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Problem-solving Potential (PsP) in the regular mathematics classroom

Dr Aidan Fitzsimons[a], Dr Eabhnat Ní Fhloinn[b]
[a] Mathematics department, Oatlands College, Dublin, Ireland
[b] School of Mathematical Sciences, Dublin City University, Dublin, Ireland

ABSTRACT

A student's Problem-solving Potential (PsP) is defined by their mindset, their mathematical resilience, and the problem-solving skills they possess. This triad construct of PsP was developed as part of a doctoral study, and investigated amongst six cohorts of highly-able mathematics Transition Year students through an educational intervention. Prior research into mindsets, largely spearheaded by Dweck (2006), found a relationship between achievement and growth-orientated mindsets; which extol the virtues of learning over "looking smart", and the value of making mistakes, among other attributes. Mathematical resilience is further subdivided into value (belief that mathematics is important), struggle (acknowledgement that struggle is a valuable part of learning mathematics), and growth (referring to growth mindsets) (Kooken et al., 2016). Prior research of mathematical resilience focussed on the development of this concept amongst low-achieving students; whilst our research investigated its development within highly-able students. There is a popular belief amongst educational researchers that mathematics is best learnt through the construction of knowledge by the learner, and mathematical problem-solving is one approach that creates this opportunity for all learners (Mason et al., 2010). Our educational intervention utilised collaborative problem-solving, and was designed to: introduce strategies for problem-solving; encourage reflection on the problem-solving process; provide opportunities for the extension of problems; and develop communication skills. In this paper, we will highlight the benefit of developing PsP in the mathematics classroom for highly-able students by discussing the relevance of each aspect of the construct through the findings of the doctoral study; and also how the development of PsP may impact lower-achieving students.

KEYWORDS

Problem-solving Potential, problem-solving skills, mindsets, mathematical resilience, collaboration

CONTACT Aidan Fitzsimons aidan.fitzsimons4@mail.dcu.ie

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Introduction

It has long been believed that mathematics is best learnt through the construction of the learner's own knowledge, and problem-solving has been lauded as one effective means of doing so (Hyland, 2018). Numerous researchers have explored the benefits of developing the skills for problem-solving amongst students. However, to be improve your potential to problem-solve, there is more than skills required. Problem-solving skills, mathematical resilience and a growth mindset each play a role in one's Problem-solving Potential (Fitzsimons, 2021). This paper will explore PsP and the intervention designed to nurture it, while discussing how they may apply to the broader context of the regular mathematics classroom.

Background

Mathematics education in Ireland has undergone changes in the past 15 years, first through 'Project Maths' (DES, 2010), and then through the new Junior Cycle (DES, 2017). Both of these developments brought a renewed call for problem-solving within second-level mathematics (Byrne et al., 2021), although research has suggested that teachers have struggled to be given adequate time allocations or resources to allow for this to occur (O'Meara & Prendergast, 2017). Concerns were raised in the early stages of 'Project Maths' that teachers were not receiving sufficient support to prepare them for the changes required in their teaching under the new curriculum (Grannell et al., 2011); while Byrne et al (2021) suggested that teachers should be given an abundance of continued professional development for the new Junior Cycle to avoid similar failures.

One further aspect of concern with these curriculum changes was the education of highly-able students (Lubienski, 2011). These students rely on classroom differentiation and out-of-school programmes to cater for their diverse educational needs (NCCA, 2007), despite calls from researchers for greater attention for this student cohort (Riedl Cross et al., 2014). Their performances on international assessments have been stagnant and below average over the past 30 years, whilst the general student population performed consistently above average and low-achieving students showed marked improvements (Cunningham et al., 2016; McGrath, 2017). In a bid to address the additional educational needs of these students, Problem-solving Potential was developed as a construct.

Problem-solving Potential (PsP)

PsP is a triad construct developed as a part of a doctoral study that outlines how a student's potential to problem-solve in mathematics is influenced by: the skills for problem-solving they possess; their level of growth mindset; and their mathematical resilience. Dai's (2020) definition of potential reflects upon intrinsic characteristics and traits, and the order of importance they are assigned within an individual reacting to their experiences or environment; but also that this may be shaped by external factors, such as resources or teaching. The importance of problem-solving to mathematics has long been known (Schoenfeld, 1992), and its benefits to the development of highly-able students' abilities has been well-researched (Sriraman, 2003). While the 'skills' associated with problem-solving form a non-exhaustive list, which further depend on the level of mathematics being studies, they include: calculation skills, strategy selection, communication, explanation, reflection, creativity, expansion, and many more. The further engrossed in the study of mathematics a study becomes, the more skills they will encounter and indeed require. It is only through the exposure to problem-solving that they will develop these skills.

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Although there is an abundance of research in the field of mindsets, there exists a gap in this research specific to highly-able students (Esparza et al., 2014). When a student is focussed solely on their test scores, such that an experience of failure or repeated failure leads to feelings of inadequacy or stupidity, they are said to possess a fixed mindset (Yeager & Dweck, 2012). Conversely, a student who values their mistakes and effort as a means to learning is said to have a growth mindset (Dweck, 2006). Research has found that a student's mindset is strongly correlated with their performance in a subject, and may even be a good predictor of a(Blackwell et al., 2007) future decline or incline in grades . With regards to highly-able students, while they may not experience failure commonly, there are fears that they develop negative traits of perfectionism, and seek to hide mistakes rather than to learn from them (Mofield & Parker Peters, 2018). Further concerns have been raised about praising students' performance rather than their effort (Boaler, 2013). Although some students may exhibit fixed mindsets, multiple research projects have shown that a student's mindset may be altered through intervention (Blackwell et al., 2007; Esparza et al., 2014).
Kooken et al (2013) defined mathematical resilience through the domains of value (the extent to which a student values the study of mathematics in their life), growth (in reference to growth mindsets) and struggle (the extent to which a student identifies struggle as a part of the learning process). Students who were highly mathematically resilient were found to show smart strategy selection in problem-solving, to crave discussion, to question mathematical ideas, and to exert the attributes of a growth mindset (Johnston-Wilder & Lee, 2010). Much of the early research into this field has focussed on two areas – how mathematical resilience may be improved, and how it impacts upon low-achieving students. With regards to the former, research has found that the development of "*coaches*" for mathematical resilience may have a positive impact on a student's beliefs (Johnston-Wilder et al., 2013). The doctoral study completed by the author was the first investigation of mathematical resilience amongst highly-able students.

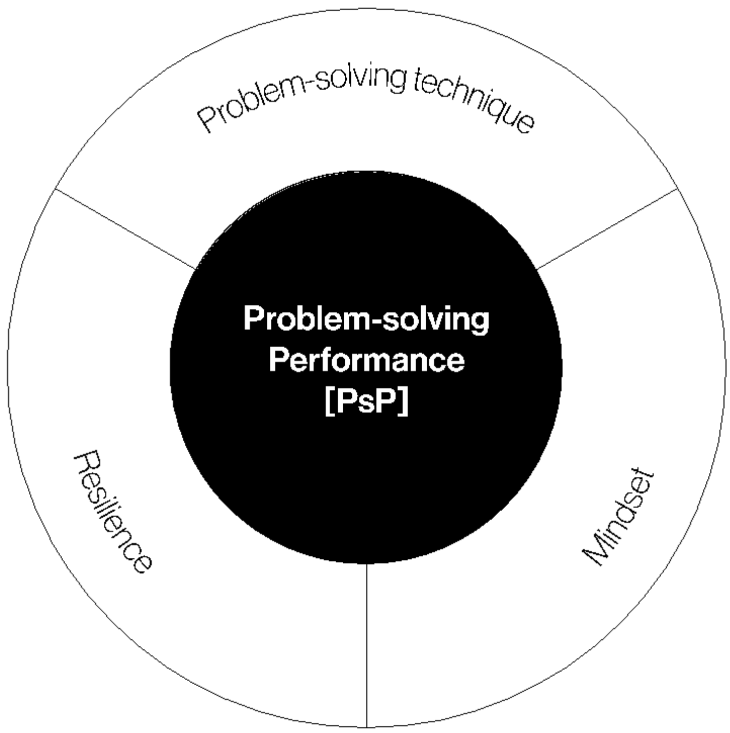

Figure 1 Problem-solving Potential (PsP)

Mathematics Intervention

The mathematics intervention was developed following a rigorous literature review, and implemented across six cohorts of highly-able, transition year students through the Centre of Talented Youth, Ireland (CTYI). Transition Year was deemed as the most-appropriate time for the intervention for a number of reasons: there is no set assigned curriculum for students; other years are highly time-sensitive due to the vast curricula to be covered by teachers, etc. Two time-variations of the intervention were utilised: a 14-week programme, one day per week for 14 weeks; and a 3-week programme, for 14 days over this period. The students completed a two-hour class and a one-hour tutorial each day. While the layout of the class and tutorial were largely identical, the students were required to complete a written diary reflection for one problem during the tutorial. Each aspect of the mathematics intervention was designed under the aim of developing the PsP of highly-able students.

Instructional Design

Problem-solving heuristics have been prevalent in research for decades, with Polya's steps of problem-solving still widely referenced to this day (Polya, 1945). Mason et al (2010) developed 'stages' of problem-solving, which built upon the work of Polya, and created a system of "Rubric writing" to navigate the problem-solving process. One notable addition was also the extension of problems, allowing for the problem-solver to display their mathematical creativity and restart the problem-solving cycle. While the problem-solving process has been well-researched and advanced through the years, there has been no clearly defined model specific to collaborative problem-solving, despite repeated reports of collaborative problem-solving as an essential skill in the 21st Century (OECD, 2017). Hence, the CoPs model was designed to bridge this gap in research. This model represents the problem-solving processes encountered when working collaboratively.

Collaborative problem-solving encourages the development of individual problem-solving skills, combined with the skills of collaboration, such as communication and the verbalisation of reasoning. It has also been found that highly-able students work effectively in a group of their peers who share similar motivations. Groups of 3 or 4 students were created on the first day of each cohort of students and remained unchanged for the duration.

Problem-solving strategies are essential to the process of solving a problem (Posamentier & Krulik, 2015). Early in the development process, the decision was made to introduce seven strategies organically to students through carefully-chosen problems (Fitzsimons & Ní Fhloinn, 2019). These strategies were chosen due to their prominence in previous literature (Krulik & Rudnick, 1989; Mason et al., 2010; Schoenfeld, 1982): visuals, patterns, generalising & specialising, conjectures, assumptions & questioning, structure, and working backwards.

The role of the facilitator within a problem-solving classroom has been well-outlined in previous literature (Dolmans et al., 2005), and the intervention utilised this role to: ask probing questions of students throughout the process; ensure all students were participating; provide a scaffold for the process where necessary; and to monitor progress of individuals. Furthermore, the facilitator provided positive messages of both resilience and growth mindsets throughout the intervention. As the intervention progresses, the role of the facilitator diminishes, as the students become familiar with the procedures required of them.

Intervention Content

Due to the vast number of resources available online or in literature, the decision was taken to carefully-select existing problems, rather than to create new ones. These problems were curated under themes for each day of the intervention - the first seven themes being those of the aforementioned problem-solving strategies; followed by four contextual themes after the four strands of Junior Cycle mathematics (DES, 2017); and finally three days of general problem-solving. The 'diary' problem for each tutorial followed the same theme as the class. As the participants of the intervention were transition year students, all chosen problems were solvable through Junior Cycle mathematics.

Classroom Design

The classroom was prepared each day before the arrival of the students. Four individual tables were combined in a square pattern for each group. The problems were introduced through a presentation broadcast by a projector. Groups completed their workings on A1 size sheets of paper that were distributed by the facilitator, with extra sheets provided when necessary. While the application of student roles in groupwork may be utilised in research to ensure all members of a group are motivated and participating (Huss, 2006), the 'house rules' and dynamic of the CoPs model require this of the students on this intervention, and thus the introduction of student roles was not deemed necessary. After observation of six cohorts of students participating on the intervention, the author believes this was the correct decision.

Regular Classroom

As previously mentioned, Transition Year (TY) was deemed the most appropriate year for a mathematics intervention to occur for the doctoral study, and this remains true for any school-based intervention. The TY guidelines offer suggestions as to what should be taught to students, but also firmly state that curricula and teaching strategies are to be decided by each individual school (DES, 1994). The guidelines do, however, also emphasise that TY should not be equipped as a third year of study towards the Leaving Certificate (DES, 1994, p. 2). It is also known that both the Junior Cycle and the Leaving Certificate are time-constraining for teachers (NCCA, 2013). TY serves as the ideal stage of second-level education for the existence of an intervention for highly-able mathematics students. The flexibility within the TY guidelines would also allow schools to adopt such an intervention to fit their needs, while not placing pressure on schools by introducing one as mandatory.

A problem-solving intervention in TY would also build upon the unifying strand in the Junior Cycle (DES, 2017), and explore the previous three years content in greater depth. As a part of the doctoral study, the intervention was implemented in two time-variations, and the results of each variation statistically analysed. No statistically significant differences were found between the variations. Further research is now needed as to how effective the intervention can be in a regular school setting, although the individual features are easily transferable.

Less-Able Students

Thus far, PsP has been investigated as a singular construct in the context of highly-able students only. There is further research needed to explore it's benefits to all students. However, an examination of previous research yields the benefits of each aspect separately.

Mathematics anxiety is an issue of great concern amongst less-able students in the study of mathematics (Ashcraft, 2002). Mathematics anxiety can lead to a mental block for students, whereby they disengage from the subject and develop intrinsic barriers to their learning (Gabriel & Barthakur, 2020). Johnston-Wilder et al (2015) found that developing a student's mathematical resilience through the use of "*coaches*" and positive messaging helped those who had developed the 'growth zone model' to illustrate how a student must navigate outside their comfort zone to learn, but may be guided by a coach to avoid entering the anxiety zone.

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Figure 2 *Growth Zone Model*

Further to this, the development of a growth mindset may also be beneficial in tackling mathematics anxiety amongst less-able students (Boaler, 2018). Hwang et al (2017) found that fixed mindsets amongst low-achieving students were a greater predictor of declining results over time than fixed mindsets amongst high-achieving students.

As previously discussed, the role of the facilitator in collaborative problem-solving is multi-faceted. The facilitator may scaffold the learning for their students, and can be as invasive as necessary in the process. For less-able students, the facilitator may become more hands-on than usual to help the students to navigate through a problem, without becoming too overwhelmed by the problem.

Conclusion

The mathematics intervention designed is a purpose-built programme for highly-able mathematics students in Transition Year in Ireland, which is of great importance to these students. The intervention has possible implications for the encouragement of mathematical talent and potential amongst highly-able students in schools. Participation on the intervention can provide students with stronger problem-solving skills that may benefit their study of mathematics in the Leaving Certificate.

With regards to general or less able students, while PsP has not yet been researched with cohorts of these groups, each individual aspect of the construct has clear and defined benefits for these students. The application of this construct within the regular classroom requires research in the near future.

Faulkner et al (2021) recently found that a sample of undergraduate students in Ireland did not display a level of improvement in problem-solving that was expected following the overhaul of the second-level mathematics curriculum. Further to this, they outlined how students may be experiencing "*helplessness*" when they encounter unfamiliar problems. The development of PsP within students has the potential to alleviate concerns such as these when they move beyond second-level education.

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