks. Together these yielded an average of one hour of audiotape for each child. I transcribed all of the question-focused interviews. In preparation for the second set of interviews I prepared individual response sheets on which I entered each child's responses as I conducted the task-based interviews. I later integrated these with transcribed comments from the tapes that related to that set of interviews. In each case, I stapled these to the transcript of the first interview (See Appendix 4 for a sample of pages from a data set). The interviews generated a considerable amount of data so my first concern was to reduce the data into manageable chunks.

Interview 1 Transcripts

In relation to Interview 1, the following were the steps that I took in order to reduce the data:

Firstly, in order to derive information related to children's views on the purposes of number, I prepared templates related to each of the seven questions that I had used as a guide to the interview. I then read and reread the transcripts of each interview carefully, identifying those segments of text that related directly to specific questions. As outlined in a previous discussion (See Chapter 3: 43-44), I took great care to ensure that as far as possible, subjective equivalence was established. At the analysis level, I similarly attended to the possibility that differences in views may represent variations in interpretations of the questions asked. Because of the fluid nature of the interviews, and the fact that the questions were a guide rather than a strict format, not every child was presented with every question. In the left-hand margin of the transcript I labelled each segment with the number of the corresponding question. Sometimes there was just one section of the transcript that was relevant. However, sometimes during the course of the interview, either the child or I returned to particular questions or topics and in those instances these segments of discussion were also relevant and had to be taken into account when doing the analysis pertaining to a particular question. When all of the interviews were read, all of the marked sections were brought together into one large '*pool*' of statements (Marton and Saljo, 1984). While agreeing in principle with Bowden (1996) that it is preferable to deal with the whole transcript in order to preserve meaning, I found that the assembly of these piles of statements helped me to get a sense of the range of views of the children and to sort the similarities and differences between them. I used the full transcript to help with interpretation wherever the statements were in need of contextualisation, or indeed in cases, where individual children referred back to other parts of our conversation.

The next step consisted of the bringing together of the children's statements that were similar and identifying those statements that were different from the others. In this way a number of categories of children's views were derived from the data. Children often expressed a number of different views

about an issue depending on the context, and all of these were taken into account (See Appendix 5). Finally, the categories were exemplified by appropriate quotations from the data in order that the reader can see how I formed the categories. This analytic process owes much to that described by phenomenographers (Dall'Alba and Hasselgren, 1996). Children's views and their accounts of their experiences provided a window on the interdependent individual, interpersonal and cultural processes at work in the development of their number sense. In the 'pooling' of statements, these details were all preserved, thus ensuring that this analysis is still in line with the approach outlined by Rogoff (1998) and discussed earlier (See Chapter 1: 6-8).

Interview 2 Transcripts

In relation to Interview 2, the following was the procedure that I took in reducing the data:

I prepared ten summary sheets, one for each of the nine tasks and one additional one on which to enter data related to the discussion regarding magnetic numerals. The majority of the data were such that they transferred well onto summary sheets. This process generated a number of tables that were useful as a means of retrieving information quickly (e.g., See Appendix 6).

As a result of the processes described above, there were a total of seventeen bundles of data, seven from Interview 1 and ten from Interview 2.

Some general observations

At this point, I noted some general observations about the data:

- Since the girls spoke more and were more forthcoming with experiences and opinions, the data from them was more extensive than that from the boys;
- The boys seemed to be much more curious about the immediate environment of the room that the interview was held in. Perhaps it was somehow related to the fact that the site for interviewing the boys was in the main school building whereas their classroom was a

prefabricated building on the grounds of the school but away from the main building. In contrast the girls were interviewed in a room on the second floor of the main building, in a room directly above their own classroom. Perhaps the fact that the boys were interviewed during the initial two weeks of school while the girls interviews were in the third and fourth weeks was significant. Certainly, it can be argued that I was more practised at eliciting information from the children by the time I interviewed the girls. Also, perhaps some of the initial curiosity of the children about the process of schooling had been satisfied by weeks three and four but was more pressing in weeks one and two. It could also be argued that the environment at my disposal in the girls school was less distracting for the children since it was a storeroom, and consequently, contained little that seemed to be of interest to the children. Also, that room was out-of-the-way of the general school traffic and so there were no interruptions at all (in contrast to the boys' school where there were a number of interruptions);

- The boys were certainly much more likely to question me in relation to the environment, my purpose, my identity, my role in the school, and the roles of others also. The girls, on the other hand, were more likely to get into discussion about the issues I raised and to relate their experiences more freely;
- The data demonstrated the huge range of experience, of interests and of knowledge of the children in relation to number;
- Some children professed to love counting and numbers, while a few actively avoided the questions related to numbers;
- Some children were very noticeably assessing their own strengths and learning needs as they talked to me;
- Although not specifically a focus of the study, a number of children talked about their experiences and abilities in writing numbers;
- The reference points used by the children to talk about number were very much related to affect, i.e., age, play and relationships;
- Children interviewed, in general, appeared to think of number in one of two ways: it related to the act of counting and/or it related to the act of writing numerals;

- I had a considerable amount of rich data in relation to number sense in young children.

Findings related to aspects of young children's number sense

In the previous section, I discussed the curriculum documents with reference to each of the four aspects of number sense under consideration. As a result of my analysis of all of the data arising from my empirical work, I derived a considerable amount of findings related to these aspects of number sense in young children starting school. However, such was the extent of these findings that it was necessary to prioritize them in some way. Below, I present findings related to two of the four aspects of number sense: children's ideas of the purposes of number and aspects of children's quantitative thinking. Both of these were explored in some depth in the literature earlier since they emerged as important in the pilot study. They are also highlighted in the curriculum so the findings have particular significance for practice. The (mainly) sociocultural perspective that I have adopted in relation to this study makes it essential that the affective aspect of the children's number sense, i.e., that related to children's interest and pleasure in number, is reflected in the findings. The importance of affective factors in relation to children's number sense was an issue that constantly re-occurred as I interviewed the children, and it is very striking in the data. In that sense, the affective aspect of children's number sense is implicitly addressed as I present my findings below.

One of the biggest analytic challenges I faced in relation to the data was to reconcile individual children's number sense with the need for a more general understanding of number sense in young children starting school. I decided to overcome this by presenting data related to all children initially (See this Chapter) and then to take a more in-depth case study approach for two children in order to capture the uniqueness of individual children's number sense (See Chapter 5).

Findings related to young children's ideas about the purposes of numbers

In preparing the initial pilot study for publication (Dunphy, 2004), the advice with respect to data reduction and display from the peer review

process was that I should strive to use the minimum number of tables and to ensure that they were as informative and as easily read as possible. This advice proved invaluable as I struggled with the challenge of presenting the data from the main study to best advantage. Having followed Robson's (2002) suggestions regarding matrix data entry, I am confident that the tables as finally presented will inform readers as well as facilitate them in drawing their own conclusions from the data.

What children said about the purposes of number

Children were asked what they thought numbers were for, if they thought that numbers were useful and about the ways in which they had observed people use them. They were also asked where they might see numbers in the environment (See Appendix 1a). The findings are presented in Table 4.2 below. This exploration of children's ideas related to purpose enabled me to examine this aspect of their number sense.

I classified the functions that children attributed to numbers as generic, communicative, count or label. This classification was informed by others whose work in this area I had reviewed (Sinclair and Sinclair, 1984; Munn, 1994). These were categories that, in my view, best described how the children in the study appeared to view the purpose of number in social activity. The classification was not always straightforward or easy. In each case, and especially where there might be any ambiguity about the purpose that children attributed to number in a given context, I reviewed the child's comments within the wider context of the extended conversation around a particular topic. For example Kate reported that she had seen the number ninety-nine written on her sister's trousers, but it was difficult to judge whether she understood the number to be communicating something specific:

Kate: *Just put one on Lorrie's trousers*

Liz: *On Lorrie's trousers?*

Kate: *Yes ... and I have one and ... eh ... one and two on my pink trousers*

Liz: *On your pink trousers ... there's a one and a two*

Kate: *Yes*

Liz: *Are they on the trousers or is it on the label?*

Kate: *It's on the end of the trousers*

I used the term *generic* to classify responses where children said that they thought that numbers were useful or 'important' but in a broad, non-specific way. Those responses mostly referred to numbers being important for learning at school and for adult's work. They were very similar to the views expressed by the children I interviewed in the initial study (Dunphy, 2004). While children recognised that numbers served a purpose, their comments here were not related to aspects such as those dealing with quantity or order or any of the other usual purposes for which number is used. Some children responded to my probing for more specific information with the response '*I don't know.*' These were usually noted as generic responses.

I used the term *communicative* to classify responses where children referred to specific instances in which number fulfilled a communicative purpose. [In one sense, it is possible to argue that all purpose is communicative but I found it useful to differentiate this function from others where children didn't appear to recognise/be aware of a communicative intent]. Ten children suggested that numbers fulfilled an important *communicative* function. However, there was considerable confusion amongst the children in relation to their understandings of how numbers are used to communicate with others, and in terms of their understanding of the nature of information that can be communicated using numbers. For instance, Bob's grasp of the role that numbers on the calendar can play seems to be considerably less developed that that of Síle (See Table 4.2). Others seemed to have derived some understanding of the less obvious ways in which number may be used in communicating different kinds of information. For instance, Maura suggested that numbers played a role in conveying information on birthday invitations and Síle suggested that they convey information '... *on the shopping packets ... on the dates.*' (See Table 4.2)

I used the term *count* to classify responses where children identified that number has a counting function either generally, for them, for specific people or in specific circumstances. Ten of the children suggested *counting* as a function of number. In all but two of these instances, children appeared to be referring to the recitation of the number string rather than any

quantifying purpose. Munn (1994) concluded on the basis of her findings that for many four-year-old children, counting was a linguistic rather than a functional activity. While my findings appear to support that claim, they also suggest that it might be more precise to say that the linguistic function is more visible to children than the quantifying function, perhaps because of its prominence in their interactions about number with adults and peers.

I used the term *label* to classify responses where children identified the function of number as a label or a description of something. While the range of *labelling* functions referred to by the children included ages, house numbers, phone numbers, clothing labels, credit cards and money, these were very infrequency referred to (See Table 4.2). This suggests that, in general, children attach little importance to these purposes of number unless they have been specifically guided to do so (See Chapter 5: Sonia:115-123). On the other hand, eleven of the thirteen children who were asked to do so could represent their age using the correct symbol chosen from those available. Eight of the children choose the correct numeral/numerals to indicate their house number. I conclude then that 'personal numbers' such as age, and to a lesser extent house number, are generally important and comprehensible to young children. However, in general children don't mention such purposes spontaneously, and so possibly don't consider such functions to be of much significance or interest, either generally or to me.

Table 4.2: Children's ideas of the purposes of number in everyday life

Name	Generic	Communicative (to convey specific/general information)	Count	Label
Bob *(M) 4y 8m**	They [boys] need to go to school ... to learn numbers Because they just are [useful] It just helps you [to know about numbers]	We have a calendar in the kitchen...to tell what time it is ... Em ... I don't know		

Name	Generic	Communicative (to convey specific/general information)	Count	Label
Shay *(M) 4y 11m*	She [his Mam] has to work in her office ... she has lots of papers ... and I can't read	Because we were going on our holidays to France Because you can tell other people Clocks are very useful ... I have a clock on the oven	You can ... eh ... count... You can count eh... count... eh ... how many rainbows	For people's ages
Terence *(M) 4y 3m*	Mam used to go to work ... before the baby came When he [his Dad] goes to work ... I don't know [what he does with numbers]		Numbers help you to count	
Tom *(M) 4y 11m*	They help Dad ... he reads them and he learns them ... em ... he goes to school That's why you need to get something in your head. You need the important things [i.e., numbers] I don't know [how Mam uses numbers]	Because letters and writing and stuff Re his Mam ... she reads them in the car, she reads something with them on ... something important but I don't know... Re his dad ... he likes doing reading with the numbers	Re his brother... He counts them like me	
Jamie *(M) 4y 9m*	Numbers are useful ... in lots of ways		Because I can count	Well ... my house is 139
Con *(M) 4y 9m*		Numbers are fun ... I like to turn around the clock ... I like that part... nothing else	Counting ...	Ten ... ten pence

Name	Generic	Communicative (to convey specific/general information)	Count	Label
Owen *(M) 4y 6m*	I don't know [if numbers are useful for anything else] You have to ... and then do work [at school]	Because to write on your copies Re Dad ... I think he writes them at his office	Re the Train Station ... they're there on the bridge [numbers] ... for to count the trains	
Jerry *(M) 5y 1m*	I don't know [why people write numbers]	Re Dad ... he works on his computer and he learns ... all numbers come up ... but they're not ones or twos ... But there's a lot... they're money... money ... he looks at money. It's 11 o'clock ... because I saw eleven		
Maura *(F) 4y 5m*		Re the clock ... to go to sleep and to go and get my brother ... because my Mammy said that Re invitations... for going to my birthday and my brother's birthday		
Sonia *(F) 4y 11m*		Re Mam ... she writes stuff to remember what you have to buy for shopping... she writes some numbers down Re road signs ... They have numbers on the signs ... I think they're for people to know some stuff ... Like how you ... em ... not to drive too fast	Counting ... playing Hide-and-Seek and Hopscotch	I know how to spell my phone number I was looking it in a credit card or something Some people's phone numbers

Name	Generic	Communicative (to convey specific/general information)	Count	Label
Síle *(F) 5y*		Re shopping ... I seen them once by the dates ... the numbers all around the place ... on the packets on the dates ... I hear the prices I write down numbers as well ... he [Dad] writes down ... em ... the dates ... the birthdays... on the calendar Re Dad measuring her sister's bedroom wall ... he uses his measuring thing ... for measuring things ... the bedroom wall		
Mary *(F) 4y 6m*	I learned them in playschool I don't know [if numbers are useful for anything else besides counting]		You just learn them for counting	I saw once a sticker ... it was on ... we got a new kitchen ... it was on a door ... It had 2 ... Mammy rubbed it off
Kate *(F) 5y*	You just learn them	When I want them ... then Mammy just writes them	Dad ... he knows them ... but he doesn't want to count them	I have one ... eh ... one and two on my pink trousers
Lara *(F) 4y 1m*			Counting ... Well you have to do a hAon, a Dó*** and I'm really bored ... My teacher says that and I don't really want to say it	

*M denotes male, y denotes years and m denotes months
**F denotes female
*** This corresponds to the counting string One, Two, in English.

Table 4.2 provides an overview of the extent to which children spontaneously call up different functions of number, sometimes by talking about their participation in number–related experiences. The data suggest that the counting function is the most transparent and therefore the most important to children at this age. They also suggest that children recognise the importance that is generally attached to numbers, and that they understand that numbers can be used to communicate information of various kinds. Previous research had noted that four-year-old children offered only the vaguest explanations of the meaning of numerals, presented in particular contexts (Sinclair and Sinclair, 1984). Most children in my study also displayed limited understanding of the specific information that numbers might be used to communicate.

Children's understandings related to what is intended to be communicated by numbers in specific instances may also differ markedly from adult understandings and intentions. For instance, Owen's working theory was that the numbers on the bridge at the train station had a counting function, whereas the adult intention is that these indicate maximum permissible speed for trains passing through. Síle's theory was that as a result of measuring the bedroom, her Dad knew that '*it's much bigger*'. After measuring the sitting room she suggested that he then knew that '*... it's a bit big ... em ... and it's small ... Yes. It was small and big.*'

We can see that Maura's understanding of the purpose of numbers on the birthday invitation was also in the process of development: '*Em ... em ... she said ... somebody wouldn't really know between our houses and their houses ... em ... em ... say... the people say how many days is ... em ... is there a birthday in ... and Maura's birthday ... in six or five days.*'

I think it is striking that almost all the boys described numbers as useful but in a broad, generic sense. In contrast half of the girls referred to very specific experiences that illustrated, for example, the communicative function of number. Half of the boys explicitly stated that they didn't know when pressed to be more specific about function. Only one girl responded in this way. Also significant, I think, is the fact that the girls were more likely

to respond to such probing by talking about a specific experience that, from their perspective, conveyed purpose.

Most children identified at least two, and usually three, functions in relation to the purpose of numbers in everyday life, showing their developing understanding of different purposes of number and a concurrent development in their ability to shift amongst meanings (Fuson, 1988). However, where children identified one particular function only, it cannot be assumed that this implies that they were unaware of other functions. My data illustrate how the discussions were sometimes dominated by individual children's reflections on particular experiences that they chose to focus on when asked about number. For instance, Jamie's interest at the time of our discussions was very much on the order of the numbers. When I asked him about his knowledge of counting, he appeared to relate counting to the numbers on people's houses and responded by talking about number in the context of door-numbers. For instance, he suggested that Sharon could count, and then remarked *'... her house is in 30 ... she used to be in 133* [...] *my house is 139 ... em ... there's a friend next door to me ... on that side ... the next number is 138.'*

Lara was just beginning to learn Irish dancing. Consequently, on two occasions, my attempts to discuss number with her prompted her to show me her dancing. For instance, when she said her Daddy could count, I asked her how he learned this:

Lara: *Well ... I learned him that*

Liz: *Did you? And who learned him one, two, three, four, five?*

Lara: *One, two, five, four..* [She sang this] *Do you want to see my Irish dancing?* [She jumped off the chair and stood with her foot out.]

Liz: *Oh ... because you say one, two, three in the Irish dancing? Okay, do a little piece for me there.*

She danced her steps as I kept time for her by counting. When I stopped she stopped, and she sat down ready to resume our discussion.

My findings suggest that while seeking children's understandings of the purpose of numbers by asking them about the ways in which they/others use

numbers may be informative, the data are in no way comprehensive. Children, at any particular point in time and for whatever reason, choose those experiences that they discuss, but the range from which they can select is determined by sociocultural factors. Piaget (1929) argued that young children neither spontaneously seek nor are able to communicate the whole of their thought. Pramling (2004: 11) reminds us that in discussion, we only get to share what she refers to as '... *fragments of a child's world or experiences.*' Taking into account these insights, I think it is likely that at least some of these children understood considerably more about the purpose of numbers than that which was elicited in the discussions reported above. Additional aspects of children's understandings are implicit in their accounts of the particular experiences that they perceived to be related to number and I present my analysis of these below.

Children's accounts of how people (they or others) use numbers

As might be expected, children in the study described a wide range of experiences that in their view were about using number and these are displayed in Table 4.3 below.

Table 4.3: Number-related experiences cited by children

Name	Experiences related by children
Bob *(M) 4y 8m **	Singing the Crocodile and Alphabet songs, Using the calendar, Counting with Mam on a Journey, Play with siblings, Taking Medicine, Playing a game, Magnetic numbers
Terence *(M) 4y 3m*	The new baby, Magnetic numbers, Game of Penguin Chuckers.
Shane *(M) 4y 11m*	Counting in French/Irish, Counting with his brother, A present of Chalk from his Grandparents, The clock on the oven, Magnetic numbers, *Snakes and Ladders* Game.
Tom *(M) 4y 11m*	Writing numbers at Playschool, Listening to his brother saying numbers, Magnetic numbers.
Jamie *(M) 4y 9m*	Observing and discussing numbers as labels on the doors of the houses of various people he knows.
Con *(M) 4y 9m*	Dad explains relative value of coins about money, Reading the clock and moving the hands, Counting and stories related to counting, Measuring size/height of family members with parents, Magnetic numbers, Dice games.

Name	Experiences related by children
Owen *(M) 4y 6m*	Sesame Street, Writing numbers, Watching Mam using the calendar, Gardening with Dad, Watching the trains at the Train Station, Shopping .
Jerry *(M) 5y 1 m*	Counting with Dad on holidays, Playing with friends, Watching Dad working on the computer, Reading the clock with Mam, Grandad explains about the watch with no numbers, The ages of children in class, Games, Magnetic numbers (on fridge).
Maura *(F) 4y 5m ***	Counting at school and on the way home, With Mam at the 'Exercising Shop', Talking about the clock with Mam, Writing birthday invitations, A poster depicting the story of Beauty and The Beast, Naming the Letterland characters.
Sonia *(F) 4y 11m*	Making Cards with Mam, Hide and Seek at school, Chalking with friends, Playing Hopscotch with Granny, Writing Numbers, Calendar, Shopping Lists, Talking to Mam about numbers in her address/phone book. Playing with friends at the Creche, Playing with her 'toy' computer, Describing numbers used on road signs.
Síle *(F) 5y*	Counting/Listening to family members counting, Playing Mams and Dads, Measuring with Dad, Using the calendar to mark birthdays, Reading numbers on packages in the shop, Writing numbers, Playing the game Snakes and Ladders.
Mary *(F) 4y 6m*	Playing School with Grandad, Shopping with Daddy and sister, Discussing ages with her sister, The new kitchen presses, Playing Snakes and Ladders.
Kate *(F) 5y*	Playing with her sisters, Writing numbers, Describing the appearance of her trousers, Playing a board game.
Lara *(F) 4y 1m*	Counting in Irish, Irish Dancing, Listening to a choir practising in the church, Writing numbers.

*M denotes male, y denotes years and m denotes months
**F denotes female

Some of the experiences for example, reading and talking about the clock and the calendar, are ones that previous observational studies have found to be very common ones in families with young children (e.g., Young-Loveridge, 1989). These were, for most of the young children in this study, transparently about number. Nine of the eleven children asked said that they had magnetic numerals at home. In contrast to this finding, Young-Loveridge (1989) reported that only one child of the six in her study in New Zealand had such items at home. There are a number of possible explanations for the difference in incidence in the two studies. Magnetic numerals are now widely available and relatively inexpensive. Also some parents may now be more sensitive to the contribution that these can make to children's mathematical development. In my study, children's familiarity

with number symbols was ascertained by asking them about the plastic shapes. When shown these, and asked what they were, three-quarters of the twelve children asked labelled the shapes as numbers, while the remaining quarter of them thought they were letters. When asked how they used these shapes at home, six children said that they used them for 'saying' the numbers, one said he played with them, while four of them said that they put them on the fridge. Ten of the fourteen children had little difficulty naming/reading the numerals.

Young-Loveridge commented on the fact that no child in her study used a computer, although some did play with calculators. Remarkably, I think, not even one child in my study mentioned an experience with calculators. The only mention of computers was by Tom who had observed his brother look at numbers on the computer. He also reported that he had observed his Dad and Mam look at money on the screen. Neither did television-viewing feature in children's accounts of their experiences with number. The only exception to this was Owen's reference to the characters from the children's television show *Sesame Street* whom he cited as examples of people he knew who could count. A number of children referred to experiences related to the writing of numbers. In fact, the data suggest that for quite a few of the children, knowing numbers was about knowing how to write the numerals (See Table 4.2). Indeed Terence, in reply to the question of how he learned to count, claimed that the learning occurred '[I]*n my pencil, my brown one*', suggesting perhaps that the writing of the numbers was in his mind synomonous with knowing numbers. Five of the six girls spoke about experiences revolving around learning to write numbers, while only two of the eight boys did so. It seems that writing numbers is very significant in the eyes of some children, in particular girls. I think this illustrates the extent to which some children appreciate that the ability to write these symbols in some way bestows power on the writer.

Discussion

Table 4.3 illustrates that when young children relate their experiences with number, they do so in a holistic way, making reference to a range of aspects of human experience. The cognitive, affective and social dimensions of learning about number are seen in the data to be inextricably linked and

interwoven. Children's views about the purposes of number are seen to be embedded in their experiences which were often explicitly purposeful, for example, measuring the rooms in the house; seeking the best-before date or price labels on items in the shop; and writing birthday invitations.

The findings presented here demonstrate that, for most children, there are a variety of opportunities in everyday life for them to learn about numbers and their purposes. However, as a result of her case-study research, Young-Loveridge (1989) concluded that the process of learning arithmetic from everyday experiences did not happen automatically. Based on my findings, I concur with that conclusion, since it now seems to me that the opportunity to discern purpose is not sufficient to enable most children do so. The data clearly support Rogoff's (1995) contention that, for some children, the possibility of learning about purpose from their experiences is greater than it is for others. She argues that '*... social interactions will benefit children's learning under some circumstances and not others.*' (p. 157) Her claim that the issue of communication with the novice is one of the centrally important factors in determining the extent of the learning that takes place appears to be supported by the findings of this study. For example, with reference to the purpose of numbers on clocks, Jerry remarked:

> *I know that ... I know the one that ... that ... has no numbers ... I know ... I know ... the ones that doesn't have any numbers ... I know them ... I know them watches too Because ... eh ... see ... eh ... why ... my ... em ... my Granddad told me it.*

His understanding seemed to incorporate the idea that whether or not numbers could be seen on the clock face, they still could be referred to in telling the time. In contrast, Bob described his kitchen clock as one without numbers on it, but he explained that while he couldn't read it, his Mam could. When I asked him how she knew the time if there were no numbers on it, he replied that she knew '*... because they're fake numbers.*'

It is not possible to detect from these accounts whether or how the adults in these situations communicated, fostered or structured the children's attention. However, Bruner (1996) has argued that it is through discourse,

collaboration and negotiation that young children's naive theories are brought into congruence with those of parents. In this study, children's accounts of their experiences often gave a vivid description of collaborative activity with adults in particular situations. Chapter 5 gives details of such accounts in relation to two of the children.

I have already observed above that, in general, the girls were more forthcoming with details about their experiences and generally more talkative in the discussions. Perhaps they were more concerned to please me than were the boys. Or perhaps the girls were more experienced and consequently more comfortable with the type of collaboration that our discussion entailed. For instance, Síle reported how she wrote numbers, but how her Dad did so for a different reason. According to her '*... he writes down ... em ... the dates ... the birthdays ... on the calendar.*' This suggests to me a good deal of effort on the part of the adult to communicate and structure the situation to enable understanding. These children clearly demonstrate the role that the adults in their lives have played in advancing their understanding through conversation.

While the persistence of young children in such conversation with adults have long been recognised (e.g., Tizard and Hughes, 1984), from a sociocultural perspective the extent of children's management of the interactions and the activity also merits discussion and analysis (Rogoff, 1990; 1998). For instance, some children may seek out answers to their questions. Perhaps that was what Sile was doing when she spoke below about her experiences of numbers:

Sile: *I seen them once by the dates*

Liz: *The dates?*

Sile: *Yes*

Liz: *On what?*

Sile: *On ... on their packages and ... I hear the prices ... and ... and I see ... em ... the numbers all around me ... all around the place ... on the packets on the dates*

Liz: *Do you ... and what do the dates say?*

Sile: *Em ... Sometimes ... sometimes a zero or a one*

Liz: *And do you read all those dates?*

Síle: *Yes*

Liz: *And who helps you to read them?*

Síle: *Sometimes my Mam helps me and sometimes my Dad helps me*

Others appear to use adults to show them how to do something. Kate tells us that, in relation to numerals '*Well ... when I want them ... Mammy just writes them.*' Lara ensures active participation for herself in the situation when her Mam writes numbers: '*... and then I say 'Can I help you Mam' and then see ... I do it with her.*'

In fact, a striking finding was the fact that five of the six girls referred in their accounts of their experiences, to the assistance in their learning that they received from a parent, usually their mother. Mary was the exception. She commented as follows when asked if her Mam used numbers: '*I never ask her ... I don't know ... she's nosy too like my Daddy.*' We can see how her participation is circumscribed and defined by the parameters set by others and she has learned to do what she thinks is expected of her (Lave and Wenger, 1999b). For Mary, asking questions is associated with nosiness or inquisitiveness and is apparently not valued in her family culture. None of the boys spoke of seeking any type of assistance with their learning, although some, for example Jerry, described how his Granddad had explained about the watch face with no numbers on it. I referred above to research that focused on the conversations of four-year-old girls in nursery schools and at home with their mothers which revealed that these children pursued, through conversation, their search for meaning in a persistent and logical way. Analysis of the data in that study revealed that these conversations were characterised by certain features, including questioning, and that as a result, they allowed children opportunities to build up understandings of abstract topics (Tizard and Hughes, 1984). However, my data appear to indicate that such intensively interactive behaviour may be more a feature of the way in which young girls approach meaning making than it is in relation to young boys. They may be more comfortable with more peripheral participation (e.g., Tom's observation of his Dad and his brother using the computer) and with not asking questions or explicitly

seeking assistance with learning. In relation to understanding the purposes of number, the result of this may be that boys are more likely to proceed on the basis of their naïve theories rather than by developing a shared understanding of purpose of number with adults. This possibility seems to be substantiated by the finding that boys appear more likely to suggest generic purposes for number rather than more specific purposes, while girls appear more likely to talk about specific purposes for number.

In summary, it appears that the nature of the children's understanding of numeric purpose differs, with boys understanding appearing to be more generic and general, while girls understanding is more detailed and specific. The metacognitive framework for number and quantity, i.e., the very broad knowledge about the role and purpose of number (Munn, 1994) which is being constructed by children during the preschool period appears, from children's own accounts, to be differently constructed by girls and boys. The girls appeared to engage more actively with adults in numerically related experiences than did boys.

Findings related to aspects of young children's quantitative thinking

In Chapter 2 (p. 28), I defined quantitative thinking as that which children engage in while responding to variations on the how-many question.

In seeking to establish how children responded to selected tasks (See Appendix 1: Tasks 4,5,7,9), I found that sometimes their quantitative strategies were overt and discernible directly from what they said or did in response to a task. At other times children's strategies were more covert and I sought to discern these by interpreting their responses. In order to assist me in making these interpretations, I derived a categorisation system very loosely modelled on the more extensive one to be found in Ginsburg *et al.* (2001).

The categories I use to describe children's strategies are as follows:

- *Relates Numbers*: The child focuses on subgroups within a quantity as a way of finding a solution, or in some other way relates numbers to other numbers;

- *Guesses*: The child's response is a wild guess, apparently unrelated to the numbers in the task;
- *Instant Response*: A response to the question is offered immediately but it is incorrect;
- *Counts:* The counting words are used in response to a task that includes the words 'how-many';
- *Instant Solution:* Instant recognition of cardinal value of a set of objects. The child subitizes, i.e., immediately recognises the amount without overtly counting;
- *Estimates:* The child offers a solution based on logical approximation. The child's response makes sense in relation to the numbers in the task.

Task 4: Counting objects within 10 (Groups of 2,6,7,5,4 presented in sequence).

One array at a time, the children were shown a number of arrays of candles that Coco had arranged in linear form. Coco was the monkey character who featured in the birthday scenario around which I had created the tasks. They were asked to count the presented array of candles to see if that was the amount that should go on the cake for Coco's fourth birthday.

The purpose of this task was to explore children's ability to count to find out 'how-many' and all of the children displayed competence in relation to this. They all subitized, i.e., recognised instantly, the quantity two. Most children counted the arrays of 6, 7, 5 and 4 candles as was suggested to them but on a few occasions I noted a subitizing response in relation to the array of 5 candles, and even more so in relation to the array of 4. In terms of children's number sense, the really revealing aspect of children's responses was that several of them talked about a quantity under consideration by relating it to the target quantity of 4 (See Table 4.4). Different children used different types of 'relating' strategies to enable them to solve the problem. For instance, Shay appeared to use an additive strategy in solving the problem: '*He needs two more.*' In relation to the array of 6 candles he remarked: '*That's not enough ... so you only have to take out two and two ... that's four.*'

Table 4.4: Children's responses to selected tasks designed to elicit quantitative thinking

Name	Task 4 Counting objects	Task 5 How many now? (4 Items)	Task 7 How many dots? (5 Items)	Task 9 About how many pieces? (5 Items)
Bob *(M) 4y 8m*	Counts Relates numbers	Counts (4)*	Instant Response (4) Instant Solution (1)	Guesses/ Estimates
Shay *(M) 4y 11m*	Counts Relates numbers	Instant Solution (4) Relates numbers	Instant Solution (4) Relates numbers (1)	Estimates
Terence *(M) 4y 3m*	Counts	Counts (3) Instant Solution (1)	Instant Response (1) Instant Solution (4)	Estimates/ Guesses
Tom *(M) 4y 11m*	Counts	Counts (1) Instant Solution (3)	Counts (1) Instant Response (1) Instant Solution (3)	Estimates/ Counts
Jamie *(M) 4y 9m*	Counts Relates numbers	Instant Solution (4)	Instant Solution (5)	Counts
Con *(M) 4y 9m*	Counts	Counts (1) Instant Solution (3)	Counts (4) Instant Response (1)	Counts
Owen *(M) 4y 6m*	Counts Relates numbers	Counts (4)	Count (1) Instant Solution(4)	Counts
Jerry *(M) 5y 1m*	Counts Relates numbers	Counts (2) Instant Solution (2)	Instant Response (1) Instant Solution (3) Relates numbers (1)	Counts

Name	Task 4 Counting objects	Task 5 How many now? (4 Items)	Task 7 How many dots? (5 Items)	Task 9 About how many pieces? (5 Items)
Maura ***(F) 4y 5m***	Counts	Counts (3) Instant Solution (1)	Instant Response (3) Instant Solution (2)	**QNP
Sonia ***(F) 4y 11m***	Counts	Counts (1) Instant Solution (3)	Instant Solution (5)	Estimates
Síle ***(F) 5y***	Counts Relates numbers	Counts (2) Instant Solution (2)	Counts (1) Instant Solution (4)	Counts
Mary ***(F) 4y 6m***	Counts Relates numbers	Counts (4)	Counts (4) Instant Solution (1)	Estimates
Kate ***(F) 5y***	Counts	Instant Solution (3) Counts (1)	Counts (1) Instant Solution (4)	Estimates
Lara ***(F) 4y 1m***	Counts	Counts (4)	Counts (2) Instant Response (1) Instant Solution (2)	Guesses/ Counts

*This denotes that all of the responses were classified as count responses
**QNP denotes Question Not Put

Bob in contrast appeared to use a subtraction strategy, reducing the excess to produce the quantity required: '*Take two out and that's four.*' The array of seven candles provoked the following response from Terence: '*Look four and three.*' This suggests to me that he used his knowledge of composition of number to solve the problem. This was similar to the strategy invoked by Shay who commented: *'Now there's four ... there's three ... so that makes ... there's seven.'*

Four children commented on the array of five candles and related it to four. Owen suggested that we take one away and similarly Mary suggested that we should 'get' one away.

Síle's suggestion was '... *so take away the fiveth one ... so then he has four.* '

Terence articulated his way of 'seeing' four by removing one from the array with the comment: '*Not enough ... there's one in the middle there is ... Look, ah that is enough it is ... that's four it is.* '

The children's comments confirm Fuson's (1988) findings that children follow different routes to the early relationships between numbers. The task above proved to be an enabling context within which some children could display different strategies for solving the quantification problem. Thus they could display the very beginning of their understandings of how numbers relate to each other and their understandings of how they can be manipulated, an important aspect of number sense identified earlier (See Chapter 2: 19). Steffe and Cobb (1988) suggest that the creation and use by children of the above type of patterns helps them to develop abstract number and arithmetic strategies. In relation to this particular task, several children displayed their use of analytic skills as opposed to counting skills to solve the problem, the skills identified by Marton and Neuman (1990) as necessary for solving quantitative problems.

Task 5: How many now? (Groups of items of 4,7,3,5 presented sequentially).

The children were asked to count out an array of birthday presents (e.g., 4) and say how- many. They were then asked to add/remove one and say how-many then.

The purpose of this task was to explore whether children overtly counted each time.

In Fuson's (1988) terms, the concern then is with the extent to which children utilize information gained from counting to solve the task. We know that many children of this age do not seem to choose or be able to spontaneously subitize in what has been defined by the interviewer as a counting situation (Fuson, 1988). The review in Chapter 2 (pp. 34-35) suggested that as children develop their number sense, it is displayed by them in their use of a variety of strategies based on perceptual processes such as subitizing or perhaps visual or auditory patterns rather than incremental counting to respond to how-many-now tasks.

I found that there were two categories of responses to Task 5. The first category was one where the children responded by counting again, and the other was one where children straight away offered the solution. Certainly, children in this study appeared to give more instant solutions than the younger children in the Fuson (1988) study, perhaps indicating the role of experience and practise in developing quicker responses in a quantification situation. There is the possibility also that the children were drawing on other aspects of their number knowledge, for example *one before/one after*. Four children always counted the items in response to the question of how-many-now, two children never appeared to count while the remaining eight children used both counting and subitizing strategies to varying degrees (See Table 4.4). This suggests that some children were perhaps moving from an overt counting strategy to a covert one (Fuson, 1988). Or perhaps they were subitizing or combining counting with subitizing (MacNamara, 1996; Clements, 1999). Research has suggested that as children's number sense develops they acquire the ability to be flexible in relation to number (Chapter 2: 16-17): and number is seen as more than the product of counting (Fuson 1988; Campbell, 1999). From Fuson's perspective, as children attempt the task above, they can respond only with the conceptual unit items they have available. If they view the set as a group of unit items rather than as a single set, then their response to the question of how-many-now will be to count again. The children who counted again after an item had been added/removed do not appear to respond to the quantity of the initial set as a number. In Greeno's (1991) terms, children who count in response to this type of task don't appear to be able to objectify numbers, i.e., to see numbers as conceptual objects in their own right. He considers this ability as important in relation to children's number sense, since it signifies an understanding that numbers can be manipulated and are objects to construct and reason with in the domain of number.

The question of why some children count in response to *'How-many now?'* would appear, then, to be comprehensively dealt with in the literature. However Sonia's remarks below gave me pause for thought:

> *Liz:* *Well let's pretend he only got that many* [I showed four]. *How many has he got there now?* [She counted four]

Liz: *Right ... and he has four presents there and let's pretend that one got lost* [I removed one]

Sonia: *Three*

Liz: *Three ... okay!*

Sonia: *I don't need to count that*

Liz: *Why do you not need to count?*

Sonia: *Because ... I know that's three ... one, two, three ... I don't need to count but I'm practising.*

Is Sonia 'practising' perhaps to verify for herself the subitizing result by now counting the set, as suggested by von Glasersfeld (1987)? Alternatively, does she perhaps think that it might be expected that she counts thus prompting her to explain, as she did here, her lack of overt counting?

When Shay's subitized in response to the initial set of four items, I asked him if he was sure. He then counted the items before '*I was just making proof for you, it's easy for me to do it.*' He appeared to be using counting to verify his rapid response, *for me.* In fact, by asking him if he was sure, it seems very likely that I prompted him to overtly count the array. These comments by Shay and Sonia appear to substantiate Fuson's speculation that children may perceive certain strategies to be '*acceptable*' in certain instances (1988: 349).

Previous research that focused on children's counting skills early in their first school year identified social pressures on children to behave '... *in a certain manner to conform to particular kinds of teacher expectations, particularly* [...] *to count where counting was not required.*' (MacNamara, 1996: 128) Fuson suggested a developmental pattern in children's responses to the how-many question. She observed that some four-year-old children recounted when asked the how-many question again, while those who didn't re-count in response to that question appeared to have related last-word responses and counting. The literature suggested that developmental progress in relation to counting may well include the relating of covert counting and perceptual subitizing (See Chapter 2: 30-31). Findings here certainly show that the majority of children in this study do use processes other than overt counting to solve problems related to quantification. These

may include rapid covert counting, or perhaps knowledge of *one more/one less*. However, my findings also suggest that where children overtly count in response to the task, there may be other reasons besides lack of ability to objectify number (Greeno, 1991), or lack of ability to relate last-word response and counting (Fuson, 1988) for their use of counting in such contexts. Such reasons may include a desire to be co-operative and helpful and to behave in ways that they perceive to be socially desirable.

Task 7: How many dots?

Children were shown a dice with dot patterns, my birthday present for Coco. As an introduction to this task, the children and I discussed their experiences with die. I then explained what I wanted them to do. Each face was displayed for only about 3 seconds to avoid giving an opportunity for children to count. The prompt *'Look quickly. You'll need to be quick here!'* was used in this instance to prompt the children to subitize. Fuson (1988) had speculated that where children assume that a quick answer is required, this may result in the use of rapid perceptual strategies for quantification.

The purpose of this task was to explore the extent of children's ability to subitize.

Table 4.4 shows that eleven of the children responded rapidly to either five or four items. In these instances no overt/covert counting behaviour was displayed by the children. It appears then, that the majority of children can respond very rapidly when quantities are presented in a spatial pattern, especially when urged to do so. Furthermore, two of the children explicitly demonstrated the ability to group objects to quantify sets. For instance, Jerry looked at the six pattern on the dice and then remarked *'What's three plus three ... oh six'*. While he appears to be using a known fact to solve the problem, he is also drawing on his ability to subitize three items. Shay in relation to the six pattern also 'sees' something similar to Jerry as communicated in his response of '*Three and three. I have to count on my fingers.*' Shay subitized the set of three, but to solve the problem, he actually counted incrementally from one. Both of these children display the beginnings of a certain flexibility with the number six. Marton and Neuman (1990) observed that the seven-year old school starters in their study

frequently used this type of 'seeing' strategy. My results indicate that it may be discernible in some children at a much earlier age. However, neither Jerry nor Shay showed any evidence of having integrated the strategies of subitizing and counting in the way suggested by Clements (1999) or MacNamara (1996).

Walkerdine (1988) drew attention to the fact that the way in which the task and the situation is set up, and the consequential positioning of the child and the interviewer, can affect children's responses. The use of the dice appeared to introduce its own contextual tensions in this task for Síle. She talked about playing the game 'Snakes and Ladders' with her friend Lisa. While such experiences may be potentially very useful in learning to respond rapidly to collections of items, the way in which these were used by me to provide a context for this task had some unforeseen consequences for how Síle read the task. She focused on the counting process involved in the game and her response to the task was initially tempered by this focus on counting:

Liz: *Did you ever play with one of these?*

Síle: *Yes*

Liz: *When? When did you play with that?*

Síle: *When we were played snakes and ladders ... with my friend Lisa. You have to throw the dice and you have to go down ... and if you get ... em ... two you go one, two because it's two and then you shake. Lisa gets her go and it's one so she goes one*

Liz: *I'll just throw the dice now and you take a quick look and see if you know what number is on it* [I threw the dice]

Síle: *There's one, two, three, four ... so you have to go four ... that's what you need ... em ... the things that help you ... do you know the things to put the candles on cakes ... do you know ... you can use them for the things for one, two, three, four ... for ... em ...*

Liz: *Oh! For counters*

Síle: *Yes*

Liz: *Now look what I'm going to do. I'm going to throw the dice and just take a quick look and tell me*

Síle: *Em ... three ... Because ... see Lisa's dice goes one, two, three.*

The way in which I had set up the task and the way that this positioned us both determined her initial response to the task. In fact, Síle subitized in relation to each of the remaining four items, thus further supporting the argument that the task was initially framed by contextual factors. The extent of the conflict induced by the introduction of a game discourse was further underlined when she urged me, as we worked through the other items in the task, to get a board and some counters so that we could play the game she was describing. Thus, for Síle, participation in this task began with her participation in previously related activity with her friend, but the context of this particular task required a different response and her participation changed accordingly. This suggests that contextual factors may determine the strategies that children use to solve quantification tasks. The vignette above also illustrates the point made by Carr (2001) that children in pedagogical settings do not always share our purposes and the data have always to be considered as arising out of the researcher's purpose rather than the child's. Síle read a purpose into my introduction of the task. She saw it as one of explaining the game of 'Snakes and Ladders' to me. My purpose, to see if she subitized, was not the purpose that the situation had initially suggested to her.

Task 9: How many pieces?

Children were presented with a succession of small plates of 'food' items for the party (8 lemons, 5 oranges, 6 apples, 7 bananas, 9 strawberries) using 'Fruity Fun' counters. Each plate was displayed for only about three seconds to avoid giving the opportunity for children to count. The prompt '*Look quickly. You'll need to be quick here!*' was again used, in this instance to prompt the children to estimate. Note was taken of whether they appeared to estimate or whether they attempted to count the items.

The purpose of this activity was to explore children's ability to estimate quantity without resorting to overt incremental counting.

Table 4.4 shows the responses to this task. We saw that, in relation to Task 7, most children responded quickly. However, the response pattern on Task 9 was somewhat different. About half of the children appeared to estimate/guess in response to some/all of the items, while the other half of them counted.

The estimates/guesses offered by the seven children are presented in Table 4.5. below. Some of the responses of both Bob and Terence appeared to be guesses rather than estimates and they were categorised as belonging to both/either categories. Lara offered instant responses, but I categorised her behaviour as guessing because her solutions were considerably off the target number.

Table 4.5: Children's estimates/guesses in response to Task 9

Name	Set size 8	Set size 5	Set size 6	Set size 7	Set size 9
Bob *(M) 4 y 6m **	3	4	4	5	14
Shay *(M) 4y 11m*	8	5	8	6	11
Terence *(M) 4y 3m*	2	7	1	4	5
Tom *(M) 4y 11m*	6	5	8	6	9
Sonia *(F) 4y 11m ***	9	5	9	8	10
Mary *(F) 4y 6m*	7	8	6	6	10
Kate *(F) 5y*	6	4	4	6	7

*M denotes male, y denotes years and m denotes months **F denotes female
Note: Shaded responses are those within one/two of the target number

In many cases the estimates are within an acceptable range, i.e., within one or two of the target number. Some studies of young children's number understanding reported on the correctness of their estimates (e.g., Aubrey,

1997). The children in my study rarely quantified exactly, though I would argue that the point of estimation is not exactness. It is the reasonableness of children's responses that is significant in terms of telling us something about their number sense.

Of the seven estimators, all of them, save Mary, had also responded rapidly to Task 7. On reviewing the recording of the interview and the relevant notes of my discussion with Mary, I noted that while her responses to Task 7 were very rapid, I had recorded in my field notes her explicit counting behaviour during this task. Was her counting behaviour now more covert than in the previous task? Or did she use some other strategy in relation to Task 9? Reviewing the audiotape, I noted that my presentation of Task 9 was very detailed in this instance. I cannot account for this except perhaps to acknowledge that I was beginning to realise, as I worked with the children, that one of their difficulties with the task probably lay in the way it was presented to them. The following explanation introduced Mary to Task 9:

> *Look at my big bag of fruit ... here feel that strawberry ... it's a pretend one ... it's for playing games like the one we're playing here. Now what I want you to do Mary ... Look ... Here's a big bunch of strawberries. Now open your hands and hold onto them. That's it. Now in a moment I am going to ask you to put them onto his plate and when you put them on his plate don't count them but just tell me about how many strawberries.*

Mary's introduction to the task was a more elaborate explanation of what was required of her than that given to some of the others. Also the question I put to her was not how-many but *about* how-many. This may have been significant, although as we will see, it didn't seem to influence Jamie in the account below. Also the fruit pieces were put into her hand and transferred *by her* to the plate. Perhaps this gave her a better opportunity to count covertly and in some way it enhanced her engagement with the task. It is also possible that, as Mary participated in the task, her understanding developed and she changed and became more able to estimate. The extent of

the communication and collaboration during participation involved adjustments and effort on her part and on mine. Rogoff (1998: 690) describes how in such situations the participants change '... *to become involved in the situation at hand.*'

About half of the thirteen children who were presented with the task responded by counting. Four of these (Jamie, Owen, Jerry, and Síle) had generally subitized in relation to Task 7. Perhaps the arrangement of the objects in Task 9 and/or the number of items could have been a factor in relation to how they responded. Research has shown that randomly-arranged sets prohibit rapid counting, and that as the number of items in sets increase, it becomes more difficult for children to quantify them (Fuson, 1988). Owen, Jamie and Jerry were all very explicit in relation to how they perceived Task 9 and their comments provide some evidence that for them at least, at this point in time, covert counting is the key to quantification. Of course, this does not rule out the possibility that they also use perceptual processes when quantifying. Owen's response to my suggestion that he tell me how many without counting was:

> *But I don't know ... I have to count them ... I think I have to put them in a line.*

Jerry's comment was '*But if I don't count them I won't know how many*!'

Jamie's forthright response when I asked him why he had counted the objects in the first item in Task 9 was as follows:

Jamie: *I counted them very low.*

Liz: *Well the next time don't count them. Just tell me how many you think.*

Jamie: *I think nine. I didn't count them. I just counted them in your hand.*

Liz: *Without counting them tell me how many about (strawberries).* [He then counted silently].

Jamie: *EIGHT!*

Liz: *And you counted them didn't you?*

Jamie: *I counted them in my head.*

These responses suggest to me that some of the children just didn't understand the difference between an exact answer and an approximation. In fact, this is not so surprising when you consider that it is very likely that most four-year-old children will have had very few experiences of quantitative estimation. Most experiences of quantification are likely to be ones in which children are learning to count to quantify, or perhaps, are prompted to count to verify how many. It is as a result of these latter experiences that that the '*absolute reliability*' (Fuson and Hall, 1983: 68) of correctly executed counting for determining the quantity in a set is established.

Pike and Forrester (1997) have argued that what children understand about estimation is highly variable and context dependent, and that considering the conceptual demands of such tasks in decontextalised terms is problematic. They also suggest that using the phrase how-many in a context that requires children to approximate rather than quantify exactly, introduces a tension and thus confusion for some children. Sowder's (1992: 371) clarification of the relationship between counting and estimation is relevant here. She points out that '... *how-many is usually a question of numerosity and asks that the number of items in a set be found. In most cases an estimate is sufficient and perhaps all that is even possible.*' The problem with estimation from the perspective of most four-year-old children is that, of course, because of their limited life experiences they don't have that realisation.

Aubrey (1997) had also noted presentational difficulties with a similar task and she has observed children's confusion when they had only a few seconds to see the quantity and then asked to 'guess' how many. Indeed she found that most four-year children in her study preferred to count when presented with the estimation task. If we include Shay, because of his initial statement '*I have to count them*', then seven of the thirteen children presented with Task 9 either counted or indicated a preference for overt counting rather than any other strategy.

However, about half of the children appeared to be happy to indulge my request that required them, perhaps, to put aside previous experiences that provoked them to count in response to how-many. My feeling is that these

children were confident about their responses because most of them had, in fact, covertly counted the items *to their satisfaction.* Fuson (1988) observed that, in general, evidence suggested that when children begin to internalise counting, this can result in less accuracy than overt counting.

In summary, my data shows that:

- While overt counting was an important quantification strategy, the four-year-old children in this study also solved quantification problems by relating numbers to each other using a variety of relational strategies, thus demonstrating that they understand that numbers are not just the product of counting but are things that can be manipulated and reasoned with;
- Covert counting was a key quantification strategy for some children, while others were in the process of internalising counting and were adopting its use to varying degrees;
- Most children responded very rapidly when quantities were presented in spatial patterns. Children who subitized spatial patterns were more likely to estimate, but a number of children who subitized spatial patterns did not estimate, and one child (Mary) who didn't subitize spatial patterns, appeared to estimate.

In addition, the data suggest that:

- Children may perceive that counting is the expected/acceptable response to problems related to quantification;
- Children may quantify larger numbers (e.g., six) by seeing patterns (e.g., three and three) in these arrays;
- Contextual factors may determine what strategy children use to solve quantification problems;
- The differentiation between an exact answer and an approximation appears to have little meaning for children in this study.

Discussion

Looking across the tasks in Table 4.4, it is clear that one of the issues in terms of children's responses is the ability to interpret the question how-many flexibly, i.e., an understanding that these words don't always have to imply a counting process. Research with three-and four-year-olds showed

that when some children hear the words how-many, their reaction is to think in terms of counting and to develop responses based on overt incremental counting processes (Fuson, 1988). My data show that, in response to Task 5, eleven of the fourteen children choose to recount all/some of the items when asked how-many-now, after an item had been added or removed. A counting response to Task 5 is not surprising though, given that the directions as presented did suggest counting as appropriate. However, in Task 7, eight of these children responded instantly. It is possible that the removal of the dice after a few seconds in Task 7 may have had the effect of forcing children to abandon a counting strategy to determine how-many by subitizing. Indeed, we saw that hardly any of the children overtly counted in response to Task 7. But the removal of the items in Task 9 did not seem to prompt most children to estimate in that case. In fact, we have seen that most preferred to count and almost half explicitly did so.

In relation to Task 9 the sets were too big for most children to count quickly but it is likely that some of those labelled as subitizers in Task 7 counted to quantify, but in a covert way, while some children may have recognised the quantity perceptually. In fact, Jamie's comments above certainly raise such a possibility (p. 105). Tom's overt behaviour in relation to some items in the estimation task also pointed to the use of covert counting in such tasks (See Chapter 5: Tom's profile, pp. 128-131). The dice with the arrangements of dots (Task 7) was relatively easy to count quickly, or merely recognise. For instance, children were very successful with quick responses in relation to the pattern of 2 and 3, relatively successful in relation to 4 but there were more quantification errors in relation to the patterns of 5 and 6. This suggests that when there is no time/opportunity to count, as in Task 9, it is very difficult for some four-year-old children to come up with reasonable responses in so-called 'estimation' tasks. Those who do come up with appropriate responses could be engaged in what Gray (1997) described as a compression process or what MacNamara (1996) described above as a subitize-then-count process. However, we can only speculate about this since I didn't ask children about their strategies and most of the mainly four-year-old children in my study didn't articulate their approach in the way that the seven-year-old child in MacNamara's study did.

Clement's (1999) position is that conceptual subitization is a developmentally more mature quantification strategy than perceptual subitization or counting, and that it often depends on accurate enumerative skills. I interpret this to mean that children who can count and who can also subitize as in Task 7, will be in a position to estimate in response to sets of 6-10 items, set sizes most likely to be just beyond their subitizing range. According to his argument those children with high levels of accuracy (100% or 80%: See Table 4.4) on Task 7, should estimate on Task 9. However, Jamie very clearly told us why he counted in response to the items in Task 9. Owen, Jerry and Síle also counted. Sonia, Shay, Terence and Kate all estimated with ease. At the other end of the scale, Mary counted covertly in response to Task 7, but estimated with ease in relation to Task 9. While Clements (1999) and MacNamara (1996) may be correct in speculating that accurate enumeration combined with perceptual processes is an important route to estimation, the data here show that it is not a route followed by all children. The fact that a number of children provide instant solutions in relation to Task 7, but do not estimate in response to Task 9, suggests that the ability to enumerate rapidly and accurately doesn't mean that estimation follows. It is impossible to say, in relation to the four-year-old children in my study, the extent to which they depend on perceptual processes or the role such processes play in their quantitative reasoning. What is clear is that many of them can /do estimate but many of them have difficulties with estimation tasks. These may relate to one/more factors including the size of the set; the arrangement of the set; the wording of the task; and the purpose of the task. Those children who appear to estimate may have a number of strategies that they can use as appropriate.

With some children, particular tasks seemed to call forth particular strategies. For example, Jamie, Con and Sonia all appeared to give instant solutions to items in Task 5. In relation to Task 7, Con gave mostly counting responses, while the other two gave instant solutions. However Jamie and Con both insisted that a counting response was appropriate in relation to Task 9, while Sonia appeared to estimate.

It was often difficult to infer the exact strategies some children were applying to the quantification problems presented. Children of this age may

reason mathematically/numerically, but they may have some difficulty articulating that reasoning [See Pound (1999) for a review of research in this area]. However, it appears from the analysis of the data, that the strategies they used may have included internalised counting (Fuson, 1988); compressed counting (Gray, 1997), perhaps involving subitization and counting on (MacNamara, 1996); and relating numbers to other numbers (Steffe and Cobb, 1988; Marton and Neuman, 1990). The responses of the majority of the children in this study strongly suggested that they were operating with these strategies to varying degrees.

Fuson (1988) described the internalisation of counting as a feature of most children's behaviour at around five- or six-years of age. The children in this study are clearly focusing on this process, as is evidenced by the nature of their responses to some of the tasks discussed here, in particular Tasks 7 and 9. The use of covert counting strategies varies from child to child (See Table 4.4), depending no doubt, as Fuson (1988) observed, on their experiences and practise with counting. However, it is also clear from the findings here that contextual factors can determine what counting strategy (i.e., overt or covert) children choose to use. It is possible that at a certain point of development, some children experience a tension between, on the one hand, the utility and ease of covert counting, and on the other hand the perceived social desirability to count aloud to quantify. Some children in this study certainly appeared to indicate such confusion. In thinking about why children might persist with overt counting, we should also consider the possibility that there is an important affective dimension at work and that some children may enjoy the pleasure of counting in the way implied by Munn (1994). This may be reinforced by adult approval of the process. Some of them then may be loath to leave behind the affective benefits of overt counting.

What all of this implies is that estimation cannot make sense to most four- or five-year-old children simply because they have not yet internalised counting. Children's experiences to date are generally ones in which overt counting is practised. Some children in this study were observant of people who 'did it in the head' and such children seemed to have a metacognitive awareness of their own/other peoples covert/mental activity with number.

To illustrate my point, I cite Shay's comments in discussion about magnetic numerals. The discussion begins with my comment:

Liz:	*If you put them all up there and tidy them up a bit we can have a good look at them*
Shay:	*We'll have to ... Is this a one? I just want to remember*
Liz:	*That's a one*
Shay:	*And I need to have a two*
Liz:	*Okay ... oh I see what you're doing*
Shay:	*I'm trying to count. Is this a two?*
Liz:	*What do you think?*
Shay:	*It's kind of like it ... oh no it isn't ... it's a seven*
Liz:	*Is it? Okay*
Shay:	*I thought it was ... because ... no this is a seven*
Liz:	*Is it?*
Shay:	*Because I got my Mum's thoughts*

Such children, I suggest, may be in a better position to understand the estimation process than those who do not appear conscious of such metacognitive issues. From a personal perspective, this possibility reminds me very much of my son's unsolicited comments (at about five years of age) about a story which I was reading him (for maybe the second or third time) from a book with no pictures; '*Oh! I get it now. You have to make the pictures in your head*'. Could it be that young children also have to 'get' it in relation to number and understand that they can 'make' the numbers/quantity in their heads before they can estimate? What this suggests is that children will need to participate in estimation activities with more skilled partners '*... in order to allow them to internalize tools for thinking and for taking more mature approaches to problem solving.*' (Rogoff, 1990: 14)

Shifting the focus of analysis

For the purposes of presentation, in this chapter I focused mainly on what Rogoff (1995) refers to as *the personal plane*. As explained in Chapter 1 (p. 8), this implies that the focus is on the personal processes by which, through engagement in an activity, individuals change and become prepared to engage in similar activities. I presented information about individual

children's experiences and understandings in relation to number and their responses to particular numerically related tasks. This allowed for comparisons between individual children and between boys as a group and girls as a group. While focusing on the personal plane, I had interpersonal and cultural/institutional information available in the background and I used this information in a general way to enable me to construct my understanding of the children's number sense. However, I did not attend to it in the same detail as I attended to the personal plane. In Chapter 5 that follows, I present individual profiles of two children. In these profiles I focus on those aspects of the data that suggest to me the nature and importance of the roles that various adults appear to play in the development of the number sense of these two children. The profiles explore issues related to the participation of selected children in number-related activity, as recounted by the children themselves. While the focus in Chapter 5 moves to the interpersonal plane, I hold the individual and cultural/institutional information (i.e., that pertaining to issues such as the nature of the activity described, how that relates to the practices and institutions of the community, and the tools and artefacts that feature in the account) in the background, drawing on it to enrich the individual profiles.

Thus the present chapter and the one that follows can be seen as complementary, each highlighting specific aspects of the development of children's number sense. Together, they give a fuller picture of the nature of the development of number sense in young children, than either presentation could provide alone.

The presentation of findings in the way described above is very much influenced by Rogoff's (1995; 1998) ideas that personal, interpersonal and community/cultural factors interact in a mutually constitutive way and that learning and development can only be understood by attending to all three aspects (See Chapter 1: 5-8). Another factor influencing my approach to presenting my findings is the argument put forward by Sfard (1998) that different approaches are useful for explaining different aspects of learning, and can compliment each other. I am conscious that in presenting children's views and understandings alongside their responses to tasks, I am implying a different sense of learner, learning context and culture, even within the

same chapter. This is reflective of the different theoretical influences on my work. Chapter 5 that follows is an attempt to present a complimentary perspective on young children's number sense, one that focuses more on the interpersonal factors that influenced the development of the number sense of two particular children.

Chapter 5: Findings 2

The development of young children's number sense through participation in collaborative activity

One of the purposes of this study is to seek a richer understanding of number sense in young children than that currently portrayed in the curriculum documentation (See Chapter 4), one that is experience-based and takes account of children's participation in sociocultural activity. In this chapter, I present an analysis of children's number sense based on their accounts of participation in numerically related experiences

I orient my analysis in this chapter by focusing on the interpersonal aspects of the development of number sense, as opposed to either the personal (as in Chapter 4) or community aspects (not explicitly dealt with in this study). In doing so, I continue to be interested in the types of events within which development is taking place, i.e., the community plane, the ways in which the children appear to participate and how they appear to change that participation, i.e., the personal plane.

There are three questions that I address as I construct the profiles and my analysis in relation to each of them is interwoven into the profiles:

- How did children appear to participate in activities that they perceived as being related to number?

This includes a consideration, from children's accounts, of the extent of their participation, their role in the activity and the nature of the collaboration they engaged in mutually with adults or other children;

- How did children appear to be supported in their participation in numerically related activity?

This includes some attention to the role of experts in supporting children as novices, as reported by children. It also encompasses the opportunities for learning about number that children appeared to be aware of/or able to recall or recount;

- How did participation in previous events prepare children for our discussions about number-related activity?

In sociocultural theory, it is argued that any event in the present is an extension of previous events (Rogoff *et al.*, 1995) and it is from this perspective that this question is addressed.

In essence, children in the study tell the story of their participation in numerical learning. In doing so, they talk about their perspective on the guidance they receive and their collaboration with others. The extent to which they suggest that they control the learning is also relevant in a sociocultural analysis such as that undertaken in this chapter. Children's number sense is described not just in terms of their proficiency with number and related skills, but also in terms of their accounts of participation in everyday activities with other people.

The children profiled in this chapter

As I conducted the experience-based flexible and focused interviews, I found that in all instances children conveyed their sense of number in distinct and individualistic ways. Following the interviews, I drew up a number of individual profiles of selected children, some of whom demonstrated a strong sense of number and some who appeared to have a less developed sense of number. In this chapter, I present two of those profiles. The first is Sonia's profile. I selected her because she conveys a well-developed sense of number and because she is an example of a child who was able to communicate her sense of number with ease. The second profile presented is that of Tom. I selected him because he was challenging to interview and also because, while he demonstrates impressive knowledge and skill in relation to some of the aspects of number sense explored in the study, other aspects were less discernible. These profiles convey to readers the range of children's responses in the interview situation and thus enable us to appreciate how different children can demonstrate number sense. Both children featured in the profiles were 4 years 11 months at the time of the interviews. Each profile is presented in narrative form. This style of presentation is influenced by that found in Pollard and Filer (1996).

Sonia's profile

Sonia is an only child and she and her Mam form a small family unit, but Sonia's relationship with her Granny from abroad seemed to be centrally

important to her at the time she was interviewed. Her Mam's boyfriend also seemed to be a significant presence in her life and she expressed the opinion that he and her Mam might get married. Indeed marriage seemed to be on Sonia's mind. In fact, she suggested to me that Andrew, a little boy in her crèche, definitely wanted to marry her and her friend. Sonia's Mam and her Irish Granny had both attended the school that she had just started.

Sonia's pleasure and interest in number

One of the most striking features of my discussions with Sonia was her obvious enthusiasm for number-related activity. For instance, she indicated that number games were one of her chosen activities in her new environment of school:

> *Eh ... I have a counting game in school ... in big school ...like you get money ... pretend money ... and then ... there's this big counting thing ... and you put ... and then you count 1,2,3,4,5,6,7,8,9,10.*

She described her experiences of playing number-related games with her friends. Sonia indicated how her participation in such play-related activities with the bigger girls afforded her opportunities to learn about number in such a context:

Sonia: *We play Hide-and–seek out in the yard and I count*

Liz: *Here in school?* [She nodded in agreement.] *Where did you learn that game?*

Sonia: *Em ... my friends sometimes play it with me ... they show me how to play it.*

While this game was played with her friends, her Granny played an important role in assisting her learning on at least one occasion:

Liz: *When you're chalking what do you do?*

Sonia: *We colour in the squares in my driveway to make it look nice but then the rain comes and it washes it away*

Liz: *Right ... and when you're chalking do you ever do numbers?*

Sonia: *No ... but my Granny does Hopscotch*

Liz: *Where?*

Sonia: *In the driveway ... but it washed off when the rain came*

Liz: *Yes but she'd do it again if you asked her*

Sonia: *She's gone away ... but my Mam can do it for me now*

Liz: *That's right ... and what numbers did Granny put in the hopscotch?*

Sonia: *1,2,3,4,5,6,7,8 and then Home*

She relates how the goals of her participation in the activity of 'chalking' changes, and indeed how the activity itself changes with the intervention of her Granny, who provides her assistance in developing Sonia's number sense in the context of her existing interest in Hopscotch. Sonia's participation becomes that of observer and the social exchange that occurs between her and her Granny appears to enable Sonia to advance her understanding of how we use numbers in situations such as this. Indeed, Sonia explicitly identified her Granny as a source of learning '*She teaches me stuff ...*' she remarked later.

On several occasions during our discussions she expressed her enthusiasm for number-related activity with remarks such as '*I like counting ...*' and '*I like ten ... because I know how to do it.*' Indeed writing numerals was something that she seemed to enjoy and on occasions even initiate. She describes how, when her Mam is having dinner, she makes cards for her. She remarks '*... and I put some numbers in sometimes.*'

Sonia's understandings related to the purposes of number

We saw earlier (See Table 4.3) that Sonia recounted a wide range of experiences related to number. She also identified a number of specific purposes of number (See Table 4. 2). Sonia expressed the view that numbers could be used for counting and for playing games. When asked about how her Mam uses written numbers, Sonia responded :

Sonia: *Yes ... she writes stuff to remember what you have to buy for shopping*

Liz: *Oh the shopping list ... and does she ever write numbers*

Sonia: *She writes some numbers down*

She told how she had used number in this way herself with the assistance of her Mam: *'Like one day I was going to a shop and I wrote numbers down and my Mam told me how to do them.'* Sonia is clearly interested in the purpose of writing numbers in this context. Her management of the activity is evident in the way in which she involved herself centrally in the activity. Her account strongly suggests that herself and her Mam were involved in each others thinking processes through sharing ideas and intent in relation to the writing of the list in the example above.

From Sonia's account, her Mam appears to be particularly sensitive to Sonia's interests and intentions, a characteristic particularly of mothers as participants in collaborative activity (Rogoff, 1998). Below, Sonia demonstrates her learning in relation to the calendar. The role of her Mam as expert with Sonia playing the role of novice and observing the way in which this number artefact or tool can be used in everyday activity is informative for Sonia:

> *Sometimes I think she goes one, two ... there's seven days in a week ... Before Halloween ... em ... its my birthday and I'm going to have a dress-up party then ... Before Halloween its Christmas ... after Halloween ... first it's my birthday then its Halloween and then its Christmas.*

As a result of discussions with her Mam, Sonia was able to sequence a number of upcoming events in her life, thus demonstrating her ability to relate calendar to purpose in that respect.

Below Sonia provides a glimpse of the detailed observations of the numerical environment that she has made and her attempts to link numerical knowledge together:

Sonia:	*I think he* [her Mam's boyfriend] *uses numbers for his phone*
Liz:	*When he's ringing his friend ... or when he's ringing your Mammy ... he'd need to know the number wouldn't he?*
Sonia:	*Yes ... his mobile number ... but he knows that already*
Liz:	*Do you know his mobile number ... it's a long one*

Sonia: *No ... but I was looking at it in a credit card or something ... on a piece of paper or something*

Liz: *Are there a lot of numbers on a credit card?*

Sonia: *But there's writing on them ... a lot of writing*

Liz: *A lot of writing ... and a lot of numbers too ... yes ... numbers are useful aren't they?*

Sonia: *And writing*

The purpose of the numbers on the road signs were also understood by her:

Sonia: *Em ... I think they're for people to know some stuff*

Liz: *Like what*

Sonia: *Like how you ... not to drive fast*

Sonia's quantitative thinking

Counting, subitizing and estimating

Sonia used, in a flexible way, a number of different strategies to quantify. These included counting, subitizing and estimating.

She counted proficiently but suggested 15 as her ceiling, although she did count higher on occasions. Sonia seemed to move with ease between the strategies of counting and of subitizing as can be seen in the following response to one of the items in Task 5 (See Appendix 1b):

Liz: *Well let's pretend he only got that many.* [Four shown] *How many has he got there now?* [She counted four]. *Right ... and he has four presents there and let's pretend that one got lost* [I removed one]

Sonia: *Three*

Liz: *Three ... okay!*

Sonia: *I don't need to count that*

Liz: *Why do you not need to count?*

Sonia: *Because ... I know that's three ... one, two, three ... I don't need to count but I'm practising*

My interest in her statement about not needing to count had obviously struck her as significant and appeared to temper the way in which she approached subsequent activities. She sought clarification where necessary while at the same time demonstrating the extent to which she had tuned into

what she had decided were the parameters of the activity in which we were engaged. For instance when I asked her to try and figure out how many items were hidden in Task 6, her response was to ask for clarification, i.e., '*And not count?.* ' Sonia also demonstrated the expectation that she could act in an agentive way in her approach to problems. According to Bruner (1996: 93), the agentive mind is '... *proactive, problem-oriented, attentionally focused, selective, constructional, directed to ends ... Decisions, strategies, heuristics-these are the key notions of the agentive approach to mind ...*' These characteristics are clearly discernible in Sonia's response to an estimation problem (Task 9): *'Spread them out ... I think ten.* '

Relating numbers to other numbers

Sonia also displayed a rich understanding of one more and one less relationships. For instance, she could compare numbers and could suggest numbers smaller than or bigger than a given number. Interestingly, she did not order the magnetic numerals, as many of the children did, perhaps indicating that numeric order was already well established for her and no longer a focus of her attention.

Sonia was clearly assisted in her efforts to develop her facility with numbers by using her fingers as concrete referents. She explained how she had figured out some relationships between numbers, and how she uses these to think about problems related to *less than/ more than*:

Liz:	*Let's pretend it's his birthday and he got three presents*
Sonia:	*Yes*
Liz:	*And then one more. How many has he now?*
Sonia:	*Four* [She showed four fingers]. *I don't need to count*
Liz:	*Why do you not need to count?*
Sonia:	*Because ... I know by one, two three four.* [She indicated the fingers as she counts]. *I know by holding them up ... I know by holding them up ... that's five.* [She showed one hand]. *That's ten* [She showed two hands]
Liz:	*Very good!*
Sonia:	*And when you take away one that's nine*
Liz:	*So you have all that figured out?*

Sonia: *Yes*

Liz: *And you don't even need to count them?*

Sonia: *No*

Sonia demonstrated here that she knows how to use her fingers to give a concrete referent to her use of the formal language of number. Young children's use of fingers in these ways is crucially important in linking the abstract and the concrete (Hughes, 1986) and in establishing numerical understanding (Fuson *et al.*, 2001).

Sonia's awareness /understanding of numerals

I struggled with the form of a suitable question that would elicit children's awareness of written number symbols in their environment. I found that the children generally interpreted me literally when I asked them, for example, about their experiences of seeing numbers on the way home. I found it easier to establish joint understanding on this issue with Sonia than I did with many of the other children. Sonia, in contrast, seemed to catch on to what I was asking straight away. For instance, our exchange in relation to speed signs (I think) demonstrates my point:

Liz: *But ... when you're on the way home from school and you're walking ... do you ever see numbers on the footpath or on the walls or anywhere like that ... or on signs?*

Sonia: *But ... sometimes I see them on signs when I'm going in the car with my Mam*

Liz: *And what does it say on the signs?*

Sonia: *They have numbers on the signs*

Liz: *Like what kind of numbers?*

Sonia: *Two or sixes or threes ... I think*

Sonia named the magnetic numerals and using these represented her age and, although not asked to do so, her phone number. I asked her how she remembered all the numbers in her phone number and she replied

> *My Mam used to tell me ... she writes them down in her phone number so she remembers how to.*

She knew the number of her house to be 52 but didn't know how to represent that. She had no difficulty matching numerals to sets (1-10). She made reference to some numbers larger than 10 and she was beginning to acquire knowledge of what certain two-digit numbers looked like in written form. She liked to display her knowledge:

Sonia: *I know how to do eleven ... one and one ... I know how to do one more ... six six ... I think it's ... em ... you count ... em ... you get sixty-six, I think'.*

Liz: *And how do you know how to write sixty-six?*

Sonia: *Because my Mam told me you write six and another six*

Liz: *You know a lot of things about number Sonia, don't you? You must do lots of talking about numbers at home with Mam?*

Sonia: *Yes ... I just remember them.*

Sonia displayed the knowledge that *'1 plus1 equals 2 ... And 2 plus 1 equals 3 and 3 plus 1 equals 4 and 4 plus 1 equals 5'*, indicating perhaps an appreciation of the pattern inherent in this series and thus of the aesthetic aspect of number. She also remarked '*I know how to do plus is* [or perhaps pluses?]. *My Mammy told me how to do that'*. Sonia knows that number can be talked about in this formal way and she also knows something of how this formality operates. This demonstrated for me the ways in which she is becoming prepared for involvement in school mathematics. I think it is interesting that Sonia responded in this way in relation to my question about learning to count.

Sonia is being introduced to both *instrumental* and *pedagogical* uses of number (Walkerdine, 1988: 81). Instrumental uses are conveyed to her in tasks such as writing the shopping list or consulting the calendar, and the pedagogical uses are conveyed where working with numbers is the explicit focus of the task, e.g., doing 'pluses'.

Summary of Sonia's participation

The nature of Sonia's relationship with her Mam, and the extent to which she shared her Mam's life, have enabled the development of Sonia's strong sense of number. Sonia was generally centrally involved in the various

number-related activities she described. She appeared to seek out her Mam's help and assistance, and at times, to involve herself in her Mam's activity. Sonia's learning was a social activity mediated by her Mam (and occasionally other significant people). Her Mam responded to Sonia's interest, and in doing so, supported and challenged her understandings and her skills in relation to her developing sense of number, as did other significant adults in her life. It appears from the above vignettes that Sonia's Mam is a major collaborator in her learning.

Tom's profile

Fantasy was important to Tom. When I questioned him about siblings, he informed me that his was a large family. He described his family as consisting of his Mam, Dad three brothers and two sisters. Tom's teacher later told me that he had only one younger brother. During the course of the first interview he told me several 'stories' that were instantly recognisable as such, for example, when he claimed to have sixteen brothers. Perhaps he was being playful, or at least asserting his control over the interview.

The interviews with Tom took place during the second week of September. The overriding impression that I got was that he was entirely engaged with making sense of this new environment of big school. Very early in our discussions he mentioned that he had learned to count in his other school (i.e., preschool). He described it as a different school, as if marking it out from the present school. He showed me the nametag on his tie and asked '*Do you know what that's for*? Perhaps the novelty of the school tie was further enhanced for Tom by the practice of having to identify its ownership by writing on it. He sought clarification from me about aspects of school life, as for instance when he asked '*Why do we not do homework at school?*' Most telling, I think, was Tom's comment about Mrs C's class (his class) '*... that's not my really class.*' He went on to describe to me where his 'real' classroom was and who his teacher was going to be (he specifically mentioned another teacher in the school). I speculate that Tom had somehow formed the expectation that he would start school with a particular teacher and in a particular room. He seemed to consider his current position as a pupil in Mrs C's classroom as a temporary arrangement (it wasn't). I thought it significant also that Tom expressed some anxiety about the

arrangements for being picked up from school that day. He announced that Granny was picking him up, then changed his mind and said his Mammy would. He then decided that she was working but predicted a problem for his Granddad and Nana who, according to him, didn't know where his school was. In short Tom conveyed the impression of a child who was unsure about his surroundings, who was trying to reconcile the reality of starting school with the anticipated arrangements, particularly in relation to the identity of his teacher and his classroom. He appeared to be actively seeking to make sense of the practices encountered in this new place.

Tom took a proactive role in relation to the course of our discussions, sometimes explicitly suggesting a topic for conversation or a game to play. A number of times Tom sought to listen to himself on the tape. He was active and engaged throughout but, as we see below, that activity and engagement was centrally focused on establishing control over the new environment of school and only peripherally connected to my area of interest, i.e., his views and understandings related to number.

Tom's pleasure and interest in number

Throughout the first interview, I found it very challenging to engage Tom in discussion about numbers. The following exchange illustrates what I mean:

Liz: *What are numbers for anyway? What are they for?*

Tom: *Do you know a house with no stairs?*

Liz: *I do ... a bungalow ... yes*

Tom: *I have a bungalow*

Liz: *Have you ... and have you a number on your front door?*

Tom: *No ... it's on the back ... you can't really see it*

I think it is possible to partly explain this difficulty in engaging him in number-related conversation by reference to the fact that he appeared to be very distracted by the surroundings of the room in which we had the discussions. Indeed, Tom questioned me a good deal about the room and about the various artefacts there.

When he spotted a wooden number-line into which little plastic pegs could be placed, the following exchange took place:

Tom: *What are these ... what's it for?*

Liz: *It's for drawing lines ... I think. And counting, It's for putting pegs in ... you put pegs into the numbers*

Tom: *What's pegs?*

About fifteen minutes into the first interview I noticed how he surveyed the room from the chair in which he sat. '*Wow! Who painted those pictures?*' he asked.

While I am suggesting that the newness of the surroundings were distracting for Tom, it is possible that Tom just didn't respond to the situation or the questions because they were not currently of interest to him. Issues related to the environment were much more pressing for him.

The extent of his interest in finding out about the school environment can be compared with his apparent lack of interest in talking about number as an aspect of everyday activity:

Liz: *Think about it really hard ... what does Mammy do with numbers?*

Tom: *She likes numbers*

Liz: *How do you know she likes numbers?*

Tom: *I don't know ... What are they for?* [He pointed to some workbooks lying on a shelf nearby].

According to Bruner (1996: 93) '... *the agentive mind is not only active in nature, but it seeks out dialogue and discourse with other active minds.*' Tom displayed *agency* throughout the course of the interviews, in particular the first one. He did so by questioning me about a range of issues related to the immediate environment. He also suggested topics for discussion. For example, at one point he suggested *'Let's talk about my brothers now'.*

Certain aspects of Tom's general approach to participation in the first interview emerged during the second interview also, in particular his agency in seeking to establish the parameters of our discussions. For instance:

Tom: *Can we go up ... em ... to seven?*

Liz: *Seven?*

Tom: *Seven o'clock ... can we stay here?*

Liz: *Can we stay till seven?*

Tom: *Yes*

Liz: *Why do you want to stay till seven?*

Tom: *I want to*

Liz: *Well we'll stay a while anyway*

Tom: *Can I stay like ... the last week*

Liz: *For as long as you did the last day?*

From this we can see that Tom obviously enjoyed our discussions and sought to prolong them. However, it was the discussion itself he enjoyed, and the opportunities it provided to pursue issues of concern to him, rather than the topic that I sought to explore with him.

Tom's understandings related to the purposes of number

Uniquely among the children profiled, Tom recounted very few experiences related to collaboration with family members or peers in relation to learning about number.

In relation to purpose, Tom suggested '*That's why you need the important things*' as the reason for learning about numbers, thus demonstrating his understanding of the cultural value of number. In relation to his parents, he said that they both 'read' numbers:

Tom: *She* [his Mam] *reads then in the car ... she reads something with them on*

Liz: *She reads some numbers does she?*

Tom: *Yes*

Liz: *Like what?*

Tom: *Like ... em ...*

Liz: *Tell me more about that ... Mammy reading numbers in the car. What numbers does she read in the car?*

Tom: *She reads something important ... but I don't know*

Liz: *Go on ... tell me more about it*

Tom: *She likes reading and ... she always likes reading.*

Liz: *And what about Dad?*

Tom: *He likes doing reading with numbers*

Liz: *Does he ... did you see him?*

Tom: *No*

Liz: *And how do you know he likes doing reading with the numbers?*

Tom: *I saw him a little bit doing them*

Liz: *And did he say anything to you ... what he was doing?*

Tom: *No*

Adopting Rogoff's (1998: 723) definition of collaborative activity as activity in which '*... an endeavour and its thought process occurs at least partially in common ...*', then Tom and his parents can be seen to be collaborating in relation to learning about number. However, what I find interesting about the above vignettes is that Tom focuses only on the activity of his parents when relating these experiences, and even when pressed, has nothing to say about his own role or activity. Lave and Wenger (1999a) have used the notion of *peripheral participation* to describe the process whereby learners observe and begin to participate in community practices. They suggest that learning to talk, in this case about the purposes of adult activity with number, is a key to more skilled participation.

Tom's participation in numerically related activity is, I think, confirmed as peripheral when his level of understanding of the purposes of number is considered. For instance, when asked why children learn these numbers, he replied that he didn't know. I pursued this with him:

Liz: *Is there any reason for children to learn numbers ... what will they do with all the numbers they learn?*

Tom: *They put them on the mat and they do something with them*

I speculate that he refers here to the large sponge mats with press out numbers commonly found in preschool and schools in Ireland and sometimes used as a floor activity for children.

It seems that Tom's experiences with number to date have led him to conclude that numbers are physical objects that can be handled, but he is unclear as to how they relate to anything else beyond that particular context. From the above it is clear that Tom understands that one of the functions of number is communicative but he appears unaware of what might be communicated in this way. He related no other experiences or opinions

about the use of numbers in everyday life. However, he did talk about age in relation to number and on several occasions he attached specific numbers to the ages of his (imaginary) brothers, indicating that he was aware of the use of number as a label.

The significance of numbers does not appear to be at all transparent to Tom and furthermore the cultural practices in which Tom's learning is taking place do not make the numerically related meaning of what is being learned at all transparent. Lave and Wenger (1999b) argue that using artefacts (e.g., numbers) and understanding their significance interacts to become one learning process. The significance of numbers in everyday life was not understood by Tom, possibly because he appears not to become centrally involved in experiences that would make such uses transparent to him.

Tom's quantitative thinking

Counting, subitizing and estimation

Tom could recite the number words in the conventional order (to at least 20) and he counted (skilfully) to quantify the number of items in various sets (See Table 4.4). He appeared to subitize smaller sets (See Table 4.4) to determine quantity. In relation to Task 7, he subitized correctly the dot patterns for 3, 5 and 2. He gave a response of 3 for the group of 4 dots but he explicitly counted the group of 6 dots.

In response to Task 9, he looked at the plate of 8 lemons and responded that there were 6 there. His response to the display of 6 apples was 8. However, in response to both of the other items presented, Tom's overt behaviour suggested to me that he was silently counting the set, although he didn't agree that he was counting. Previous research (Munn, 1994) suggested that children's understandings of what counting means often differ from that of adults. I suggest that the fact that the quantification process was carried out silently possibly meant that from Tom's perspective, he wasn't counting, but doing something else. From my perspective he was counting covertly to quantify (See also Chapter 4: 105-107).

Relating numbers to other numbers

Tom could identify numbers *bigger than* a given number. The phrase *smaller than* was more problematic for him. With the *one more than, one less than* task (Task 8) some difficulties emerged. A change of wording from *one more than* to *and another one* appeared to facilitate his understanding of the focus of one aspect of the task. However, he had great difficulty with judging how many remained if one item was lost from a given set. I think it is likely that the reasons for his difficulties were quite complex. They appeared to be related, to some extent at least, to his reading of the task as story. The context within which the task was presented appeared to introduce '*a competing discourse.*' (Walkerdine, 1988: 46) The effects of this competing discourse on his responses to the tasks were illustrated in the following:

Liz: *And then another day he looked at his presents and he had only three*

Tom: *Where are the other ones gone?*

Liz: *He must have put them away somewhere ... but he has only three presents on the table and then Granddad gave him one more ... how many has he now?*

Tom: *Four. Do you know all Coco's friends?*

Liz: *I know some of them ... and then he got another one*

Tom: *From who?*

We see here how the 'story' context seemed to impede Tom's ability to attend to the numeric aspects of the task. Certainly Walkerdine's (1988: 47) argument that '*... far from the story providing an enabling and 'meaningful context' it provides a complex and potentially bewildering confusion*' appears to hold true for Tom in relation to how he interpreted the various tasks.

Certainly, part of Tom's difficulty seemed to lie in the hypothetical nature of the events and his consequent need to understand and explain various non-numerical aspects of the tasks to his own satisfaction:

Liz: *He had six on the table ... and one fell off*

Tom: *Were they under the table?*

Liz: *They were on the table and one fell off... six of them*

Tom: *Did one? Was there a little hole in the table?*

Liz: *No it just fell off the side*

Tom: *Yes ... and he counted all them there*

Liz: *Yes*

Tom: *And one fell down and then it went under there*

Liz: *Yes ... so how many were left on the table?*

Tom: *Six*

As I reviewed Tom's responses to earlier tasks in the second interview, it became more and more obvious that in fact, quite early in the second interview, the fictional events had become the focus of the discussion for Tom, rather than the aspect of number that I had intended. For instance, in Task 5 (See Appendix 1b) where I introduced some little birthday parcels his response was:

> *What's in it? Oh ... I think I see ... no ... its paper. Are you going to open them now?*

Similarly, in relation to Task 4 (See Appendix 1b), where I introduced birthday candles for the cake, Tom responded in the following way:

Tom: *Are you only pretending?*

Liz: *Yes ... we're pretending*

Tom: *Are you not lighting them up?*

Tom could only make sense of the purpose of the candles in this task in relation to the ways in which he has experienced candles being used in the past. His comments here alerted me to the fact that it was highly likely that, until then, Tom had not had any experience of purposeful artefacts such as these being used in this hypothetical way. Pramling (2004: 3) observed that

> *With respect to the child's perspective one always has to bear in mind that children's experiences of meaning is always taking place in a certain situation, a specific context, based on the child's earlier experiences and capacity to express him or herself.*

I think that this insight has some important implications for the ways in which we think we 'concretise' number problems for young children. From Tom we learn that when quantification tasks are presented as hypothetical situations, this may impede rather than assist some children's quantitative reasoning. It seems possible that some children's number sense may be obscured by the type of situations that we choose to use as perspectives on that number sense. I will return to this topic in Chapter 6 when considering the curriculum implications of the findings presented here.

Tom's awareness/understanding of numerals

Tom could read numerals (1-20) without any difficulty. He demonstrated this as he placed pegs into each of the numerals on the wooden number line that he had spotted on a shelf nearby. He also matched numerals to representations of quantity without any problems (See Task 10, Appendix 1b). He reported that he had magnetic numerals at home and he used them, according to himself, to say the numbers. He identified the numeral 5 as indicating his age. He was within ten days of this fifth birthday so it seems likely that he now identified himself as five. He also identified the numeral 5 as the number of his house. This agreed with school records. As he handled the numerals he sought to resolve for himself two apparently different ideas that he had encountered about what comes 'after five':

Tom: *Look at that* [He selected two fives and placed them side by side]. *After five I'm going to be that age ... fifty-five*

Liz: *After five you're going to be fifty-five?*

Tom: *No ... after five I'm going to be six*

I think this incident shows the extent to which Tom is pursuing meaning in relation to written numbers/spoken numbers that he has encountered and seeking to reconcile apparently contradictory information.

I observed that for some of the children, usually the girls, knowing number seemed to be about knowing how to *write* the symbols. This was true of Tom:

Tom: *Some boys when I was in ... eh ... playschool ... and girls ... I done something and they all know their numbers*

Liz: *How do you know they know their numbers?*

Tom: *I saw them doing them*

Liz: *And what about Mam ... does she know any numbers?*

Tom: *Yes ... sometimes*

Liz: *How do you know she knows numbers?*

Tom: *That's why I saw her doing them*

Overall, the sense I got from Tom was that he had formed an understanding that numbers can be read and written and that they carry 'important' information that is discernible to some. However, I got no sense from what Tom said that there was any attempt by other participants in the situations he describes to communicate, foster or structure his attention. This doesn't mean that there were no such attempts, just that they were not a significant aspect of the experience as Tom related it. I think that it is significant that he doesn't report any interaction with other participants in the number-related situations that he describes. He seems to accept without question that other people do things and know things that he doesn't understand or know about.

Rogoff (1998) concludes that learning to collaborate may be easier for people if others in positions of responsibility have treated them in a collaborative fashion. Based on what he has told us, it might be tempting to conclude that Tom may not have had many opportunities in which to collaborate in learning about number. However, I offer an alternative interpretation. I suggest that it is possible that Tom chose not to collaborate to any great extent in these experiences because they were not of sufficient interest to him. During the interviews we saw how the numerical focus was not of any great interest to him and how other non-numerical aspects were of greater interest. While in one sense Tom collaborated well with me, it was mainly he that set the agenda for our discussions. Consequently, issues related to his affective state (his feelings of uncertainty in his new environment) and to the world of fantasy (the story of Coco's birthday) dominated the discussions.

Summary of Tom's participation

In Tom's accounts of his number-related experiences, he cast himself mainly in the role of observer who was peripherally engaged in collaborative activity with other participants about number. This is in contrast with the very different role that Tom appeared to take on in relation to his discussions with me. During these discussions his role was interactive, agentive and appeared to be focused on making sense of the immediate environment, his new experiences related to school, and of the interview itself. From Tom we can learn that where the contexts are novel, or puzzling for any reason, a child's participation is likely to be circumscribed by their need to make sense of the current situation.

Aspects of Tom's number sense emerge under particular conditions but not in other situations. During our discussions when Tom was dealing directly with the magnetic numerals or with visible quantities, his skills, knowledge and reasoning emerged quite clearly. In these circumstances, his sense of number was discernible. However, with the introduction of contextual factors, either in story form (pretending or hypothesising) or in the form of reference to everyday activity, his sense of number was much more difficult to discern, and in some instances (related to purpose of numbers) hardly visible. Talk of everyday experiences and story situations appeared to introduce a bewildering array of contextual factors that he sought to understand and explain. Consequently, any number–related dimensions of these situations appeared to recede into the background for him.

Conclusions

In this chapter, I presented in-depth profiles of two children in relation to their number sense. The impression that Sonia's gives is that of a child who is centrally engaged and participating in many aspects of her Mam's life. She displays a strong sense of number across each of the aspects explored. She displayed evident pleasure and interest in number and she had an understanding of numerals that encompassed a number of uses. She appeared to have an extensive knowledge of how numbers are used in different situations and for different purposes. Her quantitative skills were based on a flexible and fluid knowledge of number relations. She was a

strategic and skilled counter and she used a range of skills to quantify and estimate various sets of objects.

Tom, on the other hand, presents himself as a child who has relatively little shared interaction with the adults in his life, and one for whom interactions with such adults are not of much significance. Tom displayed relatively little interest in numbers, as evidenced by his reluctance to engage in number-related discussion and his vagueness about the role of number in everyday activity. He reported very few experiences with number. On the other hand, his quantitative thinking appeared to be well developed. He counted skilfully and he utilized a number of sophisticated counting strategies to quantify sets of various sizes. He displayed considerable skill at naming numerals and matched these to quantity. All of these skills will be very useful to Tom at school. We saw in Chapter 4 how the ability to demonstrate quantitative thinking (by using a range of strategies to quantify) was an aspect of some children's number sense but not of others. From Tom we learn that a child's ability to demonstrate quantitative thinking is not sufficient to conclude that that child's number sense is well developed. In other words, the ability to demonstrate quantitative thinking is a necessary but not sufficient indicator that a child has developed a strong sense of number.

Tom was challenging to interview in that it was difficult to keep him focused on my interests. I worked hard to draw him out and it was often difficult to get him to explain or elaborate on his statements. Sometimes his 'stories' seemed to intrude on the interview and presented me as interviewer with a real challenge in terms of how to proceed with the interview. Initially, establishing intersubjectivity was also challenging with Tom since he had his own purposes in relation to the interview and these stayed very much to the fore throughout. However, he did ask lots of questions and in that sense it was possible to follow his train of thought. Sonia, in contrast, spoke freely and at length about the issues raised. She was forthcoming, clear in her explanations and often justified her statements. She stayed 'on task' and it was relatively easy to establish intersubjectivity with her. She appeared to be able to tune in easily to the discourse that was part of our discussions.

Sonia is arriving at school eager, willing and able to engage in number-related activity. She has a well-developed sense of number that includes considerable skill and understanding in that domain. While she often takes control of her own learning, she is prepared to collaborate with others and follow a direction in discussion/activity as suggested to her. In summary, Sonia is easy to work with from a teacher's perspective. On the other hand, while Tom has demonstrated that he is quite skilful in relation to several aspects of number sense, he is also very agentive in relation to his learning. He may pose a challenge to teachers who may try to engage him in a pre-determined programme without consideration of his interests and preoccupations. I can imagine that some teachers might conclude from the evidence above that Tom had problems concentrating and was uninterested in number. Perhaps they might conclude that his tendency to fabricate stories about his background indicated a lack of maturity. I would argue that none of these judgements could be justified on the basis of the evidence. What is clear is that the transition to school is not completely smooth for Tom. He appears to have little experience of collaboration with adults in relation to learning about number and so may be slow to become fully engaged in such activity. Despite previous research (Walkerdine, 1988) indicating that interactions at home tend to be collaborative and participative, Tom's profile raises the possibility that such experiences, in the domain of number, may not be equally familiar to all children, and especially not to boys.

The profiles of Tom and Sonia clearly suggest that different families might have different social norms which in turn may influence the direction of development. From Tom's accounts, there appears to be relatively little communication about number between Tom and his parents, although he observes a lot of things in relation to number. In Sonia's family the energy seems to be focused on maintaining a close relationship between Sonia and her Mam. From a sociocultural perspective, Barratt-Pugh (2000: 16) argued that '*Family shapes the purpose, variety and frequency of literacy practice, as well as the value that is placed on different practices.*' Based on the profiles above, the same can be said in relation to numerically related practices. Tom and Sonia can be seen to participate in a range of differing

experiences with different people, thus determining the sequence of the development of their number sense and its individualistic nature. The cognitive independence displayed by Tom is not something that will necessarily be valued at school (Hendy and Whitebread, 2000). Also in Tom's family, it would appear that collaboration in the practices of the community are not that central. In Sonia's family, perhaps because of the way in which Sonia's Mam seems to relate to her, numerically related practice appear to be both visible and accessible for Sonia. The profiles presented here would appear to suggest distinct pedagogical styles of supporting young children's early number sense. In Chapter 4, we saw how boys and girls appear to interact differently with adults about number, with girls suggesting that they engaged more in interactions than boys. It maybe that girls and boys are treated differently in social and domestic contexts.

The documentary analysis carried out earlier (See Chapter 4: 54-73) revealed that issues related to affect received scant attention in the documents considered. The profiles here clearly illustrate the centrality of issues related to affect for children's learning, and specifically how some children, such as Sonia, display a disposition towards number. Others, such as Tom, appear not to be particularly interested in this aspect of learning. The ability to ascertain which children would benefit from particular attention to the nurturing of their numeric disposition, an important aspect of number sense (Chapter 2), is particularly important to teachers. The importance of identifying and developing children's disposition to engage in mathematics and to learn in the domain of number is increasingly seen as critical, and as an essential outcome of early education (Anning and Edwards, 1999).

Chapter 6 that follows brings together the findings from the documentary analysis, the data analysis and the profiles to explore the implications for pedagogy, for curriculum and for teacher education.

Chapter 6: Reviewing The Study: Conclusions and Recommendations

In relation to number sense, findings from this study show that:

- It is grounded in, and shaped by, everyday experiences;
- It is holistic in nature and has affective, cognitive and social dimensions which are so tightly bound together that it is almost impossible to separate them;
- Although holistic in nature, it has a number of aspects that, for the purposes of exploration, need to be considered separately. However these aspects must be understood as parts of the same whole;
- In some children it appears to be more developed than others. Some tasks, i.e., those related to quantification, seem to be good discriminators of number sense in young children;
- It is a complex notion, children do not simply have it or not. It is linked to the context within which it is observed and individual children can demonstrate different levels of number sense depending on the task/context;
- Explorations at the point of entry to school can provide teachers with information on which to plan the further development of young children's number sense.

Summary of findings related to the research questions

The number sense demonstrated by young children as they start formal schooling in Ireland: Understandings of the different purposes of number

The data analysis revealed that

- Young children's understandings of the purpose of numbers were influenced by the contexts in which they experienced number, i.e., their everyday experiences. Based on children's accounts, it appeared that these experiences differed from child to child depending on family culture, although there were some 'common' experiences that a number of children described. My findings confirm Young-Loverage's (1989) observational findings regarding the variety and extent of early learning in the domain of number;

- When children talked about numerically related experiences, numbers were generally not foregrounded within these experiences. Where children conveyed their interest in, and purposes for, engagement with numbers affective dimensions were very much to the fore. Social dimensions, in particular children's relationships with others and in some cases, particular cultural experiences, were also to the fore as children talked about their experiences. This strongly suggests that number sense in young children is very much the holistic construct suggested by Sowder (1992);
- Affective aspects of children's learning about number can be explored using a methodology such as the one adopted here;
- Children suggested generic, communicative, count and label purposes of number. The counting purpose was the most transparent to the children. Most of them displayed limited understanding of the specific information that number might be used to communicate. These understandings often differed markedly from adult understandings and intentions, in much the same way that Munn's (1994) children had beliefs about counting that differed from those of adults. The children in my study attached little significance to the label function. However, in relation to the personal labels of age and house number most children chose the appropriate symbols from amongst a collection of plastic number symbols. Boys were more likely to suggest generic purposes for number, while girls were more likely to suggest specific purposes;
- The writing of numbers was an important aspect of purpose for some children, in particular the girls in the group;
- There were differences in the ways in which girls and boys were constructing their metacognitive framework in relation to number. Specifically, there were differences in the ways in which young girls and young boys appeared to participate in numerically related activity with the adults in their lives. From their accounts, girls appeared to engage in more intense interactive behaviour with adults about number, in much the same way as those reported by Tizard and Hughes (1984). The girls appeared to ask more questions of those adults and to seek more assistance with for example, learning

to write numbers. Young boys, on the other hand, appeared from their own accounts to participate more peripherally. They did not appear to ask many questions related to aspects of numerical activity, nor did they appear to seek assistance with tasks related to numerical learning, but instead they seemed to construct their understandings in a more solitary and less social way than did the girls. Munn (1994) suggested some of the ways in which teachers need to structure the environments that enable children to develop metacognitive frameworks in relation to numeracy. This study adds some new perspectives to this debate in that it identifies aspects of participation that may need to be addressed in structuring appropriate environments. Specifically, boys may need to be encouraged to be more interactive and participative, and to engage more in discussion about the purposes of number. They will also need to be encouraged to question adults more about numeric purpose. Boys may also need to be encouraged to pursue joint (social) interests arising from collaboration with others.

The number sense demonstrated by young children as they start formal schooling in Ireland: Quantitative thinking

The data analysis revealed that:

- Some children demonstrated an impressive ability to think quantitatively, and in doing so, they appeared to use a variety of counting strategies selectively to solve quantitative problems;
- Children's understanding of how numbers related to each other was easily discernible when responding to particular tasks (e.g., Task 4);
- Children's responses to some tasks, e.g., Task 5, clearly indicated the importance of overt incremental counting for quantifying sets. But they also indicated that many children often made use of increasingly sophisticated counting strategies such as internalised counting to solve such tasks;
- The quantitative thinking displayed by most children was very much linked to the specific nature of tasks (See Table 4.4: pp. 129-131). Some children who had apparently developed a variety of counting/quantification strategies did not always use these in

response to the question *How many?/How many now?* These children mostly counted incrementally from one in response to any question containing these words;

- Where children overtly counted incrementally from one to quantify, we cannot assume that they had no other strategy for quantifying. There may be complex social reasons for children choosing incremental counting in these situations. For instance, some children indicated that they felt that a counting response was what was expected of them;
- Where quantities were presented in a spatial pattern, the majority of children appeared to use more sophisticated counting strategies which permitted them to respond rapidly to the question of *how many?*;
- Some children, understandably, seemed to find it difficult to differentiate between an exact answer and an approximation. This is not surprising since their experiences to date were probably about counting in an ordered fashion to reach a 'correct' amount;
- Just over half of the children did not appear to understand the idea of 'estimation'. They counted when asked to estimate. This resonates with Aubrey's (1997) report that her children seemed confused by the request that they 'guess';
- Covert counting was a key strategy in relation to children's responses to estimation tasks.

The congruence between the findings of this study in relation to children's number sense and the statutory curriculum in mathematics for the first year of school in Ireland

My analysis revealed that:

- Many of the elements of number sense identified in my framework are explicitly referred to in the lead curriculum documentation that pertains to all curriculum areas, and not specifically to mathematics. This suggests a good match between the 'number sense' construct and the general thrust of the curriculum;
- Although the extent of children's informal learning in the domain of number is known to be extensive and an important foundation for

later learning (See Chapter 2: 12-15), the teacher guidelines do not address either of these issues in sufficient depth;

- While the importance of the affective and social aspects of learning in the domain of mathematics are acknowledged, the teacher guidelines contain no specific guidance for teachers in relation to how such issues might impact on learning and teaching. My study provides evidence of the interrelatedness of affective and social issues and also suggests several avenues worth exploring in seeking to ensure that affective and social dimensions are addressed when seeking to develop young children's number sense at school;
- In the area of quantification, the curriculum guidelines stress the importance of the introduction of the idea of sensible 'guessing' or estimation from the first year of school. In light of my observations, further research on estimation is certainly warranted. My findings suggest that this area of learning needs careful analysis and planning in order that children are enabled to understand what estimation means, and its purpose and utility.

The elements of originality in my research

While there has been some interest in the explication of the notion of number sense in general (See Chapter 2: 15-21) at a particular point in time, i.e., at the point between the mainly 'informal' learning of home and the sometimes more 'formal' learning of school. My report thus provides a detailed explication of number sense as a form of reasoning and thinking as it might be observed and recognised in four-year old children. This detail was not previously available in the literature. It also provides a detailed explication of how number sense can be developed and constructed by children through their interactions in everyday experiences with other people and through learning to use and interact with numbers.

My exploration of young children's number sense involved the application of my particular variant of the clinical interviewing technique to a previously unresearched topic. While a (relatively small) number of researchers have used clinical interviewing with young children, the EBFF interview methodology that I developed differed in a number of ways. In September 2004, I presented a paper at the European Early Childhood

Educational Research Association (EECERA) Conference in Malta, where I outlined my particular approach to interviewing and my observations on its use with young children and the response was very positive (Dunphy 2005a).

The findings that I present in relation to young children's understandings of the purposes of number in everyday life are derived from a synthesis of children's explicit and implicit ideas about purpose as recounted in discussion and through their accounts of their experiences with number. This results in a more holistic and a more reliable picture of children's understandings in the domain of number than one based only on children's knowledge of formal aspects of number or their ability to solve number-related tasks. Thus our understanding of number sense in young children is extended accordingly.

The findings that I present in relation to aspects of young children's quantitative thinking also extend understandings in that area. They strongly suggest that there are considerable challenges involved in explaining the idea and purpose of estimation to very young children, many of whom are still developing understandings and skills related to counting.

The analysis of the number-related aspects of the revised *Primary Curriculum: Mathematics* (Government of Ireland, 1999b) for children in their first year at school and the exploration of the congruence of that curriculum with young children's number sense, is unique in the context of Irish education. This work will be of benefit to teachers, to curriculum designers and to teacher-educators. Firstly, my analysis identifies critical aspects necessary for the development of young children's number sense and clearly demonstrates that these are not given appropriate attention in the teacher guidelines. Secondly, estimation emerged as a key aspect of number sense and was seen to be somewhat problematic for many children. The assumption that the introduction of *estimation* 'fits' in a curriculum for young children during the first year of school is challenged and it is suggested that it may need further consideration.

My study is also unique in that it suggests ways in which we can establish starting points for the development of number sense at school. It

demonstrates how, and with what care, explorations of learning must be undertaken at this sensitive point. In this way, my study demonstrates a method that teachers could use to get to know children reasonably quickly and also to help to inform subsequent practice. A number of the tasks used in this study proved to be good discriminators of number sense, eliciting as they did some really informative responses (e.g., See Chapter 4: 96-97). These could prove really useful to teachers wishing to carry out their own enquiries in relation to aspects of children's informal learning in relation to number. It is my intention to develop materials and supporting documentation related to these tasks so that teachers have access to them.

In Chapter 2, I discussed number sense as a holistic understanding related to number. The particular view of knowledge implied by this definition meant that I was committed to explaining children's learning and the development of their number sense in a particular way. My exploration yielded a rich and, to my mind, compelling account of early learning in the domain of number. Others interested in number sense in young children may research the construct from their own theoretical perspective. Depending on the theoretical stance adopted and the particular definitions/descriptions used, different, but equally valid stories can be told in respect of young children's number sense. The fact that this study is based on the experiences, ideas and responses of a small group of children needs to be borne in mind when considering the findings. However, this does not detract in any way from its relevance for developments in mathematics curricula, teacher education and pedagogy.

The contribution of my study

Gifford (2004) drew our attention to the fact that while most early years research in mathematics has focused on children's competence, there has been relatively little research attention focused on pedagogy. Earlier, arising from the analysis of the curriculum documentation (See Chapter 4: 64-73), I identified a number of areas of pedagogy that should be explicitly addressed in any revisions of the teacher guidelines, and in the Framework for Early Learning being planned by the National Council for Curriculum and Assessment (NCCA, 2004). This is essential in order to ensure that the key aspects of number sense are addressed and further developed during

children's first year at school. I will address these in my work in teacher education. I will also continue to publish my research. The findings from this study will also be used to advise the NCCA in relation to the proposed Framework for Early Learning and to ensure that due consideration is given to the issues highlighted in this study.

I discuss ideas related to the development of each of these areas of pedagogy below.

Engaging young children in discussion about number

Pound (1999) advises practitioners of the need to ensure that teacher-directed mathematical activity for young children includes stories and games. While I think this is very good advice, I also think that teachers need to be aware that there are always contextual factors operating even with teaching approaches that appear to actively engage children and facilitate their collaboration and participation. The children in the study sometimes responded in very unexpected and surprising ways to the tasks, games and/or story contexts that I presented to them. The introduction of the story context and the game context sometimes set up unexpected barriers to ascertaining children's number sense. For instance, in Chapter 5 (See Tom's Profile), we saw how using a story context resulted in details of the story becoming the focus for Tom in our discussions, rather than the numerical aspect as intended.

Children's reactions to the strategy of presenting number tasks in the guise of games was considered in Chapter 4 (pp. 93-111). Such reactions provide some important pointers for teachers. Task 6 (See Appendix 1b) was designed to explore children's reasoning with numbers. This task was one in which children were invited to play a game with me. To begin, we checked by counting, and then agreed, that there were five presents on the table in front of us. I explained that the game involved me hiding some presents under a small basket and that the task for the child was to say how many were hidden. The children's reactions were very interesting from a number of perspectives. I had not anticipated the extent to which the children would apply the norms of games, as they had experienced and understood them. For instance, as soon as I mentioned a game with the presents Tom

suggested one '*Will we play pass the parcel?*' He seemed happy enough to go along with my suggestion of a hiding game. However, in reviewing my field notes, I found that I had noted Tom's apparent lack of engagement in the task as it progressed. In retrospect, I conclude that Tom passively withdrew from my game!

The comments and behaviour of a number of other children also alerted me to the fact that, from their perspectives, children thought it strange that this game didn't conform to the usual turn-taking conventions that usually applied in games, since I was getting all the turns to be in charge. This situation was tolerated by some but others announced their turn when it became obvious that I wasn't offering them one. Jamie was typical in that after the third item he announced '*It's my turn now.*'

I was also struck by the way in which a number of children sought, as they might when playing with peers, to change the game after a couple of turns. For instance Con (after three turns) asked '*Can we build a tower?*' and similarly Maura suggested '*Will we play a different game?*' Prior to the interviews, I had not thought about the game context from a child's perspective nor had I thought about the expectations that the context would introduce. These now seem to me to be really important pedagogical considerations that teachers need to be aware of when planning the use of similar strategies. My study shows that while stories and games are really useful pedagogical devices, young children can also become enthusiastically involved in intensive and prolonged discussions. The implication of this is that we need not always 'dress' our inquiries with children in the clothing of games and stories. Foregrounding children's experiences and interests and encouraging them to discuss these is a really important strategy for ensuring children's participation in assessment or inquiry type situations.

Assessing the affective and cognitive areas of learning in relation to number

There is no suggestion of assessment on entry in any of the documentation related to the curriculum. However, I see appropriate assessment as an important pedagogical tool to be used at various points in children's learning. In my opinion, there is a very strong case for teachers to carry out

assessments of children's interests, motivations and abilities in relation to number in much the same way as I have done in this study. In essence, I am suggesting that EBFF interviewing is not just a very effective research instrument, but with some modifications and guidance for teachers, it can also be a very effective form of assessment for learning that can be used by teachers. Furthermore, one of its strengths is in its ability to assess both cognitive aspects and affective aspects of children's understandings in the domain of number. In my study, children's responses to number tasks are complemented by their descriptions of their experiences and views related to number. Likewise in the classroom, teachers can use a combination of these to ensure that both cognitive and affective/ social dimensions are probed. They must do so in order to plan how best to further promote children's number sense. Pramling (2004: 8) argues convincingly that to create opportunities to work with children at '... *the intersection between each child's experiences and perspectives and the intentions of society stated in curricula*', then knowledge derived from the process of finding out about children's experiences, views and understandings is essential.

Another advantage of EBFF interviewing is that it is relatively efficient in assessing children's learning in the domain of number, in a relatively short amount of time. This means it is within the reach of teachers in terms of the time commitment required. This is an important consideration since previous research has found that methods that required lengthy observations didn't work for busy practitioners (Fleer and Richardson, 2004). While it may take time to perfect the technique of interviewing children, I believe that this pedagogical practice has a great deal to offer teachers of young children, in particular in relation to assessing their learning and in clarifying goals for learning. Consequently, as I work with both my undergraduate and post-graduate students who are specialising in early childhood education, I intend to introduce them to significant aspects of this type of inquiry. Recently, I invited Masters in Education students to compare my use of the clinical interview methodology with the uses suggested by Ginsburg (1997). The responses of the students, all practising teachers in early childhood settings, were very positive and I have encouraged them to consider the use of such a methodology in their own research/pedagogy in the coming year.

Equally positive were the reactions of practitioners and researchers who attended my recent keynote address at the annual conference of OMEP Ireland (World Organisation for the Education of Preschool Children). In my address, *Current Issues in Early Childhood Education and Care: The Challenge of Assessing to Support Learning,* I presented my arguments related to the potential of the EBFF interview for assessing for learning (Dunphy, 2005b).

Identifying and building on children's informal number-related learning

The fact that the curriculum gives particular consideration to the social implications and relevance of mathematics is to be welcomed (See Table 4.1). We have seen that such aspects of number are part of the experiences of preschool children. It is also clear from the data that children's sense of the role that number plays in everyday activity is generally vague, and not uniformly developed for all children. In Chapter 4, we saw clearly that most aspects of, for example, the communicative function of number are not apparent to young children from their everyday interactions at preschool level. This verifies Young-Loveridge's (1989) contention that the process of taking mathematics from the environment doesn't happen automatically for young children. My findings indicate that understandings related to issues such as the purposes of number in everyday life and those related to the rules we impose on the numeric system are not things that children can work out for themselves. A mathematical way of looking at everyday situations is a particular way of seeing and one that children need to be guided towards. So even where children have particular experiences that are potentially rich in relation to developing a number sense, they will need those experiences mediated by numerically-aware adults in order to develop a strong sense of number.

At school level, in giving particular consideration to the social aspects of young children's number sense, the teacher will first need to discern the range and type of experiences that the children have had with number. These experiences will need careful analysis in terms of the purposes that children might discern from them. It has been claimed that most young children come to school using number words and prior to this use these

words '... *confidently, consistently and correctly.*' (Orton and Frobisher, 1996) However, my findings indicate that while four-year-old children might use number words and appear to recognise different contexts for their use (e.g., as with numbers on the clock face), for many young children their understanding is confined. They generally will not have an understanding of the functional use of the numbers in contexts such as on best-before labels or speed-signs in train stations. We have seen that many of them will have their own theories about the role of numbers in such contexts. Children will need to become involved in activities that test these theories. They will need help in discussing and comparing their ideas with those of other children. They will need to be encouraged also to suggest and evaluate plausible alternatives to those that are seen by them, as a result of investigation, to be invalid. For example, Owen's idea that the number at the train station is to count the trains would provide a very interesting environmentally based mathematical inquiry that children could undertake with the guidance of the teacher.

Providing young children with opportunities to construct and apply numerical understandings and skills in contexts drawn from their own experiences, interests and environments

Bowman and colleagues (2001: 117) identify early childhood education as, among other things, '... *a process of gradual transition from cultural and family patterns to the expectations of a new social context.*' They consider it critical that a child's background and experiences are understood and respected. Gifford (2004) too argues that practitioners need to become aware of the significance, diversity and complexity of home learning environments. As demonstrated in this study, teachers can appraise these from discussions with children. The challenge then for teachers is to help children make connections between these experiences and the purposes of learning about number in school, and in this way to extend their informally acquired number sense. Walkerdine (1988: 105) reminds us that most of the experiences that preschool children have with numbers are ones where usage of the number is different from that contained in school practices and from that expected in the discourse of early mathematics education. She states '*Their experience of, and understanding of number, is therefore not limited to those usages contained in school mathematics.*' The primary

school curriculum emphasises a very specialised and as yet unfamiliar (to young children) usage of number, one that emphasises the labelling of numbers as objects. Children's informal experiences have enabled them to construct a number sense incorporating a familiarity with number that is quite different to that presented in school mathematics. Walkerdine (1988: 119) suggests that what is required is for children then to '*suppress*' or '*forget*' these experiences or meanings in order to understand the abstract nature of number in school. Rather than requiring children to 'suppress' the meanings they have constructed from informal learning experiences, I advise that teachers should foreground these experiences and meanings and use them as a basis from which to nurture children's sense making in the domain of number. The fact that children talk about particular experiences signifies their interest in these. Activities that children in my study spoke of, for example, writing invitations; measuring rooms for carpets; sowing plants in rows; 'reading' numbers on food packets; and using calendars to anticipate future events; all constitute contexts that could be used by practitioners both to access children's current understandings and interests and to extend their number sense. Such contexts are rich in potential for developing children's pleasure and interest in number, their quantitative thinking, their understandings of purpose, their awareness of numerals and for introducing more formal aspects of number as they arise. Using children's accounts of their experiences to infer their interests is one key to integrating informal and formal learning in the domain of number (See Base (1992) for an excellent exemplar of mathematical inquiry with young children).

Identifying children's numerical interests appears to be more problematic when relying on observations of children's play activity, than it is with experience-based, flexible and focused interviewing. According to some authors, their observations suggest that children rarely use numbers in independent play (See Gifford (2004) for a review of the evidence in this regard) and Cook (1996) offers a suggestion as to why this might be the case. Others such as Ginsburg and Golbeck (2004) cite naturalistic observations that, in their view, show that children exhibit a spontaneous interest in significant mathematical activity during play. Either way,

teachers need to have a variety of strategies for ascertaining children's learning. Walkerdine (1988) suggested that some children may not recognise mathematics in everyday activity. Therefore discussions of such activity provides the teacher with opportunities to introduce conventional mathematical meanings where appropriate. Such meanings will need to be made explicit in discussions with children. The teacher's role in such discussion is to stimulate thinking and mental activity leading to the construction of meaning. Various theorists suggest that thinking and mental activity can be stimulated by having children discuss and compare their different ideas about the purposes of number in specific contexts. In that view, shifts in children's perspectives take place as a result of dialogue between children, particularly if the teacher guides the dialogue (The Open University E386 Study Guide: 67-9). Through the extension of their understanding of the purposes of number in this way, children's number sense can be developed.

Thus providing children with opportunities to construct and apply numerical understandings and skills in contexts drawn from their own experiences, interests and environments will involve a number of steps. These include ascertaining children's experiences with number and from these and other available information discerning their interests. The teacher can then use specific experiences and interests to ascertain the understandings that children have constructed, and instigate discussion to enable the social construction of knowledge. From the information gleaned, it is then possible to plan future activities that will engage the children, extend their existing number sense and introduce ideas about number that they are not yet aware of.

My findings suggest that some children will need a lot of help with participation in such activities and discussions, i.e., in particular the boys and those girls (i.e., Mary) whose experiences may mitigate against their effective participation in the learning curriculum. The teacher's task will include enabling such children to move towards more intensive participation (Lave and Wenger, 1999b) in such activities.

Over two decades ago the Cockcroft report (1982) suggested that teachers should pay more attention to mathematics in real situations. Hughes (1986) argued the need to design situations within which to present mathematics which children find interesting and that makes sense to them. Echoing earlier ideas of Bruner, he talked about '... *the immense capacity of young children to grasp ideas if they are presented to them in ways that interest them and make sense to them.*' (p. 184)

My thesis is that we don't have to design such contexts, we just need to listen to children talk about their experiences and interests. From these, we can elicit the contexts within which we can work with young children to introduce them to those aspects of numeracy which are the specific concern of schools, and extend their number sense accordingly.

Developing newly emphasised number skills in young children (e.g., estimation)

The analysis of the curriculum documentation revealed the emphasis that was placed on children thinking and communicating quantitatively. One of the ways in which it was suggested that quantitative thinking could be developed was through the teaching of estimation skills from the earliest days at school onwards. There is also explicit mention in the guidelines (Government of Ireland, 1999c) of teaching children specific related strategies including sensible 'guessing' and the subitization of small numbers (See Chapter 4: 64-73). In relation to subitization of small numbers, it would appear from my findings that young children easily subitize small numbers in the range suggested, i.e., 1-5. There would appear to be limited value in insisting that children practise this skill if it is already well established. One of the interesting aspects of my findings is that they appear to support the idea that subitization is in fact very rapid counting (Fuson, 1988). The implication of my research is that children may benefit from opportunities to practise rapid counting on progressively larger sets. Indeed MacNamara (1996) found that for the children in her study (aged seven plus), subitizing/estimating did improve with such practise.

The suggestion that children should be encouraged to 'guess' the amount in a set has previously received some attention in the literature. For instance,

Aubrey (1997) found that her four-year-old children appeared to be confused when prompted to 'guess'. Some researchers have argued that it is the use of the term 'guess' in an estimation context which confuses the children since it somehow trivialises the process (Andrews, 1995). Most children in this study clearly indicated that, as they entered school, the dominant strategy for finding out *how many* was to count the items in the set, although most children had a variety of sophisticated ways of making use of counting and information derived from counting to quantify. They appeared not to be aware of a response that wasn't an exact quantification. This is not so surprising when we consider the attention that most adults ask children to pay to counting as *the* way to quantify. In early childhood, learning this skill is a very important and powerful indication of proficiency with the number system and one that is highly valued by adults as they interact with young children on a day to day basis. Any suggestion to children that they should abandon the '*absolute reliability*' of counting (Fuson and Hall, 1983) is I think, bound to be met by them with considerable confusion. Indeed, both the literature review (See Chapter 2) and my findings (See Chapter 4) indicate that estimation may not make sense to most young children for a number of reasons.

It follows from this that if estimation is to be introduced to young children, then ways will need to be found to introduce it in such a way that it makes human sense to them, and that they can differentiate it from counting/exact quantification.

In essence, there are some issues here for curriculum planners, teacher educators and practitioners. Sowder (1989) identifies two reasons why estimation might be in the mathematics curriculum. Firstly, she suggests that estimation is seen as helpful in developing conceptual structures for number, and learning to estimate is seen as a useful way of learning something about number size and other such structural characteristics. Secondly, she argues that estimation is useful in everyday life. It is clear from my finding related to children's understandings of the purpose of numbers that it is generally unlikely that within their everyday experiences, this purpose of estimation will be conveyed to children. It may well be possible to convey the utility of estimation to young children but it would

require some imaginative teaching. Sowder (1992) has suggested that estimation skills will need to be directly taught. However, in the early years, beginning with instruction is in Pound's (1999: 93) words ' ... *rarely a good starting point.* ' She argues that children's ideas, their experiences and their illustrative stories are far more effective. An important pedagogical task when teaching estimation is to derive situations where children see the need to estimate. Sowder (1992) has observed that these are easier to generate in the area of measure than in the area of quantity. For these reasons, an explicit emphasis on teaching the skills of estimation from the time children enter school may not make sense pedagogically. It may be more appropriate firstly to ensure that children have opportunities to develop and extend skills in relation to counting increasingly larger sets, and then to introduce them to the utility of estimates in such contexts. Discussion and practise of silent counting and related metacognitive processes would also appear to be important areas of activity for young children.

The research agenda arising from this study

Arising from this study, the following are issues for further research:

- Differences in the ways in which boys and girls participate in interactions/activities. For instance, research might investigate the following issues: how young boys/girls interact with adults in their participation in everyday activities related to number; how young boys/girls interact with teachers during their first year at school; whether patterns in interactional behaviour persist across time and across different settings;
- Young children's reactions to different types of estimation tasks;
- Appropriate ways of conveying the utility of estimating to children at four/five years of age.

The action agenda arising from this study

Arising from this study, a reforming of teacher guidelines to include advice to teachers on

- The importance of recognising and building on children's informal learning related to number;

- How to move from home knowledge and informally acquired number sense to school based number sense and number knowledge;
- Key dimensions of number sense in the young child;
- Ways of developing children's number sense, taking cognisance of affective and social issues.

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Appendix 1: Interviews

Appendix 1a: Interview 1

Ice breakers (depending on the child): *Tell me some things that are good about school; Tell me about your favourite thing at school.*

I usually used the following comments/statements to introduce the topic to children: *'I heard that children starting school sometimes know how to count/know about numbers... is that true?'*

I used the following questions to guide the initial interview

- Do you know how to count? Will you show me how you count? How did you learn that? Tell me about learning to count. Did anyone help you?
- Do you know anyone else who can count? Does your Mam/ Dad/ brother/sister/friend know how to count? How do you think they learned how to do that?
- So you know lots of numbers to count with. Tell me anything else you know about numbers? How did you learn this?
- Does your Mam/Dad/brother/sister/friend know anything else about numbers? What do they know? How do you know they know about...? How do you think they learned/know that?
- What are numbers for? What do people use them for? Are numbers useful?
- Is it important to learn about numbers? Is it important for (e.g., Mam, children, you) to learn about numbers? Why?
- You know lots of numbers. How did you learn all about numbers? Did anyone help you? Tell me about it.

Prompts used: *'Think hard now...', 'Think about this for a minute...', 'Are you sure?'*

Appendix 1 b: Interview 2

The following tasks were presented in the order outlined below and provided the structure for the second interview.

Introductory activity

Children were presented with some magnetic numerals, jumbled together on a magnetic board, and encouraged to tidy them up so that they could be seen properly. I then asked them to tell me about any previous experience with these objects.

Task 1 Naming numerals

Children were asked the following questions about the numerals:

Do you know the names of any of these? Tell me the ones you know. The purpose of this activity was to explore children's ability to name numerals (1-10).

Task 2 Using numerals to convey information

Children were asked the following questions and, where appropriate, encouraged to use the magnetic numerals to show their responses:

What age are you? Can you show me your age using one of these numbers? What number is your house? Can you show me that using one of these numbers? What else could people use numbers for? Where else do people put numbers?

The purpose of the activity was to explore children's familiarity with number symbols.

The magnetic numerals are now removed and Coco the Monkey is introduced.

Task 3 Relative size of numbers

Children were asked if they could suggest any number that was bigger than a given number or smaller than a given number.

The purpose of this activity was to explore children's understanding of relative size of numbers.

Task 4 How Many? Counting objects within 10

Do you remember earlier you showed me how you could count? Let's pretend it's Coco's birthday. Could you help Coco count the birthday candles for his cake?

One array at a time, the children were shown a number of arrays of candles that Coco had arranged in linear form, and asked to count them to see if that is what should go on the cake. A small cake (made from playdough) was then placed on the table and children were encouraged to arrange the candles on it around the edge (i.e., in a circle). They were then asked again if there were four candles on the cake.

The purpose of this task was to see the extent of children's understandings of counting and of cardinality.

Task 5 How many now? (Objects present)

children were asked to count out an array of birthday presents (e.g., 4) and say how many. They then were asked to remove one and say how many now. They were also asked to add one to another array (e.g., 5) that they had counted and state how many now.

The purpose was to ascertain if children needed to count again each time.

Task 6 The Hiding Game

Children were shown Coco's presents (5) and invited to play a hiding game with these. They first counted the items to confirm that there were five. Children closed their eyes while I hid some under a large tub and left some visible. They then figured out how many were under the cup and were asked to explain their answer. He/She then checked to see if they was correct. (All six combinations discussed but task was discontinued after three successive incorrect responses).

The purpose of this task was to explore children ability to demonstrate understanding of part- part-whole relationships in this type of situation.

Task 7 How many dots?

Children were shown a dice with dot patterns (1-6). This was one of Coco's presents. They were questioned on what he might use it for and then we

discussed its role in board games. We examined the dice and children were encouraged to say how many dots on each face. Each face was displayed for only about three seconds to avoid giving the opportunity for children to count. The prompt 'Look quickly. You'll need to be quick here!' was used in this instance to prompt the children to subitize.

The purpose of this task was to explore the extent of children's ability to subitize.

Task 8 One more than /One less than (No objects present)
Children were presented with the following scenario questioned as follows:
Coco had four presents and he got one more. How many now? (Set sizes of 3, 7, 6, 9 displayed in sequence) Coco had 7 presents and he lost 1. How many now? (Set sizes of 3,5,6,8 displayed in sequence)

The purpose of this task was to explore children's ability to add and subtract one or two items.

Task 9 How many pieces?
Food for the fruit party! Children were presented with a succession of small plates of 'food' items for the party (8 lemons, 5 oranges, 6 apples, 7 bananas, 9 strawberries) using 'Fruity Fun' counters. Each plate was displayed for only about three seconds to avoid giving the opportunity for children to count. The prompt 'Look quickly. You'll need to be quick here!' was used in this instance to prompt the children to estimate.

Children were shown collections of various sizes from 2 to 10. Note was taken of whether they appear to estimate or attempted to count the items.

The purpose of this activity was to explore children's ability to estimate quantity without resorting to overt incremental counting.

Task 10 Matching numerals to concrete representations of set (using magnetic numerals)
Children were shown a set of birthday cards without numerals but with the candles on each cake (1-10) and the following directions were given:
Look these are some birthday cards and I would like to pick one for Coco. But look, there are only candles on them but no numbers. Will you put the

number on each card? Children were asked to match the appropriate numeral to the various arrangements of candles on pictures of birthday cakes (linear arrangement of candles).

The purpose of this activity was to see if children matched numerals to sets.

Appendix 2: Access and ethics

Appendix 2a: Issues related to access, relationships and ethics

Securing access

My research was carried out in the small town in which I taught as a primary teacher for nineteen years. This town has about eight schools that serve the population of children in a large dormitory town (population c. 30,000) near the capital city of Dublin. I used two of these schools as settings for my research: the first a large all-boys school catering for about 700 children (where I once taught) and the other a large all-girls school catering for about 750 children. The age range of children attending these schools was from 4 to 13 years. As in all primary schools in Ireland, children entering school start in Junior Infants and progress through the school year-by-year with their age group until they reach Sixth Class. The children attending these two schools come from a variety of socio-economic backgrounds. In general the children in these schools are well supported in their learning by their families, many of whom become involved in their child's school at various levels. Recent government initiatives in relation to learning support for children with special needs have resulted in a substantial increase in ancillary personnel working with teachers and consequently most of the classes in these schools have classroom assistants working alongside the teachers. In recent years, because of migration of people from Europe and Africa, there has been a large increase in the number of children from different ethnic backgrounds attending the school.

I chose these schools because I was confident of the co-operation of the school community since I used to teach in the area. My first step in gaining access in both schools was to prepare a letter for parents/guardians informing them about the nature of my study and seeking their permission to interview the children (See Appendix 2b). With this in hand, I met with the Principals of the schools on separate occasions and explained my research to both of them. They secured the co-operation of the school authorities. They also secured the co-operation of the teachers involved and I subsequently met each of the teachers and explained my research to them. I felt that because of the necessity to spend a considerable amount of time in

each of their classroooms, it was essential that the co-operating teachers were put at ease in relation to my focus and interest. As an introduction to my work, I gave each of them a copy of the conference paper that I had prepared in relation to the pilot study that I had carried out the previous year. The paper outlined and discussed the findings from the pilot study and my intention was that reading it would serve to reassure the participating teachers that it was the children's number sense that interested me and not their teaching practices. Both teachers indicated that they were happy to co-operate with the research. The principal teachers wrote an introductory letter that was sent with my letter to the parent/s of each of the children selected to participate in the study (See Appendix 2c). These letters were accompanied by a request for consent from the parents in relation to the child's participation. The teachers distributed these to the parent/s of the children selected to participate in the study and they also collected the signed consent forms.

Ethical considerations

Coady (2001) argues that informed consent is the key to ethical research. With this in mind, consent was sought after the school authorities and parents had been fully briefed about the nature and purpose of the study, the ways in which the data was to be collected and ways in which the findings would be used. Following the advice of MacNaughton and Rolfe (2001), the preparation of an appropriate plain language statement about the research and each persons role in it, was addressed early in the planning stage (See Appendix 2b). It has been suggested that the issue of informed consent, i.e., the process whereby someone voluntarily agrees to be part of a research project, based on full disclosure of pertinent information, may be problematic as it applies to children (Morrow and Richards, 1996). I sought *permission* and *assent.* This involved a parallel process of parent or guardian agreeing that a child may participate and the child assenting to be a subject in the research. However, as can be seen from the dialogical nature of the interviews, children in this study were very much active participants. Being open and honest with all participants from the beginning was an important principle and the necessity for me to make and keep certain records, including field notes and audio-recordings was explained to the

participants. I sought written consent from parents to interview and audio-tape each of the children.

The professionals involved will be offered feedback on my analysis when it is complete, perhaps again in the form of a copy of any papers that I prepare arising from the research. Since the parents/guardians had been told that the focus of the study was children's perceptions of number and of learning about number, and did not pertain to individual children as such, I didn't consider it necessary to offer them feedback.

The research settings

The first school

In order for the children in the boys' school to become familiar with me, and for me to establish rapport with them, I visited the classroom on their first and second day at school and stayed for the duration of the school day (three hours each day). I introduced myself by my first name in order to differentiate myself from their teacher and other authority figures. I told them that I was interested in talking to them about school and about some of the things that children had learned before they came to school. I set about establishing relationships with the various children. I talked with them while they were playing, eating their lunches and doing various other activities that their teacher had organised for them during this period of transition to school. I also read the children a story each day. I made a particular point of being present as the parents collected the children from the classroom each day. I thought that it was important that parents were aware of my presence in the classroom and that this awareness would reassure parents should children mention me at home when talking about school. In particular, I felt that it was important that when the letters related to the research went home parents recognised the sender as a presence in their children's lives rather than a complete stranger. I felt that this would increase the likelihood that parent/s would give consent for their child to participate in the research.

As a result of my interactions with the children I selected, in consultation with the teacher, a purposive sample of children (ten) whom I felt would talk to me in a one-to-one situation out of the classroom context. An important consideration was that there was the likelihood of mutual

understanding in language terms, i.e., that the children selected understood me and that I understood them. The parents of these children were given details of my study (See Appendix 2b). I also explained to parents that not all the children for whom I had consent would be interviewed. The principal sent an accompanying letter that indicated to parents that they were free to withhold consent if they had any reservations about their child's involvement (See Appendix 2c). Permission was obtained in all instances and I interviewed eight children in total (age-range 4 years 3 months to 5 years 1 month) in that particular school.

I returned to the children's classroom at the beginning of the second week of term and began the interviews later that morning. A pattern of work was established on that day and worked well for the remainder of the week. Each day, I mingled with the children for a while, I did some individual interviews away from the classroom and towards the end of the morning I returned to the classroom to read a story for the children before they went home. Each morning I arrived with the children and chatted to them as they settled in. Usually the morning began with play activities. As the children played, I focused on a small group in which one of the children that I intended to interview was playing. As I chatted with the children, I watched for an opportunity to raise my dilemma with the child in question, i.e., that of needing help with my work. I invited the child to come a quieter place where we could talk and they could help me. Usually the child was agreeable. One child declined *'No, thank you'* he said, while another declined my invitation *'Not today'* was his reply.

The pre-fabricated classroom where the children spent their days was a considerable distance from the small room that I had been allocated to carry out my work. The journey to the interview room took us across the yard where a digger was moving mounds of topsoil to relay a small lawn, through the school general-purpose room (generally referred to as the hall) and then along the corridor into the main building. This proved to be a really useful opportunity to further establish rapport with the child outside of the classroom context. Often the opportunity to stop and watch the digger for a couple of minutes was the impetus for discussion between the child and myself. The vastness of the all-purpose hall and the children's admiration of

the basketball nets anchored at various points on the wall provoked some discussion and questions in the case of several children. For some, being in the main school building provoked stories and comments about other occasions in which they had been there, and about brothers and friends in various classes in the school.

The room in which I interviewed the children was a small one with a dual usage. It served as a shop first thing each morning and after the breaks, and children could buy school items such as copybooks and pencils from the school secretary. For part of the day the resource teacher also used this room. These uses were reflected in the contents of the room. There was a desk and a few assorted chairs in the room, there were also two computers, neither plugged in. Two of the walls were lined with shelves and they held the various items of school stationery that children could buy. There was a notice board directly behind the desk and there was a calendar on this and there was also a large clock on the wall. There was another calendar (not turned to the correct month) on the wall just inside the door. A glass panel in the door permitted a view, at adult level, in or out of the room. I invited each child to choose a chair on which to sit. It is important to note that these 'beginning' children had no experience of this particular area of the school.

Conditions during the first morning of interviewing were not ideal. During Jerry's interview (just after the morning break), there were eight interruptions. I perceived such interruptions as a potential threat to securing the type of conditions that I felt I needed to obtain authentic data. I discussed the situation with the principal and the difficulty was resolved. What was impossible to control were the various noises around the school, including vacuuming, talking in the corridors, and the delivery of art supplies to an area just outside the door of the room. These events were of course of great interest to the children who wanted to know the details related to who, what, why and what for. Discussions related to these became part of the text of the interview and so part of the context. I found that such was their interest in the interview and in helping me that it was relatively easy to attend to the children's questions about noises and so on, and still bring them back to the issues that I was interested in. As I had found during the pilot study, the environment of the room was also of great interest to the

children and some of them questioned me a lot about it. Indeed sometimes they appeared to use the environment to frame their replies to some of my questions, thereby lending support to the idea of the interview as a highly contextualised event (Scheurich 1995: 250). For example, Con asked me whose picture was featured on the calendar on the wall and then informed me that he knew all the numbers on the calendar.

The second school

In the all-girls school that I chose for my research I followed similar procedures to those that I used in the first setting. I used the same process to select the children to be interviewed, and again, I sent letters to the parents of ten children. In this instance one parent declined to allow her child to participate and one parent indicated verbal approval but did not complete the consent form. Of the eight children for whom I had written consent to interview and audio-tape, I interviewed seven initially (age-range 4 years 1 month to 5 years). However, one child was ill and could not do the second interview. This gave me six complete sets of data from this school.

The children's classroom was on the ground floor of the school and the room that I was using for the interviews was a small storage room off the Computer Room on the floor above the classroom. As in the other research setting, the journey to the interview location proved to be a very useful context to further establish rapport on an individual basis with the children. We walked to the landing at the turn in the stairs and there we paused to look out at the playground below. Sometimes we saw some children outside playing games. Sometimes there were parents or other adults crossing the yard and we chatted about what we saw, sometimes speculating about their presence there. When we reached the Computer Room, I explained the purpose of the room to the child, pointing out the computers and the pictures on the wall. Sometimes the children asked questions or made comments. I always encouraged them to step up on the chair that I had positioned at the window on the opposite wall. This window had a different perspective than the one on the landing and looked out at the side of the church in the adjoining yard. Usually the children had some comment to make, either about the church or about the Main Street, which was visible in the distance.

After this I usually invited the children to come into the small room situated off this larger one to talk.

The room in which I interviewed the children was lined on one long wall with shelves on which various resources were stored. These were mainly books and sheets of music. There were also one or two pieces of spare furniture stacked against the window. There was a small table that I had pushed against the wall and I placed the tape-recorder and my notebook on this. There were also two different chairs and a stool. I invited each child to choose which of these she wished to sit on.

In this instance, the environment was quite a stark one and held very little of interest to the children, though some of them were curious about its purpose, since they were unfamiliar with this area of the school.

The environment was very conducive to interviewing. There were no interruptions and the room was relatively soundproof, although we did occasionally hear the church bells in the distance and at least one child (Maura) commented on them.

Establishing and maintaining rapport with the children

I was conscious from the outset of the need to establish conditions under which children would talk freely and openly and in a relaxed manner about the issues that I wanted to raise with them. Because of my experience as a teacher, I had detailed insights into the structure and content of the children's daily lives during their transition to school. This was of enormous benefit to me in establishing rapport with them. However, I agree with Gollop's (2000: 22) view that '... *the most important factor in an interview's 'success' is the establishment and maintenance of rapport with the child.*' As outlined above, I took a number of steps to ensure that rapport was established and maintained with the children who participated. I felt that it was important to establish rapport with the children at both a collective level and an individual level. To achieve this, I worked with the group of children (e.g., by reading and discussing stories with them), and with individual children (e.g., when chatting in the classroom and also as we walked to the interview location). I gave considerable thought to the issue of ensuring that

the children were well-informed about the research and that they understood that they could choose whether or not to participate.

Firstly, I asked each child prior to leaving the classroom with me if they would agree to help me with my work by talking with me. When we reached the interview location I checked again in relation to children's understanding of the situation, usually by asking them if they knew why I had asked them to talk to me. Since I hadn't made any recommendation to parents as to whether or not they should discuss my consent request with their child, I was conscious that some children might only just have become aware that I wished to talk to them. I wanted to ensure that the children being interviewed understood why they were with me in a different area of the school, and not in their classroom as was their experience of school to date. In most cases they said something to the effect that they understood that I needed some help and that they were in a position to give it to me. Where children seemed to be unsure about my purpose or were unwilling to express their thoughts, I repeated once again the fact that I was interested in finding out about children and learning. I asked for assistance from them since they would know about this. My approach was something akin to that used by Piaget when he treated children as experts in particular issues that he was interested in.

To increase the transparency of the process for children, I showed them the tape-recorder and explained its role. I explained how it would be very difficult for me to remember all the things we had talked about and how taping their voices would enable me to have a record to listen to later. I explicitly asked for their consent to do this. I then suggested that they say something while recording and I played this portion of the recording for them. With some children I did this a number of times. I felt that it was important that their initial curiosity about the tape was satisfied and I reasoned that they might be less interested in its presence once our discussions began. Some researchers adopt the procedure of playing the taped interview for children at the end (e.g., Doverberg and Pramling, 1993). I found that the children sometimes asked, during the course of the discussions, to hear themselves on tape. Where this happened, we listened to the most recent portion. A number of children listened to themselves a few

times and then we resumed the discussion. I found it more democratic to accede to children's request when it was made rather than ask them to wait till the end of the interview. This was also an important concession in relation to ceding some control of the interview to the child (Tammivara and Enright, 1986).

In fact the process of children listening to themselves on tape became part of some children's efforts to control aspects of the interview situation.

Interestingly, the request to listen to the tape was something that was confined to the discussion-based interview. It didn't arise in relation to the task-based interview and I speculate that the structured character of that second interview was such that children were very absorbed with the tasks and materials in hand. Perhaps also, the novelty aspect of the taping situation had receded by the time the second interview took place.

Appendix 2b: Letter to parents/guardians and consent form

In the letter below the names of the teachers/principals have been substituted by pseudonyms. A similar letter to the one here was sent to the boys' parents.

St. Patrick's College of Education,
Drumcondra,
Dublin 9.
September 8th 2003
Phone 8842057
Email elizabeth.dunphy@spd.dcu.ie

Dear

Now that your daughter has started school I am sure it will prove to be an exciting and wonderful experience for her and also for you as you watch her develop and learn in her new environment. I am very interested in talking to her about her ideas about learning. This interest arises out of my work as a lecturer in early childhood education. Currently, I am researching young children's ideas about mathematically-related learning and one of the ways in which I propose to do this is through informal one-to-one discussions with children.

I would like your permission to speak with your daughter on one or perhaps two occasions during the next few weeks about mathematically related situations and issues. It may not be necessary to talk to every girl in the class. The decision about which children to speak to will be made on a day to day basis in consultation with [Ms Y] and will depend on a number of factors, the most important of which will be children's willingness to participate. Individual children will be asked if they would like to have a chat with me, and they will be free to decline if they so wish. It will be necessary for me to audio-record all discussions with the children I talk to for later analysis. The discussions will take place during the normal school day and have been designed to appear very similar to the kinds of activities that children partake in during the first weeks of school. These discussions

and the information that I derive from them will provide the basis for my study. The findings will be shared with my colleagues in the early childhood education community.

As a former teacher I am very aware of the sensitivity of young children and in particular of the significance to them of everything that happens during these first impressionable weeks. Consequently you can be assured that as I meet with your daughter, both in the classroom and individually, an important priority for me will be ensuring her comfort and ease in talking with me.

I am grateful to the principal [Ms X], to the Board of Management and to [Ms Y] for their permission to contact you and for their enthusiastic support for this study.

I would appreciate if you would sign the **consent form** below and return it to [Ms Y] **as soon as possible**. Thank you for your co-operation. If you have any questions related to this request I will be glad to answer them.

Yours sincerely,

_____

Liz Dunphy
Lecturer in Education (Early Childhood Education)

Consent Form
September 2003

Liz Dunphy has my permission to have informal discussions with my daughter _____ about topics related to mathematical learning in young children. She also has permission to audiotape these discussions and to use the audiotapes in analysis and explanation of the findings.

Signed _____

Appendix 2c: Text of letter from principal teachers to parents

Dear Parents,

I am writing about the research being carried out by Liz Dunphy in Junior Infants.

Liz worked in [name of school] for many years and is now lecturing in St. Patrick's Teacher Training College, specialising in early childhood education. She is a consummate professional and I am only too delighted to facilitate her research. However, if you have any reservations whatsoever, please feel free to act accordingly.

Many thanks,

[Ms X]

Appendix 3: Content analysis of primary school curriculum

Content Analysis of Primary School Curriculum in Relation to aspects of Number Sense

Document	Sections	Unit of analysis
Primary School Curriculum: Introduction (Document A)	Defining features of the curriculum (pp. 10-11) Early childhood education (pp. 30-31) Mathematics (pp. 47-48) Conclusion (pp. 74-75)	Sentences Sentences Sentences Sentences
Primary School Curriculum: Mathematics Curriculum (Document B)	**Infant Classes:** Skill development Content Assessment (pp.114-121)	Objectives Objectives Sentences
Primary School Curriculum: Mathematics: Teacher Guidelines (Document C)	Mathematics in the primary school curriculum (pp. 2-4) Mathematical language and methodology (p. 19) Approaches and Methodologies (pp. 30-31) Estimation strategies for number (p. 32) Early Mathematical Activities (pp. 40-41) Appendix: Overview of skill development (p. 68) Appendix: Symbols, numerals, fractions and terminology (p. 70)	Sentences Sentences Sentences Sentences Sentences Sentences Sentences

Appendix 4: Sample of data

Sample of data set derived from the interviews

Interview 1: Tom
Date: 9th Sep 03 **Age 4 years 11 months**

L	*Can you count?*
T	*Yes*
L	*Let's hear you*
T	*1,2,3,4,5, ...*
L	*Where did you learn to count like that?*
T	*I went to a ... when I was four ... when I was four ... when I was four ... I went to a different school*
L	*Did you?*
T	*And I learned my numbers there and my letters*
L	*What numbers did you learn?*
T	*Em ... well you know the ones that I just counted there*
L	*Oh yes! You learned all those numbers did you.*
T	*Now I'm on five*
L	*Tell me is it important to learn your numbers?* [He nods]
L	*Why? Why is it important to learn your numbers?*
T	*That's why you need to get something in your head?*
L	*Something in your head ... Why?*
T	*That's why you need the important things*
L	*Right ... it's an important thing ... why are numbers important?*
T	*That's why you need something*
L	*Do you know anyone else that can count?* [He nods]
L	*Who?*
T	*Some boys when I was in ... eh playschool ... and girls ... I done something and they all know their numbers*
L	*How do you know they know their numbers?*
T	*I saw them doing them*
L	*You saw them doing their numbers and they all know their numbers...and what about say ... your Mam ... does she know about numbers?*

T *My Mam ... I know these numbers* [He picks up a 1-20 pegboard line lying on a shelf near him]

L *Do you know those numbers?*

T *What are these ... what's it for?*

L *It's for drawing lines ... I think, and counting. It's for putting pegs in ... you put pegs into the numbers.*

T *What's pegs?*

L *Those little plastic things over there ... . do you see those? You put those into the number. See...*[I demonstrate with one peg]

T *Can I do it?*

L *Well we won't do it today ... Or later on we might do it ... leave it there for the moment. Can I ask you a question? Do you know anyone else that can count?* [He nods]

L *Who else can count?* [Silence]

L: *Have you any friends who can count? Or brothers or sisters?*

T *I have a baby*

L *And what's the baby's name*

T *Em ... I don't really know*

L *Your own baby and you don't really know the name...*

T *I don't have my own baby*

L *Oh it's not your own baby?*

T *No ... Somebody else's*

L *Oh ... somebody else's baby in your house*

T *Yes*

L *Is your Mammy minding the baby is that it? Now I have it. Mammy is minding the baby for someone else*

T *Yes*

L *And have you any brothers and sisters?*

T *Yes ... . Alan is one of my brothers ... and Cian I'm a little bit taller than him*

L *Are you?*

T *Em ... Emma*

L *Emma, yes*

T	*Katie*
L	*Yes ... Emma, Katie, Cian*
T	*... and Shane my little brother*
L	*Right ... you have quite a few brothers and sisters, haven't you? And Shane your little brother, now can he count?*
T	*No, he's a really little baby. He's five months.*
L	*Ah ... he's only five months? Will he learn his numbers?*
T	*Yes I think so*
L	*And how will he learn numbers?*
T	*He's a little bit bigger than me now*
L	*Is he? ...*
T	*He's just up to there* [He points at his shoulder]
L	*Right he's beginning to grow now, is he?*

Interview 2: Tom

Date and Time: 10 Sept 03

Introductory activity

Children were presented with some magnetic numerals, jumbled together on a magnetic board, and encouraged to tidy them up so that they could be seen properly. I then asked them to tell me about any previous experience with these objects.

L:	*Did you ever play with these before? Tell me about a time when you played with these? What are they for do you think?I want to show you all these shapes here ... do you know about these shapes?* [He nodded] *What are they?*
T:	*They're numbers ... that's not a number there* [He pointed to the plus sign that is in the set]
L:	*No that's not a number ... take that one out ... you're absolutely right ... what is that anyway?*
T:	*That's an X ... a reading X*
L:	*Did you ever play with these before*
T:	*Yes ... I have them at home*

L: *Do you play with them? And who plays with you ... when you're playing with these*

T: *Em ... my friend that lives beside me*

L: *And what games do you play with them*

T: *Buckaroo and ...*

L: *Do you play a number game*

T: *Family bumping Game and ... em ... Cowboy Reading game*

L: *Very good ... and you play a number game?*

T: *Yes*

L: *Very good ... and what number game do you play ... what do you do with the numbers with your friend?*

T: *Em ... I count them out*

L: *And what does he say ... and what do you say?*

T: *I say ... em ... do you know what I do with Buckaroo?*

L: *What ... ?*

T: *You try and put everything on Buckaroo and then BUCK*

L: *That's right . I used to play that game*

L: *And what kind of game can you play with these numbers?*

T: *You can just go like this ... eight, three, nine, six ... He selects each numeral and says its name*

Appendix 5: Sample of marked transcript

D

Question 5: What are numbers for? What do people use them for? Are numbers useful?

Bob	Em…because…we have a calendar in the kitchen And are there numbers on that calendar? *He nods* And what do you use the calendar for? What's it for? To tell which time it is? To tell which time it is…it tells you the date, doesn't it …and the day. Okay And do you ever look at the calendar? *He nods* When did you look at the calendar…tell me about it Always You always look at it.. and what does it tell you? *Eh…I don't know*…and we've got a clock in the kitchen with no numbers A clock with no numbers…so how can you tell the time? Mam knows the time How does she know if there's no numbers on it? Because…she just knows How does she know? Because there's fake numbers There's fake numbers…but you can't see them? And magic eye could see them… *Also* And when you're out playing do you ever see any numbers anywhere? Any numbers around the place? Only at TESCO Numbers at TESCO…where do you see numbers at TESCO? On the roof Yes…go on. Tell me about the numbers on TESCO's roof *Silence* Oh it says the name of the shop? *He nods* Right…and any other numbers in TESCO No…only those two numbers Only those two numbers…Right There's one outside on the roof and one inside at TESCO's One outside on the roof and one inside in the shop… right…yes…I know what you mean now Are numbers useful? 1,2,5,8,9…are they useful? Because they just are *Also* I wonder…let's think hard about that…why do you need to learn about numbers …what could we do with the numbers. I'm thinking really hard about that …are you?

Why do boys need to know about numbers?
They need to go to school…to learn numbers
[redacted]s?
[redacted]
And if they know their numbers what does that help you to do?
It just helps you
In what way?
It just does
It just does…but do numbers help Mammy
He nods
How?
They just do
When she's at home are the numbers helpful for her
They cure your germs
How do they cure your germs?
They make noise that is loud
They make noise that is loud…and the germs hear it…and they're dead?
Yes
Does Mammy do that?
Yes
When did she do it?
In Australia
Yes…What germs did you have?
The purple germs
Yes…
They came at the ??? once …I see them on the TV
Tell me about the germs again…and the numbers…how did you know you had the germs?
Because…
Yes
You drink some of the ??? drinks and then they kill your germs
Yes
If you drink these drinks they kill your germs
He nods in agreement
And what about the counting…how does that help?
It just does
Does it…so when you're taking the drink does Mammy say 'We'll count. …1,2,3…Drink it up'
He nods in agreement
That's very interesting
And did Aoran have these germs?
He nods in agreement
And what did Mammy say to him?
No…everybody has them
Oh we all have those germs do we…and Mammy says…what does she say?
Nothing

Shay *RE learning to count in French*

Yes…they…we had to
You had to? Why?
Because we were going on our holidays to France
Right…so you learned some French to go on your holidays
So we had to

Also
…is it useful to know all the numbers
Yes
Why?
Because you can tell other people
Right… that would be very useful wouldn't it?
Any other use…could you use numbers for anything else?
Yes
What?
For people's ages
For people's ages…yes…very useful for telling what people's ages are.
When I asked you about Finbarr's age you were able to tell me!
Yes
Can we think of any other use?
You can …eh…count…eh…how many rainbows (maybe referring to the picture of a rainbow on the calendar hanging on the wall).
Like…there can be lots on one day but I never saw lots

Also
Your Mammy…are numbers useful for her?
Yes
What does she do that's useful…why are numbers useful for her?
Because she has to work…in her office
And are numbers useful there for her, when she's in her office working?
I don't know
Did you ever see her working with numbers?
She has lots of papers
Right…go on tell me about them
And I can't read
You can't read what she writes, is that right?
Yes, I can't read yet

Also
Because …eh…from the clock…clocks are very useful…I have a clock on the oven
And is that useful? What's it useful for?
Because my old clock is broken
Is it? So do you use the clock on the oven to tell you the time?
Well it's not broken…but we need more room for pictures…
So you took it down…
And we put it in my Mam's office
Right…so now you have to look at the clock on the oven to tell the time?
Well…I don't know…eh…if we need more room

	Well can I ask you can you read the clock on the oven? No Can your brothers read it? Do they tell you what time it is? No…I don't have to No you don't have to at all I don't have to know But it's useful Yes… It's useful to know the time isn't it?
Terence	…why do boys have to learn the numbers? **Because your Mammy tells you** And are numbers useful…what would you use numbers for? Silence When you know all your numbers what will you be able to do? Play with Power Rangers You'll be able to play with Powere Rangers when you know all your numbers…why? Because I like power rangers And what will the numbers help you to do? **Count** To count…Okay…anything else? *Silence* Do they help Daddy…numbers? Yes How do they help Daddy? *I don't know* Think about it for a minute, think really hard…I'd like to know that…how do numbers help Daddy? **When he goes to work** Yes He's not at home He's not at home…he goes to work He's gone to work now And does he use his numbers at work? Yes What does he do at work? **I don't know** And how do you know he uses his numbers Because I do And what about Mammy…does she use her numbers Yes…she used to go to work Did she…and why did she need to know numbers at work? She doesn't any more She doesn't any more…but before the baby came…why did she need to know the numbers? I don't know
Tom	What are numbers for? What does Daddy do with numbers? He reads them, and he learns them and …em…he goes to school

…and…he works in Palmerstown
And does he use numbers at his work?
D1 I don't think so…I have never been to his work
Have you not? But you have seen him reading numbers, have you?
He nods
Where does he read numbers?
In my house
Where abouts in the house?
Silence
On a newspaper?
Yes
And where else?
I don't know

Also
Does Mammy need to know numbers?
What would she use numbers for?
What would she do with them?
B1 I don't know
Think about it really hard …what does Mammy do with numbers?
She likes numbers
How do you know she likes numbers?
I don't know…

Also
Do you need to learn numbers at school?
He nods
Why?
Because letters and writing and …
C1 You need to learn all that stuff…but why do you need to learn the numbers
Why do we not do homework at school?
I don't know…you're not doing it yet are you not?
No
But why do you need to learn the numbers at school?
I don't do them here I do them in a different school

Also
Have you…can I ask you something …why do children learn these numbers?
Learning gs.
I don't know
Is there any reason for children to learn numbers…what will they do with all the numbers they learn?
They put them on the mat and they do something with them
They put them on the mat…what does your brother do with all the numbers he knows
My brother doesn't know any numbers yet
Your big brother…what does he do with all the numbers he knows?
D1 He counts them like me
Does he…

	And does he count very high? Yes And what does Mammy do with the numbers She puts them in a (inaudible) Does she do anything else with her numbers? Yes…She puts them down on the kitchen…**she reads then in the car…** **she reads something with them on** She reads some numbers does she? Yes Like what… Like…em…. Tell me more about that…Mammy reading numbers in the car. What numbers does she read in the car?…Now I want you to think really hard about this….Mammy in the car reading numbers…What's she reading? **She reads something important…but I don't know** Go on …tell me more about it She likes reading and…she always likes reading. *Also* And what about Dad? He likes doing reading with numbers Does he…did you see him No And how do you know he likes doing reading with the numbers? I saw him a little bit doing them And did he say anything to you…what he was doing? No
Jamie	QNP
Con	…why do you need to know about numbers… Eh…I like to Right…why do like to? It's fun Numbers are fun… Yes What's the most fun thing about them? When you do what? **I like to turn around the clock** Right…so you take the clock and you turn the hands around, is that what you do? I like that part… You like that… that's very good …and what other fun things can you do with numbers Nothing else *Also* Is it useful to know about numbers? What are they for? What? What are numbers for? **Counting**

	Right…very good for counting…are they for anything else…think hard now…would numbers be for anything else? *Silence* I'm just wondering…what does Daddy do with numbers? What does he use them for? *Also* …did daddy ever tell you about numbers? **Yes…10…10 pence** Did you ever see Dad counting his money I have to get loads and loads and loads of money Right…what for ? What do you need loads of money for? To buy loads of things Does money buy you lots of things? Yes…my Daddy is going to buy me something from this..I'm going to put it in my pocket What could you buy from that? Eh…a dumper truck…or how about a gravel truck
Owen	But why do we need numbers/ **Because to write on your copies** Oh…you need the numbers to write on your copy Yes Right…do you need numbers for any other reason Yes For what To do on pages Also Why do boys …have to learn about numbers? **They just do** What are they for then? Because then they would know their numbers **And why do they need to know their numbers** Because they would know them But why they need to know them? **Because they have to and then do work** What use are they? **I don't know** When you learn your numbers what will you be able to do? I will know I'm not sure… I wonder what does Daddy do with his numbers? **I think he writes them at his office** Does he ever write numbers? Yes When…do you ever see him writing numbers? Yes…tomorrow Think really hard about this…why do the boys in Mrs C's class need to learn the numbers Because they have to…Do you not know about the numbers?

	I don't know much about the numbers Do you know letters? I know a little bit about letters…what I'm trying to figure out is why do people have to learn about the numbers…is it useful? Yes In what way? Like there He points to the counter on the tape recorder Are they useful for anything else? **Yes…I don't know**
Jerry	Excellent…You are very good at reading all those numbers, aren't you? Can I ask you something? What are they for, those numbers? What are numbers for? Silence Think about that for a moment now. Think hard for a minute? What are numbers for? **Ages** *Also* There's number one. (Pointing to my notes) That's right. I wrote that there. Is it important to write numbers? **I don't know if it is** Why do people write numbers? Silence Why do adults write numbers, and sometimes children? **Because…that's the way they do?** Why though? He shakes his head You don't know why? Shakes head *Also* Has he. Yes that's the school pens Can I ask you about Daddy's computer again? Money comes up on the screen does it? *He nods* Well then it's very useful for Daddy to know about numbers, isn't it? Can I ask you something else? *He nods* Is it useful for Mammy to know about numbers? Well… she looks at the computer too… Does she? She looks at the computer and all the numbers come up on it? And does she do anything else with her numbers? Well James looks at his numbers on his computer
Maura	*RE the clock in the kitchen* Can you read it…what does the clock tell you? **Em…to go to sleep…and to go and get my brother Jack** So how do you know when it's time to pickup Jack?

Appendix 6: Sample of tables from data

Task 3 Relative Size of Number

Name	Bigger than 3	7	2	8	10	Smaller than 4	8	3	5	12
Bob										
Shay	*	*	*	*	*	*	*	*	*	*
Terence	*	*	*	*	*	*	*			*
Tom	*	*	*	*	*		*		*	*
Jamie	*	*	*	*	*	*	*	*	*	*
Con	*	*	*	*	*	*		*	*	*
Owen	*	?	*			*				*
Jerry	*	*	*	*	*	*	*	*	*	*
Maura	*	*	*	*	*	*	*			*
Sonia	*	*	*	*	*	*	*	*	*	*
Síle	*	*	*	*		*	*	*	*	*
Mary	*	*	*	*	*	*	*	*	*	*
Kate	*	*	*	*	*	*	*		*	*
Lara	*		*	*	*	*				*

*Indicates an acceptable response
Note: No data for Bob who didn't engage with this task at all...he just shook his head when asked a question