

Datestamping the Bitcoin and Ethereum Bubbles

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Abstract

We examine the existence and dates of pricing bubbles in Bitcoin and Ethereum, two popular cryptocurrencies using the Phillips et al. [2011] methodology. In contrast to previous papers, we examine the fundamental drivers of the price. Having derived ratios that are economically and computationally sensible, we use these variables to detect and datestamp bubbles. Our conclusion is that there are periods of clear bubble behaviour, with Bitcoin now almost certainly in a bubble phase.

Keywords: Cryptocurrencies; Digital Assets; Bitcoin; Ethereum; bubbles

JEL Codes: C58, G10, G14,

1. Introduction

Are there bubbles in cryptocurrencies? If so, when do they manifest? Or are the rises due to more fundamental characteristics of the assets construction. We examine two of the largest, Bitcoin and Ethereum. The European Central Bank [2012] found that cryptocurrencies do not jeopardise financial stability, due to their limited connection to the real economy, the low volumes traded and the lack of wide user acceptance. This ECB conclusion in 2012 was associated with the caveat that the growth of cryptocurrency markets and their integration to the global economy must be monitored, since cryptocurrencies remain the potential source of financial instability. In the five years since the release of this report, the market for cryptocurrencies has evolved significantly. Figure 1 portrays the substantial price appreciation that has been observed in Bitcoin since 2013. To provide assurances and clarity with regards to broad financial stability, it is important to clarify as to whether this substantial increase in price has been predominantly driven by underlying fundamentals, or whether it can be denoted as a speculative bubble.

The aim of this paper is to test whether underlying fundamentals relating to both Bitcoin and Ethereum, denoted as the blockchain position, the hashrate and liquidity as measured by the volume of daily transactions, can be designated as drivers of price growth since the inception of both cryptocurrencies. Having derived ratios that are economically and computationally sensible we then use these measures to detect and datestamp bubbles. The three measures were selected to best represent the key theoretical components of cryptocurrency pricing structures. The first measure relates to mining difficulty reflects how difficult it is to find a new block relative to the easiest that it could be in the past. As more miners join, the rate of block creation will increase, which causes the difficulty to increase in compensation to push the rate of block creation back down. The second measure relates to hashrate which is the speed at which a computer is completing an operation in the Bitcoin code. A higher hashrate when mining increases your opportunity of finding the next block and receiving payments. Finally, the relationship between cryptocurrency returns, volatility and liquidity has been established by Donier and Bouchaud [2015] and Balcilar et al. [2017].

This paper is structured as follows: Section 2 describes the selected methodology. Section 3 describes the datestamping procedures and analyses the results of our test for bubbles in the Bitcoin and Ethereum markets. Finally, Section 4 present our conclusions.

2. Methodology

2.1. Testing for Bubbles

Bitcoin is a peer-to-peer digital asset, which claims to be decentralised and independent of monetary authority influence (Nakamoto [2008]). Transactions take place directly between users, and are verified by network nodes. Kroll et al. [2013], provides a detailed description of the mining process. Miners add verified transactions to a publicly distributed ledger, or blockchain, and are incentivised to do so by the reward of transaction fees and new bitcoins. Böhme et al. [2015] provide a detailed description of the technology behind Bitcoin, including: the blockchain, mining, mining pools, transaction fees and wallets. The authors also detail the early use cases of Bitcoin, areas of risk involved, and examine the potential for future regulation.

Despite the very large volume of commentary, scholarly research is surprisingly scant on the existence of bubbles in cryptocurrencies. Garcia et al. [2014] examines feedback loops in social media to Bitcoin price, but presupposes the existence of bubbles. Similarly, Kristoufek [2015] also pre-supposes a bubble. Both of these take their inspiration from Shiller et al. [1984]. More formal testing based on economic fundamentals does exist. Alabi [2017] adopts a very different approach, using ideas from network theory to find periodically collapsing bubbles. The idea motivating this paper is that as a network the number of users is a key determinant of value to the users. Formally speaking a bubble is where the value of an economic asset deviates, persistently, from fundamental values (Diba and Grossman [1988]). Thus to have any hope of assessing the presence or otherwise of a bubble some evaluation must take place of the fundamentals.

Cheah and Fry [2015] test for evidence of speculative bubbles in Bitcoin returns, using two assumptions for intrinsic rate of return and intrinsic level of risk. The authors find evidence to suggest that Bitcoin prices are prone to substantial bubbles. In addition, they empirically estimate the value of Bitcoin to be zero. In a more recent study, Fry and Cheah [2016] the authors uncover evidence of a spillover from Ripple to Bitcoin, which exacerbates price-falls in Bitcoin. Such an effect raises concerns about the long-term sustainability of Bitcoin, with regard to increased competition from rival cryptocurrencies. The authors also examine the effect of a number of external events on the Bitcoin market (a technical software glitch in 2013; the closure of the Silk Road website in 2013) and find that they brought about an end in the speculative bubble, a scenario which was repeatedly observed during the dot-com bubble. More recently, Corbet et al. [2017a] analyse in

both the time and frequency domain the relationships between three popular cryptocurrencies and a variety of other financial assets, finding evidence that support the view that cryptocurrencies may offer diversification benefits for investors with short investment horizons. Time variation in the linkages reflects external economic and financial shocks. Blau [2017] argued that high volatility of Bitcoin is not related to the high speculative activity in this period. The ambiguity of the results exemplifies the debates about whether the cryptocurrencies are a speculative investment asset or a currency. To be considered as a currency (i.e. money), cryptocurrencies should serve as a medium of exchange, be used as a unit of account, and allow to store value; however, cryptocurrencies are barely managing to fulfil all those properties (Bariviera et al. [2017]). Urquhart [2017] measures the efficiency of Bitcoin returns over a six-year period (between August 2010 and July 2016) using a number of tests for randomness (Ljung-Box, Runs, Bartels, Automatic variance test, BDS, R/S Hurst tests). Randomness test are utilised due to the fact that, in efficient markets, prices follow a random walk. The authors find returns to be significantly inefficient over the entire sample. When divided into two equal sub-samples, two tests indicate efficiency of returns in the latter sample, suggesting that Bitcoin may be moving towards becoming more efficient. Corbet et al. [2017b] examined the reaction of a broad set of digital assets to US Federal Fund interest rate and quantitative easing announcements providing evidence of differing volatility reactions, indicating a diverse market in which not all cryptocurrencies are comparable to Bitcoin. Baek and Elbeck [2015] find evidence to suggest that Bitcoin returns are driven by buyers and sellers internally, and not by fundamental economic factors. Using de-trended ratios, the authors determine Bitcoin returns to be 26 times more volatile than those of the S&P 500 index, suggesting that Bitcoin is a speculative investment vehicle. The authors however, determine that this classification may change as usage grows, volatility decreases and Bitcoin attracts market and economic influence. In doing so, Bitcoin may become a more balanced investment vehicle, driven both internally and externally. Cheung et al. [2015] perform an econometric investigation of bubbles in the Bitcoin market, using the Phillips et al. [2015] methodology (a technique which has proven to be robust in detecting bubbles). Using this method, the authors detect a number of short-lived bubbles, and three large bubbles (2011-2013) lasting from 66 to 106 days. The bursting of these bubbles is found to coincide with a number of major events that occurred in the Bitcoin market as the most significant of these leading to the demise of the Mt. Gox exchange.

2.2. Data

We source our data from historical API's (application programming interfaces¹) for the period between 9 January 2009 and 9 November 2017 resulting in 3,227 observations based on the fundamentals of Bitcoin. Bitcoin pricing data is used after the 18 July 2010 due to a significant number of missing observations in the dataset due to periods of reduced liquidity in the growth of the cryptocurrency. Ethereum was initially released on the 30 July 2015 and we have analysed a complete dataset between 7 August 2015 and 9 November 2017 representing 826 observations.

The cryptocurrency market is a market that operates day round, year round, so we have no gaps. Figure 1 plots the time series trajectories of Bitcoin and Ethereum. The evolution of the recent price increases in both cryptocurrencies has been significant. It is important to note that the price of one Bitcoin did not increase above \$1 until the 16 April 2011. On the 3 June 2011, it increased above a value of \$10 and traded below that level for the following year. In early 2013, there was substantial momentum evident and the price of Bitcoin first closed above \$100 on 2 April 2013. Between 30 October 2013 and 29 November 2013, the explosive nature of cryptocurrency prices were first evident as within 31 days the price of Bitcoin grew from closing at \$201.50 to that of \$1,049.35 per Bitcoin. Although the price reversed and fell to levels below \$300 in the following three years, on 3 January 2017, the price once again breached \$1,000 and a month later was closing daily above this substantial threshold. By 21 May 2017, Bitcoin closed above \$2,000. By the 14 August 2017, it was valued at \$4,000 and by 4 November 2017 it cost \$7,000 per Bitcoin. Table 1 presents descriptive statistics of the volatility and price levels of both Bitcoin and Ethereum.

2.3. The Phillips et al. Methodology

Phillips et al. [2011] proposed a recursive ex ante method that is found to be capable of detecting exuberance in asset price series during inflationary periods that is capable of acting as an early warning system. Our selected research methodology is based upon the work of Phillips and Yu [2011] who developed a methodology that analysed the house price bubble of the 2000's in the United States and Phillips et al. [2015] who developed on previous work when allowing for flexible window widths in the recursive regressions on which the testing procedures are based using a sup ADF (SADF) to test for the presence of a bubble through the inclusion of a sequence of

¹An API is a set of functions and procedures that allow the creation of applications which access the features or data of an operating system, application or other service.

forward recursive right-tailed ADF unit root tests. The next stage was to develop a dating strategy which identifies the origination and termination points of a bubble based on a backward regression technique². When multiple episodes of exuberance and collapse are included in the investigated sample which is common during rapidly changing market conditions, the generalised sup ADF (GSADF) methodology is selected to test for the presence of bubbles as well as a recursive backward regression technique to time-stamp the bubble within the data. The GSADF test extends the sample coverage by changing the starting point and end point of the recursion over a feasible range of flexible windows, becoming a right-sided double recursive test for a unit root.

We follow the asset pricing equation of Phillips et al. [2015] which accommodates other bubble-generating mechanisms such as intrinsic bubbles (Froot and Obstfeld [1991]), herd behaviour (Abreu and Brunnermeier [2003]) and time-varying discount factor fundamentals (Phillips and Yu [2011]). As a starting point for our analysis of potential Bitcoin and Ethereum pricing bubbles we use:

$$P_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f} \right)^i E_t(U_{t+i}) + B_t \quad (1)$$

P_t represents the price of the cryptocurrency, r_f is the risk-free interest rate, U_t represents the unobservable fundamentals and B_t is the bubble component. The quantity $P_t^f = P_t - B_t$ represents the market fundamental and B_t satisfies the submartingale property

$$E_t(B_{t+1}) = (1+r_f)B_t \quad (2)$$

When $B_t = 0$ there is no bubble present and the degree of nonstationarity of the asset price is controlled by unobservable fundamentals, where asset prices will be explosive in the presence of bubbles. Phillips and Magdalinos [2007] stated that no matter what unobservable fundamentals were fuelling the origins of such observed bubbles, explosive or mildly explosive behaviour in asset price can be considered a primary indicator of market exuberance during the inflationary phase of a bubble³. Model specification under the null hypothesis is important for estimation purposes. Phillips et al. [2014] studied issues in right-tailed unit root tests of the type used in bubble detection,

²Other break testing procedures such as Chow tests, model selection, and CUSUM test may also be applied as a dating mechanism

³It is this time series manifestation that may be subjected to econometric testing using recursive testing procedures such as the right-sided unit root tests used by Phillips et al. [2015]

but their analysis allowed for a martingale null hypothesis with an asymptotically negligible drift to capture the mild drift in price processes using the following:

$$y_t = dT^{-\eta} + \theta Y_{t-1} + \varepsilon_t, \varepsilon_t \sim^{iid} (0, \sigma^2), \theta = 1, \quad (3)$$

where d is a constant, T is the sample size and the parameter η is a localising coefficient that controls the magnitude of the intercept and drift as $T \rightarrow \infty$. The model specification is usually complimented with transient dynamics in order to conduct tests for exuberance similar to that of standard AFD unit root testing against stationarity. A rolling window ADF style regression is implemented on such a system as in Phillips et al. [2015]. We start the rolling window regression from the r_1^{th} fraction of the total sample (T) and ends at the r_2^{th} of the sample, where $r_2 = r_1 + r_w$ and $r_w > 0$ is the window size of the regression. The empirical regression model can then be written as:

$$\Delta t_t = \hat{\alpha}_{f1,f2} + \hat{\beta}_{f1,f2} \cdot y_{t-1} + \sum_{i=1}^k \hat{\psi}_{f1,f2}^i \Delta y_{t-i} + \hat{\epsilon}_t \quad (4)$$

where k is the transient lag order. The ADF statistic (t-ratio) based on this regression is denoted by ADF_{f1}^{f2} . We then use rolling regressions of this type for bubble detection. The test relies on repeated estimation of the ADF model on a forward expanding sample sequence and the test is obtained as the sup value of the corresponding ADF statistic sequence. The window size r_w expands from r_0 to 1, where r_0 is the largest window fraction in the recursion. The starting point r_1 of the sample sequence is fixed at 0, so the endpoint of each sample (r_2) equals r_w and changes from r_0 to 1. The ADF statistic for a sample that runs from 0 to r_2 is denoted by ADF_0^{f2} . The test proposed by Phillips et al. [2015] is then a sup statistic based on the forward recursive regression and is simply defined as:

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{f2} \quad (5)$$

We develop upon the GSADF test and repeat the ADF test regressions of Equation (4) on subsamples of the data in a recursive fashion. We define the GSADF statistic to be the largest AFD statistic in this double recursion over all feasible ranges of r_1 and r_2 and we denoted this statistic by $GSADF(r_0)$, namely:

$$SADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} ADF_{f1}^{f2} \quad (6)$$

The policy implementation of this methodology is concerned with the provision of date-stamped evidence of financial exuberance in our selected cryptocurrencies, specifically as to whether any particular observations such as $\tau = [Tr]$ belong to a bubble phase in the overall trajectory. Since it is possible that the data $I_{[Tr]}$ may include one or more collapsing bubble episodes where the ADF test may result in finding pseudo stationary behaviour. To counteract this issue, we use a double recursive test procedure that Phillips et al. [2015] denoted as a backward sup ADF test on $I_{[Tr]}$ to enhance identification accuracy. The backward SADF test performs a sup ADF test on a backward expanding sample sequence where the end point of each sample is fixed by r_2 and the start point varies from 0 to $r_2 - r_0$. The backward SADF statistic is then defined as the sup value of the ADF statistic sequence over this interval:

$$BSADF_{f2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{f1}^{f2} \quad (7)$$

The backward SADF test provides more information and improves detective capacity for bubbles within the sample because the subsample that gives rise to the maximum ADF statistic may not have the same generating mechanism as other observations within the full sample from $r_1 = 0$ to r_2 which offers greater flexibility in the detection of multiple bubbles. Phillips et al. [2015] impose a condition that for a bubble to exist its duration must exceed a slowly varying quantity such as $L_T = \log(T)$. This requirement helps to exclude short lived blips in the fitted autoregressive coefficient and can be adjusted to take into account the data frequency. The data estimates are then delivered by the crossing time formulas:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : ADF_{r2} > cv_{f2}^{\beta_T} \right\} \quad (8)$$

$$\hat{r}_f = \inf_{r_2 \in [r_e + \delta \log(T)/T, 1]} \left\{ r_2 : ADF_{r2} < cv_{f2}^{\beta_T} \right\} \quad (9)$$

where $cv_{f2}^{\beta_T}$ is the $100(1 - \beta_T)\%$ critical value of the ADF statistic based on $[Tr_2]$ observations. The significance level β_T depends on the sample size T and it is assumed that $\beta_T \rightarrow 0$ as $T \rightarrow$

∞ . Under the new identification strategy, inference about potential explosiveness of the process at observation $[Tr_2]$ is based on the backward sup ADF statistic $BSADF_{f_2}(r_0)$. We define the origination date of the bubble as the first observation whose backward sup ADF statistic exceeds the critical value of the backward sup ADF statistics. The termination date of a bubble is calculated as the first observation after $[Tr_e] + \delta \log(T)$ whose backup sup ADF statistic falls below the critical value of the backward sup ADF statistic. For a bubble to be defined it is assumed that its duration should exceed a minimal period represented by $\delta \log(T)$, where δ is a frequency-dependent parameter. The origination and termination points of a bubble are therefore calculated according to the following crossing time equations:

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) > cv_{f_2}^{\beta_T} \right\} \quad (10)$$

$$\hat{r}_f = \inf_{r_2 \in [r_e + \delta \log(T)/T, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < cv_{f_2}^{\beta_T} \right\} \quad (11)$$

The SADF test is based on repeated implementation of the ADF test for each $r_2 \in [r_0, 1]$. The GSADF test implements the backward sup ADF test repeatedly for each $r_2 \in [r_0, 1]$ and makes inferences based on the sup value of the backward sup ADF sequence $BSADF_{r_2}(r_0)$. Hence, the SADF and GSADF statistic can, be written as:

$$SADF(r_0) = \sup_{r_1 \in [r_0, 1]} (ADF_{f_2}) \quad (12)$$

$$GSADF(r_0) = \sup_{r_1 \in [r_0, 1]} (BSADF_{f_2}(r_0)) \quad (13)$$

The new date-stamping strategy may be used as an ex ante real-time dating procedure, whereas the GSADF test is an ex post statistic used for analysing a given data set for bubble behaviour⁴.

⁴See Phillips et al. [2015] for a thorough explanation of the asymptotic distributions of the ADF and SADF statistics and the processes for identifying no bubbles, one bubble and multiple bubbles

3. Datestamping Cryptocurrency Bubbles

Shown in Figure 2 are indicators of normalized statistics for Bitcoin and in Figure 3 for Ethereum. These are normalized as being the ratio of the backwards SADF calculated statistic to the simulated critical value, all less one to centre at zero. This has the ease of interpretation of a bubble being suggested when this value is > 0 . We construct a dummy variable for ease of graphical presentation. A bubble, recall, is where an asset price diverges from fundamentals. In this context then a bubble would be indicated when the price series is identified as a bubble but the fundamental drivers are not so identified. We observe very few consistent periods of bubbles indicated. Bubbles, as indicated by this methodology, are fleeting. Most bubbles are indicated as from the price alone. However, we have some periods where each of the other fundamental drivers show a bubble.

There are two distinct periods when some bubbles appear. Bitcoin shows possible bubble like behaviour around the 2013/2014 turn of the year. This comes after the seizure of large numbers of Bitcoin from the Silk Road website, the declaration by a US federal Judge (fifth circuit) and the German tax authorities that Bitcoin shared characteristics with functional currencies. What is intriguing is that the recent explosive growth in Bitcoin prices has not been accompanied by sustained bubble signals. Since Bitcoin broke through the \$1,000 barrier in early 2017, there have been distinct periods denoted as bubbles. However, in Figure 2 we also observe time-periods when the price of Bitcoin was above \$2,000 and the methodology denoted this price growth to be not a bubble. It would be reasonable to conclude from this that there is indeed a bubble in the present Bitcoin price.

For Ethereum, we observe even scantier signals of bubbles. Such as they are are concentrated in the early 2016 and mid-2017 periods. The hard fork in mid-2017 appears not to have signalled as a sustained pricing bubble during the first phase of growth in price between \$50 and \$400. After a subsequent reversal to \$160 within one month of Ethereum's lifetime high, the price subsequently doubled. This second phase of growth is shown to be best described as fundamentally-driven price appreciation. The bursting of the Ethereum bubble has resulted, it seems, in a price dynamic which is not prone to explosive bubbles.

4. Conclusions

This paper provides insight into the relationship between the relationship of cryptocurrency pricing discovery and internal fundamental explanatory variables that can generate the conditions and environment in which a pricing bubble can thrive. Based on the above presented analysis, we conclude that there is no clear evidence of such a persistent bubble in the market for both Bitcoin or Ethereum. This does not imply that the price is "correct", merely a statistical indicator being absent. Considering the theoretical interlinkages between the price of both Bitcoin and Ethereum and their relationship with blockchain position, hashrate and liquidity respectively, we can state that there are distinct short-term time period in which each fundamental influences the price dynamics of both cryptocurrency, however, these effects dissipate quickly. However, we do find evidence that supports the view that Bitcoin is currently in a bubble phase and has been since the price increased above \$1,000.

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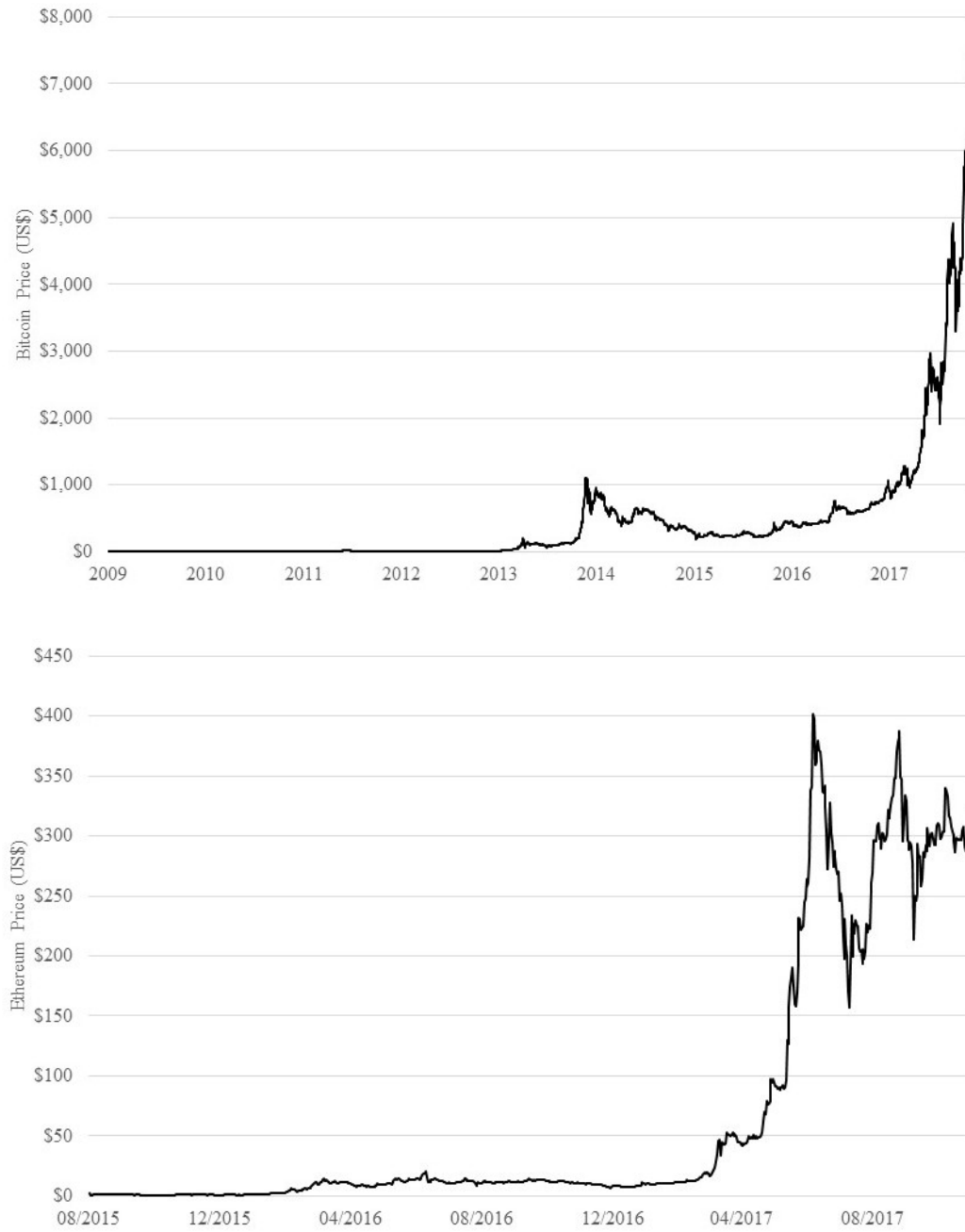
Table 1: Descriptive Statistics for Bitcoin and Ethereum

<i>Bitcoin</i>				
	Daily Volatility (%)	BTC Hashrate (Th/s)	BTC Block Size (bytes)	No. of Transactions (per day)
Mean	0.0058	753,372.25	287,508.56	83,739.22
Min	-0.3365	0.00	215.00	4.00
Max	0.3895	12,999,790.28	998,175.24	369,098.00
Standard Deviation	0.0522	1,737,311.08	331,473.01	96,725.38
Skewness	1.3114	3.2773	0.9367	1.0856
Kurtosis	16.3402	11.6487	-0.6071	-0.0865

<i>Ethereum</i>				
	Daily Volatility (%)	BTC Hashrate (Th/s)	BTC Block Size (bytes)	No. of Transactions (per day)
Mean	0.0092	20,388.16	3,924.89	96,746.27
Min	-0.7280	89.61	618.00	1,329.00
Max	0.5103	111,176.92	22,603.00	546,837.00
Standard Deviation	0.0805	32,286.69	5,179.59	124,614.69
Skewness	0.2245	1.7060	1.9890	1.7732
Kurtosis	13.4198	1.3902	2.7978	2.0910

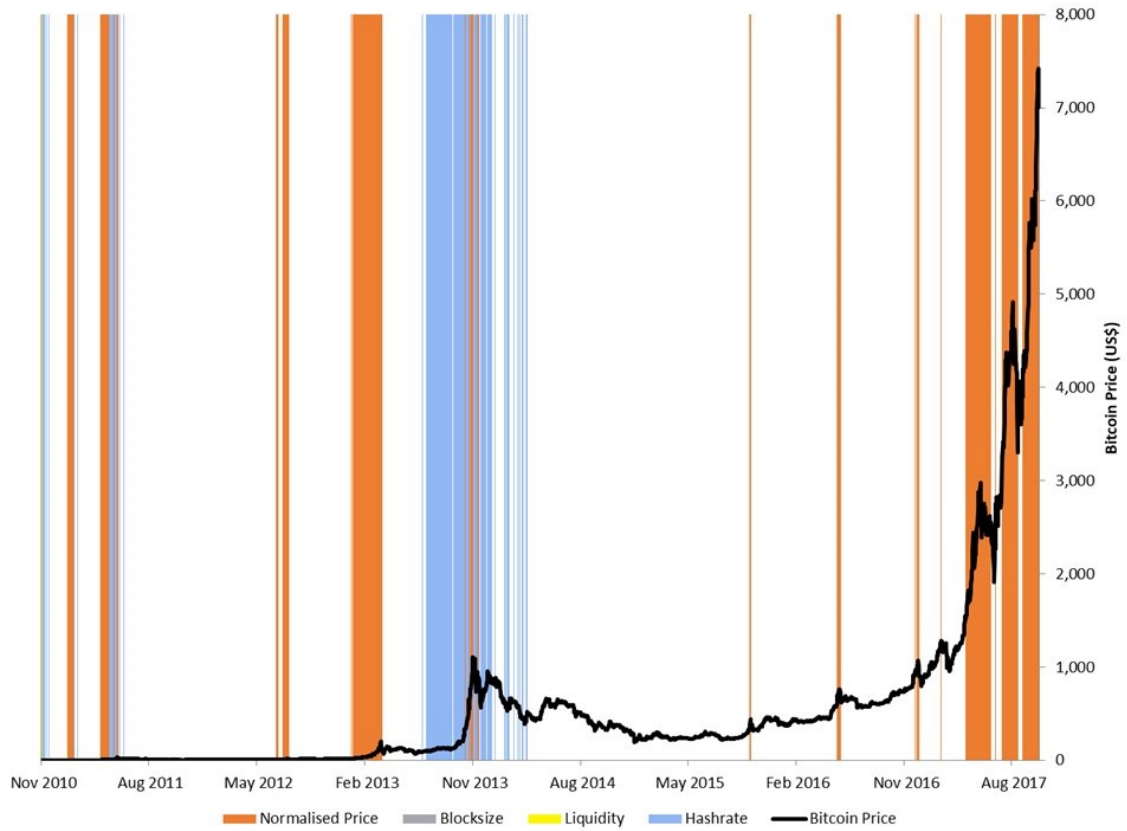
Note: Descriptive statistics of series under investigation include that of Bitcoin for the period between 9 January 2009 and 9 November 2017 resulting in 3,227 observations. Bitcoin pricing data is used after the 18 July 2010 due to a significant number of missing observations in the dataset due to periods of reduced liquidity in the growth of the cryptocurrency. Ethereum was initially released on the 30 July 2015 and we have analysed a complete dataset between 7 August 2015 and 9 November 2017 representing 826 observations.

Figure 1: Bitcoin and Ethereum Prices (US\$)



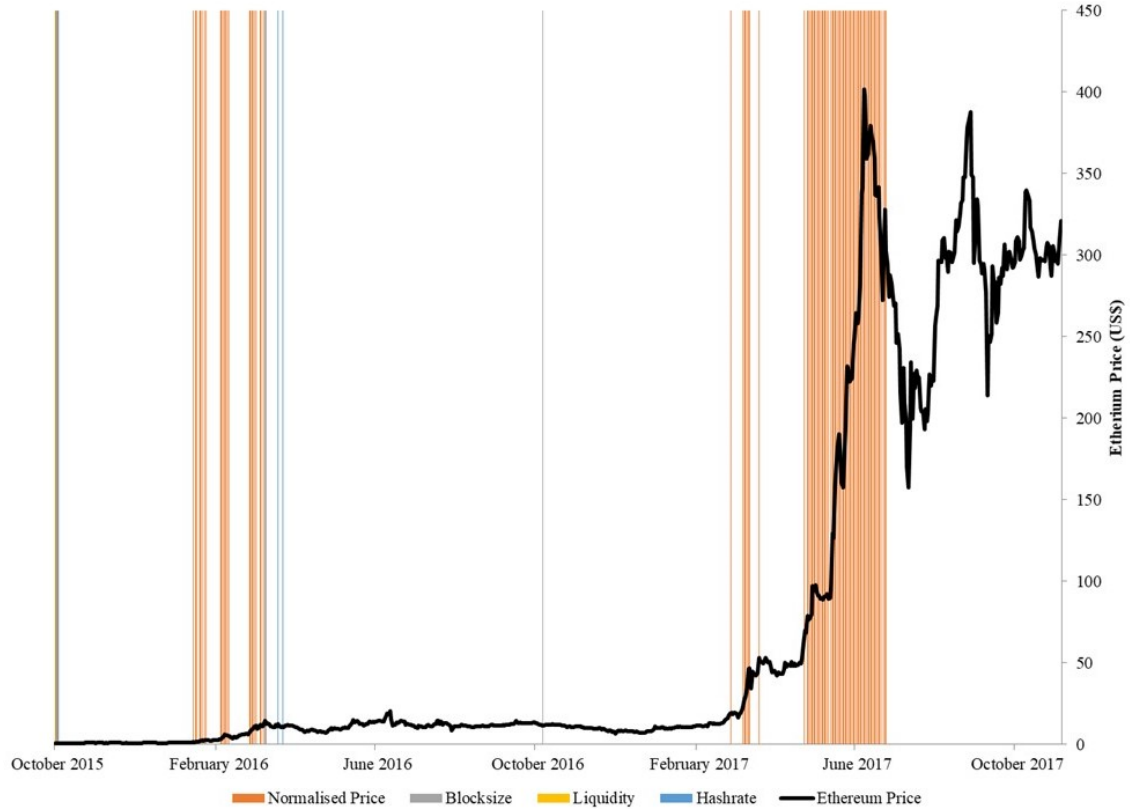
Note: Bitcoin prices (top-panel) are presented for the period between 9 January 2009 and 9 November 2017. Ethereum prices (bottom-panel) are presented for the period between 30 July 2015 and we have analysed a complete dataset between 7 August 2015 and 9 November 2017 representing 826 observations.

Figure 2: Datestamping the Bitcoin Bubbles



Note: This graphic shows the evolution of the Bitcoin price, overlaid on a series of dummy variables. These take the value of 1 when the ratio of [calculated BSDAF statistic / simulated critical value -1] exceeds zero. The dummy variables are calculated on the tests for the price alone, *Price_Normalized*; blocksize, *Blocksize_Normalized*; Volume, *Volume_Normalized* and Hashrate, *Hashrate_Normalized*

Figure 3: Datestamping the Ethereum Bubbles



Note: This graphic shows the evolution of the Ethereum price, overlaid on a series of dummy variables. These take the value of 1 when the ratio of [calculated BSDAF statistic / simulated critical value -1] exceeds zero. The dummy variables are calculated on the tests for the price alone, *Price_Normalized*; blocksize, *Blocksize_Normalized*; Volume, *Volume_Normalized* and Hashrate, *Hashrate_Normalized*