Measures in the senior primary classes

Commissioned research paper

Dr Siún Nic Mhuirí

Dublin City University

NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

June 2020

Introduction

Measurement involves quantifying aspects of the physical environment and is richly connected to other mathematical domains. Watson, Jones and Pratt (2013) argue for a repositioning of measurement in upper primary and secondary curricula as a key component which connects to all others. Length, area, and volume are spatial measures (Curry & Outhred, 2005). Time and money are connected to the domain of number, while exploration of relationships between measures and measurement formulae link to algebra (Watson et al., 2013). Probability is a measure of uncertainty and certain statistical methods can be considered as measures (Ibid.).

Measurement connects mathematics to the real world. It is directly applicable to and necessary for engagement in everyday experiences and is also a core practice in many workplaces (Smith, van den Heuvel-Panhuizen & Teppo, 2011). The human origin of standard measurement units reveals mathematics to be part of human culture (Hersh,1997). Measurement data is often used to inform social decisions (Van den Heuvel-Panhuizen & Buys, 2008). For example, measurement data contributes to how mathematics is used to describe, predict and communicate about climate change (Barwell, 2013). For these reasons, measurement is critical in children's mathematical learning. Unfortunately, research highlights issues in children's achievement in measure (Smith et al., 2011). This includes a gender achievement gap in favour of boys in the Irish primary context (Shiel et al., 2014).

Key concepts

Piaget's work, which aimed to categorise children's development of measurement concepts into different stages, has long been influential (c.f., Stephan, 2003). In research report 17 (Dunphy et al., 2014), which was prepared to underpin a redeveloped primary mathematics curriculum, new research described directions growing from critique and reinterpretation of Piaget's ideas. Liu et al. (2017) argue that attention to developmental stages has ignored whether experiences encourage children to express their intuitive understandings in quantitative ways. Nonetheless, Piaget's view that children's measurement knowledge is more complex than application of procedures remains relevant (Lehrer, 2003; Piaget et al., 1960).

Foundational ideas

Early measurement involves qualitative exploration, direct comparison of objects or attributes and verbal expression of the result (van den Heuvel-Panhuizen & Buys, 2008). Children progress to quantitative approaches where iterated units are used to measure. Table 1 summarises foundational measurement ideas described by Clements and Sarama (2009). Although this work focuses on early learning trajectories, the concepts are foundational for later understanding and pertinent in the context of measures, like volume, which may be introduced in senior primary classes. Many of the powerful mathematical ideas discussed by Dooley (2019) are inherent in measurement, in particular the idea of unit, iteration, composition and decomposition.

Table 1. Foundational ideas in Measurement. Adapted from Clements and Sarama (2009)

Understanding the attribute	Understanding is generally developed by engaging in comparison and discussion using appropriate terminology.
Conservation	An object may be manipulated or changed in some ways without changing the measure.
Transitivity	Two measures may be compared indirectly using a third object.
Equal Partitioning	The activity of mentally or physically partitioning an object or attribute of an object into equal-sized pieces.
Units and Iteration	This involves thinking about how a unit might be repeated without gaps or overlaps so that it matches exactly what is being measured.
Accumulation and additivity	Accumulation refers to the idea that iterations of a unit can be counted to find the measure. Additivity refers to the idea that measures can be decomposed and combined again to give the same measurement.

Relation between number and measure	The larger the unit, the fewer the number of units needed to measure the object.

A measurement is effectively a ratio comparison between unit and object to be measured (Watson et al., 2013). It involves the coordination of a continuous quantity and number (Smith et al., 2011). Continuous quantities can always be divided into smaller amounts (Clements & Sarama, 2009) so measurement is only ever approximate with smaller units, or a greater number of digits to the right of the decimal point, providing better accuracy (Watson, et al., 2013). Length, area, weight/mass, volume and angle are measured on a ratio scale. Ratio scales have a non-arbitrary zero and it is possible to make ratio comparisons between points on the scale, e.g., 40cm is twice as long as 20cm (Clements & Sarama, 2009). Such comparisons are not possible for time or temperature which are measured on an interval scale (Watson et al., 2013). Across both interval and ratio scales, equal differences on the scale represent equal differences in the measurement.

The *Model for Mapping Measurement* (ACME, 2011) provides an overview of learners' needs in measurement across the age range 5 - 19 years (Figure 1). ACME advise that students need to see that measurement is embedded in the big ideas listed in the top margin. The transition from discrete to continuous number takes time and multiple experiences. The green lines show these connections. The purple lines illustrate connections to other mathematical ideas. ACME maintain that proficiency in measurement is developed when students work in the contexts and using the practices identified in the right-hand column.

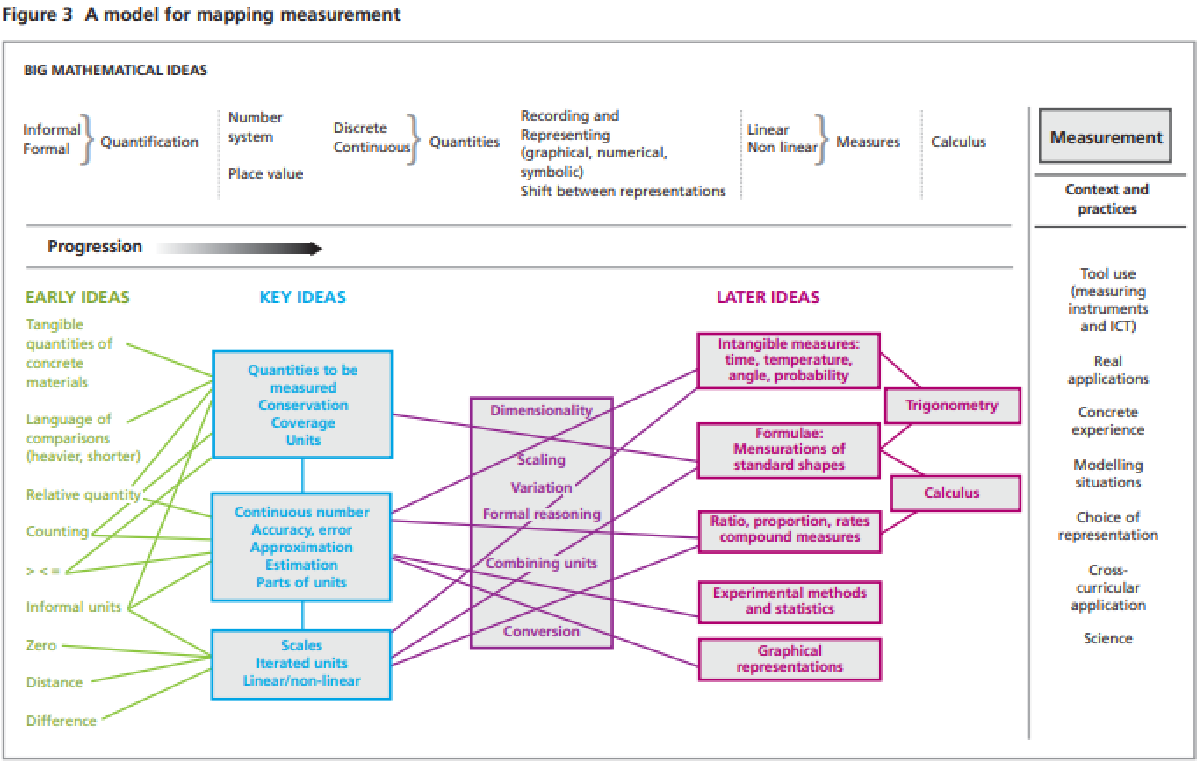


Figure 1. An exemplar progression for Measurement. Reproduced from ACME(2011)

Geometric measurement

There appears to be more research on geometric measures than other measures (c.f. Cheeseman et al., 2014; Earnest, 2017, Lehrer, 2003; Smith et al., 2011). There are spatial aspects to the unit structure of length, area, and volume measurement. Unit structure is “the pattern formed when the units fill the object to be measured” (Curry & Outhred, 2005, p. 265). To measure length, the unit is iterated along one dimension. To measure area, the unit must be iterated in two dimensions to create a rectangular array. Outhred and Mitchelmore (2000) contend that the idea of an iterable row is essential for understanding the structure of an array as a method of area measurement. Area understanding is also underpinned by a relational understanding of length measurement (Ibid.)

Capacity is the amount of liquid a container holds while volume is the amount of space an object takes up though these terms are sometimes used interchangeably (Foster, 2011). The internal volume of a container is equivalent to its capacity. This space might be ‘filled’ by iterating a fluid unit which takes the shape of the container (Curry & Outhred, 2005). Effectively the unit is iterated against the height of the container and the unit structure is one-dimensional (van den Heuvel-

Panhuizen & Buys, 2008). Alternatively, the container might be 'packed' with a three-dimensional array (Curry & Outhred, 2005). Understanding of the unit structure for area provides the foundation for understanding measurement of *volume-by-packing* as a child must extend the same unit iteration ideas into the third dimension (ibid.)

Weight

'Weight' is regularly used in non-scientific discourse to refer to what might more correctly be called 'mass' (Liu et al., 2017). Mass is the amount of matter in an object while weight is the force of gravity on an object though this distinction may be of greater relevance at second level (Newell, 2017). Existing research focuses mainly on the early years (e.g., Cheeseman et al., 2014). Recent research by Liu et al. (2017) focused on proportional relations and examined how 7 - 11 year olds investigated whether objects might be made of the same material by using ratios of measures of weight and size.

Time

Understanding time is crucial to successful functioning in the world. Time is a complex notion consisting of two different concepts, succession and duration (Fraisse, 1984). Succession involves the sequential ordering of two or more different events while the concept of duration applies to the interval of time between the two events (Ibid.). While very young children can perceive both aspects of time, they are unable to productively combine the two ideas until later. Thomas et al. (2016) propose a research-based framework which details key ideas underpinning the concept of time. They suggest that this framework can provide a foundation for the teaching and learning of time. The four components of the framework are *awareness of time*, *succession*, *duration and measurement of time*. Building a*wareness of time* involves using significant times, dates or events as meaningful reference points. It involves exploration of temporal patterns, routines and cyclical events as well as developing the language of time. Thomas et al. (2016) describe how the four components are interrelated and how *measurement of time* can actually help develop conceptual understanding. For example, when the clock reads 5:10 or ten minutes past five, it is measuring the time since twelve o' clock. This is a *duration* of five hours and ten minutes and the time has moved into the sixth hour (*succession*). Thomas et al. contend that children's reading of clocks may be meaningless without an understanding of succession and duration. This is supported by research which details children's difficulties in coordinating hours and minutes to calculate elapsed time (Kamii & Russell, 2012; Earnest, 2017).

Money

Research details the historical development of money as a measure of value and the many links to number concepts (Radford, 2003; Van den Heuvel-Panhuizen & Buys, 2008). In contrast with other measures, money can be considered a discrete physical quantity rather than a continuous quantity (Van den Heuvel-Panhuizen & Buys, 2008) and many of the foundational ideas above do not apply. With electronic payments very prevalent, the direct exchange of money is becoming less common. Instead, money is used primarily as a calculating unit for payments (Ibid.). Existing research highlights the importance of competence in calculations (Lusardi, 2012) and finance-related knowledge and skills to understanding money transactions, particularly in the context of electronic payments (Aprea et al., 2016). Decimals, estimation, and problem contexts are likely to be the main areas of linkage to other measures.

The limited existing research often has a focus beyond the teaching of mathematics. Some studies emphasize how understanding of money might inform financial decisions (e.g., Martin & Olivia, 2001).Others focus on teaching 'money skills' to students with special educational needs (Browder et al., 2008; Hordacre, 2016). A newer line of research focuses on opportunities for social justice education (c.f., Tanase & Lucey, 2015). For example, Twohill and Ní Shuilleabháin (in press) critique the 'business-centric' objectives of the 1999 Primary School Mathematics Curriculum (Government of Ireland, 1999). They argue that content objectives focused on unit price or profit and loss are not balanced with goals which consider distribution of wealth or the impact of prioritising profits over living wages.

Measurement in senior primary classes

At this stage, children need to understand how iterations of units can be represented on a standard scale and how to read scales marked in increments greater than one (Newell, 2017). Measuring scales effectively transform measurement of different attributes to a length measurement task whereby a length or height is read from a linear scale (Van den Heuvel-Panhuizen & Buys, 2008). Precision and accuracy is of increased importance in senior classes (Watson et al. 2013). As children are introduced to new measuring units, they must continue to build a system of personal reference measures, or benchmarks, with which measures can be estimated and results can be interpreted and evaluated (Van den Heuvel-Panhuizen & Buys, 2008). An understanding of the decimal structure of the metric system and of the connections between various metric units must be developed (Watson et al., 2013). The New Zealand Ministry of Education (NZME) specifies the application of multiplicative thinking to measurement as a key idea for upper primary (NZME, n.d.). Multiplicative thinking requires simultaneous coordination of number at a group and composite level (Clark & Kamii, 1996) and is fundamental to carrying out conversions between units (Earnest, 2017), a key goal for this age group. Understanding of ratio, which is embedded in measurement, is crucial to making the transition from additive to multiplicative thinking (Barrett et al., 2011; Sowder et al., 1998). Barrett et al. (2011) recommend that children are encouraged to explain the ratios that are implicit in any measurement, for example, noting that a board of length two metres is twice as long as a metre. Measures content often involves a progression from linear measures through square and cubic measures to compound measures, such as speed, which rely on two different types of measures (Watson, et al., 2013).

Measurement through reasoning alone becomes more prevalent in senior primary (van den Heuvel-Panhuizen & Buys, 2008). These authors note that questions such as, 'approximately how high is a building of six stories?' involve children using reasoning to develop solution methods. Children's reasoning will also come into play as they split complex shapes into component parts in order to calculate lengths, perimeter and area. Dissecting and reassembling shapes may help children explore, investigate and develop short-cuts for geometric measurement that might involve measurement formulae.

Tools and modeling

Tools are understood to encompass more than common measuring instruments and may also include physical materials, tables, pictures and symbols (Stephan, 2003). Thoughtful use of tools should be central to all mathematics teaching and is vital in the teaching of measurement, where competent use of specific measuring instruments is likely to be an instructional goal. Digital technologies are increasingly likely to be utilised and understanding of decimal fractions is necessary to interpret the measurements these tools provide (Watson et al., 2013). Tool-use does not automatically imply mathematical understanding (Dooley, 2019). Instead, tools become meaningful for children through joint activity (Askew, 2016). Children should have opportunities to select appropriate tools but the affordances and constraints of the tools made available must also be considered. Outhred and Mitchelmore (2000) discuss how tiles used to cover spaces in area measurement tasks can prestructure space, thereby obscuring relevant mathematical ideas such as the need for non-overlapping units and rectangular arrays. They suggest that there may be benefits to representing through drawing. This is supported by the recent work of Cullen et al. (2018) who found that recording the structure of 2-dimensional arrays by drawing organised rows and columns of equal-sized unit squares best supported children in understanding spatial structuring. In the context of time, Earnest (2017) describes how relationships between units are represented differently on various tools and details the mediating role of tool properties in relation to children's solving of elapsed time tasks. He recommends careful consideration of which tools to make available with attention to when, why, and how they will be introduced. ACME (2011) identify use of tools as one of a range of practices which students must develop to become proficient in measurement. Other notable practices within which tools might be used include real applications and modeling situations.

Modeling is one of the meta-practices identified in RR17 (Dunphy et al., 2014) as significant for the development of children's mathematical understanding. Measurement tools may themselves serve as models. For example, a ruler can be considered as a model of iterating a measurement unit. Length measurement can provide a meaningful context within which productive connections can be made to the empty number line, a model for reasoning about mental computation (Gravemeijer et al., 2003). On a related matter, Earnest's (2017) recent work posits that children's understanding of number line properties may prepare them for understanding features of the analogue clock where duration is reified as length and the treatment of unit and interval on the clock is consistent with that of a number line. Measures problems also offer scope for children to generate, reflect on and refine their own models in line with English and Sriraman's (2010) ideas (elaborated in Dooley, 2019). Siriman and Lesh (2006) maintain that Fermi problems can be used as a way to initiate mathematical modeling and to cultivate critical thinking. Fermi problems are called after the Nobel prize winner, Enrico Fermi, who was known for posing open problems which could only be solved by giving a reasonable estimate, e.g., 'How many piano tuners are there in Chicago?' (Peter-Koop, 2005). They are open, non-standard problems where the problem-solver must make assumptions about the problem situation and estimate quantities before carrying out calculations (Ärlebäck, 2009). Fermi problems offer potential for meaningful mathematisation and real-life connections (Peter-Koop, 2005). Fermi problems involving measures may include making estimates of water and oil consumption or mass of waste produced. Such problems have rich mathematical potential as well as possibilities for education about sustainability (Sriraman & Knott, 2009).

Key processes in measurement

In relation to the process of *understanding and connecting*, foundational ideas of measurement have been outlined above. The literature emphasises the importance of building relational understanding as well as procedural knowledge. As stated previously, there are many connections between measurement and other areas of mathematics. For example, see Lehrer et al., (2011) or English and Watson (2011) for meaningful linkage with statistics education. Facilitating children to make these connections will foster a rich, coherent understanding of mathematics. Possible connections between measurement and the child's environment, and between measurement and subject areas (integration) are also abundant. For example, measurement is fundamental to Science (Smith et al., 2011) but also essential to aspects of Geography and Visual Arts amongst other subjects. Enabling children to make such connections develops a sense that mathematics is meaningful and relevant, which contributes to a productive disposition. For further detail on the process of connecting, and mathematical processes more generally, see research report 18 (Dooley et al., 2014).

The process of *communicating* encompasses a range of different modes: oral, visual, textual, digital, pictorial, and symbolic (Dooley et al., 2014). Children must communicate the results of measuring activities using appropriate units and fractions/decimal fractions as necessary. Digital tools may be used or measurements might be recorded and/or presented using digital technologies, e.g., spreadsheets to record and analyse measurement data or dynamic geometry software to explore spatial measures. Oral communication continues to be an important way to develop and express understanding in senior primary school. Symbolic recordings might be accompanied with pictorial, textual or other modes that more fully document the measurement and/or problem-solving process.

The process of *reasoning* is central to meaning-making in mathematics. In measures contexts, reasoning comes into play when children use personal references or benchmarks to estimate, interpret and evaluate the reasonableness of measurements (van den Heuvel-Panhuizen & Buys, 2008). The role of reasoning in senior primary was described more fully above.

The process of *applying and problem solving* should be central to teaching measures as children should measure as a means to achieve a goal rather than as a goal in its own right (Clements & Sarama, 2009). RR18 (Dooley et al., 2014) suggests that play, modeling activities, project work and open-ended tasks might be used as contexts for problem-solving. Measurement, with its many connections to other mathematical domains as well as other areas of learning, is rich with opportunities for meaningful problem solving.

Key messages

The Primary School Mathematics Curriculum (Government of Ireland, 1999) divided the Measures strand into six strand units. Many of the key mathematical ideas underpinning the strand units of Length, Area, Weight and Capacity overlap. For this reason, the reimagining of this strand as consisting of Measuring, Time and Money (NCCA, 2017) is justified. There are particular challenges to the teaching of time and attention to the foundational ideas identified by Thomas et al. (2016) is recommended. The teaching of Money should build on and extend children's understanding of number concepts. Meaningful problem situations that acknowledge broader social implications are recommended.

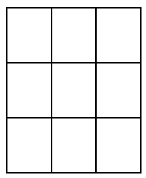
Some research challenges the traditional approach of using informal units for extended periods of time before introducing standard units. Cheeseman et al. (2012) document children's relatively quick transition to using standard units for mass (Dunphy et al., 2014) and discusses similar findings for length. However, recent research by Kobiela and Lehrer (2019) deliberately used a mixture of standard and non-standard units to problematise children's understanding of area as a product of lengths. It seems that non-standard units may serve to develop an understanding of the attributes and associated unit structure for some geometric measures. Further research across different domains is needed.

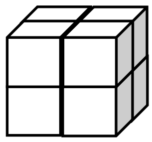
The progression from linear to square and cubic measures is generally supported by research (Watson et al. 2013). Capacity, or *volume-by-filling,* has been found to be more easily understood by children due to the fact that this essentially equates to a linear measurement task (van den Heuvel-Panhuizen & Buys, 2008). This supports the current practice of introducing capacity before *volume-by-packing* measurement. The importance of orchestrating opportunities for children to engage in spatial structuring, the construction and coordination of units and iterable units, is emphasised by many (Barret et al., 2009; Clements & Sarama, 2009; Curry & Outhred, 2005; Kobiela & Lehrer, 2019; Outhred & Mitchelmore, 2000). Such activities, along with a growing awareness of multiplicative structures, may facilitate conceptual understanding of measurement formulae. Furthermore, emphasis on the processes and practices of measurement (e.g., ACME, 2011) is appropriate. For this reason, specified curriculum content must be considered in the context of the mathematical processes described above.

Further implications for enactment might be gleaned from Cobb's (2003) design experiment on the teaching of linear measurement. Cobb proposes four interrelated means for supporting children's understanding of measurement: the instructional tasks, measurement tools, the nature of classroom discourse, and the classroom activity structure. In relation to instructional tasks, a problem-solving approach, linkage and integration are key. Problems which integrate with other subjects or link to other areas or mathematics or real-life will make measures relevant and also serve to promote opportunities for children to engage in key mathematical processes. Well-chosen problems, including child-generated problems, may also offer opportunities to connect the teaching of mathematics to broader educational aims including social justice and/or sustainability issues. Teachers must strive for more than procedural understanding of measurement instruments and must be alert to the ways in which tools shape children's opportunities to engage in mathematical thinking. Measures problems should also be used as opportunities for mathematical modeling. Aligning with the mathematical processes emphasised in the draft specification (NCCA, 2017), Cobb (2003) emphasises the importance of conceptual discourse where reasoning about measurement tasks is shared but suggests that simply sharing mathematical reasoning is not enough. Instead the classroom activity structure should incorporate a problem-solving approach in which the teacher strives to capitalise on ideas developed during individual or group work to lead a whole class discussion which focuses on mathematically significant ideas that advance the pedagogical agenda. In recognising mathematically significant ideas and setting the pedagogical agenda, teachers must have a good sense of the foundational measurement ideas and wider mathematical connections outlined above. If these are understood, then teachers will be better placed to work with children's developing ideas.

Glossary

Array: An array is an ordered arrangement of objects. A two-dimensional array consists of ordered rows and columns. A three-dimensional array consists of an ordered arrangement of objects in three dimensions.





Two-dimensional array ***Three-dimensional array***

Continuous Quantity: A continuous quantity is one which can always be divided into smaller pieces.

Conservation of Measure: An object may be manipulated or changed in some ways without changing the measure.

Iteration: This refers to repetition. In the context of measurement, the same unit must be repeated or iterated a number of times without gaps or overlaps so that it matches exactly what is being measured. An iterable unit is one which can be used in a repetitive fashion to find a measurement. For example, the squares and cubes shown in the diagrams above could be considered iterable units for area and volume measurement respectively.

Spatial Structuring: This refers to ways in which space might be structured or ordered. In the context of measurement, this structuring generally involves analysing space to consider how a unit might be iterated to find a measurement.

Temporal Patterns: Temporal patterns are recurring sequences of events. They include patterns noticeable in daily events such as the rising of the sun and patterns observable on a longer time-scale, such as seasonal patterns or yearly cycles of birthdays and special events.

Transitivity: Two measures may be compared indirectly using a third object.

References

Aprea, C., Wuttke, E., Breuer, K., Koh, N. K., Davies, P., Greimel-Fuhrmann, B., & Lopus, J. S. (2016). Financial literacy in the twenty-first century: An introduction to the international handbook of financial literacy. In *International handbook of financial literacy* (pp. 1-4). Springer: Singapore.

Askew, M. (2016). *Transforming primary mathematics: Understanding classroom tasks, tools and talk (revised edition).* Abingdon, Oxon: Routledge.

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast*, 6(3), 331–364.

Advisory Committee on Mathematics Education. (2011). *Mathematical needs: The mathematical needs of learners*. London: The Royal Society.

Barrett, J.E., Cullen, C., Sarama, J., Clements, D.H., Klanderman, D., Miller, A.L. & Rumsey, C. (2011). Children's unit concepts in measurement: a teaching experiment spanning grades 2 through 5. *ZDM, 43*, 637–650.

Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1-16.

Browder, D. M., Spooner, F., Ahlgrim-Delzell, L., Harris, A. A. & Wakemanxya, S. (2008). A meta-analysis on teaching mathematics to students with significant cognitive disabilities. *Exceptional children*, *74*(4), 407-432.

Cheeseman, J., McDonough, A. & Ferguson, S. (2014). Investigating young children's learning of mass measurement. *Mathematics Education Research Journal, 26*(2), 131-150.

Cheeseman, J., McDonough, A. & Ferguson, S. (2012). The effects of creating rich learning environments for children to measure mass. In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 178–185). Singapore: MERGA.

Clark, F., & Kamii, C. (1996). Identification of Multiplicative Thinking in Children in Grades 1-5. *Journal for Research in Mathematics Education, 27*(1), 41-51.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach* (1st ed.). New York, NY: Routledge.

Clements, D., Wilson, D. C., & Sarama, J. (2002). Young children's composition of geometric figures: A learning trajectory. *Mathematical Thinking and Learning*, *6*(2), 163-184.

Cobb, P. (2003). Investigating students' reasoning about linear measurement as a paradigm case of design research. *Journal for Research in Mathematics Education*, *12*, 1-16.

Cullen, A. L., Eames, C. L., Cullen, C. J., Barrett, J. E., Sarama, J., Clements, D. H., & Van Dine, D. W. (2018). Effects of three interventions on children's spatial structuring and coordination of area units. *Journal for Research in Mathematics Education*, *49*(5), 533-574.

Curry, M., & Outhred, L. (2005). Conceptual understanding of spatial measurement. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building* *connections: Theory, research and practice (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia)* (pp.265-272). Sydney: MERGA.

Dooley, T. (2019). Learning and teaching primary mathematics: An addendum to NCCA research reports 17 and 18. Retrieved from: https://www.ncca.ie/media/4087/primary_maths_research_addendum_2019.pdf

Earnest, D. (2017). Clock work: How tools for time mediate problem solving and reveal understanding. *Journal for Research in Mathematics Education*, *48*(2), 191-223.

English, L., & Watson, J. (2015). Exploring variation in measurement as a foundation for statistical thinking in the elementary school. *International Journal of STEM Education. 2*(1). doi: 10.1186/s40594-015-0016-x.

Foster, C. (2011). Productive ambiguity in the learning of mathematics. *For the Learning of Mathematics*, *31*(2), 3-7.

Fraisse, P. (1984). Perception and estimation of time. *Annual Review of Psychology*, *35*(1), 1-37.

Government of Ireland (1999). *Primary school curriculum: Mathematics*. Dublin, Ireland: Stationery Office.

Gravemeijer, K. P. E., Bowers, J., & Stephan, M. L. (2003). A hypothetical learning trajectory on measurement and flexible arithmetic. *Journal for Research in Mathematics Education, 12*, 51–66.

Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.

Hordacre, A.L. (2016). *Understanding everyday money skills for young people with disabilities*. Adelaide: Australian Industrial Transformation Institute, Flinders University of South Australia.

Kamii, C., & Russell, K. A. (2012). Elapsed time: why is it so difficult to teach? *Journal for Research in Mathematics Education*, *43*(3), 296-315.

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179–192). Reston, VA: National Council of Teachers of Mathematics

Lehrer, R., Kim, M.J., & Jones, R.S. (2011). Developing conceptions of statistics by designing measures of distribution. *ZDM*, *43*, 723-736.

Liu, C., Carraher, D. W., Schliemann, A. D., & Wagoner, P. (2017). Learning to quantify relationships among weight, size, and kind of material. *Cognition and Instruction*, *35*(1), 20-54.

Lusardi, A. (2012). Numeracy, financial literacy, and financial decision-making. *Numeracy 5*(1), 2-14

Martin, A., & Oliva, J. C. (2001). Teaching children about money: Applications of social learning and cognitive learning development theories. *Journal of Family and Consumer Sciences*, *93*(2), 26-29.

Newell, R. (2016). *Big ideas in primary mathematics*. London, UK: Sage.

New Zealand Ministry of Education, (n.d.). Key mathematical ideas of measurement levels 3 and 4. Retrieved from: https://nzmaths.co.nz/key-mathematical-ideas

Outhred, L. N., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education*, *31*(2), 144-167.

Peter-Koop, A. (2005). Fermi Problems in Primary Mathematics Classrooms: Fostering Children's Mathematical Modelling Processes. *Australian Primary Mathematics Classroom*, *10*(1), 4-8.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry*. New York: Basic Books.

Radford, L. (2003). On the epistemological limits of language: Mathematical knowledge and social practice during the Renaissance. *Educational Studies in Mathematics, 52*(2). 123-150.

Shiel, G., Kavanagh, L., & Millar, D. (2014). *The 2014 National Assessments of English Reading and Mathematics. Volume 1: Performance report*. Dublin: Educational Research Centre.

Smith, J. P., van den Heuvel-Panhuizen, M., & Teppo, A. (2011). Learning, teaching, and using measurement: introduction to the issue. *ZDM*, *43*, 617-620.

Sowder, J., Armstrong, B., Lamon, S., Simon, M., Sowder, L., & Thompson, A. (1998). Educating teachers to teach multiplicative structures in the middle grades. *Journal of Mathematics Teacher Education*, *1*(2), 127-155.

Stephan, M. (2003). Reconceptualizing linear measurement studies: The development of three monograph themes. *Journal for Research in Mathematics Education. 12*, 17-35.

Sriraman, B., & Lesh, R. A. (2006). Modeling conceptions revisited. *ZDM*, *38*(3), 247-254.

Sriraman, B., & Knott, L. (2009). The mathematics of estimation: Possibilities for intedisciplinary pedagogy and social consciousness. *Interchange*, *40*(2), 205-223.

Tanase, M., & Lucey, T. A. (2015). Interdisciplinary connections: teaching mathematics for social justice and financial literacy. *Journal of Mathematics & Culture*, *9*(1), 108-118.

Thomas, M., Clarke, D. M., McDonough, A., & Clarkson, P. (2016). Understanding time: A research based framework. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia)* (pp. 592-599). Adelaide, Australia: MERGA.

Twohill, A., & Ní Shuilleabháin, L. (in press). Connecting social justice education with the teaching of mathematics. In A. M. Kavanagh & F. Waldron (Eds.), *Teaching for social justice and sustainable development across the primary curriculum*. Abingdon, Oxon: Routledge.

Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based guidance for ages 9-19*. Oxford, UK: Oxford University Press

van den Heuvel-Panhuizen, M., & Buys, K. (2008). *Young children learn measurement and geometry: A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school*. (Vol. 3). Rotterdam, The Netherlands: Sense Publishers.

NCCA