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Developing second level students' understanding of the inverse square law and electric fields

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Abstract. There are specific mathematical tools involved in building an accurate model of introductory electric field theory. Algebra operations, vectors, field lines, proportional reasoning and the inverse square law are all integral parts of gaining a complete understanding of an electric field. In this paper we present a small body of research, taken from a case study with a group of 14 upper second level students, in which they developed their understanding of the inverse square law, using a pre-test-tutorial-post-test tutorial lesson model. Students struggle to understand the inverse square law unless they are repeatedly exposed to it. Using the context of intensity, our students develop their understanding of the inverse square law using a variety of representational forms, such as diagrammatic, tabular / graphical and calculations using formulae. Using our pre-test and post-test results, our students showed gains in their reasoning used to explain the variation of intensity when an object is moved various distances from a source, which we attribute to their reasoning developed in the tutorial lessons. Additionally, students completed quantitative problems involving the inverse square law in the context of Newton's gravitational law. Six weeks after the completion of the inverse square tutorial, our students completed a tutorial lesson, in which they applied their understanding of the inverse square law to Coulomb's law and the electric field. Our results show that our students could apply the inverse square law to these contexts when guided, but some difficulties still remained, such as proportional reduction / increase based on variation of the distance, and transfer between representations, such as algebraic to graphical.

1. Introduction

Marzec and Arons have noted that learners persistently display difficulties in their understanding of dimensional scaling [1,2]. For example, in the absence of formulae, learners struggle to explain how increasing the sides of a sphere by a factor of 3 results in an increase in the area by a factor of 9, as illustrated in figure 1. In the absence of area / volume formulae, students struggle to tackle questions related to scaling [2] or applying the inverse square law to an electrostatics context [3]. Arons suggests that learners attempt to memorise questions and as a result do not develop any conceptual understanding or proportional reasoning related to dimensional scaling [2]. This affects the learners' ability to develop coherent mental models of phenomena that follow an inverse square law, such as Newton's gravitational law, Coulomb's law, light intensity, sound intensity, radiation and Gauss' law. They suggest that



repeated exposure to both dimensional scaling and the inverse square law is required to enable students to develop a coherent mental model of the law.

To promote student understanding and exploration of the inverse square law, many approaches have been developed. Bohacek and Gobel suggest the use of a light sensor and bulb, in which students record the light intensity as it varies with distance [4]. In the absence of such equipment, Hestenes and Wells suggest students can model the relationship by drawing graphs of prepared data sets and analyzing the graph [5]. While these approaches may be mathematically intuitive and more accessible to students than using an exploration of the relationship using algebraic methods, they do not provide a conceptual model to explain the behavior. To promote conceptual understanding, Hewitt uses a context of droplets of paint, emitted from a spray paint can, spreading over an increasing surface area as the distance from the can to the surface increases [6]. Bardini *et al* suggest that when students explore mathematical functions, the use of real-world contexts enables students to make sensible use of mathematical symbols, form generalisations, and interpret and relate graphical representations to algebraic equations [7]. Using all these influences, the students can both qualitatively and quantitatively describe and explain behaviours that follow the inverse square law.

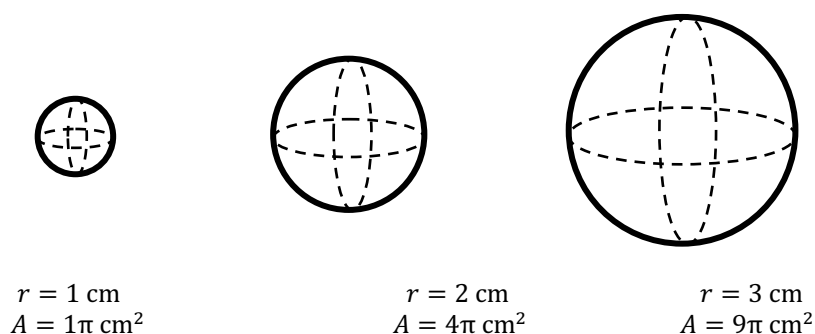


Figure 1. Diagrammatic model of area scaling as the radius of a sphere increases [1].

2. Description of participants and lessons

The participants of this study were a group of 14 students in the 16-17 years old range. The class was mixed gender (female = 4, male = 10) and mixed ability. They participated in tutorial style lessons developed in the style of Tutorials in Introductory Physics [8]. The lessons consisted of a pre-test, a tutorial worksheet and a post-test. The students completed tutorial worksheets designed to promote conceptual understanding of the inverse square law in groups of four in a structured inquiry [9] learning environment. The pre-test probed the students' initial understanding of the inverse square law and the post-test was designed to assess it post-instruction. While the tutorials are patterned after Tutorials in Introductory Physics [9], they were designed to be accessible for a second level student. To aid in this design, exercises and contexts were adapted from Conceptual Physics [6].

The study presented in this paper is a part of a larger study of students' understanding of Coulomb's law, electric field, work and potential difference. The tutorials were implemented over the timeframe of November to January, in the academic year of 2017/2018. As of 2017, the school these students attend was designated as a socio-economically disadvantaged school. All students had completed lower level mathematics and science courses, but were not familiar with the inverse square law, nor inverse mathematical functions.

This study seeks to answer the follow research question: To what extent does the use of a structured inquiry approach develop student's conceptual understanding of the inverse square law? To answer this question, the follow learning objectives were drafted to gauge the students understanding, in which upon completion of the tutorial, the students would be able to:

1. Accurately sketch and switch between graphical and algebraic representations of the inverse square law [4–5, 7]).
2. Apply a diagrammatic model utilising intensity to explain the behaviour of the inverse square law, and make predictions based on the model [6].
3. Demonstrate proportional reasoning using the inverse square law [1–3].

Data was collected in this study by scanning all student artefacts, conducting teaching and learning interviews, recording student discussions during the tutorial lessons and teacher reflections. These were analysed and interpreted to present the results shown in this paper. The results were analysed in terms of occurrences of conceptual change, in which conceptual instances of exchange and extension are noted [10].

3. Student performance in the tutorial lessons

The tutorial on the inverse square law was designed using an analogy of spray paint spraying over an increasing area, allowing student to tangibly model the inverse square law. This model was adopted from Conceptual Physics [6] and expanded.

The tutorial presents a spray can emitting 100 drops of paint per second over a given area, and the students calculated the number of paint droplets landing on a uniform area of 1 m^2 per second. The model is then expanded to increase the area covered by the paint when distance from the can to the area is increased, as depicted in figure 2. This engages the students to develop conceptual understanding of area scaling, in which they explore how the area of the frames increases quadratically, instead of linearly, as the distance increases.

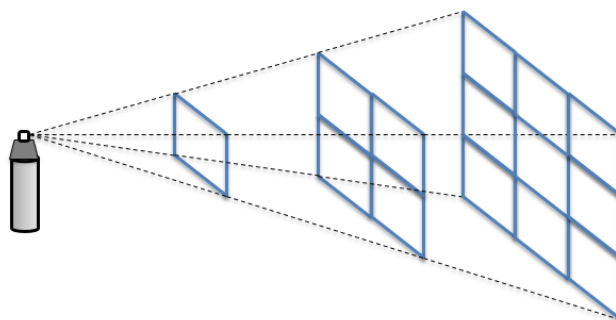


Figure 2. Inverse square law modelled using spray paint.

The students were required to explain why the area of the second frame was four times the size of the area of the first frame, when the distance from the can is doubled. The students then determined how many droplets of paint pass through the individual smaller frames of the second setup, if 100 droplets passed through the first in 1 second. By determining that there were 25 drops passing per second in an individual frame in the second setup, the students were required to determine how this showed that the spray paint “intensity” was following an inverse square law. Many discussions took place within groups to develop this reasoning, generally taking somewhere in the region of 15 minutes for the student groups to develop the reasoning to explain how the intensity drops.

While the students were developing understanding of scaling in the inverse square law they struggled to articulate why the area of the frames grew quadratically with the increase in distance between the nozzle of the paint can and the frame. The students were asked to consider both the increase in the width and height of the frames, and to consider how both these increases could explain the quadratic growth observed in the diagram. Difficulties were also encountered when the students needed to determine the number of paint droplets passing through each frame when they were presented with four and nine

frames. The students tended to consider the total area of all the frames, instead of looking at them individually.

Upon completion of this question, students were asked to consider the frame that was three times as far from the can as the initial frame. By completing this task, the students demonstrated how the growing distance from the can decreases the number of droplets passing through an individual frame.

Student 4A: *Doubling the distance and the height, fits 4 plates in.
Distance triples and height triples, fits 9 plates in.
The drops are being divided (through the frames) as it grows.*

Student 4D: *The farther away from the can, the bigger the area is because the lines are expanding, meaning more boxes (frames) can be filled in from each side.
As the distance from the pain can increases the drops per second decreases because the same number of drops pass through each part but distributed equally into each frame.*

Having used a diagrammatic model to a conceptual grounding in the inverse square law, the tutorial then evolved into a graphical treatment, in which the students graph the data for the paint “intensity” at various distances between the can and the frame, to show the inverse pattern, inspired from Ref. 5. The students used 2 sets of data points for form a ratio when the distance from the can is increased by a factor of 2, and then again by a factor of 5.

This enabled the students to observe an inverse square law using data from the graph as a verification process. A sample of work, both written and graph produced, from student 4I is shown in Figure 3, to illustrate the reasoning used by students to use the graph to demonstrate the inverse square law from the graph.

Student 4I: *It's a quadratic graph, that is decreasing, a slope that gets smaller.
1 m, it's 100. At 2 m, it's 25.
If you move 2 m away, it will cause a decrease of 4 times the intensity.
5 m, it's 4.
If you move 5 m away, it will cause a decrease of 25 times the intensity.*

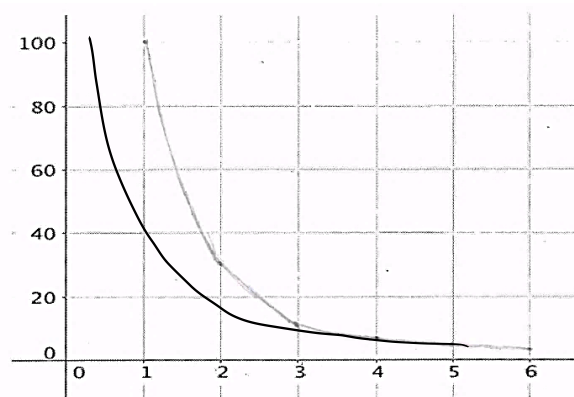


Figure 3. Graphical representation of the inverse square law, from student 4I’s tutorial lesson.

4. Results of Tutorials – Pre-test / post-test comparison

The pre-test and post-test questions were designed to elicit student understanding of the inverse square law by using graphical representations, diagrammatic representations and mathematical representations. The pre-test and post-test graphical questions are shown in figure 4.

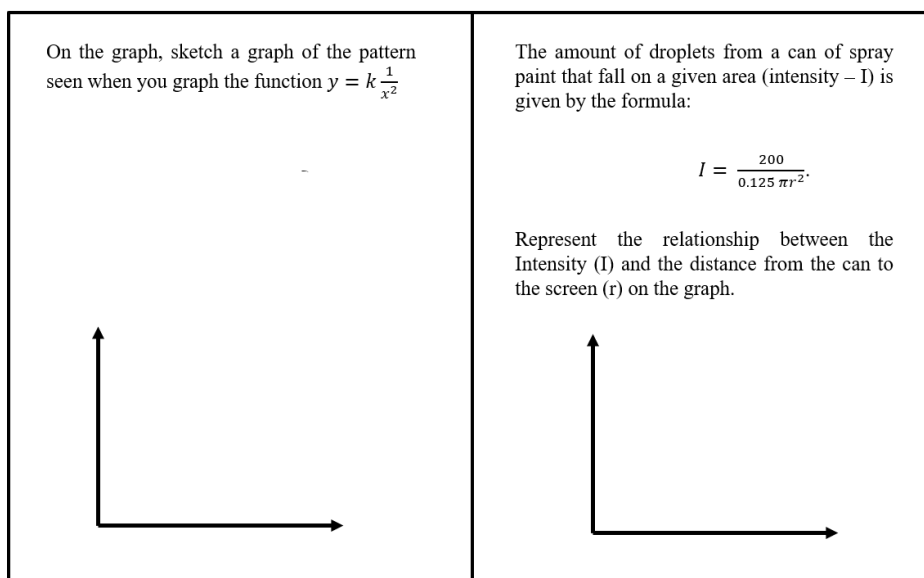


Figure 4. Pre-test (left) and post-test (right) questions eliciting students' ability to represent the inverse square law graphically.

The pre-test results (figure 5) indicated that the students were unaware of the shape of the graph of an inverse square law. In the tutorial lesson, the students were guided in mapping an inverse square law relationship graphically. The post-test results show an increase in the number of students who could transfer the mathematical formula to a graph, represent the function using the correct shape, and provide justifications for the graph choice. The gains seen in the students' responses are in line with the findings of Bardini *et al*, who found that guiding students through a function in context can help them develop an understanding of the equation and its transfer to a graph [7]. Several difficulties were seen in the students' pre-test submissions. There were no difficulties that occurred more frequently than any others, and thus all difficulties were considered for conceptual change.

A shift in the students' ability to correctly represent an inverse square law is evidenced in figure 5, with an increase of 3 to 9 students correctly drawing an asymptotic curve. In both tests, the most common persistent incorrect response was students drawing an increasing curve, representative of a $f(x) = x^2$ quadratic curve in the positive domain. This indicates that conceptual exchange and extension occurred, as the students demonstrated they could transfer the inverse square law from one representation to another, and then extend their understanding to develop an intelligible method to analyse the data on the graph [11].

The second pre-test and post-test questions were designed to elicit student understanding of the inverse square law by using diagrammatic scaling representations. The questions were similar in nature, as shown in figure 6.

Comparison of student’s responses for representing the inverse square law graphically.

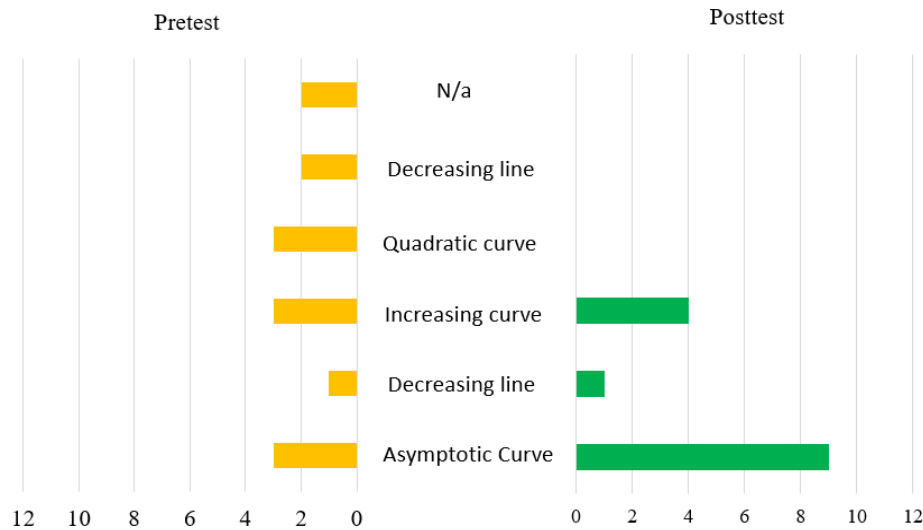


Figure 5. Comparison showing similar responses between pre-test and post-test question involving a graphical representation of the inverse square law.

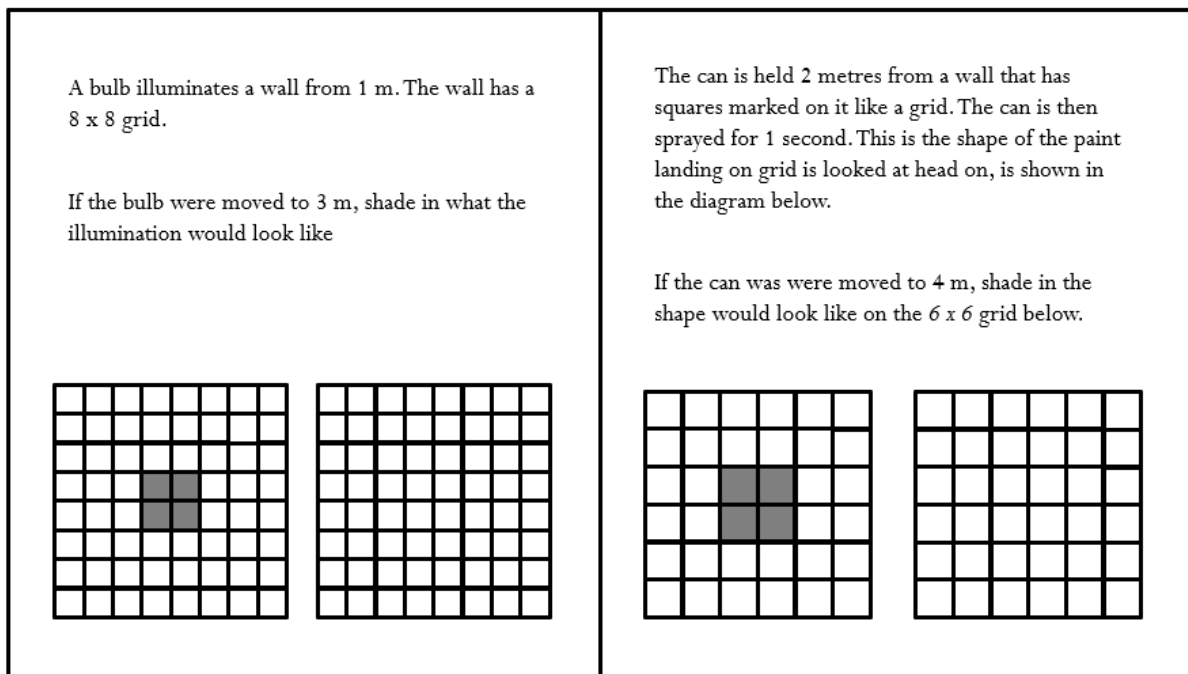


Figure 6. Pre-test (left) and post-test (right) questions eliciting students’ ability to represent the inverse square law graphically.

The development of students' understanding of the area change due to scaling is presented in figure 7. In both pre-test and post-test, it was observed that students could correctly determine the increase in the area illuminated when a light is moved back from a wall. While it was a positive outcome that the students could predict the change of the area, this increase did not correlate to students' understanding of a concept like intensity, in which a quantity is spread out evenly over this area. This would indicate that the teaching and learning sequence should not just be about dimensional scaling, but also applying the scaling area to other quantities and explaining how to apply it to concepts like intensity, in which paint / energy is "spread evenly" over the increasing / decreasing area [1,2]. During the tutorial lesson, it was found that the model adopted of using spray paint passing through square frames [6] helped students visualise the inverse square law in a relatively simple tangible context. In the post-test, a small number of the students still considered the overall area, while the remaining students to focused on the change in dimensions of the shape, indicating that conceptual exchange occurred the understanding of these students [11].

The mathematical pre-test and post-test questions are presented in figure 8, in which the students were required to apply the inverse square law mathematically. A difference between both questions not shown in the figure is that in the post-test, the students were provided with the equation shown in figure 4 that could be applied to the mathematical post-test question; this was not provided in the pre-test. A comparison of the pre-test and post-test results is presented in figure 9. The mathematical pre-test question, in which students needed to apply the inverse square law, they used either a linearly inverse law or a general reduction. In the post-test, the students were presented with a formula for intensity and were asked to prove that intensity followed an inverse square law, using the same skills developed in using ratios as completed in the tutorial when completing the graphing exercise.

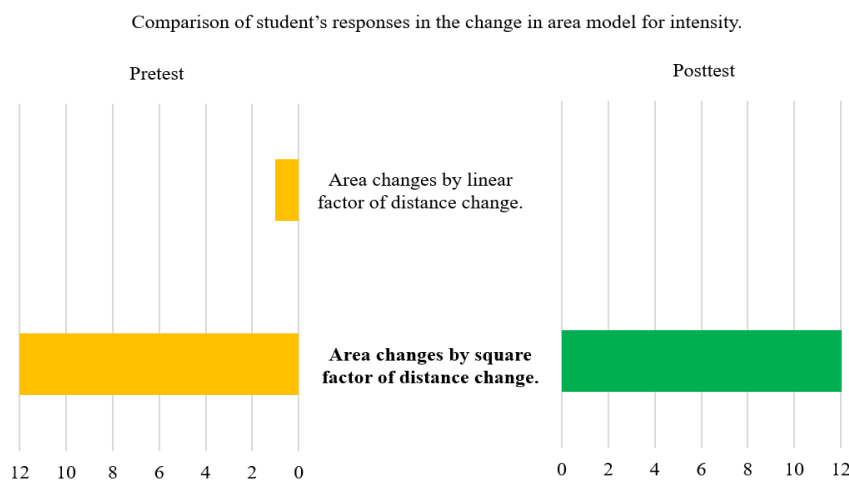


Figure 7. Comparison showing similar responses between pre-test and post-test involving a diagrammatic model of the inverse square law.

<p>A bulb is held 2 m from a light sensor. The sensor record the light intensity to be 100 Wm^{-2}.</p> <p>If the bulb is moved so that it is 4 m from the light sensor, what reading will the light sensor read?</p>	<p>The can is place 5 m from a wall. Calculate the paint intensity, using the formula on the first page.</p> <p>If the distance from the can to the wall is doubled, what is the new “paint intensity” Explain how you got your answer.</p> <p>Explain how your answer from (ii) shows this is an example of an inverse square law.</p>
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Figure 8. Pre-test (left) and post-test (right) questions eliciting students’ ability to apply the inverse square law mathematically.

It was observed that 12 of the 14 students used a formula to produce at least one correct set of results, but only 5 students calculated a ratio, as they were directly instructed to do. These five students demonstrated transfer of understanding between representations, and their consideration of the overall inverse square law in a task unseen from the tutorial. This suggests conceptual exchange occurred [11]. This also suggests that the difficulty for the remaining students was not the mechanics of using the mathematical operations, but how to utilise and extend their calculations to demonstrate an inverse square law. It was observed that nine students could produce values using an equation that involved an inverse square law, but only five could use their calculations to demonstrate the relationship. A later tutorial, based on Coulomb’s law, addressed this issue.

The tutorial itself did not address exploring the inverse square law mathematically. However, the students practised qualitative problems involving calculations involving of Newton’s gravitational law between the tutorial lesson and post-test. Therefore, we can attribute the increase in understanding of the inverse square law demonstrated mathematically to be a combination of representational transfer utilised in parallel and solving qualitative problems.

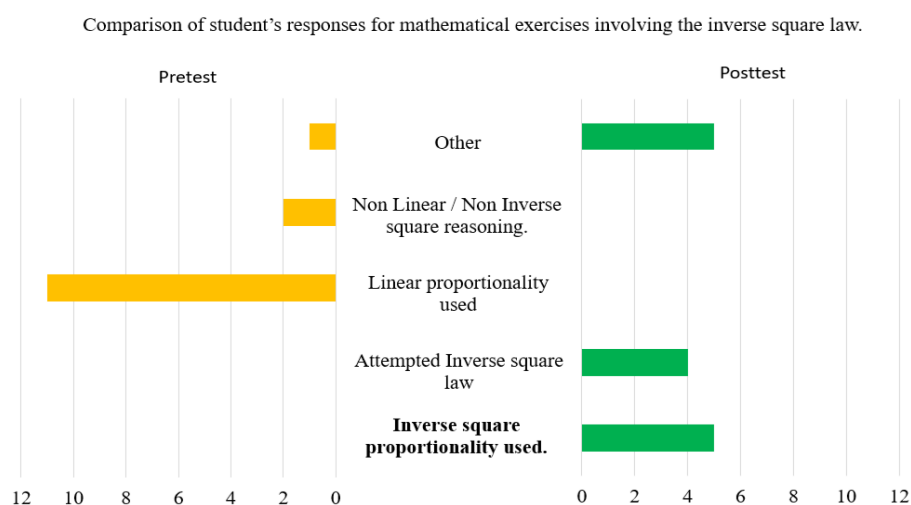


Figure 9. Pre-test / post-test comparison showing change in students’ responses for mathematical exercises using the inverse square law.

5. Application of the inverse square law to electric fields

Further tutorials used the same diagrammatic, mathematical and graphical exercises to apply and explore the inverse square law using the context of Coulomb's law and electric fields. This afforded students the opportunity to revise and practise using these representations. The electric field post-test question, shown in figure 10, was used to determine their final understanding of the inverse square law.

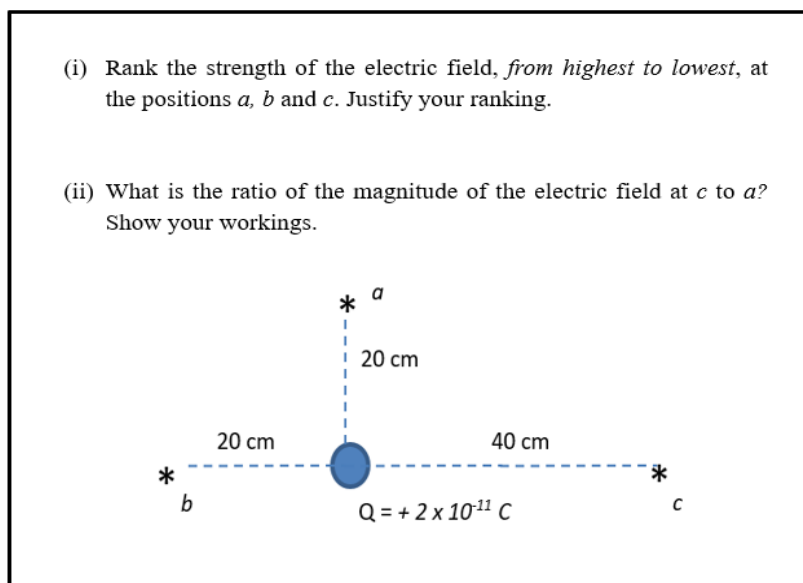


Figure 10. Inverse square law post-test question using an electric field context.

All students produced a ranking “A = B > C.” This indicates all students could qualitatively apply the inverse square law to the electric field given. However, quantitatively, 11 students found the ratio to be $\frac{1}{4}$. Six of these students used inverse square proportional reasoning, without explicitly referencing the use for formula, and likely developed the ratio mentally; the other 5 used formulae, simplified by three and left unsimplified by two. The remaining three students found a ratio of $\frac{1}{2}$. Of these students, two used a formula to determine electric field strength at point “a” but divided the magnitude of this value by 2 to determine the magnitude of the field strength at “c.” The other student calculated the ratio of both distances (20/40) but did not simplify. In these three cases, it is evident that inverse proportional reasoning was still being used by these students.

6. Conclusions

The approach adopted in this research enabled our second level students to model an inverse square law, using scaling. This was achieved using diagrammatic, graphical and mathematical presentations. The results indicate that conceptual exchange and extension occurred for students with regard their ability to interpret a symbolic representation of an inverse square law and transfer it to a graphical representation, which supports the findings of Bardini *et al* [7].

Our students developed their understanding of the behaviour of a quantity that obeys an inverse square law, as shown in analysing both the tutorial lesson worksheets, but the pre-test/post-test comparison indicates that the students require instruction in both dimensional scaling of shapes and its application in contexts in which phenomena follow an inverse square law.

Additionally, familiarity with the inverse square law showed gains in applying it using mathematical representations, even though it wasn't a focus in the initial tutorial. There was a notable shift towards use of inverse square law reasoning observe in electric fields post-test question, with a reliance to primarily use calculations to apply the law was observed by nine of the students.

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