DESIGNING MATHEMATICAL THINKING TASKS

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Abstract: Research has shown that the types of tasks assigned to students affect their learning. Various authors have described desirable features of mathematical tasks or of the activity they initiate. Others have suggested task taxonomies that might be used in classifying mathematical tasks. Drawing on this literature we propose a set of task types that are deemed appropriate for undergraduate students and which foster mathematical habits of mind. These are: generating examples; analyzing reasoning; evaluating mathematical statements; conjecturing and/or generalizing; visualizing; and using definitions. We give rationales for our choices and examples of each type of task suitable for use in an introductory calculus course.[1]

Keywords: task design, mathematical habits of mind, calculus, homework

1 INTRODUCTION

Mathematical tasks are devices for initiating mathematical activity. Research has shown that the types of tasks assigned to students highly influence the kinds of thinking and processes in which they engage, their level of engagement, and, thus, the learning outcomes achieved. In fact,

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Stein, Grover and Henningsen [22] contend that tasks with which students engage go beyond driving what content the students learn, and may determine "how they come to think about, develop, use and make sense of mathematics"(p. 459). As Mason [15] puts it

> in a sense, all teaching comes down to constructing tasks for students . . . This puts a considerable burden on the [teacher] to construct tasks from which students actually learn (p. 105).

Exposure to familiar or routine tasks alone affects students' ability to reason, to answer unfamiliar questions and to transfer their knowledge [8]. In a study of effective calculus programs in the US, Ellis et al. [12] found that assignments which contained novel and cognitively demanding tasks were important for the development of student understanding and confidence. However, studies carried out internationally (for example in the UK [20] and in Ireland [14]) have found that undergraduates in introductory mathematics courses are mostly required to carry out procedural calculations and only seldom need to use any higher-order thinking skills.

We report here on a study which aimed to steer students away from thinking about mathematics in a solely procedural or instrumental manner and to introduce them to the ways of working and habits of mind of a research mathematician. To this end, having consulted the literature, we selected a set of six types of tasks that would engage students in particular practices of mathematicians thereby promoting effective mathematical thinking. These were: generating examples; analyzing reasoning; evaluating mathematical statements; conjecturing and/or generalizing; visualizing; and using definitions (to classify a mathematical object, for instance). We do not claim that this is an exhaustive list of the types of tasks that provide opportunities for effective mathematical activity. In particular, we did not include tasks that ask students to prove a result here, not because we do not think these tasks are useful, but because we feel that they are already present in those mathematics courses where they are appropriate.

Initially in this project we focused on our Differential Calculus courses, creating a bank of questions that span the six task types we identified. Since this initial work we have expanded our efforts, using the same task types to design homework problems for Linear Algebra, Number Theory, and Introductory Analysis. We believe the task types are appropriate for many early undergraduate mathematics courses.

In this paper we will first of all briefly survey the education literature on effective practices and task design. We will then detail the task types that we have selected, and give a rationale for, and example of, each type. The examples have all been designed by us. Finally we will discuss the introduction of these task types in undergraduate modules that we have taught, and outline some directions for future research.

2 WHAT THE LITERATURE SAYS ABOUT EFFECTIVE MATHEMATICAL ACTIVITIES

If specific tasks motivate specific mathematical activity, what type of activity do we, as mathematics lecturers, wish to motivate? Many mathematics educators and researchers agree that the mathematical practices and thinking to be encouraged in learners of mathematics should mirror the practices of professional mathematicians. Students should be given the tools to make mathematics for themselves. For instance, Cuoco, Goldenberg and Mark [9] believe that students should conjecture, experiment, visualize, describe, invent, generalize, and use mathematical language precisely. Ahmed [1] asserts that students should be involved in speculating, hypothesis making and testing, proving or explaining, reflecting, and interpreting in order to ensure they are immersed in rich mathematical activity. He holds that engagement in such activity would involve students being encouraged to be inventive, being invited to make decisions, and being encouraged to ask themselves "what if?" and "what if not?" questions.

Mason and Johnston-Wilder [16] go a step further in terms of providing guidance for mathematics teachers when writing tasks. They give a detailed list of words they believe denote processes and actions that mathematicians employ when they pose and tackle mathematical problems: "exemplifying, specialising, completing, deleting, correcting, comparing, sorting, organising, changing, varying, reversing, altering, generalising, conjecturing, explaining, justifying, verifying, convincing, refuting" (p. 109). They then propose that questions posed to students should draw on these words to engage them in mathematical thinking.

Much has been written about task design and the features desired in an ideal task or set of tasks. Stein, Grover and Henningsen [22] discuss the importance of engaging students in thinking, reasoning and sense-making. The features of a mathematical task they identify as promoting these activities are its potential for multiple representations, the existence of multiple solution-strategies, and the extent to which the task demands explanations and/or justifications from the students. Swan [23] focuses on promoting conceptual understanding in secondary school students and identifies five types of tasks he deems suitable for this purpose: classifying mathematical objects; interpreting multiple representations; evaluating mathematical statements; creating problems; and analyzing reasoning and solutions.

Taking a different perspective, Sangwin [20] categorizes the types of mathematical tasks actually used in university modules. He describes the development of a taxonomy with eight classes of mathematical questions, distributed over two levels, as shown in Table 1.

1. Factual recall	5. Prove, show, justify (general argument)
2. Carry out a routine calculation or algorithm	6. Extend a concept
3. Classify some mathematical object	7. Construct an instance
4. Interpret situation or answer	8. Criticize a fallacy

Table 1. Sangwin's (2003) mathematical question taxonomy

Tasks falling into categories 1-4 (Table 1) are said to be those of "adoptive learning"; such tasks involve students applying well-understood knowledge in bounded situations and behaving as a "competent practitioner". On the other hand, tasks which typically require higher cognitive processes, such as those described by categories 5-8 (and sometimes 4), are deemed to require students to behave in a more sophisticated way mathematically (like an "expert") and are characteristic of "adaptive learners".

3 TASK TYPES TARGETING UNDERGRADUATE MATHEMATICS STUDENTS IN INTRODUCTORY COURSES

Having reviewed the literature on the design and classification of mathematical tasks, we sought to identify a set of task types that would engage our undergraduate students in effective mathematical thinking and activity. A number of considerations influenced our final choice of task types. Firstly, we wanted to keep the list of task types short in order to make it reasonable to aim to incorporate a task of each type on every homework set. Secondly, we aimed to choose tasks types that were unfamiliar to our incoming undergraduate students. To accomplish this, we drew on our experience of analyzing tasks in Irish secondary school mathematics textbooks [18], and first-year undergraduate Calculus modules [14]. Finally, we were aiming to select task types which would be suitable for many early undergraduate modules and good preparation for more rigorous modules such as Analysis or Abstract Algebra.

We then redesigned a set of homework tasks for use with mixed-ability first-year undergraduate students taking Differential Calculus modules in Irish universities. The emphasis in this type of introductory calculus module is often on procedures and calculations and not on rigorous proof. However, our aim was to assign tasks that would provide students with opportunities to gain experience of behaving like an " expert" as well as a "competent practitioner" [20]. In both institutions, each homework assignment contained a mix of routine procedural questions as well as questions designed for this project, and students were expected to work on them independently.

In the following we explain further our rationale for the particular task types we chose and we give some examples of tasks using the topic of continuity.

3.1 Example Generation

Many benefits of example generation have been outlined in the research literature. For instance, Selden and Selden [21] explain how students usually have no pre-learned algorithm to show them the "correct way" to look at a mathematical object in terms of its properties in order to come up with examples: thus, such a task requires students to develop and use different cognitive skills than those on which they often rely. Hassan and Zazkis [13] maintain that the decision-making opportunities provided by example generation tasks are important, and that learning occurs when students practice making judgements and come to terms with the freedom inherent in such tasks. Moreover, example generation is a tool employed by mathematicians; mathematicians use examples to help them understand a statement or definition, to help them generate an argument, and to help decide whether or not a statement is true [2].

Thus, not only does an example generation (or constructing an instance) task introduce students to an "expert" way of working and mirror the practices of mathematicians, it also incorporates many of the useful features described earlier in section 2. An example generation task lends itself to multiple solution strategies, and invites students to experiment and to make decisions. For further discussion of this task type see [7]. In the particular example generation task shown below, the students were asked for a set of examples, with constraints added sequentially in order to provide an opportunity for them to explore which features they could change and in what way (possibly by asking themselves "what if" questions).

Example Generation

(i) Give three examples of functions that are continuous everywhere.

(ii) Give an example of a function, f, that is not continuous at 4 because $f(4)$ does not exist.

(iii) Give an example of a function, f, that is not continuous at 4 because $\lim_{x\to 4} f(x)$ does not exist.

(iv) Give an example of a function, f, that is not continuous at 4 because $\lim_{x\to 4} f(x) \neq f(4)$ (where both exist).

3.2 Analyzing Reasoning & Evaluating Statements

The thinking processes involved in working on analyzing reasoning and evaluating statements problems are closely related. Therefore we have decided to discuss and exemplify both of these task types in this section.

For instance, tasks which ask students to analyze reasoning or to evaluate statements require students to make decisions about the correctness of a piece of mathematics. Mason and Johnston-Wilder [16] believe this to be important as it requires students to be assertive and active rather than taking a passive approach to learning. Referring to Mason and Johnston-Wilder's [16] list of prompts, both types of tasks involve students in the activities of justifying, verifying and refuting.

Evaluating statements often take the form "is it always, sometimes or never true that ..." and encourage students to specialize, exemplify, explain and convince. As Swan [23] reports, students "devise examples and counterexamples to defend their reasoning"(p.3). One such task we presented to our students before they met the Intermediate Value Theorem is the following:

Evaluating Statements

Consider the following statement and decide whether it is true or false, justifying your answer.

Suppose f is continuous on $[a, b]$ and $f(x) \neq 0$ for all $x \in [a, b]$. Then there must be some z in $[a, b]$ for which $f(z) < 0$ and some $w \in [a, b]$ for which $f(w) > 0$.

On the other hand, analyzing reasoning tasks have the advantage of drawing learners' attention to mistakes. They can be used to confront difficulties rather than avoiding them and to expose common misconceptions. (Note that finding mistakes in supposed proofs or critiquing a line of reasoning is termed criticizing a fallacy by Sangwin [20].) The analysis of reasoning may invoke correcting, completing and deleting in accordance with Mason and Johnston-Wilder's list [16]. Swan [23] also used this label to denote activities in which students compare different methods for doing a problem, thereby enabling them to recognize that there are alternative routes through a problem. However, the analyzing reasoning tasks designed in the project reported here required students to examine a proposed proof of a statement or a worked solution. One such example is shown below.

Analyzing Reasoning

Consider the following argument. Decide whether the reasoning is satisfactory, justifying your answer.

Statement Let f and g be functions that are continuous everywhere, then $\frac{f}{g}$ is continuous everywhere.

Proof Let c be a real number. Since f and g are continuous at c we know that $\lim_{x \to c} f(x) = f(c)$ and $\lim_{x \to c} g(x) = g(c)$. Thus $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$ and so $\frac{f}{g}$ is continuous at $x = c$. Since c was arbitrary, $\frac{f}{g}$ is continuous everywhere.

3.3 Conjecturing and/or Generalizing

We have already noted that Cuoco et al. [9], and Mason and Johnston-Wilder [16] included conjecturing and generalizing in their lists of practices of mathematicians, as did Bass [5] and Stein, Grover and Henningsen [22]. Indeed, Bass [4] describes the progress of most mathematical research as starting with exploration and discovery, followed by conjecturing and finally culminating with formal proof. He identifies two phases of reasoning here: *reasoning of inquiry* which incorporates the exploration and conjecturing steps; and *reasoning of justification* which is rooted in proof. Mason and Johnston-Wilder [16] advocate the cultivation of a "conjecturing atmosphere" in order to encourage students to participate in the reasoning of inquiry and so to develop their mathematical thinking skills. An International Commission on Mathematical Instruction (ICMI) survey of aspects of the transition from school to university [24, p. 111] reported that exploration and conjecturing activities can be helpful to students as a preparation for rigorous proof.

Conjecturing is usually the first step in generalizing. Mason, Burton and Stacey [17] underline the importance of the latter:

> Generalizations are the life-blood of mathematics. Whereas specific results may in themselves be useful, the characteristically mathematical result is the general one (p. 8).

Dreyfus [10] defines generalizing as "to derive or induce from particulars, to identify commonalities, to expand domains of validity"(p. 35). He notes the vital role that generalizing has in the process of abstraction, in moving from a particular instance to a generality, and the difficulty that many students have with generalization. Swan [23] includes the process of identifying general properties of a concept in particular cases of it as one in which a student must be able to engage in order to come to truly understand a concept.

We have recently embarked on a project in which we are using our selected task-types to design online interactive tasks. The use of dynamic geometry software offers possibilities to create opportunities for students to experiment and to make and test hypotheses (as advocated by Ahmed [1]). For example it would be easy to create online versions of our conjecturing and generalizing tasks below.

Conjecturing

Suppose f is a continuous function on $[a, b]$ and suppose that $f(a) < 0$, and that $f(b) > 0$. What can you say about the number of times that the graph of f crosses the x-axis? What about the number of times the graph of f touches the x-axis? Explain your answer.

Generalizing

The function $f(x) = (x^2 - 1)/(x - 1)$ has a removable discontinuity at $x = 1$.

(a) Describe a family of functions each of which has a removable discontinuity at $x = 1$. (What can change and what must stay the same?)
(b) Describe two families of functions for which each function has a removable discontinuity at $x = c$.

3.4 Visualizing

Arcavi [3] defined visualization as "the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (p. 217). The use of visualization as a tool for making connections and deepening understanding has been the object of study for many years (see [19]). Dreyfus [10] sees visualization as one way of making a mental representation of a mathematical object (another might be to generate an example of it), and he stresses the importance of creating a rich collection of mental images in order to be able to think flexibly about concepts and problems. Couco et al. [9] advocate that students should be able to visualize space, data, relationships, processes, change and even calculations.

In the task below, we asked students to visualize the effects of different types of discontinuity on the graph of a function.

Visualizing
Draw a rough sketch of the graphs of the following functions:

1. A function f which is continuous everywhere except at $x = -3$ and $x = 4$.
2. A function g which is continuous everywhere except for removable discontinuities at $x = -3$ and $x = 4$.

3. A function h which is continuous everywhere except for a removable discontinuity at $x = -3$ and a non-removable discontinuity at $x = 4$.

3.5 Using Definitions

Definitions are a fundamental part of mathematics. However research has shown that some students do not appreciate their importance and often do not make use of definitions in their own arguments [11]. For this reason, it is important to emphasize the role of definitions even in a non-rigorous course. For Bass *theory-building practices* are vital in mathematics education and he includes in these "the creative acts of recognizing, articulating, and naming a mathematical concept or construct" [5]. Zandieh and Rasmussen [25] consider defining to be a mathematical activity and highlight the role that the actions of creating and using definitions can play in the development of mathematical understanding. Asking students to take part in this kind of mathematical activity gives them an opportunity to engage in practices such as comparing, sorting, and organizing as advocated by Mason and Johnston-Wilder [16], as well as engaging them in the use of precise mathematical language [9]. Indeed, some of the task taxonomies seen in section 2 also include activities such as the classification of mathematical objects ([20], [23]).

In our example below, we ask students to attend to different parts of the definition of continuity at a point.

Using Definitions

Let f be a function with the following properties:

$\lim_{x\to 3} f(x) = 6$;	$\lim_{x\to 4} f(x) = -6$;	$\lim_{x\to 5} f(x) = 5$;
$f(3) = 6$;	$f(4) = -5$;	$f(5)$ is not defined.

Is the function f continuous at $x = 3$? What about at $x = 4$? How about at $x = 5$? Explain your answer.

4 DISCUSSION

Swan [23, pp. 8-9] lists design principles for teaching practices which aim to develop conceptual understanding; these include using rich tasks, developing mathematical language, using higher-order questions, confronting difficulties rather than pre-empting them, and encouraging reasoning rather than "answer getting". Swan's principles relate to classroom-based tasks appropriate for collaborative work in a secondary school setting, and although some university classrooms might provide a similar environment, the reality is that most student work and study at this level takes place outside of the clasroom [12]. For this reason, the design of *homework* tasks which take account of Swan's principles is important.

Bearing in mind these principles, we have proposed a set of task types that could be used in a typical (non-proof-based) introductory course. We claim that these task types can be used to design effective homework assignments in order to provide opportunities for students to develop mathematical thinking skills and understanding. While each individual task could not hope to engage students in all of the desirable practices of mathematicians (like conjecturing, justifying, exemplifying, and making decisions [1], [9]), students can be given opportunities to engage in a range of meaningful mathematical activity by using a variety of tasks from the six types outlined earlier. Through their engagement with such activity, we hope that students will come to think about mathematics in a way that reflects the true practices of mathematicians.

From our own perspective as instructors we found that tasks of these types, such as the samples shown earlier, seemed to engage our students. Also, because these tasks often lent themselves to having multiple solution methods or required students to explain their thinking, we found that students were less likely to be able to find complete solutions to the questions online and so had to produce original work. Moreover, the design framework helped us to make sure that we assigned tasks of different types in our modules and mitigated against us unwittingly assigning similar questions each week of our favorite task type. One goal of our project was to move away from a situation in which the vast majority of tasks assigned to students could be successfully completed by routine procedures and without the use of higher skills (as found by [14], [20]). Having this framework to guide us ensured that we had a constructive means of achieving this goal.

Having trialed the tasks we asked an independent researcher to interview five volunteer students from each institution for the purpose of discerning their views of the tasks. There seemed to be some evidence from the students' responses that the tasks were having the desired effects in a general sense. For example students recognized that the tasks designed were different from those with which they were familiar from school and they appreciated that the non-routine nature of the tasks made them "think more" ([6], [7]). They also reported being engaged in meaning-making [22] by making links between concepts and seeking justifications for assertions. In the future we hope to conduct a more extensive trial of the tasks designed. In order to determine whether individual tasks are eliciting the specific mathematical activity (e.g. example generation) targeted, we plan to hold task-based interviews using a think-aloud protocol with a sample of students.

Recall that, on the homework sets assigned by us, questions of the types discussed in this paper were accompanied by others that aimed to give students opportunities to practise skills. Both types of questions are important and it would be interesting to explore where the balance between them should lie. It seems clear, however, that if we do not ask students to work on non-routine problems then there is a danger that the types of mathematical thinking skills that we value will be more difficult to develop [8]. Furthermore, we believe that not only are the task-types selected in this article suitable for introductory courses such as Calculus, but that they can help ease the transition to more rigorous courses and in particular to proof [24].

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BIOGRAPHICAL SKETCHES

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