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Pedagogical Strategies, Approaches and Methodologies to Support Numeracy in Early Childhood

A Review of K□□ Literature

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Summary

- The *everyday nature of mathematics* should guide learning in ECEC settings. As young children explore and make sense of the world around them, they discover mathematical concepts in meaningful and developmentally appropriate ways.
- *Play* is a central learning process for young children. Thus, it is important that early childhood educators recognise the potential for mathematical ideas to be explored in play situations and that they engage with and extend children's understanding and learning during play activities. *Intentional teachin*g should also be utilised to provide children with varied early numeracy experiences such as opportunities to engage in real-world problem-solving based on active and hands-on experiences.
- The *role of the adult* in the ECEC setting is fundamental in early mathematics learning. Early childhood educators require knowledge of children's mathematical development and an understanding of how to communicate mathematical ideas in a relevant and meaningful way to young children (Ginsburg, 2016). The maths content knowledge (MCK) of the educator is also central to mathematics teaching and learning as it influences the educator's identification of a child's current level of understanding and supports them in devising an appropriate pathway for future learning. MCK also impacts how an educator responds to or enhances children's play in a mathematical way.
- Educators in ECEC require access to *professional development* (PD) in mathematics teaching and learning to enhance their mathematical knowledge. In particular, training in the use of learning trajectories in mathematics may support educators in noticing, interpreting and enhancing young children's mathematical ideas.
- Engaging young children in *mathematical talk and discussion* promotes mathematical thinking and provides children with opportunities to construct and communicate their mathematical reasoning. It may also support children in developing a range of higher-order thinking skills such as justifying and analysing.
- The use of *children's literature* provides young children with opportunities to engage with mathematical ideas and may have a positive impact on children's mathematical discourse and achievement.
- *Babies and toddlers' innate* mathematical abilities should be informally developed through daily experiences, interactions, and routines.
- *Numeracy early interventions* of shorter duration are more effective (Nelson & McMaster, 2018; Charitaki et al., 2021), but for those learning English as an additional language, longer interventions show higher effect sizes (Arizmendi et al., 2021). One-to-one instruction is beneficial for children including those with MD and learning English as a second language (Wang et al., 2016 & Arizmendi et al. 2021).

Key Recommendations

Curriculum and the Learning Experience

- Approaches to early numeracy must be compatible with ECEC pedagogy where holistic learning, child-led approaches and play are central. Embracing the everyday nature of mathematics will lead to an informal building up of children's innate skills, rather than an academic push down.
- Play is a central learning process for babies, toddlers and young children and play situations can enable children to explore math concepts. Both free play *and* playful learning guided by adults is required. Intentional teaching should be utilised to provide children with varied early numeracy experiences.
- A broader range of mathematical topics and concepts should be included in ECEC curricular documents.
- The Literacy and Numeracy strategy should be expanded to include children from birth to six years. Precursor concepts (Chen et al., 2017) should form the foundation of early numeracy approaches for babies and toddlers.

Teachers and ECEC CPD

- Level 5 to 8 ECEC programmes should include compulsory math content. Math content knowledge (MCK) including the *big ideas of early math* and a broad range of mathematical topics and concepts; Pedagogical content knowledge (PCK) and strategies for intentional teaching including math talk, use of picture books, enabling environments and multimodal approaches to mathematics; and finally the sequence in which children learn mathematical ideas including learning trajectories, math documentation and assessment and planning appropriate pathways for future learning, should be included in all initial education.
- ECEC educators require access to PD in math teaching and learning. It is recommended that within ECEC settings a pedagogical lead in math should be appointed to enhance staff Mathematical Content Knowledge (MCK), Pedagogical Content Knowledge (PCK)and understanding children's math's learning trajectories, particularly for those who do not have these concepts in initial education.

Students with Additional Learning Needs

- Early numeracy interventions ease existing deficits for children at risk of maths difficulty in later schooling (Charitaki et al., 2021). The use of two or more instructional strategies is beneficial to learning outcomes (Charitaki et al., 2021) and through targeting a single content strand (Wang et al., 2016). Children learning English as an additional language benefit from longer math interventions using one-to-one, traditional supports (Arizmendi et al., 2021).

Assessment and Evaluation

- Educators in ECEC require an in-depth knowledge of foundational mathematics concepts, a knowledge of mathematical development and knowledge of teaching and learning of mathematics in ECEC contexts. Possession of such knowledge enables educators to notice, interpret and respond to or enhance children's mathematical activity (Gasteiger & Benz, 2018). Learning trajectories coupled with the 'learning story' format of documentation form a strong framework for educators assessing mathematical activity in ECEC contexts (Perry & Dockett, 2013; Gillic, 2020).

Research Question
What pedagogical strategies, approaches and methodologies support early numeracy development for all children in early childhood education and care settings?

Key Search Terms
S1. DE "STEM Education" (as a Subject Heading accessed via the thesaurus);
S2. "STEM Education" (as a free text search, title search, abstract search as an OR search);
S3. Combine S1 and S2 with OR;
S4. DE "MATHEMATICS Education" (as a Subject Heading accessed via the thesaurus);
S5. "MATHEMATICS Education" (as a free text search, title search, abstract search as an OR search);
S6. Combine S4 and S5 with OR;
S7. Combine S3 and S6 with OR;
S8. "numeracy" or "math* literacy" or math* (as a free text search, title search, abstract search as an OR search);
S9. Combine S7 and S8 with OR;
S10. Combine S9 with "meta-analysis" or "systematic review" using AND;
S11. Combine S10 with "early childhood education" OR "preschool" OR "Pre-K" OR "nursery" OR "kindergarten" OR "daycare" OR "early years" OR "early childhood education and care" OR "foundation stage" with AND.

Key Data Sources Consulted
Date limit 2011 onwards, limited to 'peer review' and limited to English language where necessary.
Three databases were searched (a) EBSCO Education Research Complete (b) EBSCO ERIC (c) Scopus. 'Grey' literature was identified through hand searches.

Introduction

There is growing recognition that young children can, and do, explore mathematical ideas from a very young age. The world is a mathematical place, and children's natural curiosity and wonder enables them to discover and investigate mathematical concepts simply by participating in everyday experiences. Framing situations mathematically by using mathematical language and concepts to analyse and explore situations, supports children to become confident and competent problem solvers and mathematicians. This is the starting point to help children develop an interest in mathematics.

A distinction can be drawn between the terms mathematics and numeracy. Mathematics is a knowledge domain, while the term numeracy stresses the socio-cultural perspective involved for discovering, thinking about, and applying mathematical knowledge in children's everyday lives. Numeracy emphasises the context, purpose, and usefulness of a mathematical approach in solving problems and encourages the meaningful application of mathematical concepts (MacDonald, 2018). While Aistear defines numeracy as "developing an understanding of numbers and mathematical concepts" (National Council for Curriculum and Assessment [NCCA], 2009, p.56) a broader definition will be used here encapsulating the breadth and depth of early numeracy.

> "Numeracy is the knowledge, skills, behaviours and dispositions that students need in order to use mathematics in a wide range of situations. It involves recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully" (State of Victoria Department of Education and Training, 2018, p.6).

Research Question
What pedagogical strategies, approaches and methodologies support early numeracy development for all children in early childhood education and care settings?

Role of the adult

Educator Beliefs towards Mathematics in Early Childhood Education and Care

Beliefs about mathematics have been identified as a critical component of educator competence, in conjunction with subject content knowledge and pedagogical content knowledge (Dunekacke et al., 2016). Early childhood educators' feelings or beliefs about incorporating mathematics in their classrooms can impact the learning experiences that children receive. Beliefs can influence pedagogical practice (Chen et al., 2014; Fives & Beuhl, 2016), impact on daily decisions (Anders & Rossbach, 2015), affect curriculum implementation (Platas, 2015), and may also impinge on a practitioner's ability to see mathematical concepts in children's play (Anders & Rossbach, 2015; Oppermann et al., 2016).

Linder and Simpson's (2018) systematic review of 1141 studies focusing on early mathematics education and early childhood educators found that 31 (19%) of the studies examined related to teacher beliefs or dispositions towards mathematics in ECEC and how their beliefs contribute to classroom practice. The connection between educator beliefs and practice is complex in nature as beliefs are shaped by many influences such as previous experiences with mathematics (e.g. Lake & Kelly, 2014), mathematics anxiety (e.g.Chen et al., 2014) and curricular requirements and accountability (e.g. Gujarati, 2013). However, despite this complexity, the majority of the 31 studies examined by Linder and Simpson (2018) report that early years educators hold a positive belief in their self-efficacy in teaching and learning of mathematics in ECEC settings.

Linder and Simpson's (2018) analysis of practicing teachers showed that a firm belief in child-centred, developmentally appropriate early childhood practice was evident among early childhood educators. Their analysis also showed that some educators believe that other aspects of child development such as social, emotional and language skills should be the dominant focus in preschool rather than mathematics, with some educators holding the view that mathematics is a constant set of rules, skills and knowledge to be learned. Clements and Sarama (2018) suggest that these beliefs have held a long-lasting effect on early childhood pedagogy, where mathematics is often associated with teacher-led, direct instruction and the use of didactic equipment, strategies that are in contrast with the enduring beliefs and firmly established practices of child-led free play and play pedagogy. Pollitt et al. (2020) assert that these beliefs continue to influence the incorporation of mathematics in early childhood contexts, despite increasing research which shows that young children are mathematically competent and capable of engaging with complex mathematical concepts. McCray and Chen (2011) concur, suggesting a commitment to developmentally appropriate practice, coupled with a belief about what mathematics education is, facilitates the assumption that it is 'developmentally ***in***appropriate' to engage young children with mathematics. Consequently, Platas (2015) identified the effect of beliefs on the implementation of mathematics instruction in ECEC settings as a major challenge facing the early childhood education (ECE) field regarding the support of mathematical development.

Mathematical Content Knowledge

Linder and Simpson's (2018) and MacDonald and Murphy's (2019) systematic reviews of research in early childhood mathematics education identified the issue of educator content knowledge as one of the dominant themes in the literature. Linder and Simpson (2018) found the majority of studies indicated that the mathematical content knowledge (MCK) of pre-service educators was lacking in both procedural and conceptual understanding in certain mathematical areas, for example, geometry. Further, the review highlighted that educators struggle when evaluating and understanding student explanations and misconceptions. Gasteiger and Benz (2018) contend that only when educators possess high levels of MCK can they identify a child's individual capabilities and devise pathways for learning effectively (Ginsburg, 2016; Nguyen et al., 2016). MCK is also found to be a prerequisite for identifying mathematics in children's play and the ability to respond to or enhance activity in a mathematical way (Dockett & Goff, 2013).

Researchers (Dockett & Goff, 2013; Gasteiger & Benz, 2018; Lee, 2017) agree that early childhood educators must be able to notice, interpret and enhance young children's mathematical activity in their informal everyday interactions with people and environments. To do this effectively, educators need to possess three types of knowledge: knowledge of children's mathematical development, mathematical content knowledge and how to communicate mathematical ideas in ways that are meaningful and that make sense to children (Lee, 2010; Gasteiger & Benz, 2018).

Big Ideas of Mathematics

A general consensus exists in the early childhood mathematics research literature that early mathematics education ought to concentrate on key, fundamental ideas of mathematics

(Brownell et al., 2014; Dunphy et al., 2014; Sarama & Clements, 2009) and cover a broad range of mathematical topics. Clements and Sarama (2021) propose that these *Big Ideas of Early Mathematics* should meet the three following benchmarks: they must be mathematically coherent, build on children's informal mathematical knowledge, and lay foundations for future mathematical development and understanding. Brownell et al. (2014) have identified 26 *Big Ideas* across the following nine mathematical topics: sets, number sense, counting, number operations, pattern, measurement, data analysis, spatial relationships and shape. It is clear from this list that the range of mathematical topics that young children should engage with is broad, and equally the MCK of early childhood educators should reflect this in both depth and breadth. This is important as research (Björklund & Barendregt, 2016; Gillic, 2021; Lee & Ginsburg, 2009) has shown that traditionally, early childhood educators have narrowly focussed on numbers, counting, shapes and measures in preschool practice, despite a growing body of research demonstrating that young children are capable of exploring an understanding a wide range of mathematical concepts (Clements et al., 2017; Clements & Sarama, 2021).

In conclusion, effective early childhood teachers of mathematics need to possess a solid understanding of the mathematical content associated with the *Big Ideas.* A deep and broad understanding of these foundational mathematical concepts facilitates educator ability to notice, interpret and enhance young children's engagement with mathematical ideas (Dockett & Goff, 2013; Lee, 2017), and to devise individual pathways for learning (Gasteiger & Benz, 2018) in an effective manner.

Observation, Documentation and Planning

Observation of children's development is a long established practice in early childhood education. Early childhood pioneers, such as Froebel, Montessori and Isaacs each encouraged their teachers to observe children's development and to meet children at their own individual level. Observing or 'noticing' young children's mathematical activity during play and everyday natural learning situations is an emerging field of research (Papandreou & Tsiouli, 2020). Researchers (e.g. Dockett & Goff, 2013; Gasteiger & Benz, 2018; Lee, 2017; Perry & Dockett, 2013) attest that reflective observation of children's emerging mathematical ideas and language is a key strategy in supporting a child's mathematical development. However, in order to identify and respond to mathematical activity observed, early childhood educators need to be knowledgeable in several areas: child development (Priestly, 2021), mathematical content knowledge (Dockett & Goff, 2013) and pedagogical content knowledge (Lee, 2017).

The ‘learning stories’ framework (Carr, 2001) is one of the most common types of pedagogical documentation in ECEC. The three components of learning stories (noticing the learning, interpreting the learning and planning further learning) map well onto Lee’s (2017) three-part framework for noticing mathematics in early childhood settings (Gillic, 2020). However, despite this apparent compatibility, the use of learning stories to document young children’s mathematical activity during play is not commonplace. A study conducted by Anthony et al. (2015) noted that when mathematical activity was documented, educators tended to focus on mathematics observed during explicit mathematical situations rather than mathematics that occurred during children’s play. Thus, they contend that the richness of children’s everyday mathematical activity was not documented or deeply reflected upon and that detailed, clear plans for enhancement were not evident in the documentation. The concern here is that selective documentation of young children’s mathematical activity prevents their mathematical competencies and interests being recognised and developed, and hinders their developing identities as mathematicians.

Initial Education and Professional Development

As previously noted, effective provision of mathematics in ECEC contexts is dependent on educators’ ability to identify, interpret and enhance children’s mathematical activity observed in during play, everyday routines and adult-led activities (Lee, 2017; Björklund & Barendregt, 2016; Gasteiger & Benz, 2018). However, traditionally pre-service courses have not included a strong focus on early childhood mathematics and in some courses it does not feature at all (Gasteiger & Benz, 2018; Ginsburg, 2016). In the Irish context, the study of/modules on early childhood mathematics education is not mandatory for degree-level early childhood education courses (DES, 2019).

Linder and Simpson (2018) identified that professional development (PD) using a workshop format is insufficient, and that long-term PD programmes are more beneficial. The review found that training for both pre-service and in-service teachers had positive effects on teacher levels of MCK, child-centred teaching practices and beliefs towards discovery orientated pedagogies (Polly et al., 2014). Linder and Simpson (2018) cautions however, that training does not always translate into an increase in time given to mathematics in settings

(Piasta et al., 2015) and that further studies are required to determine the long-term effects of training on practice (Linder & Simpson, 2018). Other studies have shown that training during PD programmes can be beneficial in developing educators' self-efficacy belief (Chen et al., 2014; Cohrssen & Tayler, 2016; Sancar-Tokmak, 2015), their math content knowledge (Knaus, 2017), appropriate pedagogy (Knaus, 2017) reducing math anxiety (Lake & Kelly, 2014) and in helping to develop educator ability to notice and respond to children's mathematical activity (Perry & Dockett, 2013).

With reference to developing the skill of educator noticing and enhancing children's mathematical activity, the use of developmental (learning) trajectories of children's mathematical thinking are increasingly being recognised as an essential tool for noticing and enhancing young children's engagement with the *Big Ideas of Mathematics*. However, many educators are unfamiliar with learning trajectories as a strategy to support young children's mathematical development (Ginsburg, 2016). According to Clements and Sarama (2021), learning trajectories are research-based developmental progressions in early mathematics. Clements and Sarama (2021) maintain that educators "who understand these developmental progressions…and base their instruction on them, build math environments that are particularly developmentally appropriate, effective and meaningful" (p. 3). Learning trajectories have three components: a mathematical goal, a developmental pathway to reach that goal and teaching practices matched to that level of thinking. Using this three-component strategy as part of pre-service teacher training, Cohrssen and Tayler (2016) found that there was an increase in educator understanding of children's mathematical development. A small study by Gillic (2020) using a progression continua approach to developing spatial awareness in preschool found that the approach helped in developing educator knowledge of this strand of mathematical development and in enhancing children's spatial knowledge and language through playful activities.

Pedagogy

Everyday Nature of Mathematics in ECEC

Approaches to early numeracy must be compatible with pedagogy in ECEC, where young children's learning takes place in the context of holistic learning experiences and in circumstances that are part of their day-to-day lives (Dooley et al., 2014). Young children use math concepts to make sense of their world, although they are often not recognised or referred to in this way (McCray et al., 2014). They compare quantities, find patterns, navigate space, and struggle with real problems such as balancing a block tower or sharing food equally with a sibling. Research recognises that young children are interested in mathematical ideas especially as they present in their everyday play, routines and interactions (MacDonald & Murphy, 2019). Math is evident in children's play; they group toy animals; cows, sheep, and horses by noticing attributes and classifying them accordingly; toddlers expertly weave in and out of toys, peers and equipment outdoors skilfully negotiating space; children can predict that snack follows outdoor play in their ECEC setting, demonstrating a basic understanding of time and sequencing in their daily routine.

A goal of early numeracy in ECEC is to instil a positive attitude towards mathematics and provide a sound mathematical foundation (Thiel & Jenssen, 2018). Practitioners who see the world through a math lens are alert to the math opportunities present in day to day life. Using a math lens or 'mathematising' is a key strategy in supporting development of math concepts (Björklund, Magnusson & Palmer, 2018; Dunphy et al, 2014). By providing opportunities to see how maths applies to everyday experiences (such as measuring ingredients when making a birthday cake; using math language such as more, less, overflowing, too much, biggest bowl; discussing the temperature and timings for the cake) educators provide meaningful opportunities for children to think about math, rather than planned activities that have no connection their lives.

Play

Play is a cornerstone of ECEC and early childhood curricula and traditions have evolved valuing play and playfulness. Unsurprisingly then, play is considered to be the basis for learning and development in early childhood (Wood, 2019). "Given the importance of play as a learning process for young children, it is essential that good mathematics pedagogy recognises this fact, honours it and harnesses its power" (Dooley et al., 2014, p.46). Aistear's principles (NCCA, 2009) stress the importance of *play and hands-on experiences* and *active learning*, positioning play as central to any approach to early numeracy.

Math is abstract by nature and therefore concepts need to be represented in many ways for young children, starting with the use of concrete materials and moving on to pictures, symbols and language. Aligned with the concept of multimodal learning, Brownell et al. (2014) advocate for the practice of 'putting' math into children's eyes, ears, hands and feet. A study by Franzen (2014) found that educators of infants believe talking about mathematical concepts is insufficient for children at this age and that concepts need to be experienced bodily. By engaging multiple sensory and action systems children are provided with many different opportunities to grapple with math ideas. The more modes in use, the deeper the learning will be; building fluidity into children's mathematical thinking and preparing them for more complex mathematics to come (Brownwell et al., 2014). Concrete, hands-on experiences and the provision of 'numeracy' resources such as blocks, counters and measuring jugs does not guarantee an understanding of math concepts. Math is about more than manipulatives and requires support from adults to make the connection between math opportunities that present themselves in play or daily routines, and the *Big Ideas of Mathematics*. Thus, educators require a sensitivity to mathematical content in play situations and the ability to engage with and extend children's understanding and learning.

Intentional Teaching of Mathematics

ECEC acknowledges the integrated nature of learning, crucial role of context and requires educators to develop and use skills that differ from those in later schooling (Hayes, 2019). In traditional ECEC conceptions of play, the child, her interests and needs are central concerns and the child's enjoyment, control and independence are paramount. Hence, the introduction of mathematics in ECEC can be perceived by some as a change in its main function from providing children with rich holistic learning opportunities to preparing them for school. Research related to how early math knowledge can be supported in ECEC reveals the tension between the intentional teaching of mathematics and traditional child-led play pedagogy (Björklund et al., 2018). However, much recent research (for example, see Dunphy, 2018; Helenius, 2018; Thiel & Jenssen, 2018; Thiel & Perry, 2018; Broström, 2017; Knaus, 2017) proposes the introduction of intentional math teaching but not as a replacement for play. Instead it is the introduction of intentional and active early numeracy experiences such as real-world problem-solving, reasoning and explaining, based on active and hands-on experiences.

In Piagetian theory, children develop naturally through play when left to their own devices (Broström, 2017) limiting the educators' role to one of following the child's lead. These conceptions are still strong in practice and are connected to *laissez-faire* approaches where educators provide opportunities for free play, observe children's natural development, but do not intervene in play (Wood & Hedges, 2016). The concept of finding and using ‘teachable moments’ in children’s free play has been heavily criticised as such practice requires an inordinate amount of skill and understanding on behalf of the educator, as well as luck in timing and physical presence. Educators rarely involve themselves in children’s free-play activities (McInnes, 2019) and prefer larger groups when organising planned activities (Vogel, 2013) making the identification of ‘teachable moments’ challenging. Clements and Sarama (2014, 2018) claim that *depending* on this approach presents ‘serious problems’ such as the extensive time in careful observation that would be required to identify teachable moments, and the difficulty in seeing opportunities for multiple children in the group. Even in free play contexts, intentional, planned teaching is more effective than laissez-faire approaches, or teaching based on “teachable moments" (Clements & Sarama 2021)

Piaget’s cognitive developmental theory underpins the notion that adults simply support children’s learning endeavours, where Vygotsky’s conception of the ZPD gives primacy to the role of the adult. Fleer (2014) has written extensively about the educator’s role in reconciling children’s math concept formation through play. Drawing on Vygotsky’s cultural-historical theory (Vygotsky, 1987) she argues that development occurs through interaction with people and experiences which increases motivation, interest and participation further than children might do alone. Using this lens, children’s cognitive learning is externally generated making the mediating role of the adult crucial for learning. This requires a new way of conceiving the role of the educator. Fleer suggests that educators reclaim their professional expertise as active agents in children’s learning and ‘not be seen as passive providers of materials to foster developmental milestones, where the latter role… de-emphasises their place in children’s learning’ (Fleer, 2014, p. 41). Intentional teaching can enable children to make transition to new and more complex forms of learning without utilising didactic methods (Chesworth & Wood, 2017). This can be described as an amplification of development, where the adult role is to enrich and expand the content of play, rather than a forced acceleration one might expect in school readiness approaches.

There is no empirical evidence to suggest that the thoughtful and strategic support and extension of early numeracy is inappropriate (Cohrssen et al., 2013). When compared to free play only, direct individual instruction has been linked with higher gains in math (Chien et al., 2010). However, there isn’t a one size fits all approach to support math learning. In general, it appears that creative play-based approaches correspond with higher learning gains for most children, but children from lower socioeconomic backgrounds benefitted from more structured approaches and formal teaching (Vogt et al., 2018). The dominant discourse at present is that children require both free play *and* playful learning guided by adults (Clements & Sarama, 2014).

Mathematical Talk and Discussion

A strong theme emerging from the systematic review studied is that mathematical talk and discussion is of critical importance in supporting young children's mathematical thinking (Linder & Simpson, 2018; MacDonald & Murphy, 2019) and that this pedagogical strategy is extremely beneficial to young children with additional needs, for example, English language learners (Arizmendi et al., 2021). Research shows that one of the major indicators of early mathematical understanding and future mathematical success, is mathematical language ability (Purpura et al., 2011). Math talk has been identified as a key pedagogical strategy in enabling young children to communicate their mathematical thinking and ability to articulate their mathematical reasoning, arguments and justifications (Clements & Sarama, 2021; Dooley et al., 2014).

Linder and Simpson's (2018) systematic analysis of 1411 studies relating to both pre-service and in-service early years educators, showed that in 28% of 164 of the studies reviewed relating to teacher's adapted practices, a variety of 'math talk' strategies were used by practicing teachers. These included open-ended questioning and dialogue and purposeful pauses. High levels of mathematical talk and discussion lead to elevated student outcomes in terms of mathematical confidence, mathematical knowledge and positive dispositions towards mathematics (Björklund & Pramling-Samuelsson, 2013; Jung & Reifel, 2011).

Research carried out by Cohrssen et al. (2014) demonstrated that when educators pause during conversations with children, this allows children to think about, formulate, and articulate their mathematical thinking during play-based activities. Such mathematical interactions allow children to reconstruct mathematical understandings and to reflect on these understandings as new knowledge is constructed (Cheeseman, 2015). A study by Ryoo et al. (2018) showed that when educators engaged young children in mathematical conversations to problem-solve and aided children in making connections between the mathematical concepts discussed and real-life contexts, increases in children's maths scores were recorded.

Gesture

Martinez-Lincoln et al. (2019) conducted a systematic review to investigate the use of gesture in mathematics instruction by both teachers and students. They reviewed 35 research articles and concluded that the use of gesture had a positive impact on children's learning of mathematics content. This was largely attributed to their impact on working memory, leading to a reduction in cognitive load for the child as they explore complex ideas. A number of the studies reviewed also recognised the use of gesture as an active learning tool which may increase engagement, particularly when encountering new or challenging content (e.g., Kim et al., 2011; Logan et al., 2014). While gesture has also been associated with the development of mathematical skills, it must be noted that in one particular study relating to counting skills, the use of gestures did not improve counting accuracy in preschool children (Nicoladis et al., 2010). In this study it was found that children's counting skills were assisted by mapping words onto objects, resulting in less counting errors than when mapping number gestures onto objects. Martinez-Lincoln et al. (2019) concluded that this suggests that gesture may not assist in learning across all mathematical domains and they called for further research across a diverse range of content areas.

Children's Literature

The findings of a systematic review conducted by Edelman et al. (2019) on the use of children's literature in mathematics teaching and learning indicate that the integration of children's literature and mathematics may have a positive impact on children's achievement, engagement, and mathematical discourse. In particular, in the early years of schooling, reading aloud to children and the use of picture books have been found to contribute to the development of more robust understanding of mathematical concepts (McGuire et al., 2020; Van den Heuvel-Panhuizen & Elia, 2012). However, it is important to recognise that all picture books do not offer the same opportunities for mathematics learning and teachers must be mindful of the characteristics of effective mathematical stories when selecting picture books for use with young children (Trakulphadetkrai et al., 2020). A framework was proposed by Van den Heuvel-Panhuizen and Elia (2012) in which they identified five learning supportive characteristics of picture books: mathematical processes and dispositions, mathematical content domains, mathematical-related themes, ways of presenting the mathematical content and the quality of presentations. The utilisation of this framework may assist educators in identifying the characteristics that picture books should have to enhance young children's learning of mathematics.

Mathematical abilities of babies and toddlers

While a growing body of literature indicates that babies are born hardwired with innate abilities associated with mathematics, this systematic search indicates that there is still little investigation into how early math capacities and knowledge can be supported for infants and toddlers. Infants are able to extract meaningful information about their environment from birth and show a variety of basic numerical skills, including a rudimentary understanding of quantity, or numerosity; ordinal value; and the effects of addition and subtraction on quantity (Geary, 1994). With babies and toddlers, these informal ideas are preverbal and are very different from common skills supported in ECEC. In this way, the focus is on a supportive building up of innate skills rather than an academic push down (Chen et al., 2017).

A meta-analysis by Christodoulou et al. (2017) investigated the replicability of Wynn's seminal 1992 study where infants as young as five months were found to have an innate understanding of arithmetic. The procedure involves presenting a single doll to an infant, raising a screen blocking the infants' view of the doll as the researcher places a second doll behind the screen within the child's eye line. The screen is then lowered, showing either one or two dolls. Wynn reported that infants looked longer at the incorrect numerical solution than at the correct solution (i.e., infants preferred looking at one doll after the addition events and at two dolls after the subtraction events), concluding that infants have innate arithmetical abilities. Christodoulou et al.'s meta-analysis (2017) evaluated the extent to which original findings were reliably reproduced in other laboratories. Statistically significant summary effects were generated in replications and extensions of Wynn's study involving infants of various ages suggesting that the phenomenon Wynn originally reported is reliable. It should be noted that tests that used screens rather than real objects did not exhibit the Wynn effect reliably.

Moreover, recent research suggests that infants recognise counting as numerically significant. Wang and Feigenson (2018) found that counting aloud directs infants' attention to numerical aspects of the world, indicating that they identify counting as numerically relevant years before understanding the meanings of count words. Working with 14 to 18 month olds, Wang and Feigenson measured infants' ability to remember different numbers of hidden objects that either were, or were not, counted by an experimenter before hiding. The study found that infants remembered more hidden objects when the objects were counted before hiding, suggesting that while babies cannot understand number words (two, three, four) a more basic understanding of counting might be present earlier than previously thought.

Impact of Educator Education

A heightened awareness of mathematics can enhance opportunities provided for toddlers to explore mathematical concepts and principles. A study by Björklund (2012), emphasized the impact of educators' awareness of mathematics and how this can enable educators to detect opportunities for exploring math concepts. This articulated awareness was demonstrated in interactions with the toddlers empowering educators to use the children's initiatives as starting points for planned education, using a mathematical frame. Similarly, Blesesa et al. (2019) found that an intervention providing teachers of toddlers with supportive tools to be more explicit and intentional in their interactions with children resulted in positive, medium- to large-sized effects on targeted math skills. A study by Johnston and Degotardi (2020) found around 8% of the talk that occurred during mealtimes with infants and toddlers was mathematical and Chernyak (2020) notes that in other studies where the focus is on specific numeracy-based experiences, educator mathematical talk can rise to 25%. In line with other studies (Degotardi et al., 2016; Degotardi 2017) Johnston and Degotardi (2020) posit that increasing educators' awareness of the potential of mealtimes and other day-today experiences will positively affect the quantity and quality of mathematical talk that occurs with very young children.

Possible Content Areas

The research base for pre-numeracy is sparse (Susperrguy, 2016) and activities that can support the development of numerosity with babies and toddlers has yet to be identified Evans & Gold, 2020). There are limited suggestions for supporting infants beyond practicing skills such as basic counting and talking about numbers. Crucially however, four 'precursor mathematical concepts' have been identified as important for babies and toddlers; attribute, comparison, pattern, and change (Chen et al., 2017). These precursor concepts come before more defined mathematical ideas, such as number or measurement, but influence the development of more sophisticated math skills. These concepts are deemed a necessary ingredient for the development of several foundational early math concepts. For example, the ability to identify or classify attributes is required to recognise or make sets. and shapes are classified and defined by attributes. Educators require additional training to enable them to foster precursor mathematical understanding in infants and toddlers.

Math Interventions

Five meta-analyses (Wang et al., 2016; Nelson & McMaster, 2018; Charitaki et al., 2021; Arizmendi et al., 2021; and Yang et al., 2020) reported on the outcomes of maths interventions in early childhood. The studies had a varied foci including children learning English as an additional language and those deemed at risk of math difficulty. Corresponding and divergent findings are noted.

Firstly, Wang et al.'s (2016) meta-analysis found that specially-designed, 'developmentally appropriate' ECEC mathematics programmes were highly effective in enhancing children's levels of mathematical understanding. There was a tendency for programmes to produce larger effects when they targeted a single content strand; presented content 2-2.5 hours per week; were designed specifically for the prekindergarten environment; and used one-to-one instruction. Repetitive activities that present mathematical content via cards, and which do not focus on foundational mathematical concepts were not as effective as a consistent programme approach with dedicated time given to mathematics instruction every day. Wang et al. recommend that developmentally appropriate mathematics programmes can, and should, be developed for pre-kindergarten classes and implemented on a national scale (US).

Nelson and McMaster (2018) examined the effectiveness of early numeracy interventions including students with disabilities or those at risk for math difficulty. 34 studies with 54 treatment groups were included with a moderate average overall effect size. Results indicate larger treatment effects for interventions that included counting with 1-to-1 correspondence and were 8 weeks or shorter in duration. On average, interventions were more effective for students with lower levels of risk of maths difficulty and less effective for students with higher levels of risk for maths difficulty according to screening criteria and risk according to low socioeconomic status when compared to typically achieving students.

While focusing on children in the upper age limits of ECEC (5-8 years), a meta-analysis by Charitaki et al. (2021) highlighted the effects of interventions for those deemed at risk of math difficulty. The interventions in the review included instructional strategies such as Concrete-Representation-Abstract (CRA) methods, explicit instruction and corrective feedback; the use of concrete manipulatives; and visual representation. Moderate effect sizes were found for these instructional strategies. However large effect sizes were yielded for studies that employed two instructional strategies. The results suggest early numeracy interventions eased existing deficits for children at risk of maths difficulty in later schooling. Findings indicate larger treatment effects for short-term interventions (defined in this study as 1 to 9 sessions).

Arizmendi et al. (2021) focused on children learning English as an additional language who had been diagnosed with math difficulty. This study focused on the teaching of math language and vocabulary to develop and further mathematical understanding. Findings suggest that one-to-one instruction is more effective than small group strategies. Early intervention is more beneficial than interventions in later grades (however, authors argue that language and concepts are easier in early childhood for children to grasp than those introduced in later grades). Traditional instruction yielded higher effect sizes than computer-based interventions. A focus on numeracy skills rather than mixed math skills was highly effective. Time is also a factor, with longer interventions showing higher effect sizes than shorter ones.

Finally, Yang et al. (2020) reviewed diverse spatial training programmes for those working with children from birth to eight years. Strategies included hands-on exploration, use of visual prompts, and gestural spatial training. Young children's spatial skills could be significantly improved if they are given specific training. Findings suggest that spatial skills are more malleable in early childhood than later in life. Most of the effective spatial training used video games, play, hands-on exploration, spatial tasks, or classroom-based courses as the intervention or stimuli with the common thread of actively practicing spatial skills in various activities. It should be noted that gender was a factor in this study. Early spatial skills training leads to greater effect for girls ($g = 0.909$) than boys ($g = 0.686$). Therefore, Yang et al. (2020) suggest girls should be given the priority to engage in spatially enriched experiences.

Conclusion

The question “what pedagogical strategies, approaches and methodologies support numeracy development for all children in early childhood education and care settings?” guided this report. It is clear that educator beliefs, knowledge and understanding, and initial education and PD are crucial to supporting early numeracy in ECEC. One of the key priority actions from the Interim Review of the National Literacy and Numeracy Strategy (DES, 2017) was to support ECEC educators “to gain a deeper understanding of numeracy concepts, the sequence in which children learn early mathematical ideas and identifying and providing materials and activities which further promote learning in this area’ (p. 21). This review reinforces the crucial role of educators in supporting early numeracy. The complexity of this task should not be underestimated. MCK, including procedural and conceptual understanding of mathematics, the *big ideas of mathematic*s and the broad range of mathematical topics that young children should engage with, need to be made compulsory content in initial ECEC education. Further, PCK including learning trajectories, assessment and planning should be included. Similar professional development opportunities should be provided for those already working in, and managing, ECEC settings.

The literature points to the importance of enhancing babies’ and toddlers’ innate math understanding and the positive impact PD for those working with this age group can have on learning outcomes. In previous iterations, the Literacy and Numeracy Strategy focused on educators working in ECCE rooms only. The omission of babies and toddlers from the strategy needs to be addressed. Structures should be put in place to highlight the importance of pre-cursor concepts (Chen et al., 2017) to educate those working with babies and toddlers.

Finally, the need to raise awareness of the importance of mathematics in ECEC is paramount and should be framed as a building up of children’s innate abilities rather than an academic push down. This must be achieved to increase not only the amount of mathematics in ECEC, but also the quality of mathematical experiences for babies, toddlers and young children (Clements & Sarama, 2021).

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Córa Gillic (ORCID iD 0000-0003-2976-1893) *is an Assistant Professor in Early Childhood Education in the School of Language, Literacy and Early Childhood Education in DCU's Institute of Education. Córa lectures in early childhood mathematics education on the BEd programme. Córa's previous research focused on the beliefs and self-reported practices of preschool educators towards mathematics in ECEC. She has explored the use of progression continua as tools for CPD in mathematics in ECEC. Córa's doctoral studies focus on teacher noticing of mathematics during play. As part of the Maths4All project Córa developed CPD material for preschool educators to develop children's spatial awareness through play.*

Mary Kingston (ORCID iD: 0000-0003-0817-6230) *is an Assistant Professor in Mathematics Education in the School of STEM Education, Innovation and Global Studies in DCU's Institute of Education, Dublin City University. She is currently lecturing in the area of mathematics education on the Bachelor of Education, Professional Master of Education (Primary) and Master of Education programmes. Mary is undertaking a PhD focused on the development of young children's mathematical thinking and she is establishing frameworks of growth points that describe how mathematical thinking develops over time in relation to the topic of probability. Prior to joining DCU, Mary worked as a primary teacher.*

Appendix

Research Question
What pedagogical strategies, approaches and methodologies support early numeracy development for all children in early childhood education and care settings?

Key Search Terms
S1. DE "STEM Education" (as a Subject Heading accessed via the thesaurus);
S2. "STEM Education" (as a free text search, title search, abstract search as an OR search);
S3.Combine S1 and S2 with OR;
S4. DE "MATHEMATICS Education" (as a Subject Heading accessed via the thesaurus);
S5. "MATHEMATICS Education" (as a free text search, title search, abstract search as an OR search);
S6. Combine S4 and S5 with OR;
S7. Combine S3 and S6 with OR;
S8. "numeracy" or "math* literacy" or math* (as a free text search, title search, abstract search as an OR search);
S9. Combine S7 and S8 with OR;
S10. Combine S9 with "meta-analysis" or "systematic review" using AND;
S11. Combine S10 with "early childhood education" OR "preschool" OR "Pre-K" OR "nursery" OR "kindergarten" OR "daycare" OR "early years" OR "early childhood education and care" OR "foundation stage" with AND.

Key Data Sources Consulted
Date limit 2011 onwards, limited to 'peer review' and limited to English language where necessary.
Three databases were searched (a) EBSCO Education Research Complete (b) EBSCO ERIC (c) Scopus. 'Grey' literature was identified through hand searches.

Prisma Chart Generated by Covidence

Tabulation of Main Findings

Paper Title/ Author/ Date	No. of studies	Effect size (If available)	Numeracy area	Age range	Findings
Arizmendi, G., Li , J., Van Horn, M., Petcu, S. D. & Swanson H.L. (2021). Language-Focused Interventions on Math Performance for English Learners: A Selective Meta-Analysis of the Literature. DOI: 10.1111/ldrp.12239	35		Instructional methods - EAL learners	Pre-school – 12th grade	Focus on teaching of math language/vocab to develop and further mathematical understanding in ELs, particularly those who had been diagnosed with math difficulty (MD).
Charitaki, G., Tzivinikou, S., Stefanou, G. & Soulis, S. (2021). A meta-analytic synthesis of early numeracy intervention for low-performing young children, SN SOC SCI, 1 (5) 105. DOI:10.1007/s43545-021-00094-w	20	moderately effective g = 0.61, but heterogeneity was large.	Impact of numeracy interventions for low performing children	5-8 years	The interventions reviewed included instructional strategies such as Concrete-representation-abstract (CRA) methods, explicit instruction, concrete manipulatives, corrective feedback and visual representation Results acknowledged a modest difference in early numeracy attainment between children who received the intervention and children of the same age who did not.
Christodoulou, J., Lac, A., & Moore, D.S. (2017). Babies and math: A meta-analysis of infants' simple arithmetic competence. DOI: 10.1037/dev0000330.	12		Infants arithmetic competence	Infants	This study looks to test the replicability of Wynn's (1992) study after 25 years. Wynn concluded that infants have innate arithmetical abilities. The primary objective of this meta-analysis was to accumulate empirical evidence to evaluate the extent to which Wynn's original findings were reliably reproduced in other laboratories.
Linder, S. & Simpson, A. (2018). Towards an understanding of early childhood mathematics education: A systematic review of the literature focusing on practicing and prospective teachers. DOI: 10.1177/1463949117719553	1141		Practicing and prospective EC teachers' practice	Early Childhood	This review concentrates on three aspects of teacher practice in the field of EY mathematics: content knowledge, disposition and transformation.

MacDonald, A. & Murphy, S. (2019). Mathematics education for children under four years of age: a systematic review of the literature. DOI: 10.1080/09575146.2019.1624507	103		Math for babies/ toddlers	Children under 4	This review focuses on the critical role of educators, and the ways in which educator knowledge, attitudes, and strategies shape the mathematical learning opportunities available to children.
Martinez-Lincoln, A., Tran, L., & Powell, S. (2018). What the hands tell us about mathematical learning. A synthesis of gesture use in mathematics instruction. DOI: 10.1075/gest.17014.mar	35		Gesture in math instruction	Pre-school-12th Grade	In this synthesis, the researchers analysed how educators and students use gestures within the teaching or learning of mathematics, stating that the connection between gestures and mathematics is natural for several reasons.
Nelson, G. & McMaster, K. L. (2018). The Effects of Early Numeracy Interventions for Students in Preschool and Early Elementary: A Meta-Analysis. DOI:10.1037/edu0000334	34	The average weighted effect size was moderate (g _ 0.64)	Effectiveness of early math interventions for those with maths difficulty (MD)	Pre-school-first grade	Results of the final metaregression model predicted larger treatment effects for interventions that included counting with 1-to-1 correspondence and were 8 weeks or shorter in duration. On average, interventions were more effective for students with lower levels of risk of maths difficulty (MD) according to screening criteria compared to typically achieving students
Wang, A., Firmender, J., Power, J. R. & Byrnes, J. (2016). Understanding the Program Effectiveness of Early Mathematics Interventions for Prekindergarten and Kindergarten Environments: A Meta-Analytic Review, DOI: 10.1080/10409289.2016.1116343	29	Overall moderate to large effect size.	Effectiveness of Early Mathematics Interventions	Pre-K and Kindergarten	Specially designed developmentally appropriate early mathematics programmes are highly effective. Early Interventions positively affect children's levels of mathematical understanding.

Yang, W., Liu, H., Chen, N., Xu, P. & Lin, X. (2020). Is Early Spatial Skills Training Effective? A Meta-Analysis. DOI: 10.3389/fpsyg.2020.01938	20	Average effect size for training relative to control was g = 0.96	Spatial skills in infancy and early childhood	0-8 years	Diverse training strategies/ programmes including hands-on exploration, visual prompts, and gestural spatial training significantly foster young children's spatial skills. Hands-on exploration, visual prompts, and gestures are used to support the process of actively practicing spatial skills in various activities