Understanding and Using Mathematical Representations: A Case Study of Prospective Primary School Teachers in Lesotho

This dissertation is submitted for the degree of Doctor of Philosophy

Nkosinathi Mpalami

Supervisor:

Dr. Dolores Corcoran

St Patrick's College

Dublin City University

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Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of the degree of Doctor of Philosophy is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed: nmpalami

(Candidate) ID No.: 59266767

Date: 1st April 2013

This thesis is dedicated to

my daughter,

Thandi Mpalami

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LIST OF ABBREVIATIONS

ACL	Anglican Church of Lesotho
AME	African Methodist Episcopal
BNP	Basotho National Party
CECE	Certificate in Early Childhood Education
COSC	Cambridge Overseas Syndicate Certificate

CSR	Country Status Report
DelPHE	Development Partnerships in Higher Education
DEP	Diploma in Education Primary
DES	Diploma in Education Secondary
DRT	District Resource Teachers
DTEP	Distance Teacher Education Programme
ECCD	Early Childhood Care and Development
ECoL	Examinations Council of Lesotho
FPE	Free Primary Education
GCSE	General Certificate of Secondary Education
GDP	Gross Domestic Product
JC	Junior Certificate
KQ	Knowledge Quartet
LCD	Lesotho Congress for Democracy
LCE	Lesotho College of Education
LEC	Lesotho Evangelic Church
LHWP	Lesotho Highland Water Project

LSMTA	Lesotho Science and Mathematics Teachers' Association
MK*f*T	Mathematics Knowledge for Teaching
MKiT	Mathematical Knowledge in Teaching
MoET	Ministry of Education and Training
NCTM	National Council of Teachers of Mathematics
NTTC	National Teacher Training College
NUL	National University of Lesotho
PCK	Pedagogical Content Knowledge
PSLE	Primary School Leaving Examinations
PTC	Primary Teachers Certificate
RCC	Roman Catholic Church
REC	Research Ethics Committee
SAARMSTE	Southern Africa Association for Research in Mathematics, Science and Technology Education
SCK	Specialised Content Knowledge
SMK	Specialised Mathematical Knowledge
SPSS	Statistical Package for the Social Sciences
TP	Teaching Practice

UK	United Kingdom
USA	United States of America

ABSTRACT

A decade has passed since the introduction of free primary education in the year 2000 in Lesotho. Studies such as the Needs Analysis (Lesotho College of Education, 2006) that were carried out in order to inform the Ministry of Education and Training about progress made in the teaching of core subjects in schools during the decade of free primary education reveal that there are multiple problems associated with the teaching of primary school mathematics. The reality of overcrowded classrooms impinges on the effective teaching of mathematics. Overcrowding is a generic problem facing many schools in Lesotho. However, other problems are both content and pedagogically oriented.

Learning to teach mathematics is a complex process. It involves among other things, gaining Pedagogical Content Knowledge (PCK) (Shulman, 1986). The purpose of this study is to explore prospective elementary teachers' understanding and use of mathematical representations. By mathematical representations I refer to instructional resources, through which teachers make mathematics comprehensible to learners. The three modes of representation (enactive, iconic, and symbolic) provide contexts for both learners and their teacher(s) to construct, negotiate, and re-construct meaning for mathematical concepts, procedures, and processes. The choice and use of representations is a key component of teachers' PCK.

This study is designed in four distinct tiers, spread over three years, to explore the development of understanding and use of mathematical representations for teaching, by a group of prospective elementary teachers in Lesotho. The main analytical framework was the Knowledge Quartet (KQ) (Rowland, Huckstep and

Thwaites, 2005). The findings indicate that participants' mathematical knowledge (*foundation*) influences their choice of representations and their use in instruction. I found that participants with strong mathematical knowledge are able to choose appropriate representations and are also able to use them effectively. It is recommended that for the improvement of mathematics teaching in Lesotho, the KQ be incorporated into mathematics courses studied by prospective elementary teachers at Lesotho College of Education. The model will assist student teachers to learn to plan and teach mathematics topics collaboratively, and therefore act as a tool for teacher development.

1. Rationale and Overview of the Study

In this introductory chapter, I begin by stating the aim of this research project, followed by a section on the research questions that guide this study. I then discuss the background to the study, which covers the current socio-economic and educational status of Lesotho. In this study, student teachers were asked to work out solutions to mathematics tasks, in order to explore their understanding of representations at the entry point of the three-year Diploma in Education Primary (DEP) programme offered at the Lesotho College of Education (LCE). Five student teachers were observed teaching mathematics lessons in different primary schools located in the Maseru district. In order to situate my study within the educational trends internationally, I briefly discuss the primary school curriculum of Ireland and England with the main focus on the teaching of mathematics. This chapter concludes by giving an outline of the subsequent chapters.

Aim of the Study

The aim of the study reported in this thesis was to explore student teachers' understanding of mathematical representations and the way they chose and used representations when they were practising teaching mathematics in Lesotho primary schools. The notion of representation in this study is taken to refer to the three modes of representation as identified by Jerome Bruner in his work on the cognitive development of children. He proposed and identified the following modes of representation: "enactive representation (hands-on/action-based), iconic representation (visuals/image-based), and symbolic representation (abstract and language-based)" (Bruner, 1966, p. 11). Representation is taken in this study as a means through which mathematics content might be taught meaningfully and in an exciting way to learners at all stages of schooling.

In Bruner's point of view, the three modes of representation are the means through which information or knowledge is stored and encoded in a learner's memory. Mathematics teachers in particular have to be conscious of these three modes of representation when they plan and teach mathematics especially at primary school level. In Lesotho, even primary school mathematics teachers have a tendency to focus more on symbolic representations than the other two modes of representation. This assertion can be confirmed by reference to the three books of Lesotho Primary Syllabus (National Curriculum Development Centre, (NCDC) 1997, 1998, 1999) and to current textbooks supplied to schools (e.g., Barry and Dugmore, 2001). Perhaps this limited focus on symbolic representation has contributed in part to what is perceived as a crisis of high failure rates among learners in the national mathematics examinations (Standard 7). In this terminal examination learners are expected to answer 50 multiple-choice questions for 50 marks. See Appendix 1 for a sample page from a Primary School Leaving Examination Standard 7 mathematics paper (2003). A recent newspaper headline attributes poor Standard 7 results to "unfair marking strategies" (Mohaeka, 2012). Dissatisfaction with the influence that terminal examinations exert on curriculum has led to a policy decision that has been taken by the Ministry of Education and Training (MoET) to abolish the Primary School Leaving Examinations (PSLE) and replace it with "national assessment" for the school year 2014 (MoET, 2009). The Curriculum & Assessment Policy document states that:

> At grade 7, national assessment will be used for two purposes: to check attainment of competencies for individual learners in individual learning areas and their level of readiness to proceed to grade 8 (Form A), and to monitor the progress of the education system. Consequently, **statements of success** indicating attainments of candidates in key areas will be available at the end of grade 7 (p. 23).

It is impossible to predict how this proposal will play out in the future, and what influence if any it will have on Basotho learners' mathematical achievement. In the following paragraphs, I will explore the meaning of Bruner's three modes of representation and their implications for teaching.

Enactive representations involve tasks that call for action (hands-on activities) on the part of learners. For example in a mathematics lesson, learners may be asked to form various numbers using concrete objects such as counters, matchsticks, and stones in early years of schooling in order to assist young learners to understand early mathematical ideas and operations on number. These physical objects prove to be valuable especially in the early years of schooling because learners at this stage are not yet conversant with the four basic mathematical operations namely addition, subtraction, multiplication and division. So the use of models might be considered to play an important role of helping learners to physically see why, for example, this number sentence is true: $8 - 3 = 5$.

Iconic representations are visuals that both learners and teacher can refer to in class, in order to facilitate effective learning and teaching of certain mathematical concepts. Iconic representations are resources that can act as scaffolds for learners' indecisive thinking and strategies for mathematical operations. The following teaching aids together with the guide on how they are used, were supplied to all primary schools: metre sticks, trundle wheels, simple balance, pin-boards, rubber bands fraction boards, abacus, *makhonatsohle*[1], squared boards, b/board protractors, set squares and b/board campasses (*sic*) (NCDC, 1998, p. 61). This support for the primary curriculum was a once off grant of equipment only and one item of each piece of equipment is of limited value to schools. Examples of specifically iconic representations commonly found in many Lesotho primary schools include: the number-line, the number square, a number fan, a number track, place value cards, multiplication squares, and multiplication arrays. While most of these representations are produced commercially, student teachers in Lesotho are taught during their training, ways of improvising and constructing these resources using recycled cardboard, plastic, and other readily available materials. Given the economic state of Lesotho, it is logical to assume that many primary schools cannot afford commercial teaching aids. But irrespective of financial constraints of the country, teachers in all schools have an educational obligation to teach mathematics well, in order to promote learners' understanding. The use of iconic representations is meant to assist learners' in developing strong and independent mathematical strategies for calculation, and in my opinion, once this stage of autonomy is reached then iconic representation becomes redundant.

[1] A traditional board game played by learners to improve their proficiency with the four arithmetic operations (addition, subtraction, multiplication and division).

Symbolic representation can be considered as being at a higher level and more abstract compared to the other two (enactive and iconic). This is a more adaptable form of representation than actions and images, which have a fixed relation to that which they represent. Symbols are flexible in that they can be manipulated, ordered and classified, and so the learner is not constrained by actions or images. In the symbolic stage, knowledge is stored primarily as words, mathematical symbols/numerals, or even in other symbolic systems (Bruner, 1966).

The three modes of representation identified by Bruner are not hierarchical in nature but also intertwined (Rowland, Turner, Thwaites, & Huckstep, 2009). Rowland et al. (2009) have cited an example of an empty number line to substantiate their point that the three modes of representation are not only to be used hierarchically but also in an intertwined fashion. They argue that an empty number line is an iconic representation by its nature, but as learners make 'hops' or 'jumps' on it, they are using it in an enactive way. Yet the operations demonstrated by such hops and the answer reached are symbolic in nature. All three modes of representation for teaching mathematics are mentioned in the Lesotho primary syllabus books and during their training course, student teachers are expected to acquire and develop a web of connected representations for various mathematical concepts, which they can draw on in lessons to help learners understand mathematics.

This study was therefore planned to explore first year student teachers' understanding of mathematical representations at the entry point of a diploma course at the Lesotho College of Education (LCE). I then followed up some student teachers in the second year, when they were on teaching practice, in order to explore their choice and use of representations when teaching mathematics at primary schools in

Maseru area of Lesotho. I must emphasise that Bruner's notion of three modes of representation for teaching mathematics constitute the starting point for my understanding of mathematical representations for teaching and I aim to learn more. My conviction on this topic of representations is that the frequent and effective use of enactive, iconic, and symbolic representations by teachers can help learners to participate more fully in learning mathematics and thus develop richer and more solid understanding of abstract mathematical concepts and processes. I want to find out if student teachers in Lesotho share this conviction and how they can be helped to develop their use of representations in making mathematical ideas accessible to learners.

Participants in this study were students who had registered for the Diploma in Education Primary (DEP) programme. DEP is one of the programmes offered at the Lesotho College of Education. A total of three hundred DEP 1 students, registered at the college in 2009 were invited to participate in this study. Two hundred and twelve students accepted the invitation. This study aimed at exploring the multiple representations that these student teachers might employ when on teaching practice, and investigating what informs their choices and use of such representations. Below are the broad research questions that guided this research.

Research Questions

The study sought to provide answers to the following questions:

1. What mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?

2. How do Lesotho student teachers on teaching practice use mathematical representations in lessons?

3. What factors influence Lesotho student teachers' choices and uses of mathematical representations in teaching primary mathematics?

Why this Study?

I learned to do mathematics through representations in the early years of my life as a young herd boy in a rural area in Lesotho. In the 1970s there were not many primary schools in Lesotho especially in the highlands district of Mokhotlong. As a result, one school would serve many villages situated around it. At the time, the rural areas were the least developed places. For example, there were no proper roads built to connect villages and school and there were no buses to carry learners to school. Learners had to walk long distances, such as 10 kilometres or more, daily to get to the school. There were no bridges constructed over rivers, so that during rainy seasons it was almost impossible to ensure full attendance in schools. Because of all these factors, children, especially boys, had to wait until they were aged at least ten years before they could be sent to school. In the years before formal schooling, just like other boys, I looked after cattle and a flock of sheep. It is during this time that I learned from older herd boys the ways of counting sheep. We used to put stones equal to the number of sheep in a bag. Each stone was representing one sheep, and every morning we would stand next to the kraal's exit, to allow the sheep to go out one by one. As each sheep went out of the kraal, we would take out one stone from the bag. When all the sheep had gone out and some stones, say two, were left in the bag, that meant two sheep were missing. In a case where the number of sheep exceeded the number of stones, we knew that somebody else's sheep had joined ours, and we then had to look carefully throughout the flock for such extra sheep. The search for the additional sheep was made easy by the signs marked on each sheep's ears.

It is interesting that this form of use of stones as representations for numbers has been observed in other traditions as well. For instance, Choat (1978, p. 29), when discussing the ways in which learners acquire number sense argues that:

> Number, it is believed, originated with the efforts of primitive man to meet his needs. The caveman, by naming his possessions, was using a one-to-one correspondence that had an immediate value. The shepherd, telling (*sic*) his sheep with sticks, stones and fingers, evolved counting with names derived from natural objects to determine the size of his flock; in such ways were tallying, counting, grouping, comparing, and combining invented. The necessity for a written record of the numbers obtained by counting led to symbols and the development of systems of notation. These were unwieldy and based on a variety of principles until the Hindus, around 500 B.C., stumbled upon the idea of zero. This led to the present Hindu-Arabic system of notation.

My experiences of learning mathematics in Lesotho at primary school level, and of lecturing in mathematics education to student teachers that are training to become primary school teachers were the driving force for me in wanting to carry out this study. Focusing back during the first two years of my primary schooling, I still have in mind a vivid picture of how we (learners) worked on mathematics tasks, involving addition and subtraction of numbers, through use of small stones and bundles of sticks. The use of such concrete materials helped us, as learners, to understand how numbers are added and subtracted and in general, made learning of mathematics fun and meaningful.

Part of my interest in wanting to carry out this study was also triggered by the following factors:

- ❖ A 2006 Needs Analysis Study that was commissioned by the Minister of Education and Training of the Lesotho Government;
- ❖ My participation in the Development Partnerships in Higher Education (DelPHE) project.

The two factors are discussed in the paragraphs that follow. The needs analysis report raised issues that challenged me, as a mathematics educator who is directly involved in the training of primary school teachers. As a consequence, I became interested in studying what knowledge of mathematical representations student teachers hold when they join the DEP programme and the way they would choose and use representations when they are on their one-year-long teaching practice. The answers to my research questions are directly related to my professional work; therefore, I felt it was imperative to locate this study at the teacher training college.

Needs Analysis Report

The first factor that motivated me to work on this study are some issues raised in a report of a school-based research study, which was done by the Lesotho College of Education (LCE, 2006) for the Ministry of Education and Training (MoET). The issues raised in the report are serious challenges facing primary school teachers, in relation to the teaching and learning of mathematics at primary school level. The needs analysis report was commissioned, partly in response to publication of the South African and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ II) report (Mothibeli and Maema, 2005), which examined the teaching of key subjects at primary level. The research team participating in the study was lead by the then Director of Planning and Research, Dr. James Urwick, at the Lesotho College of Education. Other members of the research team were lecturers at the college who held masters degrees in education. In this report, the following issues were raised as challenges calling for immediate attention:

- lack of teacher confidence in the teaching of mathematics;
- lack of learning and teaching support materials appropriate to the teaching of mathematics;
- lack of teacher subject knowledge which restricts styles of teaching in the classroom;
- pedagogical limitations within the schools;
- presentation of mathematics in the classroom lacking imagination;
- lack of mathematical communication in class.

The report appears to direct part of the blame, among other factors, towards institutions of higher learning that train primary school teachers:

> Teachers seem to be having a number of problems in the teaching of mathematics. Some teachers are aware of their problems while others are not. The problems seem to be attributable partly to deficiencies in their training and partly to the situation in the schools (LCE Report for MoET, June 2006, p. 48).

It was envisaged that this study would explore prospective teachers' understanding of mathematical representations as resources that potentially could enhance the teaching of mathematics in Lesotho schools and learners' mathematical proficiency.

The DelPHE Project

The Development Partnerships in Higher Education (DelPHE) project was a consortium of three institutions of higher learning; Durham University-UK, Lesotho College of Education (LCE), and National University of Lesotho (NUL) - Lesotho. These three institutions collaborated in the exchange of expertise in education and worked on four strands; numeracy, literacy, special needs education, and environmental education. The DelPHE project had duration of three years (2008 – 2010) and was funded by the British Council. According to the DelPHE website, the title of the project was: Curriculum development for effective and relevant teacher education in Lesotho. Its Summary/Aim is listed as: To improve academic programmes and train primary level English and numeracy teachers for sustainable development and to assist learners with Special Education Needs.

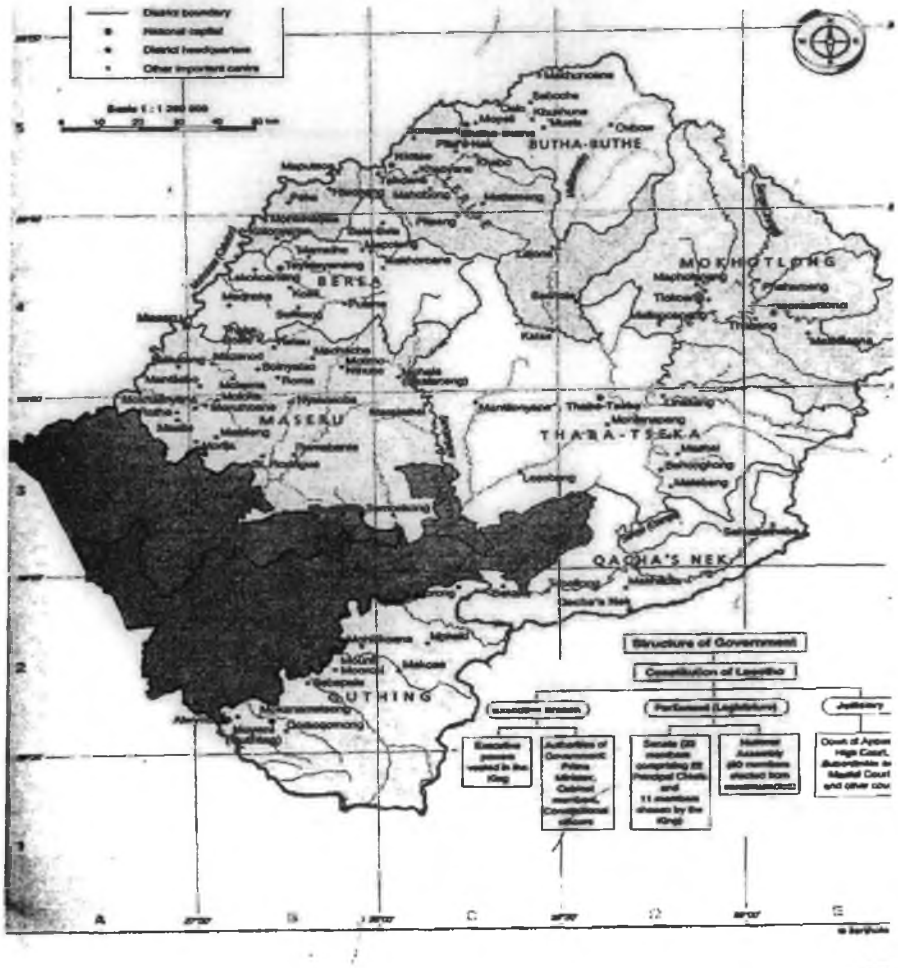
I was a 'participant observer' (Labaree, 2002) in the DelPHE project - numeracy strand, in the sense that the DelPHE project and my doctoral work took place concurrently. I was observing the impact of DelPHE initiatives on student teachers at the same time as I was preparing to carry out my own doctoral investigative project. In the numeracy strand of the DelPHE team there were five members; two from Lesotho College of Education, two from National University of Lesotho and one from Durham University. The core goal of the numeracy strand within the DelPHE project was to improve on the programmes for primary school teacher trainees, offered at both LCE and NUL, so that such courses would equip student teachers with knowledge and skills necessary to become mathematically proficient teachers. Our colleague from Durham University shared with us in Lesotho College of Education, the work a team of researchers had done with primary school learners in the UK, where they used representations such as a number line and five by five arrays of dots to teach addition, subtraction, multiplication and division of whole numbers. However, I was aware that materials that work effectively in one culture could be problematic in another culture (Delaney, 2008). For instance, most representations our colleague from Durham proposed for use by student teachers were computer based in nature and could not be readily applicable to the Lesotho situation where most schools are without electricity and computers therefore we had to adapt only those that student teachers could construct manually using locally based materials such as cardboard, planks, and plastic. Kaput (1998, p. 271) refers to instructional materials as "boundary objects" shared between discourse communities. He argues that "how these objects are moved from one place to another, how they are overlaid, modified, and discussed in a social context" is of great importance and needs to be borne in mind at all times by mathematics educators. His point was relevant to my study.

During the DelPHE project I got interested in the idea that "representation is a process, an essential component of both teaching and learning, a way to model mathematics and a way for students to show their thinking about mathematics" (Fennell and Rowan, 2001, p. 292). In order to contribute to a better grounding in the teaching of mathematics methods courses in the LCE, I wanted to study student teachers' knowledge of mathematical representations at the entry point of the Diploma in Education Primary (DEP). I also felt that it would be interesting to explore student teachers' choice and use of representations when on teaching practice. The teaching practice arises after students have spent a first academic year at the College. The contents of the two mathematics courses that focus specifically on pedagogy, which are offered to DEP student teachers during year 1 of their study include, among others, the use of multiple representations in teaching primary mathematics. This means that by the time students are placed in schools, they have already been introduced to the idea of mathematical representations and their role in enhancing the communication of mathematics ideas to learners. It is logical to assume that the mathematics content and pedagogy learned during the first year at the College could have an influence on student teachers' selection and use of key representations when on their teaching practice year. However, the impact that the mathematics education courses offered in DEP year 1 had on student teachers in relation to their choice and use of representations was not the immediate focus of the study. Where peripheral data relating the impact of such courses on participants' understanding and uses of mathematical representations became available it was analysed and reported. In what follows I describe the context of the empirical field of this study.

Background to the Study

Lesotho is a largely mountainous country, divided into ten districts namely: Berea, Butha-Buthe, Leribe, Mafeteng, Maseru, Mohale's Hoek, Mokhotlong, Qacha's Nek, Quthing, and Thaba-Tseka. Each district has its own town. The capital town is Maseru, and this is where the central government of Lesotho is located. In each of the ten districts there is an education office, built for officers who are responsible for all education issues and activities that occur at the district level. The Lesotho College of Education has two campuses. The main campus is located about 5 km away from Maseru city centre, and the other campus is located within the town of Thaba-Tseka in the highlands of Lesotho. At the moment (2011), the two campuses are offering equivalent diplomas. The following section focuses more on details about the Lesotho College of Education.

Figure 1.1: Map of the Kingdom of Lesotho



Adopted from Primary Atlas for Lesotho (2001, p.19)

The Lesotho College of Education

The Lesotho College Education (LCE) was established in 1975, under its previous name of National Teacher Training College (NTTC), to train both primary school teachers and secondary school teachers. According to LCE – College calendar (2009/2010, p. 5) the then Lesotho Government led by the Basotho National Party (BNP) "decided to establish this public institution of higher learning (NTTC) to replace three denominational teachers' colleges, owned by the Roman Catholic Church, the Lesotho Evangelical Church, and the Anglican Church of Lesotho". The amalgamation of the teacher training colleges gave birth to a new college NTTC, which since then became a department of the Ministry of Education and Training of the Government of Lesotho. Prior to the establishment of NTTC, the three denominational training colleges were producing teachers mainly each for its own church schools.

There were of course pros and cons to phasing out denominational colleges. The newly established college (NTTC) would serve as a public institution and also as a symbol of national unity where anybody could freely either become a student or a lecturer, irrespective of their religious sect. For equity purposes, the new college would also ensure that all teachers in the country receive similar training. It was also going to be cost effective for the Lesotho government to run one national college for teachers. However, the disadvantage of having one such college is that with its establishment, there would no longer be the competitive spirit of producing best teachers that existed with denominational colleges. Competition is considered a good catalyst for working towards quality education. Without competition it might be difficult to achieve the highest level of quality.

The three mentioned churches namely the Roman Catholic Church, the Lesotho Evangelical Church, and the Anglican Church of Lesotho continue to enjoy equal representation in the college to date. Each of the three churches is represented by one member as a Religious Education Lecturer in the college. That the relevant church authority nominates member and the college's responsibility is simply to check that the nominated person is qualified enough for the lecturing duties at the college level. In the year 2002, the College (NTTC) was granted autonomy by the Lesotho

Government led by the Lesotho Congress for Democracy (LCD) party and was given a new name: the Lesotho College of Education (LCE). The LCE calendar states:

> The autonomous College (LCE) reorganized its pre-service academic section into three Faculties and eight Departments. The new Departments contain staff from both the former pre-service Divisions (Primary and Secondary), in the interest of more efficient deployment. The In-service Division developed into the Distance Teacher Education Programme (DTEP), whose academic staff continues to operate, for most purposes, as a separate unit (LCE – Calendar 2009/2010, p. 5).

Participants in my study were student teachers who had registered for the three-year Diploma in Education Primary (DEP) programme for the academic period 2009 – 2011. DEP is a pre-service programme designed for primary school teachers who will qualify as generalists. As such, DEP students take courses from all departments at the college. My research project began when the participants were in their first year in 2009. All DEP student teachers in Maseru campus were invited to take part in this study. The Maseru campus students were a more convenient sample than the highland Thaba-Tseka DEP students, because I am also located at the Maseru campus as a mathematics lecturer and Thaba-Tseka is difficult to access by road.

As stated earlier, all student teachers who register for DEP programme are trained to become generalist teachers in primary schools, unlike their counterparts who pursue a Diploma in Education Secondary (DES) programme. However, the entry requirements for both diplomas (DEP and DES) are the same, which are credit passes (A – C)[2] in any four subjects and a pass (D or E) in English in Form E examinations. The current situation at the Lesotho College of Education (LCE) is that student teachers spend the first year of their study at the college doing both content and pedagogical courses. In the second year they go for Teaching Practice (TP), which lasts for one academic year. When on TP, a common practice is that each student teacher is allocated a class to teach - say Standard 3 (Year 8) - and a mentor teacher in the school, whose role is to assist the student teacher to cope with his/her work such as lesson-planning and teaching in general. The college lecturers also visit schools occasionally to observe students. In the third year, student teachers spend another year at the college, again doing content and pedagogical courses. Students' ages at entry usually range between eighteen (18) and forty (40) years.

Student teachers who register for DEP are mainly female. They are usually a homogeneous group that consists of students who are Lesotho citizens and speak the same home language, which is Sesotho. Attainment in mathematics in Form E is generally low in Lesotho and students who present for teacher training often appear to lack confidence in their mathematical ability. A recent World Economic Forum report rated Lesotho 119 out of 144 with regard to quality of math and science education (Schwab, 2012). This finding reflects a worrying drop of 5 places since 2010. However, it may not mean that Lesotho is performing badly, but that other countries are improving more. Another measure the country's mathematical performance is available from the South African and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ, 2010). The most recent SACMEQ study indicates

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[2] In Lesotho, students' performance in all subjects in Form E is classified as follows: credit when a learner scores A, B, & or C. A pass is given if a learner scores D and E, and an F denotes a fail.

that Lesotho is one of three countries in the region where levels of achievement for both reading and mathematics were observed in both 2000 and 2007 as substantially below the SACMEQ average for Grade 6 pupils. However, there are signs for hope for the future. The SACMEQ achievement trends indicate that Basotho Grade 6 pupils' reading levels increased by 16.7 points over the same period (2000-2007) and Basotho pupils' mathematics scores increased by 29.7 points. This indicates, I think that while the Lesotho mathematics education scene remains challenging, teachers, learners and the education system are working to improve.

DEP 1 Mathematics Courses

Students who enrol for the DEP programme register for two mathematics courses. The two courses are taken over two semesters. In the first semester, students register for the course entitled: 'Foundations and Teaching Methods for Primary Mathematics 1' (MAT 1400P – A). This course addresses both content and pedagogical components. In the second semester of year 1, students register for 'Foundations and Teaching Methods for Primary Mathematics 2' (MAT 1402P – B). Again, this course is made up of both content and methodology. According to 2010 DEP 1 mathematics course outlines, the main aim for both courses (MAT 1400P – A & MAT 1402P – B) is to:

> Equip student teachers with mathematical content knowledge and pedagogical skills that will enable them to be proficient mathematics teachers at primary school level. Table 1.1 below, presents a brief summary of objectives stated in each of the two courses. In both courses (MAT 1400P – A & MAT 1402P – B) students are assessed in such a way that there is 40% for course work and 60% for examination work.

Table 1.1: Objectives of two maths courses covered in DEP 1

COURSE	OBJECTIVES
MAT 1400PA	**By the end of the course student teachers should be able to:** o Represent numbers in various ways as they do the four basic operations o Represent and manipulate fractions in all forms o Define sets and represent them symbolically and by means of Venn diagrams o Form mathematical expressions and equations from word problems o Demonstrate an appreciation of problem solving as a teaching method o Solve simple simultaneous and quadratic equations o Link learning theories and processes to implications of teacher's and learner's roles in class o Demonstrate an appreciation of contributions of some societies in the development of mathematics
MAT 1402PB	**By the end of the course student teachers should be able to:** o Demonstrate skills and techniques for solving problems involving geometrical figures and commercial arithmetic o Represent numbers in various ways as they do four basic operations o Construct and select appropriate teaching aids for use when teaching arithmetic o Construct a logical Lesson Plan o Demonstrate different teaching and assessment techniques in mathematics education

The structuring of the two courses in this way is meant to be helpful because there is an assumption that when participants have successfully completed these two courses they will be in a position to choose and use representations in their respective lessons when on school placement. The assumption is based on the observation that both courses encourage the use of representations.

Lesotho's Economy

In this section, attention is paid to the current socio-economic and educational status of Lesotho. Lesotho is landlocked by the Republic of South Africa and has a total population of about 1.8 million. As such, Lesotho's economy depends largely on South Africa's economy so that its currency, the Maloti, is aligned with the South African Rand. That means 1 Loti = 1 Rand. According to the Country Status Report (CSR) (World Bank, 2005) Lesotho enjoyed high Gross Domestic Product (GDP) averaging 4% in late 1980s to early 1990s due to the Lesotho Highland Water Project (LHWP), which began in 1987 as a partnership between Lesotho and South Africa. The report states that LHWP generated many jobs for Basotho (citizens of Lesotho), which boosted the country's economy to some extent. Secondly, as a result of LHWP, Lesotho is able to generate its own electricity, and at the same time exports water to South Africa. Lesotho also exports diamonds mined from Letšeng Diamond Mine located in the Mokhotlong district. Other exports are wool and mohair, though these exports have been negatively affected by highly organised theft of animals especially in the highland districts (Mokhotlong, Thaba-Tseka, and Qachasnek). The CSR further indicates that

> The negative GDP growth registered in 1998 was primarily due to the political unrest that resulted in massive destruction of infrastructure, especially in the

> Maseru area. In the late 1990s Lesotho's economy was further weakened by reductions in the amount of Basotho labour needed in the South African mines (p. 16).

Since 1998, poverty has been rife in Lesotho, especially in the rural areas. The HIV/AIDS pandemic has contributed largely to the country's economic decline. As parents die, many children are left destitute. According to the CSR (2005, p. 15) the impact of the HIV/AIDS pandemic on the education sector is also significant. The report states that in 2003, out of all teachers who opted for testing, 30% were found to be HIV positive, and that "it is reasonable to expect that the overall prevalence rate among non-volunteers would be higher". Many children have become young parents to their sisters and brothers, a responsibility that no doubt is affecting them psychologically. However, the government of Lesotho through the Ministry of Education and Training (MoET) is making some attempts to bring this situation under control. In 2003, an HIV/AIDS Coordination Unit was established and staffed with one full-time coordinator and two counsellors. The unit developed an Action Plan focusing on voluntary testing, counselling, and treatment for all education sector staff (CSR, 2005). The previous description is intended to give an idea of the situation under which this study is done. The factors discussed also affect the participants in this study. In what follows, I focus on schools in Lesotho and the education system.

Schools and Education

As mentioned earlier in this chapter, churches in Lesotho did not only own teacher training colleges but also owned and continue to own both primary and secondary schools. The majority of schools in Lesotho are owned by churches despite recent efforts by the government to build state-owned schools. The prominent churches that own schools are as follows, in order of percentage of schools owned: Roman Catholic Church (RCC), Lesotho Evangelic Church (LEC), Anglican Church of Lesotho (ACL), and African Methodist Episcopal (AME). According to Lesotho Education Sector Strategic Plan (2005 – 2015), in 2003 the statistics of schools ownership was as follows: RCC owned 38% of all primary schools and 35.4% of secondary schools, LEC owned 35.9% of primary schools and 31.9% of secondary schools, ACL owned 13.2% of primary schools and 14.2% of secondary schools, AME owned 1.8% of primary schools and 2.2% of secondary schools, the Government owned 5.3% of primary schools and 3.3% of secondary schools, 3.6% of primary schools and 4.9% of secondary schools were owned privately, and 0.1% of primary schools and 0.4% of secondary schools were owned by "unknown" authorities. All church schools are under the management of their respective church authorities. However, it is worth mentioning that despite this pattern of ownership of schools, the government of Lesotho takes the responsibility of providing for teachers' salaries in almost all officially registered schools. But, there are a few private schools that pay their own teachers from school fees. On the whole, the Ministry of Education and Training (MoET) is responsible for the management and regulation of education in Lesotho. This discussion is relevant and important in this study in that it sheds light in relation to the protocol to be followed when access to schools is sought. The MoET has to play an important role of formulating and implementing rules and regulations on how researchers should work with schools. Unfortunately, the MoET to date has not set up an ethics committee to address issues of research activities that are carried out in school. The process of negotiating and gaining access into schools where participants practised teaching is discussed in-depth in Chapter 3 of this thesis.

The current formal education in Lesotho is such that, the first seven years of schooling are spent in primary school (standards 1 – 7), followed by three years of junior secondary school (Forms A – C), followed by two years in senior secondary school (Forms D and E), and then 3 or 4 years of tertiary education, depending on various institutions. Up until now, there have been national examinations taken at the end of every level. For example, at the end of Form C learners sit for Junior Certificate (JC) Examinations, and at the end of Form E learners sit for Cambridge Overseas Syndicate Certificate (COSC) Examinations, which corresponds to the British General Certificate of Secondary Education (GCSE). All national examinations are the responsibility of the Examinations Council of Lesotho (ECoL).

After COSC if a person chooses to come to Lesotho College of Education (LCE) for example, they will spend 3 years working for a diploma programme (DEP or DES), but if they choose to go to one of the two universities in the country for a degree programme they will spend 4 years studying there. Participants in this study are people who have gone through this education system and after the COSC they chose to come for a teaching diploma programme (DEP) offered by the Lesotho College of Education, which upon completion will give graduates a certificate to teach from Standard 1 to Standard 7.

In the year 2000, the MoET introduced Free Primary Education (FPE) beginning with Standard 1. Prior to the year 2000, all learners were paying school fees, buying school uniforms, and paying for meals especially in those schools where lunch was served for learners. As a result, access to education for many children was impossible, due to poverty that had hit many families. However, with the introduction of FPE learners in Standard 1 in the year 2000 no longer paid for fees, books, lunch meals and school uniforms. According to the Country Status Report (World Bank, 2005, p.150) "the Ministry of Education and Training (MoET) covers the following: [school] fees, book rental fee, stationery, feeding and maintenance under FPE".

Table 1.2: Enrolment numbers and primary schools (Boys, Girls), 1999 – 2000

Primary Enrolments	**1999**	**2000**
Total	364,951	410,745
Males	176,365	202,760
Females	188,586	207,985
Number of Schools	1,272	1,283

Adapted from the Lesotho Education Sector Strategic Plan 2005 – 2015 (p. 40)

The introduction of FPE posed serious challenges in Lesotho's educational system: there was a high influx of learners in Standard 1 in 2000 in all schools that implemented FPE. Table 1.2 adapted from Lesotho Educational Sector Strategic Plan 2005 – 2015 indicates the changes in learners' enrolment in primary schools and number of primary schools between the years 1999 and 2000. With the introduction of FPE in 2000, there were more boys (26,395) than girls (19,399) admitted to schools. These high enrolment numbers meant there was a great demand for qualified primary school teachers. Secondly, many classrooms had to be built in order to accommodate many extra learners in schools, especially given that only 11 new primary schools were built in 2000 as indicated by the table. In his address delivered at the 12th Graduation Ceremony (4th December 2009) held at the Lesotho College of Education, the LCE – Rector Dr. Oliphant (2009, p. 2) acknowledged that the implementation of

FPE in 2000 “resulted in an unprecedented rise in primary school enrolments and the need for more teachers”.

The FPE policy has never been binding on primary schools; however schools were encouraged by the MOET to buy into the idea. Those schools that opted for FPE would get increased support from the Ministry, especially with regard to the offering of more grants for qualified teachers and the building of more classrooms for some schools. On the other hand, those schools that had decided not to implement FPE would from the year 2000 receive limited support from the Government of Lesotho through the MoET, such as providing for salaries to only a few teachers in such schools. There are some few, especially privately owned schools that decided not to implement FPE. The current situation with regard to the implementation of FPE in schools is that the cycle is complete because it has by now covered seven years of primary schooling, and the numbers of enrolled learners in schools are still high.

Against this background, it is important for this study to explore how teacher trainees learn to teach mathematics well in these large classes during the school placement period. In what follows, attention is paid to primary school mathematics curricula of two European countries. The curricula of these countries are discussed briefly here so that the context of the Lesotho educational system can be better understood.

An International Comparison

In this section I discuss the Lesotho educational system in relation to the Irish and British education systems for primary schooling, not for the purposes of comparison, rather in order to establish international trends on the teaching and learning of mathematics at primary school level in developed countries. The reason for choosing Ireland is mainly for convenience sake, because this research is supervised and sponsored in Ireland, and as such it is at least partially influenced by mathematics education policy in Ireland. The British mathematics curriculum is chosen mainly because Lesotho was a British colony up until 1966 when she gained independence. Since independence, Lesotho has been practising the then inherited British educational system. Hence, English has been the principal language of learning and teaching in Lesotho schools, colleges, and university.

According to the Lesotho educational system, levels/classes are officially called 'standards' in primary schools. By law, teachers are officially expected to conduct lessons in Sesotho from Standard 1 to Standard 3, and from Standard 4 up to Standard 7 lessons are to be strictly conducted in English. The duration for primary schooling in Lesotho is seven years. Up until 2014, at the end of the seventh year of primary schooling learners sit for national examinations known as Primary School Leaving Examinations (PSLE). Learners are usually expected to complete primary schooling at the age of 12. However, this is not always the case, because some students fail and repeat classes so that by the time they eventually get to standard 7 they are over twelve years of age. Again with the introduction of FPE some learners are first enrolled in schools at an age way above six years.

In Lesotho, learners begin formal schooling at the age of six in Standard 1. It is however worth mentioning that there are many privately owned Reception Schools, best known in Lesotho as Early Childhood Care and Development (ECCD) institutes that offer pre-primary schooling for children aged 3 to 5 years. At the moment there is no official national curriculum for the ECCD schools. Each school has its own private curriculum and as a result many of them if not all opt to use English as a medium of instruction for political reasons. In order for learners who have completed the ECCD schooling to get into primary schools, most schools would give them an oral test conducted in English and not in Sesotho. Many parents too like their children to be introduced to speaking English as early as possible. The oral tests for learners at the entry point to primary schooling are commonly based on English and Mathematics skills. For many years, ECCD schools did not have qualified teachers. However, in 2007 the Lesotho College of Education introduced the Certificate in Early Childhood Education (CECE) Programme as an intervention to address the problem of shortage of qualified teachers in ECCD schools. The CECE is a two-year programme done in distance mode. CECE students are people who are already teaching in ECCD schools. The students only come to the college during winter and summer school vacations. The first cohort of CECE students graduated in the 12th Graduation Ceremony of the Lesotho College of Education held on 4th December 2009. This information provides a base for understanding the calibre of learners as they enter Standard 1 in Lesotho primary schools.

In contrast to Lesotho, free compulsory schooling for every child extends from six to sixteen years in Ireland with an optional extra two years (4-6 years) taught in 'Infant' classrooms, which almost every child takes up. In the UK there is free compulsory schooling from five to fifteen years with plans to extend this to eighteen years by 2015. Most important is the fact that free primary education has been available in Ireland since 1831 (Coolaghan, 1981) and in the UK since 1870 (Gillard, 2009). This means that unlike Lesotho, these societies have a long tradition of free primary schooling with high literacy and numeracy rates among parents, grandparents and great grandparents who have already come through the education system with varying degrees of success. The duration for primary schooling in Lesotho is seven years, and eight in the other two countries. It appears from studying the primary mathematics syllabus for each of the three countries, that at a superficial level at least the curricular content is quite similar. However, the culture dimension of any comparisons of schooling cannot be ignored (Andrews, 2010). The emphasis in Lesotho seems to be on teaching rudimentary formal mathematical structures and procedures. There appears to be an emphasis on mathematics process skills and problem solving, at least in the rhetoric of the Irish curriculum, and the English mathematics curriculum is remarkable in outlining indicators for multiple performance levels in different mathematical domains at each Key Stage. It is interesting to note that while in Lesotho primary school learners do not use calculators, because they do not have them, in a developed country such as Ireland, learners are recommended to use calculators for checking answers to mathematical operations. It is also interesting that in the Lesotho syllabus there is a mention of "Pictorial Representations" as part of content to be covered in mathematics lessons, which does not feature in developed states where integration of ICT with mathematics teaching is taken for granted. There is an emphasis on integration of mathematics concepts within mathematics and even with other subjects in Ireland and in the UK. In all the three countries, teachers are expected to use realistic problems in teaching mathematics lessons.

Another difference between Lesotho schooling and that of Ireland or England is the fact that in Lesotho, one teacher is responsible for teaching all learners with mixed-abilities; while in the other two countries, the class teacher is assisted by a Learning Support teacher whose role is to look after slower learners in the same classroom. With overcrowded classrooms in primary schools in Lesotho, teachers' work is inevitably challenging. It is logical to assume that these teachers would desire to have a similar arrangement to that of Ireland and England in having assistant teachers in their classrooms. While the notion of dividing learners into groups on the basis of their learning abilities might be enticing to teachers in Lesotho, it has been criticised for labelling learners in England. In this context, Boaler (2009, p. 32) argues particularly with reference to high school teaching that:

> One of the biggest problems with maths teaching in England is the desire of many maths teachers to label children, assign them a level, and prejudge their achievement. Too many maths teachers think that their role is to find the chosen few who are really good at maths, assigning the rest to low level sets and giving them low level work for the rest of their school lives. Schools in England have always had this tendency but the last ten years of government directives, with instructions to target and label children in order to track their progress, have pushed the situation out of control.

This suggests that giving learners cognitively low-level tasks on the assumption that they are mathematically weak might disadvantage such learners. Again, putting such learners on their own, denies them their right to learn from learners who are labelled 'more intelligent'. In Lesotho there is a need for assistant teachers, given the big numbers of learners that each teacher has to handle. But the purpose should not be mainly to assist 'weak' learners. Assistant teachers would also assist with providing information for learners with special educational needs, such as the blind. With the growing number of unemployed teachers in Lesotho, it should be possible to have qualified assistant teachers. During the 2009 graduation ceremony the spokesperson for graduating students mentioned that many of them were faced with the problem of unemployment and requested the Minister of Education and Training to create more teaching posts. The idea of teaching assistant posts could be part of the solution.

Acknowledging Developing Status

Researchers in the UK and Ireland, in common with much of the developed world have commented on mathematical knowledge for teaching and found it to be lacking in depth and substance for the task among pre-service teachers (Rowland, Barber, Heal, and Martyn, 2003; Corcoran, 2005). Remedies are suggested promptly (Corcoran, 2008). The biggest difference between these societies and a developing country like Lesotho is the scale of the problem of providing well-qualified mathematics teachers able to deal with the limitations of overcrowded and poorly equipped schools. These circumstances are also found in South Africa, where "historical disadvantage is at its most acute" (Adler and Davis, 2011, p. 159). There is a need here for immediate response and remedy in terms of mathematics teaching at all levels of schooling. Sells (1978) called level of high school mathematics achievement a "critical filter" in preventing minorities and women students from progressing in the US. Moses (1995) also in the US has argued that algebra is a human right for black students. Education has been called a humanitarian response to the problems of Sub-Saharan Africa (Brock, 2012) and the preparation of well-qualified teachers of primary mathematics is an imperative in Lesotho at this time, if the opportunity afforded by free primary education is to be maximised.

Outline for Subsequent Chapters

Chapter 2 focuses on the review of literature relevant to this study. I reviewed literature that focused mainly on the teaching and learning of mathematics through the use of representations. The other part of my literature review focused on the notion of mathematics *for* or *in* teaching. Chapter 2 also discusses the theoretical framework underpinning this study. In Chapter 3, I discuss the methodology employed in this study. I followed a four-tiered design, through which I worked with a cohort of DEP student teachers. Chapter 4 focuses on the findings obtained from the analysis of participants' work on mathematics tasks that were given to them in Tier 1. In Chapter 5, I report on the findings obtained from semi-structured interviews with ten participants in Tier 2. In Chapter 6, I give an account of findings obtained from the analysis of the five lessons taught by the participants in five primary schools in Tier 3. The lessons were taught in schools located in the Maseru area. In Chapter 7, I discuss the findings that emerged from video-stimulated recall interviews, conducted with five student teachers that took part in Tier 4. This happened when participants were in the third and final year of the DEP programme. Chapter 8, being the final chapter, addresses the concluding remarks and general recommendations.

2. Literature Review

In the previous chapter, I made an introduction of the study and provided an overview of the entire study. In this chapter, I discuss theoretical orientations for researching teaching and learning, with more focus on the teaching and learning of mathematics through multiple representations. Different theoretical perspectives present different lenses through which research of this nature could be done (Siegler; 1995, Hatano, 1996; Smith, DiSessa, and Roschelle, 1993; Lave and Wenger, 1991). Some researchers (Brodie, 2005, Sfard, 1998) have found it useful to employ more than one theoretical perspective in their investigative work instead of relying on just one theory. Teaching and learning are both highly complex processes that are better understood when explained from various theoretical perspectives (Lerman, 2000). It is through exploration of such theories that we begin to comprehend and explore teaching and learning processes. By this I suggest that defining teaching and learning of mathematics from a single perspective is limiting and therefore insufficient in a developing country like Lesotho, where issues of poverty and communication have hindered the spread of 'newer' or 'reform' notions of mathematics teaching. I argue that cognitive and situated perspectives are important and helpful to me as a researcher and mathematics educator for widening and enriching my understanding of prospective teachers' learning and teaching processes.

In what follows, I discuss the way in which the notion of teaching and learning of mathematics is explained from cognitive, situative and sociocultural theoretical orientations. Later in this chapter I will pay closer attention to methods of teaching mathematics because of the focus of my study, which is mainly on the use of representations in teaching arithmetic in primary schools in Lesotho. The significance of the teacher's role in students' learning of mathematics will be discussed in the light of teacher traits that facilitate learners' success. My discussion on teacher knowledge for teaching will be in the context of what Shulman (1986) terms 'pedagogical content knowledge', which serves as an overarching framework for this study. In this context, the use of mathematical representations is considered. First the nature of mathematics itself is discussed.

What is Mathematics?

Questions concerning the nature of mathematics and why it should be taught have engaged many scholars from Plato to the present generation (Huckstep, 2007). The philosophical beliefs about mathematics as a body of knowledge surely have an influence on how mathematics is taught in schools. The debates about whether mathematics exists independently of human beings and therefore has to be discovered or whether mathematics develops with time as a result of human invention have been going on for centuries (Lakoff and Núñez, 2000). Lakoff and Núñez (ibid.) criticize and reject the existence of 'Platonic mathematics' for failing to conform to scientific proof for its existence. They argue that the existence of this type of mathematics remains a matter of mere faith. They further argue from a cognitive viewpoint that:

> As with the conceptualization of God, all that is possible for human beings is an understanding of mathematics in terms of what the human brain and mind afford. The only conceptualization that we can have of mathematics is a human conceptualization. Therefore, mathematics as we know it and teach it can only be humanly created and humanly conceptualized mathematics (p. 2).

However, it could also be argued that the work of teaching mathematics involves setting scenes that afford learners opportunities to conceptualise and construct mathematical ideas that already exist. The construction of mathematical facts might happen at both individual and social levels. In order for elementary teachers to be successful in teaching mathematics concepts, they might need representations to mediate between the actual concepts and their metaphors (Lakoff and Núñez, 2000). Lakoff and Núñez (2000) argue that "mathematical thought also makes use of conceptual metaphor, as when we conceptualize numbers as points on a line" (p. 5). The issue here is that the points are not the actual numbers, but a number line as an iconic representation serves as the resource that assists learners to construct the concept of order of numbers. It could also be used to assist learners to perform operations such as addition and subtraction on the basis of points standing for numbers. The effective use of representations by the teacher in class might assist learners to cognitively develop structures for mathematical concepts, and socially through participation in the practice of learning mathematics to negotiate and reach a shared meaning of mathematical concepts (Corcoran, 2007). In what follows, I focus on two theories of learning.

Cognitive Theory

According to cognitive theory, knowledge develops through the construction of schemas about a concept under discussion within an individual's mental structures (Cobb and Bowers, 1999). The construction of mental structures of a concept is enhanced by interaction with the social world. The social world here would include for example working with other persons, reading a book, working individually on a mathematics task, and construction of a representation such as a graph or table. Arguing from a cognitive viewpoint, Sfard (1998) refers to an 'acquisition metaphor' in order to describe the learning of mathematics. In this way mathematics is thought of as an entity and/or a commodity that can or cannot be acquired and possessed, depending on the mental capabilities of an individual learner. Through interactions with facilitative environments this entity can be attained. Again, by environments I make reference to situations that would include people, textbooks, and multiple representations as thinking tools (Diezmann and English, 2001).

Mathematics in the School Curriculum

The current Lesotho Lower Primary Syllabus (1998) gives no definition of mathematics. Rather it has outlined a general aim of teaching mathematics from which the influence of constructivism is apparent. The syllabus states that:

> The general aim of teaching mathematics is to enable the pupil to develop clear and logical thought and acquire a working knowledge of mathematics as a tool in various subject areas and for use in everyday life. In addition, mathematics aims at the development of the mathematical abilities of individuals (p. 61).

Mathematics in the Lesotho syllabus is viewed as a 'tool' that can be used in other school subjects and also outside school to solve real world problems by those who have managed to acquire it during mathematics lessons in school. From the quotation above, it is tempting to conclude that the emphasis on the development of individual mathematical ability suggests that the syllabus is influenced by a constructivist theory of learning. This is a chicken and egg situation, in my opinion. Constructivist approaches to mathematics learning that mathematics curricula developers attempt to put in place in schools are important in shaping students' perceptions of mathematics and its application in various situations outside school. Yet these approaches are supposed to have come from research into how people learn mathematics (Hatano, 1996). Teachers are the mediators in this situation and they will need considerable support in understanding how they themselves learn mathematics if they are to implement a constructivist approach to teaching (Glasersfeld, 1995). This is a huge task in a developing country.

Interestingly, the British Mathematics K-6 Syllabus (2007, p. 7) takes a view that:

> Mathematics is a reasoning and creative activity employing abstraction and generalisation to identify, describe and apply patterns and relationships. It is a significant part of the cultural heritage of many diverse societies. The symbolic nature of mathematics provides a powerful, precise and concise means of communication. Mathematics incorporates the processes of questioning, reflecting, reasoning and proof. It is a powerful tool for solving familiar and unfamiliar problems both within and beyond mathematics.

According to this syllabus, mathematics is a human activity that involves processes of thinking, reasoning, conjecturing, generalising and communicating. I find it interesting that just like in the Lesotho syllabus mathematics is viewed as a 'powerful tool' that can be used in solving both mathematical and real world problems. This implies that those who possess mathematical knowledge can use it in different contexts to solve various problems.

When addressing the question of 'what is mathematics?' Boaler (2009) points out that many school learners when asked this question say that mathematics is a list of rules and procedures that need to be remembered. This view comes as a result of students' experience with the school mathematics, which is portrayed to learners as about calculations and formulae. Boaler (2009) takes a view that in order for us to understand what is meant by mathematics we must explore what mathematicians do, and that teachers must make efforts to align school mathematics with mathematics used in various sectors of life such as at a workplace. She argues that "mathematics is not about numbers, but about life. It is about the world in which we live. It is about ideas. And far from being dull and sterile, as it is so often portrayed, it is full of creativity" (p. 26). Boaler stresses the point that mathematicians succeed in problem solving through the use of a range of representations such as "symbols, words, pictures, tables and diagrams, all used with extreme precision" (p. 29). In what follows I make reference to some studies that indicate the value of relating mathematics teaching to real world contexts.

Cobb and Bowers (1999) relate a story of a study that was conducted in Nepal by Beach, where 13 school learners were apprenticed into shopkeeper work and 13 shopkeepers were attending adult education classes. The focus of the study was to compare the arithmetic thinking and reasoning of the two groups. According to Cobb and Bowers (ibid. p. 7) "Beach found that the shopkeepers' arithmetic reasoning was more closely related in the work and school situations than was that of the students". This study suggests that the students, as opposed to the shopkeepers, could not see relevance and immediate use for school mathematics in the workplace.

Boaler (2002) tells a story of two high schools she worked with in the UK that had two different approaches to teaching mathematics. At one school, Phoenix Park, mathematics teachers used a project-based approach to teaching mathematics. In this school, learners were presented with real world problems in the form of projects. Learners used their mathematical knowledge and skills to solve these problems. Describing this study in a later book, Boaler makes a point that "the Phoenix Park students came to view mathematical methods as flexible problem solving tools" (2009, p. 58). According to Boaler, in the other school, Amber Hill, mathematics teachers followed a typical traditional approach. Learners worked on tasks assigned to them by teachers. Boaler describes mathematics classrooms at Amber Hill as peaceful and quiet. She then concludes that because of this approach (repetition of algorithms) "students at Amber Hill came to believe that maths was a subject that only involved memorizing rules and procedures" (p. 59). She further argues that students at Amber Hill developed the idea that their school mathematics knowledge had fixed "boundaries or barriers surrounding it that kept it firmly within the mathematics classroom". These students could not use their school mathematics in any situations other than in school.

Drawing from these examples, it is evident that the way in which learners develop mathematical knowledge is directly related to how they might make use of such knowledge in various situations. If school mathematics is presented to learners as rules, procedures, and definitions to be memorized, then learners might find it hard to apply such knowledge in contexts other than the school setting. On the other hand, if learners learn mathematics in school as a tool to be used in solving realistic problems, then it might become easier for them to use the mathematical knowledge they possess flexibly, in various contexts even outside the school situation. In the next section I focus on a situated theory of learning.

Situated Theory

According to situated theory, mathematics is viewed as a set of practices and activities that mathematics users do (Sfard, 1998). Other researchers in mathematics education have termed mathematics practices as "authentic activities" (Brown, Collins, and Duguid, 1989). According to Brown et al., authentic activities are defined as the ordinary practices of a culture. The term 'culture' here refers to the ways and means in which people use mathematics or mathematicians work. It is important to remember that these ways and means can differ from society one society to the next (Andrews, 2010; Adler & Davis, 2011). The RAND Mathematics Study Panel (2002) defines mathematical practices as what mathematicians and proficient mathematics users do. These practices are performed in well-defined tasks and draw on various resources such as human and material resources. The RAND Mathematics Study Panel identified three core practices of doing mathematics as, "representation, justification, and generalization" (p. 31).

The key idea here is that the situations or environments in which mathematicians work are essentially linked to the representations they use. Thus according to situated perspectives, knowledge cannot be regarded as what an individual possesses, rather knowledge is broadly viewed as being distributed within a community of practice and the resources of the community (Greeno, 1991; Leinhardt and Greeno, 1986). In a school situation, students could be exposed to environments that encourage them to embark on practices of justification and generalization through processes of reasoning within various mathematical representations. These can be quite narrow as in Amber Hill School (Boaler, 2002) or 'real world' and contextually situated as advocated by the Dutch Realistic Mathematics Education (RME) movement (Drekker, 2007).

The Learning of Mathematics

I begin this section with an explanation of constructivism as a theory of learning.

Constructivism

According to a constructivist theory, knowledge is acquired through construction and restructuring of mental structures (Hatano, 1996). The processes of construction and restructuring of knowledge involve the interaction of the learner's existing mental structures and the new structures. The amalgamation of these structures into a unitary construct is a complex cognitive process. Siegler (1995) argues that Piaget identified three processes through which interaction of new and old knowledge occurs, and those processes are *assimilation, accommodation* and *equilibration*. The processes of assimilation and accommodation are subsets of equilibration in that the two processes could be used in explaining how a learner reaches a state of equilibration. Equilibration is a process of the interaction between the known and as yet unknown forms of knowledge. By the known, I refer to already existing mental structures, schemas, and thinking about a certain phenomenon. On the other hand, the unknown refers to that which is new, unfamiliar, and has potential to challenge the known thereby causing 'cognitive conflict' in the mind of a learner (Smith, DiSessa, and Roschelle, 1993). The cognitive conflict would be the disagreement between the known and the unknown. A well-known slogan in the teaching of mathematics at primary school level refers to multiplication of whole numbers, where learners are taught that "multiplication makes bigger". Later on, when the same learners do the multiplication of fractions they come to realize that the slogan does not hold, and that fact causes cognitive conflict. Smith et al. (1993) refer to this disagreement as a state of disequilibrium. Learning occurs when the state of equilibration has been reached. I would then argue that learners would be said to have learned multiplication when they can confidently make distinctions between multiplication of whole numbers and multiplication of fractions. In contrast to this theory of the individual construction of knowledge, one may pose the question: How then is learning explained from a socio-cultural theoretical orientation?

Socio-cultural Theory

The socio-cultural theorists contend that learning is part of social practice. Lave and Wenger (1991) argue that learning takes place within a community of practice. The communities of practice consist of newcomers and old-timers. Newcomers join the community by working through less challenging activities on the periphery within the practice, and learn to interact with other members of the community. Through such interactions, old-timers scaffold newcomers' actions and thinking. As they gain confidence in the culture of the community, newcomers increase their participation. Shard (1998) elaborates on the notion of learning from the situated perspective by referring to what she terms the "participation metaphor". The participation metaphor emphasizes a need for all learners of mathematics to take part in the activities of a community pursuing mathematics. Cobb and Bowers (1999) point out that, such activities include perceiving, reasoning, and talking. Therefore, manifestation of such activities implies that learning of mathematics is taking place. From this perspective, it could be argued that the more learners participate in the activities, the more they gain expertise in the practices of mathematics (Part and Rajas, 2007).

Each participant (learner) in any community of practice has a specific identity that conforms to the norms of that community. Again each member of the community has a specific role to play, which is in harmony with the known practices of the community. So the metaphor here as highlighted by Shard (1998, p. 6) is that "the identity of an individual, like an identity of a living organ, is a function of his or her being or becoming a part of a greater entity". Various parts of the human body play different roles for the proper functioning of the entire body. For example, eyes are appropriate for sight, tongue for taste and speech, and legs for walking. All these parts function together in harmony for better functioning of the body. Similarly, the participation metaphor suggests that learners are like parts of the body that function individually and collaboratively for and within a culture of using or 'doing' mathematics. According to the participation metaphor, learners become knowledgeable through actively participating regularly in well-planned mathematics activities. For any community of practice to function well and efficiently, it must have a common goal, clearly articulated to all members of the community. Therefore, the role of the teacher is to predetermine lesson goals for learners to work towards. The participation metaphor therefore suggests that knowledge takes a form of action (participation) and is elaborated clearly by Rogoff (1998).

The theoretical perspectives discussed above are important as lenses through which we perceive mathematics and learning. In this study, the cognitive theory provides a basis through which I, as a researcher, can come to understand how each participant (student teacher) manifests his/her internal mathematical representations on paper as they solve arithmetic tasks, and also in their teaching activities when on school placement. From a situated perspective, the culture of mathematics teaching in Lesotho schools can be highly challenging; traditionally, it followed a pattern of transmission of rules and procedures, which were demonstrated and then memorised through repetition and choral answering. On the same note, the socio-cultural theory helps me to recognize that participants in this study are less experienced teachers. They join the community of practising teachers, with limited experience and knowledge of teaching mathematics. It could be said in socio-cultural terms that the prospective teachers are at the periphery and are learning to master the activities of the community. However, it is worth noting that these perspectives are mainly theories of learning. I turn now to exploring how learners can be said to 'know' mathematics, which in turn leads to different notions of teaching.

Conceptualizing Understanding

There are two types of mathematical understanding, coined more than thirty years ago by a psychologist Richard Skimp (1976) namely: 'instrumental' and 'relational'. According to Skimp, instrumental understanding refers to rigid and inert knowledge of rules, procedures, algorithms that helps one to obtain correct answers but without understanding why and how one got them. Skimp refers to this kind of understanding as "rules without reasons" (p. 2). On the other hand, he described relational understanding as "knowing both what to do and why" (p. 2), and the process of learning relational mathematics as "building up a conceptual structure" (p. 14). In other words, the 'structure' here refers to a web of interrelated mathematics concepts.

When building on Skemp's notion of understanding, Suggate, Davis, and Goulding (2006) argue that teachers who possess relational understanding are armed with a 'cognitive map' of relevant mathematical concepts that they can use flexibly in various situations. It is important though to note that both types of understanding are crucial in teaching and learning of mathematics. Knowing algorithms can be extremely useful and efficient in some mathematical situations. However, such procedures need to be rooted within a rich relational understanding of the concepts studied. The key idea to note is that students who have developed relational understanding of mathematics are capable of making connections between various concepts and procedures. Unlike those with instrumental understanding only, they are able to make use of various modes of mathematical representations as resources to help them reconstruct a forgotten concept. Ma (1999) has identified such mathematical knowledge as part of what she termed the 'profound understanding of fundamental mathematics' and commented on the rich knowledge packages which Chinese teachers in her study held about each mathematical concept they sought to teach.

> The Chinese teachers' profound understanding of the meaning of division by fractions and its connections to other models in mathematics provided them with a solid base on which to build their pedagogical content knowledge of the topic. They used their vivid imaginations and referred to rich topics to represent a single concept of division by fractions (p. 78).

Sierpinska (1992) identifies five constituents of understanding namely: identification, discrimination, generalization, synthesis and using. According to Sierpinska these five categories of acts of understanding are fundamental in acquiring mathematical knowledge. Learners understand what a concept is if they are able to identify that concept amongst other concepts; when they are able to differentiate between two or more concepts; and if they could provide examples and non-examples, instances and non-instances of the mathematical concept under discussion. Learners should be able to generalize about the concept, and they must be aware of the possibilities of extending it into a range of applications. They should be able to make connections between isolated facts, results, properties, relations, and concepts so that they are organized into consistent wholes. Learners should be able to apply the concept effectively and appropriately in many situations. They should also be able to make connections between the concept and any other related concepts. Therefore, in this study, student teachers that had good understanding of mathematical concepts were expected to make connections between different possible concepts through the use of multiple representations, such as in the case of dealing with operations for addition, subtraction, multiplication and division of natural numbers.

When elaborating on the notion of conceptual understanding as one of the five strands of mathematical proficiency, Kilpatrick, Swafford, and Findell (2001) suggest that conceptual understanding incorporates comprehension of mathematical concepts, operations, representations and relations. Kilpatrick et al. (ibid. p.118) point out that "although teachers often look for conceptual understanding in students' ability to verbalize connections among concepts and representations, conceptual understanding need not be explicit". They further argue that:

> A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students' conceptual understanding is related to the richness and extent of the connections they have made (p. 119).

It follows from the above quotation that students' conceptual understanding could be recognized in their ability to make connections between various mathematical representations. Such connections could be made verbally or in writing (mathematical communication). Bills and Gray (1999) in the UK, also posit that learners' talk, writing, drawing or actions all indicate the nature of their understanding of a mathematical concept. I argue that it is through mathematical reasoning that connections between two or even more external representations are made. Mathematical thinking and reasoning as one of the crucial means through which mathematics is taught and learned, develops and demonstrates understanding of mathematical concepts and processes (Boaler 2009; Mason 2005; Rowland et al, 2009). Therefore, I feel challenged when I try to explain teaching from the perspectives outlined above. In the following section I turn to more appropriate theoretical frameworks that primarily focus on the knowledge required for teaching.

Pedagogical Content Knowledge

The theoretical framework within which I conducted my analysis in this study is what Shulman (1986; 1987) referred to as pedagogical content knowledge (PCK). Shulman argued that for decades there were two groups of teachers. First, he identified teachers who had rich knowledge of content and yet were unable to teach it well to learners. The other group of teachers had skills to teach well, although they had weak knowledge of content. The question then and probably even now is; which type of teachers do schools need in order to teach mathematics proficiently? A more important question in relation to this study might be how can both types of teachers be combined to provide the best opportunities for mathematics teaching and learning?

This study is situated within the PCK framework that focuses on teaching as the daily work of teachers and also as the substance of the teachers' profession. Shulman, (1987) argued that for effective teaching, teachers should acquire what he termed "a knowledge base". According to Shulman, the knowledge base aspect includes: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK), and knowledge of learners and their characteristics. He argues that PCK is the most crucial of all categories because "it represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). PCK is what teachers need to acquire or develop in their training and through experience gathered over years of teaching. When researching with student teachers that were preparing to become primary school teachers in Ireland, Corcoran (2008) refers to pedagogical mathematics knowledge as mathematical knowledge in teaching (Kit). Her emphasis is on the opportunity for teachers to develop their mathematical knowledge in the act of teaching as they closely engage with children's ideas, responses and representations. Although PCK is a construct that could be spoken of in all school subjects, in what follows I focus on the PCK in mathematics.

When building on the notion of PCK, Ball, Lubienski, and Mewborn, (2001) argue that it is a unique kind of knowledge that intertwines content with aspects of teaching. They suggest that:

> Such knowledge is not something a mathematician would have by virtue of having studied advanced mathematics. Neither would it be part of a high school social studies teacher's knowledge by virtue of having teaching experience. Rather, it is knowledge special to the teaching of mathematics (p. 448).

In South Africa, a study was conducted that was concerned with issues of what mathematical content is constituted and privileged in higher institutions of learning and accounts for mathematics knowledge for teaching (MK*f*T), and how much mathematics and mathematics-methods courses should be covered in teacher education programs (Adler, Davis, Kazima, Parker, and Webb, 2005). The study focused on the nature of mathematics knowledge for teaching relevant to in-service teachers and on the ways of knowing it. Adler et al (ibid.) found considerable disparity between teacher education institutions in South Africa and deficits in teacher knowledge across institutions. It seems reasonable to suggest that the situation of mathematics teacher education in Lesotho should be open to similar investigation. With regard to PCK, Hill, Rowan, and Ball (2005) suggest that:

> Teachers of mathematics not only need to calculate correctly but also need to know how to use pictures or diagrams to represent mathematics concepts and procedures to students, provide students with explanations for common rules and mathematical procedures, and analyse students' solutions and explanations (p. 372).

The above quotation points out that there is a need for teachers to practise effective use of their mathematical knowledge for teaching in order to represent mathematical concepts. However, Hill et al (ibid.) argue that making use of mathematical knowledge in the practice of teaching is challenging. In a previous study, Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992), report an example of a middle school student teacher in their study, Ms. Daniels, who despite having taken advanced mathematics courses was unable to provide a 'correct representation' for division of fractions in class. Their argument ties up well with the findings that emerged from a study with teacher trainees in the UK conducted by Carré and Ernest, (1993). In this study, they found no difference in effectiveness of teaching mathematics between PGCE teacher trainees who had specialised in mathematics for their degree programs and those who did not specialise. It emerged that when classroom teaching performance was assessed there was virtually nothing to distinguish mathematicians from others in the cohort. This finding is encouraging in the Lesotho context where mathematics achievement among teachers is generally perceived to be limited and weak.

The Knowledge Quartet

One very powerful framework for conceptualising mathematical knowledge in teaching builds on Shulman's notion of PCK. This framework was devised by researchers in the UK and is known as the Knowledge Quartet (KQ) (Rowland, Huckstep and Thwaites, 2005). According to Rowland et al. the Knowledge Quartet (KQ) has four dimensions namely: foundation, transformation, connection, and contingency. Each of these four dimensions is associated with a certain number of codes or indicators that can be identified with the teacher's activities in planning and teaching a mathematics lesson. See Table 3.5 on page 93 for a list of these indicators. The KQ can be used for at least two purposes, namely as a teacher development model, and as an analytical tool. The KQ as an analytical tool is used in educational research for purposes of analysing student teachers' or even teachers' lessons on mathematics. In this case, the researcher observes the lesson in the light of the KQ dimensions. He or she identifies incidences where each of the four dimensions is manifested during instruction. The focus here is mainly on the teacher and not necessarily on learners. For example, the research focus might be on how the teacher responds to learners' contributions, which is part of the *contingency* dimension. Or the research focus could be on the extent to which the teacher refers to the textbook and relies on procedures, or heeds learners' errors. All of these 'indicators' are part of the foundation dimension of the KQ. The teacher's beliefs about mathematics and attitudes to teaching mathematics are also part of the foundation dimension. However, the interconnectedness of the four dimensions is manifested in class during instruction, with transformation being important, in that a representation, or an example, or an analogy used by the teacher might trigger learners' thinking, and the teacher could then begin to deal with their contributions along the contingency dimension. The teacher could refer to a representation and draw learners' attention to some patterns or links between concepts (the connection dimension). Potentially, this could lead to a more open learning environment for mathematics. However, there are tensions between researchers who use KQ in their work, with regard to the dimension that precedes others. For example, Corcoran (2008) identifies contingency as the most central dimension.

These researchers' notion of PCK emerged as a result of studies that were done with teacher trainees, so I find their theories of teacher knowledge relevant in this study. In this study I too am conducting my research with student teachers. I argue that trainees' understanding, choice, and use of representations in doing arithmetic activities is part of mathematical knowledge for teaching. Goulding, Rowland, and Barber (2002) contend that being able to make explicit links between different representations (e.g. verbal, concrete, numerical, and pictorial) is part of classroom mathematical discourse.

Mathematical Representations

The notion of mathematical representation(s) in this study is taken to refer to all resources that a teacher deliberately uses in class to help learners to have access to the intended mathematics content and be in a position to communicate mathematical ideas well to others. According to Van de Walle (2001) the National Council of

Teachers of Mathematics (NCTM) in the United States and Canada came to realise the importance of representations and the significant role they play in the teaching and learning of school mathematics in the year 2000. The notion of representation since then became an important part of the 'process standards' in the NCTM document known as the *Principles and Standards for School Mathematics*. Van de Walle (ibid. p.7) states that the other four 'process standards' are "problem solving; reasoning and proof; communication; and connections". He argues that:

> The five process standards should not be regarded as separate strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics and therefore should be seen as integral components of all mathematics learning and teaching.

It is clear from the quotation above that representations as part of the process standards play an important role in helping learners comprehend school mathematics content such as number and measurement. If the notion of representation has been identified as crucial in the developed countries like the United States, then its importance in the teaching and learning of mathematics in African countries such as Lesotho is inevitable. The citation above also shows that *representation* should not be regarded as part of the school mathematics content; rather it is the means by which mathematics content should be taught and learned. It is interesting that the NCTM considered the role of *representation* in the teaching and learning of mathematics so important that the title of the 2001 NCTM yearbook was 'The Roles of Representation in School Mathematics' (Cuoco and Curcio, 2001).

Several researchers have made efforts to investigate the notion of representation in the field of mathematics education. According to Elia, Gagatsis, and Demetriou

(2007) a representation is any configuration of characters, images or concrete objects that stand for something else. These can be both teacher and learner generated. To emphasise and elaborate more on the notion of representation Kaput (1998, p. 268) argues that,

> We treat regularly symbols in place of what we know they stand for ... we know that a picture of a person is different from the person, or the drawn figure is not really a circle, the toy car is not a real car, the square we gesture in space with our hand is not a real square.

This suggests that a representation is something that stands for a mathematical construct, although the representation itself is not the real mathematical construct it stands for. Elia et al. (2007) take a view that a single representation cannot describe fully a mathematical construct and the fact that each representation has different advantages, "using multiple representations for the same mathematical situation is at the core of mathematical understanding" (p. 659). Certainly, the business of teaching and learning of mathematics calls for the use of mathematical representations as powerful resources that enhance mathematical understanding and communication in lessons. Goldin and Shteingold (2001, p. 3) also define a mathematical representation as "a sign or a configuration of signs, characters, or objects ... it can stand for (symbolize, depict, encode, or represent) something other than itself". They make an example of the numeral 5 and argue that 5 can represent a particular set containing five objects determined by counting. Goldin and Shteingold argue that mathematics teachers and learners represent situations all the time in teaching and learning mathematics respectively.

Teachers of mathematics are concerned with making mathematics comprehensible to learners. In order for learners to understand abstract mathematics concepts, they have to construct mental images of the concept in question. The learner should also be able to manifest his or her understanding by physically demonstrating the internal mental structure of the concept. Hence Goldin and Shteingold (2001) talk about mathematical representations as both 'external and internal'. External representations refer to all forms of the mathematical concepts that can be seen, touched and talked about, things that can be pointed at and brought to somebody's attention. On the other hand, internal representations refer to mental images and structures that learners construct for mathematical objects and processes (Cobb, Yackel, and Wood, 1992). Goldin and Shteingold (2001) view internal representations as systems of internal cognitive depictions that take various forms such as verbal, imagistic, formal notation, strategic and heuristic, and affective systems of representation.

Pape and Mourat (2001) take this notion of internal and external representations further and argue that:

> Within the domain of mathematics, representations may be thought of as internal abstractions of mathematical ideas or cognitive schemata that are developed by a learner through experience. On the other hand, representations such as numerals, algebraic equations, graphs, tables, diagrams, and charts are external manifestations of mathematical concepts that act as stimuli on the senses and help us understand these concepts (p. 119).

Pape and Mourat (ibid. p.119) identify what they refer to as "the zone of interaction of internal and external representations", which they argue takes place during the process of teaching and learning. While the 'zone of interaction' may seem to be taking place at an individual level, on the contrary, Pape and Mourat maintain that this interaction often takes place within social interaction. The social interaction may refer refer to the classroom context(s) where a mathematics teacher carefully chooses and uses appropriate external representations to stimulate learners' thinking. A learning context where learners challenge one another's 'representational thinking' (Pape and Mourat, 2001) is encouraged. According to Pape and Mourat (2001: p. 120), representational thinking is regarded as the "learner's ability to interpret, construct, and operate (communicate) effectively with both forms of representations, external and internal, individually and within social situations". Internal mathematical representations cannot be seen directly, rather, learners' internal representations are made manifest as they interact with external representations. There is evidence of a shift in thinking here. Instead of viewing mathematical representations "in agentless terms" (Gravemeijer, Lehrer, Van Oers and Verschaffel, 2010, p. 2), in the new instructional approach, representations are viewed as dynamic entities that facilitate evolvement of meaning through learners' personal involvement. Such an improvisational approach to teaching would require more thoughtful and committed teachers. Goldin and Shteingold (2001) argue that experienced and skilled teachers pay attention to their students' words, written work, use of external representations or calculators and computers and try hard to understand individual's conceptions and misconceptions.

Learning from Students' Errors

There is some evidence of this approach expressed in print by a Lesotho high school teacher (Kokome, 1987). In volume 20 of the *Lesotho Science and Mathematics*

Teachers Association Newsletter (LSMTA), she details numerous errors that her students make when doing fractions, percentages and decimals even in forms D and E and says "I do not know how to correct these errors at this stage" (p. 29). As a mathematics education researcher working in Lesotho I was delighted to find evidence of a thoughtful and active teaching profession publishing material such as this. However, its existence only in a print museum at Morija, belonging to the Lesotho Evangelical Church, points to the historically divided influences on education along denominational lines. The MOET funded free primary education initiative appears to be starting without awareness of resources such as this professional newsletter. I can only imagine the potential for teacher professional development of practising teachers gathering to discuss how to deal with students' errors that Ms Kokome highlighted. In the next section, I focus on the roles of representation in teaching.

Use of Mathematical Representations in Teaching

Research on the role of representations in making mathematical ideas accessible to primary school learners, especially in problem solving, has been well documented (Klein, Beishuizen and Treffers, 1998; Fennell and Rowan, 2001; Harries and Barmby, 2008). The choice and use of representations in the teaching of mathematics is one of three contributory codes to the 'transformation' dimension of the Knowledge Quartet (Rowland, Huckstep and Thwites, 2005), already mentioned as a way of conceptualising PCK. In identifying the transformation dimension thus, Rowland et al. argue that to be successful in class, a teacher has to be proficient in transforming his/her mathematics knowledge into something accessible to learners because, "in order to present ideas to learners, the teacher must find ways of representing what they themselves already know" (p. 30). They view such representations as taking the form of analogies, illustrations, examples, explanations and demonstrations. Rowland et al (ibid.) cite an example of teaching the concept of 'positive whole numbers' and point out that in order to be successful a teacher may choose to use the following representations: "objects, base-10 blocks, place value cards, number line, and grids" (p.31).

Many scholars agree that representations are useful resources that have an inherent objective to enhance learners' mathematical proficiency (Barmby, Harries, Higgins, and Suggate, 2009; Kilpatrick, Swafford, and Findel, 2001). In Sesotho culture there are several inherent mathematical representations that assist learners to comprehend counting. For example, counting from one to ten is recited as follows; *'ngoe* (1), *peli* (2), *tharo* (3), *'ne* (4), *hlano* (5), *ts'ela* (6), *supa* (7), *robeli* (8), *robong* (9), and *leshome* (10). Basotho use the fingers of their hands as representations for numbers when counting. What is interesting is that the word for 'six' in Sesotho is *ts'ela*, which means 'cross over'. The literal meaning is 'cross over to the next hand'. Then the word for seven *(supa)*, means 'point at'. *Robeli* (8) means 'bend two fingers', and r*obong* (9) means 'bend one finger.' When conceptualising the notion of 'embodied arithmetic' Lakoff and Núñez (2000, p. 51) argue that our fingers as humans have 'ordering capacity' and as such they:

> come in a natural order on our hands. But the objects to be counted typically do not come in any natural order in the world. They have to be ordered – that is, placed in a sequence, as if they corresponded to our fingers or were spread out along a path.

In Lesotho, learners in beginning classes (Standards 1 – 3) are taught mathematics in Sesotho. Therefore, teacher trainees have to be conscious of mathematical representations inherent in Sesotho culture, so that they can use them to make mathematics accessible to all learners. Primary school mathematics teachers have representations in abundance to choose from and therefore, should be vigilant in selecting key representations for teaching various mathematics topics.

Choice of Representations

While research shows that mathematical representations play a pivotal role in enhancing learners' conceptual understanding, the main challenge mathematics teachers are faced with is the selection of key representations for teaching a certain concept effectively. Harries and Barmby (2008) argue that in teaching addition and subtraction at early primary mathematics lessons, a number line is the key representation for demonstrating the essential characteristics of these operations. Elia, Gagatsis and Demetriou, (2007, p. 659) concur and further argue that "in elementary mathematics, the number line is a representation that is widely used for teaching of basic whole number operations and arithmetic in general". A substantial body of work associated with the Freudenthal Institute promotes the use of the empty number line as a powerful representation which can accommodate shifts in learners' growing sophistication in understanding number operations (Klein, Beishuizen and Treffers, 1998). As such, the empty number line is an example of a powerful and dynamic mathematical representation. Klein et al argue that such instructional models are ideal and suitable for communicating mathematics ideas effectively to others.

These scholars add that the empty number line is a key representation for teaching number because:

- it is well-suited to link up with informal solution procedures because of the linear character of the number line;
- it provides the opportunity to raise the level of the students' activity;
- it gives students freedom to develop their own solution procedures;
- it fosters the development of more sophisticated strategies (progressive mathematization);
- it allows students to express and communicate their own solution procedures and also facilitates those solution procedures;
- marking the steps on the number line functions as a kind of scaffolding that is, it shows which part of the operation has been carried out and what remains to be done;
- the empty format stimulates a mental representation of numbers and number operations (addition and subtraction);
- it is suitable for the representation and solution of nonstandard context or word problems;
- it helps learners to cognitively get involved in their actions that is, learners concurrently solve the computation task within a problem while drawing jumps on the number line;
- Learners become aware of the closeness of the numbers (e.g. difference between 61 and 59) by marking these numbers on the empty number line (p. 445).

Murata (2008) makes a strong case for the manner in which Japanese teachers and Japanese textbooks make progressive use of a variation of the number line in the form of tape diagrams. She defines tape diagrams as tape-like representations that visually illustrate key relationships among quantities in a problem. According to Murata (2008, p. 400)

> Tape diagrams, as other representations, provide and facilitate students discussing their ideas based on relationships (among different quantities, operations mathematics domains) in the problem. It helps to maintain the meanings grounded in the original problem in the discussion as solution methods are presented.

She argues that "as learners come to use particular representations in learning activities, the representations help guide the learning process and become a part of the learners' cognition" (p. 376). Murata makes reference to the mediating role that 'tape diagrams' as key representations in Japanese curricula play in making mathematical concepts accessible to Japanese learners.

Pirie (1998) makes reference to visual representation and argues that it is a powerful means of mathematical communication in mathematics lessons. She cites an example of a teacher drawing a pair of Cartesian axes on a chalkboard and talking of y (vertical) and x (horizontal) axis. A teacher in class may choose to use a graph as a representation in order to help learners understand the relationship between the two entities. The choice of representation is important in that the teacher must decide which representation is most appropriate to use in teaching a particular concept and be able to recognise and link the chosen representation with those being constructed by learners. If the representation is not well chosen for a particular mathematical concept, then instead of its being a resource, it will impede learners' understanding. Therefore she argues that teachers of mathematics have to be careful when choosing representations.

Haylock (2006) poses an interesting question to his readers; "can you explain the distinction between solving a problem and representing it?" As discussed earlier in this chapter, both the *problem solving* and the *representation* according to NCTM are the process standards through which mathematics content must be meaningfully taught to learners at all stages of schooling. Mathematical representations are powerful resources that a teacher and students can utilize to enhance learning of mathematics. Haylock (ibid. p. 228) distinguishes between symbolic representations in arithmetic and in algebra, as a means through which a problem like the one below could be understood:

> A plumber's call-out charge is £15, and then you pay £12 an hour. How many hours' work would cost £75?

Haylock shows that in arithmetic the representation might be $75 - 15 = 60$, then $60 \div 12 = 5$ while in algebra the representation might be $12x + 15 = 75$, and then followed by solving for x. Arithmetic involves numbers and a box (□) while algebra involves numbers and variables and/or unknowns such as x. In Chapter 1, I pointed out that in the Lesotho primary mathematics syllabus algebra is not included. Although Sets are introduced in Standard 2, algebra in the Lesotho schooling system is formally introduced in Form B (the second year of secondary schooling). In primary schools, teachers teach arithmetic. However, it is important to note that arithmetic is by no means less mathematics than algebra because whatever is taught in algebra it is also taught in arithmetic. I will give two examples to substantiate this point: in algebra teachers teach learners to *factorize* expressions such as $x^2 + 3x$ and in arithmetic primary school teachers teach learners to *factorize* expressions like 10. The answer for the first expression is $x(x+3)$ and for the latter arithmetic expression is 2×5 or 10×1. In algebra, mathematics learners solve for the indicated unknown or variable (e.g. x) in a given equation, for example $12x + 12 = 75$. Similarly in arithmetic, learners solve equations, but instead of using a letter, a box is used for an unknown and learners solve for the box. For example, $79 + \square = 280$. I wish to argue here for primary school teachers to become aware of the algebraic structure of arithmetic, which repeats a claim made by Domoney and Price (2004) in the UK.

When focusing on the notion of functions, Thompson (1994) contends that it is important for learners of mathematics to make connections between various representations and realise that the concept represented remains invariant. Thompson further warns that the situation being represented must be clear in students' minds, otherwise if students "do not see something remaining the same as they move among tables, graphs, and expressions, then it increases the probability that they will see each as a topic to be learned in isolation from the others" (p. 23). Thompson's point is important in that learners must not consider representation as content to be learned, rather as a means (process) toward understanding the content. If the connections that learners make between two or more external representations might demonstrate their level of understanding of mathematical content to the teacher, then by implication, teachers need to be encouraged to construct several connections among representations of the same content. Recent research would indicate that Thompson's perspective (1994) has been developed into what he calls "didactic objects" and "didactic models" (Thompson, 2010, p. 214). By *didactic object* he means 'a thing to talk about' in teaching mathematics. A particular representation (accompanied by key discussion points) becomes a *didactic object* "in the hands of someone having in mind a set of images, issues, meanings, or connections affiliated with it that focus on interpreting it … and which the teacher realises must be discussed explicitly" (Thompson, 2010, p. 205). By *didactic model* he means a "scheme of meanings, actions and interpretations that constitute the instructor's or instructional designer's image of all that needs to be understood for someone to make sense of the didactic object in the way he or she intends" (Thompson, 2010, p. 215). He also states that "a didactic model" is for teachers and instructional tool designers "of what they intend students will understand and how that understanding might develop" (p.216). I am challenged as a college lecturer in mathematics education and as a researcher to think in terms of representations as didactic objects and to formulate didactic models for their use with student teacher learners.

Making Connections between Representations

Recent studies (Thompson 2008; Harries & Barmby 2008; and McLeay 2008) on teaching and learning of multiplication at the primary school level highlight the importance of mathematical representation in enhancing learners' mathematical proficiency. Harries, Barmby, Suggate, and Higgins (2008) maintain that representational systems are important to the learning and teaching of mathematics because of the 'inherent structure' contained within each representation. They further argue that the structure of a representation has potential to either give learners access to mathematical understanding or possibly to constrain learners' conceptual understanding. This appears to contradict the more flexible understanding of representations in teaching already mentioned (Gravemeijer, Lehrer, Van Oers and Verschaffel, 2010) seems open to a possible *learning paradox* where the mathematical ideas and relationships supposed to be inherent in the representation may not be accessible to the learner, who has not yet made the necessary connections (Cobb, Yackel, and Wood, 1992). However, in general, it can be said that mathematical representations provide a context for both learners and teacher to talk about, which is a more powerful approach than the traditional ways of teaching algorithms at primary school level, such as in the division of fractions. Nonetheless, mathematics teachers still have to be cautious of the structure of representation they choose to use in class. The representation chosen should be cognitively at the level of learners. Stein, Smith, Henningsen, and Silver (2000) argue that if the cognitive demands of a mathematics task are too high for learners, then there is a high possibility that learners will resist engaging with the task, and confusion and disinterest in learning mathematics could be the result. This also holds with the representation. For example, if a teacher chooses to use a *multiplication square* in a Standard 3 class for doing multiplication, the representation might be so confusing for the learners that they may even lose interest in such a lesson. But a *multiplication array* might be more suitable for teaching multiplication at this level (Standard 3) than a *multiplication square*. Alternatively, posing a 'real world' problem (for example, how many different ways might 24 sweets be arranged in a box?) could lead to learners modelling the problem with objects or drawings, thus producing multiple representations for discussion to facilitate building understanding (Hersh, Fosnot and Cameron, 2005).

It appears to me that the work of Harries et al (2008), on representations in teaching and learning of mathematics at primary school level resonates well with Thompson's work on representing numbers, in that both studies seem to emphasize a need for learners to make connections between different representational activities (Thompson, 2008) or representational systems (Harries et al., 2008) as part of developing conceptual understanding. The appropriate use of representations in teaching includes an ability to choose tasks that have contextual relevance to learners and have potential to encourage learners to make connections between multiple representations as they work out answers. Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, and Ball (2008), when addressing the importance of teachers making connections among multiple representations, argue that:

> The links must be mathematically significant—for instance, pointing out connections between representations in ways that allow students to grasp how representations are alike or different, how pieces of one relate to pieces of another, or what one representation affords that another does not (p. 506).

Marshall, Superfine & Canty (2010) also argue that the instructional value of making connections between multiple representations is that it helps learners see mathematics as a web of connected ideas and not as a collection of arbitrary, disconnected rules and procedures. Contextual tasks provide meaningful settings for learners to realise the connection between school mathematics and its use in the real world for solving problems (Boaler, 2009). It would seem thus far that *representation* as one of the 'process standards' makes mathematics content comprehensibly meaningful to learners through tapping onto other 'process standards' namely: problem solving, reasoning, communication, and connections (Van de Walle, 2001). What emerges from the reviewed literature in this section is that in order for learners to become proficient with mathematics content and its application in various contexts, teachers have to set up appropriate problems for learners at all grades so that learners can communicate mathematically, think and reason mathematically and make various mathematical connections. All this is made possible through well-chosen representations.

Semiotic Activity and Emergent Modelling

So far, my literature review has followed mainstream work on theories of mathematical learning and teaching of primary school arithmetic. It has led me to think of mathematical knowledge for teaching and how this might be conceptualised (Shulman, 1987; Rowland et al, 2005). This brought me to work of authors involving the use and meaning of representations in teaching mathematics (Klein, Beishuizen and Treffers, 1998; Thompson, 2010; Murata, 2008). The nature of mathematics has two forms; the mental ideas that constitute the deep structure of mathematics and the many words and symbol which people use to represent and communicate the mental ideas. Regardless of cognitive or situated, constructivist or socio-culturalist theoretical framework, the studies I have mentioned are based on particular notions of mathematics teaching where responsibility for 'handing-on' the mathematical content rests mostly with the teacher and the society, aided by the use of representations "as embodiments of mathematical concepts" (Gravemeijer, Lehrer, Van Oers and Verschaffel, 2010, p. 1). However in recent times, this paradigm has been challenged on a number of fronts leading to what is claimed may eventually prove to be "the abandonment of a representational view" (ibid. p. 2). I will now try to trace this paradigm shift and relate it to my proposed study.

The Role of Semiotics in Mathematics Education

Semiotics is the study of signs, symbols and signification thus it can be said to include communication and meaning making. A 'sign' can be a word or a sound or a visual image. The French philosopher Saussure's work on linguistics (cited in Whitson, 1997) gave rise to a to 'a two-sided psychological entity' by which he defined a combination of a concept and a sound pattern which together form a 'linguistic sign', meaning that for example, the word 'chair' comprises the idea of a seat with a back and four legs (concept) and the symbolic sound which a particular language group or society, has assigned to it (sound pattern). The order of the pair of words was changed to '*signifier* and *signified*' in order to emphasise that the concept or 'signified' was not of any more significance than the 'signifier' or word/sound. This led to the notion of a dynamic, reflexive 'chain of signification' where when people are in communication, one 'sign combination' becomes the 'signifier' in a new sign combination. Gravemeijer (2010) explains it this way:

> The initial meaning of the preceding sign that originated in relation to certain concerns and interests is in fact replaced by a different meaning when the succeeding sign is constituted and used in practices that are motivated by different concerns and interests (p.17).

He demonstrates the application of this theory in the candy factory instructional sequence (McClain, Cobb, and Bowers, 1998), where learners start with unifix cubes to represent candy, and then start using pictures to represent collections of candies. This way, one representation has come to be substituted for the other, but there is more to the signifier/signified chain than a change of representation. The new representations bring some properties to the fore and relegate others. In this case, size and colour of the stacks of cubes are not important while the focus now is on numerical quantities. The meaning of the sign changed when it became the signifier in a new sign combination but the meaning can also change with use. This way of thinking has significance for the relative importance teachers attach to internal and external representations and whether they value representations as static entities or dynamic processes, which they share with learners or elicit from them.

From Models to Modelling

The Realistic Mathematics Education (RME) movement has made the notion of 'mathematizing' commonplace in mathematics education internationally. According to Hersch, Cameron and Fosnot (2004) "the teacher's goal is to enable children to mathematize – to act on, and within situations mathematically (p. ix). This means that mathematics learning is seen as constructing solutions to realistic problems in a social situation. The RME researchers talk about mathematical models as being essential to mathematizing. By mathematical models they mean "mental maps of relationships that can be used as tools when solving problems" (Fosnot and Dolk, 2001, p. 95). This definition is very similar to my initial understanding of mathematical representations (Bruner, 1966). It is different however, in that RME stresses the evolving nature of learners' construction of mathematical ideas and relationships. RME has grown from its beginnings in Freudenthal's idea of 'mathematics as a human activity' (Gravemeijer and Stephan, 2010, p. 146) with research into how children learn mathematics and publication of professional development materials for teachers both in Europe and the US. Thus, theoretical underpinnings of domain-specific pedagogy relating to each mathematical model are constantly evolving. RME teaching has been called 'guided reinvention' (ibid, p. 164). For learners, Gravemeijer and Stephan

(2010) state that the meaning associated with a model shifts over time, as they reason with it and reorganize their mathematical activity with it. By this they mean that "it is the dialectical process of symbolizing and development of meaning that underlies the constitution of new mathematical reality and the corresponding model-of/model for shift" (p.167). By this socio-constructivist way of thinking, RME models are linked to Thompson's notion of didactic models. They imply that teaching mathematics is a highly complex task, saying it "can be characterized as a journey across a landscape' (Hersch, Cameron and Fosnot, 2004, p. ix). These researchers cite Ball (1993, p. 159): "Teachers must simultaneously maintain a complex and wide-angled view of this territory, all the while trying to see it through the eyes of the learner exploring it for the first time." It is in this spirit of maintaining a 'complex and wide-angled view of the territory that I approach my research project.

Conclusion

In conclusion, I wish to argue that representations serve a "multiplicity of roles" (Meira, 2010, p. 87) in mathematics education. Knowledge and use of representations (external and internal) are necessary for teachers in understanding the mathematics they propose to teach and are helpful in promoting and enhancing learners' understanding of mathematical concepts and procedures. Representations, both those presented by the teacher (for example, a multiplication square) and emergent, informal representations (for example, an invented algorithm generated by a learner) also provide useful situations in mathematics lessons through which learners and teachers can talk, think and reason mathematically.

3. Methodology

In this chapter, I discuss methodological issues addressed in this research project. I adopted a four-tiered approach (Lesh and Kelly, 2000) in order to conduct this study, because this approach allowed me as a researcher to work with participants over a period of three years. I also combined mixed methods research (Burke Johnson and Onwuegbuzie, 2004) to obtain insights into student teachers' understanding of mathematical representations and the ways they use such representations while teaching mathematics in primary schools in Lesotho. The four tiers are outlined below.

In Tier 1, two hundred and twelve first year student teachers voluntarily participated in filling in a survey instrument that required general information about the participants and also comprised mathematics tasks that sought to establish participants' understanding of mathematical representations. In Tier 2, ten student teachers were invited from the two hundred and twelve participants in Tier 1 to participate in what I hoped would be clinical interviews (Ginsburg, 1997) where each participant was asked to elaborate on their responses to the survey instrument in Tier 1. Tier 3 involved five student teachers when they were on teaching practice (TP) in schools during the second year of the diploma in education primary (DEP) programme. These five participants, who were invited to volunteer from among the ten in Tier 2 were each observed once teaching a mathematics lesson. Four of the five lessons were taught to Standard 4 learners in four different schools located within the Maseru region. In Tier 4, the five participants who took part in Tier 3 were again invited to share their thoughts with me as the researcher in video-stimulated recall interviews (Pirie, 1996). The video-stimulated recall interviews were conducted with the five participants reflecting on their respective lessons taught during the teaching practice session. These interviews were carried out while the participants were in the third year of the diploma programme (DEP) at the Lesotho College of Education (LCE). The overall project involves multiple case studies (Stake, 2005). The procedure employed in identifying and engaging participants in each of the four tiers is outlined in this chapter. I also discuss how the data in each tier were collated and analysed. Later in the chapter, I discuss how I attended to ethical issues in the whole project.

Methodology

There are several ways through which a study of this nature could have been done. However, I chose to follow a multiple-case study design (Stake, 2006) because it appears to suit my purpose best. Stake's discussion on the notion of multiple-case study suggests that the approach is suitable for large projects that involve different cohorts of participants in different settings over a stipulated time frame. While in this study I was focussing mainly on one cohort of student teachers, multiple cases emerged from the four tiers. Stake (2005, p. 446) argues that individual cases in the *collection* are usually "chosen because it is believed that understanding them will lead to better understanding, and perhaps better theorising about a still larger collection of cases". From this, I take permission to choose particular participants in tier 1 for further study in tier 2, and again, particular participants in tier 2 for further study in tiers 3 and 4. In this way I hope to build a case study of the collection of DEP students in relation to their understanding and use of mathematics representations over their three years on the course.

Mouton (2001, p.149) defines case studies as "studies that are usually qualitative in nature and aim at providing an in-depth description of a small number of cases". However, Bryman (2008, p. 53) on the contrary argues that "there is a tendency to associate case studies with qualitative research, but such identification is not appropriate". He presents a broader position of a case study and strongly contends that case studies are sites for the employment of both quantitative and qualitative research. Either of these methods could be used to gather data from case studies. According to Bryman (ibid.) case studies should not be confined to empirical sites such as an institution (school). He makes the following point:

> I would prefer to reserve the term 'case study' for those instances where the 'case' is the focus of interest in its own right. (p. 53)

What transpires from this quote is that the researcher must pay attention to the focus of the study when he or she identifies cases. For instance, if the study is concerned with how a certain teacher teaches fractions, the case should not be the school where the teacher works; rather it must be the teacher and his class where learners are being taught fractions. Therefore, in this study, the college (LCE) and the five schools where the five participants in Tier 3 practised teaching are not regarded as cases *per se*. What remains the case is the DEP cohort of students who agreed to participate in this study. Opie (2004) concurs with Bryman that a case study might involve one student or a class of a hundred students. The number of participants here is not considered so important; what seems crucial is the in-depth description of what emanates from the cases concerned.

Table 3.1 below is intended to provide a picture of the whole data gathering process:

Table 3.1: The research design

Research Question(s)	Source(s) of Data	5 students	10 students	212 students
1. What mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?	Survey Instrument **(Tier 1)** & Semi-structured interviews **(Tier 2)**		X	X
2. How do Lesotho student teachers on teaching practice use mathematical representations in lessons?	Lesson Observations **(Tier 3)**	X		
1. What factors influence Lesotho student teachers' choice of mathematical representations?	Video stimulated Interviews **(Tier 4)**	X		

The choice of a mixed methods (qualitative and quantitative) approach in this study has been influenced by my epistemological orientation that knowledge, like mathematics, is socially constructed and co-produced within communities of practice (Lave and Wenger, 1991). When interrogated from multiple perspectives, a mixed methods approach allows the researcher to provide a fuller description and a more complete explanation of the phenomenon being studied, by providing more than one perception of it. According to Lesh and Kelly (2000, p. 197) when a multi-tiered study is conducted, “each tier can be thought of as a longitudinal development study in a conceptually enriched environment”. The idea behind tiers is therefore to focus on developing ideas. In this study, it was critical to involve a large number of participants at the beginning of the project (in Tier 1) to complete a survey instrument, in order to give as broad and as widely representative a base as possible from which to draw the participants in tiers 2, 3, and 4. Lincoln and Guba (1985) advise that “maximum variation sampling will usually be the sampling mode of choice.” They mention that this is because, “in naturalistic investigations … the purpose of sampling will most often be to include as much information as possible” (p. 201).

The survey instrument that I used for data collection in Tier 1 generated both qualitative and quantitative data as will be seen in the next chapter (Chapter 4). It was envisaged that the mixed methods approach would yield a broader picture of student teachers’ understanding of mathematical representations across the broadest possible sample. According to Stake (2006, p. 23) “an important reason for doing the multi-case study is to examine how the program or phenomenon performs in different environments”. In this study, I worked with the same cohort of pre-service primary school teachers at different stages and contexts over a period of three years (2009 – 2011). In Tier 1, the case comprised a cohort of two hundred and twelve (n = 212) student teachers who were registered for Diploma in Education Programme (DEP). In Tier 2 the case was made up of 10 purposively selected student teachers (Denscombe, 2007). According to Denscombe (ibid. p. 17) the idea of purposive sampling is:

> Applied to those situations where the researcher already knows something about the specific people or events and deliberately selects particular ones because they are seen as instances that are likely to produce the most valuable data.

This seemed to be the case in my study, because of the good response rate to Tier 1. In Tier 3, the cases were 5 student teachers that were a convenience sample chosen from participants in Tier 2. Convenience sampling saves time, money or effort (Lincoln & Guba, 1985, p. 201). In this instance, I worked only with student teachers on school placement in or near Maseru for these three reasons. In this tier, I chose to focus on each individual participant in their respective lessons when on school placement, rather than to focus on them as a group. I later conducted video-stimulated recall interviews with each one of the five in Tier 4 reflecting on their mathematics lessons in order to synthesise and extrapolate findings.

In the following section I explain how data gathered in each tier were analysed and used to respond to the three research questions. I now pose and answer three questions in relation to each tier of my study. This strategy is intended to structure my description of the methodology I followed. The three questions relating to each tier are: a) From who did I want to gather data? b) What data did I want to gather? And c) how did I propose to analyse these data?

The 'Who' of Data Collection in Tier 1

All participants in the four tiers were student teachers who were registered for Diploma in Education Primary (DEP) at the Lesotho College of Education over the academic years 2009 – 2011. In this study, all 300 registered year 1 DEP student teachers in Maseru campus of the Lesotho College of Education (LCE) were invited by me as a researcher to fill in a questionnaire (survey instrument) just before they attended any formal lectures in January 2009. It was important to gather this information before participants could be influenced in any way by the mathematics courses at the College (LCE).

Selection of Participants for Tier 1

It was important to have as many participants as possible at this stage in order to form a picture of the general mathematical background of DEP candidates. The general mathematical background involved participants' reported mathematical performance in Form E and an outline of their experiences of learning mathematics in schools. Again at this stage, I also wanted to explore participants' mathematical expectations of the DEP courses that they had registered for. Therefore, at an orientation lecture in mathematics education, I invited all 300 year 1 DEP students to participate in Tier 1. After explaining the purpose of my study and my intention to work with all of them, I asked those who were willing to take part to sign informed consent forms for me (See Appendix 3). There was a response rate of just over 70%, because a considerable number of students ($n = 212$) accepted the invitation. They were then invited to meet in the same lecture hall at an agreed time the following day and given the survey instrument to fill out.

Accessing Participants in Tier 1

All participants in tier 1 were adults aged 18 and above. Gaining access to this cohort of students meant first, seeking permission from the LCE senior management to conduct a study of this nature with the students. Second, I had to get consent from the students themselves expressing their willingness to take part in this study. Being a member of the college community, I realised that the lack of an LCE ethics committee posed some challenges to me as a researcher who wished to carry out an ethically sound study.

First, I submitted an application letter, accompanied by a written research proposal, to the college Rector. My proposal stated all the details about what I wanted to do with the students who wished to accept the invitation to participate. I also clearly mentioned that the students would be free to withdraw their participation at any stage of the project without any prejudice held against them by me as a researcher and their tutor. The proposal also had a copy of the 'informed consent' form that the students would fill and sign if they agreed to participate. There was also a 'plain language statement' sheet that I was going to read for participants before they would participate in the project. I attached these documents to my application letter to the Rector, in order to help the Administration in the decision to either grant or deny me access to researching the students. The decision to allow me to work with the students was communicated to me by the Rector by means of a letter (see Appendix 4).

The second level of entry involved my asking students to take part in my study. It was during the orientation week when the DEP students were gathered in a hall that I introduced my study to them for the first time. I explained in detail how they could participate and how I was going to use the information generated in the process of interacting with them. I extended my invitation to the whole 300 students and explained that their participation was going to be voluntary and they would be free to pull out of the study at any time without prejudice. I also made it clear to them that their participation or their not participating in this project was by no means going to affect their performance in the two mathematics courses they were about to study. I read the plain language statement (see Appendix 2) to them and requested those who wished to participate to complete consent forms and to bring them back to me on the next day, when I was going to ask them to work on the survey instrument. As mentioned earlier, 212 of them signed the consent forms and participated in filling in the survey instrument, which took them in general 30 to 45 minutes to complete.

A preliminary data analysis of the spread of participants across home districts, gender and COSC mathematics results was conducted at the initial stages of the research project. This is reported here, rather than later in chapter 4, because it was used to inform how the participants in Tier 2 were chosen from the two hundred and twelve participants in Tier 1. Table 3.2 which follows shows the number of participants who came from each of the ten districts of Lesotho.

Table 3.2: Home and gender of participants

Home District	Gender		
	Male	Female	Total
Berea	5	23	28
Butha-Buthe	8	25	33
Leribe	12	28	40
Mafeteng	2	28	30
Maseru	9	31	40
MohalesHoek	5	13	18
Mokhotlong	0	2	2
Qachasnek	1	12	13
Quthing	2	5	7
Thaba-Tseka	0	1	1
Total	44	168	212

As I expected, a large number of respondents come from lowlands districts (Berea, Butha-Buthe, Leribe, Mafeteng, Maseru, and MohalesHoek) with the highest number

$(n = 40)$ being from Leribe and Maseru. The highlands districts (Mokhotlong, Qachasnek and Thaba-Tseka) have the lowest number of students in this cohort with Mokhotlong and Thaba-Tseka respectively affording the least numbers $(n = 2; n = 1)$. However, all ten districts are represented among the respondents.

Table 3.2 also shows that the cohort comprises a majority of females $(n = 168)$. This suggests that fewer males opt for a teaching course that would lead them into becoming primary school teachers. From my observation of other DEP cohorts, this is a common pattern of always more women than men and reflects a situation also found in the developed world (Government of Ireland, 2005). Essentially, the cohort in this study is fairly typical of other pre-service teacher cohorts and of the 2009 entry DEP cohort as a whole.

Table 3.3: Participants' home districts and performance in form E

Home District		Mathematics Result in Form E			Total
		Fail	Pass	Credit	
	Berea	16	9	3	28
	Butha-Buthe	23	6	4	33
	Leribe	26	12	2	40
	Mafeteng	20	9	1	30
	Maseru	26	10	4	40
	MohalesHoek	8	9	1	18
	Mokhotlong	1	1	0	2
	Qachasnek	11	1	1	13
	Quthing	4	3	0	7
	Thaba-Tseka	0	1	0	1
Total		**135**	**61**	**16**	**212**

Table 3.3 above indicates participants' performance in the COSC (Form E, i.e., end of high school) mathematics examination and shows that two districts namely

Botha-Bothe and Maseru have the highest number $(n = 4)$ of students who obtained a credit. The best grade a candidate can obtain in COSC is credit. If a student fails to obtain a credit, he/she can either get a pass (average), or a fail (lowest grade). According to Table 3.3, the general performance in mathematics in all districts had been very poor. 16 participants obtained a credit, 61 obtained a pass, and about 64% (135) failed mathematics. This information sheds light with regard to the participants' performance, as measured by the COSC examinations in mathematics. At first, I planned to select three or four students for tier 2 from each of the credit, pass or fail groups. However, as I continued to engage with preliminary data analysis of tier 1 a different plan for selection of participants emerged. This will be outlined later.

The 'What' of Data Collection in Tier 1

In order to gather data in tier 1, I developed a survey instrument, the second part of which is presented in Figure 3.1. The survey instrument was first administered to eight third year DEP student teachers as a pilot study. These were the students who took Year 1 Mathematics courses before the review of the DEP mathematics program through the DeLPHE project mentioned in Chapter 1. This means that in terms of awareness of the use of particular representations in solving mathematics tasks the pilot study students were not that different from the first years I was proposing to research. The instrument could not be piloted with the first years because my aim was to invite all of them to participate in Tier1 of this study. The purpose of the piloting was to make sure that the questions were clearly understandable to the participants and also to establish the estimated time it would take participants to complete the whole survey instrument. According to Bryman (2008, p. 247) piloting "has a role in ensuring that the research instrument as a whole functions well". It was crucial for me to pilot the instrument prior to its use for data collection in the actual study. I was satisfied that the instrument would function well during data gathering. I found that it took between 30-45 minutes to complete and there did not appear to be any problems.

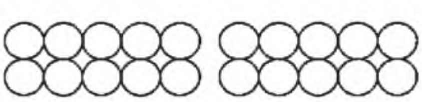
The survey instrument used in Tier 1 comprised two sections. In Section 1 (see Appendix 5), participants were asked for general information such as: gender, age, home language, home district, name of last school attended, and performance in mathematics (symbol obtained) in Form E, which has already been reported (See Tables 3.2 and Tables 3.3). There were two other general questions in Section 1, both aimed at gathering qualitative data about the respondents. The first question invited participants to write a short account of their experiences as learners of mathematics from primary school (Standard 1) to high school (Form E). The second question invited participants to mention their expectations of the DEP programme with regard to mathematics courses they had registered for at the college. Section 2 of the survey instrument is presented next.

Figure 3.1: Mathematics Tasks Given to Participants

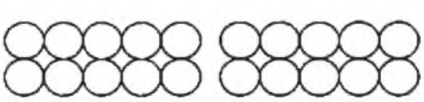
Section 2: Mathematical Operations

1. Colour in the counters to show the operation and find the answer

a. 17 - 5 =

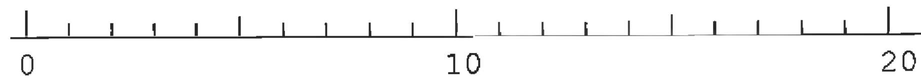


b. 12 + 6 =

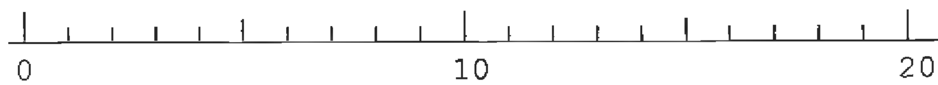


2. Draw blocks or jumps on the number line to show the operation and find the answer

a) 8 + 9 =

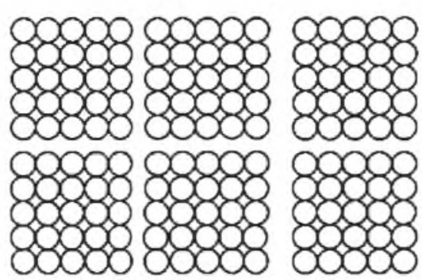


b) 11 - 6 =



3. Show the following calculation on the diagram provided and write the answer

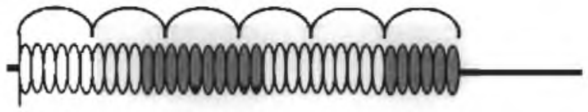
5 × 12 =

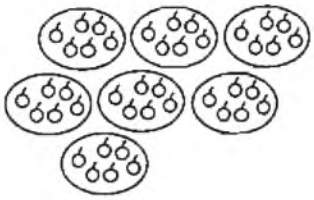


4. What calculations are shown in the following representations?

Write the calculation and the answer

a)

b) 

c) 

5. Work out the following, and show how you got your answer

201 + 79

6. If you have M44 to spend, how many Exercise Books at M4 each can you buy? (**Show clearly the representation you would use to explain to a child how you got the answer**).

7. There are nine people at a party. Each person shakes hands once with each of the other people. How many handshakes are there in all? (**Show clearly the method that you used to get to the answer**).

The 'How' of Data Analysis for Tier 1

Survey items in section 2 are numbered 1 – 7. Some are subdivided to make a total of eleven tasks. As indicated, eight tasks were accompanied with some form of iconic representation such as an array of circles, or a number line, a string of beads or a picture of groups of objects. The first four tasks offered both symbolic and iconic representations, which respondents were invited to reconcile in some way. Task 4 consisted of iconic representations only and respondents were invited to provide a possible symbolic interpretation for each of three representations. In each case, more than one mathematical meaning was possible (for example, division or multiplication). One task consisted of symbols only, while two others were in the form of word problems and participants were invited to generate their own representations of the situations and to show how they might use these to work out or explain solutions. These tasks were carefully chosen and included on the survey instrument in order to look into participants' understanding of representations and their possible role in helping learners to solve mathematics problems.

Provenance of Section 2 Tasks

In chapter 1, I mentioned the numeracy strand of a DelPHE project in which I participated. It involved workshops for college personnel in both Durham and Lesotho. The Durham team was promoting the use of particular mathematical representations. The array of circles in groups of twenty-five was a new way of representing operations on number for teachers in Lesotho. So was the use of the number line for addition and subtraction although the number line is recommended in the Lesotho curriculum for teaching multiplication. (See Appendix 6 for a sample worksheet from the DelPHE numeracy project). For this reason, some tasks used in the survey instrument were adapted from materials used by Durham University researchers in their work with primary school learners. I decided to explore whether beginning student teachers in Lesotho would recognise these representations and how they might associate them with the four mathematical operations.

I used Statistical Package for the Social Sciences (SPSS) to help me analyse participants' responses to the survey instrument employed in Tier 1. I used codes adapted from Ma (1999) for categorising participants' responses. In her study of Chinese and American elementary mathematics teachers, Ma coded teachers' responses as *correct*, *incorrect*, *complete*, and *incomplete*. In this study participants' responses were categorised as: *correct answer*, *incomplete answer*, *incorrect answer*, and *not done*. An answer was categorised as *correct* if the participant had given a correct answer to the symbolic part of the task and reconciled this with a meaningful use of some form of representation. For example, this answer to task 1 was coded as a *correct answer.*

Figure 3.2: An answer to task 1 that was coded correct

1. Colour in the counters to show the operation and find the answer

a. 17 - 5 = 12

An answer was coded as an *incomplete answer* if the participant had provided a correct answer to the symbolic part of the task, without a clear indication of meaningful use of the accompanying representation. Again, an answer was coded as *incomplete* if the participant provided an incomplete answer to the symbolic part of the task but with a correct representation.

Figure 3.3: An answer to task 1 that was coded incomplete

1. Colour in the counters to show the operation and find the answer

a. 17 - 5 =

An answer was coded *incorrect* if the answer offered to the symbolic part was false and was accompanied by an unsatisfactory explanation or use of a representation. For example, the following answer to task 1 was coded *incorrect.*

Figure 3.4: An answer to task 1 that was coded incorrect

1. **Colour in the counters to show the operation and find the answer**

a. 17 - 5 =10 

In order to establish participants' relationships with the representations accompanying tasks in Section 2, I employed Mason (2005)'s hierarchical typology of exploring learners' understanding of mathematical representations. The typology comprises the following constructs:

- gazing;
- discerning details;
- recognizing relationships;
- perceiving properties;
- reasoning on the basis of properties.

When describing typological analysis, Hatch (2002) argues that typologies are usually predetermined, however they could be generated from theory, common sense, and research objectives. The main objective at this stage was to establish participants' connecting of mathematical representations with the tasks they solved. Since some tasks were accompanied by some form of representation, Mason's typology was helpful in guiding me with regard to participants' level of understanding of these external and possibly unfamiliar representations (Goldin and Shteingold, 2001).

After coding all the answers to the 2332 items in total (212×11), I studied the spread of data carefully and felt there was more to be gained by probing some participants further in order to establish the thinking behind their use of some representations. This was in preference to a random selection along COSC results lines as previously planned. I purposefully wanted to follow only ten individuals who were locally accessible to me, and whose responses displayed potentially interesting uses of mathematical representations. The challenge was therefore to come up with valid criteria to follow in selecting a purposive sample of ten participants for in-depth interviews from a total of two hundred and twelve respondents.

As I was studying the pattern of how participants worked out each of the seven items in Section 2 of the survey instrument, I came to realise that many respondents had obtained 'incorrect answer' for the 'hand shake' task (Task 7). I became interested in the eight (8) fully correct answers to this task. I found that the eight respondents who had provided 'correct answers' had also provided interesting and useful representations to accompany them. I then decided to invite these participants for interviews together with any other two participants whose work displayed potentially interesting use of representations.

The 'Who' of Data Collection in Tier 2

The ten respondents were invited from the two hundred and twelve participants to take part in individual semi-structured interviews, where each participant discussed some of their responses to the survey instrument in Tier 1 and were asked to say how they thought they would work out the tasks at the time of the interview. The semi-structured interviews were conducted towards the end of first year diploma course (10 months after the date of participation in filling in the survey instrument). While it would have been desirable to invite all participants from Tier 1, it was not practical because at this stage I wanted to have a small sample (ten participants) that would allow me to do an in-depth analysis of participants' work. So the number 10 was purposively predetermined in my design of the study.

Gaining Access to Participants in Tier 2

The participants in this tier came about as a result of the analysis of data in Tier 1. Bryman (2008) refers to this kind of approach as 'iterative' and argues that "this means that analysis starts after some of the data have been collected, and the implications of that analysis then shape the next steps in the data collection process" p. 539). When the analysis of the data collected in Tier 1 was completed, I invited the ten selected students mentioned earlier in this chapter to participate further in the project. All ten student teachers agreed and signed the necessary consent forms. They were then each invited for semi-structured interviews.

The 'What' of Data Collection in Tier 2

The interviews in Tier 2 were semi-structured (Bryman, 2008). I had hoped to conduct clinical interviews with the selected students (Ginsburg, 1997). According to Ginsburg (ibid, p. xi), "The clinical interview is a powerful but not yet sufficiently appreciated method for both researchers and practitioners concerned with entering the minds of children". He mentions that it is 'difficult', 'poorly understood and can be used badly'. Some of the conditions of a clinical interview as outlined by Ginsburg were present in my work, for example, initial standardization of the task (the survey instrument), use of objects around which the task revolves (selected tasks) and a "how did you do it?" or "Why?" question (Ginsburg, 1997, p. 34). But the clinical interview requires a degree of flexibility to the subject matter of the interview that my approach as researcher in tier 2 did not allow.

> The interviewer, observing carefully and interpreting what is observed, has the freedom to alter tasks to promote the child's understanding and probe his or her reactions; the interviewer is permitted to devise new problems, on the spot in order to test hypotheses; the interviewer attempts to uncover the thought and concepts underlying the child's verbalizations. The clinical interview seems to provide rich data that could not be obtained by other means (Ginsburg, 1997, p. 39).

In the light of Ginsburg's challenging words, I decided to conduct semi-structured interviews about their responses to the survey instrument with the ten participants. Bryman (ibid, p. 699) views the notion of semi-structured interview as:

> A term that refers to a context in which the interviewer has a series of questions that are in the general form of an interview guide but is able to vary the sequence of questions ... also, the interviewer usually has some latitude to ask further questions in response to what are seen as significant replies.

Opie (2004, p.118) concurs with Bryman and further argues that semi-structured interviews are "a more flexible version of structured interviews, which allows for a depth of feeling to be ascertained by providing opportunities to probe and expand the interviewee's response". A semi-structured interview allows for deviation from a prearranged schedule and also allows for the interviewer to change the wording of questions or even the order in which they are asked.

Mason's original typology (2005) does not have indicators of each stage and as such presented challenges to me when trying to use it for analysis and in planning my semi-structured interviews. These have been slightly more elaborated by Watson (2009).

Table 3.4: Mason's structures of attention and indicators of same

Gazing: looking at the whole; *identifying two or more representations; naming representation(s); stating physical appearance of each representation;* proclaiming how the object/operation looks. *Key Question: What do you notice about the images/representations?*
Discerning details: *awareness of mathematical calculations, procedures, and concepts embedded within representation(s); acknowledge* more than one way of representing the object/operation? *Key Question: What are the characteristics of the various representations?*
Recognizing relationships: *establishing similarities between representations; realising differences in representations;* explaining how representations show the same calculation/idea. *Key Question: What is the same and what is different about the various representations of the same object/operation?*
Perceiving properties: *explaining how each representation helps to perform some calculation; mentioning why each representation is useful in understanding the operation, concept, and/or procedure;* identify the most useful characteristics of a particular representation. *Key Question: What aspects of the structure of the operation are emphasised by the representations?*
Reasoning on the basis of properties: *making connections between representations; deciding on the key representation for a particular operation; justifying how and why representations work.* *Key Question: How do we move from one representation to another?*

Therefore, I had to develop some indicators of how the typology might be applied to analysing the survey instrument. I also planned to ask what I considered to be a key question relating to each of the five stages. I therefore prepared a semi-structured interview schedule for Tier 2 interviews (see Appendix 13) that I used flexibly during the interview depending on each participant's response to general questions. I also had the manuscripts participants wrote on when responding to Section 2 tasks in Tier 1 as shown earlier in this chapter. Each participant was requested to talk through his or her own responses to some selected mathematics tasks for the interview. The interviews were not always conducted on the same tasks. I had taken time (approximately 10 months) to carefully study participants' responses to each of the seven tasks in Section 2 of the survey instrument and had selected some responses where I felt further discussion with the participant would be helpful in understanding their thinking and their choices. The semi-structured interviews were conducted with respect for participants, in the privacy of the PhD researchers' room. I had a heavy responsibility to create a relaxed and conducive environment for the interview sessions. I had to ensure that the interviews were conducted in a quiet environment where there was a minimal amount of noise. This also made sure that participants were free to give comments and to ask questions. Each interview lasted for 10 to 20 minutes and was audio recorded and transcribed. When such transcripts were ready, I invited each participant to read through his or her transcript to check that my accounts were true reflections of what they had said during interview sessions. This was for *triangulation* of data (Denzin, 1997).

The 'How' of Data Analysis for Tier 2

The main aim at this stage was to find out what participants would say about the representations they had used as they were solving tasks, and whether they thought they would do things differently about ten months later, after taking two DEP mathematics courses (MAT 1400P – A and MAT 1402P – B).

Analysis of the transcripts of the data gathered in Tier 2 is dealt with in Chapter 5. In what follows, I discuss methodology used for data collection and analysis for the subsequent tiers (3 and 4). The analyses of transcripts from Tier 2 were used to answer research question 1(what mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?).

The 'Who' of Data Collection in Tier 3

In Tier 3, a self-selecting group of five participants from the subgroup of 10 participants in Tier 2, while on school placement during second year of their study, agreed to teach a mathematics lesson based on arithmetic operations. These five participants in Tier 3 were a 'convenience sample' (Lincoln and Guba, 1985) in that I invited student teachers who were practising teaching in primary schools located in Maseru district that I could access easily and economically. This phase of the study took place at the time when participants were on school placement in the second year of their Diploma in Education Primary (DEP) programme.

Gaining Access to Participants in Tier 3

In order to gain access to the schools, I had to apply for permission from school principals where the student teachers were placed for teaching practice. I had to write formal letters to the chosen student teachers requesting them to continue their participation in my study. I then wrote letters to parents of primary school learners that were to be taught by participants in Tier 3, asking for permission to video record a mathematics lesson taught by the student teacher. Letters to parents were written in such a way that the concerned parties did not by any means feel obliged to allow their children to participate. For example, the letter stated that learners should feel free to withdraw at any stage of the study and would not be victimized in any way, and that confidentiality, privacy and anonymity for all participants would be preserved.

The 'What' of Data Collection in Tier 3

Each of the five participants was observed teaching a mathematics lesson on arithmetic operations from the Lesotho primary school curriculum. All five lessons were video-recorded, transcribed and analyzed. Data gathered in Tier 3 were used to respond to research question 2 (how do Lesotho student teachers on teaching practice use mathematical representations in lessons?). The lessons were observed in five different primary schools located in the district of Maseru in the months of February and March 2010. It accidentally happened that four of these lessons were in Standard 4 (learners usually aged 8 to 9) and one lesson was in Standard 6 (learners usually aged 11 to 12). Some student teachers and learners might have felt uncomfortable doing things as normal in the presence of the researcher, and this despite the fact that they had agreed to participate. In order to overcome this hindrance, I had to visit classrooms several times before the actual date of data collection. The aim of my visits was to make participants and learners familiar with me as a researcher.

The 'How' of Data Analysis for Tier 3

The lesson transcripts were analysed through the use of the Knowledge Quartet (KQ) (Rowland, Huckstep, and Thwaites, 2005). The KQ is a typology that emerged from a grounded approach to data analysis of primary mathematics teaching in the

UK. Table 3.5 adopted from Rowland and Turner (2007, p. 111) clarifies what each of the four dimensions of the KQ entails in terms of the contributory codes that can be used as indicators of mathematical knowledge in teaching. The KQ identifies the manner in which the student teachers' mathematical knowledge impacts on a mathematics lesson along four dimensions namely; foundation, transformation, connection, and contingency.

Table 3.5: The Knowledge Quartet

DIMENSION(S)	INDICATORS
FOUNDATION	*Adheres to textbook; awareness of purpose; concentration on procedures; identifying errors; overt subject knowledge; theoretical underpinning; use of terminology.*
TRANSFORMATION	*Choice of examples; choice of representation; demonstration.*
CONNECTION	*Anticipation of complexity; decisions about sequencing; making connections; recognition of conceptual appropriateness.*
CONTINGENCY	*Deviation from agenda; responding to children's ideas; use of opportunities.*

While the four interconnected dimensions are all useful in looking at the mathematics teaching, the *transformation* dimension lends itself particularly well to this study, in that it focuses the eye of the researcher on the use of representations by student teachers during mathematics teaching. In this study, the KQ was used as an analytical tool with the main focus on the transformation dimension, specifically on the choice and use of representations by student teachers. However, the four dimensions are intertwined, so even though my research interest is in transformation dimension of each lesson, the other three dimensions featured in my analysis in one way or another.

Accessing Participants in Tier 4

This part of the study occurred at the time when student teachers had just started their third year of the DEP programme and were back in college. I then invited the students who had taught the five lessons for the interview, and when they agreed we did not have to fill in further consent forms because this was a continuation of their participation in tier 3.

The 'What' of Data Collection to Tier 4

Video stimulated interviews with the five participants were conducted in Tier 4, where participants were shown some video clips of the lessons they had taught to refresh their minds. The five participants were asked to talk about how they each chose and used representations in their respective lessons. Data gathered in Tier 4 were used to respond to research question 3 (What factors influence Lesotho student teachers' choices and uses of mathematical representations in teaching primary mathematics?).

The 'How' of Data Analysis for Tier 4

Tier 4, data emerged from video-stimulated interviews held with the five student teachers. These interviews were conducted a year after the lessons were observed. This is when the five participants were in third year of the DEP programme. I needed this period (March 2010 – March 2011) to study each lesson carefully so that I could identify key clips for the subsequent interviews in Tier 4. This time span also allowed me to transcribe the video recorded lessons. The research value of video-stimulated recall has been well documented (Hall, 2000; Pirie, 1996). The limitations to such research and the danger of bias in video research have also been described (Roschelle, 2000). One disadvantage of video recording is that there is a likelihood of missing out on important aspects of the lesson because the camera focuses on one point at a time. Opie (2004: 123) argues that video recording has a lot of technical problems such as "focusing and ensuring good sound quality". This was a real problem in some classrooms. Another drawback of observation is that it is all absorbing and time consuming to observe a lesson and at the same time try to take notes. That is why I had to request Mr. Putsoa[3] (the laboratory technician at the college) to provide assistance with video- recording of lessons in Tier 3. I was therefore free to focus on taking field notes of class proceedings while Mr. Putsoa was busy video recording. Rowe (2009) proposes that the power of selection of the video clips to be discussed be shared with participants as 'consultants'. However, while, I attempted to establish a 'clinical partnership' with my student participants (Wagner, 1997) over the course of the research, cultural practices and power issues acted as a constraint on the work. The following section focuses on the ways and means through which ethical issues were addressed in this study.

Ethical Considerations

In social research, ethical issues are about a process of gaining access by the researcher into the empirical settings of research. They are also about power relations that exist between the researcher and other concerned parties (Setati, 2005). Bryman (2008, p. 131) argues that gaining access into places of research is a political process

[3] Pseudonym used.

because such access is "mediated by gatekeepers, who are concerned about the researcher's motives". Therefore, it is the responsibility of the researcher to 'negotiate' and to 'renegotiate' with the gatekeepers to find out what may be permissible and what is not allowed.

In her work with schools in South Africa, Setati (2005) identifies two forms of 'power' that every researcher must bear in mind when attempting to negotiate access to schools. She talks of Power (upper case P) to refer to hierarchical structures that exist in most schools. This might include school owners, school principal, head of department, etc. Setati (ibid.) again refers to another form of power (lower case p) and argues that individual teachers have this kind of power, which they can exercise on the researcher to deny him or her access into classrooms. If a teacher does not wish to participate in a project, the researcher will not be able to collect data even if for instance the principal of the school has agreed.

In this study, issues of process involved me, as a researcher, formally applying for ethical clearance from the St. Patrick's College Research Ethics Committee (REC) because my study involves humans. The REC had to ensure that all the proposed steps to be taken in this study would not in any way harm the participants' dignity, rights and feelings. I had to give a written declaration to REC that participants' identities and interests would be protected throughout the project. I also had to guarantee REC that the confidentiality of information given to me by participants during the research process would likewise be fully protected. As part of the requirements of the REC, I had to obtain written permission from the Rector at the Lesotho College of Education to work with the DEP students. I sought permission from the college rector, because the college had not formed an ethics committee.

Issues of Power

Issues of power are mainly about the relationship between me as a researcher and the participants as the researched. That relationship that existed between me as a staff member of LCE and the student teacher participants is an example of both kinds of power is an example of both kinds of power (Setati, 2005). The fact that I am a lecturer to the student teachers in a way gave me authority over them, which was likely to influence their decision on whether to take part or not. In order to overcome this, I explained to them in a letter that their participation was absolutely voluntary and guaranteed that those who decided not to participate were not going to be victimized in any way. The fact also that I came to these students with my own agenda in the form of research questions and overall research design gave me a certain authority over the participants. In other words, the participants had little or no power to force me to deviate from my own predetermined agenda. However, this is not to say that the participants were powerless, because they too had their own less obvious agenda which influenced each one of them to take part as argued by Setati (2005). Such agendas include personal and professional gains and formed the basis of my claim to establishing a clinical partnership with them (Wagner, 1997).

At the Lesotho College of Education (LCE), lecturers are responsible for assessing students' performance when on teaching practice. The teaching practice coordinator organizes school visits for the purposes of observing student teachers' lessons. Each student teacher is observed and assessed at least three times in a year. As a researcher and an employee of the LCE, I had to seek permission from LCE management to be excluded from all teaching practice activities. As a researcher, I had to visit participants in schools for data collection only, which was outside the schedules prepared by the teaching practice coordinator. This means I chose not to grade any students, but only visited schools for research purposes for the whole duration of data collection. I regretted however, that it was not possible for reasons of time and resources to include all volunteers in tiers 2, 3, and 4 of this research project. Given that there is a hierarchical college structure, of lecturers like me, heads of departments, deans, deputy Rectors, and the Rector himself, the success of my request lay fully in the hands of these powerful people. Exercising their power over me if they wished could mean either my failure or success in doing this project ethically.

Validity and Reliability

Validity and reliability are conceptions that have been extensively employed in quantitative studies to ensure rigor in research. According to Bryman (2008) reliability refers to the degree to which an instrument used in research is stable and produces similar results over a period of time. On the other hand, validity is a notion that deals with the integrity of the findings and conclusions that are generated from a piece of research. Opie (2004) argues that validity and reliability in qualitative studies are properties of the whole data process; this means it is the responsibility of the researcher to ensure that his or her study is trustworthy and credible.

In order to ensure rigor in this study, I employed Bryman's (2008, p. 377) criteria for evaluating a qualitative research project. Bryman's criteria involve two main concepts namely "trustworthiness" and "authenticity". According to Bryman, trustworthiness has four components: credibility; transferability; dependability; and confirmability. Authenticity also has its own criteria namely: fairness; ontological authenticity; educative authenticity; catalytic authenticity; and tactical authenticity.

In order to ensure credibility in this study, I appreciated the fact that my interpretation is subjective and as such, bound to differ from somebody else's. Therefore, in this study I asked participants to check my interpretations against theirs in order to avoid possible misunderstandings of my observations. Bell (1993, p. 64) takes a view that triangulation refers to:

> cross-checking the existence of certain phenomenon and veracity of individual accounts by gathering data from a number of informants and a number of sources and subsequently comparing and contrasting one account with another in order to produce as full and balanced a study as possible.

The key components of my study were covered in Tier 1 (mathematics tasks) and Tier 3 (mathematics lessons). However, in order for me to ensure that my accounts in these two tiers were valid and reliable, I chose to cross-check them with interviews that were carried out in tiers 2 and 4. Tier 2 (semi-structured interviews) came as a follow up on Tier 1, and Tier 4 (video stimulated recall interviews) is a follow up on Tier 3.

Taking cognisance of the fact that I was undertaking this study with a specific group of student teachers, I made an attempt to write accounts of participants and situations so that readers could have 'thick description' (Geertz, 1973) upon which, they could decide the possible transferability of my findings to other contexts, such as in other African countries or in institutions of higher learning. By so doing, I was attempting to address the issue of transferability in this study.

Taking my work on a yearly basis to the Southern Africa Association for Research in Mathematics, Science and Technology Education (SAARMSTE) Research School to be read and scrutinized by other experts in mathematics education, other than my supervisor, has proved beneficial. In the years 2009, 2010 and 2011 part of my work (research proposal, drafts of some chapters, instruments, and some transcripts) was given to two experts for audit, in order to ensure that proper procedures were followed at every stage. At these forums the experts posed questions that helped me to clarify some of the issues that I was taking for granted. I have also engaged some of my colleagues at LCE to discuss some of the issues relating to my research work. In October 2010, I was summoned to present before the PhD review viva panel held at St. Patrick's College Drumcondra (SPD). My presentation was based on the part of my work that comprised a draft of the literature review, methodology and initial analysis. After my presentation, I received constructive comments from the panel that I used to shape up subsequent work. This has contributed to dependability in this study.

Confirmability is concerned with ensuring that while recognising that objectivity is impossible in social research, the researcher can be shown to have acted in 'good faith'. This means that a social researcher must not allow his beliefs and personal values, or theoretical inclinations to influence the conduct of the research and findings deriving from it (Bryman, 2008). In order to ensure confirmability in this study, I interpreted participants' views on the basis of their own words and utterances. I further conducted interviews with participants at different stages of this study (in tiers 2 and 4) in order for them to help me understand their meaning of events that transpired, for example in lessons they taught. Analyses made in this project were based on participants' own sayings and evidence gathered from the survey instrument and lesson observations. Where necessary, interpretations are backed up with extracts from data.

Another element of great importance in ensuring rigour in any social research is *authenticity*. Authenticity deals with political issues relating to the impact of research on participants (Bryman, 2008). This project had no apparent risks for participants. On the contrary, the research had a potential for enhancing participants' knowledge and skills in choosing and using multiple representations in teaching primary school mathematics. During the semi-structured interviews in Tier 2 the ten participants were challenged to think and reason through their previous work on the survey instrument. That process could be viewed as an additional opportunity for participants where they engaged with the mathematics. Also in Tier 4, the reflective interviews helped participants to reflect on parts of the lessons they taught and specifically on the representations they used during their lessons.

Conclusion

This chapter addressed issues of methodology. I explored the research design employed in each of the four tiers. I set out to describe methodology under three headings: the 'who' of data collection; the 'what' of data gathering; and the 'how' of data analysis. I discussed the procedures followed in selecting participants in each tier, together with ethical issues involved. Discussion on data analyses procedures was also put forth. Transcription of lessons and interviews is time consuming in that it takes time to transcribe them especially when there is much data collected. It was even more demanding in cases where the interviewees had given their comments in Sesotho, and as a researcher I felt it was necessary to translate such sentences to English. Translating from one language to another is a challenge that requires special skill, and without training translating becomes a problem. After translating participants' utterances I requested some colleagues to double-check my translation against the original recordings. I did this in order to make sure that I had translated all the phrases accurately and well and for triangulation of data (Denzin, 1997).

The following chapters are devoted to data analyses and discussion of findings.

4. Tier 1 – Participants' Profile and Responses to Mathematical Tasks

In this chapter, I present an analysis of Tier 1 data collected by means of the survey instrument that was briefly presented in Chapter 3. I begin by presenting participants' general profile that includes a snapshot of their attitudes to learning mathematics and their expectations of the mathematics education courses at the college. I then focus on the analysis of mathematics tasks that participants solved.

The survey instrument had two sections. Section 1 comprised tasks 1 – 7 that required participants to provide general information. Tasks 8 and 9 in Section 1 (see Appendix 5) required participants to write their school experiences of learning school mathematics, and their expectations of the Diploma in Education Primary (DEP) programme they were registering for at the Lesotho College of Education (LCE).

The information gathered by means of Section 1 of the survey instrument was mainly intended for gathering general background of the participants, while Section 2 of the instrument was aimed at exploring participants' understanding of particular mathematical representations before they took any mathematics courses offered at the college. I chose to explore participants' understanding of mathematical representations at this stage in order to avoid any influence from the mathematics courses offered at the college so that participants' responses would only be interpreted in light of their previous knowledge.

Participants' Profile

Participants in Tier 1 are a sample of two hundred and twelve student teachers drawn from a total of three hundred students who had registered for the diploma in education primary in 2009 academic year. The students were in year 1 of their three-year diploma programme.

Only sixteen participants came from the highlands districts. Out of two hundred and twelve participants only sixteen obtained a credit in mathematics and over sixty per cent of them failed the mathematics examination in Form E. This suggests that the majority of the participants in this study did not master mathematical skills and techniques necessary for achieving either a pass or a credit. This evidence confirms my general observation over the years that the majority of students who register for DEP programme have failed mathematics in Form E, and as such, they have weak understanding and knowledge of mathematics. A typical COSC mathematics examination paper set at the end of Form E is characterised by tasks, which call for procedures and recall of and use of algorithms in a range of topics. See Appendix 7 for one sample page each, from mathematics examination papers 1 & 2, COSC 2009, as an indication of the assessment standards which students are expected to meet. It was important to obtain information relating to participants performance in Form E as mathematical proficiency is often considered to be a problem for a large percentage of DEP candidates. The information also helped me to have a better picture of the level of participants' relationships with representations and mathematics in general.

I was also interested in establishing the age of all participants in my sample because it would be easy to infer from such information whether each participant joined the college (LCE) straight from high school or had been teaching for some years before enrolling for DEP programme. Usually students who are in their twenties come from high school while those in the late thirties and forties would be in possession of a Primary Teachers Certificate (PTC), which used to be offered by the

LCE in the past. So these older participants are in most cases experienced teachers who have come back to the college to upgrade their teaching qualification with the diploma (DEP) programme.

The Frequency Table 4.1 below presents participants' ages in this study and shows that they range from eighteen to fifty. The table shows that over 57% of the participants are between ages twenty-one and twenty-five. This means a majority of participants are in their early twenties, which then suggests that most DEP candidates come straight from high school.

Table 4.1: Frequency Table showing Ages of Participants

Age	Frequency	Per cent
20 and under	33	15.6
21 – 25	121	57.1
26 – 30	28	13.2
31 – 35	14	6.6
36 – 40	9	4.2
41 – 45	6	2.8
46 – 50	1	.5
Total	212	100.0

Thus participants' responses in Section 2 of the survey instrument largely depend on their high school mathematics. On the other hand, participants aged thirty-six and above are few and are likely to possess a teacher's certificate. For these participants, responses to the tasks in Section 2 of the survey instrument arise from both their knowledge of school mathematics and their experience of teaching primary school mathematics.

In order to gather additional background information about the participants in Tier 1, I qualitatively analysed their responses to tasks 8 and 9 of Section 1 of the survey instrument. I read all two hundred and twelve participants' responses and as I was reading and studying these I made note of the common issues that participants were mentioning. I wrote a list of emerging themes for each question. I counted the number of occurrences of each of the themes. In order for a common issue to qualify as an acceptable theme it had to have been mentioned by at least twenty participants. The counting was not only necessary for identifying acceptable themes but also to enable me to develop a deeper understanding of participants' experiences of being mathematics learners in Lesotho schools.

Table 4.2 shows that in general participants experienced mathematics as a challenging venture $(n = 92)$ and also as a school subject that requires effort and thinking from the learner $(n = 85)$. These two themes are dominant and therefore it could be argued that in general, participants perceive mathematics as a difficult and cognitively challenging subject requiring effort and a lot of thinking from learners. It is worth noting, but not surprising, that a considerable number $(n = 56)$ of participants hold a view that mathematics as a body of knowledge involves learning operations, numbers, figures, symbols, place value, and formulae. This finding is characteristic of many school learners in the United Kingdom and the United States of America (Boaler, 2009).

Table 4.2 suggests that participants did not only experience difficulty in learning mathematics because of the demands of mathematics *per se*, but also because they experienced unpleasant relations with their mathematics teachers $(n = 55)$. Consider for example the following story told by one of the participants (Participant No. 143) in the following figure 4.1.

Table 4.2: Themes emerging from data and their frequencies

Emerging Themes from Data	Frequency
Mathematics as enjoyable subject at primary but not in secondary schools	48
Mathematics as a challenging venture	92
Mathematics as problem solving activity	26
Mathematics as a subject that requires effort and thinking from learners	85
Mathematics as knowledge of operations, numbers, figures, symbols, place value, and formulae	56
Referring positively to mathematics teacher(s)	38
Referring negatively to mathematics teacher(s)	55
Positive and negative experiences	34

Figure 4.1: Amelia's response to Q8 (Section 1)

8. **Write a brief story about your experiences as a learner of mathematics from primary schooling to high school level.**

When I was a learner in a primary school, I used to get lower marks especially when I was in class 6 and 7. I think the reason for this, was that, My teacher always beat us when getting the answers wrong, so I began to hate the Subject (mathematics) At High school level, I still got difficulties in passing the Subject. I end up not knowing what are the X and Y's.

Mathematics results of this participant for all three national examinations (Standard 7, Form C, and Form E) bear witness to this story. Amelia, a female aged 45, got a third class pass in maths in Standard 7, obtained an E symbol which is close to a fail in Form C and failed mathematics in Form E. She makes reference to her teacher who used to smack learners for failing to comprehend what rules and calculations meant. As a result, learners developed negative attitudes towards mathematics. Corporal punishment is still rife in many schools in Lesotho (de Wet, 2007). De Wet (ibid.) conducted a study in Lesotho where she explored perceptions and experiences of school learners with regard to school violence. To indicate the seriousness of corporal punishment in schools in Lesotho, I make reference to some comments made by participating learners in de Wet's study. She cites the following: "Some students are whipped as if they are not human beings. Teachers do not like the students …" (p. 681). It is unfortunate that this sort of situation is not uncommon despite the fact that the Lesotho Education Act (2010) declares it illegal. The Act states that "a learner shall not be subjected to cruel, inhuman and degrading punishment" (p. 164). Perhaps teachers continue to hit learners because of the Basotho culture that beating a minor by an adult is viewed culturally as part of reinforcing discipline in the hearts of young ones. I am afraid it follows that if learners continue to be beaten in mathematics lessons, it remains an empty dream to think that Basotho learners will by chance love and enjoy learning mathematics and hence obtain good results in mathematics.

In analysing item 9 (*what kind of training do you think would help you in order to teach mathematics well at primary school?*), I followed the same procedure as in item 8. In response to this item, participants stated that they expect the programme

(DEP) to equip them with knowledge and skills as follows. Table 4.3 below represents the issues they raised:

Table 4.3: Participants' expectations and frequencies

Emerging Themes from Data	Frequency
Use teaching materials (e.g. pictures, figures, diagrams, examples, experiments) to help pupils learn math	55
Explain mathematics well to learners	30
Develop understanding of mathematics	68
Guide/teach pupils how to pass mathematics	23
Learn how pupils learn mathematics easily	62

According to Table 4:3, it seems reasonable to conclude that in general participants were expecting the programme to empower them with skills and knowledge that would enable them to have a better understanding of mathematics $(n = 68)$. They also have an ambition to learn how to make mathematics teaching and learning easy for pupils $(n = 62)$. If participants view mathematics as a difficult subject, then learning how to teach it effectively and give meaning to mathematical ideas seems to be a key aim for their courses. That is, the mathematics courses offered at the college would have to devise means through which student teachers' attitudes could be changed so that they could be innovative and resourceful in learning effective ways of making mathematics accessible to primary school learners. This is a very big challenge. I hope this study can help mathematics teacher training. In what follows, I focus on the analysis of participants' responses to tasks in Section 2 of the survey instrument.

Overall Responses to Tasks in Section 2

The seven mathematics tasks in Section 2 can be put into four categories. Tasks 1 – 3 are similar. Each one is accompanied by a potential iconic representation. Participants are expected to link the representation with their working out of solutions. Task 4 (a – c) belongs to another category because here participants are offered some iconic representations and expected to identify and mention operations which they might represent. In the third category, task 5 involves a statement in symbols, most often solved by use of the standard algorithm for addition. Fourth category tasks 6 and 7 are such that participants were asked to arrive at solutions to word problems and show how they did so. Item 6 asks specifically for a representation the student teacher might use to explain to a child how the solution was achieved. In what follows, I present analysis of participants' responses to tasks in terms of highest and lowest facility and then I focus on each of the above-mentioned categories.

Table 4.4: Ranking Tier 1 tasks according to the highest and lowest facility

Highest facility	Task description	Lowest facility	Task description
Task 6 (n = 204) **96.2%** correct	Buying exercise books (division)	Task 7 (n = 8) **3.8%** correct	Handshake problem
Task 5(n = 197) **92.9%** correct	Addition of a 2 and a 3 digit number	Task 4(b)(n = 78) **36.7%** correct	Linear multiplication (*string of beads*)
Task 2(a) (n = 192) **90.6%** correct	Addition on a number line	Task 1(a) (n = 127) **59.9%** correct	Subtraction with counters
Task 1(b) (n = 188) **88.7%** correct	Addition with counters	Task 4(a) (n = 156) **73.5%** correct	Multiplication array (*no statement in symbols*)
Task 2(b) (n = 179) **84.4%** correct	Subtraction on a number line	Task 3 (n = 164) **77.3%** correct	Multiplication array (*statement in symbols*)
Task 4(c) (n = 177) **83.5%** correct	Multiplication as a collection of discrete sets		

The above table provides a summary of items and information on how participants performed on each. The ranking of the tasks is according to the number of participants who gave completely correct answers to each item. It appears that the great majority of participants found most of the tasks quite easy while almost all students found Task 7 quite challenging. The table shows that participants found Task 6 easiest of all. 96.2% of participants gave a fully correct answer to this task. Item 6 involves the context of buying exercise books and it is a routine textbook type task with 'low level cognitive demand' (Stein, Grover, and Henningsen, 1996). Participants were at liberty to suggest the representation of their choice that would help them to explain to a child how the answer was obtained. The task might be performed by division ($4\overline{)44}$ to obtain the answer of 11 exercise books) or, by repeated subtraction (44 – 4 – 4 – 4 – 4 – 4 – 4 – 4 – 4 – 4 – 4 – 4) or, as many participants opted to represent the problem, by repeated addition (4+4+4+4+4+4 +4+4+4+4+4 = 44). It should not be a surprise that respondents performed well on the symbolic part of this word problem because it is very similar to a question that is found in the textbook for Standard 4 (NCDC, 1997, p. 23). However, the part of task 6 that asks for use of a representation to explain the operation to a child requires PCK and might be expected to have been challenging for respondents. Yet they nearly all could do it correctly.

Task 5 was found to be the task with the second highest facility. 92.9% of participants gave fully correct answers. It involves addition of a 2 and a 3-digit number, an operation that calls for automatic recall of facts and the use of a standard algorithm. In this item participants were only asked to show how they worked out the answer and most used the predictable, vertical approach.

The task with lowest facility (task 7) yet having a 'high level of cognitive demand' (Stein et al, 2000) appeared to be inaccessible to the great majority of participants. Only 3.8% of respondents offered a correct answer to this task (handshake problem). It is a non-routine mathematical task that participants would not be expected to have met before and it could not be solved with a standard algorithm.

Task 4(b) is the task with second lowest facility where 36.7% of participants gave a fully correct answer. The task involves linear multiplication represented by means of a string of beads. In this task participants were challenged to think of a suitable symbolic representation. This diagram could be said to represent multiplication ($6 \times 6 = 36$) or division ($36 \div 6$). Some participants offered repeated addition as an interpretation, which was acceptable.

There was an interesting disparity between the numbers of participants who gave fully correct answers to items 1(a) and (b) and 2(a) and (b). This warrants closer study. Task 2(a) is the item with the third highest facility with 90.6% of respondents offering correct solutions. This task involves addition as counting all or counting on from the first addend on a number line, a task with which most respondents appeared familiar. The item with fourth highest facility is task 1(b) with 88.7% correct solutions. This task involves an addition sum to be linked with a representation of counters arranged in a static structure of two groups of 5 by 2 (see figure 3.2, p. 84). While all participants might be expected to be familiar with the use of counters such as stones and sticks, it is noted here that they found it somewhat easier to demonstrate addition on a number line (Task 2(a)) than to demonstrate addition with fixed counters in Task 1(b). This might be due the manner in which these counters are arranged which might be unfamiliar to a majority of participants. One of the respondents, Beatrice who was one of the ten participants in tier 2 (see pp. 139-142) recognised the array as “these pictures”, which she didn’t name. Her responses to the survey instrument will be reported in more detail in Chapter 5. A second student, Sebu, called them ‘apples’. Her responses are also to be found in Chapter 5.

Task 2(b) came fifth in facility ranking with 84.4% participants offering correct answers. This item required respondents to represent subtraction (11 – 6) on a number line. Given that the number of correct responses to task 2(b) is lower than those to task 2(a) suggests that some participants (about 4%) found it harder to do subtraction as ‘take away’ on a number line than to do addition.

Task 4 (c) is ranked sixth in facility with 83.5% respondents offering correct answers. This item involved multiplication as a collection of discrete sets. Perhaps, participants were more challenged by the demand of this task than others because of the fact that they were asked to indicate the symbolic sentence represented by the diagram, instead of being asked to supply an iconic representation for a symbolic statement.

Task 1(a) that involves subtraction with counters is the item with third lowest facility with 59.9% respondents giving correct answers. This suggests that 40% of participants found it hard to perform subtraction on the array, perhaps because of the ‘foreign’ nature of the iconic representation, or because they find the representation of subtraction difficult.

The item with fourth lowest facility is Task 4(a) with 73.5% of respondents giving a correct solution. This task involves an array of counters intended to represent multiplication but with no statement in symbols. It appeared to be alien to more than a quarter of the participants.

Finally, Task 3 is an item with 77.3% of respondents giving a correct solution. The task involves marking a representation for multiplication on the array to match a statement in symbols. The accompanying symbolic statement might have been expected to make it easier for participants to obtain the solution but over 20% of the respondents appeared unable to complete the task.

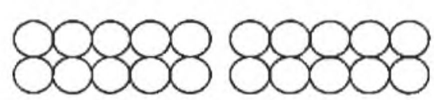
Investigation of Category 1 Tasks

Task 1(a) requires participants to demonstrate how the subtraction of a one-digit (5) number from a two-digit number (17) could be represented on the provided iconic representation (array of counters). Task 1(b) requires the combination of a two-digit number (12) and a one-digit number (6) to be represented using a similarly arranged array of counters.

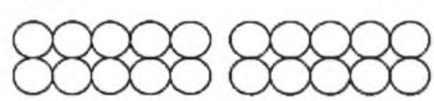
Figure 4.2: Task 1

1. *Colour in the counters to show the operation and find the answer*

a. 17 - 5 =



b. 12 + 6 =



Task 1(a) proved to be more challenging to participants compared to Task 1(b). I therefore, choose to focus on participants' responses to Task 1(a) in order to establish their degree of understanding of the array accompanying the task. Table 4.5 below shows that one hundred and twenty-six participants managed to demonstrate the operation successfully on the diagram and wrote the correct answer.

Table 4.5: Participants' responses to Task 1(a) in relation to COSC results

		TASK 1(a)			Total
		Incorrect	Incomplete	Correct	
COSC Result	Fail	1	56	78	135
	Pass	0	23	38	61
	Credit	0	5	11	16
Total		1	84	127	212

Despite the fact that in general, this might be considered a straightforward task for people who have completed high school mathematics, eighty-six $[n = (1+85) = 86]$ of them failed to demonstrate the link between the diagram and the task. The positioning of subtraction before addition might have also been misleading but since only one student was coded incorrect on this item, it is seems to indicate that 40% of respondents could perform the operation of subtraction successfully but could not link it with a fixed array of counters.

Eighty six participants might be considered to have 'gazed' at the symbolic representation (17 – 5) and the iconic representation of twenty counters arranged in two sets of 5 by 2 and did not appear to make meaningful connections between the two forms of representation, although many students coloured twelve counters to represent the correct symbolic answer only, which was coded incomplete. (See Figure 3.3, for an example of such a response). However, while eighty-five of them wrote the answer as 12, one participant, Rose (see Figure 3.4 for Rose's response) offered 17 – 5 = 10. Her use of the iconic representation may give a clue as to why she thought (incorrectly) that the answer might be 10. Rose appears to have coloured/miscounted 16 discs instead of 17. She then appears to have subtracted 6 discs leaving 10 as the answer. It appears likely that she miscounted/miscalculated and then wrote the wrong answer 10. An alternative interpretation that Rose does not know her basic number facts and then coloured the 16 discs with 6 crossed out to incorrectly match her miscalculation is unlikely.

It is probably not surprising that 68.7% of respondents with a COSC credit grade in mathematics were coded correct on task 1(a). 62% of the respondents with a COSC pass grade were also coded correct, while almost 58% of the respondents with a COSC fail grade were coded correct on task 1(a). However, the picture changes when one realises that only 7.5% of the respondents were in the COSC credit category. Less than 30% were in the COSC pass category. The majority (64%) earned a COSC fail in mathematics. Despite the unfamiliarity of the array of counters and the weak mathematics scores in COSC, almost 60% of the student teacher participants linked the two representations successfully. I therefore conclude that these students are well advanced in Mason's typology of structure of attention. Not alone did they show evidence of discerning details and recognizing relationships, but they also succeeded in reasoning on the basis of perceived properties. However, more than a quarter of the 'credit' students were coded incomplete on this item. More than a quarter of the 'pass' students were also coded incomplete. Somewhat less than half of the 'fail' students appeared unable to demonstrate the subtraction task linked with the array of twenty counters. Eighty-four participants whose responses were coded incomplete appeared to focus on the symbolic representation while considering the accompanying representation to have little connection with the operation performed. Some might argue that perhaps the task was too simplistic for them, i.e., some participants shaded in twelve discs to represent the answer only. I argue that these participants are at the *gazing* stage, where they attend to the whole twenty counters and cannot think how to represent the operation, so shade in 12 to represent the answer.

Subtraction Structures

Many scholars have researched the process of representing addition and subtraction operations for learners (Carpenter and Moser, 1984; Leinhardt, Putnam and Hattrup, 1992). Two structures for the operation of subtraction are described in the literature: subtraction as 'change', 'partition' or 'take away' and subtraction as 'comparison' or 'difference' (Rowland et al, 2009, p. 162). Presented symbolically, tasks 1(a) and 2(b) could belong to either structure, although different problem structures would be represented differently. I expected that participants might treat them both as 'take away' problems. Task 1(a) could be said to require participants to actually double shade part of the array. That is, they had to shade seventeen (the minuend) and then out of the seventeen, double shade five (the subtrahend), and count the remaining discs in order to obtain the answer (difference = 12). The problematic nature of this subtraction strategy (Rowland et al, 2009, p. 164) is represented by the requirement for double shading and is equivalent to the multiple counting inherent in subtraction calculations (Fosnot, Dolk, Cameron, and Hersch, 2001).

In what follows, I focus on tasks 2(a) and 2(b) and participants' responses to them. These tasks are each accompanied by a number line calibrated in units and labelled in tens. Task 2(a) involves addition of one digit numbers (8 and 9) while Task 2(b) involves the subtraction of a one digit number (6) from a two digit number (11). A number line is one of the highly recommended iconic representations in Lesotho primary schools. In line with international good practice, the mathematics syllabus considers a number line as a key resource for addition and subtraction of numbers throughout primary schooling (NCDC, 1997). In Task 2(a), strategies participants might be expected to use include identifying eight on the number line as the starting point and then making nine jumps (hops) to the right if they count in ones, or alternately, recognising the commutative property of addition, starting with 9 and making 8 jumps. However, if they realised that nine is equivalent to ten minus one, they could possibly make one bigger hop to the right denoting (+10) to get to eighteen, and then one small hop to the left denoting (-1) to get to the answer seventeen.

Different Strategies Used by Participants

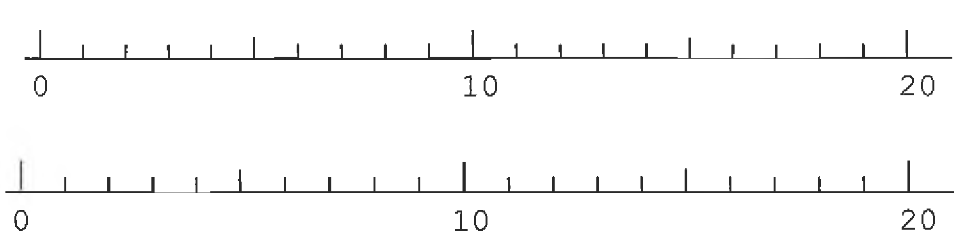
In Task 2(b) participants were expected to identify eleven on the number line and to move six steps to the left to get to the answer, which is five. Both item 2 tasks were aimed at eliciting participants' understanding of a number line as a possibly suitable thinking tool for performing addition and subtraction operations with one and two digit numbers.

Figure 4.3: Task 2

2. Draw blocks or jumps on the number line to show the operation and find the answer

a) $8 + 9 =$

b) $11 - 6 =$



The number line is an established model for teaching number that is said to be a schematic representation of a string of beads. The number line can be shown 'complete' (with ones), 'half complete' (with tens only marked) or 'empty' (without numbers (Van den Heuvel-Panhuizen, and Senior, 2001, p. 82). The empty number line is now used internationally. It is closely connected with RME. This item, borrowed from the DelPHE project, invited students to use a complete number line. While the two tasks (a & b) involve small numbers, they call for different operations with subtraction arguably more difficult than addition, depending on the problem structure represented and the strategy used. For example, at a basic counting level, the hops on the number lines might move in different directions. In Task 2(a) (addition) the hops might start at 8 and move 9 steps to the right. In Task 2(b) (subtraction) the hops might start at 11 and move 6 steps to the left. It might then be argued that task 2(b) has more cognitive demand than 2(a) so I am here presenting a table showing how participants performed on Task 2(b) only.

Table 4.6: Participants' responses to Task 2(b)

		TASK 2b				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC Result	Fail	0	4	17	114	135
	Pass	1	0	10	50	61
	Credit	0	0	1	15	16
Total		1	4	28	179	212

Table 4.6 shows that most participants ($n = 179$) succeeded in obtaining the correct answer for Task 2(b). 84% of participants were able to associate the number line with a satisfactory representation of a subtraction task. Further investigation of the 33 responses that were coded either 'not done' (n = 1), 'incorrect' (n=4), or 'incomplete'

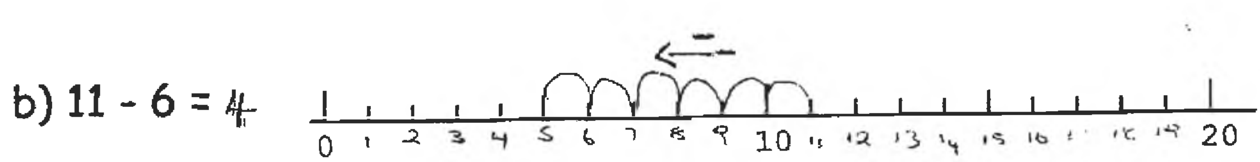
(n= 28) provides interesting evidence of different structures of attention paid to the task by different candidates.

Snapshots of Responses to Category 1 Tasks

One young student, Mary aged 20, correctly filled in the digits 1to 19 on the number line for item 2(b), but although she successfully completed 2(a), did nothing further with either the iconic or the symbolic representation. Mary might be said to have discerned the detailed calibration of the number line but did not proceed further, so her response was coded 'not done'.

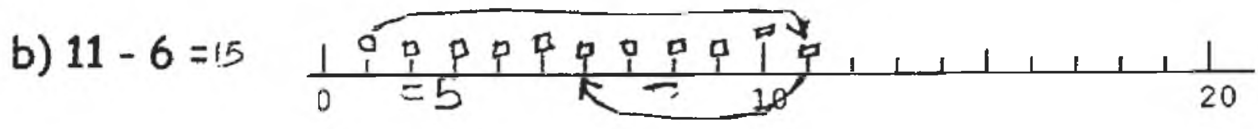
Of the four participants whose answers to item 2(b) were coded 'incorrect' one treated the item as if it were addition (11 – 6 = 17) with appropriate jumps forward on the number line. She appears to have read the minus sign as a plus. Two other students correctly indicated six jumps back from eleven on the number line but wrote the answer as 11 – 6 = 4. One of these two was more sophisticated. She placed a directional arrow to the left labelled it minus (–) but appeared to have counted the remaining spaces as 4 instead of 5 (see Figure 4.4 below). I suggest that she, like the other mentioned respondent, started counting the 'difference' either to or from the next 'free' numeral, i.e., 4. It is tempting to suggest that inviting participants to make an iconic representation confused these two students instead of helping them.

Figure 4.4: Student's response to Task 2(b)



Another of the four students that were coded incorrect, Bella provides interesting insight into levels of familiarity with the use of the number line. Her iconic representation shows a jump, starting (incorrectly) at 1 to 11. A second arrow goes towards the left from 11 to 6. This jump is labelled with a minus (–) sign. However, the individual calibrations are not labelled as numerals. Instead, eleven little boxes are drawn. The difference between 11 and 6 is labelled "equal 5". For the symbolic representation, Bella wrote 11 – 6 =15, which suggests that she was relying on the number line to find the answer to the task and misread 5 as 15.

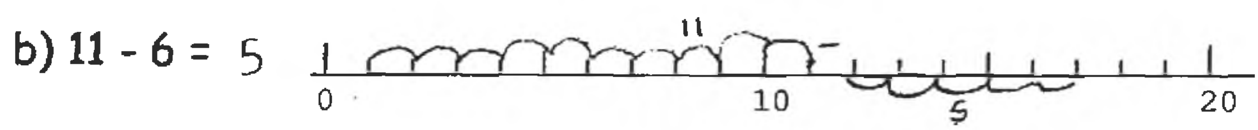
Figure 4.5: Bella's solution to Task 2(b)



Bella's use of the number line might be said to be technically 'incorrect' because she starts her jumps at 1 instead of 0 and only made 5 jumps to the left instead of 6. However, she relied instead on the counters she drew to correspond with digits 1 to 10 and consequently her calculation (= 5) was correct. Bella appears to have recognised a relationship between the given representation, a number line, and stones or counters, with which she is more familiar. She then reasoned correctly on the basis of the perceived properties of the counters and tried (incorrectly) to transfer this reasoning to the number line. Her subtraction representation could be said to be either a 'change' or 'compare' structure.

Twenty-eight responses to item 2(b) were coded 'incomplete'. Of these, fourteen respondents gave the correct answer to the symbolic representation. The other half did not address the symbolic representation at all. Of the former students whose symbols were correct but who were less successful with the iconic representation, seven of those indicated five jumps between 0 and 1 on the number line. One drew little boxes on the calibrations 1 to 5 without any further inscriptions. Another drew 11 boxes to the left of the symbolic representation and crossed out 6 of them leaving correctly a difference of 5. This student ignored the number line completely and substituted a meaningful representation that made more sense to her. Another student, Adoro drew 10 jumps on the number line starting at 1, called this 11, inscribed (–) minus in the next space, then drew a further 5 jumps below the line. Consider Adoro's response in Figure 4.6 below:

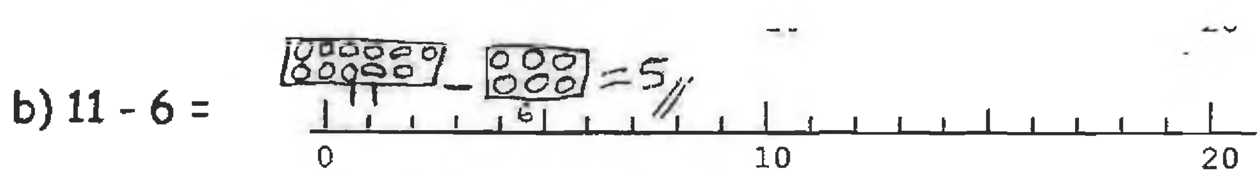
Figure 4.6: Adoro's response to Task 2(b)



Adoro has done much more than 'gaze' at the representation. Her detailed inscriptions constitute her own representation of subtraction where in fact she has represented 11 and added 5 jumps leading to total of 17 because one space is occupied by the minus symbol. This putting out of both the minuend and the subtrahend will work for the subtraction as comparison structure when the two amounts are matched one-to -one and the 'difference is deemed. In this case, it is a representational error. Her iconic representation indicates that Adoro was struggling to represent without success, a subtraction fact that she 'knows'. The use of a number line for subtraction becomes challenging here because "high school students will have not used counters or number lines for some time" (Van de Walle, 2001, p. 425).

With regard to the other fourteen who did not regard the symbolic representation at all, one might expect that they focussed on representing the subtraction operation on the number line. One started at 0 and made 5 jumps to the right indicated by a directional arrow. Two others appeared to do the same but without the directional arrow. The remaining 11 were more sophisticated in the correct use of the iconic representation. Three correctly perceived the properties of the number line and inscribed 6 jumps between 5 and 11without labelling the jumps. Six others were more detailed. One made 11 jumps and shaded 6 on the right leaving difference of 5. One drew jumps from 5 to 11 presumably representing the subtrahend 6. Another four participants gave detailed correct representations of the subtraction operation $(11-6=5)$, each slightly different in detail but all equally transparent. Two remaining students gave an iconic representation only. One drew 11 boxes above the unlabelled calibration and crossed out 6 of them leaving a difference of 5. Another drew 11 counters and put a border around them, then drew 6 and put a border around them. Refer to Tillie's Figure 4.7 below:

Figure 4.7: Tillie's response to task 2(b)



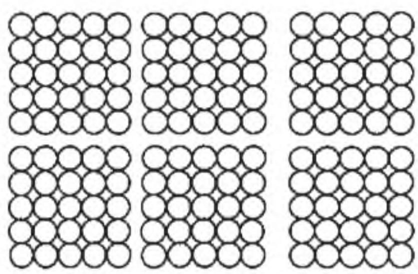
Tillie's response illustrates a dilemma discussed by Rowland et al (2009, p. 163). She has drawn 17 objects, but not matched them one-to-one to show a 'compare' structure. She has not treated the task as a 'partition' structure either. I perceived the participants drawing of counters as introducing an alternative form of representation they were more familiar with (such as stones). Much later, I learned that the instruction for task 2, to "draw blocks or jumps on the number line" confused some respondents.

Multiplication Arrays

Figure 4.8: Task 3

3. Show the following calculation on the diagram provided and write the answer

$5 \times 12 =$



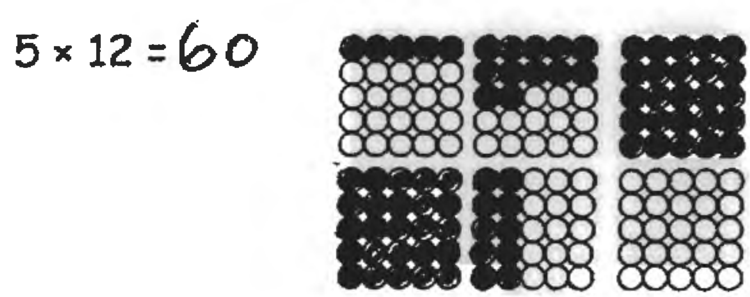
Task 3 is a multiplication task, which is accompanied by an array of 150 counters arranged in six sets of twenty-five. Participants were expected to demonstrate their calculation on the array and write the answer (60).

Table 4.7: Participants' Performance on Task 3

		TASK 3				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC Result	Fail	1	5	24	105	135
	Pass	0	1	12	48	61
	Credit	0	1	0	15	16
Total		1	7	36	168	212

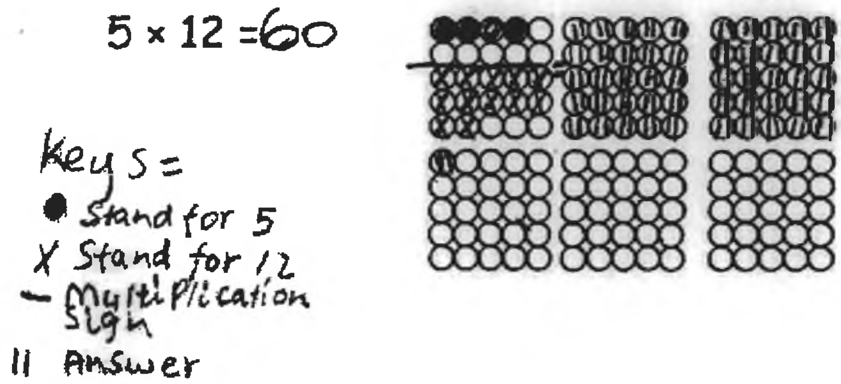
The majority (almost 80%) of student teachers performed this task correctly. There was considerably more success among the credit students, than among the pass students with this task. There was a greater spread and slightly less success among the fail students. Participants' responses were coded *incomplete* if the shading on the array diagram appeared to represent more or less than 60, or was not done at all, and yet the answer for 5×12 was given as 60. In some cases the answer for 5×12 was not written and yet the diagram was correctly shaded. On closer inspection, of the 36 responses coded *incomplete*, 16 could be called 'fuzzy' because while of 16 students each gave the correct answer to the symbolic statement and included an array of 60 shaded counters, this was preceded by a separate attempt to represent either the multiplier (5) and/or the multiplicand (12).

Figure 4.9: Student's response to Task 3



The response of this student is typical of 11 of her colleagues. Three of the sixteen showed only 5 and 12. These students appeared to miss the point of the counters as a multiplication array. Two warrant for closer analysis. One student Matau coloured 5 down and also 12 across. If she had completed this array it would have given her 65 (5×13) but Matau correctly filled 60 counters in the lower array and wrote the symbols correctly $(5 \times 12 = 60)$. The remaining student Libu was interesting for another reason. She coloured in 4 discs, she then coloured in 12 lower down on the same 25, and attempted to colour in 60 although her inscriptions only appear in 51 discs. Libu however offered a key to her work.

Figure 4.10: Libu's response to Task 3



From the keys (sic) it appears that Libu was thinking to represent the 5x12=60 in a sequence: 5 (actually 4 circles are shaded) followed by a line to represent the 'multiplication sign' followed by 12 crossed circles followed by the equals sign, followed by 51 striped circles to represent (incorrectly) the answer.

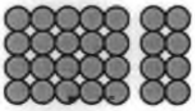
My analysis for item 3 contends that participants who gave an incorrect answer and those who left the answers incomplete are stuck at the *gazing* level. They see no way to make a connection between 5×12 and the array next to it. While Table 4.7 shows that the majority ($n = 168$) got correct answers for Task 3, some of such answers suggest a possibility that some participants worked on the task using an algorithm first and later showed the answer on the array. That was reflected in representations where the answer, sixty (60) was distributed as 5 discrete collections of 12 circles coloured on the array. Thus it would be more difficult for such participants to show that their diagrams represent five rows of twelve circles (5×12). On the other hand, participants who shaded 12 circles in the first row and 5 circles in the first column so that their diagrams showed '60' shaded circles were able to *recognise relationships* between the array and the expression '5×12'. They could flexibly move between the two representations. This means such participants would have gone beyond the stages of *gazing at the representations* and *discerning details*. Watson (2009) contends that this kind of attention can be analytical.

Category Two Tasks

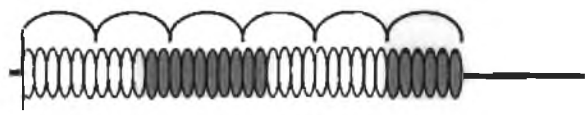
Task 4 has three subsections; 4(a), 4(b), and 4(c). The three tasks require participants to figure out the possible mathematical operations demonstrated on each of the three task- accompanying diagrams. The iconic representations shown on each of the three tasks could be said to stand for the mathematical operation of multiplication, although different iconic representations are used. However, other interpretations are possible. For instance, Task 4(a) presents part of a multiplication array that is intended to show either $7\times4=28$ or $(5\times4)+(2\times4)=28$ or $20+8=28$; Task 4(b) presents a picture of a string of beads that demonstrates the following operations $36=6\times6$ or repeated addition, $6+6+6+6+6+6=36$ or division, $36\div6=6$. Task 4(c) presents a set of seven plates each containing a group of six strawberries. Symbolically, the diagram might be read as either 6 × 7, or $7\times6=42$ or $(6+6+6+6+6+6+6=42$. Again, 42 ÷ 6 = 7 or 42÷7 =6 are possible interpretations. Given the fact that unlike the first task, tasks 4(b) and 4(c) suggest addition, multiplication and division operations, in what follows I focus on how participants performed on them.

Figure 4.10: Task 4

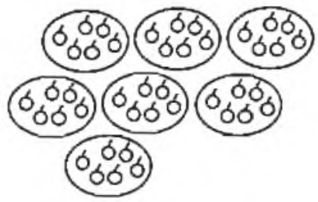
4. What calculations are shown in the following representations? Write the calculation and the answer

a) 

b)



c)



Looking at the correct answers for task 4(b) provided by the seventy-eight participants, who were coded 'correct', the popular answers came as: $6+6+6+6+6+6=36$; $6\times 6=36$, and $36\div 6=6$. Table 4.8 below presents participants' overall performance on Task 4(b).

Table 4.8: Participants' Performance on Task 4(b)

		TASK 4(b)				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC Result	Fail	4	81	2	48	135
	Pass	2	30	5	24	61
	Credit	0	9	1	6	16
Total		6	120	8	78	212

According to the information provided by Table 4.8, many participants $(n=120)$ could not suggest a correct answer for Task 4(b) while six did not attempt to work out an answer. Spread over each of the three grade categories, credit, pass and fail, more than half the respondents were coded 'incorrect'. This means for one hundred and twenty six participants, the representation given (beads) did not suggest any direct link with the mathematics operations (addition, subtraction, multiplication, or division). These participants gazed at the representation and failed to discern the mathematical details of the diagram. Almost 60% is a significant number of the participants who got this task wrong, which might appear a surprising result for students who have completed high school mathematics. It suggests that the iconic representations are all unfamiliar to these beginning student teachers. They appeared unable to assign mathematical meaning to them.

Category Three Tasks

In the following section, I focus on Task 5, which requires participants to construct their own representations as they work out solutions to 2 and 3 digit addition calculations.

Figure 4.11: Task 5

5. Work out the following, and show how you got your answer

201 + 79

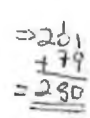
While this task may seem easy to do, it was worth including in the survey instrument in order to establish if respondents could demonstrate the answer using other methods besides the usual algorithm of adding units, tens, and hundreds. According to Table 4.9, almost all participants ($n = 197$) got the correct answer for Task 5. However, it is worth noting that all of these participants used the traditional algorithm to work out the answer 280.

Table 4.9: Participants' Performance on Task 5

		TASK 5				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC	Fail	0	9	2	124	135
Result	Pass	1	2	1	57	61
	Credit	0	0	0	16	16
Total		1	11	3	197	212

Below is an extract of one participant's work on this task:

Figure 4.12: Participant's Response to Task 5



After obtaining the answer (280) some few participants attempted to draw some arrays, which they used to illustrate their solutions. No participant appeared to have used the strategy: $201+79=200+1+79=200+80=280$. None of the participants used a number line to illustrate the answer. Again, it seems reasonable in this task to conclude that participants were still at the first stage of Mason's 'structure of attention, namely *gazing*. They looked at the task as an independent whole, which could not be associated with anything other than the standard algorithm. This kind of 'image-making' attention is known as holding wholes (Watson, 2009, p. 219).

Category Four Tasks

Task 6 is different from all the other tasks in that it has an explicit pedagogical element, which calls for explaining a strategy to learners. Even though at this stage participants were about to begin training to become primary school teachers, the task requires them to explain to the learner how they got the answer. This means it would not be enough for a participant to simply write 'eleven books'. The task also requires participants to use some form of representation to illustrate their workings.

Figure 4.13: Task 6

6. If you have M44 to spend, how many Exercise Books at M4 each can you buy? (**Show clearly the representation you would use to explain to a child how you got the answer**).

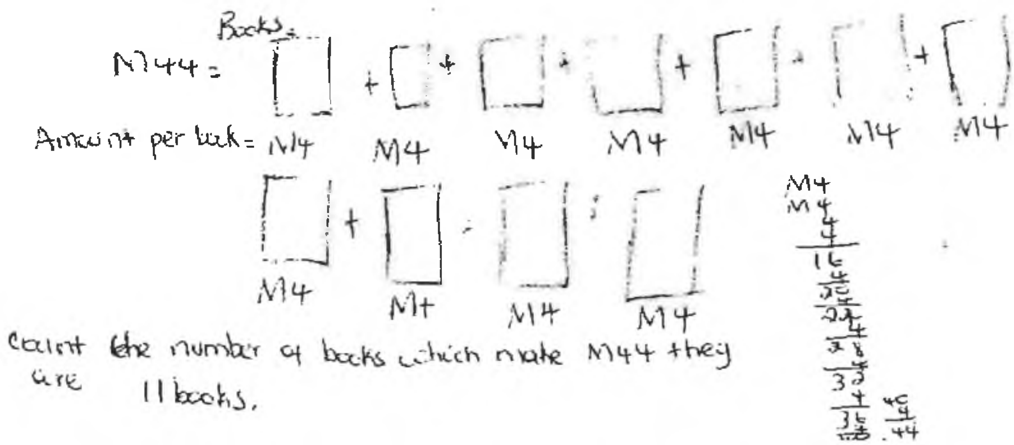
The following table displays information about participants' performance on this task.

Table 4.10: Participants' Performance on Task 6

		TASK 6				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC Result	Fail	1	2	2	130	135
	Pass	0	3	0	58	61
	Credit	0	0	0	16	16
Total		1	5	2	204	212

Two hundred and four participants offered a correct representation for the task. This implies that in addition to the correct answer they were also able to explain the method they followed to arrive at such an answer. Below, I present an example of answers provided by one participant on this task.

Figure 4.14: Tšeli's Response to Task 6



This participant, Tšeli drew pictures representing books and wrote the price of each book below it. It is interesting that she places the plus (+) sign between diagrams instead of between prices (M4 + M4). This use of repeated addition would seem reasonably adequate to serve the purpose of explaining how eleven books were bought from an amount of M44. Looking at responses to this task, it could be argued that the majority of participants $(n = 204)$ were more able to discern the relations between their own self-constructed representations (diagrams) and symbolic representations $(M44 \div M4 = 11)$ than in situations where a symbolic representation is accompanied with a given form of representation. In this case, participants came up with various diagrams to represent the given scenario of buying 11 books with forty-four Maloti.

A Non-routine Problem

The following is a popular task known within the mathematics education community as the 'hand shake problem' (Rowland 2003). This task like Task 6, gives participants an opportunity to construct their own representations and use them to solve the problem. However, from the analysis of participants' responses, it seems the non-routine hand-shake task is far harder than typical word problem Task 6. Task 7 calls for problem solving skills and innovative strategies.

Figure 4.15: Task 7

7. There are nine people at a party. Each person shakes hands once with each of the other people. How many handshakes are there in all? (**Show clearly the method that you used to get to the answer**).

Table 4.11: Participants' Performance on Task 7

		TASK 7				Total
		Not Done	Incorrect	Incomplete	Correct	
COSC Result	Fail	1	127	3	4	135
	Pass	3	55	0	3	61
	Credit	1	14	0	1	16
Total		5	196	3	8	212

The table shows that out of two hundred and twelve participants only eight of them got correct answers for Task 7. This means only 4% of the participants successfully solved this non-routine task. It is interesting to note that out of those eight, there are four people who failed the mathematics examination in Form E. Again it is worth noting that out of sixteen participants who had obtained a credit in Mathematics in Form E, only one got a correct answer for Task 7. Participants' responses to Task 7 show that a majority of them tried to draw various diagrams in an attempt to work out the correct answer (36) but most of these were in vain. With only nine people shaking hands participants might easily have experimented to find an answer. I suggest the following table as an example:

Table 4.12: Possible Table to Support Reasoning on Task 7

Number of People	1	2	3	4	5	6	7	8	9
Number of Handshakes	0	1	3	6	10	15	21	28	36

Discussion of Participants' Representations in Section 2

The analysis of participants' responses to tasks in Section 2 of the survey instrument shows that in general, student teachers who register for DEP programme are more comfortable with following algorithms used for addition, subtraction, multiplication, and division of numbers. When a task is accompanied by a diagram such as a number line or an array, they do not always recognise the possibilities of using such a representation and simply work out the answer by following a standard algorithm. In some instances, participants found the answer first and then attempted to show such an answer on the diagram, which was not always successful. For tasks that required participants to construct their own representations, it was interesting to see that many student teachers made an attempt to draw diagrams and write sentences as efforts towards establishing a correct answer. The data also show that almost all participants are weak on problem solving skills. Reference is made to the hand-shake task, which could not be associated with any known formula or taught algorithm. Only eight participants obtained the correct answer to this problem, out of a total of two hundred and twelve participants.

Conclusion

The data analysed in this chapter suggest that most respondents represented the routine textbook problem correctly, thus demonstrating a degree of PCK. Respondents were more able to associate the number line than an array with addition or subtraction. They also found it easier to represent the operation of addition than of subtraction. In terms of my analysis thus far, it seems reasonable to conclude that the many participants who performed the two subtraction items 1(a) and 2(b) were able to make a meaningful relationship between the symbolic representation and the iconic representation provided. Their work on these tasks could be interpreted as being a reflection of their understanding of subtraction of numbers and the meanings they hold of the iconic representations, including ones they drew themselves, intended for this operation (subtraction).

It seems from the data that at the stage of entry into the DEP programme, some students are likely to engage in gazing at representations that accompanied the tasks while failing to discern the details that would help them to connect the two mathematical representations (e.g. symbolic and iconic). This is probably due to their experiences of doing high school mathematics tasks. It has also been noted that at this stage, students tend to stick to algorithms for working out solutions rather than exploring possibilities with mathematical representations. This is perhaps also a result of their high school learning experiences of mathematics. At the entry point to the DEP programme, students seem to be more comfortable with symbolic representations than with any other representations. I also observed that students who failed mathematics in COSC performed relatively better in a problem (Task 7) that did not call for specific algorithms than students who obtained a credit in mathematics. I then conjecture that there is a likelihood that those who do well in COSC are those who are good at following rules, yet that does not necessarily mean they are good at solving mathematics problems.

In the following chapter, I focus on the analysis of Tier 2 data in the form of transcripts that were constructed from semi-structured interviews held with ten participants. The ten participants were reflecting on their responses to tasks in Section 2 of the survey instrument in Tier 1. Data in Tier 2 are also intended to respond to the research question: What mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?

5. Tier 2 – Semi-Structured Interviews

In the previous chapter, I focused my discussion on the general analysis of Tier 1 data. In this chapter I analyse Tier 2 data in order to provide greater depth in terms of the participants and their answers to the survey instrument as an to answer to the research question: what mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?

The main aim at this stage was to find out what participants would say about the representations they used as they were solving tasks, and their responses would therefore supplement their written work in helping me to understand their approach to mathematical reasoning with representations. The interviews were conducted towards the end of Year 1, after participants had taken two mathematics courses (MAT 1400PA and MAT 1402PB). The two courses include both mathematical content and pedagogy. This suggests that at this stage participants might have matured academically and hence have been more mathematically and pedagogically aware than at the beginning of the year when they participated in Tier 1.

As mentioned in Chapter 3, the ten participants in Tier 2 were such that five had obtained a pass in the mathematics terminal examination in Form E, two had failed, and the remaining three had obtained a credit. This means that all participants in Tier 2 came from the three categories namely *credit, pass*, and *fail*, as determined by participants' performance in high school mathematics. Each was given a pseudonym. Looking back at the home districts of the whole cohort, I note that the purposive sample represents the five districts, which are home to over 80% of the participants. The age profile of the purposive sample is 20 – 26 years, with a mode of 24 years.

More than 75% of the whole cohort comes within this age range. Thus, I claim they are a sample of "typical cases" (Lincoln and Guba, 1985, p. 200)

Table 5.1: Ten participants interviewed in Tier 2

Participants in Tier 2	**Gender**	**COSC grade**	**Participant In Tier 3**
Beatrice	Female	Pass	
Mosha	Female	Pass	
Nomsa	Female	Pass	❖
Sebu	Female	Pass	❖
Tšeli	Female	Pass	❖
Litha	Male	Credit	❖
Motati	Female	Credit	
Thandi	Female	Credit	❖
Mable	Female	Fail	
Minah	Female	Fail	

Analysing Participants' Responses

It was established in Chapter 4 that many student teachers considered mathematics as a 'challenging venture' that demands effort and critical thinking from learners. Because this perception of mathematics emerged as a theme with the greatest weight in Chapter 4, it was important to consider this perception when analysing the transcripts. It was again observed in Chapter 4 that students expect the mathematics courses offered at the college to equip them with skills that will enable them to 'develop an understanding of mathematics' and 'learn how pupils learn mathematics easily'. These ideas that were mentioned by the majority of participants in Tier 1 are useful in looking at the extent to which they influence students' thinking as they talk about the answers they gave to tasks in Section 2 of the survey instrument. In order for me to perform detailed analysis I choose to focus on each of the ten students as a particular case by studying one transcript at a time so that all ten transcripts can be reported thoroughly and a composite picture drawn.

Beatrice's Case

At the beginning of this project in Tier 1, Beatrice was twenty-six years old. Beatrice is a woman from Leribe district. The record of her performance in school mathematics shows that she has achieved a relatively good grade in mathematics in national examinations. For instance, in Standard 7 examination she obtained a first class (1st class) in mathematics, in Form C she obtained a symbol C, and in Form E she obtained a pass. From an analysis of Beatrice's work on Section 2 of the survey instrument (tasks) I realise that it is only task 1(a) that is incomplete; and all other tasks are correct. During the interview, Beatrice was asked to give comments on her work on tasks 1(a), 6, and 7, and she was further asked about representations she could use in her class when on teaching practice.

The following excerpt presents the conversation I had with Beatrice with regard to task 1(a):

Beatrice: I really didn't know what to do in this problem, but I think if I could use like … when I look at it closely now I think I can have ways of struggling with it.

Researcher: OK, how would you do it?

Beatrice: If I can say this is a sort of … what these pictures … I remember I did something like this in mathematics lectures … as a sort of a number array. Having been taught how to use the number array I think I can tackle this problem now.

Researcher: Alright, what would be the answer?

Beatrice: Can I please work it out?

Researcher: OK …

Beatrice: My first number is 17 and we subtracted 5 from it. What I will do is just to shade these pictures differently and get my number. (*Initially Beatrice had shaded just 12 counters*). I'll shade this one (*referring to 17*), as I have already shaded it in this way and this one (*referring to 5*) would just mark the process on it and get the answer …

According to the excerpt above, Beatrice had encountered some problems when solving this task in Tier 1 because she could not recognise the relevance of the diagram accompanying the task. She also omitted to write down the answer (12). However, at the time of interview Beatrice was well able to make connections between the two representations. This is because as she puts it, she "did something like this in mathematics lectures".

Beatrice was then asked to comment on task 6, which involved money to be spent on buying exercise books. In Tier 1, Beatrice had provided the following textual explanation:

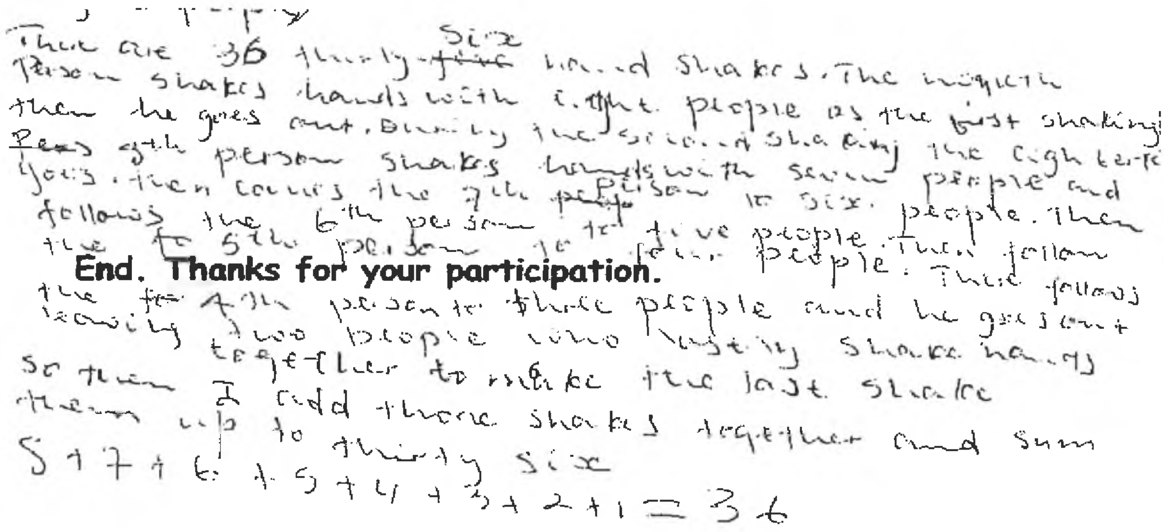
Figure 5.1: Beatrice's response to Task 6

I would take 44 stones that is little stones and
tell the child to take them as his or her money. Then
I tell a child to put them in groups of four meaning
that each group represents one book this will
make eleven groups. So I tell a child that
he has got eleven book.

This explanation ties well with what she said during the interview. Unlike most of her peers who solved this problem with repeated subtraction or addition, Beatrice recognised it as division with a grouping structure. She suggests "small stones" as counters that would be used to represent money for learners in buying books. These could be used as an enactive representation of the problem. It is interesting that while Beatrice suggests this representation for item 6 she did not recognise the iconic array in item 1 as counters that could be used to represent the operations. Perhaps the static nature of the diagrams presented in tasks 1(a) and (b) was not helpful to her thinking, since she did not recognise the counting properties of the two representations as common or possible for both.

When working out the solution to Task 7, Beatrice gave a detailed account in the form of a text that enabled her to obtain the correct answer (36). The following extract presents Beatrice's work:

Figure 5.2: Beatrice's response to Task 7



There are 36 thirty-~~five~~ six hand shakes. The ninth
person shakes hands with eight people as the first shaking
then he goes out. During the second shaking the eighth
~~Pers~~ 8th person shakes hands with seven people and
goes. then comes the 7th ~~peop~~ person to six. people. Then
follows the 6th person to five people. Then
the ~~to~~ 5th person to four people. Then follow
End. Thanks for your participation.
the ~~for~~ 4th person to three people and he goes out
leaving two people who lastly shake hands
together to make the last shake
So then I add those shakes together and sum
them up to thirty six
$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$

During the interview, Beatrice was asked how she had come up with the correct answer. It is interesting that like some other participants she was assisted by remembering a practical, church context for shaking hands. The following excerpt presents what Beatrice said:

> Beatrice: As for the answer with the handshakes I just thought of the people. Usually what we do when we are in church as we vacate the place, we always greet each other. I thought of such a thing and counted … it was a very demanding task. I really don't recall what I did at that time but I only remember that I used the system of greeting each other in the church.

Beatrice's answer was interesting to me because it was in the form of a text, which was unique from the answers provided by other participants. The main connection that Beatrice made is between the church context and the sum of natural numbers (N = {1, 2, 3, 4, 5, 6, 7, 8}). It could then be said that Beatrice competently reasoned on the basis of the real life church experience to obtain the correct answer.

Mosha's Case

Mosha is a male student from Leribe district. He is twenty-two years old. Looking at Mosha's performance in mathematics in the previous school examinations one concludes that he has a weak mathematics background. For example, in Standard 7 national examination Mosha obtained a second class pass, and failed mathematics in both Forms C and E. When asked to write a brief story about his experience of learning school mathematics in Tier 1 he wrote the following:

Figure 5.3: Mosha's account of learning school mathematics

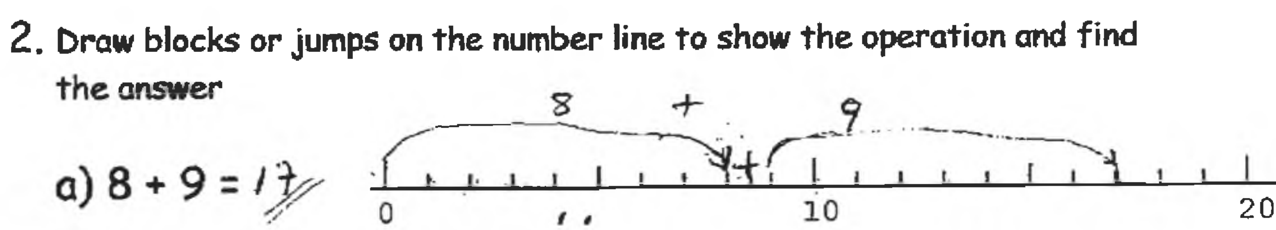
I have experienced that there is no such big difference between primary and high school level Mathematics because only four mathematical signs are used in both levels. e.g +, −, ÷ and X

This extract is brief and reflects Mosha's personal belief about mathematics. It sheds light on how Mosha had experienced school mathematics, which he reduced to understanding the four operations namely addition, subtraction, multiplication and division.

Three of Mosha's answers on tasks in Section 2 of the survey instrument in Tier 1 are marked *incomplete*. Task 1(a) is incomplete; 1(b) is correct; tasks 2(a) and 2(b) are both incomplete; task 3 is incomplete; tasks 4 (a) and (b) are incorrect; task 4(c) is correct; tasks 5, 6 and 7 are correct. All responses that were marked incomplete are situations where Mosha had written correct answers to the symbolic representation but without having matched them meaningfully on the accompanying iconic representations. This may be an indication that because Mosha learned an 'instrumental' form of mathematics at school (Skemp, 1976) he is less able to make connections with the representations presented. However, it is worth noting that Mosha obtained a correct answer to the 'hand-shake' problem, which appears to contradict the instrumental approach. During the interview, Mosha was asked to comment on his answers to tasks 2(a); 4(a) and (b) and 7.

The following extract presents Mosha's actual work on this task:

Figure 5.4: Mosha's response to Task 2(a)



According to Figure 5.4, Mosha started from 0 and made one big jump to the right and stopped at 8, correctly denoting 8 single steps. He then put a plus sign in between 8 and 9 on the number line. From there, he started at 9 and made one big jump again to the right denoting 8 single steps instead of the 9 he had written, and stopped at 17. In my opinion, the writing of (+) on the number line might have led to the incorrect hop from 9 to 17, which should have started from 8 and stopped at 17 in order to represent 9 single steps. While strictly incorrect, Mosha's work here demonstrates positive thinking about the use of a number line, which he attempted to manipulate to represent his thinking meaningfully..His strategy would work well on an empty number line, provided he knew the addition facts. Therefore, it could be concluded that Mosha perceived properties of the two representations (iconic and symbolic) and made connections between the two but was only let down by the error of writing (+) on the number line.

When asked to say how he got wrong answers for tasks 4(a) and (b), Mosha seemed surprised that he had failed to recognise what the representations stood for then and at the time of interview the two tasks appeared easy for him. Mosha was then invited to talk about Task 5, for which he had obtained a correct answer. His work looked as follows, was almost identical to the work of another student presented in Figure 4.12.

Figure 5.5: Mosha's response to task 5

201 + 79 = 280//

$$\begin{array}{r} 2\,^{1}01 \\ +\ 79 \\ \hline 280 \end{array}$$

Mosha said that he followed the standard algorithm in order to obtain the answer (280). He was then asked if at the time of the interview he could suggest other strategies for doing this task. Consider the following excerpt:

Mosha: I think I will use a number line.

Researcher: OK, do you want to use it here? Show me how you can use a number line to get the answer.

Mosha: (*Pause*) I think I'll draw it from 0 and count 79 steps, then from 79 we need 201 steps.

Mosha struggled to produce a number line that would help him get to the answer. As part of his effort Mosha produced the following number line:

Figure 5.6: Mosha's use of the number line during interview

This diagram manifests Mosha's attempt to use a number line for addition of numbers. He placed a smaller number (20) on the right of the bigger number (79) on the number line. I suspect he may have meant to write 200. In the following excerpt where Mosha talks about his work, the difference between my approach as researcher and a clinical interview (Ginsburg, 1997) becomes apparent. I did not attempt to discover the logic behind his response and I did not provide any hints to help him:

Mosha:	I will count 200 steps, and then from 200 count 80 steps then … are 80 steps.
Researcher:	Are you comfortable in using a number line to do addition?
Mosha:	I'm not that much comfortable.

It could be said here that Mosha might be more competent in using the standard algorithm learned in school mathematics than in using the number line encouraged in DEP mathematics courses for doing tasks such as (201 + 79). The shift from well known algorithms to other more flexible ways such as the use of the number line is admittedly demanding. However, as a future teacher, Mosha might be expected to teach primary school mathematics competently and flexibly through the use of thinking tools such as an empty number line to model addition and subtraction operations (van den Heuvel-Panhuizen and Senior, 2001).

Mosha was then asked to comment on how he obtained an answer to Task 7 (hand-shake problem). The following extract presents Mosha's working on this task:

Figure 5.7: Mosha's response to Task 7

Num of P	Num of Shake
1 =	8
2 =	7
3 =	6
4 =	5
5 =	4
6 =	3
7 =	2
8 =	1
9 =	0
	36 handshakes

There will be 36 handshakes

In order to appreciate of Mosha's work here consider the following excerpt:

Mosha: OK, I counted … there were 9 people at the party, so each person shakes once so the first one, we will not count him, we'll count those who will shake the hands with him, OK … then reduce 1, until the last person.

Mosha constructed a table where he showed that the first person will make eight handshakes with the rest of his friends, and the second person will make seven handshakes, third one makes six handshakes, and so on till the 9^{th} one who makes zero handshakes because he would already have touched everybody. In my opinion, this is a powerful table, which led Mosha to getting a correct answer. However, the first column might be labelled 'person's position' rather than "number of people" because this first column yields ordinal numbers and not counting numbers. It is worth noting that in spite of Mosha's weak mathematics background he was creative enough to produce a table that led him into obtaining a correct answer for a non-routine task, which must be recognised as having implications for basing mathematical learning in real life situations or contexts.

Nomsa's Case

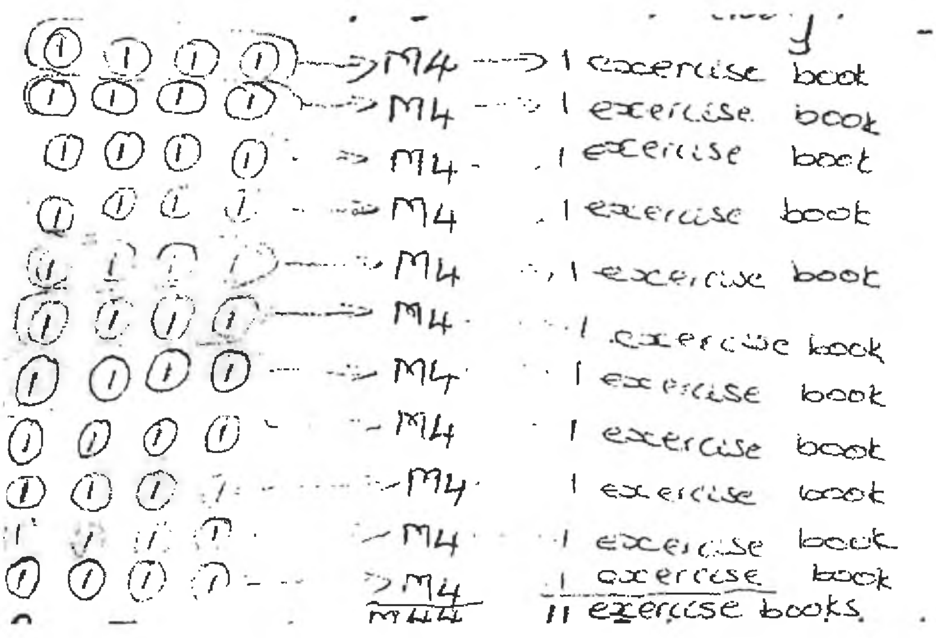
Nomsa is a twenty-four years old woman from Mafeteng district in the southern part of Lesotho, who came to the college from high school. Her mathematics performance has been average from primary schooling all through to high school. She obtained a second class pass (2)[4] in mathematics at the end of primary schooling (Standard 7 national examinations) See Appendix 1 for sample examination papers. In Junior Certificate (Form C) national examination she obtained a D pass in mathematics. Nomsa obtained a Pass in Form E. As such, Nomsa could be classified as an average mathematics learner. When asked to explain how she felt about the tasks in general, she said the tasks required participants to think quickly and critically in order to obtain correct answers. This might mean that Nomsa saw the original survey situation as a pressure, which is quite understandable and might have influenced her performance.

Nomsa completed tasks 1 – 5 in Section 2 of the survey instrument in an exemplary manner. So I asked her to begin with her comments on Task 6, which involved the buying of books. The following diagram shows Nomsa's actual working on Task 6 where she drew pictures of money each representing 1 Loti, and in each set as shown on the diagram there are 4 Maloti. In her picture she has shown that the 4 Maloti would be used to buy one exercise book. The diagram shows that there are eleven sets of 4 Maloti, which could be used to buy eleven books. Nomsa made an effort to demonstrate that M4 =1Loti ×4, the fact which was implicit in the task itself.

[4] The grades in descending order are 1, 2, 3, and F (fail)

Nomsa might have chosen to be so elaborative in her illustration because she was asked to explain it to a child.

Figure 5.8: Nomsa's response to Task 6



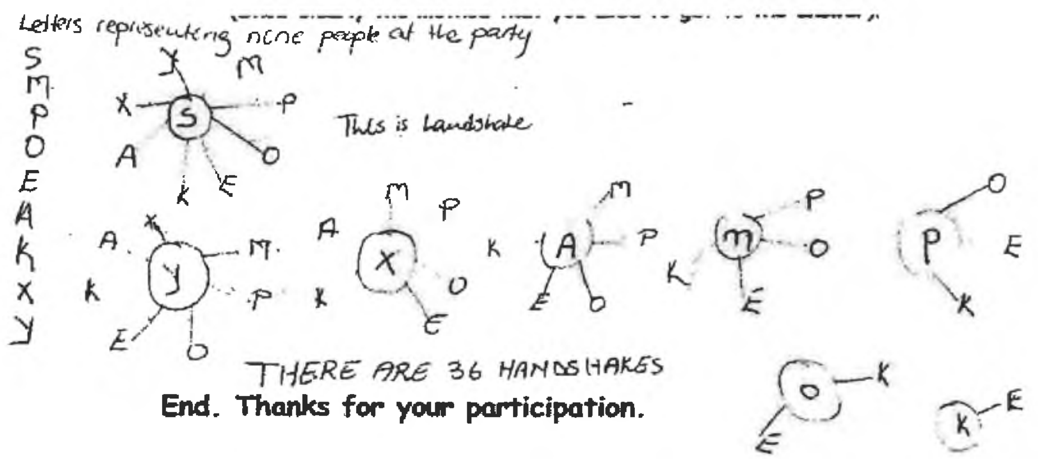
In the following excerpt she explains the underlying reasons for her work:

Nomsa: So I said the total of 4 Maloti correspond to 1 exercise, therefore 44 Maloti will correspond to 11, so I thought if I present this information like this to a learner, it will be easy for a learner to understand.

Nomsa had drawn a detailed diagram to represent the way in which M44 could possibly be distributed equally to purchase eleven exercise books mainly because she had a learner in her mind that needed to be shown this operation. It appears that Nomsa mentally divided 44 by 4 in order to be able to make her child-friendly representation of repeated addition. She is convinced that with the aid of her diagram, it would be easy for the learner to understand this as a repeated addition task. This is in harmony with the themes that emerged in Tier 1, that because mathematics is perceived as a difficult school subject, participants are concerned with finding ways of making it easy, accessible and understandable to learners.

Focusing on this particular episode, I realise that Nomsa is able to explain how each representation helps to perform some calculation, and why her self-invented representation is useful in enabling learners to understand the division operation (44÷4) by translating it into a corresponding repeated addition operation. I also conclude that Nomsa has demonstrated the highest stage of understanding representations, namely ‘reasoning on the basis of properties’. My argument is based on the fact that Nomsa was able to decide on the key representation (diagram) she was going to use for this particular operation (4+4+4+4+4+4+4+4+4+4+4 = 44). She is able to justify how and why her representation works in this particular situation. By means of her diagram she is able to make clear a connection between representations (her diagram and related symbols).

Figure 5.9: Nomsa’s response to Task 7



Nomsa was also asked to comment on what she did on task 7 involving nine people each shaking hands once at a party. When solving this problem, Nomsa represented the situation by the diagram above. This diagram is interesting to me because Nomsa is the only one who chose to use letters as representing real persons in order to help her find the correct answer. During the interview, Nomsa explained how the diagram came about. Consider the following extract:

Nomsa: Therefore, I showed that all these people will shake hands once with S.

Researcher: Alright and then how did you come up with 36 handshakes?

Nomsa: I took … I counted the handshakes here, and then added the handshakes.

Researcher: How many are they here?

Nomsa: They are 1, 2, 3, 4, 5, 6, 7, 8 and here they are 1, 2, 3, 4, 5, 6, 7, here they come 6, here 5, here 4, 3, 2 and 1.

Nomsa reasons clearly about how she eventually got to the answer. She first assigned letters for nine people (S, M, P, B, E, A, K, X, Y). She realized that the first person would touch eight people, the second person would touch seven people, and the third person would touch six people, and the sequence would carry on like that till the last handshake. Nomsa has manifested her ability to 'mathamatize', i.e., represent real world situations successfully (Hersch, Cameron and Fosnot, 2004). Despite her average achievement in the Form E mathematics examination (Symbol D), she demonstrated her ability to solve realistic problems through representations and constructing patterns, which is from my viewpoint a useful skill for teaching. Again, this shows that Nomsa is at the 'reasoning on the basis of properties' stage in relation to mathematical representations.

Sebu's Case

When the research project began in Tier 1, Sebu is twenty years old. Sebu is a young woman who lives in Maseru. She appears to have a relatively weak mathematics background. In Standard 7 examination, she obtained 3rd class in mathematics, in Form C she obtained an E symbol, and in Form E she obtained a pass. When asked to write about her experiences of learning school mathematics, Sebu wrote the following text:

Figure 5.10: Sebu's account of learning school mathematics

During my primary level, Mathematics was easiest subject
that I enjoyed and favourite one. I experience that Maths
need to be ~~per~~ practise time by time not taking it easy as
it was. At high school, Mathematics was not much
complicated but teaching methods at this level is
limited. Less examples were used and teachers based
their ~~for~~ teaching methods on text books only, while at
primary level they use natural things such as coins
apples even the chart to illustrate and empower kids
in their understanding. Again, in high schools I had
seen that Mathematics need to be ~~per~~ practise for a
long time, not for an hour or half an hour, so this
gave me hard time because there ~~is~~ was other subjects to go
over with them so hours must be shared. to revise
one subject at a time.

It could be argued from Sebu's account that the teaching materials that her teachers used at primary school level were useful in helping her think mathematically. Sebu's description of her experience of learning mathematics also shows that her secondary mathematics teachers might not have used a variety of teaching resources, and as a result the subject (mathematics) became difficult for her. Many learners share this experience.

The overall performance of Sebu in Section 2 of the Survey Instrument is as follows: the answer for task 1 (a) is incomplete. Task 1 (b) and task 2 answers are correct. The answer for task 3 is incomplete; the answers for tasks 4, 5 and 6 are correct. The answer for task 7 is incorrect. During the interview Sebu was asked to talk about numbers 1(a), 2(b), 3, 5, and 7. In the following excerpt Sebu explains how she worked the answer for task 1 (a):

Sebu: OK, I simply counted these apples (*referring to the circles*).

Researcher: How many did you count?

Sebu: Seventeen.

Researcher: OK.

Sebu: I say two, four, six, eight, and ten. Two, four, six, and seven… the question says (17 – 5) seventeen minus 5, then I … my concern was on this circle. I counted this is ten and then here we have seven, so I have to remove five in order to get the answer because …

Researcher: OK, alright.

Sebu: Then I subtracted, I count five: one, two, three, four, and five … then I observed that I'm left with twelve thus my answer.

While Sebu here explains well how she got the answer, on paper she had written twelve as the answer but did not demonstrate the operation (17 – 5) on the diagram. That is why her answer was coded *incomplete*. I now learn from the interview that she understood the potential representation perfectly and actually showed the correct answer only. She did exactly the same with tasks 1 (a) and (b). In the case of the addition task, it was necessary to shade both given quantities to represent the 'new' total. In the subtraction task, Sebu represented the only 'new' quantity (12) as difference between the two 'given' quantities.

Task 2(b) is another subtraction activity, where Sebu was asked to comment on how she came up with the answer.

Sebu: OK, then I … the question says eleven minus six (11 – 6), so I counted eleven, here we have eleven. Then I go back because it says minus.

Like other participants, Sebu first counted eleven hops from zero and then counted six steps correctly to the left to get to five. This suggests that Sebu successfully discerned details of the two representations namely 11 – 6 and the number line and successfully made a meaningful connection between them to represent the entire operation.

Sebu was then asked to comment on her answer to Task 3, which was incomplete because although the answer (60) was provided the work on the array was not. She mentioned that the diagram confused her. She said that she did not know what to do because she was seeing it for the first time. I then asked her how she would do it ten months from when she first did it. Consider the following conversation I had with Sebu:

Sebu: Now, I would count 12 across, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 and stop here. And five downward; 1, 2, 3, 4, 5

Researcher: Alright!

Sebu: Then I will move like this, and count these …

Researcher: OK, how many are they?

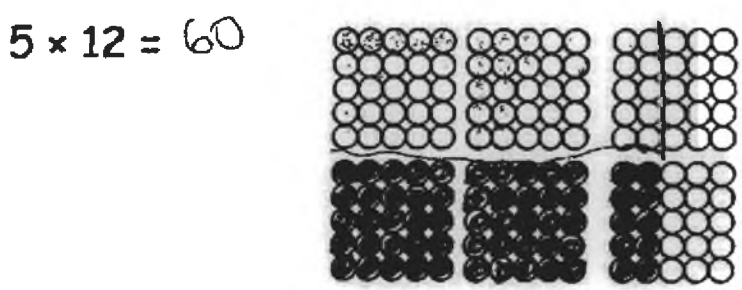
Sebu: 50, 25 + 25 = 50, 50 + 10 = 60

Researcher: So it would be...

Sebu: Sixty

The following diagram shows how Sebu thinks she would do it this time around. First, she marked the twelve by five blocks then shaded an appropriate part of the array.

Figure 5.11: Sebu's response to Task 3



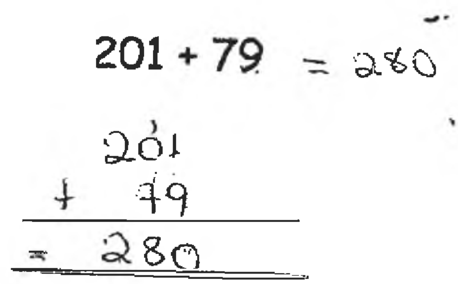
The array is presented in six blocks of twenty-five circles which appeared to be counterproductive in suggesting multiplication to these students. It does seem like at this point in time (ten months from the date of filling in the survey instrument in Tier 1) that Sebu is now competent enough to demonstrate a multiplication operation on an array, while she had no idea how it worked at the beginning of the Diploma programme. This is perhaps due to the fact that at the time when participants filled in the survey instrument, this type of array was quite out of the ordinary whereas at the time of interview participants were now familiar with this kind of diagrams from the DEP mathematics courses.

When asked about her answer to Task 5, Sebu responded as follows:

Sebu: I added them in a normal way, I wrote 201 + 79 then I said 1 + 9 = 10, I put zero and carry 1. 1 + 7 = 8, 2 + 0 = 2

The following diagram presents Sebu's work on Task 5:

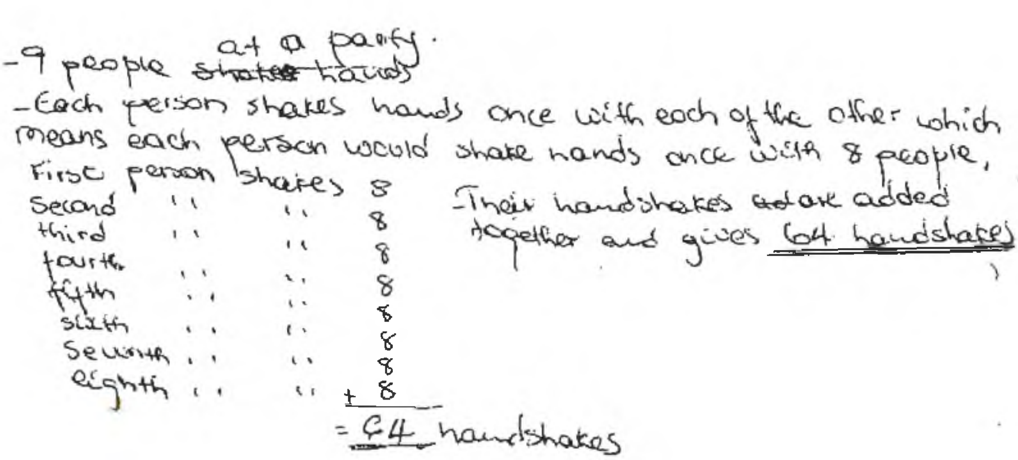
Figure 5.12: Sebu's response to Task 5



Many participants got the correct answer for this task by means of the standard algorithm just like Sebu did. The algorithm might be described as follows: the sum of 1 and 9 is 10, which means one ten and no units. Put 0 under the units and carry one 10 to tens. The next activity is to add that one ten to the seven tens that are in the number seventy-nine. So that 10 + 70 = 80, and then add the two hundreds in order to obtain the final answer (280). Sebu executed the algorithm perfectly. It may be difficult to convince her that there is a value in teaching children other more flexible ways of thinking mathematically as opposed to "the normal way" which works so well for her. During the interview, when asked if there are other ways in which this task could be done other than through the algorithm, Sebu suggested a number line. When asked to demonstrate how she could do that on a number line she experienced some difficulties. This was despite the fact that the use of a number line as a thinking tool had been dealt with in both mathematics courses (MAT 1400PA and MAT 1402PB) that Sebu had done in her first year of study at the College. While this problem might be a result of multiple factors, as a mathematics education lecturer at the college I feel there might be a need for us (mathematics education lecturers at the college) to examine the manner in which we present and teach lessons that are meant to help student teachers to be proficient with the use of various teaching aids such as a number line.

Sebu obtained an incorrect answer (64) to Task 7. When asked to say how she arrived at the answer, she explained that she realised that the first person would shake hands with the other eight, which makes a total of eight hand-shakes. She then made the mistake of concluding that each of the remaining people would also make eight hand-shakes. The diagram below shows what Sebu did on this task:

Figure 5.13: Sebu's response to Task 7


-9 people ~~shake hands~~ at a party.
-Each person shakes hands once with each of the other which means each person would shake hands once with 8 people,

First	person	shakes	8
Second	"	"	8
third	"	"	8
fourth	"	"	8
fifth	"	"	8
sixth	"	"	8
Seventh	"	"	8
eighth	"	"	+ 8
			= 64 handshakes

-Their handshakes ~~are~~ are added together and gives 64 handshakes

Given that this task is a non-routine problem, Sebu's effort of constructing this table out of her creative imagination might be interpreted as a reflection of her determination to solve the problem and her application of a heuristic to do so. Sebu's work suggests Vygotsky's Zone of Proximal Development (ZPD). Vygotsky (1978) refers to the "distance between actual mental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). If Sebu were working on this task together with any of the previous three students I feel sure she would have reached the correct solution.

Tšeli's Case

Tšeli is a young woman aged twenty-one at the beginning of this study. She resides in Maseru. Tšeli's mathematical knowledge is fairly good when judged on her previous performance in mathematics national examinations. She obtained a second class pass (2^{nd}) in Standard 7, a "B" symbol in Form C, and a Pass in Form E.

When asked to write down her expectations of the Diploma in Education Primary (DEP) programme as she joined the college in Tier 1, Tšeli wrote that she would like to be trained on how to use materials such as stones so that learners can see why two plus two is four. Tšeli seems to already know how stones could be used as enactive representations in mathematics lessons. The legitimate expectation then might be to learn how other more modern or organised materials could be used.

During the interview in Tier 2, Tšeli was asked to comment on her answer to Task 1 (a) that required participants to subtract five from seventeen [17 – 5] and demonstrate the operation on the given diagram. Like Sebu, Tšeli had coloured twelve counters only. This suggests that Tšeli found the difference (twelve) and saw no need to show the minuend or subtrahend on the diagram. The following excerpt presents Tšeli's reasoning:

Tšeli: Ehm … I subtracted 17 - 5 and then I got 12.

Researcher: OK!

Tšeli: Yes … and then this is 2, 4, 6, 8, and 10. OK, all of them are 20 (*referring to the counters*).

Researcher: So after getting twelve you showed it there?

Tšeli: Yes, yes … I thought especially for the primary kids.

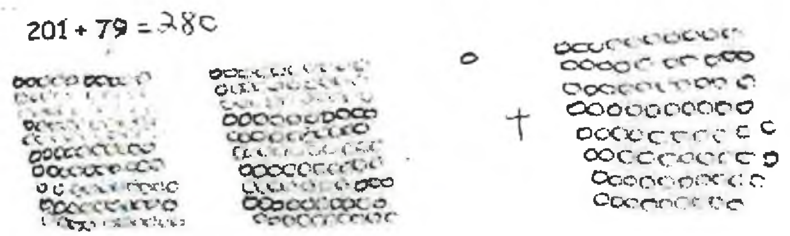
It follows that Tšeli didn't recognise that the counters accompanying the task were meant as a resource to be used in demonstrate the operation of subtraction. It could be inferred that Tšeli lacks an in-depth understanding of the different structures of subtraction and the difficulties that 'primary kids' might have in understanding them.

Differences between potential representations emerge when one realises that Tšeli's work on item 2 is similar to other participants' work in that they counted in ones from zero and successfully represented both operations on the number line. The number line is obviously more familiar and proves more useful in making comprehensively meaningful representations for both addition and subtraction operations. Tšeli was then asked to comment on what she had done on Task 5 (201 + 79). The following transcript presents what transpired during the interview:

Tšeli:	This is supposed to be 79 plus 100 and another 100 plus 1 then which makes all of this to be 201.
Researcher:	How would you do it now?
Tšeli:	I think I would use a number line.
Researcher:	A number line … OK, can you do it for me?

On her script, Tšeli had demonstrated the operation (201 + 79 = 280) with the following diagram:

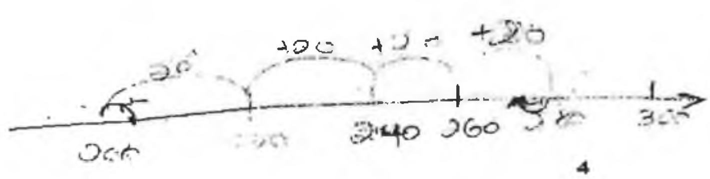
Figure 5.14: Tšeli's response to Task 5



When asked to explain the meaning of the arrays that she had drawn, Tšeli provided a clear explanation saying "actually I get it and then getting the answer, after that I just confirm by; I just confirm by … whether I wrote it correctly or something.

Tšeli suggested the use of a number line an alternative representation. In order to demonstrate to me how she could use a number line to do this operation (201 + 79), she drew the following number line:

Figure 5.15: Tšeli's response to Task 5 during the interview



Explaining her use of the number line appeared to challenge Tšeli and the following excerpt provides evidence of that:

Tšeli: Uh… ha… ha… ha… (*laughs*), OK, I'm … now, now I won't do it; I won't do a number line. I think it is giving me a little ... or something, really I cannot demonstrate it to young children.

Tšeli had comfortably used the algorithm to add a two-digit number (79) to a three-digit number (201) and obtained the correct answer (280). She was then able to demonstrate the operation by means of an array, drawing 280 individual circles which made full sense but was a bit unwieldy. When Tšeli resorted to the use of a number line she had difficulty in convincingly explaining how her diagram was showing the operation and the answer correctly. However, it is interesting that Tšeli could reason successfully and flexibly on the number line but had difficulty in articulating her thinking. She appeared to add 80 in jumps of twenty starting at two hundred, to which she had also added a single hop, labelled "+1". Her total was then correctly, two hundred and eighty, but of course she had only needed to add seventy-nine so she also showed one hop to the left from 280 and labelled it "– 1". This was a fair attempt to reconcile (200+1)+(80 −1) but Tšeli appeared to lack confidence in articulating it fully and had learners in mind as she explained that she would not feel competent to use a number line in a mathematics lesson. It could be concluded from her work that although Tšeli drew a variety of diagrams successfully, she appeared to lack confidence in explaining how she used them to confirm the answer which she was already certain of, through following the algorithm.

Tšeli's attempt to get a solution for Task 7 (hand-shake problem) was less than successful. She multiplied 9 by 8 and did not attempt an iconic representation. In contrast to this, when asked about the use of representations when teaching mathematics Tšeli said that it is important to use them because learners can see and touch them. The following excerpt presents the conversation held with Tšeli:

Tšeli: I think, I think I love them, I think they are very interesting and they are very good for the young brains because the fact that they can be seen and be touched … We make mathematics real to children …

Researcher: OK and we make them learn mathematics from things they know?

Tšeli: We teach them what they don't know through what they know, I think it's interesting.

Researcher: So which one, which representation would you use in your class?

Tšeli: So far almost, I think it's a multiplication square, especially because multiplication can be …

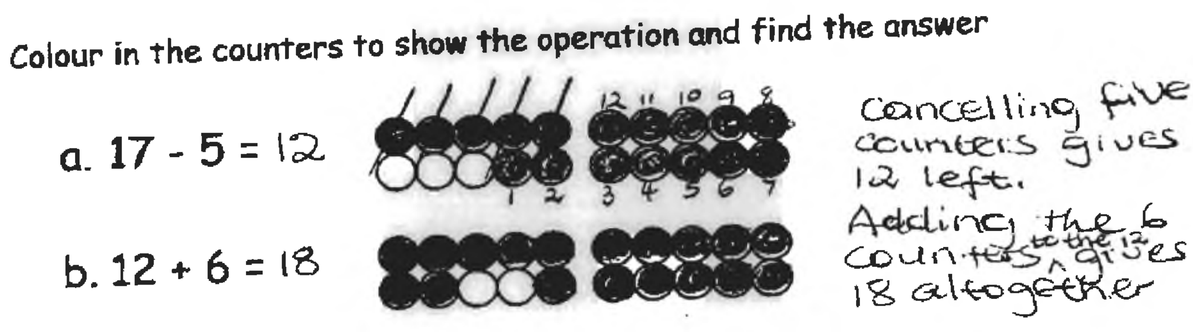
It is worth noting that Tšeli argues that through familiar representations learners could be taught new mathematics concepts. She mentions the use of a multiplication square. This shows that at the end of Year 1 of study, participants had learned some other representations that could be used to enhance teaching and learning of mathematics other than stones frequently mentioned by people in Lesotho who have not undertaken a teacher training course. However, knowing of these representations is not enough, the teacher needs to be able to be able to use them with confidence and ease and be able to communicate their thinking processes as they do so.

Three *Credit* Students

Litha's Case

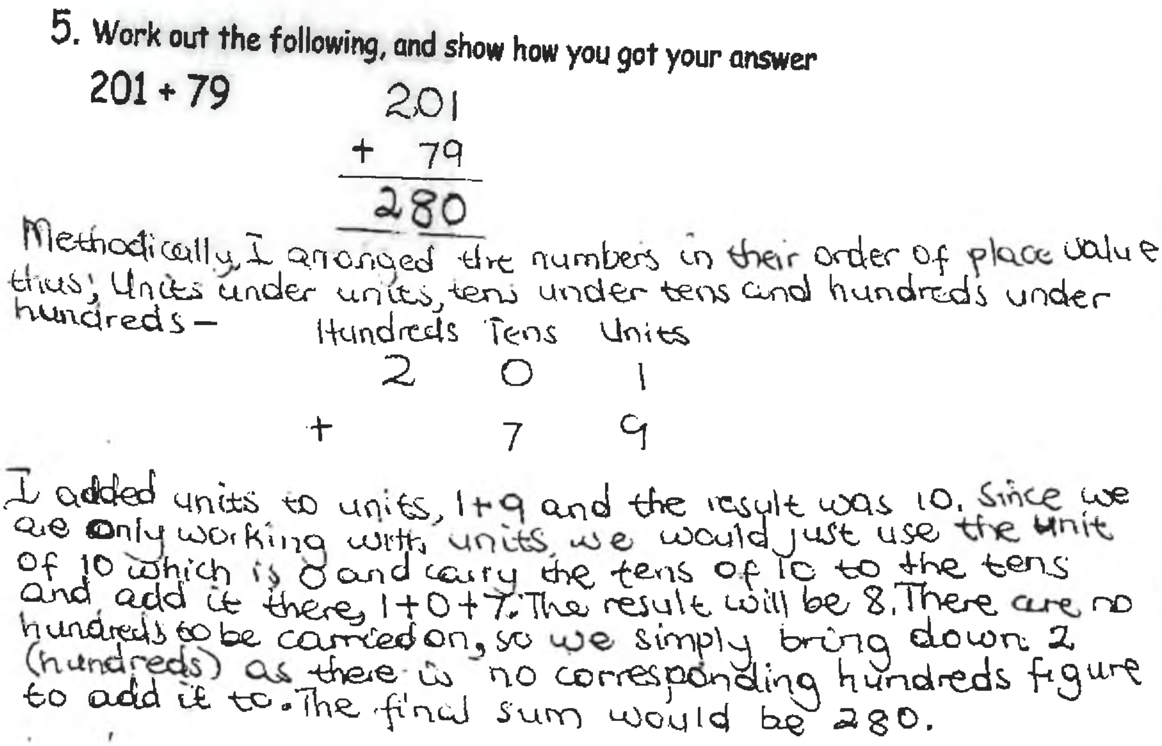
Litha is one of the few students who has obtained a mathematics credit in Form E national examination and opted to pursue a primary school teacher programme. Litha is a young man aged twenty-one. Although his home is Maseru he went to school in one of the schools in Leribe district. Litha's performance in school mathematics has been quite good throughout. For instance, in Standard 7 he obtained a second class (2nd class), obtained symbol B in Form C, and a credit in Form E. It then follows that Litha's mathematical background is fairly good compared to many other participants in the cohort. The following diagram presents Litha's work on task 1:

Figure 5.16: Litha's response to Task 1



The diagram shows that Litha was able to make meaningful connections between the symbolic representations (17 – 5, 12 + 6) and the accompanying iconic representations (arrays). Litha went further than other participants who obtained correct answers on this task by explaining by means of text how he arrived at the answers. He is able to reason on the basis of properties of various representations. Again, on Task 5, Litha unlike other participants provided a detailed textual description of how he obtained the answer. He also included the digits used in counting the answer, or difference between 17 and 5. Consider the following extract:

Figure 5.17: Litha's response to Task 5


5. Work out the following, and show how you got your answer

201 + 79

$$\begin{array}{r} 201 \\ +\ 79 \\ \hline 280 \end{array}$$

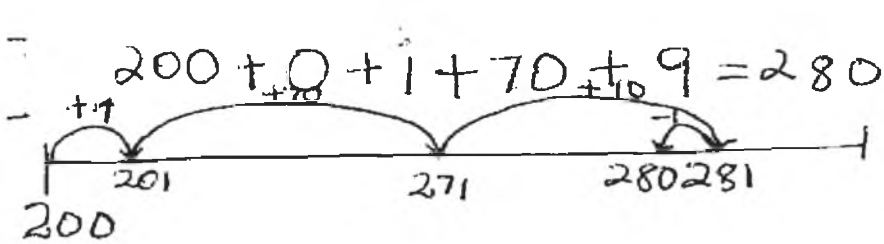
Methodically, I arranged the numbers in their order of place value thus; Units under units, tens under tens and hundreds under hundreds –

	Hundreds	Tens	Units
	2	0	1
+		7	9

I added units to units, 1+9 and the result was 10. Since we are only working with units, we would just use the unit of 10 which is 0 and carry the tens of 10 to the tens and add it there, 1+0+7. The result will be 8. There are no hundreds to be carried on, so we simply bring down 2 (hundreds) as there is no corresponding hundreds figure to add it to. The final sum would be 280.

During the interview, Litha was asked if he could think of another way of doing this task. Litha suggested a number line and he was asked to draw it and show how the task could be done. In the diagram, Litha used an empty number line successfully and it must be said that amongst the ten participants in Tier 2, Litha is the only one who confidently and flexibly used the number line to represent both addition and subtraction [(201 + 70 + 10) – 1] = 280.

Figure 5.18: Litha's use of a number line during the interview



It is interesting that Litha represented addition and subtraction on the same number line because the task itself required addition. This shows number fluency (Grey and Tall, 1997). When asked to comment on how he obtained an incorrect answer (72) in Task 7 (the hand-shake problem), Litha responded as indicated in the following excerpt:

Litha: I said one person has two hands and if each person shakes hands once to each other, this means the first person shakes hands to all others there will be eight handshakes. This means that eight handshakes multiplied 9 times. So I multiplied 8 by 9 and then I got 72.

Litha's comment here is in line with his initial working on this task as presented by the following extract:

Figure 5.19: Litha's response to Task 7

Firstly, how many hands each person has? They are two. There are nine people. Each person shakes hands once with each other. This means as the first person shakes hands to all others there will be 8 handshakes. This means that there will be 8 handshakes 9 times. Thus there will be 72 times.

Despite his good performance in other tasks, Litha had not yet got the correct answer to this task. When probed further, Litha worked out the answer successfully.

1.	Litha:	Therefore; it starts … it's going to be eight …
2.	Researcher:	The person is the 8^{th} one, but the remaining people will be how many?
3.	Litha:	7 … OK, it is going to be … is going to be 8, 7, 6, 5, 4, 3, 2, 1 and then we add.

Talking about his thinking for the handshake problem appeared to be a ZPD for Litha where my single prompt in turn 2 appears to have been enough to lead him to a correct solution. During the interview he could not think of constructing any representation that could have possibly assisted him to arrive at the correct answer. His initial thinking that a person has two hands is irrelevant in this context because a person uses one hand and not both in the process of shaking hands. However, if he had halved 72 he would have got the correct answer. By talking and thinking towards a solution he arrived at the sum of 8+7+6+5+4+3+2+1 handshakes.

Matoti's Case

Matoti is a young woman aged twenty-three years. She lives in Botha-Buthe district. Matoti's mathematical background appears quite good for the reason that in Standard 7, she obtained a 2nd class in mathematics, a symbol C in Form C, and a credit in Form E. When asked to write a story about her experiences of learning school mathematics, Matoti mentioned that it is crucial to know the four mathematical operations (addition, subtraction, multiplication, and division) at the primary school level. She said the following with regard to high school mathematics:

Figure 5.20: Matoti's account of learning school mathematics

In High School level, what is concerned ~~is~~ are the formulas ~~&~~ if one knows about the formula ~~of~~ it is easy to get to the answer.

According Matoti, high school mathematics is about knowing "formulas" for various problem solving tasks where the knowledge of formula is important in enabling one to get answers easily. This may explain why Matoti obtained good results in school mathematics examinations. It might be she was good at memorising and using formulae for calculations.

During the interview, Matoti was asked to make comments on her answers to tasks 1, 2, 5, and 7. Responses to tasks 1 and 2 were marked incomplete because Matoti had not written symbolic answers and yet she had demonstrated the operations on the diagrams accompanying the tasks. She was asked to explain why she had not written the answer. Consider the following excerpt:

Matoti: I'm not sure but … here we are subtracting and by shading these circles it seems that all the circles were 17. Now I'm going to subtract 5, then these 5, I coloured differently from these ones because they have …

Researcher: Did you double shade?

Matoti: Yes, I shaded them twice.

Researcher: So what is the answer?

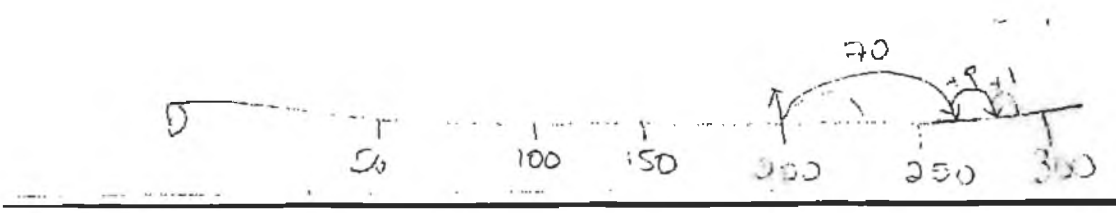
Matoti: The answer is that one which has the straight lines.

Researcher: Oh! What is the answer?

Matoti: The answer is 12.

Drawing from the excerpt above it can be said that Matoti knew the answer although she did not write it down at the time. It might be a mere omission in not writing answers. She repeated the same error on Task 2. Like other participants, Matoti got a correct answer for Task 5 using the algorithm. She then drew a full 280 circles. During the interview, Matoti was asked if she could think of another way of doing this task. She suggested a number line. She was then asked to draw it. Consider Figure 5.22 below:

Figure 5.21: Matoti's use of number line during the interview



This diagram presents an example of an empty number line. Matoti marked 200. She then added 70, + 9, + 1 to the 200 demonstrated by arrows pointing to the right. While the answer is obviously 280 Matoti did not write it on the number line. She wrote other numbers (0, 50, 100, 150, 250, and 300) on the number line as indicators. This time around Matoti used the number line correctly for this operation (201 + 79), but still she did not write the answer. After taking some mathematics courses at the college, Matoti has deepened her understanding of connections between symbolic representation (201 + 79) and more than one iconic representation (the array of dots and a number line). Her explanation of what she does on the number line also confirms Matoti's mathematical understanding of addition. Consider the following excerpt:

Matoti:	It will be 200 + 70, and 270 + 9 is 279, plus 1 gives me 280.
Researcher:	Why didn't you say that 9 + 1 is just 10?
Matoti:	Yes, I can make it, 9 plus 1 is 10, then 1 will …
Researcher:	Tell me why didn't you use number line instead of these dots?
Matoti:	I did not know …

Researcher: Did you know how to use a number line then?

Matoti: Before?

Researcher: Yes like, when you're doing this, did you know how to use a number line?

Matoti: No!

It follows from the excerpt above that Matoti in Tier 1 was not familiar with how a number line works. This evidence shows that there is the possibility for some learners to complete high school mathematics without knowing how to use a number line for addition and subtraction of whole numbers. This is surprising considering that the number line has been a recommended representation for arithmetic operations in the Lesotho primary mathematics syllabus, books 2 and 3 (1998).

When asked to comment of her answer to Task 7, Matoti said she was not sure about her answer – "there are 18 handshakes". She drew 9 people in circle holding hands, which may have lead to the answer of 18. Despite the fact that Matoti had performed well in school mathematics, she made a drawing that could not lead her to finding the correct answer to this Task. Perhaps this is because this task could not be solved following an algorithm already known to her. Participants had to be creative enough to be able to construct self representations with mathematical properties, which allowed them to reason towards a solution. At the end of year 1, when asked about mathematical representations that she would comfortably use in her class when on teaching practice (TP), Matoti mentioned that she would use games and other teaching aids that she could not mention by name, despite mathematics education lectures in the use of multiple representations. This indicates to me that she may still understand mathematics primarily in terms of procedures and formulae.

Thandi's Case

At the beginning of this study, Thandi is twenty three years old. Thandi is a woman who resides in a township situated about 20 kilometres south of Maseru. Thandi's background in mathematics appears relatively strong. In Standard 7 terminal examination she obtained third class (3) in mathematics. In Junior Certificate (JC) national examinations she got an E symbol in mathematics. In Form E examinations Thandi obtained an E symbol in her first sitting and a C symbol in her second sitting[5], which is a credit. This suggests her ambition to improve. In the following extract Thandi tells a story about her experience of learning school mathematics:

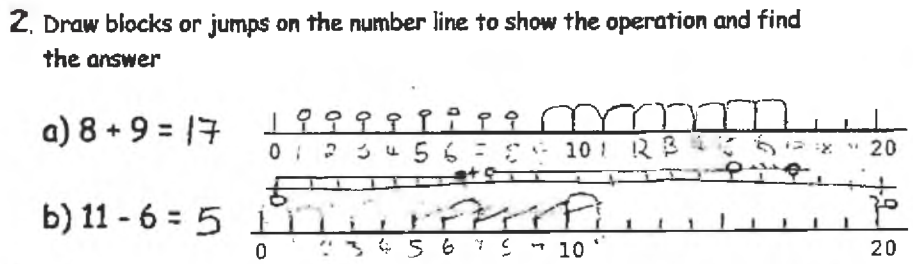
Figure 5.22: Thandi's account of learning school mathematics

In primary level mathematics was enjoyable to me because we used sticks and balls to count, add and subtract every work given, but in High School level there was no sticks nor balls all we had was counting numbers: and signs notivications to use and formulars to memorize and then it turned to be a monotonous subject for it had so much in it, more especially the use of the signs and expressions with indices.

[5] When a learner has failed or got a pass in his/her first sitting, he or she can attend private mathematics lessons conducted by various people in order to improve his/her grade in a second sitting of the examination, which at the moment takes place in June and in December. These repeat examinations like the normal national examinations (COSC) are prepared and monitored by the Examination Council of Lesotho (ECoL).

According to Thandi, concrete materials make learning mathematics an enjoyable experience for primary school learners. High school mathematics on the other hand as Thandi puts it is 'monotonous', boring, and hard to remember because of more focus put on formulas, signs, and algebraic expressions.

Figure 5.23: Thandi's response to Task 2



In Task 2(a) Thandi drew 8 small circles on the number line from 1 to 8 and also inserted missing numbers on the number line from 1 to 19. She then constructed 8 single hops from 9 to 17. This might suggest that after correctly completing the symbolic statement 8 + 9 = 17 Thandi wished to represent the answer (17) on the number line though with different forms of drawings (i.e. circles and hops). This might also suggest that Thandi was not familiar with the way in which addition could be done on a number line at that time. In 2(b) she made 11 hops from 0 to 11 and then cancelled 6 hops from 11 backwards, which yields the correct answer 5. The third number line that appears in between 2(a) and 2(b) might have been unnecessary. During the interview Thandi said the following:

Thandi:	OK, in No. 2, I just counted the numbers on the number line and then I presented …
Researcher:	Oh, how many did you count?
Thandi:	Up to 7 … I just counted what was asked and then …

Researcher: So you counted from 0?

Thandi: Yes, I counted from 0 …

Researcher: To 8 and then?

Thandi: Then again from 9 to 20, I counted from 0 to 20 then I deducted what was written, on the … to get the answer.

Researcher: OK, how would you do that now, if you were given it now?

Thandi: Now? I would do it easily because I …

Researcher: Can you do it now?

Thandi: (*silent*)

Researcher: So the answer will be …

Thandi: It will be 17.

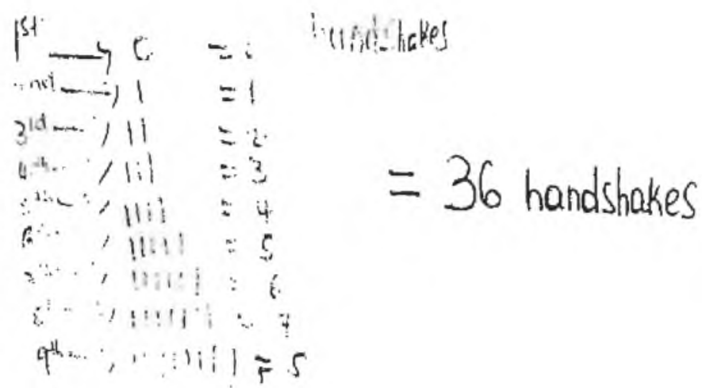
In the above excerpt Thandi explains that she counted in ones on the number line, which is fair enough given that both tasks 2(a) and 2(b) involved addition and subtraction of very small numbers respectively.

When asked to comment on how she obtained an answer to a 'hand shake problem', Thandi mentioned that she imagined being in church where they all shake hands. In the following extract, Thandi elaborates on how she reached the answer:

Thandi: OK… all I had …that is, when we greet each other in church someone just waits outside and we… all of us come out on a line and we just greet each other. After greeting, the first person stands aside. Then the second person does the same and stands aside. Also, the third person will come and greet and stands aside… that is how I did it to get to the answer.

Thinking of the real church situation of shaking hands helped Thandi to work towards the correct answer. The following picture shows Thandi's work on Task 7:

Figure 5.24: Thandi's response to Task 7



In this diagram, Thandi has shown that for the first person there is zero number of handshakes. For the second person there is one hand shake, and for the third person there will be two handshakes. Thandi's thinking generated the above table, which shows a connection between counting numbers from 0 to 8, tally marks that appear in the middle row, and the ordinal numbers from 1 to 9. She then added the counting numbers to get the sum of thirty-six.

Looking at Thandi's work closely, one could conclude that in tasks (2(a) & (b)) that involved the use of a number line she successfully demonstrated the answers on the diagrams even though on the first instance she drew both the circles and the hops, which could be evidence of confused thinking. Thandi creatively managed to establish some connections between symbolic and iconic representations. Again, on Task 7 (hand shake problem), I realise that Thandi successfully used the familiar church context as opposed to the 'party situation' to work out the answer. This has been an efficient use of context. It then follows that when Thandi is given an opportunity to use her own representations, she is able to use context to make connections between various types of numbers. Based on this evidence, it is reasonable to conclude that Thandi is able to reason on the basis of properties of self-invented representations.

Two *Fail* Students

Mable's case

Mable is a young woman aged twenty-four. Mable's home district is Botha-Bothe. Mable's performance in mathematics in national examinations has been extremely weak and there is a contrast in her primary and secondary experiences. In Standard 7, Mable obtained a first class (1st class), yet she failed mathematics badly in both Form C and Form E. In Tier 1, Mable gave the following explanation as by way of elaborating on her school experiences of learning mathematics.

Figure 5.25: Mable's account of learning school mathematics

From primary schooling Mathematics was the best subject I like because I was able to understand what was being taught. So due to that I managed to pass it. Coming to high school level, I found it difficult because the way we were taught differed from Primary level. I did not ~~not~~ understand the high school teachers, then I began to fail mathematics.

This account goes to show that how mathematics is being taught at these two levels in Lesotho namely primary school and high school may influence students' attitudes and performance. It is obvious from Mable's testimony that there is something that some high school teachers are not doing well, which makes high school mathematics difficult for many Basotho learners.

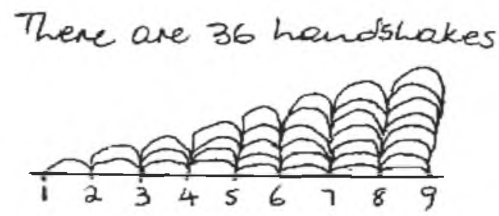
Focusing on Mable's work on section 2 of the survey instrument, the analysis reveals that tasks 1, 3, 4, 5, 6 and 7 are marked as correct. Task 2 was marked as an incomplete answer for both parts (a) and (b) because Mable had written correct answers for (a) and (b) as 17 and 5 respectively and yet the number lines accompanying the two tasks were not used at all.

During the interview in Tier 2, Mable was asked to comment on her answers for tasks 5 and 7. She volunteered to talk about Task 7 before I could ask her about Task 5. The following excerpt presents our conversation:

Mable: I … I imagined, maybe there is the first person … Let me make an example here in the church I'm attending; Lesotho Evangelical Church, at the end of the service we … we shake hands … then I imagine the first one maybe … I count … he doesn't shake; he is the first one, yes the second person follows. I count their shakes at the first one. The third person will be the three handshakes because the third person shakes the hands of the first and the one for the second one …

It is interesting that like some other participants, Mable used her church experience of shaking hands in this situation. Surely this context played an important role in assisting Mable to work towards the correct answer (36). However, what I find to be even more interesting is Mable's diagram demonstrating the hand-shakes. Consider the following extract:

Figure 5.26: Mable's response to Task 7



The numbers shown on the number-line represent the positions held by each of the nine people in a party (church). Each of the hops represents a single hand-shake so that the total number of hops is equal to the number of hand-shakes (36). I find Mable's diagram an interesting representation, which is unique and powerful in making the solution accessible to Mable. She was able to reason on the basis of the properties of the diagram and at the same time makes connections between the diagram, ordinal numbers (1^{st}, 2^{nd}, 3^{rd}..., and 9^{th}) and the real life context of the church experiences. I offer this as another good example of mathematization.

Minah's case

Minah is a woman who is twenty-four years old. Her home district is Berea. Minah's mathematics background is weak. In Standard 7 Minah obtained a second class (2^{nd} class) pass, in Form C she scored a D symbol, and in Form E she failed mathematics. When asked to comment on her experiences of learning school mathematics Minah gave the following account:

Figure 5.27: Minah's account of learning school mathematics

Mathematics is that deals with our daily lives
activities. At primary level mathematics was very easy
because we used our fingers, stone, sticks and other
materials that helped a lot in learning. But at high
school level mathematics did not change since we
still dealt about addition, subtraction, and division
and multiplication in a bit difficult and broad
way but the concepts are still the same. It is
one of those enjoyable subjects that needs practising
as it is like a free country because you can
use different ways of working out the answer
as long as the answer is correct the method
does not matter.

The most important point that Minah is making here is that the use of "fingers", "stone", "sticks", and other materials for teaching is what makes primary school mathematics different from high school mathematics. In Minah's own point of view, these materials help learners to learn mathematics well. She seems to be holding the thinking that mathematics does not have only one method of getting to the correct answer. This appears to be a positive approach in that it will likely allow her to employ various methods and multiple representations when teaching mathematics.

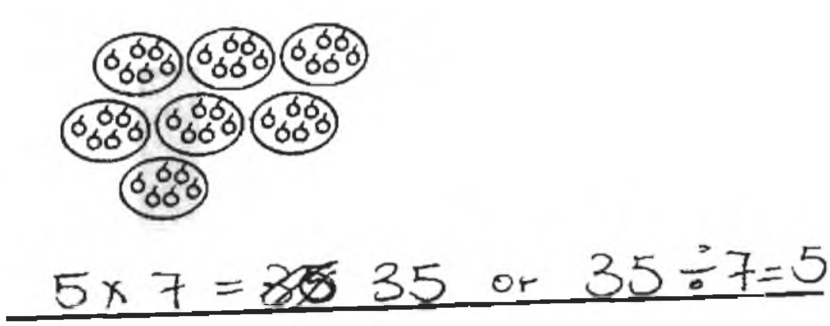
The analysis of Minah's answers to tasks in Section 2 of the survey instrument in Tier 1 shows that she made an effort to think carefully about each task and provided some interesting answers. During the interview, Minah made some comments on tasks 2(a), 4(c), 5, 6, and 7. On Task 2(a) (17 – 5), Minah explained that she shaded seventeen circles and then from the seventeen she counted five and drew a line so that the remaining circles are twelve (12). Refer to the following extract where she represented the partition structure of subtraction:

Figure 5.28: Minah's response to Task 2(a)

Minah successfully made connections between the symbolic representation (17 – 5) and the iconic representation (array). She was able to reason on the basis of the two representations.

Minah argued that for Task 4(c) she realised that the diagram showed both multiplication of 5 and 7 and the division of 35 by 7. Consider Minah's actual working in the following extract:

Figure 5.29: Minah's response to Task 4(c)



Here, Minah did not only make connections between the diagram and the operation but she also made an important connection between multiplication and division, although her actual counting of fruit on the plate is incorrect. It could then be argued that based on this particular example Minah is able to reason on the basis of the properties of the two representations. Her work however, raises an important issue. Is Minah's error in counting the sets of six as sets of five a simple 'slip' or is it an indication of lack of understanding in mathematics? I am inclined to think it is the former.

Finally, Minah was requested to talk about Task 7 on handshakes. She mentioned that at the time when she did this task she was confused and she could not get the answer, but she thought it would be nine, nine times. Unfortunately she felt that even at the time of the interview she could not immediately figure out how to obtain the correct answer. It is worth mentioning here that Minah did not make any attempt to construct any form of representation, and her suggested heuristic was to "count people". However, she appeared unwilling to follow through on this.

Conclusion

In this chapter, I focused on the analysis of participants' responses to semi-structured interview questions, which were mainly based on the representations that participants had associated with tasks in Tier 1. The research question that guided this analysis is as follows: what mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?

At the start of this study, I argued that trainee teachers' understanding, choice, and use of representations in doing arithmetic activities is part of their mathematical knowledge for teaching. It was also argued that such mathematical knowledge is weak or lacking in Basotho teachers (LCE, 2006). The findings that emerge from the analysis reported in this chapter tell a different and more complex story. Many trainee teachers have a weak background in school mathematics. All engaged fully with the tasks and expressed ambition to be good teachers of primary mathematics. My thinking about the correct use of representations was challenged as I analysed participants' responses. Of the first four items (tasks 1 and 2 (a) and (b), the task requiring respondents to represent a subtraction problem (17-5) on an array of twenty counters had the lowest number of correct responses. Yet there was only one response that was coded incorrect. 59.9% were fully correct. The remaining 39.6% were coded incomplete. When these incomplete responses are probed it emerges that almost all respondents were making an attempt to represent the subtraction structure in a manner that was meaningful to them. The true purpose of mathematical representations is to structure and communicate mathematical ideas in the minds of the makers and users.

At a surface level, I observed that the arrays of circles used in some tasks were foreign to participants and because of that many participants in this tier were challenged in making connections between them and the symbolic operations they were meant to represent. In my opinion, most participants were able to produce mathematically meaningful diagrams but on the other hand found it harder to verbally explain what they did in each case. This may be a language issue because these students have all had their early learning mathematics experiences in Sesotho, not in English. Language is said to be a *representation of thought* and eventually serves as a *tool for thought* (Fosnot and Dolk, 2001, p. 11). I argue therefore that some of these students may not have enough mastery of English terminology to properly represent their mathematical thinking on the survey instrument. I found it quite challenging to categorise a single participant at a fixed level of Mason's typology, because depending on each task the participant in question performed differently and made different connections. The semi-structured interviews taught me that the tasks on the survey instrument that appeared more difficult for participants were ones of proximal development for many respondents (Ginsburg, 1997). Watson (2009) makes the same connection with Mason's structures of attention and the role of a teacher.

> The structuring of attention indicates that teachers and learners always have a choice of what possibilities to pursue, what to bring into the public arena, how to direct attention to different ways of seeing objects and experiences, and hence afford different kinds of sense –making. The role of a knowledgeable teacher is critical in constructing sequences of examples, situations, and mediating devices, which give students access to new ways of attending (p. 220).

In other words, it doesn't matter which level or levels the participants appear to be at. In Minah's words "Mathematics deals with our daily lives activities". It is a very rich cultural activity where teachers and learners of mathematics must cooperate to build shared meanings and use them to solve problems. It was not particularly helpful to present 'foreign' representations (e.g., Task 4) and expects respondents to 'see' the mathematics in them without help. A familiar context is a good starting point for mathematical activity. All of the ten participants, with different mathematical backgrounds, were able to do Task 5 (explaining a simple mathematical problem to a child) perfectly. Four participants out of the ten interviewed mentioned that they used the real life context of a church experience of handshaking to solve a task that proved extremely challenging to many participants. It was also clear that for tasks that required participants to use their own self-constructed representations, participants drew quite interesting tables, pictures and diagrams that they used to make meaningful connections between symbolic and iconic representations.

As a researcher, I am puzzled about when an error is wrong mathematics or just a 'slip' and part of the meaning making process For example, I started out by considering Adoro's answer to Task 1(a) as 'wrong mathematics' (see Figure 3.4). However, I was impressed by Minah's answer to Task 4 (c) (see Figure 5.29 on page

178) and considered her mis-counting as a 'slip.' Both errors would seem to indicate that it is helpful when learning and using mathematics to be working with another person (ZPD). Findings from this chapter challenge me in two ways: to accept that mathematical knowledge is social, situated and distributed (Lave, 1988) and to acknowledge that changing practices is always difficult (Fullan, 2007). I will return to these issues later.

6. Tier Three – Teaching Mathematics Lessons

The analysis of lessons in this chapter is aimed at providing answers to the research question: *how do Lesotho student teachers on teaching practice use mathematical representations in lessons?* The lessons taught by these participants in Tier 3 were observed and video recorded during the second year of their study that is spent on teaching practice in schools. Five lessons were observed in five different primary schools located in the district of Maseru within the months of February and March 2010. Four mathematics lessons were taught to learners in standard 4 and the fifth lesson was taught to learners in standard 6. The standard 4 lessons were taught by four female student teachers while the standard 6 lesson was taught by a male student teacher. When student teachers are on teaching practice they are allocated classes and subjects to teach by the principal of the school, under the direction of the co-operating (class) teacher. The co-operating teacher is expected to help the student teacher with the planning of the lesson and to assist during the lesson not only with maintaining discipline but also with teaching. However, the student teacher is still expected to do more preparation and take a lead in teaching what is planned. The co-operating teacher also observes some selected lessons and grades the student teacher. At the end of the teaching practice session, the co-operating teacher is expected to prepare a report based on his/her observed lessons. This report is then sent to the college (LCE) and it is used towards scoring the final TP mark for the student teacher in question. Thus co-operating teachers may have had a considerable but 'silent' influence on lessons observed in this tier of my study. Each lesson was video-recorded and later transcribed. Each lesson is analysed in the light of the Knowledge

Quartet (Rowland, Huckstep, and Thwaites, 2005) with a main focus on looking for evidence of the transformation dimension of mathematical knowledge in teaching.

Table 6.1: Five participants who taught lessons in Tier 3

Participants pseudonym	**Gender**	**COSC grade**	**Topic taught**	**Standard Taught**
Thandi	Female	Credit	Addition and subtraction	Standard 4
Nomsa	Female	Pass	Multiplication	Standard 4
Tšeli	Female	Pass	Multiplication	Standard 4
Sebu	Female	Pass	Sets	Standard 4
Litha	Male	Credit	Measurement	Standard 6

Analysis of the Lessons

In any given lesson there are multiple activities taking place at the same time, therefore it was important for me to identify a theoretical lens through which I could focus my eye primarily on the teaching/learning of mathematics. The Knowledge Quartet (KQ) proved to be a useful tool for this purpose. According to Rowland et al (2005) the Knowledge Quartet (KQ) is a typology that emerged from a grounded approach to data analysis of primary mathematics teaching in the United Kingdom. It has since then been used at the University of Cambridge for the development of student teachers within the pre-service programme for prospective primary school teachers (Rowland, 2007). It is also used in Ireland at St. Patrick's College of Education as part of the Lesson Study protocol within a mathematics education elective course for pre-service teacher development (Corcoran and Pepperell, 2011). The KQ identifies the manner in which student teachers' mathematical knowledge plays out in a mathematics lesson along its four main dimensions namely: foundation, transformation, connection, and contingency (discussed in Chapter 3). While all four dimensions are interconnected and all are useful in looking at mathematics teaching, the transformation dimension lends itself particularly well to this study because it focuses the eye of the researcher on the choice and use of representations and examples that each student teacher uses when teaching, in order to make mathematics concepts accessible to learners. In the following two chapters, I present discussion of each lesson. I begin my discussion by first focusing on three mathematics lessons taught in Standard 4. I will include the participant's responses in Tier 4 as part of my discussion of these lessons.

The Teaching of Addition and Subtraction of Money

Every mathematics teacher has to make choices in planning and delivering a lesson. The choices involve among other things selecting key representation(s) for the concept or procedure intended to be taught. For instance, while a number line might be used to aid the teaching of all arithmetic operations, it might be considered a key representation for addition and subtraction of multi-digit numbers. During lesson preparation, a teacher has to think carefully about the examples, illustrations, and contexts that he/she will use in class to make concepts, procedures, and core vocabulary comprehensible to learners. Rowland et al (2009) identify contributory codes in the transformation dimension of the Knowledge Quartet as, *choice of representations*, *teacher demonstrations*, and *teacher's choice of examples*. In my analysis of Thandi's lesson I will focus on the first of these, choice of representations.

In what follows, I present Thandi's lesson synopsis.

Table 6.2: Thandi's lesson synopsis

Student Teacher	Standard	Lesson Duration	Class size	Lesson Topic
Thandi	4	40 min	80	Money-addition and subtraction
Descriptive synopsis of Thandi's lesson:				
At the beginning of the lesson, the cooperating teacher asks learners to stand and sing a song. This appears to be a good starter for the lesson because it wakes learners up and prepare them well for the lesson. Thandi then asks learners to take out their money. Learners take out their fake money made from paper. Thandi mentions that the operation of subtraction happens when one buys certain items in a shop and receives some change. She then places pictures of different fruit on the wall in front of the learners. She asks six learners to come to the front of the class and gives them fake money to buy items placed on the wall. She asks each learner to say how much money they have. Learners are then asked to buy items of their choice and say whether they would have change or not, and if they do, to say how much change they have. This process goes on until all the chosen learners have used their money. Learners are then asked to go back to their seats. The teacher, Thandi then distributes textbooks to learners and 2 learners are asked to share a textbook. Learners are asked to turn to page 42 and do Exercise 4(a). The lesson ends at this moment. My initial impressions of the lesson: Thandi appeared organised, confident and well prepared for the lesson, attributes that I deem necessary for the work of teaching. In the previous lesson she had shown learners how to construct money out of paper and had asked them to prepare their own monies and to bring them to this lesson. From my observation of the proceedings of this lesson, learners enjoyed participating in this lesson.				

In her lesson, Thandi chose to use the buying and selling of fruit as a context in order to scaffold learners' skills of addition and subtraction of whole numbers and decimal numbers. Figure 6.1 below presents part of Thandi's lesson plan (see Appendix 8 for the full lesson plan). Thandi's teaching objectives were that at the end of the lesson, children would be able to add and subtract money. Her proposed lesson plan specifies pupils' activity as a) watch carefully how to add, and b) do the exercises in their books.

Figure 6.1: Part of Thandi's lesson plan

Objectives By the end of the lesson pupils should be able to:
- add different sum of money and
- subtract the money used from the money they had.

Teaching material: money
improvised money.
Number line

Teaching method:
Demonstration
Socratic

With regard to the teaching materials that Thandi had intended to use for this lesson, it is clear that she had planned to use 'improvised' money and the number line. She had planned to employ the following teaching methods; 'demonstration' and 'Socratic' method. This meant that she proposed to demonstrate the operations (addition and subtraction) as learners watched carefully, and then to question the learners leading them to greater understanding of the operations in the context of money. Her proposed teaching methods are in line with cultural practices in Lesotho, where teaching can be expected to be done by teacher demonstration, questioning, chorus answers and repetition. This is how Thandi would have learned mathematics.

When the lesson started Thandi demonstrated to learners what she meant by decomposing numbers:

Thandi: Yes, when we decompose a number we break it into pieces, *ha ke re* (is that so)?

Pupils (*chorus*): *E-ea 'm'e* (yes madam).

Thandi: If you break it up into pieces, we just take out any numbers that can add up to fifty, *ha ke re* (is that so)? So my own number… I can extract M20, *Nka etsa* (I can make) 20 + 20 + 10 = M50.00 (*writing on the chalkboard*). We add up to M50.00. So which other numbers can we decompose fifty Maloti into? Which numbers can we decompose fifty Maloti into? Tefo!

Tefo: M10 + M10 + M10 + M10 + M10 = M50.00

This demonstration that Thandi made appeared helpful in aiding learners to comprehend the meaning of decomposing numbers. Tefo's response in the excerpt above may be considered evidence of this. However, this opening exercise of decomposing numbers might be coded also as 'concentration on procedures' an element of the foundation dimension of the KQ (Rowland et al, 2005). Thandi accepted M30 + M20 as a decomposition of M50, which technically is incorrect. There is no M30 note in Lesotho or South African currency. She was more interested in getting learners to think of decomposing 50 than she was in retaining the true context of available notes. Later in the lesson when learners were struggling to subtract decimal numbers from whole numbers, Thandi encouraged them to go further than their improvised notes into using other coins in the buying and selling context:

Motsamai: The banana is 1 Loti[6] and 50 cents. We subtract M1.50.

Thandi: We subtract 1.50 Loti. It's 150 *lisente* (cents), from M5.00 he has, we subtract 1.50 *lisente ha ke re* (isn't it so)?

---

[6] The currency used in the Kingdom of Lesotho is Maloti. 1 Loti is equal to 100 lisente. The word Maloti is the plural for Loti. As explained in Chapter 1, the South Africa's Rands and Maloti are used interchangeably in Lesotho. 1 Rand from South Africa is equal to 1 Loti from Lesotho.

Pupils (*chorus*): Yes madam.

Thandi: So what is the change? What is Makoro's change? So how much is he going to get as the change for Makoro? How much is he going to get? Use your fingers, use our money … just think, think, use your fingers, your head, whatever! What do you think is going to be Makoro's change? What do you think is going to be Makoro's change, when we subtract 1.50 from the M5.00 he has? Nkele!

Nkele: Makoro's change is going to be M3.50.

Here Thandi asked learners to use their fingers and their heads to think about the correct answer for the change. The excerpt suggests that she believed that the use of any of these representations (fingers or/and heads) would afford the learner (Nkele) strategies that would make it possible for her to obtain the correct answer (M3.50). It is possible that Nkele uses some of these strategies in her daily buying to determine her change. I would like to argue that Thandi's choice of the selling and buying context assisted learners to manage their strategies for subtracting decimal numbers from whole numbers. This could have been a more cognitively challenging situation if it was only presented symbolically as $5.00 - 1.5 = \square$. Thandi used improvised money to help learners to understand and practise the process of addition and subtraction of both whole numbers and decimals.

Money as a Context for Teaching Arithmetic

The following long quotation by Luciano Meira on this topic (money) appears relevant to Thandi's lesson:

> Because money is a cultural artefact of great familiarity among students, the pedagogical use of money in school mathematics should supposedly work to create meaningful situations for the learning of many mathematical domains (e.g., arithmetic). However, students' activities with money outside school (e.g., spending and saving) involve ways of doing that differ radically from the ways in which money as a knowledge domain may be broken down and presented in school. For instance, school arithmetic uses money (rather, money representations) as a context for the acquisition of oral and written computational skills. (Meira, 1998, p. 139)

According to Meira money is commonly used in mathematics lessons to set a scene for both the teacher and learners to engage in discussions and more importantly it serves as a representation that gives learners an opportunity to think and do calculations in arithmetic. However, it is clear from the quotation, that in Meira's point of view, this kind of use does not necessarily match the way money is used in real life contexts outside school where people spend and save money and so has limited value as a representation, unless it is connected to learners' everyday practices. My analysis of Thandi's lesson contends that she chose and used representations of the buying and selling context including the use of improvised money in her lesson as a stimulus rather than as a thinking tool to make mathematical concepts more accessible to learners. When discussing the notion of "Specialised Content Knowledge" (SCK) in mathematics education Ball, Thames and Phelps (2008) emphasize the key role representations play in making mathematics concepts understandable to learners. Ball et al (ibid., p. 393) further argue that because some representations are more powerful than others in affording learners access to mathematics concepts, teachers who have developed rich SCK choose and use "appropriate representations" that make content comprehensible to learners. It is hard to argue that the fake money was an appropriate representation, yet in the context of this lesson Thandi challenged learners to engage with addition and subtraction tasks involving whole numbers and decimals to two places. The class of eighty learners appeared to enjoy the money context, because they came to class with their own fake notes already made. The actual context of buying and selling that Thandi chose for her may have been the best starting point for the activities she wanted her learners to practise.

Thandi used some pictures of fruit, which were marked with appropriate prices. Figure 6.2 below shows some of the pictures namely a banana and an apple with prices R2.00 and R1.00 respectively.

Figure 6.2: Pictures of fruit used in Thandi's mathematics lesson

This is how she explained why she chose the pictures she used:

Thandi: It's because those things I used are things that pupils are familiar with, like fruits, and in order to enhance understanding of the things that they know and compare them with the things that they do not know, so I wanted to start with the background they had and move to what they don't know.

Thandi had put two things into consideration in her planning of the lesson namely, a familiar context and learners' background. She wanted to use the context of fruits, which learners in her class are familiar with. It is a common practice for learners in Lesotho to buy fruit not only in supermarkets but also from hawkers with makeshift counters on the streets and at the school gates. Many people in Lesotho earn a living by selling fruits in various places including school surroundings. Mendick (2006) might refer to what Thandi is doing here as 'scene setting'. According to Mendick (2006, p. 162) "scene setting creates a range of possible meanings for the maths that learners do, provides an alternative language that they can use to discuss it and makes these meanings and this language legitimate within the classroom". It also follows from the above excerpt that Thandi believes that learners' background, when incorporated in lessons, is important in introducing learners to mathematical concepts they do not know. So Thandi's consciousness of learners' background influenced her to produce the pictures of fruit that set a meaningful scene for learning. This would appear to be a move towards choosing a context as the basis of 'mathematizing' in the RME literature (Fosnot and Dolk, 2001). This is defined as "a process of constructing meaning"...where students are 'immersed' in "an investigation grounded in context" (p. 6). However, the teacher –led question and answer nature of the lesson may have hindered learners' opportunities to 'mathematize' fully, which would involve doing mathematics by "constructing their own strategies and defending them" (ibid, p. 9).

The picture below is a sample of the money that learners had constructed as homework before the lesson. A South African 20 Rand is approximately equivalent to €2, which in Lesotho is a large amount for any child to play around with.

Figure 6.3: Picture of fake money constructed by learners

Consider Thandi's comment below:

Thandi: I decided to choose the fake money because it was the one that was available at the time because pupils could not bring real money. And there was going to be a clash between them and parents when they demand a lot of money, that is why I improvised with fake money for real money.

Rollnick, Bennett, Rhemtula, Dharsey, and Ndlovu (2008) regard teachers' knowledge of students' context as an important domain for Pedagogical Content Knowledge (PCK). Rollnick et al (2008) argue that 'knowledge of context' includes:

> all contextual variables influencing the teaching situation, e.g. availability of resources, class size, students' socio-economic background, curriculum, the situation in the country, classroom conditions, and time available for teaching and learning (p. 1381).

So Thandi shows that she is sensitive to the economic situation in Lesotho and the difficulties her pupils may experience. While the use of real money might have been more realistic and possibly more instructive, it would be impossible to obtain, so I think the use of money constructed by learners was meaningful to them and supported their thinking. It is always important for elementary teachers to take cognisance of the context within which they operate when planning mathematics lessons.

The video of Thandi's lesson shows that at one stage of the lesson learners were finding some difficulties in mentioning the correct change, so Thandi encouraged them to use their heads and fingers to help them calculate. Thandi commented on why she wanted learners to use fingers in response to my question:

Researcher: Here you asked learners to use their heads, to use their fingers, their hands and tell me about that.

Thandi: I asked them to use everything they could use so that when they have a problem they can use whatever they have that can bring them close to the answer, which is why I asked them to use their fingers so that they see from what somebody has.

I suggest that this might be a limitation to Thandi's lesson because she did not mention the development of explicit mental strategies to help learners with number facts within a hundred, needed for calculating change in cents. Unlike other student teachers who were participating in tiers 3 and 4, Thandi happened to be the only one who used the textbook. I was then interested in what had informed her choice of task from the textbook. This was a Standard 4 class so the level of calculation was relatively easy and the textbook activity Thandi used reflects this. Figure 6.4 below presents the task that Thandi asked learners to do in class as part of the lesson:

Figure 6.4: The task given to standard 4 learners

4 A. I Loti + 25s = I.25 Maloti or M I.25
Remember: M means Maloti.

How much is:

(a) I Loti + 50s = ☐
(b) 50s + 25s = ☐
(c) I0s + I Loti = ☐
(d) I Loti + I0s = ☐
(e) 50s + I0s + 25s + 50s = ☐
(f) I Loti + I0s + 25s = ☐
(g) I Maloti + 90s + 50s = ☐

Taken from page 42 of the New Primary Mathematics for Lesotho – Pupil's Book 4

Thandi might have chosen this task with the understanding that it would create an opportunity for her to assess correctness of learners' answers, hence be in a position to decide if learners had understood the concepts well. However, the task is interesting in that it involves the addition of numbers through the use of Maloti (100 cents) and cents. The decimal structure for representing money in written form is used, which reinforces Thandi's teaching objectives. Its use might on the other hand be limiting in that it does not include subtraction activities. Typically, textbook activities are graded in difficult step by step. Given that the lesson involved both addition and subtraction of numbers so for assessment purposes one might expect to see an activity that would encourage learners to use both operations. Interestingly, the Standard 6 textbook includes a lesson on how to calculate change in a money context by counting on from the amount to be paid (Barry and Dugmore, 2001, p. 84).

The Teaching of Multiplication of Whole Numbers

Multiplicative thinking and reasoning is an important mathematical aptitude that needs to be developed in the early years of school mathematics. It is first presented in the Lesotho primary mathematics curriculum in Standard 2. In her lesson, Nomsa chose to make multiplicative thinking the focus of her Standard 4 lesson.

Table 6.3 Nomsa's lesson synopsis

<table>
<tr><th>Student Teacher</th><th>Standard</th><th>Lesson Duration</th><th>Class size</th><th>Lesson Topic</th></tr>
<tr><td>Nomsa</td><td>4</td><td>40 min</td><td>128</td><td>Multiplication of whole numbers</td></tr>
<tr><td colspan="5">Descriptive synopsis of Nomsa's lesson:</td></tr>
<tr><td colspan="5">The lesson commences with Nomsa greeting learners. She then writes the four symbols (+,−,×,÷) on the chalk board. She again writes 6 □ 2=12 and asks learners to say which symbol must be placed between 2 and 6. She then places a chart on the wall. The chart shows pictures of sweets packed in groups of four. She explains to learners how the sweets are grouped. She then gives learners a worksheet to work on in groups of 3 to 4 based on the number of learners per desk. The worksheet shows groups of fruit: pineapples, pears, oranges, and apples (see Figure 6.5 for a copy of worksheet). Learners work on the tasks while the teacher tries to walk from one group to another, which is not easy because the classroom is fully packed. She then asks learners to stop working in their groups and opens whole class discussion. This discussion is mostly about identifying different the types of fruit on each plate and counting the number in each group on each table. After this discussion Nomsa collects the worksheets and the lesson ends.

My initial impressions of the lesson: Nomsa was well prepared for the lesson. She chose a worksheet that had pictures of various fruits and used it well in class to teach the concept of multiplication. The worksheet also helped set a platform for learners to engage in productive discussion. Nomsa looked competent before the class. The cooperating teacher was in class but not actively involved in the lesson.</td></tr>
</table>

Multiplication

Three models of multiplication are recommended in the literature, discrete sets of objects; an array model and a linear model (Van de Walle, 2001). See Task 4(c), Task 4(a) and Task 4(b) of Section 2 of the survey instrument for an example of each model respectively (Appendix 5). The number line is the recommended model in the Lesotho primary mathematics curriculum (NCDC, 1998), but from findings in Chapter 4, its use may not be as widespread in schools as might be expected.

Nomsa began the lesson by writing this statement; $2 \square 6 = 12$ on the chalkboard and asked learners to decide on the appropriate sign to be placed between 2 and 6 in order to make the sentence true. Learners said the appropriate sign would be ($\times$), which means two times six is equal to twelve. She begins the lesson with the symbolic representation $2 \times 6 = 12$ in order to help learners recall and distinguish between mathematical operations. Symbolic representation accompanied by the English terminology could be problematic to learners but the fact that Nomsa chose simple single digits 2 and 6 appears to make the representation appropriate for standard 4 learners. The chart that Nomsa placed on the chalkboard in front of the class drew learners attention to what was about to follow in the lesson. The following excerpt presents what happened in that moment:

Nomsa: Thank you, I have seen that you know the multiplication sign. So let us look at the chart. Can you all see the chart on the board or can you see what is drawn on the chart?

Pupils (chorus): Yes teacher

Nomsa: What are these pictures that are drawn on the chart? They are the picture of what? Yes … Keitumetse! You! Yes … they are the picture of what?

Keitumetse: Sweets …

Nomsa: She says they are the pictures of the sweets. Are they sweets?

Pupils (chorus): Yes, they are the pictures of sweets.

At this particular moment Nomsa engages learners in the process of identifying the pictures shown on the chart. In this case the chart as an iconic representation is used to help learners to pay attention and focus on what is to be taught, which might have not been the case, given the size of this class (n = 128) and learners' ages that varied from 8 to 19. It would have been very easy for learners to lose focus on the purpose of the lesson in these circumstances. As the lesson progresses, Nomsa brings learners' attention to the groups of sweets shown on the chart.

Nomsa: How many groups of sweets do we have? How many groups of sweets do we have or drawn on the chart?

Pupil: Three groups

Nomsa: *O re* (she says) they are three groups of sweets. Is she correct?

Pupils (chorus): Yes teacher

Nomsa: We have 3 groups of sweets.

Nomsa: How many sweets are there in one group? Yes, Renang!

Renang: They are 15

Nomsa: He says we have… eh…that they are 15. Is he correct?

Pupils (chorus): Yes teacher

In this incident, Nomsa tried to draw learners' attention to the multiplier 3 and then the multiplicand 15 in each group. The grouping of sweets here is a positive start for promoting understanding of multiplication. The big idea here is that each of the three groups comprises an equal number of sweets, which is 15. This process is called 'unitizing' (Fosnot and Dolk, 2001, p.11). It is a shift in perspective from using number to count objects to thinking of groups as the 'new' units to be counted. The following dialogue presents yet another way in which the student teacher (Nomsa) uses this one example 15×3 in this lesson:

Nomsa: 15×3 (*writing on the board*), what do we mean? *Re itse re bolelang ha re re* (what did we say we mean by) this number multiplied by this number (*pointing to numbers15 and 3 on the board*)? *Re itse re reng ha re re* (what did we say we mean by) 15×3? Who can explain for us? Yes, talk aloud!

Pupil: 15 *makhetlo a mararo* (15 three times)

Nomsa: *O re* (she says) 'we mean 15 *makhetlo a ma kae* (how many times)?

Pupils (chorus): *A mararo* (three times).

This excerpt shows that at this stage, Nomsa used the expression 15×3 to explore the meaning of the operation of multiplication and also to reinforce the concept of unitizing in learners' minds. It is also worth noting that in this excerpt Nomsa used learners' home language (Sesotho) more frequently than in other parts of the lesson to reinforce the concept she wanted to teach. Barwell, Barton, and Setati (2007) support "code-switching" between languages in mathematics teaching. They argue that multilingualism/bilingualism is not a disadvantage or a problem but a resource for teaching, learning and assessment. This means the use of learners' home languages in mathematical instruction is encouraged because such languages are resources that must be used to help learners gain access to mathematical knowledge. Nomsa is therefore doing well to use learners' home language in order to support learners' understanding of multiplication. This is also evident in the following excerpt:

Nomsa: *Raba rare ha re bua ka* (we said when we talk about) multiplication *re bolela ho etsa joang* (we mean to)? *Ho kopanya palo e itseng makhetlo a itseng, palo e tšoanang, ha kere* (to add a certain number a certain number of times, not so)?

Pupils (chorus): Yes teacher.

Nomsa: $15 \times 3 = 45$. So we can see that … can you see that, we said $15 + 15 + 15 = \square$

Pupils (chorus): 45

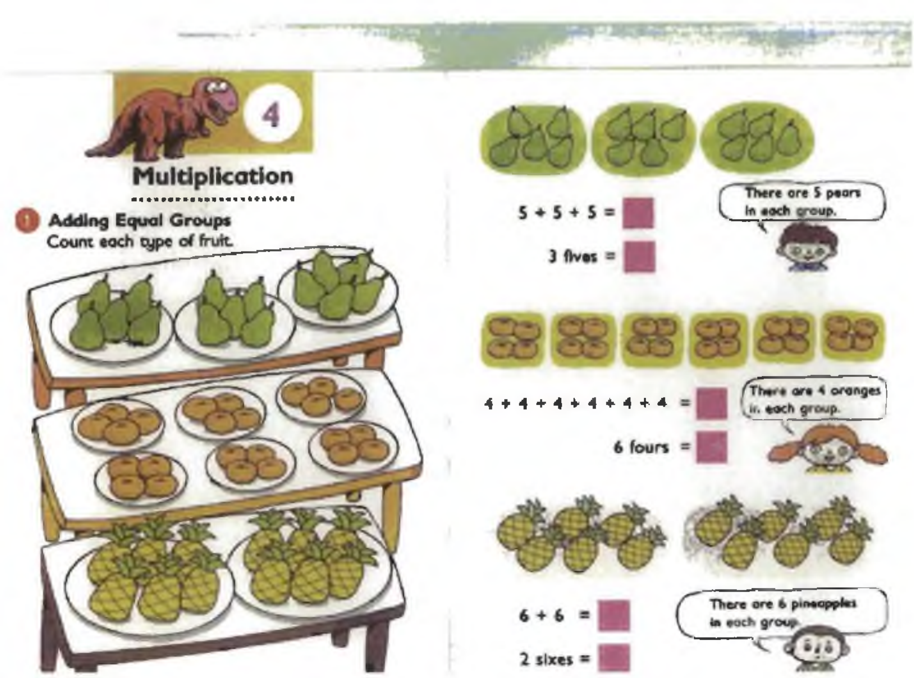
After explaining in Sesotho and discussing with learners that multiplication in the given example also means repeated addition of the multiplicand Nomsa represented the conclusion in symbolic form that $15 \times 3 = 15+15+15 = 45$. Later on in the lesson, Nomsa again emphasises the point that multiplication is repeated addition:

Nomsa: Do we all see that when we say $15 + 15 + 15$, three times is the same as saying 15×3 because we get the same answer being 45, do you see that? This means that multiplication is what? Repeated addition! *Ha ke re* (not so)? Because, *re kopanya* (we add) the same number several times, *ha ke re* (not so)?

Choice of example(s) is a contributory code of the transformation of the KQ (Rowland et al, 2005). Nomsa shows evidence of thinking about the example she would use. It is interesting here that instead of bringing up several examples, Nomsa decided to use only one example (15×3) to help learners understand the meaning of multiplication. This suggests that Nomsa was focusing on the underlying structure of the mathematical operation and knew exactly the mathematical 'big idea' she wanted her learners to understand.

As the lesson continued Nomsa distributed the following worksheet to learners and asked them to share because she did not have enough copies for everyone in the class:

Figure 6.5: Nomsa's worksheet



[Note: *Unfortunately Nomsa could not recall the source of this worksheet*].

As the following excerpt indicates the whole class discussion now revolved around this worksheet. Nomsa asked learners to mention the number of groups of each fruit (multiplier). When this was well and correctly done, she asked learners to state the number of fruit in each group (multiplicand) just like it was done with the chart earlier on in the lesson. Her work at this part of the lesson is very similar to an RME minilesson using pictures of fruit recommended to encurage skip counting (Fosnot and Dolk, 2001). Consider the following excerpt:

Nomsa: So let us look at the pears, let us look at the pears, and let us look at the pears. What can you say about the pears? Just talk about the number of pears. What can you see? Yes!

Pupil: They are 12.

Nomsa: He says that the pears are 12, is he correct?

Pupil: No teacher. They are 3

Nomsa: There are 3 groups of pears. He says they are 3. Is he correct?

Pupils (chorus): Yes teacher

Nomsa: He says that there are 3 groups of pears, *ha kere* (not so)?

Pupils (chorus): Yes teacher

Nomsa: How many pears are in each group? Yes, how many pears?

Pupil: Five

When Nomsa asks learners to mention the number of pears, one pupil says that they are twelve. This answer is obviously wrong because the total number of pears is fifteen. What is not clear is the thinking behind this answer, which appears to be a moment of contingency in this lesson. Nomsa could have made a follow up on this answer to find out from this learner why he thought there were twelve pears in that plate, and this would have shed light to Nomsa about this learner as to whether this was a mere error in counting or if the learner had difficulty in following and understanding what was being taught. The representation (a picture of three plates each holding five pears) is potentially good to explore learners' thinking but Nomsa did not use this learner's contribution productively. The transformation dimension of the Knowledge Quartet has to do with the teacher's ability to select appropriate examples and representations for a given mathematical concept. It also involves the ability to use the chosen representation or example effectively, to make the intended mathematical content accessible to all learners in the class. In order for the teacher to achieve this, she needs to be always attentive to learners' contributions that usually come in the form of a question, comment, answer, or written work.

The contingency dimension of the KQ covers this aspect of the work of teaching. Contributory codes to the contingency dimension are: preparedness to deviate from the agenda, response to learners' ideas, use of opportunities and teacher insight (Rowland, 2007). The teacher also has to be vigilant enough to take note of learners' facial expressions and reactions during the lesson when a particular representation is introduced by either the teacher or by a fellow learner in order for her to be able to follow up on such reactions or surprises. The teacher should be prepared to deviate from her main lesson agenda in order to make full use of learners' contributions. This is how the contingency dimension of the KQ ties up with transformation dimension in a lesson. However, the difficulty of implementing this kind of contingency teaching in a classroom with one hundred and twenty-eight pupils cannot be underestimated.

Looking critically into the representation (worksheet), it could be said that fruit is an enactive representation by its nature because learners could physically group and ungoup them as pieces of fruit. However, in this case as they appear on this worksheet they are iconic representations. When Nomsa asked learners to mention the number of pears in each group, the answer becomes a symbolic representation. Without a doubt, it would have been more beneficial for learners to have started by counting out three groups of five objects, but the circumstances in that classroom made such activities impossible.

Nomsa did well because after focusing the whole class discussion on the groups of pears and establishing the total number of pears, Nomsa asked similar questions for oranges and pineapples. However, it is unlikely that she has seen the full potential in her worksheet for developing skip counting strategies as a basis for multiplication attributed to such pictures by Fosnot and Dolk (2001). Instead, Nomsa asked learners to work out answers for the tasks given on the worksheet. Consider the following excerpt:

Nomsa: Can you work on this worksheet, on the right, left hand side of this worksheet? Uh … Can you fill in the … can you work out with your partner to speed up the work … Do fill up the table … you fill up the boxes with the correct answer. *Ha ke re* (not so)? Work together, you work with your partner. Work in pairs. *Re a utloanang* (do you understand)?

This form of work might be considered an advance on the repetition and transmission teaching often found in Lesotho classrooms. Nomsa has obviously given thought to engaging learners more interactively than usual. Learners' written work is useful in many ways. It helps the teacher to see if all learners understood what was taught because this cannot be easily established from learners' talk in a chorus fashion. It also serves another purpose namely to inform parents about what their children were taught at school on that particular day. In this lesson, the task required learners to produce symbolic representations from given iconic representations. This connection between the two representations (iconic and sybmolic) by learners is meant to indicate their understanding of the concept of multiplication. Nomsa's mathematical knowledge enabled her to introduce learners to the big idea of unitizing groups of objects and then treating the operation of multiplication as repeated addition. Nomsa's knowledge of mathematics also enabled her to choose and use daigrams and pictures effectively and accurately in her lesson. She also has a perception that learning mathematics is a shared enterprise (Wenger, 1998) for learners that is why she encourages them to work in groups. According to Wenger (1998) participants in a shared enterprise are involved in a collective process of negotiation of meaning. So in a class situation when learners work in groups they negotiate the meaning of the task in question and finally agree on the answer, and this is what Nomsa is promoting in her class.

Nomsa's Choice of Representations

Compared to other student teachers involved in this study, Nomsa is the one who had to handle the largest class of one hundred and twenty-eight learners. This number of learners could be challenging even for experienced teachers, given that their ages ranged from eight to nineteen. However, despite all these challenges Nomsa made an effort to teach well. Part of her teaching was characterised by demonstration and the use of various representations. During the interview, Nomsa was asked to talk about the chart that she used at the beginning of the lesson. Consider the following excerpt:

Nomsa: As my presentation was about multiplication, I thought I should show the learners that multiplication was repeated addition. So I decided to use the chart that has the drawings that would show the groups of sweets, three groups of sweets and each group had five sweets so the main idea was to show my learners that multiplication was repeated addition. I thought using a chart that has those drawings would help me to illustrate that better.

It could be argued here that Nomsa had a clear purpose for the lesson namely to teach learners that 'multiplication is repeated addition' as she puts it. She had thought carefully about the resources she was going to use to achieve this objective. She realised that the chart was going to help her to demonstrate to learners what she meant by multiplication being repeated addition. The chart served another important role, namely, making connections. She was able to make an important connection between pictures of sweets and numerical representations ($3\times5=15$). She also made connection between two mathematical operations; addition and multiplication ($5+5+5=15=3\times5$). This shows an expression of fifteen being represented by means of two different number sentences. When exploring the notion of 'procept' Gray and Tall (1994) argue that success in mathematics results from flexible ways of representing the same expression in various forms. It follows that if teachers demonstrate "proceptual thinking' that can help them clarify mathematical concepts so that they can make them more accessible to learners. Gray and Tall cite the following example:

> In this sense we can talk about the procept 6. It includes the process of counting 6 and a collection of other representations such as $3 + 3$, $4 + 2$, $2 + 4$, 2×3, $8 - 2$, and so on. All of these symbols may be considered to represent the same object, yet indicate the flexible way in which 6 may be decomposed and recomposed using different processes (1994, p. 121).

In order for a teacher to be able to do this she must have strong subject knowledge (Rowland et al, 2009), which is a contributory factor in number fluency.

It was also observed that Nomsa switched linguistic codes between English, the language of instruction in schools in Lesotho, and Sesotho the home language of learners. I was interested to find out what informed her code switching.

Nomsa: I think using both languages helped a lot because I happened to ask my learners a question in English and they seemed not getting my question at all. So that is why I decided to use both languages so that they can cope with what I was doing with them, some did not understand at all when I used English only.

In my other research work, after working with an experienced Grade 11 mathematics teacher in a multilingual classroom in South Africa, I concluded that "what came out clearly from the data gathered from the five lessons where home languages were used, is that learners actively participated in mathematical talk through the use of multiple home languages" (Mpalami, 2007, p. 77). Moschkovich (2002, p. 196), having worked in classrooms where Spanish-speaking learners were taught mathematics in English, argued that "the everyday register and students' first language can, in fact, be used as resources for communicating mathematically". So Nomsa was using learners' home language as a resource to support their understanding of mathematics concepts.

Because I thought Nomsa's lesson was successful, I wanted her to tell me of the most memorable part of the lesson. The excerpt that follows presents her response:

Nomsa: This is where I was introducing my pupils to the chart, I asked them some questions concerning what they see on the chart and the answers I got from them were what I wanted my lesson to be all about.

Nomsa mentions that the most memorable part of her lesson was the 'development'. The formal lesson plan in Lesotho schools is divided into four levels namely, introduction, development, conclusion, and evaluation. In the introduction section, the teacher tells learners what the lesson of the day is all about. This is the place where teacher tells learners the objectives of the lesson. For example, the teacher may say to learners, "at the end of the lesson you should be able to multiply two digit numbers". The development part involves the use of teaching aids that the teacher had planned to use. It is in this part that the actual teaching takes place. This is the part that Nomsa feels she did well because the learners were fully engaged as a result of the use of the chart. In the next level of the lesson, the teacher concludes by stating what learners would have learned in the lesson. Perhaps the teacher might reflect back on the objectives that were mentioned in the introduction. Finally, in order to assess if he or she has successfully achieved the objectives the teacher presents learners with tasks to do. The tasks should be based on the concepts that were taught in the lesson. Nomsa was happy that she used the chart although she did not fill in a post lesson evaluation of the lesson plan.

Finally, Nomsa was asked to say what is needed for teachers in Lesotho to teach mathematics well. Nomsa was expected to draw the answer from her year long experience of practising teaching in schools. The following excerpt presents her response:

> Nomsa: I think, for mathematics to be taught well at primary school level, we need to use either the drawings or the book materials to teach mathematics at primary level as our teaching aids, the pictures drawn on charts, the posters, and the concrete materials whereby pupils can work by touching them. I think they can understand the mathematical concepts better.

I find Nomsa's response interesting, in that just as other student teachers participating in this project mentioned, she is convinced that the teaching and learning of mathematics requires effective use of representations. I would comfortably argue that she used appropriate representations in her lesson to teach multiplication of whole numbers. Her choice of representations was to a large extent influenced by her knowledge of students, her solid mathematics knowledge, and her conviction of the value of representations in teaching mathematics.

In conclusion, it could be said that Nomsa had thought well about this lesson. She had thought well about the stages the lesson was going to be divided into and the representations she was going to use in each stage. I notice three main stages. In stage one, she introduced the lesson and put the chart on the wall in front of the class. She used this chart to introduce learners to the notion of multiplication as multiple groups of identical objects (sweets) and reinforced this with a symbolic representation 15×3. In stage two, Nomsa provided learners with a worksheet and drew their attention to the fact that the types of fruit shown on the worksheet are grouped in certain ways. Through questioning, using her chosen Socratic method, she encouraged learners to begin to develop multiplicative thinking (Clark and Kamii, 1996). In the third stage, Nomsa ordered learners to do the tasks on the other part of the worksheet. The tasks were good in that they facilated learners in making valuable connections between the diagramatic representation and symbolic representation such as in the case of orages $4+4+4+4+4+4 = \square$. The other powerful connection she made is between the symbolic representations and the textual representation (6 fours = $\square$). I consider this form of representation (textual) quite useful in bilingual classrooms such as the one Nomsa taught in. It reinforces appropriate mathematical terminology used in English for multiplication. I therefore would like to conjecture that Nomsa taught the lesson well. With the aid of the representations she chose and used, Nomsa might be said to have made the idea of multiplication accessible to learners. This was possible, in spite of the huge number of pupils and not enough sheets for everyone, through strong transformation knowldege for teaching on Nomsa's part.

The Teaching of Multiplication of Money

Another student teacher Tšeli, also taught a lesson on multiplication to a Standard 4. As her lesson synopsis shows, the lesson was on multiplication of numbers taught within the context of money. Guided by transformation as the dimension of the Knowledge Quartet that focuses on the selection and use of representations, in this section I focus on how she used the representations she had selected for this lesson.I identified two representations that Tšeli mainly used to help learners to do multiplication tasks. She invited learners to use the fingers of their hands to help them keep track as they enumerated lists of consequitive multiples of cetain numbers. Later in the lesson she placed a poster of a 10×10 multiplication square on the wall in front of the class. Tšeli drew learners' attention to it and asked them to use it as they worked out answers to tasks that she had given to them.

Table 6.4 Tšeli's Lesson Synopsis

Student Teacher	Standard	Lesson Duration	Class size	Lesson Topic
Tšeli	4	40 min	60	Money-multiplication
Descriptive Synopsis of Tšeli's Lesson:				
The lesson begins with the student teacher (Tšeli) greeting learners. Tšeli then mentions that today's lesson is planned to be on multiplication of money. She then asks learners to put up their hands. Together with learners Tšeli started reciting multiplication facts of 5. With the first finger up they shouted "five", with the second finger they shouted "ten", and then "15, 20, 25, 30, 35, 40, 45, & 50". She then writes M30 = _ × _ on the chalkboard and asks learners to work out the answer in groups. She writes another example on the chalkboard M27 = _ × _. She asks the learners to say the answer in chorus. Learners say it is 3×9. She then places a *multiplication-square* chart on the wall in front of learners. She demonstrates how to work out the answer for 3×9 using the multiplication square. She then writes the following tasks on the board: M15 = _ × _; R9 × 3 = R_; R7 × 4 = R_; M4 × 10 = M_. She then asks learners to work out the answers in groups. After 5 minutes she asks learners to say out their answers for the whole class. She then writes on the board: a) M3 × 7 = □ b) R_ × R_ = R50 and asks learners to come to the board to work out solutions. One learner comes to the chalkboard and writes R10 × R5 = R50.				
My initial impressions of the lesson: The lesson went well and Tšeli used learners' fingers and the multiplication Square. There was no cooperating teacher in this lesson.				

The following excerpt presents a moment when Tšeli used the first representation namely fingers, in the lesson:

Tšeli: We use times sign for multiplication, *ha ke re* (isn't it so)? List the multiplication of 5. Let's use our fingers and do multiplication of 5!

Pupils (chorus): 5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

It would seem that Tšeli had previously taught the learners the multiplication facts of some numbers up to ten so that in this lesson it was easy for them to recall and recite multiples of five. What is interesting about this example is that each finger represents a certain multiple of five. For instance, the little finger stands for 5 and the ring finger stands for 10, which is twice the value of the little finger though no such relationship exists in real life. Again the thumb of the first hand stands for 25, which could be interpreted as the first finger five times. This kind of use of fingers for counting multiples in this lesson is different from what I referred to as the common use of fingers for counting purposes within the Basotho culture in Chapter 1 of this thesis. It is also different from the use of fingers for keeping track of 'difference' when counting on from the smaller number in doing subtraction (Rowland et al, 2009).

Here the fingers are used as a form of unitizing to support some calculations in multiplication. That way each finger is consdered to represent a set of particular cardinality, such as five. The usefulness of this representation is likely to be seen when learners have forgotten the answer for a number bond like 7 times 5. It would be easy for the learner to simply raise her fingers and revise the manner in which she memorised multiples of five or seven. Tšeli then used this method to recall multiples of 4 and 3, and then gave the following tasks to learners to do in the whole class discussion: M6.00 × 2 = M_, M24.00 = __ × __, and M30.00 = __ × __. What remains impossible for the researcher to discover here, is whether all learners followed how the structure of multiplication works given the fact that all this part of the lesson was only done in chorus fashion.

Later in the lesson Tšeli moved away from the use of fingers and drew learners' attention to a multiplication square. See Figure 6.6 below:

Figure 6.6: The 10×10 Multiplication Square

×	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	**1**	2	3	4	5	6	7	8	9	10
2	2	**4**	6	8	10	12	14	16	18	20
3	3	6	**9**	12	15	18	21	24	27	30
4	4	8	12	**16**	20	24	28	32	36	40
5	5	10	15	20	**25**	30	35	40	45	50
6	6	12	18	24	30	**36**	42	48	54	60
7	7	14	21	28	35	42	**49**	56	63	70
8	8	16	24	32	40	48	56	**64**	72	80
9	9	18	27	36	45	54	63	72	**81**	90
10	10	20	30	40	50	60	70	80	90	**100**

In her class Tšeli used a large 10×10 multiplication square, which she had constructed using recycled cardboard and plastic for lamination. The multiplication square looked attractive as the 'spine' of square numbers on it were written using different colours, and it was large enough to be seen by all learners. The following excerpt shows the way in which Tšeli used the multiplication square in the lesson:

Tšeli: So, we have 3 × 9 = M27.00. We can look at 3 on top (*referring to the multiplication square*) and then we have 9. Here is 9 then we have 3 on top and let us look at 9 where they are going to meet, 3 × 9 = M27.00.

Pupils (chorus): 3 × 9 = 27.

Tšeli: 27 Maloti. And then we have M15.00 = __ × __ and then you take a stick there, where they will meet? Go in front and you write the numbers (ordering one child to write on the chalkboard). Is that correct?

Tšeli first made a demonstration of how the multiplication square can be used as a resource for performing multiplication calculations. Teacher demonstration is a KQ contributory code for transformation dimension and there is evidence of it in Tšeli's lesson. However, she did not mention words such as 'rows' and 'columns', which could have made learners' understanding even more clear and would have reinforced correct English terminology. After demonstrating to learners how to use the multiplication square to multiply numbers she then asked one learner to go to the board and show she could work out the factors for which the answer would be 15. The learner showed that it would be 3×5, but the word 'factor' was not used, although Tšeli used the term in her video-stimulated recall interview. After this incident, Tšeli wrote the following tasks on the chalkboard for learners to do as they referred to the multiplication square placed before the calss:

(a) M3.00 × 7 = □ (b) R__ × R__ = R50.00

There is an error here in item (b) because as it stands, it implies that an amount of money is being multiplied by another amount of money. Tšeli did not appear to recognise this error even at the time when a learner wrote R10×R5 = R50 on the chalkboard. 'Identifying errors' and 'use of terminology' are two contributory codes of the foundation dimension of the KQ (Rowland et al 2005). While my focus was on the transformation dimension of mathematical knowledge for teaching, it must be noted that there are gaps in evidence in Tšeli's foundation knowledge. The multiplication square was used to reinforce memorisation skills for multiplication facts, in chorus as a whole class activity. Some. learners were asked to demonstrate how they obtained their answers on the multiplication square.

Tšeli's Choice of Representations

Tšeli's lesson involved the teaching of multiplication of whole numbers also set within the context of money. Tšeli used some representations in her lesson to help learners comprehend the concept of multiplication. During the interview Tšeli was requested to comment on what influenced her choice of some of the representations she used in the lesson. Tšeli planned to use learners' fingers because she believed that fingers can help learners to keep track of counting multiples of numbers quickly. Tšeli also decided to ask learners to use their fingers because these are readily available resources that under normal circumstances all learners possess. The argument that the kinaesthetic involvement of learners' fingers aids them in keeping track and committing number facts to memory has some support in Ireland and the UK (Wright, Stanger, Stafford, and Martland, 2006).

Throughout the lesson, Tšeli encouraged learners to respond in a chorus fashion and she was encouraged to explain her thinking behind this approach.

Researcher: Here I see the learners are using their finger again but this time they are singing in chorus, like 3, 6, 9. Tell me more about this, why was it important for them to sing the multiplies of three in a chorus form?

Tšeli: It was for those who do not want to speak in class; I thought if they are singing [in] chorus then maybe, they will participate.

Tšeli's knowledge of her learners influences her teaching style. Rollnick et al (2008) identify 'knowledge of students' as one of the domains of teacher knowledge. In their research work of documenting PCK of science teachers in South Africa they found that knowledge of students has an influence on how individual teachers teach science. Rollnick et al. (2008, p. 1381) argue that knowledge of students includes: "appreciation of students' prior knowledge, how they learn, their linguistic abilities, and interests and aspirations". Tšeli knows that in her class there are some learners who hardly participate in mathematics lessons, possibly for linguistic reasons (Mpalami, 2007). Because she knows that it is important for all learners to take part in learning, she encourages them to participate by making them say things in a chorus form. Moloi, Morobe, and Urwick (2008) observed several primary school teachers in Lesotho using the same approach in teaching mathematics and other subjects as well such as English language. They argue that:

> The majority of pupils get the opportunity only through chorus responses; otherwise they could easily go through the whole lesson without getting the opportunity to demonstrate their understanding or lack of it (p. 618).

Tšeli's approach of encouraging learners to give answers in a chorus form is commonly used in other parts of the world as well. Watson (2000) observed teachers in Cape Town, South Africa using this method to reinforce the learning of concepts and procedures. She argues that it is "likely that the teacher's intentions are social rather than cognitive. There is an energy in a class of pupils all calling out things in unison which convinces the teacher they are participating and may energise the pupils" (p. 107). Corcoran (2008) also observed a lesson where a student teacher, Gráinne, employed the same approach in her teaching of mathematics in Ireland.

Corcoran (ibid.) argues that the approach "is firmly rooted in an oral tradition and relies heavily on children committing patterns of procedures to memory in the absence of more sophisticated teaching materials" (p. 108). I argue that Tšeli's intention to encourage chorus answering included all of these purposes.

During the lesson Tšeli had used numerical representations as well. The following excerpt discusses one such example where you she M27= _ ×_

Tšeli:	Here I was expecting the learners to solve the problem, which is to look at 27 and find the factors.
Researcher:	OK, so which factors were you expecting them to write here?
Tšeli:	I was expecting them to lift up their fingers and find the number that when you multiply a certain number by another one you get 27, e.g. I expected them to say 3, 6, 9, 12… [*Tšeli demonstrating with gestures*]
Researcher:	But were you expecting them to write two numbers like maybe 3 and 9?
Tšeli:	Yes, and I wanted them to write that number after they have used their fingers [to compute 9 sets of 3]

I asked Tšeli to talk about this example (M27= _ ×_) because in my opinion she had not thought of the difficulty for young learners to find factors for 27. Anticipation of complexity is a contributory code from the connection dimension of the KQ (Rowland et al 2005), which suggests that Tšeli had not thought deeply about the cognitive demands of the tasks she set. She might have suggested a contextual task, for example, if she had three apples in a row and asked how many rows she could make with 27 apples? Perhaps she could have dwelt more on tasks such as $9 \times 3 = \square$ and asked learners to write on their books in order for her to assess that all learners had understood the concept well. What is interesting is that Tšeli expected learners to go back to the use of fingers to get the factors of 27, without an indication of what the factors of 27 might be. It is difficult for a student who does not already know at least one factor to know where to begin.

Another teaching aid that Tšeli used in the lesson was the 10×10 multiplication square. It appears that Tšeli had intended to use the multiplication square in order to draw learners attention to multiplication facts of various numbers. Prior to the use of the multiplication square the lesson was focusing on multiplication within the context of money. So the multiplication square was introduced so as to shift from the money context into multiplication of numbers without that context. As a spatial representation of numbers, the multiplication square could have been used more to develop multiplicative facts.

In conclusion, it could be said that Tšeli was creative enough to use learners' fingers to help them to remember multiples of various numbers. It must be said that the use of fingers would be something that Tšeli had thought of on her own – it may well be how she was taught to calculate during her own school days - as opposed to the use of the multiplication square, which is an iconic representation that was introduced in mathematics education courses at the College during her first year. Mathematics education methods classes included how to construct and use a multiplication square for teaching multiplication of whole numbers. When analysing this lesson I felt perhaps Tšeli might have brought fake money to class so as to provide a context for learners to explore what it means to multiply money. That might have set a more meaningful context for symbolic activities such as M5 × 7 = □.

However, money might not have been the best context in which to teach this lesson because the commutative property of multiplication is another big idea that should have arisen in Tšeli's lesson, with use of the multiplication square. For example, it might have been useful for these learners to consider a display of seven pears acroos by five pears down (an array). This would be a very common context, and learners in Maseru would be familiar with fruit stalls set out like this.

Discussion of First Three Lessons

The analysis of these three lessons on primary school mathematics taught by student teachers while on teaching practice reveals that although the students have undergone the same college training in the first year of the DEP programme, their knowledge of mathematics in and for teaching varies. One might be tempted to claim that those students who seem to have a better understanding of mathematics are better at selecting appropriate representations and also at using them effectively in teaching. Thandi obtained a credit in her Form E mathematics examination. Her lesson on the addition and subtraction of money was stimulating for the learners because her preparation and planning involved asking learners to make improvised money and bring it to class. She caused them to engage even more with the context by providing pictures of the fruit they were buying and selling. Nomsa obtained a pass in her Form E mathematics yet she is the only participant who completed the survey instrument in Tier 1 in an exemplary manner, with very task completely correct. Her lesson in Tier 3 might be considered a 'good' example of a teacher successfully setting up a familiar context for children to mathematize. Tšeli is also a pass student in COSC and she appeared to conduct a useful lesson on multiplication facts. However, I conclude that while the use of all forms of representations (enactive, iconic, and symbolic) is advocated throughout this study, what might be critical is how effectively the chosen representation is used in lesson to help learners understand the mathematical concept taught. Student teachers' choice of representations for use in teaching seemed to be influenced more by 'knowledge of learners' and the 'context' within which they teach than the mathematics content to be taught. Of course this is no surprise given that these are novice teachers. However, in general it could be said that all three student teachers made an effort to plan well in advance of the lesson they were going to teach and thought carefully about appropriate materials that had potential to make mathematical ideas such as multiplication and money activities understandable to learners.

The next chapter focuses on a synthesis of the findings drawn from analysis of data gathered in tiers three and four with special reference to two lessons I regard as illustrative of the current challenges to mathematics teaching in Lesotho primary schools.

7. Tier Four – Two Mathematics Lessons Contrasted

In this chapter, I contrast two lessons taught during Tier 3 and I discuss the different understandings of what it means to teach mathematics that emerges from the video stimulated recall interviews with the two student teachers. There appear to be different views of mathematics afforded to learners by the two lessons. Litha, whose lesson on measuring length was taught to Standard 6 learners, obtained a credit in his Form E mathematics examination. He is twenty-one years old. Sebu on the other hand, obtained a COSC pass in mathematics. She taught a lesson on sets to a Standard 4 class. Sebu is twenty years old. Both student teachers were enthusiastic in their approach to teaching. Both had prepared appropriately detailed lesson plans. In both classrooms the lessons did not follow the plan exactly, because both Sebu and Litha appeared to act in the moment at times. Both gave honest and thoughtful responses in their video-stimulated recall interviews. However, there are interesting differences between the two lessons that are worth considering. I will describe Sebu's lesson first, followed by Litha's. I will conclude the chapter by contrasting both lessons and with a discussion of the broader issues they suggest about learning to teach mathematics in Lesotho, and the role of representations in teaching.

The Teaching of 'Sets' to a Standard 4 Class

This topic (sets) is very popular in the mathematics curriculum in Lesotho. It is taught from Standard 3 in primary mathematics level throughout the upper primary schooling and it is taught throughout all levels (*forms*) in high schools up to Form E mathematics. The topic is also taught at the College (LCE) in the first year in both the Diploma in Education Primary (DEP) and Diploma in Education Secondary (DES). For the DES programme 'sets' is taught as the prerequisite for the topic 'logic'.

Writing in 1977, Dorothy Evans contends that "most people admit that mathematics is a study of sets" (p. 57). I find it surprising therefore that such a popular mathematics topic in Lesotho does not feature explicitly in the UK or Irish primary mathematics curriculum, where 'sets' seem to be a taken-for-granted part of the discourse.

In this study, Sebu's lesson was based on the topic 'sets'. Table 7.1 below presents Sebu's lesson synopsis. In this lesson, Sebu made an effort to use various representations in enactive, iconic and symbolic ways (Bruner, 1966).

Table 7.1: Sebu's lesson synopsis

Student Teacher	**Standard**	**Lesson Duration**	**Class size**	**Lesson Topic**
Sebu	4	40 min	34	Sets
Descriptive Synopsis of Sebu's Lesson:				
The teacher greets learners and writes on the chalkboard "SETS". She then asks learners to define a set. Learners give various verbal definitions of a set. The definitions include names of people, counting numbers, bottle lids, and drawn triangles. One learner says "set boundary". Sebu accepts all answers as correct. She then puts a group of ten stones on each desk and says that these stones make a set. She makes reference to a set of trees outside the classroom. She then takes bottle lids and distributes them, but before she finishes she withdraws them and instead gives learners colourful cubic counters. The counters are in different colours. Sebu demonstrates how to form sub-sets of a set by means of cubes. She draws Venn diagrams on the chalkboard to demonstrate subsets, and also introduces the symbol ($\subset$) for subsets. The lesson ends at this stage. [Learners did not write in their books till the end of the lesson].				
My initial impressions of the lesson: In this lesson Sebu seems to be keen to use various materials in order to help learners understand the concept of sets and sub-set. She is left alone in the class, there is experienced teacher.				

When the lesson starts, Sebu writes the word "SETS" on the chalkboard. She then asks learners to say what the word 'set' means. One learner mentioned various names of people and said those were a set. Another learner mentioned that there is a set of counting numbers. Another mentioned bottle tops. Another learner offered that a set means a 'set boundary' and finally a learner said that there could be a set of drawn triangles. After listening to all learners' answers Sebu in conclusion says that all answers are correct. Mention of a "set boundary" might be seen as a contingency opportunity from the KQ (Rowland et al, 2005) but Sebu appeared unaware that there might be a misunderstanding of the terminology in English.

She then put a collection of ten stones on each desk and says that these stones make a set. She then goes on to define a set. Consider the following excerpt:

Sebu: So you have that two (*referring to two groups of stones*) and I said we call this, a set of stones. We call this, a set because it has more than one stone. A set is called a set because it has more than one object. *Ntho ha ele 'ngoe u keke ua e bitsa sete* (one object cannot be called a set). *U bona sefate se le seng ebe uso re kena le sete ea lifate* (when you see one tree and you claim you have a set of trees), no! No you call this a set because you have more than one tree. You have these (*pointing at the trees through the window*) you'll say I have a set of trees. So do you have a problem? Tšeliso and your group, are you still following?

The excerpt above indicates the intention of Sebu to use various everyday objects such as stones and trees to teach the concept of sets. However, her definition of a set appears mathematically inadequate. In this excerpt, she argues that a set is a set if and only if it has more than one element. This shows that Sebu herself has some limited knowledge of sets and because she was now on teaching practice, I know that Sebu had passed the mathematics course (MAT 1400PA) that covered the topic of 'sets'. It would seem however, that Sebu's definition of a set is based on the ordinary use of the word 'set', which simply means a collection of similar objects like the stones she had grouped on learners' desks. The example of trees she chooses also reinforces the misconception, in that she says a set must contain more than one tree. Dienes and Golding (1975) tell us that the term "set" is difficult to define, largely because there are no simpler terms to be used in the definition. They argue that "teachers should understand that a set is merely a group of things or people being considered at the same time" (p.72). They suggest there is nothing difficult about the concept of 'set' and that teachers should "let it be a word which merely 'creeps into' our mathematical language as we go along" (p.72). This appears to be the approach of teachers in Ireland and England. The textbook used in mathematics courses in LCE also defines a set as "a collection of clearly defined objects, things or states" (Croft and Davison, 2006, p. 34). It is important to note that in this definition there is no mention of the number of elements in a set.

On the other hand, in mathematics the word 'set' has a broader meaning than just being a collection of things. In mathematics, there exists a set without elements and it is called an empty set which is usually represented symbolically as Ø, and there is also a set that has one element such as {a}, both these two sets have subsets (Croft and Davison, 2006). The subset of an empty set is the empty set itself, and the subsets of this set {a} are {} and {a}. Sebu's lesson objective was that "by the end of the lesson pupils should be able to identify sets and subset and the use of Mathematical symbol '$\subset$'." (See Appendix 11 for Sebu's lesson plan). She proposed to achieve her objective by "demonstration, questions and answers and grouping" and the development of her lesson is quite detailed with columns stating teacher's activities and interesting pupils' activities. Sebu wrote a 'conclusion' to her lesson plan that appears to indicate confusion in her understanding of the mathematics she proposes to teach (see Appendix 11 for Sebu's lesson plan):

Sebu: Set is a group of elements and subset is a set that contain all elements that are found in the main set and the sign for subset is '$\subset$'

I consider that Sebu's claim that a subset would "contain all the elements that are found in the main set" is an 'overt subject knowledge' issue. This is a code in the foundation dimension of the KQ. It appears to indicate that Sebu is not quite sure of the mathematics underpinning the lesson she has so elaborately prepared. This is problematic in a classroom where the teacher (Sebu) seeks to explain and transmit a piece of formal mathematics (sets, subsets and symbols).

As the lesson continues, Sebu took out some commercially made plastic cubes[7] from a big box placed next to her table and distributed them to learners. She put ten cubes on every desk. She then went on to ask learners what they could say about these objects. Learners mentioned the colours of the cubes, which did not appear to be what Sebu wanted them to say. She wanted them to focus on the 'set' aspect. Students were discerning details and recognising relationships (Mason, 2005) but not the property

[7] As part of government's support to schools that implemented free primary policy, the Ministry of Education and Training supplied a number of primary schools with kits containing mathematics teaching aids such as the colourful cubes used in this lesson.

Sebu was interested in teaching. However, the lack of focus on the 'set' aspect did not stop her from doing what she wanted to do next. She drew a Venn diagram and set braces on the chalkboard. Consider the excerpt that follows:

Sebu: *E mong* (another one)! Oh, I'm trying to show you that if you have this ten coloured square objects. To show that this is a set you can put them in a circle (*shows them by drawing*). You can rename it, give it a name, either A, or you choose another name.

At this stage it seems that Sebu may have moved too quickly from the use of enactive representations (cubes) to iconic representations (Venn diagrams and set braces) before learners could understand the purpose of the cubes. It was also observed that Sebu called the cubes "square objects". She never referred to them as cubes throughout the lesson. This then suggests that Sebu might not be aware that a square is a two dimensional shape while a cube is a three dimensional object with square faces or she did not see a need to distinguish between them[8]. Learners at this early stage (Standard 4) of learning school mathematics should hear correct 'mathematical terminology' from teachers (Turner and Rowland, 2011). Otherwise, it might be quite difficult to convince them at a later stage that these things are not squares but cubes. However, the excerpt shows that Sebu used the drawings of Venn diagrams and set braces to demonstrate to learners the two ways of presenting sets. This was a useful demonstration because learners have to be aware that the following numbers written as: 1, 2, 3, 4, 5, 6, 7, 8, 9, are not a set. They only become a set when written as {1, 2, 3, 4, 5, 6, 7, 8, 9} or put in a Venn diagram, it is then called the set of

[8] The word for 'cube' in Sesotho is 'lebokose', which means 'box'. There is no Sesotho word for 'square'.

natural numbers from 1 to 9. So this set can be given a name, which is usually a capital letter as Sebu points out in the excerpt above.

Sebu next introduced the bottle tops as teaching aids in this lesson. It is worth mentioning here that she did not remove the stones from the desks, rather she placed the bottle tops next to the stones on learners' desks. In the following excerpt, Sebu used the bottle tops to make mathematics accessible to learners:

Sebu: Now you can say … Set A is a set of coloured square objects. They are more than one. Now here … we … but it can happen that you have more than one object or more than one set. I have another set here, a set of bottle tops; this is a set of bottle tops. Now I have two sets. Sit down (*ordering learners to sit down*)! This is a set of coloured square objects and I give it A. I name it A. The second one, a set of bottle tops.

Pupil: Set B.

Sebu: Now I have two sets, each set contains different objects … *kapa le bona ntho tse tšoanang* (or do you see similar things)

Pupils (chorus): No teacher …

Sebu: This Set A contains coloured square objects. Set B contains bottle tops. Do you see the difference?

Pupils (chorus): Yes teacher …

Sebu: You have ten stones and it is a set of ten stones. They are completely different. … Now I have a bottle tops on Set A. I have a bottle tops only in Set A. So *haese e le tjee re ka reng* (So when it is

> like this, what can we say)? What we say? … Set A is a set of coloured square objects. What about Set B? Set B have a set of bottle tops. I have blue, I have a yellow and red, both they are squares. Let's concentrate on colours. Let's concentrate on colours, let's change the objects. I have a yellow, I have blue and I have a red. Now what can you say? What can you say?

In this excerpt, Sebu used the bottle tops in order to emphasise that there are two sets that are completely different (Set A – a set of coloured cubes and Set B – a set of bottle tops). I would argue here that perhaps instead of using the bottle tops Sebu could have used the cubes only. She also makes reference to a set of stones, and it is not clear why they were brought into this lesson. The cubes alone might have been more effective in forming various sets given the fact that they are of different colours. It appears reasonable to conclude then that the stones and the bottle tops were used less appropriately to advance learners' understanding of sets. It appears that Sebu suddenly changed her mind and put the bottle tops and the stones away. She then invited learners to focus on the colours of the cubes although she continues to call them 'squares'. Based on this, I am tempted to conclude that the student teacher lacked 'insight' of how best she could have used the three representations (stones, cubes, and bottle tops) in order to translate her knowledge of 'sets' into something understandable to learners (Turner and Rowland, 2011). However, I realise that it might also be true that Sebu experienced a moment of 'teacher insight' from the contingency dimension of the KQ (Rowland et al, 2005) because she realised that the current activity was not working and decided to harness the potential for demonstrating a subset with the different coloured cubes. It appears that she suddenly switched focus from the other representations she was using. When shown the clip of this part of her lesson indicated that she had deliberately selected the cubes for the purpose of introducing the subset.

Sebu: There I was at the point to discuss sub sets. I used different colour cubes which enabled me to explain better about sub sets as you can see set A contains different colours while set B contains only red cubes so I tried to show pupils that if a set contains all elements that are present in another set, that set is a subset of that one. This means that set B is a subset of set A.

This time the property of colour was intended as a focus in the representation. The following excerpt reveals the manner in which the introduction was done:

Sebu: Yes, they are the same, the thing is that here Set A, *ho buuoa ka* (we talk about) Set B *empa ke setho* (but it's a member), same as this one (*pointing at Set A*). I said to you, you have 2 reds here and 2 reds in B, *hase li le teng li le peli tjena* (when both two are together), *empa u ntso bona hore ee e kahar'a ee* (but do you see that one is inside the other?), does it make sense? *Mona ua bona hore Set B e se e le ka hara Set A* (here Set B is inside Set A), *empa Set B e ntse na le 2 red le Set A e ntse na le 2 red* (but set B still has two red and Set A too has two red), does it make sense?

Pupils (chorus): No teacher.

Sebu used the coloured cubes to form two sets (Set A and Set B) in her attempt to introduce learners to the concept of a sub-set. She demonstrated to learners how these two 'sets' could be possibly made. The demonstration was performed on Sebu's table, placed before the class where all thirty-four learners could see. The video of this lesson shows that while learners were focusing on the practical demonstration Sebu was making, she immediately left that and went to the chalkboard to draw a Venn diagram showing Set B inside Set A. Next to that Venn diagram she wrote a symbol for sub-set ($\subset$) but it is not surprising given the conclusion in her lesson plan that she could not give a satisfactory example of its meaning. As she was drawing on the chalk board Sebu asked learners if what she was doing made any sense to them. Learners in a chorus said "no" as the above excerpt indicates. It must be said that it was only in this class that learners openly said to the teacher they were not following what was being taught. Again while this class is comparably smaller than the four classes from other schools where I collected data, it appeared that learners failed to understand the intended mathematics in this lesson.

The following excerpt presents Sebu's lesson in its final minutes:

Sebu: Now I have a statement on the board: first statement says: Set H is a set of Class 4 pupils and their names. (Matšeliso, 'Mamokete, Katleho, Paul, Tšeliso, Teboho, Moteoli, 'Mampeo, 'Mamphaka, Motlalepula, Lebohang).

Second statement says: Set G is a set of boys in Class 4 and their names are: (Motlalepula, Lebohang, Teboho, Tšeliso, Paul).

Please sit and listen. Now I want you to go and draw a set which represents those two statements. The first is H and the second one is … G. Reitumetse be in class (*meaning Reitumetse must pay attention*). Molaoli what is your problem? Mohatlane helang u etsang (Mohatlane what are you doing?)! Come on! le itse lea utloisisa (you said you understand), are you in class or what? Draw

> the sets and I'll draw mine. Hurry up! Here is my school … Class 4 students and their names are on the board. Let's use this part, as big as you can, draw big sets, as big as you can. Nthabiseng what are you doing there, you're… Hela (hey)! Stop talking 'Mamphaka.

Thus towards the end of the lesson, it could be said that Sebu introduced another example of sets to learners, where elements were the names of learners. She then asked one learner to go and represent that information in the form of a Venn diagram. He did so successfully, which means he understood the lesson. My observation of other learners is that because they were not able to follow what was going on as they earlier said, they resorted to doing other things. It could also be that they wanted to be more involved in the lesson. Sebu tried desperately to deal with the learners' inappropriate behaviour as the lesson came to an end. It could be said from Sebu's lesson that it is not many representations/teaching aids that will make learners understand better, rather it is one carefully chosen representation, effectively used, that will make mathematics concepts accessible to learners. It could also be said that Sebu had carefully planned an interesting and worthwhile lesson, with nice activities planned to engage learners in making sets and subsets but she herself was unsure of the mathematical ideas she wanted to teach. This is apparent from her written lesson plan. It seems that Sebu prepared for the lesson by collecting multiple representations rather than checking the mathematics she proposed to teach. This may be because she knew the focus of my research and wanted to comply.

Sebu's Choice of Representations

Sebu used a variety of teaching aids in her lesson to teach the concept of sets to Standard 4 learners. During the video stimulated recall interview I asked her to comment on using bottle tops to represent sets.

Sebu: I was going to represent sets using the bottle tops together with those cubes. My point was to represent sets. I wanted to show the pupils that a set can contain more than one object that is why I collected those bottle tops and those cubes.

It would seem that Sebu had a clear purpose of what she wanted to achieve in the lesson namely to represent sets. In accordance with her plan, she collected bottle tops to use in this lesson. She also wanted to use cubes for the same objective. In addition to these two forms of representation, Sebu used stones.

Sebu: I used stones because they are easily available; pupils were able to collect stones on their own that is why I used them.

Having worked with teachers who teach mathematics in the United States, Ball, Thames, and Phelps (2008, p. 392) came to a conclusion that "some representations are especially powerful; others, although technically correct, do not open the ideas effectively to learners". Therefore, the key idea is to select one most appropriate material and use it effectively in the lesson to develop learners' understanding of the concept in question. The attribute of colour in the cubes could have been used to draw learners' attention into the lesson and she could have given them an opportunity during the lesson to construct sets of their choice in their small groups. It must be said again that Sebu's class group was the smallest of all, compared to class sizes of other student teachers participating in tiers 3 and 4 of this study, with a total number of students being equal to thirty-four.

The Teaching of Measurement in a Standard 6 Class

Measurement is an important topic taught at primary school level. Unlike many mathematics topics, measurement comprises an aspect of practical skill that is important in daily life (van den Heuvel-Panhuizen and Buys, 2008). A creative primary school teacher can set scenes where learners are given practical tasks to do in mathematics lessons. This is seen as an improvement on formal 'chalk and talk' teaching.

The topic 'measurement' forms part of the mathematics content covered in primary schools in Lesotho. Litha's lesson was based on this topic. His teaching objectives were that: By the end of the lesson, pupils should be able to:

i) estimate the length of objects,

ii) measure the length of objects,

iii) estimate the height of objects,

iv) measure the height of objects.

In what follows, attention is paid to Litha's lesson on measurement.

Table 7.2: Litha' lesson synopsis (see Appendix 12 for Litha's lesson plan)

Student Teacher	**Standard**	**Lesson Duration**	**Class size**	**Lesson Topic**
Litha	6	50 min	69	Measurement-length of objects
Descriptive Synopsis of Litha's Lesson				
The lesson begins with Litha inviting learners to come to the front of the class. Five learners stand in front of the class, then Litha asks learners to say who the tallest learner is. Learners mention that some are taller than others as they compare and order the five learners. Litha then emphasises the need for accurate measurements because otherwise it may not be correct to say that one learner is taller than the other just by merely looking at them. He asks learners to mention tools that are used to measure length. Learners mention: metre ruler, ruler, and tape. One learner says kilometres. Another learner says that 1 cm is bigger than 10mm. Litha treats the comment as a contingency opportunity and continues to question the class till all learners realize that 1cm = 10mm. Litha then requests learners to measure the width of their first finger. Learners say it is about a centimetre. He says to learners "that is your finger ruler". He then says "measure length of pen using your finger ruler". Learners give these answers: 7cm, 16cm, 11cm, 6cm, and 20cm. He then asks them to measure the length of the pen by a ruler. Learners gave these answers: 16cm and 14cm. Litha mentions that finger rulers were used for the purposes of estimation. On the other hand, rulers are used more to find accurate measurements. In the next activity, Litha asks learners to measure the width of their palms with a ruler. He then asks learners to measure the length of desks using these 'hand rulers'. Learners give various answers. After that he asks them to use a ruler to measure the length of their desks. He again mentions that hand rulers provide estimates while the standard 30 cm ruler gives more accurate measurements. The lesson went on to the use of the metre ruler, and the measuring tape. A shovel which was in the classroom was also used to estimate the length of the chalkboard. Towards the end of the lesson, Litha asks learners to mention the things they learned in this lesson. Learners mention the following: accuracy, how to measure length of objects, and to estimate length of objects. He concludes the lesson by saying that the lesson was about centimetres (cm), millimetres (mm), and metres (m). He then asks: "how many centimetres are in a metre?" learners say: 100cm. My initial impressions of the lesson: The student teacher was well prepared. He looked relaxed and confident before the class. He appeared to know what he wanted to teach. The cooperating teacher was present in the classroom.				

With the initial ranking by height of the five pupils Litha might have wished to bring learners' attention to the fact that there is a need for accurate measurements in life even though estimation is also useful in making quick judgements. Focusing on the attribute to be taught might be a good first step in teaching measurement. An example of such a classroom environment as Litha's lesson is the one that Ainley (1991) described where learners were first asked to estimate the length of various objects and then to verify their estimates by accurately measuring the length of the objects. Ainley presents the scene from three perspectives: 1) the teacher who thinks children are learning to estimate measure in non-standard units and realise the usefulness of standard units. 2) The children who enjoy being actively engaged in a clearly defined not too difficult task where it is acceptable to get wrong estimates. 3) The mathematics education researcher who sees a lesson in which everyone is busy, but there is no real purpose, with very little mathematical thinking going on at all (Ainley, 1991, p. 70). She goes on to question, 'where is the mathematics in measurement? 'I ask the same question of Litha's lesson.

Kalejaiye (1985, p. 4) points out that "when teaching measurement, practical work should feature prominently and estimation of measurement should be encouraged". Litha's move to ask learners to compare the heights of the five learners served another purpose, of ensuring learners' high participation in lesson proceedings. The setting served as a good "oral and mental starter" (DfEE, 1999) for the lesson and guaranteed learner participation.

After this, Litha asked learners to look around and identify objects that could be measured, consider the following excerpt:

Litha: Now look around your classroom here. Which things can you measure their lengths? Which things in your classroom can you measure their lengths? That's my question. Which things can you measure? Yes!

Pupil: We can measure the door.

Litha: We can measure length of the door, what else?

The excerpt shows that Litha made learners aware that in their classroom there are many materials that possess the attribute of length that they could measure. Learners mentioned that they could measure the length of objects such as the chalkboard, window frames of the classroom, and the book-locker. Another learner mentioned the length of their classroom. Finally, one learner said that she could measure her own height. Litha referred to objects found in the classroom, which he appeared to use confidently as teaching aids to help learners understand the concept of measurement.

Litha then moved on and asked learners to focus on their rulers. Consider the following excerpt:

Litha: OK, that is what he says. Now we said the ruler is one of the instruments that we use, *ha ke re* (isn't it so)? Now you have your rulers, study your rulers. What can you see on your rulers?

Pupil: I can see the millimetres.

Litha: How can you see that these are millimetres? Show us on the board (*the learner goes to the board to write mm*). He says he can see the millimetres. What does mm stands for?

Pupils (chorus): The 'mm' stands for millimetres

Litha: What else can you see in your ruler?

Pupil: I can see the centimetre.

In this excerpt, Litha is using a ruler in a different fashion from its usual use namely to measure length of straight-edged objects. He asks learners to study what is written on their rulers. Learners mention that they see "mm" and "cm" and the discussion focuses on what each one of these represents. Learners mention that mm is an abbreviation for millimetres while cm is an abbreviation for centimetres. As the lesson progresses the discussion revolves around these two units (mm and cm). The issue was which one of the two is greater than the other, and some learners were making mistakes such that 1mm = 1cm. Litha followed up on each learner's answer until they all realised that 1cm = 10mm. When he was satisfied that this concept was understood by learners, Litha wanted them to participate in constructing 'hand rulers'. Consider the following excerpt:

Litha: I want us to make our own rulers, look at your fingers. Your five fingers … now test, which finger amongst your five fingers best fits into the 1cm. So that is a finger ruler. Finger ruler ee ea rona ke'ng (our finger ruler tend to be what)? It is 1cm ruler, *ha ke re* (is that so)?

Pupils (chorus): Yes sir

Litha: Now, measure the length of your pen using your finger ruler. Measure the length of your pen, using your finger ruler. How may centimetres have you found? Yes!

Pupil: 11cm

Litha: You found 11cm. That is your pen, *o re* (he says) by measurement … after measuring his pen he found what? 11cm after using his finger ruler, yes! What about yours?

Pupil: I found 7cm

Litha: He found 7, what about yours?

Pupil: I found 16cm

In this excerpt, Litha asks learners to construct improvisational rulers (finger rulers) that they would use to measure some objects. Each of the rulers was to be about 1 cm long. When learners use their 'finger rulers' they realise they obtain different measurements for pens that are of equal length. Then Litha explains that various answers were due to 'finger rulers' that simply provided approximate measurements, hence a need for more accurate measurements that can be obtained by using a 30 cm ruler.

Figure 7.1 below demonstrates what Litha meant by using a personal benchmark unit in this fashion. This activity is recommended in the prescribed textbook for this standard.

Figure 7.1: An illustration of measuring a finger

Adapted from Page 95 of Lesotho Standard 6 prescribed mathematics textbook.

Later on in the lesson, Litha asked learners to construct what he termed "hand ruler" and use this new unit to measure the length of the desk. The following excerpt presents that:

Litha: Now take your palm, your hand, *ha ke re* (isn't it so)? Take your ruler, your ruler like this (*showing them how to place the ruler on the palm*), make sure that this small finger is at the mark zero. This small finger is at the mark zero. *Letsoho la hau kaofela le felle* (your whole hand must fit) on the ruler. How long is that? *Le monoana o motona o kenelle* (even the thumb must be part of it). Even the palm, how much do you get? How many centimeters? Yes!

Figure 7.2: An illustration of measuring palm using a 30 cm ruler

Adapted from Page 96 of Lesotho Standard 6 prescribed mathematics textbook.

This activity kept all learners engaged in the classroom. Learners recorded various numbers of centimetres as lengths of their palms. Litha then asked them to use their palms to measure the length of the desk. He explained that the length of the desk could be found by calculating the iterations of the unit number of palms multiplied by the size of the palm (unit of measure). For instance, if the length of the palm is 8 cm and the number of times a learner placed along the desk is 10, then that would mean the length of the desk is 80 cm. This yields the following equation: *length of desk = length of palm × number of times the hand was used along the desk*. Learners used their ‘hand rulers’ and found various measurements. Litha explained that just like in previous example of ‘finger ruler’ these many different answers are estimates and not accurate measurements. If Litha had set his lesson in the context where real measurements mattered, like marking out plant beds in the school garden, then this point might have emerged more usefully from the children’s work, rather than from the teacher telling them.

Litha then asked learners to mention a measuring tool that they could use to measure the length of the desk in order to obtain more accurate measurements. They mentioned a metre stick. Litha took a metre stick and asked learners to name the unit of measures that the metre stick is based on. Learners mentioned millimetres, centimetres, and eventually they said that the unit for the metre stick is a metre. The metre stick is being used here to teach learners the units that are used in measurement such as mm, cm, and m and how they relate to one another. What I find most interesting about this part of the lesson is the fact that again Litha followed up on learners’ contributions (Turner and Rowland, 2011). Turner and Rowland maintain that while teachers’ actions for the lesson are pre-planned, learners’ contributions cannot be planned and that is why the teacher will have to ‘think on his feet’ as they put it. They further argue while the moments of *contingency* are difficult for teachers to handle, they are more challenging for novice teachers. Turner and Rowland (ibid., p. 202) make a point that “the quality of such responses is undoubtedly determined, at least in part by the knowledge resources available to the teacher”. It appears that Litha has strong mathematical knowledge of the material he wants to teach.

Towards the end of the lesson, Litha took a shovel that was kept behind the door in the classroom and showed it to the learners. He then asked them to estimate its length. The following excerpt presents this part of the lesson:

Litha:	Thank you very much; I also have a shovel here, how long is a shovel? The length of a shovel …
Pupil:	1cm
Litha:	1cm, 1cm, look at the 1cm on your ruler, you say 1cm is the same as the shovel? I want other hands. Another hand! Let me choose you, yes!
Pupil:	The length of the shovel is 1m.
Litha:	*O re 'ng* (what does she say)? *O re* (she says) her estimation of the shovel is about a meter. Can you see that the shovel is about a meter?

Litha used the shovel to reinforce the idea of estimation. The excerpt also shows that when the learner made a mistake, it could be said that Litha treated it as a contingency moment (Rowland et al, 2005). He referred this learner back to the ruler, which makes a nice connection between what is being done now with what was covered in the lesson earlier. The lesson came to an end when Litha had just asked learners to estimate the number of shovels that would make the length of the chalkboard. Perhaps it could have been more meaningful for learners if Litha at this stage to have used the metre stick for this purpose instead of the shovel because, prior to this he together with learners had established that the shovel's length was almost equal to the length of the metre stick.

Litha's Choice of Representations

In his lesson, Litha taught learners the concept of measurement, within which he introduced the concept of estimation. Litha used various teaching aids to enhance learners' understanding of these two concepts. He introduced his lesson by engaging learners in comparing heights.

Litha: I could not tell them exactly what my lesson is all about so I wanted them to discover from my introduction as a class.

Litha's approach to teaching is 'discovery'. He does not want to tell learners everything instead he prefers that they find out themselves as to what the lesson is based on. Van den Heuvel-Panhuizen and Buys (2008) value the skill of comparing when measurement is taught at primary school level. They refer to a practical task of comparing amount of water in two glasses. Litha's learners got fully involved in comparing the heights of the other students.

Litha: I wanted to bring reality into them, I did not want to talk about things that are outside the class that they could not see or use in measuring. The tools that were found in the classroom were readily found.

Litha wanted them to develop an understanding of the concept of estimation. Pang and Ji (2009, p. 480) also believe that "an experience of measuring through comparison should be a prerequisite for understanding length measurement, and the conceptual understanding of the units of measurement is essential". Therefore, Litha's learners were able to compare their fingers' width with a centimetre on the ruler and as Litha points out, it was then easy for them to have a better idea of how big a centimetre is.

> Litha: I think it would make it easy for the learners to see how big or small the unit is e.g. making their own 'cm ruler' they were able to see how much smaller the centimetre is and using their 'hand ruler' they were able to see how big 10cm is and I used that to build the concept of estimation in measurement.

As part of the lesson, Litha moved between the rows of the desks and sometimes talking with and focusing on a small group of learners. I asked him during the video-stimulated recall interview to explain why he did that. He explained:

> Litha: I was trying to show the pupils how long the meter stick is and since I had only one meter stick, I had to go around the class and gather them into groups to show them the meter stick for them to discover how long it is and few pupils were able to see only because we were using only one meter stick.

The excerpt shows that Litha had planned well for this lesson in order to make useful demonstrations for learners. By walking closer to learners, Litha made sure that they could easily see the units on the metre stick. Demonstration is one of the codes of the transformation dimension of the Knowledge Quartet (Rowland, Turner, Thwaites, and Huckstep, 2009). Being able to demonstrate is an important skill for student teachers to develop, because in the absence of enough resources to give everyone ruler young learners are likely to understand concepts better when the teacher has made the demonstration.

Discussion of the Two Lessons

I wish to contrast these two lessons from a number of perspectives. I wish to argue that the analysis of Litha's lesson reveals that Litha has an understanding of the mathematics he wishes to teach (measurement and estimation of length). As a result of his knowledge of these two concepts he is creative enough to use various objects in his lesson as resources to help learners understand some of the big ideas of measurement. I recognized the connection he made of 'finger ruler' and 'hand ruler' from the textbook that he used effectively to teach the concept of estimation. I realised that even though Litha did not refer to the prescribed textbook during the lesson, he had used it during the planning of the lesson. The evidence of this is manifested in the activities such as making 'finger ruler and hand ruler', which are ideas suggested in the prescribed mathematics textbook for Standard 6 learners referred to already in this chapter. The use of textbook is one of the codes of *foundation* dimension of the Knowledge Quartet. It suggests that Litha had good foundation knowledge of the mathematics he planned to teach, which is why he was able to use various representations effectively in his lesson. Litha also used a 30cm ruler and a metre stick effectively to reinforce the concept of accuracy in measurement. Ainley (1991) acknowledges that there is mathematics in measurement and emphasises estimation, the structure of the system of units, approximation and appropriate accuracy as important ideas to be learned. I suggest that Litha exposed learners to each of these ideas.

From a traditional perspective on mathematics teaching, Litha achieved his objectives. Pupils were engaged and busy. In his evaluation he commented: "Pupils were participative. The lesson was effective." However, in the light of current research on successful mathematics teaching from RME, mathematics is learned when teachers introduce well-chosen contexts and pose problems that learners can meaningfully engage with. Litha did not actually pose problems in this lesson but his approach to mathematics and to teaching mathematics appears to have basis in the learners' lives. He made repeated efforts to involve all the learners in activities related to measurement of themselves and their classroom.

In contrast, Sebu's lesson on sets and sub-sets belongs to formal mathematics. Like Litha, Sebu made efforts to connect the notion of sets to the learners' environment, by presenting them with sets of stones and bottle tops. However, Sebu felt obliged to stress the formal symbol for a subset even when she felt her pupils had failed to understand what she was trying to teach. During the video-stimulated recall interview, I asked Sebu to talk about the use of the symbol for the sub-set ($\subset$). I offer this excerpt from our conversation to illustrate Sebu's beliefs and attitudes about mathematics and how she thinks it should be taught.

Researcher: Here you introduced the symbol of subsets and then you wrote B is a subset of A. What impact do you think this had on learners in understanding the concept of subset?

Sebu: They were confused and did not understand what subset means.

Researcher: Why did you decide to introduce the symbol although they were still confused?

Sebu: Because in mathematics you do not write B is a subset of A in words. We use signs so pupils were supposed to be taught signs. I was supposed to present that sign because it is a sign for subsets so they had to know that.

Beliefs and attitudes belong to the foundation dimension of the KQ (Rowland et al, 2005) and Sebu appears to demonstrate a formalist view of mathematics. Sebu realized that learners were confused, and in my own viewpoint they were confused perhaps as a result of the way materials were used to represent sets and sub-sets. It could also be that the introduction of the concept 'subset' with its symbol ($\subset$) complicated matters even more. Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, and Ball (2008, p. 468) explored the notion of mathematical knowledge for teaching and came to a conclusion that "teachers without mathematical knowledge cannot provide explanations, justifications, or make careful use of representations". It could be that Sebu had a limited mathematical knowledge for teaching the idea of subsets. I draw this conclusion from the excerpt where Sebu argues that she introduced the symbol for subsets because "pupils are supposed to be taught signs". Mathematical concepts have to be represented symbolically. It would seem, according to her, that teachers must use symbols even if it confuses learners. Sebu felt obliged to introduce the symbol. Perhaps when she realised the confusion the symbol was causing, she could have flexibly deviated from her agenda, a code on the contingency dimension of the KQ (Rowland et al, 2009), which was to teach subsets, and take that as an opportunity for her to go back and clarify the meaning of sets. This did not happen, though at the end of the lesson one learner correctly represented a set and subset of his classmates on the chalkboard.

Towards the end of the video stimulated recall interview, Sebu was asked to mention her general impressions of her lesson. Consider the following excerpt:

Sebu: I used those teaching aids because the cubes were many and I was able to use them correctly, then I used stones and bottles tops because they are easily found.

The excerpt shows that the main reason for Sebu to use the teaching materials depended more on their availability than their effectiveness in making the concept of 'sets' comprehensible to learners. Yes, indeed the cubes were many and colourful enough to be used by all learners in an enactive way to develop their understanding and knowledge of sets. Exploring this attractive resource was a good context in which to begin learning about sets. But the mathematics is not in the resource. Good mathematics teaching requires teachers to understand the mathematics they want the learners to engage with.

Engaging Learners in Group Work

Both teachers planned to have learners engaged in doing the mathematics they planned to teach. This is evident in Sebu's list of teaching activities, which includes "grouping". She outlined her planned teacher activity as, to "put pupils in groups and give them coloured squares *(sic)* and shapes and name their set S and Q. She asks them to make Q subset of S with her help". The plan to organise group activities was ambitious and unusual in Lesotho, where class sizes are often very big. On the other hand, Nomsa's lesson was successful in this regard. Nomsa said:

> I think to allow learners to work in groups is to give opportunity to learners who didn't get exactly what I said to get that from their peers. If they explain to one another what the concept is all about they will understand better.

Group work could have been possible with just 36 learners in Sebu's class but the learners were not given any group tasks. Sebu had given out sets of stones and bottle tops but did not distribute the coloured cubes. She did not set any mathematical group task. Instead of asking learners to reason about a task, it seems that Sebu thinks the mathematics must be transmitted by the teacher. She did however invite some pupils to stand up to model a set and subset of learners.

Litha who had a class of 69 learners also selected a couple of pupils to have learners compare their heights. He also organised all learners to be actively involved in making their own finger and palm rulers. Some teachers would claim that it might have been more beneficial for learners if Litha had given even some of them an opportunity to actually measure the lengths of these objects themselves. As already mentioned he deliberately moved among the desks to let the pupils get a closer look at the metre stick.

Issues Emerging from Analysis of the Lessons

The details from all the lessons raise the same question: how is mathematics learned and how should it be taught? It is true that mathematics is an important cultural activity. Student teachers in this study appear to be striving for improvement in teaching methods (use of multiple representations) while continuing the practices for teaching mathematics they experienced themselves (teacher-led demonstration with the focus on symbols). However, following a small-scale study, Corcoran and Dolan, (2012, p. 199) argue that "mathematics education in Lesotho is not best served by thinking of mathematics in terms of 'product' to be transmitted or as a 'quantity' to be measured". From this study it appears that five issues are relevant to how mathematics is learned and should be taught.

First, there is the complex Basotho cultural context. Lesotho remains a low-income country with many challenges. As a developing country it has traditionally been dependent on external funding. Over the years, foreign aid has taken many forms from missionary activity to direct food and financial donations. This has resulted in some good initiatives that have not persisted or spread (e.g., LSMTA). Poverty and poor communication and connectivity issues mean that educational ideas that have gained wide support internationally are slower to gain acceptance in Lesotho (Urwick and Griffin, 2012). Lesotho is a close-knit, traditional, rural society, where national pride and respect for elders are important values. Recently, there have been changes in how humanitarian aid is conceived, (Brock, 2012). The CGDE funding which facilitated this study is an example of Irish aid investment in higher education. The DelPhHE project, which engaged in teacher education professional development, is an example of aid from the UK towards training of local personnel. Developing status means that recipient countries accept the aid that donors give rather than choose the aid which best meets the local needs. Like all 'boundary objects' (Corcoran, 2009), mathematical representations, like the empty number line and arrays of circles will need time and sustained use if they are to become accepted in the everyday practice of primary school teachers.

Language as Context

Language is a very important aspect of teaching and learning mathematics. Sesotho is the home language of all learners. Lessons are taught through Sesotho in standards one, two and three. English is the language of education from standard four upwards and is the language of mathematics. There may be a need when teaching Sesotho learners for even more conscious code switching than is evident in these lessons. It might also be even more important for teacher educators to work together with student teachers to develop a wide bank of key mathematical terms for use in teaching primary mathematics, where particular terms and definitions may be an extra barrier to learners working in a second language. One participant, Nomsa spoke about the bi-lingual context:

> I think using both languages helped a lot because I happened to ask my learners a question in English and they seemed not getting my question at all. So that is why I decided to use both languages so that they can cope with what I was doing with them, some did not understand at all when I used English only.

The study of meaning making through language (semiotics) is intimately connected with mathematics learning. Therefore it follows that language, as a means of communication is an important aspect of mathematics education. Mathematical skills such as questioning, explaining, conjecturing, justifying and generalising have to become part of the practice of mathematics in both languages.

Representing Ideas

Mathematics is a human activity, which has grown in depth and complexity over centuries. Researchers have used an acquisition metaphor and a participation metaphor to try to understand how mathematics is learned. Both ideas are needed and have implications for teaching (Sfard, 1997). Essentially, mathematics is about ideas and the teacher's job might be considered to be to communicate these ideas to learners. This can be achieved through a blend of explaining and providing opportunities for doing mathematics. Well-chosen representations may help build understanding, but the mathematics is not in the representation itself. Sebu's representations of sets and subsets are an example of this. So are the tasks in section two of the survey instrument. Modern research suggests that traditional 'chalk and talk,' teacher telling is not the most effective way to teach mathematics to children, since learners need to invent and (re)invent the mathematics for themselves. Commenting on the need to have pupils do written work, Tšeli said:

> It is important because I am going to see where they are not getting the concept because if I only use the chalk board, I will only go along with the ones that are fast learners and the slow learners will be left behind, now if I mark the books I will see those mistakes I did in the lesson.

This appears to be an honest assessment of the limitations of traditional 'chalk board' teaching. Tšeli's lesson on multiplication facts was aimed to teach multiplication facts but she started by multiplying money in order to draw them into the lesson. This appeared to me to be a false context, because she didn't use improvised money, only symbols.

Context for Mathematizing

Many researchers agree that a realistic context is a good starting point from which learners can progress in mathematics. The RME movement has produced research in Holland and in the USA that supports the claim that 'mathematizing' is a good basis for learning mathematics (Dekker, 2007; Fosnot & Dolk, 2001). There is evidence that all of the student teachers attempted to base their lessons in reality. In Litha's own words,

> I wanted to bring reality into them, I did not want to talk about things that are outside the class that they could not see or use in measuring. The tools that were found in the classroom were readily found.

Thandi was the participant who was probably most successful in creating a context for mathematizing. However, creating a mathematical context is not enough. Teachers need to plan rich mathematical contexts which interest and challenge children, and pose problems related to their experience. It is important to note that the item in the survey instrument Section 2 that had the highest number of fully correct responses was Task 6. This was a routine textbook problem that respondents were asked to solve and make a representation to explain their solution to a child. There was a greater than 96% correct response rate to this problem. I argue that this task was set in a realistic context even though it was not a task of high cognitive demand. In contrast, the task with the lowest response rate of correct answers (hand-shake problem, Task 7) was also set in a realistic context but less than 4% of participants could solve it correctly. It raises at least two issues. The teaching of mathematics requires providing learners with opportunities to develop strategies for problem solving. I argue that such teaching is not common in Lesotho, even among students who are considered to have been strong in mathematics in high school. Litha is such a student. His response to Task 7 suggests that he is in a ZPD where the right question or comment from a teacher or a fellow student could lead him to a correct solution. Student teachers need to learn how to recognize ZPDs in their pupils' work. First, I argue they must learn about the common strategies for structuring mathematical ideas that learners use, for example in performing operations on number (Fosnot et al, 2001; Fosnot and Dolk, 2001). It has been observed that a number of student teachers had used learners' fingers in one way or another. When exploring factors that influenced their choice of this representation it was found that student teachers chose to use learners' fingers because of the following reasons:

- they would help learners to get answers quickly;
- they would provide efficient ways of calculating multiples of numbers;
- learners' fingers are considered readily available resources;
- they would serve as non-standard units for measurement and estimation.

These might be considered strategies for helping calculation but they are not a substitute for mathematical thinking.

From Representations to Representing

I started out on this research project with a conviction that the frequent and effective use of enactive, iconic, and symbolic representations by teachers can help learners to participate more fully in learning mathematics and thus develop richer and more solid understanding of abstract mathematical concepts and processes. I wanted to find out about student teachers in Lesotho in relation to this topic of representations and how they could be helped to develop their use of representations in making mathematical ideas accessible to learners. I remain convinced that enactive, iconic, and symbolic representations are essential in learning mathematics, but I am aware of a shift in my thinking. Teachers can help learners to participate more fully in learning mathematics by starting with mathematically rich, everyday contexts and helping learners to build mathematical ideas. Learners can be encouraged to make their own representations and to discuss their work. Occasionally, the teacher will introduce powerful mathematical representations which learners will be encouraged to use and develop as they try to solve problems. The shift is from a static notion of 'representation' to a dynamic notion of 'representing', which can be a collective or an individual attempt at doing mathematics and problem-solving.

Conclusion

The findings in these two chapters show that student teachers were creative in choosing and using various representations in order to help learners to understand mathematics. Some informal representations included learners' fingers, finger rulers, hand rulers and fake money. Student teachers also used formal representations (multiplication square) that have been promoted in mathematics education courses at the college.

Litha's lesson differs from the other four lessons not only because it was taught by a male teacher but that it was in Standard 6 and was based on a different mathematics topic (measurement). However, all five lessons are similar in that each student teacher used at least one form of representation in an attempt to facilitate learning. There was variety in the contexts chosen for basing the mathematics lessons and it appears that there was greater success in teaching the mathematics where students identified with and were interested in the contexts chosen.

The findings also indicate that student teachers tended from time to time, to make reference to objects that are easily available around and within the classroom vicinity, such as the fruit, a book-locker, trees, and stones. I argue that this shows that student teachers are already beginning to be innovative in their teaching of mathematics through improvising when need arises. All the five student teachers made effort to make mathematics accessible to learners through the choice and use of multiple representations in their lessons. The teaching of mathematics is a demanding task that requires effort on the part of a teacher. This challenge is even more serious for novice teachers. In my opinion, cooperating teachers might play an important role in assisting student teachers with the teaching of mathematical ideas. This is a role that could be formalised and supported by sustained in-service.

Student teachers' choice of representations for use in mathematics lessons is influenced by various factors, possibly including the cooperating teacher. Some student teachers mentioned that 'knowledge of learners' influenced them when planning for lessons to choose teaching aids that potentially would encourage learners to participate in lessons. Student teachers' knowledge of learners' socio-economic background influences them to use locally available material to construct teaching aids, such as fake money and the multiplication square for use in mathematics lessons to develop conceptual understanding (Kilpatrick, Swafford, and Findel, 2001) of number operations. Knowledge of learners' social context played an important role in student teachers' choice of representations. For instance, knowing that in Lesotho learners are familiar with buying and selling fruit, one student teacher (Thandi) used this as her context in class to set a learning scene, while another student teacher drew pictures of fruits on the chart (Nomsa). In particular, Nomsa used a worksheet because it was relevant to the content she was going to teach prior to the use of the worksheet. Thandi also chose the task from the textbook because it was relevant to what she was going to teach.

Student teachers' determination to link mathematics to reality was also noted as one of the factors that influenced them to choose certain teaching aids. Some had thought carefully about their lessons and had chosen teaching aids that would help them to do successful demonstrations. All five students chose to use learners' home language (Sesotho) as a resource during teaching to support learners' understanding of mathematics concepts.

8. Conclusions and Implications for Practice

This study was based on the premise that the 'work of teaching' mathematics includes amongst other things, a teacher's ability to choose appropriate representations and use them accurately in a mathematics lesson (Hill, Rowan, and Ball, 2005). So the aim was first, to explore student teachers' understanding of mathematical representations, and second, to investigate student teachers' choice and use of representations in teaching. The aim therefore suggested two levels of explorative work. First it meant working with the participants to establish the meanings they attributed to certain mathematical representations associated with number operations, and second, following them when they were on teaching practice to investigate their choice and use of representations in teaching mathematics in primary schools in Lesotho. Therefore, a four-tiered research design was employed in this study in order to meet all the requirements mandated by the aim of this project. It was also important to focus on one cohort of students from their entry to the college, and this resulted in "prolonged engagement" with this cohort over a period of three years (Lincoln and Guba, 1985, p. 328). Student teachers' understanding of representations was explored by means of mathematics tasks and semi-structured interviews, and this work was covered in tiers 1 and 2. Student teachers' choice and use of representations in teaching were explored through lesson observations and video-stimulated recall interviews, which is the work that was covered in tiers 3 and 4 respectively.

Tiered Design of Study

While there are numerous ways of exploring students' understanding of mathematical ideas, simple tasks associated with early number operations were chosen in the first tier as the most appropriate method because by analysing students' responses I would be able to determine their level of familiarity with and understanding of such representations. The iconic representations used in the survey instrument were taken from work-sheets recommended for use in teaching number in Lesotho by UK researchers (DelPHE project). My decision to use these tasks as a form of survey instrument in Tier 1 was influenced by their availability and by the work of Stein, Grover, and Henningsen (1996) on the use of tasks in mathematics instruction. When they advocated for the importance of students' development of deep and interconnected understanding of mathematical ideas they pointed out that:

> Although more representations do not necessarily lead to greater understanding numerous cognitive advantages associated with the establishment of links among various ways of representing a problem have been proposed (p. 472).

It was my hope that I would get a picture of the participants' manner of establishing links among various ways of representing common operations on number.

I used a four-tiered approach because each tier was meant to achieve a particular goal. In order to supplement students' work on the tasks, I engaged them in semi-structured interviews where they would talk through their work in Tier 1. These interviews were transcribed and a copy was returned to each participant to verify that they were a record of what was said. Tier 3 happened when student teachers were on teaching practice. Each of five student teachers was observed teaching a mathematics lesson. The purpose in this tier was to explore students' use of representations in teaching. Each lesson was video-recorded and transcribed. I invited a team of colleagues to check my translation of Sesotho words and phrases used in the lessons. Each participant was given a copy of the DVD of the lesson. I later interviewed participants on how they each chose and used representations in teaching. Again, these interviews were transcribed and a copy was returned to each participant to verify that they were a record of what was said.

Based on the belief that students' understanding of mathematics can be determined by considering the connections (links) that they make as they solve mathematics tasks, I analysed the participants' responses to tasks. The findings emerging from tiers 1 and 2 data show that participants in this study were able to make sense of the routine tasks and the accompanying diagrams with which they were familiar. While in most cases they managed to make connections between operations and accompanying diagrams, where the symbolic representation was absent the relevance of such diagrams was not so easily recognised. This was perhaps due to the fact that while the tasks involved simple operations, the structure of accompanying diagrams was unfamiliar to the participants. For the tasks that required participants to construct their own diagrams, student teachers creatively produced interesting and helpful diagrams that influenced them to think of the ways that lead to either correct or sometimes 'incomplete' answers.

In situations where tasks could be solved by following a known algorithm, students comfortably obtained correct answers. On the other hand, respondents found the 'hand shake' problem extremely challenging and therefore they were much less successful in their attempts to solve it. It was also noted that some students whose performance in mathematics in Form E was poor were able to obtain correct answers for the 'hand shake' task. This is perhaps due to the fact that this task could not be solved following a standard algorithm, and therefore students had to 'invent' for themselves of ways and means that would possibly lead them to correct answers. It then follows that during training at the college student teachers might need to be given opportunities to work on many unfamiliar tasks like the 'hand shake' problem in order for them to develop problem solving skills.

Tiers 3 and 4

Student teachers' choice and use of appropriate representations for teaching a particular mathematics concept is considered to be an indication of their 'pedagogical subject knowledge' (PCK) (Shulman, 1987). This means that the ability to choose a suitable teaching material during the lesson planning phase for a particular mathematics concept depends hugely on the teacher's own knowledge of that concept and his/her beliefs regarding the ways in which such concept/process could be taught effectively (Rowland, Turner, Thwaites, and Huckstep, 2009). Again, how well and effectively the chosen teaching tools are used during the lesson depends on the teacher's 'foundational knowledge' of the mathematical concept in question (Rowland et al, ibid.). The key dimension of the Knowledge Quartet (KQ) used in this study was 'transformation' which deals with the choice and use of representations, examples and demonstrations in a mathematics lesson. According to Rowland et al, the four intertwined dimensions of the KQ are useful in helping a researcher to focus on mathematical issues in class rather than on what they call 'generic issues'. Rowland et al (2009, p. 12) argue that "generic issues include, for example, those aspects of the management of behaviour in the lesson, general aspects of the management of learning, general assessment frameworks, and so on" in which this researcher was less interested.

The findings that emerge from tiers 3 and 4 indicate that student teachers who chose appropriate representations and used them effectively in lessons had a better knowledge of the mathematical content they were teaching and their choice was influenced by their knowledge of students and context. It was also found that the student teacher that used many representations in her lesson to teach 'sets' and 'sub-sets' was less successful in translating her inadequate knowledge of 'sets' into something comprehensible to learners. On the basis of this lesson, it seemed reasonable to argue that it is not about the number of tools used in lesson that makes the teaching of a mathematical concept meaningful but it is the effective manner in which one or two representations could be used that might make learners understand what is intended for that lesson. These findings are in line with the conclusion reached by Stein, Grover, and Henningsen (1996, p. 474) that "more representations do not necessarily translate into deeper understanding. The crucial factor is whether and how representations become connected or linked to one another". More important is the opportunities provided in the lesson for discussion of the meaning learners ascribe to the representations used and the manner in which the representations relate to students experience. This implies that when representations are chosen for use in mathematics lessons, teachers have to think carefully on how the use of each representation might open up the concept or concepts to learners and help them to think of, and use the mathematics in different ways. In order to organise the summary of findings in this chapter, I use the research questions raised in the first chapter as headings in the following section. I also present the implications for school curriculum development and teacher education in Lesotho. I finally address the limitations of this study.

Summary of the Findings

What mathematical representations (if any) do student teachers in Lesotho associate with particular mathematical processes and symbols or use when solving mathematics tasks?

The findings obtained from the analysis of the data in tiers 1 and 2 suggest that at the stage of entry into the DEP programme, most students were comfortable with tasks that clearly required the use of known algorithms and they experienced difficulties in working out correct answers for an unfamiliar task (hand shake task). Perhaps as a result of their experiences of learning school mathematics, participants tended to stick to algorithms for working out solutions to tasks in tier 1 rather than exploring possibilities with the accompanying mathematical representations. A few participants, who got the correct answer for the hand shake task, used the real life context of church experience of handshaking to solve that more cognitively high-demand task (Stein et al, 1996). It was also observed that for tasks that required participants to use their own self-constructed representations, participants drew useful tables, pictures and diagrams and used those to make meaningful connections between symbolic and iconic representations. Other findings show that some participants in Tier 2 experienced difficulties with the use of the number line for addition and subtraction of whole numbers despite the fact that at the time of the interview they had already learned in DEP 1 mathematics courses how an 'empty number line' might be used flexibly for the same purpose. Therefore, this suggests that mathematics lecturers at the college might have to invest more time during the first year of DEP programme to help student teachers to master the ways of using a number line effectively as a thinking tool in teaching and learning of mathematics. It also suggests that student teachers and other learners need more opportunities to solve rich, non-routine problems. In tasks where participants were given opportunities to construct their own representations they displayed creativity and imagination in doing so. This suggests that during teacher training these important skills need to be nurtured, perhaps by engaging student teachers in solving a considerable number of this kind of tasks. In order to improve student teachers' problem solving skills rich non-routine tasks, in my opinion, need to be used frequently in mathematics lectures. In that way student teachers might be empowered to work systematically, to recognise and develop strategies, to construct multiple representations such as tables, diagrams, and graphs to help them think about and solve mathematical tasks. In the Lesotho context, this would necessitate a big change in beliefs about what it means to understand and use mathematics. It would have implications for mathematics teaching and teacher education.

Finally, I found Mason's typology somewhat useful in analysing data in tiers 1 and 2. The typology provided a theoretical window through which I studied participants' responses gathered by means of tasks and interviews. It also suggested the teacher's role in directing learners' attention to important mathematical properties. This underlines the fact that mathematics is a cultural and social activity, where 'correct' or 'incorrect' might be the wrong signifier because learners can usually solve something with help from another that they could not solve on their own (ZPD).

How do student teachers on teaching practice use mathematical representations in lessons?

The analysis of the five lessons taught by student teachers on teaching practice suggests that although the students had undergone the same college training, their ability to choose and use representations for teaching mathematics varies. All the five student teachers demonstrated their willingness to teach mathematics in an exciting and meaningful way through the use of multiple representations. When on teaching practice, the observed participants used various representations in teaching school mathematics. The cultural approach to mathematics teaching was mostly, teacher demonstration followed by whole class chorus repetition. Student teachers who appeared to have used the chosen representations effectively in their lessons were those who had made reference to the prescribed textbooks during planning of the lesson. As discussed in Chapter 7, some of such student teachers used examples taken directly from textbooks. However, during the teaching process instead of adhering totally to the textbook they appeared to have made these examples and demonstrations their own, which is a move that manifests the student teacher's foundational knowledge (Rowland et al, 2009).

Some student teachers placed teaching aids in front of class such as a chart, a multiplication square, and coloured cubes in order to teach concepts like multiplication and subsets. These student teachers made efforts to demonstrate with some examples in order to make mathematics concepts comprehensible to learners. For instance, Nomsa used examples of groups of fruit to demonstrate to learners that 'multiplication is repeated addition'. Tšeli made demonstrations for learners on how to work out multiples of numbers on the multiplication square. Sebu too used coloured cubes and some drawings on the chalkboard in her attempt to demonstrate to learners how subsets are formed.

Choice of Mathematical Representations

There are multiple factors that influence student teachers' choice of the representations they use in teaching. Some student teachers have accurate knowledge of learners and in their planning of the lessons choose representations that encourage learners to participate in lessons. Student teachers' knowledge of learners' socio-economic background influences them to use locally available material to construct teaching aids, such as fake money and the multiplication square to develop conceptual understanding (Kilpatrick, Swafford, and Findel, 2001). The findings reveal that the knowledge of learners' context played an important role in student teachers' choice of representations. For instance, knowing that in Lesotho learners are familiar with buying and selling of fruit, one student teacher used this as her context in class to set a learning scene, while another student teacher drew pictures of fruits on the chart. It was also observed that some choices were influenced by relevance of the teaching aid to the concept to be taught. For example, Nomsa used a worksheet because as she puts it, it was relevant to the mathematics she had planned to teach. Thandi also chose a task from the textbook because it was relevant to what she was going to teach. Other findings show that student teachers' determination to link mathematics to reality was another factor that influenced some of them to choose certain teaching aids, such as trees, stones, book-locker, shovel, and fruit. All five student teachers chose to use learners' home language (Sesotho) as a resource during teaching to support learners' understanding of mathematics concepts.

Profiles of Students

Another lesson that I learned from carrying out this study has to do with the profile of candidates of the DEP programme. A majority of candidates who register for DEP programme have weak mathematical background. When carrying out the analysis of participants' Form E mathematics results, it was found that most of them had failed mathematics. It was also found that the participants in this study had bad experiences of school mathematics. Participants' experiences of school mathematics led them to considering mathematics in general as a subject that is *highly challenging* and that *requires the knowledge of operations, numbers, figures, symbols, and formulae*. This suggests that the participants in this study had only experienced mathematics as a subject that involved procedures that were hard to learn and master. It was found in this study that none of the participants thought of mathematics as a means and a tool through which problems are studied and solved. It is therefore, logical to conclude that this is a result of the way school mathematics is presented to learners (as a set of rules to be memorised and recalled). Knowledge of participants' mathematical backgrounds helped me to have a clearer picture of the calibre and the needs of students I am working with. Participants' diverse mathematical attainment in Form E mathematics enabled me to interpret their work on the use of representations, with an understanding that in general they had weak mathematical knowledge as measured on COSC examinations. As a lecturer who has the responsibility of preparing DEP student teachers to become competent and proficient mathematics teachers at primary school level, I realize that I have to teach mathematics differently to these students from how they were taught school mathematics. This PhD project helped me to realize that in my teaching I have to focus more on mathematical problems that create opportunities for student teachers to use multiple representations in finding solutions to non-routine tasks. I also have to make an effort to present mathematical knowledge for teaching (MKT) to student teachers in a meaningful, painless and exciting way, through the deliberate and explicit use of appropriate mathematical representations as the theme that cuts across all mathematics courses offered to DEP students at the college. The potential of RME as a means for teaching number concepts by building on everyday mathematical contexts familiar to students is worth exploring and developing in Lesotho.

In this study, it was found that DEP 1 student teachers have difficulty in understanding the use of a *number line* for performing mathematical operations. This was noticed in Tier 1, when participants were working on tasks and in Tier 2 during the semi-structured interviews. I would therefore recommend that more time be spent on exploring with student teachers the ways in which a *number line* might be used, so that at the end of the teacher education program, student teachers would be proficient with the use of an "empty number line" (Klein, Beishuizen, and Treffers, 1998). According to Klein et al (1998) the empty number line helps learners to think flexibly about numbers.

This study has also shown that student teachers' social contexts (such as church situation) play a crucial role in helping them to produce necessary diagrams for solving unfamiliar mathematics problems, like the *hand-shake* task. The context in this case became a stimulus that triggered students' thinking and helped them to construct appropriate diagrams for solving the problem. This suggests that as an educator, I will have to think carefully about the choice and use of contexts that might encourage student teachers to construct representations for solving problems. Boaler (2009) takes a view that contexts must be used 'sensibly and responsibly'. She admits that "contexts may be also used to give a visual representation, helping to convey meaning" (p. 48). The analysis of the student teachers' lessons has revealed that some of them used various contexts such as *selling and buying of fruit* sensibly, in order to make mathematics concepts and procedures (addition and subtraction of whole numbers and decimals) accessible to learners. RME researchers claim that the mathematics is in the context itself and learners learn to use mathematics as they try to organise and make sense of rich contexts.

Trustworthiness and Transferability

Just like any other researcher, I worked hard to ensure that my study was ethical in every step, and addressed the research questions carefully. However, as a scholar I take it to be my primary responsibility to be critical of my own work before giving it to others for appraisal. First of all, I observe the fact that the findings that emerged from this study cannot be generalised to other situations, because by its nature a case study is not interested in generalisations. Rather it aims to provide a detailed analysis and "thick description" of the phenomenon under study. In this study, I worked with prospective teachers who came straight from high school, and were registered for a diploma programme. I acknowledge that this cohort of prospective teachers is unique in its own right; hence their performance on the tasks used in tier 1 and the manner in which they chose and used representations in lessons in tier 3 is specific to this group. While the 212 respondents to the survey instrument were fairly representative of the entire cohort, by age, gender, home district and achievement in mathematics, the five students who participated in tiers 3 and 4, were younger and from only five of the home districts. Therefore, I cannot conclude that the findings obtained from working with this cohort are typical of all prospective teachers in Lesotho or beyond. In the words of Lincoln and Guba (1985, p. 329) my study "can at best persuade" readers that I have uncovered current understandings and uses of mathematical representations for teaching early number concepts among this cohort of pre-service primary school teachers in Lesotho.

The Knowledge Quartet and Teacher Development

The Knowledge Quartet (KQ) as the analytical framework is useful in directing researcher's eye to mathematics content taught during the lesson. While the four dimensions of the KQ are intertwined, *foundation* appeared to be central in determining student teachers' choice and use of mathematical representations. Student teachers with fairly considerable mathematical knowledge were able to choose appropriate representations for lessons. They were also able to use the chosen representations effectively in lessons to make mathematical concepts accessible to learners. Again, student teachers that appeared knowledgeable about what they were teaching also displayed ability to make use of learners' contributions (*contingency*) during lessons. This also applies to the *connection* dimension. Student teachers with reasonably good foundation in mathematics (e.g. Thandi) were found to link concepts well and with greater ease. Corcoran (2008), when working with (prospective) teachers in Ireland found that contingency opportunities that present themselves in a classroom, and the collaborative reflection on lessons with the student teachers helped participants to develop other dimensions of the Knowledge Quartet.

Given that only the transformation dimension of the KQ was used in this study only for the purposes of analysis of student teachers' mathematics lessons, I recommend that it might be helpful to employ it in the future in the DEP programme at the Lesotho College of Education as the teacher development model. I suggest that student teachers be introduced first to the model's (KQ) four dimensions (foundation, transformation, connection, and contingency) as part of the pedagogical courses during year 1 of their study. Student teachers could then plan lessons together on a given mathematics topic (e.g. Number). In planning lessons student teachers could be encouraged to bear in mind the four dimensions of the KQ and consciously choose representations that are more likely to make the concepts comprehensible to learners. Student teachers could also discuss how they might make connections between a series of mathematics concepts, procedures and practices. While it is difficult to plan for contingent moments, it could also be useful at this stage for student teachers to think of possible reactions from learners and agree on how they would deal with such moments. In this way, student teachers might be empowered through the use of the KQ to become proficient mathematics teachers (Rowland, Turner, Thwaites, and Huckstep, 2009). A similar model for teacher development based on promoting and studying children's mathematizing strategies is recommended for use in the US by Fosnot and Dolk (2001)

Facilitating Mathematics Teacher Development

Once student teachers have collectively completed their lesson plans, the mathematics lecturer concerned might create a platform where students present their lessons. After every lesson, student teachers might be encouraged to learn to give feedback to their colleague who presented the lesson. The feedback should be structured to pinpoint moments of each of the four dimensions of the KQ. I suggest the inclusion of the KQ in the DEP mathematics programme because of my conviction that it might help student teachers to be more prepared for teaching practice than they are at the moment. The inclusion of the KQ into the DEP mathematics programme has potential to equip student teachers with mathematical knowledge and skills for teaching. With its strong attribute of focusing the eye of a lesson-observer more on mathematics content taught, than on classroom generic issues, the KQ is an ideal teacher development model to be incorporated within teacher education programs. I consider this model (KQ) to be a good replacement for the micro teaching model, which has been phased out at the Lesotho College of Education LCE). As part of their training, student teachers have to learn experientially how to teach some if not all, mathematics content before they are sent to schools. In the past the micro teaching model served this purpose at LCE but was dropped when the college phased out certificate programmes and introduced diplomas. Recent research in teacher education for professional practice emphasises a need for participation in "pedagogies of enactment" (Grossman, Hammerness, and McDonald, 2009). Implementing the KQ framework in this manner would constitute pedagogy of enactment.

The Lessons Learned and the Way Forward

My understandings of representations and teaching have shifted over the years of the study. The structured teaching and learning of mathematics through representations happens within social contexts such as a school. Learners interact and construct the meaning of mathematical concepts cognitively at individual level and socially at the collective level. The construction of meaning occurs in the interaction. Representations play an important role in lessons in helping teachers to communicate mathematics ideas to learners and also in making mathematics concepts and procedures accessible to learners. Representations as thinking tools help both a teacher and learners to think and reason mathematically in lessons. Learners' self-constructed mathematical representations are valuable thinking tools and to be encouraged and shared. Student teachers' foundational knowledge of mathematics too plays a central role in choosing and using of representations in teaching. The student teachers that appeared to have a secure knowledge of the mathematical concepts they were going to teach also appeared to have chosen appropriate teaching tools and used them effectively in lessons. It might be argued that to a large extent it is this knowledge combined with the knowledge of learners that influences student teachers' choice of representations during lesson preparation and their ability to use such representations effectively during the lesson. It was also found that student teachers that appeared comfortable with the mathematics they were teaching also managed to deal well with learners' contributions. Reference is here made to Nomsa's and Litha's cases.

Ethical Issues

My involvement in this study where I worked with prospective primary school teachers in Lesotho, a less developed country, taught me important lessons. One of the lessons learned involved issues of ethics. It is the responsibility of all social studies researchers to conduct ethically sound studies. However, it was extremely challenging for me to obtain ethical clearance from all the institutions involved in Lesotho. The institutions referred to here are the Lesotho College of Education and the five primary schools in the Maseru district, where student teachers in Tier 3 practised teaching. In the absence of an Ethics Committee to oversee and monitor research processes, I found it difficult to gain entry to research sites in Lesotho. St Patrick's College Research Committee was the body that granted me ethical clearance.

Limitations of the Study

I acknowledge that there are some limitations to this study. The purposive sample (Tier 2) did not contain any representatives from the five more remote highlands regions. This may have skewed the findings somewhat, since disparity in primary education in Lesotho is said to relate among other things to location within the country (Urwick and Griffin, 2012). Second, the convenience sample (Tier 3) did not contain any participant above 25 years. That means that student teachers that may have had previous teaching experience were omitted.

I was not confident or experienced enough as an interviewer to conduct clinical interviews with students in tier 2. Instead, I accepted answers without probing more deeply. I now believe there was more to be learned about participants' understanding of representations by treating many answers in tier 2 as 'zones of proximal development'. My focus on tier 4 was also limited to why participants had chosen to use particular representations in the lessons. I now believe there was more to be learned in some cases by exploring the participant's personal responses to the lesson itself and to the meaning they attributed to each teaching activity.

One of the challenges I had to deal with was an issue of time. I was sponsored to carry out this study within a given timeframe, at a time when the world had been hit by an economic recession. Against a background of increasing austerity in the donor country, the project work had to be completed within the given time (2009 – 2012). Because of this, I could not include other components that I initially deemed important to the study. Such components would have included: exploring primary school learners' use of self-constructed representations in doing mathematics tasks; exploring the impact of first year mathematics courses on DEP 1 students; and an evaluation of local instructional documents with the main focus on representations. These are the issues that I plan to research further within the context of Lesotho after this project. I also plan to investigate how experienced and successful elementary school teachers choose and use mathematical representations in instruction.

Possibilities for Further Research

While this study has shed light on the notion of student teachers' choice and use of representations in teaching elementary school mathematics in Lesotho, there is room for further research on this topic. There is a need to follow up on the five student teachers when they begin teaching, in order to find out how they develop as teachers of mathematics as they broaden their experience and use representations to teach various topics. It would also be important to investigate the manner in which experienced primary school teachers choose and use multiple representations in teaching. Given the fact that different mathematics topics call for different representations, it is also crucial to conduct case studies that focus on other mathematics topics. The use of the Knowledge Quartet in researching and theorising the teaching of mathematics at primary school level in Lesotho needs further exploration. Both novice and experienced elementary school teachers could be encouraged to engage in action research where they employ the KQ as a framework for improving their own practice. The role of the co-operating teacher in training student teachers on TP is very important and not often highlighted. Again, given that the ministry of education and training (MoET) of the government of Lesotho is already making efforts to provide in-service workshops for primary school teachers, in order to improve their teaching skills of English and Mathematics, I realise that my future contribution in this could be to work with the TP co-operating teachers and District Resource Teachers (DRTs) to introduce the KQ and to explore with teachers how it could be used to enhance their teaching of mathematics.

Conclusion

Over the three years of data collection, I developed deep respect for the participants in my study, especially the four young women and one man who took part in tiers 3 and 4. I single out these five participants because of their agreeing to participate further in my research meant there were more opportunities for me to get to know them better and learn about their choice and use of mathematical representations. In tiers 3 and 4, I observed these student teachers as they began to "craft identities" as teachers of primary mathematics (Corcoran, 2011). As the research progressed into Tier 4, most of the five participants were already referring themselves as mathematics teachers and were well able to articulate their thinking about mathematics teaching and were beginning to reflect on the consequences of their actions. This suggests that the student teachers at this stage might have gained confidence to teach mathematics and perhaps their beliefs about mathematics 'as a challenging subject' have also begun to change.

At this juncture, I would like to cite a quotation derived from one of the participants in the study. The quotation is intended to show how the participants in this study felt about mathematics for teaching at the end of the project.

> I think, for mathematics to be taught well at primary school level, we need to use either the drawings or the textbook or materials for teaching at primary level as our teaching aids, the pictures drawn on charts, the posters, and the concrete materials whereby pupils can work by touching them. I think they can understand the mathematical concepts better (Nomsa).

This realisation expressed by Nomsa acts as a challenge to me and my colleagues in the LCE to work hard with student teachers to provide mathematically rich learning environments, to encourage and direct their structures of attention towards efficient and elegant strategies for working mathematically and to strive for maximum shared understanding in mathematics education workshops.

In order to conclude my story, I reflect back and briefly narrate the whole trajectory relating to this project. It all began with my participating in the DelPHE project – the numeracy strand which focussed on improving mathematics courses taken by students who train to become primary school teachers. At the time I shared a view that numerous representations introduced to student teachers during training were wonderful materials that would overcome the problems such as the ones raised in the needs analysis report (Lesotho College of Education, 2006). However, as the needs analysis report indicated mathematics teaching remains "challenging" in Lesotho. In observing that group work was not well understood and little used, my findings agree with the needs analysis report. I learned from my interpretative study that while standard representations can be useful thinking tools, mathematics does not reside in them and as such, these representations cannot automatically make mathematics accessible to learners. The complexity of choosing and using appropriate representational materials needs to be understood within cultural factors such as the language (Sesotho), and a socio-cultural approach to mathematics teaching where mathematical thinking is developed in the context of learners' life experiences. This study indicates that such creative mathematical thinking may be rare but is certainly possible. With sensitive teacher support and a focus in lessons on progressing learners' thinking, self-constructed representations and problem solving skills, such as those displayed by Beatrice, Mosha, Nomsa and Thandi could become mainstream.

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APPENDICES

Appendix 1: Standard 7 Examination Paper (2009)

1. 682 + 316 =

A 998
B 978
C 888
D 878

2. Which of the numbers is the smallest?

A 8909
B 8900
C 8109
D 8099

3. What is the sum of 14 and 2?

A 28
B 16
C 12
D 7

4. Round off 58.48 to the nearest whole number.

A 57
B 58
C 59
D 60

5. 5^3 is the same as which of the numbers?

A 5 + 5 + 5
B 5 x 5 x 5
C 3 + 3 + 3 + 3 + 3
D 3 x 3 x 3

6. What is the lowest common multiple of 3, 4 and 5?

A 15
B 20
C 30
D 60

7. Mpho wants to take the biggest share of a cake. What fraction of the cake should she take?

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{1}{5}$

8. The diagram below shows the picture of a school jersey. How many lines of symmetry does it have?

A 1
B 2
C 3
D 4

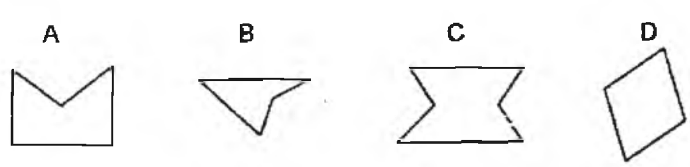
9. 64 x 8 =

A 64 + 8
B 8 + 64
C 8 x 64
D 64 – 8

10. The number "Two hundred thousand and four" written in figures is ....

A 2004
B 20004
C 200004
D 200 000 4

11. Which of the following shapes is a pentagon?



12. What is ten percent of 50?

A 5
B 0.5
C 0.05
D 0.005

13. What is 0.5 kilograms in grams?

A 50g
*B 500g
C 5000g
D 50000g

Appendix 2: Plain Language Statement

ST PATRICK'S COLLEGE DRUMCONDRA

Introduction to the Research Study

This study is aimed at exploring prospective primary school teachers' understanding, choice, and use of mathematical representations. This is a case study with a cohort of student teachers registered for the Diploma in Education Primary (DEP) (2009 - 2011) at the Lesotho College of Education.

Principal Researcher: Nkosinathi Mpalami, Lecturer in Mathematics Education at the Lesotho College of Education, Maseru, Lesotho.

Contacts: +266 5884 2058 (phone),
nkosinathi.mpalami2@mail.dcu.ie

Details of what involvement in the Research Study will require

The proposed project is a four-tiered study with student teachers. The whole cohort of first year DEP students is invited to take part in Tier 1. The main research instrument in Tier 1 is a survey instrument that consists of tasks that are aimed at unpacking participants' understanding of mathematical representations. The instrument is also aimed at gathering background information of participants. Such information will help the researcher to make further sampling of participants for tiers 2, 3, and 4. In Tier 2, a sample of about 10 students will be interviewed and such participants will be invited to take part in reflective meetings to discuss their answers to the initial survey instrument. In Tier 3, a small self-selecting number of participants will be observed teaching a mathematics lesson while on teaching practice (TP). This will be observed by the researcher with and aid of video recorder. Finally in Tier 4 participants will be shown carefully selected clips of their lessons and be interviewed individually.

The maximum time needed for the completion of the survey instrument in Tier 1 is about 50 minutes. Each semi-structured interview in Tier 2 will last less than 30 minutes. Given the official time allocated for each lesson in Lesotho primary schools (30 minutes/lesson for standards 1, 2, and 3 and 40 minutes/lesson for standards 4, 5, 6, and 7) student teachers' participation (teaching a lesson) in Tier 3 will take a maximum of 50 minutes. The envisaged video stimulated recall interviews in Tier 4 will take a maximum time of 30 minutes.

There is no potential risk to student teachers. All DEP students are over 18 years and therefore regarded legally as adults. All are invited to participate in this project. They are all free to give or withhold consent.

Student teachers participating in this project will benefit from experience they will gain from doing mathematics tasks given in Tier 1. Participants will have an opportunity to reflect on their work (tasks in Tier 1) in Tier 2 during the semi-structured interviews, which will contribute toward their mathematical knowledge. Participants in tiers 3 and 4 will be helped to reflect on and talk about their respective lessons taught during teaching practice session. This experience will only be gained by participants in this study.

Participants' real names will not be used in any part of this study. Pseudonyms will be used whenever reference is made to a specific individual participant. School names will not be used either so that participants' identities are concealed from readers. Where I use video-clips for conferences or for teaching in the future, I will only do so with the express permission of the participant in the clip and will make every effort to disguise the faces electronically.

All data in the form of audio and video recordings will be destroyed by fire after 5 years upon completion of this study, as will all e-data and completed survey instruments. This will be done to prevent loss or misplacement of such documents.

Student teachers will be asked to sign up for Tier 1 only at the beginning of the project. Participants in the subsequent tiers will be self-selecting and keen to participate students will be asked to contact me. Participation in this study is entirely voluntary. Participants are free to withdraw from the study at any point and there will be no penalty for withdrawing before all stages of the study are completed.

All student teachers participating in this study can be assured that I will not be grading their work in any form while at the college and when on teaching practice.

If participants have concerns about this study and wish to contact an independent person, please contact:

The Administrator,

Office of the Dean of Research and Humanities,

St Patrick's College, Drumcondra,

Dublin 9.

Tel 01-884 2149

Signed: Nkosinathi Mpalami (Researcher)

Appendix 3: Informed Consent Form

ST PATRICK'S COLLEGE DRUMCONDRA

Research Study Title:

Understanding and using mathematical representations: A case study of prospective primary school teachers in Lesotho

Purpose of the Research:

The purpose of this study is to explore student teachers' understanding of mathematical representations and the way they choose and use such representations when teaching in Lesotho primary schools.

Requirements of Participation in Research Study:

Tier 1. (*The whole year cohort is invited to participate in Tier 1*)
You are invited to complete a survey form indicating a) your mathematical achievement profile to date and b) your familiarity with and preferences for certain methods of representation of simple mathematical operations.

Tier 2. (*A group of about 10 students selected from Tier 1 will be invited to participate in Tier 2*)
You are invited to participate in a reflective (clinical) interview to discuss your answers to the Survey Instrument used in Tier 1.

Tier 3. (*A sub-group of about 5 students who volunteer to teach a mathematics lesson will be invited to participate in Tier 3*).
If you agree, you will be observed teaching a mathematics lesson at a primary school during your teaching practice session on a date that suits the school authorities, yourself and the researcher.

Tier 4. (*A group of 5 students who participate in Tier 3 will also be invited for video-stimulated recall interviews*).

You are invited to participate in an interview where we will discuss the mathematics lesson you taught while on teaching practice.

Confirmation that involvement in the Research Study is voluntary

Any participant may withdraw from the study without prior warning at any time. There will be no penalty for withdrawing before all stages of the Research Study has been completed. Data in the form of audio- and/or video recordings will be destroyed by fire after 5 years upon completion of this study.

Participant – Please complete the following (Circle Yes or No for each question).

Have you read or had read to you the Plain Language Statement?	Yes/No
Do you understand the information provided?	Yes/No
Have you had an opportunity to ask questions and discuss this study?	Yes/No
Have you received satisfactory answers to all your questions?	Yes/No

Signature:

I have read and understood the information in this form. The researcher has answered my questions and concerns, and I have a copy of this consent form. Therefore, I consent to take part in this research project.

Participant's Signature: ______________________________

Name in Block Capitals: ______________________________

Witness: ____

Date: _____________________________

Appendix 4: Letter from the Rector-LCE

LESOTHO COLLEGE OF EDUCATION
P.O. BOX MS 1393, MASERU 100, LESOTHO
TELEPHONE: (+266) 22312721 : TELEGRAM ADDRESS: BOSUOE

LCE/PUB/5

31st August 2009

Mr.N.Mpalami
P.O. BOX 1393
MASERU 100

Dear Sir

RE: APPLICATION FOR PERMISSION TO DO RESEARCH WITH STUDENT TEACHERS AT THE COLLEGE

Thank you for your letter dated 31st August 2009, bearing the heading captioned above. My office is happy to grant you permission to do research with the Diploma in Education Primary students as you propose. By copy of this letter the office of the DRAA is informed of this permission.

Good luck in your research, ensuring as much protection as possible of your research subjects.

Yours sincerely,

JOHN N.OLIPHANT (DR.)
RECTOR (a.i) -LCE

Appendix 5: Tier 1 Survey Instrument

This task is designed for research purposes only.

1. Answer all questions as honestly as possible
2. If the space is not enough, feel free to ask for an extra paper.

Section 1: Personal Details

1. Name (*in full*)

   ____

2. Gender

   ____

3. Age

   ____

4. Home Language

   ____

5. Home District

   ____

6. Name of last school attended

   ____

7. **Performance in Mathematics**

   a) Symbol in STD 7

      ____

   b) Symbol in Form C

      ____

   c) Symbol in Form E

      ____

   d) Symbol in other level (*specify if any*)

      ____

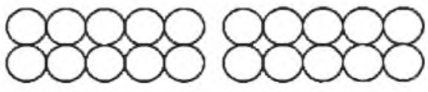
8. Write a brief story about your experiences as a learner of mathematics from primary schooling to high school level.

9. What kind of training do you think would help you in order to teach mathematics well at primary school?

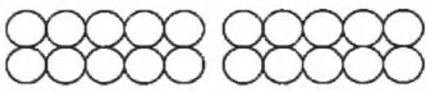
Section 2: **Mathematical Tasks**

1. Colour in the counters to show the operation and find the answer

c. 17 - 5 =

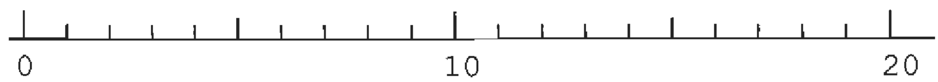


d. 12 + 6 =

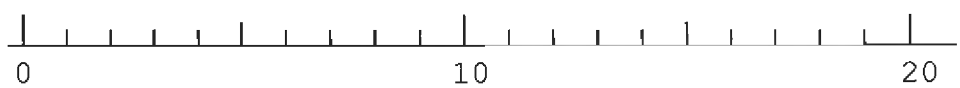


2. Draw blocks or jumps on the number line to show the operation and find the answer

a) 8 + 9 =

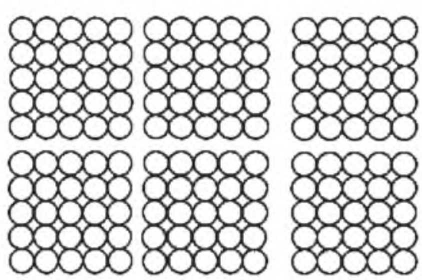


b) 11 - 6 =



3. Show the following calculation on the diagram provided and write the answer

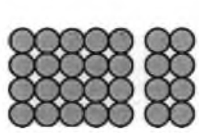
$5 \times 12 =$

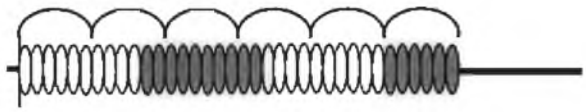


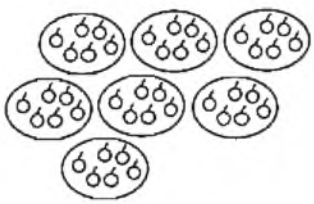
4. What calculations are shown in the following representations?

Write the calculation and the answer

a)



b) 

c) 

5. Work out the following, and show how you got your answer

201 + 79

6. If you have M44 to spend, how many Exercise Books at M4 each can you buy? (**Show clearly the representation you would use to explain to a child how you got the answer**).

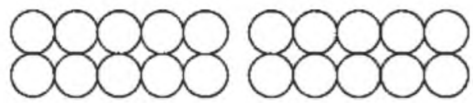
7. There are nine people at a party. Each person shakes hands once with each of the other people. How many handshakes are there in all? (**Show clearly the method that you used to get to the answer**).

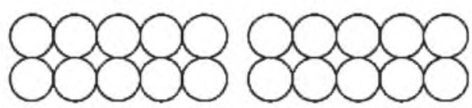
End. Thanks for your participation.

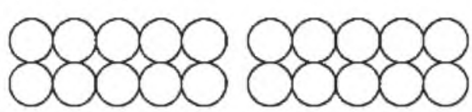
Appendix 6: Durham University Worksheet

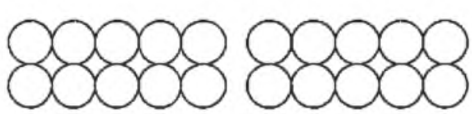
Worksheet 6: Addition and Subtraction up to 20

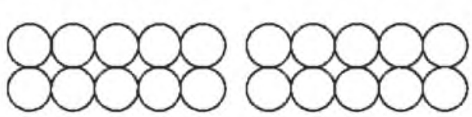
Colour in the counters to show the sum and find the answer

1. $17 - 5 =$ 

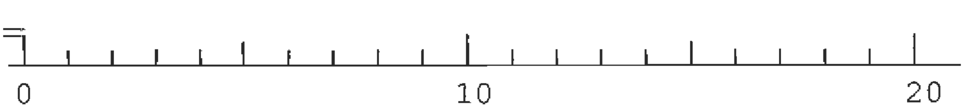
2. $12 + 6 =$ 

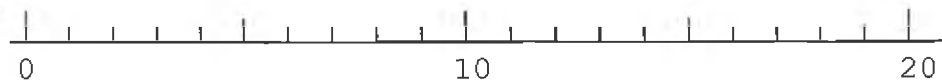
3. $6 + 9 =$ 

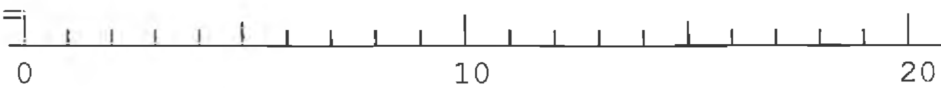
4. $19 - 7 =$ 

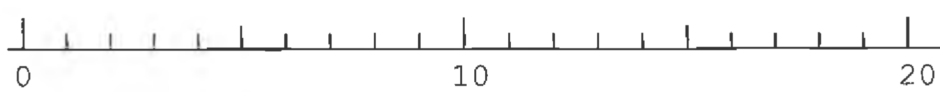
5. $12 - 4 =$ 

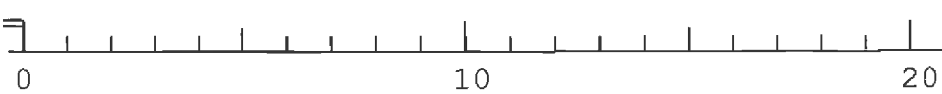
Draw blocks or jumps to show the sum and find the answer

6. $12 + 7 =$ 

7. $8 + 9 =$ 

8. $11 - 6 =$ 

9. $9 + 4 =$ 

10. $17 - 6 =$ 

Appendix 7: Samples of Mathematics Examination Papers (COSC)

November 2009, Paper 1

* 229637002 *

NEITHER ELECTRONIC CALCULATORS NOR MATHEMATICAL TABLES MAY BE USED IN THIS PAPER.

For Examiner's Use

1 (a) Evaluate $\frac{2}{3} - \frac{4}{7}$.

Answer (*a*) ....[1]

(b) Evaluate $1\frac{1}{3} \times \frac{5}{8}$, giving your answer in its simplest form.

Answer (*b*) ....[1]

2 (a) Add brackets to the equation in the answer space to make it correct.

Answer (*a*) $4 + 6 \times 7 - 5 = 16$ [1]

(b) Find the value of 27×0.002.

Answer (*b*) ....[1]

November 2009, Paper 2

Section A [52 marks]

Answer **all** questions in this section.

1 Solve the equations

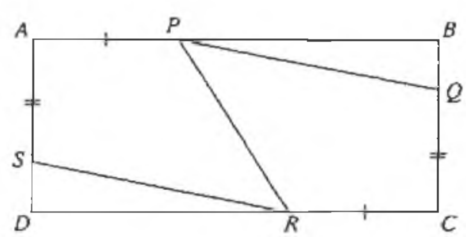
(a) $2^y = 8$, [1]

(b) $3p + 4 = 8 - 2(p - 3)$, [2]

(c) $\frac{18}{q} - \frac{16}{q+2} = 1$, [3]

(d) $5x^2 + x - 7 = 0$, giving each solution correct to 2 decimal places. [4]

2



ABCD is a rectangle.
Points *P*, *Q*, *R* and *S* lie on *AB*, *BC*, *CD* and *DA* respectively such that $AP = CR$ and $QC = SA$.

(a) Giving reasons, show that

(i) $PB = RD$, [1]

(ii) triangle *PBQ* is congruent to triangle *RDS*, [3]

(iii) $R\hat{P}Q = P\hat{R}S$. [3]

(b) State the special name of the quadrilateral *PQRS*. [1]

Appendix 8: Thandi's Lesson Plan

Subject: Mathematics
Topic: Measurement
Sub-topic: Addition of money and subtraction

class: 4
Date: 08-03-10
Duration: 80 mins
Class size: 80

Objectives By the end of the lesson pupils should be able to:
- add different sum of money and
- subtract the money used from the money they had.

Teaching material: money
improvised money
Number line

Teaching method:
Demonstration
Socratic

Introduction: We are going to add and subtract money

Development:

Stage	Content	Teacher activity	Pupils activity	assessment	conclusion
1	addition of money	shows how to add money up to 50	they watch carefully how to add	they add money on the chalkboard	addition of money is easy
2	Subtraction of money	Shows how subtraction takes place.	they subtract in their own groups	they do the exercise in the textbook	subtraction is what you have used

Conclusion: Addition of money is you have to add all the cents you have. Subtraction is to remove what you have used from what you have.
Assessment: Make an improvised money at home
comments: It went went because pupils responded effectively.

Appendix 9: Tšeli's Lesson Plan

LESSON PLAN
CLASS: 4
SUBJECT: MATHEMATICS
DATE: 09/03/10
TOPIC: MEASUREMENTS
DURATION: 80min
SUBTOPIC: MULTIPLICATION OF MONEY
CLASS SIZE: 66

OBJECTIVES: By the end of the lesson, pupils should be able to: multiply money up to R50/M50
~~multiply~~ solve multiplication problems of money.

TEACHING METHODS: Demonstration and Discussion
TEACHING/LEARNING MATERIALS: Money, Multiplication chart

INTRODUCTION
Copy and complete:
a) R5.50 + R2.50 = □
b) M11.00 − □ = M5.00
c) List multiples of 5.

5 DEVELOPMENT

Step	Content	Teachers Act	Learners Act	Assessment	Conclusion
1	multiplication of money	Instruct learners to multiply money up to R50/M50 a) M10 x 6 = □ b) R25 x 2 = □	They will multiply these amount and find the total amount. a) M10 x 5 = □ b) R25 x 2 = □	Find the total amount of: a) R30 = — x 2 b) M16 x 3 = □	we have learnt how to multiply money up to R50/M50
2.		Demonstrate how to solve multiplication problems of money. a) M5 x — = M15 b) R27 = — x — c) R30 = — x —	They will multiply the following amounts a) M5 x — = M15 b) R27 = — x — c) R30 = — x —	Copy and complete. a) M18 x 2 = b) M50 = — x 5 = c) R24 = — x —	we have learnt how to solve multiplication problems of money

CONCLUSION

We have learnt how to multiply money up to R50/M50 and also how to solve multiplication problems of money.

ASSESSMENT

1. Find the total amount for the following

a) M9.00 x 3 = □

b) M3.50 x 2 = □

c) M35 = R7 x _ = □

COMMENTS/EVALUATION

The lesson was successful because most learners were able to multiply money up to R50/M50 and also they were able to solve multiplication problems of money.

Appendix 10: Nomsa's Lesson Plan

Subject: Mathematics
Date: 23-02-2011
Topic: Numbers
Class: 4
Subtopic: Multiplication of whole numbers
Class size: 128
Duration: 80m

Objectives: By the end of the lesson pupils should be able multiply a 2 digit number by a number ≤ 10

Teaching Methods: Groupings, Demonstration, Socratic

Teaching Materials: Chart and work sheets

Introduction: Review their background knowledge by asking them the Mathematical operation should be placed in this sentence to make it true

Lead in statement: Today we are coming to learn about multiplication of 2 digit number by a number ≤ 10.

Development

Steps	Content	Teacher's Activities	Pupils Act	Assessment	Concl.
1	multiplication of 2 digit # by a # ≤ 10 using chart	Asks learners to look at the chart and say what they see	They look at the chart and say that they see different groups of slates	work out 11+11+11+11 = □ 4 elevens = □ 11 x 4 = □	we l[...] learn[...] that multi[...]cation is rep[...] additi[...]
		- Assist learners to work on the given operation	- They work on the operation through the help of the teacher 15+15+15 = □ 3 fifteens = □ 15 x 3 = □		

Multiplying using work Sheets	Teacher asks learners to look as the work sheets and they discuss about what they see on the work sheet	They look at the work sheets and discuss what they see with the teacher. - They set pairs and work on the work sheet	fill in the boxes on the work sheets

Conclusion: We have learnt that multiplication is repeated addition. ~~and it is~~

Assessment: Work out the following

i. (a) 12 + 12 + 12 = $\square$

(b) 3 twelves = $\square$

(c) 12 x 3 = $\square$

Comment/Evaluation

Appendix 11: Sebu's Lesson Plan

SUBJECT : MATHEMATICS
TOPIC : NUMBERS
SUBTOPIC : SETS
CLASS : 4
DATE : 15 - 03 - 2010
DURATION : 80 MIN
CLASS SIZE : 36
OBJECTIVE : By the end of the lesson pupils should be able to identify set and subset and the use of Mathematical symbol "⊂".

TEACHING METHODS : DEMONSTRATION, QUESTIONS AND ANSWERS, GROUPING
TEACHING MATERIALS : Bottle tops, Stones, shapes Coloured squares, chart.

INTRODUCTION : What is a set?

LINK IN - STATEMENT : We are going to Learn about sets; Subset of and the use of sign "⊂".

DEVELOPMENT

STAGE	CONTENT	TEACHERS ACTIVITIES	TEACHERS PUPILS ACTIVITIES	ASSESSMENT	CONCLUSION	SUMMARY
1	WHAT IS A SET?	TEACHER ASKS PUPILS TO COLLECT 10 STONES AND PUT THEM ON THEIR DESKS AND ASKS THEM WHAT WE CALL THAT 10 STONES. SHE HELP THEM	PUPILS COLLECT 10 STONES AND PUT THEM ON THEIR DESKS AND ANSWER THE QUESTION. THIS IS A SET OF TEN STONES.	TEACHER ASKS PUPILS TO CHOOSE ANY OBJECTS FOUND IN CLASS THAT CAN BE CLASSIFIED AS A SET, FOR EXAMPLE BOOKS	SET IS A GROUP OF OBJECTS AND SET CAN CONTAIN DIFFERENT OBJECT.	
2	SET WITH A SUBSET	TEACHER CHOOSE SOME BOYS AND GIRLS IN CLASS TO COME AND MAKE A SET, CHOOSE BOYS SET AND RENAME IT SHE NAME BOYS [illegible] SUBSET OF MAIN [illegible]	PUPILS WATCH THE TEACHER AND TELL HER WHAT BOYS SET IS CALLED THE SUB SET OF THE MAIN SET	TEACHER GIVE PUPILS STONES AND BOTTLE TOPS AND ASKS THEM TO MAKE ONE SET THE SUBSET OF OTHER.	SET M IS A SET OF CLASS 4 PUPILS AND IN IT WE HAVE SET OF 6 BOYS SO B IS A SUB SET OF M BECAUSE IT HAS ALL ELEMENTS IN M.	
3	SUB SET AND SIGN $\subset$	TEACHER PUT PUPILS IN GROUPS AND GIVE THEM COLOURED SQUARE AND SHAPES AND NAME THEM SET S AND Q SHE ASKS THEM TO MAKE Q SUBSET OF S WITH HER [illegible]	PUPILS NAME SET OF SHAPES S AND COLOURED SQUARE SET Q THEY MIX THEM AND MAKE Q SUBSET OF S WITH THE HELP OF TEACHER	TEACHER ASKS ONE MEMBER FROM EACH GROUP TO COME IN FRONT AND SHOW Q AS SUBSET OF S.	Q IS A SUBSET OF S BECAUSE IT CONTAINS ALL S ELEMENTS AND WE USE $\subset$ AS SIGN OF SUBSET $Q \subset S$.	

Appendix 12: Litha's Lesson Plan

SUBJECT; Mathematics **DATE;** ______

TOPIC; Measurement **CLASS;** 6

SUB-TOPIC; Length and Height **CLASS SIZE;** 69

DURATION; 40 min

OBJECTIVE(S); By the end of the lesson, pupils should be able to;

i) estimate the length of objects.
ii) measure the length of objects
iii) estimate the height of objects
iv) measure the height of objects.

TEACHING METHODS; Demonstration, Role-play

TEACHING/LEARNING MATERIALS; Metre stick, cm/mm rulers, tape measure, string, Pupils

LEAD-IN-STATEMENT(Optional); To measure how high something is, we say we are measuring its height. To measure how long something is, we say we are measuring its length. We are going to measure length and height

INTRODUCTION; Teacher calls any three pupils to the front and ask? The pupils are of different sizes. Who is tall? Who is taller? Who is the tallest?

Teacher then takes a piece of string and then ask? How long do you think is this string?

DEVELOPMENT;

S/N #	CONTENT	TEACHER'S ACTIVITIES	PUPIL'S ACTIVITIES	ASSESSMENT	CONCLUSION
1	Estimating and measuring length.	Look carefully at your cm ruler. Which of your finger best fits into a 1cm space on your ruler. • That is your finger ruler. Use it to measure the length of your short exercise book. • Now use your ruler to measure accurately the length of your exercise book.	• Pupils use their fingers to see which best fits into a cm space. Pupils use their finger rulers to measure their exercise books. • Pupils measure their exercise books using their rulers.	• Make a ten centimetres ruler with your hand. Use it to measure the length of your desk, and record Use a ruler to measure the length of your desk, and record.	We have seen that to measure very short lengths we may use a ruler and give units in cm and use a metre stick for larger lengths giving units in metres
2	Estimating and measuring heights.	• Tell pupils to come to the front. Estimate their heights. • Now guide pupils to measure the heights of the pupils in front • What units do you use to give the height of the quantities you have measured? • Metres are the standard units of length and height.	Pupils estimate the heights of the ones in front. Pupils measure the heights of the pupils using metre sticks and tape measures • Pupils respond (metres)	Estimate the height of your classroom door. What is the standard unit of length and height?	We have seen that a metre is the standard unit of length

CONCLUSION; Length is a measure of how long something is. It is measured in mm, cm, m, km. The standard unit of length is therefore metres. Height is a measure of how high something is. It is measured mostly in metres, which is the standard unit.

EVALUATION; ____ is the standard unit of length and height.

What do these words mean: Length?

Height?

• Measure the length of your Maths book.

• Draw a line that is 28cm long.

COMMENTS/REMARKS: Pupils were participative. The lesson was effective.

Appendix 13: Tier 2 Interview Schedule

(Participants are shown audio-recorder and how to press stop-button)

Welcome [name of participant]. Thank you for agreeing to participate in this interview. If at any time you want to end the interview or to stop the tape recorder please do so. There is no right or wrong answer, so feel free to say whatever you consider relevant in this discussion. Stop me if you don't understand something and I will rephrase the question. If there is something you want to ask, you can ask.

Are you happy for me to go on with the interview?

You will recall that about 10 months ago you filled up this form, which had some tasks to be solved. Please read silently through the document so that you can refresh your memory about what you did.

My interest is on how you worked out solutions to some tasks and how you arrived at the answers. I also wish to know how you felt as you worked out solutions.

Tell me about your answers to Part B of the Survey Instrument:

I see you came up with a solution to [name of mathematical task]

- Can you tell me about the solution?
- What helped you to arrive at this solution?
- How did you feel when you arrived at the solution?
- If you were given this task now, how would you work out the answer?
- Which diagrams/pictures do you think were most helpful when working out answers to [specific tasks]? Why/why not?
- Do you like using mathematical representations when solving tasks? Why/why not?
- Can you describe how you feel about the role of mathematical representations in solving tasks?
- Is there any other thing that you would like to say before we end our discussion?

Once again, thank you for agreeing to take part in this interview.