DIAGNOSTIC TESTING IN DCU: A FIVE-YEAR REVIEW

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Diagnostic testing in mathematics is in widespread use across many third-level institutions in order to identify, and hopefully redress, areas of particular mathematical weakness at an early stage in a student's university career. Since September 2004, first-year service-mathematics students in Dublin City University have taken such a test upon entry. The test consists of fifteen, multiple-choice questions on a range of basic mathematics topics. Students who score 50% or less are deemed to be at-risk of failing their mathematics module and are advised to attend "refresher sessions" held during the first two weeks of semester. During the course of the past five years, it has been considered necessary to introduce certain modifications to the test; we discuss in detail the rationale behind these changes as well as the likely impacts on overall test results. In addition, we highlight the questions which were most poorly answered and look at possible relationships between examination grades and performance in the diagnostic test. Finally, we discuss the relative merits of module-specific diagnostic tests, where the difficulty of the test is related to the level of mathematics the student will encounter in their module.

INTRODUCTION

The poor core mathematical skills of a large number of students entering third-level education continue to be a cause of growing concern for many mathematics educators. This concern has been expressed in numerous journal articles and conference proceedings, and inquiries have been undertaken to ascertain the mathematical accomplishment of these students. In Ireland, studies were being undertaken as early as 1985, when Cork Regional Technical College concluded that their incoming undergraduates were deficient in basic mathematics (Cork Regional Technical College, 1985). Numerous other universities and institutes were soon reporting similar findings (Hurley and Stynes, 1986; Brennan, 1997; O'Donoghue, 1999). By 1995, in the United Kingdom, the London Mathematical Society (LMS), in collaboration with the Institute of Mathematics and its Applications (IMA) and the Royal Statistical Society (RSS), had produced a report entitled "Tackling the Mathematics Problem" (LMS, 1995), which investigated concerns amongst mathematicians, scientists and engineers in third-level education about the mathematical preparedness of new undergraduates. This was followed up by a report by the UK Engineering Council which showed strong evidence of a "steady decline" in basic mathematical skills and "increasing inhomogeneity in mathematical attainment and knowledge" (Savage, Kitchen, Sutherland & Porkess, 2000). One of the main recommendations of this report was that "students embarking on mathematics-based degree courses should have a diagnostic test on entry.” However, the report was also at pains to point out that diagnostic testing is a means to an end:

> Diagnostic testing should be seen as part of a two-stage process. Prompt and effective follow-up is essential to deal with both individual weaknesses and those of the whole cohort. (Savage et al, 2000, p. iii)

Without some category of support structure in place for students identified as being at-risk of struggling substantially with their mathematics module, there is a risk that diagnostic testing may not be of major benefit to these students:

> In situations where students are simply told their test result and advised to revise certain topics on their own, there is little evidence that this happens. (Lawson et al, 2003, p. 8).

DIAGNOSTIC TESTING

In 2003, the U.K. Learning and Teaching Support Network (LTSN) MathsTEAM project produced a detailed collection of case studies of diagnostic testing throughout the UK, which highlighted the range of testing being undertaken, as well as the results obtained, possible barriers to test execution and general recommendations (LTSN MathsTEAM, 2003). They concluded that:

> Diagnostic testing provides a positive approach to a situation. For the student it provides a constructive method, which leads to ongoing support, and for the academic it is an indication of “what is needed” in terms of teaching and curriculum changes. As the number of institutions implementing these tests increases it is becoming an integral part of mathematical education for first year students. (LTSN MathsTEAM, 2003, p. 7)

Diagnostic tests fall into two main delivery types: paper-based and computer-based. The optimal choice is generally dependent upon internal resources within each university, with a lack of sufficiently reliable computing facilities in some locations (including Dublin City University) meaning that paper-based is the only sensible choice, particularly at the start of the academic year when many students may not yet be set up on the computing system. Although a large number of these tests are multiple-choice, to enable speedy return of marks to the students involved, some universities, such as the University of Limerick, have opted instead for open-ended questions, as this provides them with greater information about the mathematical deficiencies in question:

> The test was designed for marking by hand so that one could investigate the specific errors that students make and identify where the gaps in student knowledge lie…As the students were provided with rough work areas, it was possible to determine why students were making the type of mistakes they were. (Gill & O’Donoghue, 2007, p. 228).

The majority of diagnostic tests are given during orientation week or in the first couple of weeks of the academic year (LTSN MathsTEAM, 2003, p. 4); in some cases, such as the Institute of Technology in Tralee, the tests are repeated several weeks later to assess students' improvements (Cleary, 2007).

DIAGNOSTIC TESTING IN DUBLIN CITY UNIVERSITY

The Maths Learning Centre in Dublin City University (DCU) opened in February 2004, and before the start of the 2004/2005 academic year, developed a diagnostic test for incoming first-year service mathematics students. The test consists of fifteen, multiple-choice questions on a range of basic mathematical skills, including percentages, fractions, numerical and algebraic manipulation, and solving linear and quadratic equations. It is paper-based and was initially conducted during the first mathematics class of the year, but in the past two years has been done during orientation week instead. There are two versions of the test, to prevent students who take the test earlier in the week passing it on to those taking it later in the week, as the solutions are given to students once they have completed the test. Both copies of this test can be found in Appendix A.

Student who receive below a certain grade on the diagnostic test are deemed to be at risk of failing their mathematics module and are advised to attend refresher sessions which take place during the first two weeks of semester, revising basic mathematics, and to make frequent use of the Maths Learning Centre during the year.

Over the past five years, approximately 700 students per year have taken the diagnostic test upon entry to the university (with the exception of 2006, when about 500 students took the test, due to a once-off alteration in the orientation programme for the business faculty). This amounts to roughly 70% of the first-year service mathematics cohort who have sat the test. The students involved are from six different service mathematics modules: Mathematics for Computing, Mathematics for Scientists, Mathematics for Physicists, Business Mathematics, Accounting Mathematics, and Engineering Mathematics.

MODIFICATIONS MADE

Introducing any modifications to a diagnostic test means that a full comparative study of results with previous years is no longer possible, and as such, the benefits of any such changes must be carefully weighed up against this loss of comparative data. However, bearing in mind the principal aim of the test, which is to identify those most at risk, occasionally changes are necessary in order to improve the accuracy of the information being obtained.

One example of this in the DCU diagnostic test was in regard to the most poorly answered question on the test, based on fractional indices (see Appendix A, question 3). Last year, a discrepancy was noticed between the two versions of the test: in one test, students were asked to simplify $\left(\frac{16}{9}\right)^{\frac{5}{2}}$, a question involving only fractional indices, whereas the corresponding question in the second test asked students to simplify $\left(\frac{100}{9}\right)^{-\frac{3}{2}}$, involving both fractional and negative indices. To rectify this oversight, the second test was altered to include only fractional indices (students were asked to simplify $\left(\frac{9}{100}\right)^{\frac{3}{2}}$ instead), and the results for the question showed a substantial improvement, with 48% of students answering correctly compared with 26% the previous year. Clearly, this is still an area of extreme difficulty for students, but it would seem likely that the inclusion of negative indices skewed the results for this question, given that they only appeared on one version of the test.

Another modification introduced two years ago was in regard to the marking scheme. For the first three years, negative marking was not used, as it was felt that this might be overly intimidating for students at the start of their university experience; however, in 2007, it was decided that the introduction of negative marking was necessary for more accurate results and to avoid random guessing of answers. In addition, negative marking provides a greater level of knowledge about the students' perceptions of their own learning gaps, as it would seem likely that, on the whole, students who leave questions blank are at least aware that they are unsure of that area, while students who answer incorrectly think they know how to solve the problem and as such, may be unlikely to seek further help for that topic. Therefore, as there were five options given for each question, students received 4 marks for a correct answer, and lost 1 mark for an incorrect answer, while receiving no marks for an unanswered question. Students were made aware of this fact and advised to leave blank any questions about which they were unsure.

MOST COMMON AREAS OF DIFFICULTY

Among the fifteen questions on the test, there are several questions which are invariably poorly-answered, with little variation shown on these from year-to-year. Figure 1 below shows the results for all six modules for each question of the diagnostic test in 2008.

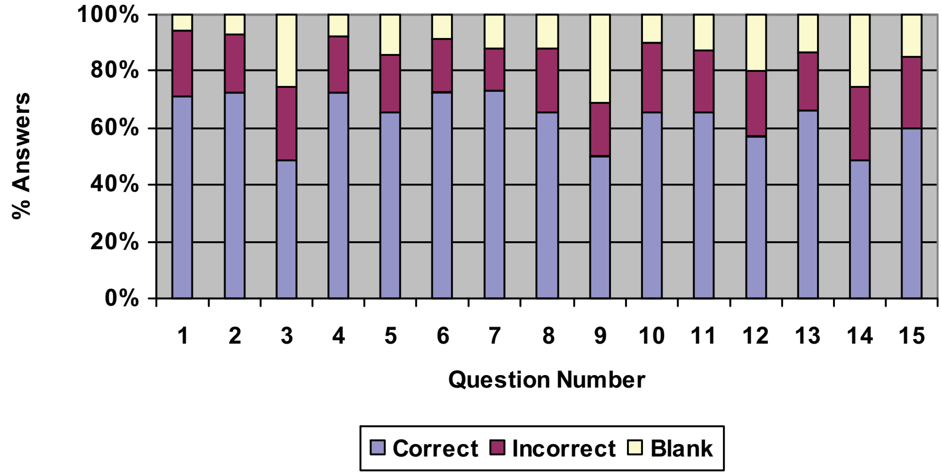

Figure 1: Results from all modules for each question of the diagnostic test in 2008.

The question which causes the most difficulty for students every year is based on fractional indices (see Appendix A, question 3). This is closely followed by questions on algebraic indices (question 9), solving a partially-factored cubic (question 12) and inequalities (question 14). In some modules, such as Mathematics for Computing, the percentage of correct answers for indices questions such as 3 and 9 can drop lower than 25%. It is also worth noting that a large number of students leave these particular questions unanswered.

DIAGNOSTIC RESULTS

As a result of the introduction of negative marking in 2007, students now obtain a percentage mark for the test, whereas pre-2007, they received a mark out of 15. The pass mark for the test is now 50%, whereas previously, it was 8 out of 15 (53.3%). Table 1 below shows the mean mark and standard deviation for 2007 and 2008, once negative marking was introduced.

Year	**2008**	**2007**
# Student Responses	726	730
Mean Mark (%)	58.22	62.52
Standard Deviation (%)	23.63	22.08

Table 1: Number of student responses to diagnostic test, along with mean mark and standard deviation for 2007 and 2008. Negative marking was used in these years.

Table 2 shows the results before negative marking was introduced. However, for greater clarity, all marks are given in percentages, instead of as marks out of 15.

Year	**2006**	**2005**	**2004**
# Student Responses	474	694	728
Mean Mark (%)	64.07	69.93	68.60
Standard Deviation (%)	21.73	28.47	19.67

Table 2: Number of student responses to diagnostic test, along with mean mark and standard deviation for 2004-2006. Negative marking was not used in these years.

If these results are compared with students' Leaving Certificate mathematics grades on a student-by-student basis, the resulting correlation is consistently low. There are several possible reasons for this: one is that students are given no prior warning about the test, and as such, have no opportunity to revise any material, and it has been at least three months since they last looked at any mathematics. Some students have naturally better recall than others. In addition, many students are trained to answer questions in state examinations and as such, do not cope as well with a new format of test. This theory is supported in countless instances of classroom observation (Lyons et al, 2003), as well as in the comments of the State Examination Commission:

> (E)xaminers have been commenting on a noticeable decline in the capacity of candidates to engage with problems that are not of a routine and well-rehearsed type. (SEC, 2005, p. 73)

Also, the structure of the marking scheme for Leaving Certificate mathematics means that it is possible for students to make small errors in basic components of questions and still obtain high "attempt marks" for that question, as they lose one mark for numerical slips, or three marks for mathematical errors or omissions (SEC, 2009 b, p. 2), whereas any such error would result in an incorrect answer in the diagnostic test. For all of these reasons, the diagnostic test identifies students whose grasp of some basic mathematical concepts is not as good as is needed to cope in their course, and as such, is a valuable add-on to their Leaving Certificate grade in identifying at-risk students as early as possible.

FUTURE WORK

The Leaving Certificate Mathematics examination can be taken at three different levels: Foundation (F), Ordinary (O) and Higher (H). Within each level, fourteen possible grades can be awarded, as laid out in Table 3 below.

Result, r (%)	Grade	Result, r (%)	Grade	Result, r (%)	Grade
$90 \leq r \leq 100$	A1	$65 \leq r < 70$	C1	$40 \leq r < 45$	D3
$85 \leq r < 90$	A2	$60 \leq r < 65$	C2	$25 \leq r < 40$	E
$80 \leq r < 85$	B1	$55 \leq r < 60$	C3	$10 \leq r < 25$	F
$75 \leq r < 80$	B2	$50 \leq r < 55$	D1	$r < 10$	NG
$70 \leq r < 75$	B3	$45 \leq r < 50$	D2		

Table 3: Percentage range for each grade awarded at Leaving Certificate (SEC, 2009).

The standard required for first-year service mathematics modules in DCU varies considerably, depending on the course involved. Indeed, the minimum mathematics requirement for Engineering Mathematics is a HC3 in Leaving Certificate mathematics (namely, 55% or greater in the Higher Level paper), whereas for many of the other modules, it is a HD3 or OC3. Even apart from this, however, the level that students are expected to reach by the end of their module is very different. If we then look at the diagnostic results for 2008, in which the mean mark was 58.22 ± 23.63 when all modules were taken together, we see that the mean for Mathematics for Computing, for example, was as low as 50.83 ± 22.99, while the mean for Engineering Mathematics was 74.74 ± 17.15. As a result, only 8.5% of the Engineering students who took the diagnostic test were identified as being "at risk" of failing their module, while 51.9% of the Computing students were similarly classified. However, based on anecdotal evidence, continuous assessment results and observations within the Maths Learning Centre, a significant proportion of the students in Engineering Mathematics will struggle with their mathematics module every year. Therefore, there is a real danger that the diagnostic test currently being used is not suitable for some of the more demanding modules.

However, the advantage of having all service mathematics students take the same diagnostic test is that it allows a direct comparison between students in different modules. Therefore, the optimum response is to produce two different tests, one a subset of the other, and give the more demanding test to students in the more difficult modules. As a result, a modified test is being designed which will hopefully be introduced in the coming year for three of the modules (Mathematics for Physicists, Accounting Mathematics and Engineering Mathematics), based on the original test, but with a number of additional questions, in order to better identify those at risk in such modules.

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APPENDIX A: DCU DIAGNOSTIC TEST

Question 1

$\frac{2}{3}+\frac{2}{5}=?$

A) $\frac{4}{8}$ B) $\frac{2}{8}$ C) $\frac{16}{15}$ D) $\frac{2}{15}$ E) None of these

$\frac{3}{4}-\frac{1}{3}=?$

A) $\frac{2}{1}$ B) $\frac{2}{12}$ C) $\frac{5}{12}$ D) $\frac{2}{4}$ E) None of these

Question 2

$\frac{2}{3}\div\frac{1}{5}=?$

A) $\frac{2}{15}$ B) $\frac{10}{3}$ C) $\frac{10}{15}$ D) $\frac{3}{15}$ E) None of these

$\frac{3}{7}\div\frac{2}{3}=?$

A) $\frac{9}{14}$ B) $\frac{2}{7}$ C) $\frac{2}{21}$ D) $\frac{6}{21}$ E) None of these

Question 3

Simplify $\left(\frac{9}{100}\right)^{\frac{3}{2}}$

A) $\frac{1000}{27}$ B) $\frac{300}{18}$ C) $\frac{81}{1000}$

Simplify $\left(\frac{16}{9}\right)^{\frac{5}{2}}$

A) $\frac{40}{9}$ B) $\frac{40}{22.5}$ C) $\frac{1024}{9}$

D) $\frac{1000}{90}$ E) None of these	D) $\frac{1024}{243}$ E) None of these

Question 4

What is 1.5 expressed as a percentage of 2?	What is 6 expressed as a percentage of 15?
A) 75% B) 1.5% C) 15% D) 150% E) None of these	A) 37% B) 40% C) 75% D) 250% E) None of these

Question 5

A car bought for €12,500 loses 15% of its value in one year. What is the car worth at the end of this year?	A house increases it value by 8% each year. If a house cost €325,000 last year, how much will it cost this year?
A) €12,485 B) €11,000	A) €333,000 B) €26,000
C) €10,625 D) €1,875	C) €585,000 D) €348,000
E) None of these	E) None of these

Question 6

$-(2x+4y)-2(-x-2y)=?$	$(3x-6y)-5(-x+3y)=?$
A) x B) $-4x+8y$	A) $8x+9y$ B) $-2x+9y$
C) $-4x-8y$ D) 0 E) None of these	C) $8x-21y$ D) $8x-9y$ E) None of these

Question 7

Expand & simplify $(x-3)(2x+1)$	Expand & simplify $(4+x)(-3x+1)$
A) $2x^2-6x-3$ B) x^2-5x+3	A) $-12x^2+x+1$ B) $-3x^2-11x+4$
C) $2x^2+x-6$ D) $2x^2-5x-3$	C) $5-2x$ D) $-12x^2+4$
E) None of these	E) None of these

Question 8

Write $\frac{2}{x}-\frac{1}{1+x}$ as a single fraction.	Write $\frac{1-k}{2k}+\frac{k}{1-k}$ as a single fraction.
A) $\frac{2+x}{x(1+x)}$ B) $\frac{2}{x(1+x)}$	A) $\frac{1}{1+k}$ B) $\frac{1}{2k(1-k)}$ C) $\frac{1}{2}$

C) $\dfrac{2+2x}{x+x^2}$ D) $\dfrac{1}{-1}$ E) None of these

D) $\dfrac{1-2k+3k^2}{2k(1-k)}$ E) None of these

Question 9

Simplify $a^4\left(\dfrac{a^5}{a^3}\right)$

A) a^{17} B) a^8 C) a^6 D) a^{12} E) None of these

Simplify $b^{-2}\left(\dfrac{b^3}{b^4}\right)$

A) b^{-3} B) b C) b^3 D) b^{-2} E) None of these

Question 10

Find the value of s if $\dfrac{2}{s}-1=3$.

A) $s = 2$ B) $s = 8$ C) $s = 4$ D) $s = -8$ E) None of these

Find the value of Q if $-\dfrac{6}{Q}+5=8$.

A) $Q=2$ B) $Q=\dfrac{1}{2}$ C) $Q=-2$ D) $Q=-\dfrac{1}{2}$ E) None of these

Question 11

Find all values of x such that $2x^2-x-3=0$

A) $x=\dfrac{3}{2}$, $x=1$ B) $x=\dfrac{3}{2}$, $x=-1$ C) $x=-\dfrac{3}{2}$, $x=1$ D) $x=-1$, $x=3$ E) None of these

Find all values of x such that $3x^2+5x+2=0$

A) $x=\dfrac{2}{3}$, $x=1$ B) $x=\dfrac{2}{3}$, $x=-1$ C) $x=-\dfrac{2}{3}$, $x=-1$ D) $x=-2$, $x=-3$ E) None of these

Question 12

Find all values of t such that $t(t^2-1)=0$

Find all values of r such that $r(r^2-9)=0$

A) $t=1$ B) $t=0,\ t=1$ C) $t=0,\ t=1,\ t=-1$

D) $t=0$ E) None of these

A) $r=0,\ r=3,\ r=-3$ B) $r=0,\ r=3$ C) $r=3$

D) $r=0$ E) None of these

Question 13

Let $y=(2x-1)^2+\sqrt{-8zx^3}$. If $x=-1$ and $z=2$, what is the value of *y*?

A) $y=-5$ B) $y=5$

C) $y=3$ D) $y=13$ E) None of these

What is the value of $a-(b\sqrt{a+c})$ when $a=10$, $b=12$ and $c=15$?

A) –10 B) 50

C) 70 D) 10 E) None of these

Question 14

Find all values of *x* for which $-2x+7>3$

A) $x<2$ B) $x=2$

C) $x>2$ D) $x<8$ E) None of these

Find all values of *x* for which $2x+4<-2$

A) $x=-3$ B) $x>-3$

C) $x>-6$ D) $x<-6$ E) None of these

Question 15

At what point do the lines $y=2x+4$ and $y=-4x+1$ intersect?

A) (–0.5, 3) B) (1.5, –5)

C) (0.5, 3) D) (–3, 3) E) None of these

At what point do the lines $y=3x-2$ and $y=-x+6$ intersect?

A) (–2, 4) B) (2, 4)

C) (2, 2) D) (0, 6) E) None of these