**Does it go without saying?**

**Mathematical thinking in whole class discourse**

This thesis is submitted for the degree of Doctor of Philosophy

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**2012**

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed: Sinéad Nic Mhuirí ID No.: 58262911 Date: 10-12-12

## Acknowledgements

Undertaking a PhD is a long process which is demanding on many levels. My sources of support and inspiration have been many. Firstly, I would like to thank my supervisor Dr Dolores Corcoran who treated me as an equal long before I could have hoped to deserve that privilege. Her unfailing enthusiasm sustained and encouraged me throughout the research process. I am also grateful to Dr Thérèse Dooley who provided many words of wisdom, support and encouragement along the way.

There would be no research to report were it not for the Boards of Management that granted me permission to access schools and the teachers, students and parents who consented to my research. I am grateful to all of these people. Much of the depth of this research comes from the teaching experiment I carried out in my own classroom. This would not have been possible without the support of the Board of Management, parents and students and of the different principals I served under during the course of the research. I am thankful to my current principal and Board of Management who also granted me leave to allow me finish my thesis. It is to my students that I perhaps owe the biggest debt of gratitude. It is possible that I learned more from them than they ever did from me and I am grateful for their honest, genuine engagement. I am also grateful to the Teaching Council who recognised the potential of this research and eased some of the financial pressures of undertaking it when they granted me a bursary.

I must finally acknowledge colleagues, friends and family whose friendship supported and encouraged me. Distraction is a powerful motivator and looking forward to my 'breaks' with you motivated and reenergised me more than you realise. Also to Joey, the first professor and source of inspiration. A special word of thanks to Mum and Andy for proof-reading, constructive advice and support. Also to Jörg for his practical support, understanding and patience.

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## List of Abbreviations

| | |
|---|---|
| CAO | Central Applications office |
| CPD | Continuous Professional Development |
| CR | Collaborative Reasoning |
| DES | Department of Education and Skills |
| DEIS | Delivering Equality of Opportunity in Schools |
| EMT | Explaining Mathematical Thinking |
| IRE | Invitation, Response, Evaluation |
| LANDS | Literacy and Numeracy in Disadvantaged Schools |
| NA | National Assessment of Mathematics and English Reading |
| NAMA | National Assessment of Mathematics Achievement |
| NCCA | National Council for Curriculum and Assessment |
| NCTM | National Council of Teachers of Mathematics (US) |
| NUI | National University of Ireland |
| OECD | Organisation for Economic Co-operation and Development |
| PDST | Professional Development Service for Teachers |
| PGCE | Postgraduate Certificate in Education |
| PISA | Programme for International Student Assessment |
| RFL | Responsibility for Learning |
| RME | Realistic Mathematics Education |
| RUDE | Read, Underline, Draw, Estimate |
| SES | Socioeconomic status |
| SMI | Source of Mathematical Ideas |
| TIMSS | Trends in International and Science Study |

## Transcript Conventions

These conventions were based on Dooley (2010).

| | |
|---|---|
| ... | A pause or hesitation shorter than three seconds |
| [...] | A pause longer than three seconds |
| ( ) | Inaudible speech |
| (word) | Speech that was difficult to hear exactly but likely to be as transcribed |
| [ ] | Lines omitted from the presentation of the transcript based on perceived irrelevance of content. |
| // | Encloses overlapping utterances. |
| (*word*) | Brackets contain the researcher's/transcriber's comments. |
| Teacher | The teacher/researcher (myself). |
| Student | A student that I could not identify. In all other cases pseudonyms are used. |
| Students | Two or more students speaking at the same time. |
| Word- | Indicates interruption by another speaker |

**Abstract**

My research examines the role of mathematical thinking in whole class discourse. Initially a study exploring the nature of discourse in some Irish primary mathematics classes was carried out. Six recordings of mathematics lessons from four different teachers were analysed using the Math Talk Learning Community framework (Huffed-Ackles, Fuson & Sherin, 2004) and Boaler and Brodie's (2004) teacher question categories. Student thinking shared in whole class discourse in these lessons seemed to be limited by the teachers' central role as evaluative authority and source of mathematical ideas.

The aim of the second stage of my research was to create a discourse community in my own classroom. This involved positioning students as mathematical authorities capable of evaluating mathematical thinking. It was envisaged that the resulting classroom discourse would be progressive and mirror discourse at domain level (Bereiter, 1994). Thirty-one recordings on fractions, decimals and percentage topics were collected, fourteen of which were transcribed. Five of these lessons were analysed in the same manner as at stage 1. This analysis showed that student thinking became an object of discussion in the teaching experiment lessons as students commented on and evaluated the thinking of their peers. The participation trajectories (Dreier, 1999; 2009) of ten students were tracked across the fourteen transcribed lessons with a view to documenting students' use of discourse community practices such as explaining and justifying thinking, or disagreeing with previous contributors. Despite initial concerns over the participation of lower achievers, this analysis showed that all students participated in the practices of the discourse community to some extent. However it also highlighted differences in the nature of the experience for different individuals. I also interrogated my own experience as teacher-researcher to investigate the issues involved in facilitating a discourse community. This analysis suggested that many of the dilemmas experienced by the teacher are managed rather than resolved (Lampert, 1985).

# Chapter 1: Context and Rationale

'May you live in interesting times.'

Chinese Curse (origin contested).

The origin and authenticity of this Chinese curse cannot be verified but the sentiment it expresses, of a man cursed to live in changeable times, seems an apt way to begin. Whether it is a blessing or a curse, these are interesting, changeable times and it is an exciting, if daunting, time both in Ireland and beyond, to undertake research in mathematics education. Change is occurring in many places at many levels, some of it voluntary and some forced by economic circumstance or institutional reform. From an economic perspective, the recession has led many people to question Ireland's future as a knowledge-based economy, that is an economy that uses information or technology to create jobs. Such questions often focus on the nature of mathematics teaching and learning and link mathematics education to employment opportunities and financial concerns. Another influence of the recession on the educational sector in Ireland is the reduction in capitation grants and loss of allowances for teachers who complete Masters or PhD programmes. These constraints stand in contrast to the reform effort that is occurring at second level with the phased introduction of Project Maths, which began nationally in September 2010 (www.projectmaths.ie). Project Maths is itself a reflection of the changing perspectives of mathematics as a subject and of mathematics education in particular. For these reasons and others, we live, teach and research in interesting times.

In this chapter I will present an overview of this context. I will discuss the role of mathematics in modern society. Then I will discuss the differences between school mathematics and mathematics as a discipline with reference to practitioners of mathematics. Following this, I will discuss media reports of 'failure' in mathematics as well as reports that have been carried out on mathematics achievement in Ireland. Finally, I will present the research questions that have grown from my awareness of these issues and my own teaching experience and describe the chapters that follow.

## The Role of Mathematics in Today's Society

### Mathematics and the Knowledge Economy

Apple suggests that "no analysis of education can be fully serious without placing at its very core a sensitivity to the ongoing struggles that constantly shape the terrain on which the curriculum operates" (2000, p. 244). In Ireland, the widespread public and media debate about the teaching and learning of mathematics is one influence on the terrain. The economic recession has led many people to question Ireland's future as a knowledge-based economy. New voices have entered the debate on the teaching and learning of mathematics. For example, the professional body Engineers Ireland, representing engineers and those in related professions, has produced a report on the teaching of science and mathematics (Taskforce on Education of Science and Mathematics at Second Level, 2010). Also, in a speech to both politicians and academics in February 2010, the former Chief Executive Officer (CEO) of Intel – a multinational computer manufacturing company, Craig Barrett spoke about falling educational standards. Barrett surprised many by saying that while high educational standards was one of the reasons Intel had originally come to Ireland more than twenty years ago, educational standards are now only average (Kennedy, 2010). His statement was given extensive coverage in print media, on radio and on television. Improving mathematics education was point one of his ten point plan for the future recovery of Ireland's economy. Mathematics is seen as vitally important because it is a 'high-yield' subject, one which is central to the many scientific and technological advances that are occurring at pace in our globalised world (Conway & Sloane, 2005; Lyons, Lynch, Close, Sheeran & Boland, 2003; Steen, 2001). Thus it is generally accepted that society, and Barrett would argue economies, need mathematically able citizens.

**Mathematics and Democracy**

Ernest (2000) highlights issues of power when he suggests that even in a society where mathematics underpins many systems and everyday technology, this mathematics is often hidden from the general public. He suggests that higher level mathematics is only needed by the 'elite' minority who create and control the systems that underpin our everyday financial and technological systems. The majority of the population interact with these systems without using higher level mathematics. For example, it is not necessary to understand the complex mathematics underlying internet security measures to shop or bank online. Gutstein (2007, p. 10) discusses the "stratified labour force" and how mathematical literacy can be conceived differently at different strata. He suggests that mathematical power is inherent in conceptions of mathematical literacy for knowledge workers but a more limited mathematical literacy is expected of workers at different strata. Democratic mathematics education is an approach that opposes such notions of mathematical elitism. This perspective views democratic access to powerful mathematics as central to the development of successful citizens in a democratic society (Ellis & Malloy, 2007; Malloy, 2002). Steen discusses how quantitative or mathematical thinking is used in different jobs and also how it has become ubiquitous in every aspect of society:

> The world of the twenty-first century is a world awash in numbers. Headlines use quantitative measures to report increases in gasoline prices, changes in SAT scores, risks of dying from colon cancer, and numbers from the latest ethnic war. Advertisements use numbers to compete over the costs of cell phone contracts and low-interest car loans. Sports reporting abounds in team statistics and odds on forthcoming competitions. (2001, p.1)

Malloy (2002) notes that because mathematics permeates society, it is impossible to be a democratic citizen without being proficient in mathematics. However the democratic mathematics education movement goes beyond developing mathematical proficiency and is more akin to teaching mathematics for social justice (Gutstein, 2007). It encompasses ideas of equity within mathematics education and also proposes that mathematics education can serve as a vehicle for change within the larger society (Ellis & Malloy, 2007; Gutstein, 2007). Hannaford argues that teaching mathematics well also teaches democracy:

> Mathematics cannot be politically neutral ... Only a little training in good mathematics gives people more confidence in democracy. This is because good mathematics teaches pupils to listen, to think, and to argue more effectively; to respect others always and to accept ideas which at first they do not understand; and even to accept decisions which they do not like or respect. Democracy depends on attitudes like these … If children are taught mathematics well, it will teach them much of the freedom, skills, and of course the disciplines of expression, dissent and tolerance, that democracy needs to succeed. (1998, p. 185 - 186)

Moses and Cobb (2001) suggest that because high level mathematics achievement can be associated with power, access to high level mathematics should in fact be a civil right. This statement holds particular meaning for minority groups whose low mathematical attainment often correlates with disadvantaged status within the larger society in terms of both financial gain and social standing (Every Child a Chance Trust, 2009). The review of research presented by Every Child a Chance Trust (2009) suggests that poor numeracy restricts the opportunities open to people and is linked with antisocial behaviour even when other factors such as home background and general ability are accounted for.

**Creative Mathematics and Higher Order Skills**

Arguments about both mathematics for the knowledge economy and concerns about equity are audible in the debate about the teaching and learning of mathematics. There is also another issue that could perhaps be considered as an off shoot of both these perspectives. It connects with the idea of supporting and advancing the knowledge economy and also with the 'powerful mathematical ideas' element that is present in the democratic mathematics movement. It is suggested that because of the pace of change in society, influenced by growth in technology and globalisation amongst other issues, the skills that will be needed by students in their future lives are different from the skills prioritised in traditional curricula (Boix Mansilla & Jackson, 2011). For this reason, it can be argued that there should be a focus on meaningful problem solving, effective communication and higher order thinking skills that help develop flexible, creative thinking in students so that they can adapt and succeed into the future. These areas are not always prominent in traditional approaches to mathematics education.

## Mathematics as a Discipline, in Everyday Life and School Mathematics

### My Experience of the Discipline of Mathematics

When considering mathematics as a discipline, I am referring to the types of mathematical activities that are carried out by professional mathematicians or those who study mathematics at university level. Unlike the main cohort of primary school teachers, I had the good fortune to study mathematics to Masters level in the National University of Ireland (NUI) Galway before finding my way into primary teaching via a PGCE at the University of Exeter. Though much of the specific course content that I learned in NUI, Galway is forgotten now, the mathematical ways of working are not. These included peer discussion as well as lecturer-led whole class discussion. My engagement with mathematics also involved long periods of solitary struggle, intense concentration, and persistence. It involved, almost exclusively, tasks that were cognitively demanding. These tasks and methods of working demanded creativity, flexibility and logical thinking. Persistence, creativity, motivation and even joy would seem to be common themes that emerge when mathematicians speak about their work (Boaler, 2009; Hersh, 2005; Pólya, 1945/1990).

**Mathematics in Everyday Life**

Mathematics in everyday life can be more difficult to identify. There are some situations where the mathematics in everyday practice is obvious. For example a carpenter measuring planks of wood or a person in a supermarket counting her change. Other situations where mathematical thinking is used are less obvious and often involve some kind of estimation or a problem in context. For example, in the case of decorating a house, one may do some straight forward mathematical activities like taking measurements or money calculations to inform budgeting choices but decisions will also be informed by practicalities and features of the local environment. In real life, problem solving is highly situated and the context of a problem can both form the problem itself and suggest a solution method (Lave, 1988). This contrasts with mathematical problem solving as it is experienced by students in classrooms. There is evidence to suggest that mathematics as it is carried out in everyday life is very different from the mathematics of the classroom and also that the procedures used to solve problems in real life vary immensely from the procedures used to solve problems in the classroom (Brown, Collins & Duguid, 1989; Carraher & Schliemann, 1985; Lave, 1988).

**Differences between Mathematics as a Discipline and School Mathematics: the Traditional Approach**

One would suppose that having experience of the discipline of mathematics would make it easier to approach school mathematics but in my own case, I have not found this to be true. In fact at times they have seemed almost completely unrelated. Features of working mathematically such as patterns of dialogue involving making conjectures and examining the mathematical thinking of others and justifying ones' own mathematical opinions seem central to the mathematical activities of those working with mathematics as a discipline (Lampert, 1990; Pólya, 1945/1990). However these features are often absent from mathematical activities carried out at school level (Boaler, 2009). School mathematics often consists of traditional style lessons where the teacher models a procedure or strategy and then students practice examples of this type of task often from a textbook. Boaler states:

> In many maths classrooms a very narrow subject is taught to children, that is nothing like the maths of the world or the maths that mathematicians use. This narrow subject involves copying methods that teachers demonstrate and reproducing them accurately again and again. (2009, p. 2)

As a result the cognitive challenge, persistence, creativity and often the joy experienced by mathematicians are not often experienced by students. Lampert (1990) notes that instead of emergent mathematical thinking being shared in the form of conjectures, school mathematics is instead associated with certitude and mathematical authority rests with the teacher and textbook. Lampert's comments about the differences between school mathematics and mathematics as a discipline were written almost twenty years before Boaler (2009), who with experience of both the UK and US systems, reported that the traditional approach is still common.

**The Situation in Irish Mathematics Classrooms**

Evidence from studies carried out at secondary level suggests that this traditional approach of teacher exposition followed by pupil practice is also common in Ireland. Lyons, Lynch, Close, Sheeran and Boland (2003) conducted a video study of twenty mathematics lessons involving second year students in ten different schools. They found that all twenty observed mathematics lessons followed the traditional approach outlined above. The results of the 2009 National Assessment of Mathematics and English reading (NA 2009) provide some insight into the situation at primary level (Eivers, Close, Shiel, Millar, Clerkin, Gilleece, & Kiniry, 2010). Mathematical assessments were carried out on almost 4000 second class students and a similar number of sixth class students from a variety of schools. Second class students are generally 7 – 8 years old and sixth class students are generally 11 – 12 years old. Relevant contextual details were gathered by means of questionnaires completed by students, parents, class teachers and principals. These questionnaires indicated that textbooks were widely used by teachers both as a planning tool and as the source of mathematical activities students undertake in lessons. The teachers of 82% of second class pupils and 94% of sixth class pupils indicated that the main class textbook had been used in planning lessons the previous week. Textbooks and workbooks or worksheets were also used in the majority of classes with almost all pupils in classes where teachers indicated that textbooks were used most days or at least once or twice a week (98% at second class level and 99% at sixth class level). Eivers at al. state that these results combined with data on the prevalence of group work and use of concrete materials or technology suggest that the traditional approach of "whole class, textbook-based teaching still predominates in Sixth class maths lessons" (2010, p. 64). The use of textbooks was also found to be very prevalent in the 1999 and 2004 National Assessments of Mathematics Achievement (NAMA) which were carried out on over 4000 fourth class students (Shiel & Kelly, 2001; Shiel, Surgenor, Close & Millar, 2006). Fourth class students are generally 9 – 10 years old. Eivers et al. (2010) note that although the Inspectorate reviewed textbooks in the past, no agency is presently responsible for carrying out this work. They suggest a review of textbooks should be considered for future research. The influence of the traditional approach on students' beliefs about mathematical authority in the classroom will be discussed in Chapter 2.

## Reports of Mathematics Achievement

### Media

I have already noted that speculation about achievement levels in mathematics and possible future economic opportunities for the country has been widely discussed in the Irish media. Another issue that regularly makes headlines is that of Leaving Certificate and Junior Certificate mathematics results and Project Maths, the new approach to the teaching of mathematics at second level. For example consider the following extract from an article which was published after last year's Leaving Certificate results were released.

> In a now familiar trend, 10 per cent of students failed maths at ordinary level. Overall, 4,367 students failed maths across all levels, making them ineligible for many third-level courses. Results were marginally better among the 1,900 students who took the new "user-friendly" Project Maths course in 24 schools. But the Government will be disappointed by the poor take-up for higher level maths in these schools. Only 16 per cent took the subject at higher level, the same number who took the mainstream exam. (The Irish Times, 17/8/2011)

The low numbers taking higher level mathematics as well as the failure rate at ordinary level in the Leaving Certificate is often discussed in the media. Project Maths and adjustments to the Central Applications Office (CAO) points system which controls entry to third level courses, are often suggested as approaches to tackling these problems (NCCA, 2005).

**Programme for International Student Assessment (PISA)**

The results of the Programme for International Student Assessment (PISA) shed further light on mathematics achievement at second level. PISA is carried out every three years by the Organisation for Economic Co-operation and Development (OECD). In each assessment the literacy of 15 year old students from various countries is assessed in reading, mathematics and science to "assess students' preparedness for the reading, mathematical and scientific demands of future education and adult life" (Eivers, Shiel & Cunningham, 2008, p. 1). The three areas of reading, mathematics and science are assessed in every round of PISA with one area being assessed in depth, while the other two are assessed more broadly. Mathematics was the focus area in PISA 2003. It was found that in this year, as in PISA 2000 and 2006, students in Ireland achieved a mean score just above the OECD average but not significantly so. In each of these years, students from Ireland performed well above average in reading.

Most countries tend to achieve the same levels across the subjects but Ireland showed differences of achievement between literacy and mathematics (Cosgrove, Sofroniou, Zastrutzki, Shortt & Shiel, 2004). However in PISA 2009, Irish students scored significantly below the OECD average in mathematics and Ireland was rated as 26th out of the 34 OECD countries (Perkins, Moran, Cosgrave & Shiel, 2010). It is possible that students from Ireland are not fully prepared to sit the PISA mathematics tests. PISA mathematics has grown from the Realistic Mathematics Education (RME) movement which has significant differences from the traditional Junior Certificate curriculum. RME is a movement that focuses on the importance of solving mathematical problems in real world settings and has influenced the development of the Project Maths syllabus which is currently in initial stages of implementation. RME will be discussed in more detail in Chapter 2. Analysis of the distribution of mathematics results from PISA 2003, 2006 and 2009 suggest that the overall average score may be relatively low due to the comparatively low performance of high achievers (Cosgrove et al., 2004; Eivers, Shiel & Cunningham, 2007; Perkins et al., 2010). Perkins et al. (2010) also note that although mathematics achievement scores declined overall in PISA 2009, there was a somewhat greater decline in the scores of higher achieving participants. This amplifies the concerns about the low number of high achievers in mathematics at Leaving Certificate level.

**National assessments of Mathematics Achievement (NAMA/NA)**

In relation to mathematics at primary level, the main source of data comes from NAMA 1999 and 2004 and NA 2009. In both NAMA 1999 and 2004 which were carried out at fourth class level, it was found that pupil achievement was higher in tasks that involved the lower-level processes such as 'understanding and recalling' and 'implementing mathematical procedures' and students performed least well on higher level skills such as 'applying and problem-solving' (Shiel et al, 2006). There was a significant improvement in achievement however in tasks involving the higher order skill of 'reasoning' from 1999 to 2004. NA 2009 which was carried out at both second class level and sixth class level found that the 'apply and problem solve' skill category was the most difficult for both sets of pupils with 47% and 42% correct at second and sixth class level respectively (Clerkin & Gileece, 2010). Eivers et al. (2010) note that most 'apply and problem solve' items were set in measures contexts and this was the curriculum content strand in which students performed least well. In NA 2009, pupils at sixth class level scored relatively well on reasoning items.

Both NAMA 1999 and 2004 recommended that schools and teachers should put a greater emphasis on the teaching of higher-order mathematics skills. In fact, Shiel et al. recommend that:

> Schools and teachers should place a stronger emphasis on teaching higher-order skills, including Applying and Problem Solving, to all pupils by implementing in a systematic way the constructivist, discussion-based approaches outlined in the Guidelines accompanying the 1999 PSMC. (2006, p. 155)

They further recommend that pilot projects linked to problem based approaches to the teaching of mathematics such as RME should be supported and note that such projects would help to inform those involved in mathematics education. Similar recommendations are made in respect of NA 2009:

> Classroom practice should reflect advances in the teaching of problem-solving. Pupils should spend more time solving substantial problems, analysing and discussing problems with other pupils and their teacher, and acquiring improved understanding of the concepts and skills involved. Teachers should ensure that pupils meet a range of problems across curriculum strands, including complex problems embedded in real-life contexts and those of a non-routine nature. (Eivers et al., 2010, p. 93)

It was also recommended that continuous professional development be offered to teachers in this and other areas and that schools should identify areas for continuous professional development at both school and individual level. A recommendation for future research also calls for observational data on classroom discourse and problem solving. My research is aligned with some of these recommendations as can be seen from the outline which I will present at the end of this chapter.

**Mathematical Achievement in Schools Designated as Disadvantaged**

The history of how schools were designated as disadvantaged is complex with no unified approach until relatively recently (Weir & Archer, 2005). Generally approaches included identifying indicators of socio-economic status and levels of education in the population of parents. Indicators included unemployment levels, types of housing and information on basic literacy and numeracy. The current scheme in place in schools designated as disadvantaged is known as the Delivering Equality of Opportunity in Schools (DEIS) scheme. DEIS aims to ensure that the educational needs of students from disadvantaged backgrounds are met. At school level it implements a School Support Programme (SSP) which supports intervention measures such as Ready Set Go Maths, First Steps, Maths Recovery and Reading Recovery (Weir, Archer, O'Flaherty & Gileece, 2011).

Large scale studies, such as NA 2009, report lower achievement scores for students with low family socioeconomic status (SES) scores and from schools with low SES enrolment (Eivers et al., 2010). These results are echoed by the findings of NAMA 1999 and 2004 (Shiel & Kelly, 2001; Shiel et al., 2006). It was found that the mean score of children who attended designated disadvantaged schools was significantly lower than their counterparts in non-designated schools for both NAMA 1999 and NAMA 2004.

The *Literacy and Numeracy in Disadvantaged schools* (LANDS) report explored literacy and numeracy achievement in twelve designated disadvantaged schools (DES, 2005).

An examination of the mathematics results achieved on standardised tests in some of the schools involved in the evaluation suggest that a majority of the students had very poor numeracy skills with 64% of students scoring at or below the twentieth percentile and only 2.7% between the eightieth and hundredth percentile (DES, 2005). These mathematics achievement scores are significantly lower than the literacy achievement scores. There was variation across schools with some schools faring significantly better than others in mathematics. This part of the study involved only nine out of twelve schools which were selected from among the one hundred schools with the highest reported levels of disadvantage in Ireland. In these schools, there appeared to be a further decline in students' mathematical achievement in the senior classes with almost three quarters of fifth and sixth class students (73%) at or below the $20^{th}$ percentile and only 6 pupils out of 479 (1.25%) at or above the $80^{th}$ percentile in the standardised tests.

It would be interesting to know if decline in mathematical achievement of such a scale would be found in a larger sample of designated disadvantaged schools but large scale studies such as NA 2009 do not present this information. The NA 2009 assessment shows differences between the achievement levels of second class students and sixth class students for students from all school types but the results are mixed. Second class students performed substantially better on 'understand and recall' items than sixth class students but sixth class students performed better on reasoning items. In terms of overall percentage correct scores for the different content strands, the second class results ranged from 49% for measures to 73% for shape and space and the sixth class results ranged from 38% for measures to 64% for data. The percentage of correct responses to measures and shape and space items decreased from second to sixth class but increased for items on data. Previous national assessments such as NAMA 1999 or 2001 were focussed at fourth class level so varying achievement levels through the primary school were not tracked. The new form of national assessment presented in NA 2009 will allow for such tracking and for trend data to emerge (Eivers et al., 2010).

A review of the impact of the DEIS scheme has recently been published (Weir et al., 2011). A cross-sectional comparison of achievement on assessments carried out in 2007 and 2010 found that average mathematics scores increased across class levels with the greatest increase at second class and the smallest at sixth class level. Also the proportions of students scoring below the tenth percentile were lower in 2010 and there was an increase in proportion of students scoring at or above the 90$^{th}$ percentile in 2$^{nd}$, 3$^{rd}$ and 6$^{th}$ class, with the increase being lowest at 6$^{th}$ class level. Weir et al. (2011) note however that despite the improvement in achievement levels, average test scores for students attending disadvantaged schools are still below the norm average. Interestingly, when the data was interrogated to investigate the achievement levels of Travellers, a minority group even in DEIS schools, although increases were found in achievement levels for reading at all class levels, there was no corresponding increase in mathematics. In fact at sixth class level in 2010, 70% of Traveller students scored at or below the 10$^{th}$ percentile.

As it stands, the widening gap in achievement of students as they move through the senior classes of designated disadvantaged schools reported in LANDS, is a cause for concern (DES, 2005). It is a serious challenge for teachers to meet the disparate needs of the pupils in their classes. The dwindling numbers of high achieving students as pupils progressed through the senior classes also raises questions. Is this finding unique to the small group of schools studied in the LANDS report or might similar findings emerge if other groups of schools were examined? Is it due to the nature of the curriculum which grows in complexity as students progress through the senior classes? What other factors are at play? How much time is spent learning new procedures such as algorithms for long division or addition of fractions versus time spent in solving problems? Do teachers, faced with mixed ability classes, value computation and procedures more than 'real' cognitively demanding mathematics that might develop the higher order skills of all students and the skills of potential high achievers in particular? Attempting to address the lack of achievement outlined above, the LANDS report recommends "differentiated approaches" and "revision and consolidation of learning … assisting pupils in linking concepts and knowledge between the strands of the curriculum" (DES, 2005, p. 64). Also it is recommended that the development of higher-order thinking skills should be pursued in both numeracy and literacy. Another inspectorate document that details effective approaches to teaching and learning in designated disadvantaged schools recommends strong school leadership, strategic planning practices, collaboration between teachers and a commitment to continuous professional development (DES, 2009). Whether schools and teachers have the necessary understanding, resources and supports to meet this challenge is another question.

**DES Response**

The Department of Education and Skill (DES) has responded to reports of underachievement in literacy and numeracy by producing a draft national plan to improve literacy and numeracy in schools (DES, 2010). Some of the targeted areas for improvement include school leadership and professional development. Literacy and numeracy are to be prioritised above other curricular areas and are to be allocated more teaching time. There is an emphasis on target setting and a drive for continual improvement "by improving radically the assessment and reporting of progress at student, school and national level and by focussing school self-evaluation and school inspection on literacy and numeracy" (DES, 2010, p. 13). While the "relentless focus" (DES, 2010, p. 12) on improvement may be warranted, there is a danger that a relentless focus on assessment data may lead to a 'teaching to the test' approach rather than 'teaching for understanding'. Hiebert et al. comment on a similar testing regime in the American system and ask "how can *processes* be improved by inspecting only their *outcomes*?" (2005, p. 112, original italics). Rather than focussing on the outcomes, my research aims to investigate the process of teaching. It is outlined below.

**Outline of Research**

It is hoped that this chapter has provided an overview of the context of mathematics education at this time in Ireland and a rationale for continued research. My research questions are:

- What is the nature of the discourse students in some Irish primary classrooms engage in during mathematics lessons on number strand topics?
- What is the nature of student learning in a discourse community?
- What is the nature of the experience for a teacher attempting to facilitate a discourse community?

Discourse community should be understood to mean a community where students engage in sharing and justifying their mathematical thinking and play a central role in the evaluation of the mathematical thinking of their peers. Such a community is intended to reflect authentic mathematical practices and envisaged to position students, rather than teacher, as mathematical authorities. This conception of a classroom discourse community is discussed in detail in Chapter 2.

The first question was explored through the gathering of audio recordings of mathematics lessons on number strand topics. Analysis of these recordings is valuable in its own right but also sets the scene for the teaching experiment that I undertook to explore the second and third question. The aim of the teaching experiment was the facilitation of a discourse community. It was undertaken at fifth class level, where students are 10 – 11 years old, in a designated disadvantaged school.

## Overview of Thesis

In the following chapter I will present a literature review. This literature review will highlight areas relevant to my research and will also constitute a rationale for the chosen research questions. In chapter 3, I will present details of the research methodology. In chapter 4, I will present analysis and findings of the first stage of the research, which involved gathering audio recordings of primary mathematics lessons. Chapter 4 is intended to give a sense of the 'lie of the land' in relation to mathematical discourse in some Irish primary classrooms. In chapter 5, I will present a similar analysis of a selection of lessons carried out in my own classroom during the teaching experiment. This chapter will give some insight into the nature of teaching and learning in a discourse community. These issues will be examined further in Chapter 6, which will detail some of the issues faced by me as teacher during the teaching experiment. Chapter 6 will also include details of some students' trajectories of participation through the course of the teaching experiment. Chapter 7 will examine the nature my own participation trajectory as teacher researcher. Chapter 8 will consist of summary notes, conclusions and directions for further study.

# Chapter 2: Literature Review

The title of my thesis asks if mathematical thinking goes without saying in whole class discourse. This literature review aims to explore this and related questions about the role of shared mathematical thinking in developing mathematical understanding in whole class discourse. The chapter is divided into three sections. The first section will explore different perspectives of the nature of mathematics. The second will discuss theories of mathematical learning. The final section will review the literature of mathematics education from the teacher's perspective and aims to explore the place of the teacher in mathematics education research. Also the question 'how does a teacher facilitate a discourse community?' will be discussed. This question in central to reform orientated teaching practice.

## What is Mathematics?

### Moving towards Fallibilism

Some of the issues discussed in chapter 1 raise questions about the 'true' nature of mathematics. There is wide divergence between mathematics as it is experienced by professional mathematicians or people in everyday life and student mathematicians in traditional classrooms. It can be argued that these experiences reflect different conceptions of mathematics. One perspective involves a positivist epistemology where knowledge is understood to be is "hard, objective and tangible" (Cohen & Mannion, 1985, p. 7). This view of mathematics supports forms of teaching based on the transmission model, where teachers are perceived as transmitting knowledge to students (Ernest, 1994) and mathematics is understood to consist of a fixed body of definite facts and procedures (Schoenfeld, 1989). This absolutist perspective perceives mathematics as black and white, consisting of right or wrong answers (Mendick, 2006) and procedures to be memorised. Mathematics, as it is experienced by professional mathematicians, people in everyday activity or students in more progressive classrooms, is of a different character. Such mathematics requires flexibility and creativity. From this perspective, mathematics is not a rigid body of facts to be memorised or transmitted from teacher to student (Lave, 1988; Hersh, 2006). Instead it emerges by engagement in problems. This conception of mathematics is more akin to the fallibilistic view discussed by Ernest (1999). In recent years, there has been a move away from absolutism in mathematics and positivism in general in the literature of mathematics education. However the research of Lyons et al. (2003) illustrates that a didactical traditional approach is still common in some second level mathematics classes in Ireland.

The perspectives that have come to replace the absolutist/positivist perspective tend to focus on the nature of mathematics and the practices of mathematicians, both as they are carried out today and from a sociohistorical viewpoint (Ernest, 1994). Ernest states that this movement is "fallibilist in its epistemology" (1994, xi). Fallibilism sees mathematics as ever evolving, with no mathematical truths that cannot be subject to revision. This perspective demands a relativistic approach as it is impossible to view mathematical truths as existing independently either of the individual learner or of the wider culture. It is this later perspective of mathematics that has become more prominent in recent times as summed up below by Gravemeijer:

> In the community of mathematics educators, the view of mathematics as a system of definitions, rules, principles, and procedures that must be taught as such is being exchanged for the concept of mathematics as a process in which the student must engage. (1994, p. 443)

## Centrality of Discourse

### Reform

Reform has been a central theme of the literature of mathematics education for a number of years. Skott (2004) notes that reform cannot be considered as 'unidirectional' or indicative of a single theoretical perspective but suggests that there are considerable differences in past approaches to mathematics education and what is currently emphasised in the field. For this reason he considers it appropriate to discuss the 'reform movement'. In response to changing conceptions of knowledge and changing conceptions of mathematics, educational reform efforts have been made in many countries, including Ireland. In 1989, The National Council of Teachers of Mathematics (NCTM) issued a document that called for widespread reform of mathematics education practices in North America (NCTM, 1989). Similarities exist on this side of the Atlantic where the Cockcroft report (1982) precipitated various reform measures in the British system. In Ireland, reform of primary mathematics education took place with the introduction of the *Primary Mathematics Curriculum* in 1971 and the introduction of the revised curriculum in 1999 (NCCA, 1999). The reform efforts continue at second level with the changes currently being introduced through Project Maths (www.projectmaths.ie). This involves the introduction of revised syllabuses for both Junior and Leaving Certificate mathematics. Changes are being made to content and assessment procedures with greater emphasis on problem solving skills and increased use of meaningful contexts and applications of mathematics. The phased introduction of the reformed syllabus is already underway. [1] On the cusp of this major reform of mathematics education at second level, it is interesting to reflect on the nature of the reform movement at primary level, more than ten years after the introduction of the revised curriculum. From the nature of some of the evidence cited in chapter 1, it may be inferred that lessons at primary level still have features of traditional teaching, particularly a reliance on textbooks (Eivers et al., 2010; Shiel & Kelly, 2001; Shiel et al., 2006).

**Mathematics as Progressive Discourse**

On examination of the pedagogy demanded by newer perspectives of mathematics, there is much reference to discourse, communication or talk (see for example Cobb, Boufi &

[1] The timeline for the introduction of the revised syllabus can be viewed at http://www.projectmaths.ie/overview/

McClain, 1997). In this context, discourse is taken to mean all forms of communication or "the totality of communicative activities" (Sfard, 2000, p. 160). Sfard conceptualises learning as "gaining access to a certain discourse" (2000, p. 160) and by extension defines learning mathematics as developing a discourse. She notes that each community, including the mathematical community, can be characterised by the distinct features of the discourse created by it. In fact, the differences between mathematics as it is experienced by traditional school communities and mathematics experienced by mathematicians themselves or practitioners of mathematics, are evident when we examine the discourses these various communities produce (Lampert,1990; Lave, 1988; Richards, 1991). This view of learning mathematics necessitates the idea that thinking is a form of communication and discourse with self. Pólya described thinking as "mental discourse" (1945/1990, p. 133), where the thinker is in conversation with himself. Sfard argues this case too, suggesting that thinking is a special form of communication and concludes that "becoming a participant in a mathematical discourse in tantamount to learning to *think* in a mathematical way" (2001, p. 5, original italics).

Indeed, the overarching idea connecting these observations is that mathematics itself is progressive discourse. This view follows the arguments put forward by Bereiter for science as progressive discourse (1994). Bereiter suggests that "there is no knowledge beyond discourse" (1994, p. 5). In fact, Sfard argues that the change to viewing knowledge as discourse leads to the conclusion that knowledge is a human construct and with regard to the communicative aspects of thought outlined above, the construction of knowledge must be social in nature (2000). One of the points made by Bereiter to support the perspective of science as progressive discourse is the notion of the process of dialectic, where those with opposing views engage in a discourse which leads to new understanding, for one or other, or perhaps both sides. This is similar to the view of mathematics put forward by Lakatos (1976), where mathematical understanding emerges when the learner follows a zig-zag path between proof and refutation. The work of Lakatos is a dialogue between teacher and students discussing various historically significant mathematical proofs. Lakatos implies that this process of refuting and refining mathematical ideas, occurs both across mathematics as a domain and at an individual level. Bereiter notes that judgements of progression are subjective but that "we can see a recursive pattern emerging in which new criticisms and alternatives keep being brought into the discourse, thus enlarging with no inherent limit the circle of those for whom the discourse represents progress" (1994, p. 6).

Considering the discourse of mathematics through the ages one cannot but notice elements of progression. Bereiter insists on the importance of the progressive aspect of the discourse and suggests that it is this progressive aspect which differentiates scientific discourse from other forms of discourse. He says that in the long term, participants must be committed to progress. He lists the "moral commitments" that are necessary to facilitate progressive discourse: the mutual understanding commitment, the empirical testability commitment, the expansion commitment and the openness commitment (1994, p. 7). These commitments involve respectively a willingness to work toward common understanding, a willingness to pose questions and propositions so that they can be tested by others, a willingness to expand the set of collectively accepted propositions, and a willingness to subject any belief to criticism in order to advance the discourse (1994, p. 7). Bereiter's descriptions of the four commitments are similar to the qualities of courage and modesty that are called for by Pólya (1954) who suggested that mathematicians need such qualities in order be able to examine their assumptions and refine them when necessary.

**Discourse in the Mathematics Classroom**

If one accepts this view of mathematics as progressive discourse, what are the implications for mathematics classes? What shape might mathematical discourse take in the classroom and where does it fit within the broader picture of mathematics as a discipline? Bereiter, speaking of science, attempts to answer these questions:

> We may think of science as a continuing discourse that went on before our time and that will continue after it: a discourse in which various people become involved in various ways and degrees and for various lengths of time, some of them altering and advancing the discourse noticeably, but most participating with no discernible effect except on themselves.
>
> On this view, classroom discussions may be thought of as part of the larger ongoing discourse, not as preparation for it or as after- the-fact examination of the results of the larger discourse. The fact that classroom discourse is unlikely to come up with ideas that advance the larger discourse in no way disqualifies it. (1994, p. 9)

Such an understanding of classroom mathematics is exciting. The discourse of school lessons *is* mathematics. Learners, at the appropriate levels, can engage with mathematics in the same manner as mathematicians and practitioners of mathematics do. Teachers do not have to wait until students are older or know the 'basics'. If one accepts the perspective of mathematics as progressive discourse, then one must accept that the classroom discourse should be mathematical in nature. The work of Lakatos (1976), taking a mathematical viewpoint and others such as Lampert (1990), working from a mathematics education viewpoint, would suggest that this would mean students arguing various positions, justifying these arguments and refining their solutions if challenged by others in the course of the mathematical discussion. The work of Bowers, Cobb and McClain (1999) would suggest that various classroom, sociomathematical and mathematical norms be in place such that moral commitments mentioned above become 'taken as shared' classroom practice. The work of Freudenthal also supports the perspective of mathematics as progressive discourse in that

RME aims for learners to actively reinvent the conventional mathematics of mathematicians by mathematising their own activity and engaging in mathematical discourse (1973).

In this way, the discourse of the classroom would mirror discourse at domain or practitioner level. The fact that presently, in a lot of cases, it does not, can also be explained by elements of Bereiter's perspective. It seems likely that the progressive element of discourse is missing in some traditional classrooms particularly those in which the tripartite initiation-response-evaluation sequence described by Mehan (1979) commonly occurs in response to 'known answer' questioning by the teacher. It is likely that this approach is influenced by teachers' beliefs that rather than reflecting mathematical discourse, classroom discourse should be "as preparation … or as after- the-fact examination of the results of the larger discourse" (Bereiter, 1994, p. 9). If one accepts this view of mathematics as progressive discourse, many questions remain, the most pertinent of which to my mind, at least, is how best to translate this theory into effective classroom practice.

**Theories of Mathematical Learning**

In general, current theories of mathematical learning fall into two large categories, either theories created from a constructivist viewpoint or theories created from a sociocultural viewpoint though many theorists see value in both perspectives (Cobb, 1994; Ernest, 1994; Jaworski, 1996). It has been suggested that these perspectives represent respectively a view of learning as cognitive self-organisation or enculturation into established practices (Cobb, 1994). Often, the divisions between these perspectives are not always absolute and theorists may borrow from both in developing theories such as social constructivism (See Ernest 1994, chapter 6). In this section I will give a brief introduction to constructivist views of learning but will focus more on sociocultural theories as these are more relevant to my research.

**Constructivism**

The central principle of constructivism, that "knowledge is actively constructed by the learner, not passively received from the environment" (Jaworski, 1996, p.1), is widely accepted (Phillips, 1995) and it has been claimed that no modern mathematics educator would admit to believing otherwise (Kilpatrick, 1987). Constructivism raises a challenge to traditional teaching approaches by suggesting that simple transmission of knowledge from teacher to student is impossible. In traditional mathematics teaching, a common objective is that students become competent at various procedures reflecting a view of teaching as training (Steffe & Thompson, 2000). This is in conflict with the constructivist view of teaching for understanding (Kilpatrick, 1987). Related to both the notion of teaching as training and teaching as transfer, it has been pointed out that when teachers are over explicit, it can lead to the "excessive algorithmatization of mathematics and the disappearance of conceptual meaning" (Cobb, Yackel & Wood, 1992, p. 5). For this reason, constructivism raises questions about the types of activities that are used in teaching. Theories of constructivism suggest that solving non-routine or challenging problems must be part of the students' experience of learning mathematics and that the discourse of the classroom should be a discourse based on inquiry (Richards, 1991).

**Socioculturalism**

The main principle of the sociocultural perspective, is to paraphrase Lave, that cognition is a complex social phenomenon which is highly situated (1988, p.1). This can be linked with perspectives on the nature of knowledge. Brown, Collins and Duguid state: "knowledge is situated, being in part a product of the activity, context and culture in which it is developed" (1989, p. 32). Learning is seen to occur when participating with others in cultural practices. For socioculturalists, the focus is not just on the individual learner, but on the learner in action in society (Lerman, 2001). This holistic view of the relationships between the individual learner and culture as they exist in wider society is at odds with the positivist epistemology that has been dominant in the past. From the sociocultural viewpoint, the knowledge domains of mathematics, philosophy or science are seen in terms of culture and learning in any of the domains is seen as a process of enculturation.

Examples of early studies carried out from the sociocultural perspective include those of Lave (1988) and Carraher, Carraher and Schliemann (1985). Lave has conducted studies which examine the mathematical practices of people in everyday situations. One of these studies was the Adult Maths Project. It consisted of observations of and experimental investigation on adult participants carrying out mathematical tasks as part of their lives in grocery shopping and in the context of Weight Watchers diet restrictions. The findings suggest that there are discontinuities between individuals' performances in real life or work situations and in school-like testing situations. The methods of working in one situation, such as the algorithms used in the school setting, are rarely transferred to the real life setting. Instead, people use features of the social setting and the environment to help solve their problems. Lave observed that in real-life situations, the context and environment often co-create both the problem and the solution.

The sociocultural perspective has implications for teaching in two main areas, that of the context of learning activity and of how learning relates to the associated knowledge domain. It seems imperative that teachers be aware of how context sensitive learning is. According to Brown, Collins and Duguid, "it is common for students to acquire algorithms, routines, and decontextualised definitions that they cannot use and that, therefore, lie inert" (1989, p. 33). This statement is supported by the findings of Carraher et al. (1987) who found, similar to Lave (1988), that often algorithms learned in school were not used to solve problems in real-life settings. Furthermore, they found that students tended to be more successful when using informal oral methods than written computation procedures. The implications of these findings are complicated further when it is acknowledged that in an effort to bridge the gap between the classroom and the real world, many students are introduced to word problems. These word problems are intended to be an approximation of a problem in real life but often are considered by children completely independently of real life knowledge (Greer, 1997). It is considered that the word problems students are asked to solve have a syntax and diction common only to mathematics word problems and thus learners begin to "rely, in important but little noticed ways, on the features of the classroom context, in which the task is embedded" (Brown et al., 1989, p. 34). Beyond the language features of word problems, other features of the classroom context that learners begin to rely on include subtle social clues, such as the facial expressions of the teacher and other students, and also the contributions of the students perceived as 'good' or 'bad' at mathematics (Stein, Engle, Smith & Hughes, 2008). When considering the context of mathematical activity in the classroom, one must be aware of how mathematical tasks are supported and constrained by the environment and culture of the classroom and how, if at all, classroom mathematics relates to real life activity.

The second major implication of the sociocultural perspective for teaching is connected with the relationship between the knowledge domain of mathematics and mathematics as it is carried out within the classroom. From the sociocultural perspective, the knowledge domain of mathematics is constituted by "the ideas and traditions growing out of centuries of mathematical exploration and invention" (Ball, 1993, p. 375). Access to this mathematical heritage is gained through a process of enculturation into the practices that are shared by the community (Cobb, 1994). However the mathematics practices of the classroom community can differ greatly from the mathematical practices of the community of mathematicians and practitioners of mathematics such as engineers or financiers (Boaler, 2009; Brown et al., 1989; Lampert, 1990). They also differ from the community of "just plain folk" (Brown et al., 1989, p. 35). Some of the differences between these communities have been discussed above and also in chapter 1. Brown et al. (1989) note that the mathematical activities of ordinary people are similar to those of practitioners of mathematics, with both groups dealing with emerging or ill-defined problems and negotiating meaning in the social setting. However, the activities of students are different because traditionally they engage with well-defined problems, act with symbols and meaning is fixed and non-negotiable. If the aim of teaching is to facilitate the process of enculturation, the activities in classrooms should be more reflective of the activities common to both mathematics practitioners and people in everyday life. Brown et al. (1989) suggest that one way of facilitating enculturation is through the cognitive apprenticeship model. In this model, learners engage in 'authentic' activity i.e. activity which is also part of the culture of practice of practitioners. Scaffolding is provided through apprenticeship or coaching and as self-confidence develops learners begin to collaborate with others and take an active part in the culture (which is in effect assumed to be modelling practitioner culture). Some of the work of Lampert (1990) and Schoenfeld (1980) can be seen to follow this model.

## The Mathematical Aspects of Mathematical Learning

Above, I have discussed constructivist and sociocultural theories of learning in relation to mathematics as that is the primary area of my research. Naturally, these theories can be applied to other areas. In this section, I will discuss concepts and theories that are particular to the learning of mathematics. Specifically, I will address instrumental and relational understanding, and the processes of reification and mathematisation.

### Relational and Instrumental Understanding

Skemp was one of the first to write extensively on different forms of mathematical understanding. He described two types of mathematical understanding, relational and instrumental understanding (1976, 1989). He describes instrumental understanding as "rules without reasons", whereas relational understanding is "knowing what to do and why" (1976, p. 2). He further elaborates that although relational understanding may be perceived as somehow 'better' than instrumental, a number of teachers and students, supported by their textbooks, are actually aiming for instrumental understanding. Some of the reasons for this are that when teaching instrumentally, results are more immediate and often instrumental understanding can appear 'easier' to achieve. This approach also fits with the notion of a fixed body of universal mathematical truths and rules and is compatible with traditional, absolutist views of mathematics. The negative side of learning mathematics in this manner is that the rules learned are not applicable in new situations. For each new problem type, a new rule must be learned. This can be a burden on the memory and can result in disenchantment with a subject perceived as full of arbitrary rules. The arguments for teaching for relational understanding include its adaptability and flexibility in new situations, the reduced load on memory and the fact that it can be a motivating factor in its own right. Skemp, writing more than quarter of a century ago, argued that the difference between the two types of understanding is so great that, in effect there are two different forms of mathematics being taught in schools. This is very similar to the arguments made in chapter 1 about the current difference between mathematics as it is experienced by practitioners and as it is experienced by students in traditional classrooms. Traditional forms of teaching generally support instrumental understanding, while more progressive forms aim to create opportunities for relational understanding which is the type of understanding most used by mathematicians.

**The Process of Reification**

The contrasting conceptions of mathematics noted by Skemp are reflected in the work of more recent writers, some of whom argue that certain dualities present in the learning process can account for some of the difficulties learners experience in mathematics. Sfard (1991) suggests that mathematical concepts can be conceived both operationally and structurally, and that in a hierarchy of conceptions, operational conceptions precede structural. She suggests that the development from the former to the later occurs through a process of interiorisation, then condensation and eventual reification. Reification seems vital to mathematical development as it allows a person to consider an abstract concept as if it were real. Sfard describes this structural understanding as follows:

> Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing. It means being able to recognize the idea "at a glance" and to manipulate it as a whole, without going into details. (1991, p. 4)

She cites empirical evidence from the history of mathematics to support this theory. For example, she explores the development of the concept of number through time and states that it was originally, and for quite a long period of time, associated with the measuring processes in which it originated. What once could only be understood at an operational level became reified, conceived as an abstract structure, without further reference to the operational processes that were the original basis for understanding. Sfard further argues this process does not just occur on the broad historical level but that this model can be used to describe the learning process of the individual. Boaler (2009) argues that mathematics is a compressible subject because of this reification process. When a concept is reified, it can be compressed and used flexibly and automatically.

***The procedural 'cul de sac'***

Gray and Tall (1994) argue that some learners never manage to reach the reification stage whereby the product of a process becomes reified. In particular, symbols in mathematics can be seen as representing either a process or a stand-alone concept ('structure' in Sfard's terms). For example, $\frac{3}{4}$ stands for both the process of division and the concept of fraction and the authors argue that flexibility in the interpretation of symbols in this manner is at the heart of successful mathematical thinking. They discuss a study carried out by Gray in

1991, which investigated the methods children used to solve simple arithmetic exercises. They found that the lower achieving students (as identified by their teachers) were more likely to use procedural techniques based on counting and suggest that "their persistence in emphasising procedures leads many children inexorably into a cul-de-sac from which there is little hope of future development" (p. 133).

Such theories may have relevance to the findings of the LANDS report, which documented a decline in some students' mathematical achievement as they progressed through some designated disadvantaged schools (DES, 2005). Is it possible that these students became stuck in a procedural rut and never developed conceptions that were fully reified and thus manipulatable structures? Gray and Tall argue that in order to make progress in mathematics, learners must move through the condensation and reification stages described by Sfard and the consequence of not passing through the phases is that "the mathematics of the more able is conceived in such a way as to be, for them, relatively simple, whilst the less able are doing a different kind of mathematics which is often intolerably hard" (1991, p. 1).

**Mathematisation**

Mathematisation is another theory of learning in mathematics that deserves attention. In discussing this idea, it is necessary to discuss RME which I introduced briefly in chapter 1. RME is the Dutch approach to mathematics education and was originally initiated in the seventies by Hans Freudenthal. Freudenthal believed strongly that in teaching mathematics, the direction should be from the environment to the mathematics and not the other way around (1981). Often, the direction taken by textbooks and teaching is from the mathematics to the environment, with learners practising examples of basic application after being exposed to the formal mathematics. For example, in many textbooks the beginning of a chapter and the learner's introduction to the topic is the formal mathematics. The bulk of a chapter will be devoted to practice of formal algorithms or procedures followed by a selection of word problems based on the algorithm or focus procedure at the end of a chapter. This scenario embodies the notion of anti-didactical inversion to which Freudenthal strongly objected (Freudenthal, 1973). Gravemeijer described this as the process whereby "the end results of mathematicians are taken as the starting point for mathematics education" (1999, p. 116). Freudenthal objected to this on the premise that it is unfair to expect learners to start at a point it has taken the history of mathematics generations to arrive at. Instead, he argued for mathematics as an activity rather than mathematics as a fully formed system (Freudenthal 1968, 1973; Gravemeijer, 1999). His vision for mathematics education and a central premise of the RME movement includes the use of context problems, which are problems where the context is experientially real for the student. The idea is that learners mathematise the context problem by describing it in mathematical terms. Then, in reflecting on this process, learners should be encouraged to mathematise their own mathematical activity (Freudenthal, 1973; Gravemeijer 1999). This gives learners the opportunity to reinvent the formal mathematics so that they "acquire their cultural heritage by their own activity" (Freudenthal, 1973, p. 58). From this statement it would seem that Freudenthal theories and by extension RME fulfil aspects of both constructivist and sociocultural implications for teaching. Furthermore, Freudenthal's ideas of mathematisation connect with Sfard's explanation of the reification of processes into objects. As Gravemeijer notes, Freudenthal speaks of the operational matter on one level becoming the subject matter on the next level (Gravemeijer 1996; Freudenthal 1971). Freudenthal contends that it is through the mathematisation of our mathematical activity that such progress is made and perhaps it is this activity that is lacking when children find themselves in the operational "cul-de-sac" described by Gray and Tall (1994).

## The Literature of Mathematics Education and the Classroom Teacher

In the two previous sections, I have discussed perspectives of mathematics and theories of mathematical learning. In the section that follows I will explore issues related to the literature of mathematics education and the classroom teacher. Initially, I will explore the role of teachers in educational research. Subsequently, I will attempt to make sense of recent themes of mathematics education research from the perspective of the classroom teacher. Specifically, I will address the question, 'how do I facilitate a discourse community in my classroom?' This question is relevant to a teacher aiming to reform his/her practice and will be discussed with reference to relevant mathematics education literature.

**The Role of the Teacher in Educational Research**

**Dissemination of and Access to Research**

Hiebert, Gallimore and Stigler point out that "archived research knowledge has had little effect on the improvement of practice in the average classroom" (2002, p. 3). Burkhardt and Schoenfeld echo this sentiment when they state: "in general, education research does not have much credibility- even among its intended clients, teachers and administrators. When they have problems, they rarely turn to research" (2003, p. 3). This is disappointing considering that much relevant research has been done and is being carried out presently. Burkhardt and Schoenfeld (2003) discuss various models of research to practice transfer and five out of their six models employ an intermediary between the teacher and the actual educational research. Some of these models include professional development, summary guides of research produced by professional bodies and policy or systemic change. Perhaps the biggest obstacle to teachers themselves being able to read and interpret research literature is the language and format educational research takes. Wertsch, Del Rio and Alvarez (1995) discuss the issue of social scientists struggling to understand the particular language of disciplines that may be related to their own. This reflects the discourse theory mentioned previously. To learn psychology or anthropology, one must gain access to the discourse of psychology or anthropology. However this makes engaging in the discourse of educational research extremely complex, as education research is not just written by educationalists but by psychologists, sociologists, mathematicians or by professionals in various branches of either these disciplines, related sub-disciplines or others. Take for example the book, *18 Unconventional Essays on the Nature of Mathematics,* edited by the mathematician Reuben Hersh (2006). It is unconventional because it collects in one place essays by philosophers, mathematicians, an anthropologist, sociologists and a computer scientist, all of whom have interesting points to make on the nature of mathematics and related issues in mathematics education. A survey conducted by Sfard (2005) in conjunction with other mathematics education researchers examined how research has been informing practice in the field of mathematics education. Many researchers noted a lack of coherence in terminology and a tendency to invent new terms. Even if physical access to relevant books and articles were unproblematic, accessing research ideas would require teachers to learn a whole new vocabulary and attempt to assess theories that often overlap and interweave with each other (Corcoran, 2009).

A further deterrent to teachers attempting to read research first hand is that even if successful when identifying theories from the research relevant to particular classroom challenges, it remains to re-translate these theories back into practice. It has been noted that "translating research into practice is a decidedly nontrivial task" (Burkhardt & Schoenfeld 2003, p. 4). Even if it is clear exactly how the research should relate to practice, time pressure can present major problems to the teacher interested in pursuing new approaches. McKernan (1987) notes from his experience within the Irish setting that as much as five hours must be spent planning and developing new materials for every hour of instructional time.

Teacher professional development programmes present an opportunity for research findings to influence practice. Hiebert et al. suggest that there is a growing consensus that effective professional development should be "long-term, school-based, collaborative, focused on students' learning, and linked to curricula" (2002, p. 3). Both Corcoran (2011a) and Kennedy (2008, 2010) have demonstrated how effective this approach to teacher professional development can be in the Irish context. Kennedy describes a collaborative relationship with teachers in which a "professional learning community" was established (2010, p. 5). This allowed the teachers to develop a balanced literacy framework suitable to their own particular context that was informed by the relevant research base. This two year intervention resulted in increased student engagement and motivation as well as achievement in literacy. Corcoran (2011a) describes her engagement with the staff of a designated disadvantaged primary school over a two –three year period which appeared to result in increased staff ownership of mathematics professional development over time. This intervention was based on Japanese lesson study, which has been recommended by Hiebert et al. (2002) as means of facilitating suitable professional development opportunities for teachers. Corcoran (2008) discusses possibilities for wider application in the Irish context. She suggests that school structures or the continuous professional programme itself could be re-examined to enable effective implementation of the lesson study approach. However, further examples of this type of professional development seem to be lacking in the Irish primary context which consists, for the majority of teachers, of elective summer courses with three extra personal vacation days as an inducement to participate (Corcoran, 2008). Corcoran (2008) and Delaney (2005) have noted the lack of mathematics focussed courses offered to teachers on these summer programmes although this has been addressed somewhat by recent DES recommendations (DES, 2010). Another meeting of practice and research occurs when teachers undertake university courses such as masters or doctorates in education. The fact that currently only qualifications earned before 5th December 2011 (DES, 2011) will be recognised for the calculation of paid allowances may discourage teachers from undertaking such study, particularly when the expense of undertaking these courses is generally met by the individual teacher. This also serves to downgrade the public perception of the knowledge that is required to teach by suggesting that qualifications beyond degree level are not necessary or desirable for teachers.

### The Difference a Preposition Makes: Research *by*, *with* and *on* Teachers

Cochran-Smith and Lytle writing in 1990 noted that teachers were often the objects of research, but that their voices were absent from the end product. Over time there has been a move from conducting research *on* teachers to conducting research *with* teachers as collaborators. Wagner (1997) notes that educational research can be viewed as a social intervention and all forms of research demand cooperation of some sort. He suggests that different forms of cooperation between researchers and teachers support different conceptions of researcher and practitioner and notes that different research questions may be more suitable to diverse cooperative practices. Shulman (1990) describes the ethical dilemmas for the researcher when deciding how to acknowledge valuable teacher contribution while protecting anonymity of school, district, or pupil.

Cochran-Smith and Lytle (1999) note that the different traditions in which research *by* teachers has productively developed are all similar to the extent that teachers are conceptualised as knowers and agents for change. The action research movement is one such tradition (Cochran-Smith & Lytle, 1990). Stenhouse helped drive the development of the action research movement in Britain. He makes a case that teacher research is important for both teachers and researchers:

> The basic argument for placing teachers at the heart of the educational research process may be simply stated. Teachers are in charge of classrooms … Moreover, there is in the research field of education little theory which could be relied upon by the teacher without testing it … The application of insights drawn from naturalistic case studies to a teacher's situation rests upon the quality of the teacher's study of his home case. Using research means doing research. The teacher has grounds for

> motivation to research. We researchers have reason to excite that motivation: without a research response from teachers our research cannot be utilized. (Stenhouse, 1981, p. 109 – 110)

Stenhouse's approach is radical as he calls for a change in the dynamics of power between teachers and educational researchers. He suggests that "researchers must justify themselves to practitioners, not practitioners to researchers" (1981, p. 113). The potential power of teacher research to upset the traditional roles of teachers and researchers has been noted by others (Cochran-Smyth & Lytle, 1999; Lampert, 2000).

Though it is accepted by many that teacher research is worthwhile, the forms such research should take is still debated. Cochran-Smyth and Lytle (1991) argue that that there is always an underlying comparison to university-based research which results in the exclusion of teacher research from research on teaching. Lampert (2000) notes that teacher research raises questions about the intended audience. She asks: "Is it meant to produce knowledge for teachers? Or for those who prepare teachers? Or for those who control teachers' working conditions?" (2000, p. 87). She notes that the change from a perspective of research carried out *on* teachers to research carried out *by* teachers has been influenced by the growing acceptance of qualitative approaches to research and identifies three issues that teacher research raises for the qualitative approach:

> … the potential for teacher research to change ideas about who is responsible for producing professional knowledge, the benefits and dangers of inserting self into social science, and the challenges of presenting the problems of practice from inside that practice. (Lampert, 2000, p. 88 – 89)

Like Stenhouse (1981), Lampert (2000) suggests that teacher research may redefine the power relations between practitioners and researchers and affect conceptions of what is appropriate applied research. However, she notes that it is difficult to distinguish between reflective practice and teacher research. A further difficulty highlighted by Lampert, is the dearth of a professional language of practice. This may be related to the lack of opportunity for collaboration, reflection or structured research in the average teacher's day (Delaney, 2005). Though teacher practitioner knowledge is recognised as valuable, the fact that it is context bound and not made public limit the scope of its application (Hiebert et al., 2002). Efforts to publicise and represent professional teacher knowledge and teacher research are complicated due to the insertion of self into the research effort which leads to questions of analysis and representation (Hiebert et al., 2002; Lampert, 2000). These issues are addressed more fully in the context of my own research in chapter 3.

A further rationale for teacher research can be found in Mercer's (2008) calls for a temporal analysis of classroom discourse. Mercer argues that research must pay attention to "the cumulative quality" of the education process (p. 3). He states:

> In order to understand how classroom education succeeds and fails as a process for developing students' knowledge and understanding, we therefore need to understand the temporal relationship between the organization of teaching-and-learning as a series of lessons and activities and how it is enacted through talk. To put it another way: as learning is a process that happens over time, and learning is mediated through dialogue, we need to study dialogue over time to understand how learning happens and why certain learning outcomes result. (p. 5)

He suggests that the lack of research in this area may be due to the substantial time commitment researchers must give to pursue this issue. Teachers are present daily in the classrooms in which this research may be carried out and have a vested interest in understanding these issues. It seems natural that they, whether independently or in collaboration with outside researchers, should play a role in answering Mercer's call.

**The Research Answers a Teacher's Question**

In this section, I will explore aspects of the literature of mathematics education relevant to a teacher attempting to teach in the spirit of reform. First, I will first discuss the reform context at primary level in Ireland. Then, I will address a question relevant to my own study and relevant to reform orientated teaching practice in general: how do I facilitate a mathematical discourse community in my classroom? The notion of a mathematical discourse community will be explained more fully with references to mathematics education literature which explicates the mathematical discourse community in action.

**Reform Context**

Much of the content of the revised *Primary School Mathematics Curriculum* (DES/NCCA, 1999a) is consistent with a discourse based inquiry approach. Problem solving is a central part of the mathematics curriculum. *Applying and problem-solving* is one of six identified skills that students should develop through engagement with the content of the curriculum (DES/NCCA, 1999a, p. 4). *Communicating and expressing* is another skill identified as central to the planned curriculum for all class levels. Problem solving is emphasised in *Mathematics Teacher Guidelines* (DES/NCCA, 1999b) which reminds teachers that although "problems in mathematics have often been seen as textbook examples at the end of a section on a particular topic" (p. 41), there can be more than one solution/strategy to solve a problem, that problems can be open or closed and that class discussion is a valuable methodology for the sharing of mathematical language and reasoning. Using guided discussion as a teaching methodology is explicitly recommended (DES/NCCA, 1999b).

The revised curriculum builds on the initial reform introduced with the 1971 curriculum. The fact that the revised curriculum was also issued in the spirit of reform can be inferred from the support and training teachers and schools received towards its implementation. This training largely focussed on novel teaching methodologies (Delaney, 2005). At both primary and second level, there is an intended curriculum focus on relating mathematics to real life and on solving problems in realistic contexts (DES/NCCA, 1999a; www.projectmaths.ie). Research into the implementation of reform of primary mathematics has been carried out. *Primary Curriculum Review Phase 1* (NCCA, 2005) included information on teachers' and students' experiences of the mathematics curriculum. Results were generally positive and teachers were broadly welcoming of the increased emphasis on real world mathematics. However, the same report suggests that students spend more time on lower order activities and have fewer opportunities to develop higher order skills such as integrating and connecting, applying and problem solving and implementing and reasoning. Teachers themselves report spending most time doing activities that focus on understanding and recalling rather than the above mentioned higher order skills. Activities based on understanding and recalling are unlikely to be genuine problem solving situations and the mathematical thinking and discourse stemming from such activities is likely to be limited to some extent. It is for this reason that Surgenor, Shiel, Close and Millar recommend a stronger focus on higher order skills and a systematic implementation of "the constructivist, discussion-based approaches" recommended by the revised curriculum (2006, p. 37). Similarly, as I mentioned I chapter 1, Eivers et al. (2010) recommend an increased classroom focus on analysing and discussing demanding mathematical problems.

***Word problems and textbooks***

To prompt mathematical discourse and the mathematical thinking involved in problem solving there is a need to present students with problems that are cognitively demanding. Over three-quarters of the teachers surveyed in NAMA 2004 report using word problems at least a few times a week (Shiel et al. 2006). If this is an attempt to incorporate problems into teaching, it may fall short of the inquiry orientation suggested by *the Primary*

*School Mathematics Curriculum* (NCCA/DES, 1999a) and called for by Surgenor et al. (2006). The nature of word problems was alluded to earlier in this chapter where it was pointed out that they are often poor approximations of real-life problems and children tend to solve them independently of real life knowledge (Greer, 1997). Greer describes the peculiar nature of word problems where in general cues in the text direct children to choose one of the four basic arithmetic operations and apply it to the numbers in the problem statement. Students can in effect become experts in this routine and experience some success but may not be able to adapt their method to unfamiliar situations or messy real life problems. Stein, Grover and Henningsen warn that some tasks are "simply a disguised way to have them practice an already-demonstrated algorithm" (1996, p. 456). These authors also note that as well as shaping the types of thinking students have opportunities to engage in, mathematical tasks can influence student beliefs about the discipline. This can also be exacerbated by the way in which word problems are presented in textbooks (Burton, 1980). As mentioned in the section on mathematisation, some Irish textbooks at primary level follow a format of presenting sequences of computational based work involving a particular operation or procedure followed by one to two pages of word problems based on the same procedure. In effect, students take their cue from their previous computational work and can disregard challenging aspects of the problem statement and real life considerations. This is in direct contrast to the RME approach which attempts to build on context problems as starting points for mathematical exploration as discussed earlier (Dekker, 2007).

As I noted previously there is evidence to suggest that a large proportion of Irish teachers are heavily reliant on mathematical textbooks. Eivers et al. have highlighted the need for future research on Irish textbooks (2010). Points of interest include to what extent textbooks are used for setting homework, in conjunction with concrete materials or as a planning guide. The most important question is possibly to what extent Irish primary textbooks reflect the intended curriculum as set out in the *Primary School Mathematics Curriculum* (DES/NCCA, 1999a) and what view of mathematics is presented in textbooks. Indications from international studies suggest that the textbooks and other resources used by teachers can vary enormously and "high mathematical achievement, as indicated by TIMSS, PISA or both, appears linked to exercises with steep gradients of difficulty, while average or low attainment seemed linked to shallow gradients" (Andrews & Sayers, 2006, p. 36). This is of course subject to the manner in which such resources are employed. A further complication can arise if the layout of texts encourages problem solving activities as extension work for higher ability students (Herbel-Eisenmann, 2007). A knock-on effect of such a layout is that lower achievers may only ever experience repetitive computational exercises and never be exposed to problem solving activities. For all of these reasons as well as the increased availability of online mathematics resources, for example www.primaryresources.co.uk; www.seomraranga.com; and www.nrich.maths.org, it seems vital that teachers become aware of some issues to consider when choosing mathematical tasks for their students and how to enact these tasks in practice so as not to lessen the cognitive demands on students (Stein et al., 1996).

### How do I Facilitate a Mathematical Discourse Community in my Classroom?

This question is central to my research and is pertinent to any teacher attempting to teach in the spirit of reform. First, I will review the literature with a view to developing an understanding of what is meant by mathematical discourse community. Then I will present a description of the Math Talk Learning Community (MTLC) framework (Hufferd-Ackles, Fuson & Sherin, 2004). This framework describes specific teacher and student actions that shape classroom discourse and highlights four vital areas or components of a discourse community: *questioning*, *explaining mathematical thinking*, *source of mathematical ideas* and *responsibility for learning*. These components will be discussed in detail with respect to the MTLC framework. I will then discuss some of the challenges of implementing this approach in action.

***Understanding what is meant by mathematical discourse community***

By a mathematical discourse community, I mean a community that engages in discourse of a mathematical nature. The complexity of this apparently simple concept becomes obvious when it is acknowledged that much of the mathematics focussed discourse that occurs at school level is not truly mathematical in nature but more like a form of 'number talk' (Richards, 1991), based on lower-level questions and an adherence to the invitation-reply-evaluation format (Mehan, 1979). Such discourse does not reflect authentic mathematical practices as it is not progressive (Bereiter, 1994) and it values answers rather than mathematical thinking. Wood (1994) discusses how differences in the patterns of interaction that occur in the discourse of teachers and students can result in different settings for learning. Her main contention is that "differences in the cultures of mathematics classrooms are realized in the existing patterns of social interaction among the participants" (1994, p. 150). She contrasts a funnel pattern of interaction with a focussing pattern of interaction. In the funnel pattern, the teacher effectively takes on the cognitively demanding aspects of the task in an attempt to guide the student to the right answer but by so doing limits the student's opportunities to engage in any meaningful mathematical thinking of his/her own. In the focussing pattern of interaction, the teacher aims to focus the student's attention on the critical mathematical feature of the problem but still leaves the student with the responsibility for its solution. In this way, attention is given to student thinking rather than the elicitation of the correct answer.

There is more to the facilitation of a mathematical discourse community than mere avoidance of the funnel pattern of interaction. Teaching experiments carried out by Lampert (1990), Ball (1993) and Dooley (2010) are helpful for showcasing classroom discourse that does not follow the traditional format. Lampert (1990) attempted to make the meaning of knowing mathematics in school closer to what it means to know mathematics at discipline level. She did this by initiating and supporting changes in patterns of social interaction such that student were engaged in making mathematical conjectures and arguments. Such an approach, similar to that taken by Ball (1993), is authentic practice in the discipline of mathematics and facilitates the creation of a local mathematical discourse community; discussing, justifying, refuting, negotiating and hopefully eventually agreeing on the definitions and mathematical 'truths' that can be accepted by the community. Dooley (2010) investigated the construction of insight by students and showed how students built on each other's ideas in whole class discourse.

Both Lampert and Ball explicitly mention the moral qualities that are needed both to teach and to learn mathematics in this manner. Lampert (1990) in particular refers to Pólya's (1945/1990) assertions about the required qualities of intellectual courage, intellectual honesty and wise restraint. Pólya's description of these moral commitments was discussed earlier and related to Berteiter's (1994) notion of the four commitments necessary to produce progressive scientific discourse. Not only should a discourse participant be open to challenge from others, a person should be able to justify his own mathematical opinion such that others understand or can test his approach. This is related to the notion of mathematical authority and taking personal responsibility for the verification of what is mathematically 'true' rather than relying on teacher or other mathematical expert for ratification. Effectively a discourse community aims to work toward common understanding satisfactory to all. This democratic principle facilitates progressive discourse but also implies a responsibility to attempt to understand the views of others and a willingness to justify and amend one's own beliefs if faced with compelling evidence.

Assuming teachers find such an aim worthwhile, is there a practical means of supporting the development of such practices in the classroom? Returning to Wood's assertions about the connection between patterns of social interaction and classroom culture (1994), it seems necessary that if a classroom culture exists that supports and encourages these moral commitments, it will be evident in the discursive patterns of interaction in that classroom. This seems to be supported by the work of Ball (1993) and Lampert (1990). It is further supported by the work of Cobb and various colleagues (Bowers, Cobb & McClain, 1999; Cobb, Boufi, McClain & Whiteknack, 1997; McClain and Cobb, 2001). Cobb, Stephan, McClain and Gravemeijer (2001) discuss their view of how the normative practices of a classroom community are constituted by the on-going interactions of teacher and students. It is in paying attention to what exactly these normative practices are that a mathematical discourse community may be realised. In these articles, Cobb and his colleagues discuss both a social perspective that focuses on the development of the group and a psychological perspective that focuses on the development of the individual. Under the group or social perspective, attention is given to classroom social norms, sociomathematical norms and classroom mathematical practices. Examples given by the authors of social norms include explaining and justifying solutions as well as an obligation to attempt to make sense of the solutions of others. Examples of sociomathematical norms are norms relating specifically to mathematics such as what counts as a different mathematical solution, an efficient mathematical solution or an elegant mathematical solution. Classroom mathematical practices are described as the "normative ways of reasoning mathematically" that emerge as students engage in mathematical tasks (Cobb, 2000).

The establishment of suitable norms in the classroom community would go some way toward fulfilling the moral commitments called for by Bereiter (1994). In light of this, a teacher querying how to facilitate a mathematical discourse community might ask what classroom social norms; sociomathematical norms and mathematical practices create the conditions for mathematical discourse? Cobb et al. (2001) do warn that a teacher cannot establish or specify a social norm on her own by the power of her authority alone. Instead, "she expresses that authority in action by initiating, guiding and organizing the renegotiation of classroom social norms" (p. 123) in interaction with her students. It is for this reason I refer to the act of *facilitating* rather than *creating* or *establishing* a discourse community.

***The Math Talk Learning Community framework***

The Math Talk Learning Community (MTLC) framework is an example of research that has successfully explored a discourse community in practice (Hufferd-Ackles, Fuson & Sherin, 2004). The researchers conducted an in-depth yearlong study in an urban Latino third-grade classroom in America. The focus classroom teacher successfully changed from traditional teaching approaches to more reform-orientated practice. She was supported by a reform-orientated teaching programme, reform focussed school management who facilitated regular meeting with colleagues, and weekly feedback from the researchers. Hufferd-Ackles et al. (2004) tracked the progress of the classroom community and created the MTLC framework which describes developmental trajectories for both teacher actions and student actions across the areas of questioning, explaining mathematical thinking (EMT), source of mathematical ideas (SMI), and responsibility for learning (RFL). These developmental trajectories track changes in teacher and student actions as the classroom community moved from operating as a traditional classroom community to a discourse community.

An overview of the levels of the MTLC framework is shown in table 2.1. There are four levels of teacher and student actions ranging from level 0 to level 3. Level 0 describes traditional teacher-directed classroom discourse and level 3 describes a mathematical discourse community. To understand how the components of this framework can be used to characterise classroom discourse, it is necessary to consider each of the components in turn and note their interplay with the other components and the general norms of the classroom community. Each of the components is discussed individually below and as the MTLC framework is an element of my framework of analysis, the developmental trajectories for teacher and students in each of the four component areas is discussed further in Chapter 3.

Table 2.1
*Overview of the levels of the MTLC framework.*

| Level | Description |
|---|---|
| Level 0 | Traditional teacher-directed classroom with brief answer responses from students |
| Level 1 | Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community |
| Level 2 | Teacher modelling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. Teacher physically begins to move to side or back of the room. |
| Level 3 | Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral role (coach and assister) |

*Note.* Adapted from Hufferd-Ackes et al., 2004, p. 88 – 90.

*Questioning*

Although Irish primary teachers are advised that "discussion, rather than just questioning, should be the basis of the interactions between teacher and child" (DES/NCCA, 1999b, p. 30), teachers must still play a role in posing appropriate questions to facilitate this dialogue. For example, teacher questions may introduce student thinking to the public discourse for other students to comment and build on (Hufferd-Ackles et al., 2004). Effective teacher questioning can also help teachers attend to student thinking, a practice which has been found to be effective (Fennema et al., 1996; Franke & Kazemi, 2001).

The relationship between higher order teacher questions and student achievement is complicated and though research findings are generally positive, there is variation in findings (Hiebert & Wearne, 1993). Boaler and Brodie (2004) suggest that focussing on the nature of the cognitive demand of the initial teacher question ignores the importance of other question types that probe student thinking and guide students through the mathematical terrain of a lesson. Timing, quality and frequency of questions must be considered as well as the instructional approach in which they are embedded (Hiebert & Wearne, 1993). Martino and Maher (1999) cite international and cross cultural research which found that open ended questions about conceptual features of problems and problem solving strategies may result in students constructing more sophisticated mathematical understandings. In one of these studies, Perry, Vanderstoep and Yu (1993) discuss how the higher order questions that were commonly used in Japan and China gave the impression that teachers expected students to face challenging conceptual questions, but American teachers did not use such questions.

The art of posing suitable questions is complicated and effective questioning by teachers can take years to develop as knowledge of both mathematics and students' ways of learning mathematics is required (Martino & Maher, 1999). The complexities that teachers negotiate when choosing questions have been complicated by reform efforts. Traditional teaching relies on an initiation-response-evaluation pattern where the classroom discourse generally does not deviate from a plan laid out by the teacher in advance (Meehan 1979; Van Zee, 1997). Wells (1993) notes that in reform orientated inquiry teaching the third part of the three part conversation structure mentioned above is generally a follow up move rather than an evaluation. Teacher follow up moves, such as *elicit* or *press* have been described by Brodie (2008). An elicit move occurs when a teacher aims to obtain a specific response from student and functions much like Wood's (1994) description of funnelling. A press move involves a press for conceptual thinking rather than a specific response (Brodie, 2008; Kazemi & Stipek, 2001). Teachers are under pressure when making these follows up moves to be both accountable to their own objectives for the lesson as well individual thinking (Ball, 1993). Productive decisions made in contingency moments may involve the recognition of mathematically significant student contributions and strong pedagogical knowledge concerning how best to capitalise on the contribution (Rowland, Huckstep & Thwaites, 2005). Reform orientated practice also calls for genuine student contributions to whole class discussion rather than simple statements of answers and more student-to-student discussion. For this reason, in reform settings, teacher questions may include asking one student to restate and explain another student's contribution or asking students to use their own reasoning about another student's mathematical suggestion (e.g. do you agree or disagree and why? See Chapin, O'Connor, Anderson, 2009).

Though much of the literature of reform is concerned with creating communities of inquiry (Goos, 2004) the area of student questions in mathematics lessons does not have the same amount literature as teacher questions. Many of the references in this paragraph refer to student questions in science lessons but for the most part the arguments and findings are also relevant to mathematics lessons. Chin and Osbourne (2008) reviewed the existing research on student questions and suggest that in general students asked few questions, the majority of which were factual or procedural. Student questions can function as a link between teaching and learning whereby students, attempting to align their previous knowledge with new stimulus, may be prompted to ask a question during the process of new knowledge construction (Cuccio-Sharripa & Steiner, 2000). Such questions may "enable adjustments to the teaching explanatory structure" (Aguiar, Mortimer & Scott, 2010, p. 174). Chin and Osbourne suggest that:

> For students learning science, their questions have the potential to (a) direct their learning and drive knowledge construction; (b) foster discussion and debate, thereby enhancing the quality of discourse and classroom talk; (c) help them to self-evaluate and monitor their understanding; and (d) increase their motivation and interest in a topic by arousing their epistemic curiosity. (2008, p. 3)

Eagle and Conant (2002) also note links with motivation as they propose problematizing content as a key means of fostering productive disciplinary engagement and by which students may develop questions of their own. Boaler and Brodie (2004) describe how in their research, when teachers began asking more conceptually orientated questions, students began to do likewise. Areas of teaching that student questions may have a potential to influence positively, including diagnosis of understanding and formative assessment that may feed into future teaching plans (Chin & Osbourne, 2008). Chin and Osbourne (2008) discuss the type of barriers that may prevent students from asking questions. These barriers include personal issues such as self-confidence and external features such as the nature of the classroom environment and the type of reaction they may receive from their teacher and peers.

*Explaining mathematical thinking*

As stated previously the notion of using mathematical discussion as an approach to teaching mathematics and higher order skills such as problem solving in a collaborative environment has been discussed and promoted in DES, NCCA and Education Research Centre literature (DES, 1999; DES, 2005; NCCA, 2005a; Eivers et al., 2010; Shiel & Kelly, 2001; Shiel et al., 2006). For productive mathematical discussion to be realised, students must have opportunities to explain and share their mathematical thinking. In reform classrooms, there is a focus on the process of solution and students may compare and contrast multiple solution strategies (Heaton, 2000). They may share their mathematical thinking and engage in justifying and defending their mathematical reasoning, effectively mirroring authentic mathematical practice (Boaler 2009; Lampert 1990). When students engage in public EMT, teachers have the opportunity to develop their understanding of students' mathematical thinking. To create opportunities for significant mathematical thinking to be shared tasks must be chosen wisely as different cognitive processes are necessary for different tasks. Henningsen and Stein argue that if students are to experience opportunities for high level mathematical thinking then they must have regular access to "dynamic mathematical activity that is grounded in rich, worthwhile mathematical tasks" (1997, p. 525).

Examining EMT in the classroom also involves a focus on the teacher's role in explaining mathematical thinking. Traditional approaches position the teacher as 'teller' of mathematical truths (Dooley, 2011a). Attempting to reform practice involves a reformulation of this role. However, negotiating what this means in practice involves much more than simply refraining from telling. It can also involve tensions around recognising when it is judiciously wise to take a directly explicit approach (Ball, 1993). Ball (1993) discusses three dilemmas of content, discourse and community that became apparent to her when attempting to teach so that her students would make conjectures in an attempt to solve mathematical problems. She notes the inherent tension between respecting students' mathematical thinking consisting of invented non-standard methods and teachers' duties as laid out in curricula that specify the standard tools and concepts of the wider mathematical community that must be taught. This is echoed by Sherin (2002) who discusses the 'balancing act' of developing a mathematical discourse community while ensuring that discussions are mathematically productive. Dooley (2011a) notes that many teachers feel obliged to tell, or explain the mathematics, thereby keeping the 'mathematical power' that could otherwise be devolved to students. In the literature of reform in America the dilemma over whether teachers should tell or not has been debated with authors such as Lobato, Clarke and Ellis suggesting that the debate cannot be reduced to a simplistic argument for or against and that telling as a teaching practice must be understood in terms of context and teacher intent (2005).

*Source of mathematical ideas*

Traditionally the classroom teacher, textbook or both, has been the source of mathematical ideas in mathematics lessons. This positioning of the teacher as SMI has links to the notion of mathematics as a rigid body of facts and procedures that may be transmitted from teacher to students. As noted previously, reform orientated instruction involves the conception of mathematics as a process in which students must engage (Gravemeijer, 1994). In this approach, student ideas are valued and students are positioned as SMI. However this can be complex for a teacher to facilitate productively in practice. Facilitating a student or students to become the source of mathematical ideas in a lesson can be problematic because students' ideas may lead classroom discourse off topic and deviate from the teacher's overall plan. This can lead to a perceived lack of efficacy on the part of the teacher (Smith, 1996). It is also complicated by negotiating how to respect and value student thinking that contains errors. Teachers' decisions about how to utilise students' ideas effectively are taken in contingency moments and require high standards of subject knowledge as well as pedagogic knowledge to be capitalised on productively (Rowland et al., 2005).

The issues around positioning students as the source of mathematical ideas are linked to those discussed above for *Explaining Mathematical Thinking*. A natural tension arises between the teacher's wish to position students' as a source of mathematical ideas and her obligation to teach a mandatory curricular program (Ball, 1993). In the American reform movement, many early efforts at including students' mathematical ideas in lessons were 'show and tell' activities where students explained how they completed a problem (Stein et al., 2008) without necessarily making any deep conceptual connections between the different solution strategies that were presented. This is discussed further below.

*Responsibility for learning*

In traditional classes students often assume low levels of responsibility for their own learning. It is often not necessary for students to engage with mathematics at a deep conceptual level because the focus on correct answers often correlates with a focus on procedural rather than conceptual thinking. In such environments, teachers focussed on eliciting the correct answer may funnel students toward this answer without directing them to the underlying conceptual issues (Woods, 1994). This component of the MTLC framework is connected with the notion of mathematical authority. The traditional approach whereby mathematical authority rests with the teacher stands in contradiction to a reform based approach where authority is devolved to students who verify what is mathematically correct in collaboration with peers (Hamm & Perry, 2002; Lampert, 1990). In reform approaches, an attempt is made to foster student agency and RFL. Boaler (2003) describes the 'dance of agency' between student, teacher and the discipline of mathematics itself and describes some of the complexity involved in developing student agency. One aim of reform approaches is to develop students' relationships with the discipline of mathematics so that they use their own agency and the agency of the discipline to verify mathematical conjectures rather than the traditional practice of relying on the judgement of the teacher.

The RFL component is the foundation for the other components of the MTLC framework. It is this feature that prompts students to ask questions of each other or of their teacher; to explain their thinking in such a way that others can understand; or to become a source of mathematical ideas by taking the responsibility to create or extend the mathematical ideas under discussion. If students take responsibility for their own learning and the learning of their peers seriously, then many of the moral commitments described by Bereiter (1994) that are necessary for progressive discourse may be realised in the classroom community. In this way, the discourse community approach can be understood to have saliencies with Boaler's (2006) descriptions of the contexts in which relational equity was promoted.

***Reform in Practice***

The call for reform in the areas of mathematical discourse and problem solving has resulted in some teachers attempting to make these features central to their teaching approach. This has been described and analysed by a corresponding response from mathematics education researchers. The MTLC framework is an example of research on teaching practice in which positive changes were noted (Hufferd-Ackles et al., 2004). Other researchers report challenges for teachers in implementing reform practice. For example, Nathan and Knuth (2003) describe a mathematics teacher's efforts to change to reform orientated practice after partaking in a professional development programme. Initially whole class discussions in her classroom were teacher centred and almost all of the interactions involved her. Communications by the students were most often directed at the teacher and not to the whole class. On attempting to change her practice to discourse based teaching, the teacher was successful in facilitating more student-student interactions but lacking clear focus of exactly what her new role should be, provided only social scaffolding and the class discussion lacked the mathematical precision she had previously offered. Such findings are similar to those of Corcoran (2008) who found that individualistic planning approaches and disjointed mathematical thinking among student teachers resulted in hybridised forms of mathematics teaching.

Ansell and Forman (2001) discuss related issues in their commentary on the multiple voices distinguished in a mathematics classroom community where the distinct voices represented the "an irreconcilable tension between the goals of the reform movement … and those of traditional instruction" (p. 137). The issue of authority is explored by Hamm and Perry (2002). They argue that the reform of mathematics education, which encourages teachers to create opportunities for students to take an active role in the creation and verification of mathematical concepts, has implications for authority. In classrooms operating according to the traditional format, the teacher and textbook are the recognised mathematical authorities but Hamm and Perry suggest that this need not be the case. In their study of six teachers, they describe one who was at least partially successful at granting authority to her students through offering repeated opportunities for students to share their mathematical thinking.

Stein, Engle, Smith and Hughes, writing in 2008, almost twenty years after the first call for reform by the NCTM (NCTM,1989), reflect on the progress that has been made toward the inclusion of productive mathematical discussion in lessons. They discuss a 'typical' reform based lesson as consisting of three parts; firstly a launching problem that is mathematically significant and solvable in multiple ways, then an exploration phase where students work on the problem in pairs or small groups and finally a third phase where the lesson concludes with a whole class discussion of the problem. Stein at al. note that in the initial reform efforts, teachers were often uncertain of how to facilitate productive mathematical discourse and often the discussions became more of a 'show and tell' where students shared their solution strategies but gained little in the way of connecting to powerful mathematical ideas and discovering links between related representations and concepts. They note the teacher's critical role in ensuring not just that the discussion occurs, but that it is productive mathematically. This can be a significant challenge due to teachers feeling a loss of control in addition to the actual mathematical and pedagogic knowledge needed to connect the classroom discussions with underlying mathematical concepts. In an attempt to address this problem, the authors present five practices that will help the novice facilitate productive mathematical discussions:

> Specifically the five practices are: (1) anticipating likely student responses to cognitively demanding tasks, (2) monitoring students' responses to the tasks during the explore phase, (3) selecting particular students to present their mathematical responses during the discuss- and-summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make the

mathematical connections between different students' responses and between students' responses and the key ideas. (Stein et al., 2008, p. 321)

In this manner, it is intended that the 'novice' teacher attempting to facilitate productive mathematical discourse in her class increases the opportunities for discourse around key mathematical ideas in the style of the 'experts' documented in the literature such as Lampert (1990), Ball (1993) or Dooley (2010).

**Approaches with similarities to the discourse community**

There are aspects of the discourse community approach that have similarities to other teaching approaches currently being promoted, for example, the notion of *Dialogic teaching*. Alexander describes five criteria of dialogic teaching which would appear to be met by a fully implemented discourse community approach:

> Such teaching is:
> · *collective*: teachers and children address learning tasks together, whether as a group or as a class;
> · *reciprocal*: teachers and children listen to each other, share ideas and consider alternative viewpoints;
> · *supportive*: children articulate their ideas freely, without fear of embarrassment over 'wrong' answers; and they help each other to reach common understandings;
> · *cumulative*: teachers and children build on their own and each others' ideas and chain them into coherent lines of thinking and enquiry;
> · *purposeful*: teachers plan and steer classroom talk with specific educational goals in view (2005, p. 14, original italics)

Alexander notes that cumulative and purposeful classroom talk appears to be particularly hard to achieve. This approach is also similar to *Collaborative reasoning* (CR), a teaching approach that aims to engage students in group discussion about controversial topics that arise from their reading (Reznitskaya et al., 2009). Student participation in CR lessons can be considered similar to students in a discourse community because they are expected to provide reasons for their positions and listen to and evaluate the reasoning of their peers. The description of a CR community as one in which "teachers and students can see themselves as co-inquirers, exploring complex concepts, improving their judgments, and discovering new meanings" (Reznitskaya's et al., 2009, p. 33) has saliencies with the discourse community.

The links between the discourse community approach, dialogic teaching and CR are particularly relevant in light of some of the findings of research carried out in these areas. Mercer (2008) discusses research findings in these and related areas. He suggests that "evidence supports the view that focused, reasoned, sustained dialogue amongst peers not only helps children solve problems together, but can promote the learning and conceptual understanding of the individuals involved" (2008, p. 8). Mercer reports on the findings of research into the *Thinking together* intervention programme which aimed to support students' use of dialogue as a tool for learning. Findings support the initial hypothesis that "reasoning is fundamentally dialogical … the use of language as a cultural tool used for collective reasoning could be expected to shape individual reasoning" (Mercer, 2008, p. 10). These findings are relevant to the discourse community approach as they suggest that appropriate activity at group level may support the reasoning of individual students. In addition, Alexander reports the beneficial effects of dialogic teaching which included the increased participation of low achievers and "quiet, compliant, attention-resisting children 'in the middle'" of a class (2003, p. 21). The discourse community approach also has saliencies with classroom discourse based on 'Accountable talk' principles where students are held accountable to the learning community, standards of reasoning and common knowledge or accepted facts (Michaels, O'Connor & Resnick, 2007).

**My research in relation to these issues**

This final section seeks to summarize some of the themes discussed so far in the context of my own research. Both sociocultural theories of learning and fallibilistic views of mathematics have resulted in a reform movement that emphasises discourse and authentic mathematical practices in classroom mathematics. In effect, this means making student mathematical thinking an object of discussion in whole class discourse. This deceptively simple concept is so powerful that rather than simply engaging in mathematical thinking, Pratt (2002) suggests that in following this approach, students should come to accept mathematics *as* thinking. This issue is at the heart of my research. It seems suitable to re-present the research questions here with a view to relating them to the issues discussed in this chapter.

- What is the nature of the discourse students in some Irish primary classrooms engage in during mathematics lessons on number strand topics?
- What is the nature of student learning in a discourse community?
- What is the nature of the experience for a teacher attempting to facilitate a discourse community?

The first question addresses the discourse of Irish primary mathematics classes and implicitly asks if the reform movement is evident in classroom discourse practices. The second and third questions address the issue of pursuing reform methods in practice. Though literature exists on this subject, the practicalities of implementing such an approach still remain complex. Stenhouse's declaration that "using research means doing research" (1981, p. 109) provides a rationale for teacher research in general and this teaching experiment in particular. In effect, my research was carried out in an effort to understand what it means to try to implement research findings about mathematical discourse in practice. The teacher-researcher perspective is not only necessary for this research but adds to the uniqueness of the research product.

## Chapter 3: Methodology

The first section of this chapter deals with the methodology or the theoretical perspective and conceptual framework which guide the research. In particular I will discuss the sociocultural perspective proposed by Lerman (2001), the notion of participation trajectories (Dreier, 1999; 2009), Wenger's (1998) conceptions of trajectories of identity in a community of practice and the conception of learning as transformation of participation that connects these concepts. In the second section, I will detail the research methods that were followed. I will discuss the data gathering procedures and main data sources. I will also introduce the framework for analysis which consists of the Math Talk Learning Community (MTLC) framework (Hufferd-Ackles et al., 2004), Boaler and Brodie's teacher question categories (2004), Wenger's (1998) conceptions of identity in communities of practice and the key practices of a discourse community. Finally, I will give an overview of the research approach, present the steps in the analysis and detail how the analysis will be presented in the rest of this thesis.

### Methodology

#### Perspective

In this research project, I have adopted the sociocultural perspective described as cultural, discursive psychology by Lerman (2001, p. 87). The basic principles of socioculturalism were discussed in chapter 2, including the idea that from the sociocultural perspective, the knowledge domain of mathematics is seen in terms of culture. Learning is seen as a process of enculturation or participation in authentic practices (Brown, Collins & Duguid, 1989). The major part of this research was a teaching experiment aimed at the facilitation of a mathematical discourse community. This involved encouraging students to elaborate on, defend and justify their mathematical thinking and to position themselves as mathematical authorities. It could be described as an attempt to engage students in authentic mathematical practices and is premised on a sociocultural approach to teaching mathematics.

From Lerman's (2001) perspective, the contributions of individual students are not simply viewed as opportunities to interpret the individual's thinking but can also be understood in terms of their effect on other members of the community, for example as possible prompts for the participation of others. Acknowledging this aspect of classroom discourse is particularly pertinent in the present context of a design experiment aimed at the facilitation of a discourse community. Lerman describes this research approach as "a particular focusing of a lens, as a gaze which is as much aware of what is not being looked at, as of what is" (2001, p. 90). Recognition of this point is also pertinent in my research. The complexity of describing and analysing the discourse of a classroom community through time necessitates a certain 'focussing of the lens' that foregrounds some issues while acknowledging contextual detail. Lerman suggests that when working from this perspective, a researcher must consider issues of culture, history and power. He states:

> Each person is the unique product of a range of socio-historical cultural communities and practices, of unconscious drives and desires, as well as propensities by virtue of genetic make-up and socio-cultural location. As a result, each person is positioned in any situation differently from any other person. (2001, p. 94)

Participants may adopt different positions depending on their prior experiences, resources or goals. Also social practice within lessons may position participants in a certain manner. For example, the teacher is generally positioned as mathematical authority from whom students are understood to receive knowledge in traditional classrooms (Hamm & Perry, 2002). It is essential to consider positioning within the classroom community in this teaching experiment in which teacher and students are expected to take on non-traditional roles.

## Conceptual Framework

Maxwell describes the conceptual framework of a research methodology as "the system of concepts, assumptions, expectations, beliefs, and theories that supports and informs your research" (2005. p. 33). He suggests that a conceptual framework should help in the refinement of research questions and goals. Three notions from the literature of sociocultural studies seemed to be particularly relevant to my work: the conceptions of participation trajectories (Dreier, 1999; 2009), the notion of negotiation of identity within communities of practice (Wenger, 1998), and the concept of learning as transformation of participation in social practices (Lave & Wenger, 1991, Rogoff, Matusov & White, 1996).

### Participation Trajectories

Dreier argues that we need to "conceptualize subjects as participants in structures of on-going social practice" (1999, p. 5). He presents four reasons for this approach. First, he suggests that by making participation central to the theory, subjects are always conceptualized as taking part in social practice. Second, he argues that using participation as a concept implies that the social practice of the local situated context must be taken account of. Third, he suggests that the concept of participation allows for and expects differences between the actions of individuals and also that individual actions must be understood as "partial phenomena in relation to social practice" (p. 6). Finally, he argues that *accepting social practice* or *acting to change current practice* can be understood as modes of participation whereby people participate in ways that reproduce the current state of affairs or contribute in ways aimed at changing this.

A novel aspect of Dreier's (1999) concept was the development of the notion of a trajectory of participation. This is echoed by Lerman who emphasises how social factors and practices are constitutive of learning and suggests that "learning is about becoming, it is about participation in practices … But people react differently in those practices, and perform their own trajectories through them" (2001, p. 88). Dreier's conception of trajectory encompasses ideas of both time and space. The temporal dimension encompasses both historic and possible future events and the spatial dimension accounts for the many different contexts an individual may experience. Dreier (1999, 2009) developed the idea of the life path of a person as a personal trajectory of participation in social practices across contexts. Over time, the nature of the individual's personal experience with different social practices in different contexts leads him/her to reflect on the similarities, differences and the interrelationships between experiences. Individuals adopt stances to "compose and structure their complex social practices" (Dreier, 1999, p.14). The complexity and diversity experienced by an individual in complex social practice may raise personal conflicts and these "conflicts raise personal issues of critique and change and turn personal stances into dynamic ones, siding for or against change" (1999, p.17). These personal stances orientate individuals and help guide their participatory actions across contexts such that participatory actions in any one context are not disadvantageous to personal concerns or goals in another context.

Rasmussen (2005) has extended Dreier's presentation of the concepts and developed the idea of participation trajectories in her research on project work using information and communications technology. She addresses the notion of how student participation is managed and suggests that the issues around studying learning over time can be addressed by considering participation trajectories. This is supported by Mercer (2008) who calls for a temporal analysis of classroom discourse. The notion of participation trajectories is used in my research in two main areas: as a means of a conceiving of students' participation in the discourse community over time, and as a means of conceiving of my own experience as teacher-researcher.

**Communities of Practice and Identity**

A related concept and one that has grown in prominence with the recognition of the sociocultural perspective is that of a community of practice (Lave & Wenger, 1991; Wenger, 1998). The three defining features of a community of practice are mutual engagement, joint enterprise and a shared repertoire (Wenger, 1998). Engagement with the joint enterprise requires negotiation and "creates among participants relations of mutual accountability that become an integral part of the practice" (Wenger, 1998, p. 78). The community may also develop a shared repertoire of resources for negotiating meaning which may include "routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice" (Wenger, 1998, p. 83).

Graven and Lerman (2003) note the potential of Wenger's theory for educational settings but stress that although this theory reconstructs learning, teaching has not been reconstituted. They note that Wenger's theory of learning built on earlier collaborative work with Lave on learning as legitimate peripheral participation (Lave & Wenger, 1991) where learning was situated in apprenticeship contexts. In such contexts, teaching is often incidental and cannot be easily compared with formal instructional settings where teachers must be accountable for learning outcomes (Goos & Bennison, 2008). Staples also addresses this issue noting that the theories about communities of practice "do not attend significantly to how an individual can shape culture, particularly in a deliberate manner, or how a community ultimately generates new practices that appear quite distinct from former ones" (2007, p. 194). Despite these shortcomings, the concept of community of practice has been used productively in mathematics education research particularly in areas involving teacher education and professional development (cf. Goos & Bennision, 2008; Corcoran, 2011).

The concept of identity is central to theories of participation in communities of practice. Lave suggests:

> Developing an identity as a member of a community and becoming knowledgably skilful are part of the same process, with the former motivating, shaping, and giving meaning to the latter, which it subsumes. (1993, p. 65)

This contrasts with common understandings of identity which are based on fixed notions of who a person is and which have connotations of being eternal and unalterable rather than constituted by a person in communication with self and others (Sfard & Prusack, 2005). Wenger suggests that because identity is constantly negotiated and renegotiated in practice, identities form trajectories within given communities of practice and across communities with a "coherence through time that connects past, present and future" (1998, p.154). This has links with Dreier's (1999) notions of participation trajectories.

Any student entering a classroom can be understood to have a multi-dimensional mathematical identity consisting of "knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions related to mathematics and mathematics learning" (Grootenboer & Zevenbergen, 2008 p. 244). Grootenboer and Zevenbergen (2008) suggest that the process of developing a mathematical identity is constituted by the relationships between teacher, students and the discipline of mathematics and posit that the teacher's role in developing the mathematical identities of students involves building relationships between the students and the discipline. They suggest that to be effective toward the goal of developing students' mathematical identities, teachers themselves must have a well-developed mathematical identity with a positive attitude toward the subject. For example, Corcoran (2008) provides examples of changes in teacher mathematical identities through participation in a lesson study community of practice. Teachers must be willing to be "vulnerable to the ways students may transform the teacher's relationship with the subject" (Grootenboer & Zevenbergen, 2008, p.

246). Such transformation may not always be positive and teachers who are aware of their own struggles to teach mathematics effectively may as a consequence face issues in their own mathematical or teacher identities. For example, McLoone (2011) describes teachers' reports of emotions such as guilt and frustration arising out of their experiences of teaching mathematics to primary students.

Because identity is developed in participation with others, the teacher and the classroom community are key influences. However, the involvement of the teacher and the classroom community is only central for a limited time in a student's life (Grootenboer & Zevenbergen, 2008). It is a student's identity and relationship with the discipline of mathematics that will remain an influence on the student's learning over time. For this reason, the student's experience of the discipline should reflect genuine mathematical practice. The aim of the discourse community was to create such an experience for students. It can be argued that within the discourse community, there is a different understanding of what it means to be 'knowledgeably skilful' (Lave, 1993) than in traditional mathematics classrooms. Competence in traditional classes is often associated with correct answers but in a discourse community mathematical thinking, whether correct or incorrect, is valued. For this reason, it was envisaged that engagement in the practices of the mathematical discourse community might have an influence on students' mathematical identities. In a similar manner, my own engagement in the processes of teaching and research can also be expected to influence my relationship with mathematics.

**Learning as Transformation of Participation in Social Practices**

Inherent in the sociocultural approach and in the concepts discussed above is the notion that learning occurs when participating with others in social practice. Learning can be understood as transformation of participation in social practice. Here participation should be understood to mean more than simply taking part and should be understood to be closer to the idea of becoming part of, or becoming a full participant in a practice (Sfard, 1998). This is related to Sfard's (2000) notion of learning mathematics as gaining access to the discourse of mathematics, or participating fully in the discourse. Rogoff et al. describe an approach where learning is perceived as "a community process of transformation of participation in sociocultural activities" (1996, p. 389). Stephan, Cobb and Gravemeijer (2003, p. 67) describe a research project where "students' learning is seen as participation in the local emerging mathematics practices." Lave and Wenger (1991), perhaps the first to formally present ideas on this conception of learning, describe learning as legitimate peripheral participation in communities of practice. This concept is explained as newcomers or learners in a community holding peripheral positions initially and becoming more central participants as learning occurs over time. The notion of learning as evolving participation in social practice has much support in the literature.

**The Conception of Learning in this Research**

In chapter 2, I presented a perspective of learning that viewed learning mathematics as becoming a participant in the discourse of mathematics (Sfard, 2000) and argued that mathematics can be understood as progressive discourse (Bereiter, 1994). In this way, it is possible to conceive of learning mathematics as becoming a participant in progressive discourse. Combining this notion with the conception of learning as transformation of participation in social practices (Rogoff et al., 1996), learning mathematics can be understood as transforming participation in practices in ways that are that are consistent with progressive discourse. This is similar to Boaler and Greeno's suggestion that "participation in social practices is what learning mathematics is" (2000, p. 172). In this manner, it is possible to conceive of the 'transformation of practices' involved in learning mathematics in the discourse community, as students' increased participation in authentic mathematical practices which facilitate progressive discourse

**Design Research**

As design research is an emerging trend, terminology and definitions are sometimes debated (Van den Akker et al., 2006). There are numerous variations in even the names of such studies varying from design research (Mehan, 2008) to design-based research (The Design-Based Research Collective, 2003; Anderson & Shattuck, 2012) to design experiment (Cobb, Confey, diSessa, Lehrer & Schauble, 2003). However there is broad agreement about the central features of design research which generally consists of the testing of a design in a situated context using multiple iterations, which lead to the evolution of design principles (Anderson & Shattuck, 2012; Cobb et al., 2003). Design research builds on the ethnographic tradition of attempting to describe and interpret events, objects and participants from within the community (Mehan, 2008) and as such is situated in a real educational context "with context being a core part of the story and not an extraneous variable to be trivialized" (Barab & Squire, 2004, p. 3).

My research proceeded in two stages. The first stage involved gathering audio recordings of mathematics lessons in other teachers' classrooms. The second stage involved a teaching experiment that I undertook in my own classroom. I had initially formed the opinion that an action research methodology was suitable for this part of the project. The aim of action research is to solve problems in context. I was attracted to the paradigm because it aims to cross the divide between theory and practice (McKernan, 1987). However on considered reflection, it seems that the emerging methodology of design research described above, which also aims to increase the relevance of research for practice, is more suitable (Van den Akker, Gravemeijer, McKenney & Niveen, 2006). In an attempt to explain the difference between action research and design research, Anderson and Shattuck (2012) quote Barab and Squire who state that in design research, the intervention or "design is conceived not just to meet local needs, but to advance a theoretical agenda, to uncover, explore, and confirm theoretical relationships" (2004, p. 5). This feature is present in my research to the extent that my focus was not just on the investigation of the discourse community approach in the local context of my classroom. Instead, I was also concerned with how this approach might facilitate student learning on a more general level.

This dual purpose has been recognised as one of five essential characteristics of design research by Cobb et al. (2003). These authors suggest that the purpose of design research is twofold: to develop a set of theories about the process of learning as well as theories about the design(s) that have been used to facilitate the learning. This element is present in my research to the extent that the aim was to explore how students learned mathematics in a discourse community and how the discourse community approach helped students learn mathematics. The second crosscutting feature identified by Cobb et al. (2003) is the highly interventionist nature of this methodology. Because design research tends to involve innovation of some sort, there is often a significant contrast between features of the intervention and features of teaching and learning typically occurring in educational settings. This element is also present in my work with the planned discourse community contrasting strongly with traditional mathematics lessons. In fact, a rationale for researching discourse in mathematics lessons outside the teaching experiment classroom was to explore to what extent the teaching experiment intervention differed from other lessons.

The third feature identified relates to the two faces of design research: prospective and reflective analysis (Cobb et al., 2003). In planning for an intervention, the designer conducts a preliminary thought experiment in which she conjectures the possible learning trajectory of students as they engage with the design activities (Gravemeijer, 1994; Cobb, 2001). These conjectures are provisional and designers must be willing to adapt, refine or even abandon their conjectures in light of reflective analysis of the actual learning that occurs when the design is implemented in the classroom. Indeed often the initial conjectures about learning trajectories or ways of supporting learning will lead to more specialised conjectures which can then be tested (Cobb et al., 2003). This element is also present in my work which in prospect conjectured that the discourse community approach was an effective way to teach mathematics. During the experiment and subsequent analysis many other questions and hypotheses arose which are detailed in chapter 6.

The fourth important feature of design research is the iterative nature of the design-analyse-redesign process. The "feed-back loop" (Kelly & Lesh, 2000, p. 198) and the "cycles of invention and revision" (Cobb et al., 2003) lead to systematic iteration. This feature was also present in my research. The teaching intervention was directed at fractions, decimals and percentages topics which occurred at different stages throughout a school year (See page 79 for more details of the teaching intervention and appendix 8 for the recording collection dates). The teaching of each of these mathematics topics can be considered as an iteration of the research project with the experience and analysis of the teaching of one topic informing the design and teaching of the next. Some of this on-going analysis was formal analysis using the analytic framework described in this chapter. However, as my research was in the specific context of a classroom teaching experiment (McClain, 2002; Dooley, 2010) where I operated as teacher-researcher, time constraints effectively meant that the majority of the formal analysis occurred on completion of the teaching experiment lessons. There are similarities between my research and McClain's (2002) descriptions of the "cycles of invention and revision" (Cobb et al., 2003) that are involved in considering how iteration occurred during the teaching experiment. McClain suggests that conjectures about how to support students' mathematical development in relation to the overall goal of the experiment are "continually being modified against the background of informal daily analyses of students' ongoing mathematical activity" (2002, p. 92) and this daily analysis informs decisions about appropriate next steps. This process would seem to be a feature of reflective, investigational teaching and Ball (2000, p. 368) describes how "the iterative process of design, experimentation, and analysis" was central to her own teaching efforts. Reflection, observation and early efforts at analysis were recorded in my teaching journal and informed decisions and planning in teaching experiment lessons on a daily basis. Anderson and Shattuck jokingly refer to this feature of design research as 'research through mistakes' and suggest that "design-based interventions are rarely if ever designed and implemented perfectly; thus there is always room for improvements in the design and subsequent evaluation" (2012, p. 16). This feature affords opportunities for the evolution and improvement of the design.

The fifth feature relates to the nature of the theories that arise from design research. These theories tend to have a practical bent and are not often generalised theories of learning on a grand scale but tend instead to take into account particular contingencies that have arisen during the research (Cobb et al. 2003). In this manner, design research "does not strive toward context-free generalizations" (Van der Akker, et al. 2006, p. 5) but aims to account for the instructional dynamic at play in educational settings. Ball and Forzani (2007) describe the instructional dynamic as the multiple interactions among teachers and students. They describe these interactions as active processes of interpretation and suggest that for educational research to be truly 'educational', it must pay attention to these dynamics. This was also a feature of my research with many of my findings specific to my class and my teaching practice. The wider import of the research is related to the insight it provides into the instructional dynamics of reform orientated teaching in an Irish primary context.

***A note on teaching experiments***

Dooley (2010) details how design research has been applied to the classroom situation in the form of the 'classroom design experiment' (Cobb, Gresalfi & Hodge, 2009) which has its roots in the teaching experiment approach to research. She details the history and evolution of this notion which is prominent in the work of Cobb and various colleagues (Cobb & Yackel, 1996; Cobb, 2000; Cobb, McClain & Gravemeijer, 2001; Cobb, Confey, DiSessa et al., 2003). The advantages of the teaching experiment over classical research methods include opportunities to study student learning in the context of an individual's participation in a local community and opportunities to study student progress over time (Dooley, 2010; Steffe, Thompson & VonGlasersfeld, 2000). Steffe et al. also note that teaching experiments provide a means of crossing "the chasm between the practice of research and the practice of teaching" (2000, p. 270).

In my own research, the aim of the teaching experiment was both the facilitation of a discourse community and the study of this instructional design. In this way, my research differs from the example of the design research based teaching experiment described by McClain (2002). She describes a teaching experiment focussed on the testing of "an instructional sequence that is designed to support students' mathematical development in a particular content domain" (2002, p. 91). The instructional design involved in my approach was somewhat broader in that it involved the testing of an *approach* to teaching rather than specific teaching activities and lessons.

**Design for learning**

My design for learning is that of the discourse community. This has been previously introduced and described in chapter 2 using descriptions from the MTLC framework (Hufferd-Ackles et al., 2004) to define different levels and components of classroom discourse. The MTLC framework proved a useful tool in both developing the design for learning as well as providing a means for analysing classroom discourse. The role it played in analysing discourse is discussed later in this chapter. In terms of specifying a design for learning, the descriptions of teacher and student actions at levels 2 and 3 of the framework are consistent with a discourse community. In this sense, although I as teacher could not specify student actions, by attempting to follow teacher actions at the higher levels of the MTLC framework, I attempted to facilitate and develop a discourse community. Table 3.1 shows MTLC framework descriptions for teacher and student actions at levels 2 and 3.

Table 3.1
*Teacher and Student Actions at levels 2 and 3 of the MTLC framework.*

| | Level 2 | | | |
|---|---|---|---|---|
| | Questioning | Explaining Mathematical Thinking | Source of Mathematical Ideas | Responsibility for leaning |
| Teacher Actions | Teacher continues to ask probing questions and also asks more open questions. She also facilitates student- to-student talk e.g. by asking students to be prepared to ask questions about other students work. | Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies. | Teacher follows up on explanations and builds on them by asking students to compare and contrasts them. Teacher is comfortable using student errors as opportunities for learning. | Teacher encourages student responsibility for understanding the mathematical ideas of others. Teacher asks other students questions about student work and whether they agree or disagree and why. |
| Student Actions | Students ask questions of one-another's work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions | Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing *fuller descriptions* and *begin to defend* their answers and methods. Others students listen supportively. | Students exhibit confidence about their own ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson. | Students begin to listen to understand one another. When the teacher request they explain other students' ideas in their own words. Helping involves clarifying *other* students' ideas for themselves and others. Students imitate and model teacher's probing in pair work and in whole-class discussions. |

| | | | | |
|---|---|---|---|---|
| | | Level 3 | | |
| Teacher Actions | Teacher expects students to ask one another questions about their work. The teacher's questions may still guide the discourse. | Teacher follows along closely to student descriptions of their thinking, encouraging students to make their answers more complete. Teacher stimulates students to think more clearly about strategies. | Teacher allows for interruptions from students during her explanations; she lets students explain and "own" new strategies. (Teacher is still engaged and deciding what is important to continue exploring.) Teacher uses student ideas as the basis for lessons or min-extensions. | The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. She supports students as they help one another sort out misconceptions. She helps and/or follows up when needed. |
| Student Questions | Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to the responses/ Many questions are "why?" questions that require justification from the person answering. Students repeat their own or other's questions until they are satisfied with answers. | Students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. Students realise that they will be asked questions from other students when they finish so they are motivated and thorough. Other students support with active listening. | Students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons | Students listen to understand then initiate clarifying other students' work for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors. |

*Note.* Adapted from Hufferd-Ackes et al., 2004, p. 88 – 90.

Hufferd-Ackles et al. (2004) describe teacher and student actions in four components of classroom discourse and the teacher actions described in these areas are useful for imagining and planning how a teacher might act in a discourse community. On closer examination, there seem to be two major features that define teacher actions in a discourse community. The first is the expectation that students should "be responsible for co-evaluation of everyone's work and thinking" (Hufferd-Ackles et al., 2004, p. 90). It is this feature that may provoke genuine mathematical discourse between students and lead to devolution of mathematical authority from teacher to students. In this sense, a major part of the design for learning is the postponement or abandonment of a teacher evaluative move so as to create opportunities for students to take an evaluative role.

The expectation that students take on the role of evaluating mathematical thinking also implies a respect for student thinking and for students as active agents in their own learning (Boaler, 2003; Steffe et al., 2000). In the discourse community approach, there is an obligation on the teacher to respect student thinking. For this reason, the teacher is more likely to ask questions probing thinking and to use the airing of misconceptions as opportunities for the learning of both the individual and the group. This respect for student thinking also involves an obligation to create opportunities for students to share their thinking which may involve refraining from explicit teacher explanations at times to create opportunities for students to take on this role. This involves a willingness to operate in contingency moments (Rowland et al., 2005) and to choose when to pursue student ideas that may lead the lesson from its planned path.

**Research Questions**

The research questions were presented previously. They are presented again here with the intention of discussing the influence of the conceptual framework on the methods followed when exploring them.

1. What is the nature of the discourse children in some Irish primary classrooms engage in during mathematics lessons on the number strand?
2. What is the nature of student learning in a discourse community?
3. What are the issues faced by a classroom teacher attempting to facilitate such a community?

### Influence of the Conceptual Framework

The conceptual framework influenced my understanding of these questions and has implications for my approach to analysis. There are implications for all three of these questions but particularly for questions 2 and 3 which address the learning and teaching of mathematics in a discourse community. These questions were explored by a teaching experiment. Although the community of practice model is not without criticism, some of which was mentioned above, the classroom community in which the experiment was implemented can be viewed as a community of practice. In terms of the teaching experiment, the aim of the facilitation of a discourse community can be conceptualised as the facilitation of student participation in authentic practice with an intended effect on student mathematical identities. Learning mathematics in a discourse community is understood to be students increased participation in authentic mathematical practices such as evaluating the thinking of their peers. In this sense my examination of learning within the discourse community is not focussed on the specific content of a lesson or a sequence of lessons. Instead my examination of learning is based on a more holistic focus on *what might be learned from the discourse community approach* and how the nature of student participation in whole class discourse may change over time. The notion of participation trajectories allows for a temporal aspect to the analysis.

## Methods

In this section I will outline the data collection procedures at stage 1 and stage 2 of the experiment and detail the data collected at each stage. I will also describe the framework for analysis of the data and the steps followed in performing the analysis. Finally, I will indicate where the findings of the analysis are presented in this thesis.

## Data Collection Procedures

### Stage 1

Stage 1 of my research involved considering my first research question: What is the nature of the discourse children in some Irish primary classrooms engage in during mathematics lessons on the number strand? This phase of the research involved the collection of audio recordings and student work from number-focussed mathematics lessons at the senior end of primary schools. The decision was made to limit the focus to lessons on number topics for various reasons. The content strands of the *Primary School Mathematics Curriculum* are number, algebra, shape and space, measures and data (DES/NCCA, 1999a). Due to the fact that number as a topic links to all of the strands, I felt that asking teachers to record a lesson focussed on number should not cause them to deviate much from their own plans. In *Primary Curriculum Review Phase 1*, (NCCA, 2005a), it was reported that the strand that received most 'approval' from teachers across class levels was the number strand. For this reason, it was felt that teachers may be more comfortable teaching number lessons than lessons in other strand areas which received less 'approval' such as data. I also felt that confining the lessons for analysis to one strand unit might allow greater scope for comparison and contrast of teaching approaches. This was also one of the reasons for confining the data collection to the senior half of primary schools. Most of my teaching experience has been in the middle to senior classes of primary school and I knew that I would most likely undertake the teaching experiment in a senior class. To maximise opportunities to apply my own teaching experience and to maximise possible linkages between the content of recorded lessons, I decided to confine data collection to senior classes, 3$^{rd}$ class to 6$^{th}$ class where students are generally 8 – 12 years old.

Seven audio-recordings were gathered from five different teachers teaching senior classes in two different schools. Full details of class levels, lesson topics and recording length are given on table 3.2. Two teachers provided additional data in the form of screenshots of board work carried out on the interactive whiteboard during their lessons. Both schools are designated disadvantaged primary schools in Dublin. It was initially hoped to collect data in a larger number of sites with contrasting socioeconomic backgrounds. However the invitations I made to target schools were not accepted resulting in a small sample. However, even if this sample size were increased, I could never hope to gather a representative sample of recordings given the scale of my research. Instead, analysis of this selection of recordings provides insight into the types of whole class discourse that occur in these particular classrooms and provides a contextual background for comparison with the mathematical discourse community that was the aim of the teaching experiment.

Table 3.2
*Overview of recordings collected at stage 1 by school, teachers, class, lesson, topic and recording length.*

| School | Teacher | Class | Main Topic | Recording length |
|---|---|---|---|---|
| St. Eithne's boys' school | John | 4th Class (lower)[2] | Decimals | 53mins 6sec |
| | Liam | 4th Class (upper) | Multiplication | 44mins 3sec |
| | Áine | 3rd Class | Time | 31mins 5sec |
| St. Ita's girls' school | Anne | 6th Class | Decimals<br>Decimals | 39mins 45sec<br>38mins 41sec |
| | Joan | 3rd Class | Fractions<br>Fractions | 56mins 5 sec<br>47mins 21sec |

The audio-recordings were collected during the 2009-2010 school year on an Olympus VN-6800PC digital voice recorder. Participant teachers were chosen on a volunteer basis after I made an initial invitation for participation to the whole staff of various schools,

[2] Fourth Class in St. Eithne's school were split for mathematics lessons into higher and lower achieving groups.

some of which I have a personal or professional connection with. All recordings and invitations to participate were carried out according to the ethical guidelines laid down by St. Patrick's College. Plain language statements for teachers, students and their parents were provided and participant teachers were asked to read a plain language statement to their students. Consent forms for teachers, students and their parents were also provided. All documents stressed that there was no obligation to participate. Copies of these documents are provided in appendices 1 – 7. I met with all teachers prior to the recordings, discussed the project and invited questions. I also met with them on completion of the recordings to provide them with an opportunity to discuss issues arising from the lesson. I also offered to provide them with transcripts of the recordings.

I was not present during recordings due to the constraints of my own teaching position and must acknowledge the limitations of this form of data collection. The audio recordings provide a form of observation but without being physically present to take field notes and without the benefit of video recordings, there is much detail that has been lost. However the reasons for choosing audio-recordings above video recordings are related to the ethical and access issues described above. Firstly, acquiring consent to participate required teachers, parents and students to give their consent. I felt that parents may not be as likely to consent to video recordings of their children. Parental reservations about video recordings have been detailed by other researchers (Dooley, 2011b). Additionally as I noted above, I could not be present in the classrooms for these recordings and therefore would not have been able to provide the teachers with support when using the video-recording equipment. I felt it was too much to ask teachers to undertake video-recording of their own lessons unsupported and felt the unobtrusive, easily operated audio-recorder was the most suitable option. Examples of student work were also collected.

**Stage 2**

The data relating to stage 2, the teaching experiment is extensive. As in stage 1, I decided to focus on number based topics for the teaching experiment. In particular I decided to focus on fractions, decimals and percentages. There were a number of reasons for this decision. Fractions, decimals and percentages have often been acknowledged as problematic for teachers and students and the teaching of fractions and decimals were mentioned by teachers as a priority area for professional development in the 2009 National Assessments (Eivers et al., 2010). Also in the LANDS report, the inspectorate describes students' poor understanding of the decimal system and the links between fractions, decimals and percentages (DES, 2005). It was envisaged that in the areas of fractions, decimals and percentages, cognitively demanding tasks with multiple solution strategies and ways of representation could be created that would motivate students to engage in whole class discourse. The opportunity also existed to set these tasks in contexts that were experientially real for students in the manner of the RME tradition discussed in chapter 2.

The design of the lessons was undertaken using the approach of learning trajectories. Clements and Samara (2009) describe learning trajectories as consisting of three parts "a specific mathematical goal, a developmental path along which children develop to reach that goal, and a set of instructional activities that help children move along that path" (p. ix). In terms of the teaching experiment, the idea of a learning trajectory operated on two different levels. A specific learning trajectory was envisaged for each of the three topics, fractions, decimals and percentages but there was also a more holistic overarching trajectory envisaged for the whole teaching experiment. The specific mathematical goal of the overall project was that students would participate in a mathematical discourse community, proposing, explaining and justifying their mathematical ideas and questioning the mathematical ideas of others thereby taking responsibility for determining what is mathematically correct. The details of the mathematical goals, developmental pathways based on key understandings and the instructional activities were developed with reference to research findings in fractions, decimals and percentages. In particular the work of Streefland (1991), Simon (2002), Tzur (1999), Simon and Tzur (2004) and Steffe (2004) were useful in planning the instructional activities on fractions. In planning the instructional activities for decimals and percentages I hoped to capitalise on the links with fractions and student understandings of same. I also hoped to foreground links with other mathematical content areas such as measures.

Lessons were recorded using the same Olympus VN-6800PC digital voice recorder as in the stage 1 recordings. At times this was supplemented with other recorders of the same type to record different pairs in conversation. In this stage of the research project, some of the disadvantages of the audio recording method were mitigated by the many other sources of data collected. For example, for most recorded lessons, a digital record of the board-work was kept. The classroom was equipped with an interactive whiteboard that runs Smart Notebook software. This software facilitates teacher and student writing as on a normal blackboard but also allows for the importation and manipulation of images. Each Notebook file can contain a number of pages with writing and images which can be saved digitally for future reference. These Notebook files are effectively a digital record of board work and proved useful when attempting to understand student contributions as many of the representations that students refer to have been recorded. These files also contain samples of students' work and students' own representations of fractions, decimals and percentages as it was common practice to have students present their work to the whole class on the board. Samples of such pages can be seen in the figures in chapter 5. In some cases, student activity sheets that I had designed or otherwise sourced were collected. These proved useful for assessment of student understanding on a day-to-day basis throughout the experiment but were not used during the analysis phase of the research due to the focus on whole-class discourse rather than individual or pair written work.

Not all lessons were recorded. The data that could be gained from recording all lessons would be interesting but overwhelming for a one-person research team. Instead lessons or sections of lessons were recorded based on their perceived potential for interesting classroom discourse. Interesting in this case should be understood to mean relevant to the research project because of the participation patterns of students in whole class discourse or their mathematical contributions to the same.

The full list of recordings is shown in appendix 8. There were 31 recordings in total collected on different days. This accounts for roughly one sixth of the total primary school year of 183 school days (DES, 2007). Considering that not all lessons on the topics of fractions, decimals and percentages were recorded and as such are not counted above, this may seem like a substantial amount of time to devote to these topics. However, often the lesson content spanned strands and fractions, decimal or percentage lessons also addressed measures topics. Similarly the majority of the recordings took place during the teaching input on fractions and in fact more time was spent on the fractions topic than on either decimals or percentages. This reflects my understanding that a strong grounding in fractions was necessary to prepare students to tackle the decimal and percentage topics. Furthermore, the teaching experiment was in its early stages at this point and I felt it important that recordings be collected at this time to have the opportunity to track the nature of students' participation. During the teaching of the decimals topic, I recorded a number of students engaged in pair work, just one representative of which is counted in the 31 recordings and listed in appendix 8. The recordings of 21/2/11 listed in appendix 8 refer to short recordings of three student pairs engaged in a mathematical task in a decimals lesson. I considered that recordings of students working in pairs might give further insight to their mathematical thinking processes.

Such recordings of pair work undoubtedly do give insights into students' thinking and styles of communicating. However the relationship between students' participation in whole class discourse and participation in pair work is complicated and would benefit from study its own right. For this reason and because of my research focus on whole-class discourse, I decided that my efforts would be better focussed on whole class settings.

In addition to this data on student participation, I kept a teaching journal in which I recorded my observations and reflections of the teaching experiment in progress. It functioned as my own personal 'feedback loop' (Kelly & Lesh, 2000) in that I often used it to reflect on the lessons and to plan necessary adjustments to the coming lessons. I often described the lessons that I had taught and noted features of students' participation that were interesting from either a teaching or a research perspective. I also sometimes used the journal to tease out my understanding of issues identified in the literature and how I saw these issues manifesting in my classroom. For example the issue of the negotiation of classroom norms was one area that I discussed in my journal. Writing the journal was an act of introspection where I engaged in dialogue with self (Holly & Altrichter, 2011). The journal includes observational data, contextual data and reflections and was a means of recording preliminary analysis. Holly and Altrichter suggest that a research diary is essential in cases such as mine where "the eye of the beholder is a variable in the research" since diaries facilitate researchers in attempting to "become spectators of their own observations and interpretations" (2011, p. 46).

***Ethical issues of the dual role***

The ethical issues involved in undertaking research in one's own classroom are complex. Regarding my role as class teacher, the lessons recorded as part of this research were planned as part of a scheme of work for mathematics in $5^{th}$ class (Students are generally 10 -11 years old in fifth class). As such, they were an integral part of the planned programme of work for all students. However, the researcher's responsibility involves giving participants free choice of whether to participate or not with no penalty for withdrawal. Therefore, all pupils (and their parents) must have full freedom to choose whether they want to be recorded or not. It is suggested that because of the special relationship that exists between teacher and student and also teacher and parents, either parent or child may feel under pressure to partake in the project (Hammack, 1997; Nolen & Vander Puten, 2007; Pritchard, 2002). Zeni warns that "children in classrooms are always vulnerable – especially if their families have little money or education" and encourages the teacher-researcher to reflect on how their research demonstrates mutual respect and justice (1998, p. 14). As it happened no parents or children opted out of the project. Perhaps it is naïve to hope that agreement to participate was connected more with Zeni's notions of respect and justice than a power imbalance in my favour. However, this teaching experiment had the principles of mutual respect, justice and equity at its heart. The mathematical discourse community that I aimed to facilitate can be described as one in which the classroom norms around social interaction position students, rather than the teacher, as mathematical authorities. By centralising student thinking, there was a redressing of the power imbalance in favour of the teacher and community responsibility for learning was encouraged. However, while my teaching may have demonstrated democratic practice, it is likely that for various reasons my research practice did not (Waldron, 2006). This is discussed further below. The letter to the board of management, consent forms of parents and students and the plain language statements are included in appendices 9 - 14.

Some of the literature about participant research in educational settings describes conflict inherent in the dual teacher-researcher role. Hammack (1996) warns that ambiguity between the roles of teacher and researcher as enacted by the one person may confound teachers' abilities to enact both responsibly and that in fact they may not even perceive any conflict between the roles. However, there are some advantages to the dual role and Baumann (1996), writing from a participant research perspective, highlights some of the positive influences research may have on teaching. For example, he describes how his teaching journal, which was initially kept with a view to its long term importance in his research effort, was a powerful source of ideas for instruction. Similarly tests that had been planned as a data gathering exercise also informed and possibly benefitted his teaching efforts. He describes time as the ultimate tension and test of his dual role in that he consistently did not have enough time for both his research duties and his teaching duties. He freely admits that often activities associated with teaching by necessity took priority. I believe this must always be the case and throughout the experiment the demands of my role as teacher always took precedence over those of my role as researcher.

***Constraints of the dual role***

In some ways my research practice was constrained by role as teacher. Constraints were both practical and ethical. For example, there is no doubt that my research would have benefitted from follow up interviews with students. Such interviews could have involved discussion of specific lesson events or more general discussion about students' attitudes toward the nature of mathematics lessons or beliefs about mathematics. On a practical level, the issue of when such interviews should occur was not negotiable as I was obliged, not just to teach mathematics but also all other subjects of the primary curriculum. There was no time in which to conduct such interviews during the school day. Ethical considerations prevented me from asking students to conduct these interviews outside the school day as the power imbalance in my favour may have obliged them to do so. I also felt that such practice may have altered our relationship as teacher and student in a way that the unobtrusive recording of mathematics lessons did not. This is not to suggest that I wished to hide my researcher role. Students appeared to understand this well for example asking questions about the cost of the digital recorders and whether they could choose their own 'fake names' or pseudonyms in any reports I would write. Instead I wanted them to feel secure in their relationship with me as teacher with no 'hidden surprises' that would break their trust. This was not an attempt to deny my researcher role but an attempt to fulfil what I understood to be my duty as teacher.

In fact, my duty as teacher would seem to have militated against involving students as co-researchers. While the teaching approach demonstrated "respect for children as social agents and recognition of the value of children's voices" this is not reflected to the same extent in the research practice (Waldron, 2006, p. 85). Constraints of my teaching position meant that it was difficult to involve students in the research process in any meaningful way beyond their position as 'research subjects' on Waldron's continuum of children's participation in research (2006). In retrospect, I realise that the informal feedback I gave students on my initial analyses while the experiment was still in progress could have benefitted from being presented in a more formal manner. This might have helped students recognise the difference between 'researcher' feedback and 'teacher' feedback. Feedback at this time consisted mainly of general comments about the nature of participation and communication in the classroom. This issue is discussed further in Chapter 8.

**Framework for Analysis**

Early attempts at analysis involved focussing on patterns of communication within the discourse (NicMhuirí, 2011a). While this form of analysis paid attention to the structure of the discourse, I felt it paid little attention to the content which became increasingly important in the teaching experiment lessons. Instead a framework for analysis was developed which is discussed below. An overview of the research by stages, details of data and analytic approach is given in table 3.3 to help orientate the reader. The framework for analysis consists of four elements: the Math Talk Learning Community (MTLC) framework (Hufferd-Ackles et al., 2004); Boaler and Brodie's teacher question categories (2004); Wenger's types of identity trajectories (1998); and the key practices of the discourse community. In this section I will describe these elements and give details of how they were used in my analysis. Because Boaler and Brodie's (2004) teacher question categories apply to the questioning element of the MTLC framework, this will be discussed within the MTLC section as will my approach to the analysis of student questions.

Table 3.3
*Overview of research by stages, details of data and of analysis*

| Stage | Details of data collected | Details of recordings transcripts and analysis carried out |
|---|---|---|
| Stage 1 | Seven audio-recordings of mathematics lessons collected from five different teachers. (Table 3.2, page 77) | Seven lessons transcribed.<br>Six lessons were analysed using the MTLC framework and Boaler and Brodie's teacher question categories. |
| Stage 2 | Thirty one lesson recordings collected in my own classroom. (See appendix 8) | Descriptive synopses were written of all recordings. Fourteen recordings transcribed (appendix 15).<br>Five transcripts were selected and analysed with MTLC framework and Boaler and Brodie's teacher question categories (table 3.9).<br>Participation trajectories of ten students were charted in the thirteen transcribed lessons with reference to the key practices of a discourse community and Wenger's descriptions of trajectories of identity. |
| | Teaching journal | Examined for insight into issues for teacher in implementing the teaching approach of the discourse community.<br>Examined for overarching theoretical issues that affected practice.<br>Examined to chart the participation trajectory of the teacher researcher and for issues of reflexivity. |

**Math Talk Learning Community (MTLC) framework**

As discussed in Chapter 2, Hufferd-Ackles, Fuson and Sherin (2004) conducted an in-depth yearlong study where the classroom teacher succeeded in developing a discourse community. Student and teacher actions were charted and developmental trajectories for teacher and students in the four key areas of questioning, explaining mathematical thinking, source of mathematical ideas and responsibility for learning. In this section I will present the details of these developmental trajectories and discuss the changes that occurred in each of the four areas. One of the reasons I chose this framework for data analysis purposes, was its potential for both informing teaching practice and analysing the same. It has been used in initial teacher education, in - career professional development (Hufferd-Ackles et al., 2004) and in the analysis of afterschool mathematics enrichment projects (Smith & Piggott, 2009).

***Questioning in MTLC framework***

The MTLC developmental trajectories for teacher and student questioning are shown in table 3.4. The feature that differentiates level 1 teacher questioning from level 0 is a focus on student thinking rather than answers. At both of these levels, it is unlikely that students will ask questions. At level 2, the teacher may ask more open questions and encourage students to ask questions of each other. At level 3, students will ask questions of each other of their own volition and these may be questions requiring justification.

There are some obvious links between teacher and student questioning actions and actions described by other components of the framework. For example the type of questions posed by the teacher may determine the manner in which a student explains his or her mathematical thinking e.g. with a single word answer or with a fuller explanation and possible justification. Similarly, teacher questions may create opportunities for students to be the source of mathematical ideas. At level 3, the descriptors for questioning have implications for responsibility for learning and hint at the underlying norms for whole class discussion. The level 3 descriptor implies that students are expected to listen to the explanations of others and ask clarifying questions of each other.

Table 3.4

*Levels of the MTLC framework for questioning: Action trajectories for teacher and students*

| Participant Actions | Level 0 | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Teacher Actions | Teacher is the only questioner. Short frequent questions function to keep students listening and paying attention to the teacher | Teacher questions begin to focus on student thinking and focus less on answers. Teacher begins to ask follow-up questions about students' methods and answers. Teacher is still the only questioner. | Teacher continues to ask probing questions and also asks more open questions. She also facilitates student- to-student talk e.g. by asking students to be prepared to ask questions about other students work. | Teacher expects students to ask one another questions about their work. The teacher's questions may still guide the discourse. |
| Student Actions | Students give short answers and respond to the teacher only. No student to student math talk. | As a student answers a question other students listen passively or wait for their turn. | Students ask questions of one-another's work on the board, often at the prompting of the teacher. Students listen to one another so they do not repeat questions | Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions and listen to the responses/ Many questions are "why?" questions that require justification from the person answering. Students repeat their own or other's questions until they are satisfied with answers. |

Taken from Hufferd-Ackles et al., 2004, p. 88 - 90.

*Boaler and Brodie's categories for teacher questions*

Boaler and Brodie's (2004) teacher question categories were developed to analyse practice in reform and traditional classes. They note that analysis of teacher questions using these categories gives some insight into the nature of opportunities for students' mathematical thinking but acknowledge limiting factors. For example, in this approach to analysis, issues of the sequencing and intent of teacher questions are ignored (2004, p. 781). However, this possible loss of detail is balanced by the usefulness of the categorisation in examining and exploring teacher questions across lessons. The categories for teacher questions and descriptions and examples are given in table 3.5.

Type 1 questions aimed at gathering information or leading students through a method tend to be associated with traditional teaching. While they are also commonly used in reform classes, teachers in reform classes are more likely to use a larger range of questions (Boaler & Brodie, 2004). These question types can also be linked to some of the descriptors for teacher and student actions in the MTLC framework. For example, a teacher is more likely to use type 4 questions probing student thinking at levels 1 and above than at level 0. Similarly, one might expect a higher proportion of type 5 questions aimed at generating discussion in a discourse community. If a teacher asks a number of type 6, linking and applying questions or type 9, establishing context questions, this may reflect the influence an RME approach on the lesson. Similarly, type 8 questions aimed at orientating and focussing students stand in contrast to type 1 questions in much the same way as the focus pattern of interaction contrasts with the funnel pattern of interaction (Wood, 1994).

*Student questions*

Off topic student questions were not considered. The remaining student questions were coded as 'questions seeking clarification about mathematics being discussed' or 'questions seeking organisational clarification'

Table 3.5
*Boaler & Brodie's categories of teacher questions with descriptions and examples*

| Question Type | Description | Examples |
|---|---|---|
| 1. Gathering information, leading students through a method | Requires immediate answer<br>Rehearses known facts/procedures<br>Enables students to state facts/procedures | How many apples did he have? |
| 2. Inserting terminology | Once ideas are under discussion, enables correct mathematical language to be used to talk about them | What is this called? Can you tell me another word for that? |
| 3. Exploring mathematical meanings and/or relationships | Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations | Describe the pattern. What would the next term be? |
| 4. Probing, getting students to explain their thinking | Ask student to articulate, elaborate or clarify ideas | Why do you think that? |
| 5. Generating discussion | Solicits contributions from other members of class | Do you agree or disagree with that suggestion? |
| 6. Linking and applying | Points to relationships among mathematical ideas and mathematics and other areas of study/life | Where else have we used this? |
| 7. Extending thinking | Extends the situation under discussion to other situations where similar ideas may be used | Would this work with other numbers? |
| 8. Orientating and focusing | Helps students focus on key elements or aspects of the situation in order to enable problem solving | What is the problem asking you? |
| 9. Establishing context | Talk about issues outside of math in order to enable links to be made with mathematics | Have you shared pizzas with your family? |

Taken from Boaler & Brodie (2004, p. 77).
Note. Some examples were adapted to suit a primary school context.

***Explaining mathematical thinking (EMT) in the MTLC framework***

The MTLC framework trajectories for teacher and students in explaining mathematical thinking (EMT) are shown in table 3.6. As a classroom community moves through these levels, a certain amount of responsibility for learning and mathematical authority is devolved to students and while the teacher may tell answers and provide explanations at level 0, students take on this responsibility at higher levels. This is linked with the idea of students, rather than teacher or textbook, as source of mathematical ideas. The descriptors at level 3 imply classroom norms where students are expected to explain and justify their reasoning. With the scope to discuss multiple strategies there may be the opportunity to develop the mathematical norms described by Bowers, Cobb and McClain (1999) associated with mathematically different solutions.

***Source of mathematical ideas (SMI) in the MTLC framework***

The MTLC developmental trajectories for teacher and students in SMI are shown in table 3.7. As with the explaining mathematical thinking component, the progression through the levels for SMI, reflects the devolution of responsibility for mathematical ideas from teacher to students. This is not a complete devolution of authority and even in communities operating at level 3, the teacher is the one who decides which ideas should be explored and how best to explore them. For example, for communities operating with a high SMI level, one would expect a larger number of Bolar and Brodie's (2004) type 5, generating discussion questions. Also similar to what was discussed above, the descriptors for level 3 in SMI imply a mathematical norm of comparing solutions to check for mathematical similarity and difference. The level 3 descriptor also has implications for patterns of interactions within the discourse that occurs in the classroom. At level 3 students may interrupt the teacher and interject their own ideas. This necessitates a pattern of interaction which is in contrast with the traditional invitation-response-feedback pattern (Meehan, 1979). The level descriptors also make explicit reference to the exploitation of student errors as opportunities for learning.

Table 3.6
*Levels of the MTLC framework for explaining mathematical thinking: Action trajectories for teacher and students*

| Participant Actions | Level 0 | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Teacher actions | No or minimal teacher elicitation of student thinking, strategies or explanations; teacher expects answer focussed responses. Teacher may tell responses. | Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in explanations herself. | Teacher probes more deeply to learn about student thinking and supports detailed descriptions from students. Teacher open to and elicits multiple strategies. | Teacher follows along closely to student descriptions of their thinking, encouraging students to make their answers more complete. Teacher stimulates students to think more clearly about strategies. |
| Student Actions | No student thinking or strategy-focussed explanation of work. Only answers are given. | Students give information about their mathematical thinking usually as it is probed by the teacher (minimal volunteering of thoughts). They provide brief descriptions of their thinking. | Students usually give information as it is probed by the teacher with some volunteering of thoughts. They begin to stake a position and articulate more information in response to probes. They explain steps in their thinking by providing fuller descriptions and begin to defend their answers and methods. Others students listen supportively. | Students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. Students realise that they will be asked questions from other students when they finish so they are motivated and thorough. Other students support with active listening. |

Taken from Hufferd-Ackles et al., 2004, p. 88 – 90.

Table 3.7

*Levels of the MTLC framework for source of mathematical ideas: Action trajectories for teacher and student.*

| Participant Actions | Level 0 | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Teacher Actions | Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math. | Teacher is still the main source of ideas, though she elicits some student ideas. Teacher does some probing to access student ideas. | Teacher follows up on explanations and builds on them by asking students to compare and contrasts them. Teacher is comfortable using student errors as opportunities for learning. | Teacher allows for interruptions from students during her explanations; she lets students explain and "own" new strategies. (Teacher is still engaged and deciding what is important to continue exploring.) Teacher uses student ideas as the basis for lessons or min-extensions. |
| Student Actions | Students respond to math presented by the teacher. They do not offer their own math ideas. | Some student ideas are raised in discussions, but are not explored. | Students exhibit confidence about their own ideas and share their own thinking and strategies even if they are different from others. Student ideas sometimes guide the direction of the math lesson. | Students interject their ideas as the teacher or other students are teaching, confident that their ideas are valued. Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons |

Taken from Hufferd-Ackles et al., 2004, p. 88 – 90.

### *Responsibility for learning (RFL) in the MTLC framework*

The MTLC developmental trajectories for teacher and students in RFL are shown on table 3.8. The progression through the levels is characterised by the devolution of responsibility for evaluation of mathematical thinking from teacher to students. At level 0 and level 1 the teacher is the ultimate arbiter of mathematical truth and is the mathematical authority of the community. However at levels 2 and 3 this authority is shared by students and students are expected to be responsible for listening to and evaluating the work of others. Students take responsibility for their own learning by asking questions. The descriptor for level 3 implies classroom norms where students are expected to explain their mathematical thinking in a manner that can be understood by others. The other students are expected to actively listen to explanations and ask clarifying questions or state why they disagree. The evaluative action of teacher is gradually taken over by students. In terms of analysis, examining the lesson recordings for the evaluative action of the teacher helped in determining the appropriate MTLC level for RFL.

### *Connections with participation trajectories concept*

The MTLC framework, derived from empirical research, can also be connected with the more theoretical concept of participation trajectories. It describes possible trajectories of participation for both teacher and students moving from a traditional classroom community to a reform orientated mathematical discourse community. However the trajectories described in the MTLC framework are not exhaustive and there are many other possible ways of participating for students including choosing not to participate. Lerman (2001) notes that in educational settings in particular, participation in social practice is not always carried out in a willing manner. Also, in practice the actual implementation may fall short of the community described at the higher levels of the MTLC framework.

Table 3.8

*Levels of the MTLC framework for responsibility for learning: Action trajectories for teacher and students*

| Participant Actions | Level 0 | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Teacher Actions | Teacher repeats student responses (originally directed to her) for the class. Teacher responds to students' answers by verifying the correct answer or showing the correct method. | Teacher begins to set up structures to facilitate student listening to and helping other students. The teacher alone gives feedback. | Teacher encourages student responsibility for understanding the mathematical ideas of others. Teacher asks other students questions about student work and whether they agree or disagree and why. | The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. She supports students as they help one another sort out misconceptions. She helps and/or follows up when needed. |
| Student Actions | Students are passive listeners; they attempt to imitate the teacher and do not take responsibility for the learning of their peers or themselves. | Students become more engaged by repeating what other students say or by helping another student at the teacher's request. This helping mostly involves showing how they solved a problem. | Students begin to listen to understand one another. When the teacher request they explain other students' ideas in their own words. Helping involves clarifying other students' ideas for themselves and others. Students imitate and model teacher's probing in pair work and in whole-class discussions. | Students listen to understand then initiate clarifying other students' work for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors. |

Taken from Hufferd-Ackles et al., 2004, p. 88 – 90.

### Wenger's Trajectories of Identity

Wenger (1998) describes possible trajectories of identity within communities of practice. These trajectories are not intended to indicate a definite destination but instead to capture the coherence of identity through time and reflect the positioning of the person within the community of practice. Wenger (1998, p. 154 –155) suggests five types of trajectories: *peripheral, inbound, insider, boundary* and *outbound* trajectories. A peripheral trajectory suggests less than full participation in community practices. An inbound trajectory may indicate current peripheral participation but a commitment to future full participation. This is linked with the idea of the legitimate peripheral participation of a newcomer transforming into fuller forms of participation over time (Lave & Wenger, 1991). An insider trajectory indicates full membership. Despite gaining full membership, evolution of practice and negotiation and renegotiation of identity will continue. A boundary trajectory indicates that identity is located in the nexus of communities of practice and Wenger notes sustaining an identity that spans boundaries is a challenge but that some individuals are drawn to such experiences. An outbound trajectory indicates outward movement from one community to another. As such negotiation of identity in this context involves the development of new relationships and the repositioning of self in respect of the original community.

### Key Practices of a Discourse Community

Wenger's conceptions of trajectories of identity were used when exploring student participation trajectories in the teaching experiment data. To identify students' trajectories of identity it was necessary to determine to what extent they engaged with the practices of the discourse community. This in turn meant specifying the key practices of a discourse community. In doing this, my reasoning was based on theoretical practices rather than identifying the practices from the teaching experiment lessons. Using theory rather than empirical study to extrapolate the key practices of a discourse community was necessary because “practice emerges from rather than results from design” (DePalma, 2009) and I could not be sure that the ‘designed’ community of the teaching experiment would show all the practices that were envisaged in the design.

The central practice of the intended discourse community was to take responsibility for mathematical learning by engaging in determining what is mathematically correct. The practices that this implies includes presenting possible solutions; explaining and justifying one’s thinking; evaluating the solutions or strategies of others; building on suggestions of others; asking questions of teacher or peers requiring clarification or justification (Hufferd-Ackles et al., 2004).

**Steps in Analysis**

**A Note on the Transcription Process**

In the sections below I will provide details of decisions I made about which data to transcribe. I would like to make a general note here on other issues related to transcription. The transcription conventions presented on page xiii are based on Dooley (2010) and were chosen for their usefulness in presenting materials in an easily comprehensible manner. An issue that arose in transcription at times was that sometimes I could not identify the students who spoke or could not determine the exact wording of their contributions. At stage 1, identifying individual students was difficult unless the teachers used the students’ names. The majority of these lessons proceeded in an orderly fashion with teachers calling on students to contribute so it did not prove as challenging as it might have been. Also the disconnected nature of the mathematical questions meant that identifying individual students was not as important as in other contexts where teacher questions might probe and thinking and press students to develop their mathematical thinking. At stage 2, identifying students proved less problematic due to my knowledge of and experience with my students. Where it was an issue was normally in the case of short interjections (e.g. oh yeah!, what?) or where overlapping contributions occurred. In the transcriptions of both stage 1 and stage 2 recordings, I used words to represent mathematical numbers and symbols as using the mathematical symbols instead might influence interpretations of the transcript (Gee, Michaels & O'Connor, 1992).

**Stage 1 Data**

Stage 1 data were collected with a view to exploring the nature of the discourse students in some primary classes engage in during mathematics lessons on number strand topics. The MTLC framework was deemed a suitable tool for exploring and describing the nature of such discourse and was used in the analysis of both stage 1 and stage 2 data. The seven recordings collected at stage 1 were all transcribed. A descriptive summary was written for each lesson and six out of the seven lessons were chosen for analysis. As discussed earlier, I had decided to limit the investigation to mathematics lessons on number topics. All supporting documentation had stated this clearly. However the main topic of Áine's lesson was time. This may have been a simple misunderstanding on her part. Equally the many natural links that exist between the topics of time and number may have influenced her decision to present this lesson. I decided to discount the recording of Áine's lessons from analysis as I felt that confining the analysis to the other lessons that focussed strictly on number would allow for greater opportunities to compare and contrast lesson activities and teaching approaches.

The remaining six recordings were analysed using the MTLC framework incorporating analysis of teacher questions using Boaler and Brodie's categories (2004). There are methodological issues in counting questions. For example, often teacher prompts which do not have the form of a question function as questions while other statements in the form of a question do not actually function as questions. I followed Boaler and Brodie's method of including "utterances that had both the form and the function of questions, and which were mathematical" and only counting repeated questions once (2004, p. 776). A similar approach was followed in the counting of student questions. The results of the analysis of the stage 1 data are presented in chapter 4.

### Stage 2 Data

#### *Thick description*

Out of the thirty-one lesson recordings, fourteen recordings were transcribed with a view to being representative across mathematics topics and over time. Appendix 15 contains details of which lessons were transcribed. A descriptive synopsis was written for every lesson. These descriptive synopses, taken in conjunction with the teaching journal in which I reflected on the progress of the experiment, constitute a 'thick description'. Ponterotto (2006) discusses the origins of the term 'thick description' which is often credited to the American anthropologist Geertz though it originated in the work of British philosopher Ryle. Ponterotto (2006) notes that the term has become ubiquitous in qualitative research though an exact definition can be elusive. He synthesises the work of others on thick description and offers a definition in which context and meaning are central:

> Thick description refers to the researcher's task of both describing and interpreting observed social action (or behavior) within its particular context … Thick description accurately describes observed social actions and assigns purpose and intentionality to these actions, by way of the researcher's understanding and clear description of the context under which the social actions took place. Thick description captures the thoughts and feelings of participants as well as the often complex web of relationships among them. (Ponterotto, 2006, p. 543)

It is my belief that my data as outlined above meets this notion of thick description. However the extent to which this thick description might be used required careful ethical consideration.

### *Ethics of thick description*

Davis (1991) describes the dilemmas involved in trying to present rich cases in detail while protecting the anonymity of participants. Her research was carried out in a medical setting but many of the same concerns arise in an educational setting. Davis writes:

> ... when I began to describe the cases on paper I realized that I was confronting a powerful paradox: the very details that needed to be falsified were just those that gave the cases their integrity and usefulness. (1991, p. 12)

She refers to issues such as gender, age, immigrant status and other identifying factors that cannot be changed to protect the identity of the participants without significantly affecting the nature of the case to be presented. In my own research, this issue is further complicated by the fact that the 'thick description' I have written about my students and our work together contains references to information gleaned in my privileged insider position as teacher. I may not have had access to this information as outsider researcher. The fact that my researcher understanding has been enriched by my position as teacher would be unproblematic if I could share and describe openly my insider knowledge. For example, in my teaching journal, I could comment that a particular student's non-engagement in lesson activities may be related to his past experience in school, specific issues in his home life or perhaps a suspected learning difficulty. However in presenting my interpretations of his choice of participation to a wider audience, I must take care to protect his anonymity and privacy and present only the information that I have explicit consent to do so. Beyond that, I must also ensure to protect the right to privacy of my colleagues. The case of my teaching colleagues is particularly interesting because background knowledge that has helped shape my interpretations of student actions was gained either in informal conversation with colleagues of through our shared prior teaching experience. My colleagues were and are participants in the educational experiences of my students but I am not free to describe my understanding of those experiences to protect the right to privacy of all parties.

***MTLC analysis***

The data collected at stage 2 was intended to explore the nature of teaching and learning in a discourse community. The MTLC framework was used to explore the nature of teacher and student practices during the teaching experiment. The level descriptors also facilitated reasoning about the relative strength of the discourse community. In choosing what lessons to analyse in depth, I considered the following issues. I decided that they must be chosen in such a way that they represented different time periods as well as different mathematical topics. As can be seen from the data list in appendix 8, the teaching input on fractions occurred before the input on decimals. This in turn was ahead of the input on percentages. Many decimal and percentage lessons focussed on links to the previously covered topics. In choosing lessons within topics, I decided to choose lessons that best showed the features of the emerging discourse community and some of the challenges for the teacher. It must be acknowledged here that not all of the lessons in the data list above could be considered successful in terms of the lesson objectives or the overall goals of the experiment. These 'failures' I reflected on in detail in my teaching journal and I will discuss further in Chapter 6. It is also necessary to point out that the lessons chosen for analysis are not intended to be models of exemplary practice. Instead they are chosen for the insights they may provide into what it means to learn mathematics in a discourse community and what it means to attempt to facilitate such a community. The lessons were also chosen with a view to connections that can be traced across time either in mathematical concepts or forms of student participation.

In the end, five lessons were chosen for MTLC framework analysis, two from the fractions topic, one from the decimals topic and two from the percentages topic. The lessons chosen for analysis are labelled as fraction, decimal or percentage lessons in table 3.9. The table also shows the date the recording was made, the length of the recording, a brief description of the content and some initial points of interest. The results of the analysis of these lessons are presented in Chapter 5.

Table 3.9
*Stage 2 recordings analysed using the MTLC framework by mathematics topic, date recorded, length, brief description and points of interest.*

| Name (Mathematics topic) | Date recorded and length of recording. | Brief description | Points of interest |
|---|---|---|---|
| Cutting Pizzas (Fractions) | 29/11/2010 34 min 1s | The main activity of this lesson involved discussing how best to share a set number of pizzas between children and what fraction of a pizza individual children would get in each case. | Real life concerns affected how some students approached mathematical problems. |
| Fraction problems (Fractions) | 10/12/2010 18 min 35s | Students worked in pairs or groups of three on a worksheet showing a fictional computer game character with two rectangular bars representing his energy and his ammunition. The students worked together to find out what fraction of his ammunition and energy he had left. They also worked on word problems involving finding fractions of metric quantities. The recording contains the whole class discourse around their solutions. | Student question diverted discussion from planned agenda. Positioning within the community and the social issues of the discourse approach. |
| Dienes' decimals (Decimals) | 18/2/2011 3 min15 s; 26 min 37 s (paused for interruption by visitor to class) | In this lesson Dienes' blocks were used to represent decimal numbers in the context of points won by a classroom group. The starting number of 0.44 was increased in increments of a tenth and the resulting decimal numbers were discussed. Then we considered a group of four decimal numbers and discussed which would be the largest. After this students worked on similar exercises in pairs. | Student thinking did not become an object of discussion to the same extent as other teaching experiment lessons |

| | | | |
|---|---|---|---|
| Equivalence challenge (Percentages) | 28/3/2011<br>12 min 56 s (introduction)<br>5 min 16s; 3 min 21s (whole class correction paused for interruption for by visitor) | The lesson began by challenging students to find different ways to write 10% in fraction and decimal form. A similar activity was carried out for 20%. Then the students worked on a worksheet which involved identifying which fractions and decimals were the same as a given percentage. This was then corrected in a whole class session with the main discussion here centring on whether 25% = $\frac{1}{4}$ was also equal to $\frac{1}{25}$. | The pace and mathematical level of the discourse in this lesson appeared to inhibit the participation of lower achievers. |
| Percentage present and absent (Percentages) | 31/3/2011<br>24 min 3s | The main activity of this recording was considering what fraction of the class was present and absent and how to convert these fractions to a percentage. | Students showed responsibility for learning both in their mathematical contributions and in their demands for explanations from their peers. |

*Analysis of participation trajectories of students*

In a bid to explore the nature of learning in a discourse community at an individual level, student participation trajectories were explored. There are methodological issues relevant to examining student participation trajectories through time. As Mercer (2008) has pointed out the temporal aspects of learning are under addressed in research literature. Although Dreier's notion of participation trajectory (1999) can be used to conceptualise learning over time, suitable data interrogation techniques are hard to find. Simplistic quantitative approaches counting the number of contributions to class discourse are insufficiently detailed. I trialled using Nvivo to track the nature of student contributions across lessons. However even on the most basic coding task of coding student contributions according to whether they were invited by the teacher or not, I realised that data coded in a specific way, were not particularly similar. For example contributions that could be coded as unprompted by the teacher could vary enormously in terms of context. In fact, such contributions could represent students proposing their own mathematical ideas, commenting on other students' contributions or defending their own mathematical argument in the face of objections from their peers. Furthermore, Mercer's (2008) assertion that "the same act repeated cannot be assumed to be "the same" act in repetition, because it builds historically on the earlier event" creates problems about using coding schemes that do not acknowledge the temporal nature of discourse. For this reason, it should be no surprise that I could not devise any systematic coding scheme that circumvented the embedded nature of student contributions in specific times and contexts.

Instead, I re-examined the data of lesson transcripts in order to write a thick description of student participation trajectories. This involved charting the nature of participation of students in thirteen of the fourteen transcribed lessons with reference to the key practices of the discourse community. The one excluded transcription consisted of pair-talk rather than whole class discourse. The participation of all students was not explored but because of concerns about the nature of the experience for lower achievers, all lower achieving students were tracked. The trajectories of the six identified lower achievers, two middle achievers and two higher achievers were tracked across the thirteen transcribed lessons with reference to the key practices of the discourse community. The categorisation as higher, middle or lower achiever was based on results achieved on a standardised mathematics test carried out the previous school year. Attention was paid to the nature of the contribution and the nature of how the student came to speak. Facets of the nature of the contribution that I examined were whether the contribution was mathematically correct or incorrect; the degree to which it was confidently and coherently stated; whether it contained a question for me or another student; whether it built on the solutions of others or came to be built on by others and whether it gave an indication of ability or emotion. In this way, a description of the nature of student participation over time was created. I then linked the nature of the student's participation with Wenger's (1998) descriptions of different trajectories of identity. The results of this analysis are presented in chapter 6.

***Analysis of the participation trajectory of the teacher-researcher***

The nature of my experience as teacher-researcher was interrogated to explore the issues for the teacher in aiming to facilitate a discourse community. This phase of the analysis was also necessary to address the issue of reflexivity and the complexity of the dual role as teacher and researcher. The data for this analysis were drawn from the teaching journal which I kept throughout the experiment and from a close reading of the descriptive synopses of the teaching experiment lessons. The teaching experiment was an attempt to trial theory in practice and by reflecting on these issues I was in fact meditating on the tensions of implementing theory in practice and the practice based problems that resulted. Reflecting on my reflections on completion of the teaching experiment gave rise to another layer of analysis that identified overarching theoretical issues that frame the practical issues. Both the practical issues I faced during the teaching experiment and the overarching theoretical issues are discussed in chapter 6. Issues of reflexivity and self-identity are addressed in chapter 7 where I explore my own personal trajectory through the teaching and research experience.

**Questions of Validity**

Kelly (2006) discusses the issue of quality criteria within design research in relation to the 'commissive spaces'. He explains that a commissive act is one which is in accordance with certain background assumptions. He explains:

> Communities of practitioners develop shared commitments. These commitments – to background assumptions, acceptable verbal moves, adherence to standards of evidence, warrant, data and technique – constitute the space in which research conversations can occur. (2006, p. 111-112)

Kelly notes that design researchers violate many of the assumptions of the commissive space of researchers working with randomized field trials which he describes as "confirmatory and conservative" (2006, p.113). That "exploratory and ambitious" design research does not operate in the same commissive space as other research paradigms is not a problem as long as design researchers do not make claims more suited to the commissive spaces of other paradigms (Kelly, 2006, p. 114). For example, Kelly (2006) suggests that the making of strong causal claims is not appropriate within design research.

In a similar manner, the concepts of reliability and validity, which are rooted in the positivist tradition, can be understood differently depending on whether qualitative or quantitative research is being carried out (Golafshani, 2003). Golafshani (2003) suggests that because post positivist research approaches conceive of knowledge as a social construction, conceptions of reliability and validity that imply an objective knowledge or truth are unsuitable. Instead often in the qualitative perspective terms such as credibility, transferability and trustworthiness are more likely to be used. With Rubin, I believe that:

> The quest should not be for the fool's gold of objectivity, but for the real goal of self-awareness. For it is not our subjectivity that traps us, but our belief that we can somehow be free of it. (1981, p. 103 quoted in Jansen & Peshkin, 1992, p. 703)

Subjectivity is especially relevant in participant research such as mine. Peshkin argues that researchers "should systematically identify their subjectivity throughout the course of the research" (1988, p. 17) and notes that his efforts to be alert to his own subjectivity involved an alertness to the emotions which the research experience aroused. He argues that this alertness helps in the search for genuine data rather than data which the researcher's "untamed sentiments" seek out (p. 20). Lather argues that there are "no formulas to guarantee valid social knowledge" and that researcher's best tactic is to engage in "vigorous self-reflexivity" (1986, p. 267). This vigorous self-reflexivity seems particularly important for the practitioner researcher and I have attempted to address it with a study of my own personal participation trajectory in Chapter 7.

**Representation of Self in Research**

The conception of reflexivity most obvious in my research is that of reflexivity as introspection, where the researcher's reflections, intuitions and thinking are used as primary evidence (Finlay, 2002). This process has been followed in my research when the observations made in my teaching journal became a source of data in later stages of the analysis. However Finlay notes that personal revelations are not an end in themselves and should be used only as springboards for further insight. This observation is similar to Behar (1996) who suggests that vulnerable research writing which includes the researcher's own experience should only include the personal elements if through doing so it brings the reader to somewhere he/she may not have got to without it.

This is connected with how to represent the self and acknowledge the emotional impact of the research in a way that is acceptable by academic standards. Behar (1996) suggests that the emotion has just recently been acknowledged by 'the academy' but the limit of what is acceptable or necessary in academic writing is still very much in question. She discusses the challenges of writing vulnerably:

> Writing vulnerably takes as much skill, nuance, and willingness to carry through on all of the ramifications of a complicated idea as does writing invulnerably and distantly. I would say it takes yet greater skill. The worst that can happen in an invulnerable text is that it will be boring. But when an author has made herself or himself vulnerable, the stakes are higher: a boring self-revelation, one that fails to move the reader, is more than embarrassing, it is humiliating … Efforts at self-revelation flop not because the personal voice has been used, but because it has been

> poorly used, leaving unscrutinized the connection, intellectual and emotional, between the observer and the observed (1996, p.13 – 14).

The challenge this represents is immense. Behar suggests that in considering how to present the self in research, one must consider both the real intellectual and emotional connection with research participants and potential connections with future readers. In my case, this is complicated by the fact that I am both observer and observed, observer of my students and observer of myself. The connections between my students and myself are easier scrutinised now that they are no longer in my classroom and I have some sense of faux-distance. The emotional connection with these students, like other groups that have passed through my classroom, fades in intensity with time. However, this research has documented the connection forever, transforming it even as it is transcribed; reifying our real community into turns of speech where my students have pseudonyms. In many ways, this cleanses the teaching experiment of its emotional aspects and hides my very real worries about the experiences of lower achievers and our many shared joys about mathematical successes.

Considering 'emotional and intellectual' connections with the possible audience of the research is not straightforward either. The intended audience is primarily academia but I would like to believe that this text might be accessible and even useful for teachers too. The emotional and intellectual response these different audiences may have to this work complicates my decisions about self-representation. Aspects of self-revelation may be useful for the academic audience in understanding my personal trajectory through the process of teaching and research but may not be palatable. Linking emotions and research can be seen as 'anomalous' because of the way reason is seen as in opposition to emotion in traditional positivist approaches (Holland, 2007). Newer perspectives of knowledge acknowledge "the impossibility of the detached researcher on a scientific quest for objectivity and truth"

(Holland, 2007, p. 196) but the tradition of emotionless reports written in third person still persists in some areas (Hyland, 2002).

It is hoped that that some of the research experience, presented in emergent form, may connect with teachers who encounter dilemmas in their own practice. By this, I mean that by including questions I asked and reflected on in my teaching journal throughout the experiment, I am not representing research findings. Instead, these extracts are reflections on the experiment in progress. Some of these are presented in chapter 6 and chapter 7. The fact that such 'work in progress' data are not often presented leaves the impression that the results of research findings emerge in an obvious manner as undisputed truths from the research process. Such representations hide all traces of the creative process of invention and the doubts and wrong turns made by the inventor along the way (Gravemeijer & Doorman, 1999). The act of including information on my doubts is intended to provide a fuller description of the research process and also to provide a record of reflection in teaching and research practice.

**Summary**

In this chapter, I have presented details of my conceptual framework which is based on a sociocultural view of learning and Drier's conception of participation trajectories. The idea of the negotiation of identity in communities of practice is also central to my research. I have presented details of the data gathering procedures which were followed at stage 1, a gathering of audio-recordings from five primary mathematics classrooms; and stage 2, a teaching experiment I carried out in my own classroom. I then presented the four major elements of my framework for analysis the MTLC framework; Boaler and Brodie's teacher question categories; Wenger's descriptions of trajectories of identity and the key practices of a discourse community. Table 3.3 (page 86) shows an overview of the data and the details of the analysis followed at different stages.

## Chapter 4: The Lie of the Land

In this chapter I will present analysis of data from the first stage of my research. Stage 1 involved consideration of the question, what is the nature of the discourse children in some Irish primary classrooms engage in during mathematics lessons on the number strand? It was carried out with the intention of developing a sense of 'the lie of the land' in relation to discourse in some Irish primary mathematics classrooms. Audio-recordings were gathered from five different teachers teaching senior classes in two different schools. As I discussed in chapter 3, six of these recordings were analysed using the MTLC framework (Hufferd-Ackles, et al., 2004) and Boaler and Brodie's (2004) teacher question categories. First I will present details of John's and Liam's lessons which took place in St. Eithne's boys' school[3]. In St. Eithne's school, streaming had been enacted in senior mathematics classes and John's taught the lower achieving fourth class group, while Liam taught the higher achieving group of the same class level. The discussion of Liam's lesson will be presented in summary form. Further details of this lesson and early attempts at analysis based on discourse patterns are described in NicMuirí (2011a). I will then present the analysis of Anne's and Joan's lessons which took place in St Ita's girls' school, at fifth and third class respectively. Both schools were designated disadvantaged (DEIS band 2).[4]

### Recording 1: John's Lesson

**Dienes' Blocks**

In this decimal lesson John used Dienes' blocks, which are often used to teach whole number place value concepts, to represent decimal fractions. Figure 4.1 shows how the different blocks can be used to represent whole numbers values. To use the same materials to represent decimal fractions it is necessary to consider the cube that represents a thousand in

[3] The names of schools, teachers and students have been changed to protect the identity of participants.
[4] The DEIS scheme for designated disadvantaged schools was discussed in chapter 1. Disadvantaged schools are classed according to the levels of disadvantage in the population of parents. DEIS band 1 schools are considered relatively more disadvantaged than band 2 schools and qualify for favourable teacher-pupil ratios in the junior end of primary schools.

figure 4.1 as a one instead. Then the 'flat' that represented one hundred could be considered to be a tenth and the 'rod' that represented ten for whole number values could be considered as one hundredth. The small cube that had the value of one for whole number representation would then be taken to be a thousandth. Thousandths were not considered in this lesson as they are not part of the fourth class curriculum (DES/NCCAa, 1999).

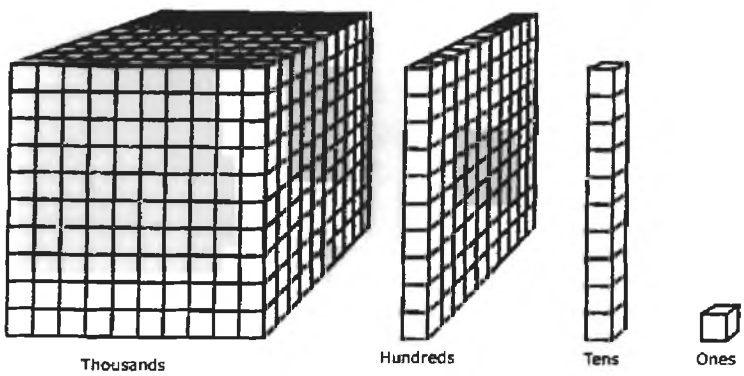



Figure 4.1. Dienes' materials used to represent whole-number values. Illustration taken from Lesh, Post, & Behr (1987), http://www.cehd.umn.edu/ci/rationalnumberproject/

**Descriptive Synopsis**

This lesson occurred at fourth class level where students are 9 – 10 years old. This group was with the lower achieving group and the lesson was just over 53 minutes long. The students were talkative and off topic conversations were audible on a number of occasions. John introduced the lesson with an activity where students used Dienes' materials to show units, tens, hundreds and thousands. Then he invited students to use the same materials to show units, tenths and hundredths. Some students struggled to do this correctly. John then presented a selection of Dienes' blocks and asked students how to write the numbers. In this task the Dienes' materials were intended to represent units, tenths and hundredths. Some students did not seem to grasp what was being asked of them and many students gave incorrect answers sometimes based on the whole number understandings of the Dienes' blocks representation. John directed the students to an exercise in their textbooks, *Mathemagic 4* (Barry, Manning, O'Neill & Roche, 2002), involving writing fractions in decimal form (figure 4.2). In the first exercise, fractions were in the range to $\frac{1}{100}$ to $\frac{9}{100}$. John circulated and attempted to keep students on task. He corrected the work of students who had finished the question and directed them to proceed to the next question which involved writing decimals in fraction form where decimals ranged in value from 0 to 0.09.

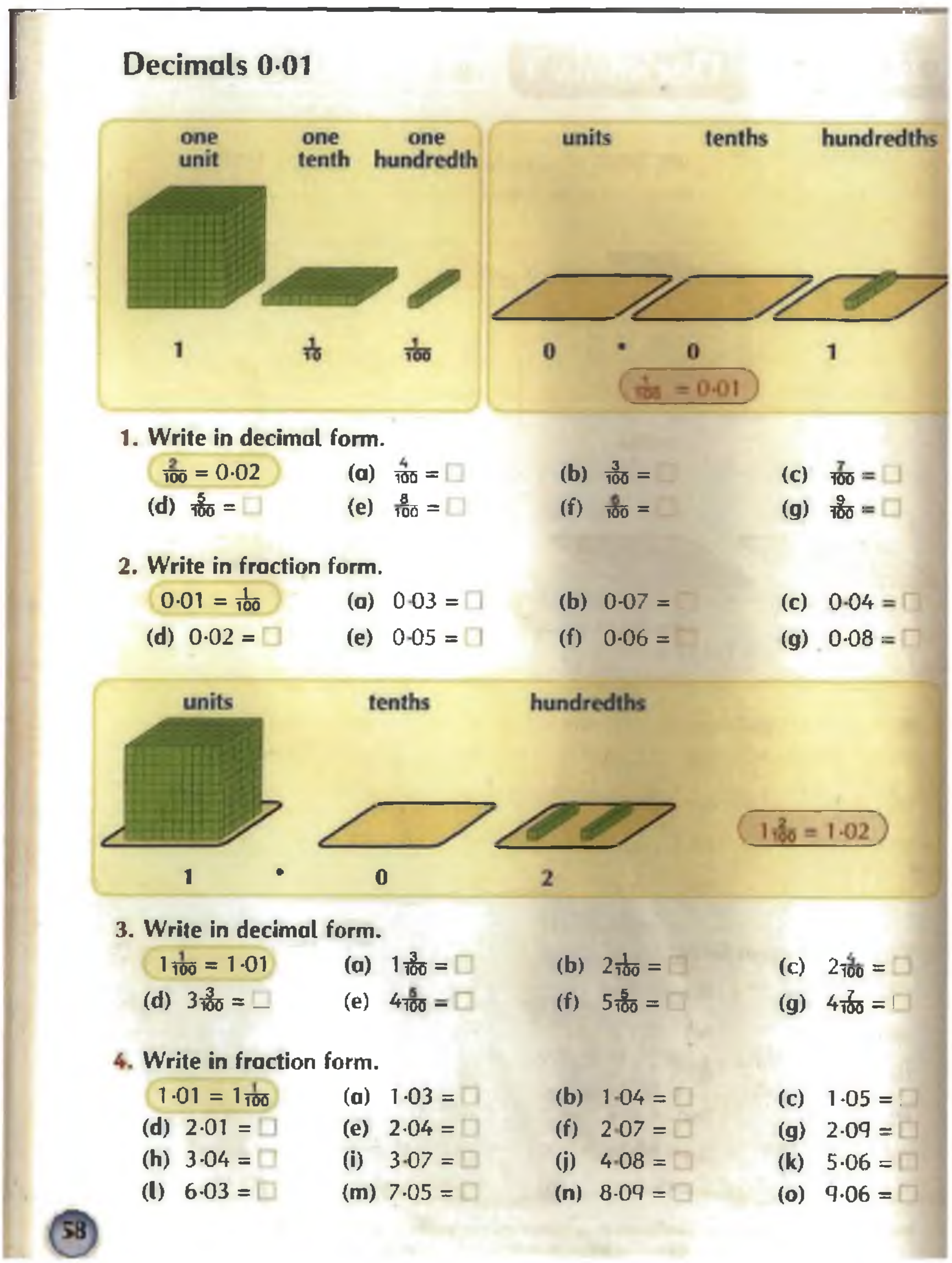



Figure 4.2. Mathemagic 4, page 58 (Barry, Manning, O'Neill & Roche, 2002). The students completed exercises from this page throughout John's lesson.

After eleven minutes, questions 1 and 2 were corrected in a whole class setting. Then John directed students to begin question 3. Question 3 involved writing mixed fractions in decimal form where the value of the fractional part was no more than $\frac{9}{100}$. He worked with individuals who were experiencing difficulties. Students who completed question 3 were directed to move on to question 4. Question 4 involved writing decimals greater than 1 as fractions where the fractional part was no more than $\frac{9}{100}$. After almost ten minutes, the process of calling out answers and correcting work was repeated.

**Discourse Community Analysis**

I confined my analysis to the sections of the discourse that were whole-class centred. These sections make up the majority of the discourse, 436 out of 583 turns. The discounted sections include students talking to each other while completing their written work. Some of these exchanges were off topic conversations. Some of John's interactions with individuals or groups of students were also recorded. For the most part, these consisted of John encouraging students to finish their work or correcting completed work.

**Questioning**

***Teacher questions***

Teacher questions were counted and coded using the question categories developed by Boaler and Brodie (2004). In assigning categories to John's questions, questions of the form "what's that number?" were considered carefully. I debated whether this should be classed as a type 1 question as it requires an immediate answer and may allow students to state a known fact. However considering that this lesson was only the second lesson on decimals and also that students had trouble saying decimal numbers correctly, I decided to treat them as type 2, inserting terminology questions. Straight forward questions such as "how many units?" were categorised as type 1, as were all invitations to students to call out answers to their written work. Questions regarding place value or how to write a number were categorised as type 3. The results of this analysis are shown in table 4.1 and in pie chart form in figure 4.3.

There is a fairly narrow range of questions asked here with the majority consisting of type 1 questions. The lack of type 9, establishing context questions and type 6, linking and applying questions, reflects the fact that the decimal numbers discussed in this lesson were discussed in a context free environment with no links made to real life decimals or to other mathematics topics in which decimals occur such as measures. This mirrors the nature of the textbook questions which formed a large part of this lesson. One might have expected more questions focussing on mathematical relationships and representations (type 3) in a lesson concerning writing decimal numbers and the conversion of fractions and decimals.

***Student questions***

Student questions were also counted and coded using the codes 'questions seeking clarification about mathematics being discussed' or 'questions seeking organisational clarification'. Table 4.2 shows the results of this analysis. Only mathematical or task related questions were counted. Despite some obvious evidence of confusion, the students as a group did not ask many direct questions about the mathematics. This low level of mathematics focussed questions may reflect low levels of responsibility for learning on their part or may be due to features of the classroom culture.

**Explaining Mathematical Thinking (EMT), Source of Mathematical Ideas (SMI) and Responsibility for Learning (RFL)**

While it is not difficult to identify lesson events in which the notions of EMT, SMI and RFL are relevant, it is difficult to discuss one component independently of the others. For this reason they will be discussed together in this section. The discussion will be organised under the following headings: effects of teacher questions, teacher EMT, student contributions as possible SMI, treatment of errors and the evaluative action of the teacher.

Table 4.1

*Analysis of teacher questions in John's lesson by type and number with examples*

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 175: How many units had you Adam?<br>Turn 237: Does the decimal point ever go anywhere? | 39 |
| 2. Inserting terminology | Turn 43: What's that called?<br>Turn 193: Two … now who can say that number? | 12 |
| 3. Exploring mathematical meanings and/or relationships | Turn 43: What's going to be one step smaller?<br>Turn 78: Because? How many of them would go into Conor's unit? | 12 |
| 4. Probing, getting students to explain their thinking | | 0 |
| 5. Generating discussion | Turn 123: Who can get this? Who can figure this one out for me?<br>Turn 169: Is he right? | 11 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 74 |

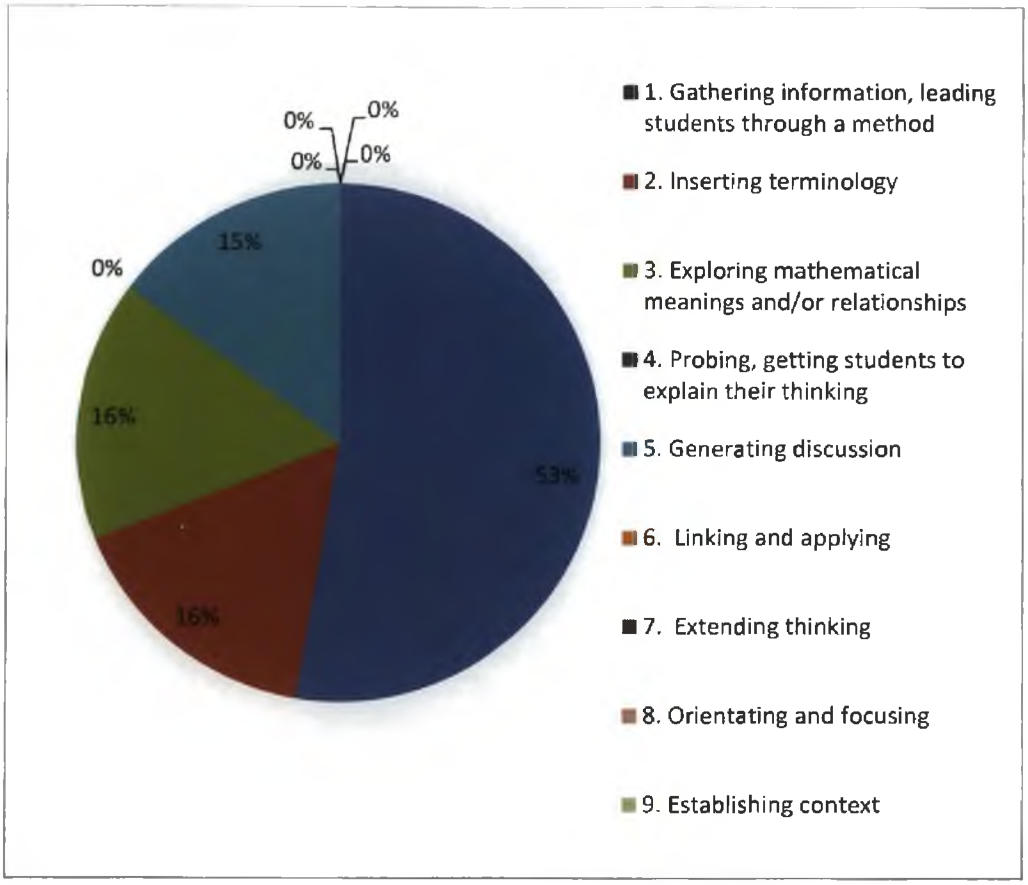



Figure 4.3. Types and percentages of teacher questions in John's lesson according to Boaler and Brodie (2004) question categories.

Table 4.2

*Analysis of student questions in John's lesson by type and number*

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Edward: The unit's getting smaller?<br>*Turn 37, the student asked this question in relation to the new application of the Dienes' materials to represent decimal fractions.*<br><br>Student: Like a spelling? Spell it?<br>*Turn 60, the student asked this question when John asked how to write one tenth.*<br><br>Cathal: Is that stuck together?<br>*Turn 111, the student asked this question in relation to the Dienes' materials)* | 3 |

| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Ciaran: Any colour? Rory: Which pen?<br>*Turns 127 and 141 respectively. The students questioned which colour marker to use on the whiteboard.* | 2 |

### *Effects of teacher questions*

Over half of John's questions were type 1 questions aimed at gathering information or leading students through a method and there were no questions probing student thinking. Type 5 questions aimed at generating discussion can be associated with reform methods. John asked 11 of these questions. The majority of these questions were variations of the phrase "who can do this one?" and functioned more as a means of moving the focus onto the next task rather than provoking discussion. On three occasions, John asked a variation of "is he right?" or "do you agree?" On two of these occasions the student in question was right and it could be guessed fairly easily from the preceding dialogue which was leading to the correct answer, that the student was right. In the last case, a hint was given to the student that he must "fix" the Dienes' blocks before writing the corresponding decimal fraction on the board. Presumably this meant placing units, tenths and hundredths in groups and in place-value order before writing the number. Here John's "is he right?" question seemed to be asked more as a hint for the student who had made an error to change his solution than a genuine call for students to evaluate his suggestion. John's questions did not appear to create opportunities for student EMT or to position students as a SMI.

### *Teacher explanations of mathematical thinking (EMT)*

As this was only the second lesson in the decimal topic, it might be expected that John, as teacher, would play a role in explaining mathematical concepts. This could be considered particularly important in this lesson because of the use of the Dienes' materials in a way that was novel for students. Students had previously used them to represent whole number values. John acknowledged the first and more familiar representation in an introductory activity where he asked students to hold up thousands, hundreds, tens and units. His transition to the new use of the Dienes' materials to represent decimal fractions is summed up in the following statement:

35 John: Now, so that was all well and good if you're getting bigger, but now our numbers are getting smaller, so what way … we're going the opposite way, so who can, who can show me a unit now? Now what is a unit when we're getting smaller?

The transition to the novel use of Dienes' materials caused confusion and students seemed to react by guessing. Due to the limited number of options some eventually discovered the right answer but it is not clear what mathematical understandings were emerging. Consider the episode which followed John's invitation above.

37 Edward: The unit's getting smaller?

38 John: Uhhuh. *(Appears to express agreement)* Show me which one of those is going to be a unit?

39 Student: No.

40 John: No. Have to change that … C'mon.

41 Edward: I think 'cause you said the unit's getting smaller, so I thought that it'd be the smallest?

42 Students: We're doing it a different way, Sir!

43 John: Ok, but we're going the other way now, remember, so when it, when, when we're getting bigger, it's units, tens and hundreds. Now we're going to be going the other way, so ... well done.

Edward eventually identified the 'new' unit and John evaluated his choice (turn 43). Students' beliefs about the nature of mathematics may be affected by the seemingly arbitrary changes of 'rules' the teacher is making (as perceived by the students) and the possibilities for mathematical misconceptions arising from these issues of representation are many.

### *Student contributions as possible source of mathematical ideas (SMI)*

Using students' contributions as a SMI was not a feature of this lesson. In general, students made suggestions to each other or to the whole class by way of calling or shouting out. There was one example where a student made a suggestion directly to John. John had just talked through the first written exercise where students had to write fractions in decimal form for fractions in the range to $\frac{1}{100}$ to $\frac{9}{100}$.

| | | |
|---|---|---|
| 297 | John: | Well done, nothing difficult now about these. One a to g. |
| 298 | Joe: | Sir I know what ten w- |
| 299 | John: | Quickly |
| 300 | Joe: | Sir I know what ten will be |
| 301 | John: | What? |
| 302 | Joe: | Zero point one zero |
| 303 | John: | Right get to work |

Joe's assertion that ten-hundredths is 0.10 is quite significant but John did not pick up on it. Ten-hundredths can be problematic to represent in decimal form because to some degree it necessitates knowledge that it is equivalent to one-tenth. The making of such connections should be at the heart of a sequence of lessons on decimals. This makes the potential of Joe's contribution particularly potent because it could start a classroom discussion around important decimal and fraction equivalences. Whether Joe has made this leap, or just made a lucky guess is not explored. The opportunity for meaningful classroom discussion arises only if John recognises the mathematical and pedagogical potential of Joe's statement (Rowland, Huckstep & Thwaites, 2005). It is possible that John did recognise Joe's contribution as potentially interesting but did not wish to explore it in order to stick to his plan for the lesson. In either event, students' ideas did not seem to be valued or viewed as opportunities for further exploration of mathematical ideas.

***Treatment of errors***

*The funnel pattern*

Some parallels can be drawn with the funnel pattern of interaction described by Wood (1994) when examining sections of dialogue where John deals with student errors. For example consider the following section of dialogue where John had arranged a selection of Dienes' blocks and asked James to write the correct decimal value.

| | | |
|---|---|---|
| 233 | John: | James, so what number do you think is there? … What number do you think is there? … O.K., well have you any units there James? … Is there any units there? Have you put in that you have units? ... So maybe, think about it again. |
| 234 | Student: | No. |
| 235 | John: | O.K., start with the units. How many units have you? |
| 236 | James: | Three. |
| 237 | John: | They're the units, O.K., does the decimal point ever go anywhere? |
| 238 | James: | No. |
| 239 | John: | Well done. That's fine. Yeah, you're right the decimal point stays exactly where it is. How many tenths? |
| 240 | James: | Zero. |
| 241 | John: | How many tenths? |
| 242 | James: | Three. |
| 243 | John: | How many hundredths? … What is that number? |
| 244 | James: | Thirty three. |
| 245 | John: | Is it thirty three? |
| 246 | //Students: | No, no// |

247 John: What about this, all this is important?

248 James: Zero point thirty three

249 John: Well done. You can't, now that's where you have to be really careful. Right just because there's no units, you can't just forget about it. That's the number, it's not thirty three, it's zero point three three. Zero point three three.

Here John is essentially leading the student to the right answer by asking a series of type 1 questions. James gives John an incorrect answer at turn 240 and John repeats his question. Given that in this case the numerical options are only zero and three, James can come up with the answer through a process of trial and error without really understanding why his final answer is right and the reasons for his confusion remain unexplored and unaddressed. Also the relative size of 33 and 0.33 was not discussed.

*The evaluative action of the teacher*

Further examples of student errors and of John's use of the evaluative action can be seen in the following extract. He arranged a selection of Dienes' materials and then asked for a student to write the number.

100 John: Who can write that number on the board?

101 Student: Ooooh!

102 John: Adam

103 Adam: Two point eh, two point five.

104 Ronan: Six!

105 Brian: It's five (*sounds of writing*)

106 John: Uh uh (*appears to indicate that an error has been made*)

107 Adam: Mmmm

108 //Kevin: Use the tens and units

109 Jake: Two thousands//

110 Adam: Oh yeah.

111 Cathal: Is that stuck together?

112 Kevin: No, it's separate.

113 John: Ok, now but we're doing decimals and we've been doing, we're doing tenths and hundredths.

114 //Students: Tenths, hundredths ( ) //

115 John: So I'm going to say, I'll say this time, that's units

116 Student: Aaahh.

117 John: They're not hundreds this time, they're units, so now, what do you think it is?

118 Student: Aaah.

119 //Kevin: Units ( ) thousands

120 Joe: I know, I know the answer!//

121 Adam: I'll just do it all backwards

122 Jake: Backwards?

123 John: Incorrect. Who can get this? Who can figure this one out for me?

124 Student: Aaah!

125 John: Sit down (*Shuffling noise*), Ciaran.

127 Ciaran: Any colour?

128 John: Any colour ...

129 Ciaran: I don't know.

130 Student: No, ah-ah, Incorrect.

131 John: Sit down.

132 //Student: Oh!

133 Student: One small one, one half big yoke and one yellow

134 John: It seems people aren't getting it. // I might eh, give you a little hand-

135 //Student 1: Sir I know, I know.

136 Student 2: Five thousands, two hundreds and twenty tenths

(*sound of writing*)

137 Students: Aaah! Yeah! Aaah //

138 John: Who can do it now, do they think?

139 Student: Aaah.

140 John: Rory ....

141 Rory: Which pen?

142 John: Black ... Sit down.

143 //Student: It's all backwards.

144 Student: Oooh //

The value in question was actually 2.25. While it is difficult to make sense of this extract or understand student thinking without seeing the errors that have been made on the board, it is obvious that some students were struggling with the novel use of Dienes' blocks to represent decimal fractions. Elements of the classroom culture highlighted in this extract include the students' habit of calling out and John's use of evaluation. His use of the formal "incorrect" as an evaluation of students' answers is a strong teacher action and the fact that a student later picked up on it and used it in response to the contributions of his peers is noteworthy (turn 130). It would suggest that the classroom culture is not one which would be supportive of risk taking. "Sit down" also effectively worked as an evaluation of two of the students' efforts in the extract above. These evaluations were given with no questioning of motives for the error and no discussion of why the students' solutions were wrong. The only feedback students received was that they had made an error. The only option open to them was to make another guess. Opportunities for mathematical thinking were limited in that some students may have made guesses simply by manipulating the numbers and might not have made connections with the underlying place-value concepts.

Some students took responsibility for their learning in this lesson. They were vocal in their ideas and by offering suggestions, tried to help each other from time to time. However there was no formal structure evident for this helping and instead it seemed to occur as students shouted out answers or suggestions for solutions.

**Discussion**

The written activities completed by students in John's lesson were exercises from the class textbook *Mathemagic 4* (Barry et al., 2002). John told me that this was the second lesson on decimals and the page the students worked from during the lesson was the second page of the chapter on decimals. This suggests that the students may have worked through the first page of decimal exercises during their first lesson. It seems likely that the textbook influenced John's planning actions and also his choice of representation as the textbook uses Dienes' materials to represent decimal fractions (figure 4.2). The nature of the textbook exercises, which involved the conversion of fractions and decimals within strict value limits, meant that students may have successfully completed the exercises by following a pattern rather than making any conceptual links. For example, in the first question all the fractions are hundredths less than or equal to $\frac{9}{100}$. Effectively this means that the solutions will all start with 'zero point zero' and students must merely fill in the missing last number in the hundredth place. Not all students successfully completed this exercise. For example during the whole class correction of questions one and two, Adam admitted to not getting question one, part b done because he "was stuck". The arrangement John made for students who completed their work was to come to him to correct it. For this reason, students who were slow to settle to work or who were struggling like Adam, may not have received his attention. All of the textbook exercises involved repetitive patterns. For example question 3, involved writing mixed fractions in decimal form where the value of the fractional part was no more than $\frac{9}{100}$ so the appropriate answer is of the form 'number point zero number'. It is difficult to distinguish between students who genuinely understood why the zero was in the tenths place and students who merely followed a fairly obvious pattern. There was no explicit discussion of the reason for inserting this zero as place-holder. Neither was there any attempt to connect decimal values with examples of real life decimal numbers. The decimals considered were not compared to each other either and there was no discussion of their relative size.

The mathematical relationships represented by Dienes' blocks are clear to the expert but how they serve to support the understanding of learners is less clear (Cobb, 1994). There is evidence that the use of Dienes' materials as representation confused some students rather than supported them. The transition from the familiar use of Dienes' materials to represent whole number values to the novel representation was not discussed in any detail or rationalised in any meaningful way. The struggle to switch to and understand the new use of the materials should have been predictable, particularly for this group of lower achieving students. Questions arising from John's choice of representation include whether John himself fully understood or had considered in depth the mathematics inherent in the changing use of the Dienes' materials and if he had considered the pedagogical complexities involved in facilitating this change. Also activities that Dienes' considered necessary to support meaningful interaction with these materials were not present in this lesson (Lesh, Post & Behr, 1987). In particular, students had little direct contact with the materials as they were used mainly for teacher demonstration and no effort was made to develop links with other representation such as a three pronged abacus or notation board.

John's used both direct negative evaluations and implicit negative evaluations where he ignored what a student had said and called on another students to answer instead. These evaluations often occurred without follow-up questions or direct instruction. This left students guessing answers in some cases. These actions marked John out as an evaluative authority and though students regularly called out suggestions and comments there was little genuine opportunity for them to function as mathematical authorities and evaluate what was mathematically correct.

**Discourse in this Community**

A summary of findings in relation to the components of the MTLC framework is given in table 4.3. In this lesson, many students struggled to understand the basics needed to participate effectively in whole class discourse. John's questions were focussed on answers rather than on mathematical thinking. This lack of attention to student thinking suggests a level 0 description on the MTLC framework. Students attempted to participate without a context to connect the decimal fractions to, a model for decimal fractions that was problematic and with little explanation from John. Considering the nature of the experience from their point of view, the possibility exists that the whole class discussion could be viewed as an exercise in which students learned what they were expected to say rather than the mathematical concepts that may have been the intended objective (Bauersfeld, 1995).

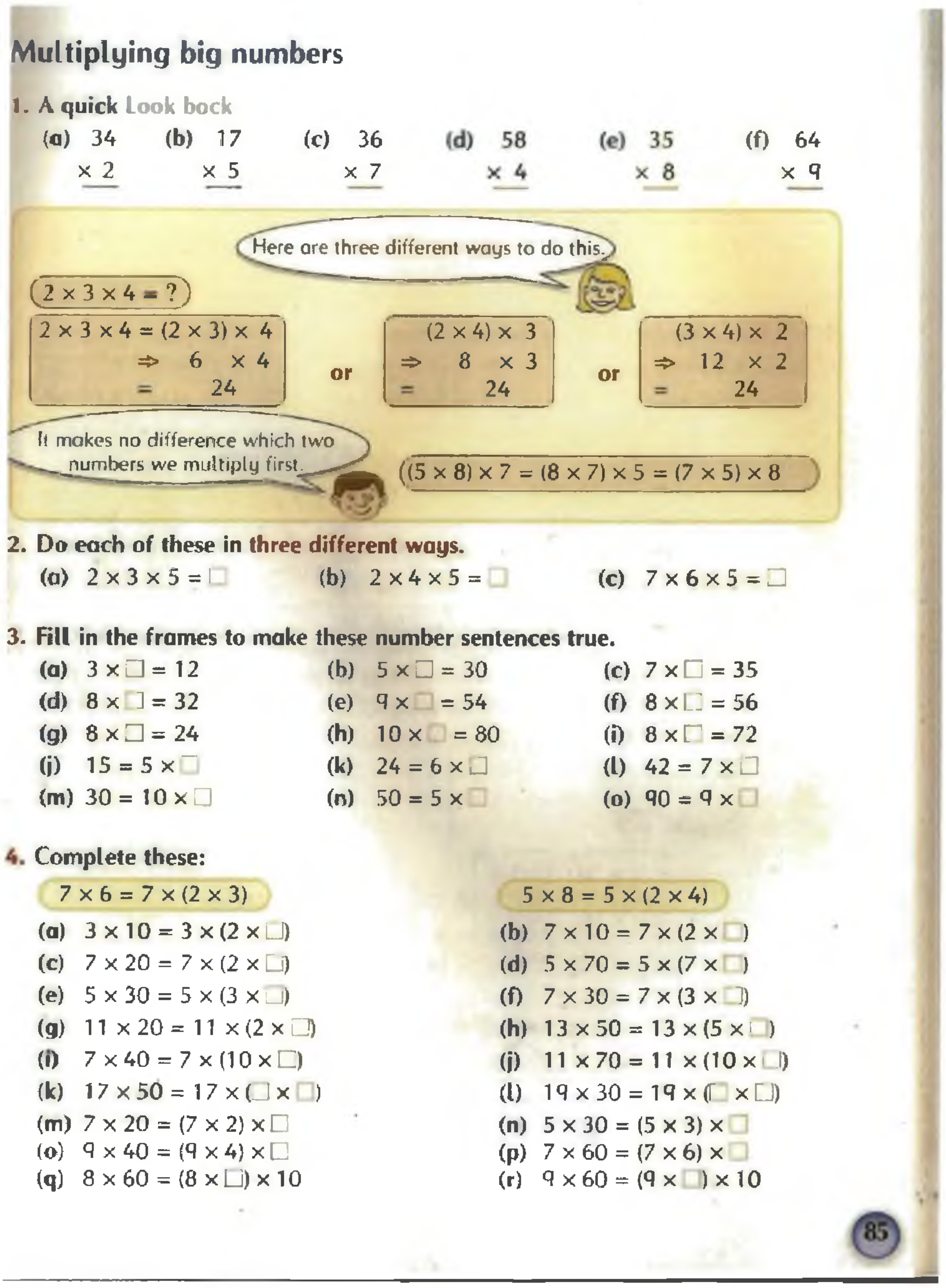
Table 4.3
*Summary descriptions of MTLC framework components in John's lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher as main questioner with questions generally focussing on answers rather than student thinking. |
| EMT | Low levels of both teacher and student explanations of mathematical thinking. |
| SMI | Teacher as primary source of mathematical ideas. |
| RFL | Teacher-centred classroom with students appearing to show low levels of RFL. For the most part, students were not involved in evaluating mathematical ideas. |

## Recording 2: Liam's lesson

### Descriptive Synopsis

Liam's lesson will be presented in summary form consisting of a descriptive synopsis and a discussion. This lesson occurred in the higher achieving fourth class group in St. Eithne's school and the topic was the distributive and associative properties of multiplication. It was a calm, orderly class at all times. Liam began by presenting the class with an array of three rows of fourteen dots on the interactive whiteboard. He stated that fourteen times three does not occur in "our tables" and it could be split up. At the suggestion of a student he attempted to split the array into two equal sets of seven groups of three before realising that the array was actually a 13 x 3 array. Liam noted the error and a student suggested splitting it into 6 x 3 and 7 x 3. Students solved 7 x 3 and 6 x 3 separately and then mentally added the answers together to solve for the total. Liam then completed a similar whole class activity for a 14 x 4 array. Homework was then corrected as a whole class with students calling out the answers when prompted by the teacher. The homework, exercise 3, questions a - i on page 85 of *Mathemagic 4* (Barry et al., 2002), can be seen in figure 4.4. It consisted of missing factor questions such as 3 x ___ = 12, with all numbers within the range of the multiplication tables generally learned by heart. After this was completed, Liam asked the students to finish exercise 3 from the textbook while he checked the homework of individual students. The questions were similar to the previous work but written with the product first e.g. 15 = 5 x ___. This form of representation seemed to confuse some students. Liam also worked with two lower achieving students on a one to one basis on differentiated material during this time. These students were later collected by the learning support teacher. The activities involved solving simple multiplication word problems.



## Multiplying big numbers

**1. A quick Look back**

(a) 34 × 2 (b) 17 × 5 (c) 36 × 7 (d) 58 × 4 (e) 35 × 8 (f) 64 × 9



**2. Do each of these in three different ways.**

(a) 2 × 3 × 5 = □ (b) 2 × 4 × 5 = □ (c) 7 × 6 × 5 = □

**3. Fill in the frames to make these number sentences true.**

(a) 3 × □ = 12 (b) 5 × □ = 30 (c) 7 × □ = 35
(d) 8 × □ = 32 (e) 9 × □ = 54 (f) 8 × □ = 56
(g) 8 × □ = 24 (h) 10 × □ = 80 (i) 8 × □ = 72
(j) 15 = 5 × □ (k) 24 = 6 × □ (l) 42 = 7 × □
(m) 30 = 10 × □ (n) 50 = 5 × □ (o) 90 = 9 × □

**4. Complete these:**

7 × 6 = 7 × (2 × 3) 5 × 8 = 5 × (2 × 4)

(a) 3 × 10 = 3 × (2 × □) (b) 7 × 10 = 7 × (2 × □)
(c) 7 × 20 = 7 × (2 × □) (d) 5 × 70 = 5 × (7 × □)
(e) 5 × 30 = 5 × (3 × □) (f) 7 × 30 = 7 × (3 × □)
(g) 11 × 20 = 11 × (2 × □) (h) 13 × 50 = 13 × (5 × □)
(i) 7 × 40 = 7 × (10 × □) (j) 11 × 70 = 11 × (10 × □)
(k) 17 × 50 = 17 × (□ × □) (l) 19 × 30 = 19 × (□ × □)
(m) 7 × 20 = (7 × 2) × □ (n) 5 × 30 = (5 × 3) × □
(o) 9 × 40 = (9 × 4) × □ (p) 7 × 60 = (7 × 6) × □
(q) 8 × 60 = (8 × □) × 10 (r) 9 × 60 = (9 × □) × 10

85

Figure 4.4. Mathemagic 4 page 85 (Barry et al., 2002). The students completed part of question 3 for homework and finished this question and question 4 in class.

Afterward the set work was corrected in a whole class setting with students calling out answers at the invitation of the teacher as before. Then Liam presented the class with the task: 3 x 4 x 5 and stated that brackets must be put in so that two of the numbers could be multiplied first. Some examples were worked through. In some cases, the final answer was found i.e. the class talked through the following solution 3 x 4 x 5 = 3 x (4 x 5) = 3 x 20 = 60. In other cases where the numbers were larger, the final solution was not considered e.g. 5 x (6 x 5) was only brought as far as 5 x 30. The overall solution was not discussed. Then the strategy of splitting a number into its factors in the context of a multiplication exercise was discussed. For example, after some discussion 3 x 20 was broken down into 3 x 4 x 5. Various examples were discussed. Some examples had only one obvious solution e.g. 3 x 70 = 3 x 7 x 10 (within the range of tables based factors of 70 and ignoring non-tables based factoring solutions such as 70 = 2 x 35). Other examples such as 6 x 20 had more than one obvious solution, 6 x 4 x 5 or 6 x 2 x 10. In only one case, different factoring possibilities were discussed. After this students completed question 4 on page 85 (figure 4.4) where questions were of the form A x B = A x (C X __), where C is a factor of B. In most cases, B was a multiple of ten and the 'missing number' was in fact ten. Students worked on these while the teacher consulted with some individuals who were struggling to understand how to complete the written exercise. Then they were corrected in a whole class setting as before.

**Discussion**

The lesson activities for this lesson were drawn mainly from the students' textbook *Mathemagic 4* (Barry et al., 2002). The first activity was not related to the textbook or later student tasks. Liam presented arrays on the interactive whiteboard to explore the distributive property of multiplication. It is unclear whether he saw any conflict between these introductory activities where partitioning occurred along additive lines and was supported by visual representation and the later focus on the associativity of multiplication. The activities on associativity were completed without support from concrete materials or visual representation. Also because students did not solve many exercises in full they did not verify for example that 7 x 10 = 7 x (2 x 5) (Question 4b, figure 4.4).

For some of the whole class discussion and all of the written exercises, students were essentially factorising products but Liam did not use the term 'factorise' and instead referred to the need to "open up" the numbers. The students completed part of question 3 on page 85 of their textbook for homework and completed question 4 the next day. This would suggest that the textbook was a strong influence on the planning of Liam's lesson. Liam's interpretation of what learning objective could be pursued using this particular page of the textbook is hard to imagine given his introductory focus on the distributive property and that he did not mention factoring specifically when discussing associativity. A further question arises in relation to Liam's sensitivity to the possibilities of multiple possible solutions. Some of the factoring questions he posed had multiple possible solutions but for the most part these were not discussed. Instead Liam focussed on the manipulation of symbols and lesson activities were not situated in any context.

Liam, as teacher, was the source of mathematical ideas and it was generally only he who explained his mathematical thinking and evaluated what was mathematically correct. Incorrect or incomplete answers from students resulted in him elaborating on mathematical points or repeating simplified questions. There was only one case where Liam asked students "isn't that right?" (turn 213). He posed this question after explaining that $3 \times 4 \times 5 = 3 \times 20$ and it functioned more as a rhetorical question than a genuine call for students to evaluate the mathematics he had just presented. The high levels of teacher EMT and strong teacher controlled evaluation procedures created little opportunity for students to pose ideas of their own or to take responsibility for their own mathematical learning.

**Discourse in this Community**

The discourse of the lesson fits with a level 0 description on the MTLC framework (Hufferd-Ackles et al., 2004). It seems that opportunities for high level mathematical thinking were limited in this lesson and that the discourse was not truly mathematical but more a form of 'number talk' (Richards, 1991). A summary of the findings in relation to the components of the MTLC framework is given in table 4.4.

Table 4.4
*Summary descriptions of MTLC framework components in Liam's lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher as main questioner with questions generally focussing on answers rather than student thinking. |
| EMT | High levels of teacher EMT; Low levels of student EMT. |
| SMI | Teacher as primary source of mathematical ideas. |
| RFL | Teacher-centred classroom with students appearing to show low levels of RFL. For the most part, students were not involved in evaluating mathematical ideas. |

**A Note on Achievement Levels**

It is impossible to speak of the achievement levels of the students in John or Liam's class in any detail given the nature of the collected data. However it is important to address this issue given that that these classes were identified as lower and higher achievers respectively. The practice of streaming by ability levels has been questioned in recent years with findings suggesting better opportunities for growth in mathematical understanding for lower achievers in mixed settings (Boaler, 1997; 2006). However the gap between higher achieving students and lower achieving students has been observed to widen as students' progress through some schools (DES, 2005) and some teachers view streaming by ability as a practical means of attempting to meet the disparate needs of pupils (McLoone, 2011). Because the pupils in Liam's class had been identified by teachers as higher achievers their errors and misconceptions are all the more interesting as it provides some illustration of the expectations of the wider school community in relation to mathematical achievement. More than one student was confused by the less common representation of the factoring exercises in the form 15 = 5 x ___. Whether this is an indication of achievement or of lack of exposure to such questions is hard to tell. Similarly the type of mental mathematics methods necessary to solve exercises such as 5 x (6 x 5) = 5 x 30 = 150, which generally could be reduced to a single digit multiplied by a multiple of ten, did not seem to be within the grasp of many students. Some did attempt these calculations but Liam discouraged them from doing so, focussing on manipulation of symbols rather than overall answer. It is difficult to tell whether the student errors that surfaced in these calculations reflect contributions of lower achieving students or lack of practice in this area of mathematics.

Although this class was identified as the higher achieving class, Liam spent some time working on differentiated material with two students. The demands of attending to the lower achieving students cannot be underestimated. Liam's time and attention was split between two groups and it is possible that his planning had to involve easily completed exercises for the larger group so that he could give his attention to the lower achieving students. These students were collected by the learning support teacher mid-way through the lesson.

There were many observable differences between the nature of student participation in Liam's lesson and in John's lesson. During Liam's lesson, the students worked quietly and the whole class discussions were conducted in an orderly manner with Liam calling on students to invite them to participate. In John's lesson the students were more talkative both in whole-class discussion and in off-task conversations amongst themselves. The whole class discussion in John's lesson was less orderly, with many students calling out suggestions both to the whole class and to individuals they were sitting near. The culture of both classes was surprisingly different considering that they were both of the same class level within the same school and that the students worked only in these groups for mathematics lessons. For the majority of their time in school, students worked in different groupings that would have contained a mixture of students from both John's and Liam's mathematics classes. Whether the difference observable in the recordings is related to the identified achievement levels of the different pupils, the expectation of teachers or features of the mathematical activities students were engaged in is hard to tell.

These students were streamed for mathematics but not for other subjects. One would expect that if there was such a large gap in mathematics achievement levels that teachers felt impelled to stream classes, then there might also be an achievement gap in literacy. If such a gap did exist it is interesting to consider why teachers felt they could meet the disparate needs of students without streaming in literacy, but not in numeracy. On the other hand if an achievement gap did not exist in literacy but did in numeracy, this raises questions about the conditions that created such a contrast in achievement for the same pupils in the same school.

**Recording 3a and 3b: Anne's lessons**

Two recordings were collected in Anne's fifth class in St. Ita's girls' school on consecutive days. In fifth class, students are generally 10 – 11 years old. Both lessons focussed mainly on decimals and were similar in terms of structure and mathematical activity. For this reason, I will present only the first lesson here. Anne used a target board in both lessons. This is a number grid that can be used to develop mathematical language in mental mathematics activities. Teachers may ask students different questions depending on their teaching objectives. For example, students may be instructed to combine numbers using different operations to get a specific target or to compare numbers on the board.[5]

**Descriptive Synopsis of Anne's First Lesson**

The lesson was almost 40 minutes long and began with mental mathematics exercises. Anne called on students for oral answers but also invited them to present written computations on the board. Questions included taking a single digit number from a four digit number, a division word problem and a two-step money word problem. After these questions were completed Anne moved on to working with a notation board on the interactive

[5] A video showing the use of a target board in an Irish primary classroom can be seen on the PDST website at http://ppds.ie/index.php?option=com_content&task=view&id=417

whiteboard. She wrote various numbers in decimal form and invited students to represent them on the notation board and to write them in fraction form. An example of a notation board from Anne's second lesson is shown in figure 4.5.

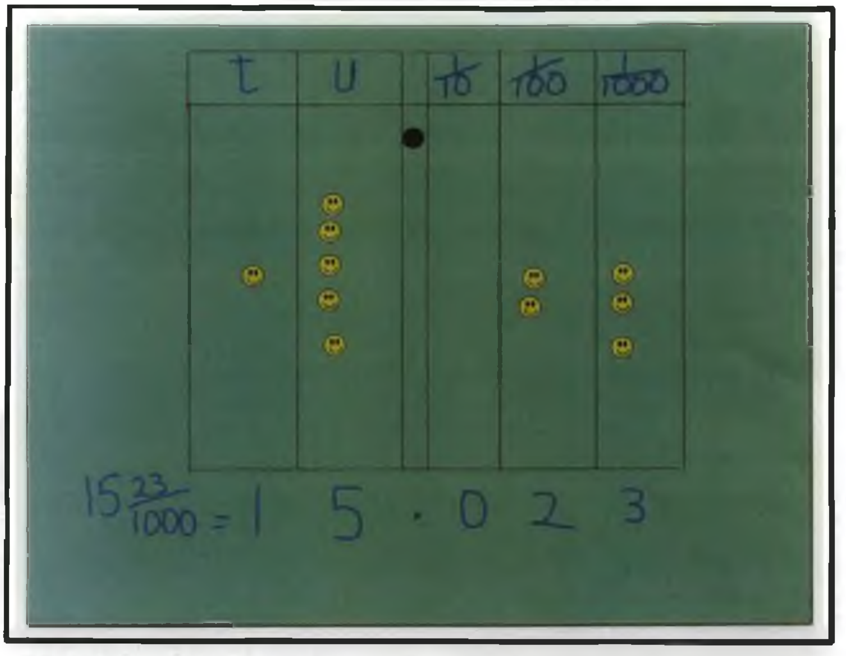



Figure 4.5: Notation board used to represent decimal numbers on the interactive whiteboard in Anne's second lesson. Anne wrote numbers in decimal form and students represented these on the notation board and wrote the equivalent fraction.

Students then worked in pairs with their own notation boards. One student represented a number with counters on the board and her partner had to say this number in decimal and fraction form (i.e. if one partner showed 2.034 on the notation board, her partner would say "two point zero three four or two and thirty four thousandths"). Anne circulated and talked with various pairs. After about three minutes, she called the group together again and asked some groups to call out the numbers on their notation boards in fraction and decimal form. Some students struggled to say these numbers correctly. Anne then wrote numbers in either fraction or decimal format and asked the students to represent these on their notation boards. She questioned students about the place value of various digits in the numbers. Then she wrote up examples for students to write into their copies, converting fractions into decimals and decimals into fractions. On completion of the written exercise, Anne presented a target-board that showed decimal numbers and the class discussed the greatest and least numbers in various rows and columns.

**Discourse Community Analysis**

As before, I confined my analysis to the sections of the discourse that were whole-class centred. These sections make up the majority of the discourse, 368 out of 391 turns in this lesson (387 of 409 turns in the second lesson). These discounted sections of the transcripts consist of student pair work or Anne's interaction with various pairs. Due to the nature of the recording method, some of these exchanges are unclear.

**Questioning**

***Teacher questions***

Teacher questions were categorised using Boaler and Brodie's categories (2004). Questions where Anne asked for the name of a fraction or decimal number were classed as type 2, inserting terminology questions. The alternative of categorising them as type 1 questions did not seem appropriate as for many students the answers were not 'known facts' (Boaler & Brodie, 2004). Questions about the place value of digits were classed as type 3, exploring mathematical meanings and representations. These results are shown in table 4.5 and in figure 4.6.

The type 3 questions, exploring mathematical meanings and representations, often referred to place value or the conversion of decimal fractions to decimal notation or vice versa. The one orientating and focussing question (type 8) was posed during the discussion of a division word problem which was one of the mental mathematics tasks. It asked, "A minibus carries twelve people. How many minibuses are needed for seventy children?" Midway through the solution process Anne asked, "Why can we not say that we'd need five buses?" (turn 61) which was classed as a type 8 question.

Table 4.5

*Analysis of teacher questions in Anne's first lesson by type and number with examples*

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 27: Now, you're taking away what?<br>Turn 323: How many have you in the last one? | 42 |
| 2. Inserting terminology | Turn 220: What number is that?<br>Turn 327: Read it as a decimal | 14 |
| 3. Exploring mathematical meanings and/or relationships | Turn138: Which is the biggest value of these three Jenny?<br>Turn 180: O.k. And what does the zero stand for please, quickly | 41 |
| 4. Probing, getting students to explain their thinking | Turn 90: How did you do it?<br>Turn 286: Do you understand it Leah, do you see where you went wrong? | 4 |
| 5. Generating discussion | Turn 33: … Right who can help me … anyone know the answer? | 1 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | Turn 61: Why can we not say that we'd need five buses? *(In relation to minibus problem mentioned above)* | 1 |
| 9. Establishing context | | 0 |
| Total | | 103 |

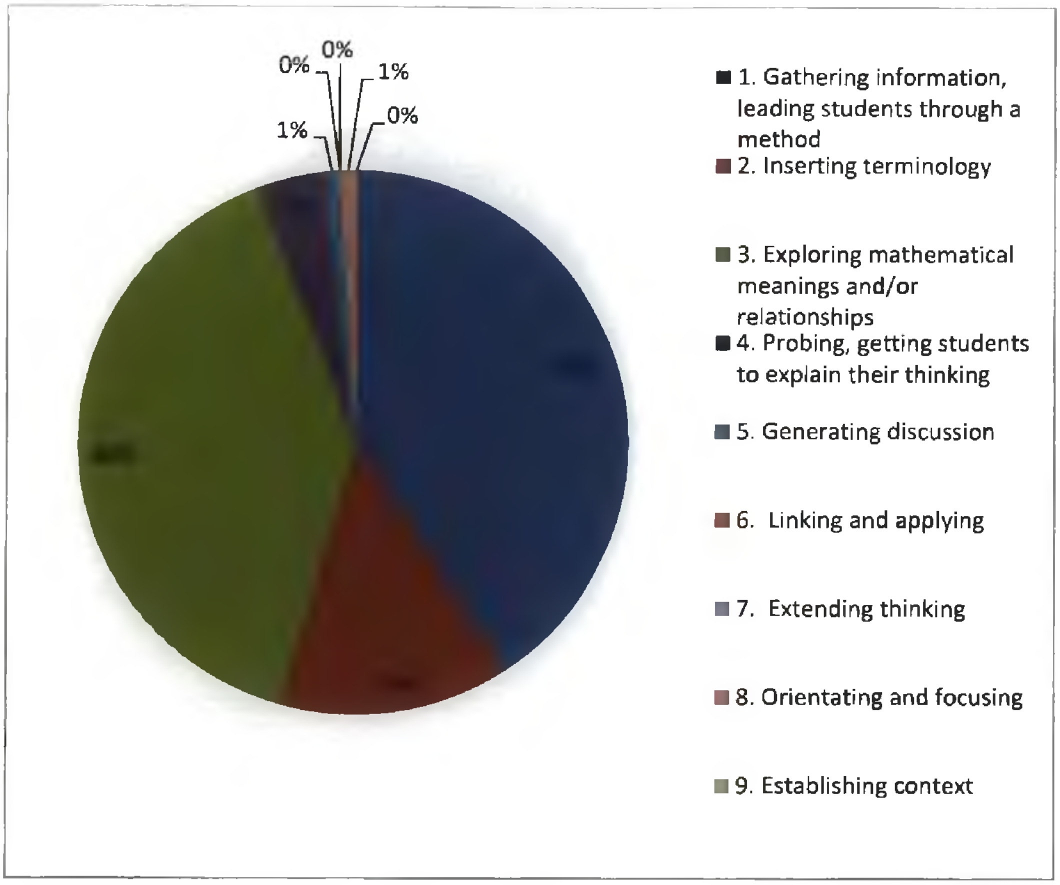



Figure 4.6. Types and percentages of teacher questions in Anne's first lesson according to Boaler and Brodie (2004) question categories.

***Student questions***

Student mathematical or task orientated questions were also counted and coded using the codes 'questions seeking clarification about mathematics being discussed' or 'questions seeking organisational clarification'. The few questions asked by students that can be deemed as seeking mathematical clarification appeared to seek basic understanding or simply clarification. At no point did students ask questions of each other in whole class discourse. Whether this low level of mathematics focussed questions reflects low levels of RFL on the part of the students or reflects features of the classroom culture would require more lesson recordings to investigate. The results of this analysis are shown on table 4.6.

Table 4.6
*Analysis of student questions in Anne's lessons by type and number.*

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: A fraction?<br>*Turn 18, Anne instructed the student to write the number shown on the notation board as a decimal and the student asked this question.*<br><br>Student: Oh, is it a decimal?<br>*Turn 281, Anne asked the student to say the number 23.051 as a fraction and the student asked this question.* | 2 |

| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: On the side?<br>*Turn 144, the student was instructed to write a decimal number and she posed this question querying where she should write it* | 1 |

**Explaining Mathematical Thinking (EMT), Source of Mathematical Ideas (SMI) and Responsibility for Learning (RFL)**

Due to the difficulty of discussing these components separately they will be discussed together as I did in the discussion of John's lesson. Following a similar format, the discussion is organised under the following headings: effects of teacher questions; teacher EMT and SMI; and treatment of errors.

***Effects of teacher questions***

The teacher question category analysis showed a range of question types but a reform classroom might be expected to have more type 4 questions probing student thinking and type 5, generating discussion questions. In some sections of dialogue it seemed that Anne's questions were posed with the aim of leading students to a certain point. For example, consider the following section of dialogue which highlights the dual focus on mental arithmetic and formal written algorithms.

| | | |
|---|---|---|
| 68 | Anne: | We're going to the shop this time and a bar costs … forty five cent, so there's the drawing of my bar and it's forty five cent. I bought four bars … what change do I have out of two euro … O.K., I'm going to ask a few questions. First of all do I know the price of the bar? |
| 69 | Students: | Yes. |
| 67 | Jade: | Of one. |
| 68 | Anne: | Pardon? |
| 69 | Jade: | Of one bar. |
| 70 | Anne: | I do know the price of one bar, what is it Jade? |
| 71 | Jade: | Forty five cent. |
| 72 | Anne: | Good girl. Do I know the price of four bars? |
| 73 | Students: | No. |
| 74 | Anne: | Do I know how much I spent? |
| 75 | //Students: | No. Yes.// |
| 76 | Anne: | Do you know how much you spent? |
| 77 | Students: | No. |
| 78 | Anne: | No. How much money had I going into the shop? Lámha suas.[6] How much money? |
| 79 | Amy: | Two euro. |
| 80 | Anne: | Two euro. Do I know my change? |
| 81 | Students: | No. |
| 82 | Anne: | No, now I've a few things to find out here. I know they asked me what my change is. What must I find out first, Clare? |

[6] 'Lámha Suas' is the Irish instruction meaning 'put your hands up'.

83 Clare: The price of the four bars.

84 Anne: The price of the four bars. Sara.

85 Sara: One eighty.

86 Anne: How did you do it?

87 Sara: I just thought four fives are twenty … I just write it down.

88 Anne: Right come up and do it yourself here. Now I could do it in my head; a quick way of doing it in my head … while Sara is just coming up. Áine.

(*In turns 88 – 100 the class discussed doubling 45 and then doubling the result to find the result of €1.80*)

100 Anne: One eighty. Now, that's what I spent (*writing*). Can anyone tell what was I asked?

101 Clare: What change you get out of two euro.

102 Anne: What change will I get Gráinne out of two euro?

103 Gráinne: Twenty cent.

104 Anne: Good girl and that's an easy one but what could I do if it wasn't as easy? I know you could do two euro take away one eighty in your head. Supposing you couldn't, what would you do? …

105 Grainne: Write it down and take it away.

106 Anne: Good girl. Come up and do it for me.

Anne led her students through the process of solving this word problem. This could be interpreted as Anne modelling appropriate procedures for her students and this may in fact have been her intention. She broke the solution process down into steps by asking leading questions which also served to lessen the cognitive demand on students. At turn 90, Sara seemed to be explaining that she completed 45 x 4 by mentally manipulating numbers as one would in the written multiplication procedure. The aim of solving these problems using mental methods may be undermined by completing the written algorithm as well.

***Teacher EMT and SMI***

The mental mathematics section that began the lesson contained more student EMT than other parts of the lesson. The different solutions that were discussed in this phase of the lesson generally consisted of one mental mathematics approach and one written algorithm. In the main body of the lesson, Anne was more likely to explain her mathematical thinking than to pursue the thinking of her students. Her EMT often took the form of non-negotiable statements. For example when some students were struggling with what exactly Anne expected when she asked them to say the decimal number, she said “when you’re doing a decimal, you don’t say the tenths, I just want it, just as a decimal first” (turn 243). Similarly, when a student called out 0.75 as “zero point seventy five”, Anne corrected her saying “no, zero point *seven* five is what we say when it comes after it” (turn 377). These statements function almost as rules for students to follow and through them Anne positioned herself as mathematical authority. She was also the only source of mathematical ideas during these lessons and students played no role in evaluating the ideas of their peers. For these reasons students can be considered to display low levels of responsibility for learning as described in the MTLC framework (Hufferd-Ackles et al., 2004).

***Treatment of errors***

Much like the pattern of leading questions in the last excerpt, Anne’s approach to errors or incomplete mathematical statements was generally to ask more guiding questions or to give hints. Consider the following section of dialogue. The class had been working in pairs representing numbers on their notation boards and Anne stopped the activity and began to question Sorcha and Kate about the number they had created.

| | | |
|---|---|---|
| 262 | Anne: | I see one girl here and I'm just going to ask them what they have. Hands behind the back. |
| 263 | Sorcha: | Three hundred and five point zero eight two. |
| 264 | Anne: | Well done … Now, How many tenths have you Sorcha? |
| 265 | Sorcha: | Zero. |
| 266 | Anne: | How many hundreds have you? |
| 267 | Sorcha: | Eight |
| 268 | Anne: | No, hundreds. |
| 269 | Sorcha: | Oh, three |
| 270 | Anne: | Can you give it to me as a fraction Kate? |
| 271 | Kate: | Three zero- |
| 272 | Anne: | No, give me the number … Three hundred? |
| 273 | Kate: | Three hundred and five point- |
| 276 | Anne: | No, no. That's a decimal |
| 277 | Kate: | Aah |
| 278 | Anne: | I want a fraction now… Do we use a decimal point when we're talking about fractions? |
| 279 | Kate: | No. |
| 280 | Anne: | No. |
| 281 | Kate: | Three hundred and five point- |
| 282 | Anne: | No, no point. |
| 283 | Kate: | Oh, three hundred and five eighty two. |
| 284 | Anne: | Eighty two what? |
| 285 | Kate: | Thousandths |

Kate's uncertainty is evidenced in some of her errors. When at turn 273, she began to mistakenly say the number in decimal form, Anne corrected her and asked "do we use a decimal point when we're talking about fractions?" (turn 278). Kate answered this correctly in the negative but it is hard to know if this helped her understand what was expected of her because she repeated her error again at turn 281. At Anne's more explicit hint regarding the decimal point, ("no, no point", turn 282), Kate simply repeated what she had said previously, this time leaving out the decimal point. It was only at Anne's prompting of "eighty two what?" that she completed her answer correctly. In this situation as in all others, no students were invited to comment on or correct the errors of their peers and there was little attempt to pursue the thinking that had led to the error.

**Discussion**

There is some evidence that factors such as collaborative planning or a whole school approach to the teaching of mathematics may have influenced Anne's lessons. Many of the same elements were also to be found in Joan's lessons. The common elements include mental mathematics exercises, concrete materials and the use of target boards. This idea is returned to in more detail in the discussion of Joan's lesson. Some of the mental mathematics questions were situated in a specific context but the majority of the tasks were focussed on decimals and these were generally context free. Anne's style of EMT varied at different stages of the lessons. In the initial task involving mental mathematics questions, she was more likely to ask prompting questions and lead students through the stages in solving a problem. In the later phases of the lesson when the class was focussed on decimals, Anne's EMT generally took the form of statements of fact or rules.

The activities of the lesson appeared at times to have contradictory aims in that some of the activities seem to fit a reform agenda but Anne's actions often suggested a traditional approach. For example, the mental mathematics questions were tackled both as a mental mathematics exercise and as an exercise in practising formal algorithms. Similarly, Anne's use of concrete materials would suggest reform orientated practice but her actions of stating formal mathematics 'rules' would suggest a traditional community with the teacher firmly positioned as mathematical authority. There was no attempt to include activities that were genuine problem solving opportunities for students. These contradictions contribute to the mixed voice of the lesson. Forman and Ansell (2001) discuss the multiple voices of classroom discourse in reform settings and note how institutional factors combined with the past experiences of participants as well as their expectations for future events influence the different voices that may emerge in classroom discourse. This could also be described as hybridised practice where some elements of novel practice have been adopted and adapted to the teacher's own beliefs (Corcoran, 2008).

**Discourse in this Community**

This class was at all times quiet, orderly and polite. Students contributed to the whole class discussion at the invitation of their teacher. Some sections of this lesson could be described as 'number talk' containing for the most part answer-focussed responses (Richards, 1991). Students did not appear be positioned as a SMI in this lesson. Despite elements of reform such as concrete materials and pair work, there are many elements present associated many traditional approaches. For example the large proportion of type 1 questions, the low number of student questions and the fact that Anne rather than her students gave the mathematical explanations and verified what was mathematically correct. It is possible that this is a classroom community in transition. It would seem that this community is working somewhere between level 0 and level 1 on the MTLC framework. A summary of the findings in relation to the components of the MTLC framework is given in table 4.7.

Table 4.7
*Summary descriptions of MTLC framework components in Anne's lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher as main questioner with questions generally focussing on answers rather than student thinking. |
| EMT | High levels of teacher EMT; Low levels of student EMT. |
| SMI | Teacher as primary source of mathematical ideas. |
| RFL | Teacher-centred classroom with students appearing to show low levels of RFL. For the most part, students were not involved in evaluating mathematical ideas. |

## Recordings 4a and 4b: Joan's Lessons

Two recordings were collected in Joan's third class on consecutive days. In third class students are generally 8- 9 years old. Both lessons focussed mainly on fractions and were similar in terms of structure and mathematical activity. For this reason I will present only the first lesson here.

### Descriptive Synopsis of Joan's First Lesson

The lesson began with some mental mathematics exercises. Questions covered time, addition, subtraction, a division word problem, shape, and odd and even numbers. Joan's questions drew attention to aspects of the underlying mathematics not directly probed by the formal question. For example she discussed both digital and analogue approaches in the time question which only presented an analogue example. She also asked questions about angles and other features beyond the scope of the basic textbook shape question. Then Joan introduced the main topic of the lesson by asking students to recall what they already knew about fractions. A series of shapes with unit fractions shaded ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{1}{8}$ respectively) were shown on the interactive whiteboard. The class discussed how many equal parts the shapes were split into, how many of these were shaded and what the numerator and denominator represented, though the formal terms were not used. Two examples were also presented where the dividing lines between the fractional pieces were not seen. The example of $\frac{2}{3}$ is shown in figure 4.7. It was discussed how best to split these to get equal pieces and figure out what fraction was represented. Then Joan presented the class with a target board where students had to find pairs of numbers where one number was half of the other number. Students explained how they figured out the pairs. A similar activity was carried out to find pairs of numbers where one number was a quarter of the other. The activity was then modified slightly when finding pairs of numbers where one is an eighth of the other. Here, Joan accepted suggestions for appropriate pairs from students and checked the solutions by dividing the suggested number of counters into eight groups on a mat which showed a circle split into eighths. Then students were given their own mats and counters. One mat showed a circle divided into halves, the other showed a circle in quarters. In a whole class setting, with Joan calling out the questions, students used these mats and counters to find half of a given number and then quarter of the same number. They were encouraged to make predictions before dividing the counters. Students practised more examples in pairs. Joan finished the lesson by asking the students to recall what they knew about fractions.

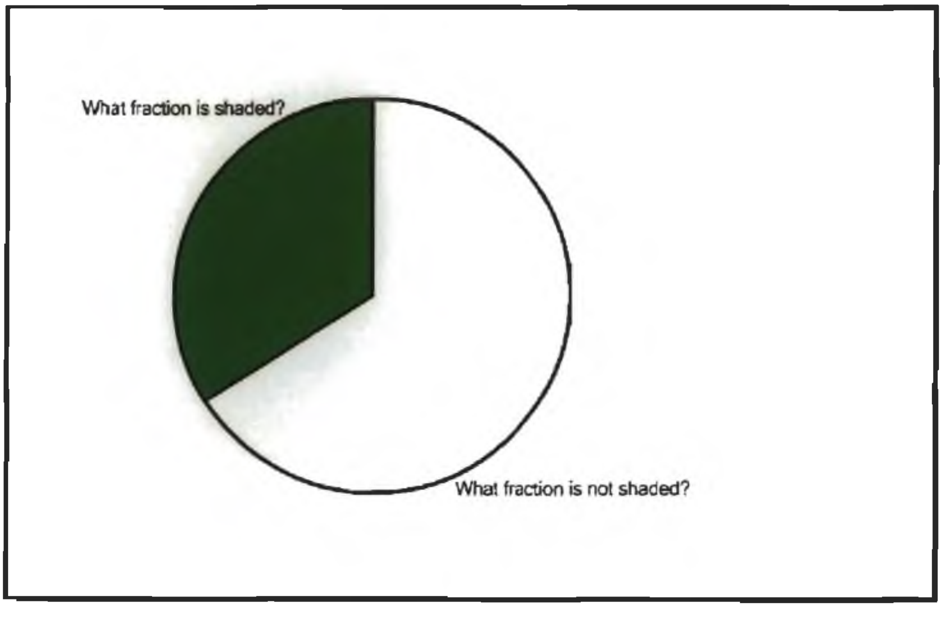



Figure 4.7. Sample of fraction identification task used on the interactive whiteboard in Joan's first lesson.

## Discourse Community Analysis

Some pair work was carried out in this lesson but it appears that Joan paused the recorder at this time. In some ways this is unproblematic because of my focus on whole class discourse but it does raise concerns about the limits of this form of data collection and what data has been lost through the researcher not being present during recordings. Even with the pair work excluded, the recording is over 50 minutes long and consists of over 700 turns of dialogue. For this reason and due to the nature of Joan's approach which contrasted with the other teachers discussed so far, I did not want to exclude it from the analysis.

### Questioning

#### *Teacher questions*

Teacher questions were counted and categorised using Boaler and Brodie's categories (2004). All questions where Joan asked for the name of a fraction were classed as type 2, inserting terminology questions. Questions about the denominator and numerator or questions about the fractional parts of numbers were categorised as type 3 questions, exploring mathematical meanings and relationships. Joan also posed a number of type 5 questions aimed at generating discussion. In five of these questions, she appeared to create opportunities for students to evaluate the thinking of their peers by asking whether they agreed or disagreed with previous contributors. Unlike some of the other lessons in which similar questions were posed, Joan appeared to give no hint as to whether the contribution was correct or not in the discourse preceding her question. The results of this analysis are shown in table 4.8 and in pie chart form in figure 4.8. The focus of the lesson is reflected in the totals for different question categories. For the first part of the lesson, Joan focussed on correctly identifying fractions and discussed the significance of the numerator and denominator. This is reflected in a high number of terminology questions (e.g. what is that fraction?) and type 3 questions exploring mathematical meanings and relationships.

Table 4.8
*Analysis of teacher questions in Joan's first lesson by type and number with examples.*

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 24: Where's our big hand?<br>Turn 73: What are we adding? | 88 |
| 2. Inserting terminology | Turn 678: What fraction is that going to be?<br>Turn 356: What fraction is each part? | 34 |
| 3. Exploring mathematical meanings and/or relationships | Turn 566: So eight is what fraction of thirty two?<br>Turn 700: What's half of twelve? | 34 |
| 4. Probing, getting students to explain their thinking | Turn 501. Why is that?<br>Turn 761: Why do you think that? | 26 |
| 5. Generating discussion | Turn 209: What do we know already about fractions?<br>Turn 711: Does anyone agree with her? | 17 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 199 |

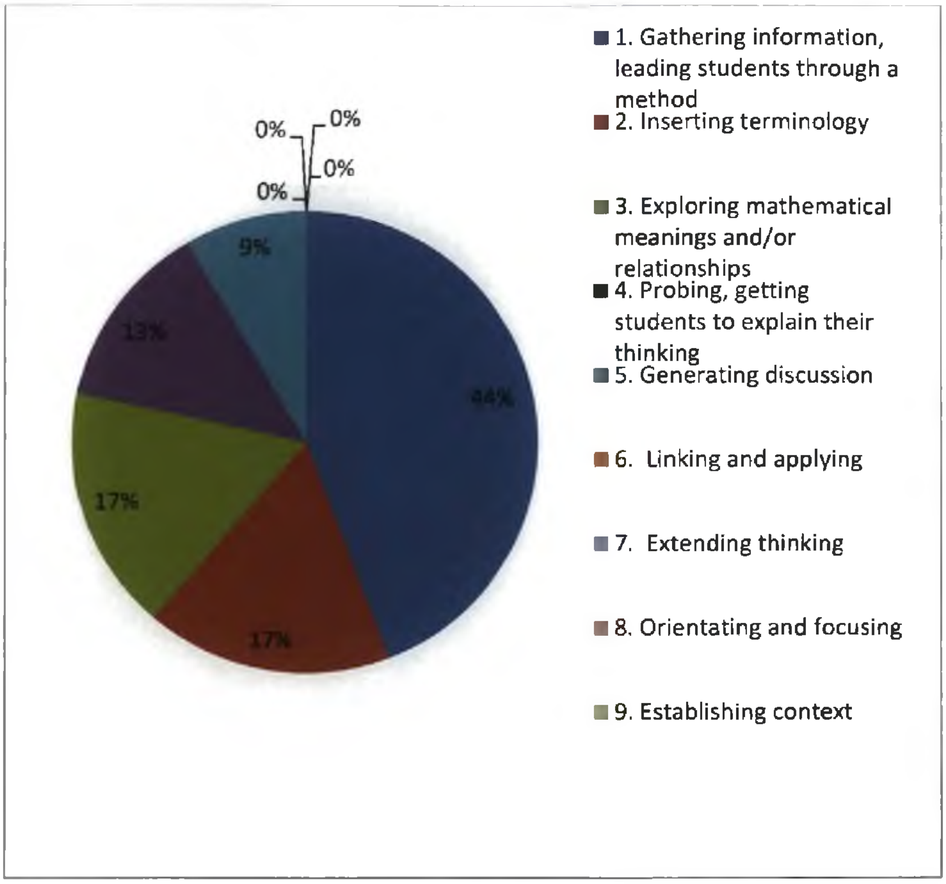



Figure 4.8. Types and percentages of teacher questions in Joan's first lesson according to Boaler and Brodie (2004) question categories.

*Student questions*

There were no student questions recorded in any of the whole class discussions in either lesson. Pair work was not included in the analysis and neither were any teacher-student conversations. It is possible that students asked questions during these times. As stated previously, whether this low level of mathematics focussed questions may reflect low levels of RFL on the part of students or might be due to features of the classroom culture would require more lesson recordings to investigate.

**Explaining Mathematical Thinking (EMT), Source of Mathematical Ideas (SMI) and Responsibility for Learning (RFL)**

These components will be discussed together as I did for the analyses of the previous lessons due to the difficulty of discussing one component independently of another.

Following the same structure as before, the discussion is organised under the headings: effects of teacher questions; teacher EMT and SMI; and treatment of errors.

***Effect of teacher questions***

Joan's question profile shows some features that are normally associated with reform classes, in particular a sizeable proportion of questions probing student thinking. Joan asked more questions probing thinking than the teachers in the other recordings and students consequently did regularly explain their thinking. However, the extent of the mathematical thinking that was shared would seem to be limited to calculation procedures, possibly due to the nature of the mathematical tasks. For example, consider the following extract that occurred when the class was using a target board.

| | | |
|---|---|---|
| 331 | Joan: | I'm going to give you an example. I'm going to choose four and sixteen because four is a quarter of sixteen and the reason I know four is a quarter of sixteen is …(*writing*) if I have four plus four plus four plus four or if I have four multiplied by four, it'll make? |
| 332 | Jenny: | Sixteen |
| 333 | Joan: | Good. Two numbers where one is a quarter of the other please. Chloe. |
| 334 | Chloe: | Ten and forty. |
| 335 | Joan: | Good. Why? |
| 336 | Chloe: | Because half of, twenty is half of forty and then a qua, ten is a quarter of forty. |
| 337 | Joan: | So you got a half of forty first and then you got a half again to get a quarter. Good girl. Rachael. |
| 338 | Rachael: | Two and eight. |

| | | |
|---|---|---|
| 339 | Joan: | Why? |
| 340 | Rachael: | Because two multiplied by two is eight. |
| 341 | Joan: | Two multiplied by? |
| 342 | Rachael: | Four. |
| 343 | Joan: | By four is eight so two is a quarter of? |
| 344 | Rachael: | Eight. |
| 345 | Joan: | Someone is playing with their cubes, stop please ... Niamh |
| 346 | Niamh: | Eh, five and twenty. |
| 347 | Joan: | Right, why? |
| 348 | Niamh: | Because five plus five plus five plus five is twenty |
| 349 | Joan: | And what fraction is five of twenty? |
| 350 | Niamh: | One… |
| 351 | Joan: | One what? |
| 352 | Niamh: | One-quarter |
| 353 | Joan: | Good girl it's one-quarter of twenty. So when I say why, you say five is a quarter of twenty. |

Each student gave an answer and Joan asked each one to explain why their answer was right. In this section, Joan's repeated use of the why question occurs so often, it almost functions as a type of drill. Students generally interpreted this question as an invitation to share their solution method. Each girl gave a slightly different answer focussing on her own solution method. However it seems, from this extract at any rate, that perhaps some of Joan's why questions may not have been probing student's thinking. Instead these questions may have been probing for generalities in the underlying mathematical relationship, particularly in light of Joan's statement at turn 353, which is almost a rule or a least a guideline for pupils, that encouraged general mathematical statements rather than statements based on computation methods. This has some links with Gray and Tall's (1994) discussion of procedural and conceptual knowledge. It would seem that the students were comfortable carrying out the procedures necessary to find the correct solution but may not yet have attained the more generalised conceptual understanding that Joan is pushing for. Although Joan regularly invited student contributions and pursued students' mathematical thinking, the thinking that was shared generally described computation methods.

***Teacher EMT and SMI***

Joan often presented explanations or elaborations in the middle of a sequence of questions to students. For example consider the following section of dialogue that occurred when the class was discussing fractions. Joan had presented a representation of one-third on the interactive whiteboard when this discussion occurred.

| | | |
|---|---|---|
| 246 | Joan: | Anyone know what that fraction is, now we haven't done that one … But would anyone be able to read it I wonder … would you Alice? |
| 247 | Alice: | Three. |
| 248 | Joan: | Um, you're sort of right but that's not exactly what we call it. |
| 249 | Students: | Oh! Ah! |
| 250 | Joan: | How many parts is that circle divided into? |
| 251 | Alice: | Two. |
| 252 | Joan: | No. |
| 253 | Alice: | Three. |
| 254 | Joan: | You're right. It would be divided into three parts. Now we only have one of those parts … if we had the other two it would make the full circle. Does anyone know what that fraction is called? Cathy. |

255 Cathy: I think it's called one-third or something.

256 Joan: Excellent. What's it called everyone?

257 Students: One-third.

258 Joan: Now this one is trickier again. It's a smaller part isn't it? Dearbhla, what's that one called?

259 Dearbhla: One-tenth.

260 Joan: Excellent. How many equal parts is that circle divided into Gemma?

261 Gemma: Ten.

262 Joan: Good girl, because the number on the bottom is-it tells us that it's divided into ten equal parts but this is a much smaller part. Why is it a smaller part? ... Emily.

263 Emily: Because it's em, it's a higher number on the bottom.

264 Joan: It's a higher number on the bottom, so it means we're sharing it-

265 Emily: Into ten

266 Joan: Between ten people or ten parts. So you have to get a smaller part each. It's like if you have a birthday cake, and if you were only sharing it with yourself and your mammy both of you would get a huge part each, wouldn't you? You'd get half and half. But if you had a birthday cake and you're sharing it between ten people, you'd only get a small slice each wouldn't you?

Here Joan asked a series of questions and elaborated or explained mathematical features at suitable times during the discussion. The student answers show various levels of mathematical understanding with Alice struggling to identify one-third but Emily able to explain that one-tenth is smaller than one-third because of the larger denominator even if she does not use the precise mathematical vocabulary. Liam also used a similar type of presentation that involved inserting information or elaborating on mathematical points in the middle of a sequence of questions (NicMhuirí, 2011a). In terms of the nature of SMI in this lesson, it is clear that Joan herself is the main source of mathematical ideas. Considering the extract above, although students contribute to the discussion, it is Joan's questions that direct the discussion and her explanations or elaborations that punctuate the dialogue. In contrast the student contributions are generally shorter and largely concerned with providing a straight forward answer. Joan's focus on student thinking appeared to be limited by the mathematical concepts she seemed to view as important in the lesson and the classroom discourse did not seem to stray from her pre-planned agenda.

***Treatment of errors***

In many ways, Joan's treatment of student errors is similar to Anne's approach. Errors were generally met by prompting or leading questions. In one case, her leading questions in pursuit of a specific answer seemed to invalidate reasonable contributions. Consider the following piece of dialogue in which the students were presented with a shape with one-quarter shaded.

| | | |
|---|---|---|
| 307 | Joan: | Look at this circle. How many parts is it divided into? Nicole |
| 308 | Nicole: | Four Q- |
| 309 | Joan: | Four? |
| 310 | Nicole: | Quarters |
| 311 | Joan: | Well four parts and each one is called a- |
| 312 | Nicole: | Quarter |

| | | |
|---|---|---|
| 313 | Joan: | Right, now there's how you write one-quarter, one over four. What does the four, the number on the bottom tell us? Emily |
| 314 | Emily: | How many we have |
| 315 | Joan: | How many? |
| 316 | Emily: | How many we have altogether |
| 317 | Joan: | How many what we have? How many? |
| 318 | Emily: | Quarters |
| 319 | Joan: | Or, no. How many? |
| 320 | Student: | Ah! |
| 321 | Emily: | How many slices? |
| 322 | Student: | Ah! |
| 323 | Joan: | Slices or? |
| 324 | Emily: | How many equal parts? |
| 325 | Joan: | How many equal parts. |

In this section of the lesson, Joan was concentrating on identifying fractions and establishing what the denominator and the numerator mean. In this case, the students were considering $\frac{1}{4}$ and when Joan asked Emily what the 4 represented, Emily provided an imprecise answer ("how many we have", turn 314). Joan's follow up question of "how many?" at turn 315 prompted Emily to give a more complete answer and Emily suggested, "How many we have altogether". Joan's next question, "How many what we have? How many?" suggests Emily's amended answer was still has not precise enough for Joan. Emily replied, "Quarters" in turn 318. At this stage it probably appeared to Emily that she had given a perfectly reasonable mathematical answer, that the 4 in $\frac{1}{4}$ tells how many quarters there are in the shape. However Joan suggested that Emily's idea was incorrect and used a prompting question again. The reasons Joan did this are unclear. Perhaps she considered the language used until this point.

'How many quarters we have' may actually suggest what the numerator represents and not the interpretation of how many quarters there are altogether (which is supported by Emily's earlier attempt at turn 316). It is also possible that Joan was aiming for a more generalised description of what the denominator means across all cases and not specific to this example. This may also link with Gray and Tall's (1994) notions of different levels of understanding whereby Joan is aiming for generalised reified understanding but students like Emily are focussed on specific examples.

**Discussion**

There are similarities between Joan's lessons and Anne's lessons in relation to lesson structure and resources used. Like Anne, Joan used concrete materials in her lessons and students used counters to find fractions of quantities. This choice of materials is in line with the reform suggested in the revised curriculum (DES/NCCA, 1999a). Both teachers began their lessons with mental mathematics questions and both teachers used target boards. As I suggested earlier this may indicate collaborative planning on the part of the teachers. Or given that the teachers taught at different class levels and may be unlikely to plan together, it may reflect a whole school approach to mathematics.

In this lesson students did engage in EMT but as I discussed above, the nature of the mathematical explanations was limited to descriptions of computation processes and did not involve students in making conjectures, justifying their opinions or evaluating the ideas of others to any great extent. Instead, Joan was the source of mathematical ideas and authority responsible for determining what was mathematically correct. The students' participation in whole class discourse seemed to have very definite limits. Student thinking, in the form of the sharing of computational strategies, was encouraged but no student ideas capable of diverting the whole class discourse from its predictable track were shared. Students did not ask any questions in the whole class setting.

**Discourse in this Community**

The elements of reform that have been adopted by this teacher do not appear to have resulted in increased student agency. Instead the discourse of these lessons was still teacher-centred. It would be interesting to see this class and this teacher work on more open ended problems because it is possible that more features of a discourse community would become evident with more cognitively demanding open problems. As in Anne's lessons, there are mixed elements of both traditional and reform approaches and it is possible that with regard to the MTLC framework, this classroom community was in transition between level 0 and level 1. A summary of the findings in relation to the components of the MTLC framework is given in table 4.9.

Table 4.9
*Summary descriptions of MTLC framework components in Joan's lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher is the only questioner. Some questions focussed on student thinking. |
| EMT | High levels of teacher EMT; Students engaged in EMT more than in other lessons but explanations generally confined to computation processes. |
| SMI | Teacher as primary source of mathematical ideas. |
| RFL | Small efforts made to enable student evaluation of the thinking of their peers but generally teacher positioned as mathematical authority. |

## The 'RUDE' approach to problem solving

A feature common to both Anne and Joan's second lessons was the use of the RUDE approach to problem solving. This involves using the acronym RUDE to help support students engaged in problem solving to remember certain steps: read, underline, draw, and estimate. It is intended that the student reads the word problem, underlines key words, draws or models the problem situation and estimates before solving. Other acronyms that may support students in following steps in problem solving include ROSE: read, organise, solve and evaluate or LUVCC: look, underline, visualise, choose numbers and check.[7] These acronyms can be viewed as a way of making a defined procedure out of the problem solving process. This procedural approach may support the solving of word problems where the solution process, being the straight forward application of one or more numerical operations, is relatively obvious. It is less likely to support the solving of more complex mathematical problems where the solution process is not obvious unless students have developed some skill in the modelling stage ('draw' or 'visualise'). RUDE was not mentioned in the lesson analyses above as it occurred in the second lessons of both teachers. It was the only significant difference between the first and second lessons of both teachers. For this reason, I feel that is worthwhile to discuss it here. It also gives an insight these teachers' attempts to incorporate problem solving into their teaching. In the case of Anne and Joan this problem solving was limited to word problems. In the case of John and Liam it did not feature at all.

Consider the following extract from Joan's second lesson, where students used the RUDE acronym to tackle a word problem.

| | | |
|---|---|---|
| 552 | Students: | There were four apples on a tree. John eats half of them. How many does he eat? |
| 553 | Joan: | O.K., with your partner whisper and don't underline anything, just whisper to your partner for about ten seconds about which words you're going to underline. |
| [ ] | | |
| 554 | Joan: | O.K., hands up, hands up … well Linda. |
| 555 | Linda: | Em, |
| 556 | Joan: | What words do you and Jane think we should underline? |

[7] Posters for these problem solving acronyms and others can be found on the government supported Professional Development Service for Teachers (PDST) website:
https://sites.google.com/a/pdst.ie/region5problemsolving/home/additional-materials-1/problem-solving-strategies-and-role-cards

557 Linda: Four, half and how many.

558 Joan: Four, because it's four apples, half because we've to get half of them and how many. Right, does anybody know how to do that problem?

559 Student: Oh!

560 Joan: Well first of all, what will we draw? Because we've to draw next. What would you draw Melissa?

561 Melissa: I would draw … a tree … with loads of apples

562 Joan: Loads of apples? How many apples?

563 Melissa: Four.

564 Joan: You've got four apples. And then what would you do?

565 Student: Draw John.

566 Joan: Draw John. Draw John eating them, good. Now, what fraction are we talking about here, what fraction did he eat? ... Yeah

567 Student: Half.

568 Joan: Half. So how many groups do we have?

569 Urusla: Two.

570 Joan: Two. So if we split the four in two ... does anybody know how we would write that? First of all, our estimate, we're going to estimate what our answer is going to be ... what do you think the answer night be? Karen

571 Karen: Two.

572 Joan: You think it might be two. So will we do it and find out

573 Karen: Yeah.

574 Joan: How would I write that as a division sum? Paula

| | | |
|---|---|---|
| 575 | Paula: | Four. |
| 576 | Joan: | Good, divided by? |
| 577 | Student: | A half. |
| 578 | Joan: | By? … Not divided by a half. We're getting a half so we divide by? |
| 579 | Student: | Two? |
| 580 | Joan: | We divide by two. Good. Now, here's my two circles and I can divide out the four. Count. |
| 581 | Students: | One, two, three, four |
| 582 | Joan: | How many did each one get Lisa? |
| 583 | Lisa: | Two. |
| 584 | Joan: | What's four divided by two? |
| 585 | Lisa: | Two. |
| 586 | Joan: | Two, so was your estimate correct Karen? |
| 587 | Karen: | Yes. |
| 589 | Joan: | Yes, well done. |

Following the draw and estimate steps of the RUDE acronym is somewhat superfluous in this case given the scale of the numbers involved. In fact, the drawings the students propose do not tackle the mathematical aspects of the problem and it is Joan herself who draws two circles and models dividing the four apples into two groups (turn 580). In this way, both the nature of the word problem and the procedural RUDE approach serve to limit opportunities for students' mathematical thinking. Similar issues were obvious on the use of the RUDE acronym in Anne's class.

## Limits of Stage 1 Research

The limits of this type of research are many. The fact that I as researcher was not present in the classroom and must rely only on the audio-recordings is a limiting factor. Further issues around the collecting of data by audio recording have been discussed in Chapter 3. The fact that only one or two recordings were collected in each classroom means that little can be said in general about discourse in these classrooms and comments and interpretations must relate only to these specific lessons. It must also be acknowledged that with this small non-representative sample size, it is impossible to make claims about discourse in Irish primary schools in general. Instead these cases are intended to be illustrative. They provide insight into the types of whole class discourse that occurred in these particular classrooms during the given lessons. They also provide a contextual background for comparison with the mathematical discourse community that was the aim of the teaching experiment. The insight provided by these recordings limited though it is, is not available in large scale national assessment studies such as NAMA1999, NAMA 2004 or NA 2009 (Eivers et al., 2010; Shiel & Kelly, 2001; Surgenor et al., 2006;). For this reason, it provides a valuable insight into the processes of teaching and not just the outcomes (Hiebert et al., 2005).

## Summary

In this chapter I have presented details of four lessons from four different teachers in two different schools. John's fourth class decimal lesson involved activities from a textbook. His use of Dienes' blocks to represent decimal fractions may have been influenced by the textbook. This representation seemed to have been problematic for students. Liam's fourth class multiplication lesson also revolved around textbook activities and the classroom discourse revolved around traditional answer focussed questions. Anne's fifth class decimal lessons involved mental mathematics activities from a textbook, students' use of notation boards and target board activities. Her approach seemed to be informed by both traditional and reform agendas which resulted in a form of hybridised practice (Corcoran, 2008). Joan's third class fraction lessons involved mental mathematics activities from a textbook, students' use of concrete materials and target board activities. The similarities of activities in Joan's and Anne's lessons may suggest joint planning or a whole school approach to the teaching of mathematics. Joan asked a higher proportion of questions pursuing student thinking than the other teachers. These questions generally focussed on student computational methods rather than emergent mathematical thinking or conjectures.

The lessons varied across the two schools. Liam's and John's lessons were quite traditional textbook based lessons while Anne's and Joan's lessons contained some features of reform such as use of concrete materials and a mental mathematics focus. John's lesson also used Dienes' blocks as concrete materials but only to aid teacher demonstration not for student manipulation. Activities were similar across schools in that most activities were context free, though a number of word problems were considered in both Anne's and Joan's lessons, some of which were tackled using the RUDE approach.

The lessons were also similar in that the teacher was positioned as the mathematical authority who evaluated students' mathematical contributions. Students' thinking rarely became an object of whole class discourse and students rarely evaluated the contributions of their peers. Pair work was carried out in both Anne's and Joan's lessons but there was no evidence of student-to-student talk in whole class discussions.

Admittedly the nature of the study reported here is limited but this chapter gives an indication of how the land lies in relation to mathematical discourse in primary classrooms. Analysis using the MTLC framework (Hufferd-Ackles et al, 2004) shows that where efforts at mathematics reform have been adopted in these classrooms, it has not resulted in positioning students as mathematical authorities or the source of mathematical ideas. This is an interesting finding considering that from a traditional perspective, the majority of the lessons might be considered as examples of 'good' teaching. For the most part, they contained clear teacher explanations and structured activities which appeared to engage students. The fact that these examples of 'good' teaching fare poorly on the MTLC framework highlights how radical the shift to more reform orientated practice actually is for both teachers and students. It also highlights the need for sustained professional development for teachers to guide and support them through the reform process. The questions raised by this part of my research are discussed more fully in chapter 8.

## Chapter 5: Teaching Experiment Lessons

This chapter contains analysis of five lessons that occurred during the teaching experiment. These lessons were chosen with respect to their mathematical content and features of the discourse community that they illustrate. Issues around this choice were discussed more fully in chapter 3. The lessons were analysed in the same manner as the stage 1 lessons. The analysis of two lessons will be presented in full with the other three in summary form consisting of descriptive synopsis followed by discussion. The lessons I have chosen to present in detail are the *Fraction Problems* and *Equivalence Challenge* lessons. These lessons highlight complexities of the teaching approach including social issues and the tension in attempting to maintain a balance between mathematical challenge and accessibility for students of different abilities.

I will begin the chapter with a discussion of the classroom context in which the teaching experiment took place. Then I will discuss the five lessons in the order they occurred chronologically: the *Cutting Pizzas* lesson, the *Fractions Problems* lesson, the *Dienes' Decimals* lesson, the *Equivalence Challenge* lesson, and the *Percentage Present and Absent* lesson (for dates of recording and recording lengths see table 3.9, pages 102 - 103). Finally I will discuss issues arising from this analysis and highlight where these will be addressed in the remainder of this thesis.

### Classroom Context

The teaching experiment took place in fifth class. This is the penultimate year of primary school in Ireland and pupils are generally 10 -11 years old. The school was a designated disadvantaged boys' school (DEIS band 2). In total, there were twenty four boys in the class. A standardised mathematics test carried out in June 2010 showed a range of achievement levels with a number of students scoring at or below the tenth percentile. Such students are considered 'priority' cases for learning support intervention (DES, 2000). A timetable was devised so that these lower achieving students would receive instruction at levels appropriate to their needs from a learning support teacher while I taught the fifth class curriculum to the remainder of the class. Seven students were withdrawn daily for this instruction. In effect, this arrangement is similar to the approach of streaming by ability. There is little data available to suggest how widespread this approach is at primary level in Ireland but McLoone (2011) discusses one group of teachers who enacted streaming of mathematics classes in an Irish disadvantaged primary school, in an attempt to address the disparate needs of sixth class pupils. The students who attended learning support from my class did engage in some mathematics activities in whole class settings for example, daily mental mathematics tasks, mathematics games, tables practice and activities directed at revision and consolidation work. However, the majority of the teaching experiment lessons occurred without these students present.

This should not suggest that the remaining seventeen students were all middle or higher achievers. There remained in the teaching experiment group a number of students who had scored below the twentieth percentile in the standardised test carried out in June 2010 and who had significant difficulties with mathematics at fifth class level. There was also within the class a group that could be described as 'macho lads' (MacAnGhaill, 1994), who appeared more concerned with how they were perceived by each other than how they were perceived by me as teacher (Ashley, 2003). MacAnGhaill's use of the term 'macho lads' refers to a group of students with low academic achievement levels whose key social practices were " 'looking after your mates', 'acting tough', 'having a laugh', 'looking smart' and 'having a good time'" (1994, p. 56). In my own classroom, these key social practices appeared to be shared by students of different achievement levels.

**Recording 1: Cutting Pizzas**

This lesson is presented in summary form consisting of descriptive synopsis and discussion. The results of the analysis of teacher and student questions are shown in appendices 16 and 17 respectively. This lesson was also discussed in NicMhuirí (2011b).

**Descriptive Synopsis**

After completing a fraction identification activity, we considered the question: "Three children shared two pizzas. How much did they get each?" This was shown on the interactive whiteboard with a picture of two pizzas and three children (figure 5.1). Suggestions were solicited from students about how to "organise the pizzas". Edward suggested cutting them into thirds. Anthony suggested cutting them into sixths and letting the children have four slices each. Kevin suggested cutting both pizzas in half, giving the children one-half each and giving the extra slice "back to the man". Steven asked who the oldest was and suggested giving her more. He struggled when drawing his solution on the whiteboard and was assisted by his peers who commented on his work and gave suggestions on how to complete it.

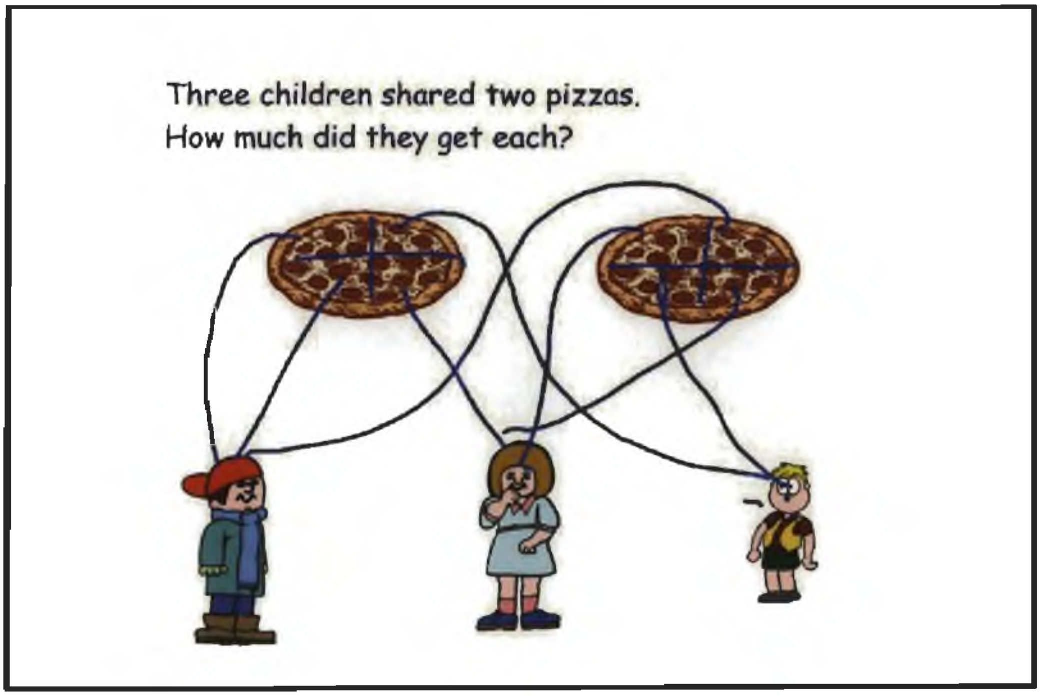



Figure 5.1. Michael's solution to the first pizza sharing task in the *Cutting Pizzas* lesson. Michael shared two pizzas between three children by splitting into quarters and allocating portions unevenly.

When I continued to press for more suggestions, Steven asked me directly "do you know?" Michael suggested twelfths and Andrei suggested cutting each pizza into twenty one pieces and giving the children fourteen slices each. Darragh commented that these pieces would be very small. Michael came to the board and drew out a solution that was similar to Steven's suggestion. His solution of cutting both pizzas into quarters and distributing them unevenly amongst the children is shown on figure 5.1.

At this point, I reintroduced Edward's original suggestion of thirds. I used the Notebook representations of thirds to give the children in the picture two slices each (figure 5.2). I asked for suggestions of what fraction they got each and how we should write a third and a third. I also wrote $\frac{1}{3}+\frac{1}{3}$ on the board. Alan suggested that it was two-sixth. When prompted, Alex said he disagreed and suggested one-sixth but could not explain why. Jared suggested that Alex might be right as you could add the bottom and add the top of the fractions. Darragh suggested that it could be both two-sixth and two-third. Jake suggested that it could not be two-sixth as this would mean we would have cut the pizzas into sixths initially. When prompted he repeated his explanation and added more detail. To help students follow his explanation, I drew circles cut into sixths to represent the pizzas. This diagram can be seen in the top right hand corner of figure 5.2. Darragh revised his previous contribution and suggested that "Two-sixths is equivalent to one-third which means that it's the same as one-third". He explained that the fraction in question could be two-thirds or four-sixths. Alan suggested cutting the pizzas into eighths but realised that this would result in an unequal share. Edward then asked "Wait, can you go up over one-twelfth?" He seemed to be asking if there are fractions with denominators higher than twelve. Darragh, Andrei, other students and I gave answers to his question with reference to decimal fractions. Steven asked about the meaning of the word simplify which Darragh had used in some of his contributions and linked it with a similar word from the Harry Potter series of books (Rowling, 1997).[8] I explained it. Then Darragh noted the multiplicative pattern between two-third and four-sixth. Luke noted that he had done this in a previous lesson too. I repeated his explanation and represented it on the board (figure 5.3.)

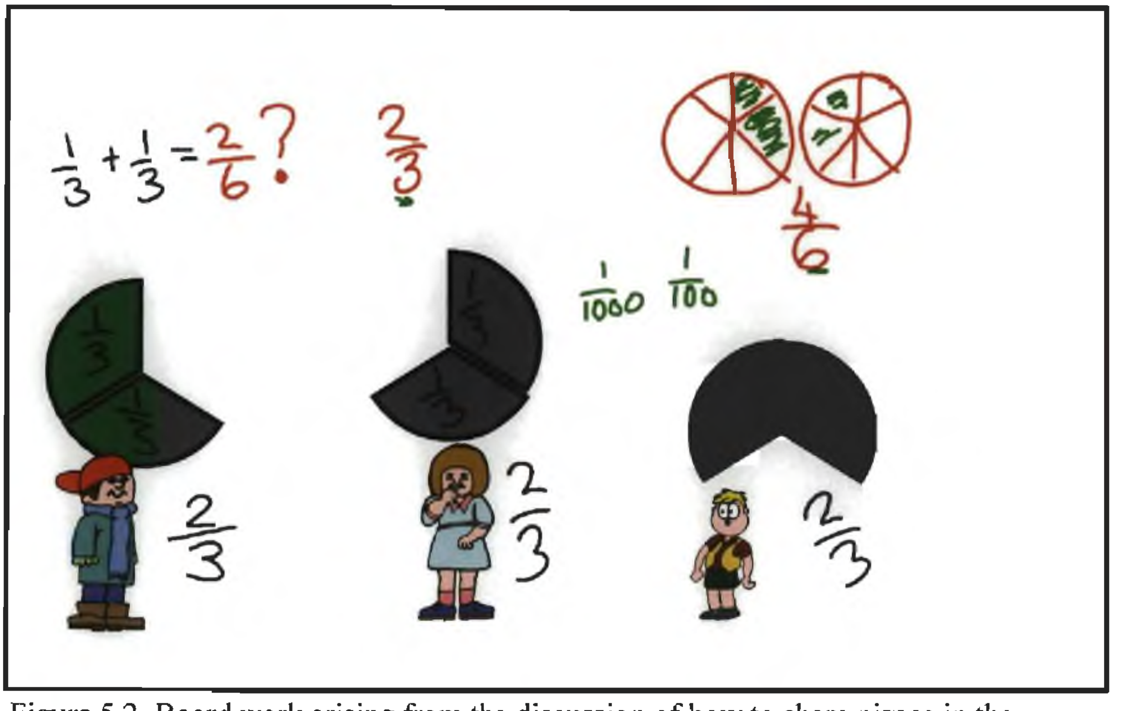



Figure 5.2. Board work arising from the discussion of how to share pizzas in the *Cutting Pizzas* lesson. The discussion focussed on how to share two pizzas among three people evenly and how to represent two-thirds in fraction form.

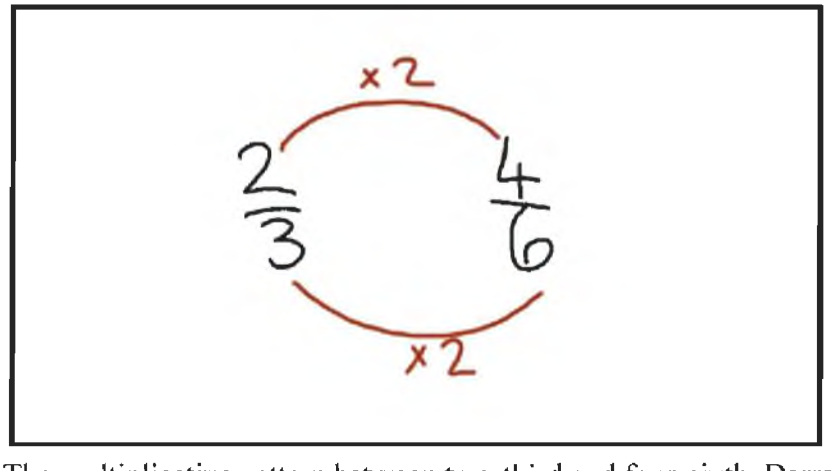



Figure 5.3. The multiplicative pattern between two-third and four-sixth. Darragh explained this to the class.

[8] Steven appeared to link the word 'simplify' with the word 'stupefy' which is used in the Harry Potter books as an incantation to stun enemies (Rowling, 1997)

The students were then given some time to work in pairs on a similar problem sharing three pizzas between four children and considering if they would get more or less than a half. After some time I invited Alex to come to the board and share his solution. He solved it by cutting two pizzas into halves and the remaining pizza into quarters (figure 5.4). Some students commented on how well he had explained it. When questioned he could not say what fraction each person got in total. I encouraged the students to consider what a half and a quarter would look like together. Darragh shouted out the correct answer of six-eighths. Alan suggested it would be "three something because if you have that and you add them together it's a half and a quarter and there's only one piece missing." With suggestions from other students he came up with the correct solution of three-quarters.

At this point, I asked if other students agreed or disagreed and Darragh said "Eh, no, I'm not going to confuse people but you could make it six-eighths." Anthony suggested splitting all three pizzas into quarters and came to the board and showed that this solution method also gave each child three-quarters. Many groups said that this was how they solved it too, but Steven said that he and his partner split the pizzas into thirds. When he tried to show this on the board, he copied from the way he had solved it in his copy and split the pizzas into four pieces. I questioned him about this and then finished up the lesson.

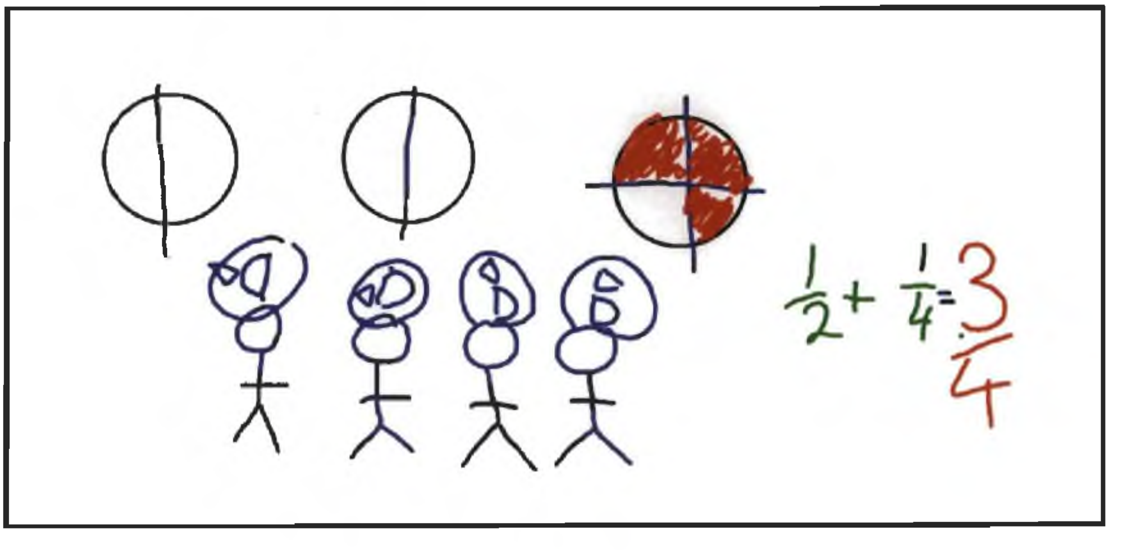



Figure 5.4. Alex's solution to sharing three pizzas between four people.

**Discussion**

The idea of this lesson was to have students explore fractions in a context that was imagined to be meaningful for them. The fact that multiple solutions were possible created opportunities for a variety of different contributions. The Boaler and Brodie (2004) analysis of teacher questions showed that over 50% of the teacher questions in this lesson were either questions probing student thinking or questions aimed at generating discussion. These results can be seen in appendix 16.[9] There was also a sizable amount of student questions (appendix 17). Individual mathematical questions were discussed in detail often with multiple student contributors. For example, we began discussion of the problem of sharing two pizzas between three people at turn 69. In the following turns I did not explain or present an approach of my own, but attempted to position students as the SMI by soliciting suggestions from various students. As explained in the descriptive synopsis, Edward, Anthony, Kevin and Steven all contributed suggestions. By this point, we had spent almost seventy turns of dialogue discussing possible solutions and Steven asked me directly, "do you know?" (turn 136). It is possible that he was unfamiliar with working on questions with multiple possible solutions and understood the lesson process as a search for a single 'right' answer. The absence of teacher explanation seemed to make him question whether I knew 'the' answer at all. As researcher, I recognised his question as a significant event immediately but within the complex rush of the lesson, I was unsure how to respond as teacher. In the end, I did not respond at all. It seems obvious in hindsight that this would have been an opportune moment to discuss the nature of the experiment and begin an explicit discussion of classroom norms for the devolution of mathematical authority from teacher to students.

The lack of teacher explanation combined with teacher questions aiming to probe thinking and generate discussion seemed to create an opportunity for students to explain their

[9] For the lessons presented here in summary form, I have also included the results of the analysis of teacher and student questions as appendices. This applies to the *Cuting Pizzas* lesson, the *Dienes' Decimals* lesson and the *Percentage Present and Absent* lesson.

mathematical thinking. Some students were more comfortable in this central role than others. For example, Darragh and Steven were very vocal throughout the lesson. Jake also displayed high levels of RFL when he disagreed with previous contributors to present his own understanding of the problem situation. Student EMT was not always coherent or readily comprehensible. It sometimes contained errors or was incomplete from a mathematical perspective possibly reflecting emerging rather than reified mathematical understandings. At times, students seemed to struggle with the language needed to express themselves clearly. Because of this, the teacher's role in re-voicing student suggestions or asking students to restate contributions was important (Chapin, O'Connor & Anderson, 2009; Dooley, 2010).

Kevin and Steven both presented mathematically naïve solutions or solutions based on real life considerations. I did not address these errors directly. It would perhaps have been better to explicitly discuss the differences between a real-life situation involving pizza sharing and the mathematical context. Zevenbergen and Lerman note the tendency of working class students to apply real life knowledge to mathematical situations because they "fail to recontextualise the everyday task into a mathematical task, instead offering an (incorrect) response to the question" (2001, p. 573). This tendency may be exacerbated by their lack of experience with mathematical problems set in real life contexts. The problem we considered is noticeably different from the word problems students would have encountered previously in their textbooks which often have only one solution.

In retrospect, I feel that I could have been more proactive about teaching the necessary fraction language to help students participate more fully in these discussions. As it happened, it was mainly Darragh who introduced fraction terminology. One of the issues that emerged for me as teacher after this lesson was the range in achievement levels of the students. This range may not have been so obvious in a lesson where student thinking was answer focussed and not made public to the same extent.

**Discourse in this Recording**

The discourse of this lesson was not the answer focussed 'number talk' found in some stage 1 lessons (Richards, 1991). Instead, student ideas formed the content of the discussion and students themselves evaluated what was mathematically correct. It was not a traditional lesson and therefore not at level 0 of the MTLC level descriptors. The student-to-student engagement that characterises level 3, though present at times, cannot be considered robust or regular enough to consider this community at level 3. The choice that remains is between level 1 and level 2. The strongest argument for describing this lesson as an example of a community operating at level 2 of MTLC framework is the large role that students' multiple solution strategies played in the course of the lesson. This is a feature of the level 2 descriptors for EMT, SMI and RFL in particular. Table 5.1 shows a summary of findings.

Table 5.1
*Summary descriptions of MTLC framework components in the Cutting Pizzas lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher questions included questions probing thinking and generating discussion. Students also asked some mathematics focussed questions. |
| EMT | Low levels of teacher EMT. High levels of student EMT. |
| SMI | Students appear to be positioned as SMI. |
| RFL | Students involved in evaluating mathematical ideas. |

## Recording 2: Fraction Problems

**Descriptive Synopsis**

Before the recording began, students worked on a teacher-designed worksheet (appendix 18). This shows a fictional computer game character with two rectangular bars representing his energy and his ammunition. Students worked together to find out what fraction of energy and ammunition the character had left. They also worked on fraction word problems. This recording is of the whole-class discussion about the correction of the worksheet. The fractions represented on the energy and ammunition bars were discussed briefly and students seemed to answer these questions with relative ease. The first word problem was: "Three boys need ribbon to finish off Christmas art work. There is only one piece of ribbon left. Show where they should cut it. What fraction of the strip do they get?" When called to the board, James initially approached this by drawing lines to represent cutting the ribbon into two pieces but was corrected by his peers. The next question was: "Four boys must share 1m (100cm) of ribbon for their Christmas art work. Show where they should cut it. What fraction of the strip do they get and how many cm of ribbon does each get?" (figure 5.5). Steven came to the board and partitioned the strip into four pieces. The discussion was interrupted when John announced his tooth had fallen out. After dealing with this, I asked Steven to explain how he figured out the problem.

We then discussed the next question: "Mr Hunt organised a trial for the athletics team. Boys had to race 1000m. Kevin gave up half way through. Jim made it $\frac{3}{4}$ of the way and Tom finished it. Can you mark the point where they finished on the empty number line and fill in on the grid how many metres they ran?" (figure 5.6). Alan presented his solution on the board and I questioned him about how he knew to position the three-quarter mark (turn 131). Anthony and Edward added comments about the length of the race and the winners. Then Darragh referred to the question I had earlier posed to Alan about the three-quarter mark and suggested that a quarter was half of a half (turn 174). At this point I restated Darragh's contribution and labelled the points on the number line: $\frac{1}{4}, \frac{1}{2} = \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$. Figure 5.6 shows the initial solution efforts of Alan in black hand-writing and my notation in green hand-writing. Jonathan then asked a question about this labelling; effectively wondering why one-quarter is smaller than a half when four is bigger than two. I asked Jake to restate his question.

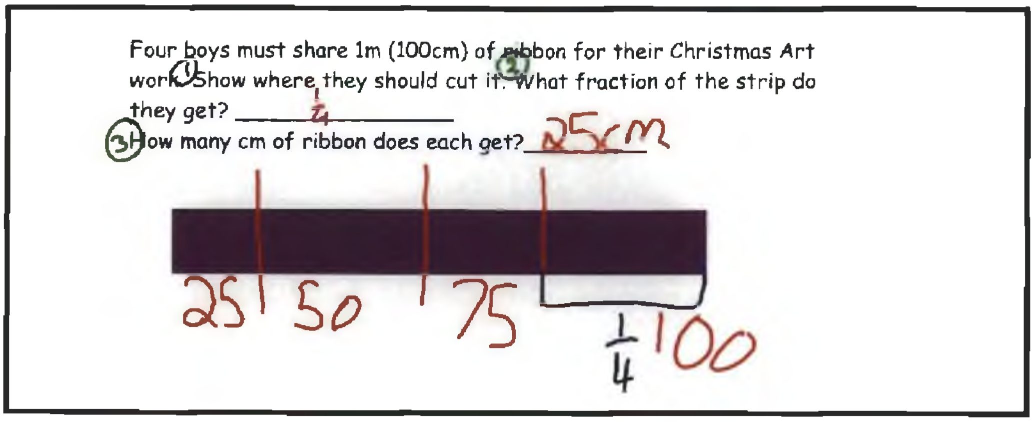



Figure 5.5. Task involving sharing 100cm of ribbon between four people. The figure shows Steven's solution.

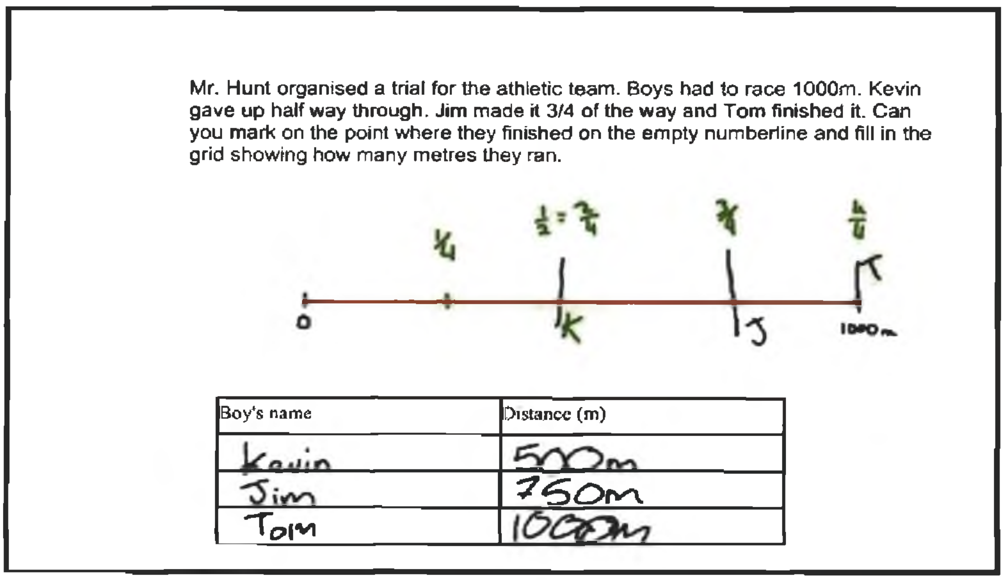



Figure 5.6. Task involving finding common fractions of 1km. This shows Alan's solution and the notation I made marking the points on the fraction number line.

In response to Jonathan's question, Alex came to the board. He split one circle into quarters and another into halves and explained that because the circle that showed quarters was split into four pieces, they would be smaller (figure 5.7). Jonathan indicated that he was still unsure and I asked him to choose somebody else to explain it. He called on Aidan. Aidan referred to taking away lines so that two-quarters would be the same as a half but then began to joke and was asked to sit down. Alan came to the board and explained that two-quarters were the same as a half so one-quarter had to be smaller than a half (figure 5.8). The students attending learning support arrived back to class during this discussion. When the class was settled again, Darragh explained that because quarters mean a whole was cut into four pieces, they are smaller than halves, which means the whole was only cut into two pieces. I restated Darragh's proposition using the idea of cutting a cake into quarters or halves and said that the halves would be bigger slices. I then finished the lesson.

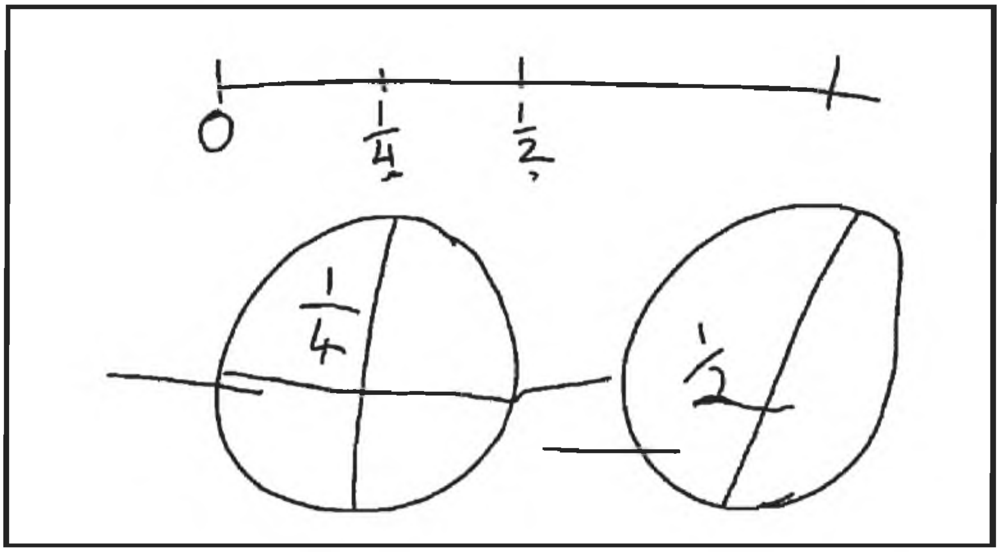



Figure 5.7. The diagram that accompanied Alex's explanation of why $\frac{1}{4}$ is smaller than $\frac{1}{2}$. This shows my drawing and positioning of $\frac{1}{4}$ and $\frac{1}{2}$ on the number line and the diagrams Alex drew in attempting to explain why $\frac{1}{4}$ is smaller than $\frac{1}{2}$

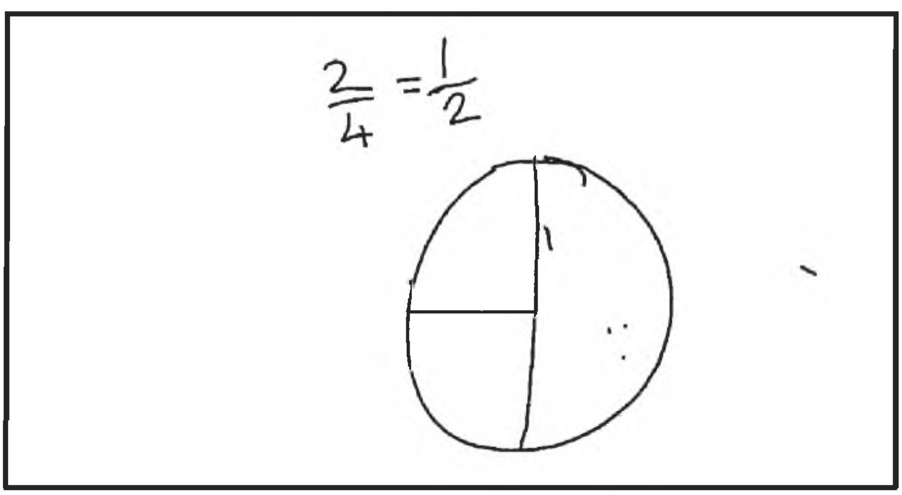



Figure 5.8: The diagram that accompanied Alan's explanation of why $\frac{1}{4}$ is smaller than $\frac{1}{2}$. Alan used the argument that because $\frac{2}{4}$ is equal to $\frac{1}{2}$, $\frac{1}{4}$ must be smaller than $\frac{1}{2}$.

## Discourse Community Analysis

This recording consisted entirely of whole-class discourse (320 turns). A similar process of analysis was followed as in the stage 1 lessons and the presentation of the analysis will follow a similar format.

### Questioning

#### *Teacher questions*

Teacher questions were classified using Boaler and Brodie's (2004) question categories as before. This recording was 18 minutes 35 seconds long and is substantially shorter than other recordings analysed. This, as well as the methodology of not counting repeated questions, may account for the relatively low number of teacher questions. The results of this analysis are shown in table 5.2 and in figure 5.9. Type 4 and type 5 questions, aimed at probing students' thinking and generating discussion respectively, were the primary questions I employed in this recording. This question profile may imply that students were not engaged in exploring mathematical meanings (type 3 questions) but this would be misleading, as will become clearer as the data are examined in more detail.

#### *Student questions*

Student questions were counted and coded using the codes 'questions seeking clarification about mathematics being discussed' or 'questions seeking organisational clarification' as before. Table 5.3 shows the results of this analysis. Jonathan's question about the relative size of a half and a quarter (turn 182) is a question that explores mathematical relationships and representations. It is interesting to note that in this lesson the lack of teacher questions focussed on exploring the nature of mathematical relationships is balanced in some way by this question of Jonathan's which prompted 132 turns of dialogue and accounts for more than 40% of the total 320 turns. Jonathan's question is a fundamental one and exposes a common fraction misconception (Newstead & Murray, 1998).

Table 5.2
*Analysis of teacher questions in the Fraction Problems lessons by type and number with examples*

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 6: Is there eight altogether? | 1 |
| 2. Inserting terminology | Turn 35: Do we say two-threes? | 1 |
| 3. Exploring mathematical meanings and/or relationships | | 0 |
| 4. Probing, getting students to explain their thinking | Turn 16: One-third? Why's that Aidan?<br>Turn 129: How did you know that that was three-quarters Alan? | 10 |
| 5. Generating discussion | Turn 48: Somebody who's making those noises, do you want to tell James why you think that might be the wrong idea?<br>Turn 82: O.K., I could hear some people saying there, when it says what fraction of the strip do they get, were some people saying thinking it should be a different number? | 9 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 20 |

Table 5.3
*Analysis of student questions in Fraction Problems lesson by type and number.*

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: Two threes?<br>*The student seemed to question Jonathan's attempt to say two-thirds.*<br><br>James: Oh there's three boys?<br>*James made an error when sharing a ribbon between three boys and he questioned or clarified Alan's explanation of his error.*<br><br>Student: What?<br>*The student questioned Darragh's suggestion that a quarter is half of a half.*<br><br>Jonathan: Look it, why is that a two? One four and one two, that's like you have more.<br>*Jonathan attempted to ask why a half is greater than a quarter when two is smaller than four.*<br><br>Jake: Eh, why is one-fourth- oh- it's why is one-fourth bigger than – oh smaller than a half when four is bigger than two.<br>*Jake was asked to repeat Jonathan's question.*<br><br>Jonathan: Anyone have a better answer?<br>*Jonathan indicated that he was not satisfied with Alex's attempt to answer his question and asked other students for contributions.*<br><br>Jonathan: Teacher they're all quarters aren't they?<br>*Jonathan asked this question about one of the diagrams drawn by Alex shown in figure 15.* | 7 |
| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
| Steven: Which one?<br>*Steven asked which whiteboard marker to use.* | 1 |

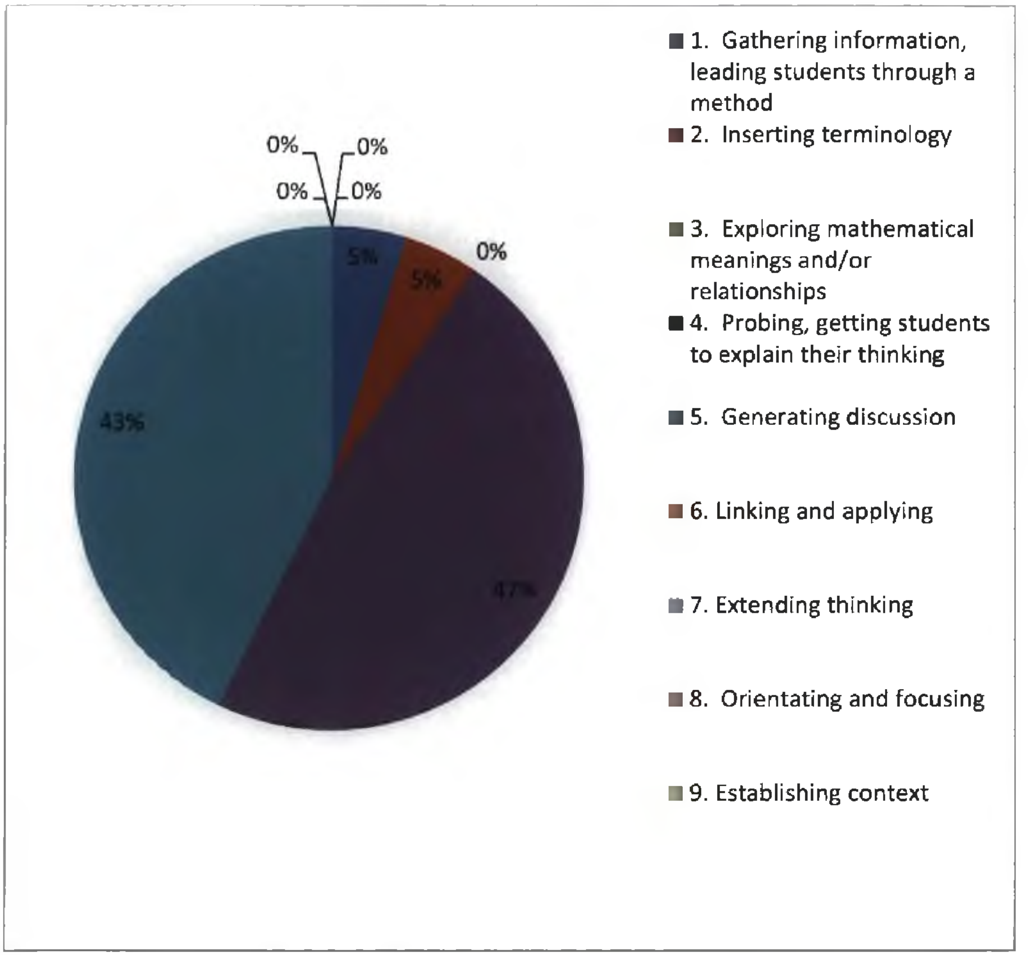



Figure 5.9. Types and percentages of teacher questions in *Fraction Problems* lesson according to Boaler and Brodie (2004) question categories.

**Explaining Mathematical Thinking (EMT)**

In this lesson, I did not often present mathematical explanations. Almost half of the teacher questions in this recording were questions probing student thinking and as a result, students regularly explained their mathematical thinking. At times, the language of student EMT was striking. For example, consider the following extract where Steven explained how to do the question about dividing 100cm of ribbon between four boys (figure 5.5).

| | | |
|---|---|---|
| 103 | Steven: | Well because, what happens is you have a quarter so this is twenty five … fifty, seventy five |
| 104 | Andrei: | A hundred |
| 105 | Steven: | One hundred |

106 Teacher: O.K.

107 Steven: So that's how I got twenty five because it goes up in twenty fives.

Steven did not mention division and it is unclear whether his conception of counting up in a given number was linked with his conception of division. Alan gave a similar counting up explanation for his solution to the race question (figure 5.6). Again, it is unclear whether he recognised the counting-up process as division. This issue becomes more critical when students progress to more advanced problems that do not have such 'friendly' numbers.

**Source of Mathematical Ideas and Responsibility for Learning**

I will discuss lesson events that relate to the two components SMI and RFL in the same section as student contributions often show evidence of both components and it is difficult to discuss one component independent of the other. This discussion will be organised under the following headings: *treatment of errors* and *Jonathan's contribution*.

***Treatment of errors***

In this lesson various students made errors that were corrected by their classmates. For example consider the following section of discourse where James made an error when attempting to partition a ribbon among three people.

46 Students: Oh-Oh. No!

47 //Teacher: Wait a second, what-

Andrei: There's three boys!//

48 Teacher: There're too many noises here. Somebody who's making those noises, do you want to tell James why you think that might be the wrong idea.

49 Steven: Yeah!

50 Teacher: Off you go Alan.

51 Alan: 'Cause there's three boys and you can't cut it into half.

52 Teacher: Yeah, you've only cut it into two.

53 James: Oh, there's three boys?

54 Teacher: Yeah, there's three boys

55 James: Oh.

Here James made a simple error which his classmates took an active role in evaluating. In situations of peer evaluation, I always asked students to explain why they disagreed with the suggestions of their peers "so that their challenge took the form of a logical refutation rather than a judgement" (Lampert, 1990, p. 40). It was intended that students would understand the process of peer evaluation as their mathematical thinking being judged rather than them personally. Using student errors as learning opportunities is a recognised feature of a discourse community (Hufferd-Ackles et al., 2004).

***Jonathan's contribution***

Jonathan's question about the relative size of a half and a quarter was the main question that was discussed for a large part of this recording. His question, as he originally phrased it, was not particularly easy to understand. It is doubtful that I would have understood it without the contextual clues of the board work and the discussion we had just been having. In fact, the context of how he came to pose this question is also interesting. Alan came to the board to solve the 1000m race problem (figure 5.6). When I asked him how he knew where to position the line for the three-quarter mark, he explained that he used a counting in twenty fives scheme as I referred to earlier, although he did adjust his final answers to 500m and 750m correctly. I posed this question to Alan at turn 131. He answered and completed the question. Edward added a comment about who came first, second and third and Anthony noted that one thousand metres is a kilometre. Then at turn 174, more than forty turns later, Darragh returned to the question I had addressed to Alan.

174 Darragh: It's just, do you know the way that you were asking Alan how that's three-quarters of the way, just halve half way.

175 Student: What?

176 //Teacher: Halving half way, you're right, because –

Student: A half half way? (*chatter*)//

177 Teacher: Can you listen to this, if we're saying half of a half, well that's the first quarter isn't it? And a half is the same as the second quarter and half of this one then is the third quarter and then we go to the end, it's the-

(*labels points on empty number line* $\frac{1}{4}, \frac{1}{2} = \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ *as can be seen on figure 5.6*)

178 //Students: Fourth quarter ( )//

[ ]

182 Jonathan: Look it, why is that a two? One four and one two, that's like you have more.

183 Teacher: Jonathan's asked a good question. I'm just going to do it on a new page.

184 Darragh: Can I answer it?

185 Alan: A fourth isn't bigger than a two.

186 Teacher: Jonathan said there, we have this, we started off and we had the half. I stuck in a quarter there and Jonathan said or Jonathan you say. You say it out to the class there and explain it to me.

187 Jonathan: Why is t- why is four

188 Student: Not bigger than two.

189 Jonathan: Yeah.

190 Steven: Because it wants to be.

| | | |
|---|---|---|
| 191 | Teacher: | No, give him a chance Steven, because it's a good maths question and I'd like somebody to give him the answer. Jonathan says … try and say it again please and put it into a question. |
| 192 | Jonathan: | I can't remember. |
| 193 | Teacher: | You can remember. |
| 194 | Steven: | I can! |
| 195 | Jonathan: | Oh! Four is eh, four is no- |
| 196 | Teacher: | Yeah you're right. Four is bigger |
| 197 | //Teacher: | Than two |
| | Jonathan: | Than two// |
| 198 | Teacher: | Yeah. |
| 199 | Jonathan: | Then why is |
| 200 | Teacher: | A quarter |
| 201 | //Jonathan: | Smaller |
| | Student: | Smaller // |
| 202 | Jonathan: | Smaller than two |
| 203 | Teacher: | Smaller than a half. |
| 204 | Jonathan: | Oh yeah. |
| 205 | Teacher: | That's a really good question. |

[ ] (*Teacher organises for Jake to repeat Jonathan's question)*

| | | |
|---|---|---|
| 209 | Jake: | Eh, Why is one-fourth- oh, … it's why is one-fourth bigger than – oh smaller than a half when four is bigger than two. |
| 210 | Darragh: | Everyone knows. |

This extract illustrates some of the difficulties and opportunities that are created using the discourse community approach. On one level, Jonathan, a lower achieving student, is being positioned in such a way that his idea or question becomes the focus for the whole class. Such a central positioning is envisaged to be powerful for Jonathan but also involves risk. He asked his initial question fairly confidently even if it lacked precision but he struggled when I asked him to repeat it and seemed to want to withdraw from the activity altogether when he claimed he could not remember (turn 192). This occurred directly after Steven's joking remark at turn 190. Steven and Jonathan generally had a friendly relationship. My opinion, both at the time and in retrospect, is that Steven's joke was made for classroom comedy rather than as a means of poking fun at Jonathan. However if Jonathan was feeling vulnerable, he may have taken both Alan's contribution at turn 185 and Steven's joke as personal criticism. Alan's contribution suggests that he did not fully understand Jonathan's previous remarks. For the teacher, the decisions made in these contingency moments are complex (Rowland et al. 2005), on one hand attempting to facilitate some kind of mathematical narrative while taking account of the social aspects and the vulnerabilities of individual students on the other. I asked Jake to restate the question as both times Jonathan posed it, it lacked clarity and I was not sure that the other members of the class understood. Darragh, however, seemed to understand Jonathan's question from the very beginning and even offered to provide an explanation (turn 184). His experience of the drawn out process of the repetition of the question must have been different from the way in which students who did not understand the question the first time around experienced the same dialogue. Some of his impatience can be sensed in his comment at that "everyone knows" (turn 210).

Three students presented answers to Jonathan's question. Alex explained by drawing pizzas and splitting them into halves and quarters (figure 5.7). He began explaining "That's in halves, that's in fourths because a fourth is smaller because it's in four pieces and one person only has one, and a half -" (turn 232). He was interrupted by Steven calling out "I get it!" (turn 233). This is interesting because it would seem from Steven's reaction that this is an aha! moment, or a moment of insight (Dooley, 2010) and that he has understood something he did not previously understand. In this respect, Jonathan's question and the discussion it provoked take on a new importance. It would seem that the situation facilitated learning for Steven, who may never have articulated the question as Jonathan did and could have remained unaware, as I would have been, of this gap in his knowledge. Steven's exclamation provides an indication of his presumed understanding. Questions remain about the nature of the lesson event experience for students who remained silent.

After Alex's explanation, Jonathan indicated that he still did not understand and I asked him to invite another boy to give an explanation. He chose one of his close friends Aidan to explain. Aidan used the diagram Alex had drawn to suggest that if you "take away" a line, the quarters join together to make a half (figure 5.7).

| | | |
|---|---|---|
| 248 | Aidan: | See if you take away that line then that's a half, oh |
| 249 | Teacher: | Yeah, |
| 250 | Aidan: | That'd be a half of it and that'd be- |
| 251 | Jonathan: | Teacher they're all quarters aren't they? |
| 252 | Teacher: | Yes |
| 253 | Aidan: | Yeah and that's a half and that's bigger than this so that's… |
| 254 | Student: | Teacher! Miss! |
| 255 | Aidan: | So say you have another brother and two sister |
| 255 | Student: | Yeah |
| 256 | Aidan: | And you only get a quarter and your brother gets a quarter and your sister gets a quarter and your sister gets a quarter and your ma and da make a pancake … |

Unfortunately his interesting mathematical reasoning was lost in the joking way that he continued with his presentation. In the end, I asked him to sit down and chose someone else to explain. His joke may have been based on the pizza sharing activities from earlier lessons where students sometimes imagined sharing pizzas between family members. The reasons why Aidan chose to continue his explanation in the joking manner are impossible to judge. His participation in lessons appeared to be inconsistent. Sometimes he engaged well and he was capable of good mathematical thinking as evidenced in his interesting way to compare halves and quarters. At other times, he disengaged and made little effort. He always seemed very tuned in to the social aspects of the classroom, being an example of a 'macho lad' for whom peer attachment seemed to rank above maintaining a positive relationship with teacher (Ashley, 2003). I will discuss below how he may have viewed the opportunities to make thinking public as opportunities for social embarrassment rather than success.

The next student to volunteer an explanation was Alan (figure 5.8). His reasoning seemed to be based on his knowledge that $\frac{2}{4}$ is equivalent to $\frac{1}{2}$. It was unclear whether Jonathan understood Alan's argument. Darragh had repeatedly indicated that he wished to explain so I invited him to.

| | | |
|---|---|---|
| 295 | Teacher: | Mmm, Darragh could you say it please. |
| 296 | Darragh: | Eh, it's just eh, see the way quarters are cut into four pieces and halves are only cut into two |
| 297 | Teacher: | Yes … Jonathan this is important listen to it. |
| 298 | Jonathan: | I know |
| 299 | Teacher: | He said quarters are cut into four pieces- |
| 300 | Jonathan: | I know them |
| 301 | Darragh: | Quarters are cut into four pieces and halves are cut into two |
| 302 | Student: | Two |

| | | |
|---|---|---|
| 303 | Darragh: | So halves are bigger because quarters are cut into four so they're smaller but halves are cut into two so they're bigger. |
| 304 | Jonathan: | Yeah but like- |
| 305 | Teacher: | So if you had a cake, Jonathan, |
| 306 | Aidan: | He knows |
| 307 | Teacher: | Aidan, you're interrupting. |
| 308 | Jonathan: | I'd keep the whole thing but |
| 309 | Aidan: | *(Giggles)* |
| 310 | Teacher: | If you had a cake and you cut it into two pieces, which are the halves |
| 311 | Student: | Halves |
| 312 | Teacher: | Well they're going to be much bigger pieces than |
| 313 | Darragh: | Quarters |
| 314 | Teacher: | If you get that cake and you cut it into four pieces |
| 315 | Andrei: | It doesn't really make a difference. |
| 316 | Alan: | Teacher, they're not going to be that much bigger because there's only four slices. |
| 317 | Jonathan: | It's still going to even taste the same |

Darragh's explanation is similar to the way in which Alex originally explained it. Jonathan stresses that he "knows them" and I am interested to know what he may have said at turn 304 if I had not interrupted him. "Yeah but like" sounds like the start of either a disagreement or a question. My own insistence that he pay attention was probably due to a suspicion that he still did not understand combined with his general concentration difficulties. Aidan's statement that "he knows" at turn 306 may be as a result of impatience on his part or an effort to protect his friend from what he considers to be further social embarrassment.

Jonathan suggested that he would keep the whole of the fictional cake that I described being partitioned into fractions in turn 308. It is hard to judge whether this statement is made in jest or results from mathematically naivety or a combination of the two. This statement may have been 'playing for laughs' and made in an effort to deflect perceived social embarrassment. Even if this was his motivation, the fact that he followed it with a statement that it would taste the same either way suggests some mathematical naivety on his part. This is in stark contrast with the sophistication in mathematical reasoning that is evident in other students. For example the thinking that Alan and Andrei display in the extract above is quite sophisticated. Not only have they engaged and understood completely the ideas behind the partitioning of the fictional cake, they argued that there will not be such as great difference between the size of $\frac{1}{2}$ and $\frac{1}{4}$ because "because there's only four slices" (turn 316). It seems likely that they have made links with the discussion in the *Cutting Pizzas* fraction lesson when Andrei suggested cutting a pizza into twenty one pieces and Darragh commented that these pieces would be very small. Quarters (of the same unit) would be relatively large compared to such pieces. The participation of Aidan and Jonathan in this lesson appears similar in ways to Barnes' (2000) observations about a group of male students she termed 'the mates'. These students were neither underachieving nor "antischool" but Barnes suggests aspects of their class participation was a form of performance. Like Aidan and Jonathan, this performance:

> … seemed to be primarily for the benefit of others in their group ... It was their means of establishing and maintaining group membership … The pose of not taking work too seriously, and not putting in too much effort, can be seen as a defence against failure: if they did not try too hard, they could be said to have failed, and could always claim that if they had tried, they would have succeeded brilliantly" (Barnes, 2000 p. 162 – 163).

**Discussion**

This lesson highlights a number of issues that arise in attempting to facilitate the discourse community approach. Firstly in prioritising student ideas, I had to be willing to deviate from my own planned agenda (Ball 1993, Lampert 1990). This was not too problematic here as the question Jonathan raised related to a common fraction misconception so there was a valid reason to explore it. However, this section of the lesson also highlights the social aspects that must be navigated so that students feel comfortable taking a central role. Ball speaks of "keeping an eye on the mathematical horizon" (1993, p. 373) and Lampert speaks of navigating the "mathematical terrain" (1990, p. 41) but equally important is how to navigate the social terrain. This was far from straight forward in a teaching experiment which effectively co-opted students into non-traditional roles. As teacher, I made subjective judgment calls based on how far I could 'push' students mathematically while being sensitive to their social vulnerability. Jonathan's question put him centre stage and there is some evidence to suggest that this may have been a mixed experience for him. It is also possible that Aidan's choices around the nature of his participation were affected as much by social as mathematical concerns.

This lesson also highlights the complexity of communication and mathematical thinking in the classroom environment. A question I posed to Alan about how he knew to position the three-quarter mark on the number line (figure 5.6) was referenced by Darragh over forty turns later when he suggested that a quarter is half of a half. This in turn led me to label the fractions on the number line which triggered Jonathan's question. The discussion around the question appeared to provoke understanding for Steven. This web of student thinking and teacher and student action is visible because these students made contributions to whole class discourse. Whether the thoughts of students who did not contribute were provoked to the same extent is hard to tell.

Students engaged in pair work on fraction word problems set in metric contexts. The idea of these activities was to link fractions with magnitudes in contexts that were experientially real for students as suggested by Streefland (1991). I hoped that this approach would facilitate genuine conceptual understanding instead of following a 'divide by the bottom, multiply by the top' procedure. However, the flaw in this logic may be that some students' weak conceptions of division may have prevented them from applying the instinctive methods used here in similar tasks with more complex numbers.

For the most part, I refrained from providing a direct explanation of mathematical features. Many of the student contributions, particularly in the section of the lesson where they attempted to answer Jonathan's question, were mathematically correct and clearly explained. I took a more active role in providing an explanation to Jonathan's question at the end of this lesson than I had previously. Why exactly I decided to step in at this time and not before is unclear but may have been due to a feeling I had that he did not actually understand the explanations presented. This is discussed further in chapter 6.

**Discourse in this Recording**

The teacher and student question profile indicate that this recording is not at level 0 of the MTLC framework descriptors (Hufferd-Ackles et al., 2004). It is also clearly not at level 3 either, as the requirements of student-to-student discussion of mathematical thinking unprompted by the teacher is not in evidence. A rationale for describing this as a level 2 lesson rather than a level 1 lesson includes the fact that students did not passively listen as others explained. They commented on the explanations of others and asked questions of their own. Their ideas and Jonathan's question in particular were central in determining the overall content of the lesson. A summary of the findings is given in table 5.4.

Table 5.4
*Summary descriptions of MTLC framework components in the Fraction Problems lesson.*

| Component | Description |
|---|---|
| Questioning | Teacher questions focussed on probing student thinking and generating discussion. Students also asked some mathematics focussed questions one of which was pursued in depth. |
| EMT | Low levels of teacher EMT. High levels of student EMT. |
| SMI | Students appear to be positioned as SMI. |
| RFL | Students involved in evaluating mathematical ideas and explaining mathematical ideas to their peers. |

## Recording 3: Dienes' Decimals

This lesson was preceded by two introductory lessons that explored Dienes' representations of decimal fractions. The use of Dienes' blocks to represent decimal fractions was discussed in chapter 4 (page 104). Like John's students, my students had previously used Dienes' materials to represent whole number values. This lesson will be presented in summary form consisting of descriptive synopsis and discussion. Results of the analysis of teacher and student questions are shown in appendices 19 and 20 respectively.

### Descriptive Synopsis

The lesson began by reviewing some of the work we had covered in the preceding days. In particular the Dienes' block representation for decimal fractions was reviewed. I explained that because we wanted to study decimal fractions we would be cutting a unit up into many pieces so we had to use the larger cube block as a unit or one rather than the smaller cube which had been used to represent a unit previously. I said, "Because at least if I use that one as a unit, you can still see the things that I'm cutting up. Whereas if I used that one as a unit and I chopped it into ten pieces, or I chopped it into a hundred pieces … Or if I chopped it into a thousand pieces- you're right. They'd be far too small" (turns 3 – 5). I was interrupted by Darragh saying, "They'd be microscopic" (turn 4). I reviewed what fraction each piece of material symbolised by asking students questions. The questions mainly involved the consideration of the physical Dienes' materials but a representation of these materials that I had used previously was displayed on the whiteboard. This is shown in figure 5.12. The figure also shows a decimal number frame in the top right hand corner that we sometimes used to help in the writing of decimal notation.

From the beginning of the school year, I had split the class into four groups based on their location in the classroom and students in those groups worked as a team to earn points. Good behaviour such as sitting properly and listening well was rewarded with group points. Groups competed to have the highest points at the end of the week. I regularly changed the method of point allocation, for example counting up in eights on a week where we were focussing on the eight times tables or in combinations of units, tens, hundreds and thousands when focussing on place-value (i.e. give points in units on Monday, tens on Tuesday etc.). On this particular week I had assigned points in hundredths on one day and in tenths the next. The points were shown on the standard whiteboard with place-value labels as per the decimal number frame shown in figure 5.10.

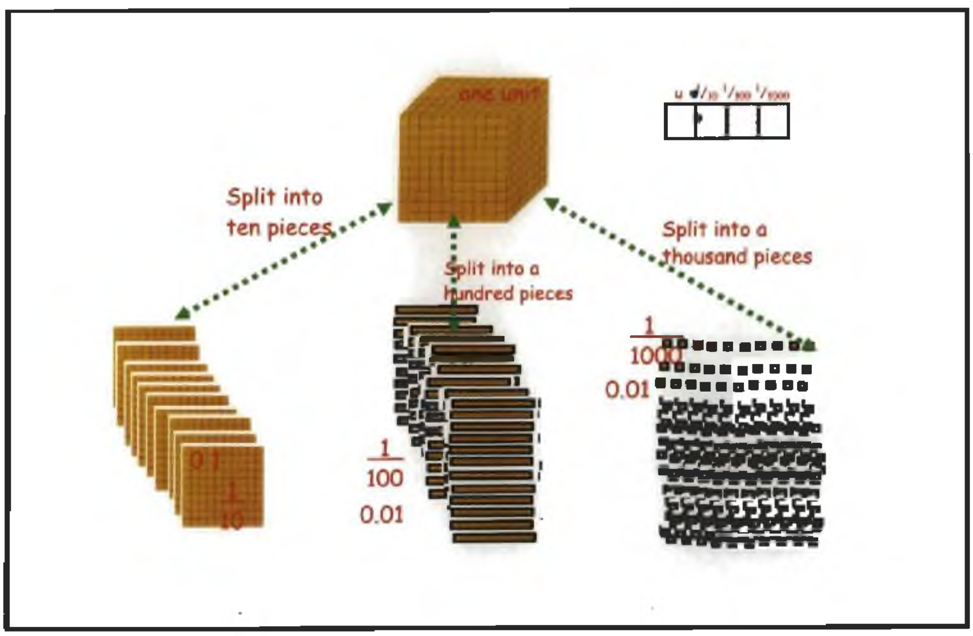



Figure 5.10. Dienes' materials used to represent decimal fractions. This was shown on the whiteboard while we worked with physical Dienes' materials.

At this stage, I realised that not all students understood this decimal based points system but I hoped it would motivate students and provide a reason to discuss the relative value of various decimal numbers. Group A were on 0.44 having received 4 tenths and 4 hundredths in the days leading up to this lesson. I asked Darragh, a member of group A to come to the front of the room and represent their points using the Dienes' blocks. He chose the correct materials but Jonathan disagreed and seemed to suggest using the thousandths. The recording was interrupted for a couple of minutes by a visitor to the class. Afterward, I asked Darragh to explain why he had chosen his materials. He said, "Because it says four tenths and four hundredths," possibly referring to the place value labels. Jonathan replied that he understood it now, that he had been doing it another way. I asked him if he meant that a selection of the thousandths would be the same: "Right, so you were saying that, Jonathan, you were saying that the bag of them could be the same as this?" Darragh said, "So you could use four hundred and forty thousandths." Luke asked, "How?" I explained that you could use four hundred thousandths to replace the four-tenths and forty-thousandths to replace the hundredths. I referred to a similar activity we had completed previously and as a class we read aloud the points of all the groups in terms of the number of thousandths each had.

I then asked the students to imagine that Group A were being very well behaved that I was rewarding them in tenths. I asked students what their points would go up to. Andrei gave the answer of "zero point five forty" and I added a tenth to the Dienes' block representation of their points. I continued adding tenths in this manner and various students told me the correct number of points, 0.540, 0.640, 0.740 and 0.840 respectively until we got to 0.940. I asked what would come next. Steven replied, "Zero point nine fifty." Edward laughed at Steven's error, was reprimanded and asked to leave. I asked if there was anybody with a different opinion to Steven. Jake suggested the correct answer: "It's one point zero four 'cause it goes up to hundredths for tenths." Steven said that he was confused and Luke called out an incorrect answer, “It could be nine four one.” I built on Jake’s solution saying, “So you’re saying, I had, what we had was nine, we had, if you think of it, remember units, tenths, hundredths, thousandths *(pointing to the blocks)* … We had nine of those tenths and then I gave them because they’re so good and they’re sitting properly and they’re putting their hands up, I gave them an extra one,” while adding an extra tenth to the pile of materials. I then asked how many tenths they have now and Steven said, “A unit.” I agreed and said that I could swop the ten tenths for a unit. Luke added, “And then zero tenths and four hundredths.” I asked Steven what he had increased group A’s points by in his initial answer. He replied, “I, I thought it would have been nine fifty because you were going up in tenths the whole time so I thought it would have went *(sic)* up in fifty.” Darragh added, “You thought we were going up in hundredths.” Andrei noted that it was like swapping or trading.

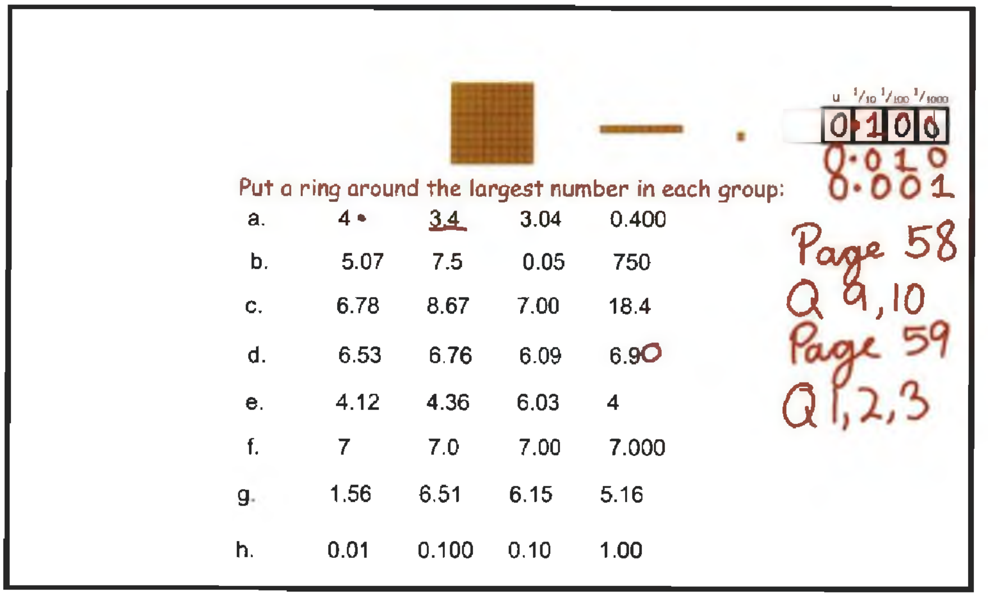



Figure 5.11: Task involving finding the highest decimal number. This task was presented to students after initial work involving using Dienes’ materials to represent decimals numbers.

I then showed the students the questions shown on figure 5.11 with all but the first group of numbers covered. I explained that these were almost like the points for four groups and asked students to find the biggest number. Kevin suggested that it was 4. Alex said that he was not sure how to do them but thought it might be 3.4. I asked Kevin to respond to Alex. Kevin explained that in his number the four stood for four units. I attempted to show this by manipulating the images on the interactive board but it did not work and I used the physical Dienes' blocks instead. I asked Alex to explain what his number was. He did and I noted that it only had three units. Andrei added that we could write 4 units as 4.000. I asked the students if they agreed with him and asked Luke to explain why this was true. He explained that it was "'cause, 'cause there's no tenths and there's no hundredths and there's no thousandths" (turn 155). James suggested that I was trying to trick them. I then set the students to work on other examples of this kind and moved around supporting those who were having trouble.

**Discussion**

In this lesson, decimal fractions and notation were explored through the use of Dienes' materials. The context used was that of class points in decimal fraction values and while this may have been motivating, it did not provide support for student thinking. Many of the elements Dienes felt were necessary for the effective use of these materials were missing (Lesh, Post, & Behr, 1987). In particular, students did not have access to the materials themselves. Some students seemed to link the materials with the place value labels but not all students seemed to understand these links and Steven seemed to incorrectly apply reasoning about whole number values to decimal fractions. I played a more active role in directly explaining the mathematical concepts than in previously presented lessons. However, students also engaged in explaining their mathematical thinking. In fact when Alex disagreed with Kevin about the largest number in the group, it was Kevin that gave the initial explanation of why his suggestion was right. I followed up on this, using the materials to demonstrate each number. Edward's actions in this lesson are in contrast to the willingness to engage he displayed in other lessons. My reaction to his laughing at Steven's error was to send him out of the classroom, one of the strongest actions a teacher can take. I felt it was necessary to take a strong approach because I hoped to facilitate a community where students would be supportive of each other's thinking and Edward's action was in opposition to this.

**Discourse in this Recording**

The discourse in this recording is of a different nature to some of the recordings already presented. In this recording, student ideas' did not become the central topic of conversation. In many ways, the discourse of this lesson followed a more traditional format than other recordings presented in this chapter possibly due to the nature of lesson activities. The content was not presented in problem form (Engle & Conant, 2002) and the context was artificial. This lessons raises questions regarding the nature of whole class discourse in relation to different stimulus and the functions of different types of discourse at various times throughout the teaching cycle. For example, should the nature of whole class discourse change depending on whether a teacher's aim is to introduce, build or consolidate a topic? This issue will be discussed further in chapter 6.

Table 5.4
*Summary descriptions of MTLC framework components in the Dienes' Decimals lesson.*

| Component | Description |
|---|---|
| Questioning | Some teacher questions focussed on student thinking and generating discussion. |
| EMT | Relatively high levels of teacher EMT and low levels of student EMT compared to previously discussed lessons. |
| SMI | Teacher as SMI. At times, students were positioned as SMI (e.g. Kevin). |
| RFL | Students involved in evaluating mathematical ideas and explaining mathematical ideas to their peers but fewer opportunities were created for such actions than in other stage 2 lessons. |

Though more traditional that other stage 2 lessons, a MTLC level 0 description is not appropriate. The real debate is whether this should be classed as level 1 or level 2. The main argument for classing it as level 2 are the efforts made to facilitate student-to-student conversation. The main argument for classing it as a level 1 involves the predominance of teacher-exposition rather than students explaining their mathematical thinking. A summary of the findings in relation to the components of the MTLC framework is given in table 5.4.

**Recording 4: Equivalence Challenge**

This lesson occurred after five preliminary lessons exploring percentages. One of these introductory lessons involved finding common fractions as percentages and must be described briefly here as it was referred to by myself and students during the *Equivalence challenge* lesson. In the earlier lesson, students were given copies of blank hundred squares and were challenged to cut them up into various fractions and find the equivalent percentages. They then stuck these fractional pieces on card and made a poster of the fraction-percentage equivalences. A poster completed by Darragh is shown in figure 5.11.

A discussion arose when Alex accidently cut a blank hundred square into many pieces each with five little squares representing $\frac{5}{100}$ or 5% instead of cutting the hundred square into five pieces or fifths as I had asked students to. I spoke with him and his partner Jared about this error and asked Aidan who also sat near them to explain what he thought. Eventually the students agreed that Alex's 'mistake' actually represented twentieths and that $\frac{1}{20}$ is equivalent to 5%. This fact was then discussed with the class along with the original focus fact of $\frac{1}{5} = \frac{20}{100}$ = 20%. This discussion and Alex's initial error is referred to in the *Equivalence Challenge* lesson that will be presented in this section.

**Descriptive Synopsis**

I showed the students the question shown in figure 5.12. Darragh was invited to the board to highlight 10%. I then introduced the challenge of finding other ways to write this. I asked Kevin first and he suggested one-tenth. Darragh suggested 0.1 and Luke asked if it was two-fifth. I started explaining and Darragh called out two-twentieth. I explained by stating that one-fifth is the same as 20% so two-fifth would be 40%. I returned to Darragh's suggestion of two-twentieth and asked if others agreed with it. Alan said he was not sure and Edward attempted to say something but got confused. Andrei explained that it was equivalent by referring to the task we had done earlier in the week when Alex mistakenly cut his hundred square into twentieths. I reviewed this episode and the marks I made to indicate some of the twentieths can be seen in figure 5.12.

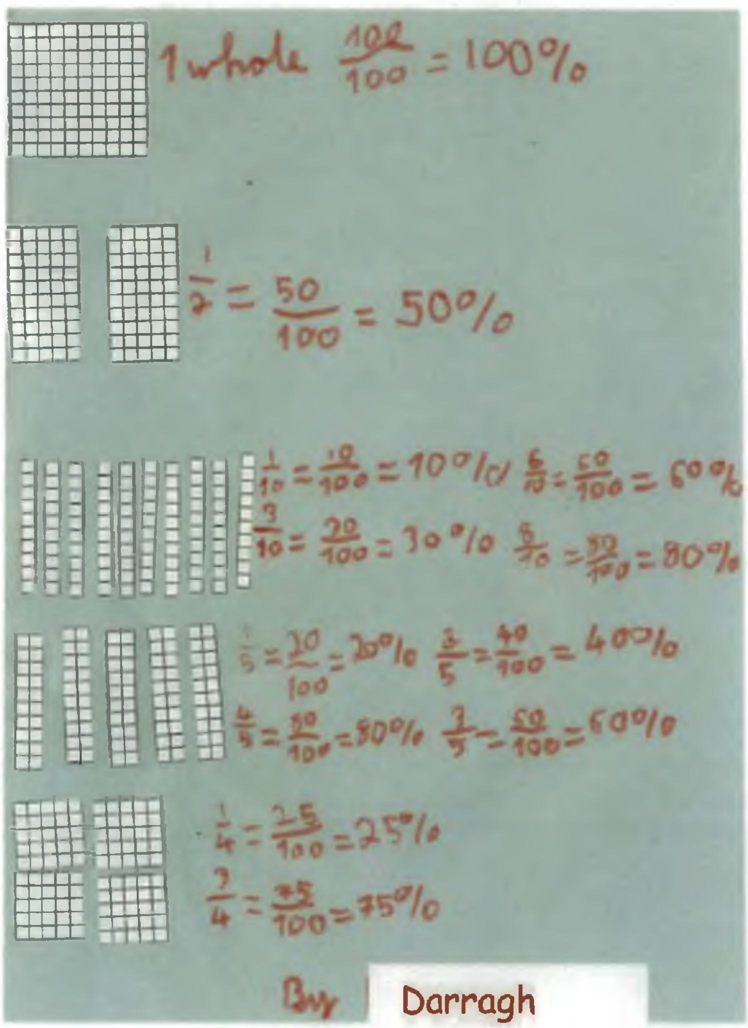



Figure 5.11. Darragh's poster showing common fraction and decimal equivalences. In a lesson that occurred prior to the *Equivalence Challenge* lesson, students cut copies of a blank hundred square into common fractions and made a poster detailing fraction and percentage equivalences. The scan has been altered to protect Darragh's real identity.

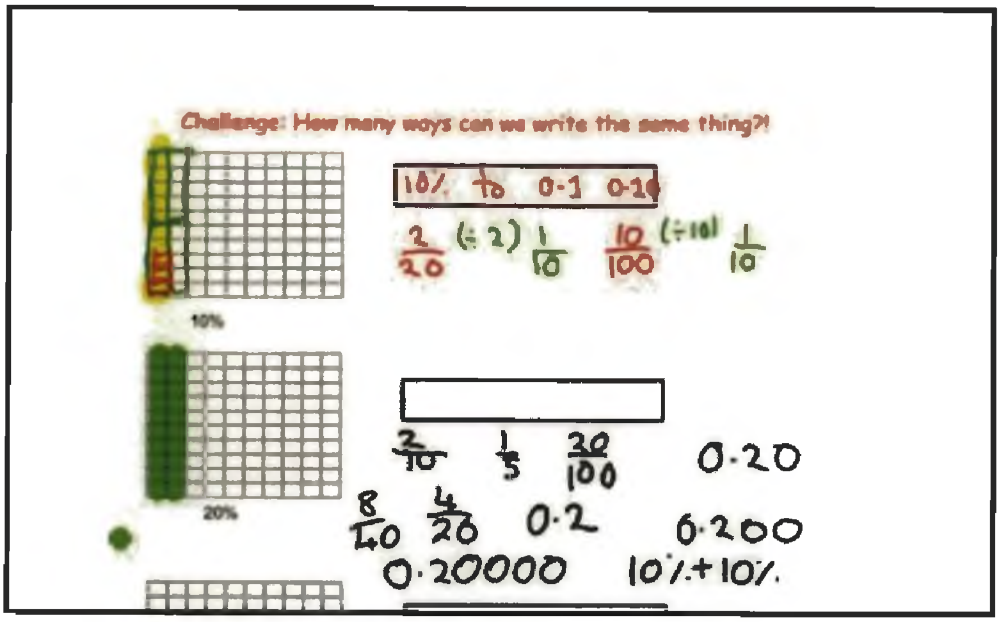



Figure 5.12: The first equivalence challenge task. The figure shows the challenge presented to students to write 10% and 20% in different ways and the solutions they suggested that I wrote on the board during whole-class discussion.

I then completed a formal multiplication conversion to show that they were equivalent fractions. As I completed this, Darragh began calling out other fractions "Ah fortieths! … Four-fortieths … And six-sixtieths" (turns 69 – 76). Andrei suggested one hundred-thousandths. I said that it was a great pattern and asked Alan if he had another one. He suggested ten-hundredths. Steven suggested eleven-hundredths and Darragh said one hundred-thousandths. I returned to the suggestion of ten-hundredths and spoke about links with the hundred square representation, noting that ten out of the hundred squares were coloured. I also completed a formal division conversion showing ten-hundredth and one-tenth are equivalent fractions. After some further suggestions, Kevin said that if we coloured in the whole square it would be one hundred-hundredths or one unit.

We then moved on to the challenge of finding equivalences for 20%. Aidan gave the first suggestion of two-tenth. Edward said that it would be the same as one-fifth. Again, I did the formal division conversion procedure to show $\frac{2}{10}$ and $\frac{1}{5}$ are equivalent. Darragh called out, “four-twentieth”. Anthony said, “twenty-hundredth,” and Josh suggested four-twentieth. Andrei, Edward and Anthony suggested 0.2, 0.20 and 0.200 respectively. Jake then suggested 0.2000 and I commented that you could have any amount of zeros. Edward suggested 10 % + 10%. Andrei suggested eight-fortieth. While I was restating his answer in terms of multiplying numerator and denominator, Darragh said, “So would it be twelve-sixtieths then?” I then assigned the students some work to do in pairs. The worksheet was later corrected on the interactive board (figure 5.13).

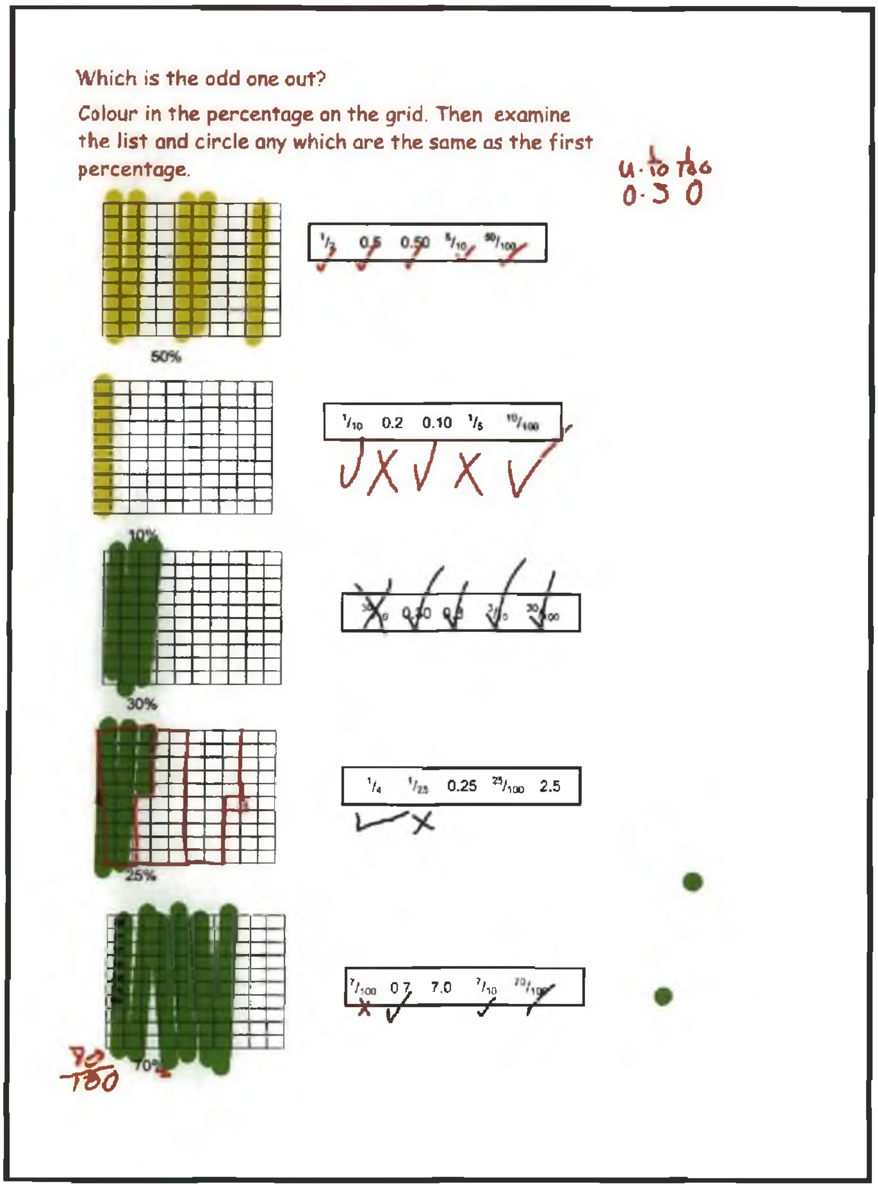



Figure 5.13: Find the odd one out fractions, decimals, percentage equivalence challenge. The figure shows the worksheet given to students and the notes made by the students and I during whole-class correction.

When the worksheet was corrected in the whole class setting, the discussion proceeded without incident until there was a dispute over whether 25% was the same as $\frac{1}{25}$. I noted that some students thought a quarter was the same as $\frac{1}{25}$. I posed the related question "is one twenty-fifth, twenty five squares?" (turn 260). Andrei gave an initial answer to this question but was interrupted when I was called to speak to the learning support teacher. When I returned, I restated the problem and asked Andrei to restate his reasoning. He did this and refined his answer. Some students seemed to follow the explanation; others seemed to have lost interest. The remaining question was corrected and the lesson was ended.

## Discourse Community Analysis

### Questioning

#### *Teacher questions*

Teacher questions were counted and coded as before. An issue arose over how to treat the teacher questions focussed on the central challenge of the lesson, that of representing percentages in different ways. Simple equivalences would be expected to be known facts and would therefore imply that corresponding teacher questions should be classed as type 1. For example, one would expect that students should know equivalences such as $10\% = \frac{1}{10} = 0.1$. However when students were pressed for equivalences beyond the basics, it seems better to describe this as an exploration of mathematical relationships and representations (type 3 questions). It is problematic to draw a line between type 1 and type 3 questions in this context. For this reason all teacher questions of this kind were categorised as explorations of mathematical relationships and representations (type 3). Similarly, I pressed students to come up with many different equivalences asking variations of "can you tell us another one?" (turn 143). The variations on this question could be considered as a repetition of the question and following the methodology of Boaler and Brodie (2004) should not be counted. However, each variation of the question was posed after another solution was suggested by the students.

In this way the problem posed changed and so the teacher questions aimed at finding further equivalences could be considered a different question. For example a teacher question challenging students to find another equivalence when only $10\% = \frac{1}{10} = 0.1$ have been considered is qualitatively different from the same question posed when $10\% = \frac{1}{10} = 0.1 = \frac{2}{20} = \frac{10}{100}$ have already been presented. For this reason each variation of teacher question challenging students to find new equivalences was counted. It could be argued that these questions were posed with the intention of generating discussion by soliciting different contributions from individual students (type 5 questions). This is true to some extent. However, the overall aim was to explore mathematical relationships and so they were classed as type 3. The results are shown on table 5.6 and also in pie chart form in figure 5.14.

Many of the type 1 questions involved my leading students through the division method of verifying equivalent fractions. It seems strange that there were no type 2, inserting terminology questions. Through the process of analysis, I have come to realise that I do not put enough emphasis on terminology. This is discussed more in chapter 8. This lesson contained a larger proportion of questions exploring mathematical representations and relationships and a smaller proportion of generating discussion questions than in previously presented lessons. This is probably related to the question categorisation process described above. Many of the questions categorised as exploring mathematical relationships also served to solicit different contributions from students. The one type 6, linking and applying question, related to work carried out in a previous lesson.

Table 5.6

*Analysis of teacher questions in the Equivalence Challenge lesson by type and number with examples*

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 64: And we can look at these and we can say is there any number that divides into both of them? Turn 93: Could we do the same thing here, what number could I divide into the two of them? | 7 |
| 2. Inserting terminology | | 0 |
| 3. Exploring mathematical meanings and/or relationships | Turn 165: Right Luke, have you another way to tell us? Turn 260: Right, right, this is the question then. Is one twenty fifth, twenty five squares? | 16 |
| 4. Probing, getting students to explain their thinking | Turn 198: Eight fortieths. How did you get eight fortieths? Turn 139: How did you know it was one fifth? | 7 |
| 5. Generating discussion | Turn 41: Right Darragh is suggesting two twentieths. Can any- hands up boys who agree with him or disagree with him? Turn 50: Right, Andrei do you agree? | 5 |
| 6. Linking and applying | Turn 271: Do you remember what we said at the very beginning about what Alex did last week? | 1 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 36 |

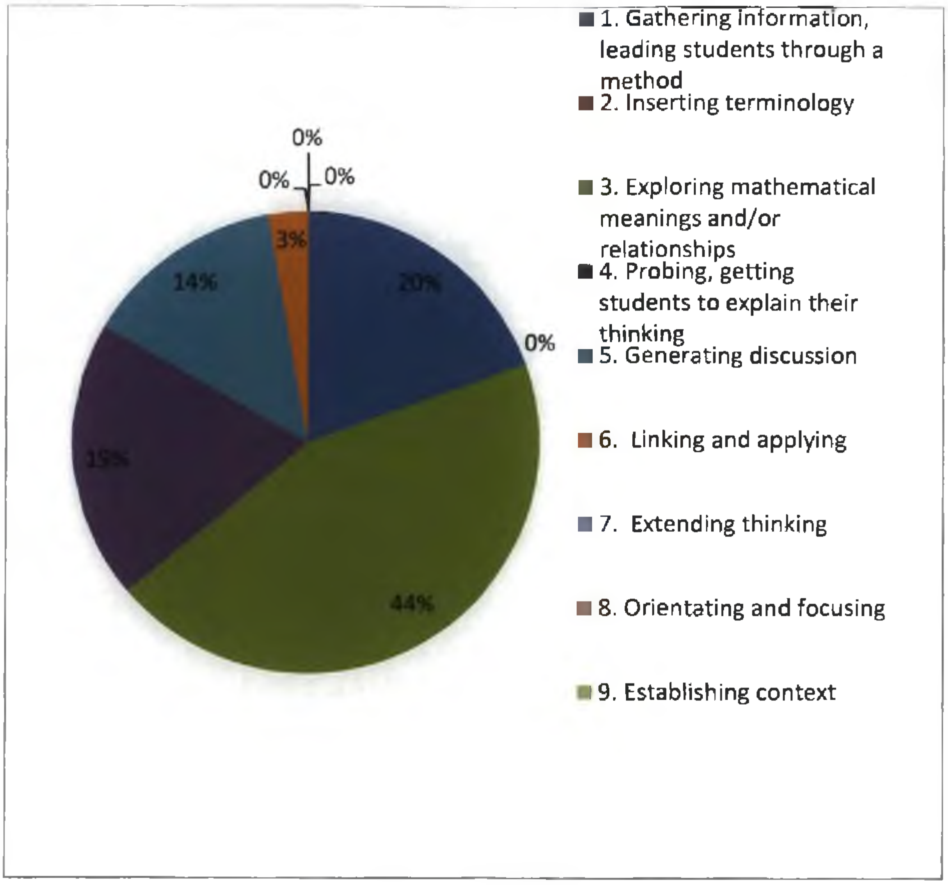



Figure 5.14. Types and percentages of teacher questions in the *Equivalence Challenge* lesson according to Boaler and Brodie (2004) question categories.

***Student questions***

Student questions were counted and coded as before. The results are shown on Table 5.7. The first question that is significant mathematically is Andrei's question. Andrei's question functioned not as a question in the truest sense of the word, but more as a linguistic device to introduce into conversation the ideas of the previous lesson (similar to my own type 6 question noted in table 5.6). Andrei used the fact discussed in the prior lesson that $\frac{1}{20} = 5\%$, to show that Darragh's suggestion that $\frac{2}{20} = 10\%$ was correct. Andrei referred to this idea again later in the lesson and it would seem that he was developing strong conceptual understanding of twentieths. The second question that is significant mathematically is Darragh's question about twelve-sixtieths. Darragh's question or suggestion that twelve-sixtieths is equivalent to 20% built on Andrei's previous contribution, which in turn built on an initial contribution of Darragh who suggested that 20% was equal to four-twentieth. Andrei suggested that this was equal to eight-fortieth and when prompted he explained that he figured this out when he "timesed" (*sic*) the four-twentieth i.e. multiplied the numerator and the denominator by two. Darragh then built on this again asking "So would it be twelve-sixtieths then?" (turn 203). These equivalences are not trivial. In fact with the exception of hundredths and thousandths, the Primary Mathematics Curriculum directs that fifth class students explore fractions with denominators less than or equal to twelve (DES/NCCA, 1999a). It was a challenge for me to immediately identify correct suggestions from incorrect suggestions as the pace of student contributions at various stages was very fast.

Table 5.7
*Analysis of student questions in Equivalence Challenge lesson by type and number.*

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Luke: Is it two-fifth?<br>*Turn 30, Luke asked this question about possible equivalence with 10%.*<br><br>Andrei: Do you remember Alex cut them into fifths (bits?) and he cut them into fifths(bits?) of twenties?<br>*Turn 54, Andrei's question connects work under discussion with work done previously. Brackets show other possible interpretations of Andrei's words.*<br><br>Steven: What about one hundreds? One hundred-one thousandths?<br>*Turn 80, Steven posed this question about possible equivalence with 10%*<br><br>Darragh: So would it be twelve-sixtieths then?<br>*Turn 203, Darragh asked if this is equivalent to 20% after Andrei has suggested that he multiplied* $\frac{4}{20}$ *to get* $\frac{8}{40}$ *as equivalent to 20%.*<br><br>Steven: Will I show you what ones I did?<br>*Turn 224, Steven asked if he can show how he coloured 10% on his worksheet.* | 5 |
| **Questions seeking organisational clarification**<br>*Notes on context of question where appropriate.* | **Total** |
| Luke: Can me and him work together? (*sic*)<br>*Turn 205, Luke asked about a partner for pair work.* | 1 |

**Explaining mathematical thinking**

Some opportunities were created for students to share their mathematical thinking through the use of type 4 and 5 questions probing thinking and generating discussion. There were also times where I directly explained the mathematics or engaged in 'telling'. In particular, I referred on a number of occasions to formal ways of multiplying or dividing the numerator and denominator to find equivalent fractions. In all cases this method was used to check equivalence in fractions already suggested by students. For example in the following section of dialogue, I used formal division procedures to show that $\frac{2}{20} = \frac{1}{10}$.

| | | |
|---|---|---|
| 61 | Teacher: | You're right, and also back a long time ago do you remember when we were doing fractions on their own? |
| 63 | Student: | Yeah. |
| 64 | Teacher: | And we can look at these and we can say is there any number that divides into both of them? |
| 65 | Steven: | Yeah two tens. |
| 66 | Teacher: | A two will divide into both of them, wouldn't it? |
| 67 | Steven: | Yeah you'll get ( ) |
| 68 | Teacher: | And if I divide that by two it's t, divide by- |
| 69 | Darragh: | Ah fortieths! |
| 70 | Teacher: | Two, it's one, I divide this by two it's- |
| 71 | Student 1: | Ah ten. |
| 72 | Student 2: | Ten. |
| 73 | Teacher: | So two-twentieths is just the same as one-tenth. |
| 74 | Darragh: | Four-fortieths. |
| 75 | Teacher: | O.K., that's- |
| 76 | Darragh: | And six-sixtieths and- |

| | | |
|---|---|---|
| 77 | Teacher: | Yeah perfect. |
| 78 | //Andrei: | So what- |
| 79 | Darragh: | Eight-eightieths// |
| 80 | Andrei: | -about one hundreds? One hundred-one thousandths? |
| 81 | Teacher: | A great pattern. |

In this section of dialogue I attempted to revise a formal method for finding equivalent fractions. Whether Darragh built on this or something else in finding his set of equivalent fractions $\frac{1}{10}=\frac{2}{20}=\frac{4}{40}=\frac{6}{60}$ is hard to tell. The fact that he did not suggest $\frac{3}{30}$ may suggest that he doubled the numerator and denominator to give $\frac{4}{40}$ but a doubling strategy would not then give him $\frac{6}{60}$. The fact that this pattern is then built on and he suggests $\frac{8}{80}$ indicates that he may be building on a pattern of counting in multiples of two. The next fraction in the sequence initiated by Darragh would be $\frac{10}{100}$ but this was not introduced into the classroom discussion until turn 82, a couple of turns after Andrei's suggestion of $\frac{100}{1000}$. I did not recognise Andrei's suggestion as not matching the pattern of Darragh's suggestions at the time.

There is evidence to suggest that some students were more comfortable than others when explaining their mathematical thinking. For example Edward struggled when attempting to explain why $\frac{2}{20}$ is the same as $\frac{1}{10}$. When I asked Steven to explain how he knew 10% wasn't the same as a fifth he replied that he had copied the answer from his partner. This occurred during the period of whole class correction and as it is a question about the basic equivalence relations that students must eventually know automatically, it raises questions about Steven's overall understanding of fraction-percentage equivalences. In contrast Darragh often interrupted other students and me to share his mathematical thinking.

**Source of Mathematical Ideas and Responsibility for Learning**

It should be evident from some of the extracts and descriptions above that students contributed ideas to the whole class discourse. As noted above, there were times where I took full control and gave explanations about specific methods. At other times, I attempted to position students as SMI and made them responsible for determining what was mathematically correct for example asking them if they agreed or disagreed with other students (turns 41, 49).

The main mathematical question that arose in the last part of the lesson involved identifying which out of the set $\frac{1}{4}$, $\frac{1}{25}$, 0.25, $\frac{25}{100}$ and 2.5 are equivalent to 25% (figure 5.13). The extract below begins with Andrei commenting on how 0.25 and $\frac{1}{25}$ are equivalent.

| | | |
|---|---|---|
| 257 | Andrei: | I know why it is! That's twenty five and that's zero point twenty five |
| 258 | Teacher: | Yeah? |
| 259 | Darragh: | 'Cause how about the one-twenty fifth? It is one-twenty fifth because one-twenty fifth is twenty five squares. |
| 260 | Teacher: | Right, right, this is the question then. Is one-twenty fifth, twenty five squares? |
| 261 | //Students: | Yes |
| 262 | Students: | No// |
| 263 | Teacher: | Somebody tell me why yes or why no … Is one-twenty fifth, think about, remember when we cut up these squares last week. Is one-twenty fifth, is it the same as twenty five squares? … Darragh, are you thinking about it? |
| 264 | Darragh: | Yeah |
| 265 | Teacher: | Andrei? |
| 266 | Andrei: | One-twenty fifth would be like threes because one-twentieth was five and it had to be less than five. |

267 Teacher: It has to- aah, that' s actually a very good description, say it again.

Initially, Andrei referred only to the question about equivalence with decimals (turn 257) and it was Darragh who directed his attention back to the question of $\frac{1}{25}$ and who suggested that it must be equivalent as it was twenty five squares (turn 259). Then I stepped in and asked "Right, right, this is the question then. Is one-twenty fifth twenty five squares?" (turn 260). I see now that even this phrasing of the question may not have been obvious to students and that perhaps I should have developed it more. I was also perhaps too quick to suggest the connection with the previous task. Andrei gave an explanation that while not complete in terms of mathematical vocabulary explained very clearly his reasoning. Omitting the terms 'hundredth' or 'percent', he suggested that because $\frac{1}{20}$ was five, $\frac{1}{25}$ must be less than that. He suggested that $\frac{1}{25}$ could be "like threes" (turn 266). This is strong mathematical reasoning on his part and my reaction suggests that it took me a moment to understand (turn 267).

Unfortunately, we were interrupted at this stage by the students returning form learning support and the teacher wished to speak to me. I spoke with her briefly (three minutes approximately) and attempted to restart the conversation from where we left off.

269 Teacher: Before the interruption there we were talking about, is one-twenty fifth the same as twenty five squares? Andrei, can you say again because there was some boys didn't follow that, can you say again what you said.

270 Andrei: One twenty- just an example one-twentieth would be five so one-twenty fifth can't be bigger than five so it can't be twenty five percent.

271 Teacher: Do you remember what we said at the very beginning about what Alex did last week?

272 Andrei: Cut them into fifths

273 Teacher: He cut, he got twenty pieces by cutting them all into fives didn't he? He got twenty different pieces.

274 Student: Who?

275 Teacher: Alex, when he cut the-

276 Aidan: I told him that.

277 Teacher: But if we want to get one-twenty fifth, how many pieces would we need to cut the thing into?

278 Jonathan: Does Edward have his boots?

279 Teacher: I am not talking about football now. We are talking about maths

280 Darragh: Twenty five.

281 Teacher: Yeah and that's what Alex is saying. If I cut it into twenty pieces, there's five of them each. If I cut it into twenty five-

282 Andrei: It's fourths.

283 Teacher: They have four. There'd only be four little squares.

284 Student: Fourths.

285 Teacher: Do you get that Steven, yeah? So is this, is this big one that we coloured in, is that the same as one-twenty fifth?

286 Darragh: No.

287 Andrei: No.

288 Teacher: Because if I have them, I'd only have … how many pieces of these? *(pointing at 25% that were coloured in on figure 5.13)*

289 //Student: ( )

290 Student: *(coughing)* //

291 Teacher: John.

291 John: Four.

292 Alan: Four.

293 Teacher: Four exactly. There's one set. There's another twenty-five hundredths, there's another twenty-five hundredths

*(Marks the 25% = $\frac{25}{100}$ sections on hundred square, figure 5.13)*

294 Andrei: That's fourths

295 Teacher: There, it's fourths. So, the important thing here is it's not one-twenty fifth. It's not one-twenty fifth

296 Darragh: That's a big discussion for a little question.

Andrei's language use is interesting here as he seems to refer to 5% or the $\frac{5}{100}$ pieces as fifths (turn 272). In turn 276, Aidan referred to the fact that when Alex made the mistake of cutting the square into twentieths rather than fifths in the previous lesson, it was Aidan who helped him figure it out. Jonathan's off topic question at turn 278 suggests that he had lost interest and was possibly waiting for the class to end. Andrei again misused the language of fractions when he referred to the 4% or $\frac{4}{100}$ one would get on cutting a hundred square into twenty five pieces as fourths (turn 282). Despite the incorrect language this is also a step forward from his initial estimation of "threes" (turn 266). I continued explaining and asked Steven if he understood. I think I was unsure if students other than Andrei and Darragh were following the discussion, though the correct answers given by John and Alan at turns 291 and 292 may indicate that they were. The conversation provoked by Andrei's idea was long but I felt it was mathematically significant and worth having. Whether students who were not actively participating felt the same way is hard to judge. Darragh's comment that it was "a big discussion for a little question" is accurate (turn 296). Whether it was mathematically productive for all students is another question that requires "big discussion." Such dilemmas were central to my teaching during the experiment and will be discussed further in chapter 6.

**Discussion**

The activities were intended to facilitate discussion of fraction, decimal and percentage equivalence. The opening task was designed to be open ended and because there were multiple possible solutions, many students gave suggestions. I had considered that students might use equivalent fractions in this lesson but had not predicted that the speed of contributions of the higher achieving students might make it difficult for other students to access the dialogue. For this reason, my repeated use of formal multiplication or division equivalence checks served both to slow down the pace of the dialogue, and to attempt to revise the methods covered during teaching input on fractions. I also hoped that this might trigger students to use multiplication to produce more fraction equivalences. The pace of the contributions made the lesson hard to follow and it may have been more worthwhile to have students work in pairs on the introductory equivalence challenge questions before discussing ideas in the whole class setting.

In some ways student EMT was less prevalent in this lesson than in previously presented recordings particularly during the opening task where higher achieving students dominated by shouting out equivalences without providing explanations. The latter stages of the lesson in which Andrei explained why 25% is not equal to $\frac{1}{25}$ contained more student EMT but it is not clear whether all students could follow his explanation. This is an example of a situation where I could simply have provided an explanation myself. I believe that passing this responsibility to Andrei was beneficial for him if not also for some of his peers.

The most noteworthy thing about the discourse in this recording was its pace. In the initial stages the pace was set by Darragh and Andrei and it was even difficult for me to follow at the time. I feel that even in descriptive synopsis format the sheer number of suggestions can be overwhelming and difficult to process. The pace and content of student contributions was also hard for me to follow and illustrates the mathematical knowledge demands on the teacher (Rowland et al., 2005). Worthwhile mathematical questions were discussed but it would be disingenuous to claim that the understanding reached by Andrei, Darragh and possibly John and Alan at the end of this lesson was shared by all students. This highlights the issues involved in attempting to facilitate a discussion that is challenging mathematically but also accessible to students of different abilities. In this case, while the discourse was challenging and worthwhile mathematically, it was not accessible for lower achievers and appeared to lead to students like Jonathan 'switching off' and opting out of attempting to participate in the whole class discourse. A central tension of the teaching experiment, discussed in chapter 6, was a concern that it was disadvantaging lower achievers.

**Discourse in this Recording**

In some parts of the lesson, I explained mathematical ideas and methods but students were also a significant source of mathematical ideas. They interacted with each other and built on each other's ideas without my explicit prompting. Such actions would suggest that a level 2 or 3 descriptor may suit this recording. Questions remain however over whether all students were displaying actions consistent with such a level or whether it is just some able, vocal students that were participating at the higher levels. A summary of the findings in relation to the components of the MTLC framework is given in table 5.8.

Table 5.8
*Summary descriptions of MTLC framework components in the Equivalence Challenge lesson.*

| Component | Description |
|---|---|
| Questioning | Most teacher questions directed at exploring mathematical meanings. |
| EMT | Medium levels of teacher EMT with repeated reference to formal checks for fraction equivalence. Some, but not many, students engaged in EMT. |
| SMI | Students appear to be positioned as SMI but opportunities might have been confined to higher achieving students only. |
| RFL | Students involved in evaluating mathematical ideas to some extent. Some students commented on the contributions of their peers and showed high RFL. Other students were not as engaged and displayed low levels of RFL. |

## Recording 5: Percentage Present and Absent

This lesson is presented in summary form consisting of descriptive synopsis and discussion. The results of the analysis of teacher and student questions are shown in appendices 21 and 22 respectively.

### Descriptive Synopsis

Students were presented with the spelling test shown in figure 5.15. They worked in pairs to decide what fraction and percentage of spellings were correct. Conor explained that $\frac{7}{10}$ were right and Jake explained that this is the same as 70%. I introduced the idea of repeating the test ten times to see how many would be correct out of a hundred. Darragh noted that Ryan would have to do ninety more spellings and Andrei noted that he would get 30 wrong. I attempted to copy the test ten times using the Notebook software but it did not respond quickly enough so the demonstration was abandoned.

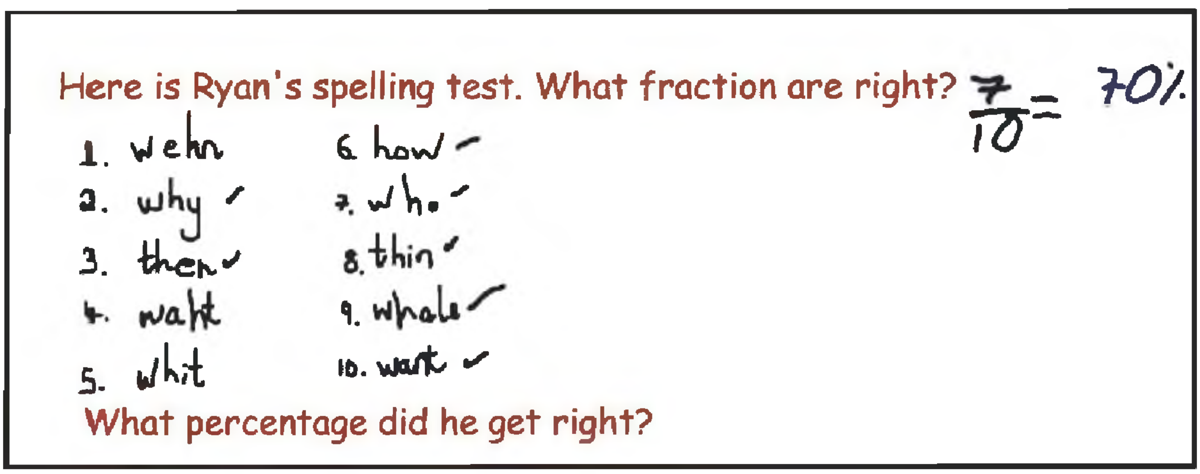



Figure 5.15. Task presented to students about the fraction and percentage correct in a spellings test. The figure shows inscriptions I made during the whole class discussion.

I then presented students with the problem shown in figure 5.16 which asked them to consider the fraction and percentage of our class that were present and absent. I explained that we should consider that there are 25 children in our class. Whether to consider our class total as 24 or 25 had come up previously in a fractions lesson due to students' confusion over one particular individual who was still officially on roll but had not returned to school since the previous school year. The students were given roughly two minutes to consider this problem in pairs before I called the whole class back together again. Anthony suggested $\frac{22}{25}$ present. Steven asked if the fraction of those who were absent was $\frac{3}{25}$. I asked for suggestions on what percentage this might be. Alan suggested 70% present "'cause if you just imagine a hundred people and you take away three and they count as tenths that'd be seventy and then there's thirty people out". I asked the other students what they thought of his answer. Darrragh said, "No". I asked if they would like to hear it again. Many said that they would and Aidan suggested that Alan should not do it "speedy." Alan repeated his suggestion and this time Darragh agreed. I asked Anthony if he agreed with Alan or if he had an idea of his own. Anthony suggested 97% present and 3% absent, "'cause like if the whole class was in that'd be a hundred per cent and three people are out so that's taking like three off." Jonathan stated that he understood now and Alan suggested that both his answer and Anthony's answer might be right. At this point I told the class that neither answer was right.

Jake then suggested: "Twenty one per cent and ten-tenths are in and three per cent are out". I was not sure how to represent his suggestion on the board with the previous suggestions and asked him about it. He agreed that it should be written as $21\frac{10}{10}$ % present and 3% absent as shown in figure 5.15. He explained the 3% was for the three children who were absent and when I asked him if $21\frac{10}{10}$ was another way to write 22, he agreed. I asked Andrei what he thought and he replied with the correct answer of 88% present and 12% absent. Michael said "oh!" and Darragh laughed. Alex, sounding like he might not believe it, asked "eighty eight in the class?" As I wrote Andrei's suggestion on the board some of the other students made comments. Darragh said, "Oh, yeah it's by four" and "Three by four is twelve." Alan said, "Oh, yeah I get it." I asked Andrei to explain it. He said, "'cause it's twenty five so count in four up to a hundred ... And since that's up in fours you put twenty two up in fours and you get eighty eight." I restated his suggestion using multiplication and the formal multiplication equivalence procedure. Students joined in as I explained and I asked the class what they thought of it. Edward said that he thought it was "brilliant" but another unidentifiable student said he "sort of" understood it. Anthony suggested that it was sort of like his way. Alan said that it was a good way of figuring it out but Alex said that it was a bit confusing and Michael said that he did not understand.

I suggested that maybe somebody else would like to explain Andrei's idea. Darragh volunteered to explain and said, "Twenty two by four is eighty eight and three by four is twelve. Add them together and you get a hundred per cent." I suggested that he might have skipped one of the most important things and asked him how he knew to multiply by four. He said that he just guessed. I asked Andrei the same question and he said, "Four times twenty five is a hundred." Darragh said that he understood and asked for a chance to explain again. He said, "By four is a hundred and that's you're whole hundred per cent so eh ... Then what you do the bottom you do the top so twenty two by four and then what's left out of a hundred per cent so eighty eight take away a hundred is ... twelve." He then appeared to ask either Steven or Michael of they understood his explanation. Steven indicated that he had not understood and Michael added that Darragh had spoken too quickly. Darragh then gave a more elaborate explanation: "Right you have to turn the twenty two- twenty fifths into hundredths to find what per cent it is and if four twenty fives is a hundred ... So you multiply by four. What you do to the bottom you have to do to the top so you have to multiply twenty two by four which is eighty eight. So that's eighty eight hundredths so that's eighty eight per cent." During this phase of explanation Michael interrupted him and added his own comments. Then I set the students some written work to do from their textbooks. The original question as presented to students and the inscriptions made on the board during the whole class discussion are shown in figure 5.16.

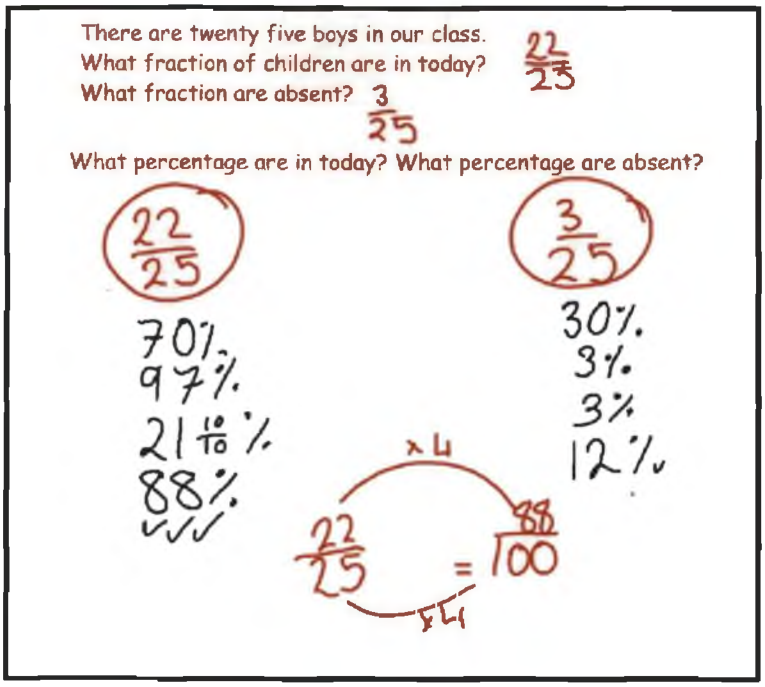



Figure 5.16. Task presented to students about the fraction and percentage present and absent. The figure shows inscriptions I made during the whole class discussion

**Discussion**

The lesson activities for this recording consisted of a preliminary starter activity that was intended to prepare students for the harder challenge of finding the percentage present and absent in the whole class. Both activities were set in a context that was familiar to students. I had hoped that the first activity, finding the numbers of spellings correct out of ten and discussing how this was expressed in hundredths, might help students make this same connection in the second task. However, most students failed to make the connection and even when Andrei explained his solution, an able student like Darragh still seemed to miss the importance of the conversion to hundredths step. Students participated well and most seemed to enjoy the challenge. Edward admitted that he had not thought of Andrei's solution but still proclaimed it 'brilliant'. This would seem to show an appreciation of the mathematics itself as well as the inventiveness of his peer. Students were encouraged to explain their reasoning and to comment on the reasoning of their peers. It was the students rather than me as teacher who played the largest role in evaluating the mathematical reasoning. The strongest evaluative move I made was when I stated that neither of the first two solutions suggested by Alan and Anthony was right. This move was intended to motivate students to offer other solutions or at least to generate discussion.

Andrei's central role here should not eclipse the active participation of many other students. Students like Anthony displayed confidence in explaining and defending their mathematical ideas even when these were incorrect. The students also show a willingness to take risks as Jake's unexpected suggestion indicates. There was also a notable ease with the possibility of multiple possible solutions and Alan suggested (incorrectly) that this was a possibility. Students who did not actively suggest solutions to the problem showed active participation by asking questions and admitting that they did not understand.

**Discourse in this Recording**

The fact that students struggling to understand admitted this and criticised the speed of the explanations of their peers suggests an expectation that student contributions should be understandable by all. This in turn suggests an obligation on the contributor to do his best to communicate his ideas and suggests high levels of RFL within the community. Much of the student-to-student discussion that characterises level 3 in the MTLC framework was present in this discussion. Student ideas were central to the development of the lesson and many students showed high levels of responsibility for learning by attending to and commenting on the contributions of their peers. Many middle and lower achieving students displayed RFL by questioning the mathematical ideas they did not understand. A summary of the findings in relation to the components of the MTLC framework is given in table 5.9.

Table 5.9
*Summary descriptions of MTLC framework components in the Percentage Present and absent lesson.*

| Component | Description |
|---|---|
| Questioning | A range of teacher questions including questions focussing on thinking and generating discussion. |
| EMT | Low levels of teacher EMT. High levels of student EMT. |
| SMI | Students appear to be positioned as SMI. |
| RFL | Students involved in evaluating mathematical ideas. Some students commented on the contributions of their peers and showed high RFL. Other students showed RFL by admitting to not understanding. |

**Summary**

The activities carried out in the five lessons varied. The *Cutting Pizzas*, *Equivalence Challenge* and *Percentage Present and Absent* lessons focussed on whole class problem solving that was planned in advance. In the *Fraction Problems* lesson, the mathematical issue that was discussed in depth was not planned in advance, but one that arose from a student question. The *Decimal Dienes'* lesson varied from the others because the content was not presented in problem form and this appeared to have a knock-on effect on the nature of the discourse. All lesson activities were similar in that there was an attempt made to position students as the source of mathematical ideas. In the *Cutting Pizzas,* the *Fraction Problems*, the *Dienes' Decimals* and the *Percentage Present and Absent* lessons, there was an attempt made to make the contexts experientially real for the students. The *Cutting Pizzas* lesson would seem to have been the most effective of these as the context supported students' attempts at mathematical reasoning.

Student explanations of their mathematical thinking were central to all lessons and one of the main issues I had as teacher was trying to determine when it was best that I should take the central role in explaining mathematical reasoning rather than my students. This issue is complicated by the nature of student reasoning which was sometimes incomplete, incoherent, incorrect or a combination of all three. This issue was thrown into sharp focus when considering if lower achievers could access and follow the whole class discourse particularly in the *Equivalence Challenge* lesson. Because I wanted students to take responsibility for their learning and the learning of their peers, I tended to shy away from direct evaluation of their reasoning, preferring that other students would share their thoughts on why suggestions might be correct or incorrect. I believe that the lack of direct teacher evaluation created the opportunity for students to take on an evaluative role in their own right. It appears that the lack of direct teacher evaluation and low levels of teacher EMT created a space for student thinking to become more prominent. The strong teacher emphasis on student EMT served to recognise student EMT as valuable and a central component of discourse in our classroom community.

Issues arising from the teacher question category analysis, including the obvious lack of certain categories of questions, will be discussed in chapter 8. In chapter 8, I will also comment on comparisons of lesson content and discourse with the stage 1 recordings. In chapter 6, I will discuss other issues related to the teaching experiment including the individual participation trajectories of students and my concerns about the nature of the experience for lower achieving students.

## Chapter 6: The nature of teaching and learning in our fledgling mathematical discourse community

In Chapter 5, I presented an analysis of a selection of lessons that illustrated some of the opportunities for the sharing of mathematical thinking that arise in a discourse community. The lessons also illustrate the different ways in which various students participated in lessons and some of the dilemmas that arise for a teacher aiming to teach in this way. In this chapter, I will first discuss the participation trajectories (Dreier, 1999, 2009) of a selection of students throughout the teaching experiment in an attempt to explore the nature of learning in a discourse community. Learning mathematics in a discourse community is understood as students' increased participation in authentic mathematical practices. The participation of students was examined for evidence of key discourse community practices such as evaluating mathematical thinking. It was envisaged that this exploration would document students' use of discourse community practices and give an indication of their learning. In this sense, the learning of specific mathematical content was not being examined. Instead, the focus was on what practices students might learn from the teaching approach over time (Lampert & Ball, 1999). In the second section of the chapter, I will consider issues of mathematics and teaching that arose during the experiment. Finally, I will discuss the overarching questions that unite these issues and suggest possible answers.

### Participation Trajectories of Students

In chapter 5, I explored the nature of whole class discourse in some teaching experiment lessons using the MTLC framework (Huffered-Ackles, et al., 2004). This was effectively analysis at a community level. However, the nature of the discourse community experience was not uniform for all students. The analysis presented in this section is focussed on the individual. The analytic method, explained briefly above, was discussed in more detail in chapter 3. A description of the participation trajectories of seven students will be presented here. This is almost half of the whole group and includes a mixture of students in terms of their achievement levels and participation styles. Students' participation trajectories will be presented in conjunction with their achievement levels because I questioned possible links between achievement and the nature of student participation during the experiment.

**Higher Achievers**

Four students were identified as higher achievers because they scored between the $70^{th}$ and $100^{th}$ percentile on a standardised mathematics test in summer 2010 (prior to the teaching experiment). Descriptions of the participation of Darragh and Anthony will be presented here because although both were vocal and active in all lessons, there are some apparent differences in the style of their participation.

**Darragh**

Darragh consistently contributed ideas to whole class discourse. His many contributions were confidently and coherently stated and he was correct mathematically more often than he was incorrect. He regularly commented unprompted on the solution efforts of his peers, sometimes to agree, sometimes to disagree (T1, T3, T4, T5, T11)[10]. He questioned students about their strategies (*Cutting Pizzas,* T1) and also directly questioned me on occasion (T3, T5). He often displayed willingness to contribute and introduced into the discussion significant mathematical ideas and vocabulary (T1, T3, T5), sometimes building on the suggestions or errors of his peers (T8, T12, T13). On one occasion he changed his mind about ideas he had previously suggested were correct (T1, *Cutting Pizzas*). On another occasion, he followed up on a question I had posed to another student (T4, *Fraction problems,* page 165). He also noted and commented on similarities in activities across lessons (T9). In the *Cutting Pizzas* lesson (T4), he seemed to take his responsibility for his peers

[10] Transcript X is shortened to TX. A full list of the transcribed lessons is given in appendix 15. Where issues discussed in chapter 5 are relevant to the points made in this section, links are made explicit between transcription number and the lesson names used in chapter 5.

seriously (page 159). It appeared that that he had an awareness of his own role and ability when he referred to not wanting to "confuse people" (turn 441).

In Wenger's (1998) terms, the nature of Darragh's participation could be described as an *insider trajectory*. Darragh's mathematical ideas played a central role in whole class discourse and he participated fully in the practices of the discourse community from the start. In fact, Darragh's central role and the nature of his participation meant that he demonstrated ways of acting in a discourse community to his peers. His ways of acting coincided with many of the desirable student actions for a discourse community. This meant that in some ways he could be considered an *old timer* from whom other students may have learned ways of acting in a discourse community (Lave & Wenger, 1991). The role of the teacher cannot be described in quite the same way as teacher actions are not necessarily suitable models for student actions. For example, in the discourse community approach, the teacher will often refrain from explaining her mathematical thinking and evaluating the contributions of her students. Instead, students are expected to take on this role. This makes the nature of the participation of 'model' students like Darragh even more important. I discussed in chapter 2 how researchers have commented that the role of the teacher has been under theorised in conceptions of school based communities of practice. While the observations made here do not address this issue, the possibility of viewing students with certain participation styles as acting as old-timers goes someway to applying the apprenticeship model of learning in classroom contexts (Lave & Wenger, 1991).

**Anthony**

Anthony consistently displayed a willingness to participate and contributed confidently in most lessons. His regular mathematical contributions were stated clearly and confidently and were generally accurate. On a number of occasions he presented alternative strategies or solutions than had been presented previously by peers (T1, T3, T14). This suggests high levels of RFL on his part and an active engagement in the unfolding lessons. An interesting situation arose in the *Percentage present and absent* lesson (T12), where it was determined that $\frac{22}{25}$ pupils were present and we were attempting to express that as a percentage (page 207). Anthony suggested that 97% were present and 3% were absent. When the correct answer of 88% present and 12% absent was discussed, Anthony suggested that he was still right in some way. It may be that he was referring to the fact that the successful problem solver, Andrei, had worked out the answer of 12% absent through complements to 100, as Anthony had. However, Anthony's strong insistence that his solution was valid too suggests that being successful in mathematics was important to him and that he may associate success with the production of correct answers. Although Anthony was not as vocal as Darragh, the nature of his participation in the practices of the discourse community also indicates an *insider trajectory*.

**Middle Achievers**

The group of eight students identified as middle achievers all scored between the 20$^{th}$ and the 50$^{th}$ percentile in a standardised mathematics test in summer 2010. (No students of the class scored between the 50$^{th}$ and the 70$^{th}$ percentile on that year's test). Of these eight students, notes on the participation trajectories of two students will be presented here. Jake's case is presented because he was observed to make progress throughout the year. The nature of John's participation is slightly different and for this reason I present it also.

**Jake**

Jake often showed a willingness to contribute to whole class discussions. At times his contributions, though frequently mathematically correct, were faltering and hard to follow. This is particularly true of his contributions to lessons in November and December 2010. His increased confidence and ability to articulate his thinking after this time may be due to the experience gained in teaching experiment lessons. It may equally reflect a greater competence with the post-Christmas lesson topics. On a number of occasions Jake referenced other students' work sometimes to agree with it or to suggest a new approach when a peer made an error (T8, T10, T13). On other occasions, he offered suggestions that went against contributions previously made by his peers (T13, T14). Some of Jake's contributions were significant both in relation to their mathematical content and their role in shaping the classroom discourse (see for example *Cutting Pizzas* (T1) page 159, *Percentage Present and Absent* (T14) page 207). This pattern of participation would suggest a genuine engagement in the discourse community. It is tempting to describe the nature of Jake's participation as indicative of an inbound trajectory because of his growing confidence observable across the course of the year. However, he engaged in many of the practices of the discourse community right from the start, for example by giving a suggestion that ran counter to those previously contributed by his peers in the *Cutting Pizzas* lesson, the very first lesson of the teaching experiment (T1). This suggests that a description of *insider trajectory* is more suitable.

**John**

Just one question from John was recorded over the thirteen transcripts. This question was about the difference between plastic Dienes' blocks and wooden Dienes' blocks (T8). John's contributions to whole class discourse stemmed from a mixture of direct teacher invitations and occasions where he showed a willingness to contribute. The majority of occasions where a desire to contribute was observed occurred after January 2011. This may indicate that it took him some time to adapt to the discourse community approach. These occasions are interesting as his statements suggest that he had been reflecting on the discourse to that point and aiming to make it relevant to himself or to understand it better. For example, when I displayed a selection of circles divided into various fractions and asked if the students could 'see' that three-sixth was the same as a half, John replied, "Teacher, I get the idea, you're kind of saying how many sixths would cover the area of a half. And then you cover it and it'd be three and then in fours it'd be two and eighths it'd be four" (turn 238, T2). Not only did he restate my question in terms that were understandable to him, he then applied the same approach to the other shapes which showed quarters and eighths. On another occasion he built on a suggestion Kevin made about number patterns observable in equivalent fractions (T7). Similarly in a decimal lesson that focussed on necessary and unnecessary zeroes, John likened necessary zeroes to price-tags in a shop saying: "Teacher that's like em, em, you go into Xtravision and you have like a sticker for a game that's ninety euros but you rip off the zero, Then it'd only be nine euro" (Turn 24, T10). John spoke less frequently in whole class discourse than some other students but when he did speak, his contributions were generally stated confidently and clearly and on most occasions he was mathematically correct. This pattern of participation suggests a genuine engagement in lessons. John did not participate in all practices of the discourse community at first but increased his participation as time went by. This pattern of participation suggests a description of *inbound trajectory* is appropriate.

**Lower Achievers**

The six students identified as lower achievers scored between the 10$^{th}$ and 20$^{th}$ percentile in a standardised mathematics test in summer 2010. The participation of all lower achievers was examined in depth. Details of three of these students are presented here. These particular three were chosen because of apparent differences in their participation styles.

**Kevin**

Kevin regularly displayed a willingness to contribute. His contributions were generally understandable but were not always mathematically correct. For example, he sometimes used real life ideas and language which lacked mathematical precision. On a pizza sharing task, in order to achieve a fair share he suggested giving one piece "back to the man" (*Cutting Pizzas,* T1, turn 92, page 159). On another occasion he suggested an alternative solution than that which had been presented by a peer and said, "You can put one slice in a half to get the same way… but not like one big half" (Turn 319, T3). It would seem that in this case, he was referring to cutting a half in half but lacked either the mathematical knowledge or language to identify what the result would be. He sometimes commented on the ideas of others, generally to agree with them rather than suggest alternative solutions (T3, T6, T14). On two occasions his contributions appeared to influence the ideas of others. On the first occasion he described a pattern he noticed in a group of equivalent fractions equal to half which John then built on in later contributions (T7). On the second occasion in a decimal lesson, he correctly identified the largest of a group of numbers and when Alex suggested that it may be a different number, Kevin successfully explained his reasoning to him and Alex accepted this (*Dienes' decimals,* T8, page 184).

Though Kevin's ideas lacked the mathematical complexity and precision of some other students, his pattern of participation indicates a genuine engagement in the lessons. Whether this engagement extends to more than just surface level features of discourse community practices is questionable. As I noted above, Kevin sometimes presented his own ideas and agreed with the ideas of others but at no stage did it appear that he built on or added to the thinking of his peers in whole class discourse. At no time, did he disagree with his peers or ask questions of their methods or my explanations. Because of this, I would suggest that although Kevin was observed to engage in some of the key practices of a discourse community, there is not enough evidence to describe his participation as an insider trajectory. Instead I would suggest that he may be on an *inbound trajectory*.

**Steven**

Steven contributed regularly to whole class discussion both on my invitation and unprompted. He appeared to state his opinions confidently and coherently but was not always correct and on a number of occasions he struggled with correct fractions language (T1, T2, T5, T8). He also inappropriately applied real life reasoning to produce an unequal sharing scheme in the *Cutting Pizzas* lesson (T1, page 159). Making errors in a whole class setting did not seem to faze him and in fact he seemed to enjoy being the focus of the class's attention. On one occasion, he volunteered to come to the board to complete a subtraction sum to find the difference between the heights of a girl and her dog (T11). The task and Steven's attempt at solution are shown in appendix 23. The larger height was $1\frac{1}{4}$ m and the smaller was 57cm. Steven ignored the 1m component of the girl's height and wrote the numbers in a vertical format without changing the $\frac{1}{4}$ m into 25cm or to a decimal representation. Despite hints and questions from his peers, he insisted on attempting the normal subtraction procedure using renaming. I asked him if he wanted any help or wanted to ask a question but he continued with his method. Steven did not seem in the slightest bit discomfited by the situation. When I eventually called on another student to explain the process and write it out correctly, Steven joked "My way is still better!" (turn 178, T11). This could be interpreted as an attempt to save face but given the fact that I gave him several opt out opportunities while he was at the board, my understanding was that he enjoyed the attention. Steven asked questions in whole class discourse in eight out of the thirteen lessons studied for this part of the research project. In the *Cutting Pizzas* lesson (T1), Steven asked seven out of the fourteen recorded mathematically orientated student questions (appendix 22). This mode of participation is very different from other students in the class, the majority of whom posed few if any questions. Steven's willingness to pose questions may be related to his willingness to share his ideas regardless of whether they were right or wrong. It seems that he did not attach meaning to the commonly perceived social risk of asking questions or making mistakes. Steven often admitted to not understanding explanations of mine or of his peers (T1, T2, T3, T7, T9). These frequent admissions of incomprehension may suggest that he was struggling with mathematics at fifth class level. This is probably a factor because of his baseline achievement levels. However, it may also indicate that he was expecting a more traditional teaching approach involving direct explanations. If this was a factor, then his articulations of incomprehension may have been made in the hope of persuading me to provide explicit direction and lessen the cognitive load (Stein et al., 1996). On a positive note, they also suggest that he was following the whole class dialogue and self-monitoring for understanding. This interpretation is supported by his frequent comments on other students' contributions.

Steven was very vocal in all lessons and many of his actions do suggest an active engagement. However despite this, like the case of Kevin, Steven's participation appeared to be limited to certain practices of the discourse community. Consider for example, his actions in the decimal lesson described above. The fact that he insisted on persisting in his efforts and not accepting the input of his peers suggests that he did not view participation in whole class discourse as a community effort to negotiate mathematical understanding. In fact, it is only his many questions that reflect in any way practice of the discourse community. There is little evidence to suggest that his participation reflects an inbound trajectory either. Wenger (1998) notes that an aspect of the inbound trajectory, is that identity is invested in future full participation which seems to be the case for Kevin. Steven's participation does not indicate this and despite his high level of contributions in lessons, his lack of participation in the central practices of the discourse community suggests a description of his participation as a *peripheral trajectory* may be best.

It is interesting to consider how Steven would view the nature of his own participation and how he might be influenced to change. It is likely that he viewed himself as a central participant or insider because of his many contributions. It is possibly only by explicitly addressing the specific discourse community practices required that he may be encouraged to change the nature of his participation. This is similar to Boaler's (2006) discussion of teachers who explicitly address valuable practices in whole class discourse. In this way, teachers draw attention to specific learning practices that may prove effective for students.

**Jared**

Jared contributed much less frequently to whole class discourse than any of the students discussed so far. For the most part he contributed only when he was invited to. These contributions were generally stated fluidly and were sometimes right and sometimes wrong. On two occasions he posed clarifying questions. The first occurred in a lesson on mixed numbers and top heavy fractions when he asked, "See the two of one and two-fifths? ... Well would that be two pizzas or would it just be two slices?" (turn 121, T6). In the same lesson, when I asked what he thought of a solution we had worked out, he said that he had thought $2\frac{1}{3}$ would be bigger than $\frac{5}{3}$ and I became aware that I had inserted the greater than sign the wrong way round. On another occasion we were discussing fractions and percentages in terms of the amount of energy and ammunition a computer game character had left, Luke identified that the character had 25 % of his ammunition left and Jared asked "Teacher, isn't it just half of fifty?" (turn 47; T13). The one other occasion where he showed a willingness to contribute occurred in a decimal lesson where we were comparing various numbers and trying to put them in order of size. The numbers were 4, 3.4, 3.04, 0.004. Jared said "Teacher you were just trying to trick us out there" (turn 157, T8).

It is difficult to draw conclusions about the nature of Jared's participation due to the lack of data. On some level, the limited evidence suggests limited engagement but the nature of some of the contributions described above, in particular the questions, suggests Jared was actively engaged in those lessons at any rate. The fact that he displayed more willingness to contribute in the later stages of the experiment may suggest a development in participation and a possible shift in trajectories was underway. Ideally this would be from a peripheral to an inbound trajectory. However, it may equally be that Jared had a long history of non-participation in school settings and it is possible that throughout the teaching experiment he was negotiating a trajectory from a *marginal position*. In relation to marginality, Wenger suggests that "forms of non-participation may be so ingrained in the practice that it may seem impossible to conceive of a different trajectory within the same community (1998, p. 167). It is possible that Jared, a lower achieving student who appeared much quieter than his often boisterous peers, had been in such a position for some time and the ways of participating in the discourse community were not as easily negotiated by him as by some of his peers. For this reason, his participation trajectory, even lacking detail as it does, is interesting.

**Discussion**

Many higher achieving students seemed to be more willing and able to engage in the discourse community right from the start. For example, Darragh and Andrei played central roles in most of the lessons presented in chapter 5 and in the teaching experiment in general. In fact, their participation served as a model for other students. It is likely that previous positive experiences had influenced their sense of self-efficacy and their predisposition to participate. For students like Jared who contributed less frequently, it is interesting to question what prompted his more vocal engagement in certain lessons above others. Was it because given mathematical topics or tasks were more accessible or was it influenced by social aspects not obvious in this recording method? It would be interesting to extend the exploration of his participation to small group or pair work. Would the nature of his participation have changed if the teaching experiment were carried out for a longer period or if more attention was explicitly focussed on the desired discourse community practices?

As a teacher, reflecting on the nature of student participation was informative. Formal analysis occurred too late to inform my teaching practice during the experiment, which I feel could have benefitted from more explicit discussion of learning practices within the discourse community (Boaler, 2006). At this point I should also point out that the exploration of student participation trajectories was limited to their oral participation in whole class discourse. A more nuanced picture would emerge if dialogue in pair work or other settings was explored. Similarly other methods of data collection such as video recording would allow for observations of gesture and body language which would provide a fuller picture of engagement and participation.

Another issue that arises in considering this information is the multiplicity of identities for each individual student. Students arrive in classrooms with multifaceted identities formed through experience in many different contexts (Grootenboer & Zevenbergen, 2008). This issue is obvious when the differences in participation between those on similar trajectories are examined. For example, Darragh and Anthony, who were both described as on insider trajectories, varied in the style of their participation. Also the contextual and temporal aspects of the teaching experiment cannot be ignored. School experience would have influenced students' negotiation of identity for six years by the time the teaching experiment took place. It is worth considering how the nature of student participation trajectories might change if the discourse community approach was followed over a longer period or enacted at a different class level or in a different type of school.

As it stands, students engaged in the practices of the discourse community in different ways and while these were not fully realised in all cases, there is evidence to suggest that many students did benefit from the teaching experiment lessons in terms of an observed growth in participation in key practices over time. The relationship between students' sharing of mathematical thinking in whole class discourse and their performance on paper and pencil tasks is complicated. This is particularly true of the relationship between the type of relational thinking (Skemp, 1977) I tried to promote and the procedural thinking sometimes pursued in textbooks. However, results of a standardised mathematics test carried out in June 2011 show either improvement in or maintenance of achievement levels for all students when compared with their results from the previous year. By this, I mean that all students either scored within or above the same achievement bands discussed at the start of this chapter. Some of the improvements of individual students were dramatic changes from the standardised test carried out a year previously. Four students jumped between ten and twenty percentile points, two students jumped by more than twenty percentile points and two by more than thirty percentile points. One of the students who jumped by more than thirty percentile point was initially identified as a lower achiever and the other had initially been identified as a middle achiever. While improvements in achievement cannot be attributed directly to the influence of the teaching experiment and may in fact have been influenced by other factors, these results show at the very least that the teaching experiment did not disadvantage students in terms of their abilities to complete standardised tests.

## Questions of Mathematics and Teaching Arising from the Experiment

The aim of this section is to explore the nature of teaching in the discourse community and to discuss the issues I faced as teacher during the experiment. As I discussed in chapter 3, notes made in my teaching journal detailed my initial understanding of events and in these reflections I began to identify and begin the process of naming and describing various issues. In effect through writing in my journal, I was trying to answer or at the very least explore certain questions related to my teaching approach. In this section, I will present these questions and detail, with reference to my teaching journal and relevant research, my attempts to answer them. By including journal extracts in my discussions I am attempting to be explicit about the subjective judgements involved in some of these areas.

### How should a balance be arranged between cognitively demanding and routine tasks?

The nature of the mathematics I was aiming to teach is fundamentally different from mathematics as it is presented in traditional approaches in much the same way as Skemp defines the difference between relational and instrumental understanding (1976). Influenced by Freudenthal's notions of anti-didactical inversion (1973), I hoped that students could be creators and arbiters of their own mathematical reasoning and would engage in genuine mathematical discourse in the building of complex ideas. The nature of mathematical tasks influences the nature of the resultant thinking and discourse (Stein et al., 2000). To promote the type of discourse where students would engage in mathematical reasoning, I felt it was necessary that the tasks be challenging rather than routine for students.

These beliefs about the nature of mathematics and appropriate teaching approaches led me to prioritise activities of high cognitive demand in a discourse rich environment where students would share their mathematical thinking. These activities stand in contrast to traditional teaching activities consisting of repetitive algorithmic or procedural exercises that demand little in the way of novel student thinking (Boaler, 2009). I recognised the value of procedure focussed tasks to facilitate practice and consolidation opportunities for students but struggled to devise an appropriate balance between repetitive practice exercise and tasks designed to challenge and develop student thinking. This can be challenging in the design of a single lesson in which students of different achievement levels participate. I was engaged in designing learning opportunities over time so the issue takes on even more significance. The lack of possible models was also an issue. The only readily available schemes for teaching across time were Irish textbook based schemes, which to my mind at least, did not emphasise cognitively demanding tasks sufficiently or use such tasks as a spring board for the learning of mathematical content.

**Routine and complexity, order and disorder**

This issue was further complicated by problems specific to my own classroom. Due to the nature of the timetable negotiated with the learning support team (as discussed at the start of chapter 5), the daily mathematics lesson always took place after first break. [11] I found that when the students returned from break, they were slow to settle and attempting to engage them with complex mathematical tasks at this time proved difficult. Instead, at the start of our mathematics lessons, I generally set routine, practice orientated tasks that could be completed relatively easily. These repetitive, exercises in which the students knew exactly what was expected of them, were completed calmly and quietly by students.

This was in effect a choice between the presentation of examples to "provoke or facilitate abstraction" or practise-orientated examples that might "assist retention of the procedure by repetition" (Rowland, 2008, p. 150). I wondered if I was not being true to the mathematics by presenting tasks that were not cognitively demanding and did not require student communication. On the other hand, it seemed a valid strategy given that students seemed to need periods of calm activity particularly at this time of the school day. In the end, I felt that though these exercises were questionable because they did not reflect the nature of mathematics as I perceive it to be, they were necessary from the broader perspective of classroom management and classroom routine. They provided a 'sense of order' for both me and my students that was a necessary foundation from which the complexity of the discourse community could be negotiated. Some students appeared more comfortable with the predictable traditional activities than the more open format of the mathematical tasks in the teaching experiment lessons, perhaps because successful instrumentalists may be reluctant relationalists (Skemp, 1979).

Issues of how the discourse community approach might upset this 'sense of order' and may be perceived as disorderly were complicated by the nature of a whole school culture which not uncommonly favoured traditional teaching methods. The discourse community approach is somewhat looser and it was acceptable for students to contribute without waiting

[11] Generally, there are two breaks within the Irish primary school day where students have time to eat lunch and go outside to play.

on an invitation from the teacher. Also mathematical ideas appeared to emerge piecemeal from students and even incorrect mathematical ideas were given central stage at various times. This stands in contrast with the orderly traditional routine of direct teacher explanation followed by student practice. The possibility that students and teaching colleagues might not see value in the complexity of the discourse community approach and perceive it as 'disorderly' was a real concern and is related to the next question which I will discuss.

**Did the activities of high cognitive demand or the discourse focussed approach foster the disengagement of some pupils?**

Ball notes that asking students to grapple with problematic mathematics for themselves "may result in frustration and surrender rather than confidence and competence" (1993, p. 377). Students schooled in a traditional approach of didactical procedure based instruction have clear ideas about mathematical authority (Lampert, 1990; Hamm & Perry, 2002) and may be reluctant to take on the responsibility for determining appropriate solution methods themselves. The unstructured nature of cognitively demanding tasks can result in students urging the teacher for more explicit guidelines on how to tackle the problem (Stein et al., 1996). This move may result in a pressure for the teacher to 'tell' mathematical aspects that would otherwise have to be grappled with by students. At various stages during the teaching experiment, some students reacted to the increased challenge of the tasks by disengaging or becoming high-spirited and engaging in off topic activities. For example in my teaching journal on the 20th September I noted that Aidan, Steven and Jonathan had disengaged from the task and become disruptive. It seems that in the early days of the school year, I was worried that not all students were participating, or more precisely that some students were choosing non-participation as a form of engagement (Wenger, 1998). For example consider this teaching journal entry from September 23rd. The class had worked on a selection of problems from New Wave Mental Mathematics 5 (Krajcar, 2002) and we had discussed their solutions in a whole class setting.

> The question asking "Which is better value 9 sweets for 60c or 5 sweets for 40c?" seemed initially problematic but over half the class offered to come to the board and show their solutions and there was some significant mathematical input. The dominant argument seemed to be that if you could get ten sweets for 80c through the second offer, the first offer was better as you would only get one sweet less for 20c less. Andrei 'proved' this when he drew bags of sweets to show that you can buy 18 sweets for €1.20 in the first offer and only 15 sweets for €1.20 in the second. Two children actually applauded him when he presented this on the board as if recognising the difference between this 'proof' and the other 'reasonable argument'. However other boys still wished to contribute (though Steven was heard to groan when I called for more contributors). Michael, usually quiet unless completely sure of his facts, (up to this point I had believed he relied on Alan for most of his work) suggested finding the unit price. He did not do formal division but instead suggested that in the first offer, the sweets were about 8c each and in the second, they were "about 6c or something" so it gave me a chance to 'insert' about direct division...
>
> In a short 40 - 45 minutes a lot happened some of which I remember and have recorded. What worries me now is what I don't recall, those children whose contributions were smaller or non-existent. I know that I had to intervene to monitor the attention of Steven but what of John, Alex, Conor, Jonathan? Is it enough to take three-quarters with you?

Criticisms of mental mathematics schemes like the one I used include that the repetitive nature of tasks renders them unproblematic. This is undoubtedly true but as this lesson occurred at the start of the school year and students were encountering this type of task for the first time, it was genuinely problematic for them. The dilemma for me as teacher was that I felt the whole class discussion around the solving of the question was worthwhile mathematically but I recognised that not all students contributed. Of course, lack of contribution may not indicate that the student is not following the whole class discussion but disengagement was a concern. As noted above, I began to wonder about the ethics of my teaching approach. To paraphrase the last sentence of the journal entry above, I wondered if a teaching approach that in this case engaged only three-quarters of the students could be considered valid. Of course, it must be acknowledged, that it is quite possible that some of these students might also have disengaged in a more traditional lesson.

On examination of the evidence of the student participation trajectories presented at the start of this chapter, it seems that my concerns for the most part were unfounded. Non-participation as a form of engagement was a feature of some students' participation in some lessons over the course of the experiment. However exploring the student participation trajectories was an attempt to address the temporal aspects of this issue. I found that while there was evidence that individual students may not have engaged in specific lessons, their engagement over time showed more positive aspects of participation in the practices of the discourse community. The question of possible student disengagement leads to the next question on which I deliberated in some depth.

**Does this teaching approach disadvantage lower achievers?**

**Initial Concerns**

My worries about how the teaching approach may result in the possible disengagement of students fed into a concern that the teaching approach may not meet the needs of lower achieving students. Consider my teaching journal entry from December 8th.

> Though the snow and frost continues, we are back at school. Attendance is down but I decided to continue with our fractions work anyway. Yesterday children worked in

pairs/groups on the dividing pizzas problem and I was surprised by how eager and able most were. Much of the discussion veered into equivalent fractions territory and many boys talked confidently about it. However I worked with Aidan and Steven and I am unsure how much they took out of it. Steven still misnames fractions at times and his answers don't seem to have any discernible pattern to me, just guesswork or return to the common fractions.

I was very aware today that even in the smaller group, there was an uneven pattern to the contributions and I am afraid that I am developing some kind of tier system with Anthony, Darragh, Andrei and Alan contributing much valid and well- reasoned mathematics with others taking a back seat waiting for an answer … I don't know if the free talk format suits them. I know I am inconsistent regarding the hands up rule for example letting Darragh shout out because he's contributing to the mathematical discussion but silencing Steven because he's often contributing to the mayhem. I am conscious that Steven may not understand my rationale, underdeveloped and unspecified as it is, and may feel punished in the wrong.

I am excited about the emerging maths and emerging confidence but am worried I see it only in a handful of boys and only in the stronger ones at that. Am I further disadvantaging the lower achievers with this methodology?

There are a number of issues raised in this extract some related back to the previous topics of disengagement and classroom management. I also note the issue of low attendance which has been associated with underperformance in disadvantaged schools (DES, 2005). It would seem that at this stage, in informal teacher-researcher observation, I was noticing the effect of the discourse community approach but only in a limited number of students. I worried that pursuing a teaching approach that seemed to be favouring higher achievers was inappropriate in a class with a relatively large number of students with low mathematical achievement.

However, the relatively poor performance on previous standardised tests would suggest that the traditional approach was not particularly effective for the achievement of lower attaining students either. My question about whether the "free talk format" was suitable for this class is related to my feeling that the students were unsure of the boundaries or norms for behaviour and communication in the novel discourse community approach. The freedom from a strict invitation-response-evaluation (IRE) based form of interaction left some students unsure of how to communicate and I admit that I was sending mixed messages in this regard. Because I also attempted to devolve mathematical authority to students, they were placed in a position they were unfamiliar with. The question "am I further disadvantaging the lower achievers with this methodology?" was the major dilemma that confronted me throughout the teaching experiment.

**Refocusing the Lens - The Bigger Picture**

Attempting to understand the nature of the experience of lower achieving students led to a 'refocusing of the lens' (Lerman, 2001). Take for example the following journal entry written on December 10th which also focussed on the nature of participation of lower achieving students and challenging mathematical activities.

> I am torn because part of me wants to see students truly engage with the maths and perhaps struggle from time to time. There is no real drive toward attempting to understand without willingness to struggle or a certain 'stuckness' from time to time. However some of my class seem so used to not understanding that there is no internal drive toward attempting to understand or independent response that drives them to ask for help … They somehow lose a whole 15 minutes, not asking me for help, or their partners, not drawing attention to themselves in anyway by bad or distracted behaviour. Is this a coping mechanism? "I can't do this, so I'll just wait it out, wait until she moves on"? The reality of how they experience their school lives is very

different from the way I expect or hope they will. I don't believe the tasks are too hard, but they do. I believe that they can gain from them but must teach them how to do that. Have higher expectations of them? Explore again the reasons why we come to school? I feel in some ways that these boys failure to engage, observable across subjects other than maths, is a systemic failure and much bigger than our mathematics lesson and these few individuals.

I noted here that the reasons for some students' non-engagement might have been influenced by their previous experiences and not just be as a result of the teaching approach adopted in the teaching experiment. The influence of their concurrent experiences in other contexts must also be an influence. Perhaps the most important point that arises from this journal entry is the acknowledgment that this teaching experiment did not take place in a vacuum. In no way could the conditions for the experiment be considered 'neutral'. In the next chapter I will present a detailed account of my own background. Beyond the fact that this experiment took place in fifth class in a designated disadvantaged school, ethical issues prevent me from giving further details of my students' backgrounds. However their previous history, their beliefs, preferences and prejudices influenced the manner of their participation as my past experience influenced my participation. In referring to "systemic failure" in the journal entry above, I was attempting to articulate how some students seemed to have been 'processed out' of the educational community. Downes and Downes suggest that "we are all a processed people. We are either processed into the mainstream cultural mix of society or processed out of it to languish and struggle for survival on its margins" (2007, p. 24). These students appeared to be alienated from the rest of their school community who engaged with classroom activities in a different manner and appeared to have different expectations of their school experiences. Acknowledging the larger context seems vital considering that the possible future trajectories of students are limited by their present forms of participation and attempting to change trajectories requires attention at a systemic as well as individual level.

It may seem that in blaming the wider system, I was giving up my responsibility to encourage the active engagement of my students. In fact, I would argue that this was not the case as the teaching approach, if implemented successfully, creates opportunities for the increased participation of all members of the class. In particular, there was an attempt to design multidimensional lessons (Boaler, 2006) where success did not depend on mathematical correctness as there was an attempt made to value all thinking. In traditional approaches, procedural fluency is valued highly and even small errors can lead to perceived failure. It was also intended that students sharing and justifying mathematical thinking combined with their role in the evaluation of peer reasoning would create not just a responsibility for their own learning, but a communal responsibility for the learning of the group. The areas of multidimensionality and student responsibility are features of the Complex Instruction approach to group work suggested by Cohen (1994) and recognised by Boaler (2006) as contributing to equitable outcomes and high achievement levels in Railside, a low SES urban school in North America. There are many differences between the case of Railside and this teaching experiment, notably the prominent attention given to other aspects of the Complex Instruction approach such as assigning roles in group work and publicly assigning competence to low status students (Boaler, 2006). Also and perhaps critically, the teaching approach followed at Railside was a whole school approach where teachers followed Complex Instruction guidelines across all subjects. Unfortunately this teaching experiment was not part of a whole school approach and it is likely that it stood in contrast to students' previous experiences. In this regard, attempting to teach in this manner was an act that endeavoured to change the established system.

**Negotiation**

The question of whether the teaching approach was disadvantaging lower achievers was one that I deliberated on intensively and returned to repeatedly in my journal. In the following journal entry from January 10th, I discuss the issue of negotiation and how lower achieving students may be disadvantaged by the perceived incoherence of the teaching experiment lessons.

> The 'negotiation' that is always trumpeted in learned articles is a tricky thing to grasp at any time. In the situation of a class where some children's behaviour issues bring power relations to the fore it is trickier still. I worry too about the lower achievers. I don't know that this teaching approach does them any favours. I don't know that old school teaching does either but the way I am trying to work with them may introduce even less lesson coherence. And I have never been more aware of my responsibility for the curriculum I present in the classroom.

This journal entry serves to highlight the responsibility I felt as lone implementer of the curriculum. It was my personal beliefs about mathematics and education that provoked the choice of activities and teaching approach and I felt solely responsible for the learning of my students in a way I did not when using the textbook based traditional approach that was followed by the majority of my colleagues. This fed into my concern about the under achievements of lower achievers. Their long history of poor performance prior to the teaching experiment did little to lessen this burden.

The reference to negotiation in the extract should be understood to encompass the social and socio-mathematical norms and the classroom mathematical practices that this teaching experiment aimed to change (Bowers, Cobb & McClain, 1999; Cobb, 2000). At a basic social level, norms for patterns of interaction between teacher and student were being negotiated. This is why I worried about the over representation of higher achievers in terms of contributions. I was worried that we were creating norms whereby lower achievers contributed less often than higher achievers. Black has demonstrated how a student of whom the teacher had high expectations had his "status as a high ability pupil re-confirmed to him, the teacher and other pupils in the class" repeatedly through productive interactions with the teacher in whole class discussion (2004, p. 357). It would appear that such interactions are not uncommon. Andrews (2011) reports that teacher interactions with able students commonly occurred in whole class settings in a case study of four Finnish teachers. He describes the choice of student contributors who are likely to make meaningful contributions as the "teachers' exploitation of the confident child" (2011, p. 3) which suggests an agenda on the teacher's part. There was also an on-going negotiation of mathematical authority, or a dance of agency (Boaler 2003). I was attempting to institute a norm whereby students rather me as teacher would determine what was mathematically correct (Bowers, Cobb & McClain, 1999). The nature of negotiation of both mathematical and socio-mathematical norms is complicated by issues of power. In this lively class issues of power sometimes arose, for example Edward's laughing at Steven's error in the *Dienes' Decimals* lesson (page 184).

The discourse community approach also involved negotiation on a larger scale. Students were effectively been challenged to negotiate new roles or identities for themselves within the discourse community. The positioning of students as mathematical authorities within the discourse community is the antithesis of students' positioning as 'received knowers' in traditional approaches (Boaler, 2003). The negotiation of trajectories of identity may be trickier for students who may be negotiating changes from marginal or peripheral positions (Wenger, 1998) like the case of Jared discussed at the start of this chapter.

**Coherence**

Returning to the notion of lesson coherence I referenced in the journal entry above, Fernandez, Yoshida and Stigler (1992) describe how students learn mathematics from classroom instruction by first forming a coherent mental representation of the lesson events and then use this representation to construct knowledge. The teacher's goals may determine the lesson activities but it is the clarity of the links made between the goals and the activities that will affect whether students construct a coherent understanding of events that is linked to the underlying mathematical concepts. Fernandez et al. note that students may not always comprehend the relationship between different lesson events and highlight the teacher's role in bridging the gap, usually by talk, when students are unable to make the requisite links independently. They argue that lower achieving students may struggle more than higher achieving students because lower achievers may not make links or see interrelationships between events. This is particularly true in lessons with low levels of coherence. Baxter, Woodward and Olson discuss the commonly held view that lower achievers may benefit most from "the use of a clear set of procedures when teaching mathematics to reduce ambiguity" (2001, p. 530). In the teaching experiment lessons, many student ideas were raised and discussed before being dismissed as mathematically inaccurate. Then the process would begin again. It may have been difficult for lower achievers to use these events to build the same conceptual knowledge that higher achieving students did. Interestingly, Fernandez et al. conjecture that the different educational settings of Japanese and American students may result in different expectations for lesson coherence. They suggest that Japanese students may expect coherence while American students may not have such expectations and that the difference in expectations bears an influence of what is learned. It is likely that the difference in the nature of the learning experience for lower and higher achieving students over time may have the same influence on their expectations of coherence in lessons.

If some students struggle to comprehend the relationship between lesson events, this must also be true in the context of related events in a series of lessons on a given topic. It was my experience that some students operating at higher achievement levels often grasped these links even when they remained implicit while lower achieving students struggled to make these links even when they were explicitly discussed. This is illustrated by some of the contributions in the *Equivalence Challenge* lesson (page 190), where some students made links with previous lesson activities. The idea that the same lesson could be experienced by higher and lower achievers in vastly different ways is also illustrated in my observations in a journal entry from March 21st.

> Today we began our first lesson on percentages. There were large disparities in even the common knowledge around what percentages are. Darragh, as usual, displayed strong conceptual understanding and spoke of how 50% equals a half. Jake displayed good thinking skills and a certain independence of thought and inquiring mind when he spoke of how only 10, 20, 30 etc. hundredths have equivalents in tenths and most boys seemed to enjoy it as it was not too demanding for them. That said while the higher achievers made some of these links like Jake above, I'm not sure that it was anything more than a colouring exercise for some of the others … As usual there were many interruptions- children being collected for learning support, returning from learning support, a message on the intercom, a boy about a lost jumper … It was hard for me to focus and keep track, I don't think it's any easier for my students.

Interruptions for various reasons occurred frequently throughout the teaching experiment lessons and are an area over which I had no control. It is generally accepted that interruptions will also affect the perceived level of coherence in lessons (Fernandez et al., 1992). The percentage lesson described above began with an introductory activity where students explained what they already knew about percentages and spoke about real life contexts for percentages such as examination results and sales. Then I formally introduced the idea that $1\% = \frac{1}{100} = 0.01$ and represented this as one coloured square on an empty hundred square. I questioned students about various percentages and asked about common fraction and percentage equivalences. It was at this stage of the lesson that Darragh and Jake made their contributions. Jake's suggestion was that when considering percentages or hundredths, only the multiples of ten have equivalences in tenths, e.g. 20 % = $\frac{20}{100} = \frac{2}{10}$ but 22% = $\frac{22}{100}$ cannot be expressed in tenths. His reasoning seemed to be based on the representation of the coloured percentages on the hundred square, where 10% = $\frac{1}{10}$ can be represented as one 'full strip'. The lower achieving students all completed the activity sheet successfully probably by simply counting the appropriate number of squares and few engaged in the same level of conceptual thinking that Jake did.

**Summary**

I would like to conclude this section by summarising what has been discussed above and to make some concluding remarks on this issue. As a group at risk of failure, I would have had concerns about lower achievers regardless of what teaching approach I followed. In the early stages of the experiment my concerns were heightened because of the observed lower levels of participation of some lower achieving students. At this stage, it seemed to me that with the exception of some very vocal lower achievers such as Steven and Jonathan, most did not contribute to the lessons to the same extent as their higher achieving peers. I noted that this lack of engagement may be a function of their past trajectories and their participation in other contexts beyond the mathematics classroom. It is also likely that the on-going renegotiation of mathematical and socio-mathematical norms left some students unsure of how exactly they should participate and what might count as a successful contribution in the new context. The participation trajectories presented at the start of this chapter show that all students negotiated this experience in their own way. I have also outlined some observational data which suggests that the way lower achievers experienced lessons in the teaching experiment may have been qualitatively different from the way higher achievers experienced it. This observation aside, I do not believe that lower achievers were disadvantaged by this teaching approach. As I noted above the teaching experiment lesson had many of the elements of multidimensionality which has been associated with equitable outcomes (Boaler, 2006). Also the participation trajectories of lower achieving students showed their active engagement in authentic mathematical practices to varying degrees. Furthermore results from students' performance on standardised mathematics tests conducted before and after the experiment would seem to indicate maintenance or improvement of achievement levels as measured by standardised tests over the course of the year.

The various issues encountered in the teaching experiment were not easily resolved. Instead negotiating these issues in practice involved the management of various tensions and conflicting aims. Lampert describes the dilemmas encountered in teaching as "an argument between opposing tendencies within oneself" (1985, p. 182). My aims of maintaining cognitive demand and the active engagement of all students seemed in conflict at times but could not easily be resolved by simply choosing one over the other as "the conflicted teacher is her own antagonist; she cannot win by choosing" (Lampert, 1985, p. 182). Lampert puts forward a case that these inner tensions are a 'tool of the trade' for teachers and it is the teacher's personal sense of identity that comes to the fore when managing them. A teacher's understanding of who she is influences how she will act in relation to any given conflict. Lampert suggests that in practice teachers cope with rather than solve dilemmas. In short, they find a way to manage dilemmas and tensions in practice.

In the case of the teaching experiment it is true that certain tensions were managed rather than resolved. With the goal of both summarising and clarifying, I developed the list shown in figure 6.1. As noted above, teacher identity is a factor in how these tensions are identified and managed (Lampert, 1985). For this reason, it should be clear that this is my own understanding of the tensions involved in facilitating a discourse community and were another teacher/researcher to carry out a similar experiment, it is possible that different issues would be identified. Similarly, although the presentation of these tensions may suggest a choice between 'dichotomous alternatives' (Lampert, 1985), in practice the picture is much more nuanced and there are few categories that can be considered as mutually exclusive.

| | | |
|---|---|---|
| Cognitively demanding tasks | --------- | Routine tasks |
| Participation of higher achievers | --------- | Participation of lower achievers |
| Community | --------- | The individual student |
| Coherent lessons | --------- | Making student thinking an object of classroom discourse |
| Teacher responsibilities | --------- | Student agency |

Figure 6.1: Managing tensions as a feature of facilitating the active engagement of all students in a mathematical discourse community.

I do not see teacher agency in tension with student agency. Even in teaching approaches that promote student agency, teachers retain their own agency. Carlspecken and Apple suggest that human agency exists in "patterned ways" (1992, p. 510) due to the social structures within which people operate. The pattern of enactment of teacher agency in a discourse community context is different from the pattern of enactment in traditional approaches. At times in my own case this involved choosing when *not* to act, such as when to refrain from providing explanations or evaluations. In my efforts to promote student agency, I acted in ways that I hoped would support students' relationships with the discipline of mathematics (Grootenboer & Zevenbergen, 2008). In this way I hoped they would engage in a 'dance of agency' with the discipline (Boaler, 2003) but I was still an active agent at all times. For this reason, I do not perceive teacher and student agency to be in conflict but I did experience conflict between promoting student agency and my responsibilities to teach a prescribed curriculum (Ball, 1993). This tension also arose when I perceived a tension between my focus on an individual student and my responsibility to the whole group.

## Overarching questions

The questions discussed above arose for me as teacher-researcher during the teaching experiment. Reflecting on these reflections with the experiment complete adds another layer of analysis. This analysis has helped me formulate two more questions that I have come to believe are central to the whole endeavour of trying to facilitate a discourse community. The first question considers the role of teacher talk in reform mathematics teaching. The second question is concerned with the role of instrumental understanding in learning mathematics. I do not claim to have answers to these questions but have found them central to the enterprise of attempting to teach mathematics in a reform orientated manner.

### What is the role of teacher talk in reform mathematics?

The debates over teacher telling or direct explanation of mathematics are evident in the literature of reform. While teaching approaches such as guided discussion are recommended (DES, 1999) direct telling of mathematical facts is discouraged in an effort to promote student thinking/agency. Researchers highlight the dilemmas for the teacher in making decisions about direct teacher telling (Chazan & Ball, 1999; Dooley, 2011a). I admit to reading such articles before the teaching experiment and noting that it was a complex issue. However, I felt that between my teaching experience, mathematical background and research interest, I might be well placed to deal with such issues in my own practice. The reality of the teaching experiment was somewhat different and in retrospect questions around when a teacher should 'tell' seem to have been the elephant in the room for much of the duration of the research.

In attempting to value and promote student thinking, I often refrained from giving direct explanations myself particularly in the early stages on the experiment. At times, as in the *Fraction word problems* lesson (page 165), after various students had provided an answer to a question raised by Jonathan, I also gave a direct explanation. In the analysis, I implied that I felt that Jonathan still did not understand the mathematical reasoning and in a way I am admitting to acting on intuition. Intuition, even if it is based on detailed knowledge of the student and non-verbal cues such as body language, does not seem like a valid way of determining when a teacher should tell or not. Dictionary definitions describe intuition as "quick and ready insight; immediate apprehension or cognition" (http://www.merriam-webster.com). A teacher's decision during whole class discourse requires this type of 'immediate cognition' with no time to rationally consider alternatives. If intuition can be considered to be influenced by teaching experience, detailed knowledge of students' abilities and the social context of the classroom as well as knowledge and beliefs about mathematics, then under the pressure of facilitating a whole class discourse that is meaningful for all students, using intuition in determining when to 'tell' may be all a teacher can do. This pragmatic view on teacher telling is similar to that of Chazan and Ball who advise against prescriptive directions against teacher telling and suggest that teacher moves "are selected and invented in response to the situation at hand, to the particulars of the child, group or class and the needs of the mathematics" (1992, p. 7).

Questions about teacher telling can also be related to the notion of coherence. It seems natural that traditional approaches involving direct teacher exposition of mathematical ideas and procedures may be perceived as more coherent than approaches based on working with student thinking. However, there is much evidence that despite its perceived coherence, the traditional approach has not resulted in high student achievement at primary level in Ireland (Eivers et al. 2010). It would seem that some form of blended approach is necessary, where student thinking can play a central role in classroom discourse but direct teacher explanations would punctuate the discourse where necessary to aid coherent conceptions of lesson events.

Coherent lessons have been described as like a story with a definite structure and interrelated events or activities (Chen & Li, 2009; Fernandez, Yoshida & Stigler, 1992; Shimizu, 2009). The mathematical story of a lesson may not be perceived easily in a reform lesson where student ideas play a central role in the narrative and mathematical understandings emerge gradually through community negotiation and evaluation. In such settings direct teacher telling and evaluation is limited in a bid to increase student agency as evidenced in the descriptions of teacher and students' actions in the MTLC framework (Hufferd-Ackles et al., 2004). Perhaps unsurprisingly, the aim of maximising student agency can be misunderstood as a diminishing of the role of the teacher. Boaler (2003) describes the delicate balance between student agency and disciplinary agency in reform mathematics lessons where students develop a relationship with the discipline of mathematics. She notes that reform mathematics is sometimes misunderstood as imprecise and not reflective of mathematics as a discipline because it is assumed that only student agency is developed. The role of the teacher in developing the relationship between the student and the discipline is unappreciated in this conception of reform mathematics. However in reality teacher talk still plays a vital role in maintaining instructional coherence across lessons and across time.

Anthony and Ding (2011) note that much of the literature that exists on instructional coherence results from cross cultural comparison studies or studies carried out in Asian countries. An exact definition of instructional coherence is hard to come by but the term is often used when discussing coherent lessons on a longer timescale or within a specific curriculum or teaching system. It can be understood to encompass both coherent mathematical content as well as discourse that connects mathematical topics and lesson activities with the goal of enabling students to make meaningful connections and develop coherent understandings (Anthony & Ding, 2011; Chen & Li, 2009; Valentin, 2011). Some research on this topic points to elements of lesson structure as a form of coherence (Stigler &

Perry, 1998; Hiebert et al., 2003) and the importance of a plenary section in a lesson where the main mathematical points are highlighted and connections made (Shimizu, 2009). Shimizu notes that "experienced teachers in Japan typically highlighted and summarized the main points at particular phases of lessons to have their students reflect on what they have learned" (2009, p.312). Shimizu suggests that this particular lesson event, known as 'Matome' in Japanese, serves the purpose of creating an opportunity for reflection or for setting a context for the following tasks as well as for creating opportunities to make connections across topics and time. Mercer (2008) notes that the 'recaps' used by teachers he studied were of two types: either literal statement of fact or a rewriting of history to suit the teacher's current pedagogic concerns. He notes the impact of teacher talk and suggests that through talk:

> ...Teachers invoke common knowledge and highlight the continuities of educational experience, trying to draw students into a shared, cumulative and progressive understanding of the activities in which they are engaged (2008, p. 8).

The tension arises in managing this goal while also positioning students as mathematical authorities and valuing student thinking which may be incorrect or incoherent (Ball, 1993).

Reflecting on my own practice, I feel that at times my emphasis on positioning students as mathematical authorities overwhelmed my smaller attempts at instructional coherence. Emphasising instructional coherence through teacher talk was is some ways a casualty of my attempts to reform my practice and given my concerns about lower achievers, particularly important in my classroom context. Also in terms of the MTLC framework, this would seem to imply that aiming to facilitate classroom discourse at level 3 at all times is not necessary or desirable. It seems likely that the more traditional forms of discourse described by the lower levels of the framework may be necessary to ensure instructional coherence. Further exploration of and emphasis on this function of teacher talk within the reform approach may address some of the issues of teacher efficacy beliefs (Smith, 1996) in problem based teaching methods.

To this point, I have discussed some of the dilemmas around the use of teacher talk and teacher telling 'in the moment' in a whole class discussion. Considering that such decisions are made in contingency moments makes them almost impossible to plan for (Rowland et al., 2005). However it is likely that the reflective practice of the Japanese teachers referred to above, who engaged in lesson study, may better prepare them to plan for reflective moments and teacher recaps (Shimizu, 2009). Another issue arises if the question of teacher telling and teacher talk is considered on a longer timescale. This issue is related to my earlier discussion of achieving a balance between cognitively demanding and routine tasks so that mathematical goals can be perceived coherently across different contexts for learning. When designing a scheme of work, how can one plan for instructional coherence across classroom discourse and mathematical tasks where students are positioned as mathematical authorities? This question could also be phrased in terms of the teacher actions or classroom discourse levels outlined in the MTLC framework. How should one devise a scheme of work compromising a combination of discourse types that will maximise instructional coherence and student learning? In many Irish textbooks, if a balance between teacher telling and student invention is attempted, the weight of tradition seems to favour an introduction to a new topic by direct exposition and an after the fact student exploration through problem solving activities or word problems. The fact that textbooks are central to most mathematics lessons at primary level in Ireland (Eivers et al., 2010) would suggest that this approach may also be adopted by teachers. There are few models or resources available to Irish primary teachers to support an alternative form of practice where new topics may be introduced by means of a problem although this approach has been proposed and used in other areas (Engle & Conant, 2002, Gravemeijer & Doorman, 1999, Nasir, Hand & Taylor, 2008; Stein et al., 2008). Because of the complexity of maintaining instructional coherence across mathematical topics and across time, it seems imperative that teachers have appropriate models to build on, particularly in a reform context.

**A note on the refinement of my design for learning**

In the above section, I have discussed the role of teacher talk in reform mathematics teaching. It is this feature of the discourse community approach that was refined most throughout the teaching experiment. The design for learning I presented in chapter 3 used the description of teacher actions at the higher levels of MTLC framework as a basis for teacher actions within the teaching experiment. However, in pursuing such actions in practice, I identified problems with maintaining instructional coherence in lessons and across sequences of lessons. For this reason, I began to alter my design for learning to include direct teacher explanations or teacher recaps at times. This is somewhat in conflict with my initial understanding of my role as mathematics teacher as a facilitator of classroom discourse at level 3 of the MTLC framework. However, it seemed that in order to maintain instructional coherence it was also necessary to facilitate discourse described by other levels of the MTLC framework where the teacher plays a more central role. This is a more nuanced approach than my initial understanding of the MTLC framework. In some sense, my initial design was based on hierarchical understanding of the framework where level 3 discourse might be understood as 'better' than lower levels. Instead, as discussed in the section above, the challenge is to balance many opportunities for level 3 discourse with segments of classroom discourse at other levels in order to facilitate instructional coherence.

**What is the role of instrumental understanding in learning mathematics?**

As I stated previously, teaching experiment activities were planned with the intention of developing students' relational understandings of mathematics (Skemp, 1979). I regularly asked students 'why' questions and attempted to create an expectation that mathematical reasoning would accompany mathematical statements or explanations of solution methods. For example, when focussing on finding fractions of quantities, I avoided the often used 'divide by the bottom, multiply by the top' rule. Instead I expected students to explain the connection between their division and multiplication computations with respect to the original fraction. For example, when finding $\frac{2}{5}$ of a quantity, a student might explain that he would divide by five then multiply by two because fifths mean that the whole is split into five pieces and we must find the value of two of those pieces. Such explanations were given by me and by many students and were supported in the representations used in this phase of the teaching experiment. However some students did not seem to make the expected conceptual links and shared very procedural descriptions of their thinking instead. After observing such thinking in Steven's approach to finding fractions of quantities, I wrote the following in my teaching journal on January 11th:

> I note again problems with basic mathematical operations by the lower achievers (Steven with division). When I asked why they were dividing by five, Steven replied in a really algorithmic way saying something like "the five from there goes there". I wonder how this process has developed for him. Is this what's happening in the group work? That some children explain it to each other in these terms? Or is this a self-devised strategy, perhaps he has observed what's gone on and devised these general rules for himself?

Here I note that not only did Steven struggle with the actual process of completing a short division sum, he also did not provide a mathematical explanation as to why he and his partner divided by five to find a fifth of a quantity. Instead he seems to have developed a procedure of his own for solving such questions noting the denominator of the fraction becomes the divisor in the sum that is written to solve the question. Such procedural thinking may have helped him successfully complete questions that he did not understand conceptually. This observation links to my concerns expressed about the *Fractions problems* lesson (page 165) that some students' weak conceptions of division may have hindered their understanding of fraction concepts. In a traditional answer focussed approach the gap in his conceptual knowledge may not have been so obvious. In this case, although Steven's procedural thinking may not have advanced his conceptual understanding it served a purpose in allowing him to successfully complete these particular problems. The fact that it is not transferable to other situations in the way that conceptual understanding might be matters little to the student if he/she understands successful participation as successful completion of exercises.

More generally, I began to wonder about the role of instrumental understanding in learning mathematics and whether student preferences for tasks that supported instrumental understanding were influenced by factors beyond previous experience and the perceived unchallenging nature of the tasks. In my journal entry of March 20th I wrote:

> Again reflecting on these transcriptions I feel that some of the lower achievers missed the point of the harder more complex lessons and would have been more comfortable with a more traditional straight forward teach and drill step-by-step algorithm … I wonder now if the lower achievers have the appropriate skills to access the more complex evolving, dynamic mathematics? … I feel in my classroom, there is some kind of two-tier system evolving where the higher achievers function at a more complex mathematical level and indeed access and develop more complicated mathematical structures. While the lower achievers struggle to make sense of these complexities and grasp for an algorithmic step by step approach or else turn off from maths entirely.

I infer that some of the lower achievers did not develop the conceptual understanding that was the intended objective of certain lessons. Instead, like the case of Steven above, some developed their own procedural ways of thinking about the mathematics that was presented to them. The question of whether "the lower achievers have the appropriate skills to access the more complex evolving, dynamic mathematics" is not intended to insinuate a deficit model where it might be considered that lower achievers might never access the more complex mathematics. Instead the question was intended to imply a link with the work of Gray and Tall (1991) who showed significant differences in the way lower and higher achievers understand certain concepts. They argue that some learners never manage to reach the reification stage whereby the product of a process becomes reified and can be considered by the learner as an abstract structure. A reified concept can be used flexibly and automatically (Boaler, 2009). For lower achievers, the automatic and flexible use of mathematical concepts may be inhibited and thus their ability to access and gain from a mathematical discussion is limited compared to those for whom the reification process is complete or nearing completion. This highlights again how the same lesson activities can be experienced very differently by higher and lower achievers. However the work of Gray and Tall would suggest that it is exposure to tasks that develop conceptual understanding that is important for these learners to avoid becoming stuck in the 'procedural rut.' In summary, in reform approaches where mathematics can be understood as progressive discourse (Berieter, 1994), it seems likely that those with reified understandings of mathematics may find it easier to access and gain from the discourse. Investigation is needed into how those with instrumental understandings can be supported in their efforts to access the same discourse and also how such discourse may transform their understandings.

## Summary

In this chapter, I have explored the nature of student participation and teaching in our fledgling discourse community. First, I presented an analysis of students' participation trajectories using Wenger's (1998) notion of identity as trajectory. Students' trajectories showed differences with some students acting as old timers and others taking more peripheral roles. Questions arising from this analysis include how students' multiple identities influence their participation and how the results may differ if the experiment was carried out for a longer period or at a different class level. I also presented the questions that arose for me as teacher when attempting to facilitate the discourse community. The issues discussed are connected to the sometimes conflicting teaching practices required when aiming to develop student agency while maintaining coherence in lessons and across time. A central tension was encouraging the participation of all members of the class while maintaining cognitive demand and instructional coherence. The issue of when a teacher should 'tell' is complicated in the case of a single lesson and also in the context of developing schemes of work that promote the development student agency. I would suggest that Irish primary teachers should have access to suitable models and examples of such activities. The question of the role of instrumental understanding in learning mathematics is also connected as instrumental understanding is usually associated with traditional settings in which teacher telling or exposition is the dominant approach. I have presented some questions and some answers. That many of these questions require further study and exploration is reflective of their significance to the enterprise of reform orientated teaching.

## Chapter 7: The Participation Trajectory of the Teacher-Researcher

The primary aim of this chapter is to explore my participation trajectory (Dreier, 1999, 2009) and by doing so, demonstrate reflexivity in the research process. The participation trajectory (Dreier, 1999) concept stretches across contexts and across time. For that reason, I will begin this chapter with some background information about my previous experiences with mathematics and teaching. I will follow this with notes on my own role as I perceived it during the teaching experiment and research process. I will then address the issue of the representation of self in research and speculate on what this thesis might represent as the end product of the research process.

### Charting my Participation Trajectory

#### Mathematical and Teaching Background

As a student I always enjoyed mathematics. I studied physics, chemistry and mathematics at honours level for my Leaving Certificate but also enjoyed languages and art. One of the reasons I decided to study for a Bachelor of Arts degree at NUI, Galway, was because I could combine the study of mathematics with other Arts subjects such as languages. At that time in first year in NUI, Galway, Arts students took four subjects and continued with only two of these in the final two years of the degree. In the initial weeks of first year before confirming our subject choices, we were obliged to meet with a designated member of staff with whom we could talk through our decisions. I was sure that I wanted to study mathematics and Irish but was unsure about the other two subjects. My designated supervisor was a lecturer in one of the language departments. Mathematics was the one subject that he attempted to talk me out of undertaking, claiming that it was very difficult. At the time, I seriously considered taking his advice. I am grateful that with encouragement from friends and family I ignored his opinion. His view would seem to have reflected latent personal prejudice rather than any institutional bias. Whether his prejudice was directed at mathematics as a subject or at women in mathematics is hard to tell but it certainly challenged me. It provoked my first critical review of my relationship with mathematics. It seems likely that I had understood mathematics prior to this as a 'sanctuary' but now, with my previous high achievement being called into question, I began to perceive of mathematics as a possible 'source of anxiety' (Black, Mendick, Rodd, Solomon & Brown, 2009).

In retrospect, much of my university experience with mathematics involved issues of identity and belonging although I might not have described the situation in this way at the time. It was not common for Arts students to study higher level mathematics after first year. There seemed to be a divide between us and the science students who studied mathematics with whom we often shared lectures. There was a perception, held by myself if no one else, that somehow they were more authentic students of mathematics than we were and that the scientific nature of the rest of their studies raised them above those of us who also studied 'Béaloideas', 'an Nua Fhilíocht' or 'Gaeilge na hAlban' (folklore, modern poetry or Scots Gaelic). Although I still enjoyed mathematics, the cultural aspects of my studies in Irish were fulfilling and felt more relevant to my experience in a way that some of the mathematics courses did not. This may relate to research findings which suggest that "deep, connected understanding" appears to appeal to female learners (Boaler, 1997; Boaler & Greeno, 2000) However, issues of identity and community were also significant in my Irish classes. NUI, Galway is renowned for the emphasis placed on the Irish language and many of the students studying Irish there were from Gaeltacht areas where Irish is spoken as a first language. No matter how much my language skills improved, I felt that the Gaeltacht students would always belong in a way that I did not. The opposite felt true for mathematics. I believed that insider status might be gained in mathematics circles through achievement and that the more I achieved the more I could demonstrate my right to belong in the mathematics department. This relates to Wenger's suggestion that "membership in a community of practice translates into identity as a form of competence" (1998, p.153). In the Irish group, I felt that I would always remain on a *peripheral trajectory* but by focussing on mathematics, I was choosing what I understood to be an *inbound trajectory*.

After I completed my Bachelor of Arts degree, my results allowed me to apply for and receive a fellowship to study mathematics at Masters level in NUI, Galway. It was a taught Masters programme. I doubt I would have had the confidence to undertake a research programme at that time. This is certainly the way I felt upon completion of the Masters degree. I never seriously considered undertaking a PhD in pure mathematics. During the Masters year I had worked at understanding all that was presented to me in lectures but even after achieving first class honours, lacked the confidence to believe I could genuinely create something novel at PhD level. Black et al. (2009) discuss the how vast the difference can be between the external reality of conventional mathematical achievement and subjects' perceptions of their own achievement. They discuss the case of Nikki, a mathematics graduate with first class honours who perceived herself to be bad at mathematics. She seemed to split mathematics into 'good mathematics', the unlearnable, creative aspects of the discipline; and 'bad mathematics' which can be 'easily' learned and regurgitated (p. 23). This is uncannily like own understandings of the time where I would have classed myself as good only at 'bad' mathematics. This issue returned to me during the teaching experiment and will be discussed further later in this chapter.

After completing my studies, I drifted into teaching more by accident than design. My first experience was as a mathematics and Irish teacher in a fee paying secondary school. Like the majority of the population, I had not attended a fee-paying school as a student. While I enjoyed the experience, my lack of teacher training hampered my effectiveness. The relentless examination focus also limited the teaching topics and teaching approaches that I could chose to employ. I then spent two years teaching in a disadvantaged primary school.

My experience over these two years focussed my mind on primary teaching. I enjoyed teaching the broad range of subjects of the primary curriculum and I found the camaraderie of the staff was a great support for me as a beginning teacher.

Having decided that my future was in primary teaching I travelled to the University of Exeter to complete a PGCE with a subject semi-specialism in upper primary mathematics. My previous unqualified teaching experience helped me to make the most of my learning experience. With this year completed I travelled to Colombia and worked for a year in a bilingual school where I taught subjects through English to children whose first language was Spanish. On return to Ireland, I taught in a Gaelscoil in Dublin where all teaching is carried out in the Irish language. Here we played a delicate game, teacher and students both speaking in their second language. I began working in my current school six years ago and have taught a variety of classes during this time. My experience of teaching mathematics in this context influenced the research questions I decided to explore. Having experienced some of the professional development that was associated with the introduction of the revised curriculum, I have always been aware of gaps between the aims of the reform and the reality in classrooms. It is this gap between reform theory and practice that informed the choice of subject of my research.

**The Teaching Experiment**

**Ethics**

Ethical issues, whether implicit or explicit, are often associated with the dual teacher-researcher role. In chapter 3, I discussed how my responsibilities as teacher limited to the extent to which I could incorporate democratic research practices. Also in chapter 6, I discussed how I considered whether the teaching approach was disadvantaging lower achievers. This was a real ethical dilemma for me as I was solely responsible for the implementation of the teaching programme. Another ethical issue arose in relation to data gathering protocols in the classroom. Novotná (2003) discusses repeating a teaching activity in a class because sufficient detailed documentation for research purposes was not produced by students the first time. A similar dilemma occurred for when I conducted what I considered was a good lesson where students used Dienes' equipment to model fractions and find decimal equivalences. Unfortunately, through some error of my own I failed to record what I considered to be an interesting whole class discussion on the activity. I debated presenting the same lesson again and ensuring to record the resulting discussion. I decided against this in the end because from a teaching point of view it was not worthwhile and I considered my teaching responsibility to come before researcher interest. This incident highlights the situated nature of ethics and the fact that ethical decision making cannot be fully mandated by prescribed 'abstract statements of intent' in the manner in which ethical guidelines are normally presented (Piper & Simons, 2011).

**Identity**

Undertaking the roles of both teacher and researcher, while researching my own practice, was complex. There were competing demands for my time, and competing directions for my thoughts. By this I mean that after a significant lesson event, such as when an obvious student misconception was aired or a student contribution took the lesson off track, I had both the teacher's duty to fulfil in planning the appropriate action to take and the researcher's duty to explore how and why the event occurred. Such competing demands were evident in moment-to-moment decision making such as the incident of Steven's question to me in the *Cutting Pizzas* lesson (Page 159). They were also evident when I tried to analyse my teaching approach and adapt my plans in response to the day's lesson while noting issues relevant to the research.

This teacher-researcher divide was compounded by the fact the colleagues and friends I had always held informal conversations with about teaching matters, now appeared to refrain from discussing mathematics teaching in my presence. It is becoming more common for teachers to pursue Masters degrees but is still relatively rare for practising teachers to undertake PhD research in Ireland. It can be argued that the act of undertaking a PhD was a political act that identified me as not content with the status quo in the school setting. In fact amongst teachers, 'PhD' could be considered a loaded word carrying with it connotations of genius, great exertion and perhaps a sense of distance from real life concerns. This is not to suggest that these descriptions are what I understand by 'PhD' but it seemed that by undertaking a PhD, I was viewed as different from my colleagues. Wenger suggests that "what our communities pay attention to reifies us as participants" (1998, p.150). Possibly because of the weight attached to the concept of PhD, my teaching colleagues paid attention to the academic aspects of my study. The teaching aspects of the research were unexamined and so it served only as a marker of difference. In effect, the informal collegial collaboration I enjoyed in other subject areas was now out of bounds for mathematics teaching.

A slight 'refocusing of the lens' (Lerman, 2001) may provide more insight into this situation as the teaching experiment was carried out in a time of great change for the education sector in general and our school in particular. Our school community, like many others, had to deal with an exodus of senior teachers in the time coming up to February 2012 before new budget cuts would affect pension entitlements (INTO, n.d). At the same time, teachers' salaries were being affected by the pension levy[12] and they were trying to understand what the Croke park agreement[13] meant for them in terms of increased working hours (Department of Public Expenditure and Reform, 2010). In this context, I did not feel that is was appropriate to ask colleagues to give up their personal time to become involved in my research work.

[12] The pension levy for all public service workers came into effect on March 1st 2009. More details can be found on http://www.education.ie/home/home.jsp?pcategory=31472&ecategory=47136&language=EN

[13] The Croke Park agreement is an agreement between the Irish Congress of Trade Unions and the Government to implement changes to public services with a view to reducing costs and numbers of public servants without further affecting pay. It was agreed in 2010 and is expected to run until 2014.

In effect I was operating at the periphery of the community of practice of my teaching colleagues. However, despite the best efforts of my supervisor and others involved in mathematics education in St. Patrick's college, I was not a full member of the local mathematics education community of practice either. The nature of my full time teaching position made this impossible. It is likely that all teacher-researchers 'walk the line' between communities of practice to some extent. Where research occurs as part of a planned programme of study such as a Masters degree, the possibility exists that teachers are in fact students and are part of a class community of practice which 'walks the line' together. This was not true in my case and I 'walked the line' alone. It was at times a lonely experience.

The following journal extract, from March 19th, addresses directly the issue of emotion. In particular I discuss the issue of hearing my own voice when listening to lesson recordings to make transcriptions. It was my habit to mark my turns of dialogue as 'T' to indicate teacher. I do not recall when or why I chose to do this rather than use my initials as I did for student turns. It may have been an unconscious act made in an attempt to take the self out of the script, thereby attempting to simplify my role as researcher.

> The muddle that is caused by the overlapping roles of teacher and researcher has grown harder still. I now have to deal with my emotional reaction to the voice of 'T', my disembodied voice, that I try to re-hear through ears that are not my own. It's O.K. when the lesson is going well but at times I wonder what I was thinking, basic errors, missed misconceptions of students, explanations that I myself find hard to follow. And I feel sorry for 'T' when I hear the tension grow in her voice as the clock ticks on and the best planned lesson dissolves in a fit of giggles at the word "wholes". I feel farther away than ever from the 'expert' teacher-researchers that are depicted in the literature.

This journal entry is interesting not least because it describes some of the stress and emotion attached to the teaching experiment. The reference to the 'expert' teachers of the literature like Lampert (1990), Ball (1993) or Dooley (2010) is also interesting because it is a statement of identity. It seems that it is with these 'expert' teachers that I was aiming to identify. As discussed above, by undertaking the PhD my identity as teacher was called into question. It seems natural that I would measure myself against the 'expert' teachers as these are the only models for combining PhD and teacher identities. It would seem that the negative emotions arose when I judged that my teaching was not up to what I perceived to be their high standards. In the teaching journal I was often critical of my own efforts and sometimes journal writing, as a dialogue with self, was like 'dialogue with a cruel partner' (Canetti, 1981 quoted in Holly & Altrichter, 2011, p. 44). The attempt to negotiate an identity that spans both the research community and my community of teaching colleagues corresponds with Wenger's notion of identity as 'nexus of multimembership' where "we define who we are by the ways we reconcile our various forms of membership into one identity" (1998, p. 149).

Undertaking the research involved not just a renegotiation of identity but also involved a reappraisal of my relationship with mathematics. In the following journal entry of 11th January, I reflect on how the experience of teaching and researching in the discourse community had cast new light on my previous experience of mathematics.

> I think of the role of mathematics in my life and how I turned from it after doing the masters. My standard answer to why I never pursed mathematics at PhD level at that time has always been that I found it dry and abstract and too removed from real life. Of course this is true but I never dwelt on the other factors, the male dominated department, the lack of role models for that sort of career within my own life. It is only with this recent examination of self that I can admit to another reason. I was afraid of PhD level work at that time in my life because I did not believe that I could

actually produce something new and novel and worthwhile. And I wonder now that if I had been taught mathematics in the way that I am aiming to teach it now, creatively, giving students ownership of the mathematics, would I have felt differently? Would I have had the confidence to pursue pure mathematics if I had believed then what I believe now about the nature of mathematics, if I had not lived the experience of 'anti-didactical inversion'?

My comments about 'anti-didactical inversion' reference Freudenthal's argument that mathematics should not be presented to students as a ready-formed system (1973). In the teaching experiment, I aimed to position students as mathematical authorities and the source of mathematical ideas in a way that I was never positioned in my own studies of mathematics. Despite my apparent success at university level mathematics, it is likely that I had positioned myself as 'received knower' (Boaler, 2002) rather than source of mathematical ideas or mathematical authority, an experience I hoped to avoid for my students

As may be inferred from the information presented above, the experience in the classroom was intense. It went beyond the intensity of a normal discourse focussed teaching experience where the teacher is challenged to act in contingency moments (Rowland et al, 2005). The dual teacher-researcher role meant that I responded to students queries, comments and disinterest sometimes, with one "eye on the mathematical horizon" (Ball, 1993, p. 373) and the other on the digital recorder, knowing I would relive each moment, dwell in the data, reflect on both success and failure. The fact that I knew I would have to present these successes and failures created a pressure all of its own, all the more powerful because the new identity of researcher is at stake. In the end I decided that "I must somehow get used to this horrible and very public vulnerability" (Teaching Journal, 6th April). This 'horrible' vulnerability stems from the representation of self in research and creates issues in the formal writing of the research which were addressed in chapter 3.

## Locating this Research

In many qualitative research studies there is an effort to locate the researcher in the research (Dooley, 2010; Du Prez, 2008). This has hopefully been accomplished above. Because of the possibility of different audiences for this research, it remains to 'locate' the research itself. This research was carried out in a school based community of practice but is being written for a university based audience, the community of practice of mathematics education researchers. Even at school level, the communities of practice within which this research was undertaken include the classroom community of practice comprised of myself and my students and the community of my teaching colleagues. In undertaking this research I was participating in multiple communities of practice at once. Wenger (1998) suggests that one of the possibilities of participating in multiple communities of practice is the complex work of brokering. He states "brokers are able to make new connections across communities of practice, enable coordination, and – if they are good brokers – open new possibilities for meaning" (1998, p. 109). As a teacher, I utilised concepts from the shared repertoire of mathematics education researchers and attempted to use these to inform my teaching practice which in turn influenced the practices of my classroom community. In this sense brokering work was being carried out in at least one direction. The fact that it did not appear to impact the community of my teaching colleagues suggests that the opportunities for 'new possibilities for meaning' were limited to myself and my students. The possibilities for my research to enable new possibilities for meaning within the community of mathematics education researchers lies for the most part with how well this thesis is received. As such a possibility exists for brokering work that has not quite been realised yet.

## Chapter 8: Summary and Conclusions

In this chapter I will present a summary of the research that was carried out at both stage 1 and stage 2. I will then relate the findings to the original research questions, discuss the value of the research and discuss links with a critical orientation. Finally, I will discuss possibilities for further study and possible applications of this research to teacher education and professional development.

### Summary of Stage 1

Audio recordings of six mathematics lessons carried out by four teachers in two different schools were analysed using the MTLC framework (Hufferd-Ackles et al., 2004) (See table 3.2, page 77). The nature of the discourse and the teacher and students' actions in these lessons indicate either level 0 or level 1 descriptions from MTLC framework. Level 0 is described as: "Traditional teacher-directed classroom with brief answer responses from students" (p. 88). Level 1 is described as "Teacher beginning to pursue student mathematical thinking. Teacher plays central role in the math-talk community" (p. 89).

Questioning, the first of the four components of the MTLC framework, was analysed using Boaler and Brodie's (2004) teacher-question categories. Student questions were also analysed using the codes 'questions seeking clarification about mathematics being discussed' and 'questions seeking organisational clarification'. The analysis of teacher questions showed that type 1 questions, which are posed to gather information or lead students through a method, were the most common in all lessons. With the exception of Joan's lessons, relatively few questions were posed that probed students thinking (type 4). Although a number of questions were categorised as type 5, generating discussion questions, some of these functioned more as rhetorical questions or as way of moving the dialogue along (page 111). In general, questions of types 6, 7, 8, and 9 were relatively rare across both stage 1 and stage 2 data. This is discussed more fully in the section on stage 2 data below. Students asked very few mathematically orientated questions (5 in total in the lessons presented in chapter 4, and 3 in Liam's lesson).

Regarding the other three components of the MTLC framework, EMT, SMI and RFL, little if any mathematical authority was devolved to students to enable them to contribute high level explanations of mathematical thinking or become a source of mathematical ideas. In general, the mathematical discourse in all of the lessons was limited and student thinking was rarely an object of discussion in whole class discourse. Where student thinking was pursued, as in Joan's lesson for example, teacher questions generally focussed on computational strategies. It was common for the teachers to directly explain features of the mathematics and all teachers appeared to position themselves as mathematical authorities and played a strong role in evaluating student thinking. The high levels of teacher EMT and evaluation left little scope for student agency and students appeared to be positioned and to position themselves as 'received knowers' (Boaler, 2003, p. 5). Features that varied across the two schools include the extent to which textbooks and concrete materials were utilised as well as the extent to which textbooks or whole-school approaches to mathematics had influenced the planning of individual teachers. With the exception of some word problems used in Anne's and Joan's lessons, most activities were not set in context.

**Answering the Research Question**

Stage 1 was an attempt to explore the question: What is the nature of the discourse students in some Irish primary classrooms engage in during mathematics lessons on number strand topics? I must acknowledge that this was a small scale study and that results are not generalizable to the wider population of Irish primary teachers. However, this part of the research was carried out in order to get a sense of 'the lie of the land'; of the nature of discourse in some Irish primary classrooms more than ten years since the introduction of the revised curriculum (DES/NCCA, 1999a) and the discourse focussed methodologies associated with it (DES/NCCA, 1999b). The discourse of the lessons analysed at stage 1 was similar to a form of 'number talk' (Richards, 1991) and focussed mainly on numerical answers or computation strategies. Possibilities for discourse may have been limited due to the lesson activities which contained few genuine problem solving tasks or opportunities for students to make mathematical conjectures. There were few tasks with multiple possible solutions and where these existed teachers often pursued only one method (e.g. Liam's lesson, page 121). For the most part, the teachers at stage 1 retained strict control over the direction of the lessons.

However, as I indicated in chapter 4, the majority of the recordings might from a traditional perspective be viewed as 'good' lessons, featuring clear explanations from teachers and structured activities for students. In this way, the analysis using the MTLC framework has highlighted the gap between traditional and reform understandings of 'good' mathematics teaching. It also raises questions about what kinds of professional development may be necessary to support teachers in bridging this gap. It seems likely that the long term, school-based professional development which Kennedy (2008, 2010) has already used to good effect in the Irish context for developing the teaching of literacy may be most effective in supporting teachers to develop new ways of teaching.

**Directions for Further Study**

Two of the teachers in this study seemed to use the textbook for the planning of their lessons. All student tasks in John and Liam's lessons were exercises from the class textbook. This finding is not surprising in the context of the research of Eivers et al. (2010) who found that textbooks were widely used by Irish primary teachers both as a planning guide and a source of mathematical tasks and activities. This suggests that the content of textbooks should be analysed to explore how well it supports teachers in the planning and delivery of the mathematics curriculum using the teaching approaches recommended in *Mathematics Teacher Guidelines* (DES/NCCA, 1999b).

The other two teachers studied seemed to have adopted some methodologies associated with the revised curriculum (DES/NCCA, 1999b). It would be interesting to explore to what extent professional development courses or whole school planning policies had influenced their teaching approaches. It would also be interesting to find out what further professional development opportunities in the teaching of mathematics all four teachers would like to be offered.

**Summary of Stage 2**

Stage 2 involved a teaching experiment carried out in my own classroom with the aim of facilitating a discourse community. Audio-recordings were made of thirty one lessons on fractions, decimals and percentages topics and a digital record was kept of board work in the majority of cases (Appendix 8). Fourteen of these lessons were transcribed (Appendix 15). A teaching journal was also kept throughout the experiment. In effect, four levels of analysis occurred on this data.

1. MTLC analysis: Five lessons were analysed using the same method as for stage 1 data i.e. the analysis used the MTLC framework (Hufferd-Ackles et al., 2004) incorporating Boaler and Brodie's (2004) teacher question categories. The lessons were chosen for perceived interesting aspects of teacher and student actions that might provide insight into what it means to learn mathematics in a discourse community and what it means to attempt to facilitate such a community. The MTLC analysis represents analysis at group or community level.
2. Student Participation Trajectory Analysis: The participation trajectories (Dreier, 1999; 2009) of ten students were charted across fourteen lessons with reference to the key practices of the discourse community and Wenger's (1998) descriptions of

trajectories of identity. The aim was to explore the nature of student participation in a discourse community. The student participation trajectory analysis represents analysis at the level of the individual student.

3. Identification of issues and tensions involved in facilitating a discourse community: The transcripts of lessons and the teaching journal I kept throughout the experiment were examined to identify the issues involved in facilitating a discourse community. From these issues I developed a list of tensions that must be negotiated when attempting to manage student engagement in a discourse community (figure 6.1, page 242).
4. Teacher-Researcher Participation Trajectory Analysis: My own participation trajectory (Dreier, 1999, 2009) was examined to address issues of reflexivity.

The results of each of these four levels of analysis will be summarised and discussed below.

**MTLC Analysis**

The results of the MTLC analysis of teaching experiment lessons suggests that due to the focus on student thinking, all lessons were at level 1or higher of the MTLC framework (Hufferd-Ackles et al., 2004). Some of the lessons displayed elements of a level 2 community: "Teacher modelling and helping students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases" (Hufferd-Ackles et al., 2004, p. 89). In the *Percentage Present and Absent* lesson (page 207) in particular, there were elements of level 3 present: "Teacher as co-teacher and co-learner. Teacher monitors all that occurs, still fully engaged. Teacher is ready to assist, but now in more peripheral role (coach and assister)" (Hufferd-Ackles et al., 2004, p. 90).

**Questioning**

Regarding questioning, the first component of the MTLC framework, I would first like to address the issue of student questions. The number of mathematically orientated questions varied from lesson to lesson with the highest total being 14 in the *Cutting Pizzas* lesson (appendix 22) and the lowest total being 4 in the *Dienes' decimals* lesson (appendix 25). The nature of student questions also varied from questions about basic misconceptions or procedural issues to questions regarding 'big' mathematical ideas such as Jonathan's question in the *Fractions Word Problems* lesson (page 165). Some students appeared more willing to pose questions than others. The student questions also included some instances of direct student-to-student questioning and sometimes indicated that students were monitoring their understanding. The student questions, limited in number though they were, helped develop my understanding of student thinking. They also created opportunities for me to organise student to student discussion.

There are some interesting issues arising from the teacher question category analysis. In general, there was a relatively low proportion of type 1 questions, leading students through a method. There was also a very low proportion of type 2, inserting terminology questions. I am disappointed to uncover this aspect of my teaching practice. It seems that I did not put enough emphasis on mathematical terminology. This area is particularly important given the emphasis on developing mathematical discourse in a disadvantaged school setting. However mathematical discourse, constituted as it is by mathematical thinking, involves much more than precise terminology and many students managed to engage in genuine mathematical discourse despite limitations of vocabulary and terminology (for example Andrei in the *Equivalence Challenge* lesson, page 190). My approach was to value thinking regardless of how this was expressed and I felt that if I insisted on precise mathematical vocabulary, it may have in some way inhibited students' willingness to share their thinking. However I should have attempted to balance this consideration with efforts to develop more precise terminology that meets the standards of the discipline. Many type 3 questions, exploring mathematical meanings and relationships, were explored in great depth with multiple student contributors to the same conceptual question. In this way, the relatively low number of type 3 questions (compared to stage 1 data) does not reflect poor attention to mathematical meanings. Instead it is related to the methodology of not counting repeated questions and the sustained discourse on individual mathematical meanings in lessons. The relatively high number of type 4 questions probing student thinking and type 5 questions aimed at generating discussion, reflect the aims of the teaching experiment in centralising student thinking for peer evaluation.

Boaler and Brodie's type 6, 7, 8 and 9 questions were barely in evidence either in the teaching experiment lessons or in the stage 1 lessons presented in Chapter 4. A type 6 question is described as one which "points to relationships among mathematical ideas and mathematics and other areas of study/life" (2004, p. 776). I originally considered that questions around fraction, decimal and percentage equivalence might be described as focussing on "the relationship among mathematical ideas" but I think these are best regarded as type 3, exploring mathematical meaning questions which make "links between mathematical ideas and representations" (2004, p. 776). The conceptual ties that bind fractions, decimal and percentage topics together are too strong to consider the areas solely as 'linked' rather than mutually constitutive. In the *Fraction Problems* lesson, all of the word problems were set in measurement contexts but I asked no question that probed this linkage directly. The context of other lesson activities also provided scope for links between mathematics and real-life but no question was recorded that probes this relationship. This may imply that simply by providing the context I assumed that students might infer the links. Direct teacher questioning may have been more successful at making these links explicit.

The aim of type 7 questions is to extend "the situation under discussion to other situations where similar ideas may be used" (Boaler & Brodie, 2004, p. 776). Boaler and Brodie give the example "Would this work with other numbers?" Ideally teachers would aim to extend thinking in every lesson. It is hard to provide a justification for the fact that not one single question of this type arose in the teaching experiment. It is quite possible that I simply missed the opportunities for posing such questions, by not keeping 'an eye on the mathematical horizon' (Ball, 1993). However, often it seemed that the nature of the activities in the teaching experiment lessons was already of a high cognitive challenge. As I discussed in detail in chapter 6, one of my concerns was about the nature of cognitive challenge and how challenging tasks might possibly foster disengagement. Also discussed in chapter 6 was my concern about the nature of the experience for lower achieving students. It is possible that these concerns limited to some extent my willingness to pose questions to further extend student thinking.

The aim of type 8 questions is to orient and focus the student on crucial aspects of the problem. The examples given by Boaler and Brodie are "What is the problem asking you?" and "What is important about this? (2004, p. 776). These seem like introductory questions or questions a teacher may pose when a student is having difficulty in developing a solution strategy. It would appear that focussing questions are part of my repertoire but perhaps I use them more often in one-to-one settings rather than in the whole class context.

Type 9 questions occur when a teacher talks "about issues outside of math in order to enable links to be made with mathematics" (Boaler & Brodie, 2004, p. 776). Similar to the comments made about type 6 questions above, there were real-life contexts to many of the activities of the teaching experiment lessons but no questions were recorded that linked the two. This may suggest that I assumed students would make links without explicitly questioning them in this area.

**The Remaining Components: EMT, SMI, RFL**

During the teaching experiment, I was successful to some extent in devolving mathematical power to students. This involved valuing their mathematical thinking;

positioning them as a source of mathematical ideas; and encouraging their responsibility for their own learning and for the learning of their peers by positioning them as mathematical authorities capable of evaluating mathematical thinking. Attempting to achieve this often involved my refraining from providing them with mathematical explanations or evaluations and passing that responsibility back to them thereby attempting to foster student agency. It seems that the curtailment of teacher explanation and the postponement or abandonment of an evaluative teacher move created a space for different mathematical possibilities to be considered and for students to begin building their own mathematical ideas or building on the contributions of their peers. The key discourse community practice of positioning students as mathematical authorities seems to be a necessary condition for ensuring that classroom mathematical discourse is progressive (Bereiter, 1994).

**Comparisons with Stage 1**

There are some obvious contrasts between the stage 1 recordings and the recordings collected during the teaching experiment. It would seem that in most of the stage 1 recordings, with the possible exception of Joan's lessons, the focus was on instrumental understanding rather than relational understanding of mathematics (Skemp, 1976). In the teaching experiment, activities were generally cognitively demanding for all students and student thinking became an object of discussion for the group. Individual mathematical questions were discussed in a lot of depth often involving an explicit or implicit 'why' question arising out of the students' role in evaluating mathematical ideas or 'truths'. This implies a focus on relational understanding and contrasts with an answer focussed approach in most of the stage 1 lessons where student discourse was constrained to some extent by the teacher's agenda and the teacher's position as mathematical authority and SMI.

The open nature of the discourse of the teaching experiment lessons facilitated student contributions to the whole class discourse without the need for students to wait for teacher invitation. In this manner, some misconceptions became apparent that may otherwise have gone unnoticed by me as teacher. Similarly, gaps in conceptual understanding were exposed that may have remained hidden in a more traditional answer focussed context. Student errors were treated as opportunities for learning and other students often offered guidance or comments on the errors of their peers. This contrasts with the controlled manner of lesson delivery in stage 1 lessons where student errors were often 'managed' by the teacher by funnelling (Wood, 1994) or direct exposition of relevant mathematics.

Differences can also be seen between the teacher question analyses carried out at stage 1 and stage 2. Most of the stage 1 lessons contained a high proportion of type 1 questions and with the exception of Joan's lesson, a relatively low proportion of questions probing student thinking (type 4). In general, there was also a higher proportion of questions aimed at generating discussion (type 5) in stage 2 lessons. The difference between the proportion of type 3 questions aimed at exploring mathematical relationships is explained by the methodology of not counting repeated questions more than once and the fact during the teaching experiment, conceptual questions were discussed in depth. At stage 1, there were more answer focussed responses and less time spent on individual questions.

The differences between stage 1 and stage 2 recordings might also be understood in terms of the teacher's beliefs about what constitutes 'good' mathematics teaching. As I stated above, from a traditional perspective, the majority of the stage 1 lessons might be considered 'good' lessons featuring clear teacher explanations. The teaching experiment lessons would fare poorly if judged on the same criterion. In this sense, the analytic framework used in this research undoubtedly favours my own lessons as these were purposely designed with the higher levels of the MTLC framework in mind while it is likely that other concerns influenced the teachers' lesson design at stage 1. However, it is not intended that the teaching experiment lessons should be viewed as 'better' than the stage 1 lessons. In fact, the overall findings of the research suggest that the levels of the MTLC framework should not be understood as a hierarchy where level 3 discourse might be considered as 'better' than level 2. Instead it is likely that discourse of all levels is necessary at different times to ensure effective teaching and learning (page 255).

**Student Participation Trajectories**

The participation trajectories (Dreier, 1999; 2009) of ten students were tracked across thirteen lesson transcripts (appendix 15). The chosen students included representatives of higher, middle and lower achievers as grouped by their performance on a standardised mathematics test carried out in June 2010. The participation of all students identified as lower achievers was examined. The transcripts were examined for evidence of the key practices of the discourse community (page 90) such as proposing, questioning and justifying mathematical ideas. The participation trajectory analysis supported my initial observations which suggested that the nature of student participation and the nature of the experience of the teaching experiment varied from pupil to pupil. The participation of certain vocal high-achieving students such as Darragh appeared to be consistent with an insider trajectory from the very beginning of the experiment (Wenger, 1998). Darragh and other students on insider trajectories could be conceived as acting as old-timers, modelling ways of acting in the discourse community (Lave & Wenger, 1991). For other students such as Jared, who was a lower achiever and participated to a much lesser extent, the possibility exists that due to patterns of past participation, he was negotiating a trajectory of participation from a marginal position. Steven's pattern of participation contrasted with both Darragh's and Jared's. Steven was very vocal in all lessons but when his participation trajectory was examined, there was little evidence of the key practices of the discourse community with the exception perhaps of his attention to monitoring his own understanding. This would indicate a peripheral trajectory. Comparison of scores achieved in a standardised mathematics test conducted in the June prior to the teaching experiment with one carried out in the June after the teaching experiment, shows maintenance or improvement of achievement levels relative to the descriptions of lower, middle and higher achievement bands described in chapter 6. Some improvements were dramatic. In the case of one lower achiever and one middle achieving student, their improvements consisted of a jump of more than thirty percentile points. Although these improvements cannot be said to be caused by the teaching experiment and may have been influenced by other factors, they are useful for showing that the teaching experiment did not have adverse consequences for students in relation to their performance on standardised tests. This point may be important for teachers who might consider that a more traditional approach would result in greater achievement in examinations. It is also pertinent given the Government's current focus on promoting the assessment and reporting of achievement levels (DES, 2010).

**Identification of some of the issues and tensions involved in facilitating a discourse community**

The identification of issues and tensions involved in facilitating a discourse community was carried out through an interrogation of my own experience as teacher. This interrogation was carried out by reflecting on the notes, descriptions and reflections I had recorded in my teaching journal in conjunction with a close reading of the descriptive synopses of the teaching experiment lessons.

An initial issue I faced was how to arrange a balance between cognitively demanding and routine tasks. This was complicated by ancillary notions of order and disorder and the practical issues of the timing of the mathematics lesson directly after first break. A second issue, was that prioritising cognitively demanding tasks might result in the disengagement of some students. This concern appears particularly relevant to the *Equivalence Challenge* lesson (page 190) where Jonathan seemed to opt out of engaging in the task possibly due to its complexity. The balancing act of maintaining cognitive demand while ensuring the discourse was accessible to all students was a central tension in my attempting to facilitate the discourse community. It is also related to an issue I faced during the teaching experiment when I considered whether the discourse community approach was disadvantaging lower achievers. My initial concerns were based on my observations that lower achievers appeared to participate in whole class discourse to a lesser extent than their higher achieving peers. I considered how their ability to access the discourse may be a factor in low participation levels in terms of the cognitive challenge of tasks. I also considered 'the bigger picture' and how their previous experiences both in the school context and beyond may have influenced their patterns of participation. I was concerned that lower achieving students were left unsure of how to participate in mathematics lessons due to the renegotiation of classroom mathematical and socio-mathematical norms. I also worried that the teaching experiment lessons, which consisted primarily of student ideas and student EMT, lacked coherence and the mathematical ideas discussed within these lessons might not have been readily understood especially by lower achievers.

The teaching experiment lessons could be characterised as multidimensional (Boaler, 2006) with a strong focus on student responsibility for learning. Boaler (2006) has identified these areas as important factors in high achievement and equitable outcomes in Railside, a low SES urban school in North America. For this reason and based on the analysis of their participation trajectories, I would argue that despite my initial concerns, the teaching experiment was a worthwhile experience for most lower achieving students. I would also argue that where results are less clear, as in Jared's case for example, the effect of prior experiences must be taken into account and to truly examine the possible impact of the discourse community approach on lower achievers, it would be necessary to examine the enactment of the teaching intervention at an earlier stage in the pupil's school career.

The issues of facilitating the active engagement of all students in the discourse community were not easily resolved in practice and in examining my experience during the course of the experiment, it seems apt to say that many dilemmas were managed rather than resolved (Lampert, 1985). Acknowledging the subjective judgments involved in identifying these dilemmas and Lampert's (1985) suggestion that it is the teacher's identity that is key in attempting to manage them, I presented a summary of the central tensions I experienced in figure 6.1 (page 242).

In analysing the stage 2 data, another issue became apparent. My tendency to refrain from direct telling meant that many issues remained unaddressed that possibly should have been addressed more directly. For example, it may have been more effective to explicitly address aspects of the experiment such as my expectations of students and specific learning practices (Boaler, 2006) that I hoped to institute. My struggles over when to be explicit and how much I should directly tell students, link with the idea of lesson or instructional coherence (Anhony & Ding, 2011; Shimizu, 2009) and the debates over teacher talk and teacher telling (Dooley, 2011a).

**Teacher-Researcher Participation Trajectory analysis**

This analysis was carried out to demonstrate reflexivity and with a view to addressing issues of subjectivity in the research. Engaging in the teaching experiment as both teacher and researcher caused me to re-examine my own relationship with the discipline of mathematics (Grootenboer & Zevenbergen, 2008) and reflect on past experiences and understand them in a new light. Issues of my own identity, not without emotion, came to the fore during the research and attempting to represent these honestly without violating academic norms requires equal parts skill and courage (Behar, 1996).

**Answering the research questions**

The teaching experiment was carried out in order to explore the following research questions.

- What is the nature of student learning in a discourse community?
- What is the nature of the experience for a teacher attempting to facilitate a discourse community?

The second question was addressed in the section on the *Identification of some of the issues and tensions involved in facilitating a discourse community* above and does need to be addressed further here. Addressing the initial question, I must first acknowledge that the focus on student learning was not on the learning of specific mathematical content but instead encompassed the notion of what students may have learned from the teaching approach in general. In particular, students' participation in terms of their use of key discourse community practices was examined. The MTLC analysis suggests that the nature of student participation in the discourse community involved students being positioned as mathematical authorities and as a possible source of mathematical ideas. In practice this meant that they explained their mathematical thinking, shared their ideas and evaluated the thinking of their peers by correcting, extending or agreeing with it. Their mathematical thinking became central to lessons and an object of discussion for the community. The analysis of teaching experiment lessons using the MTLC framework illustrated the complex web of thinking and communication in the classroom community over time. For example, through talk, mathematical connections were made across lessons (Alan and Andrei, *Fraction problems,* page 165; Andrei and Aidan; *Equivalence Challenge*, page 190). The complexity of communication and its effect on student thinking is illustrated by examples of interaction that occurred in the *Fraction Problems* lesson (page 165): Darragh shared his thinking about a question I had initially posed to another student and the discussion around a question raised by Jonathan appeared to provoke understanding for Steven. These examples of student thinking would seem to highlight that Darragh and Steven's thinking has been stimulated by discourse in which they were not originally participating. The evidence of their thinking only exists in this analysis because of contributions they made to whole class discourse. In this way, it is possible but not provable, that students, who practiced thinking as 'discourse with self' (Polya, 1945/1990; Sfard, 2001) rather than in communication with the classroom community, did engage in mathematical thinking that was not detailed by this analysis.

The student participation trajectory analysis supports my initial observations that the nature of the experience was different for individual students. Higher achievers appeared to engage in the practices of the discourse community from the very beginning, although it was not only higher achieving students that were vocal from the start. Students at other achievement levels gradually came to participate in ways that were consistent with the practices of the discourse community to some extent. Patterns of participation cannot be explained solely by links with achievement levels however and it seems likely that many other factors effect students' ways of participating. The influence of past school experiences as well as experiences in other contexts cannot be discounted.

In the analysis of issues involved in facilitating a discourse community, I considered the issues that arise for the teacher in encouraging the active engagement of all students but particularly lower achievers. In considering the results of this analysis in conjunction with the MTLC framework analysis and the analysis of student participation trajectories, I began to consider that the nature of the experience was very different for higher achievers and lower achievers. Their past experience at school conditions students to hold certain expectations of mathematics lessons (Fernandez et al., 1992). Higher achievers generally experience success and even if they must struggle through periods of incomprehension, hold an expectation that they will eventually understand. Lower achievers, due to the nature of their experiences, may not have such expectations for mathematics lessons. The previous positive experiences and high expectations of higher achievers may better equip them to face the challenge of a change in mathematical focus from instrumental to relational understanding (Skemp, 1976). Although the discourse community approach is multidimensional (Boaler, 2006) and as such creates multiple opportunities for students to be successful, lower achievers who do not have high expectations of personal success may not engage to a sufficient extent to make success possible. They may be negotiating a trajectory from a marginal position (Wenger, 1998) and may require a longer lead in time before they are willing to act in ways that are consistent with the discourse community approach. I also considered how the nature of students' mathematical understanding may facilitate or inhibit their comprehension of whole class discourse. This involves considering not just which particular facets of mathematics are understood by students, but also takes into account *how* that mathematics is understood. Those students with reified understanding of mathematical concepts must find it easier to access and engage in whole class discourse than students whose mathematical understandings are based on operational understandings (Boaler, 2009; Gray & Tall, 1994; Sfard, 1991).

Another issue that affected student participation was their perception of the social risk involved in participating in discourse community practices. This was a feature of the *Fraction Problems* lesson (page 165). Some students appeared to have higher regard for how they were perceived by their peers than how they were perceived by me as teacher, an observation which has been perceived to be feature of some male students' participation in educational settings (MacAnGhaill, 1994; Ashley, 2003). Grootenboer and Zevenbergen (2008) discuss how the teacher's relationship with students is only transitory. This is often in contrast with students' peer relationships. In Irish primary schools, students often spend one school year with a given teacher but complete the eight years of their primary school experience with the same peer group. The extent to which any teacher can affect the renegotiation of peer-peer relationships, particularly within a community which contains a group of 'macho lads', after significant amounts of time (six years in the case of the teaching experiment) is debateable (MacAnGhaill, 1994). It is for this reason that I suggest that for the full possibilities of the discourse community to be explored it should be trialled with younger students.

In summary, the nature of student participation in our discourse community was not uniform. At a group level, it was characterised by taking responsibility for evaluating what is mathematically correct and was consistent with the student actions described in higher levels of the MTLC framework (Hufferd-Ackles et al., 2004) such as explaining and justifying mathematical thinking and questioning of teacher and peers. On an individual level, despite some similarities across bands of achievement, student participation in the teaching experiment was a diverse as the students were.

**Underlying Aspects of a Critical Approach**

In developing my research questions and research approach, my initial focus was on developing student thinking. It was only as the research progressed that I became aware of an underlying critical orientation. As a professional middle-class female, originally from 'the country' and teaching in an urban disadvantaged boys' school, I was not unaware of issues of inequity. However I was initially unaware of the connections between my focus on student thinking and the possibilities for democratic mathematics teaching or teaching mathematics for social justice (Ellis & Malloy, 2007; Gutstein, 2007). The discourse community attempted to institute democratic principles (Hannaford, 1998) where each student's thinking was respected and a form of relational equity (Boaler, 2006) was promoted.

The shape my research took grew largely from the constraints of my teaching position. By this, I mean that conducting teacher research was not an ideological decision on my part. It was only much later in the research process that I recognised it as a "creatively subversive activity in the field of education" (Erickson, 1986, p. 158). In this way issues of power and the renegotiation of how power is distributed have permeated all aspects of my research.

***Conflicts with democratic research practice***

Given the elements of the critical approach outlined above, I must address further the issue of democratic research practice with children (Waldron, 2006). Waldron proposes a continuum of children's participation in research characterised by the child's access to decision making. She suggests that in research a child may be positioned as a research subject, contributor, participant or co-researcher depending on his access to decision making during the research process. The nature of a child's decision making in the research process depends to some extent on his access to the evolving information that is gathered and produced throughout the process. In this way the notion of 'informed consent' becomes much larger than introductory explanations of the research and grows with the research project, so that the subject may make an informed decision on whether to withdraw from the project at any stage and may even participate in decisions about outcomes of the research. I discussed in chapter 3 how my feedback to students during the course of the teaching experiment may have been too informal and they may have interpreted it as teacher comment rather than 'researcher feedback'. Also, due to the constraints of my teaching position and the complexities of the dual role, there was little opportunity to invite students as collaborators in the research process. A further difficulty arose on completion of the research in considering how if at all, students should be informed of findings. Practically I could ask for permission from my principal and their new teacher to speak with them during school time. However the extent of the information that I could ethically share with them may limit this exercise to a superficial sharing rather than an example of democratic practice (Waldron, 2006). For example, I imagine that I could find ways to speak to students about the nature of how mathematical thinking was communicated in our classroom perhaps with reference to transcripts or audio-recordings. However it would be inappropriate to comment on the nature of participation of higher and lower achievers and other findings of the research making such an information sharing session strictly superficial.

The issues involved in demonstrating democratic research practice are non-trivial and the extent of children's participation is influenced by the research questions. Wagner (1997) suggests that some research questions are more suitable to different types of cooperation between researcher and researched which in turn suggests that democratic research practices with students as co-researchers may not be suitable for some projects. To some extent my dual teacher-researcher role militated against such practice in this research project. This is regrettable and seems incompatible with the underling critical orientation outlined above.

**Validity**

Traditional conceptions of validity must be reformulated in interpretive research. Erickson suggests that the primary validity criterion for qualitative research should be '*the immediate and local meanings of actions,* as defined from the actors' point of view" (1986, p. 119, original italics). Eisenhart and Howe suggest that "this criterion applies to the audience as well as the subjects of research" (1992, p. 649). For Erickson, general quality criteria such as clarity and appropriateness are incomplete, if the research is not also useful to its audience (Erickson, 1986; Eisenhart & Howe, 1992). Despite the complications of a possible dual audience of academics and teachers for this research, I would like to think that I have met this version of validity. How subjectivity was addressed in this research is discussed in chapter 3.

**Value of this research**

Limitations to this study include that audio-recordings were the primary mode of data collection rather than video; the small sample size at stage 1 and issues of the lack of generalizability of the results at both stages. However the value of this research does not arise from its generalizability, as any claims I make are limited by my design research methodology (Kelly, 2003). Instead the value of this research lies with its recording, detailing and examination of the practices of classrooms. What has been presented in this research is not exemplary practice but 'examples of practice' (Ball & Lampert, 1999). The teaching practices of the stage 1 lessons contrasted with the teaching practices of stage 2, perhaps because like Ball and Lampert, I wanted to use my classroom as an example of "a serious effort to teach elementary school mathematics for understanding and as a site for developing new ways to investigate teaching and learning" (1999, p. 374). I do not claim that the resulting teaching practice is exemplary just that it may be an interesting, perhaps even a useful example of practice for other teachers and researchers (Erickson, 1986). The research presented here consists of analysis at finer grain size than that which is available in large scale national assessments of student achievement (Eivers et al, 2010). Instead of the common focus on the outcomes of teaching, I focussed instead on the process or practice of teaching (Hiebert et al., 2005).

In addition, my use of the MTLC framework (Hufferd-Ackles et al., 2004) to both guide my actions during the teaching experiment and to structure the analysis of classroom discourse strengthens and extends the use of this construct. The components of the MTLC framework; questioning, EMT, SMI and RFL, have been shown to characterise classroom discourse of various levels. Teacher and student actions related to the levels of discourse in a lesson have been shown to be consistent with the descriptions of the MTLC framework.

## Directions for further study

I see three main areas for study stemming from this research: further exploration of the possibilities of discourse community approach; opportunities for teacher education and professional development using records of practices and the MTLC framework; the investigation of Irish textbooks and the possibilities of developing support materials for teachers that might aid the implementation of the discourse community approach.

**Further possibilities for exploration of the discourse community approach**

As I stated above, to explore the true potential of the discourse community approach, it would be interesting to see the intervention carried out in different school settings, at a younger student age group and over a period longer than a school year. Carrying out this research in different settings would create an opportunity to explore the effects of issues such as gender and SES status on intervention outcomes. Similarly, research in a Gaelscoil context might explore related language issues.

Any such exploration would undoubtedly be improved by a collaborative rather than independent approach. By this I mean collaboration between both teacher and researcher; and researcher and students (Wagner, 1997). Some limitations to my study arose because of my dual-role as teacher-researcher which I felt limited my opportunities for follow-up with students and militated against a truly collaborative approach where students might feedback on the nature of the experience. Some of these limitations might be overcome by a research project in which a teacher and researcher collaborated. In such a context, it might also be possible to extend data collection to instances of pair and group work. Other data that might be gathered includes details of prior achievement and details of achievement during the study. Such a study might combine my focus on what is learned from the teaching approach over time with a more traditional study of the learning of specific mathematical content. Also further data might be gathered on student experiences by way of interviews. This may allow for a fuller exploration of how students' mathematical identities are negotiated within the discourse community. This research context would also allow for a temporal analysis of how learning occurs through dialogue in classroom communities (Mercer, 2008). The opportunity also exists for democratic research practice (Waldron, 2006) on a teaching intervention based on the discourse community approach. Such research could be carried out using an explicitly critical methodology that would explore issues of power (Carlspecken & Apple, 1992).

**Opportunities for Professional Development and Teacher Education**

The methods followed in this research suggest future directions for teacher professional development and education. Ball and Cohen argue that:

> Participation in modal staff development is the professional equivalent of yo-yo dieting for many teachers. Workshop handouts, ideas, and methods provide brief sparks of novelty and imagination, most squeakily practical. But most teachers have a shelf overflowing with dusty vinyl binders, the wilted cast-offs of staff development workshops. Since professional development is rarely seen as a continuing enterprise for teachers, it is only occasionally truly developmental. (1999, p. 4)

They argue that for teacher education or professional development to be truly 'developmental' and effective against the 'apprenticeship of observation' (Lortie, 1975), it must be grounded in practice. The pace of events in the average classroom allows little time for teacher reflection so quality records of practice consisting of video recordings, student work, teachers' lesson plans and other materials could be used. Collaborative discussion of records of practice is necessary to extend the discourse of teachers "past exchanges of judgement and opinion" (Lampert & Ball, 1999, p. 373). Similar to ethnographic research methods, teachers investigating practice would take an inquiry stance, "*make the familiar strange*" and problematize the commonplace (Erickson, 1986, p. 121). Ball and Cohen (1999) refer to the need for teachers investigating records of practice to experience 'disequilibrium'. In this sense contrasting records of practice like stage 1 and stage 2 data should be considered. The possibility also exists for teachers to investigate records of practice using the MTLC framework and in this way 'make the familiar strange'. Ball and Cohen (1999) argue for collaborative exploration that would "intervene in the isolation of practice, in which the only material for learning is one's own practice" (1999, p. 15). It would seem that introducing these methods in teacher education is particularly important as it makes "systematic study and analysis of learning and teaching the core of professional education" (p. 16).

Finding a productive way to share the records of practice documented here may be one way of disrupting both the isolation and familiarity of practice. A wider scale application of an approach to professional development based on teachers investigating records of practice may involve adaptions to current structures of continuous professional development (CPD). A reformulation of the twenty hour summer CPD course as blocks of CPD over the school year may be necessary to meet the recommendations of Hiebert et al. (2002) that CPD should be long-term, school-based and collaborative. Of course the possibilities of newer technologies mean that facilitating online teacher communities discussing and investigating records of practice would not have to be overly expensive or require major restructuring.

**Research on Textbooks and Support Materials**

I have already mentioned the issues around the use of textbooks in Irish primary classrooms and the need for an examination of the opportunities for learning offered by the textbooks currently in use (Eivers et al., 2010). Teachers are responsible for teaching a prescribed curriculum (DES/NCCA, 1999a) and to accomplish this goal, there is an undisputed need for instructional coherence over time (Anthony & Ding, 2011; Shimizu, 2009). If they are to be expected to adopt reform orientated practices such as the discourse community approach, they should have access to sequences of reform-orientated lesson plans that facilitate instructional coherence over time. For the population of Irish primary teachers who have been shown to be largely reliant on mathematics textbooks, this may involve textbooks and ancillary materials that would support reform orientated teaching. While I acknowledge that this alone is may not be enough to encourage and support changes to teaching practices, unless teachers can envisage how long term learning may occur in reform lessons they have little reason to engage in the more novel teaching practices. This is especially true in the context of the increased emphasis on examinations of achievement within the Irish educational system (DES, 2010). Providing teachers with resources that illustrate how the discourse community approach may be used to mediate learning over time may inhibit the tendency to see 'problem-solving' as a discrete activity rather than an approach to learning in which more than just the content is learned (Lampert & Ball, 1999).

## Final words

This research has given an indication of both 'the lie of the land' in relation to discourse in Irish primary mathematics lessons and the potential of the discourse community approach. It has detailed teaching practices and created records of two contrasting forms of practice with the potential to create 'disequilibrium' in readers, whether they are teachers or academics. The discourse community approach is founded on the principle that each member plays a role in evaluating the propositions suggested by their peers. In this way new 'truths' are eventually developed and accepted by members and the discourse can be considered to be progressive (Bereiter, 1994). It is based on a similar understanding, that I offer this research for the consideration of my peers. It is offered in the hope that it may add to the continuing discourse of all those concerned with developing teaching and learning practices in mathematics education.

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**Appendix 1: Letter to principals (stage 1)**

[Date]

Dear Principal/ Chairperson Board of Management,

I am currently teaching fifth class in ( ) and have been working as a class teacher here for the past three years.

Last year I enrolled in St Patrick's College, Drumcondra to begin working on a PhD in Mathematics Education. As part of this I will be obliged to carry out research in the classroom and would like to ask permission to carry this out in some of the classes of our school.

The exact details of the research are included in the attached information sheet. One important point is that participant teachers will be recruited on a volunteer basis and the children in their classes should not be affected by the research as it will take the form of an ordinary mathematics lesson. Also there will be no need to disclose children's names or other personal information to the researcher and every effort will be made to ensure the anonymity of participating teachers. It is hoped that the research would be undertaken in January.

I would be very grateful if you would consider my request and if possible circulate the enclosed information sheet amongst the relevant teachers. I will contact you by phone in the coming week and I would be more than happy to meet with you or your staff members to discuss this further. If you have any queries, please do not hesitate to contact me.

Yours sincerely,

Siún McMorrow[14]

[14] In teaching I have always used the English version of my name and used it here so as not to cause confusion.

**Appendix 2: Plain language statement for teachers (stage 1)**

I am currently doing a PhD in Mathematics education in St. Patrick's College of Education. I am particularly interested in challenging children's mathematical thinking and the role of talk in mathematics lessons. At the moment, I am organising a small research study examining these areas as they are experienced by children in 3rd, 4th and 5th class and I would be grateful for volunteers who teach these classes that might be willing to help.

Volunteers would be invited to teach a mathematics lesson on any number topic (addition, place-value, fractions etc.) and tape record the lesson. I would then like to collect from the volunteers the recording, photocopies of the resources used and samples of children's work. They could also provide lesson objectives/plans.

A meeting will be arranged with anybody who is interested. At this meeting I can answer any questions you may have about the nature of the study and distribute recording equipment. Any teacher who wishes to comment further on the lesson after it has been recorded will be invited to make a written commentary or to speak with me personally in a recorded interview. Further anyone who has a query at any stage of the study should feel free to contact me at any time.

Every effort will be made to ensure confidentiality subject to legal requirements. Any teacher who wants to check the transcription of their recording may do so. When the recordings are transcribed, pseudonyms will be given. Any part of this data used in my PhD thesis will use these pseudonyms and every effort will be made so that neither teacher nor student can be identified. All data will be held securely by the researcher while conducting the study and disposed of safely after five years.

Involvement in this research study is on a voluntary basis. Participants may withdraw from the study at any point.

Thank you for considering my request.

Siún McMorrow

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**If participants have concerns about this study and wish to contact an independent person, please contact:**
The Administrator, Office of the Dean of Research and Humanities, St Patrick's College, Drumcondra, Dublin 9.
Tel 01-884 2149

## Appendix 3: Plain language statement to be read to students (stage 1)

To be read to children by their class teacher

There is a teacher I know who is very interested in the way we talk and the way we think about maths.

She has asked me to do a special favour for her.

She asked me to record one of our maths lessons so that she can hear how we speak about maths and maybe learn a little bit about how we are thinking.

She can't come here herself because she is busy teaching her own class.

She is also interested in seeing some of the work you are doing so if you would like to help her out, and if your parents give permission, we will record our Maths lesson on (day) and afterwards I will collect some of your work to show her.

She might write about our maths lesson or talk to other people who are interested in teaching maths, but if she does she will not use our real names and will not even give the real name of our school.

You do not have take part and if you decide not to, you will still stay here when we are recording the lesson because you cannot miss your maths class but I won't give her any examples of your work.

She gave me a letter to give to you. We'll read it together and then you can sign the form if you want.

## Appendix 4: Plain language statement for students (stage 1)

Dear Pupil,

I teach 5th class in ( ) school and I am really interested in trying to find the best ways to teach maths in primary schools.

Your teacher, (name), has agreed to help me with my work. I am interested in how teachers and students talk about maths and think about maths so (name) will be recording one of your maths lessons soon.

It will be really helpful for me to listen to this and to see some of your work. I would also like to tell other teachers about the work you do but I will not use your real names when I am doing this.

I would be grateful if you would complete the form to show that you agree to take part but you do not have to take part if you do not wish to.

Thank you,

Siún McMorrow

## Appendix 5: Consent form for students (stage 1)

Please tick the Yes or No box:

- I understand what the project is about.
  Yes ☐ No ☐

- I know that our maths lesson will be recorded.
  Yes ☐ No ☐

- I know that a sample of my work might be taken
  Yes ☐ No ☐

- I know that my real name will not be used
  Yes ☐ No ☐

- I know that I can drop out if I want to
  Yes ☐ No ☐

- I want to take part
  Yes ☐ No ☐

Signed ________________________________________

## Appendix 6: Plain language statement for parents (stage 1)

Dear Parent/Guardian,

I am a primary teacher in ( ) school and I am currently doing research in St. Patrick's College of Education, Dublin. I am interested in the role of talk in relation to mathematical thinking. Your child's teacher, (name of teacher), has kindly agreed to be part of my research project.

This will involve a once off audio-recoding of your child's daily maths lesson. On the day (Teacher's name) will teach the mathematics lesson as normal. No video or photographs will be taken but samples of the children's work will be collected. I will not be present in the classroom and the only difference to any other day's mathematics lesson is that it will be recorded.

In any reports on this project, children's names will changed to protect their anonymity.

I would be grateful if you could complete the form below giving permission for your child to participate in this research.

Thanking you in advance.

Yours sincerely,

Siún McMorrow

**Appendix 7: Consent form for parents (stage 1)**

I have read about the research project on the role of talk in relation to mathematical thinking and understand what is involved

I agree to let my child take part in the project

Yes ☐ No ☐

Signed

**Appendix 8: List and details of recordings and transcriptions collected at Stage 2 by date recorded, lesson topic, recording length, other data collected. (Table 18)**

| Lesson No. | Date Recorded | Lesson Topic | Recording Length | Additional Data collected | Transcription made: yes or no |
|---|---|---|---|---|---|
| 1 | 26/11/10 | Fractions (Brainstorm) | 16.33 | - | No |
| 2 | 29/11/10 | Fractions | 34.01 | Digital record of Notebook file | Yes |
| 3 | 6/12/10 | Fractions | 27.43 | Students' activity sheets | Yes |
| 4 | 7/12/10 | Fractions | 38.35 | Digital record of Notebook file | Yes |
| 5 | 9/12/10 | Fractions | 34.04 | Digital record of notebook file | No |
| 6 | 10/12/10 | Fractions | 18.35 | Digital record of notebook file<br>Student's activity sheets | Yes |
| 7 | 15/12/10 | Fractions | 42.24 | Digital record of notebook file | No |
| 8 | 20/12/10 | Fractions | 20.31 | Digital record of notebook file | No |
| 9 | 12/1/11 | Fractions | 35.32 | Digital record of notebook file | Yes |
| 10 | 13/1/11 | Fractions | 35.13 | Digital record of notebook file | No |
| 11 | 14/1/11 | Fractions | 45.59 | Digital record of notebook file<br>Selection of scans of student work | No |
| 12 | 17/1/11 | Fractions | 25.23 | Digital record of Notebook file | Yes |

| | | | | | |
|---|---|---|---|---|---|
| 13 | 18/1/11 | Fractions | 26.48 | Digital record of Notebook file | No |
| 14 | 19/1/11 | Fractions | 39.18 | Digital record of Notebook file | No |
| 15 | 20/1/11 | Fractions | 18.27 | Digital record of Notebook file | No |
| 16 | 21/1/11 | Fractions | 34.54 | Digital record of Notebook file | Yes |
| 17 | 24/1/11 | Fractions | 34.46 | Digital record of Notebook file | No |
| 18 | 25/1/11 | Fractions | 25.57 | Digital record of Notebook file | No |
| 19 | 27/1/11 | Fractions | 31.37 | Digital record of Notebook file | No |
| 20 | 31/1/11 | Fractions | 44.14 | Digital record of Notebook file | No |
| 21 | 18/2/11 | Decimals | 26.37 | Digital record of Notebook file | Yes |
| 22 | 21/2/11 | Decimals | 4.45; 4.24; 4.21 Paired Talk | | Yes |
| 23 | 23/2/11 | Decimals | 26.43 | | Yes |
| 24 | 28/2/11 | Decimals | 18.55 | Digital record of Notebook file | No |
| 25 | 3/3/11 | Decimals | 26.46 | Digital record of Notebook file | Yes |

| | | | | | |
|---|---|---|---|---|---|
| 26 | 21/3/11 | Percentages | 10.24; 30.49<br>Pause for interruption | Digital record of Notebook file<br>Student activity sheets | No |
| 27 | 22/3/11 | Percentages | 38.49 | Digital record of Notebook file<br>Student activity sheets | No |
| 28 | 28.3.11 | Percentages | 12.56 ; 5.15 ; 3.21<br>Pause for interruption | Digital record of Notebook file<br>Student activity sheets | Yes |
| 29 | 29.3.11 | Percentages | 23.01 | Digital record of Notebook file<br>Student activity sheets | Yes |
| 30 | 30.3.11 | Percentages | 14.33 | Digital record of Notebook file<br>Student activity sheets | No |
| 31 | 31.3.11 | Percentages | 24.03 | Digital record of Notebook file | Yes |

## Appendix 9: Letter to principal and Board of Management (Stage 2)

[Date]

Dear Principal/ Chairperson Board of Management,

This year is my fifth year in (     ). I am currently teaching fifth class and am studying for a PhD in Mathematics Education in St. Patrick's College. I was recently awarded a bursary from the Teaching Council toward this project. I would like to ask permission to undertake some research within my own class.

It is hoped to make audio-recordings of 4-5 mathematics lessons a month for the remainder of the school year. All of the students and their parents/guardians will be given information about the study and may choose not to participate if they do not wish. I attach the information sheets and consent form that I will provide them with.

Every effort will be made to ensure confidentiality subject to legal requirements. When the recordings are transcribed, pseudonyms will be given. Any part of this data used in my PhD thesis will use these pseudonyms and every effort will be made so that students cannot be identified. All data will be held securely by the researcher while conducting the study and disposed of safely after five years.

If you have any queries, please do not hesitate to contact me.

Yours sincerely,

Siún McMorrow

## Appendix 10: Plain language statement to be read to students (stage 2)

My favourite subject is maths. I loved doing maths when I was a child and I love teaching it now that I'm an adult. I love puzzles and problems and I love finding out about what children think about when they're solving maths problems. Two years ago, I went back to college in St. Patrick's college in Drumcondra to learn more about how children think when they're solving problems.

I'd like to do some problem solving and puzzles this year in our class and I'd like to record what we talk about in these lessons. I'll use a small Dictaphone to record some maths lessons, like the one we've used before for Irish.[15]

I will be writing about some of our maths lesson but when I do, I won't use your real names and will not even give the real name of our school.

You do not have to take part and if you decide not to, you will still stay here when we are recording the lesson because you cannot miss your maths class but I won't take any samples of your work or record your voice.

I have a letter to give to you. We'll read it together, then you can bring it home and discuss it with your parents. If they agree, you can then sign the form if you want.

[15] Dictaphones had been used previously for the recording of Irish poems and songs.

## Appendix 11: Plain language statement for students (stage 1)

Dear Pupil,

We are going to do some problem solving in our maths classes this year and I'd like to record what we talk about in these lessons.

It will be really helpful for me to listen to this and to see some of your work. I would also like to tell other teachers about the work you do but I will not use your real names when I am doing this.

I would be grateful if you would complete the form to show that you agree to take part but you do not have to take part if you do not wish to. [16]

Thank you,

Siún McMorrow

[16] This letter was read aloud in class with students but the consent form shown in appendix12 that follows was sent home for students to decide if they wanted to sign it with their parents.

## Appendix 12: Consent form for students (stage 2)

Please tick the Yes or No box:

- I understand what the project is about.
  Yes ☐ No ☐

- I know that our maths lesson will be recorded.
  Yes ☐ No ☐

- I know that a sample of my work might be taken
  Yes ☐ No ☐

- I know that my real name will not be used
  Yes ☐ No ☐

- I know that I can drop out if I want to
  Yes ☐ No ☐

- I want to take part
  Yes ☐ No ☐

Signed ________________________________________

## Appendix 13: Plain language statement for parents (stage 2)

Dear Parent/Guardian,

I have always been interested in mathematics and am currently doing a PhD in mathematics education in St. Patrick's College of Education, Drumcondra. I hope to provide the boys in fifth class with some mathematical problem solving activities this year and to encourage them to talk and write about their mathematical thinking. It is hoped that they will become better and more confident in mathematics as time goes by.

As part of my studies in St. Patrick's college, I would like to audio record some of our mathematics lessons. This will allow me to listen back to them and figure out ways to improve the lessons. All of this is intended to help your child improve in mathematics and enjoy the subject more.

In any reports on this project, the identity of the school will not be revealed and every effort will be made to preserve confidentiality so that children's names will not be revealed in any manner.

Of course you may chose not to let your child participate or to withdraw from the project at any time. If you do decide to let your child participate, that is to be audio recorded during some maths lessons and to have their written work analysed as part of the research, I would be grateful if you could complete the form below giving permission for your child to participate in this research. If you have any questions, please do not hesitate to contact me.

Thanking you in advance.

Yours sincerely,

Siún McMorrow

*This study has been part funded by the Teaching Council.*[17]

[17] This statement was not included in Stage 1 documents as funding had not been secured at that time.

## Appendix 14: Consent form for parents (stage 2)

I have read about the research project on the role of talk in relation to mathematical thinking and understand what is involved

I agree to let my child take part in the project

Yes ☐ No ☐

Signed ______________________________

**Appendix 15: List of stage 2 transcripts of recordings by date, lesson topic, recording length and additional data.**

| Transcription no. *Name used in chapter 5 where appropriate* | Date Recorded | Lesson Topic | Recording Length | Additional Data collected |
|---|---|---|---|---|
| T1 *Cutting Pizzas* | 29/11/10 | Fractions | 34.01 | Digital record of Notebook file |
| T2 | 6/12/10 | Fractions | 27.43 | Students' activity sheets |
| T3 | 7/12/10 | Fractions | 38.35 | Digital record of Notebook file |
| T4 *Fraction Problems* | 10/12/10 | Fractions | 18.35 | Digital record of notebook file Student's activity sheets |
| T5 | 12/1/11 | Fractions | 35.32 | Digital record of notebook file |
| T6 | 17/1/11 | Fractions | 25.23 | Digital record of Notebook file |
| T7 | 21/1/11 | Fractions | 34.54 | Digital record of Notebook file |
| T8 *Dienes' Decimals* | 18/2/11 | Decimals | 26.37 | Digital record of Notebook file |
| T9 | 21/2/11 | Decimals | 4.45; 4.24; 4.21 Paired Talk | None |
| T10 | 23/2/11 | Decimals | 26.43 | None |
| T11 | 3/3/11 | Decimals | 26.46 | Digital record of Notebook file |
| T12 *Equivalence Challenge* | 28.3.11 | Percentages | 12.56 ; 5.15 ; 3.21 Pause for interruption | Digital record of Notebook file Student activity sheets |

| | | | | |
|---|---|---|---|---|
| T13 | 29.3.11 | Percentages | 23.01 | Digital record of Notebook file<br>Student activity sheets |
| T14<br>*Percentage Present and absent* | 31.3.11 | Percentages | 24.03 | Digital record of Notebook file |

**Appendix 16: Analysis of teacher questions in the *Cutting Pizzas* lesson by type and number with examples.**

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 24: So how many pieces are in it John?<br>Turn 290: If we had eight Alan, and eight in one pizza and eight in the other so how many slices altogether? | 15 |
| 2. Inserting terminology | Turn 38: So what are those pieces called Steven?<br>Turn 427: Well what are those little pieces? | 6 |
| 3. Exploring mathematical meanings and/or relationships | Turn 201: So how would write that though?<br>Turn 410: What would a half and a quarter look like together? | 11 |
| 4. Probing, getting students to explain their thinking | Turn 214: You think it's one sixth. Why do you think it's one sixth?<br>Turn 412: Two eighths? Where are you getting two eighths? | 18 |
| 5. Generating discussion | Turn 209: Do you agree with him or is there a different way?<br>Turn 238: Is it two sixth or does anyone have a different example? Jake, what do you think? | 24 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 74 |

**Appendix 17: Analysis of student questions in the *Cutting Pizzas* lesson by type and number with examples.**

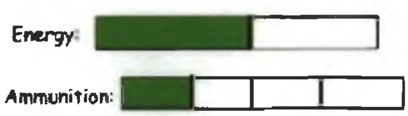
| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: What's the third one ... the third one?<br>*The student posed this question about the fraction Jonathan had written on the board in the introductory fraction labelling task.*<br><br>Steven: So all I've to do is do a one and then a five?<br>*Steven asked how to write* $^1/_5$ *in the initial fraction labelling task*<br><br>Steven: Eh I think, who would be the oldest?<br>*Steven asked this question about the pizza sharing task before proceeding to share the pizzas unequally giving the 'oldest' a larger share.*<br><br>Student: What?<br>*The student questioned Steven's solution strategy described above.*<br><br>Darragh: What does she get Steven?<br>*Darragh questioned Steven's solution strategy.*<br><br>Steven: Do you know?<br>*Steven asked me if I knew the solution. This question is discussed in detail below*<br><br>Steven: What?<br>*Steven questioned Alex's suggestion about cutting the pizzas into twenty one pieces.*<br><br>Darragh: But what's left?<br>*Darragh questioned Michael's uneven pizza sharing strategy shown in figure X.*<br><br>Steven: What's equivalent?<br>*Steven questioned Darragh's suggestion that two sixth is equivalent to one third.*<br><br>Edvard: Teacher, it should be higher than one sixth because you can't go higher than one twelves … Wait can you go up over one twelfth?<br>*Edvard appeared to ask if it is possible to have denominators greater than 12.*<br><br>Steven: What does simplify mean?<br>*Steven questioned the term simplify which was used once by Darragh over seventy turns previously.*<br><br>Steven: Hey what's that word again?<br>*Steven asked what equivalent means.*<br><br>Andrei: Then quarters?<br>*Andrei predicted the second half of Alan's solution shown in figure X.*<br><br>Student: What?<br>*The student seemed to question my explanation of Anthony's solution.* | 14 |

| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Michael: Can I clear this?<br>*Michael asked if he can delete the solution drawn on the whiteboard by Steven.*<br>Student: Why is it green?<br>*The student asked why the whiteboard marker is writing in green.* | 2 |

**Appendix 18: Teacher designed worksheet that students completed in pairs prior to the *Fractions problems* recording.**

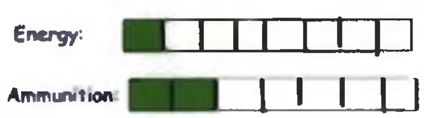
Name: _______________ Date:_____________

Mark in what fraction of energy and ammunition that Xeno has left.




a.


b.




c.


d.


e.


f.

**Appendix 19: Analysis of teacher questions in the *Dienes' Decimals* lesson by type and number with examples.**

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 102: So how many tenths do they have now?<br>Turn 143: Andrei, what's your number that you choose got? | 3 |
| 2. Inserting terminology | Turn 5: ... what were those ones called?<br>Turn 145: Three point four- what is that? | 4 |
| 3. Exploring mathematical meanings and/or relationships | Turn 32: How many of them will you need to make one of these?<br>Turn 128: Which is the largest one? | 13 |
| 4. Probing, getting students to explain their thinking | Turn 28: Right what do you think it should be? … tell me why.<br>Turn 154: Why? | 3 |
| 5. Generating discussion | Turn 150: Andrei has given us a different way of looking at his number. Do you agree with him, that this number is the same as that?<br>Turn 134: Kevin thinks it's four. Is there anybody who thinks it's a different number? Alex? | 7 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 30 |

**Appendix 20: Analysis of student questions in the *Dienes' Decimals* lesson by type and number with examples.**

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Conor: Blow it up?<br>*Turn 2, Conorasked this question in relation to my statement about the need to increase the size of the unit block to see the decimal fractions*<br><br>John: What's the difference between them and them?<br>*Turn 19, John questioned the difference between different bags of thousandth blocks*<br><br>Luke: How?<br>*Turn 51, Luke asked this questions after Darragh's comment that four hundred and forty thousandths could be used to represent 0.44.*<br><br>Steven: I'm confused, how?<br>*Turn 97, Steven asked this question after Jake's explanation of what happens when one tenths is added to 0.940.* | 4 |

| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: Why not me?<br>*Turn 21, the student asked why he wasn't chosen to come to the board.* | 1 |

**Appendix 21: Analysis of teacher questions in the *Percentage Present and Absent* lesson by type and number with examples.**

| Question Type | Examples | Total |
|---|---|---|
| 1. Gathering information, leading students through a method | Turn 14: Right, could you count up again which ones, how many has he right?<br>Turn 16: Seven. So do you need to change it? | 5 |
| 2. Inserting terminology | | 0 |
| 3. Exploring mathematical meanings and/or relationships | Turn 19: And Conor how did you know it should be seven tenths because I saw some people who had just written down seven?<br>Turn 218: How did you know to multiply by four? | 4 |
| 4. Probing, getting students to explain their thinking | Turn 7: Right, would you explain to us there Conor, what are you thinking about this?<br>Turn 82: What did you, how did you come up with that number or that fraction? | 16 |
| 5. Generating discussion | Turn 37: Do you agree with him?<br>Turn 189: Now what do you think of that solution? | 8 |
| 6. Linking and applying | | 0 |
| 7. Extending thinking | | 0 |
| 8. Orientating and focusing | | 0 |
| 9. Establishing context | | 0 |
| Total | | 33 |

**Appendix 22: Analysis of student questions in the *Percentage Present and Absent* lesson by type and number with examples.**

| Questions seeking clarification about mathematics being discussed.<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Steven: How do you know?<br>*Turn 3, Steven asked this question about the first task of finding the fraction and percentage correct of a spelling test.*<br><br>Edward: Teacher is this our class?<br>*Turn 67, Edward asked if the percentage present and absent question referred to our class.*<br><br>Andrei: What like, explain it?<br>*Turn 175, Andrei asked this question clarifying what I had asked him to do.*<br><br>Darragh: Turn it into the hundredths?<br>*Turn 218, Darragh asked this question when I pointed out that he had left out an important part of his explanation of how to turn* $^{22}/_{25}$ *into a percentage.*<br><br>Darragh: Can I try now?<br>*Turn 233, Darragh asked if he could attempt his explanation again.*<br><br>Darragh: Do you get that?<br>*Turn 244, Darragh asked Michael if he understood his explanation.* | 6 |

| Questions seeking organisational clarification<br>*Notes on context of question where appropriate.* | Total |
|---|---|
| Student: What page?<br>*Turn 264, the student asked which page of the text book I had set work from.* | 1 |

**Appendix 23: Board work from 3/3/11 (transcript 11) showing word problem and Steven's incorrect solution attempt in bottom left corner.**