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The journey or the destination? An investigation into the beliefs of pre-service post-primary mathematics teachers regarding problem-solving

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ABSTRACT

It has been widely reported that the affective domain is an important contributor to problem-solving behaviour among students. Cognitive resources available to students are related to the students' beliefs around what they consider useful in learning maths (Schoenfeld, 1983). Problem-solving holds a key position in both Junior Cycle and Senior Cycle curricula in Ireland. Given that much research has shown that the teachers' beliefs about problem-solving play an integral role in building positive attitudes to problem solving among their students, it is essential to investigate the beliefs of prospective mathematics teachers. The aim of this study was to investigate the affective domain of pre-service post-primary mathematics teachers in Ireland. This study was conducted in a university setting and involved the implementation of both quantitative and qualitative measures; the existing Indiana Mathematical Belief Scale (IMB), and open-ended questionnaires. Participants in the survey were enrolled in a module on mathematical problem solving. The open-ended questionnaire asked participants to describe how they felt at different stages during their attempt to solve a mathematical problem. This was then analysed using an inductive approach. A statistical analysis of the IMB (n=151) showed that students strongly believed that an increase in effort can have a positive influence on mathematical ability. However, it was concerning to find that students believe that problem-solving involves learning step-by-step procedures. Another positive finding of the IMB was that students strongly value the understanding of mathematical concepts over memorization of procedures. In contradiction to this, it was found through the analysis of the open-ended questionnaire that students had a greater focus on achieving an answer rather than on the problem-solving process. We discuss the implications for the design of the module, which seeks to support the development of the capacities required for the successful teaching of mathematical problem solving.

KEYWORDS

Mathematical problem-solving, affective domain, prospective post-primary teachers

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Background

Problem-solving has always been a part of mathematics, but the formal study of problem-solving has a shorter history with a prominent role in the earliest stages taken by Polya (1945). Schoenfeld (1985, p. 69) states that Polya's work on problem-solving "is held in high regard by both mathematicians and mathematics educators". Schoenfeld (1992) attests that there is a wide variety of meanings for the terms "problems" and "problem solving": this has been highlighted more recently by Lester (2013). It has been highlighted that with the focus on developing problem solving, there is confusion regarding the actual definition of problem-solving (Stanic & Kilpatrick, 1989). Acknowledging the lack of clarity on the definition of problem-solving but recognising the need for clarity, from a review of the literature there appears to be consensus on the following aspects of problem-solving (Chamberlin, 2008; Lester & Kehle, 2003):

1. Problem-solving includes a goal;
2. It is not immediately clear how to achieve the goal;
3. The problem-solver must organize prior knowledge to generate reasoning towards achieving the goal.

Throughout this paper, we will refer to these as the Three Key Characteristics of problem solving.

The lack of agreement described above must be set against the widespread acknowledgement of the importance of problem solving in the mathematics curriculum both nationally and internationally (Conway & Sloane, 2006). In Ireland, problem-solving is one of the six elements of the Unifying Strand of the Junior Cycle specifications and is specifically mentioned in each strand of the Senior Cycle mathematics syllabus. Similarly, this privileged position that problem-solving holds is evident internationally (Cheng, 2001; DfE, 2013; Mullins et al., 2016).

The role of the affective domain on problem-solving

Lester (2013) notes that it is widely agreed that the development of students' problem-solving capabilities is a main goal of mathematics instruction. The realisation of this goal involves multiple factors such as metacognition and beliefs along with factors associated with the teacher (Schoenfeld, 1992). It has been widely reported that the affective domain is an important contributor to problem solving behaviour (Lester & Kroll, 1993). McLeod (1988) set out to provide a theoretical framework for investigating the affective factors that are associated with problem solving. McLeod (1988) defines *affect* as a term used to represent "all of the feelings that seem to be related to mathematics learning" (p.135). He highlights that a variety of emotions can be expressed while a person is trying to solve a non-routine mathematical problem. When failure to reach a solution occurs, he states that the emotions can include frustration and panic (McLeod 1988). These emotions can become increasingly intense over a prolonged period of time, particularly for novice problem solvers with little experience of problem solving.

As stated in the Three Key Characteristics of mathematical problem-solving, it is not immediately clear to the problem solver on how to achieve the goal that the problem sets out . McLeod (1988) highlights that this ambiguity in how to approach a non-routine mathematical problem is precisely the situation that can lead to emotions arising. McLeod (1988) explains that the reaction to problem-solving can be different in every person with both negative and positive emotions being possible results of a variety of stimuli or experiences while working on the problem, for example interruptions (as discussed in detail by Mandler (1984)). McLeod (1988) explains that when the majority of a students' mathematical experience involves doing routine exercises then the inevitable consequences to interruptions during problem-solving are intense emotional reactions.

The beliefs of teachers have a direct influence on the beliefs of their students because when many students are confronted with a mathematics problem, they may have low intrinsic motivation to work through the problem and depend on extrinsic motivation (Marcou & Philippou, 2005) which primarily comes from the teacher. Schoenfeld (1983) highlighted that beliefs influence behaviours when attempting mathematical problems. Furthermore, he claims that if students do not value understanding as a goal of mathematical learning, then they may not be able to access stored information that would otherwise be of use to them in carrying out mathematical work.

When entering teacher education programmes, future teachers possess ideologies on what being a teacher entails based on previous experiences (Taguchi, 2007). Kayi-Aydar (2015) highlights that prospective teachers' identity is not based on the attributes they desire to possess when teaching once qualified. However, a teacher's identity is greatly influenced by the teacher education they have undergone and is reflected in their teaching (Chapman, 2014). Teacher education programmes are viewed as a critical stage in teachers' development (Teaching Council of Ireland, 2017). During teacher education programmes, prospective teachers' beliefs regarding teaching and learning should be considered and challenged as they will bring these beliefs forward into their professional practice (Teaching Council of Ireland, 2017). Philipp (2007) suggests that teacher education programmes must promote positive beliefs in prospective teachers in order to develop mathematical proficiency and ultimately help equip graduates of the programme to create positive mathematical learning environments for their students.

The Study

This study aims to present findings about the affective domain of pre-service mathematics teachers (PSMTs) relating to problem-solving, motivated by the capacities outlined by Chapman (2015) as being a key to effectively teaching problem-solving.
The participants in this study are PSMTs undertaking a concurrent initial teacher education programme. The participants are students of two different programmes of study. Students of both programmes were taking a module that includes the study (and practice) of mathematical problem-solving. Graduates of the relevant programmes are qualified to teach mathematics to Leaving Certificate level in Ireland, and typically go on to do so, and so preparing the PSMTs for the task of teaching problem-solving is a key concern of the programme team. All the participants completed their second-level education in the Irish system, and thereby completed the Leaving Certificate curriculum.

Methodology

The methodology for this study is a mixed methods approach. Creswell & Garrett (2008, p. 2) describe mixed methods as an approach to inquiry that involves the researcher connecting quantitative and qualitative data in some way in order to make a 'unified understanding of a research problem'.

The quantitative element of this study was done through the implementation of a survey, the *Indiana Mathematics Belief Scale (*IMB), developed by Kloosterman and Stage (1992).

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Kloosterman and Stage created an instrument to measure the beliefs of secondary school and college students which they extensively analysed and reviewed to ensure validity using statistical software. The IMB has since been used by researchers to test the mathematical beliefs of students (Mason, 2003; Prendergast et al., 2018).
This instrument consisted of five scales with six items in each scale totalling in thirty items. Of these thirty items, twelve questions were with a negative valence and eighteen with a positive valence. Each item was graded in a Likert- scale fashion whereby the following numbers indicated the respondents' level of agreement or disagreement with each item; 1 = strongly disagree, 2 = disagree, 3 = undecided, 4 = agree, 5 = strongly. In tabulating the results below, the scales are reversed (where necessary) so that in every case, a higher mark corresponds to a more positive disposition. Each of the scales are described below as defined by Kloosterman and Stage (1992):

1) I can solve time-consuming mathematics problems [Difficult Problems]
The first scale involved investigating the persons' perceived ability to solve time-consuming mathematics problems. According to Schoenfeld (1985), students who give up on any problem which cannot be completed in five minutes or less believe that problems must be solvable in five minutes or less.

2) There are word problems that cannot be solved with simple step-by-step procedures [Steps]
This scale involves the use of procedural skills and formulae to solve problems. Given that the definition of a mathematical problem involves having no clear path or procedure readily available, it is evident that a problem solver must be motivated to solve problems for which there are no memorized procedures to employ (Lester & Charles, 1992).

3) Understanding concepts is important in mathematics [Understanding]
This scale measures the degree to which the respondent believes in the importance of understanding the concept. This involves understanding why an answer is correct, and how the solution was obtained. High scores on this scale are associated with motivation to learn and solve mathematical problems.

4) Word problems are important in mathematics [Word problems]
This scale involves investigating the respondents' beliefs about the importance of word problems compared to computational or procedural skills. It has been shown that those who believe that computational skills are more important than word problems will be less motivated to solve problems.

5) Effort can increase mathematical ability [Effort]
Like the other scales, belief that effort can increase mathematical ability is associated with motivation in problem-solving. This scale is used to provide an insight into the respondents' attitude towards their ability to improve their mathematical skills by putting in effort.

These beliefs were chosen specifically for secondary school and college level students in relation to their motivation to learn to solve mathematical problems (Kloosterman & Stage, 1992). Scales 1, 3 and 5 measure the beliefs of the respondent as a learner of mathematics while scales 2 and 4 measure the beliefs about mathematics.

The qualitative element of this research involved participants completing an open-ended question after attempting a mathematical problem. The participants were asked to describe how they felt at three different stages of their problem-solving attempt, namely; the start of the problem, the middle of the problem, and the end of the problem. The students were prompted to refer to how they felt if they were stuck and if they were making progress. This was done at three different points during the module while participants worked on three different mathematical problems. The first problem involved the topic of area, the second problem involved number and algebra, and the third problem involved the topic of trigonometry.

The responses were analyzed using a general inductive approach (Thomas, 2006). This involved the coding and identification of categories from iterative reading of the raw data by the researcher. The data was analysed to categorise the statements made by the respondents at the start, middle and end of each problem attempt. The themes were identified and the number of statements in each category was counted. The data was then analysed to convert the counting of statements into the counting of participants' perspective.

Results

Table 1 below shows the mean result for each of the five scales of the IMB as described above. Each mean score is out of a maximum of 30. The mean for each cohort and the overall mean is shown below.

Table 1. *Table 1 shows the results of the IMB of each of the four cohorts of participants*

	Cohort 1 (n=30)	**Cohort 2 (n=44)**	**Cohort 3 (n=30)**	**Cohort 4 (n=47)**	**Total (n=151)**	**Overall mean**
Difficult Problems	22.9	22.7	17.1	13.4	76.1	19.025
Steps	17.2	16.6	16.8	16.0	66.6	16.65
Understanding	20.9	21.5	19.4	20.7	82.5	20.625
Word Problems	19.2	19.3	14.3	20.5	73.3	18.325
Effort	26.7	27.0	25.5	19.9	99.1	24.775

From the general inductive analysis of the qualitative data the following categories were identified. These were; neutral, negative, positive, both positive and negative, answer positive, answer negative, process positive, process negative, negative answer but positive process, both answer and process positive, and both answer and process negative. The term 'positive' refers to statements of a positive disposition, the term 'negative' refers to statements of a negative disposition, and the term 'neutral' refers to statements which are neither positive nor negative. The analysis of the data from the end of each problem produced the categories involving the problem-solving process and the positive or negative answer categories. Space precludes provision of representative quotes in this paper.

Table 2 shows the number of respondents who were classified in the previously mentioned categories at the start, middle and end of each problem. SP1 indicates the start of problem one, MP1 indicates the middle of problem one, and EP1 indicates the end of problem one. This notation is also used for problem two and problem three.

Table 2. *Table 2 indicates the number of participants in the appropriate classification across start, middle and end of each of the three problems.*

Classification	SP 1	MP 1	EP 1	SP 2	MP 2	EP 2	SP 3	MP 3	EP 3	Total
Neutral	10	4	2	6	3	3	4	11	1	44
Negative	16	12	4	14	15	2	11	14	2	90
Positive	16	16	3	11	8	1	20	9	0	84
Both Positive and Negative	1	9	1	1	6	1	5	6	0	30
Answer positive	0	0	5	0	0	6	0	0	7	18
Answer negative	0	0	9	0	0	10	0	0	12	31
Process positive	0	0	8	0	0	3	0	0	1	12
Process negative	0	0	1	0	0	0	0	0	1	2
Negative answer but positive process	0	0	6	0	0	3	0	0	2	11
Both answer and process positive	0	0	4	0	0	1	0	0	8	13
Both answer and process negative	0	0	0	0	0	0	0	0	2	2
Missing values	7	9	7	18	18	20	10	10	14	113
Total	50	50	50	50	50	50	50	50	50	

Discussion

The IMB has been used by researchers to investigate beliefs about mathematical problem-solving. One such study was conducted by Mason (2003) which involved implementing the IMB with Italian high school students (n=599). Similarly, Prendergast et al. (2018) conducted the IMB with post-primary students but in Ireland. The IMB was distributed to nine secondary schools with a total of 975 questionnaires completed and returned. The participants in this study are PSMTs who all completed post-primary education in Ireland and the focus of this study is to investigate their beliefs using the IMB through the perspective of prospective teachers rather than students.

The analysis of the results of the IMB from the four cohorts of participants is discussed below. The combined results of the four cohorts of the IMB showed that the scale with the highest mean score was *Effort* with a score of 24.85/30. This scale had the highest mean score in Cohorts 1, 2, and 4 and was second highest in Cohort 3. This shows that participants positively agree that effort and working hard can have a positive impact on mathematical ability (Kloosterman & Stage, 1992). This finding is in line with the findings of a study conducted by Prendergast et al., (2018). It is interesting to note that the results of the study conducted by Prendergast et al. (2018) could be seen to be representative of the same group of participants that were involved in this study since participants in this study all completed post-primary education in Ireland. This is a positive finding as there is an implication that participants demonstrate aspects of a growth mindset. Dweck (2008) states that students who have a growth mindset are at a significant advantage to students who are of a fixed mindset. In research conducted by Dweck, it was found that students with a growth mindset cared more about learning and also demonstrated a greater belief in the influence of effort on their grades than

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students with a fixed mindset. Similarly, it was found that those having a growth mindset reacted in a more positive manner to setbacks than those with a fixed mindset.
The scale which had the lowest overall mean was *Steps* with a score of 16.645/30. This scale had the lowest mean score in each cohort of participants. This result is of concern as it is indicative of the belief that rote learning and procedures are adequate to solve mathematical problems. This lowest mean aligns with the lowest mean other studies (Kloosterman & Stage, 1992; Mason, 2003; Prendergast et al., 2018).

The analysis of the qualitative data showed that there was a strong focus on achieving an answer in each of the problems. From the 150 responses regarding the end of each of the three problems, 84/150 referred to achieving or not achieving an answer. This showed that there was a greater focus on finding an answer rather than on the problem-solving process. Of these 84 answer-focused responses, 31 consisted of negative comments in relation to not being able to achieve an answer; 18 participants provided positive comments.

Forty out of the 150 respondents referred to the problem-solving process, with four of these responses reported negative feelings towards their problem-solving process while 44 out of 150 responses reported negative feelings toward achieving an answer. This shows that there may be a stronger negative association with not achieving an answer than on the problem-solving process. We also note that the number of participants that referred to the problem-solving process declined between each of the problems. Simultaneously, there was an increase in the number of participants who referred to achieving or not achieving an answer.

From the results of the IMB the scale, *Understanding*, had the second highest overall mean with a score of 20.61/30. A high score in this scale demonstrates that there is a greater value on understanding a mathematical concept rather than achieving a correct answer. This involves the understanding of why an answer is correct and how a procedure works. One negatively worded question in this scale is; *It doesn't really matter if you understand a maths problem if you can get the right answer*. This suggests that the PSMTs place a greater importance on understanding rather than achieving an answer. This is in contrast to the results of the qualitative data which showed that there was a greater focus on achieving an answer than on the process. A potential reason for this is participants reporting what they believe they should say rather than their actual behaviours.

From the analysis of the scale; *Understanding*, Prendergast et al., (2018) found that Junior Cycle students demonstrated stronger levels of agreement than Senior Cycle students. The authors hypothesize that this could be due to an increased focus on obtaining a correct answer in examinations. This hypothesis could explain the strong focus placed on achieving an answer by the PSMTs.

Conclusion

From benchmarking against other studies, the results of the IMB showed that; the *Effort* scale was higher in this study, the *Understanding* scale was lower in this study, and the *Steps* scale was consistent with other results. This is indicative that the PSMTs have a positive disposition towards effort influencing mathematical ability. The results of the *Word Problem* scale and the *Steps* scale are relatively low given that the maximum score is 30. This suggests that PSMTs place a high value on the role of computational skills and previously learned procedures rather than on problem solving skills.

The qualitative data indicates that there appears to be a greater focus on achieving an answer than focusing on the problem-solving process. This is contrary to the results of the *Understanding* scale of the IMB. Comparing the quantitative and qualitative results, there appears to be a discrepancy between what PSMTs report and how they behave while problem-solving. This is something that will be taken into consideration in the development of the module that the PSMTs undertake.

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