Mathematical Identity of Science and Engineering Students (MISE)

A thesis submitted to Dublin City University
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by

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Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, and that I have exercised reasonable care to ensure that the work is original, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Abstract

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Fionnán Howard

In this study, a qualitative, longitudinal research design was used to characterise participants' mathematical identity, and investigate how this changed during their transition to university mathematics education. The study extends previous research on mathematical identity in Ireland to include a previously under-researched cohort of science and engineering students.

A definition of mathematical identity was operationalised under a narrative paradigm, meaning that identity was seen as fluid and ever-changing, and that narratives were positioned as "enactments of identity, constructed in the moment" (Radovic et al., 2018, p. 29). The study included three sequential stages of data collection, which combined reflexive thematic analysis of questionnaire ($n = 32$) and focus group ($n = 5$) data with an in-depth narrative analysis of individual interviews ($n = 6$).

This research found further evidence that reflecting on their own mathematical identity helps students to engage more effectively as mathematics learners (Kaasila, 2007b) and process new mathematical experiences (Sfard & Prusak, 2005, p. 16). The results of this study are of interest to practitioners who seek to improve the overall learning experience of science and engineering students, and the efficacy of the teaching they encounter. The narrative methodology and methods used are of interest to a wider audience of researchers in affect and identity.

The key findings include that participants presented only absolutist views of the discipline of mathematics but saw the value in relational understanding and problem-solving when it comes to real-world applications. They also believed that progression through mathematics at Senior Cycle and university is based on hard work, interest, and passion, not natural ability. Over time, participants collaborated more with their classmates, and relied on sources of learning outside the classroom as well as inside. Teaching and work placements, in which participants learn the role of mathematics in their intended work environments, were found to have a significant influence on participants' mathematical identity.

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Chapter 1

Introduction

In this chapter, I introduce my PhD study: an innovative, nuanced, and holistic investigation whose aim is to empower participants in their own mathematics education, and to contribute to knowledge about their transition to university. This chapter begins with an outline of the study (section 1.1). The terminology and context associated with schooling and examinations in Ireland is introduced in section 1.2, and recent changes to this system are discussed in section 1.3. A brief introduction to mathematical identity, as well as the aims and motivation for this study are presented in section 1.4. Finally, the structure of the thesis is explained in section 1.5. In later chapters, it is argued that the "experiential knowledge" (Maxwell, 2013, p. 54) of the researcher influenced the research design, including the manner in which data is collected and analysed. An explanation of my personal motivations for undertaking this PhD study, with reference to my own experiences of teaching and learning mathematics, is presented in appendix A.

1.1 Outline of the Study

The purpose of this study is to explore the relationship of science and engineering students with mathematics, and investigate how this relationship changes and matures as they transition to university-level education. The name that was chosen for the study is Mathematical Identity of Science and Engineering students (MISE).[1] This title was chosen to reflect the focus of the study on facilitating, as well as analysing, the development of participants' mathematical

[1]"Mise" is the Irish word for "oneself," and is pronounced mish-eh.

identities, because the researcher plays an influential role in participants' identity development (see chapter 3). Doing this will broaden the scope of mathematical identity research to a new and under-explored context, that of students who undertake to study mathematics as a constituent part of a degree in science or engineering (including Science Education) in Ireland. I seek to answer the following three research questions:

RQ1. What is the relationship of science/engineering students with mathematics?

RQ2. What is the difference in mathematical identity of science students compared to engineering?

RQ3. How does the relationship of these students with mathematics change over time?

To answer the research questions, a qualitative, longitudinal study was employed, which comprised of three main stages of data collection, supplemented by two pilot studies. The data collection stages involved open-ended questionnaire responses, focus groups, and narrative interviews. The first two stages were analysed using reflexive thematic analysis, while the interviews were analysed through a narrative analytic framework called *storying stories*.

1.2 The Irish Education Context

This section provides an explanation of the Irish Education system, in order to better understand students' school context, and the possible nature of its influence on their mathematical identities (see section 1.4). There are three levels of schooling in Ireland that are relevant to this study: primary, post-primary, and university levels. In this section, I draw on a publication by OReilly et al. (2017) to explain the post-primary context in detail. Students attend primary school for eight years, approximately from the age of four until 12, whereafter they attend post-primary school for five or six years, from the ages of 12 to 17 or 18 (p. 347). Participants frequently refer to post-primary school as "secondary school," while university is often referred to as "college" (unlike in the UK where college refers to post-primary education). Most students in post-primary school sit national examinations in mathematics at two points of post-primary school. In post-primary school, Junior Cycle refers to years one to three, which culminate in a state exam called the Junior Certificate. Thereafter, students can take a fourth transition year (TY) (OReilly et al., 2017, p. 347; Prendergast & O'Meara, 2016, Table 1), which is optional in most schools (Clerkin, 2012, p. 7). Senior Cycle refers to the following two years (years five and six), after which students sit the Leaving Certificate examination. The Leaving Certificate examination awards points to students based on their results in six examination subjects, which are then used by education institutes (including universities) to determine which applicants to their programmes are successful. Although TY is a non-academic year, which focuses on students social and personal development through exploration of familiar and new areas of study (p. 2), it sometimes involves pre-emptive study of the Leaving Certificate mathematics curriculum, which should be reserved for Senior Cycle (Prendergast & O'Meara, 2016).

For the Junior Certificate and Leaving Certificate, there are three levels of examination available: higher, ordinary, and foundation (OReilly et al., 2017, p. 347). In 2017, the year that the participants in this study sat their Leaving Certificate examinations, approximately 30% of students sat higher-level mathematics, 59% sat ordinary-level, and 11% sat the foundation-level examination (State Examinations Commission [SEC], 2017, see appendix B.1). Participants sometimes refer to the higher and ordinary levels as "honours" or "pass," respectively. Students of university programmes that are accredited by the Teaching Council (2020), and thus are most likely preparing to enter the workforce as a primary or post-primary teacher, will be referred to as pre-service teachers (PSTs). It is almost universal for students, lecturers, and universities in Ireland to refer to their programmes simply as "courses."

1.3 Recent Changes to Mathematics Education in Ireland

As a result of changes in teacher education and qualification criteria, which will be described below, post-primary students are more likely than they were in the past to be taught by mathematics teachers who have experience with abstract university mathematics. Research in 2009 estimated the proportion of post-primary mathematics teachers without a mathematics teaching qualification (known as out-of-field teachers) at 48%, and highlighted this as a particularly acute problem amongst young teachers (Ní Riordáin & Hannigan, 2009). In 2012, the Professional Diploma in Mathematics for Teaching (PDMT) programme was established to educate such teachers in the areas of expertise required by the Teaching Council (2020), so that they could practice as qualified teachers of mathematics. It has been noted that approximately 1100 teachers have since graduated from the programme, with 300 further teachers still undergoing the training and accreditation (Quirke, 2022, p. 101). By 2018, the figure given by Ní Riordáin and Hannigan

(2009) for out-of-field teachers had dropped from 48% to 25% in the nine years that followed (Goos et al., 2021, p. 9).

The above changes all took place against a backdrop of transformation in post-primary mathematics curricula and examinations. A new mathematics curriculum, *Project Maths*, was implemented nationwide from 2010 to 2013 (Johnson et al., 2019, p. 3). Project Maths aimed to prioritise the understanding of mathematical concepts; promote higher-order thinking and problem-solving skills; and include more context and everyday life applications of the subject (O'Meara et al., 2011, p. 334; Department of Education and Skills [DES], 2016, p. 20). It was hoped that students would develop an understanding of mathematical theory alongside the ability to apply mathematics in practical situations (O'Meara et al., 2011, p. 334). Furthermore, the bonus points initiative was introduced in 2012, which awarded extra points towards their Leaving Certificate tally, for students who achieved at least 40% in their higher-level mathematics examination (see appendix B.3). As a result, the proportion of students sitting higher-level mathematics almost doubled, from 15.8% in 2012 to 31.5% in 2019 (O'Meara et al., 2019, p. 222). The vast majority of teachers credited the introduction of the bonus points initiative with this increase, but also disbelieved that it had brought about improvements in students' mathematical ability (p. 225).

1.4 Why Study this Problem?

Mathematics is a fundamental underpinning of Science, Technology, Engineering, and Mathematics (STEM) programmes, and the National Development Plan for the next decade in Ireland includes an intent to significantly increase the number of places available at higher education institutions in such programmes (Government of Ireland, 2021). According to a more recent report by Financial Services Ireland (2022), produced with input from the Irish Business and Employers Confederation, the technical STEM skills which are in demand by the sector specifically include skills in mathematics, computer science, computer engineering, and software engineering (p. 13). Researchers in Ireland have been concerned with the mathematical preparedness of first year students in Irish universities for some time (Breen et al., 2009; Hourigan & O'Donoghue, 2007; Lane & Walshe, 2019; Ní Fhloinn & Carr, 2010; Treacy et al., 2016). Although the landscape of post-primary education in Ireland has undergone changes recently (see section 1.3), a recent report into the Irish education system repeated concerns about students being mathematically underprepared, and lacking basic mathematical skills for university programmes, even those who had completed higher-level Leaving Certificate mathematics (DES, 2016). Despite the size of the cohorts of science and engineering students, there is little evidence of research in mathematical identity concerning these groups in Ireland. In Dublin City University (DCU), they represent a significant proportion of the undergraduate population (11.6% in 2017/18, see appendix C.1), and of students taking mathematics modules, yet they had not previously been included in research on mathematical identity.

Personal Motivation

The knowledge and broader worldview that I bring to this research is informed by my experience as an undergraduate and postgraduate student of mathematics, and my experience as a mathematics lecturer and tutor at university level. While I did not perform particularly well in written examinations as an undergraduate, nor did I enjoy the disengaging lecture style I often experienced, I excelled at self-guided project work, and I felt particularly comfortable researching functional analysis at masters level. For me, the practice of research in mathematics was not at all aligned with my undergraduate experience of the discipline, in which repetition and rote-learning featured more heavily than exploration and construction. I identified far more with Stewart's image of mathematicians jabbering and hand-waving about a problem, until a likely avenue to the solution is formulated (Stewart, 1945/1990, pp. xxviii-xxix).

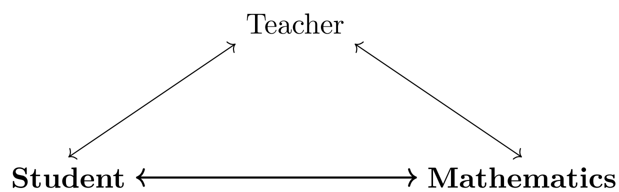
I was motivated to conduct the current research by my early teaching experience in Trinity College Dublin (TCD), during which I conducted tutorials for students in a wide range of programmes in mathematics, science, and engineering. It appeared to me that some of the TCD School of Mathematics lecturers, who communicated very well with mathematics students, struggled to engage effectively with students of science and engineering. Through my experiences as a teacher at university level, I further consolidated a perception of mathematics as a collaborative, socially communicated, and socially negotiated area of knowledge.

A definition of mathematical identity is offered in the next subsection, wherein it is argued that the notion can be used to explore how science and engineering students' negotiate their relationship with mathematics by acknowledging the influence of social and cultural factors on this relationship.

Teachers' and Lecturers' Influence on Students' Mathematical Identity

This study takes inspiration from previous research in Ireland involving PSTs, wherein mathematical identity was defined as the "multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others" (Eaton & OReilly, 2009c, p. 228). Grootenboer and Zevenbergen (2008) explained that mathematical identity can be thought of as "the relationship between the student and the discipline of mathematics" (p. 245), which is developed in the classroom community (primary and post-primary) through their teachers, but remains thereafter as the foundation for their future experiences with mathematics (p. 248), as illustrated in Figure 1.1.

Figure 1.1
The relationship between the student and mathematics, their mathematical identity, is developed under the influence of the teacher/lecturer, but remains in place long afterwards (Grootenboer & Zevenbergen, 2008, Figure 2).



Teachers and lecturers play a vital role in the formation of students' mathematical identities, in turn shaping the students' careers, as well as everyday attitudes to mathematical challenges. In other words, ideally students not only learn how to do mathematics, they learn how to be mathematicians (Boaler & Greeno, 2000, p. 188; Goldin et al., 2016, p. 14). Pedagogy in mathematics education attends not only to the effective communication of mathematical knowledge, but is also laden with views of the discipline of mathematics, which shape the identities students develop as learners of mathematics (Boaler, 2002, p. 132).

To create an effective learning environment for science and engineering undergraduates, an awareness of how these students relate to mathematics is vital. Investigating mathematical identity can help identify pedagogical issues that may contribute to marginalisation, and thus impact students' relationship with mathematics and their decision to continue, or not, their mathematical studies (Grootenboer & Zevenbergen, 2008; Solomon, 2007b). Thus, teachers influence the relationship that each student has with mathematics, and by understanding this relationship they can improve students' learning experience, and the efficacy of their own teaching. Inspired by Esmonde (2009), it has been suggested that "the role of the teacher includes fostering change for the better in students' mathematical identity" (Eaton & OReilly, 2009c, p. 235), and therefore, there is a need to understand the mathematical identity of these students as they enter, and progress through, university, in order to facilitate such changes in identity.

The Influence of Identity on Students' Learning

Although a student's relationship with mathematics persists from post-primary school as their mathematical identity, transition to university education involves many changes for students. Aside from possibly leaving home and living with others for the first time, students must adapt to different teaching and learning styles as well as new types of knowledge, reasoning, and classroom environments (Clark & Lovric, 2008). Additionally, they must negotiate the often confused and mixed messages from institutions about what it means to become a university student, and to work and succeed in this new environment (Gibson et al., 2019). The first two years, in particular, can be a time for "reorientations in outlook, ambitions and approaches" but also for "disengagement and motivational 'slump'" (Johnston, 2010, p. 5). Hernandez-Martinez et al. (2011) suggest that transition be thought of in terms of identity, since students can "see themselves developing due to the distinct social and academic demands that the new institution poses" (p. 119). In this study, I used a longitudinal exploration of students' mathematical identity during the transition to university education, to give insight into the policies and practices that universities might put in place to support students' successful transition.

For students of science and engineering, preconceptions about mathematics influence their learning, and are further complicated by the transition to university-level education, where they will be required to learn and demonstrate knowledge in new ways. It is well documented that primary and post-primary students in Ireland struggle with applying knowledge and problem solving in school (DES, 2016, p. 8), and with the kind of abstraction that is common in university mathematics (Breen et al., 2013; Clark & Lovric, 2008). Reflecting on their mathematical identity can help students engage more effectively as mathematics learners during this transition (Kaasila, 2007b), since identity guides their own learning process, and shapes the nature of the learning experience:

> When students are learning mathematics they are simultaneously developing mathematical identities, and, their mathematical identities are enabling and constraining the way they are learning mathematics (Grootenboer & Marshman, 2016, p. 116).

In the same vein, Sfard and Prusak (2005) suggest that "identity talk makes us able to cope with new situations in terms of our past experience and gives us tools to plan for the future" (p. 16). Such identity considerations are of relevance to the science and engineering participants in this study, since they are among those most at risk of non-progression through their university programmes in Ireland (Lane & Walshe, 2019). Non-completion rates for engineering have fallen in the past six years (Higher Education Authority [HEA], 2022b), but these rates have historically been higher than the national average across all programmes (HEA, 2020). Within DCU also, non-completion rates for science and engineering have historically been higher than the average across all programmes (see appendix B.2).

Identity Research

One of the aims of this study is to inspire science and engineering students to critically reflect on their mathematical experiences, and to bring about the coherency of thought (McAdams, 2006, pp. 98-99) concerning their mathematical identities which will aid them in their professional lives. It has been suggested by Eaton and OReilly (2009b) that even though PSTs study and analyse teaching methods, they experience difficulty in adopting new and unfamiliar models of teaching that they have not experienced themselves. In fact, all students emerge from schooling with "baggage" (Brown et al., 1999, p. 301; Eaton & OReilly, 2012, p. 256) related to their own ideas about the teaching and learning of mathematics as well as the nature of the subject itself (Ball, 1988b, p. 40). However, when prompted to do so, they can reflect on, and reanalyse, the experiences that have shaped their pre-conceptions, and are empowered in "deliberately enacting practices that [are] counter to those they had experienced as student" (Mewborn & Tyminski, 2006, p. 32).

Korthagen (2004) claims that determining the essential qualities of a "good teacher" is a central question in teacher education, and, as noted by Skott (2019, p. 369), concepts of teacher and professional identity have proved popular for investigating the new perspectives on mathematics that participants take from teacher education programmes, and how they put these into practice

(Brown et al., 1999; Goos et al., 2021; Hodgen & Askew, 2007). Although the concept of identity exhibits great potential as a tool for understanding students' transition to university education, developing a robust framework to achieve this goal is far from straightforward. The contested definitions and diverse range of theoretical orientations that have been used to study identity are explored in chapter 2. In section 2.5 the benefits of a narrative approach to mathematical identity are explained with regard to longitudinal studies of university students, and consequently the specific methodology for this study is laid out in chapter 3.

1.5 Layout of the Thesis

The terminology surrounding the nature of mathematics, its teaching, and learning will be introduced in section 2.1. Mathematical identity is addressed in the subsequent sections, beginning with the variety of definitions (section 2.3) and theoretical approaches (section 2.4) to the study of the concept, with a particular focus on the narrative approach (section 2.5). The narrative social constructivist philosophical stance behind this study is presented in section 3.1, which is important for understanding how the definition of mathematical identity was operationalised. The emergent research design, including data collection methods and timeline, is presented in section 3.2, and linked to the research questions to demonstrate the rationale behind the methods. The specifics of each of the three data collection stages (Data1, Data2 and Data3) are detailed in chapter 4, along with two pilot studies that informed these stages. The data analysis chapter (chapter 5) explains the approach to analysis for each stage of data collection. The themes resulting from thematic analysis of Data1 and Data2 are presented in chapter 6, while the personal narratives resulting from Data3 are presented in section 7.2. Finally, the findings and recommendations are presented in chapter 8.

Chapter 2

Literature Review

In this chapter, the definitions and concepts regarding the nature of mathematics, its teaching, and learning will be introduced (section 2.1), followed by a discussion of science and engineering students in particular (section 2.2). Mathematical identity is addressed in the subsequent sections, beginning with the variety of definitions that feature in the literature (section 2.3) and the ways they are operationalised (section 2.4). It is argued that most studies adopt a sociological view of identity but vary in their emphasis of subjective or social dimensions, and which social level(s) they take into account. Research in mathematical identity has flourished through the "social turn" in mathematics education (Lerman, 2000), which invited consideration of students not just as learners of mathematics but as humans negotiating an evolving sense of self as they adapt to new surroundings in university (Black et al., 2010, p. 56; Boaler & Greeno, 2000, p. 189). In this light, a narrative approach to identity will be presented in section 2.5 as an appropriate and productive one for studies involving university students. Previous research in Ireland, which inspired this study to take such an approach, will be presented in section 2.6, including the development of an instrument to assess mathematical identity, its use amongst different cohorts of students (mainly PSTs), and the findings in relation to those participants.

2.1 Teaching, Learning, and Understanding Mathematics

This section begins with a discussion about the concepts and terminology surrounding the nature of mathematics (section 2.1.1), including the teaching and learning of the subject (section 2.1.2).

Subsequently, an explanation of understanding in mathematics is presented (section 2.1.3), followed by a related discussion about growth and fixed mindsets (section 2.1.4).

2.1.1 Nature of Mathematics

Conceptions of the nature of mathematics itself often underpin assumptions about how mathematics should be taught and learned. Much of the seminal work with regard to how these assumptions are absorbed by teachers and transmitted through their teaching took place in the late 1980's and early 1990's and will be described below. Ernest (1988, 1991) compared the *instrumentalist view* of mathematics, where mathematics is seen as a set of unrelated, unchangeable rules and facts; the *Platonist view* of mathematics, where mathematics is seen as a static, immutable body of knowledge which exists independent of human consciousness; and the *problem-solving view* of mathematics as a cultural product, dynamic and continually expanding.

According to Thompson (1992, p. 132), the instrumentalist and Platonist views build on absolutist foundations, where mathematics is seen as pre-existing, unchanging, objectively true and discovered, rather than created (Ernest, 1988). The problem-solving view is underpinned by fallibilist assumptions, where mathematics is seen as uncertain, subject to critique and re-evaluation, and is created through human endeavour (Thompson, 1992, p. 132). In the words of Ernest (1988), under a problem-solving view, mathematics is seen as "a process of enquiry and coming to know, not a finished product." A report into STEM education in the Irish school system recommended resisting the instrumentalist view of STEM subjects, since this perspective forms a barrier against a problem-solving or "inquiry-based" approach to learning and assessment (DES, 2016, p. 68).

In addition to the instrumentalist viewpoint, which casts mathematics as rules without reasons, Ernest (1991) also described the technological pragmatist, who celebrates the practicality of mathematics as a tool which is used to understand the real world (cf. Eaton & OReilly, 2012, p. 259). The technological pragmatist philosophy regards mathematics as a fixed absolute, but the choice between the many ways in which mathematics can be applied is made with regards to utility and expediency (Ernest, 1991, p. 118). The goal of the technological pragmatist is to master "both basic skills and the higher knowledge and skills needed to solve practical problems with mathematics and information technology" (Ernest, 2019, p. 85), but mathematics itself is treated as a given truth (Ernest, 1991, p. 152). Eaton, Oldham, et al. (2011b) suggested that the type of absolutism that positions mathematics as "providing powerful tools (rather than a bag of tricks) for the solution of real-life problems" (p. 31) might be derived from a focus on applying mathematics to real-world situations. Indeed, the literature suggests that engineering students may see the role of mathematics as a "suite of problem-solving approaches" that requires relational understanding (which will be discussed next), rather than as a selection of tools to be learned (Craig, 2013, p. 1020). Thus, the technological pragmatist philosophy of mathematics may have particular relevance for science and engineering students, whose goal is not to make generalisations or abstractions, but to apply mathematics to relevant real-life situations in order to produce a result.

2.1.2 Nature of Mathematics Teaching and Learning

Literature concerning the philosophy of mathematics at the time of Ernest (1988) included the proposal that "[o]ne's conception of what mathematics is, affects one's conception of how it should be presented" and that "[o]ne's manner of presenting it is an indication of what one believes to be most essential in it" (Hersh, 1986, p. 13). Building on this viewpoint, Ernest (1988) identified three roles that teachers can embody, which he linked to different types of desired learning outcomes: teacher as *instructor*, *explainer*, or *facilitator*. An instructor values compliant behaviour from their students, who passively receive knowledge from the teacher and demonstrate that they have mastered skills through correct performance. This role often involves strict adherence to a text or marking scheme. An explainer focuses on actively constructing students' understanding of mathematics, using problems and activities to supplement learning from textbooks. A facilitator seeks to instil confidence in problem solving amongst their students, encouraging them to explore mathematical interests autonomously.

Beswick (2005), drawing on the work of Ernest (1988) and Van Zoest et al. (1994), proposed sets of connections between beliefs about mathematics, teaching, and learning, which are theoretically consistent. Firstly, an instrumentalist view of mathematics corresponds to a focus on content, and students' performance (Beswick, 2005, Table 1), which aligns well with Ernest's description of the teacher as an instructor. Secondly, a Platonist view of mathematics also corresponds to a focus on content, but with an emphasis on students' understanding (Table 1), echoing Ernest's account of the teacher as an explainer. Lastly, a problem-solving view of mathematics is consistent with a focus on learner autonomy (Table 1), embracing Ernest's vision of the teacher as a facilitator. The teaching and learning of mathematics by science and engineering students will be addressed more specifically in section 2.2.

2.1.3 Nature of Mathematical Understanding

The contrast between *conceptual* or *relational understanding* of mathematical content, and *procedural* or *instrumental understanding*, where disconnected rules and procedures are understood, has long been a focus in mathematics education research (Skemp, 1976). Instrumental understanding results in the algorithmic application of "rules without reasons" (p. 20), whereas relational understanding entails not only knowing what needs to be done, but also why it works. It is argued that a genuine understanding of mathematics involves making connections between mathematical topics, enabling the learner to extrapolate their understanding to tackle unfamiliar problems (Ma, 2020, p. 126; Skemp, 1976, pp. 23-24). Relational understanding is an important means for students to relieve themselves of the pressure to memorise unrelated information or procedures, and to leverage the benefits of "mental compression": to study a mathematical topic, "file it away, recall it quickly and completely ... and use it as just one step in some other mental process" (Thurston, 1990). To paraphrase Skemp (1976, p. 23), it is hard to learn, but easier to remember. Additionally, Machalow et al. (2020) found that opportunities to develop relational understanding led to so-called "positive" narratives among PSTs, while instrumental learning opportunities lead to fragile or "negative" narratives. In their study of 163 narratives, they found that confident PSTs who enjoyed mathematics, and were excited to share this with their students in the future, held a relational view of mathematics learning, whereas those who had less affinity for mathematics, and instead had feelings of anxiety and fear, were more likely to hold an instrumental view of learning (pp. 13-14).

Care should be taken not to simply assume that relational understanding is the universally superior way to think about mathematics (Ernest et al., 2016, p. 7). Skemp (1976) acknowledged that instrumental understanding can be used to produce the correct answer more quickly, and that mathematicians often switch between thinking relationally and instrumentally (p. 23). In other words, students can deduce rules from general mathematical relationships, but should also be familiar with, and confident in, applying the procedures to produce the correct result.

Kilpatrick et al. (2001) used the term *procedural fluency* to describe the aforementioned type of knowledge as one interwoven strand of *mathematical proficiency*, which has since been become a feature of Irish education policy at post-primary level (DES, 2015, 2017; National Council of Teachers of Mathematics [NCTM], 2014).

2.1.4 Disposition

Concepts that are used for research in mathematics education often have their origins in psychological traditions, particularly those that focus on students' self-conceptions (Goldin et al., 2016, p. 8). For teachers, disposition is increasingly considered to be involved in numeracy (Geiger et al., 2015, Table 1), and to influence teachers' knowledge and practice (Jacobson & Kilpatrick, 2015, p. 402). Under the model developed by Kilpatrick et al. (2001), which features heavily in post-primary educational policy in Ireland (DES, 2015, 2017; NCTM, 2014), a *productive disposition* is one of five interwoven stands of mathematical proficiency, which involves the perception of mathematics as useful and worthwhile (p. 131). Students who have a productive disposition believe that steady effort in mathematics pays off, and see themselves as effective learners (p. 131). A distinction between *growth mindset* and *fixed mindset*, introduced by Dweck (2006), has emerged as a simple characterisation of students beliefs about themselves as learners. This distinction is of particular importance in mathematics education, where students often develop "fixed ability beliefs," and believe that understanding mathematics is simply beyond them (Solomon, 2007b, pp. 88-90).

Belief in a fixed mindset means the learner considers themselves to possess a certain amount of ability, which cannot be changed, and, therefore, every situation calls for confirmation of intelligence, personality, or character, which creates an urgency for students to prove themselves over and over (Dweck, 2006, p. 7). Those learners with a growth mindset, on the other hand, focus on development, and believe that although people may differ in their initial ability and interest, these qualities are malleable and can be cultivated through hard work to allow them to reach their unknown potential (p. 7). A growth mindset facilitates students to embrace and persist in mathematical challenges (Meehan & Howard, 2018, p. 7), and to learn from the success of others rather than to be fearful of challenges to their own intelligence (Dweck, 2006, p. 10; Campbell et al., 2020, Table 1).

2.2 The Mathematics Education of Science, Engineering and PST Students

In this section, I describe some characteristics of the Project Maths post-primary curriculum, and subsequently discuss the teaching and learning of mathematics for university students of science, engineering, and PSTs. In the literature, students for whom mathematics is not the main focus of their programme are often grouped under the term "service mathematics," particularly with regard to programmes in science or engineering. Although a full review is beyond the scope of this thesis, the elements of teaching and learning described in this section have direct relevance for the analysis and findings presented in chapter 6, chapter 7 and chapter 8.

In Ireland, the Project Maths post-primary curriculum, which was implemented nationwide from 2010 to 2013 (Johnson et al., 2019, p. 3), has drawn influence from Programme for International Student Assessment (PISA), that is relevant to the concepts discussed in section 2.1.2. Previous curricula emphasised the development of mathematical skills in purely mathematical and abstract concepts, whereas PISA and Project Maths focus on solving problems in novel, authentic, real-life contexts (Cosgrove et al., 2005, p. 210). Thus, Project Maths diverges from the previous curriculum through its focus on "horizontal" instead of "vertical mathematising" (Freudenthal, 1991, p. 41). These terms emanate from realistic mathematics education, which proposed that knowledge of mathematical skills and concepts should be abstracted from real-life situations (horizontal) rather than from within mathematics (vertical) (Cosgrove et al., 2005, p. 210; Fitzmaurice et al., 2021, p. 3; Johnson et al., 2019, p. 8). In realistic mathematics education, real-world problems serve as both the application area of mathematics, and the primary source for learning about mathematical concepts (Schroeder & Lester, 1989, p. 33; Treffers, 1993, p. 89). Thus, mathematics learning in Ireland has moved away from a formal and abstraction approach, towards a more application-centred one (Kirwan, 2015, p. 322). Such an approach aligns with what Schroeder and Lester (1989) called teaching mathematics *through* problem solving (p. 33). The same authors distinguished this approach from teaching *about* problem-solving, which involves discussion of specific problem-solving strategies (looking for patterns, solving a simpler problem, or working backwards), which are not dependent on a particular question or mathematical idea (Lubienski, 1999, p. 254; Schroeder & Lester, 1989, p. 32). Alternatively, teaching mathematics *for* problem solving focusses on the key mathematical ideas and skills that are needed for solving problems

(Lubienski, 1999, p. 254), the goal being that students can apply such knowledge to routine and non-routine problems in a variety of contexts (Schroeder & Lester, 1989, p. 32).

For PST students, a rich tradition of research in mathematics education has brought attention to the types of knowledge that are required by mathematics teachers, and the contribution of teachers' practice to the development of such knowledge (Beswick & Chapman, 2020). Ma (2020) contends that teachers should have a "profound understanding of fundamental mathematics" so that they can exploit opportunities to reinforce basic mathematical ideas amongst their students (p. 129). A profound understanding is necessary so that through their teaching, teachers can reinforce basic ideas, connect mathematical topics, and lay the groundwork for topics to come in the future (p. 129). Ball and Bass (2009) described the latter point as a "peripheral vision" or "horizon knowledge" of the larger mathematical landscape, which allows teachers to "build bridges between [students'] thinking and fundamental ideas and practices of the discipline [of mathematics]" (p. 1). PSTs must understand how to do mathematics, but also "why it is so" (Ball et al., 2008, p. 391), since asking why in this way is the first step towards conceptual understanding of the mathematics involved (Ma, 2020, p. 115). One of the more influential frameworks of teacher knowledge, mathematical knowledge for teaching (MKT), was proposed by Ball et al. (2008, Figure 5). MKT is a practice-based theory which built on the distinction between "subject matter knowledge" and "pedagogical knowledge" (Shulman, 1986). This distinction arises because, for example, teachers are required to have knowledge of mathematically correct notation, procedures, and solutions (which is not unique to the teaching profession), but also need pedagogical knowledge to predict what students will find confusing, interesting, or motivating (Ball et al., 2008, p. 401).

Nelson et al. (2015) analysed the learner profiles of university students who were taking a set of foundational STEM modules which included programming with MATLAB, problem solving, vectors, matrices, and 2D plotting (p. 81). The majority of engineering students adopted learner profiles that did not view mathematical tasks as important for learning, nor for their emerging identities (Table 3), but simply as "stepping stones" to solving problems in engineering (Goldin et al., 2016, p. 21). In Ireland, this perspective might be expected amongst first year university students, since similar research amongst Senior Cycle students has shown that they do not see mathematics as useful in daily life, but put a logistical value on the subject as a gateway to certain university programmes and careers (McNamara, 2013, p. 209). Gainsburg (2007) argued that the mathematical disposition of structural engineers, involves deciding "how, when, and when not to use mathematics" (p. 500), but that the in-school perception of mathematics as immutable might constrain the development of such a disposition among students.

In their review of mathematics education in engineering, Pepin et al. (2021) noted that engineering students are often presented with mathematical techniques and theories in two different contexts, that involve different types of tasks, techniques to achieve them, and ways of justifying them (p. 166). These different *praxeologies* are "likely to cause misunderstandings and conceptual difficulties" (p. 167). Bingolbali et al. (2006) reported that lecturers perceived distinct goals from different departments with regard to the outcomes for each programme, and with regard to preparing students for future professions (p. 174). For example, when teaching within their mathematics/physics departments, lecturers prioritised theoretical aspects, but when teaching in science and engineering departments, they foregrounded applications in order to "meet the expectations" of those students (p. 174). Recent research from Spain has also shown that engineering lecturers can see mathematics as a modelling tool, and, therefore, place lower value on students' ability to perform calculations by hand (e.g., integration), since, in practice, technology is used for such technical work (Florensa et al., 2022, p. 2391). For example, González-Martín and Hernandes Gomes (2019) identified that different praxeologies were employed by mathematics and engineering textbooks with regard to topics in calculus. The same authors previously detailed how integrals are used in engineering, without reference to the language and rationale employed in calculus modules (González-Martín & Hernandes Gomes, 2017).

For science students too, mathematics is often taught as a separate subject, which cuts them off from meaningful learning through seeing the relationships between ideas in both contexts, and developing an identity that is more relevant to the integrated real-world in which they will practice their trade (Czerniak & Johnson, 2014). In a pre-university science programme, Hitier and González-Martín (2022) found that praxeologies in a mechanics and calculus module differed, explicit connections were not established between common mathematical content, and students developed "techniques from calculus without giving any real sense to them" (p. 8). In light of the difficulties associated with conflicting praxeologies, Schoenfeld (2020) argued that "inquiry, sense making, and exploring how things fit together" (p. 1167) are important facets of mathematical thinking that should be made available to all students (p. 1171). However, it should be acknowledged that mathematical problems that aim to provide opportunities for generalisation, abstraction, and further problem posing (p. 1168), may not be aligned with the values and practices of science and engineering departments (p. 1164).

Overall, the literature highlights that teaching mathematics to students of science and engineering, is not a simple case of transplanting the same approaches that are designed to guide students into becoming mathematicians (Goold, 2015). It is important to acknowledge that in different fields, "values are central to considering which practices, to what degree, are appropriate for students" (Schoenfeld, 2020, p. 1164). This is of particular interest in an Irish context, where students' mathematical preparedness for science and engineering programmes has been a cause for concern in the past (Hourigan & O'Donoghue, 2007; Ní Fhloinn & Carr, 2010), and has continued to demand close attention in recent years (Lane & Walshe, 2019; Treacy et al., 2016).

2.3 Definitions of Mathematical Identity

The concept of mathematical identity has enjoyed increased attention in recent years within education (Darragh, 2016, Figure 1), particularly in the last two decades, in part because of the influence of teachers' identities on their teaching practice (Goldin et al., 2016, p. 14). According to Grootenboer and Marshman (2016), the power of identity lies in its holistic multi-dimensionality and unification of interrelated dimensions, including "beliefs, values, attitudes, emotions, dispositions, cognition, abilities, skills and life histories" (pp. 27-28). However, there is no agreed-upon definition of identity (Sfard & Prusak, 2005), and Darragh (2016) found that even recent studies sometimes do not provide a definition at all (p. 24). The latter point has come into focus in recent publications as researchers have been more explicit about defining identity, but such definitions are not always operational (Graven & Heyd-Metzuyanim, 2019, p. 10).

Psychological and Sociological Framings

A comprehensive review of identity research in mathematics education, conducted by Darragh (2016), traced both sociological and psychological framings of the concept, where identity can be seen, respectively, as an action: something one does (Mead, 1913/2011), or as an acquisition: something one has (Erikson, 1968). The psychological perspective is characterised by the pursuit of a "core stable identity" (Darragh, 2016, p. 27), which expands from a fixed, internal beginning to include external factors, rather than being informed by external factors at the root (Samuel & Stephens, 2000, p. 476). The sociological viewpoint describes identity as an action or process, to evoke a sense of performance and change over time (Darragh, 2016, p. 27). Research into mathematical identity has drawn mostly from sociological perspective (Darragh, 2016, p. 27; Goldin et al., 2016, p. 14), which distinguishes identity from research concepts in affect, such as attitudes or beliefs, which are usually considered to be more stable (Hannula, 2011, p. 36; Grootenboer & Marshman, 2016, Figure 2.1). The constantly changing and evolving nature of identity has been pinpointed as the key feature of the concept (Sfard & Prusak, 2005), and, perhaps as a result, this perspective has characterised recent mathematical identity research in Europe (Hannula & Garcia Moreno-Esteva, 2017, p. 1103).

Darragh (2016) noted that the social turn in mathematics education, explicated by Lerman (2000), has simultaneously brought social perspectives of mathematics teaching and identity research to the forefront, while in the other direction, a rift has developed between studies of affect and studies of identity because of the "unanimous reliance" on the sociological perspective of identity (Graven & Heyd-Metzuyanim, 2019, p. 10). However, doubts have been expressed about the potential for the psychological perspective to function as more than a catch-all term for affect (Darragh, 2016, p. 28). Under this perspective, quantitative studies may seek to collapse identity into measurable, affective components that are argued to capture enough of the concept (Kaspersen et al., 2017), while qualitative studies run the risk of "ontological collapses" (Graven & Heyd-Metzuyanim, 2019, p. 9) associated with viewing identity as an action, but discussing it as if it is acquired, fixed, or pre-existing (Darragh, 2016, p. 28), calling into question the reliability of the findings.

Influential Authors

Darragh (2016) found that several publications concerning identity have informed research in mathematical identity more than any others: Gee (2000), Holland et al. (1998) and Wenger (1998) (which extended earlier work by Lave and Wenger, 1991). Of these, the definition of identity given by Wenger (1998) is the most prominent, with 41% of articles reported to draw from those sources, more than double the amount drawing from the next most influential authors (Darragh, 2016, p. 23). The figured worlds framework from Holland et al. (1998) is also commonly relied upon (Goldin et al., 2016, p. 14). For example, in their study of high school students' learning as a process of identity formation, Boaler and Greeno (2000) reported two figured worlds: one "structured, individualized, and ritualized," and the other "relational, communicative, and connected" (p. 178).

In Ireland, Quirke (2018), drawing from Ma and Singer-Gabella (2011) and Boaler and Greeno (2000), identified two figured worlds that described the current and preferred identities of teachers in a policy document of the National Council for Curriculum and Assessment (NCCA, 2005). Quirke found that the document offered both traditional and reform-oriented figured worlds. The traditional world is a highly ritualized one, wherein the teacher takes on the role of an instructor (Ernest, 1988). Students are positioned as helpless victims of poor teaching, and mathematics is presented as "a bag of unconnected tricks" (Quirke, 2018, p. 560). The reform-oriented world is a "web of beautiful relationships" (NCCA, 2005, p. 18), wherein the links within mathematics are foregrounded, as well as links to other subjects, and students are positioned as active learners who build a relational understanding of mathematics (Quirke, 2018, p. 561). The traditional figured world was presented as the current state of affairs, while the reform-oriented world represented the NCCA's preferred identity of teachers (p. 561).

Although influential, the definitions given by Gee (2000) and Holland et al. (1998) are dependent on the notion of "who one is" (Sfard & Prusak, 2005, p. 16), and, are not sufficiently disassociated from the fixed, psychological identity that Sfard and Prusak (2005) wished to avoid: one that exists independently of discourse or action (p. 15). On the contrary, they saw great potential in embracing the role of discourse and proposed to "equate identity-building with story-telling" (p. 21) by defining identity as "[t]hose narratives about individuals that are reifying, endorsable and significant" (p. 16). By adopting a sociological viewpoint, their definition acknowledges that identity is ever-changing and communicated best through narratives authored by the participants. Participants can draw attention to the mathematical experiences they consider to be influential, and reflect on them as they speak/write, often leading to realisations or re-interpretations of the experiences. I adopted a similar approach to operationalising the definition of mathematical identity that was used in this study, which will be discussed in section 3.1.4.

2.4 Operationalising Definitions of Mathematical Identity

The foundations of research in mathematical identity have been developed through two independent fields of expertise: studies of student/learner identity and teacher identity (Goldin et al., 2016, p. 14). Several important reviews, most of which are recent publications, have the potential to bring clarity to the contentious underpinnings of the concept, by collating the progress that has been made by independent communities of research: those in teacher identity (Lutovac & Kaasila, 2019) and student/learner identity (Darragh, 2016; Radovic et al., 2018). Of the latter two publications, Darragh (2016) analysed definitions and sources, while Radovic et al. (2018) focused on theory and methodology. In their review, Lutovac and Kaasila (2019) concluded that teacher identity literature generally fails to provide appropriate methodological grounding of their research and tends to follow conventional analytic approaches as a result (p. 513). They found that when referring to seminal works, some authors rely on "the authority of the acknowledged work, feeling no further need to clarify or justify their decisions or actions" (p. 512) with regard to methodology, and, therefore, fail to properly link theoretical foundations and data analysis. A shift in the attention of the research community, from what definitions are used, to how these definitions are employed, is vital in order solidify the methodological grounding of mathematical identity for the future (p. 506). In response to this call, section 3.1.4 will detail how the definition of mathematical identity used in this study, facilitated methodological coherence, and supported answering the research questions.

Radovic et al. (2018) found that determining the theoretical orientation of a mathematical identity study based on its foundational text was not possible, since studies employed the same definitions in different ways. They determined five categories of identity studies which viewed identity as individual attributes, as narratives, as relationship with a specific practice, as ways of acting, or as afforded and constrained by local practices (Radovic et al., 2018, Table 2). As will be discussed below, three operational dimensions facilitated the determination of these categories: stability/change, enacted/representational and subjective/social. As was addressed in section 2.3, most mathematical identity studies adopt a view of identity as dynamic, flexible, and constantly in flux. Only one of the five categories determined by Radovic et al. (2018) involved an *enacted* view of identity as exhibited through extra-discursive actions, where different observable actions are purported to indicate different identities (p. 30). For the other four categories, identity is seen as a phenomenon which is demonstrated through discourse or narrative (*representational*), including self-concepts that can only be communicated by the person themselves. These four categories can also be considered along a spectrum of *social* or *subjective* emphasis, as shown in Table 2.1. An emphasis on the subjective aspect of mathematical identity concerns participants' "private experience of who one is," whereas an emphasis on the social aspect positions identity as a social product and focuses on how individuals are recognised in social spaces (p. 26).

Table 2.1
Representational studies ordered according to whether they emphasise the subjective or social aspect of the subjective-social dimension (Radovic et al., 2018). Capital 'S' indicates the stronger emphasis.

Category	**Emphasised aspect(s)**
Identity as individual attributes	Subjective
Identities as narratives	Subjective-social
Identities as a relationship with a specific practice	Social-subjective
Identities as afforded and constrained by local practices	Social

While Radovic et al. (2018) proposed that the most social-leaning studies attend to local practices, attention should be paid not only to the extent and manner of the social emphasis, but also to the particular social levels that are the focus of each study. Martin's (2000) framework (as used by Cobb et al., 2009, p. 42, for example) included four social levels, beginning with practices local to the classroom and zooming outwards: the intrapersonal level (students), the school level (teachers), the community level (e.g., families), and the sociohistorical level. A "zoom of a lens" metaphor for social research (Lerman, 2000, p. 33) is a useful way to think about these social levels. Mathematical identity studies that involve small-scale interactions between individuals represent the most zoomed-in social level, while identity studies analysing the role of broader political, educational, or economic systems, sacrifice a detailed view in favour of a zoomed-out perspective.

2.4.1 Studies which Emphasise the Subjective Aspect

Studies of mathematical identity that focus on the subjective aspect concentrate on self-concepts, either disregarding or making assumptions about the social levels that are out of focus. Radovic et al. (2018) described that such studies favour individuals' perceptions of themselves over their perceptions of mathematics, what it means to be good/bad at mathematics, to do mathematics, or be a mathematician (p. 28). They were also less likely to explicitly communicate the fluidity of mathematical identity and veered towards the psychological viewpoint (see section 2.3) by considering participants to possess "quantifiable 'levels' of identification" (p. 28) with statements about themselves. That is, they viewed identity as something participants have, not something they do.

Kaspersen et al. (2017) adopted a quantitative approach by reducing their considerations of mathematical identity to a selection of self-concepts that themselves can be measured, and further claimed that such an approach is the only way to quantify the notion of mathematical identity (p. 165). For researchers who accept that such measures of self-concepts also measure mathematical identity, comparison across contexts would require (relative) invariance of the social conceptions of what it means to do mathematics, be mathematical, or be successful in mathematics (Kaspersen et al., 2017, p. 165; Radovic et al., 2018, p. 27). It is argued throughout this chapter that other qualitative definitions of mathematical identity aspire to capture more of the social aspect, and conceptualise identity as more than simply the sum of its affective parts. Furthermore, quantitative measures are rare in research on identity (Darragh, 2016; Lutovac & Kaasila, 2019), and mostly involve counting instances of codes (Goldin et al., 2016, p. 15) that arise within qualitative research designs.

2.4.2 Studies which Emphasise the Social Aspect

On the other extreme of the subjective-social spectrum are those studies that focus solely on the possible identities afforded by particular contexts, and how participants manoeuvre themselves within the constraints that arise there. This perspective on identity "encompasses how broader culturally organised activity and modes of reasoning inform action and sense making" (Skott, 2019, p. 470). In his influential publication, Martin (2000) argued that identity research should consider the broader contextual forces that affect the lives of participants and include several levels of social zoom.

Researchers engaged in studies of this type do not focus on students' self-perceptions but may consider their perceptions of their social contexts. For example, Cobb et al. (2009), drawing on the work of Martin (2000) and Wenger (1998), explored students' interactions in the classroom, using observational data (field notes of classroom observations and videos). Boaler and Greeno (2000) drew on the same two authors to interview pairs of students concerning their perceptions of their mathematics lessons. Hodges and Hodge (2017) employed a similar definition of identity to illustrate how two PSTs (elementary school) came to reconcile the conflicting obligations of mathematics teaching required in two different social contexts: their teaching context, and their teacher education context. All three studies focussed on the social structure of the classroom and how individuals identify with, comply with, or resist the obligations of those systems. Those who resist these obligations often believe that their contributions are not welcome and "develop an identity of non-participation that progressively marginalizes them" (Wenger, 1998, p. 203).

Darragh (2016, p. 20) and Sfard and Prusak (2005, p. 15) argued for a definition of identity that applies to a wider context than the classroom and would at least acknowledge the cultural influences on students' learning. In particular, Sfard and Prusak (2005) proposed that identities are "products of collective storytelling" (p. 21) which are filtered through their sociocultural context. They pointed to Ogbu (1992), who claimed that "communities' cultural models of understanding of 'social realities' and the educational strategies that they, their families, and their communities use or do not use in seeking education are as important as within-school factors" (p. 5).

2.4.3 Identity as a Relationship with a Specific Practice

To reconcile the distinction between social and subjective identity development, a theoretical perspective attributed to Lacan suggests that subjectivity (the individual sense of who we are) results from alienating discourses (comparing ourselves to the social world we encounter) (Brown, 2010, p. 257). More recently Skott (2019) described how teachers' professional identities are oriented by social contexts, but that manoeuvring within this space allows them agency in their identity formation (p. 470). Research based on "communities of practice" (Wenger, 1998) is often used in attending to the aforementioned distinction between social and subjective aspects, since the unit of analysis is the relation between the individual and their environment (Nardi, 1996, p. 71). As a consequence, the broader social structures move out of focus, and researchers foreground the study of identity as participation in, or identification with, a particular community of practice, and their "established ways of thinking and doing" (Jaworski & Goodchild, 2006, p. 354).

For example, in her study of mathematics undergraduates, Solomon (2007b) argued that dominant discourses in each community establish what it means to be successful in mathematics, but exclude students who interpret or hold conceptions of success that do not match those of the particular community. In particular, she found that the reward system in the undergraduate community in her study "marginalises learners who seek to participate beyond a focus on correct answers" (p. 92), and furthermore, that communities which emphasise deeper learning and engagement have rules of success that are perceived by students as opaque, and, therefore, exclusive (p. 94). One major limitation of the communities of practice approach concerns the presence of multiple overlapping communities, which embody different ways of thinking about mathematics. In the words of Solomon (2007b) "[t]here is a mismatch between the values of the wider community of practice of mathematics and those of the immediate undergraduate and classroom communities of practice" (p. 88). For this reason, shared meaning and how it is negotiated is usually considered in the context of a particular practice or enterprise, from which concrete manifestations of broader institutional practices can be analysed (Radovic et al., 2018, p. 30). Jaworski and Goodchild (2006) noted that, in general, determining the community of practice at broader social levels (e.g., school, community), and identifying the ways in which students participate or not at these levels, would be much more difficult (p. 190).

2.5 Identity as Narrative

Black et al. (2010, p. 56) distinguished identity that is tied to a practice, from the narratives we construct about ourselves upon reflection. Such narratives are altogether more subjective, and liable to diverge from the so-called real version of events, which could be observed by others (see "remembered truth" in section 3.1.4). By taking a narrative view of identity, Black et al. (2010) contended that mathematical identities are formed "upon reflection on the subjectivities we have experienced when engaging in various forms of mathematical activity" (pp. 57-58). In contrast to those studies that focus on small-scale interactions, narrative studies emphasise how "representations and senses are storied and *change* in time and not necessarily in relation to a specific shared practice" (Radovic et al., 2018, p. 34). Unlike research in affect, research in narrative identity generally does not distinguish between beliefs, values, emotions, motivations, and attitudes from the outset of the research, but rather seeks to present coherent narratives which are infused with these affective dimensions (Frade & Gómez-Chacón, 2009). For researchers in affect, terminology and theoretical models are used to differentiate between these dimensions in terms of stability, intensity, and the degree to which cognition plays a role (Wedege & Skott, 2007, p. 390). Whereas in narrative research, the process of generating narratives from data includes consideration of narrative properties that shape the final research product and findings e.g., use of language, context of data collection, and how the participant positions themselves within their social world (Polkinghorne, 1995). Some authors have sought to combine the above approaches to disentangle these dimensions from the narratives they create, and thus present findings that are consistent with the language of research in affect (Kaasila et al., 2005).

It was noted in section 2.4.3 that subjectivity (the individual sense of who we are) results from alienating discourses (comparing ourselves to the social world we encounter) (Brown, 2010, p. 257). Although an individual's social world affects which views of mathematics they encounter, research in beliefs appears to take a contrary approach to that in narrative mathematical identity regarding an individual's social context. In the former, strong beliefs about mathematics are adopted as an individual's subjective knowledge, which over time may converge towards an objective knowledge of mathematics, which is externally validated and accepted by the mathematics research community (Pehkonen & Pietilä, 2003, p. 3). So-called "positive" beliefs are those shared by the experts (Di Martino & Zan, 2011, p. 474). In identity research, a sociocultural approach is more common (Goldin et al., 2016, p. 14), where the views of mathematics we encounter are limited, and are different for each person. One's own view of mathematics stems from personal mathematical experiences and manoeuvring within the variety of views of which one is aware (Skott, 2019, p. 470). In narrative studies, our individual experiences "provide a resource—and a constraint—which we draw on in constructing stories about ourselves" (Black et al., 2010, p. 56). As will be discussed in chapter 3, a narrative paradigm facilitates researchers in paying attention to participants' social world, through their narratives about their experiences.

The most common data sources in narrative studies are written reflective autobiographies, or oral histories, collected through semi-structured interviews (Goldin et al., 2016; Lutovac & Kaasila, 2019), both of which are most commonly referred to as *narratives* or *stories*. Although these two terms are often used interchangeably, some authors have taken either term to mean something more specific, but in such cases, they usually place great emphasis on the meaning with which they use the terms. For example, Labov and Waletzky (1967) studied the structure of narratives that people used when describing their everyday experiences and defined a *narrative* to be a sequence of events, or a plot (Polkinghorne, 1995), which leads the reader from a beginning state to a final state, where something has changed (Labov, 1972). Several authors have instead opted to use the term *story* to refer to similar structures (Di Martino & Zan, 2010; McCormack, 2004; O'Kane & Pamphilon, 2016).

At primary and post-primary levels, the majority of studies considered by Radovic et al. (2018) did not emphasise the representational and subjective aspects of identity, and, therefore, relied more on observational methods and less on narrative methods. They noted that younger students may not have developed the "self-awareness and self-reflection dispositions or abilities" (p. 35) on which narrative activities rely, an observation that is consistent with older research in language, which claims that narratives given by children are "often heard as not having a point" and thus, it is difficult to make interpretive decisions about meaning from their narratives (Labov, 1982, p. 227). Methods that involve forming a social connection with participants, may be more advantageous for identity studies that involve university students, since these methods can capitalise on students' matured capability for engaging in narrative processes of self-reflection, rather than relying on observational methods. Indeed, at university level, most studies have been found to take a narrative approach (Radovic et al., 2018, Figure 1). This approach has also proved popular in studies involving PSTs, where narrating prior experiences of learning mathematics has been found to influence teaching practices, by building resilient teacher identities (Goldin et al., 2016, p. 17). For example, Eaton et al. (2013) employed narrative methods of data collection to encourage self-reflection and found that participants across a range of programmes and universities expressed significant insight into their mathematical identity, without needing to spend a long time considering their responses (p. 293).

Teacher identity studies tend to involve a smaller number of participants. Lutovac and Kaasila

(2019, p. 509) found that less than 20% of teacher identity studies involving PSTs or in-service teachers included more than 10 participants. Even in larger scale studies, there is a tendency to focus the findings on a small number of cases (p. 509), and, therefore, such studies often utilise a narrative or case study design to draw out the details of these cases. Goldin et al. (2016, p. 16) also noted a recent increase in the narrative and discursive emphasis in teacher identity, and it appears that this shift has emanated from a maturing of the field over the last two decades. Both findings are consistent with the broader study conducted by Darragh (2016) who concluded that "identity is seen as a complex concept requiring detailed descriptions of individuals rather than generic findings of larger groups" (p. 23). For this reason, most studies incorporate multiple time points for data collection, rather than relying on just one (Lutovac & Kaasila, 2019, p. 509).

2.5.1 Analysing Narratives

Some studies collect narrative data in an attempt to approximate participants' "core identity" (Gee, 2000), which exists as a hidden, inaccessible entity that is stable across contexts (Hodges & Cady, 2012, p. 113), and use other data collection methods to reinforce their claims about this core. Others follow the approach advocated by Sfard and Prusak (2005) in equating identities with narratives and as a result, the unit of analysis is most often the narrative, not the person. Although both viewpoints may draw upon narrative methods, the latter viewpoint opens the door to the integration of established methodological elements from narrative inquiry (Clandinin & Connelly, 2000, p. 77) and narrative analysis (Polkinghorne, 1995), that go far beyond merely collecting narratives as data.

A central idea in narrative inquiry is that people are not isolated in a moment, they exist temporally with a past, present, and many possible futures (Clandinin & Connelly, 2000, pp. 29-30). Radovic et al. (2018) found that authors in their narrative category tended to include a temporal property in the way they operationalise their definitions of identity (p 29). Sfard and Prusak (2005) incorporated a temporal property by distinguishing between two "subsets" (p. 18) of identity. *Actual identity* refers to the current time and is often manifested as factual assertions using terms like "I am" or "I have" (p. 18). *Designated identity*, on the other hand, refers to future hopes and expectations that are yet to be realised including those that may be absorbed subconsciously from one's social context (p. 18) i.e., they may not be personal goals, but identities that individuals have come to think are required of them. Designated identities "give direction to ones actions" (p. 18) in the sense that one's actual identity is formed in preparation for future challenges. Narrating mathematical identity has also been described as a process of comparing one's former, current, and future selves in order to construct continuity and coherence between them (Lieblich et al., 1998, p. 7). Hauk (2005) highlighted that autobiographical memory performs exactly this function since "remembrances of past events, behaviors, thoughts, and feelings is largely determined by current self-image" (p. 38). Likewise, when constructing a narrative about oneself through interview, it has been noted that past events are always seen from the perspective of the present moment (Kaasila, 2007b, p. 212; Spector-Mersel, 2010, p. 212). Therefore, the popularity of autobiographic methods of data collection can be explained by the desire to infuse a temporal property in definitions of mathematical identity.

Lutovac and Kaasila (2019, Table 3) identified three common analytic approaches to teacher identity: thematic analysis (data-driven and theory driven), discourse analysis, and narrative analysis. Thematic analysis and grounded theory coding are common analytic techniques because they are not tied to a particular philosophy or framework (Braun & Clarke, 2022, p. 2) and offer important flexibility for studies that do not follow prescribed routes. Ntinda (2020) described narrative research as a "family of approaches which focus on the stories that people use to understand and describe aspects of their lives" (p. 3). Narrative studies that emphasise social aspects are more concerned with how participants manoeuvre themselves within the available discourses they encounter (Radovic et al., 2018, p. 29; Skott, 2019, p. 470) and prefer to employ discourse and positioning analytic approaches (Lutovac & Kaasila, 2019).

Narrative analysis frequently focuses on connections between plots, characters, and themes in participants' narratives (Machalow et al., 2020, p. 4), and emphasises the relevance of the social and cultural backdrop within which these narrative journeys are shaped (Radovic et al., 2018, p. 29). Furthermore, the social dimension is considered to varying degrees but, "the main emphasis is on how these social products are used in individual storylines and plots, and not in the particular social practice that produces them" (Radovic et al., 2018, p. 34). The idea is that the interaction between the subjective and social dimensions of the plots of an individual narrative can yield "wider identity scripts" (Darragh & Radovic, 2019, p. 521) that resonate for a broad class of students beyond those in the study itself.

It is sometimes hard to discern whether studies of teacher identity are truly conducting narrative analysis, even when studies claim this is so. It has been reported that narrative analysis of teacher identity is often employed in a manner more similar to thematic analysis (Lutovac & Kaasila, 2019, p. 512). Such studies frequently used pre-determined categories for their analysis, and therefore, they overlook the particular strength of narrative analysis to attend to a holistic view of identity, which emphasises the uniqueness of experiences (p. 512). Lutovac and Kaasila (2019) issued a call to researchers in identity to look outside familiar contexts towards analytic methods in social science research that could "expand the analytical scene of mathematics education research" (p. 513). Given their observation that teacher identity studies often do not clearly communicate how they constructed narratives about their participants (p. 512), there is scope for such studies to incorporate other narrative analysis techniques, and to be more complete in describing the methodology of these approaches.

Narrative studies generally take a nuanced approach to categorising identities. Drake et al. (2001, p. 8) proposed to categorise high-school teachers' interview narratives into four types of plots, which they called "narrative arcs." Their framework was expanded to six types of learners by McCulloch et al. (2013, p.383): smooth track, minor setback, roller-coaster, consistently frustrated, positive/negative turning point. The terminology used for these narratives arcs evoke images of mathematical graphs that represent change over time, similar to the simple graphs recommended for use in holistic analysis of narratives by Lieblich et al. (1998, pp. 94-95) to describe narratives of progress, steadiness, or decline. Many analytical techniques used to investigate mathematical identity rely on transition points to aid analysis of participants' narratives. At such points "change is described with more details, thus giving more information about the possible causes" (Di Martino & Zan, 2010, p. 41). In the literature, these arise under various terms such as turning points (Di Martino & Zan, 2010), outcomes (Kaasila, 2007a), leading activities (Black et al., 2010), critical events (Eaton, Oldham, et al., 2011b, p. 38) or core events/episodes (Kaasila, 2007b). Analyses using transition points benefit from a participant telling a full sequential story of their mathematical experience and have been used to characterise types of canonical story (Black et al., 2010, p. 59), direction (Di Martino & Zan, 2010, p. 36), and narrative arcs (Drake et al., 2001; McCulloch et al., 2013).

2.6 MIST and MINT Projects

In this section, I review a series of previous studies in Ireland that provided inspiration for this study and developed an online instrument for exploring the mathematical identity of PSTs.

The authors highlighted that such research is of importance with regard to PSTs who hold conceptions of mathematics that are misaligned with the curricula they are obligated to implement in school (Eaton, Oldham, et al., 2011b, p. 29). Since then, the changing profiles of mathematics teachers, and of the post-primary curriculum in Ireland (see section 1.3), have lent renewed relevance to the mathematical identity of teachers and students alike. Recent identity projects have focussed on the progression of teachers through the PDMT training programme (Quirke, 2022) and, more generally, to investigate the identities of practising mathematics teachers who are not qualified in mathematics (Goos et al., 2020).

A small-scale study in Ireland, entitled Mathematical Identity of Student Teachers (MIST), developed a research instrument to access mathematical identity (Eaton, McCluskey, et al., 2011; Eaton & OReilly, 2009a, 2009b, 2009c). The instrument included a questionnaire which consisted of two open-ended questions: a broad opening question, and a follow-up question which included some prompts. The researchers expressed a desire to allow their nine participants to make responses that were “indicative of their personal mathematical identity” (Eaton & OReilly, 2009c, p. 229) but not leave them without any direction. The instrument also included eight Likert scale items involving students’ attitudes towards mathematics, which are reported in Eaton et al. (2013). The prevalence of self-reflection as a key part of mathematical identity became evident in MIST (Eaton & OReilly, 2009a), and as a result, in a subsequent study, entitled Mathematical Identity using Narrative as a Tool (MINT), a third question about self-reflection was added, and the questionnaire was migrated to an online tool (Eaton et al., 2013, 2014; Eaton & OReilly, 2012). In their recent review of the development of this instrument (Eaton et al., 2019), the authors noted that it had “proved stable when working with a range of students both within and outside teacher education and in different countries” (p. 1491).

Through several publications over the last 16 years, seven themes were developed which proved broadly appropriate for probing the mathematical identity of students from different programmes, universities, and schooling-systems. In MISE, I sought to use a similar data collection instrument to include students of science and engineering, and thus, it was important to consider how the findings of my study might enhance, corroborate, or contradict those in the MIST/MINT literature. The themes reflected both social and subjective aspects, and, as recommended by Grootenboer and Marshman (2016, p. 127), accounted for an affective dimension (Eaton et al., 2019, p. 1488). Descriptions of the themes were given in Eaton, McCluskey, et al. (2011), and I have expanded those descriptions below using the other MIST/MINT publications:

T1. "**Harnessing student teachers' mathematical identity as a tool for self-reflection** relates to how students' exploration of their mathematical identity, leads them to deepen their insight into learning and teaching mathematics" (Eaton, McCluskey, et al., 2011, p. 5). It was noted that on reflection, students re-evaluated their own experiences, and indicated how these could inform their teaching through "valuing perseverance, encouraging cooperative study and fostering understanding" (Eaton & OReilly, 2009a, p. 155).

T2. "**The role played by key figures in the formation of mathematical identity** focuses not only on teachers and family members, but also on peers and society at large" (Eaton, McCluskey, et al., 2011, p. 6). Students emphasised the influential role of teachers in fostering the attitudes and enthusiasm of their students through their teaching (Eaton & OReilly, 2009b, p. 297). Family members inspired students to study mathematics because of their own love of mathematics, their own struggles or weaknesses in mathematics, or because the subject opens options for future careers (p. 297). Despite taking pride in studying a difficult subject, students were aware of the negative stereotypes associated with mathematicians that were held by their peers (p. 296).

T3. "**Ways of working in mathematics** explored what students found effective in learning mathematics, and why, either through individual endeavour or through collaboration" (Eaton, McCluskey, et al., 2011, p. 6). It was found that the individual work required at post-primary level, contrasted with a more collaborative approach at university (Eaton & OReilly, 2009c, p. 232). Students reported working collaboratively with their classmates in their own studies, and wished to encourage their students to do likewise, because they "know it works" (Eaton & OReilly, 2009a, p. 154). This may have been facilitated by a perceived shift from procedural mathematics in post-primary school, to a focus on deeper meaning in university (Eaton & OReilly, 2009c, p. 233; Eaton, McCluskey, et al., 2011, p. 10).

T4. "**How learning mathematics compares with learning in other subjects** considers the particular characteristics of learning mathematics, usually at school, that distinguish the process from learning in other subject areas" (Eaton, McCluskey, et al., 2011, p. 6). Mathematics was described as more demanding and requiring more hard work than other subjects, but also more satisfying because this hard work is rewarded: "what you put in you get out!" (Eaton & OReilly, 2009c, p. 232).

T5. "**The nature of mathematics** draws from a broad range of students' perceptions touching on the philosophy of mathematics, and on what doing mathematics is about" (Eaton, McCluskey, et al., 2011, p. 6). Post-primary mathematics was viewed as a subject taught for the purposes of examinations and thus, its nature was determined by that of the examinations (Eaton, McCluskey, et al., 2011, p. 9). Furthermore, there was no real connection between the mathematical topics that students encountered at post-primary level (Eaton, McCluskey, et al., 2011, p. 9; Eaton & OReilly, 2009a, p. 155). Students reported difficulties in balancing the "abstract nature of mathematics with the importance of contextualising the subject" (Eaton & OReilly, 2012, p. 259). They expressed visions of mathematics ranging from instrumentalist to technological pragmatist through to Platonist (Ernest, 1988, 1991). It was noted that their exact placement along this philosophical continuum is less important than their awareness of the diversity of views, since the latter enables PSTs to make "informed decisions about how and why they teach and learn mathematics" (Eaton & OReilly, 2012, p. 262).

T6. "**Right and wrong in mathematics** concerns students' perception that what is important in mathematics is to find the correct answer, and also a more general notion around the unambiguous nature of mathematical truth" (Eaton, McCluskey, et al., 2011, p. 6). Procedural questions were seen as safer because students know when the answer is correct, although PSTs acknowledged the importance of encouraging their own students to make an effort without worrying too much about getting the right answer (Eaton & OReilly, 2009c, p. 233).

T7. "**Mathematics as a rewarding subject** involves the extent to which students enjoy the subject, often relating to how they persist with it or to significant moments of insight" (Eaton, McCluskey, et al., 2011, p. 6). Students noted that it takes time to appreciate and take value from challenges in mathematics but emphasised the satisfaction that they derive from reaching 'Eureka moments' when things become clearer (Eaton & OReilly, 2009c, p. 234). It was also reported that teachers with high expectations of mathematical ability of their students, were very likely to contribute to their students' appreciation of mathematics Eaton and OReilly (2009a, p. 154).

It was noted that the themes were not always "sharply distinguished" from one another, with T6 and T4 initially considered to be sub-themes of T5 and T3 respectively (Eaton, McCluskey, et al., 2011), while T1 appeared to connect to a broad range of other elements of mathematical identity (Eaton & OReilly, 2009a).

A further study in Trinity College Dublin (Eaton, Oldham, et al., 2011a, 2011b), hereafter referred to as the "bridging study," found the themes developed by MIST to be "broadly sufficient" (Eaton, Oldham, et al., 2011a, p. 158) for analysing the mathematical identity of students of mathematics, but noted variations of emphasis between and within the themes (p. 165). This was attributed to PSTs positioning themselves as "teachers-in-the-making" (p. 164), and mathematics students as "mathematicians-in-the-making" (p. 164), and focussing respectively on influential people or the subject of mathematics itself. The two themes T4 and T6 were found to play only a minor role for the mathematics students who participated in the bridging study (p. 164). The participants were found to demonstrate a broadly Platonist, absolutist conception of mathematics, with fallibilist foundations rarely evidenced, although some students acknowledged their preference for instrumental or application-focused mathematics (Eaton, Oldham, et al., 2011b, p. 40). It was emphasised that care should be taken when interpreting expressions used by students, since references to problem solving can imply an instrumental/procedural view of mathematics as answering exam questions (p. 40), rather than a problem-solving view of the discipline, as discussed in section 2.1.

As with PSTs in the earlier MIST study, students of mathematics reported inspirational teachers who ignited their passion for the subject (Eaton, Oldham, et al., 2011b, p. 37), but overall, students of non-teaching programmes were found to be less likely to acknowledge the influence of their teachers compared to PSTs (Eaton et al., 2019, p. 1490). Mathematics undergraduate participants also reported having frequently experienced teaching that was devoid of understanding at post-primary level, but highlighted the influence of specific teachers who would reacquaint them with their enjoyment of the subject by incorporating non-instrumental approaches (Eaton, Oldham, et al., 2011b, p. 39). Evidence of collaborative working groups of mathematics students were found to develop organically over time (Eaton, Oldham, et al., 2011a, p. 161), but in contrast to those in MIST, participants in the bridging study described socialising in scientific and mathematical groups such as the Young Scientist exhibition and the university mathematics society (p. 160).

The MINT study involved 99 participants including primary and post-primary teachers, and students of business and psychology (Eaton et al., 2013), who were administered the instrument online for the first time. A third, open-ended question was added to the questionnaire, which invited participants to consider how the first two questions had helped them organise their thoughts around mathematical identity, and their awareness of this relationship (p. 287). While the researchers expected participants to spend up to one hour on the questionnaire, a mean time taken of 14.5 minutes was observed, with no strong difference between the groups in this regard (p. 289). They concluded that participants expressed significant insight into their mathematical identity without needing to spend a long time considering their responses (p. 293). A further publication (Eaton et al., 2014) narrowed in on the responses of PSTs in MINT, and helped the authors discern four "aspects" of T1 from their responses, which provided a "local framework" for the discussion of the theme:

- A1. Journey across at least one transition giving rise to new perspectives and increased maturity (in relation to mathematics).
- A2. Statements of aspiration and commitment in relation to teaching mathematics.
- A3. Appreciation of the challenge of learning mathematics as the substance behind growing in confidence as a teacher-in-the-making.
- A4. "Math anxiety" and, in particular, (still) feeling poorly prepared to become a teacher.

It was noted that participants from programmes that focussed on pedagogy, rather than mathematics, and those in later years of study, were more likely to demonstrate reflection on teaching and learning in their responses and contribute to T1 (Eaton et al., 2014, p. 373). Responses categorised as A1, addressed participants' experiences of working in mathematics without understanding the process or the purpose, particularly at post-primary level. One participant noted that in university they concentrated on understanding why calculations work and discussing "different ways of doing things" (p. 372). A3 involved participants seeing challenges in learning mathematics as productive for growing confidence as a teacher, but the researchers highlighted the view that mathematical material should be “sufficiently challenging, but not too much” (p. 373), so that students can make progress without relying on the teacher to show them the way.

Chapter 3

Methodology

This study was conducted under a narrative paradigm, which is itself informed by a constructivist worldview. Together, these are the underlying set of beliefs that guided the research (Guba & Lincoln, 1989, p. 117). Throughout the chapter, I address a recommendation of Lutovac and Kaasila (2019), that identity researchers should make their methodological decisions explicit (see section 2.4). This practice allows for "better comparability of the methods in this line of research" (p. 512). In section 3.1, I will describe constructivist ontology, epistemology, and axiology. Narrative social constructivism will be presented as the theoretical perspective of the study (section 3.1.3), by which, I mean "the philosophical stance lying behind [the] methodology" (Crotty, 1998, p. 66). As recommended in the literature reviewed in section 2.4, the manner in which the definition of mathematical identity used in this study was operationalised within the narrative paradigm is discussed in section 3.1.4. In section 3.2, I will discuss how the philosophical stance presented in section 3.1 was used to guide the research design. As recommended by Hennink et al. (2020, Table 11.2), this section includes the population and context for the study (section 3.2.1), as well as the rationale behind data collection, processing, and analysis. The chapter concludes with a discussion of the measures of research quality that were used to demonstrate the rigour of the study (section 3.2.6).

3.1 Theoretical Orientation

In this section, I will describe the relativist ontology, constructionist epistemology, and axiology associated with a constructivist worldview. Figure 3.1 shows how this worldview shaped the positions which I adopted, and in turn shaped my choice of theoretical perspective and methodology.

Figure 3.1
A depiction of the narrative paradigm as described by Spector-Mersel (2010).

Worldview

Constructivism

Relativist ontology | Constructionist epistemology | Value-laden axiology

↓

Theoretical Perspective

Narrative Social Constructivism

Society and culture influence our meaning-making processes.

We understand our reality through stories.

↓

Methodology

Narrative Inquiry

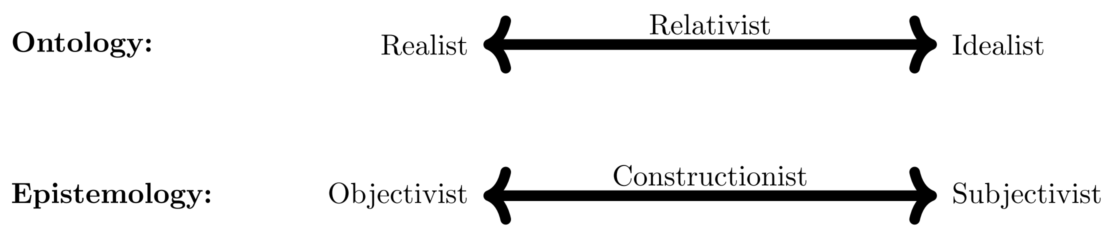
3.1.1 Ontology and Epistemology

Ontology involves the nature of reality, while epistemology describes how we gain knowledge of what we know (Crotty, 1998, p. 9; Maxwell, 2013, p. 51). The former is commonly considered on a spectrum which varies from realist to idealist ontology, with relativist ontology in the middle ground, as shown in Figure 3.2. It should be noted that this terminology is not universal,[1] and it has been argued by Miles and Huberman (1994, p. 4) and DeMarrais and Lapan (2004, p. 177)

[1] Idealism is sometimes called nominalism (Cohen et al., 2007, p. 9) or naturalism (Silverman, 2017, p. 134), while realism is sometimes referred to as positivism (Silverman, 2017, p. 134).

that most qualitative research is located away from the ontological and epistemological extremes, in the middle ground of the spectra in Figure 3.2. While a realist ontology regards reality as existing independent of the human mind (Maxwell, 2013, p. 52), an idealist ontology positions reality as a human creation, consisting only of ideas (Crotty, 1998, p. 64). Constructivists view reality from a relativist standpoint (Denzin & Lincoln, 2000, p. 158) believing that "the way things are" is really just "the sense we make of them" (Crotty, 1998, p. 64).

Figure 3.2
One way of visualising the spectra of ontology and epistemology, with a constructivist worldview located towards the centre of each of the extremes.



The literature also describes two diametric epistemological positions: subjectivism and objectivism (Guba & Lincoln, 1989, Table 3.1). A subjectivist epistemology assumes that meaning is entirely imposed on objects by the subject (Crotty, 1998, p. 9), while an objectivist epistemology takes the contrary view, that an object has meaning in itself, and that this meaning is simply discovered by the conscious mind, not created by it. In terms of research, a subjectivist epistemology asserts that the relationship between researcher and participant completely determines what the researcher can know about the participants, whereas an objectivist epistemology permits the researcher to remain a completely detached observer and discover facts that are uncompromised by the researcher's values (Guba & Lincoln, 1989, Table 3.1; DeMarrais & Lapan, 2004, p. 81). Consequently, under a subjectivist epistemology, research findings are unique to the researcher, participants, and context, and should not be used to generalise to other scenarios (Guba & Lincoln, 1989, p. 8). Constructivists support a constructionist epistemology, believing that meaning is constructed by human beings, in a way that is partially subjective (based on themselves in some way), and partially objective (using information about the object in question) resulting in the construction, and not invention or discovery, of meaning (Crotty, 1998, p. 42). Under this viewpoint, it is sensible to view research findings as a joint product of the researcher, participants, and context, and to take care if attempting to generalise the results to other scenarios (see transferability in section 3.2.6).

Ontology and epistemology are not easily distinguished within a narrative or constructivist paradigm, since constructivist ontological reality is one which is "socially and experientially based, local and specific in nature" (Guba & Lincoln, 1994, p. 110), and which is "shaped largely by the way in which we perceive it, know it, interpret it" (Spector-Mersel, 2010, p. 212) (discussed further in section 3.1.3). The relativist ontological position is the detail that distinguishes constructivism most from other paradigms in qualitative research (Guba & Lincoln, 1994, p. 111), since constructivist researchers acknowledge that their relationship with their participants shapes the research findings (epistemology), by influencing what they can know (ontology).

Positioning MISE

Guba and Lincoln (1994) frame the question of ontology as "[w]hat is the form and nature of reality and, therefore, what is there that can be known about it?" (p. 108). I adopted the view that multiple realities exist based on the sense each participant makes of their world, and, thus, I sought to understand these multiple realities (Creswell, 2009, Table 1.1). Such an understanding was co-constructed by researcher and participant as the investigation proceeded, since "what can be known is inextricably intertwined with the interaction between a particular investigator and a particular object or group" (Guba & Lincoln, 1994, p. 111). I followed a constructionist epistemology in believing that meaning is constructed, not discovered, in a way that is neither wholly subjective nor objective (Creswell, 2009, p. 8; Crotty, 1998, pp. 8-9). Under a constructionist epistemology, researcher and participants are "partners in the generation of meaning" (Crotty, 1998, p. 9), and the researcher attempts to mitigate their own influence by challenging themselves on whether their interpretive decisions are representative of participants' meanings (Morrow, 2005, p. 254).

3.1.2 Axiology

The last element of a constructivist worldview, as shown in Figure 3.1, is axiology. Based on the relativist ontology and constructionist epistemology presented previously, it is not possible for a researcher to step outside their biases and conduct research impartially as an objective observer. Therefore, I presented my conclusions as justified beliefs rather than absolute truths, thus acknowledging the value-laden nature of this research (Huberman, 1996, p. 134), and the influence of my own values on the study (Guba & Lincoln, 1994, p. 110). It is incumbent upon researchers to critically inspect their own philosophy and values, since these affect the way they collect, interpret, or analyse qualitative data (Creswell, 2009, p. 19; Strauss & Corbin, 1998, p. 115). What I brought to the study from my own background and identity, was embraced as "experiential knowledge" (Maxwell, 2013, p. 54), which informed, but did not dominate, the research design. I took this viewpoint for two reasons. Firstly, "[s]eparating your research from other aspects of your life cuts you off from a major source of insights, hypotheses, and validity checks" (p. 54). Secondly, when brought forward to the conscious, our experiential knowledge can, and should, be an advantage (Strauss & Corbin, 1998, p. 97), since the qualitative researcher is part of the analytic instrument (Nowell et al., 2017, p. 2).

3.1.3 Narrative Social Constructivism

Beyond mathematics education, in the broader field of qualitative research, *social constructivism* refers to the inseparable influence of society and culture on the meaning making processes of the human beings existing within them (Creswell, 2009, p. 8). This viewpoint acknowledges that our culture and experiences shape how we see/feel things, and indeed, whether we see them at all (Crotty, 1998, p. 59; Maxwell, 2013, p. 52). Rather than a bias or prejudice (Guba & Lincoln, 1989, p. 244), such subjectivity leads to the complexity of views that researchers often seek to understand, formed through interaction with others, and mediated by social and cultural factors (Creswell, 2009, p. 8; Silverman, 2017, p. 134). As a consequence, those who conduct social constructivist research, accept that the way they interpret their data "flows from their personal, cultural and historical experiences" (Creswell, 2009, p. 8), from which they cannot separate themselves in the name of objective interpretation.

Ontological Connection

The theoretical perspective chosen for this research was narrative social constructivism. The narrative paradigm suggests that constructivist reality is understood through stories (Spector-Mersel, 2010, p. 212), and particularly that "social reality is narrative reality" (p. 213). Drake (2006) points to several seminal authors, including Bruner (1990), McAdams (2006), and Clandinin and Connelly (2000), whose work has demonstrated that "individuals know themselves in the form of stories, but also that these stories then frame and guide the ways in which individuals understand and act on new information" (Drake, 2006, p. 581). Piaget (1937), who developed constructivism within psychology, claimed that organising one's own cognitive structures is akin to ordering the world one experiences (p. 311).[2] The concept of identity is well-placed to attend to aforementioned assumptions about reality:

> Narrative identities are stories we live by. We make them and remake them, we tell them and revise them not so much to arrive at an accurate record of the past as to create a coherent self that moves us forward in life with energy and purpose (McAdams, 2006, pp. 98-99).

Epistemological Connection

I followed Kaasila (2007b, p. 208) and Lieblich et al. (1998, p. 8) who, in line with the discussion of epistemology in section 3.1.1, regard narratives as not entirely created in the mind, but not entirely based on objective reality either.[3] It was important therefore, to facilitate participants in sharing their narratives about their mathematical experiences, in a manner that preserved their "freedom of individuality and creativity in selection, addition to, emphasis on, and interpretation" (Lieblich et al., 1998, p. 8).

Connection to Social Constructivism

Narrative social constructivism embraces the social constructivist view, by acknowledging that narratives are constructed within a "collective social field" (Spector-Mersel, 2010, p. 212), and in the context of the "cultural-meta narratives" (p. 212) available to the individual narrator. Although we depend on culture to direct behaviour and organise experience (Crotty, 1998), for a social constructivist, narrative is primarily concerned with "what goes on inside an individual's head when he or she is engaged in social interaction" (Sparkes & Smith, 2008, p. 297), and particularly with experience. Social and cultural influences are considered to be packaged into, or interwoven with, these elements (p. 297). Clandinin and Connelly (2000) agreed that people cannot

[2]Translated from French: "L'intelligence ... organise le monde en s'organisant elle-même" which translates as "intelligence organises the world by organising itself."

[3]Memories that are objective or subjective have been distinguished in the literature using the terms "historical truth" and "narrative truth" (Spence, 1982) or "life history" and "life story" (Spector-Mersel, 2010, p. 208).

be understood only as individuals (p. 2) and suggested that considering a person's experience helps us think through that person's social context. Their socially constructivist description of narrative inquiry as "a way of understanding experience" (p. 20) is echoed by Lieblich et al. (1998), who said that "by studying and interpreting self-narratives, the researcher can access not only the individual identity and its systems of meaning but also the teller's culture and social world" (p. 9).

Summary

By taking a narrative social constructivist theoretical perspective, I sought to co-construct an understanding of mathematical experiences from the participants' perspective (their socially and culturally mediated reality), since "people may construct meaning in different ways, even in relation to the same phenomenon" (Crotty, 1998, p. 9). This co-construction is important because, as mentioned in section 3.1.1, researchers are complicit in the creation of the world that they study (Clandinin & Connelly, 2000, p. 61). Given that people understand themselves and their experiences through narratives, I will follow a recommendation of Clandinin and Connelly (2000) in using narrative inquiry to attend to this understanding. Narrative inquiry is appropriate for longitudinal studies such as MISE, because it is "a collaboration between researcher and participants, over time" (p. 20) which allows constructivist researchers to "make out the extraordinary features of ordinary life" (Silverman, 2017, p. 9). This collaborative property aided me in ensuring that my own agenda did not dominate the research, by positioning the participants as co-researchers who influence the research process (Cohen et al., 2007, p. 37). This is important in research involving participants' narratives since these "don't fall from the sky ... they are composed and received in contexts" (Reissman, 2008, p. 105), and thus, researchers influence the narratives that they collect.

3.1.4 Narrative Mathematical Identity

As discussed in section 2.6, mathematical identity is defined as the "multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others" (Eaton and OReilly, 2009c, p. 228; see also Grootenboer and Zevenbergen, 2008). It is usually considered to be an ever-changing and constantly renegotiated phenomenon (see section 2.4), which people can use to make sense of their experiences with, and relationship to, mathematics (Grootenboer & Zevenbergen, 2008; Kaasila, 2007b; Sfard & Prusak, 2005). Kaasila (2007a) summarises succinctly that "(narrative) mathematical identity should not be seen as a stable entity but as something that people use to justify, explain and make sense of themselves in relation to mathematics and to other people acting in mathematical communities" (p. 374). It is from this perspective that mathematical identity emerges as a means of constructing and organising one's relationship with mathematics through autobiographical narratives.

Although some authors have argued that identity exists without narration (see "enacted" view of identity in section 2.4), I follow Sfard and Prusak (2005) and Bruner (1991) in equating identity with narrative (see section 2.5). The view that narratives construct and organise identity, rather than mirror some deeper imperceptible trait within, is consistent with the constructivist foundations of the narrative paradigm (Spector-Mersel, 2010, p. 208; Sparkes & Smith, 2008, p. 299). Many authors acknowledge that narratives are "re-constructed during the process of recollection, thus preserving identities' dynamic and fluid quality" (Radovic et al., 2018, p. 29). Under a constructivist worldview (see section 3.1.1), such mathematical identity narratives are seen as co-constructed through social interaction between the participants and the researcher (Silverman, 2017, p. 134), and thus I was an active participant in the creation of the mathematical identities of my participants.

In section 2.4, it was discussed that studies in mathematical identity attend to the concept through subjective and social dimensions. In this study, I sought to understand mathematical identity from the participants' point of view by eliciting their narratives of their socially constructed reality since "it is our *vision* of our own or other people's experiences and not the experiences as such, that constitutes identities" (Sfard & Prusak, 2005, p. 17). The accuracy of one's memories does not matter for mathematical identity since the "remembered truth" (Spence, 1982) of narrative reality is the socially constructed reality that drives the narrator's relationship with mathematics, including future understanding and action (see section 3.1.3). Narrative inquiry is an important means of accessing the fluid and ever-changing phenomenon of mathematical identity, since it includes the constructivist ontological assumption that multiple realities exist, and the same events can have different significance for different people (Ntinda, 2020, p. 2).

3.2 Research Design

The purpose of this section is to present the overall design of the study, and to demonstrate the rationale behind the data collection stages and methods, by linking these to the research questions. This study is a qualitative, longitudinal study which comprises of three main stages of data collection, informed by two pilot studies. The stages of data collection were sequential, and the research design was emergent rather than pre-determined, which is typical of qualitative studies (Creswell, 2009, p. 15). It is noted in the literature that "[q]ualitative research is not linear, as often presented in methodological literature, but dynamic and interactive" (Tobin & Begley, 2004, p. 391), and that changes to the research approach are "expected products of an emergent design" (Guba & Lincoln, 1989, p. 242). Thus, each stage of data collection and analysis informed the subsequent stages, and the full research design emerged as the study progressed (Kelly, 2000, p. 260). This study aims to answer three research questions about science and engineering students in DCU:

RQ1. What is the relationship of science/engineering students with mathematics?

RQ2. What is the difference in mathematical identity of science students compared to engineering?

RQ3. How does the relationship of these students with mathematics change over time?

As shown in Figure 3.3, the study commenced with a pilot questionnaire and subsequent data collection (*Data1*), using a questionnaire based on that developed by Eaton et al. (2013), with some modifications arising from the pilot (see appendix E.2). The rationale behind the elements of the questionnaire is discussed in section 4.1.1, and the modifications in section 4.2. After the questionnaire data was analysed, two focus groups were conducted to clarify the results, elaborate on the questionnaire responses, and to track changes in participants' mathematical identity (*Data2*). Next, a pilot interview was conducted to evaluate whether the longitudinal aspect is better assessed through interviews than focus groups. Lastly, narrative interviews were used to explore the change in participants' mathematical identity over their time in DCU (*Data3*). The data was collected over a period of four years, the timeline for which is shown in Figure 3.4. Table 3.1 shows the analytic approach to each stage of data collection (see chapter 5), and which research questions were addressed by each stage of data collection. Data1 was collected to investigate RQ1 and RQ2, while Data2 was intended to address all three research questions.

Data3 was collected to investigate RQ3, which was not adequately answered through the analysis of Data2, with some attention to RQ2 also.

Figure 3.3

Data collection stages used in MISE.

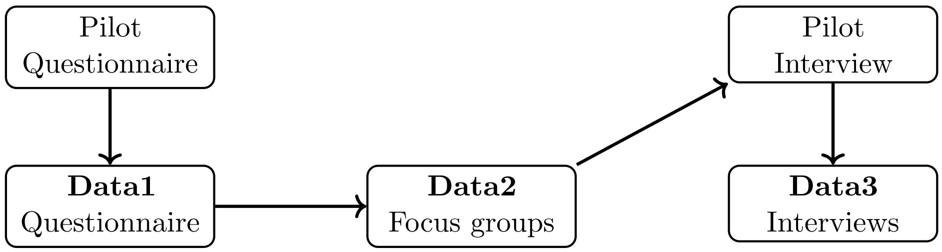


Figure 3.4

Timeline of two pilot studies and three data collection stages in MISE (Data1, Data2, and Data3).

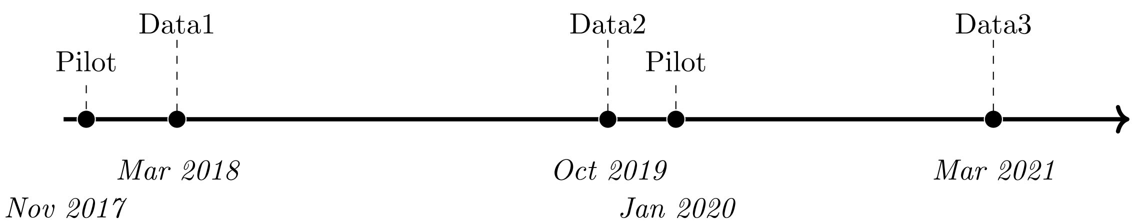


Table 3.1

Data analysis approaches used for each stage of data collection in MISE, and the research questions answered by each stage.

Data collection stage	**Data collection method**	**Analytic approach**	*Research questions*		
			RQ1	RQ2	RQ3
Data1	Questionnaire	Reflexive thematic analysis using NVivo	✓	✓	
Data2	Focus group	Reflexive thematic analysis (deductive) using NVivo	✓	✓	✓
Data3	Interviews	Narrative analysis using NVivo and Excel		✓	✓

3.2.1 Participants

Following on from section 3.1.3 and noting the discussion in section 2.4 concerning the subjective-social spectrum of mathematical identity research, studies should be attentive to differences in social contexts when attempting to compare participants' mathematical identities. In MISE, this was undertaken with regard to three groupings of participants who shared the same (zoomed-in) mathematics classroom context as they transitioned to university education: Engineering, Science I and Science II. The rationale for the groupings was also informed by broader issues related to the historic marginalisation of women from education, and the role of mathematics as a gateway to education, and, therefore, career options (Leder, 2019). Studies that investigate the differences between male and female participants' mathematical identities are rare (cf. Solomon, 2007a; Mendick, 2005), but a significant body of knowledge exists with reference to the gender differences in various affective components of mathematical identity (Goldin et al., 2016, p. 2). Martin (2012) provided a compelling argument that identity studies comparing the mathematical achievements of Black and White children may in fact serve to normalise one group and marginalise the other. He strongly believed that there was a danger of the gap between the groups being seen as a hierarchal model, which involves raising the lower group to meet to normalised standard of the upper group or, even worse, could be used by some to ask "*if*, not *how*, Black children can learn mathematics" (p. 48). Furthermore, as mentioned in section 3.1.3 with regard to narratives, mathematical identities are context-bound: each person's identity is attached to different social contexts (Kaasila, 2007b, p. 206). The influence of broader social and cultural factors on MISE participants' mathematical identity narratives should, and will, be acknowledged in analysis. I reasoned that since this study involves comparing participants of science and engineering, it was reasonable to create the groupings based on shared mathematics modules, rather than broader influences which may differ along demographic lines, such as gender or race.

Participant Profile

The population for the study consisted of 16 cohorts of science and engineering students in DCU who study mathematics in their first year (see appendix 3.2.1). The science and engineering cohorts are listed in Table 3.2 and Table 3.3 respectively, the data for which was drawn from an internal university system in February, 2018.

Table 3.2
First year Science cohorts at DCU in 2017/18 academic year, split into Science I and Science II groupings, with PST programmes in italics.

Code	Program Title	Program Code	Size	MISE
PEM	*Physical Education and Maths*	DC206	19	1
SE	*Science Education*	DC203	19	4
PBM	Physics with Biomedical Sciences	DC173	13	1
AP	Applied Physics	DC171	17	1
PHA	Physics with Astronomy	DC167	23	3
DS	Data Science	DC123	10	1
		Science I subtotal	101	11
CES	Common Entry Science	DC201	57	5
ESH	Environmental Science and Technology	DC166	23	3
AC	Chemical and Pharmaceutical Science	DC162	15	1
AS	Analytical Science	DC161	20	2
BT	Biotechnology	DC181	24	2
		Science II subtotal	139	13
		Total	**240**	**24**

Table 3.3
First year Engineering cohorts at DCU in 2017/18 academic year.

Code	Programme Title	Programme Code	Size	MISE
CE	Common Entry Engineering	DC200	80	4
CAM	Mechanical and Manufacturing Engineering	DC195	16	
ECE	Electronic and Computer Engineering	DC190	8	2
BMED	Biomedical Engineering	DC197	27	
ME	Mechatronic Engineering	DC193	15	2
		Total	**146**	**8**

Table 3.2 is split into two parts: Science I included participants who studied mathematics for more than one year, while participants from Science II were required to study mathematics only in first year. Notably, Science I included some PSTs, who could be expected to familiar with self-reflection in later undergraduate, but perhaps not at the time of collection of Data1. The Science I cohorts took modules MS126 (Calculus) and MS116 (Calculus for Teachers), while the Science II cohorts took MS125 (Calculus). The engineering cohorts are listed in Table 3.3. All five of these cohorts took EM101/EM102 (Engineering Mathematics I/II) in year 1.

Some important contextual differences between the groupings were noted from the outset of the research. Engineering students were required to take higher-level mathematics to qualify for entry to their programme, whereas Science I and Science II students can qualify for entry with ordinary-level mathematics. In DCU, mathematics modules are delivered to engineering students by lecturers in their school, and tutorials are run by the engineering lecturers themselves. On the other hand, science students are taught mathematics by members of the School of Mathematics, and tutorials are conducted by postgraduate students or staff members who are employed specifically as tutors.

First year students had the option to undertake a common entry programme in science or engineering, which allowed them to specialise in one of the other available programmes after first year. In latter years, science and engineering students are also required to participate in workplace-based learning, known as Integrated Training (INTRA). This flagship programme takes the form of internships in industry for science and engineering students, and teaching placement in schools for PSTs (Dublin City University, n.d.).

As part of support offered by the university, all students who are studying mathematics have access to the Mathematics Learning Centre (MLC): a drop-in service which has operated for almost 20 years, with the aim of supporting at-risk students in passing first year mathematics modules in particular (Jacob & Ní Fhloinn, 2019). The tutors are available for one-to-one or group help, and groups would generally be encouraged to work together as much as possible in the centre. Many of the tutors who conduct tutorials with science students also tutor in the MLC, and nationally, such mathematics support has been reported to be most popular amongst students of engineering, science, and business (Cronin et al., 2015).

Ethical Considerations

Ethical approval was sought, and granted, by the university research ethics committee. As part of the ethical approval, participants in MISE were required to be at least 18 years of age at the time of the first data collection. The first data collection was conducted towards the end of their first year in DCU to maximise the number of eligible participants. At a minimum, it is incumbent upon researchers across all disciplines to conduct research in a manner that is consistent with ethical principles that minimise the potential negative impacts on participants. Farrugia (2019) argued that in studies where the relationship between the researcher and participants is significant for the research (as discussed in chapter 3), it is most beneficial to apply flexible guiding principles that invite ongoing and careful ethical considerations throughout the study. In particular, Farrugia advocated the principles of respect (obtaining informed consent and ensuring anonymity and confidentiality for participants), beneficence (ensuring participants' safety), and justice (that the burden and benefits of the research are shared fairly) (Farrugia, 2019, pp. 49-50).

Following the principle of respect, it is vital to provide transparency to participants with regard to the requirements, risks, and benefits associated with their participation in the study (Farrugia, 2019, p. 49; Creswell, 2009, p. 89). In MISE, this was accomplished by creating a meeting script for recruiting participants and making a plain language statement (PLS) and informed consent form (ICF) available for potential participants to consider for a time before they agreed, or not, to participate. In line with recommendations by Farrugia (2019, p. 49), the PLS explained the research goals, what was required of participants, the potential benefits of participation, and the steps to be taken to ensure the anonymity of participants. Creswell (2009) also highlighted ethical issues around gaining access to participants through "gatekeepers" (p. 90), which in MISE meant asking permission of module lecturers to address their class during lecture time. The head of the Faculty of Engineering and Computing, as well as and the head of the School of Mathematical Sciences, were consulted before the recruitment of participants was undertaken, and invited to allow the relevant cohorts to participate. In discussion with lecturers, a time was identified for me to address each group of students. The lecturers were given details of the time this would take, and provided with the PLS to explain the risks, benefits, and potential impact of the study.

After due consideration of beneficence, I concluded that there were no reasonably foreseeable risks posed to participants in the study (Cohen et al., 2007, p. 55), but the population included some students for whom I had conducted tutorials in "Calculus for Teachers" in the previous semester. Thus, as is mentioned by Cohen et al. (2007, p. 63) and Farrugia (2019, p. 48), it was important that students were aware that they did not have to participate in the research because I had been their tutor, or indeed for any reason. Volunteer sampling was used to recruit participants from the three groupings described previously, and during my contact with each cohort I emphasised the voluntary nature of their participation, and their right to withdraw from the study at any point (see appendix C.5). In particular, I highlighted that this study is not a test, and would not contribute positively or negatively to their grades for the module.

To preserve the participants' anonymity, they were assigned identifiers immediately after each stage of data collection, before formal analysis took place, and a list of these identifiers was stored separately to the data itself. The participant identifiers follow two distinct formats. For interview participants, their identifiers consist of an Irish pseudonym, followed by an underscore, and lastly their grouping. Each pseudonym begins with the letters A to F of the alphabet for interview participants (e.g., Aodhán_SC2), with one exception for the pilot interview participant (Saoirse_SC1). For all other participants, their identifiers consist of their grouping, followed by an underscore, and lastly a randomly assigned number (e.g., SC1_08). To further solidify the principles of justice and respect, PST students, who share lectures with other science cohorts involved in this study, were also invited to participate. Furthermore, each subsequent stage of data collection was open to all participants who had agreed to participate in the study, which meant that every participant who volunteered to take part in any stage of data collection was included (Farrugia, 2019, p. 50). Details of those who participated in each data collection stage will be given later in this section.

Recruitment Process

Before the first data collection (Data1), I addressed each group at the end of a timetabled lecture. The PLS and ICF (see appendix C.6 and appendix C.7) were distributed during the first meeting and the ICF was collected from willing volunteers at the second meeting, which allowed students time to think about their mathematical journeys and/or formulate questions about participating in the research. The scripts for both meetings are presented in appendix C.5. Based on my experience with invalid ICFs in the pilot study, I colour coded the PLS on white paper and

ICF on yellow paper, to indicate that the yellow page had a special function. Those who had completed and returned the ICF could access the questionnaire through the MISE page on the university virtual learning environment.

I collected 125 valid ICFs, from which I received 32 questionnaire responses: 8 Engineering, 11 Science I, and 13 Science II students. Participants' demographic information is summarised in appendix C.3. The sample included only four students who had taken ordinary-level mathematics for Leaving Certificate (two in Science I and two in Science II) and included 8 female students and 24 male students. At the outset, I had hoped to recruit upwards of 100 participants to take part in the study. Although only 32 participants completed the questionnaire, they engaged very deeply with the questions therein, and with the reflective and narrative processes involved in its completion. The necessity of additional data-driven (inductive) codes to analyse their responses (see section 5.1.4), which consisted of almost 7800 words, demonstrated the depth of the participants' engagement with the process. Furthermore, as will be discussed in section 3.2.6, in this qualitative study, the adequacy of the sample is determined by the so-called richness of the collected data, rather than the sample size. In her review of identity research in mathematics education, Darragh (2016) found that the majority of identity studies involved fewer than 10 participants (p. 23), while Lutovac and Kaasila (2019) found that even in large scale studies of teacher identity, there is a tendency to focus the findings on a small number of cases (p. 509).

For the second data collection (Data2), I used volunteer sampling, and sent an invitation email to all 32 participants from Data1. To encourage participation, the invitation included an explanation of the benefits of participating in the focus group, an update on the research to date, and included a link to the MISE website (https://www.sites.google.com/mail.dcu.ie/fhoward/research) where participants could view the papers and talks related to the study, and access the invitation to participate in Data2. The initial email was followed up with a reminder 10 days later, the combination of which resulted in six responses. The sample included only one participant from Engineering and one from Science II, with the remaining participants belonging to Science I. The participants were divided into two focus groups which were stratified by their grouping, since focus groups operate better with participants who are not familiar friends (Cohen et al., 2007, p. 377). Focus group 1 (FG1) had three participants: ENG_08, Breandán_SC1, and SC1_08. Focus group 2 (FG2) had two participants: Fiach_SC1 and Aodhán_SC2. The remaining volunteer for

FG2 had a last-minute problem and could not attend the focus group.

For the final data collection, all 32 participants were invited to volunteer to be interviewed, using a similar sampling approach to Data2. Since this stage of data collection took place during the corona virus (COVID-19) pandemic, DCU students were not present on campus, and thus email was the only contact option. Interviews were conducted with six participants from Data1, three of whom had most recently contributed to Data1 (Ciarán_ENG, Dónal_SC1, Éabha_ENG), and three of whom also contributed to Data2 (Aodhán_SC2, Breandán_SC1, Fiach_SC1).

3.2.2 Pilot Questionnaire

The purpose of the pilot study was to prepare for the collection of Data1. The questionnaire, which is shown in appendix E.1, consisted of five demographic questions, three open-ended questions about participants' experiences with mathematics, and two questions to invite their feedback on completing the questionnaire. The demographic questions enquired about participants' gender, age, programme of study, highest exam level completed, and grade. These were followed by three open-ended questions: a broad opening question, a follow-up question which includes some prompts, and a final evaluative question. A broad opening question allowed participants to make responses that were "indicative of their personal mathematical identity" (Eaton & OReilly, 2009c, p. 229), and the follow-up question included some prompts "to balance the need for some direction" (p. 229). The rationale behind each of the open-ended questions in the pilot questionnaire will be discussed in section 4.1. The pilot questionnaire influenced the next stage of data collection by providing an opportunity for the researcher to run through the practicalities of recruiting participants, administering the online questionnaire, and performing data analysis. Although no changes in the overall research design resulted from the pilot study, a selection of codes were identified to contribute to a codebook for thematic analysis (see chapter 5). The codebook was expanded and refined in the subsequent stages of data collection and analysis (see chapter 4). The outcomes of the pilot study are discussed further in section 4.1.

3.2.3 Data1 - Questionnaire

The purpose of Data1 was to establish the mathematical identity of science and engineering students and to identify the similarities/differences in mathematical identity between the groupings.

The rationale for the questionnaire is given in section 4.1.1, and the data collection methods for Data1 will be discussed in section 4.2. The influence of Data1 on the research design is demonstrated below. In investigating RQ1, I found it helpful to create two questions about the codebook that had been developed through the pilot questionnaire. Firstly, which elements of mathematical identity can be identified using the codebook? Secondly, which elements of mathematical identity arise in this cohort but are not present in the codebook? Deductive and inductive thematic analysis respectively allowed me to answer these questions using a *group narrative*. As will be explained in section 5.1, participants' narratives were first considered at an individual level, by reading through each individual's questionnaire in turn and applying coding. Then thematic analysis was used to identify themes that were common across participants and develop a group narrative, in order to understand what Sfard and Prusak (2005) would call the "collective discourses" that shape participants' "personal worlds" (p. 15).

The themes developed from Data1 influenced the subsequent data collection. Some required clarification and included elements that would benefit from further elaboration to make their meaning clear to the researcher. As I began to develop a more considered theoretical stance and refined my research design, I began to think of the participants more as co-researchers who should shape the research process (Cohen et al., 2007, p. 37), as discussed in section 3.1.3. Since thematic analysis of narrative data requires the researcher to make interpretations about meaning, it is important, therefore, to provide an opportunity for these interpretations to be challenged or reinforced by the participants.

3.2.4 Data2 - Focus Groups

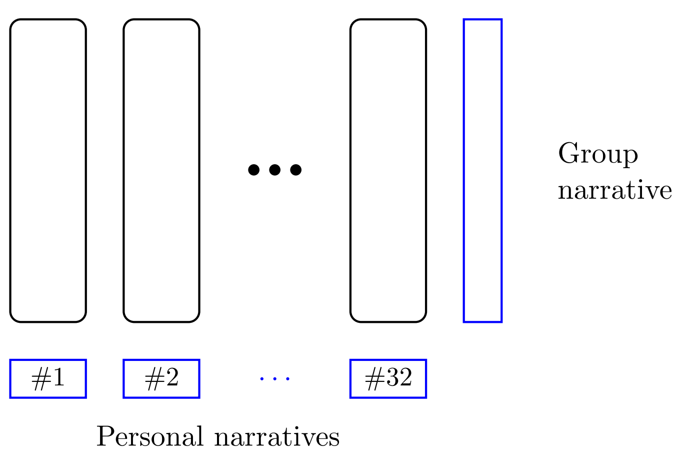
The aims of Data2 were, firstly, to verify and elaborate on the themes developed in Data1 (RQ1, RQ2), and, secondly, to examine the changes in the mathematical identity of these participants since the first stage of data collection (RQ3). The methods used in Data2 will be discussed in section 4.3, and their rationale is presented in this section. In analysing mathematical identity, Graven (2004) saw great value in combining formal written and informal spoken methods of data collection since "questionnaires enabled teachers time to organise and revise their thoughts" while interviews "may illicit more informal, 'thinking as one speaks' responses" (p. 192).

On the first aim, checking the researcher's interpretations is a means to ensure the credibility in the research process through member checking (see section 3.2.6), and has featured as a means of validation in other narrative identity studies (Lutovac & Kaasila, 2018, p. 264). According to Mauthner and Doucet (1998), the "participants' voice" can be kept alive by involving them in the data analysis "so that the analysis is more collaborative and meaning is negotiated" (p. 139), and in particular, that that the researchers interpretations do not overpower what the participants really meant. Asking participants to elaborate on certain topics from Data1 ensures that their agenda can predominate (Cohen et al., 2007, p. 376), and expands the discussion beyond what has already been identified in the questionnaire responses and thus, beyond what is already known to the researcher. Mauthner and Doucet (1998) noted that there is no perfect way to represent the voices of participants in qualitative research, but "there are ways in which we can attempt to hear more of their voices, and understand more of their perspective" (p. 140).

The focus group questions were drawn from themes in the questionnaire data, so that the analysis of participants' responses could add to, and expanded on, these themes and this collective view (Cohen et al., 2007, p. 376). As this stage progressed, I began to visualise group and personal narratives (as discussed in section 3.2.3) by thinking of each questionnaire as a column of information, and picturing the analysis as collapsing the data in one of two distinct directions. This visualisation is presented in Figure 3.5, where collapsing the data to the right produces a group narrative (using thematic analysis for example), and collapsing the data downwards results in separate personal narratives for each participant (which will be discussed in section 3.2.5).

Figure 3.5

Comparing group and personal narratives by collapsing the data (black columns) to the right, or downwards respectively. The analytic product of each approach is depicted in blue.



The focus groups were helpful for providing elaboration and clarification of some aspects of the themes developed through analysis of the questionnaire data, and participants shared many insights related to the themes that were not evident in the first data collection. However, I was met with a significant challenge in discerning changes to participants' individual mathematical identities over time. Establishing common ground in the focus group (Hydén & Bülow, 2003, p. 305) appeared to limit opportunities for participants to reflect on their mathematical identities in the context of their own journey, rather than in comparison to the other focus group participants, and perhaps did not allow enough time and space for them to consider how their relationship with mathematics had changed since first year. Furthermore, if mathematical identity is thought of as a process which involves interaction with others and sharing of narratives, acknowledging the co-constructed nature of mathematical identity (i.e., that it is influenced by the people with whom one interacts), then separating one voice from that of the researcher and other focus group participants strips the narrative of its context. Thus, epistemological issues arose when analysing the focus groups with regard to the third research question, since the focus group data represented "more than the sum of its individual participants" (Hydén & Bülow, 2003, p. 311).

3.2.5 Data3 - Interviews

Analysis of the focus groups gave insight into the group narrative, but not the individual view, and thus did not adequately answer RQ3: to investigate the change in participants' mathematical identity over time, as they transition to university level education. Thus, the rationale for the interviews was informed by the previous stage of data collection and analysis and is discussed below (for a discussion of the methods, see chapter 4). The same rationale applies to the pilot interview, the details of which are presented in section 4.4.

In order to address RQ3, I was drawn to narrative interviewing because such interviews seek individuality rather than standardisation (McCormack, 2001, p. 70), and allow space for participants to tell stories that are important to them (Kaasila, 2007b, p. 207). Narrative interviewing is a method of data collection that supports the co-construction of meaning between interviewer and participant (Mishler, 1991, p. 52) (see section 3.1.4). Participants are allowed time and space to "hold the floor" (Coffey & Atkinson, 1996, p. 56) and direct the course of the interview more than usual (Mishler, 1991, p. 117), rather than this power resting mostly with the interviewer (Cohen et al., 2007, p. 377). Such concerns are not prioritised in traditional semi-structured interviews where “there is usually not enough scope for recounting narratives” (Kaasila, 2007b, p. 207). To empower participants to do this, one must change the traditional interviewer-interviewee relationship to one of listener-narrator .

The modest number of participants in MISE allowed me to consider data collection and analysis methods that were focused on the individual. This meant that although Data1 and Data2 were used to explore experiences that were common across several participants, for Data3, I could concentrate further on distinctive experiences that were unique to a participant, thereby broadening the ultimate picture of mathematical identity that would emerge from the study. Parts A and B of the interview were designed to elicit a year-by-year journey through participants' mathematical experiences in DCU. Parts C and D were designed to evoke specific comparisons between the participants' past and current selves, highlighting how their mathematical identity has changed over time. The full details of the interview parts will be presented in section 4.5.1.

3.2.6 Assessing the Research Quality

According to O'Reilly and Parker (2012, p. 194), to conduct quality research researchers must “address their theoretical position” (see section 3.1), “evidence the congruence between methodology and methods” (see section 3.2), and “highlight the strategies they used to establish rigour” (see the current section). In this section, the traditional positivist criteria for determining research quality will be discussed, along with their qualitative counterparts proposed by Guba and Lincoln (1985). Thereafter, I will present the criteria used in this study to establish trustworthiness, transferability, validity, and saturation.

The so-called “scientific holy trinity” of reliability, validity, and generalisability are constructs that are classically used to demonstrate rigour in positivist studies (Emden & Sandelowski, 1998, p. 207), mostly through determination of appropriate methods (Emden & Sandelowski, 1999, p. 2). Such an approach is not appropriate for most qualitative research (Lincoln & Guba, 2000, p. 178), and, thus, Guba and Lincoln (1985) proposed qualitative counterparts of the aforementioned long-standing accepted measures (see appendix D.2). Although their criteria have been very influential, they have been widely challenged (Sparkes, 2001, p. 540; Emden & Sandelowski, 1998, p. 211; Tobin & Begley, 2004, p. 392), not least because the authors have acknowledged that the definitions had their "roots and origins in positivist assumptions" (Guba & Lincoln, 1989, p. 245). If one accepts that qualitative research can be based on a wide range of philosophical and methodological approaches, and that there are no unifying elements that are common across all qualitative research (Emden & Sandelowski, 1999, pp. 3-4), it seems natural that the criteria for judging quality should be specific to the philosophical and methodological assumptions inherent in the research (Engel & Kuzel, 1992; Morrow & Smith, 2000, p. 217). Any potential criteria should be open-ended, in that they attempt to persuade the reader that the research is of quality but should not purport to prove this is absolutely true (Guba & Lincoln, 1985, p. 329; Lincoln & Guba, 2000, p. 179). Loh (2013) recommends that narrative studies use techniques to justify the trustworthiness criteria proposed by Guba and Lincoln (1985), and amend, or add to, these techniques as appropriate for the philosophy of the particular study.

Trustworthiness and Transferability

In this study, the notion of *trustworthiness*, a term originally proposed by Guba and Lincoln (1985) to parallel reliability, has been reimagined as the "willingness to act on the basis of, as well as pay attention to, a study" (Mishler, 1990, p. 419). Under this definition, trustworthiness is no longer a fixed, objective judgement, but a "continuing social process through which claims are contested, assessed, and warranted" (p. 419) by other researchers who judge whether the research is trustworthy enough to be acted upon over time. As a result, trustworthiness is determined by "the degree to which we can rely on the concepts, methods, and inferences of a study, or tradition of inquiry" (p. 419).

Transferability takes the place of generalisability in this study: a "thick description" of the context of inquiry, which allows others to determine to which other contexts it may be transferable (Guba & Lincoln, 1985, pp. 124-5). A social constructivist cannot speak to transferability without knowledge of the context to which the research is to be transferred, since this depends on the "degree of similarity" (Guba & Lincoln, 1985, p. 316) between the two contexts. Trustworthy research should enable "intersubjective agreement" (Silverman, 2017, Table 18.1) by clarifying the methods of data collection and analysis, and researchers should provide a rich description that makes judgements of transferability possible (Guba & Lincoln, 1985, p. 316).

In order to establish trustworthiness and transferability, Tobin and Begley (2004, p. 391), drawing on the work of Arminio and Hultgren (2002), detail six elements of the research process that must be embedded throughout a study, central to communication of the study, and explicit in the written report. In Table 3.4, I have noted the locations in the thesis where I have dealt with each of these elements, since Tobin and Begley (2004) advise that these criteria "cannot be limited merely to a discussion in a methodology section" but must be "an integral and embedded component of the research process" (p. 391). In the relevant sections, I have endeavoured to show that this is the case.

Table 3.4
Six elements to embed throughout a qualitative study in order to establish trustworthiness and transferability (Arminio & Hultgren, 2002; Tobin & Begley, 2004).

Element for judging a qualitative study	**Location in thesis**
Foundation (epistemology and theory): the philosophical stance which gives context to, and informs, the study methodology.	section 3.1.1
Approach (methodology): the specific grounding of the study's logic and criteria.	section 3.1 section 3.2
Collection of data (method): explicitness about data collection and management.	chapter 4
Representation of voice (researcher and participant as multicultural subjects): researchers' reflections on their relationship with participants and the phenomena under exploration.	section 3.1.3 section 3.1.4
The art of meaning making (interpretation and presentation): the process of presenting new insights through the data and chosen methodology.	chapter 5
Implications and recommendations for professional practice	chapter 8

The criteria in Table 3.4 emerged from measures of rigour defined by Guba and Lincoln (1985) but did not have their disguised positivist assumptions (Emden & Sandelowski, 1998, p. 208). Rather than being a set of post-hoc criteria for evaluating research quality (Tobin & Begley, 2004, p. 392), these guidelines are "developmental, leading to growth of understanding, surfacing of clarity, emerging of criteria and stretching of epistemologies" (p. 391). Attending to the question of *saturation*, which is discussed next, can been considered as an attempt to track the extent to which a study is actually developing a deep and comprehensive understanding.

Validity and Saturation

For qualitative researchers, establishing *validity* often centres on the way that researcher insights are interpreted and presented, which Tobin and Begley (2004) called "the art of meaning making" (p. 391). This approach to research foregrounds the importance of a valid and defensible reasoning for the interpretations made in the study, since validity cannot be determined by the methods alone, as in positivist studies (Emden & Sandelowski, 1999, p. 2; Lincoln & Guba, 2000, p. 178). It is recommended in the literature that researchers who acknowledge the social construction of knowledge (see section 3.1.1), should consider meaning and interpretation as central to validity (Mishler, 1990, p. 417). This is of particular importance for studies that employ reflexive thematic analysis (see section 5.1), since this analytic method relies on researcher subjectivity and reflexivity, rather than on codified procedures or techniques (Braun & Clarke, 2021a, p. 329).

Guba and Lincoln (1989) proposed *member checking* as one strategy for reinforcing the validity of a qualitative study. Member checking involves "testing hypotheses, data, preliminary categories, and interpretations with members of the stakeholding groups" (pp. 238-239). Although this idea appeared to be an appropriate way of involving my participant co-researchers in the study, member checking is sometimes discounted by qualitative researchers since there is no single reality to validate (Sparkes, 2001, p. 541). However, I reasoned that the idea of member checking is still applicable under the "art of meaning making" criteria, if it is carried out with respect to the constructions and interpretations of the researcher, rather than to validate the data itself. Social constructivist researchers accept that their past experiences influence their interpretations (Creswell, 2009, p. 8), and thus, these decisions about meaning should be checked for validity with the participants who provided the data in the first place (Loh, 2013, p. 6).

In qualitative research, the concept of *saturation* is defined in different ways, but generally, saturation refers to "a criterion for discontinuing data collection and/or analysis" (Saunders et al., 2018, p. 1894). According to Saunders et al. (2018), there is widespread acceptance of saturation as a vital component of a sound qualitative methodology (p. 1893), to the point where it is increasingly expected as a "generic requirement for all qualitative inquiry" (O'Reilly & Parker, 2012, p. 191). Qualitative researchers use terms like *rich data*, because the adequacy of a sample is not determined merely by the sample size, nor the word count (O'Reilly & Parker, 2012, p. 195), but more so by the "richness, depth, diversity and complexity" of the data collected

(Braun & Clarke, 2021b, p. 202). In other words, for qualitative researchers, saturation is more a question of quality than quantity. In fact, Marshall and Long (2010) argued that by not making claims about the representativeness or generalisability in their narrative methods, they could relax the need to consider saturation at all. Similarly, O'Reilly and Parker (2012) claim that when developing codes from data there is a "potentially limitless" number of themes, making data saturation an "unrealistic target" (p. 294), since there will always be the potential for more insight or knowledge to be gleaned from further data collection (Braun & Clarke, 2021b, p. 206; O'Reilly & Parker, 2012, p. 294). Hennink et al. (2017) made a distinction between two categories: *meaning saturation*, which is measured qualitatively, and *coding saturation*, which is measured quantitatively. Meaning saturation is the degree to which new "dimensions, nuances, or insights" (Hennink et al., 2017, p. 594) can be found in relation to a phenomenon, whereas coding saturation is the "point when no additional issues are identified and the codebook begins to stabilize" (p. 594).

Two measures of saturation were used in this study to allow for the consideration of both coding and meaning saturation, since "counting occurrences of themes, without also assessing the meaning of those themes" would be insufficient (Hennink et al., 2017, p. 593). Both measures applied to the analysis phase of the research, since, for a thematic analyst, considering whether themes are saturated before establishing the themes themselves would be "superficial at best" (Saunders et al., 2018, p. 1900).

Meaning saturation was measured using *theoretical saturation*, a qualitative measure which occurs in a study as less and less data is being found which contributes to the understanding of a particular category or theme (Saunders et al., 2018, p. 1895; Glaser & Strauss, 1967, p. 61). This type of saturation is appropriate for studies that seek to develop themes or theories from analysis (Saunders et al., 2018, p. 1896), and the researcher is permitted to look for new data to stretch the diversity in the categories i.e., to find more pieces of the puzzle, rather than to invalidate what has already been found (Tobin & Begley, 2004, p. 393). While themes were being developed in MISE, I followed the recommendation of Braun and Clarke (2022) and judged saturation by assessing the *coherency* (p. 99) of the narrative for each theme: what the theme is, what the theme is not, and how they represent the dataset as a whole. The resulting renaming, editing, and removal of themes is discussed in Stage 6 of the thematic analysis process in section 5.1.8.

Coding saturation was measured using *inductive thematic saturation*, which is based on the number of new codes or themes emerging through the analysis process, rather than their completeness or understanding (Saunders et al., 2018, p. 1896). The rate of development of data-driven codes, and thus of more themes, should decrease as the codebook becomes more stable (Hennink et al., 2017, p. 594). To monitor the expansion of the codebook throughout the study, inductive thematic saturation was used to examine the number of new codes created at each stage. This method focuses "more explicitly on reaching saturation at the level of analysis" (Saunders et al., 2018, p. 1900), which is useful for studies that are more data-driven than theory-driven (see section 5.1).

Summary

In MISE, the trustworthiness, transferability, and validity of the study were established mostly through the six central elements from the literature (Arminio & Hultgren, 2002; Tobin & Begley, 2004), which are detailed in Table 3.4. Considerations of validity involved attention to data collection methods and analysis, which are described in detail in chapter 4 and chapter 5, respectively. Member checking was used to establish the validity of the researcher's interpretations by providing preliminary summaries/findings to the participants (DeMarrais & Lapan, 2004, p. 243), using the focus groups to clarify interpretations from the questionnaire data analysis, allowing space for participants to communicate their ideas more fully in the interviews, and asking follow-up questions to verify the meaning of certain parts. With regard to the data however, new data gathered in Data2 and Data3 that appeared to controvert the themes established in Data1 was seen as suggesting "there is more to this theme," rather than that the theme was invalid. Thus, the development of new inductive codes in Data2, which was measured by inductive thematic saturation, was expected to relate to the comprehensiveness of the themes from Data1, but not their validity. Meaning saturation was established through the coherency of the themes, as part of the thematic analysis process (see section 5.1.8).

Chapter 4

Data Collection Methods

In section 3.2, I presented an overview of all the data collection stages and connected them to the research questions. The details of each stage will be discussed, in turn, in this chapter. Each section explains the purpose of the stage, reports how the data was collected, and concludes with the influence on the subsequent stages. In the remainder of this chapter, I will demonstrate the rationale behind the data collection stages and methods, and link these to the research questions. The data collection methods will be discussed in more detail in chapter 4.

4.1 Pilot Questionnaire

As mentioned in section 3.2.2, the aims of the pilot study were as follows: to determine if the questionnaire needed to be changed to cater for this new context, to compare two online tools for questionnaire data gathering, to trial and refine the codebook for mathematical identity (which was based on themes in the literature), and to design effective materials for distribution to potential participants. This section details the influence of the pilot study on the main data collection, Data1. The pilot questionnaire, which is included in appendix E.1 and discussed in section 4.1.1, consisted of five demographic questions, three open-ended questions about experiences with mathematics, and two questions to invite feedback on completing the questionnaire.

4.1.1 Questionnaire Content and Rationale

The three open-ended questions consisted of a broad opening question, a follow-up question which includes some prompts, and a final evaluative question. These questions, which were adapted from Eaton et al. (2013), are discussed in turn below.

Question 1: *Think about your total experience of mathematics. Tell me about the dominant features that come to mind.*

In line with the theoretical orientation of the study, this broad, open-ended first question provided participants with agency in selecting which experiences are most relevant to their mathematical identity, while asking about experience allowed me to think through their social context, and to connect with their world (see section 3.1.3). This question aimed to create space for participants to construct their own meaning about their experiences (Creswell, 2009, p. 8), which might not have been afforded to them by more targeted data collection methods.

Question 2: *Now think carefully about all stages of your mathematical journey from primary school to university mathematics. Consider: Your feelings or attitudes to mathematics, influential people, critical incidents or events, specific mathematical content or topics, how mathematics compares to other subjects, why you chose to study a course which includes mathematics at third level. With these and other thoughts in mind, describe some further features of your relationship with mathematics over time.*

While the first question focused on past and present experiences which inform mathematical identity, the final prompt in question two invited consideration of the future by asking why participants chose to study a programme ("course") which includes mathematics. Thus, the autobiographical nature of the questionnaire invited participants to construct coherency between their past, present, and future selves (McAdams, 2006, pp. 98-99), both implicitly, as discussed in section 3.1.3, and explicitly, through the prompts in question two. Hauk (2005) recommends such a "deliberate search of [autobiographical] memory" (p. 38) as a means to put ourselves back into the shoes of our past selves, in order to think about mathematical experiences. This attends to the vital temporal idea that individuals are not isolated in a moment (Clandinin & Connelly, 2000, p. 29), and should not be studied as such (see section 2.5).

Question 3: *What insight, if any, have you gained about your own attitude to mathematics and studying the subject as a result of completing the questionnaire?*

The final question provided a deliberate, reflective component for participants whose responses to questions one and two may have been more descriptive than reflective. Focusing on self-reflection has proved fruitful in previous mathematical identity research (see section 2.6).

4.1.2 Recruitment of Participants for the Pilot Questionnaire

Second year science and engineering students were selected as the sample for the pilot study. Since the main data collection involved first year students, the pilot questionnaire could safely be distributed amongst their second year counterparts without tainting the upcoming main data collection. Although the participants had entered the second year of their university education, their transition to university was ongoing (Tett et al., 2017) and thus, the questionnaire was relevant and appropriate for this sample. Of the three groupings discussed in section 3.2, the Science I grouping was represented in the pilot, with the exception of Data Science (DS) which, at the time, was a new programme without a second year cohort (see appendix 3.2.1 for the list of programmes). The engineering grouping was also represented. Unfortunately, the Science II grouping could not be represented as those programmes do not take any mathematics modules in the second semester of second year. Since most of the science cohorts were represented, and the aims of the pilot were predominately focused on logistics, this was not considered to be problematic. I used a process of recruitment which was similar to that used in Data1 (see section 3.2.1) and resulted in 18 respondents to the questionnaire: 10 Science I students and 8 Engineering students.

4.1.3 Insights from Pilot Questionnaire

One of the aims of the pilot study was to trial and refine the codebook for thematic analysis, which is detailed in section 5.1.3. I determined that some changes to the demographic questions (see appendix E.2) were required for clarity. In QD4, the word "level" caused participants to refer to the National Framework of Qualifications (Quality and Qualifications Ireland [QQI], n.d.), rather than to name their actual qualification (e.g., undergraduate or further education programme) or their level of mathematics taken at Leaving Certificate (ordinary- or higher-level).

I made the question more specific by giving participants four possible options: Leaving Certificate higher-level, Leaving Certificate ordinary-level, A-levels and Other. The “Other” option triggered a text box where participants could input their post-primary level mathematics examination. In the pilot, one participant reported previously studying a different undergraduate programme, but two older participants did not mention any previous undergraduate degrees or programmes of further education. To encourage future participants to share these details, I added QD6 which asks, “Have you previously completed another undergraduate degree or further education course?”

In the pilot study, no concerns were raised about the phrasing or clarity of the questions. However, an opportunity arose to better prepare participants for the type of questions they would be asked. Contrary to the intentions of the researcher, some participants in the pilot questionnaire reported that they had expected the questions to be more “specific”, “direct,” or “to the point,” as well as less “vague,” possibly involving “multiple choice questions” rather than “paragraph style answers.” This indicated that I had not adequately prepared these participants for the open-ended and autobiographical style of questions they would encounter in the questionnaire. To remedy this, I selected appropriate terminology and created a recruitment script for myself (see appendix C.5) to present the questionnaire to potential participants in a way that would prepare them for the style of the questions. In the script, I used the term “questionnaire” rather than “survey,” and stated that the focus of the questions is on each participant’s “mathematical journey” to prepare them for the open-ended and reflective nature of the questions. In response to questions I received in the pilot study, the meeting script included a description of the value of the study to the researcher, and to the participants.

I tested two different distribution methods for the online questionnaire: SurveyMonkey and the university virtual learning environment. I determined that the best distribution system for the questionnaire was the university virtual learning environment for several reasons. To add a participant, I could search for them on this system, eliminating the possibility of invalid email addresses being provided, of misreading the email addresses from the completed ICFs, or emails to participants being filtered as spam. Furthermore, students were familiar with the platform from accessing the online content for each of their modules.

4.2 Data1 - Questionnaire

There were 32 participants in this stage of data collection. As mentioned in section 3.2.3, the purpose of Data1 was to establish the mathematical identity of science and engineering students and to identify the similarities/differences in mathematical identity between the groups (RQ1 and RQ2). The questionnaire (see appendix E.2) consisted of six demographic questions and three open-ended questions about participants' experiences with mathematics, which are discussed in section 4.1.1. The questionnaire was administered online through the university virtual learning environment, with no time limit, and with each of the open-ended questions appearing on a separate page. The save feature was activated so that participants could view the questions, or start composing their responses, but had the option to save their response and complete it later.

4.3 Data2 - Focus Groups

There were six participants in this stage of data collection (see section 3.2.1 for their details). As discussed in section 3.2.4, the purpose of Data2 was to elaborate on the themes from Data1, to verify some researcher interpretations, and to examine the changes in participants' mathematical identity since they completed the Data1 questionnaire.

4.3.1 Questions and Procedure

Opening ice-breaker questions are recommended at the outset of a focus group, because they stimulate conversation and give each participant a chance to speak (Creswell, 2009, p. 183). In section 5.1.10, I argue that participants' personal feelings about mathematics connected to lots of other parts of the thematic map, and thus served as an effective jumping-off point, from which to begin the focus group discussion. I choose a range of quotations from the personal feelings category (see appendix F), to invite participants to consider the emotions that frame their overall experience of mathematics, in preparation for the rest of the discussion.

The final list of questions comprised of the icebreaker question, five key questions (with follow-up or sub-questions), and a closing question. The key questions (see appendix F) were developed with reference to two sources: a quantitative analysis of the important codes in Data1, and a qualitative analysis of the themes. For the quantitative analysis, I used the matrix query function in NVivo to compare the responses to Q1 and Q2 from the Data1 questionnaire, in terms of the number of codes applied, and the number of participants who mentioned each one. For the qualitative analysis, I referred back to stage 6 of the analysis of Data1 (see section 5.1.8), which investigated whether the themes "cohere together meaningfully" (Braun & Clarke, 2006, p. 91), as well as the connections and distinctions between themes (p. 91). I also decided to incorporate the wording participants used in Data1 into the focus group questions where possible, in order to probe what participants meant by certain terms or ideas. Specific terminology I decided to utilise included that students felt "unprepared" at various transition points (post-primary school, Junior Cycle, Senior Cycle, university), that "thinking outside the box" is important when doing mathematics, and that mathematics was characterised as "interlocking," with each topic "intrinsically linked."

The key questions were supplemented with several other prompts and potential question variants, as well as some general directions for moderating the discussion, and the kinds of responses to include or avoid because "the moderator takes and active role in controlling not only the topic but also the group's dynamics" (Hydén & Bülow, 2003, p. 307). To aid my decision making during the focus groups, I used a diagram to track the speakers and themes as they arose, as well as a grid to track which themes and aims had been addressed (see Figure F.1 and Table F.3). The key questions were not asked in order in either focus group. Flexibility in this regard was important because the same question could inspire responses relating to different themes in each focus group, depending on the trajectory of the discussion and the participants, among other factors.

4.4 Pilot Interview

As was noted in section 3.2.5, the main purpose of the pilot interview was to evaluate whether longitudinal change in mathematical identity could be better assessed through interviews rather than through focus groups. The pilot provided an important opportunity to develop a narrative interview process for Data3 that was consistent with the aims of the research, and to test potential narrative analytic approaches.

A single pilot interview consisting of three parts was conducted with Saoirse_SC1, who volunteered for the focus groups, but could not attend on the day. In order to explore changes, part I focussed on her university mathematical experience since she had completed the questionnaire. Saoirse_SC1 naturally started by describing her experience in first year (when she completed Data1) but did not proceed year-by-year thereafter. The conversation drifted between university level and post-primary level, often referring to comparisons between the two, and how prepared or unprepared she felt for university as a result of her post-primary school experiences. Part II addressed her earlier experiences and began with the icebreaker question quotes from the focus groups in order to invite an overall consideration of her mathematical identity throughout primary, post-primary, and university level. During part III of the interview, Saoirse_SC1 was asked to read her questionnaire submission, and reflect on her words as she read the response. Since her submission was quite long (651 words), I reasoned that it would be easier for her to comment as she read rather than to summarise at the end. Furthermore, hearing her thought process as she compared her current and former selves could point to changes in mathematical identity over that time, and indicate which parts still resonated most with her.

Saoirse_SC1 noted that she did not remember much of what she had written in the questionnaire. This was highlighted when she was asked about their Data1 response which said "when I entered senior cycle my relationship with maths completely changed. I went from being bored by how easy maths was to really struggling with it. I failed every maths exam I did in 5th year and 6th year" (Saoirse_SC1, Data1). When she was asked to characterise how things had changed, several follow-up questions were required to jog her memory. Even then, she accepted that it was believable that she once thought this, but she did not reiterate her interpretation of the change that she mentioned previously:

FH: You did say in your questionnaire that your relationship with maths completely changed in fifth and sixth class.

ID069: Did it?

FH: Well, you mightn't remember it, that's fine.

ID069: I'm trying to think what would have changed.

FH: I think you, you mentioned that you started struggling a bit more with maths after Junior Cert.

ID069: Oh, I did yeah, yeah. I see, I can see that yeah.

It is possible that the change to which she referred had become a less important feature of her mathematical identity since then, or alternatively, that she had simply forgotten what she had included in her questionnaire response. Therefore, I concluded that it would be necessary to present future interview participants with their Data1 responses, rather than relying on their memory of their submission. In fact, several participants in Data3 noted that they did not remember their Data1 response (see appendix H.9 for more details).

4.5 Data3 - Interviews

There were six participants in this stage of data collection (see section 3.2.1 for their details), who participated in narrative interviews. As discussed in section 3.2.5, the purpose of Data3 was predominantly to examine, and compare, the changes that participants felt had occurred in their mathematical identity in the three years since they completed the questionnaire. Each interview consisted of four parts, which are shown in Table 4.1, and was conducted online via the Zoom platform. Parts A and B were designed to evoke participants' narratives about their mathematical experiences in DCU, while parts C and D were intended to encourage participants to compare two stages of their journey and reflect on how they progressed from then to now.

Table 4.1
Schedule for the narrative interviews (Data3) which were estimated to last 40 minutes.

Interview part	Description	Time estimate
Part A	Discussion of participants' university experiences with mathematics year-by-year.	15 minutes
Part B	Co-construction of a key events list and graph.	10 minutes
Part C	Reading of questionnaire response from Data1 followed by clarification/elaboration.	10 minutes
Part D	Comparing participants' current mathematical identity with their former self from Data1.	5 minutes

The interviews were expected to last 40 minutes, and the questions and procedure used in each part will be described next.

4.5.1 Questions and Procedure

The interview schedule (see appendix H.1) consisted of a description of the interview parts (to be communicated to the interviewee), a set of general questions for all participants, and specific questions for each participant. Specific questions were informed by three types of analysis of participants' responses to the questionnaire (and focus groups if applicable): broad reading of the participant's response(s) to identify important issues raised, and possible key events; quantitative analysis of coding to indicate areas of focus in their response(s); and quantitative analysis of Data1 coding to identify codes and themes which were prominent in other participants' responses, but the interview participant did not address.

In part A, I requested that each participant begin with first year, and tell me about their mathematical experiences year-by-year. I reasoned that this would provide an opportunity to explore changes in mathematical identity over time (RQ3) and help reveal some key events for part B of the interview. As noted in section 2.5, analyses using transition points benefit from a participant telling a full sequential story of their mathematical experience, which did not occur in the pilot interview (see section 4.4). In Data2, it was revealed that PSTs have the option of speaking from multiple perspectives of mathematics experience: as a student, as a trainee teacher, and as a practising teacher (see section 5.2). For these participants, I followed Drake (2006), Drake et al. (2001), and McCulloch et al. (2013), and delineated the doing, teaching, and learning of mathematics more so than in Data2, by including questions involving all three contexts. For other participants, their professional context might involve internships, work placement, and/or laboratory work, depending on their programme, so questions about these contexts were included also.

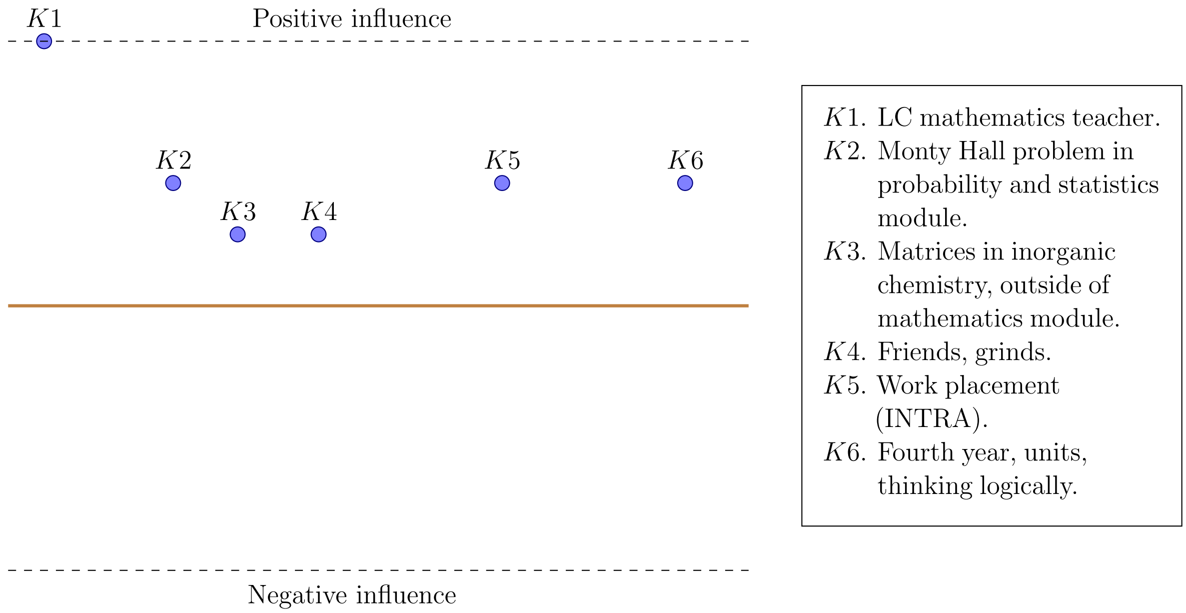
For part B, I developed a co-constructive process for creating a *key events graph* to further stimulate the discussion about participants' mathematical journeys. By using graphs, I could capitalise on the students familiarity with these objects as a mathematical concept, including turning points and slopes, while also using visual representations of qualitative data to understand their journey. This approach was inspired by the work of Drake et al. (2001) and McCulloch et al. (2013), whose terminology for narrative arcs (roller coaster, turning point) evokes images of mathematical graphs, such as those recommended for use in holistic analysis of narratives by Lieblich et al. (1998, pp. 94-95).

To create the key events graph, interviewees were asked to participate in several steps of construction, which are listed in the interview questions in appendix H.1. Firstly, I asked them about their suggestions for key events as follows: “You’ve spent the first part of this interview telling me your story about your time in DCU, and particularly your relationship with mathematics in that period. What are the key events that influenced this story?” I noted their suggestions, compared them to the key events that I had identified during our conversation, and, if there were more possibilities, I asked the participant if they would like to include any of my suggestions. Once a list of key events was finalised, I shared my screen, and used mathematical software called GeoGebra to represent the events as points which were spaced along the centre line of the graph in chronological order (the brown line in Figure 4.1). Next, I informed participants that the horizontal axis represents time, the vertical axis has an upper and a lower bound, and I asked what they thought the vertical axis might represent. This placed the decision about the meaning of this axis squarely in the hands of the participant, my co-researcher, and empowered them to use their own appropriate terminology to best describe the difference between the key events they identified in part A. Once the participant had chosen a label for the upper and lower bounds, I asked them to describe where to place each key event vertically, and I adjusted the graph until they were satisfied with the positioning. Finally, when the graph was completed, I asked each participant for their thoughts about the representativeness of key events graph with regard to their journey as follows: “This graph is one way to represent your journey in terms of key events. Knowing what you know about your own journey, have you any thoughts about how to interpret this graph?”

An example of a key events graph is shown in Figure 4.1, where the key events are organised chronologically, and the vertical axis represents positive and negative influence, as suggested by the participant. In this case, Aodhán_SC2 considered all the key events to have had a positive influence, but to different degrees.

Figure 4.1

Key events graph for Aodhán_SC2.



The key events graph was created in line with a recommendation of Osborn (2005), that diagrams should be composed of simple elements that leave enough room for interpretation to represent complex trajectories (p. 16). As can be seen in Figure 4.1, the spaces between the points were intentionally left blank, with no lines or arcs connecting them, thus avoiding the risk of misrepresenting segments of the participants' journeys that have already been communicated in narrative form in part A. This space represents a portion of the participant's narrative whose nature is potentially too complex to be depicted by the graph, and allows scope for their mathematical identities to fluctuate in between the outline provided by the key events.

In part C, the participant read their questionnaire response, which was submitted three years previously, and was invited to clarify, or elaborate, on their submission as they read it. This part was informed by the success of the same process in the pilot interview (see section 4.4). Lastly, in part D, I used the icebreaker question quotes from the focus groups (see appendix F) to bring the participant back to an overall consideration of their mathematical identity, from primary school up to university. For this part, the interview schedule (appendix H.1) also included some prompts to encourage participants to compare their current mathematical identity (as explicated in the interview), with their questionnaire response. A full evaluation of the narrative interview process was performed after data analysis and is included in appendix H.9.

Chapter 5

Approach to Data Analysis

This chapter details the analytic approach to each of the three stages of data collection conducted in MISE. Firstly, reflexive thematic analysis of the questionnaire data is presented in section 5.1, along with an additional separate analysis of a category of codes that required an alternate approach (section 5.1.10). Secondly, the transcription conventions and more deductive thematic analysis of the focus group data are presented in section 5.2. Finally, the narrative analytic approach to the interview data is explained in section 5.3.

5.1 Data1 - Questionnaire

The advantages of thematic analysis will be described first in this section, followed by the particular approach taken in this study. In conducting thematic analysis, it is important to characterise what is meant by a code and a theme, whether and how the researcher's subjectivities play a role in the interpretive process, and to engage with the "theoretical and philosophical assumptions that underlie procedures" (Braun & Clarke, 2021a, p. 329). These details will be discussed next, with connections to the philosophy laid out in chapter 3, after which the stages for thematic analysis will be presented. The names of **themes** and **sub-themes** are presented in bold font, and *codes* are presented in italics, to clarify that the terms refer to these items rather than concepts in mathematics education. In coding examples, the codes are included in square brackets to indicate where they were applied. The full list of themes, sub-themes, and codes is given in appendix G.5, and the codebook is included in appendix G.2.

Advantages of Thematic Analysis

Thematic analysis is frequently used as an effective tool for analysing narratives (Reissman, 2008, p. 53), including in research on identity (Lutovac & Kaasila, 2019, Table 3). In qualitative research, analyses that are essentially thematic are often not identified as such (Vaismoradi et al., 2013, p. 400), meaning that this type of analysis may be more prevalent than it appears (Braun & Clarke, 2006, p. 101). Thematic analysis allowed me to take a systematic approach to data analysis (Braun & Clarke, 2022, p. 27), but also provided important flexibility in deconstructing a rich dataset (see section 3.2.6) into manageable codes, which could then be used to construct theme narratives that convey the subtle and complex elements of participants' experiences (Joffe & Yardley, 2004, p. 57). It has been highlighted that thematic analysis can be used without restriction to specific theoretical assumptions, but that it is not atheoretical in its implementation (Braun & Clarke, 2021a, p. 337). Thematic analysis is a particularly appropriate analytic method for constructivist researchers who seek to generate patterns of meaning or theories from data (Creswell, 2009, p. 8), because the method is used to identify, analyse, and report patterns in rich detail (Braun & Clarke, 2006, p. 79).

5.1.1 Manner of Coding and Theme Development

The early work of Braun and Clarke (2006) brought some clarity to the process of thematic analysis, and through recent publications, the authors have distinguished between three varieties of thematic analysis based on their philosophical assumptions: reflexive, coding reliability, and codebook thematic analysis (Braun & Clarke, 2022, p. 235). I chose reflexive thematic analysis as the particular variety of thematic analysis to employ in this study. Under this approach, codes are analytic units thought to represent one idea, or one facet, of the data. The outcomes of the coding and analytic processes are themes, which are multi-faceted and exist at a broader, more abstract level than codes (Braun & Clarke, 2021b, p. 208; Braun & Clarke, 2021a, p. 342).

In their seminal paper, Braun and Clarke (2006) proposed several decisions about coding and themes in thematic analysis that should be made explicit by those who utilise the method: semantic or latent themes, realist or constructionist epistemology, rich description of the dataset or detailed account of a particular aspect, and inductive or deductive coding. These considerations are discussed, in turn, below, where I also rely on the work of Lieblich et al. (1998) to help fully describe my approach to coding and themes. Braun and Clarke (2019) recently clarified that it is possible to mix certain choices in thematic analysis e.g., semantic/latent or inductive/deductive coding (p. 592). Previously, I had highlighted that deductive and inductive coding approaches are mostly presented as dichotomous in the literature (Howard et al., 2019).

Latent Themes and Constructionist Epistemology

In this study, coding was semantic, in that codes denoted surface, obvious, or overt ideas among the data (Braun & Clarke, 2021a, p. 332). Themes, on the other hand, were latent, because they drew together data that seemed unrelated on the surface, into patterns of meaning (not just a common topic) that cohered around a central concept (p. 341). Semantic themes give descriptions of occurrences, whereas latent themes interpret the meaning of what was said, answering the question "What do these themes tell us?" (Boyatzis, 1998, p. 23). Thus, the analytic process employed in this study began with describing and organising the data using codes and progressing to interpret and theorise the "broader meanings and implications" of the themes (Braun & Clarke, 2006, p. 84). The themes were created with regard to a constructionist epistemology, under which a semantic approach to coding facilitates a focus on the social contexts and conditions that bound the participants' responses, rather than their individual psychologies (Braun & Clarke, 2006, p. 85). In reflective thematic analysis, processes are not seen as prescriptive, mechanical procedures, but as guidelines which are applied under the active influence of the researcher (Braun & Clarke, 2021a, p. 343). In particular, themes are not discovered but are "*actively* created by the researcher at the intersection of data, analytic process and subjectivity" (Braun & Clarke, 2019, p. 594). The researcher's subjectivity in identifying patterns and determining how to report them, is embraced as a resource for the generation of knowledge (Braun & Clarke, 2021a, p. 334), and therefore, themes are positioned as stories researchers tell about their data (p. 341).

Holistic Approach with Attention to Content and Form

Lieblich et al. (1998) proposed that approaches to analysing narrative materials be differentiated with regard to whether they are holistic or categorical, and whether they concentrate on the form or content of narratives (p. 12). With regard to the autobiographical data collected through the questionnaire, this particular thematic analysis took a *holistic* approach (Lieblich et al., 1998, p. 13), meaning that extracts were analysed in the context of the entire narrative (Reissman, 2008, p. 12; Kaasila, 2007b, p. 207; Vaismoradi et al., 2013, p. 399), and connections between the themes were acknowledged and analysed (Maxwell, 2013, p. 108). Data analysis was conducted with consideration for both *content* (what is said) and *form* (how it is said) (Lieblich et al., 1998, pp. 12-13). The content of a narrative includes to whom each event pertains, what the event entails, and what exactly happened, while the form of a narrative deals with how each event is described (Kaasila, 2007b). Through form, one can assess what motivates an individual to relay a story from their past in a particular way, essentially attempting to discover how a person feels about an event, rather than simply recording the facts of the event itself. Usually in thematic analysis, "primary attention is on what is said rather than how, to whom or for what purpose" (Reissman, 2008, p. 53), but attention to both content and form is advised for garnering a more complete view that may reveal hidden aspects of identity (Kaasila, 2007b, p. 212; Lieblich et al., 1998, p. 13). It is also plausible to view the form of a narrative, or part thereof, to be an "embodiment of its content," meaning that it is not always reasonable, or possible, to separate content from the way it is conveyed (Lieblich et al., 1998, p. 14). The written questionnaire data was analysed for several of the elements of form mentioned by Lieblich et al. (1998, p. 13): the sequencing/timing of events, the choice of words, and turning points.

A rich description of the whole dataset was sought in order to investigate whether certain themes generated a surprising narrative or were more prominent. This meant that themes may be proliferous across the data or more rare, but "more instances do not necessarily mean the theme itself is more crucial" (Braun & Clarke, 2006, p. 82). Instead, each theme should capture patterns of meaning in the data that are relevant to the research questions (Castleberry & Nolen, 2018, p. 812; Braun & Clarke, 2022, p. 90). Although this is a qualitative study, some quantitative measures played a role in the analysis, particularly in analysing the affective codes in Data1 (see section 5.1.10). I used these as indicators to areas where more meaning may be revealed using a qualitative lens. For example, the number of instances of a code was an indicator of, but not evidence for, extracts of meaning or importance, since such simple counts "[do] not take into account the nature of the responses nor the strength of feeling indicated" (Eaton, McCluskey, et al., 2011, p. 11). Incorporating quantitative data in a qualitative study is methodologically consistent, provided that positivist notions of objectivity, validity, and generalisability are not attributed to quantitative findings (Crotty, 1998, pp. 41).

Hybrid Inductive and Deductive Analysis

Thematic analysis can be based on codes generated from the data in two ways: Inductive (data-driven) thematic analysis is used where there are no previous studies dealing with the phenomenon, such as in grounded theory research (Braun & Clarke, 2006, p. 83), while deductive (theoretical) thematic analysis is used to test a previous theory in a different situation, or to compare categories at different periods (Vaismoradi et al., 2013, p. 401). It has been reported that research in identity has tended to focus on deductive rather than inductive coding strategies for analysis (Lutovac & Kaasila, 2019, p. 510). To analyse the open-ended responses to the online questionnaire, I used a hybrid process of inductive and deductive coding adapted predominantly from Fereday and Muir-Cochrane (2006), with influence from Braun and Clarke (2006), Boyatzis (1998), and Crabtree and Miller (1999) (see appendix G.1 for a comparison of the stages used in each publication). The rationale for this hybrid approach was to incorporate the knowledge gained from previous work on mathematical identity in Ireland, while also grounding the study in the new data I had collected. Braun and Clarke (2006, p. 90) recommend the use of visual representations for organising codes into themes and developing a thematic map. I chose NVivo to conduct thematic analysis, in part because it included a visual feature that is helpful for organising codes and themes (Castleberry & Nolen, 2018, p. 809). In NVivo, the surrounding context of any extract can be accessed with a single click, and this facilitated the researcher in making interpretive decisions that are consistent with a holistic approach.

5.1.2 Analytic Process

When conducting thematic analysis, meaning is not established at a particular stage but gradually as the analysis is cycled through its stages, since the process involves "a constant moving back and forward between the entire dataset, the coded extracts of data that you are analysing, and the analysis of the data that you are producing" (Braun & Clarke, 2006, p. 86). This aligns with the view of saturation (and of meaning) as a process that occurs incrementally (Saunders et al., 2018, p. 1901; Strauss & Corbin, 1998, p. 136). In reflexive thematic analysis, quantitative measures that reference codes, such as inter-coder reliability, should not be used to demonstrate the rigorous development of themes, as they clash with the constructionist approach described earlier in this section (Braun & Clarke, 2021a, p. 334), and may in fact over-simplify the resulting coding, analysis, and insights (Castleberry & Nolen, 2018, p. 811). Instead, in line with the criteria given in section 3.2.6, rigour was established by using a seven-stage analytic approach as shown in Table 5.1.

Table 5.1
The seven-stage approach to thematic analysis taken in MISE.

Stage	Description	Location in thesis
Stage 1	Development of a codebook from a literature review of MIST/MINT, and from the pilot study	section 5.1.3
Stage 2	Coding of a stratified sample of the main data and detailing the coding principles for analysis	section 5.1.4
Stage 3	Broad reading and summarising of entire dataset	section 5.1.5
Stage 4	Coding data using inductive and deductive codes	section 5.1.6
Stage 5	Grouping codes to develop thematic map	section 5.1.7
Stage 6	Reviewing themes and using the data to check for coherency and representativity	section 5.1.8
Stage 7	Defining and naming themes	section 5.1.9

An iterative coding process was used to cycle through stages 4-5, in which the researcher moved up and down through two layers of thematic analysis, considering the data in the context of both codes and themes respectively (Braun & Clarke, 2021b, p. 210). Stage 5 included a review of coded extracts that did not fit with the tentative themes or needed to be reconsidered in order to develop a more representative thematic map. If changes were needed, then stages 4, and 5 were reiterated, possibly several more times, before the analysis proceeded to the interpretive phase in stage 6 (Fereday & Muir-Cochrane, 2006, p. 90). Stage 6 included a search for data within themes that required additional codes. These may have been missed earlier, or they may have become more evident after the broad, but tentative, themes were developed, and the data extracts had been further dissected. I considered whether the theme itself was founded on too little data, needed to be separated or combined, or whether some of the data did not fit in the theme. I present a formal account of these steps in section 5.1.8. The iterative process starts again at this point, and ceases "when your refinements are not adding anything substantial" (Braun & Clarke, 2006, p. 92). Such a process was expected to boost inductive thematic coding saturation, since instances of a code are less likely to be overlooked. These cycles of coding were important to allow time and space for insights to manifest, develop, and mature (Braun & Clarke, 2019, p. 332), since the interpretative depth of the themes was dependent on the analyst engaging reflexively with the process (p. 342). The seven stages of the analysis will be described in turn in the subsequent sections.

5.1.3 Stage 1 - Developing a Codebook

Following a similar approach to Fereday and Muir-Cochrane (2006), I used the examples and discussion from five MIST papers, two MINT papers, and two papers from an interim "bridging study" (see section 2.6), to partition the seven themes identified by previous research into 30 codes. The codebook included a small number of *contextual codes* (primary school, post-primary school etc.) that were useful for analysis, but did not need to be included in the thematic map (see "contextual codes" in appendix G.5). I followed the advice of Fereday and Muir-Cochrane (2006, p. 85) and Boyatzis (1998) in naming the codes, providing a description, providing keywords (if possible), and giving at least one example of an instance of each code from the pilot study (Hennink et al., 2017, p. 595). After analysing the pilot questionnaire data, I updated the codebook (see appendix G.2) by adding new codes that arose in the data (+14 codes) and removing codes that did not occur (-7 codes). As summarised in Table 5.2, the resulting codebook consisted of 37 codes, 23 of which came from the literature review, while the remaining 14 were developed from the pilot questionnaire responses.

Table 5.2
Number of updates to the codebook after the pilot questionnaire.

		Source of code	
Stage	Codes	Literature	Pilot
Literature review	30	30	-
Pilot study	37(+7)	23(-7)	14(+14)

5.1.4 Stage 2 - Coding a Stratified Sample of the Data

After establishing a codebook of 37 codes from the literature and pilot study, I followed a recommendation of Fereday and Muir-Cochrane (2006, p. 85) and coded a sample of eight questionnaire participants. The sample was coded by myself and my two supervisors to assess the clarity of the codebook, and was stratified based on grouping, cohort, gender, word count, and age. It should be noted that establishing a quantitative measure of inter-coder reliability was not the purpose of this process (see section 5.1.2), but rather the clarification of the definitions and examples in the codebook. Codes were split, combined, or created as appropriate, changing the size of the codebook to 47 codes. There were 34 deductive codes (21 from the literature review, and 13 from the pilot study) and 13 inductive codes (from the Data1 sample).

Stage 2 informed the inductive coding for the full dataset, as evidence was discovered to suggest that codes should be broad enough to encapsulate a range of responses, especially where a continuum existed, e.g., mathematics is exciting/boring or hard/easy. This evidence presented in multiple ways. Often, participants expressed a combination of both positive and negative, e.g., elements they liked or disliked, and parts they found hard or easy:

> I enjoy certain parts of it (SC2_12).
>
> Certain aspects came easier to me than others (SC1_06).
>
> Leaving a hard topic and starting to learn a harder one makes the first hard topic seem very easy (ENG_06).

They also gave a middle-ground or neutral response:

> Generally ok experience (SC2_09).
>
> I found mathematics okay (SC2_11).

Affective statements may also be more nuanced than a single point along a continuum, and require further explanation e.g., I like mathematics in a certain sense but not every sense: "I don't like Maths as a subject but I like doing it" (SC2_05). Reflexive thematic analysis should take account of data which departs from the main themes (Braun & Clarke, 2006, p. 89), and "consider variation (and even contradiction) in the account that is produced" (p. 95). Thus, codes that represented a continuum of statements were included in the codebook as appropriate.

5.1.5 Stage 3 - Broad Reading of the Full Dataset

I read through the entire dataset for "close contact and familiarity" (Boyatzis, 1998, p. 45), and to make the important, but tentative, first step towards understanding the narratives therein (Braun & Clarke, 2022, p. 197). It is important that the researcher immerses themselves in the data, in order to understand "the depth and breadth of the content" (Braun & Clarke, 2006, p. 87). This immersion requires repeated reading of the data, "taking notes or marking ideas for coding" (p. 87), "searching for meanings and patterns" (Nowell et al., 2017, p. 5), as well as noting the key points made by participants (Fereday & Muir-Cochrane, 2006, p. 86). Following the approach of Nowell et al. (2017, pp. 7-8), I used memos in NVivo to summarise each questionnaire response, added notes on the aforementioned topics, and used these for comparison with coding in the next stage of data analysis.

5.1.6 Stage 4 - Coding the Full Dataset

To enact the principles detailed earlier in section 5.1, I expanded the coding process for stage 4 significantly beyond what was indicated in the literature. This stage consisted of several ways of looking at the data, which were incorporated in the coding process as described in this section. Each of the steps (a)-(e), which are discussed below, added to the exhaustiveness of the coding by illuminating more opportunities for codes to be applied, but were also useful for adding new ideas as notes under each participant's response.

(a) Apply codes using a fine-grained, line-by-line approach.

Step (a) arose because I found that participants' responses were rich, in the sense that they contained a lot of information (and codes), even in short pieces of text:

> I generally did well in maths exams [*I'm (not) good at maths*] compared with other subjects [*Comparing maths with other subjects*], which made me enjoy the subject more [*I like/dislike maths]*, and gave me confidence and a belief that I could do well [*Origin of interest in maths*] in the field [*Confidence in maths*] (SC1_04).

(b) Compare with the codes allocated to the original notes from broad reading of the data.

In step (b), I discovered that almost every initial note in the NVivo memos had been encapsulated in the coding. Of the remainder, most notes were now either obviously unfounded, or involved broader interpretations that remained to be addressed by the themes.

(c) Code each piece of interest by removing chunks of text and re-reading them in isolation, then return to the data to consider them in context.

Although extracts were to be considered in the context of the entire narrative, removing the context allowed me to think about the other ways of interpreting each extract, and interrogate these possibilities by re-introducing the context and reading again. This was an effective strategy for questioning the assumptions that were being made about the data, when read only in a wider context (Braun & Clarke, 2022, p. 198).

(d) To identify possible extra occurrences of each code:
   (i) Pick each code and read through data with that code in mind,
   (ii) Use keywords to search the data.

In step (d), I discovered that searching for keywords often brought up results from my summaries and notes from stage 3. This was useful for identifying, via a paraphrasing, whether the correct codes had been applied. For instance, searching for "pace" brought up my comment "Pace. Quicker (because I was good?)" The original statement did not mention pace: "I would be finished far ahead of everyone in my class" (Saoirse_SC1). The keywords identified for each code are listed in the codebook (see appendix G.2).

(e) Interrogate data coded as miscellaneous to determine if re-categorisation is possible.

Braun and Clarke (2006) recommend including a miscellaneous theme to "house the codes — possibly temporarily — that do not seem to fit into your main themes" (p. 90). From the start, I used an extra miscellaneous code[1] and theme, for items which did not fit any existing category. These were used to check the comprehensiveness of the codebook, and to ensure that no evidence was discounted during the coding, or analysis, process simply because it was difficult to categorise (Crabtree & Miller, 1999, p. 171).

Updating the Codebook

During the analysis of the full dataset for Data1, 20 new inductive codes were added based on the data, bringing the total number of data-driven codes to 33. The origin of each code is presented in appendix G.3. The evolution of the codebook throughout the research is shown in Table 5.3.

[1]The miscellaneous code was given number 79 to distinguish it from the other codes in the codebook.

Table 5.3

Number of updates to the codebook after inductive codes from Data1 were included.

		Source of code			
Stage	Codes	Literature	Pilot	Data1 sample	Data1
Literature review	30	30	-	-	-
Pilot study	37	23	14	-	-
Data1 sample	47	21	13	13	-
Data1	67	21	13	13	20

5.1.7 Stage 5 - Developing a Thematic Map

This is a broader, interpretive level of analysis, where the raw data does not feature. Thematic analysis is a form of pattern recognition (Crabtree & Miller, 1999, p. 82), so codes were grouped by starting with the first code and searching through the list for codes that fit a common narrative, which may develop into a theme. Codes with no instances in the data were not included in the thematic map, but they were not discarded just yet either, since further iterations of coding might require their inclusion. The inductive codes were included first, so that the thematic map could be built up from the new ideas expressed by these participants. Each cycle of this stage concluded with a list of changes to be made to codes (splitting, deleting, merging) before moving on to reinterpret the tentative themes (Braun & Clarke, 2021b, p. 207). Codes that developed as a result of splitting, or creating, codes during the analysis, were given the suffix ‘a’ e.g., 42a.

Deductive codes were added to the thematic map in groups of three or four by reviewing the content of each code, and forming some connections in the thematic map with other codes e.g., *I need it explained to me* and *I took charge* were connected because some participants said that when the teacher was bad at explaining, they took the initiative to find another source from which to get better explanations. Each connection in the map was assigned a brief phrase to explain the connection, while the memos in NVivo catalogued each connection in more detail. It was both necessary, and beneficial, to return to the data and re-read the content of codes when determining these connections (cycling through stages 4-5), rather than relying on the code names and definitions alone. Due to the interconnected nature of the codes, it was not possible to partition them into themes without any connections between the themes themselves. The aim was to cluster well-connected parts of the thematic map to minimise (rather than eliminate) the connections between themes, as was done by Maguire and Delahunt (2017, p. 3356). To this end, codes were included in the most sensible potential theme, even if connections still existed to other themes, and the miscellaneous theme was used to store codes temporarily before they were placed elsewhere. I reasoned that it would be easier to test the coherency of the themes in stage 6 if codes were included in the most sensible theme, instead of waiting for the included codes to produce a narrative that might not fit the miscellaneous codes.

5.1.8 Stage 6 - Reviewing Themes

At this stage, a narrative was created for each theme since they were now considered more stable, although still tentative. The stability of each theme was assessed by investigating the *coherency* of the narrative within each theme, the coherency of the distinctions between the themes, and the *representativity* of each theme.[2] A coherent narrative ensures that each theme has a meaningful, clear "take-home message" (Braun & Clarke, 2022, p. 197), which is built around a "strong core concept" (p. 36). There must also be "clear and identifiable distinctions between themes" (Braun & Clarke, 2006, p. 91), and any connections between them should be justified. Lastly, the themes should be representative of the data, which means that the thematic map should reflect the "meanings evident in the data set as a whole?" (p. 91), without any parts being omitted or over-emphasised. For a theme that does not exhibit coherency or representativity, I found it helpful to identify the problem as one of the following:

P1. Is the theme itself problematic?
   (a) Does the theme name not represent the narrative of the extracts within and need to be renamed?
   (b) Does the theme name (or map) say one thing and the extracts say something different? (This is a more extreme case of a.)

P2. Do some of the extracts simply not fit the theme? Do they belong somewhere else?

For example, after the first cycle of coding was complete, the theme name **Maths gets harder as you progress** was too narrow to cover the narrative of all the extracts within the theme

[2] In the past, Braun and Clarke (2006, p. 91) used the terms "internal homogeneity" (coherency within a theme) and "external heterogeneity" (coherency between themes) along with representativity, but this does not feature in their more recent publications.

(P1(a)), and thus was renamed **Transitions and realisations/reflections** in order to be more representative. The theme **Maths is a means to an end** had a coherent narrative but much of this narrative involved participants' fondness for mathematics as their reason for choosing mathematics-heavy programmes in university (P1(b)). Thus, it was renamed **Reasons for studying mathematics**, and included a sub-theme called **Maths is a means to an end**. The theme **Mindset and getting started** included extracts that did not fit the theme (P2). The code *Right and wrong* had been included because some participants expressed concerns about starting problems for which there was no single prescribed, or previously demonstrated, route to a correct answer (Schoenfeld, 1989, p. 88). However, most extracts in *Right and wrong* did not mention this link, and instead focussed on the objectivity of mathematics. Thus, this code was moved to **What is mathematics?** and the link between non-routine problems and *Getting started* was noted.

A list of the changes to codes and themes after the first cycle of coding is included in appendix G.4. After changes were enacted, the analysis returned to stage 4 and the iterative cycle recommenced. The analysis concluded with a total of five themes, and two categories which fell short of the requirements to be considered a theme, to be described in stage 7.

5.1.9 Stage 7 - Defining and Naming Themes

This stage involved defining and naming the themes, in a manner that "captures the essence of the theme and engages the reader" (Braun & Clarke, 2022, p. 112). This stage began only when a satisfactory thematic map has been developed, and no further changes to the coding or themes were necessary in stage 6. The narratives for the themes (see chapter 6) were used to identify what is interesting about each one, and why, justifying these assertions with reference to the data, since "[i]t is important that by the end of this phase you can clearly define what your themes are and what they are not" (Braun & Clarke, 2006, p. 92).

5.1.10 Analysing Participants' Personal Feelings about Mathematics

When I reviewed the potential theme **Personal Feelings about Mathematics** (see appendix G.5) for coherency within its narrative (stage 6), it was rejected as a theme because the narrative was disjointed and did not cohere around a central concept (Braun & Clarke, 2022, p. 36). Although these *affective codes* were reasonably grouped together under this heading, cycling through the stages of thematic analysis helped reveal several potential explanations for the difficulties experienced by the researcher in combining the extracts into a coherent theme narrative. Several affective codes were coded on a continuum (see section 5.1.4) and involved contributions from all 32 participants when considered together for the purposes of thematic analysis, which means the affective codes contained a wide range of diverse and contrasting responses. Furthermore, when the affective codes were positioned in the thematic map, and tentative connections to other themes were considered, it was discovered that these codes exhibited a great deal of connections to the rest of the map, since they contained feelings and emotions which could pertain to any part of participants' mathematical identities. Thus, the analytic stages that proved fruitful for the development of other themes, were not sufficient for distilling the multiplicity of connections between participants' personal feelings about mathematics and the other themes/codes.

Analysing the Connections

Since the connections were so numerous, and echoed by multiple participants, I began to consider the **Personal Feelings about Mathematics** codes as a superstructure layer over the thematic map and sought a means of deciphering the connections between this layer and the other themes. Inspired by research in attitude conducted by Di Martino and Zan (2010), I also reconsidered whether it was sensible to isolate these affective codes from the rest of the map and search for a coherent narrative given their connectedness to, and reliance on, other themes. Indeed, descriptions of one's relationship with mathematics are usually viewed in tandem with the core narratives of experience (Di Martino & Zan, 2010, p. 37) and not based on emotion alone.

Di Martino and Zan (2010, p. 29) proposed that considering positive/negative emotional disposition (statements about liking or disliking mathematics) provides a simplified manner of analysing students relationship with mathematics, which is particularly relevant for written autobiographical narratives, such as those collected in Data1 in MISE. The authors noted that affective statements are often used in essays to introduce or frame students' descriptions of their mathematical identity, indicating they have some elevated role in packaging or bounding the narrative as a whole (p. 36). It should be noted that although research in affect has drawn on emotions in the context of problem solving (Goldin et al., 2016, p. 2), for example, the perspective offered by Di Martino and Zan (2010) focuses on the role of emotions in declaring the direction of students' relationship with, and choices around, mathematics, without making implications about their behaviour in any specific mathematical activity (p. 29). Hannula (2011) similarly suggests that emotional disposition can be thought of as a more stable affective trait, which is separate to the fluctuating emotional state that may change rapidly in the process of problem solving (p. 45). Thus, the affective codes may play an important interpretive role in participants' questionnaire responses, without which, the other themes could be misrepresented. This is especially important given the holistic approach to thematic analysis employed in this study where extracts should be read in the context of the entire narrative (Lieblich et al., 1998, p. 13).

In their study, Di Martino and Zan (2010) identified emotional disposition (like/dislike mathematics) and several other emotional dimensions that students used to describe their relationship with mathematics in written essays: like/dislike, hate, love, fear, and anger. As shown in Table 5.4, these were matched to three MISE codes: *I (dis)like or enjoy maths*, *Maths is scary or daunting*, and *Maths is stressful, causes anxiety*. Two further codes captured some other emotional dimensions that were present in the MISE data, which will be discussed below.

Table 5.4
Emotional dimensions of mathematical identity used by Di Martino and Zan (2010) compared with the affective codes from MISE.

Emotional dimension	**MISE code**
Like/dislike, love/hate	[40] - *I (dis)like or enjoy mathematics*
Fear/anger	[53] - *I find Maths scary or daunting*
	[54] - *I find maths stressful, causes anxiety*
Other emotions	[42a] - *I find maths frustrating*
	[43] - *I find maths interesting/boring*

Investigating the Causes

Di Martino and Zan (2010) found that participants' references to liking or disliking mathematics (emotional disposition) could be attributed to three causes: the teacher, vision of mathematics, and perceived competence (p. 39). Two of these causes were already represented in the analysis of the MISE data through the **Teachers/Lecturers** theme, and through the codes [39] - *I find maths hard/easy* and [41] - *I'm (not) good at maths* which represented perceived competence. Thus, some of the codes in **Personal Feelings about Mathematics** functioned as emotions, and others as causes for emotions. This distinction is discussed below, and a diagram of the causes is presented in appendix G.6.

For example, participants sometimes used the word "challenging" to mean that they found mathematics difficult, which falls under code [39] - *I find maths hard/easy*. The code *I find maths frustrating/challenging/rewarding* was split into two codes, with code [42] containing extracts from participants who relished the challenge that was offered by mathematics, while code [42a] included participants who expressed frustration with mathematics. The coding query function in NVivo was used to identify 19 participants who connected their like/dislike of mathematics (code [40]) to the difficulty of the subject (code [39]) or to their self-perceived ability in mathematics (code [41]). Extracts that were identified using coding queries sometimes indicated a causal connection between codes, and other times were separate expressions. In the quotations below, the extract from SC2_04 was interpreted as I like mathematics because it is challenging/rewarding, whereas the extract from Dónal_SC1 was interpreted as I like mathematics and I also find it frustrating:

> I enjoy doing maths [40] as it is challenging and when you get an answer correct it gives you a great feeling of accomplishment [42] (SC2_04).
>
> I like maths [40], but sometimes it just frustrates me [42a] (Dónal_SC1).

In the first case, the sense of accomplishment was presented as a cause for the participant's emotional disposition, whereas in the second case, the participant presented a positive and a negative side to their emotional disposition without attributing a cause for either.

The examples above highlight the variation in sentiment that was captured by coding on a continuum (dislike, neutral, like) which, as was noted previously, created difficulty in finding a coherent narrative among the affective codes. The same examples also demonstrate the inadequacy of quantitative coding queries for identifying a causal link between codes, since the qualitative data itself must be investigated and interpreted by the researcher to make this determination. On the other hand, the targeted nature of such queries aided the investigation of potential causal connections between participants' emotional disposition and the other themes that resulted from the thematic analysis. The results of this analysis, which are presented in section 6.1.1, included two further causes for the emotions expressed by MISE participants which did not feature in the analysis conducted by Di Martino and Zan (2010).

5.2 Data2 - Focus Groups

Determining a transcription convention is important because this process is preparation for, if not part of, the analysis process (Saldana, 2011, p. 44). Oliver et al. (2005) proposed a continuum of transcription with two poles: *naturalist transcription*, which includes every aspect of content (what was said) and form (how it was said), and *denaturalist transcription*, which includes only the content. A naturalist transcription includes every part of the interview in as much detail as possible, including pauses, breaths, emphasis, or non-verbal activity (p. 1276) which allows for an analysis of such subtle components of speech. A denaturalist transcription may remove "stutters, pauses, nonverbals" (p. 1273) and other elements that do not contribute to the "meanings and perceptions" (p. 1274) held by the participants, instead focusing on what was said, not how.

In MISE, I paid attention to both content and form, which therefore required some elements of a naturalist transcription. Drawing on a range of literature, including Oliver et al. (2005), Linde (1993), and Labov (1982), I developed a list of transcription conventions for the focus groups (see appendix H.2). Under a holistic approach to thematic analysis, where extracts should always be read in context across sentences and paragraphs (Lieblich et al., 1998, p. 13), commas or full stops in a transcript carry less meaning than they otherwise might. For ease of reading, commas and full stops were applied at the discretion of the transcriber. This was the most appropriate choice because there were frequent long paragraphs in the focus group transcription, where an explanation ran across several sentences. Participants' responses, usually by way of agreement with a simple "yeah" to statements made by other participants, or the moderator, were separated from overlapping conversation in the audio into distinct responses in the transcription.

5.2.1 Deductive Thematic Analysis

The focus group data was analysed using a simplified, and more deductive, version of the reflexive thematic analysis employed in Data1 (see section 5.1). In deductive analysis, "existing research and theory provide the lens through which we analyse and interpret data" (Braun & Clarke, 2021a, p. 331). The focus group data was coded to themes, rather than codes, since the themes were already established and, in general, coding is not appropriate for uncovering evidence of pre-determined themes (p. 332). The analytic process for the focus group data built on the themes that had already been established using the following five stages, which were adapted from those in section 5.1:

- **Stage 1**: Broad reading of entire focus group dataset and summary of ideas about themes.
- **Stage 2**: Code data using the themes developed from Data1. Re-read for development of inductive codes.
- **Stage 3:** Look for patterns amongst the coded material. Check for consistency with thematic map.
- **Stage 4:** Review themes.
- **Stage 5:** Redefine and rename themes as appropriate.

Hydén and Bülow (2003) warned that in analysing focus groups, it is important to consider "from what perspectives the participants talk" (p. 308). In Data1, participants spoke almost entirely from a personal perspective, rather than as a representative, or member, of any group. The most common occurrence of "we" referred to a post-primary class, with a smaller number referring to a primary school class. It is likely that at that early stage of their university studies, they had not yet encountered many of these groups, and it would take time for them to develop voices with which to speak authoritatively on behalf of a larger group. In Data2, however, participants spoke as members of a range of different groups, which are shown in Table 5.5. As discussed in section 3.2.4, this served as evidence to confirm that the mathematical identity of individual group members could not be reliably determined from this data.

Table 5.5
The range of perspectives from which focus group participants spoke, other than their own individual one, and their associated methods of capture in analysis.

NVivo code	Method of capture in analysis
'We' in the access programme	Create a contextual code for *Further Education*.
'We' my fellow post-primary classmates	A code exists from Data1 to capture this.
'We' my fellow professional teachers	Create a contextual code for *Teaching Placement*.
'We' my university classmates	Existing *Third Level* code will capture this.
'We' my study group	This arose in a once-off description by one participant. No code needed.

Updating the Codebook

A total of seven inductive codes were created during thematic analysis of this data, which were numbered from 70 to 76. As detailed in Table 5.6, these codes mostly dealt with aspects of form that were unique to the focus group setting, with two new codes related to the content of participants' contributions. Collaboration had not been mentioned in Data1, nor had the role for influential people, beyond that of post-primary teachers.

Table 5.6
Codes added to aid the analysis of Data2.

Type	Code added
Contextual	70 Further Education
Contextual	71 Teaching Placement
Form	72 Transitions during third level
Form	73 Thinking as I speak
Form	74 Back-channel signals
Content	75 Friends, peers and family
Content	76 Collaboration

Both content codes had been identified in the literature review and included in early drafts of the codebook, but were removed from the thematic map because they were not required in the analysis of Data1. The necessity of only two additional content codes, both of which had previously been present in the codebook, provided evidence for inductive thematic saturation of the data collected (see section 3.2.6 for the definition and discussion). In particular, this suggests the existing themes were exhaustive in categorising the contributions of the focus group.

5.3 Data3 - Interviews

Polkinghorne (1995) described two types of narrative inquiry which were distinguished, in part, by the two modes of thought proposed by Bruner (1986): studies that employ paradigmatic reasoning to narrative data to produce themes/categories represent an *analysis of narrative*, while those that employ narrative reasoning to descriptions of events and actions to produce narrative data through emplotment represent a *narrative analysis*. In Data1, I used thematic analysis to focus on elements of mathematical identity that were common across participants (Howard et al., 2019). This process gave rise to a group narrative (discussed in section 3.2.4) and constituted an analysis of narrative, since it allowed the development of themes that hold across the narrative data. To concentrate on distinctive experiences that were unique to a participant required a different method of analysis. The *storying stories* framework seeks stories as data, and generates personal narratives through emplotment, and thus, represents an analysis of narrative and a narrative analysis (McCormack, 2004, p. 220; O'Kane & Pamphilon, 2016, p. 588). Storying stories will be presented as a two-stage framework in section 5.3.2.

McCormack (2004) pointed out that the task of analysing narrative data is "daunting," and that narrative research literature has been "largely silent" about how to do this (p. 219). Furthermore, Lutovac and Kaasila (2019) issued a call to researchers in identity to look outside the context of mathematics education, towards analytic methods in social science research that could "expand the analytical scene of mathematics education research" (p. 513). Since the interviews were semi-structured, established approaches that analyse how the story is told, such as using text structure sequentialisation (Corbally & O'Neill, 2014) or key rhetoric (Kaasila, 2007a), would not have been appropriate. Such approaches would be more apt in unstructured interviews or in open-ended questionnaires.

Through his work in the 1970s, Schütze claimed that three constraints of narration (to condense, to be detailed, to close the narrative at the end) significantly limit what a person says when telling stories, and how they say it (Schütze, 2008, pp. 14-16). The goal of the narrative interviews was to elicit stories from the participants concerning the development of their mathematical identity over their time in university. The storying stories framework offers important flexibility since participants can make these connections in a variety of ways that are still identified by the framework as stories, allowing them the freedom to relay the details, context, and meaning of their experiences in any order as they reflect upon them.

The personal narratives, which resulted from applying the storying stories framework, have the capacity to involve the reader in the emotions and detail of the participant's journey, projecting the participant's voice in a manner that is simultaneously more accessible and more resonant for the reader (Bruner, 1986; Johnston et al., 2021). Presenting the results of analysis in this manner allows the reader to judge the trustworthiness of the analytic conclusions (Guba & Lincoln, 1985; Mishler, 1990), and allows the opportunity for others to read alternative possibilities amongst the data (Hannula, 2002, p. 43). In narrative studies, providing a final report that allows the reader to vicariously experience and understand the decisions and emotions of the participants, is an important element of ensuring the quality of such studies (Loh, 2013, pp. 9-10). The transcription conventions for the interviews were mostly the same as those used for the focus groups (see appendix H.2), but also included notation for identifying surprising, insightful, reflective, contradictory, and puzzling moments, as recommended by McCormack (2000a, p. 294).

5.3.1 Definitions of Narrative Processes

In this section, I describe the outcome of a literature review of *narrative processes*, how they have been defined, and how to identify each process in a piece of text. By synthesising from several authors (Schütze, 2008; Labov, 1972; Labov, 1982; Mishler, 1991; Rosenthal, 1993; Reissman, 2008; McCormack, 2000a; O'Kane & Pamphilon, 2016), I produced the table in appendix H.3, which allowed me to compile the following definitions of the five narrative processes that are used when describing experiences:

- **Story**: Identified by "recognizable boundaries – a beginning and an end" (McCormack, 2000a, p. 288). A story is required to have a sequence of linked events/actions.
- **Description**: Static structures, such as people, places, and routines (Schütze, 2008, p. 15), which reduce the "information gap" (p. 61) between interviewer and participant.

Descriptions help the listener to get a more complete picture of the other narrative processes (McCormack, 2004, p. 224).

- **Argumentation**: Abstracted elements outside the story process, which present the perspective of the present (Rosenthal, 1993, p. 66). They add meaning to the other processes.
- **Theorising**: The narrator's general orientation at the moment (Rosenthal, 1993, p. 66). Reflecting or trying to work out something (McCormack, 2000a, p. 290).
- **Augmentation**: Additions to, or expansions on, previous stories (McCormack, 2004).

Following the principles of narrative inquiry, the story is the main unit of this analysis (Clandinin & Connelly, 2000, p. 77). The other four processes may elaborate on these stories, or they may include other elements from outside the stories. As the stories are not isolated in a conversation, McCormack (2000a) introduced the augmentation process to include descriptions that arise as a follow-up to a story, or other narrative process.

McCormack (2000a, pp. 288-289), taking inspiration from Rosenthal (1993), defined a story as having a recognisable beginning and end, along with a sequence of linked events/actions which together explain why the story was told (the point of the story). These events/actions can be organised chronologically or thematically (Mishler, 1991, p. 87; Rosenthal, 1993, p. 66). By synthesising definitions in the literature, I produced the table in appendix H.4, from which I defined a story as consisting of five distinct elements, one of which is optional:

- **Orientation** (beginning): Describes the general situation before, or at, the time of the first action. *Who, what, when, where?*
- (Optional) **Abstract**: Summarises the point of the story. Substance of the story as viewed by the narrator. *What was this about? Why is it being told?*
- **Linked events/actions**: *Then what happened?*
- **Evaluation** (the point of the story): The narrator steps out of the story to explain what was in their mind at the time of the events and how they felt about what was happening. This conveys the teller's emotions and attitudes to the story as they explain why it is being told. They may compare things that occurred and what might have occurred. This is the point of the story and was used as the title for stories in McCormack's work. *So what?*
- **Coda** (end): Finishes the story and brings the listener back to the present. *Then what happened? Nothing, I just told you what happened.*

Labov (1972, p. 370) proposed each of the guiding questions in italics above. My interpretation of these five elements was also informed by three explicit examples from the literature, which demonstrate how the framework has been applied (McCormack, 2000a, p. 289; McCormack, 2004, p. 223; O'Kane & Pamphilon, 2016, p. 589).

The evaluation element of a story, although technically not obligatory, is an almost universal feature of stories told by adults (Labov, 1982, p. 226). Stories that are limited to events/actions do not make a point to the listener (p. 226), and may leave their audience asking, "So what?" (Labov, 1972, p. 370). Some authors consider narratives limited to events/actions to be unreportable (p. 227), or unnarratable (Georgakopoulou, 2007, p. 62), such is the proven importance of the evaluative element (Mishler, 1991, p. 83). Of the 35 stories identified in the MISE interviews, only one had no evaluative element, and this story referenced a time and teacher who had been discussed earlier in the interview.

5.3.2 Storying Stories

The storying stories framework consists of two distinct stages (McCormack, 2000a, 2000b), which are described in this section. Each stage reflects the parts of the framework that function as an analysis of narrative, and a narrative analysis, as described at the start of section 5.3. The first stage, which I followed quite closely, involved identifying specific features of each interview by viewing the data through different lenses, which are detailed further below. The second stage, which I adapted to fit the research questions, involved using these specific features to construct a *personal narrative* from the interview data. Storying stories represents a formal framework for analysis that takes account of several narrative properties which have featured in other mathematical identity research. Much of the approach to analysing the interviews in MISE was already established in mathematical identity literature discussed in section 2.5, albeit in fragments e.g., the identification of turning points, core events (Drake et al., 2001; McCulloch et al., 2013), unexpected moments, and searching for evaluative language or moral judgement (Kaasila, 2007b). The framework as used in MISE is presented below:

1. View the interview transcript through multiple lenses (in bold):
    (a) Identify the stories and other **narrative processes** used by the participant in the interview.

   (b) Analyse the use of **language** of the transcript. Identify the key events that were discussed in the interview, and narrative processes concerning **Transitions** (A theme from Data1).
   (c) Consider the **context** of the interview by referring to the notes taken by the interviewer during each pre-interview, interview, and reflections immediately after each one.
   (d) Identify **moments** where something unexpected happened, often indicated by a response of surprise from the interviewer (such moments were identified as “Type B” responses by Labov, 1982, p. 227).

2. Develop a personal narrative for each interview using the views highlighted through the multiple lenses:
   (a) Identify an analytic point to each story in item 1a, and each key event and transition from item 1b above, and group similar analytic points together.
   (b) Include data from other narrative processes that address the same analytic points.
   (c) Order the narrative processes (including stories) chronologically from primary school to post-primary, and into first to fourth year at university, then the present, and the future.
   (d) Compose a beginning and ending to the personal narrative: What does the reader need to know for the middle portion to come, and how did the participant present their current mathematical identity and look forward to the future?

The first stage should be seen as a categorising strategy: a means to organise and compare the interview data, whereas the second stage is a connecting or contextualising strategy: one that addresses “the influence of one thing on another, or relations among parts of a text” (Maxwell, 2013, p. 108). In narrative terminology, the second stage is known as emplotment, and it culminates in the creation of a personal narrative (Polkinghorne, 1995, p. 5). At both stages, the story and other narrative processes were the units of analysis, the definitions of which will be presented in section 5.3.1.

The second stage of the above framework differs in several important ways from the one presented by McCormack (2000a, p. 285), most notably by omitting her suggestion to use story titles (evaluations) as headings to construct the plot for each personal narrative. Since there were no particular narrative processes that dominated any of the interviews, unlike in the example given by McCormack (2000b), I determined that a more trustworthy and complete personal narrative could be derived from including all of the narrative processes. I found that the short list of story titles provided only a partial insight into the experiences communicated by the participants in their interviews. An example list of story titles is included in appendix H.8 to demonstrate how such a list does not give a full impression of the corresponding personal narrative, which is presented in section 7.2.1. The narrative processes were coded using the NVivo software programme while the subsequent tasks were conducted in Microsoft Excel. Since the third research question concerning change in mathematical identity over time was prioritised in the interviews, the data was reorganised so that discussions concerning first year appeared before those concerning fourth year, whether they occurred at the start or towards the end of the interview. These were organised using excel spreadsheets, an extract of which is shown in appendix H.5.

I called the final product a personal narrative because it diverged in several ways from the interpretive stories developed by McCormack. Firstly, a personal narrative does not chronicle the interview itself but instead refers to the participant's entire university experience, including some details from their earlier experiences during primary and post-primary school. Secondly, it is not solely written in the first person since it includes the analytic voice of the researcher and, in line with the co-constructive vision of the interview process (see section 3.2.5), results in a narrative of the same style as those produced by other researchers in mathematical identity (cf. Kaasila, 2007a). Lastly, a personal narrative shares a property with the analytic narratives that result from reflexive thematic analysis, in that they include both findings and discussion, presented through the analytic voice of the researcher (Braun & Clarke, 2022, p. 131).

Chapter 6

Findings: Thematic Analysis of Data1 and Data2

The results and discussion of the reflexive thematic analysis of Data1 and Data2 are presented in this chapter simultaneously, by means of an analytic narrative for each theme. An analytic narrative is a series of analytic points (researcher interpretations) that "make an argument" rather than simply paraphrasing or summarising the data (Braun & Clarke, 2006, pp. 93-94). The results and findings are discussed in this chapter, in the sense that they are justified by data extracts which are, in turn, linked to research questions, literature, and/or theory (Braun & Clarke, 2022, p. 129).

This chapter begins with the presentation of participants' affective statements about mathematics and their reasons for studying a programme involving mathematics in section 6.1. Although neither of these categories of codes was promoted to a theme during thematic analysis, it is argued that they include important orientation for the reader in interpreting the themes that follow. In section 6.2, the analytic narratives for five themes that resulted from thematic analysis are presented. The manner of their presentation is explained at the beginning of that section. Participants' demographic information is presented in Table 6.1, which was extracted from Table C.5 (see appendix C.3). The sample of 32 participants included 24 male participants and 27 who had taken higher-level mathematics for Leaving Certificate.

Table 6.1

Demographic information for MISE participants including their gender, age as of March 2018 when the study commenced, and contribution to Data1 and Data2.

Grouping	Participant	Gender	Age	Data1 Word Count	Data2
Engineering	ENG_01	M	19	242	
	ENG_02	M	18	449	
	ENG_03	M	20	85	
	ENG_04	M	18	223	
	Éabha_ENG	F	18	230	
	ENG_06	M	18	167	
	Ciarán_ENG	M	18	214	
	ENG_08	M	32	75	✓
Science I	Dónal_SC1	M	20	468	
	Fiach_SC1	M	19	219	✓
	SC1_03	M	18	345	
	SC1_04	F	18	685	
	Saoirse_SC1	F	19	651	
	SC1_06	M	33	108	
	SC1_07	M	19	129	
	SC1_08	M	18	114	✓
	Breandán_SC1	M	19	334	✓
	SC1_10	M	24	39	
	SC1_11	M	19	680	
Science II	SC2_01	F	19	121	
	SC2_02	M	19	110	
	SC2_03	F	18	237	
	SC2_04	M	18	126	
	SC2_05	M	18	92	
	SC2_06	M	19	164	
	SC2_07	F	20	174	
	SC2_08	M	18	602	
	SC2_09	M	19	121	
	SC2_10	F	19	96	
	SC2_11	M	18	40	
	SC2_12	F	18	116	
	Aodhán_SC2	M	19	335	✓

6.1 Setting the Scene

This section details the findings arising from two categories of codes, which did not rise to the level of themes during thematic analysis: Personal Feelings about Mathematics and Reasons to Study a Programme involving Mathematics. The former category did not exhibit a coherent narrative organised around a central concept, as discussed extensively in section 5.1.10, and thus failed point 6 of the 15-point checklist for good reflexive thematic analysis proposed by Braun and Clarke (2022, Table 9.3). A prompt in the questionnaire explicitly invited participants to consider their reasons for studying their programme of choice, and these reasons are summarised in section 6.1.2. Thus, this category satisfied point 6 of the checklist, but failed on point 7, since the data therein was more summarised and described than interpreted (Braun & Clarke, 2022, Table 9.3). Together, section 6.1.1 and section 6.1.2 play an important role in setting the scene and guiding the reader in interpreting the themes that are presented in section 6.2.

6.1.1 Personal Feelings about Mathematics

As discussed in section 5.1.10, participants' **Personal Feelings about Mathematics** were analysed by distinguishing between emotional dimensions and associated causes. To establish the various functions of the emotions referred to by MISE participants, the extracts were coded as follows: introductory statement, bookend statement, cause, no cause. This resulted in the identification of 26 introductory statements, 19 of which also had an associated cause, and 7 bookend statements, 5 of which had an associated cause. This demonstrated that although affective statements play another important role in framing the thematic narratives in the remainder of this chapter, MISE participants tended to justify them with a cause.

Firstly, causes were attributed to participants' emotional disposition, which is indicated by statements of the form "I like/dislike mathematics." Their reasons for liking or disliking mathematics were attributed mainly to the same three causes from the literature: the teacher, vision of mathematics, and perceived competence (Di Martino & Zan, 2010, p. 39). Di Martino and Zan (2011) described vision of mathematics as expressed concisely with statements of the form "mathematics is ..." (p. 476), which in this study, aligns with Theme 2: What is Mathematics?. Perceived competence refers to students' perceptions of being able, or not, to succeed in mathematics (p. 476). Two further causes emerged among the MISE extracts: mathematics is rewarding/challenging and mathematics as a means to an end. Of the 29 participants whose contributions were assigned the code *I like/dislike mathematics*, all but one provided a reasoning that fit into the above five categories as shown in Table 6.2. When all the codes in **Personal Feelings about Mathematics** were analysed, it emerged that every participant had provided a cause for at least one of their affective statements. They were categorised as shown in Table 6.2.

Table 6.2
Participants' reasons for liking or disliking mathematics with the number of extracts coded to each reason in brackets. Four participants did not give a reason and some extracts were coded to multiple causes.

Cause	*Number of participants (extracts)*	
	I like/dislike	**All Personal Feelings**
Teacher	10(13)	17(22)
Vision of Mathematics	9(18)	14(23)
Perceived Competence	15(21)	21(31)
Rewarding/Challenging	8(9)	8(9)
Other	5(6)	10(13)
Total	28(64)	32(98)

Maths is rewarding/challenging had already been identified through thematic analysis, so this approach did not uncover any further instances. By considering the other codes in **Personal Feelings about Mathematics**, the number of participants who had mentioned each cause was expanded as shown in the "All Personal Feelings Codes" column of Table 6.2.

Since the affective statements that link to these causes are important for setting the scene, and framing the manner in which participants wish their contributions to be interpreted (see section 5.1.10), these were incorporated into the other theme narratives which are presented in the remainder of this chapter. They were incorporated as follows: the role of the teacher in inspiring their students is discussed in section 6.2.1, students' views of mathematics are discussed in section 6.2.2, perceived competence and the rewarding/challenging balance in mathematics are proliferous across several themes, and are most prominent in section 6.2.4. In the next section, the discussion of the themes resulting from thematic analysis begins with participants' reasons for studying mathematics, which build on those identified in this section.

6.1.2 Reasons to Study a Programme involving Mathematics

In the Data1 questionnaire, participants were prompted to comment on their reasons for choosing their programme of study. All participants had chosen to study programmes that involved some mathematics modules, and the influence of this on their decision fell into three categories: those who were attracted to study their programme because it involved mathematics; those who were neutral about the role of mathematics in their programme; and those for whom their desire to pursue science, teaching, or engineering outweighed their negative feelings towards mathematics. These categories will be explored in turn, after which, a striking difference between the reasons given by participants from each grouping (Engineering, Science I and Science II) will be noted.

Attracted by a Mathematics-Based Degree

Participants who expressed positive emotions about choosing to studying a programme involving mathematics, identified their fondness of the subject as a main reason behind their positivity:

> I decided to study maths at third level because it really appealed to me more than any other subject - it stood out (ENG_01).
>
> It became my favourite subject and is the reason I am studying engineering (ENG_04).
>
> My exposure to maths has been nearly entirely positive for my whole life and this has caused me to want to study it at university level (Breandán_SC1).

Neutral Feelings Towards Mathematics

Other participants expressed more neutral feelings about their relationship with mathematics, but saw themselves as capable of pursuing a programme involving mathematics, and therefore, believed that it was not a barrier to studying science or engineering:

> I was never inspired to pursue a course that involved maths, I just wanted to expand my knowledge, on a topic I was good at (SC2_02).
>
> Choose to do a course with maths as I'm generally good with numbers compared to other things like language (SC2_09).
>
> I chose to study a course which includes maths because I enjoyed studying it and I was

> naturally adept at it, which gave me confidence. ... I generally did well in maths exams compared with other subjects, which made me enjoy the subject more (SC1_04).

These participants expressed a belief in themselves as learners, by identifying themselves as "good" at mathematics, with SC1_04 specifically linking this to good results in examinations. The literature suggests that such self-belief, if based on past performance or achievement, may be associated with identities of exclusion at university level. Solomon (2007b) found that students who choose to study mathematics because they are good at it, are subsequently surrounded by others in university who are similarly high-achieving, and as a result they can feel marginalised and develop identities of exclusion. Boaler (2016) further suggested that students can be led into pursuing mathematics, because the teaching of the subject rewards performance, but lose their sense of purpose when they realise that they had never developed an interest in the subject itself (pp. 157-158). This could have particular relevance for participants, like the first two in the previous set of extracts, who do not express any interest in mathematics, but are required to study it.

In contrast to the mathematics undergraduate students interviewed by Solomon (2007b), MISE participants have the option to express neutral views about mathematics, because they are more interested in the applications of the subject in their degree programmes:

> Expected as common knowledge in my course ... Maths didn't influence my choice of an engineering degree - I based it on my interest in electronics, where I already applied the necessary maths as a hobby (ENG_06).
>
> I chose to study a course which includes mathematics at third level because I believe mathematics is a necessity when studying the subject that I chose (physics) (SC1_03).

The following focus group contribution presents some relevant data from Data2 that clarified and expanded upon participants' reasons for studying their programme. The format of presentation is described more fully in section 6.2.

Focus group contribution 1.
Mathematics as a tool.

An expression of neutral emotions about mathematics, which incorporated a view of mathematics as a tool, also arose in the focus group. An engineering participant said that he does not see mathematics as something to be enjoyed or feared: “I don’t enjoy it. On the other hand, I’m not scared of it either” (ENG_08). He reasoned that the icebreaker quotes about emotions and mathematics, presented to the participants at the beginning of the focus group (see appendix F), did not apply to him:

> My attitude towards maths is I treat it as a tool. Not as a subject or anything that I like or dislike. For me it’s just a tool to solve other problems that I need to use in the workplace and in studying. So, if I want to be efficient, if I want to solve the problems then I just need to be able to use this tool ... So, it’s just a matter of getting used to it and like whatever I told, you just need to be able to use it efficiently. That’s all. (ENG_08)

His reasoning revealed elements of a technological pragmatist view of mathematics, where the purpose of mathematics is its efficient and pragmatic application to real-life situations (Ernest, 1991), which will be discussed further in section 6.2.2.

Mathematics is a Means to an End

There was a clear message from a subgroup of participants that they decided to study their programme in spite of mathematics, but certainly do not relish the obligation. For some participants, mathematics was simply a means to an end:

> I was not going to let maths put me off what I wanted to do (SC2_10).
>
> I have an unrelenting passion for science and I thought that I will only have to do maths for 1 year (SC2_08).

While others were not so embattled with mathematics, it was still seen as an obstacle to their real interest in science:

> I like science and I decided I could put up with the maths (SC2_01).
>
> Most science degrees have Maths and I just wasn’t able to not choose it (SC2_05).
>
> I knew I’d have to do maths at 3rd level but as long as I can keep up, which I seem to be, I’m happy enough (SC2_07).

This group was best summarised by Dónal_SC1, a PST, who said “I knew I’d have to do maths” but he planned on dropping it in favour of chemistry and physics as soon as possible.

Summary and Insights

A dichotomy was observed between science and engineering participants with regard to the reasons discussed in this theme. Although most participants had studied higher-level mathematics (see section 3.2.1), many of the most positive comments came from Engineering participants, who expressed their passion and fondness for the subject. On the other hand, all the extracts describing mathematics as a means to an end came from students of science (mostly the Science II grouping), many of whom looked forward to discarding mathematics and ending its interference in their degree programmes. As was noted previously, research by Solomon (2007b) and Boaler (2016), in particular, suggests that students who study mathematics because of their prior achievements alone, are more likely to develop identities of exclusion with regards to university mathematics (see section 2.4.3). However, in a significant contrast, in section 6.2.4 it is demonstrated that MISE participants did not support the belief that success in mathematics could be achieved effortlessly. In other words, they did not agree with the "dominant discourse" about fixed ability beliefs that Solomon (2007b, p. 88) noted amongst the undergraduate community in her study.

It should be noted that the influence of teachers on participants' reasons for studying their programme have not been mentioned so far, because this is explored as part of the more widespread influence of teachers on participants' mathematical identities in the next section. Experience with teachers has been reported as a common reasoning for PSTs in pursuing a career in teaching (Latterel et al., 2016), even those who experienced what they described as "poor teaching" (Eaton & OReilly, 2009b, p. 295), and wished to do better themselves. Previous research discussed in section 2.6, has found that students of non-teaching programmes (Business and Psychology in this case) were less likely to acknowledge the influence of their teachers than PST or mathematics students (Eaton et al., 2019, p. 1490). However, it will be argued in section 6.2.1 that MISE participants across both categories (PST and non-PST) frequently referred to primary and post-primary teachers as important influences on their attitude, enthusiasm, passion, and interest with regard to mathematics, as well as their views of the nature of mathematics and its teaching/learning.

6.2 Themes

In this section, the analytic narratives for five themes that resulted from thematic analysis of Data1 and Data2 are presented in the following order:

1. Theme 1: Teachers/Lecturers (section 6.2.1)
2. Theme 2: What is Mathematics? (section 6.2.2)
3. Theme 3: Ways of Learning Mathematics (section 6.2.3)
4. Theme 4: Mindset and Getting Started (section 6.2.4)
5. Theme 5: Transitions (section 6.2.5)

Presentation Conventions for the Themes

The themes are drawn from the first two data collection stages. Data1 was collected during participants' first year of university in March 2018, while Data2 was collected at the beginning of participants' third year of university in October 2019. The themes developed from Data1 will be presented below, interspersed with clarifications/elaborations from Data2, which are identified as "Focus group contribution n" in the text. FG contribution 1 is included in section 6.1.2 to introduce the style of presentation. Presenting the data in this manner allows the reader to identify the time period, and relevant part of the participants' journey, to which the data refers. Since the participants from the Science I grouping in Data2 and Data3 were all PSTs, they will be referred to as such for clarity, while the single participant from the Science II grouping in Data2 and Data3 (see section 3.2.1) will be referred to as a science student throughout. Participants' written responses from Data1 were not subject to a transcription process, and, therefore, are presented verbatim. The focus group data was transcribed according to the process described in section 5.2. The names of **themes**, **sub-themes** are presented in bold font, and *codes* are presented in italics, to clarify that the terms refer to these items rather than concepts in mathematics education. The full list of themes, sub-themes, and codes is given in appendix G.5.

6.2.1 Theme 1: Teachers/Lecturers

This theme involves the significant influence of teachers and lecturers on participants' mathematical identity. In first year, participants tended to focus on post-primary school experiences, whereas the influence of university lecturers was mostly drawn from the focus group contributions in third year. This theme adds to previous research that has demonstrated how teachers' practice in the classroom influences the kinds of identities that are constructed by students (Gardee, 2021), as well as their attitude and enthusiasm for mathematics, and their views of good and poor teaching (Eaton & OReilly, 2009b, pp. 294-297). Although some MISE participants mentioned friends or family, their experiences with post-primary school teachers featured heavily in their questionnaire responses, and thus appeared to be more influential in the development of their mathematical identities. This section begins with participants' views on the positive and negative influence that certain teachers had on their own attitude and enthusiasm for mathematics. Subsequently, participants' descriptions regarding adapting to different teachers' styles of teaching are addressed, after which it is noted that some participants moved away from relying on the teacher as the sole authority on mathematics. Lastly, the pace at which mathematics classes and lectures are conducted is discussed, with focus group participants addressing how they keep up with mathematics modules in university.

In section 5.1.10, teachers were reported to be a main cause of participants' personal feelings about mathematics, both positive and negative. SC2_05 explained how students can absorb a negative attitude from their teachers: "I don't like Maths as a subject, but I like doing it. This I think is to do with my Junior Cert teacher and his attitude towards Maths." (SC2_05). Another participant, who still finds mathematics stressful at university level, similarly noted that the disinterest of his teachers in primary school has contributed to this feeling of stress around mathematics: "In primary school I'm pretty sure none of the teachers ever wanted to teach us maths" (SC2_07). Participants pinpointed enthusiasm and passion for mathematics as valuable characteristics of those teachers that had a positive influence:

> These 2 teachers were influential because their passion for the subject shone through (Fiach_SC1).
>
> I also had a wonderful maths teacher for 4 years of secondary school - she was very enthusiastic, explained everything very well and pushed us all to keep improving constantly (SC1_04).
>
> Maths is a love of mine. ... I have been influenced by many teachers who have been passionate about maths (SC1_07).

Saoirse_SC1 was inspired to become a teacher having experienced great encouragement from her own Leaving Certificate teacher: "My influential maths figure would have to be my leaving cert maths teacher. She was incredibly encouraging for everyone. Super talented and immensely respected. She inspired me to become a teacher" (Saoirse_SC1). Meanwhile, Fiach_SC1 explained how his early interest in mathematics stemmed from encouragement from his teacher, which contributed positively to his perceived competence:

> A critical point for me regarding mathematics was in primary school when one of my teachers said at a parent teacher meeting that I had a talent in maths. Before that I never really realised it, I just done the homework etc., but after that comment it made me get a bit more interested in mathematics in general (Fiach_SC1).

Thus, his teacher's comments from many years ago affected Fiach_SC1's perceived competence, underlining the abiding influence of such encouragement on students attitude towards mathematics (Di Martino & Zan, 2010), and on their mathematical identities.

Teachers' Influence on Students' Learning

Participants mostly saw the teacher's role as an explainer (Ernest, 1988), and, therefore, placed great value on their teachers' ability to explain mathematics to their students:

> My teacher was very good at explaining different concepts (ENG_01).
>
> I believe grades are also highly dependent on the teacher/lecturer ... simply the teacher is bad at explaining, for example my [Leaving Certificate] maths teacher was not the greatest (ENG_03).
>
> If the explanation is good I'll understand it easily ... I found mathematics okay due to my teachers explanation about certain topics (SC2_11).

Teachers' ability to "explain," which was commonly referenced by participants, has been the focus of previous mathematics education research. Kinachi (2002) focussed on prospective post-primary teachers' conceptions of good instructional explanations, as opposed to procedural explanations akin to "rules without reasons" (Skemp, 1976). For example, with regard to addition and subtraction of negative numbers on the number line, the explanation " '–' means change direction and symbol '+' means same direction" was identified as a procedural "trick" rather than a good explanation (Kinachi, 2002, p. 168). The author proposed a list of criteria for explanations that promote understanding, which included determining the meaning of symbols, using different representations, using "mathematical reasons" rather than procedural "tricks," and answering the question "Why is this the way it is?" (p. 165). Subsequently, a comprehensive treatment of the critical role of examples in such explanations was given by Bills et al. (2006), while in teacher education, Charalambous et al. (2011) demonstrated how PSTs can be facilitated in learning to provide good explanations through deliberate reflection on recurrent opportunities to engage in this process during their university programmes (p. 461).

One MISE participant in particular demonstrated the importance of having a teacher who can explain things in order to help their students understand:

> I found that teacher very bad in terms of her ability to explain maths. I moved myself to higher[-level] mathematics ... That teacher was very professional and talented at explaining maths and for me it became easier to understand maths (SC2_08).

The move from ordinary-level to higher-level mathematics for this reason is remarkable given the lower participation in higher-level mathematics in the Leaving Certificate (see section 1.2), and the wide jump between both levels that was perceived by participants (discussed in section 6.2.5). In 2017, when SC2_08 sat his exams, almost twice as many students chose ordinary level compared to higher level (SEC, 2017), but he perceived that the potential for good explanations to lead to understanding, outweighed the difficulty of the higher-level curriculum.

Focus group contribution 2.
The link between good explanations and understanding.

Both participants in focus group 2 (FG2) agreed that teachers' ability to explain mathematics, is both characteristic of, and requires, understanding:

> You definitely understand it if you're able to fully explain it to someone else and they get it (Aodhán_SC2).
>
> You have to understand it, so you can explain it to a student. Like I couldn't walk in and explain trigonometry if I don't fully understand it (Fiach_SC1).

In the literature, it is similarly argued that while procedural explanations are based on memorisation (Ball, 1988a), good mathematical explanations instead promote learning that accounts for meaning, as well as students understanding of why things are the way they are (Kinachi, 2002, p. 165). Thus, there was some evidence to suggest that participants' references to good explanations were connected to relational understanding, and this connection is represented in the literature.

In section 6.2.3, participants' conceptions of what it means to "understand" mathematics will be presented as involving knowledge of the applications of the subject. Teachers were often credited for inspiring students' own interest in mathematics, because they offered them some insight into real-life applications:

> He made me very interested in the subject as he was able to show how something in mathematics can be applied to real life situations in ways you never thought of (SC1_03).
>
> My grinds teacher helped me a lot to understand the concepts. ... She explained everything through real life situations, rather than ink printed on paper (ENG_04).

Private tuition or "grinds" is a popular source of "shadow education" for Leaving Certificate students and one which has been reported to promote procedural learning over conceptual learning (Prendergast et al., 2022, p. 334).

Focus group contribution 3.
Stirring students' interest by demonstrating the applications of mathematics.

The notion that there is a focus on applications at post-primary level arose in the focus groups also. SC1_08 reckoned that there was "a lot more real-world application in secondary school" compared with the more formulaic approach in primary school where "it could be a page full of numbers" (SC1_08). Fiach_SC1 focussed on communicating the applications of mathematics, and proposed that this should be an important concern of a good mathematics teacher:

> Yeah, as a teacher if you show applications, if you show everyday examples, if you, basically if I can tell you what you're going to use this for. When, if when you're older or when, if you're in a job. It can definitely resonate with students (Fiach_SC1).

He expanded on this in great detail in his interview which is discussed in section 7.1.2, describing how he felt confident to teach mathematics because he understands the applications.

Thus, participants described that teaching that paid attention to the applications of mathematics was important for their understanding, and for inspiring their interest in the subject.

Moving Away from the Teacher/Lecturer

Participants highlighted the time it takes to transition between teachers, with the premise that "different teachers use different teaching methods which may not suit everyone" (ENG_03). In post-primary school, consistency regarding teachers was seen as an advantage, whereas frequent changing of teachers had a negative influence on students' mathematical identity:

> My secondary school teacher stayed the same for all 6 years so there was no time needed to adjust to a new teacher and their own personal ways of teaching (ENG_01).

> When I began studying maths, I really didn't like it because I had a new teacher every year in secondary school ... I didn't enjoy maths in secondary school because of my variation of teachers (Ciarán_ENG).

Another participant noted how a sudden change of teacher had a negative effect on his interest in mathematics, even though it did not affect his performance: "for first year I had a terrible teacher who got suspended halfway through the year. I didn't fall behind, but I got kinda bored of trying to do maths" (Dónal_SC1). In his interview in section 7.2.4, Dónal_SC1 explained that the frequent change of teachers at post-primary level had a negative impact on his experience of mathematics. Adapting to different styles of mathematics teaching is important for participants in Science I and Engineering in particular, since they take multiple mathematics modules each semester in their first two years of university.

In section 6.2.3, it will be noted that participants began to move away from relying on the teacher as the sole source of mathematical knowledge by searching for sources of learning outside the classroom. Some participants gave evidence that in post-primary school, they moved away from seeing the teacher as the sole authority on mathematics, and enjoyed forging their own paths to progress:

> I realized pretty quickly I was only doing bad in it because of how I was taught. Once I began teaching myself and actually understanding the concepts, I started to really enjoy it (ENG_04).

> But when I took it upon myself to improve my maths, as well as the help of my great grinds teacher, I found maths more appealing and myself more capable (Ciarán_ENG).

In the focus group, I investigated whether participants had sought out other sources of learning when they perceived that they did not understand the content in their university lectures.

Focus group contribution 4.

Engineering students move away from viewing the lecturer as the sole authority, but PSTs do not.

MISE participants mostly described learning experiences that aligned with a reception of knowledge model of learning (Ernest, 1988), where they rely on the lecturer to provide explanations, examples, and context (including applications). PSTs emphasised the importance of seeing the lecturer work through examples of applying mathematical theory:

> [Lecturer 1] would put up just reams and reams of examples on the board, or talked you through actual examples on how to use stuff ... you come out of lectures knowing what he's talking about, but if you looked at his notes without being at the lecture, you would have no idea at all (Breandán_SC1).
>
> [Lecturer 2] will explain the theory, it'll make no sense, he'll say let's do an example, and suddenly it's not too bad. So, examples and sample papers is the best thing, I think (SC1_08).

Both participants' descriptions positioned the lecturer as the main authority on mathematics, whom they rely on to demonstrate mathematics through examples. They suggested that this was the case because their online notes may not be explained well, and they do not get the same level of feedback on their own work compared to post-primary school:

> And if the notes are there, that's great and if they're not explained well, it's a bit of, it's a bit more stress (SC1_08).
>
> You kind of can't use the lecturer as a teacher as you do in [post-primary] school ... You don't get that same kind of feedback on what you're doing. And even tutorials you don't get the same level of feedback, like you get through examples and exam-style examples, but you don't get the same level of teacher feedback that you do from secondary school. (Breandán_SC1).

ENG_08 expressed the same expectation from lecturers, to provide worked examples and exam-style questions. However, rather than expressing frustration that lecturers do not always provide complete explanations or methods that are understandable, he seemed familiar with, and amenable to, working around such shortcomings by finding satisfactory resources himself:

> If the lecturer fully satisfies me in terms of how the teaching is being conducted

> in terms of lecture slides, worked examples, sample papers, maybe some other additional material, then I'll stick to the lectures and tutorials and, in general, if I don't, if I feel I don't need any additional material, I'll stick to the lecturer. But sometimes either the notes are sparse, or the method of explanation was not great, then, obviously, the books, additional websites, whatever material I can fetch to improve my skills in the module (ENG_08).

Importantly, he felt confident in evaluating whether the methods and resources presented to him by the lecturer made sense to him:

> If I'm shown a particular way, how to solve a particular problem in lecture, I might not exactly understand that that way, but there may be another method, which might make more sense to me. Because I will actually understand it, I can understand the mechanics of how it works. So, if it makes more sense to me. I'd rather choose that method, even if it's a bit longer than the one that I was shown (ENG_08).

Although differences between PST and engineering participants are evident above, a variety of perspectives for investigating this difference exist in the literature. In their research with mathematics undergraduates, Solomon and Croft (2015) concluded that university teaching that promotes internal, independent judgement, such as that described by ENG_08, is "an important component in developing students' sense of ownership of their mathematical knowledge" (p. 275). Those with such a sense, know why mathematics works, and look internally to validate their solutions, rather than relying on external authorities, such as the lecturer (p. 273).

However, through their university studies, PSTs gain knowledge of mathematical content and pedagogy, which is often understood with reference to the seminal Mathematical Knowledge for Teaching (MKT) model developed by Ball et al. (2008). For example, they should have more nuanced awareness of how different representations can communicate mathematical ideas more effectively than others or have experience with evaluating and adapting the content of textbooks for the same purpose (p. 400). Thus, Breandán_SC1 and SC1_08 may have perceived that some of their lecturers tailored their mathematics modules for their PST audience, much like PSTs expect themselves to do for their own students in the post-primary classroom. Therefore, they focussed their efforts towards understanding the lecturer's ways of representing, doing, and evaluating mathematics, whereas ENG_08 saw little variation in the mathematical content of his lectures compared to other sources e.g., books or websites.

Time Management

Several participants emphasised the time it takes to study mathematics and to build an understanding:

> It took a lot of time sometimes ... doing these maths questions (ENG_01).
>
> Leaving cert maths took up too much of my study time (SC2_03).
>
> I could spend 4 to 5 hours straight working on a maths problem (SC1_04).

One participant thought that there was no way around the "[h]ours and hours of practice" (Saoirse_SC1) that the Leaving Certificate mathematics examination requires. Several participants also pointed to difficulties in "meeting the deadline" (SC1_11) to cover the Leaving Certificate mathematics curriculum, since "there is limited time to focus on it" (ENG_02).

At university level, SC2_08 quickly discovered that the pace of lectures can be challenging: "I'm faced with a choice to either listen without taking anything down or taking everything down without listening to the lecturer, without a clue of how to do the question" (SC2_08). The frustration expressed by SC2_08 may be reflective of broader issues related to transition to university. She appeared to perceive her role as a non-interactive listener or note taker, rather than as a participant in the class who could ask questions. She may also have found that lecturers do not focus on tasks as much as a post-primary teacher would, and instead, lecturers present theory and examples which promote a shift from instrumental to relational understanding (Breen et al., 2013). Focus group participants also identified keeping up with the content of mathematics lectures as a key concern.

Focus group contribution 5.
Keeping up with mathematics lectures.

> In the focus group, a PST pointed out that adapting to the pace of each module, so that you can keep up, is an important responsibility of the student in university: "If the secondary school teacher thinks that there's a few people in the class that doesn't get a topic, they will spend extra time at it. But it's usually the case that the college lecturer just moves

on" (Fiach_SC1). ENG_08 added that lectures are for explanations, but knowing how to apply the theoretical material requires self-directed study:

> The lecturer is explaining you the material and you don't have a chance to practice it. So at the end of the lecture, you barely remember anything about the practical part of things, how to apply the rules that you've learned and theorems. It's all left up to the students in their own time (ENG_08).

In contrast with post-primary school, they also had to manage lecturers who set assignments without knowing the other deadlines or pressures that their students were under:

> Like one of our lecturers. ... She'd like, set work that we have to do each week. Whereas, I think we had a lecturer last week, he was only new. He's like quite young. And he said I know [you] have a lot of work so like, just do what [you] can and then, and I won't set any work like. But you know, he kind of understands, what we're going through a little bit more (Aodhán_SC2).

A major concern of focus group participants, therefore, was balancing the demands of modules which required different manners, and amounts, of self-directed study. As is reported in section 6.2.3, their goals involved understanding material, practising procedures, and applying this knowledge to continuous assessment and final examination questions.

Summary and Insights

In this theme, in line with previous research involving PSTs (Eaton & OReilly, 2009b, p. 297), participants reported that their post-primary teachers influenced their own motivation, attitude, and enthusiasm, with regard to mathematics. Of particular influence were those teachers who grounded the learning of mathematics in real-life applications, which participants linked to their understanding of, and interest in, the subject. The influence of post-primary teachers on participants' attitude, which was reported in section 6.1.1, was further elaborated in this theme. Participants' descriptions support the characterisation of teachers as an overarching mediating factor with respect to students' attitude towards mathematics (Di Martino & Zan, 2010, p. 43).

Although participants' descriptions indicated that they saw the role of the post-primary teacher as an explainer, corresponding to a reception of knowledge model of learning (Ernest, 1988), such beliefs have also been associated with an emphasis on understanding (Beswick, 2005, Table 1). Di Martino and Zan (2010) noted that their students exhibited different meanings of success in mathematics, with some associating success with good grades, and thus relying on the teacher to validate their work, while others identified success with understanding (p. 40). In this theme, many MISE participants described moving away from viewing their teachers as the sole authority on mathematics, and therefore, it is of importance to investigate their evolving conceptions of success and understanding, which will be addressed in section 6.2.3.

6.2.2 Theme 2: What is Mathematics?

This theme deals with participants' views about the nature of mathematics, broadly categorised into three viewpoints which were explained in chapter 2: instrumentalist, Platonist, and technological pragmatist (Ernest, 1988, 1991). The views presented in this theme have strong resonance with research in teacher beliefs, wherein beliefs about the nature of mathematics are conceptualised as "rudiments of a philosophy of mathematics" (Thompson, 1992, p. 132), which have important implications for the teaching and learning of the subject, as discussed in section 2.1. In this theme, participants' views on the nature of mathematics were conceptualised as being derived from their experience of mathematics in the classroom. This is an important point of note, since researchers in affect, for example, generally distinguish between teachers' epistemological beliefs about the discipline of mathematics and mathematics in the classroom (Depaepe et al., 2016), in a manner that students entering university may not. As mentioned in section 2.5, another major feature of such research is that teachers' or students' views are considered with regard to the degree to which they agree with established philosophies of mathematics, such as those listed above. Folland (2010) claimed that "almost all mathematicians are Platonists" in that they believe they are uncovering pieces of a universal truth, and Hersh (1997) agreed that Platonism is the pervasive philosophy of mathematics. Research in mathematical identity in Ireland has noted that although many mathematics students "take great delight in this conception" (Eaton, Oldham, et al., 2011b, p. 40), in practice, they recognised a preference for instrumental or application-focused mathematics. However, Sfard (1998) argued that mathematics education researchers have significantly diverged from the absolutist viewpoints held by mathematicians, and instead embraced a fallibilist philosophy of mathematics, resulting in a "serious conceptual gap" (p. 491) between the two communities. This gap is still the subject of attention by scholars to this day, with Thanheiser (2023) arguing that a fallibilist view is essential if students are to use mathematics to make sense of the world (p. 6). In this theme, it is argued that MISE participants appreciated the logical, objective structure of mathematics, and demonstrated absolutist philosophies of mathematics. Thus, they viewed mathematics as objectively correct, and involving the mastery of facts and procedures, which are useful in addressing real-world applications. However, they varied in whether they viewed this infallible knowledge as internally consistent and logically connected, or arbitrary and disconnected.

Rote-learning and an Instrumentalist View of Mathematics

In the cross-border MIST study, students from two different schooling systems on the island of Ireland presented distinct views on the nature of mathematics, with students from the Republic of Ireland being heavily influenced by an examination-driven focus on rote learning compared to their counterparts in Northern Ireland (Eaton, McCluskey, et al., 2011). Two MISE participants mentioned rote learning specifically, and others emphasised the influence of examinations on their view of mathematics. For example, Saoirse_SC1 believed that Leaving Certificate mathematics "requires a lot more energy and effort than any other [subject] ... there is no learning or cheating only practice. Hours and hours of practice" (Saoirse_SC1). A positive reading of this extract might be that there were no shortcuts to success in Leaving Certificate mathematics, it took hard work and dedication, which is discussed in section 6.2.4. At the same time, she may also be suggesting that rote learning was not only rewarded, but required, and a deeper understanding was not a strong feature of mathematics for Leaving Certificate, in her experience. Other participants used terminology related to learning off information and having to "memorise theorems" (SC2_09). There is a list of geometry theorems, proofs, and constructions for Leaving Certificate which is likely to be the subject of this quote, however, other participants acknowledged how most of the curriculum does not consist of convenient lists that can be subjected to rote learning techniques:

> Nothing you can learn off to guarantee you those few extra marks (Saoirse_SC1).
>
> Maths cannot be learned off by heart as in other subjects which is more challenging but allows rest from other subjects (ENG_06).

There is a juxtaposition in these extracts between the guarantee of examination marks that rote learning can bring, and the pressure to memorise large quantities of information without hitting a saturation point. Some participants felt that mathematics is about rules, and you can train yourself into following the rules, instead of trying to understand when and why they work:

> It is sometimes simple enough to follow the rules to solve a given problem. The trouble is largely in trying to remember all the rules (ENG_02).
>
> Maths never gets harder, but there is always more rules/methods (ENG_06).

Both extracts unequivocally evoke an instrumentalist image of mathematics as a simple accumulation of rules and facts, where the absence of relational understanding means there are no easy ways to remember these rules (Skemp, 1976, p. 23).

A Platonist View of Mathematics

Aside from the instrumentalist view of mathematics described above, some participants presented a Platonist view of the subject, as an organised, consistent system of truths, connected by logic (Ernest, 1988). SC1_04 referred to the "regularity and straightforwardness of the maths problems I did in primary and [post-primary] school" (SC1_04), implying that it involved practising standard exercises repeatedly, and contrasted this with her experience at university. She had come to view mathematics as a logical subject, organised around "predictable patterns" but not in a formulaic fashion: "When I think of maths I think of order, patterns and understanding" (SC1_04). She reported that the objectivity and logical nature of mathematics levels the playing field compared to other subjects: "I loved knowing that there was one right answer out there, and anyone could find it by implementing a series of logical steps" (SC1_04). Similarly, another participant described how they appreciated working within the "logical structure and clearly defined rules of the game" (ENG_08). This emphasises, as noted by Eaton, Oldham, et al. (2011b, p. 40), that care should be taken when interpreting expressions used by students, since references to rules and structure can indicate a Platonist, rather than instrumentalist, view of mathematics. Objectivity, and the existence of a correct answer was another property distinguished mathematics from other subjects, in the eyes of MISE participants:

> Maths, unlike many other fields of study, isn't subject to opinion (SC1_04).
>
> Compared to other subjects there is only one answer (ENG_02).
>
> Maths is black and white, whereas in English it's about your opinions and emotions etc. ... You're right or you're wrong, thats it (SC1_11).

Although some participants appreciated mathematics for its objective property, this positivity was not universal. Saoirse_SC1 described mathematics as "very heavy and stress inducing in comparison with other subjects," and she linked this to objectivity, suggesting that it's stressful to know that your solutions will be categorised simply as right or wrong. This is the same participant who decried not being able to learn information to guarantee exam marks, and evidently, has struggled to operate within an instrumentalist view of mathematics that required her to comply with a strict scheme of right and wrong (Ernest, 1988). Both instrumental and Platonist views of mathematics incorporate absolutist foundations, for which mathematics is seen as unchanging, objectively true, and discovered rather than created. MISE participants also presented evidence of this more general viewpoint: "There are no flaws or errors in mathematics - there are errors in human understanding of maths" (SC1_04).

Seeing the Real-World Applications of Mathematics

Rather than taking comfort in the logical, organised structure of mathematics, some participants saw mathematics as a useful tool, to be applied or used in the real world. For these participants, the purpose of mathematics is to attend to real-world situations: "take a real world problem break it down and come up with a solution" (SC2_09). Correspondingly, they wished to have mathematics explained "through real life situations, rather than ink printed on paper" (ENG_04) and they expressed a preference for tackling questions that attended to this viewpoint:

> Now I treat mathematics as a tool, which I'll use in future to solve various work related problems (ENG_08).
>
> How useful mathematics is in every day life (SC1_03).
>
> Generally ok experience when dealing with questions dealing with real life applications (SC2_09).

While Dónal_SC1 recalled being frustrated at post-primary level with the lack of application, SC1_11 noted how the nature of mathematics changed in university to focus on applications:

> It's interesting to see the difference in teaching methods when it comes to third level, where we're taught the applications and significance of what we have learned (SC1_11).

The views expressed above are strongly resonant with that of the technological pragmatist, who views mathematics as a tool to be applied according to utility and expediency, in the solution of practical problems (Ernest, 2019, p. 85).

Focus group contribution 6.
A technological pragmatist view of mathematics in engineering.

For ENG_08, the purpose of mathematics is to apply theory to solve engineering problems, a view which aligns with the utilitarian aims of industry, from which the technological pragmatist view of mathematics is drawn:

> We are covering some theoretical material, we are always told how and what and where we can actually apply that material. So, it's not just some imaginary data but we actually know how to apply those things so it makes sense to us and makes the material more interesting to learn ... You need to see a real, well, ideally a real-life example of how to apply that. Why, why do you need all that? Because then it makes sense. If you're just given a formula so this does this, and you get this as an outcome, like why do I need this? What's the point? (ENG_08).

Thus, as noted in section 6.1.2, he saw the purpose of mathematics in terms of its utility in attending to real-world applications in engineering. In section 6.2.3, it will be presented that ENG_08 conceptualised "understanding" mathematics as involving knowledge of applications (see FG contribution 8). Thus ENG_08's description above reflects a technological pragmatist view of mathematics, since under this view, constructing one's own solutions to practical problems is viewed as "additional content to be adjoined to a content driven mathematics curriculum" (Ernest, 1991, pp. 287-288).

Mathematics and Physics: Understanding the Real World

Rather than addressing the purpose of mathematics, some participants presented mathematics and physics as a powerful combination for attempting to understand the real world, indicating that both rely on each other for achieving this purpose. SC1_04 said that mathematics "allows us to make sense of the world" and of observations, allowing us to "have a better understanding of how the universe works and how certain things that appear very different are in fact intrinsically linked to one another" (SC1_04). Participants emphasised the strong mathematical basis of science subjects, but particularly physics:

> One cannot study physics without having a very good grasp of the mathematics that goes along with it (SC1_03).

> Maths helped with my physics and chemistry in school also as I had a firm knowledge of the basics (Éabha_ENG).

Physics may be considered as extending mathematics to the real world: "I loved physics as it was similar to maths but with logic and real world problems added to it" (Éabha_ENG). According to several MISE participants, the two subjects are united under their combined purpose to make sense of the world we live in:

> Mathematics and physics ... can be used to describe lots of aspects of the world and I believe that they go together when being used to describe how the world works (SC1_03).
>
> Physics is about explaining everything essentially and making sense of everything. That's why maths is interesting to me (SC1_11).

The Interlocking Areas of Mathematics

In addition to linking mathematics and physics, participants identified areas of mathematics as connected to each other. A conceptualisation of university mathematics as "linked up" has been noted in previous research in Ireland (Eaton, McCluskey, et al., 2011, p. 7). MISE participants touched on the same idea, believing that mathematics, unlike other Leaving Certificate subjects, is not compartmentalised, and its parts interlock:

> It's easy to look at say, Leaving Cert English and identify which aspects you are strong at, such as poetry or speech writing, or comprehensions. With maths, I see it all as the same, even though there are individual topics ... It might be because each topic is intrinsically linked to one another, and no piece of maths is isolated on its own (SC1_11).

Another participant made this even clearer, describing the areas of mathematics as if they are connected by threads, and when one part of one thread gets lifted upwards, each of the connected areas move upwards with it: "each part of maths interlocks with other parts ... if you become skilled in one area, the remaining areas become easier" (SC2_02). These characterisations are striking ones to hear from first year university students, since previous research has demonstrated how topics in mathematics are presented in a disconnected manner in post-primary school (Eaton, McCluskey, et al., 2011, p. 9; Eaton et al., 2014, p. 372; O'Shea & Breen, 2021, p. 36). Another MISE participant described mathematics as a "broad subject" (SC1_03), possibly for the same reason as SC1_11 who elaborated that "you have to cover and be well versed in everything to progress" (SC1_11). These extracts demonstrated that the connectedness of mathematical topics can be seen in a negative manner, since students must have broad knowledge across different areas to improve their skills in mathematics overall. The silver lining was that studying one area will benefit students when studying others.

Focus group contribution 7.
Mathematics consists of different topics which interlock together.

Breandán_SC1 explained that such understanding is particularly useful when it comes to answering Leaving Certificate exam questions:

> There are questions on the papers always that you need to have a good understanding of how to bring trigonometry into a question on differentiation, or whatever the case may be, that you need to be able to bring parts of what is, really separated (Breandán_SC1).

However, he confirmed that this connectedness was not emphasised in post-primary school: "you have trigonometry, coordinate geometry ... they're taught as if they're totally separate but ... you need to be able to go from one to the other very easily" (Breandán_SC1).

ENG_08 agreed that mathematics has interlocking parts, in that "you can't do (b) without knowing how to do (a)," but disagreed that this makes things easier. He proposed that as the material becomes more complex, it relies on more and more previous knowledge from many different areas of mathematics:

> The further you progress and the deeper you go, the more complicated the material becomes, because it involves, it relies, not only on one area of maths, but usually the material we are covering next relies on a massive amount of other methods we've learned previously (ENG_08).

Thus, Breandán_SC1 was concerned with the connections between areas of mathematics because these connections are important for answering exam questions. ENG_08, on the other hand, had developed a technological pragmatist view of the subject, where relational knowledge could be useful for applying a broad range of mathematics to solve complex problems, with regard to utility and expediency (Ernest, 1991, p. 118).

Summary and Insights

Schoenfeld (2020) argued that the key to students owning the product of their work in mathematics, is to make their thinking central to classroom discourse by allowing them to contribute to core ideas and refine their contributions in the course of classroom interaction (p. 1173). Such an approach aligns well with a fallibilist philosophy of mathematics, where students see mathematics as created, rather than discovered, and the role of the lecturer is to facilitate students in developing the skills to autonomously enquire about, and come to know, mathematics (Ernest, 1988). However, MISE participants mostly reported teaching that prioritised instrumental understanding, involving memorising and training themselves to follow disconnected rules through practising questions. Correspondingly, they presented only absolutist views of the discipline of mathematics, wherein mathematics is pre-existing, immutable, and objectively true, yet believed that its purpose is realised through application to real-life situations. Some participants saw mathematics and physics as a powerful combination for attending to this purpose, and subsequently, a technological pragmatist view of mathematics emerged more strongly through the focus groups. These results have significant implications for the teaching of science and engineering students, because their absolutist views of the subject may work contrary to pedagogies that encourage students to see themselves as active participants in the creation of mathematics in the classroom. In particular, it might be expected that such views are incompatible with the inquiry-based "discipline of exploration and sense-making" in mathematics envisioned by Schoenfeld (2020, p. 1172). Nevertheless, it will be presented in the remainder of this chapter that despite the absolutist views of mathematics evidenced in this theme, a focus on making sense of the real-world (applications of mathematics) directed many participants' attention towards relational understanding of mathematics (section 6.2.3), and towards solving unfamiliar problems (section 6.2.4).

6.2.3 Theme 3: Ways of Learning Mathematics

Since the questionnaire was administered in participants' first year of university, they tended to focus on their mathematical experiences at post-primary level. They made a distinction between "doing" mathematics and "understanding" mathematics. Participants described doing mathematics as involving instrumental understanding, with the aim of completing questions in class and in examinations, and mostly presented this in a negative light when considered on its own. Understanding mathematics was described as a separate, more comprehensive suite of knowledge, which involved a broader contextualisation of the subject, beyond simply completing questions. Their contributions appeared to align with findings from Solomon and Croft (2015), who found that second year mathematics students, across four universities, held conceptions of mathematical understanding as either relational/conceptional or involving applications (p. 271). This theme begins with a description of what it means to "understand" mathematics, by aligning participants' views with well-established research regarding relational and instrumental knowledge (Skemp, 1976). Subsequently, the role of real-world applications in understanding is developed, after which, participants' experiences with collaborative learning and seeking sources of learning outside the classroom is discussed.

The majority of MISE participants had experienced, and disliked, doing tasks without any element of understanding, and demonstrated a resistance to what Skemp (1976) called "rules without reasons." They decried being "told to do questions in the book with no explanation" (SC2_07) or learning without hearing the "reasoning behind what [they] were doing" (SC1_11). Classroom experiences that lacked these elements were viewed as hollow, frustrating, and lacking purpose, since "we just learned it for the sake of learning" (SC1_11). Both ENG_04 and SC1_03 suggested that although they were able to learn facts and to do examination questions, they subsequently sought to understand mathematics, and reported positive outcomes from the pursuit of understanding:

> Once I started to actually understand maths, rather than just do it, I began to really enjoy it (ENG_04).
>
> Understanding the maths we were studying instead of just learning off an equation. This definitely helps me when it comes to studying maths now (SC1_03).

The Meaning of Understanding

Participants presented understanding as involving relational/conceptual understanding, and/or knowing the applications of mathematics. In other words, "doing mathematics," means knowing what to do, whereas "understanding mathematics" means knowing why the process works, or why it is relevant to their studies in science, teaching, or engineering. On relational/conceptual understanding, participants suggested that understanding mathematics frequently involved a higher level of thinking, one where students are familiar with the "underlying concepts" (ENG_02), "learn through concepts rather than through questions" (ENG_04), and, almost exactly quoting the definition of relational understanding given by Skemp (1976, p. 20), understand "what to do, and why to do it" (SC2_08).

In the focus groups, participants were asked about the difference between doing mathematics and understanding mathematics. In attempting to explain the difference, participants often focused on formulaic exams as an inaccurate way to determine understanding. They expanded on the characterisations from Data1, that understanding means conceptual/relational understanding (FG contribution 8) or being able to apply mathematics to solve real-world problems (FG contribution 9).

Focus group contribution 8.
Relational understanding: seeing the connections between topics in mathematics.

PSTs in FG1 reasoned that understanding means making decisions about which mathematical concepts you wish to employ to solve a mathematical problem. In particular, one who understands is able to borrow concepts from different strands of mathematics, and decide which formulae or methods best suit the situation:

> We can take something from another section and use it here, like maybe geometry or something like that. When you're working with triangles like, there's a load of different formulas you could use to solve it, and different formulas (SC1_08).

This conception builds on the view of mathematics as consisting of interlocking parts, as discussed in section 6.2.2. For SC1_08, knowing lots of mathematical procedures is not enough to demonstrate this kind of mathematical thinking or planning. He presented an image of a student who does not understand, as one who makes uninformed or arbitrary decisions, rather than planning which steps might be most effective:

> They might just be like well, I'll try to do this step, this step, this step and see if it works. There's no understand ... they're just, oh there's this formula maybe I'll try that one or they'll run to the [formula and tables booklet] and hope for the best (SC1_08).

Fiach_SC1 similarly argued that doing questions was not enough to demonstrate understanding since "if you're able to do a question, [that] doesn't necessarily mean you can understand it" (Fiach_SC1). In particular, he proposed that this does not mean that you understand "how" or "why" your solution worked.

One PST highlighted that post-primary school teachers may not attend to students' desire for understanding, drawing on his teaching experience to say that "you don't necessarily teach for your students to have an understanding, you teach that they have an ability to answer questions" (Breandán_SC1). Lubienski (2011), writing about the Irish context during the introduction of the Project Maths curriculum, noted that examination pressure can subsume opportunities for problem solving and sense making in the mathematics classroom as the examinations draw closer (p. 40). The same pressure has been reported to consistently undermine efforts to resist methods that "teach to the test" across all subjects at post-primary school (Devine et al., 2013, p. 101). It is reasonable to conclude that Breandán_SC1 has a belief that he should teach for understanding, or that he would prefer to, but that this belief can become subordinated (Thompson, 1992, p. 135) to the pressure to prepare students for examinations which, he believes, do not reward understanding. Indeed, in his interview in section 7.2.2, the opportunity was taken to follow up with Breandán_SC1 about the role of examinations in the belief he expressed here.

For university students, moving beyond instrumental understanding of mathematics is important because they are expected to exhibit more relational/conceptual understanding and abstract reasoning compared to post-primary students (Clark & Lovric, 2008). Many MISE participants expressed an awareness of this change, and a preference for understanding mathematics. Other research in Ireland (Breen et al., 2013; O'Shea & Breen, 2021) has found that first year PSTs (primary and post-primary), who were taking a differential calculus module, were also aware of these different expectations, and that they found working on "unfamiliar tasks" to motivate and help them to develop relational understanding. The unfamiliar tasks were those for which students had "no algorithm, well-rehearsed procedure or previously demonstrated process to follow" (Breen et al., 2013, p. 2318), and such tasks will be discussed further in section 6.2.4.

Although SC2_08 expressed a preference for understanding what to do and why above, in practice he exhibited an instrumental approach to learning. He was more concerned with replicating the steps to solve questions: "I prefer every single step to be written down, without every step being written I will simply not understand how to do the equation/question" (SC2_08). Practising procedures to memorise the steps can indicate an instrumental approach to learning, because "when students learn a procedure without understanding, they need extensive practice so as not to forget the steps" (Kilpatrick et al., 2001, p. 123). Furthermore, students who see the solution to each different question/equation as isolated information to be remembered, may struggle to apply their knowledge to solve other problems that are not explicitly covered in class (Carpenter & Lehrer, 1999, p. 19). Indeed, SC2_08 acknowledged that he had experienced difficulty in tackling questions that were dissimilar to those he had encountered before:

> I could remember how to do questions that were posed, but as soon as that question became a hidden part of the problem, I could do it if I was told what to do, but figuring out what to do and where to start was challenging (SC2_08).

For SC2_08, understanding mathematics meant knowing how to apply it in "practical" situations:

> Its hard for me to visually understand calculus and its purpose ... I was always better off with project maths or practical mathematics especially the one involving geometry, like the length of timber needed for a roof, where there is only one length given and using geometry you can find the rest (SC2_08).

He further suggested that seeing "a graph of a practical use of an equation" can provide the motivation for studying such equations, but that practical uses of mathematics have rarely been presented to him by his teachers and lecturers.

Thus, SC2_08 preferred to understand mathematics in theory, but in practice he expected that he could take an instrumental approach and be shown how to apply mathematics in science thereafter, rather than preparing to make such decisions himself. As will be presented in FG contribution 9, Aodhán_SC2 described a similar viewpoint to SC2_08, with regard to knowing how to apply mathematics in science, while ENG_08 saw the value of relational/conceptual understanding because he regularly had to apply mathematics in his other modules.

Focus group contribution 9.
Understanding the real-world applications of mathematics.

As in Data1, participants were quick to point out that doing mathematics without understanding it feels hollow or pointless, with engineering and science participants being most vocal about this:

> Doing something, and not really understanding why you're doing it can be like fairly frustrating, you know ... we're never going to use this, why do we need to learn this (Aodhán_SC2).
>
> If you're just given a formula so this does this, and you get this as an outcome, like why do I need this? What's the point? (ENG_08).

For Aodhán_SC2 and ENG_08, understanding mathematics meant attending to real-world problems in science or engineering, as opposed to the mathematics problems discussed in FG contribution 8. Such questions require some agency on the part of the student, in choosing the best course of action:

> Yeah, like it's all, like, well and good to be able to like do a, say a question but once it's applied to like a real world situation, you being able to know which one, which formula or whatever to use (Aodhán_SC2).

In contrast, when it comes to preparing for examinations, Aodhán_SC2 reckoned that practice makes perfect, since "once you got the hang of a question, you could do it time and time again" and that "there's a certain way to do questions, most of the time." He hypothesised that the same mathematics questions "could be presented in a different way" if you are working on a building site for example. Thus, for Aodhán_SC2, university mathematics involved practising questions in the belief that their solutions will be useful for solving real-world problems in the future. He did not report struggling with unfamiliar questions like SC2_08, but at the same time, he suggested that he has not routinely encountered such questions in university.

ENG_08 also saw understanding as related to applications in his programme, believing that "if you don't see how you apply the theory, then you don't really understand how it works" (ENG_08). He suggested that understanding mathematics allows one to apply mathematics to solve a wide range of problems, perhaps using relational/conceptual understanding:

> I would also say there is a difference, because doing maths for us at least in our degree programme, it's a matter of getting your points and passing the exam. That's doing maths. But actually understanding it means going deeper, understanding how things work from the fundamentals. If you will have that kind of an understanding then you're going to apply the theorems or specific rules, or algorithms to solve a wider range of problems (ENG_08).

Previous research in DCU involving science education students found that they struggled with the kind of understanding described by ENG_08, with regard to differential equations, and that this could lead to difficulties in identifying and setting up such equations in a physical context, as well as interpreting their solutions (Hyland et al., 2018). ENG_08 was the only participant who argued that his examinations are designed to assess understanding, because he encounters unfamiliar questions rather than those that can be practised beforehand: "they give us unseen problems that we have to try to solve using the theory that we've been taught. ... We have to find our own way of solving them" (ENG_08). Moreover, he routinely has to engage in the process of applying mathematics in his other modules:

> We need to understand how things work because we regularly apply the same tools in other modules: electromagnetism, algorithms, computation, programming, it's applied everywhere. We can't just do it and forget about it (ENG_08).

Thus, for ENG_08, understanding mathematics is necessary in order to apply it in other modules, and to prepare for real-world scenarios by tackling unfamiliar questions.

Learning Outside the Classroom

As students move from close teacher-student relationships in post-primary school, to more impersonal, large, lecture-based teaching in university, relationships with tutors and peers become more important factors in their mathematical learning (Solomon et al., 2011). Solomon and Croft (2015) found that when they did not understand, second-year mathematics students turned to their peers for explanations or to collaborate in developing mathematical arguments together (p. 272). In first year, MISE participants presented no evidence of working collaboratively with their peers in post-primary school, or first year in university (although upon reflection in third year in FG contribution 10, Aodhán_SC2 described his Senior Cycle classes in mathematics and science as more collaborative than his university classes). Instead, some emphasised that they worked on their own, either because of a personal preference, or because they had no alternative:

> I tend to not ask for help on maths work as I prefer to get it myself (ENG_02).
>
> I had to learn a lot of it myself (SC2_10).
>
> It is between you and the numbers (SC1_11).

However, in post-primary school, some other participants availed of sources of learning outside the classroom, as well as relying on their teacher, to develop an understanding of mathematics. The most common recourse outside the school environment was private tutoring, known in Ireland as grinds. Although grinds have been reported to promote instrumental learning over relational learning (Prendergast et al., 2022, p. 334), participants reported experiencing grinds teaching which attended to either or both of these types. SC2_01 acknowledged that "maths is pretty hard and my aim for most maths exams is just to pass" and therefore, unsurprisingly, focused on learning for exams: "I did grinds for my leaving cert so that helped me improve on answering the questions" (SC2_01). SC2_08 reckoned mathematics became easier for him because his grinds teacher would explain how to do mathematics as well as why it works: "Maths became very easy only when I had a private tutor that would sit beside me and explain what to do, why to do it" (SC2_08). On the other hand, some participants reported that their grinds teacher helped with understanding concepts rather than questions directly: "My grinds teacher helped me a lot to understand the concepts, and she was an electronic engineer. That explains why I started to like it" (ENG_04).

In the focus groups, participants were asked about how they work in mathematics outside the classroom, and whether they work individually or in groups.

Focus group contribution 10.
Participants still mostly work on their own

Collaborative approaches to studying mathematics have been reported to develop organically among mathematics students at Irish universities (Eaton, Oldham, et al., 2011a, p. 161) with third year PSTs in mathematics education reporting working collaboratively themselves and wishing to encourage their students to do likewise because they "know it works" (Eaton

& OReilly, 2009a, p. 154). In the focus groups, MISE participants mostly presented the study of mathematics as an individual endeavour.

Aodhán_SC2 remarked that he works on his own by necessity rather than by choice. For him, the university experience had not been as collaborative as his Senior Cycle classes in mathematics and science: "if one person got it, he could explain it to someone else, whereas now it's just like, learn stuff off pretty much" (Aodhán_SC2). He attributed this partially to the nature of science. Biology is "pretty much rote-learning" while in chemistry "there's only so many ways you can explain it." He notably omitted mathematics from this description, which suggests that he believed collaboration in mathematics was possible, but he had not experienced this in university so far. He addressed his desire for more collaboration in his interview, in section 7.2.1.

SC1_08 described his preference for working by himself, at his own pace, in post-primary school: "I much preferred maths classes where it was like, get questions one to five done, and then you just get pleasure in yourself. You can go slowly" (SC1_08). Reflecting further on the experience of working independently with online question banks in university, he noted the benefits of the approach:

> I think maybe I got more out of it because I did more of it on my own maybe, I'm not sure ... For the first four or five I was really sort of trying them on my own and I think that helped me in the end (SC1_08).

Breandán_SC1 similarly suggested that he derived more satisfaction from figuring things out himself, rather than having it explained to him: "I like just the feeling of getting through stuff, just being able to look at something, not know how to do it at first and then just sit down, keep hammering at it, just getting out your answer." Although he mentioned working with classmates in the library, it was not as valuable to him as working alone, because they took shortcuts towards answering questions in their online question bank by comparing questions with predictable variables:

> What we did anyway, was sit in a group of probably 5 to 10 people, book one of the study rooms in the library, go in with 5 to 10 people and I'll do a question. I'll do the question get it wrong, lad beside me does it, puts in his answer, gets it wrong. You just keep going until you see that in the question there's a $3x^2$.

> And the answer that's right is that differentiated or whatever. So, you just go to everyone else's questions, put in, if the number is five instead of three, you change it accordingly (Breandán_SC1).

The little collaborative work they did undertake at this stage, was motivated by getting questions correct, as determined by the automatic online system, rather than furthering understanding. As noted by Schoenfeld (2004), structuring and supervising student interactions in a collaborative learning environment is difficult, but essential, if they are to benefit from the experience, especially if they are unaccustomed to learning in a group context (p. 272). In particular, MISE participants' comments suggest that PSTs may have preferred group work which facilitates them in making incremental progress towards understanding, with feedback or guidance from the lecturer, but also with space and time to make sense of mathematics individually. While evidence for collaborative work in mathematics was rare, the few examples that arose did not give a clear picture of students helping each other to understand, nor developing mathematical arguments together, as described by Solomon and Croft (2015, p. 272).

Summary and Insights

In this theme, participants described what it means to understand mathematics. Their descriptions varied in focus between relational/conceptual understanding of mathematics topics, and application to real-world problems in science or engineering. The notion that understanding mathematics meant relational/conceptual understanding was supported most strongly by Engineering and PST participants, who were concerned with applying mathematics to a wider variety of questions and contexts. In contrast, some Science II participants defaulted to instrumental understanding because they did not immediately see the applications of mathematics within their programme and expected this to become clear at a later stage of their studies. Although participants did not mention availing of grinds in university, the interviews in chapter 7 give some insight into their attendance of the MLC, and reliance on help from friends, neither of which featured in the themes drawn from Data1 and Data2. Upon reflection on their university experiences in the interviews, participants described other instances of collaborating with their classmates throughout their four years in university, which will be discussed in chapter 7.

The characterisations of what it means to understand mathematics that are developed in this theme, complement other research which has found that students perceive a much stronger emphasis on understanding mathematics at university compared to post-primary school in Ireland (O'Shea & Breen, 2021, p. 35). Faulkner et al. (2023) recently recommended that Irish universities continue the groundwork laid by Project Maths, and promote the concurrent development of relational understanding of procedures, alongside the procedures themselves, amongst science and engineering undergraduate students. This facilitates students in making decisions about "what procedures are needed in what situations" (Faulkner et al., 2023, p. 17), and thus in developing procedural fluency (Kilpatrick et al., 2001), rather than learning particular procedures for given situations (NCTM, 2014). The next theme builds on these ideas, by addressing participants' views of "getting started" on non-routine problems in mathematics.

In this theme, the link that was drawn between applications and relational/conceptual understanding has important implications for university practitioners in developing procedural fluency amongst undergraduates. For example, Petocz et al. (2007) argued that an instrumental view of mathematics as isolated computational techniques could be resisted with a stronger emphasis on broader conceptions of mathematics in engineering education, including abstraction, modelling, and applications to real-life (p. 456). In section 6.2.2, it was argued that students who focus on real-world applications are likely to search for real-world contextualisation when learning mathematics. Furthermore, in section 6.2.1, demonstrating the applications of mathematics was highlighted by participants as an important responsibility of teachers.

6.2.4 Theme 4: Mindset and Getting Started

This theme consists of two sub-themes: the states of mind that affect one's ability to study mathematics (**Mindset**) and the skills/abilities one needs to learn to solve problems in mathematics (**Getting Started**). Participants presented a particular mindset that they believed is productive for studying mathematics, which is discussed in section 6.2.4: Mindset. This mindset is built up over time and involved confidence, persistence, hard work, and not seeing mathematics as something to fear. There was widespread agreement that for university mathematics, natural ability is less important than developing the right mindset, and working on strategies that help when engaged in problem solving. The problems or questions to which participants referred, matched part of the definition of a problem given by Schoenfeld (1989): a task for which they do not have a “readily accessible mathematical means” to provide a solution (p. 88). In section 6.2.4: Getting Started, it is argued that participants considered it important to engage with the processes required to face such problems, including interpreting questions, and taking steps towards formulating a solution, acknowledging that there are several possible paths to a correct answer.

Mindset

Some important connections can be drawn between this sub-theme and the discussion of mindsets in section 2.1. The following participant expressed that they were motivated to study mathematics, and had confidence in their own ability, but that this requires hard work: “I know I can do it, and I want to, it’s just tough” (SC1_11). In this short statement, SC1_11 displayed elements of a productive disposition: “the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p. 5). A student’s productive disposition develops through opportunities to make sense of mathematics, by working on non-routine problems (p. 131). Such opportunities can promote positive attitudes and beliefs (p. 131), while the problems themselves may require several of the strategies which will be described in section 6.2.4: Getting Started. When students do not have opportunities to tackle challenging mathematical problems, they can develop the belief that mathematical learning is about memorising and start to see ability as fixed (Kilpatrick et al., 2001, p. 131). In this sub-theme, it will be presented that MISE participants appreciated the challenge of mathematics, and persisting with hard work, while examples of the fixed ability beliefs that have arisen in the literature (Solomon, 2007b, pp. 88-90) were hard to discern. The term **Mindset** was chosen to refer to the mental state that participants described as productive for studying mathematics. As is the case with a productive disposition, the mindset described by participants is developed gradually “over months rather than minutes” (Kilpatrick et al., 2001, p. 316), and incorporates the belief that achievement is the product of effort rather than fixed ability (p. 171).

Persistence and Hard Work

Some participants described mathematics as stressful, scary, or daunting, including some who connected the stress to the effort required:

> It is a scary subject and requires a lot more energy and effort than any other (Saoirse_SC1).

> It's the work that has to be put in to persevere is what is scary ... You have to cover and be well versed in everything to progress, which is obvious but still daunting (SC1_11).

Thus, two affective codes were included in this theme because of the direct links that multiple participants made between them and mindset: [53] *I find maths scary or daunting* and [54] *I find maths stressful, causes anxiety*. Thanheiser (2023) argued that such negative emotions can be associated with the view of mathematics as a set of abstract rules and methods, designed for the pursuit of a "right answer" (p. 4), and some participants demonstrated a resistance to this conception. Saoirse_SC1 wished to relay the importance of persistence to her future students so they can combat these feelings: "I want to encourage people to keep trying and keep practising and not to give up when your answers are wrong" (Saoirse_SC1). Another participant recounted experiencing such encouragement from his TY teacher, the "most influential person" in the formation of his mathematical identity, whom he praised for "not taking things seriously or putting pressure on students to get it right in first try" (SC2_09). Even students who were comfortable with, or enjoyed, mathematics acknowledged the effort required to continue to progress: "for me it became easier to understand maths, doing it was still effort full and hard" (SC2_08). Likewise, SC1_11 reported that he enjoys mathematics, but that the workload is "massive." He suggested that regardless of ability, everyone needs to work hard at university level, and that this is a "daunting" prospect. Viewing mathematics as requiring effort from everyone, is a viewpoint that appeared to develop over time, and participants presented a variety of degrees to which they felt they had a natural ability for mathematics at certain stages of their studies:

> It came naturally to me from an early age (SC2_02).
>
> The only reason why I got this far is that of a slight natural ability (SC2_10).
>
> I'm not naturally good at maths it seems but have just worked hard at it because I enjoy the topic (SC1_11).

Critically, participants also presented a belief that even those who have benefited from a natural ability in the past, have to work hard to be successful in university:

> I feel I have a natural ability for it compared to perhaps the average person but still struggle with the most advanced aspects (SC1_06).
>
> Everyone needs to work hard to succeed in the field of maths, as at this level it doesn't come easily to anyone unless you really are gifted (SC1_11).

Thus, participants acknowledged the notion of being naturally good at mathematics, and differed in whether they believed they possessed this ability, but indicated that university mathematics requires hard work to be successful. Focus group participants emphasised the latter point in even stronger terms, with PSTs utilising their new knowledge of mindsets to explain their reasoning.

Focus group contribution 11.
University mathematics requires effort and persistence, regardless of ability.

In FG1, the participants rejected the notion of natural ability leading to effortless success in mathematics at university level, and proposed, unprompted by the mediator, that a more relevant consideration is that of mindset. Breandán_SC1 pointed to persistence as key: "I would say that I like maths, so it wouldn't matter if I have three hours of maths ahead of me, I'll sit down and just stay at it" (Breandán_SC1). Together with SC1_08, they settled on the idea that outsiders often dismiss their hard work, believing that they are effortlessly good at mathematics, and do not realise the time and effort that is required at university level. They proposed that the idea of being naturally good at university mathematics, without much effort, is a common fallacy projected upon them by others:

> But there's still a thing in people's head about maths like. People will say "oh you're always good at maths" sort of thing. We've all heard that probably, but they don't say it about history or geography ... I just think you need to put in as much work to maths as you do for all the other subjects (SC1_08).

Fiach_SC1 said categorically that "everybody has to work hard at it," and that the key to this is a "growth mindset." Across both focus groups, all three participants who had experienced a teaching placement raised the issue of mindset, without being prompted to do so. This is a topic with which they are familiar from a second-year module, which is provided to Science Education students by the School of Mathematics. However, they were not alone in addressing the important elements of mindset, with Aodhán_SC2 stating "[t]alent doesn't work if talent doesn't work hard" and admitting that "you can be good at it, but it doesn't really translate too well if you don't put any effort into it."

The hard work required can be made easier with the right mindset: "I don't think it's a natural ability, might be a natural affinity, like that you'd like to think like that you approach it, that you think that you can do it" (Breandán_SC1). In particular, the ability to persist is related to confidence, and to self-belief:

> So, for somebody like me for example you take significantly more time to cover the same aspect, as compared to the person who is smarter than me, in plain English language. But I can do it as well. It's just, it will take me more time to do it" (ENG_08).

In FG2, participants similarly pointed towards self-confidence and the belief that you can grow your knowledge in mathematics as more important than ability:

> I don't think it's natural abilities as such. I think it's just the growth mindset allowing you to become better or more nat- seem more natural at it (Fiach_SC1).
> I think it's more so your confidence and your ability to be able to do something (Aodhán_SC2).

Those who described mathematics as not requiring effort, referred only to primary school, or Junior Certificate. Dónal_SC1 emphasised how effortless he found Junior Certificate mathematics, while ENG_06 remarked that he "didn't study for the Junior Cert in Maths because it seemed common knowledge" (ENG_06). While they did not specifically compare the level of effort required thereafter, both participants discussed how they struggled subsequently, in terms of results. Thus, MISE participants supported the notion that mathematics was effortless for a time, because of a natural ability, but after Junior Certificate, they came to see success in mathematics as requiring hard work regardless of natural ability.

Participants' rejection of the notion of fixed ability is notable amongst literature on identity and mindsets, in which, it has been reported that "students who are successful through hard work often think that they are imposters" (Boaler, 2016, p. 148). Solomon (2007b, p. 89) identified a dominant belief in the undergraduate community, that those who are good at mathematics are effortlessly successful, but MISE participants thoroughly rejected this viewpoint. Instead, the clear message from students was that a natural ability in mathematics means you do not have to work as hard as your classmates, which was most often reported at primary school level, and to a lesser extent at Junior Certificate level. This transition between effortless success at Junior Cycle, and struggling with results at Senior Cycle, is discussed further in section 6.2.5.

Confidence

As reported at the beginning of this section, SC1_11 suggested that his confidence, as much as his ability, was an important influence on his capability to persist in mathematical problem solving: "I know I can do it, and I want to, it's just tough" (SC1_11). Another participant highlighted that good exam results "made me enjoy the subject more, and gave me confidence and a belief that I could do well" (SC1_04), and that this confidence inspired her to believe that she could be successful in mathematics by being persistent: "I felt that there was no reason why I couldn't solve whatever problem was put in front of me, by thinking outside the box and being stubborn!" (SC1_04). The influence of exam or assessment scores on confidence was undeniable, and participants placed value on knowing whether their work was correct:

> [I] liked simultaneous equations as I liked to know that I was right to boost my confidence during the exams (SC2_03).
>
> The online homework giving the answers as you answer each question is good as you can easily see if you are making progress on a topic (ENG_02).

Solomon and Croft (2015) reported that mathematics undergraduates derived confidence in their work from several different sources: the lecturer, their peers, or themselves. They may passively receive such judgements from lecturers or peers, or they may control the authority of mathematics by developing their own understanding regarding such judgements (pp. 272-273). In FG contribution 4, it was presented that PSTs valued learning through examples, while feedback from their lecturers was important in determining whether they were making progress towards correctly implementing what they had seen in lectures. SC2_03 and ENG_02 above, described sources of confidence that were independent of the lecturer, and may be helpful to students when they are working on mathematics outside the classroom.

Focus group contribution 12.
Confidence can come from practising questions, or from the teacher.

> Confidence in answering questions was mentioned in both focus groups. Aodhán_SC2 was unequivocal that he derived confidence from practising lots of similar questions for Senior Cycle, and likewise at university: "I think it's more so I got the last one easy enough, so this one, I know the kind of process I have to work through and I know I can do it" (Aodhán_SC2). A PST disagreed with this approach, and suggested that his confidence

comes from tackling challenging problems, rather than working through lots of simple examples:

> Mindset, probably hasn't changed but approach has. Like I still come into maths thinking, I'm going to be able to understand what's going on here. ... I don't sit down and hammer through 50 easy examples first, and then move on to slightly harder, slightly harder ... rather than to go in at the bottom, keep working up, it's kind of just, go in at the top see if you can swim (Breandán_SC1).

ENG_08, an engineering student, had a different perspective on confidence in mathematics, and highlighted the influence of teachers on their students. He had attended post-primary school in another EU country, and over ten years later he applied for a further education programme in Ireland to qualify for entry into DCU engineering. He revealed that at post-primary level, since he attended a "language-based" post-primary school, he and his classmates were pressured into believing that mathematics should be avoided because they do not have the ability to do well in it. However, his experience of further education led him to change this mindset, and realise that he is more than capable, given the right support from the teacher or lecturer.

> We were always told that maths is just not for us. Ok, that's, that's the way it was being said. We were being told in plain English language, maths was just not for us, forget about it, you're not capable of doing it. But that was my mindset. That's what I was thinking, thinking of myself maybe just not capable, that's not for me. But when I went on to [university] and did that access programme, this is when I felt that this is not so, I can actually do it. So, it all depends on the teacher, or the lecturer who's teaching you (ENG_08).

It is relevant to note here that the impact of teacher expectations on students' identity is reflected in the literature. Eaton and OReilly (2009a, p. 154) found that high expectations of students are very likely to contribute to students' appreciation of mathematics, while Johnston et al. (2021) noted a similar positive influence on confidence, and that "decreases in confidence were associated with negative appraisal of the teacher" (p. 9).

Getting Started

Participants frequently referred to **Getting Started** as a springboard to solving a problem, due to the momentum generated by starting the process:

> I would find difficulty trying to start a problem but once I know how to start, I'd be able to finish it (SC2_12).
>
> I like writing out the steps when I know I'm getting near the answer, but I find it hard to start a question (SC1_08).

Reading across participants' data, it is possible to infer their understanding of "getting started" as a two-stage process which involves understanding the question being asked (analytical skills, breaking down a problem), and taking steps towards finding a solution (spotting ways to complete a question, seeing non-obvious methods, trying different methods). Firstly, one must use "analytical skills" (ENG_01) to understand what is being asked. SC1_07 observed that the "questions asked are not straight forward" (SC1_07) which suggests that he believed that there is a process of interpretation between being presented with a problem and beginning to formulate a solution by using "problem solving skills" (SC1_07).

Referring to her own struggles with getting started, SC2_12 reiterated this idea of a thinking stage: "Maths involves more thinking and trying different methods" (SC2_12). While some students may see this as shooting in the dark at first, hoping to stumble upon the correct solution, it also suggests that students may come to consider trying several methods to be illuminating with regard to understanding what is being asked. Schoenfeld (1989) highlighted that this aspect of "metacognition," which includes realising when it is worth spending more time investigating the problem before deciding on a route to its solution (p. 94), is an important part of problem solving.

After interpreting the question, participants described a second stage which involved "seeing the bigger picture. ... spotting ways to complete a question" (ENG_01) or using "lateral thinking" to "see the non-obvious method to answer a question" (SC1_07). Éabha_ENG expressed great delight in the fact that mathematics problems do not have a prescribed route: "I love how in maths there's only one correct answer but lots of different ways to get there" (Éabha_ENG). SC2_03 presented several mathematical skills that are useful for interpreting a question and probing possible entryways to a solution: "Ability to understand the question being asked, knowing how to deal with fractions. Graphs and functions are very important, factorizing equations and manipulating equations is crucial" (SC2_03).

Overall, participants reported that understanding the question being asked, and taking steps toward finding the solution, are important aspects of getting started on a mathematical problem. It is noted in the literature that there is no common definition of problem solving in mathematics education, and a variety of different types of tasks have been referred to as "problems" (Schoenfeld, 2016). For example, Lesh and Zawojewski (2007) included in their definition that problem solving is "a process of interpreting a situation mathematically" and iteratively interrogating the "givens and goals" of a problem (p. 782), which would align well with the focus that MISE participants put on "getting started." The seminal work of Schoenfeld (1989) regarding problem solving as part of "mathematical thinking," built on Descartes' ideas about "powerful mathematical practices," and Pólya's vision of problem solving as an inquiry-based vehicle for sense making, rather than a collection of strategies to be assimilated (Schoenfeld, 2020). Schoenfeld (1989) proposed six "metacognitive activities" that he used to track how experienced and naive problem solvers self-regulate their thought processes. The first stage presented by MISE participants is comparable to Schoenfeld's reading and analysing activities, which were found to feature heavily in a mathematics faculty member's approach to solving a problem but were completely absent from the approaches of two student groups (Schoenfeld, 1989, Figures 5.4, 5.5, 5.6). The second stage of "getting started" evidenced in this theme involved exploring and planning solutions, which were highlighted by Schoenfeld as two important activities for determining when to abandon a particular attempt at a solution, and reassess or try an alternative (pp. 98-99). It should be noted that MISE participants did not address these frameworks directly and did not discuss any concrete experiences of problem solving in school or university. However, as has been explored in this theme, the mathematical identity of these participants involves some awareness and appreciation of the processes required in mathematical problem solving, with particular attention to the initial stages of problem-solving models that are evident in the literature.

Summary and Insights

In this theme, participants indicated that their mindset affects their ability to get started with a question or problem, to persevere with multiple attempts, or to interpret and analyse the given information. This mindset incorporated several qualities that are indicative of a good problem-solver, including awareness of strategies for tackling problems for which they do not have a "readily accessible mathematical means" to provide a solution (Schoenfeld, 1989, p. 88), consideration of the best route to a solution, and a desire to persevere in such solutions (Schoenfeld, 2004, p. 263). Many of the factors that hindered the development of such a mindset, related to the daunting nature of mathematics, underpinned by the effort and persistence required, as discussed at the start of this section. Participants concurred that "[e]veryone needs to work hard to succeed in the field of maths" (SC1_11) especially at university level, and demonstrated what Dweck (2006) would call a growth mindset, reflecting the belief that while the effort required is daunting, anyone can reap the rewards if they persist: "one's ability in maths can be substantially increased due to practice and dedication" (Aodhán_SC2). The presence of such beliefs reinforces tentative findings in the literature that, overall, Irish post-primary students support the idea that effort can increase mathematical ability (i.e., a growth mindset, Dweck, 2006), and that older students are more likely to believe that mathematics problems sometimes require multiple strategies, which involve more than the simple application of routine procedures (Prendergast et al., 2018, pp. 1213-1215). Dweck (2015) recently felt compelled to clarify that a growth mindset is more than diligence in hard work or effort, and that students need multiple strategies to try when they are stuck to guide their efforts in a manner that allows them to learn from the experience. MISE participants duly presented their strategies for "getting started," which aligned well with the initial stages of problem-solving models that are communicated in the literature.

Working on problems is important because they help students realise the benefits of perseverance, see themselves as effective learners of mathematics, and provide important opportunities for sense making (Kilpatrick et al., 2001, p. 131). Thus, students are facilitated in developing a productive disposition, which aids the development of other strands of mathematical proficiency in the model developed by Kilpatrick et al. (2001). However, recent research involving science and engineering students at an Irish university suggests that an increased awareness and appreciation of the benefits of engaging with mathematical problem solving, has not resulted in a corresponding increase in problem-solving ability (Faulkner et al., 2023), as was intended by the Project Maths curriculum reforms (O'Meara et al., 2011, p. 334; DES, 2016, p. 20). In particular, although their participants saw the benefits of problem-solving proficiency, as was the case for MISE participants, Faulkner et al. (2023) observed that their performance on even simple problems was statistically significantly weaker compared to paired procedural tasks, which were drawn from the

Project Maths curriculum. In conclusion, MISE participants had some knowledge of early stages of problem solving, and appreciated of the benefits of the process, but further research, and/or intervention in university, is required to ensure that they can fully benefit from proficiency in this area.

6.2.5 Theme 5: Transitions

It was noted in section 2.5 that a narrative approach to mathematical identity facilitates students in identifying "critical events or episodes in their mathematical journey" (Eaton, Oldham, et al., 2011b, p. 38), and several authors have developed analytic processes that focus on such points (see section 2.5). In post-primary school, two transition points concerning examinations were evidenced by MISE participants: progression from Junior Cycle to Senior Cycle and the choice between ordinary- and higher-level mathematics for Leaving Certificate. Although different perspectives are represented, it should be noted that the majority of MISE participants had taken higher-level Leaving Certificate mathematics, as described in section 3.2.1. The transitions described in this theme, come under more precise focus in section 7.2, where a year-by-year account of six participants' university experiences facilitated attention to changes in their mathematical identity throughout that time.

Focus group contribution 13.
The spectre of examinations in post-primary school.

Previous research has highlighted the Junior and Leaving Certificate examination structure as a major contextual difference between mathematics at primary school and post-primary school and reported the strong emotional reactions that students have when discussing the "damaging role of the examination" (Eaton, McCluskey, et al., 2011, p. 8). In the focus groups, Breandán_SC1 noted the prevalence of the examination focus throughout post-primary school:

> The minute you go into secondary school, it's just hammered into you that you need to know this, you have an exam coming up at the end, at Christmas, at the end of the year, next Christmas at the end of next year into like, Junior Cert ... Leaving Cert it's the same that it's just drilled into you that you need to know it for your big exam that you don't have that at primary school level (Breandán_SC1).

From Breandán_SC1's perspective, rather than providing healthy motivation, the examinations placed pressure on students to take short routes to guarantee better results rather than to further their understanding of mathematics: "if someone is looking for good results in exams, there's always a thing in their mind if they don't understand at something straight away, that there's this kind of sense of panic about it" (Breandán_SC1). The strong focus on examinations in post-primary school can result in instrumental teaching which focuses on rote learning techniques (Eaton, McCluskey, et al., 2011, p. 8), limiting opportunities for students to understand mathematics (see section 6.2.3) or engage with problem-solving experiences that facilitate them in making sense of mathematics (see section 6.2.4).

Transition from Junior to Senior Cycle

As mentioned in section 6.2.4, participants pinpointed the transition from Junior Cycle to Senior Cycle mathematics as a difficult process, wherein mathematics became more challenging, and they reacted differently. For example, SC2_03 emphasised the predictability of the Junior Certificate exam and indicated that practising exam questions yielded good results: "The maths was pretty repetitive so once you knew how to do one question if it came up in the exam you were pretty set" (SC2_03). On the other hand, Saoirse_SC1 insisted that the standard is totally different between Junior and Senior Cycle: "the leaving cert maths course is far too much to cover, very intense and such a big jump from junior cert that it is ridiculous" (Saoirse_SC1). Although the number of students opting to sit higher-level mathematics for Leaving Certificate has increased in recent years (O'Meara et al., 2019), it has been reported that "a significant percentage of teachers ... believed that the Framework for Junior Cycle supported the transition from primary to post-primary school but in general did not believe it supported transition to Senior Cycle" (Mc Garr et al., 2022, p. 54). Indeed, one MISE participant wished that the time in transition year could have been used to prepare her for the mathematical content which followed in Senior Cycle, and perhaps feels unprepared for university mathematics as a result:

> I found that transition year [TY] could have been used to help me retain maths skills I learned previously and prepare me for what was to come in leaving cert ... When I entered third level, I expected I would be prepared from the leaving cert. However, I was

> very very wrong (Saoirse_SC1).

Although the Senior Cycle curriculum is not intended to be studied in TY, research has shown that many schools use the opportunity to get a head start on the mathematics curriculum (Prendergast & O'Meara, 2016). Interestingly, participants described responding to changes as they moved from fifth to sixth year, suggesting that as they settle into Senior Cycle mathematics, they gradually discover the differences in the requirements compared to the Junior Certificate and take their time to consider which level they should sit. For Dónal_SC1, mathematics had always "come a lot easier to [him] than other essay subjects" but Leaving Certificate higher-level presented a steeper challenge: "Higher level became difficult in secondary school as I came into 6th year so I dropped into ordinary" (Dónal_SC1). However, some participants enjoyed the higher level of difficulty and described themselves as rising to the challenge: "From 5th-6th year, I actually started to like maths as I enjoyed the challenge of figuring out problems, and also my teacher was extremely helpful" (Aodhán_SC2). The increased demands of Senior Cycle mathematics were reflected in section 6.2.4 also, where some participants described primary school and Junior Cycle as effortless, but none used the same language for Senior Cycle level. It appeared that an event horizon was crossed at upper post-primary level, where students who previously could get by with less effort, now needed to work harder to maintain good results in mathematics.

Language Barrier

Another interesting factor related to transition was described by two participants who attended Irish-speaking primary and/or post-primary schools. They suggested that learning through Irish created a language barrier for those who switched to English-speaking schools at some point of post-primary education and need to adapt:

> In fifth and sixth year at the beginning there was a language barrier as I only knew the maths terms in the Irish language (Éabha_ENG).
>
> A lot of maths were taught through Irish which I feel made it slightly harder to grasp certain topics as there was some difficulty in trying to translate the book and notes. When concepts were in English they were a bit easier to grasp. (ENG_02).

Both participants switched to English-speaking post-primary schools before entering Senior Cycle. Éabha_ENG switched schools in fifth year, and in her interview in section 7.2.5, she reckoned that by the time she went to university this language issue was no longer a factor. Although ENG_02 did not address the transition to university specifically, it can be inferred from the quotations that language was not a factor in this transition for either student who reported that they had studied mathematics through Irish in school.

Taking Control

The other points of transition or change in this theme did not refer to a specific point in time, but coalesced around participants taking charge, and making decisions about their own learning of mathematics. For example, SC2_08, referring to second year in post-primary school said, "*I moved myself* [emphasis added] to higher mathematics (2nd year)" (SC2_08). Language of the type in italics featured in other participants' descriptions also and was interpreted to indicate that they felt they had agency over their own learning, and acted on this agency. ENG_04 provided another striking example: "I was told to drop to ordinary [level]. After that, I took matters into my own hands, and came out with a H3[1] in the leaving" (ENG_04). Similar actions were noted in section 6.2.3, where participants sought out sources of learning outside the classroom, including Ciarán_ENG who said "I took it upon myself to improve my maths" by finding a great grinds teacher. In fact, in section 7.2.3 he describes how he went to several grinds teachers before settling on one who explained mathematics in a manner that, he felt, suited his learning style. Such changes were associated with participants moving away from the teacher in order to pursue the types of learning that would benefit their understanding of mathematics, or help them to answer exam questions, which is discussed in section 6.2.1.

Choosing between Ordinary- and Higher-level Mathematics

The entry requirements for university programmes played a part in students' taking control of their learning at Senior Cycle. It is important to note that a fail grade in ordinary- or higher-level mathematics, would preclude students from most university programmes. Applicants to engineering were required to have studied higher-level mathematics, unlike the Science I and Science II groupings. Participants from the two science groupings weighed up the extra points on offer, versus the effort involved in higher-level mathematics, as well as likelihood of achieving a passing grade:

[1] The grade H3 means 70-80% at higher level. The grade notation for Leaving Certificate is explained in appendix B.3.

> I dropped into ordinary before the leaving to ensure I would get a good enough grade (SC2_04).
>
> I sat higher level as the teachers in ordinary level classes were not that good and the extra 25 points for sitting higher level was a good incentive (Aodhán_SC2).

Participants noted a discrepancy between the difficulty of both levels of Leaving Certificate mathematics, suggesting that the gap is too wide: "I stuck with the higher level as I thought there was too much of a standard drop between [ordinary level] and [higher level]" (SC2_03).

Focus group contribution 14.
Difference in standard between ordinary- and higher-level mathematics.

The pressure felt by students in Irish post-primary schools to choose between ordinary- and higher-level mathematics has been reported previously (Eaton, McCluskey, et al., 2011). The literature shows that streaming classes for Junior Cycle affects students pathways for Senior Cycle, and, therefore, affects their access to further education and university (Smyth, 2018). MISE participants reported the decision between levels to have a stronger impact in fifth and sixth year (as evidenced above), and part of this struggle may have been caused by the perceived difference in standard between the levels:

> Just between higher level and ordinary level seems like a massive, massive jump in maths. More than anything else. ... with ordinary-level maths, for some it can seem way too easy, and then to the same person higher-level maths can seem way too hard (Aodhán_SC2).

Participants' choice between ordinary- and higher-level mathematics, was informed by which programmes they wanted to study after Leaving Certificate, and which careers they may want to pursue after that. Their reasons for choosing their programme are explicated in section 6.1.2.

Early Impressions of University Mathematics

Before the discussion of transition points is concluded, some attention should be paid to participants early impressions of their university studies. At university level, participants disagreed about the level of difficulty, when it comes to mathematics. Some thought that the trend of mathematics becoming increasingly demanding from Junior Cycle to Senior Cycle, had continued into university level:

> Now in [university] level education the maths level has gone up again in difficulty and a lot more study must be done to keep up to the standard needed (SC2_04).
>
> The jump in standards from secondary school to university level regarding mathematics is very challenging (SC1_11).

Two other participants reported feeling comfortable with the experience so far, because the content in first year was largely familiar from higher-level Leaving Certificate:

> So far, the majority of maths has been leaving cert standard so it's not that bad (SC2_01).
>
> In college now, I'm enjoying maths as it is relatively fine compared to higher level leaving cert maths and pretty similar in most cases (Aodhán_SC2).

As noted in section 6.2.4, the hard work and effort required to study mathematics at university level was a major factor in participants' determination of its difficulty. The interviews, which are presented in chapter 7, gave participants the opportunity to reflect on the key events and turning points that they encountered during the transition to university. Their contributions demonstrated that this transition is an ongoing process throughout their time in DCU (Tett et al., 2017), and that their imagined future after university gradually comes into view as they progress through their programmes.

Chapter 7

Findings: Narrative Analysis of Data3

Narrative interviews were employed in Data3, mainly with the aim to investigate the change in participants' mathematical identity over the course of their time in DCU. The interviews took place in participants' fourth, and final, year of study. The analytic process for developing personal narratives (see section 5.3) focussed on individual journeys, in contrast to the previous stages of data collection and analysis, which established themes that were common across many participants (see discussion of group narrative in section 3.2.3). The personal narratives are presented in section 7.2, but first, I present some observations from two cases that were not used to produce such narratives in section 7.1. In order to improve the readability of the chapter, since each section concerns a single participant, their quotations will not be referenced with their participant identifier but can be inferred from the title of each section. As was the case in chapter 6, since the interview participants from the Science I grouping were all PSTs, they will be referred to as such for clarity. The demographic information for the interview participants is shown in Table 7.1, which was extracted from Table C.5 (see appendix C.3).

Table 7.1

Gender and age of the interview participants at the commencement of the study in March 2018 (interviews were conducted approximately three years later).

Participant	Gender	Age
Saoirse_SC1	F	19
Fiach_SC1	M	19
Aodhán_SC2	M	19
Breandán_SC1	M	19
Ciarán_ENG	M	18
Dónal_SC1	M	20
Éabha_ENG	F	18

The full list of modules discussed by the interview participants is included in appendix H.7, while details of the contexts within which they learn mathematics were discussed in section 3.2.1, and for science and engineering students more generally in section 2.2.

7.1 Exceptional Cases

In this section, a discussion of the pilot interview is presented, as well as one interview from the final stage of data collection (Data3), from which I did not construct a personal narrative.

7.1.1 Pilot Interview

The main purpose of the pilot interview was to evaluate whether longitudinal change in mathematical identity could be better assessed through interviews rather than through focus groups. Since this step was exploratory, the interview protocol, as explained in section 4.4, was significantly more open-ended than the resulting protocol used for the final narrative interviews. The pilot interview was conducted during the participant's third year, and did not include a review of mathematical experiences year-by-year, nor did it feature the four-part design intended to draw-out participants' comparison of their current and former mathematical identities. For these reasons, it was not considered appropriate to compose a personal narrative for comparison with those in section 7.2. Nevertheless, a wide variety of elements relating to mathematical identity were discussed, and one particular issue was judged to be of enough importance to be presented in this section.

Saoirse_SC1 presented many negative emotions around mathematics in Data1. The key transition to Senior Cycle, described by her and other participants in section 6.2.5, involved a negative turn in her perception of herself and mathematics from one of reward, natural ability, and enjoyment to one of great struggle: "[W]hen I entered senior cycle my relationship with maths completely changed. I went from being bored by how easy maths was to really struggling with it." During her first year in DCU, this trajectory seemed to continue, and she expressed strong negative perceptions of her own disposition, ability to understand, and the amount of effort required to keep up with her classmates:

> I loved maths and now I'm afraid of it. When I entered third level I expected I would be prepared from the Leaving Cert, however, I was very very wrong. Maths now drains me. No matter how much I am trying to engage with it now I just can't grasp the basic concepts. Maths is very heavy and stress inducing in comparison with other subjects. Its daunting as a woman to study maths as the guys just seem to understand it so easily. It is a scary subject and requires a lot more energy and effort than any other.

As part of mathematics support in DCU, all students studying mathematics have access to the Mathematics Learning Centre (MLC): a drop-in service where in where students are encouraged to collaborate and make progress on their work with the guidance of the tutors (see section 3.2.1). In a large-scale study in Ireland, Ní Fhloinn et al. (2016) found that female students are more likely to utilise mathematics support, with practical concerns about the location or opening hours of the centre reported as the main obstacles to seeking such support. However, Saoirse_SC1 gave new insight into other potential barriers in this regard, which related to mathematical identity. Among her descriptions of her negative emotions around mathematics, were two references to the MLC, indicating that part of her negative feelings about studying mathematics in university were related to attending the centre:

> I find the maths learning centre very daunting as everyone in there seems to be male. ... I need to get over my fear of the maths learning centre and seek help before its too late (Questionnaire Response, Data1).

Initially, I wondered if her comments referred to the gender of the tutors from whom she would seek help. In the pilot interview, the opportunity was taken to follow up on whether she had avoided seeking help in the MLC with other classmates, and to investigate the experiences that evoked this early negative perception of a space that strives to be an inclusive and accessible environment. However, she clarified that most of the students in the MLC were male, and proceeded to explain why she found the centre "very intimidating" in first year:

> I remember sitting at a table and it was full of these, like it's just full of guys and they were all like chatting about girls and like, just the kind of attitude ... the kind of lad banter ... I would have seen in first year and I just didn't like that.

It appeared that this impression was not a one-off occurrence: "I went a few times the first year, and discovered I did not like it." Instead of attending the MLC, her classmates would collaborate via text when working on assignments. This highlights an important role for the tutors in the MLC in balancing students' freedom to discuss and collaborate in the centre, with the rules about social interactions that maintain an inclusive atmosphere for everyone who comes in for help. Over time, Saoirse_SC1 and her classmates began to use the MLC more often:

> I noticed it's changed a good bit, I think anyway.
>
> Now we can just go and we go to the maths learning centre and we collaborate there.

She had identified a particular issue she had with calculus, which stemmed from missing post-primary school during a difficult personal time. Thankfully, she demonstrated resilience in pursuing help with Calculus in the MLC despite her earlier, negative impressions of the atmosphere in the centre:

> After that calculus course I did, I kind of just went to the maths learning centre again. And ehm, I think I went in second year and again in third year and was like, look, just help me do some ... differentiation, integration.

Much like teachers must manage the dynamics of the interactions in their own classrooms, so must the MLC tutors ensure that students are aware of what is acceptable to discuss in the shared MLC space, if it is to be accessible and welcoming to students who are new to the environment, as well as to those who are comfortable attending and working there regularly. For Saoirse_SC1, this experience triggered complex issues around the perception of mathematics as a so-called male domain (Brandell & Staberg, 2008, cf. Mendick, 2005, 2006), but it is also representative of a broader problem to be addressed in mathematics support more generally. Saoirse_SC1 had previously explained in her questionnaire response that "guys seem to understand it so easily," and clarified her perception during the interview as follows: "Well I don't know if they find it easier. But I've just seen that I don't think they get stressed out about it as the rest do." The presence of male students in the shared space, who perhaps were working independently of the tutors and felt comfortable enough to socialise in that setting, may have caused her to feel alienated from the space and created an immediate perception of "not belonging" (Solomon, 2007b). Thus, although the undergraduate community of practice can be refigured using the physical space of the MLC (Solomon et al., 2010, p. 429), it should be acknowledged that facilitating both collaborative group-work and direct interaction with tutors in the same space, may create tensions between these different communities of practice. Saoirse_SC1's comments suggest that this may have particular relevance with regard to new attendees to mathematics support services who perceive that other groups have "colonised" the space, in a more negative sense than that in which Solomon et al. (2010) used the term.

The data gathered in this pilot interview demonstrated the power of the narrative interview method to uncover important details about participants' powerful experiences that affect their mathematical identities, especially those that are difficult to describe in only a few words. Through the pilot interview, some of Saoirse_SC1's positive experiences with mathematics since first year were revealed, but so were her negative experiences with the MLC, as described previously in this section. The accessibility of the MLC is an issue of broader interest to the university that frames how students interact with the resources made available to support their learning of mathematics, one which would not have been uncovered without this second point of data collection from this participant. Two of the ten stories identified in the pilot interview related to the MLC and thus, even though the interview process was still in development, important stories emerged from the pilot interview relating to mathematical identity.

7.1.2 Interview 6: Fiach_SC1

The interview with Fiach_SC1 built on his contributions from Data1 and Data2, meaning he was one of only four participants to volunteer for every stage of data collection for which they were eligible. In the notes made immediately after the interview, I observed that overall, it felt "flat" and "empty," as though we did not get to the heart of many of the topics we discussed. After I completed the analysis of this interview using the storying stories framework (see section 5.3), I noted that it had the fewest stories of all six interviews: I identified only two stories whereas the others had between five and 10. This corroborated my initial impressions which I had written in the post-interview notes. The analysis using the narrative processes described in section 5.3

showed that the interview was dominated by the description process (with much less story, argument, theorising, or augmentation) and included lots of detail about placements, projects, and modules, but little evaluation of these experiences.

While reflecting on these thoughts, I returned to the data to investigate why this interview felt anomalous among the others. I observed that Fiach_SC1 often phrased critical views of his experiences in the passive tense rather than the first person. For example, while discussing a fourth-year module in abstract algebra, I followed-up with the participant to inquire whether the passively phrased opinion matched his personal opinion, and I was surprised by his response:

Fiach_SC1: [Abstract Algebra] was probably another one of them modules where you're kinda, where you are kind of asking, you know, I'm going to be a teacher ((laughs)), I'm not going to have to teach this. But like, but at the same time there was still algebraic general algebraic ehm, rules and stuff that you need to know that we need to apply. But ehm [???]

FH: [What do you], what do you think about it yourself?

Fiach_SC1: The module?

FH: Hmm, the content. I mean, do you think, you know, it's "I won't have to teach it, so it's not very interesting" or what do you think about it?

Fiach_SC1: Ah no, I wouldn't say it's not interesting now but ehm, it was interesting, I did enjoy it, ehm (...1) Again working with, with numbers working, and it was, abstract algebra is a lot more like general terms ehm rather than (...1) rather than numbers. So, a lot more general terms you're working, generally with ehm, with scenarios and stuff so that was ehm (...1) that was interesting. It was challenging.

It emerged that his own opinion differed from the one he was putting forth, suggesting that the argument he presented was a dominant discourse amongst his undergraduate community with which he did not fully identify. Although he expressed that he found abstract algebra interesting, he did not specify why this was the case, or whether he saw the content as relevant for his own teaching. Furthermore, he seemed surprised to be asked about his own opinion, suggesting that he either thought this was beyond the scope of the interview as I described it, or that he preferred not to talk about such opinions, and to focus on the details of the module instead. Another valid interpretation of this interaction might be that Fiach_SC1 did in fact identify with the view of abstract algebra as less relevant for teaching, but that he tempered his opinion when challenged in order to appease the researcher. It is possible that Fiach_SC1 saw me more as a member of the School of Mathematics staff than a student researcher, and, therefore, was reluctant to criticise those who he saw as my colleagues.

Thus, while I felt I learned about his social context, attributing personal opinions to this participant was difficult to do with confidence. Although I attempted to proceed through the steps and compose a personal narrative from this data (described in section 5.3), the infrequent evaluations (via stories) and the unreliability of potential inferences from general statements risked rendering the process untrustworthy. However, there are some elements of the interview that touched on issues raised in earlier stages of data collection, or in the other interviews, and these will be presented in the remainder of this section.

Collaborating with Classmates

As was the case with his submission to Data1, in the interview Fiach_SC1 paid no attention to the nature of mathematics (section 6.2.2), and very little attention to the ways of learning mathematics (section 6.2.3). In parts C and D of the interview, he focussed on his experiences of online learning and teaching during the COVID-19 pandemic, as well as his views on good teaching more generally, which will be discussed below. This may have been influenced by the researcher, since I asked him more questions related to teaching because of his earlier reluctance to critically appraise his learning experiences in DCU. However, he did shine some light on the ways in which this cohort of PSTs worked collaboratively in mathematics.

As well as working on exam papers and continuous assessments in first year, when Fiach_SC1 and his classmates met in a group, "sometimes it was just general, kind of keeping up with content of modules." Although they would sometimes work in the MLC, they also used the study rooms in the library, as mentioned by focus group participants (FG contribution 10 in section 6.2.3). Fiach_SC1 described how they worked together, saying that they "always kind of studied together in maths anyway. We all just kind of studied and bounced stuff off of each other." He appreciated this outlet as the first port of call when studying mathematics, and attributed the organic development of the collaborative group to the small size of the cohort in the Science Education programme:

> I like the support network, I like bouncing off ehm, like, my friends first, if you if you like. So, it's nice that whatever there is in our, in our course. There was 16 or 17 and all of us done the maths components so it was nice having that kind of small network.

He also noted that he was less likely to go to the MLC on his own, preferring to attend with the rest of the group. In second year, they were motivated to spend more time in the MLC to work on the "tough" Linear Algebra module:

> It was toughly marked, it was tough, tough [continuous assessments] in it you know. It was, it was I just found it tough all round ... We went to the maths learning centre after, I think I spent a lot of time in there for that module alone in that particular semester.

He noted that "lots of people in the class disliked Linear Algebra" and indeed, this module also features heavily in the interviews given by other PSTs in section 7.2.2 and section 7.2.4.

Being a Good Teacher

Much of the interview focussed on what constitutes good teaching, and what teachers should do to keep students engaged in mathematics. Fiach_SC1 described how he enjoyed modules that helped him identify ways to do this with his own students in his teaching placements. He felt confident that he is ready to teach probability and statistics because of the second year module in DCU, and in particular that he is ready to explain the relevance of the topic to his future students:

> I think I enjoyed the fact that we could see the real-life applications of it. And I know as a maths teacher, you're always asked, you know, what do we need this for and all. So, it was good to have that experience of myself, appreciating the application of it in real life. So I can like, show that to students in whatever it is, in probability, in geometry you know, in coordinate geometry and stuff like that so we, I can explain, describe, and show the real life applications for it.

Feeling unprepared for teaching mathematics arose as part of a theme in previous mathematical identity research in Ireland, discussed in section 2.6. Fiach_SC1 hypothesised that as students progress through post-primary school, more and more they ask, "what do we need to do this for?" or "what'll this do for us?" He regarded it as important to have examples of where, and by whom, mathematics is used in the real world in order to bring life to textbook mathematics:

Fiach_SC1: Yeah, you're always trying to make sure you have examples, make sure you have maybe people who use this kind of work.

FH: Sure, yeah, it's much more than just knowing how to do it.

Fiach_SC1: Yeah, exactly, it's a lot more than number crunching in a calculator.

He pointed to a group STEM education project that he completed in third year as enlightening because they analysed how well they engaged their students during lessons:

> [It] gave us the chance to sit down and reflect on the methods we used, the activities we did, the student perspective on it. ... Were students engaged? Were students disengaged? And stuff like that. So that was something that we picked up on, and it was nice to get that opportunity to kind of focus in on your own teaching.

Peer-assisted tutoring and lesson study have been reported as important resources for the development of PSTs' pedagogical content knowledge (Ní Shúilleabháin & Meehan, 2022). To further underline his focus on his future as a teacher, he pinpointed the modules in teaching and assessing mathematics as experiences that helped him to develop the skills required of a teacher:

> He had real good like software of showing us how to assess students in completing their proofs. How to, how to do the proofs, how to, you know, what things to look out for with students. So yeah, that module was definitely worthwhile as a teacher.

The quotes in this section suggest that Fiach_SC1 was fully focussed on his efficacy as a teacher at this stage of his studies, and he spoke from the point of view of an informed professional who used his own learning experiences to inform his teaching.

Teaching and Learning during COVID-19

Many of the issues to do with the COVID-19 pandemic discussed in this section are reflective of themes identified by practitioners across Europe in other literature (Fitzmaurice & Ní Fhloinn, 2021). The onset of the COVID-19 pandemic put paid to many of the familiar forms of teaching and learning that could be employed by students and post-primary teachers alike. PSTs experienced online teaching as both a learner and a teacher, and Fiach_SC1 noted that the move to "blended, online learning" required MISE participants to adjust to working from home, mostly on their own: “the kinda, group aspect, the group helping, the group maths, if you like, kind of fell out the window.”

Fiach_SC1 attended some one-to-one online MLC sessions (the only type of support available from the MLC at that time) but found that this was lacking the vital interactive characteristic which, arguably, makes mathematics support so helpful:

> I preferred being among people, I prefer being, looking at whiteboards rather than looking at poor handwriting on an iPad. I liked being able to hop into the Maths Learning Centre. I know it was, it was on, you could go on Zoom and stuff, but I just liked again the group feel ... of engagement with people, we’re solving stuff out together ... as a group.

Assessment processes in the university, as was the case around the world, were modified to include more continuous assessment in place of proctored closed-book examinations, which were much more difficult to implement online. This was a particularly seismic change for students in mathematics modules, which now required students to sit assessments throughout the semester instead of the traditional final examinations: “I was happy with that kind of approach at the start. But then ... having a 15% or 20% ehm exam or [continuous assessment] every ten days, two weeks. It was kind of, it was a lot to be fair.” Understandably, although he “can see the advantages ... of continuous learning,” he suggests that a mixture of both would be fairer in mathematics modules in which more time is needed to get up to speed and understand the material.

With regard to online lectures, Fiach_SC1 appears to have disengaged quite quickly, because of the nature of their delivery through zoom:

> There was kind of, there was a lack of motivation there in fairness. ... it was easy to say that this one’s gonna record it and I can watch it later. ... I didn’t fall into that trap too often, but I definitely did do that a few times.

He also offered some insights from a teacher’s point of view, based on his own experience of online teaching during placement. Firstly, the medium of delivery presented a barrier between teacher and learner, and evoked a feeling of speaking into the void:

> It felt like you were talking to yourself. There was, although there was engagement, work was being done, there was no conversation. There was, a lot of students were reluctant to talk into the mic. They used the chat function a lot.

Secondly, he emphasised the forms of continuous, informal feedback that teachers use while teaching that were suddenly made unavailable to them through this medium:

> There was assessment, but you weren't sure how students were really understanding it. So that affected you know, the speed you went at, the plan you had. ... you couldn't gauge body language, you couldn't gauge feedback, like expressional feedback, you couldn't gauge if they understood that or did they copy the example you did on the recorded video.

Insights into Fiach_SC1's personal mathematical identity journey were limited because of a lack of evaluation in his responses, and because the narrative analysis revealed only two stories from which to extract such evaluations. The first of the two stories concerned an inspiring post-primary teacher, whom he appreciated for staying after school to help him and his classmates with mathematics, even whilst the teacher was "going through a tragic time himself." The second story involved the STEM education project mentioned previously, which allowed him a means of reflecting on his own teaching practice in terms of engaging his students. Overall, he did not appear to struggle in terms of results but characterised several mathematics modules as "tough," and indicated that working with others on such modules was of great benefit to him. He was cognisant of how his own knowledge of the applications of mathematics (probability and statistics in particular) was useful for encouraging his own students to engage in the content, and generally seemed more comfortable discussing the elements of sound pedagogy during the interview. While the comments from Fiach_SC1 discussed in this section do not shed particular light on his personal journey, they provide important social context that is applicable to this cohort of PSTs in the first instance, and spotlight several factors that affect students right across the university. The personal narratives of the other participants can now be considered in section 7.2 against this backdrop.

7.2 Personal Narratives

In this section, the personal narratives for five participants in the narrative interviews of Data3 are presented. The relevant modules for each participant were discussed in the introduction to this chapter and are included in appendix H.7. The analytic process for developing these narratives is explained in section 5.3.

7.2.1 Interview 1: Aodhán_SC2

Aodhán_SC2 is an Analytical Science student who arrived in DCU having built up his confidence outside school by attending summer courses in the Irish language, conducted in Irish-speaking (Gaeltacht) areas: "After the Gaeltacht and TY, I was able to go back and I just felt like I could raise my hand in class and actually ask the question and not feel like an idiot doing it." His new-found confidence was the catalyst for an "inward journey" where he gained more control or agency over his mathematical learning when he returned to school: "I gradually realised, I could do better ... feeling I could ask more questions, which led to me wanting to ask more questions. And then I eventually did, and I started to really enjoy [mathematics] then." This journey culminated in Aodhán_SC2 making the unusual move from ordinary-level to higher-level mathematics for Leaving Certificate, and enjoying the challenge that this afforded him:

> In terms of pushing yourself in testing, or thinking like logically about a problem, and going through it step by step, and then maybe getting wrong and looking back over and seeing oh I could have done this different, the whole kind of process behind it was good. I don't think ordinary would have challenged me as much.

Aodhán_SC2 described how he prefers more logical subjects (like chemistry and mathematics) that involve identifying errors and fixing them, since there is a "set way of where you could have improved," unlike in Irish and English. He also emphasised the rewarding feeling that comes from hard work in mathematics by "pushing yourself" to meet the challenge of the higher-level curriculum. Another participant, SC2_08, also described moving himself to higher-level mathematics in section 6.2.1. In recent years, post-primary curriculum reforms have resulted in a larger percentage of students choosing higher-level mathematics for Leaving Certificate (O'Meara et al., 2019, p. 222), but in 2017, when these participants sat their exams, the number of students who chose ordinary-level mathematics was almost double the number who chose higher-level

(SEC, 2017). Thus, the move of these two participants to higher-level mathematics highlights the changing landscape of mathematics education in Ireland, driven by Project Maths and the bonus points scheme (see section 1.2 for a discussion of these initiatives).

First Year

The year-long first year Calculus module was "very broad" and covered a lot of topics, but it shared some similarities with Senior Cycle mathematics. Aodhán_SC2 referred to his continued use of the mathematics formula and tables booklet, which students rely on for use in post-primary examinations, and which is often made available during university examinations in DCU. He mentioned the indices page of the booklet, which was useful for "the powers, the exponentials and how they multiply and divide and add together," and the differentiation page, which has direct relevance for any calculus module: "there was definitely a lot of that in the Leaving Cert." He noted that he did not experience any overlap between his first year mathematics modules and others in biology, chemistry, and physics: "it was all about like each topic in itself, there was never really any real overlap ... even physics as well there was no real overlap there at all." Although he thought that the overlap between the disciplines of mathematics and science is "pretty sizeable," he appeared to accept that mathematics modules were separated from science modules without rejecting either as irrelevant for his studies, perhaps because of his passion for both. Indeed, he paints a picture of first year mathematics as mostly forgettable: "I can't even really remember first year. I don't even remember like the classroom or the teacher, or anything." It was "more the maths from [the physics side of things] that kind of stuck with me" rather than the mathematics module itself. In these extracts, Aodhán_SC2 demonstrated a belief that he retained pertinent mathematics knowledge from his non-mathematics modules because it was more immediately useful than the content in the mathematics module.

Second Year

In second year, the mathematics modules moved away from "Leaving Cert type things" and began to overlap with other modules: "It was more branching out into what we were doing specifically." Aodhán_SC2 gave contradictory explanations about whether the content from the second year mathematics modules "came up a lot" in subsequent modules or only rarely. Contradictory answers can suggest a process of identity change, as the narrator strives to make their narrative coherent for themselves and for the listener (Linde, 1993; Watson, 2006). Such contradictions could rise to the surface if the narrator has not yet reflected on these experiences in the context of their entire journey through university. Only at the end of their journey can they explore this perspective and interrogate the connections between their past and present mathematical identities (Lieblich et al., 1998, p. 7). He reflects on second year module which was relevant to his studies in the third year section below.

Third Year

In third year, Aodhán_SC2's work placement (INTRA) involved working in a laboratory making up solutions of different concentrations and using "analytical software to interpret the results." He reasoned that this work did not involve mathematics, and also played down the role of mathematics in his laboratory work (labs): "I think you're learning new things, but you're also relearning things that you forgot before, like even the simplest things ... it's maths in a different way." This may suggest that remembering basic mathematics skills, perhaps from post-primary school or first year in university, is more directly relevant in labs than using more advanced mathematics from current or recent modules. However, his comments may be related to his perception that mathematics as experienced in the classroom was separate from the applied mathematics he used in practical situations. He explained that mathematics now arises in scientific contexts, such as mass spectrometry, rather than within subjects like geometry or calculus that "you'd associate with just pure maths." For this participant, "pure maths" ended in second year, and the mathematics he encountered in other modules in third year was difficult to identify as mathematics, because it was contextualised in science, hence he introduced the "pure maths" terminology to distinguish between them. The difference might be explained by different praxeologies (see section 2.2) that are employed by lecturers in science and engineering departments, compared to those in mathematics (Bingolbali et al., 2006; Pepin et al., 2021), as well as similar discrepancies in relevant textbooks (González-Martín & Hernandes Gomes, 2019).

While the qualifier in the phrase "just pure mathematics" could suggest a lower position for mathematics modules, further conversation suggested that this was a term of affection. Aodhán_SC2 explained how content from his second year Probability and Statistics module arose in a third-year chemistry module and "[t]hat's when we kind of realised ... it does overlap now. I see why they taught us that." Despite previously saying that he did not like the subject in Senior Cycle, he remembered the "pure mathematics" from the Probability and Statistics module fondly: "compared to what I'm doing now, it was actually one of the more enjoyable modules that I did." Despite his earlier ambivalence, after reflecting on his experiences over the entire four years, Aodhán_SC2 came to see more connections between his mathematics and science modules. He concluded that "it's rare I would have like seen something maths-based in like a chemistry module and hadn't come across it already."

Affinity for Mathematics

While discussing second and third year modules, Aodhán_SC2 summarised his mathematical identity as "I do like maths, but I don't think I'm necessarily like very good at it." This was surprising given that he had chosen to challenge himself with higher-level mathematics for Leaving Certificate and appeared to have managed university mathematics without any problems. It transpired that he was comparing himself to a PST friend, and the comparison lead him to position himself as someone who can "get by" in mathematics in comparison: "she would give me grinds every now and again, and I just kind of see like, okay, I really, maybe just out of the loop on this one. I did enjoy it though." Such comparisons can be interpreted as Aodhán_SC2 establishing a positional identity (Holland et al., 1998) which is grounded within his own community of friends who study mathematics in university (Hodgen & Marks, 2009, p. 33). He appeared to somewhat alienate himself from others who are engaged in the activity of mathematics (Solomon & Croft, 2015) for reasons that were not related to grades, and positioned himself as a "receiver" of education from his friend (Boaler & Greeno, 2000, p. 173). The bookend statement "I did enjoy it though" serves to demonstrate that he wants to be seen as a student who has an affinity for mathematics, regardless of his perceived social positioning in relation to others.

Fourth Year

Aodhán_SC2, who reported in FG contribution 10 that he had found his university experience to be less collaborative than his experience at Senior Cycle (see section 6.2.3), also gave a lonely impression of working through a science degree in fourth year. Although he had weekly group workshops, he was "pretty much just working on [his] own" and rather than a hindrance, a solo approach may actually have benefited him when it came to progressing through university: "In terms of like my own development in my own course, I think working alone definitely suits me better, yeah." This may have been magnified by the COVID-19 pandemic, during which the interviews were conducted, because students stayed away from the university and teaching was conducted almost entirely online: "you can't really ... meet up that much or really do anything." In placement and coursework, he may not have collaborated much with others, but an image of science as a collective endeavour persisted in his narrative: "I think one of my lecturers said you go faster alone but you go further together." Aodhán_SC2 mentioned in his pre-interview meeting that he would be interested in pursuing an MSc programme at some point in the future. Despite his belief that he worked best alone, when I asked him about how he envisioned his potential future studies he expressed a desire to work as part of a group:

> I think it would definitely have to be collaborative to an extent ... but I'd definitely like to, like, work with other people, even in just like a job ... And if everyone's on like the same kind of page and you're all like understanding the same concepts.

Summary and Insights

Seeking to be challenged, and rising to the occasion, was identified as a common reason behind participants' affective statements related to mathematics (section 6.1.1), and was exemplified by Aodhán_SC2, one of two MISE participants who described moving themselves from ordinary- to higher-level mathematics for Leaving Certificate. He started asking more questions in class, saw his grades improve, and began to enjoy studying mathematics as a result. With regard to changes in mathematical identity at university, in first year, he described how his science modules seemed disconnected from the mathematics modules, and that he retained pertinent mathematical knowledge from his science modules, rather than the mathematics ones, because they did not seem to overlap. In second year, however, he began to see some connections when the mathematics modules branched out to cover what he specifically needed to know. By the time we discussed third year, he re-evaluated his initial assessment, and concluded that most of the mathematics content that arose in science modules had been encountered before in mathematics modules. This gradual change could be explained by different praxeologies experienced in first year modules (Bingolbali et al., 2006; Pepin et al., 2021), or by mathematics content featuring more heavily in modules in subsequent years. In any case, it appeared that the interview process stimulated a reorganisation of Aodhán_SC2's mathematical identity as he reflected on his overall experience:

> I think maths will come into play in a certain degree whether I realise it or not. Even as I was talking to you, I kind of realised okay, maths did come up more here and there than I thought initially.

Around second and third year, he had received mathematics grinds from a friend, which was the only instance of working with others that he mentioned in the interview or previous data collection stages. In fourth year, he described that he mostly worked alone, which he had addressed in FG contribution 10 also (see section 6.2.3), although this might have been amplified in fourth year by the COVID-19 pandemic. Nonetheless, he persisted in presenting a collaborative image of science and mathematics and hoped to work as part of a team or group in the future. Although his interest in mathematics has "waned off a little bit," he is looking forward to new challenges that do not involve exams. His vision of the next step in his career has been influenced strongly by his experience on work placement, and he believes that mathematics will continue to play a role in his future: "Yeah, I think I'd definitely go into pharmaceutical again. And whether that's like quality control or quality assurance, I think maths will definitely play a part in it."

7.2.2 Interview 2: Breandán_SC1

First Year

Breandán_SC1 is a Science Education student who arrived in DCU with a passion for chemistry and mathematics that permeated beyond the classroom into his social setting in post-primary school:

> I was kind of lucky in that lads that I hung around with in school, like from first year ... if you wanted to talk about the schoolwork, it would be absolutely no problem in sitting down at lunch into a half hour discussion.

However, in first year at university, he found that rich and fruitful discussion regarding a broader interest in these subjects was not immediately accessible:

> I came out of [lectures] like, I suppose, excited would be the word that I would use like. I was like, that's really interesting, like wanting to nerd out on it, or about it with the class, like some of the others in the class. And just, no one, there was no one to, kind of reciprocate it. And it's actually something that I missed all through college. If you were

> talking about the stuff that we'd be doing it would be much more, kind of on a surface level of "how do you do it?"

Breandán_SC1 had an interest in mathematics and chemistry beyond the instrumental understanding he referenced above, and he researched videos online to satiate his desire to see the bigger picture:

> I would have went down YouTube rabbit holes of ... the stuff that ... I went on to study in college. Like, that's so far beyond what you need for Leaving Cert but I just found it interesting enough that I kept looking at it. ... if something caught my interest, I would go look for something like that, say, Numberphile or ehm, Matt Parker, or someone. Ehm, ThreeBlueOneBrown ... the same for chemistry with NileRed and [Nile]Blue.[1]

He had a like-minded community in post-primary school who self-identified, and were happy to be seen by others, as so-called "nerds." However, amongst his university classmates, he suggests that this identity is one of an outsider rather than someone who is part of the community. In first year, the Science Education programme includes students who later specialise in teaching physics and chemistry (not mathematics), and they share some lectures with students of science, many of whom may see mathematics as a means to an end (section 6.1.2) and thus may focus on passing first year mathematics exams so that they can concentrate on science modules thereafter. For Breandán_SC1, doing mathematics may always have been an individual pursuit, but it was an experience he enjoyed:

> I still wouldn't have a problem, sitting in to do into doing a chunk of maths work. But, like, I've two essays to write for education modules ... and I am dreading doing them ((laughs)). But I would have no problem if it was to write 15 pages of maths. Grand, sit down, over two evenings and just do it.

In transition year (TY) in post-primary school (see section 1.2), he had the opportunity to tutor first year and primary school students, which inspired him to pursue a career in teaching: "I didn't really know [that I wanted to be a teacher] until ehm, kind of fourth year onwards, where we started doing stuff with the primary schools [in] town." In university, he quickly developed a belief that you need to be familiar with more than just Senior Cycle mathematics content to be

[1] Matt Parker is a former mathematics teacher, and current comedian, who runs a YouTube page called Stand-Up Maths." ThreeBlueOneBrown is a YouTube channel run by Grant Sanderson, "one of the most successful math communicators on the internet" (Hershberger, 2022), who uses a graphics tool he developed to make videos about university-level mathematics topics. NileBlue is spinoff of the NileRed YouTube channel, whose stated aim is to "capture the natural beauty of chemistry in fun and interesting ways."

an effective teacher: "the content that we [did] ... you're doing a university degree, it's going to be far beyond what I was using [in the classroom.]"

In first year, Breandán_SC1 reported that Science Education students have to "derive everything" and are presented with more axiomatic mathematics compared to post-primary school. With this more generalised and abstract form of mathematics, he described that "instead of being given tools to answer questions or problems, say, you would be given tools to verify or to prove theorems, unseen theorems, stuff like that." Although the methods were the same as those at post-primary level, "you need a much deeper understanding of what they actually do rather than how to apply it" and he enjoyed the "level that it went to." His desire for "deeper understanding" rather than simply knowing "how to apply it" resonates with Skemp's characterisation of relational understanding as "knowing what to do and why" (Skemp, 1976, p. 20), as well as the definition of content knowledge for teachers: "The teacher need not only understand that something is so; the teacher must further understand why it is so" (Ball et al., 2008, p. 391).

Second Year

A deeper understanding allowed him to situate Senior Cycle mathematics within a broader field of knowledge. He presented a second year Probability and Statistics module as an example which demonstrated that such an approach is beneficial to teachers:

> Coming out of school ... I had no intuition about [probability]. Whereas having that module done, it helps ... [you to] be able to think on your feet and it helps [you] be able to explain stuff that ... just doesn't seem rooted in, much at all ... it kind of helps to be able to explain away the shortcomings [of Senior Cycle probability].

Although the content was "not far beyond" Senior Cycle, going a little bit further gives an important perspective on the material, providing what Ball and Bass (2009) called "peripheral vision" or "horizon knowledge" of the larger mathematical landscape (see section 2.2).

His desire to go "deeper" into Senior Cycle mathematics persisted into second year, even when faced with new, more abstract material. He described the Linear Algebra module from second year as "quite abstract" mathematics that does not follow directly from Senior Cycle, and therefore left him out of his "comfort zone" for the first time:

> [I] kind of just had a mental block against it ... told myself that I couldn't understand it

> ... it was kind of the first time that that had happened with any of the stuff that we were doing, and even in school.

Interestingly, he described himself as someone who had not experienced such a struggle before, even in first year. At the time, this Linear Algebra module relied heavily on calculations with matrices and Gaussian elimination (along with additional theory), none of which features in Senior Cycle mathematics since the full implementation of Project Maths. Given that there was no existing foundation on which to build an understanding of Linear Algebra, he struggled to align his approach to this module with his mathematical identity as described above: "I could answer the question fairly adequately but if the second part of the question was 'explain why it works' I was just totally out." In other words, he still wished to go further than answering the questions to pass the exam.

Unsurprisingly, he felt that "just learn[ing] how to do it" was not his preferred strategy but he had no choice, and, therefore, he took little consolation in the success of this approach: "it's applying it to stuff that you kind of don't know what it means. ... I don't like doing that myself ... but it kind of had to be done at the same time." As to why his fellow classmates would sometimes choose to do this, he suggested that since third year teaching placement involved teaching Junior Cycle students only, they did not see the value in abstract mathematics modules whose content is beyond that level. However, for Breandán_SC1, Linear Algebra was useful in subsequent modules in mathematics and chemistry: "the stuff that we [did] in third year ... even though it's beyond what you need to teach for Leaving Cert, it's helpful to have ... and I suppose, you can't do that without having linear algebra."

Third and Fourth Year

In third year, inspired by teaching placements, Breandán_SC1 began to focus on pedagogical content knowledge (Ball et al., 2008; Shulman, 1986) that could be used in the classroom, rather than mathematical content. Studying the psychology of mindsets helped him to switch his focus towards his students: "it was definitely helpful to kind of know what someone might be thinking, or how they might be thinking about it." Paying attention to this type of teacher knowledge inspired a broader change in Breandán_SC1's mathematical identity, from a mathematics/science student to a mathematics/science teacher: "It kind of changed for me from I have to do this now, and need to get passed an exam" to "working to the end point of, you're going to use some of this stuff [in the classroom]." In other words, he began to focus on his end goal of becoming an effective mathematics teacher: "I changed from maths for the sake of maths" to thinking "this is going to help me." He began to reflect on how he would use content from his modules as tools for understanding and explaining mathematics to his students, again describing pedagogical content knowledge that Ball et al. (2008, pp. 391-392) argued is vital for teachers: "this is going to be useful to be able to explain something or to help problem solve yourself ... to help understand it, to be able to explain it to others." This change coincided with crossing the halfway threshold in his degree. In second year, "my way of thinking about the degree on a whole was like 'there's no end in sight' kinda," whereas in third year "there was an end goal there that I could see, it was [within] reaching point." The change was gradual but formed a cornerstone of his current mathematical identity. He explained several different views of his identity before and after this change to emphasise the magnitude of the shift:

> I don't really know if I can put a pin in, when or, like, when my attitude changed but like first, second year, I would have been completely on this "I'm doing a science course, I'm doing science and maths rather than teaching" ... into third year, I don't know whether it was actually getting into schools or what but ... my attitude towards the course in general, turned from "I'm doing a science course" ... the emphasis for me changed from predominantly science-based to education ... the modules we were doing, I would kind of think about them in terms of what way I could try [to] use them.

Third year "changed my thinking about the degree on a whole, as well as attitudes towards what I was doing." By focussing on putting knowledge into practice through teaching placement, he realised that "some of the stuff you could nearly pick from the third-year modules and actually bring it into a fifth-year class." In fourth year, the Abstract Algebra module exposed him further to abstract mathematics that, he agreed, is not directly relevant to Senior Cycle mathematics:

> For the Abstract Algebra module this, this year... kind of the same approach to it as the linear module in that I hadn't really a clue what was happening ((laughs)). But kind of learning the exam rather than the material if that makes sense.

As was the case with Linear Algebra, the Abstract Algebra module challenged his mathematical identity, this time as a teacher who was motivated to learn how he could use this in the classroom. It is argued in the literature that abstract algebra plays an important role in teachers' understandings of connections between basic arithmetic operations (Ma, 2020, p. 119), dividing by zero, and dividing by fractions (Zazkis & Mamolo, 2011, p. 12), but Breandán_SC1 did not mention any such pedagogical connections to the material. However, this time around he felt that he handled the abstract and unfamiliar nature of the module quite well: "having a similar experience with the linear [algebra], it doesn't have as profound an effect."

Teaching for Understanding

Taking on the role of an expert, he explained that his own desire to go deeper, in order to understand and explain mathematics, is not necessarily reflected in his teaching (building on his comments from section 6.2.3):

> If you have a group of sixth years, that are higher-level ... that are getting over 70 in all of the exams, you can teach them to understand. You can try to teach to, to help them understand, which in turn helps them answer the questions. But especially if you have ... an ordinary-level class, you're not going to be able to do it.

Johnson et al. (2019) found that many teachers in Irish post-primary schools share Breandán_SC1's reservations about the demands that Project Maths places on "less-able" students, particularly in terms of literacy, problem solving, and relational understanding (p. 6). In FG contribution 8, Breandán_SC1 had expressed reservations about whether Leaving Certificate examinations reward understanding (see section 6.2.3). He repeated those reservations while discussing his own teaching, paraphrasing an observation about the predictability of examinations made by Lubienski (2011) in the time before Project Maths was fully implemented for Senior Cycle: "to understand doesn't help them to answer the three types of questions that is on the exam on that topic, because it doesn't change enough ... to actually examine understanding." In a similar vein, Berry et al. (2021) reported that mathematics teachers continue to utilise direct instruction, ahead of more unfamiliar pedagogies that might better address students' understanding, because they are not convinced that such pedagogies are as effective, nor that they will benefit students in their examinations (p. 138). Breandán_SC1 noted that for some post-primary students "to just explain the methods, they're happier," perhaps because of the examination structure.

However, he feels that he is forced into these short-cuts to exam success, and gave examples related to chemistry to justify the views about teaching for understanding in mathematics that he expressed above: "it's kind of actually annoying, in a way, when you sit down to teach, especially the chemistry ... because it's very much, here's the formula, go use it ... rather than knowing what's going on." He pinpoints the role of feedback in perpetuating this approach since you "just do an exam, and you get a score back" but "that's no good for you, just the score written on the front of your paper is absolutely no good to you." This is discordant with university chemistry where they "get you to actually show that you understand what is being done," and where the goal of feedback is to inspire improvement rather than to measure performance:

> The rubric would be up with what you've got in each thing. ... there would be a comment on each section of the breakdown of marks and why you got such whatever marks. ... Even if you got good marks in it ... there would be a comment there of how to improve it.

Campbell et al. (2020) noted that even in classrooms where students are required to focus on acquiring objective knowledge, feedback which encourages improvement, rather than simply ranking the students and rewarding those with high scores, is an effective means of encouraging a growth mindset (p. 32). It appears that the way in which feedback was provided in the chemistry modules, may promote a growth mindset, which Breandán_SC1 espoused because it connected to his views about using feedback to encourage improvement amongst his own mathematics students. Building on his comments in the focus group (see FG contribution 4 in section 6.2.1), he summarised the essence of his issue with poor quality feedback in mathematics succinctly, with reference to understanding and explanations as means of demonstrating learning:

> Every teacher in the country ... has the habit of writing "well done." What does "well done" mean? Does that mean that you showed a good understanding, or does "well done" mean you got all the answers right but if I pressed you to explain it, you wouldn't be able?

It appeared that Breandán_SC1's approach to feedback echoed his own approach to learning, where he desired to understand mathematics and explain why solutions work, in order to "be able to explain it to others."

Summary and Insights

At several points in the interview, Breandán_SC1 emphasised his view that teachers need to be familiar with mathematics content beyond the Senior Cycle, and in particular that a "deeper" understanding of mathematics is important: a conception that appeared to match what Ball and

Bass (2009) termed "horizon knowledge" or "peripheral vision" for teachers (p. 1). Throughout university, Breandán_SC1 thought about mathematics and science as much more than giving correct answers, and gradually incorporated this into his mathematical identity as a learner, and ultimately into his beliefs about best practice in the classroom. The second year Linear Algebra module (and fourth year Abstract Algebra module) placed him outside his comfort zone for the first time, because the content was unfamiliar from Senior Cycle, and because he learned to answer examination questions rather than learning how to "explain why it works." He described a monumental, but gradual, change, in which his viewpoint transformed from studying a science programme and doing "maths for the sake of maths" with "no end in sight," to a focus on education and thinking "this is going to help me [in the classroom]." It appeared that teaching placement inspired him to see himself as a teacher-in-the-making (Eaton, Oldham, et al., 2011a, p. 164), and begin to focus on pedagogical knowledge rather than on the subjects he was studying. While elaborating on his focus group comments about teaching for understanding, he endorsed the use of feedback in promoting a growth mindset. However, he echoed concerns that are present in the literature with regard to less-able students (Johnson et al., 2019, p. 6), and whether the examinations actually reward such pedagogies (Berry et al., 2021, p. 138).

7.2.3 Interview 3: Ciarán_ENG

Ciarán_ENG arrived in DCU having worked hard to achieve good results in higher-level mathematics for Leaving Certificate. The practice exams were a "wake up call" after which he put in a lot of effort, concentrating on answering exam questions sometimes for hours at a time:

> I suppose it was just the amount of effort I put in, and the fact that it was one of the few subjects that I could sit there for a few hours on end, and kind of, I wouldn't mind so much, putting so much time into it.

He persisted with higher-level mathematics because of the programmes he wanted to study in university, and over the course of the interview, he positioned himself as someone who does not struggle with mathematics, even though it takes hard work.

In section 6.2.1, I discussed how Ciarán_ENG and other participants valued consistency regarding post-primary teachers, whereas frequent changing of teachers had a negative influence on some participants' mathematical identities. Reflecting further upon his experience of frequent change in teachers at Senior Cycle mathematics, he acknowledged that different teachers have different teaching styles, and it takes time for students to adjust:

> There's different ways of doing things and like one teacher might show you, might use one way. This is the way you've learned how to do it, and then the next teacher comes in and they do a completely different way.

He took control of his mathematical learning by sourcing grinds teachers outside of school: "I missed logs and I remember trying to learn it myself. That was kind of the start of me looking for grinds then." Although one grinds teacher "wasn't very good," and another "scared you into making sure you got things right," he eventually found someone who could help him understand mathematics: "whatever way she explained things to me it was like, it's like she knew exactly what I needed to see, to learn something." After rising to the challenge of higher-level Leaving Certificate, he developed a more positive outlook on mathematics and his future studies: "I suppose when I started getting better grades, I was kind of seeing the positive side of things."

First Year

Ciarán_ENG did not remember putting engineering on his university choice form and expected to be accepted into a computer science programme. Unsurprisingly, he was not sure that engineering was the right choice for him from the outset, but he was confident that he wanted to study something involving mathematics since his choices were all "very mathematical things" including business and actuary. When reflecting on his Data1 response at the end of the interview, he wondered why he had been so neutral about choosing programmes involving mathematics (see section 6.1.2) and his fondness of the subject: "I don't know why I said that I didn't decide to study a mathematics related course. All my courses were pretty much maths based."

Given his range of programme choices, first year was partly a journey for him to discover whether he wanted to persist with an engineering degree: "I was going in kinda real pessimistic about engineering because I didn't particularly want to do it. But then very quickly learned that I really like it." In DCU in 2017, engineering students had the option to study common modules in first year that cover the basics of mathematics, materials, physics, chemistry, and biology, before specialising in one of four strands of engineering thereafter (the strands are listed in appendix 3.2.1). Ciarán_ENG availed of the common entry option in first year and, as was reported by other participants, found it difficult to assess the trajectory of the degree as a whole at this early stage:

> I considered deferring for a year because I was just so unsure what I wanted to do and while I enjoyed first year, like I mentioned, the, doing like biology, chemistry, stuff that I didn't want to do. I kind of thought that was going to be my whole four years.

In engineering, as with mathematics, "[t]here's always going to be parts of it that you do enjoy and parts that you don't" and first year gave him the opportunity to explore the different strands of engineering to determine which one to specialise in the following year:

> I chose Electronic Engineering [at the end of first year] and I went and met both the programme chairs and got transferred [to Mechatronic Engineering at the start of second year] ... I just realised that, while I enjoyed the electronic engineering more, I was not as good at it, I was a lot better at the mechanical side of things.

Like other interview participants, his memories of first year mathematics were minimal, although he does remember that "it's nothing like Leaving Cert" and certainly not what he expected, suggesting that the scope of the content was much bigger: "I remember at Leaving Cert my teacher saying to me, oh we won't go this far into this topic because that's more for university." However, later in the interview he described how some topics from Senior Cycle have remained relevant to his programme:

> Doing all like the trigonometric proofs and stuff like that. And funny enough, they pop up a lot, in college as well and one of our modules we're doing now at the moment, robotics, we're using a lot of them.

Even though he admits that his results in first year mathematics were not good, on reflection in the interview his comments about his experiences with mathematics suggested that he was a confident student who did not worry too much about results: "maths modules I never really struggled with, well, I guess, I, if I put the time in I didn't struggle with them, let's say that."

Second Year

He settled into the idea that this was the programme for him "literally as soon as I went back into second year," earlier than was reported by other interviewees. He had the same lecturer (Lecturer A) for mathematics in second year as first year, which may have aided the transition. He liked his lecturing style, because you would understand "90% of things as he was teaching them." Lecturer A was "probably one of our most interactive lecturers in the whole four years" and Ciarán_ENG appreciated the manner in which he engaged with the class and kept them on their toes:

> He'd asked you a simple maths question and if something, someone got it wrong, like he'd always ask someone that he didn't think was paying attention. And they might give you the wrong answer, and then he'd just make a laugh out of them. ... it made, like you paid attention to him when he was actually teaching you the important things, I guess.

He presented himself as a student who resisted instrumental teaching, and with reference to another module which was conducted online during the COVID-19 pandemic, he emphasised his preference for lecturers who were accessible for interaction and answering questions:

> I liked it because the lecturer was able to help you, like if you had a question. He'd know the answer straight away. He'd turn on his share screen and show you how to do it.

Third Year

In third year, he had Lecturer A for Data Analytics, which he enjoyed because of the probability and statistics basis for the module. He sees a role for this type of mathematics in his future since he would "definitely" prefer a mathematical work role: "I enjoyed ehm, all the Data Analytics and stuff, like that statistics, probability ... if I was offered a job with something like that, I'd definitely take it over, say, control systems or anything like that that I've done." He contrasted this with lecturing styles that did not suit his way of learning. In one module, there was not enough time to think about, and understand, the mathematical content during the lecture, and he felt there was a lack of resources outside of class time to aid self-study:

> Great, great lecturer, but the way Lecturer B taught things, it was like real old-school. Lecturer B used to project a piece of paper up on the screen and write everything down on the piece of paper ... Just that whole way of teaching for me wasn't, like I couldn't learn from that ... Lecturer B never uploaded, like the lecture notes that we covered ... So, it was literally, write everything down as fast as you can ... and then when the class is over, go to the library and look at it, see can you understand any of it.

The above extract is reminiscent of comments made by SC2_08 about the pace of university lectures, except Ciarán_ENG noted that the so-called "chalk and talk style" of lecturing created an additional burden when revising the material. Faced with this challenging, fast-paced module, he resorted to rote learning when there was no other option, even though he would have preferred to understand the material: “I learned off that this is what I have to, do you get me? But I never knew why I was doing it. I didn’t understand it. I just kind of had it drilled into my head.”

He also highlighted a contrast between the above approach and practising questions, which he saw as a necessary evil to get through exams: “The way I find that I learn maths is just by doing it, again and again, doing questions, different types of questions.” Although he clearly prefers to understand mathematics as lectures progress, and is responsible and curious enough to ask questions about the elements he does not understand, his focus must eventually shift to analysing how he will be expected to prove his understanding in examinations: “I ehm, went to the library and printed off this 30 or 40 page like ehm, maths questions that could come up on our exams. And tried to get through a fair chunk of them.” Thus, in this case, practising questions was not an indicator of a rote learning or a preference for instrumental understanding of mathematics in general, but simply a means for Ciarán_ENG to optimise his efforts in university examinations.

In section 6.2.3, it was noted by ENG_08 that he regularly had to apply mathematics in other engineering modules. By the end of third year, Ciarán_ENG also realised how useful the mathematical content had become in his other modules, where he often had to apply mathematical concepts in engineering contexts:

> With the engineering maths one, two, three and four it made sense that they all kind of linked together, but then even stuff that you’ve learned ... we’d use that then later on in other modules. And you wouldn’t think when you’re learning that it’d be useful to you, and then literally, you know, pops up again, whether it’s a year or two years later.

On reflection, the modules integrated together much earlier than he realised and looking back he described the impressive extent of the connectedness:

> I can’t get over how well, like some of the modules link in. As you’re going into, say, second year, third year, you’re learning stuff and you’re like “when the hell am I going to ever use this, how is this going to be useful to me?” And then you’re in third year doing a question you’re like, “oh I’ve learned this before,” like, I know this from somewhere.

He reckons that a lot of dedicated planning must go into integrating the modules since “nearly everything that you learned in each of them, you carried over to the following year.”

Fourth Year

Like Aodhán_SC2, in the second half of his degree Ciarán_ENG has come to recognise the work he does in engineering as mathematics-based but not exactly mathematics. The subjects have fused into a new form that is unrecognisable compared to mathematics in the classroom:

> Like when I think of maths, I just think of like simultaneous equations and, just stuff, similar like statistics, probability. I wouldn't think of like using maths to design control systems and just generally like that. I wouldn't con- well I wouldn't say that's maths. Obviously, it is maths but it's not what I would think of maths, if you get me.

Echoing the contributions to earlier data collection stages from himself and other participants in section 6.2.2, he described how the real-world problems they solve, may require a broad knowledge of many different areas of mathematics, and thus are distinct enough from problems presented in the classroom or examinations to be considered a separate entity:

> It's more just seeing where maths is actually implementable in the real world. Because when you're doing maths at Leaving Cert and even say, first and second year with [lecturer], it's maths. Yeah, it's good to know but you're thinking to yourself, "where is this ever going to be used?"

He credited his final year modules, project, and first year shadowing programme with demonstrating how he will put what he has learned into practice: "I'd say fourth year, maths is like almost showing you, nearly, majority of the aspects of where maths can be implemented in everyday life." Later in the interview, he repeated how influential his final year has been in demonstrating the role of mathematics in engineering: "It's become more apparent to me in fourth year with seeing how like everything is applied to more real world applications."

He shared a view that working alone in engineering can be better: "I wouldn't say that I'm a team player. Like a lot of things, I like doing it on my own or I like it being done my way." His final year project reinforced the benefits of working individually since he had been required to work together with his classmates on assignments and projects: "with the final year project I'm on my own. Everything's my way. ... Compared to other modules where you have to work as part of a team and people don't pull their own weight." While collaborative working seems to be a feature of many engineering modules, students also formed study groups to "compare answers [to assignments], just to see if we were getting them right" and this way of working has persisted throughout the degree: "we're still doing it like, I'm still kind of cooperating with some of my peers for assignments." Previous research found such groups to develop among PSTs and students of mathematics (Eaton & OReilly, 2009a, p. 154; Eaton, Oldham, et al., 2011a, p. 161), but in this study there was little evidence of student-led collaboration developing by the start of third year (see section 6.2.4). Ciarán_ENG's comments suggest that if more engineering students had participated in Data2, there may have been more discussion of collaborative working. It should be noted that cooperative groups that are not guided by a lecturer may not be beneficial for students (Schoenfeld, 2004, p. 272). Ciarán_ENG explained how sharing his reasoning to help other group members understand turned into a negative experience at times:

> The good side is obviously when you make, when you realise you're both right. Or, I suppose it is good when your friend kinda does give you the right answer or if you extend something correct but then the whole arguing side of it and, almost like competition between certain friends. ... It definitely pushed me a little bit harder. But then, like just becoming frustrated when people are telling you that you're wrong after spending a couple of hours working on something. That was the negative side of it.

Summary and Insights

After the "wake up call" of his Leaving Certificate practice exams, Ciarán_ENG took control of his mathematical learning in post-primary school (see section 6.2.5) by sourcing a grinds teacher from whom he could learn easily, and, like Aodhán_SC2, started to enjoy studying mathematics once his grades improved. Having unexpectedly ended up in an engineering programme, first year was partly a journey for him to discover whether he wanted to persist in the engineering programme. He described how first year mathematics went much deeper into Senior Cycle content but also broadened out into other areas of mathematics that were less familiar. Unlike Breandán_SC1, who detailed a gradual change in mathematical identity which culminated in him identifying as a teacher in third and fourth year, Ciarán_ENG was certain at the start of second year that Mechatronic Engineering was the programme for him. He highlighted the variation in teaching styles at university, describing one lecturer, Lecturer A, with whom he understood "90% of things as he was teaching them," and another with whom the entire lecture was spent writing "as fast as you can" and trying to understand the material afterwards. Like Breandán_SC1, Ciarán_ENG resorted to rote learning in modules where he felt that there was no other option, but he would have preferred to understand the material, and expressed a preference for lecturers who were more interactive with their students.

Ciarán_ENG noted that his third year modules utilised mathematical content from previous years, and, more generally, that everything he learned in engineering carried over from one year to the next. He particularly praised the fourth year of his programme for demonstrating how mathematics can be "implemented in everyday life," which helped him see the connections between the engineering problems he has encountered and their mathematical foundations. Ciarán_ENG saw his final year modules, project, and first year shadowing programme as particularly influential when it comes to seeing theory put into practice. He also corroborated the observations made by ENG_08 in the focus group, that engineering students are routinely required to collaborate on group assignments and projects. He described how engineering students developed collaborative groups to work on assignments together, as Fiach_SC1 described amongst the PST students (see section 7.1.2). However, Ciarán_ENG highlighted how the dynamics of such groups may cause members to feel discouraged from exploring mathematical questions or sharing their understanding with others.

On reflection at the end of the interview, Ciarán_ENG was glad that he chose to study engineering even though his overall experience was "up and down." With regard to mathematics, he gained insight into his own responses to different teaching styles: "I wouldn't say I had a great attitude towards things, but when I have a bad attitude towards something, that definitely impacts my capabilities to learn." By considering his overall mathematical journey, he re-evaluated his lukewarm comments about mathematics in Data1, described how he sees mathematics in his future plans, and realised how much he has enjoyed mathematics throughout the ups and downs:

> Just thinking back like, a lot of that would have left my mind with like the Leaving Cert teachers and stuff like that. Kinda thinking back on all of that, it was like good times and bad times. ... Like I'd know if I didn't like something straight away, but I suppose it's made me realise that I don't sit down and think like oh I actually enjoy this.

7.2.4 Interview 4: Dónal_SC1

Dónal_SC1 took a non-standard[2] entry path into DCU by enrolling in a pre-university science course after completing his Leaving Certificate. Entry to university via further education (FE) programmes has been prioritised in Ireland as part of educational policy aimed at broadening opportunities for access to university education beyond the traditional CAO application route (HEA, 2015, p. 30). At the time of the interview, he was on teaching placement in a school teaching Junior Cycle mathematics and science. I began by asking Dónal_SC1 to clarify his pre-interview comment that he had “a lot of drama with maths.” Building on his comments from section 6.2.1, he described having an unreliable teacher in post-primary school, which resulted in a period of instability: “you’d be learning from a different teacher every week because [teacher] went and got suspended again ... And they brought in a teacher who was really good. But we were so far behind right, that it didn’t even matter.” However, despite this tumultuous backdrop, he positions himself as a resilient, mathematically oriented student: “it didn’t matter for me because I was a good student in general, I knew maths.” Over the next few years, he alternated between relishing the challenge of mathematics and feeling disinterested:

> I was finally getting the challenge I wanted. And then I went from third year to fifth year where I was no longer getting the challenge I wanted, because I wasn’t being kept up to speed so it got boring again ... and it was really difficult to enjoy.

As a result, he moved to ordinary-level mathematics for Leaving Certificate and he notes that “I really forgot my love for maths until, the end of first year [in university] when I rediscovered it.”

Further Education Programme

Dónal_SC1 presented his decision to pursue a degree in teaching as last-minute, made on the final day of post-primary school, and emphasised that it took time for him to be comfortable that he’d made a good choice:

> I came in to clear up my locker, and I wanted to talk to my favourite teacher. And he was just sitting in his room as he always was, he always kept it open for students. ... I didn’t know what I wanted to do, but I wanted to do something. And I just said it on a whim, I’m going to be a teacher, and I’m stuck with it since. ... along the years I’ve both gone “yeah this is for me” and “this is not for me,” but I think that it’s a happy mix.

He acknowledged that he did not apply for a teaching programme straight away because he was not sure about this career direction. Instead, he applied for a one-year further education

[2] The terminology “non-standard” has been used in Irish education studies to include students who qualify for entrance to a university by completing another certificate, diploma, or degree (Faulkner et al., 2014, p. 653).

programme that would allow him to access the Science Education programme in DCU:

> A lot of people don't think about [programmes in further education], they think about going straight to college, you know, getting their degree. I was thinking, I don't know what I want to do with my life, I may as well start off here since I like science.

As was the case when discussing post-primary school, he reported that he did not struggle with the mathematical content of the further education programme, and had a productive working relationship with the teacher:

> It was a mix of ordinary and higher-level maths ... You had to do a certain amount of questions per chapter, but we always did them right after we did the material so it was never difficult, you know ... and the guy who was teaching it there was very good. Like, you know, great to work with and that's kind of what you want, right.

It appears that this programme reaffirmed his identity as a capable student of mathematics, and he emphasised its importance to his overall development: "It was probably the best decision of my life to ever go to [pre-university science course]" because "I met a lot of good people and like really grew into myself." Previous research has highlighted that students of science and engineering who have completed further education or access programmes, are likely to report the benefits of doing so when it comes to transition to university (Liston & O'Donoghue, 2010).

First Year

In first year, he began to doubt the utility of studying advanced mathematics that he will not have to teach in the classroom, absorbing this message from surrounding discourses within his undergraduate community:

> Well, so in one mind frame as every PST thinks why do I have to do the thing I don't want to do, right? That's, everyone thinks that. Ehm, in my case, it was much the same, where [in] first year, I lost a lot of motivation towards everything. So eventually I just started, like, I'd stopped showing up.

As a result of feeling demotivated and ceasing attending his Calculus lectures, he realised that he had relied too much on working on his own outside of lecture time, which became particularly apparent when he was preparing for exams:

> I did the exams and ehm I basically, as all students do, I crammed like, for days, and I just couldn't get the maths ... I really have to be there to understand what I'm doing,

> and I wasn't there. So, I couldn't really figure out from that point, where I was looking at stuff and you know I've done pretty well with maths, all the way up to that point. And I just, I understood what I was being asked for, but I just couldn't do it.

Now he was faced with a choice to either abandon the pursuit of a degree in teaching or readjust his learning approach and repeat the year. Again, he expressed doubt about whether this programme was the correct path for him, and although he did not detail any reasoning for wanting to become a teacher in particular, he decided that he could not justify leaving the programme:

> It was more like I was unsure that I wanted to do anything else ... I was like, if I stopped doing this now, that means I have effectively wasted two years of my life, wasted a lot of my parents' money and have more or less failed at everything I've tried to do. So that was one end and then the other end was if I keep going, I will eventually come out with a degree, which ... enables me to do a job where I make decent money and have a decent amount of time off, so it was more so I wasn't sure I wanted to do it but I was really sure I didn't want to drop it.

In his description above, he focused on the practicalities of teaching as an employment opportunity, which has been acknowledged as a motivating factor for those who choose a career in teaching, (Heinz, 2015, p. 279). His comments on himself as a teacher further on will reflect this practical focus also.

In first year, the Calculus module included some aspects of problem solving, which he considered a distraction, because it was not helpful in other modules or in his teaching experience:

> I don't remember tutorials for Calculus at all. Ehm, I remember the problem solving we did. But that hasn't been relevant anywhere, and I don't mean that out of badness, I mean, nowhere I need to look and solve a maths puzzle right unless, as I said, it's a hobby.

He is rather dismissive of problem solving as a recreational endeavour for mathematics enthusiasts (Schoenfeld, 2016, p. 5), which has entertainment value, but no practical value for his own studies, nor for his students in the post-primary classroom. This issue may have particular relevance for teacher education given recent research by Faulkner et al. (2023), which found that incoming undergraduate students' problem-solving skills have continued to suffer in favour of procedural skills, contrary to the aims of the Project Maths post-primary curriculum (O'Meara et al., 2011, p. 334).

Aside from the content, Dónal_SC1's experiences in first year differed from his previous experiences with mathematics in terms of the type of support available from the lecturer, and the manner in which solutions were expected to be presented. He identified a problem with mathematical language as a source of frustration for him in first year:

> Basically, they had decided that it wasn't actually saying what I thought it was saying. And that my mathematical language was off, right ... So, what annoyed me was, and this has happened since, that we were being judged on a module we have yet to be doing. ... I wouldn't blame [Calculus lecturer] because he has a fair point in a way, where I, you know, technically I didn't. But ... we hadn't actually done the module to be held to that standard for.

Adapting to the formal and abstract language of university mathematics is an important factor in the successful transition to university (Clark & Lovric, 2008, p. 28), and it has been reported in the literature that students see such language as more cumbersome, rather than more precise (Bergsten & Jablonka, 2015, p. 2053). At the time, mathematical language was the focus of a second year module called The Mathematical Experience, which Dónal_SC1 had yet to take. This has since been included in first year for Science Education students which might be considered an acknowledgement of a weakness in the previous programme structure. However, it should be noted that Dónal_SC1 subsequently agreed that the module that covered mathematical language was interesting, but of limited use, drawing a comparison with his earlier description of problem solving as recreational: "It really was like, hey, if you like maths as a hobby, this is the module for you." Although he was quite magnanimous in not blaming the lecturer for his problem with mathematical language, I was concerned by his description of the issue as a technical hurdle, and the overall impression that the problem was never fully resolved because he did not view this as an essential part of his future mathematical learning.

For repeat examinations he received one-to-one help from his lecturer to surmount this problem, and appreciated that a staff member went to these lengths to help him since this was not what he expected of a university lecturer:

> It sort of ends up as an overall positive just because he was willing to sit and work with me right. Take time out of his day to just sit and work with me and sort of go over stuff and revise and, I do remember a few things I might have not understood that I asked him about I was like, "how do you do this? I don't understand this." And he would sit, and

he would work with me on it.

He gave a striking description of this experience, in which he emphasised the impact that receiving this degree of support from his lecturer had on his mathematical identity: "That was the first time a DCU staff member has ever really helped me ... I was just a face, but he still remembers my name." Given his previous expressions of doubt about whether he wanted to pursue this programme, or a career in teaching, his characterisation of himself as "just a face" is a powerful statement about his perceived isolation in navigating the transition to university mathematics. Such a statement indicates that first year mathematics was experienced by Dónal_SC1 as a "life crisis" (Clark & Lovric, 2008, p. 26), followed by a separation phase, which he described previously, in which he felt "'removed' or isolated from the rest of the community" (p. 26). This was further emphasised by his characterisation of the possibility of leaving the programme as "wasting two years" of his life and would be akin to admitting that he had "more or less failed at everything I've tried to do."

Having previously described his pre-university course lecturer as "great to work with," the opportunity to do similarly with a DCU lecturer, and establish a personal connection, appeared to reinvigorate him, and allowed him to work collaboratively to pinpoint and question mathematics content that he had struggled to understand by himself. Positive relationships with peers and staff members have been found to significantly influence the transition to university for students arriving from further education institutions (Tett et al., 2017). Hernandez-Martinez et al. (2011) pointed out that students who experience challenges to mathematical identity, such as resitting examinations or dropping out, can retrospectively view their successful navigation through such experiences as an affirmation of who they have become (p. 128). Dónal_SC1's descriptions suggest that he received the "right connected support" (p. 128), which facilitated him in turning an identity crisis, and possibly dropping out of university, into an opportunity for progression because of the positive relationship he established with a member of staff. Further on, he will mention some other sources of one-to-one help that he sought out in subsequent years, which suggests that he became aware that such sources were both available and beneficial to him, in part because of his first year experience.

Second Year

In second year, he began to see more of the practical elements that he wanted to see. He described the module on teaching and assessing Junior Cycle mathematics as more useful, but more basic. It was about building up the background to the more basic methods that students have come to know, and seemed to broadly address the pedagogical content knowledge and subject matter knowledge needed for teaching (Ball et al., 2008, Figure 5):

> They build upon the initial methods, and they get to the final method, which works amazing, and you don't need to think about how you did it at the beginning. So, you forget all about what you did at the beginning, right. So that module is about building that back up, and you know ... it refreshed us on what we, you know, had long forgotten.

Using the language of Ma (2020), this module can be described as encouraging Dónal_SC1 to refocus on the roots of fundamental mathematics, having spent several years attending to the advanced branches in his recent studies (pp. 122-123). Ma (2020) contends that profound understanding of such fundamental mathematics allows teachers to exploit opportunities to reinforce basic mathematical ideas amongst their students, and to build longitudinal coherence between topics that students have seen before and will see in their future studies (p. 129). Dónal_SC1 explained that this module demonstrated a "new angle" on mathematics which was "not even theoretical, it's very practical." He explained that it is useful, for example, when introducing students to negative numbers for the first time and, like Breandán_SC1 (see section 7.2.2), he appreciated that "you can literally bring this into a class and use it, if you need to." The reorientation of his learning towards "practical" knowledge rekindled his passion for mathematics: "So, yeah up to ... the end of semester one in second year, I was really falling in love with maths in a way." Indeed, he described himself as enthusiastic and motivated at the start of second year: "I literally was there every day. And, you know, I had the motivation to be in and do stuff right."

However, as the above quotes suggest, there were some challenges to come in second year, which echoed an issue from his first year. Regarding Linear Algebra, he framed his approach as follows: "I knew that this was probably the last time I was going to do maths as a module. Just wanted to get through it." It appears that he was ready to move on from learning mathematics and put what he had learned into practice in the classroom instead. The crux of his issue with Linear Algebra was the inflexibility of the lecturer with regard to credit for solutions: "He gives his exams, if you don't do it his way, you're not getting marks. ... His ways worked but he wasn't fond of other people's ways that may be effective, I guess." As was the case in first year, Dónal_SC1 felt that his approach was reasonable but differed from what was expected. In retrospect, he sees this experience a little differently to his previous struggles with mathematical language. Firstly, he puts more responsibility upon himself: "So I saw these things that I was like, I think I know how to do it, but realistically, I didn't do it. Well, I didn't know how to do it I guess." Secondly, he noticed the limitations of his approach to learning mathematics in this case: "he changed a lot of things and asked questions he never asked before, and you know that always trips up people who study like I do, by practising." The bottom line is clear though, in his opinion there was not enough room for finding your own valid ways of understanding or answering the questions: "There's one way to do it, you do it any other way, no matter if it's right, you're not getting marks. I didn't like that." The influence of these experiences on his own beliefs about teaching is discussed further below.

He makes it clear that the teaching he experienced in this module is what lead to his disengagement, yet the subject itself was quite interesting to him: "dealing with matrices, cross products, all of that stuff was very interesting, right. And I loved it, in the sense that it was very logical." Thus, he disengaged from the Linear Algebra module, but having learned from first year, he searched for help in the MLC (see section 3.2.1), knowing that he would have to make up for not attending the lectures:

> I was slipping back into that habit of getting lazy and you know coming in sporadically. So, at, sort of at the end of the semester I had to teach a lot of it to myself, and you know, you, and like other tutors, taught me a lot of it. ... That is a much more positive experience right because we were able to go in and we got the help we needed. But then you take it with the negative, that we were in a place where we were supposed to learn it [in lectures] and we didn't right, that's the other side of that.

Third Year

Dónal_SC1 noted that he was one of the few students in his programme who chose not to specialise in mathematics in third year, deciding on physics and chemistry instead. However, Linear Algebra was useful for his science modules because they had learned the mathematics behind "paths travelling through space," while Probability and Statistics was relevant for everyday life, and for video games that involve choosing actions based on probability of success. He suggested that mathematics and statistics also played an important role in his lab work, but only infrequently, perhaps (as described by Aodhán_SC2 in section 7.2.1) requiring specific basic skills that could easily be learned as you go:

> As you have to do with every lab, you have to take data, right. A lot of this data was numerical you had to go graph it, which *in a way* [emphasis added] is very mathematical right because you have to graph stuff. So, it's *sort of relevant* [emphasis added] in that stance, you have to calculate your what do you call it, standard error, all that stuff, so ... It is relevant, right, like *I wouldn't say it's completely irrelevant but* [emphasis added] it's not you know, if you didn't have that you wouldn't be able to do this stuff.

Given his previous focus on the relevance of his university modules to teaching, it is likely that the qualifiers he uses here (in italics) serve to emphasise that even if mathematics was somewhat relevant in laboratory work, this did not translate into his teaching practice in third and fourth year. He contrasts this with the mathematics modules which, although more complex than Senior Cycle mathematics, had their foundations in concepts he saw as relevant for his future teaching: "Maths was sort of always relevant because you were building off stuff that would be useful in the classroom, right. So, in a way, I got bored with chemistry, maybe I wouldn't have gotten bored of maths."

Returning to the topic of module sequencing in his programme, Dónal_SC1 described how choosing physics and chemistry left him at a disadvantage:

> Where I know other students had done a maths module, which they covered some of the stuff we covered in Quantum Physics beforehand. ... I, wasn't lost, but it was my first time seeing this thing which, everyone else now has a step up on. So, it was probably intended that people in my course, who do physics and maths, touch on it twice.

Even then, the connection between the chemistry he was studying and his classroom practice was also tentative, and he suggested that those modules are designed more for students who intend to practice as scientists because they studied "fun stuff" which was "interesting in a way, but it's not relevant to someone who's not doing pharmaceuticals or, you know, medicine." By taking the less common option to study science modules in a teaching programme, he seemed to fall between two stools: those who specialised in mathematics teaching, and those who were studying science with a view to working in industry or laboratories, rather than intending to teach the subject.

By the start of third year, he had begun to rely on mathematics support services in the university to help him through physics and mathematics modules: "Since the end of second semester in second year, I've been going to the Maths Learning Centre when I don't understand mathsy physics. I know I can probably get help there." There is evidence that he relied on the MLC to bridge the gap between the mathematics others were learning in third and fourth year and the physics modules in which this mathematics featured:

> Physics got very mathsy, and I didn't understand a lot of it. So, I went in [to the MLC] and I was like, [I don't] understand this, and it was very mathematics, so I was like, I can probably get help here ... Yeah, I used to pop in for Quantum Physics. Because [tutor] there did physics and he knew that stuff, so he was really helpful with it.

The MLC appears to have played an important role in facilitating him to fill the gaps created by the modules, which, he felt, were poorly sequenced for someone who did not chose to specialise in mathematics. Moreover, having tutors in the MLC that are knowledgeable about physics and chemistry is important, since they will know how mathematics features in these subjects, and can contextualise them in science to help students understand.

As discussed above, his attendance of lectures was sporadic in first year, and improved in second year, but he missed out on the formation of groups of Science Education students who collaborated in mathematics:

> I was very spotty attendance. I came in for, maybe, most of the days, and maybe most of the days I missed a few hours out of them. I always missed when people were grouping up and doing these assignments together. And then when I repeated the year, I didn't know anybody, so I had to do them by myself right.

His approach in third and fourth year is a stark contrast to the above. Instead of working alone, he spent time in the MLC and found great value in working with a friend in computer science who, although not a mathematics expert, was willing to listen to explanations and participate in determining which arguments were sensible:

> He would sit and be like, yeah, I think this is how you do it, and I'm like yeah, I think that's how you do it too. So, we would work together in that sort of way ... a lot of those

> assignments, he couldn't help me with. But he sat and he's like listening to me talk about it. He's like oh yeah that does make sense ... It meant that I didn't turn it off.

This type of collaborative work resonates well with the work of Solomon and Croft (2015), who observed that students can passively receive knowledge from each other, or they can value understanding each other's explanations, and thus develop their own arguments about what is right (p. 272).

Fourth Year

Now in fourth year, he spoke with great passion about his teaching experience. At the time, teaching was still being conducted online because of the COVID-19 pandemic which began in March of the previous year. He appreciated working with a teacher who is experienced in the practicalities of the job, so he could learn how to improve his own teaching:

> The woman I'm working with she's a year head, and she has what 40 years of experience, and it really shows in the sense that one, she's old time right, she's been there for a long time, she's been there through way before Project Maths, then phasing in Project Maths, and now, I know Project Maths still in, but the new Junior Cycle maths right, and her experience clearly shows.

The qualifiers that he used when talking about the connections between mathematics modules, science modules, and teaching, are absent from his comments above, which demonstrated the clarity with which he saw the value of this experience for his own development.

What Constitutes Good Teaching

Dónal_SC1 provided an insight into his views on teaching and learning by describing the qualities of a "good teacher." First and foremost, although it may be easier to "fall into a pattern" of doing the same thing every day or reusing slides from last year, a good teacher should be interesting and creative in order to keep students engaged:

> The number one thing a teacher has to be, is interesting, right. Ehm, if you're not interesting your students ... you're gonna lose the ones who, maybe don't like the subject, you're gonna lose them, and then they might create a problem.

He acknowledged that teaching mathematics sometimes involves rote learning, which he associates mostly with older teachers in his experience. However, he is open to using such an approach in his own teaching since he has seen first-hand that this can be done in an engaging way: “Even my chemistry teacher who was brilliant, had us memorising stuff ... But the class wasn’t boring, which is probably why, you know, I think fondly of them.” When it comes to explaining mathematics to his students, he advocates for the use of a visual or hands-on approach:

> A lot of what I do with them is something you can look at ... it’s a lot easier to understand if you’re looking at it, than looking at like $x^2 + 3x + 1$. Like that’s very abstract, but if you bring it to a graph, you can look at it, you’re like “ah.”

Having spoken with great confidence about the qualities of a good teacher above, he is quick to separate himself from his image of a mathematical expert:

> I sort of learned that I love watching maths and I don’t like doing it right. Like, if it’s a sport like people like to watch football, but they don’t really like to play it sometimes. They love to, you know, watch this. I like to watch people do maths alright. You know, I sort of knew, I can’t do them, but I like watching people do them.

This quote arose during his discussion of the first year Calculus module in DCU where he struggled in the final examination. It is likely that he sees himself as a spectator compared to university lecturers who research in mathematics or appear to do mathematics for fun (as mentioned before with regard to problem solving), but also in comparison to his classmates who specialised in mathematics, rather than chemistry and physics. He expressed a belief that moving too far beyond the basis of mathematics needed to teach, could be problematic when the time comes to put learning into practice:

> I think I’m gonna have to really get back to basics ... it almost backfired, having us do maths in college, because we have gone so far, and I can only imagine how much like people who did maths as a subject [speciality] ... I wonder if they go back to teach will they struggle with it more than I would. Because I haven’t gone that far, if you know what I mean.

In this extract, Dónal_SC1 appeared to question whether the mathematics content covered in latter years by undergraduates who specialise in mathematics, lies within, or beyond, the “peripheral vision” or “horizon knowledge” (Ball & Bass, 2009) of mathematics that is needed for teaching. Indeed, answering this question has been the subject of recent research aimed at defining horizon content knowledge (Guberman & Gorev, 2015; Jakobsen et al., 2013). However, given his resistance to changing mathematical language in first and second year, this may indicate that he does not see the relevance of some important characteristics of university mathematics with regards to his own learning and teaching. In research by van Putten et al. (2014), a participant named "Thandi" presented herself as a good mathematics educator because she was knowledgeable about education theory and not mathematics itself, but classroom observations revealed related deficiencies in her interactions with students (p. 383). Similarly, Dónal_SC1 appeared to position himself as a good teacher relative to his classmates because he had a lesser knowledge of, and had remained untainted by, advanced mathematics. Although classroom observations were not undertaken in this study, and it would be of interest to see how Dónal_SC1's mathematical identity affected his teaching practice, he gave an example about solving quadratic equations that provided an interesting insight into his teaching approach. Although his earlier comments were dominated by teaching for understanding, going "back to basics" appeared to mean deference to teaching that prioritises procedural or instrumental understanding:

> You know the method solving quadratics by roots? And it only works for the ones in the book because they set it up so they specifically work for that right. Doesn't work on real ones, or most of them, 90% of them like. Knowing that, right, we have to teach students that. I can't for the life of me, I'm like well if the minus b formula[3] always works, why not just teach them that?

A comparison can be drawn between Dónal_SC1's comments and the work of Ma (2020). As part of her discussion of profound understanding of fundamental mathematics, the author noted that the group of Chinese teachers in her study sought to "know how, and also why" (p. 115) a particular algorithm works, a quote which echoes the definition of relational understanding given by Skemp (1976, p. 20). Ma (2020) claimed that asking "why" in this way is the first step towards conceptual understanding of the mathematics involved (p. 115). In the case of quadratic equations of the form $ax^2 + bx + c = 0$, asking why factorising this equation makes sense mathematically involves understanding the relationship between its coefficients a, b, and c, whereas applying the quadratic formula to find the solutions does not.

[3] The "minus b formula" is a common colloquial term for the formula which gives the roots of a quadratic equation.

Summary and Insights

Dónal_SC1 recalled what he called "drama" with regard to the frequent change of his mathematics teacher in post-primary school, but nevertheless wished to be seen as a resilient, mathematically oriented student. Indeed, he spoke fondly about his pre-university programme and reported that he did not struggle with mathematics until he came to university. The personal connection and one-to-one support that he received from his Calculus lecturer in first year, appeared to shift his narrative of struggle towards one of success in overcoming adversity, and may well have made the difference between him continuing, or not, his programme in Science Education. In the first half of his programme (first year and second year) he slipped into a habit of missing lectures and concentrated on working by himself, with the exception of one-to-one help from his Calculus lecturer, while in third and fourth year, he described working with friends and frequently attending the MLC for support.

He appeared to reach a point where pedagogical knowledge took precedence over content knowledge (Ball et al., 2008), and he saw more value in continued study of the former over the latter. He appreciated a second year module which helped him deconstruct mathematics he had come to know and suggested that other students who continued to study advanced mathematics in third and fourth year may struggle to stay grounded in the post-primary school content they will be required to teach. While describing the relevance of mathematics modules to his studies in science, and his teaching practice, he used qualifiers such as "sort of relevant," "in a way," and not "completely irrelevant." However, the absence of such qualifiers from his description of learning from an experienced teacher during teaching placement, demonstrated that he placed higher value on this knowledge for his continued development as a teacher. In other words, the question of "horizon knowledge" (Ball & Bass, 2009) appeared pertinent to much of the interview discussion. It should be noted that Dónal_SC1 described himself as a "Junior Cycle teacher" who will not be qualified to teach at Senior Cycle, which would affect his conception of the relevance of horizon knowledge to his teaching.[4]

Dónal_SC1's descriptions of problem solving as a "hobby" indicated that he saw little relevance

[4] The Teaching Council (2020) rules to qualify as a mathematics teacher exclude Dónal_SC1, because he specialised in physics and chemistry in third year. His self-description as a "Junior Cycle teacher" may relate to the reality that post-primary schools in Ireland can choose to appoint "out-of-field" mathematics teachers. In section 1.3, it was discussed that such appointments are less common now, compared to previous years, but are still a regular occurrence (Goos et al., 2021, p. 9).

in this process for aiding mathematical learning, instead describing his own role as an explainer in the classroom, with a focus on student engagement. However, he also demonstrated an amenability towards using rote learning in the classroom, and the potential to slip into less sound pedagogical approaches that limit avenues for conceptual understanding among his students. He presented himself as student who, from the beginning of his university studies, used approaches to mathematical tasks that made sense to him, and demonstrated a resistance to simply adopting those presented to him by others. For instance, when studying with his friend in computer science, he would develop and explain his own mathematical arguments rather than passively receive explanations (Solomon & Croft, 2015, p. 272). This is an aspect of his mathematical identity that he has brought into his teaching: "I do break down stuff, into really simple like understandable things, but I tend to think of my method and why it works for me, and of course I try to communicate that."

7.2.5 Interview 5: Éabha_ENG

Éabha_ENG is an engineering student who specialised in Biomedical Engineering in second year. Her "first realisation" of her talent and passion for both problem solving and engineering occurred when she was quite young, and she fixed the stabilisers on her brother's bike. As we discussed important influences of mathematical identity, she realised that the idea of working autonomously to come up with practical solutions to physical problems was greatly encouraged by her family:

> I would have been, you know, hearing a lot about the pharma industry and like how machines work and that kind of thing, at home ... we were never kind of given the answer for things. We were always like "have a think about that now" you know, and come back to me with a solution, it was all problem solving at home. Actually yeah, now that you're saying it, it really was heavily like that though.

She credited her Junior Cycle teacher for instilling her with confidence in her own ability in mathematics, building on the sentiments expressed by other participants in section 6.2.1:

> I had an amazing Junior Cert teacher for maths and then, from that onwards I kind of, I really really liked maths and I knew that it didn't matter how hard it was going to get, I was actually going to enjoy it and like it.

However, she experienced a big transition in post-primary school when she changed from an Irish-speaking to an English-speaking school and began to relearn mathematical terminology:

> My Junior Cert was done completely in Irish ... even very small things like simultaneous equations I didn't know what they were, because I only knew the Irish term for it. Or even if someone was like, "oh you divide that by two and plus that" then it'd take me a second to be like ok, "roinn é sin" and like kind of do it through Irish.

Although Irish-speaking primary or post-primary schools exist in Ireland, to the best knowledge of this author, there is no option to study engineering at university level through Irish. Éabha_ENG was glad to have made the language transition to English before the terminology became even more complex: "I'm very happy that I did it for the Leaving Cert so that it wasn't a big issue in college." She was one of two MISE participants who mentioned changing to an English-speaking school in section 6.2.5, and she gave some insight as to why this is a difficult change that goes beyond learning new vocabulary, and affects the way one thinks about mathematics:

> I had to sit down and think about things rather than just being naturally good at it. It was, I think a little bit ehm, it was a bit of a shock to the system, I think. So that's when that kind of started for the first time.

It was discussed in section 6.2.4 that some MISE participants relied on natural ability at primary school and Junior Cycle, but much less so at Senior Cycle or university level.

Physics, Mathematics, and Problem Solving

She connected physics, engineering, and problem solving in her reasoning for choosing Biomedical Engineering, so I asked what problem solving means to her. She presented an interesting dichotomy between the problem solving she undertakes in labs and engineering modules, and the contrived versions that she encountered in post-primary school:

> Like, the Leaving Cert maths situation where you knew there was a right answer but it's, it's frustrating in the sense that they're kind of seeming to phrase it in a way where they're kind of making it seem like it's problem solving but really it just kind of seems like a roadblock in the way of being able to do the maths that you actually want to do. So, I think the problem solving that I was more so interested in, was like the physical hands-on problem solving that we saw in the labs.

She contrasted this with university, where mathematics lectures focused on developing the tool box explicitly: "I preferred maths in college, definitely than Leaving Cert maths, because it was pure maths." Vinner (2014, p. 122) and Schoenfeld (2016, pp. 4-5) have both suggested that school examinations tend to feature routine or disguised routine problems, which students are expected to solve using strategies from their existing tool box, that have been demonstrated to them previously. This participant appeared to describe such disguised routine problems, and saw the fabricated context as an unhelpful barrier to the “pure maths” basis of the question. She has come to recognise problem solving as identifying a real-world engineering problem and “breaking it down to the simplest form” of “pure maths” so that the solution can be found.

On the role of mathematics in science at Senior Cycle, she reckons that biology and chemistry make instrumental use of mathematics (as did Breandán_SC1), whereas physics stands out by requiring students to analyse situations, a possible reference to getting started by interpreting a question/situation as discussed in section 6.2.4:

> I'm very interested in like biology and chemistry, but it wasn't hitting the kind of maths craving that I had where it was like, analytical thinking more so than just ‘”here's the chemical formula, figure it out” kind of thing.

First Year

Éabha_ENG had already mentioned how changing to an English-speaking school for Senior Cycle, meant that she could not continue to rely on being naturally good at mathematics. Now at first year in university, she acknowledged that everyone must work hard at this level, making reference to understanding mathematics (section 6.2.3) and the belief that being successful at mathematics requires hard work (section 6.2.4):

> I had a natural ability all the way up to the Leaving Cert and then I went to college and it's a big smack in the face then we have to actually work to understand maths. It's a difficult adjustment and it takes, it takes a minute I think to realise ok, I can't just sit back and understand this, I have to sit down and learn it. So, I think to a certain degree, to a certain capacity I have a natural learning ability but then as it got harder, no one can just understand it naturally. I don't think I think everyone, there's a bit of learning to do for everyone.

She described first year engineering mathematics as “really fast-paced” and “a step up from Leaving Cert.” The lecturers set a high standard in the amount of work required:

> I think, for engineering specifically they try to wean out the weaker ones and people who

> maybe, you know aren't going to be as serious about the learning side of things, or like... they were trying to kind of scare people off a little bit and make sure that anyone who was sticking with engineering was passionate about it.

Engineering applicants are required to have studied higher-level mathematics, and the extract above hints at an intimidating atmosphere in engineering mathematics, which Éabha_ENG explained thereafter. She felt that the high standard of mathematics expected from them, was complicated by a pressure to appear comfortable with these requirements in front of their classmates: "No one wanted to seem stupid because in engineering you think everyone's very smart. So, I think particularly in maths, they expected that no one had any questions, and to just kind of accept and get on with it." She perceived the message that if you do not understand mathematics from lectures and tutorials, you are just not working hard enough. She characterised first year mathematics lectures as "a bit of a daunting situation" and indicated that she would have preferred a more interactive lecture style:

> I think in first year particularly, it's very important to feel like you can ask questions, and I think there were ehm, I think there was a scary side to some of the engineering maths lecturers.

There are similarities here with identity research that has addressed disparities between male and female students. Solomon (2007b) noted that female students were more likely to express a preference for working in a group, because it provides reassurance that they are not alone in not understanding (p. 92). She also uncovered a connection to classroom interaction, in that students fear that they will "look stupid" (p. 92) if they ask questions about simple ideas in lectures. Aside from the high standard expected from students, Éabha_ENG regarded some of her lecturers as unapproachable because they represented an extension of the competitive atmosphere that she experienced among her engineering class:

> Yeah, especially in engineering like I feel like there's a bit of an upperosity about some of the lecturers. They're like "oh well like I have a PhD in engineering, I'm obviously the smartest thing going," but like, come on we're all, we're just learning like ((laughs)).

This description might pertain to first year mathematics in particular because lecturers, perhaps understandably, viewed the material as basic compared to what was to come in later years for these students. Her perception of some lecturers as all-knowing, was not mentioned with regard to modules in subsequent years of study, and indeed she will discuss an important experience to the contrary in third year.

Thankfully, at the end of first year, her hard work paid off. Unsurprisingly, she mostly worked on her own because of the aforementioned pressure to appear knowledgeable to the rest of the class:

> Ehm (elongated) actually yeah, it's more so by myself I think that I managed to piece [the maths modules] together. I, because it's just very high paced in the actual lectures and again especially in first year, you didn't want to be asking what people would deem as like a stupid question.

It appears that working alone was enough for her to bring everything together to her satisfaction in examinations:

> I think towards the end of all the modules everything kind of came together for maths particularly. Like I think throughout it you haven't a clue what's going on but then when you sit down with all of the content at that the end of the module, we kind of realised that they all piece together quite well.

However, she did not only work alone, and explained that other students formed study groups, and suggested that there was more comradery to be found among the first year engineering class with regard to working together in mathematics compared to other modules: "maths in particular was actually one that kind of brought the year together I think. A lot of people formed friends so they could figure stuff out in the library." Despite giving an isolated impression of first year mathematics, Éabha_ENG has come to see collaboration as a vital part of working in her field: "I think for engineers especially, it's very important to kind of like have a group of people working on things rather than just working on things individually." This perspective seems to have developed after her first year experience although, as demonstrated above, the utility of collaborating in mathematics broke through the competitive atmosphere to some extent, even then. In line with the collaborative approach taken by Dónal_SC1, Éabha_ENG espoused the benefits of multiple minds figuring out the same problem, valuing different perspectives contributing to a broader understanding over correct explanations from an all-knowing authority (Solomon & Croft, 2015, p. 272):

> Since first year, everyone has kind of said engineering is one of those degrees that you need to be able to talk to people about to actually pass and do well in. Because no one person has all the information ... So, like someone's going to get something more so than you get it, and then vice versa, because your minds are going to work different. And then

> having a few people together, being able to kind of figure out, then one maths question you end up under- understanding a lot more than just one person standing up and kind of teaching you that way so.

Having described an intimidating atmosphere in mathematics lectures in first year, Éabha_ENG explained how she reacted positively to her favourite lecturer, who motivated students while maintaining an approachable demeanour:

> [Lecturer A] is so good at keeping you engaged. He's scary enough to make sure you do your work but also very approachable ... he was very, really good at his job I felt, he was very good at, you know, making something that seems a bit scary and daunting, very interesting and then as well, really doable.

Similarly to Ciarán_ENG, she described how Lecturer A kept students engaged by interacting with them during the lectures in a manner that kept them alert:

> He was so funny in the actual lectures but then no one would be caught dead being picked on. As in, you know, ah being asked the question and not knowing it because you would be shamed ((laughs)), named and shamed in the actual lecture. So, I think he did a really good job lecturing and I think he did, like he was a really good college lecturer, I think.

It may be that a more competitive environment in engineering, where every student had passed higher-level mathematics, allowed this lecturer to inspire students to work harder by putting them on the spot in class, rather than demotivate those who did not understand. In any case, this approach was well received by both interview participants from Engineering.

Second and Third Year

A strong growth mindset (Dweck, 2006) emerged from this participant's descriptions of her experiences, and she indicated several times that hard work is required to progress through the mathematics modules more than natural ability, and that those who are interested and dedicated enough can meet the standard required. She hypothesised that a main purpose of the first year engineering mathematics module was to "weed out" those who were not willing to put the work in. Figures extracted from data compiled by the HEA (2022b) show that the non-progression rate for engineering programmes in Éabha_ENG's first year (in 2017-18) was quite high, at 14% (see appendix B.2). The language used by Éabha_ENG in the following extract indicates that the "weaker" students, who did not progress through the first year mathematics modules, were better off finding out at that early stage, rather than in second year where things get more difficult:

> It's a big step up from first year. Second year is kind of when you know the people who know they kind of have an interest in it are still staying on. The weaker people have kind of backed out after first year ... first to second year because first year you're really covering your bases, with second year you start your engineering.

A key feature of teaching that, Éabha_ENG felt, helped promote this growth mindset involved an interactive student-lecturer relationship in class. This stemmed from her earlier descriptions of working collaboratively with classmates in first year, where she championed the benefits of multiple engineering minds contributing different knowledge to a discussion over a single authority disseminating a single correct explanation. In second year, she expected her engineering mathematics modules to include meaningful interaction with knowledgeable lecturers, but when Lecturer A was on sick leave, the replacement lecturer, Lecturer C, took a different approach. She commented that Lecturer C "didn't seem to have a lot of knowledge about [the module] ... didn't seem to be, you know, interested in making sure everyone knew what was happening" and "talked at you" rather than to you. Instead of attending these lectures, Éabha_ENG would work in the library with her classmates but acknowledged that they missed out somewhat because "if you're not going to a certain class because of a lecturer, there's no guarantee that you're going to work on that specific thing during that hour. Like we'd be working on other things during that hour sometimes as well."

In contrast, she encountered a more comfortable style of teaching from Lecturer A, which suited her view of mathematics very well. in third year, she praised him for presenting his own knowledge of Data Analytics as fallible, and open to correction, which made her feel like they were all in the same boat, working together to understand the area of study as best they could:

> He was very transparent in being like, saying that it wasn't something that he was very, you know, knowledgeable about. ... he was very open with being like, look, we're all just learning, this is the basic stuff I'm teaching you, but I'm no pro either. Like he was really, he's very honest, I think. He was really good at being like you know you might struggle with this but that's ok because this is all new and you know I'm not unreal at it either so just try your best kind of thing.

This description certainly imagines this classroom experience as one that promoted a growth mindset, wherein students focussed on furthering their knowledge and learning from their mistakes, rather than worrying about whether they are right or wrong (Boaler, 2016; Dweck, 2006). The fallibility demonstrated by the lecturer is a stark contrast with first year mathematics where Éabha_ENG noted an "upperosity" amongst some lecturers. She emphasised how this influenced her mathematical identity and self-image as an engineer, again touching on issues raised by Solomon (2007b, p. 92) which were discussed previously in the first year section:

> I suppose it was nice seeing [Lecturer A] being like, I'm not too good at this either. I thought that was kind of a nice thing to hear as an engineer being like, oh ok, that doesn't make me feel stupid.

In section 6.2.3, another engineering student described how he would find his own methods to solve questions that made sense to him, rather than blindly follow the methods demonstrated by their lecturers. Éabha_ENG illustrated how she and her engineering classmates were encouraged to take this approach. This view of learning aligned with her depiction of problem solving as involving different approaches to get to the correct solution, each of which comes from a valuable perspective that may contribute to one's own understanding of the problem:

> Yeah, there's, there was, they taught you one way but then they'd be like, you can also do it this way and this way but I'm teaching you this way. ... I think that was good as well that they understood that there's various different people coming into the course. I'm going to teach you this way because that's how my brain works, but you can also do it this way and this way. So yeah, no that definitely happened a lot.

Corroborating a point made by ENG_08 in FG contribution 9 (see section 6.2.3), Éabha_ENG noted that engineering students "end up using the end product of the maths that [they] learn" because they regularly have to apply it across their other engineering modules. She described the utility of simultaneous equations, differentiation and integration, particularly in fluid mechanics and mechanics of machines, noting that "they were taught heavily in the second year modules and the first year modules so I suppose they would have been featuring a lot in most of the other modules after that." The importance of seeing her coursework put into practice came out strongly in this interview, as did her conception of problem solving as tackling real-world problems:

> I think internships are a real good one for being like, oh ok, this that we're seeing in college, it's being applied ... I was just presented with a problem, and they were like "fix it, what are you going to do to fix it?" ... it was eye-opening to see that the stuff that

> you're learning is actually, it's in practice as well.

She further explained how she likes to see engineering and mathematics in action, and how this inspires her to learn more:

> Open days and stuff when you're brought into the labs. ... even going into the industry to have those internships like, it's all very inspiring and ... it gives you more of an interest when you see that it's applicable in the real world, I think.

Seeing the fruits of her labour cemented her passion for "hands-on, problem-solving situations" and she took satisfaction in seeing the process to a conclusion: "I, you know, had the experience of getting to draw that up, and send it off to someone, and get it designed, and then seeing the actual product in my hand." Again, she distinguishes this type of real-life problem solving from the deficient version that was presented to her in mathematics classes:

> You have to think about materials if you are going to be looking at anything or you have to think about, you know, there was a machine that wasn't working properly, and you had to understand like the actual mechanics of it to see what was happening, and it was that kind of problem solving. But I suppose like, the base layer of all of that stuff is maths.

Fourth Year

In third/fourth year, the work placement (INTRA) that usually takes place was cancelled because of the COVID-19 pandemic, and despite having experienced two summer internships already, her enthusiasm to see and learn more about the industry demonstrated how highly she valued practical experience: "It's a bit of a shame ... that would have been another six months of kind of seeing, does that apply somewhere else in the industry that I just haven't had a chance to see yet." When teaching moved online, this had some repercussions for the atmosphere among her class, who, by now, were used to collaborating:

> That's the huge difference between COVID now and first and second year is that it's, you can't get together and work things out. Which I think is one of the most important things in engineering.

It has been documented in the literature that a lack of engagement and interaction in online lectures was a notable limitation of this form of teaching for both lecturers and students (Gilbert et al., 2021; Ní Fhloinn & Fitzmaurice, 2021). Éabha_ENG had previously disengaged from lecturers who were less interactive and appreciated those who presented their knowledge as fallible, and whose classes involved active construction of knowledge. Online lectures moved back towards the former type:

> I was much better in second year, but over Zoom I just feel like I may as well be sitting looking at YouTube like, I'm learning nothing. It's, it's "interactive" in inverted commas. ... you're just looking at screen ... I didn't go to half as many classes as they would have this year, than what I would usually be going to.

I brought up the interactive classes she had described with great fondness earlier on, and she explained how different her online lecture experience had been:

FH: Hmm. I can't imagine a class like [Lecturer A]'s taking place on Zoom.

Éabha_ENG: Yeah, me neither. I feel awful for the first and second years, like not getting to interact with [Lecturer A] ((laughs)).

FH: Yeah. They don't get the full experience.

Éabha_ENG: Yeah, exactly. Ehm, but yeah, even like no one's gonna pipe up in a zoom class and ask a question. Like, there hasn't been anything in classes that I've been in that people, unless it's at the very end, [they] will muster up the courage to ask a question or they'll type it in the chat. And you know yourself like, actually yeah, most people just type in the chat, instead of saying it but that's not, you may as well be emailing them then after a lecture.

Previously, it was noted that Éabha_ENG was motivated to work hard by herself, and with her classmates, because she had lecturers who she considered to be intimidating and unapproachable. Her final year project supervisor fell into this category:

> My fourth-year project, I was distraught by it at the start. My supervisor was [supervisor], one of the ones that we had in first year, so my perception of him was unapproachable, don't even ask him any questions, just kind of get on with it.

The effects of COVID-19 moving teaching online, and requiring students to work from home, further isolated her from seeking help when she needed it:

> Because the COVID, I'm alright now, but at the start the thoughts of a zoom call just gave me so much anxiety, I hated it. I didn't want to interact with anyone over zoom. So, I put off any interactions with them instead of asking them for help.

Although this was a temporary situation, and in-person learning has since resumed, the insight given by Éabha_ENG into the student experience of online learning is important for preparing for a future where this medium is expected to feature more strongly in university education.

Summary and Insights

In this personal narrative, Éabha_ENG exhibited an alternative conception of problem solving, which appeared to align well with her preference for teachers who filled a facilitator role but did not express a problem-solving view of the discipline of mathematics, as described by Ernest (1988). Firstly, on problem solving, she bemoaned Leaving Certificate physics questions that, she felt, bore no resemblance to the real world, and disguised routine tasks that were presented as problems in mathematics lessons (Schoenfeld, 2016; Vinner, 2014). She described how problems have no prescribed route to a solution, and that one must work autonomously to construct such solutions (Schoenfeld, 1989), but that her focus was on practical, physical problems. This conception bears some similarities with "getting started" as presented in section 6.2.4, particularly the idea of breaking down a problem, but aligns more closely with a technological pragmatist's view of mathematics (Ernest, 1991), than a problem-solving view (Ernest, 1988): "I'm still a firm believer that you know the world goes round because of maths and stuff like that ... it's breaking it down to the simplest form I think." She gave examples which included the chemical equations behind happiness (biomechanics), the way the human eye works, adding that "it's all maths to me ... that's kind of how I'd see the world."

As part of her conception of problem solving, she appreciated collaborating with her classmates to reap the benefits of multiple minds figuring out the same problem, because she felt that different perspectives contribute to a broader understanding, more than explanations from an all-knowing authority. As was the case for Dónal_SC1, this type of collaborative work resonates well with the work of Solomon and Croft (2015), who observed that students can passively receive knowledge from each other, or they can value understanding each other's explanations, and thus develop their own arguments about what is right (p. 272). A complicating factor in her personal narrative arose in the form of the intimidating backdrop in first year mathematics, where she perceived her lecturers as mostly unapproachable or "scary," and highlighted that students were resistant to ask questions in class for fear of appearing "stupid" in front of their classmates. Comparable perceptions have been evidenced in the literature by Solomon (2007b), who argued that students feel they may "look stupid" (p. 92) by asking questions about simple ideas in lectures because they do not realise that they are not alone in not understanding.

On the role of the lecturer, Éabha_ENG expressed a preference for those who were approachable and interactive, and contrasted this with a particular lecturer who "talked at you" not "to you," and whose lectures she avoided as a result. Overall, she felt that engineering students were encouraged to autonomously pursue methods to solve questions which made sense to them, and that they "end up using the end product of the maths that [they] learn." Unsurprisingly, online lectures during the COVID-19 pandemic lacked the engagement and interaction that Éabha_ENG appreciated in previous years, and discouraged her from seeking help when she needed it, particularly with regard to completing her final year project. She also expressed disappointment that work placements (INTRA) were cancelled at this time, and despite having experienced two summer internships already, her enthusiasm to see and learn more about the industry demonstrated how highly she valued such practical experience. On reflection at the end of the interview, she underlined how her interest in mathematics has always been in the background of the experiences she has had in engineering (both in class and during the internships), and will continue to do so in the next stages of her career:

> I think maths have played quite a big part in my life and what I'm interested in, and what I find interesting in the world, and that kind of thing. ... my future job is probably going to be heavily based around maths and engineering.

Chapter 8

Conclusion and Insights

This study presented a qualitative investigation of the mathematical identity of science and engineering students and consequently, as discussed in section 3.2.6, did not seek to generalise results beyond the context considered here. However, I claim that the theme narratives (chapter 6) and personal narratives (chapter 7) that were developed through analysis, are resonant for a wider class of students than are included in the study, although the elements of each that any particular student might find applicable to their own identity will vary. To answer the research questions, a longitudinal study was employed, which comprised three main stages of data collection, supplemented by two pilot studies. The data collection stages involved open-ended questionnaire responses, focus groups, and narrative interviews. The first two stages were analysed using reflexive thematic analysis, while the interviews were analysed through a narrative analytic framework called storying stories.

Through thematic analysis, I considered the meaning of participants' identities within their own narratives and within the context of the entire dataset, to establish the elements of mathematical identity that were common across the data (Braun & Clarke, 2006, p. 81). Thematic analysis provided important flexibility in deconstructing a so-called rich dataset (see section 3.2.6) into manageable codes, which could then be used to construct theme narratives that convey the "subtlety and complexity of a truly qualitative analysis" (Joffe & Yardley, 2004, p. 57). In particular, the hybrid inductive-deductive approach to thematic analysis (see section 5.1) allowed me to incorporate the knowledge gained from previous work on mathematical identity in Ireland, while also grounding the study in the new data contributed by a previously under-researched group of participants. Participants' personal narratives gave insight into the nuanced ways in which they came to develop particular mathematical identities, through an investigative and analytic approach which attended to the complexity and individuality of this process. A personal narrative has the capacity to involve the reader in the emotions and detail of the participant's journey, projecting their voice in a manner that is simultaneously more accessible and more resonant for the reader (Bruner, 1986; Johnston et al., 2021). A powerful advantage of taking a narrative approach among a smaller cohort of students is that it allowed me to present not only what aspects of mathematical identity they embraced, but also the reasons why these aspects were significant for their identities.

8.1 Contribution to Knowledge

In this section, I will detail the contribution to knowledge made by the data, findings, and methodology as part of an innovative and extensive investigation into the relationship of science and engineering students with mathematics.

8.1.1 Contribution of the Data and Findings

This study provides a nuanced and multifaceted account of the mathematical identity of science and engineering students which gives insight into the changes in mathematical identity that occurred during their time in university. The longitudinal aspect of the study allowed differences between groupings (Science I, Science II, Engineering) to emerge more strongly over time. A rich, longitudinal picture of the mathematical identity of science and engineering students, adds a new perspective to the existing literature in Ireland, by presenting the changes that occur in the mathematical identity of this under-studied group of participants as they transition to university.

As will be discussed in section 8.2, I discovered that MISE participants' mathematical identities were characterised by an absolutist view of mathematics (with Platonic and problem-solving views seldom evident) and a belief in the necessity of hard work and persistence at university level, regardless of natural ability (see section 8.2.1). Furthermore, a focus on the real-world applications of mathematics directed many participants' attention towards relational understanding and towards problem-solving competencies. In participants' first year, differences between the groupings were explored through analysis of their questionnaire responses, the strongest of which can be characterised as follows: While some participants appreciated the reward of rising to the challenge in mathematics, others simply saw their mathematics modules as a means to an end (see section 8.2.3), with the latter viewpoint most prominent amongst the Science II grouping.

The findings of this study suggest that science and engineering students, including PSTs, would benefit from teaching which emphasises relational as well as instrumental understanding (procedural fluency), and which incorporates learning mathematics through problem solving. Moreover, the findings provide evidence of an opportunity for university practitioners to provide more effective support for students in their transition to, and successful progression through, university education. In section 8.3, I recommend that such support should entail more one-to-one contact and should build on the aims of primary and post-primary school curricula to foster a productive disposition among students.

It is beyond the scope of this study to make inferences about teacher identity separate to participants mathematical identity. However, participants' relationship with mathematics was considered to influence their real-world practice as teachers, scientists, and engineers, as was evidenced in chapter 7. Through analysis of the interviews, I reasoned that Science II and Engineering participants were more likely to acknowledge a perceived disconnect between classroom mathematics and their real-world practice, a phenomenon which is well-established in literature concerning engineering education (Gainsburg, 2007, p. 478; Goold, 2015, p. 7). PST participants, on the other hand, readily provided examples of direct connections between their studies and practice, with several participants acknowledging that what they had learned in certain lectures could immediately be implemented to improve their own teaching. It is worth noting that three of the PST interview participants appeared to view the increased difficulty, complexity, and workload associated with Leaving Certificate and/or university mathematics as a catalyst for changes in their mathematical identity over time. For Saoirse_SC1, the apparent confidence and ease with which male students approached the study of mathematics caused her to feel isolated when seeking mathematics support through the university, but working with a close community of classmates helped her navigate through her own struggles with the subject. Dónal_SC1 recounted experiences in post-primary school where his natural ability in mathematics shone through, despite the variation in teachers, and attributed his struggles for Leaving Certificate to a lack of interest or motivation, rather than ability. Although he found university mathematics quite challenging at first, it appeared that a positive, supportive experience with a DCU lecturer helped him re-establish a mathematical identity as a capable student, and subsequently as a capable teacher.

The experiences of Saoirse_SC1 and Dónal_SC1 impacted on their mathematical identities and although they did not explicitly link every part of their narrative to their teaching practice, the totality of their experience (informing their mathematical identity which, in turn, persists long after the experiences themselves) undoubtedly influenced their teacher identity. The intertwined nature of mathematical identity and teacher identity can be discerned even more clearly in Breandán_SC1's interview narrative. As his mathematical identity shifted from being a student of mathematics to a teacher-in-the-making, he embraced the concept of a growth mindset, largely rejecting the idea of fixed ability, and espoused the benefits of teaching for understanding, and using constructive feedback to encourage incremental improvement amongst his students. While Goldin et al. (2016, p. 25) observed that identity studies are often detached from studies concerning emotions, attitude, or beliefs, my study demonstrates that conceptualising mathematical identity as participants' relationship with mathematics initially as a student, and, over time, as a developing professional, can give insight into aspects of this relationship that are relevant to both contexts.

8.1.2 Methodological Contributions

The study has methodological implications for future research regarding how the concept of mathematical identity is framed, and which methods of data collection and analysis are used in its exploration. An important product of this study is the list of codes that were used to capture elements of participants' mathematical identities (see appendix G.2). These codes were drawn from the literature, a pilot study, and two stages of data collection/analysis, and thus, are of interest to other mathematical identity researchers who implement analytic approaches that build on a "start list" (Miles & Huberman, 1994, p. 58) of deductive codes.

By employing a narrative paradigm, I operationalised the definition of mathematical identity in a manner that was methodologically consistent with the methods of data collection and analysis, addressing concerns expressed by Sfard and Prusak (2005) about identity research. In particular, this paradigm brought methodological coherence to the assumption that narratives and identity are equated (Sfard & Prusak, 2005), and that narratives construct and organise identity, rather than mirror some deeper imperceptible trait within the individual (Spector-Mersel, 2010, p. 208; Sparkes & Smith, 2008, p. 299). Establishing methodological coherence early in the process was of particular importance because of the emergent design of the study (see section 3.2). The approach to coding, which is described in section 5.1, was guided by the constructionist epistemology of the narrative paradigm and aligns well with recent publications concerning reflexive thematic analysis (Braun & Clarke, 2022), which were not available at the outset of this study. The use of a narrative paradigm also facilitated the incorporation of the storying stories framework (McCormack, 2000a), answering a call made by Lutovac and Kaasila (2019, p. 513) for identity researchers to investigate new analytic methods from social sciences within their research.

In terms of data collection, this study highlights the importance of preparing participants for the narrative mode of thought (Bruner, 1986) that is required of them when writing, or talking, about their mathematical identity. The four-part narrative interview method presented in section 4.5, placed prime focus upon the creation of space for participant reflection, and allowed them agency with regard to their personal narrative, and is thus of interest to other researchers who prioritise these aspects. In the interview, the philosophy of co-construction was put into practice through the creation of a key events graph, which proved effective for encouraging participants to step back and reflect on the overall picture of their journeys.

With regard to analytic methods, a hybrid inductive-deductive approach to thematic analysis proved to be a successful means of incorporating the knowledge gained from previous work on mathematical identity in Ireland, while remaining grounding in the new data. Although it was not clear in their original work, Braun and Clarke (2022) recently clarified that thematic analyses “can have elements of both orientations” (p. 56), and this study provides evidence of the utility and effectiveness of the combination. In tandem with the use of thematic analysis to investigate patterns across the data, I subsequently implemented a narrative analysis to probe the individual mathematical identities of participants, and thus developed a group narrative and personal narratives respectively. The distinction between analytic methods that emphasise a student’s experiences in the context of the group, and within the context of their own journey (see Figure 3.5), could aid methodological consistency in identity studies more broadly. In particular, this study demonstrates that the storying stories framework is an effective means of attending to longitudinal changes in identity, and should be considered by those who seek a detailed picture of a smaller group of participants.

8.2 Addressing the Research Questions

The three research questions will be addressed in turn in this section, with reference to the three groupings of participants (Engineering, Science I, Science II) which were described in section 3.2.1. RQ1 and RQ2 were answered mainly through thematic analysis of the questionnaire (Data1) and focus group (Data2) responses in chapter 6, while RQ3 was addressed through narrative analysis of the interviews (Data3) in chapter 7. The findings, detailed below, draw from many of the points made in the "summary and insights" sections that were presented throughout chapter 6 and chapter 7. While every effort was made to distinguish the findings between the three research questions, some differences between the mathematical identity of participants in each grouping were revealed over time as the study progressed, and thus were of relevance to RQ2 and RQ3. To take this into account, RQ2 will be presented last, after RQ1 and RQ3.

Since the narrative paradigm utilised in this study acknowledges multiple valid realities held by individual participants (see section 3.1.4), the credibility of the findings depends on implementing data analysis procedures whose outputs can adequately represent these multiple realities (Guba & Lincoln, 1985, p. 296). For the purposes of presenting this audit trail with clarity, the two categories, five themes, two exceptional cases, and five personal narratives from chapter 6 will be abbreviated as shown in Table 8.1.

Table 8.1

Abbreviations used in the findings section for categories, themes, exceptional cases, and personal narratives.

Abbreviation	Category, theme, exceptional case, or interview	Location in thesis
C1	Personal Feelings about Mathematics	section 6.1.1
C2	Reasons to Study a Programme involving Mathematics	section 6.1.2
T1	Theme 1: Teachers/Lecturers	section 6.2.1
T2	Theme 2: What is Mathematics?	section 6.2.2
T3	Theme 3: Ways of Learning Mathematics	section 6.2.3
T4	Theme 4: Mindset and Getting Started	section 6.2.4
T5	Theme 5: Transitions	section 6.2.5
INTP	Pilot Interview	section 7.1.1
INT6	Interview 6: Fiach_SC1	section 7.1.2
INT1	Interview 1: Aodhán_SC2	section 7.2.1
INT2	Interview 2: Breandán_SC1	section 7.2.2
INT3	Interview 3: Ciarán_ENG	section 7.2.3
INT4	Interview 4: Dónal_SC1	section 7.2.4
INT5	Interview 5: Éabha_ENG	section 7.2.5

8.2.1 RQ1: What is the Relationship of Science/Engineering Students with Mathematics?

In their questionnaire responses, I found that MISE participants attributed their personal feelings about mathematics mainly to their own perceived competence, experiences with teachers, and their vision of mathematics (C1). While these influences are in line with previous research (Di Martino & Zan, 2010), MISE participants also emphasised the rewarding feeling of rising to the challenge of working in mathematics (C1, INT1, INT3), as well as an additional cause related to their reasons for choosing their programme, which will be discussed in section 8.2.3. Participants' recollections of their post-primary teachers (T1) supported the characterisation of teachers as an overarching mediating factor with respect to their attitude towards mathematics (Di Martino & Zan, 2010, p. 43), and also with respect to their motivation and enthusiasm for mathematics. In comparison with the Irish mathematical identity literature reviewed in section 2.6 (which did not include students of science and engineering), MISE participants placed much stronger emphasis on seeing the applications of mathematics, and reserved particular praise for those teachers who grounded the learning of mathematics in real-life applications. They linked such teaching to their understanding of, and interest in, the subject. An emphasis on applications of mathematics might be expected in science and engineering programmes, but in the focus groups and interviews, Fiach_SC1 was particularly vocal about the necessity for post-primary teachers to demonstrate the applications of mathematics to inspire their students (INT6). He reported feeling confident to teach mathematics because he can "explain, describe and show the real life applications for it" (Fiach_SC1).

Although participants' descriptions indicated that they saw the role of the post-primary teacher as an explainer, corresponding to a reception of knowledge model of learning (Ernest, 1988), I reasoned that participants also saw success in mathematics as involving relational understanding. Many participants described moving away from viewing their teachers as the sole authority on mathematics and emphasised the importance of developing an "understanding" of mathematics (T3), although some differences were revealed with regards to the meaning of "understanding" between the groupings (see section 8.2.3). Participants drew connections between knowledge of applications and relational understanding of mathematics. They acknowledged the use of instrumental understanding for passing examinations (T3, INT2, INT3, INT4), but believed that their ability to do examination questions should be supplemented with understanding: knowledge of why the procedures work, and why they are relevant to them. Since MISE participants' mathematical identities showed evidence of integrating the notion of "understanding" mathematics in the manner described above, I propose in section 8.3 that university practitioners can, and should, avail of the pedagogical opportunities that relate to their students' mathematical identities. In particular, I suggest that practitioners ensure that students not only know how to conduct mathematical procedures, but also when and why they work, so that students can carry out such procedures "flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 5).

MISE participants described how they mostly experienced an instrumental approach to teaching mathematics in the post-primary classroom, which resulted in learning that mostly involved memorising and training themselves to follow disconnected rules through practising questions

(T2). Correspondingly, their mathematical identities were build around absolutist views of the discipline of mathematics, wherein mathematics is pre-existing, immutable, and objectively true (Ernest, 1988; Thompson, 1992), yet mostly believed that its purpose is realised through application to real-life situations. On the surface, the absolutist view would appear to be incompatible with the inquiry-based “discipline of exploration and sense-making” in mathematics envisioned by Schoenfeld (2020, p. 1172). More broadly, the absolutist view may be problematic in classrooms where students are expected to make and refine contributions to core mathematical ideas through classroom discourse and are therefore positioned as active participants in the creation of mathematics (p. 1173). If mathematics is fixed and absolute, why would students concern themselves with its (re)creation in class? However, it emerged that despite their absolutist views of mathematics, a focus on real-world applications directed many participants’ attention towards relational understanding (T3) and towards problem-solving competencies (T4).

Some participants saw mathematics as requiring a daunting amount of persistence, effort, and hard work in comparison with other subjects (T4). Solomon (2007b) identified a dominant belief in the undergraduate community, that those who are good at mathematics are effortlessly successful, and that “you can either do it or you can’t” (p. 89). MISE participants thoroughly rejected this viewpoint, and there was widespread agreement that while they might have been able to rely on natural ability before now (see section 8.2.2), in university everyone needs to work hard to be successful in mathematics. Part of the effort required concerned the large amount of time that is necessary to study mathematics, and the need to balance understanding in lectures with self-directed study outside of lectures (T1). Ciarán_ENG highlighted the variation in these demands between teachers, describing one lecturer with whom he understood “90% of things as he was teaching them,” and another with whom the entire lecture was spent writing “as fast as you can” and trying to understand the material afterwards (INT3).

Participants saw success as easier with the right mindset, which is built up over time, and involves confidence, persistence, and not seeing mathematics as something to fear (T4). They indicated that their so-called mindset affects their ability to start a question or problem, to persevere with multiple attempts, or to interpret and analyse the given information. They also demonstrated some familiarity with the language of problem solving (T4), and duly presented their strategies for “getting started,” which aligned well with the initial stages of problem-solving models that are communicated in the literature (Lesh & Zawojewski, 2007; Schoenfeld, 1989, 2020). Such strategies are an essential tool for students to learn through making decisions and tackling roadblocks when engaged in problem solving (Dweck, 2015), and thus are important for reaping the benefits of a growth mindset (Dweck, 2006). Since participants exhibited mathematical identities in which evaluating how to start mathematical tasks was crucial, in section 8.3 I recommend that university practitioners should be cognisant of the aims of primary and post-primary curricula in Ireland and enact pedagogies that advance students' problem-solving skills through opportunities for the exploration and interrogation of mathematical concepts.

8.2.2 RQ3: How does the Relationship of These Students with Mathematics Change Over Time?

Changes in Mathematical Identity at Post-Primary Level

With regard to transition points in post-primary school, participants noted challenges associated with the breadth of the Senior Cycle curriculum, and the greater effort required compared to Junior Cycle (T5). Their strategies for navigating this transition coalesced around "taking control" of their mathematical learning, by establishing sources of learning outside the classroom (mostly through private tuition known as grinds) and adapting their own approach to learning mathematics. Making the decision to take ordinary- or higher-level mathematics for Leaving Certificate provided a catalyst for this change, and participants' comments reflected how they viewed this as a decision that was within their own control. Several participants noted the difference in teaching styles, their intended programmes of study in university, and the perceived difference in standard between ordinary and higher-level as factors in their decision.

As mentioned in section 8.2.1, participants strongly endorsed the notion that university mathematics requires hard work from everyone, regardless of natural ability. However, participants spoke about relying on natural ability in their early years at school. Those who described mathematics as not requiring effort, referred only to primary school, or Junior Cycle (T4) whereas several participants described struggling with mathematics for the first time at Senior Cycle or in university (T5, INTP, INT2, INT4, INT5). It appeared that participants took control of their learning at Senior Cycle because many of them found mathematics challenging for the first time in their schooling, and changed their minds about the balance between hard work and natural ability in mathematics. This finding resonates with research from the UK, which reported that while students felt that they could achieve good grades at GCSE level with little effort, A-level mathematics required more hard work and understanding (Hernandez-Martinez et al., 2011).

Changes in Mathematical Identity at University Level

Although some MISE participants noted an increased level of difficulty or workload in first year at university, many more saw the mathematical content as largely familiar from higher-level Leaving Certificate (T5, INT3, INT5). In MISE, the latter two stages of data collection and analysis were used to investigate changes in mathematical identity in later years of study, when the mathematical content has broadened further beyond Leaving Certificate mathematics.

In INT1, Aodhán_SC2 repeated his earlier impression that his science modules did not overlap with his mathematics modules. However, in second year he described how he started to see some connections, and by the time we discussed third year, he re-evaluated his initial assessment, and concluded that most of the mathematics content that arose in science modules, he had encountered before in mathematics modules. He highlighted that this realisation was brought about, in part, through narrating his mathematical identity in the interview. In contrast, Ciarán_ENG commented that everything in engineering "carried over" from one year to the other (INT3), and he particularly praised the fourth year of his programme for demonstrating how mathematics can be "implemented in everyday life" (Ciarán_ENG).

PST interview participants presented differing views with regard to the connections between mathematics and education in their programme. Breandán_SC1 described a monumental, but gradual, change from seeing himself as a science student to seeing himself as a teacher-in-the-making, prompted by teaching placement in second and third year (INT2). He highlighted that going "deeper" into mathematical content in university gives PSTs an important perspective on the material which features at post-primary level, a description which resonates well with "horizon knowledge" as defined by Ball and Bass (2009). Similarly, Dónal_SC1 highlighted his appreciation of the opportunities to learn that were provided by teaching placement, and of modules that attended to pedagogical content knowledge (Ball et al., 2008; Shulman, 1986) in the latter years of his programme (INT4). However, as a PST who did not specialise in mathematics, and viewed mathematics as a means to an end (C2), he perceived himself to be a good teacher relative to his classmates precisely because he had a lesser knowledge of, and had remained untainted by, advanced mathematics. The influence of seeing their future career in action was evident in the personal narratives of Ciarán_ENG and Éabha_ENG also, both of whom saw their internships as formative for judging the role of mathematics in their future careers (INT3, INT5).

In the early stages of university, studying mathematics was mostly presented as an individual endeavour, and participants saw the benefits of working on their own (T4, INT4, INT5). For Aodhán_SC2, collaborating in mathematics appeared to be elusive throughout his four-year programme, and he described that he mostly worked alone (INT1). He noted that his experience at Senior Cycle was more collaborative and described how a student who "got it" would explain to their classmates (T3). Engineering participants, in contrast, were routinely required to collaborate on group assignments and projects, and some developed collaborative groups to work on assignments together in the library (T3, INT3, INT5). Éabha_ENG was most vocal in her endorsement of multiple minds working together, because she felt that different perspectives contribute to a broader understanding, more than explanations from an all-knowing authority.

PST participants also reported collaborating with their classmates (T3, INTP, INT6), but some evidence was presented which showed that some participants' early experiences of the MLC discouraged them from using the group study space (INTP). For Saoirse_SC1, the presence of male students, who perhaps were working independently of the tutors and felt comfortable enough to socialise in that setting, caused her to feel alienated from the space and created an immediate perception of "not belonging" (Solomon, 2007b), which lead to the group studying elsewhere in the university. It appeared that her perception that female students of mathematics have to work harder and manage more stress in order to keep up with their male counterparts (INTP), resulted in her early experiences with the MLC triggering complex issues related to the perception of mathematics as a so-called male domain (Brandell & Staberg, 2008). It is reported in the literature that students can use shared spaces like the MLC to reformulate their mathematical identities (Solomon et al., 2010, p. 429), and this study demonstrates that students like Saoirse_SC1, who are interested in engaging with mathematics support, can experience challenging reformulations of their mathematical identity which were not intended by the university. Furthermore, in the interviews, during MISE participants' fourth year, they appeared to have become more familiar with the value of collaboration with others, suggesting that shared spaces like the MLC could be utilised by the university to encourage a deeper appreciation of multiple perspectives and meanings among students (Solomon et al., 2010, p. 429). To achieve these aims, recommendations concerning the management of such shared spaces are given in section 8.3, which broaden the relatively small body of literature concerning best practice in mathematics support.

Since the interviews took place during the COVID-19 pandemic, this research offered some insight into the changes to the learning experiences of science and engineering students during that period. Participants described disengaging from online lectures with reference to the difficulties associated with using the online platform in an interactive way (INT4, INT5, INT6). Éabha_ENG's favourite lecturer, Lecturer A, interacted so much with his mathematics class that we both agreed that we could not imagine his lectures being conducted in an online environment. She expressed further difficulties in communicating with her final year project supervisor, in part due to fatigue from using the Zoom platform. Both engineering participants were unable to embark upon their work placements (INTRA) in third year, with Éabha_ENG expressing particular disappointment about this given her previous positive experiences of summer internships (INT5). As was noted previously, opportunities to collaborate in mathematics were barely evidenced by Aodhán_SC2, and this may have been exacerbated by the exclusion of students from campus in his third and fourth years (INT1). PSTs experienced online teaching as both a learner and a teacher, with Fiach_SC1 noting that it became much more difficult to determine whether students were engaged during his lessons, and to encourage them to interact over the online platform (INT6). At the same time, Dónal_SC1 was content to let his students work "in their own little bubble" and check in on them "once in a while" (INT4).

8.2.3 RQ2: What is the Difference in the Mathematical Identity of Science Students Compared to Engineering Students?

A dichotomy was observed between science and engineering participants with regard to their reasons for studying their programmes (C2). Although most participants had studied higher-level mathematics, many of the most positive comments came from Engineering participants, who expressed their passion and fondness for the subject, and none of whom described mathematics as a means to an end. Comments of the latter type were mostly attributed to the Science II grouping, many of whom looked forward to discarding mathematics and ending its involvement in their degree programmes.

The notion that understanding mathematics meant relational/conceptual understanding was most strongly supported by Engineering and Science I participants, who were concerned with applying mathematics to a wider variety of questions and contexts (T3). In the focus groups, PST participants reasoned that when it comes to solving mathematics questions, it is important to see the connections between different areas of mathematics (T3). Their conceptions of understanding built on the view that mathematics consists of interlocking parts which, in post-primary school, are taught as if they are completely separate (T2). Engineering participants emphasised how relational understanding is useful for applying a broad range of mathematics to solve complex problems with regard to utility and expediency (T2, INT3, INT5), embracing a viewpoint that more closely aligned with that of a technological pragmatist (Ernest, 1991, p. 118). In contrast, some Science I participants defaulted to instrumental understanding in first year because they did not immediately see the applications of mathematics within their programme and expected this to become clear at a later stage of their studies (T3, see section 8.2.2). In section 8.2.2, differences were noted between the groupings with regard to the development of collaborative groups of students. Science II participants mostly described mathematics as an isolated experience, whereas Engineering and PST participants worked together on mathematics, as was revealed in Data2 and Data3. The latter grouping was the only one to mention the MLC, but it was also clear that they used alternative study spaces and means of communication to work together, perhaps because of the negative experiences of Saoirse_SC1 on her initial visits to the centre (INTP).

Aodhán_SC2, Ciarán_ENG, and Éabha_ENG all distinguished the real-world applications of mathematics that they engage with from their classroom experiences of the subject, using terminology such as "maths in a different way" (INT1), "it is maths but it's not what I would think of maths" (INT3), and "[It's] not just maths alone, definitely not" (INT5). Goold (2015) showed that engineering students in another Irish institution perceived a gap between their academic learning and their workplace practice, which was attributed in part to the dominance of students' "objective" mathematical learning in class compared with the "subjective" mathematics competencies that are necessary in the workplace (p. 552). This suggests that over time, for these three participants, the focus on real-world applications noted in Data1, developed into a pronounced divide between classroom mathematics and its real-world application. In Éabha_ENG's case, this distinction appeared to be related to her conception of problem solving in engineering, as breaking down a real-world problem into objectively true mathematics, which can then be used to provide a physical solution (INT5).

8.3 Recommendations

8.3.1 Recommendations for Practice

Teaching through Problem Solving

Mathematics learning in Irish post-primary schools has moved away from a formal approach, towards a more application-centred one (Kirwan, 2015, p. 322), where it is intended that students learn their mathematical skills and concepts through abstraction from real-life situations (horizontal mathematisation), rather than from within mathematics (vertical mathematisation) (Cosgrove et al., 2005, p. 210; Fitzmaurice et al., 2021, p. 3; Johnson et al., 2019, p. 8). Such an approach aligns with teaching mathematics “through” problem solving, rather than teaching “for” or “about” problem solving (Schroeder & Lester, 1989, p. 33), as discussed in section 2.2. In Data1, MISE participants demonstrated some familiarity with strategies for solving problems that were not dependant on particular mathematical ideas (T4), but their concern with “getting started” on a mathematics problem aligned only with the initial stages of problem-solving models in the literature.

Further intervention in university is required to ensure that students can fully benefit from the sense-making and ownership of mathematics that accompanies proficiency in this area (Schoenfeld, 2020). To follow-on from students’ post-primary education, and ease the transition to university mathematics, lecturers should continue to teach students “through” problem solving. Therefore, a key concern of teachers and lecturers should be to provide opportunities for students to engage in problem solving that facilitates students in making sense of mathematics they may already know. This entails “understanding how and why mathematical ideas fit together the ways they do” (Schoenfeld, 2020), but also coming to see mathematics as a process of discovery which involves asking the right questions. Teaching “through” problem solving, ideally involves careful scaffolding of students’ exploration of the problem, and time for students to discuss, investigate, and reflect upon, the mathematical ideas that arise (Lubienski, 1999, p. 255; Schoenfeld, 2020, pp. 1172-1173). Initiatives that embrace these principles are already underway in DCU, with Hyland et al. (2023) reporting success in implementing a guided-inquiry approach to learning differential equations, which was built around student engagement, collaboration for sense making, and instructor inquiry into student thinking (p. 251). The findings of this study suggest that expanding such initiatives to other modules in mathematics, science, and engineering, would be of great benefit to DCU students. However, it should be noted that developing students' problem-solving skills is not a straightforward endeavour. Amongst a small group of nine PSTs in DCU, Owens and Nolan (2019) reported that when provided with a guided framework for problem solving, students nevertheless struggled to produce clear written accounts of their explorations of mathematical problems (p. 249). A further publication (Owens & Nolan, 2022), which instead focussed on affective aspects of problem solving amongst a larger sample of 151 PSTs in DCU, may offer an explanation of the challenge facing university practitioners. It was observed that students believed in the benefits of understanding mathematical concepts, as was the case in MISE, but still maintained a focus on achieving the correct answer when engaged in problem solving (p. 47). I make some recommendations for further research with regards to these findings in section 8.3.2.

Developing Relational Understanding alongside Knowledge of Procedures

The value that participants allocated to relational understanding, was based on their view of mathematics as consisting of connected, interlocking parts (T2), and was reinforced by encountering questions for which there was not a prescribed route to the solution (T4). The characterisations of what it means to understand mathematics that were developed in T3, complement other research which has found that students perceive a much stronger emphasis on relational understanding of mathematics at university, compared to post-primary school in Ireland (O'Shea & Breen, 2021, p. 35). Petocz et al. (2007) argued for a stronger emphasis on broader conceptions of mathematics in engineering education, involving abstraction, modelling, and applications to real-life, to combat the instrumental view of mathematics as isolated computational techniques (p. 456). In this study, Engineering participants appeared focussed on procedural fluency (Kilpatrick et al., 2001) because they viewed relational understanding as useful for applying mathematics to a range of real-world problems, many of which were evident from their other modules in the engineering programme (T2, INT3, INT5). In first year, although Science II participants expressed a preference for understanding the connections between the mathematics they were learning and their other modules (T3, INT1), these connections were not revealed with the same clarity that was reported by Engineering participants. Thus, I echo a recent call by Faulkner et al. (2023), who recommended that Irish universities continue the groundwork laid by Project Maths, and promote the concurrent development of relational understanding of procedures, alongside the procedures themselves, amongst science undergraduate students. For lecturers, the findings of this study suggest that students may more strongly appreciate the value of relational understanding if it is connected to the real-world applications of mathematics with which they interact in their programme, and/or if they perceive a tangible reward for such knowledge (over instrumental understanding) in their continuous assessments and examinations. This would motivate students to make decisions about "what procedures are needed in what situations" (Faulkner et al., 2023, p. 17), and thus develop procedural fluency (Kilpatrick et al., 2001), rather than learning particular procedures for given situations (NCTM, 2014).

Supporting Students' Transition to University

In DCU, students can seek direct support from their module lecturers through asking questions in/after class, attending office hours, or contacting the lecturer via email, but a question remains regarding whether DCU students feel encouraged or reluctant to pursue these avenues. Thus, to aid their transition to university, it would be beneficial for lecturers to inform students not just about the availability of support, but also to be proactive about advising students of the circumstances under which they can/should seek help from their lecturers, from tutors, or mathematics support via the MLC. If lecturers present themselves as open and willing to engage with their students in this manner, the students would be better equipped to fully utilise the supports that are already available in university. Although post-entry university supports are recommended by the HEA (2020, p. 42) to aid the progression of all students, research in Ireland and the UK has highlighted that the perceived lack of one-to-one support in university, has a particular affect on whether students from further education choose to progress to, and persist at, university level (Sartori & Bloom, 2023, p. 71; Tett et al., 2017, p. 405). Such entryways are important because access to university via further education programmes has been prioritised in Ireland, as part of educational policy aimed at broadening opportunities for access to university education beyond the traditional application route (HEA, 2015, p. 30).

Research in this area tends to focus on mature students or those returning to education, but approximately 40% of further education students are under 23 years of age (Sartori & Bloom, 2023, p. 71). Dónal_SC1 availed of this route, which allowed him to continue in full-time education and, thereafter, proceed to university in DCU. Following analysis of his interview, it appeared that the personal connection and one-to-one support that he received from his Calculus lecturer in first year transformed his narrative of struggle to one of success in overcoming adversity. This connection may well have made the difference between him continuing, or not, his chosen programme in Science Education. It is recommended by Sartori and Bloom (2023) that students' concerns around the demands of university education be combatted through mentoring from other university students who have made the step up from further education. These mentors are well placed to "demystify the university experience" (p. 71) for other first year university students, and for students in further education who might apply to university.

Fostering a Productive Disposition

Solomon (2007b) and Boaler (2016) argued that students who choose to study mathematics based on their high achievement in the subject in school, may experience difficulties in university if they do not have an interest in the mathematics itself. In MISE, many participants reported that they found mathematics to be more challenging at Senior Cycle in post-primary school, and developed a belief that rather than relying solely on natural ability, mathematics requires hard work for everyone. Such beliefs are an important element of a productive disposition (Kilpatrick et al., 2001, p. 5), whereby previously high-achieving students are not easily discouraged by results when surrounded by other high achievers in their university programmes (pp. 131-132). The post-primary curricula for Junior and Senior Cycle emphasise the importance of engaging students in mathematical experiences that are challenging, but achievable, so that students develop a productive disposition and, ultimately, mathematical proficiency (DES, 2017, p. 5; DES, 2015, p. 6). The findings of this study suggest that there is still progress to be made to cater for students who, like some MISE participants, experience Junior Cycle mathematics as effortless. The findings also endorse the aims of a new draft curriculum for primary school mathematics, which positions the development of students' productive disposition and the use of cognitively demanding tasks as key pedagogical practices (NCCA, 2022, p. 25).

Mathematics Learning Centre

Research has shown that the shared collaborative space of a MLC can play an important role in facilitating students to reformulate their relationships with mathematics, and to value multiple perspectives and understandings (Solomon et al., 2010, p. 429). It should be acknowledged that facilitating both collaborative group work and direct interaction with tutors in the same space, may create tensions between these different communities of practice in the MLC in DCU (INTP). On the one hand, the centre might be seen by some students as a social space, for building a community (Croft et al., 2011, p. 5) and availing of group learning strategies (Solomon et al., 2010, pp. 426-428). For other students, the greatest benefit of the centre might be as a study space, which they primarily use to get work done and avail of one-to-one focussed support from the mathematics tutors.

Thus, I recommend that MLC tutors be familiarised with their responsibility to ensure that students know what is acceptable to discuss in the shared MLC space. Tutors' responsibilities with regard to managing the MLC environment should be codified in the documents issued to tutors, and the purposes of managing the environment should be clearly communicated during tutor training. Such an approach would also benefit other universities who are engaged in mathematics support, as the issues raised in this study do not appear to be reflected in literature concerning best practice in mathematics support centres (cf. Croft et al., 2011; Lawson & Croft, 2011). Furthermore, it should be examined whether the existing space in DCU should be physically split in order to prioritise the foremost aim of the MLC to support at-risk and first year students (Jacob & Ní Fhloinn, 2019).

It is noteworthy that despite appearing to be required to work together more than other groupings, Engineering participants did not mention the MLC, and were not included in the DCU contribution to a recent national survey on mathematics support (O'Sullivan et al., 2014, Table 2). Collaboration between the School of Mathematical Sciences and the Faculty of Engineering and Computing, with regard to the mathematics support needs of DCU engineering students, is needed to replicate the previously reported success of the MLC in helping other "service mathematics" students to pass their first year mathematics modules (Jacob & Ní Fhloinn, 2019).[1]

[1] The modules considered to constitute "service mathematics" included those delivered to programmes in Computer Science, Science, Economics, Business, and Accounting (Jacob & Ní Fhloinn, 2019).

Such support has particular relevance for engineering programmes, where the required standard of mathematics knowledge is high, and the non-progression rates for first year students are above the university average (see appendix B.2). In another Irish university, follow-up messages to a first-year diagnostic test were used to advise students to attend mathematics support, if the test result identified them as "at-risk" (Mullen & Cronin, 2022). Integration of similar responses is possible amongst engineering students (and many other cohorts) in DCU, since they undergo diagnostic testing in their first year, and this would provide a manageable means to target mathematics support towards students who are most at risk, as early as possible.

8.3.2 Recommendations for Future Research

Further Exploration of the Findings

MISE participants described the importance of "getting started" on a mathematical problem and appreciated of the benefits of the process, but no attempt was made to assess their problem-solving skills in this study. Recent research involving science and engineering students at an Irish university has shown that although they had an increased awareness and appreciation of the benefits of engaging with mathematical problem solving, this has not resulted in a corresponding increase in problem-solving ability (Faulkner et al., 2023), as was intended by the Project Maths curriculum reforms (O'Meara et al., 2011, p. 334; DES, 2016, p. 20). It has been reported that post-primary teachers in Ireland feel less comfortable with problem-solving pedagogy and are reluctant to invest classroom time in problem solving (Berry et al., 2021). This finding is consistent with research by Owens and Nolan (2022), who found that when engaged in problem solving, PST students in DCU focussed more on producing the correct answer than on the process itself. It is of interest to determine whether students' post-primary and university experiences facilitate them in considering their own thought processes when engaged in problem solving, or mostly provide opportunities for them to reason about mathematical relationships, both of which are promoted in realistic mathematics education (Rasmussen & Kwon, 2007, p. 191). In other words, is the goal of problem solving to use relational understanding of mathematics to find solutions, or is attention also paid to students' understanding of their own thought processes (Schoenfeld, 1989, p. 94)?

Firstly, a investigation of the problem-solving competencies of incoming science and engineering students would help determine if they are arriving into DCU (or Irish universities more generally) with the problem-solving skills they are expected to learn in post-primary school. Secondly, since the Project Maths curriculum has particular relevance for PSTs, a longitudinal examination of the development of their problem-solving skills in university is needed to determine whether they are prepared for their university programme in the first instance, and subsequently whether they are prepared to implement problem-solving pedagogy in the classroom. Lastly, an examination of the types of tasks and problem-solving opportunities that are afforded to post-primary and university students would provide an insight into whether practitioners emphasise learning mathematics about, for, or through problem solving (Schroeder & Lester, 1989, p. 33).

It was noted in section 8.2.3 that over time, both Engineering interview participants came to see a pronounced divide between classroom mathematics and its real-world applications, as was reported by another study in Ireland (Goold, 2015). It is possible that this distinction is related to participants' absolutist views of mathematics, which are discordant with the subjective nature of their real-world practice in engineering. Goold (2015) reported that engineering practice involves ambiguous problems with many possible solutions, where data is not easily accessible, and variables are more complex and numerous compared to problems that students encounter in their education (p. 547). Furthermore, Pepin et al. (2021) proposed "mathematical modelling" as a core competency for engineering students (p. 168), and pointed to Faulkner et al. (2019), who found that engineering lecturers associated their students' beliefs that mathematics is abstract and irrelevant to the real-world, with shortcomings in students' modelling skills. Therefore, it would be of interest to investigate the epistemological beliefs held by final year engineering (and/or science) students with regard to the discipline of mathematics and compare with their beliefs about the application of mathematics in their field. Such a study could have important implications for students who struggle to see the connections between mathematics and engineering (or science), both for their learning at university and for their practice in the working world.

Areas Not Addressed by this Study

This study opens up several interesting avenues of investigation that are deserving of attention in the future. Now that MISE has provided some insight into the mathematical identity of science and engineering students, further studies that examine these groupings with regard to gender and mathematical identity would make an important addition to the existing literature (Mendick, 2005; Solomon, 2007a). In particular, it remains to be explored whether the themes produced by this study are more significant or relevant to students of different genders, and this should be pursued with careful attention to the meaning of gender (Mendick, 2006, p. 10). This would be possible in a large-scale study, or a small-scale study that invites participants from a single grouping or programme. Two female participants in MISE described experiences and situations that they perceived to be scary, daunting, or intimidating (INTP, INT5), and subsequent studies should investigate how female students, particularly those in STEM programmes, navigate through such negative experiences.

As reported in appendix B.2, in the academic year that this study commenced (2017/18), an average of 10% of new entrants to DCU did not proceed to second year in their programme, with a higher rate of 14% observed in engineering programmes. Since subsequent data collection in MISE did not allow for contributions from such students, a future study concerning their transition to university mathematics would improve knowledge of the issues they face which, ultimately, lead to them deferring, repeating, or leaving their programme.

For researchers who are interested in the education of PSTs, the absence of observations of their classroom practice could be considered to be a limitation of this study. While the review conducted by Lutovac and Kaasila (2019) demonstrates that a wealth of research exists with regard to teacher beliefs, teacher identity, and their influence on practice in the classroom, the PST participants in this study presented different, and nuanced, mathematical identities. Further investigation into the influence of their mathematical identities on their classroom practice would add to the complex picture of identity development, particularly if used to stimulate PSTs to reorganise their mathematical identities. Thus, a research project which performs an identity intervention with a focus on practice would be beneficial for both teacher educators and PSTs.

Since the original intention of this study was to recruit up to 100 participants, it remains to be explored whether the adapted questionnaire could be an effective tool for examining the mathematical identity of science and engineering students at such a scale, and whether a version of this instrument could be used to track changes in identity over time. While the codebook and themes developed in this study would be helpful in this regard, I recommend that a similar hybrid inductive-deductive approach to analysis be undertaken, in order to take account of issues that were not evident among the relatively small number of participants in MISE. Future longitudinal studies should address mathematical identity across a large sample of students (using thematic analysis for example), or by using more individualised, nuanced data collection and analytic methods among a smaller sample (see section 8.1.2 for more on combining such methods).

Appendices

Appendix A

Biography of the Researcher

In this appendix, I explain my personal motivations for undertaking this PhD study.

My first experience of lecturing in university came in 2015 with PSTs in St. Patrick's College, Drumcondra in Dublin. The small department of mathematics, with a strong focus on the pedagogy of university mathematics education, influenced my development as a practitioner at university level towards consideration of my own knowledge of students and teaching, and the mathematical identity of my students. Concurrent with this experience, I began working as a tutor in the MLC in DCU in 2014. The MLC is a drop-in service, which forms the main part of mathematics support in DCU, and is available to any student who wants to work on mathematics. Rather than providing solutions, tutors try to ask the right questions to help students make progress on their work, and, thereby, facilitate the learning of mathematics rather than instruct (see chapter 2). Tutors try to develop students' skills to determine which part(s) of a question or solution they do not understand, what to do when they are stuck on a question, where to go to find resources to help them progress, and, consequently, they communicate a particular view of what it means to study mathematics in university. I also worked with the Education Research Centre (ERC) on the mathematics data from Irish schools for two international studies, the PISA and the Trends in International Mathematics and Science Study (TIMSS), which gave me an insight into the quantitative methodologies that are employed for evaluating students' mathematical performance around the world.

My pedagogical approach was influenced by the belief that "[g]ood teaching starts from where the learners are" (Wiliam, 2017, p. 13), which is a particularly important principle if students are expected to be involved in the construction of mathematical knowledge. Such an approach aligns well with a model of learning that involves active construction of mathematical understanding, based on pre-existing mental objects within the learner's mind (Ernest, 1993, p. 89). I endeavoured to establish a classroom culture that supported this constructivist view of mathematics by shifting the "locus of control" (Lerman, 1994, p. 44) to the student who, guided by the lecturer, constructs their own relational understanding, mostly through classroom discourse and formative assessment. As a qualitative researcher, whose values are to be part of the research instrument, it was important to design a study that allowed me to work within and towards these values.

Appendix B

Background Context

B.1 Leaving Certificate Statistics 2017

Table B.1
Number of students who sat each level of Leaving Certificate mathematics in 2017 (SEC, 2017).

Exam level	# of students	Percentage of total
Higher level	16,394	30%
Ordinary level	32,335	59%
Foundation level	5,936	11%
Total	54,665	

B.2 Non-progression Rates in Science and Engineering

This appendix shows the non-progression rates of students from their first year into the following academic year (HEA, 2022b) nationally (Table B.2) and in DCU (Table B.3). The figures refer to entrants to any level 8 programme (QQI, n.d.) in an institute of higher education in Ireland.

Table B.2
Non-progression rates of new entrants to Level 8 programmes in Irish institutes of higher education by field of study, adapted from statistics compiled by the HEA (2022b).

Field of study	2015-16	2016-17	2017-18	2018-19	2019-20
Natural sciences, mathematics and statistics	9%	10%	10%	9%	6%
Engineering, manufacturing and construction	13%	13%	12%	10%	9%
Average for level 8 programmes	10%	10%	11%	10%	8%

Table B.3
Non-progression rates of new entrants to Level 8 programmes in DCU by field of study, adapted from statistics compiled by the HEA (2022b).

Field of study	2015-16	2016-17	2017-18	2018-19	2019-20
Natural sciences, mathematics and statistics	11%	11%	10%	13%	6%
Engineering, manufacturing and construction	14%	10%	14%	7%	3%
Average for level 8 programmes	8%	8%	10%	7%	5%

B.3 Leaving Certificate Mathematics Grades and CAO Points

Table B.4

Leaving Certificate mathematics grades and points. Note that 25 bonus points are awarded for higher-level grades H6 and above.

Result	*Higher level*		*Ordinary level*	
	Grade	CAO Points	Grade	CAO Points
90-100%	H1	100(125)	O1	56
80-90%	H2	88(113)	O2	46
70-80%	H3	77(102)	O3	37
60-70%	H4	66(91)	O4	28
50-60%	H5	56(81)	O5	20
40-50%	H6	46(71)	O6	12
30-40%	H7	37	O7	0
0-30%	H8	0	O8	0

Appendix C

Participants

C.1 Science and Engineering Students in an Irish Context

According to the HEA (2022a), as of March 1 2018, DCU included 988 students of science. Nationwide there were 10,397 students studying science at university and 15,620 studying science overall. In DCU, there were 444 engineering students as of 1 March 2018, which were further categorised into Electronics (190), Mechanics (131), and Other (106). Nationwide there were 4085 students in those categories at university level, and 8303 overall. These figures are summarised in Table C.1.

Table C.1
Undergraduate enrolments in Ireland as of 1 March 2018 from the HEA (2022a), expressed as total number of students and percentages registered for science and engineering programmes in DCU, in any university nationwide, and in any higher education institution nationwide.

Context	Science	Engineering (MEO[8])	Total students
DCU (12,263)	988 (8%)	444 (3.6%)	1432 (11.6%)
University Nationwide (87,955)	10,397 (11.8%)	4085 (4.6%)	14,482 (16.4%)
Overall Nationwide (159,823)	15,620 (9.8%)	8303 (5.2%)	23,923 (15%)

[9]Mechanical, Electrical, and Other.

C.2 Participant Reponse Rate

The response rates are summarised in Table C.2. The grouping sizes of 101 for Science I, 139 for Science II, and 146 for Engineering, were drawn from an internal university system in February, 2018. The list of programmes within each grouping was presented in Table 3.2 and Table 3.3 in section 3.2.1.

Table C.2
Questionnaire responses for Data1 (32) as a percentage of 125 students who agreed to take part in MISE by correctly completing the informed consent form (ICF).

Grouping	Responses	Questionnaire responses
Science I	35/101 (34.65%)	11/35 (31.4%)
Science II	44/139 (31.65%)	13/44 (29.6%)
Engineering	46/146 (31.5%)	8/46 (17.4%)
Total	125/386 (32.4%)	32/125 (25.6%)

C.3 Demographic Information for Participants

The demographic information for each grouping of MISE participants is presented in this appendix. Table C.3 shows which level of Leaving Certificate mathematics they took, higher level (HL) or ordinary level (OL), and Table C.4 shows the gender breakdown. The demographic information for all participants is shown in Table C.5, including their contribution to each data collection stage.

Table C.3
Leaving Certificate examination level (HL or OL) demographic information for MISE participants by grouping.

Examination Level	Engineering	Science I	Science II	Total
HL	7	9	11	27
OL	0	2	2	4
Other[14]	1	0	0	1
Total	8	11	13	32

Table C.4
Self-reported gender demographic information for MISE participants by grouping.

Gender	Engineering	Science I	Science II	Total
Male	7	9	8	24
Female	1	2	5	8
Total	8	11	13	32

[15]Completed post-primary school in another European country.

Table C.5

Demographic information for MISE participants including their gender, age as of March 2018 when the study commenced, and contribution to Data1, Data2, and Data3, where relevant.

Participant	Gender	Age	Data1 Word Count	Data2	Data3
ENG_01	M	19	242		
ENG_02	M	18	449		
ENG_03	M	20	85		
ENG_04	M	18	223		
Éabha_ENG	F	18	230		✓
ENG_06	M	18	167		
Ciarán_ENG	M	18	214		✓
ENG_08	M	32	75	✓	
Dónal_SC1	M	20	468		✓
Fiach_SC1	M	19	219	✓	✓
SC1_03	M	18	345		
SC1_04	F	18	685		
Saoirse_SC1	F	19	651		Pilot
SC1_06	M	33	108		
SC1_07	M	19	129		
SC1_08	M	18	114	✓	
Breandán_SC1	M	19	334	✓	✓
SC1_10	M	24	39		
SC1_11	M	19	680		
SC2_01	F	19	121		
SC2_02	M	19	110		
SC2_03	F	18	237		
SC2_04	M	18	126		
SC2_05	M	18	92		
SC2_06	M	19	164		
SC2_07	F	20	174		
SC2_08	M	18	602		
SC2_09	M	19	121		
SC2_10	F	19	96		
SC2_11	M	18	40		
SC2_12	F	18	116		
Aodhán_SC2	M	19	335	✓	✓

C.4 List of Participant IDs

Table C.6

List of identifiers (IDs) of MISE participants used in the thesis, organised by grouping.

Engineering	**Science I**	**Science II**
ENG_01	Dónal_SC1	SC2_01
ENG_02	Fiach_SC1	SC2_02
ENG_03	SC1_03	SC2_03
ENG_04	SC1_04	SC2_04
Éabha_ENG	Saoirse_SC1	SC2_05
ENG_06	SC1_06	SC2_06
Ciarán_ENG	SC1_07	SC2_07
ENG_08	SC1_08	SC2_08
	Breandán_SC1	SC2_09
	SC1_10	SC2_10
	SC1_11	SC2_11
		SC2_12
		Aodhán_SC2

C.5 Data1 Recruitment Script

C.5.1 Initial Meeting

Count number of students in room.

Time start/finish time.

- Fionnán Howard, PhD student in IoE here in DCU. My PhD research involves all of you!
- Mathematical identity is your relationship with mathematics including your experiences and perspectives of the subject. This perspective has been used to analyse student-teachers' mathematical identity but not among science and engineering students.
- Requirements of participants:
  - Questionnaire is part of my PhD research. Takes 10-15 mins and consists of 3 open ended questions. I want you to tell me about your mathematical journey, your experience with maths that got you to this point where you're in first year in DCU. No maths in this questionnaire and it's not a test.
  - Focus groups: a small number will be invited to take part in a focus group to elaborate on your thoughts. This will help me to more deeply understand your answers to the questionnaire.
- How will this benefit you:
  - I expect self-reflection on your own journey, learning, and perspective on mathematics to aid your learning as has been reported by past participants in MI research (student teachers).
  - By reflecting on what has happened before, you may be better able to deal with what comes next.
- Participation is absolutely voluntary.
- Read plain language statement (PLS) and email me questions you may have. I want it to be a meaningful experience for you. If there are any barriers to you participating in the research, I would like to hear about them. I want as many people as possible to take part because that will give me a much better sense of what you all are trying to say, and a much better sense of your mathematical journeys.
- Anyone who isn't here, visit website and read about the study.

C.5.2 Follow-up Meeting

- Fionnán Howard, PhD student in IoE here in DCU. My PhD research involves all of you!
- The name of study is MISE.
- Study is about mathematical identity which is your relationship with mathematics. I want to hear about your mathematical journeys. Questionnaire takes approximately 15 mins.
- Required of you: Questionnaire, Focus group (small number of people) after one year and second questionnaire at beginning of third year.
- If you agree to participate you may still withdraw at any point. If you sign this form now, you can still withdraw at any stage.

Summary of PLS finished.

- There are six items to which you must agree in order to participate.
  - Read PLS (white form) attached.
  - Must be over 18 today. Anyone turning 18 in the next two weeks can be accommodated but you cannot sign this form until you are 18.
- Use DCU email, and write clearly. Witness is the person next to you.

Summary of ICF finished.

- Any questions or concerns?
- Anyone who is not here can read about the study, and how to be included, on the MISE website.
- If you have any hesitations, come up and talk to me now while I collect the forms. Keep white page!

Follow-up contact.

- Follow up with those who complete an ICF but do not start the questionnaire, by attending tutorial groups with my business card to remind them to consider submitting a response.
- The business card includes a link to the MISE page on the university virtual learning environment (VLE), and a picture of me to remind them that I have spoken to them previously.
- Follow up with all non-respondents via email, including those who commence a response on the VLE, but have not yet submitted the response.

C.6 Plain Language Statement

Institiúid Oideachais
Institute of Education

CASTeL
Centre for the Advancement of STEM Teaching and Learning

Explanation of the MISE Research Project

What is this research project, Mathematical Identity amongst Scientists and Engineers (MISE), about?

The mathematical identity of an individual can be defined as the relationship he/she has with mathematics, including knowledge and experiences, and perceptions of oneself and others. The focus of this research is to broaden the scope of mathematical identity to students who undertake to study mathematics as a constituent part of a degree in Science/Engineering. I intend to analyse how mathematical identity can be used to explore the challenges that these students face during the transition from post-primary to third level education in Ireland.

Who is conducting this research?

This work is being conducted by Fionnán Howard, a PhD student in the school of STEM Education, Innovation and Global Studies at the Institute of Education, St. Patrick's Campus DCU, Drumcondra. The results of the study will form the basis of the PhD thesis of the researcher which will be available from DCU library.

What will involvement in MISE require of me?

You must be at least 18 years of age to participate in this research. If you agree to be involved, I will ask you to complete a questionnaire. I will ask you for some limited factual information about yourself as well as your open-ended responses to prompts relating to mathematical identity. I expect the questionnaire to take between 10 and 15 minutes to complete.

At the second stage, I will invite some students to participate in a focus group. If you agree to participate, I will ask you to elaborate on your thoughts about mathematical identity in a discussion group with some other students.

In your third year, I will ask you to complete another questionnaire. I will ask you for some open-ended responses to questions which analyse how your mathematical identity has changed over your first two years in university.

Page 1 of 3

Institiúid Oideachais
Institute of Education

CASTeL

What benefits can I expect from involvement in MISE?

As a result of your participation, I expect that your awareness of how your mathematical identity influences your learning will be sharpened, enabling you to engage more effectively as mathematics learners. I hope that this critical awareness will be of benefit to you during your studies and afterwards in your chosen career.

I expect that knowledge of mathematical identity will assist science and engineering students in the transition to third level education and, in the long run, change the way those students are taught mathematics by universities. There are no expected risks associated with this study.

Will my details of my identity be kept confidential?

All data will be gathered anonymously. Although you will not be asked to provide your name at any stage, your email address will be used to keep track of the data. These will be encrypted before being used in any data analysis to ensure that your responses remain confidential.

The focus group will be audio recorded to allow for anonymous transcription. Other participants in the group will know what you have said, but all identifying information will be removed from the transcription, including any names, schools etc. mentioned by the participants.

Any use of the data (for publication, for example) will contain no information identifying individuals, although it may contain information identifying the cohort, if appropriate. Thus, the confidentiality of the information you provide will be protected within the limitations of the law.

Is involvement in MISE voluntary?

Yes, it is. You may withdraw from MISE at any point.

Questions

Institiúid Oideachais
Institute of Education

If you have any questions or would like to know more about any part of the study please contact Fionnán Howard (fionnan.howard3@mail.dcu.ie), *Room F118, St. Patrick's Campus DCU, Drumcondra* or visit the website for the study: http://www.sites.google.com/mail.dcu.ie/fhoward/research.

If you have concerns about this study and wish to contact an independent person, please contact:

The Secretary,
Dublin City University Research Ethics
Committee,
c/o Research and Innovation Support,
Dublin City University,
Dublin 9.

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C.7 Informed Consent Form

Institiúid Oideachais Institute of Education

Informed Consent Form

Research Study Title: Mathematical Identity among Scientists and Engineers (MISE)

Purpose of the Research: MISE aims to analyse how mathematical identity can be used to explore the challenges that these students face during the transition from post-primary to third level education in Ireland.

Requirements of Participation in Research Study: You must be at least 18 years of age to participate in this research. If you agree to be involved in this research, you will be asked to complete a questionnaire, taking between 10 and 15 minutes.

Some students will be invited to participate in a focus group. If you agree to participate, you will be asked to elaborate on your thoughts about mathematical identity in a discussion group with some other students.

In your third year you will be asked to complete another questionnaire, taking between 10 and 15 minutes.

Confirmation that involvement in the research study is voluntary: I am aware that if I agree to take part in MISE, I may withdraw at any point, without penalty of any kind.

Arrangements to protect confidentiality of data: I am aware that all data will be gathered anonymously and that any use of the data (for publication, for example) will contain no information identifying me individually. I realise however that the confidentiality of the information I provide can be protected only within the limitations of the law.

Please answer the following (circling Yes or No for each question):

Are you at least 18 years of age on the date undersigned?	Yes / No
Have you read or had read to you the Plain Language Statement?	Yes / No

Page 1 of 2

Do you understand the information provided? Yes / No

Have you had an opportunity to ask questions and discuss this study? Yes / No

Have you received satisfactory answers to all your questions? Yes / No

Are you aware that the focus groups will be audio taped? Yes / No

I have read and understood the information in this form. The researcher has answered my questions and concerns, and I have a copy of this consent form. Therefore, I consent to take part in MISE.

Participant's Signature :

DCU email :

Witness :

Date :

Page 2 of 2

Appendix D

Research Quality

D.1 Saturation

Table D.1
Categories of saturation according to Saunders et al. (2018) split into coding and meaning saturation (Hennink et al., 2017).

Type of research	Quantitative Measure (Coding)	Qualitative Measure (Meaning)
More deductive	Data saturation	Apriori thematic saturation
More inductive	Inductive thematic saturation	Theoretical saturation

D.2 Trustworthiness

Guba and Lincoln defined several notions of research quality to parallel those in quantitative research. The quantitative criteria, and their qualitative equivalents are shown in Table D.2. The information in the table was synthesised from prominent publications (Tobin & Begley, 2004, pp. 391-2; Guba & Lincoln, 1994, p. 114; Lincoln, 1995, Table 1).

Table D.2
The qualitative versions of quantitative measures of rigour (Tobin and Begley, 2004, pp. 391-2; Guba and Lincoln, 1994, p. 114, Lincoln, 1995, Table 1).

Qualitative (naturalistic) research	**Definition**	**Quantitative (rationalistic) research equivalent**
Credibility (Plausibility)	Addresses the issue of 'fit' between respondents' views and the researcher's representation of them. Credibility is demonstrated through a number of strategies: member checks, peer debriefing, prolonged engagement, persistent observation and audit trails (Guba & Lincoln, 1985).	Internal Validity (Coherence)
Transferability (Context-embeddedness)	A "thick description" of the context of inquiry which allows others to determine to which other contexts it may be transferable Guba and Lincoln, 1985, pp. 124-5.	External validity
Dependability (Stability)	Research process is logical, traceable and clearly documented. Reflexivity is central to the audit trail. Should be self-critical and include the researcher's internal and external dialogue.	Reliability (Replicability)
Confirmability	The findings are not figments of the inquirer's imagination, but are clearly derived from the data.	Objectivity or neutrality (Value-freedom)
Authenticity	Can show a range of different realities (fairness), with depictions of their associated concerns, issues and underlying values.	n/a

Appendix E

Questionnaire

E.1 Pilot Questionnaire Text

Page 1

Welcome to my study: Mathematical Identity amongst Science and Engineering students (MISE).

Research study title: Mathematical Identity among Science and Engineering students. (MISE)

Purpose of the research: MISE aims to analyse how mathematical identity can be used to explore the challenges that these students face during the transition from post-primary to third level education in Ireland.

Requirements of participation in the research study: You must be at least 18 years of age to participate in this research. If you agree to be involved in this research, you will be asked to complete a questionnaire, taking between 10 and 15 minutes.

Confirmation that involvement in the research study is voluntary: All data will be gathered anonymously and any use of the data (for publication, for example) will contain no information identifying you individually. However, the confidentiality of the information you provide can be protected only within the limitations of the law.

Click next to begin the questionnaire.

Page 2

1. What is your gender? [select one]
   - Male
   - Female
   - Prefer not to answer
2. What is your age in years? [Type in input box, one line displayed]
3. What course are you studying? [Select from list]
4. Highest exam level of mathematics completed at post-primary level. [Type in input box, one line displayed]
5. Grade obtained in exam from previous question. [Type in input box, one line displayed]

Page 3

6. **Q1**: Think about your total experience of mathematics. **Tell me about the dominant features that come to mind.** [Type in input box, 10 lines displayed]

Page 4

7. **Q2**: Now think carefully about all stages of your mathematical journey from primary school to university mathematics. Consider:
   - Your feelings or attitudes to mathematics
   - Influential people
   - Critical incidents or events
   - Specific mathematical content or topics
   - How mathematics compares to other subjects
   - Why you chose to study a course which includes mathematics at third level

   With these and other thoughts in mind, describe some further features of your relationship with mathematics over time. [Type in input box, 15 lines displayed]

Page 5

8. **Q3**: What insight, if any, have you gained about your own attitude to mathematics and studying the subject as a result of completing the questionnaire? [Type in input box, 10 lines displayed]

Page 6

9. How would you rate your experience completing this survey? [Choose 1, 2 or 3 star]
10. Why did you choose the option above? Please give as much feedback as possible. [Type in input box, 5 lines displayed, not a required answer]

E.2 Data1 Questionnaire Text

Page 1

Welcome to my study: Mathematical Identity amongst Science and Engineering students (MISE).

Research study title: Mathematical Identity among Science and Engineering students. (MISE)

Purpose of the research: MISE aims to analyse how mathematical identity can be used to explore the challenges that these students face during the transition from post-primary to third level education in Ireland.

Requirements of participation in the research study: You must be at least 18 years of age to participate in this research. If you agree to be involved in this research, you will be asked to complete a questionnaire, taking between 10 and 15 minutes.

Confirmation that involvement in the research study is voluntary: All data will be gathered anonymously and any use of the data (for publication, for example) will contain no information identifying you individually. However, the confidentiality of the information you provide can be protected only within the limitations of the law.

Click next to begin the questionnaire.

Page 2

1. **QD1**: What is your gender? [select one]
    - Male
    - Female
    - Prefer not to answer
2. **QD2**: What is your age in years? [Type in input box, one line displayed]
3. **QD3**: What programme are you studying? [Select from list]

4. **QD4**: Which mathematics exam(s) have you completed at post-primary level? Select all that apply.
   - Leaving Certificate Higher Level
   - Leaving Certificate Ordinary Level
   - A-Levels
   - Other [Text input box, one line displayed]
5. **QD5**: Grade obtained in exam from previous question. [Type in input box, one line displayed]
6. **QD6**: Have you previously completed another undergraduate degree or further education course?
   - No
   - Yes. Please give details [text input box, one line displayed]

<u>Page 3</u>

7. **Q1**: Think about your total experience of mathematics. **Tell me about the dominant features that come to mind.** [Type in input box, 10 lines displayed]

<u>Page 4</u>

8. **Q2**: Now think carefully about all stages of your mathematical journey from primary school to university mathematics. Consider:
   - Your feelings or attitudes to mathematics
   - Influential people
   - Critical incidents or events
   - Specific mathematical content or topics
   - How mathematics compares to other subjects
   - Why you chose to study a course which includes mathematics at third level

   With these and other thoughts in mind, describe some further features of your relationship with mathematics over time. [Type in input box, 15 lines displayed]

Page 5

9. **Q3**: What insight, if any, have you gained about your own attitude to mathematics and studying the subject as a result of completing the questionnaire? [Type in input box, 10 lines displayed]

Appendix F

Focus Group Questions

The focus group questions, prompts, and moderator directions are presented in this appendix.

Icebreaker (round robin) Question

Read the quotations about emotions and mathematics in Table F.1. Pick one (or two) that resonate most with you and tell me why you picked them. Did anyone else pick the same one(s)? [Then return to those who haven't spoken]

Table F.1
Quotes for icebreaker question at the start of each focus group.

Quotes for Icebreaker Question (Q1_Icebreaker)
"love the feeling of solving a complex question"
"it is possible for it to feel impossible at times"
"Maths at times can be relaxing"
"I find it a chore and boring"
"Really enjoy maths"
"Maths is very heavy and stress inducing"
"It is a scary subject"
"I enjoy doing maths"
"maths was definitely my favourite"

Key Questions

1. Theme 5: Transitions: In the questionnaire some students saw a difference between their experiences of mathematics at primary and secondary school. What, if anything, do you think is significant about this transition? Q2_PrimarySecondary. Lookout for:
   - To what extent did you feel (un)prepared for mathematics in secondary school?
   - Third level transition.
2. Theme 3: Ways of Learning Mathematics: When I analysed your answers, it seemed that there was a difference between doing mathematics and understanding mathematics. Have you experienced a difference like this in your own mathematical studies? Q3_DoingUnderstandingMaths
   - How would you describe the difference between doing and understanding?
   - Can you give an example of how you could do mathematics without understanding it?
   - (track changes) Has your view on this changed since you entered third level? Is this a view you held before you entered DCU? How did you come to realise the difference?
3. Theme 1: Teachers/Lecturers: What do you think makes a 'good' teacher? Q4_GoodTeacher
   - In the analysis, it seems like students rely less on the teacher as they progress. How does that statement reflect your own experience?
   - If you rely less on the teacher, what do you rely on more? [Look out for MLC here]
   - (track changes) Do you find yourself relying less on the lecturer in third level? Q5_RelyTeacher
   - (track changes) Did this happen right away in first year or did it develop over time?
4. Theme 4: Mindset and Getting Started: In the questionnaire some students mentioned "thinking outside the box" as important. What does this mean to you when solving problems in mathematics? Q6_ThinkOutsideBox
   - (track changes) Is this thinking unique to university or did you do this in school also?
   - Do you imagine yourself using these skills after you leave university?
5. Theme 2: What is Mathematics?: From the questionnaire, some students seemed to think it's possible to have a natural ability for mathematics, but it only gets you so far. Do you think some people are just naturally good at maths or does everyone have to work hard at it? Q7_NaturalAbility
   - (track changes) Is it the same in secondary school as now in university?

- I've chosen some questionnaire responses about mathematics in Table F.2. I'd like to hear your own perspective on these ideas.

Table F.2
Quotes for key question 5 (Theme 2: What is Mathematics?).

Quotes for Key Question 5 (Q8_NatureofMaths)
"each part of maths interlocks with other parts ... if you become skilled in one area, the remaining areas become easier."
"you have to cover and be well versed in everything to progress."
"each topic is intrinsically linked to one another, and no piece of maths is isolated on its own."

Ending Question

- Are there any important aspects of your experience with mathematics that you think I've missed or anything you'd like to add?

Moderator Directions

- Use open-ended questions.
- Use "think back" when talking about past events.
- Allow silences to happen. Sometimes participants need to think, and the short silence can encourage more contributions.
- Instead of "Why?" ask about attributes (characteristics of a phenomenon) or influences (that cause an action).
- Avoid "That's good" or "Excellent" in response to participants.
- Avoid yes/no questions.
- Use prompts:
  - What influenced you to do/think that?
  - Would you explain that further/give an example?
  - What attributes of X are important to you?

Moderator Materials

Figure F.1
Framework for tracking focus group questions.

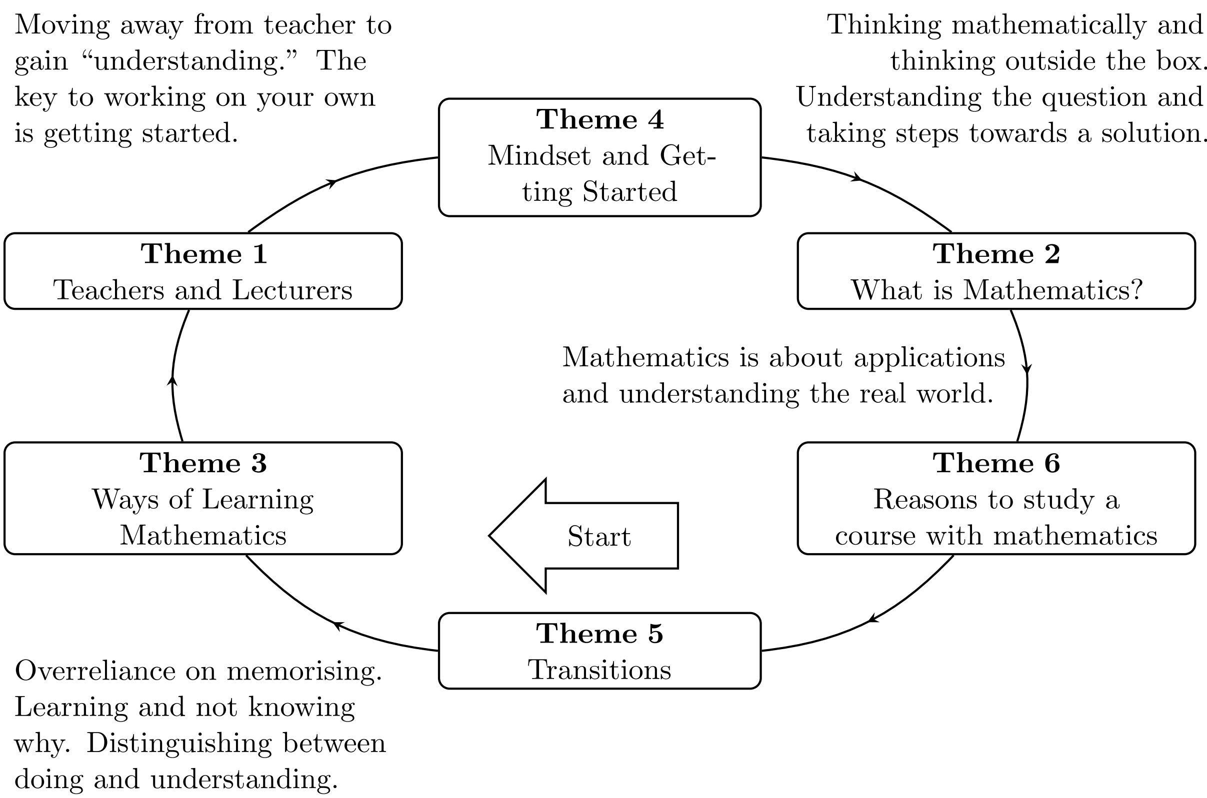


Table F.3
Table for tracking which aims, and which themes, were discussed in the focus group.

Theme	Clarify	Explore	Track changes
Theme 5: Transitions			
Theme 3: Ways of Learning Mathematics			
Theme 1: Teachers/Lecturers			
Theme 4: Mindset and Getting Started			
Theme 2: What is Mathematics?			
Reasons to Study a Programme involving Mathematics			

Appendix G

Thematic Analysis

G.1 MISE Thematic Analysis Process Compared with those in the Literature

Stages 3-7 broadly align with the five steps suggested by Braun and Clarke (2006) (see Table G.1), whose framework has been an "influential approach, in the social sciences at least" (Maguire & Delahunt, 2017, p. 3353). Stages 1-5 correspond to steps from the work of Fereday and Muir-Cochrane (2006), but with a significantly expanded fourth stage, as well as stronger interplay between the stages. Crabtree and Miller (1999) presented a simple four-step model, which combines stages 3-4 and omits stage 2 (testing the codebook), although they do go on to discuss the possibility of such a step (p. 168). The overlap between the stages used in MISE, and those aforementioned, is shown in Table G.1.

Table G.1
The overlap between the stages of thematic analysis employed in MISE and those in the literature.

Author	*Stage of thematic analysis*						
	1	2	3	4	5	6	7
Braun and Clarke (2006)	-	-	1, 2	2	3	4	5
Fereday and Muir-Cochrane (2006)	1	2	3	4	5	6	6
Crabtree and Miller (1999)	1	-	2	2	3	4	4

G.2 Codebook for Data1

In the codebook below, some colloquial terms were used by the researcher. Post-primary school is referred to as "secondary school," university is referred to as "third-level," and university programmes are called "courses."

Code number and name	Origin	Description
01 - Then and now	Literature	Comparing past with present, or two past stages of mathematical journey. "I realised that.." Can include things that haven't changed.
02 - Origin of interest in maths	Pilot 1	Explicit statements about when I started enjoying maths or when I realised I was interested in, or good at, maths. Must include a specific time/place/person/event. "I really liked maths in primary school" is not explicit and should not be included.
03 - Imagining the future as a learner of maths	Literature	The maths I learn now will help me with practical subjects next year. In the future, I won't use maths because I'll specialise in Biotechnology. Subject choice may indicate whether mathematics features in the future or not (see examples.)
04 - Imagining mathematics in future career	Literature	Scientists or engineers in the making. I will use my maths in my job for... I only study maths so I can be a civil engineer (will not use it in future career.)
05 - Primary school experience	Pilot 2	Simple, broad categorisation of the level of schooling the student is speaking about. Will often occur with code 9.
06 - Secondary school experience	Pilot 2	Simple, broad categorisation of the level of schooling the student is speaking about. Will often occur with code 9.
07 - Third-level experience	Pilot 2	Simple, broad categorisation of the level of schooling the student is speaking about. Will often occur with code 9.
08 - Reason to study current course at third level	Pilot 3	Students may choose the course because of maths or in spite of maths. Maths was not a reason I chose this course, I just put up with it. I love maths and science so I decided to study science. **Keywords:** chose; choose; choosing.
09 - Teachers / Lecturers	Literature	A teacher has influenced my mathematical journey. **Keywords:** teacher; lecturer.
10 - Parents / Relations	Literature	My family have influenced my mathematical journey. **Keywords:** mam; dad; parents; brother; sister.

11 - Peers	Literature	My peers have influenced my mathematical journey. **Keywords:** friends.
12 - Projects run by school / teacher	Pilot 1	I remember particular organised events or projects at school which influenced my mathematical journey.
13 - I work collaboratively in maths	Pilot 1	I work with others when doing maths. **Keywords:** friends, work, together.
14 - Maths should be taught like...	Pilot 1	Best practice in my opinion. This is how mathematics should be experienced in the classroom. **Keywords:** should.
15 - I need maths as a basis for my studies	Pilot 1	Maths is vital. It is a basis for my university studies. I need it for my other subjects. To work effectively in science and engineering I need a good basis in maths.
16 - How I work in maths	Pilot 2	When X happens, I learn. When Y happens, I don't learn. This code should include extracts which are clearly about how students learn or prefer to learn mathematics, that don't fit in the other blue codes.
17 - I do homework / questions to learn mathematics	Data1 Sample	When I do homework or questions I know how much I've learned. I can't learn without doing some homework. When I go to do the questions, I know how much I've really learned from class. Note that repeating questions or answering lots of questions is code 20. **Keywords:** homework.
18 - I attend lectures to learn mathematics	Data1 Sample	I need to attend lectures otherwise I won't learn. I go to all the lectures but I still don't understand. If I go to the lectures I learn a lot more. **Keywords:** lecture.
19 - I don't learn if the pace is too slow/too fast	Data1 Sample	The pace was too fast so I couldn't keep up. If the pace was slower I would have learned more. He taught too slowly so I lost interest. **Keywords:** pace.
20 - I practice by answering lots of questions	Pilot 2	I learn by repeating questions, practising exams etc. I focus on doing lots of questions or lots of past papers.
21 - I need time to learn / study mathematics	Pilot 2	I need time to understand the mathematics or time to study. I don't have enough time for study. I find it difficult to study because of other commitments. **Keywords:** time.
22 - I like to understand the maths rather than learn it off	Data1 Sample	I like to understand the concepts because then it's easier to learn/remember. Relational understanding over rote learning and instrumental understanding. The questions are easier if I understand the maths first. **Keywords:** understand, concept.

23 - I attend grinds or maths support help me learn	Data1 Sample	I got grinds and it really helped me through. I like to work in the maths learning centre where I can get help. **Keywords:** grind; tutorial.
24 - I find it hard to get started	Data1 Sample	Starting questions is difficult. If I can start the question I'm ok. **Keywords:** start.
25 - Comparing with other subjects	Literature	Can appear in small space of text as a throwaway reference to a connection.
26 - Understanding the real world	Literature	Maths is about solving real-world problems or understanding the world around us. Applications of maths does not fit here (see code 27.) **Keywords:** real world; application.
27 - Mathematics as a tool	Pilot 1	Maths is about applications. Maths is useful. Maths is useful for... Maths can be used for... **Keywords:** useful.
28 - Mathematics as a language	Pilot 1	Mathematics is a way of thinking or a language for use across all subjects. "Thinking Mathematically" should go in here if not clarified further. Implies mathematics is a way of thinking. **Keywords:** think.
29 - "Mathematical thinking"*	Data1 Sample	Creativity, seeing the bigger picture, thinking outside the box. Mathematical thinking means non-algorithmic mathematics. Contrast with code 30.
30 - Algorithmic application of methods	Pilot 2	Maths is about learning a method and repeating it or repeating questions until you learn the pattern. **Keywords:** formula.
31 - Proofs	Pilot 2	Mathematics is about proofs. **Keywords:** proof.
32 - Teaching to the test	Literature	School mathematics is about teaching to the test. Often passive voice: "It is..."
33 - Rote Learning	Pilot 2	Mathematics involves learning things off by heart. **Keywords:** rote learning; learning off.
34 - You're either good at it or you're not	Pilot 1	Fixed ability in mathematics over which the individual has little or no influence. A subject where things just click.
35 - Getting started	Pilot 2	Maths is about interpreting information to understand what you should do with it, how to start a question or having multiple approaches to get started. **Keywords:** start.
36 - Mathematics is straightforward and logical	Pilot 3	Maths is structured. Involves rules and order. **Keywords:** logical.
37 - Nature of Mathematics	Data1 Sample	Broad code for statements that are clearly about the nature of mathematics which do not fit the other green codes.

38 - Right and wrong.	Literature	Answers are either correct or incorrect. References to there being one answer: the correct one.
39 - I find maths hard / easy	Literature	I find maths easy, it just clicks with me. I always struggled with maths. Often, comments go further and should be categorised into one of the codes below. Comments about amount of effort are code 46. **Keywords:** hard; easy.
40 - I (dis)like or enjoy maths	Literature	I enjoy it. I have mostly bad memories of maths. **Keywords:** like.
41 - I'm (not) good at maths	Literature	I've always been good at maths, it is my best subject. I have no talent in mathematics, I've always been bad at calculus.
42 - I find maths frustrating / challenging / rewarding	Pilot 2	I like overcoming the challenge: anything that mentions challenge should be in here. It's nice when things finally come together and make sense (Eureka moments = rewarding). Although it is frustrating when you can't figure it out, it is rewarding if you persist and get an answer. **Keywords:** challenge; frustrating; rewarding.
43 - I find maths interesting / boring	Pilot 2	**Keywords:** interesting, boring.
44 - Maths is undervalued / under appreciated	Data1 Sample	Most people don't think maths is interesting. The average person on the street doesn't understand that maths is important.
45 - Exams, results and LC subject/level choice	Data1 Sample	Exams and results. I studied hard and got a H4. I got 99 on my secondary school entrance test. I mostly just try to pass. I did ordinary level maths to make sure I got in to this course. Note that third level subject choice should go under code 3. **Keywords:** exam, test.
46 - Effort / Persistence	Data1 Sample	I find maths is hard work (requires persistence – a growth mindset) or I do maths without much effort. **Keywords:** hard work.
79 - Miscellaneous	Literature	Anything that does not fit in the above codes but constitutes a statement that is relevant to mathematical identity.

G.3 Inductive Codes from Data1

A total of 33 inductive codes were developed during the analysis of Data1. Coding a sample of the data resulted in the creation of 13 codes (section 5.1.4), which are included in the codebook in appendix G.2, and coding the entire dataset resulted in the creation of 20 inductive codes (section 5.1.6), which are listed below. When coding the entire dataset, inductive codes were numbered starting at 48. The list below shows the inductive codes along with the question number and the participant who inspired the creation of the code. LCO/LCH refer to the ordinary- and higher-level Leaving Certificate examinations.

48 – Interlocking parts. Maths not compartmentalised (Q1 SC2_02).

49 – (Classroom) Maths is cumulative. You need the previous material to advance (Q1 ENG_02).

50 – I learn if it's explained well to me (Q1 ENG_03).

51 – Teacher transition. The teacher's teaching methods need to suit their students. Or students need to adjust to teachers' methods, including new teachers (Q1 ENG_03).

52 – I taught myself. I worked on my own (Q1 ENG_04).

53 – I find maths scary/daunting (Q1 Saoirse_SC1).

54 – I find maths stressful, causes anxiety (Q1 SC2_07).

55 – Reference to definitions or terminology. Maths has a specific language associated with it which is part of learning the subject (Q1 ENG_02).

56 – Specific important topics. Possibly students think these are so-called gateway topics (Q1 SC2_03).

57 – Primary/Secondary transition (Q1 Breandán_SC1).

58 – Transition to third level (Q1 SC2_04).

59 – Junior Cycle/Senior Cycle transition (Q1 Saoirse_SC1).

60 – Natural ability (Q1 SC1_06).

61 – Language barrier – Irish (Q2 ENG_02).

62 – To understand I like to visualise (Q1 SC2_08).

63 – LCO/LCH difference in standard (Q2 SC2_03).

64 – Confidence (Q2 SC1_04).

65 – Doing something or learning how to do something without knowing why. What purpose? (Q2 SC2_07).

66 – In lectures it's difficult to write and listen simultaneously (Q2 SC2_08).

67 – To understand I need to see the steps (Q2 SC2_08).

G.4 Changes to Themes after Stability Analysis

Below is an example of the changes made to the themes, codes, and thematic map after analysis of each theme for coherency and representativity, as described in section 5.1.8:

- Renamed **Mindset and strategies** to **Mindset and getting started.**
- Made *Getting started* a sub-theme of **Mindset and getting started.**
- *Natural ability* moved in to **Getting started** sub-theme.
- **Maths gets harder as you progress** renamed to **Transitions and realisations/reflections**.
- *I took charge* and *LC subject choice* moved to **Transitions and realisations/reflections**.
- **Maths is a means to an end** changed name to **Reasons for studying mathematics**.
- Moved *Right and Wrong* to **What is Mathematics?**
- Moved *Parents* to **Miscellaneous**
- Moved *Personal Feelings on Mathematics* out of **Miscellaneous** to form its own theme.
- Moved *Teachers/Lecturers* out of **Miscellaneous** to form a theme called **Teachers/Lecturers** with the codes from **Learning from the Teacher**.
- Removed **Maths anxiety** subtheme and collected this under *Barriers* in **Mindset and Getting started**.

I also noted two questions from the new thematic map: Why is *Natural ability* not in **What is mathematics?** and why is *anxiety* not in **Personal feelings**? These questions would be dealt with in the next cycle of coding recommending at stage 4.

G.5 List of Themes and Codes

The themes, categories, and codes that resulted from thematic analysis of Data1 and Data2 are presented below. Codes are numbered, whereas themes and sub-themes are not.

- Personal Feelings about Mathematics (section 6.1.1)
  - 25 Comparing personal feelings about mathematics with other subjects.
  - 25a Comparing personal feelings about mathematics and science.
  - 39 I find maths hard/easy.
  - 40 I (dis)like or enjoy maths.
  - 41 I'm (not) good at maths.
  - 42 Maths is rewarding/I enjoy the challenge.
  - 42a I find maths frustrating.
  - 43 I find maths interesting/boring.
- Ways of Learning Maths (section 6.2.3).
  - 23 I attend grinds or maths support help me learn.
  - Learning to do maths.
    - 16 How I work in maths.
    - 17 I do homework/questions to learn mathematics.
    - 65 Doing something without knowing why.
    - 67 I need to see the steps.
  - Learning to understand maths.
    - 22 I like to understand the maths rather than learn it off.
    - 62 To understand I like to visualise.
- Teachers/Lecturers (section 6.2.1). [Code 09 promoted to a theme]
  - 14 Maths should be taught like...
  - 18 I attend lectures to learn mathematics.
  - 19 I don't learn if the pace is too slow/too fast.
  - 21 I need time to learn, study, understand mathematics.
    - 66 In lectures it's difficult to write and listen simultaneously. [Code 66 absorbed into code 21]
  - 50 I need it explained.

  - 51 Teacher transition.
- Mindset and getting started (section 6.2.4).
  - 29 "Mathematical thinking."
  - 46 Effort/persistence.
  - 53 I find maths scary or daunting.
  - 54 I find maths stressful, causes anxiety.
  - 60 Natural ability.
  - 64 Confidence in maths.
  - Getting started.
    * 24 I find it hard to get started.
    * 35 Getting started.
    * 52 I work on my own.
- What is Mathematics? (section 6.2.2)
  - 25an Comparing the nature of mathematics and science.
  - 25n Comparing the nature of mathematics and other subjects.
  - 26 Understanding the real world.
  - 27 Mathematics as a tool.
  - 30 Algorithmic application of methods.
  - 33 Rote Learning.
  - 36 Mathematics is straightforward and logical.
  - 37 Nature of mathematics.
  - 38 Right and wrong.
  - 44 Maths is undervalued/under appreciated.
  - 48 Maths has interlocking parts.
  - 49 Maths is cumulative.
  - 56 Specific important topics.
- Transitions (section 6.1.2).
  - 01 Then and now.
  - 52a I took charge.
  - Transitions.
    * 57 Primary to post-primary transition.

    - ∗ 58 Transition to third level.
    - ∗ 59 JC to LC transition.
    - ∗ 61 Language barrier – Irish.
    - ∗ 63 LCO - LCH difference in standard.
    - ∗ 72 Transitions during third level.

- Reasons to study a course involving mathematics (section 6.1.2).
  - 04 Imagining mathematics in future career.
  - 08 Reason to study current course at third level.
  - Maths is a means to an end.
    - ∗ 03 Imagining the future as a learner of maths.
    - ∗ 15 I need maths as a basis for my course.
- Miscellaneous.
  - 10 Parents/Relations.
  - 45 Exams/results.
  - 75 Friends, peers, and family.
  - 76 Collaboration.
  - 79 Miscellaneous.
- Contextual codes.
  - 05 Primary school experience.
  - 06 Secondary school experience.
  - 07 Third level experience.
  - 70 Further education experience.
  - 71 Teaching placement.

G.6 Emotions and Causes

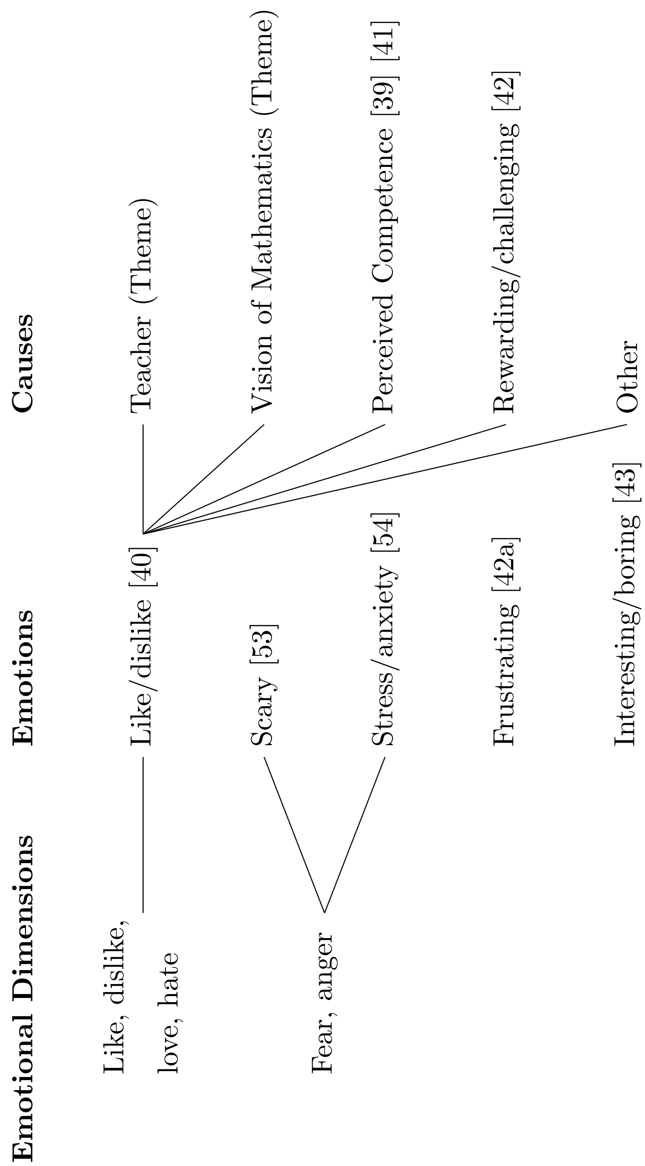


Figure G.1
Emotional dimensions considered by Di Martino and Zan (2010) with regard to attitude, matched with associated emotions and causes identified in MISE, with relevant code numbers in square brackets.

Appendix H

Interviews

H.1 Interview Questions

Most recent participation:

List of potential key events:

Here's what I want to do:

- We will talk mainly about your experience of mathematics in DCU, year-by-year, focusing on the key events or influences on your mathematical identity.

Mathematical identity is defined to be an evolving collection of narratives, based on an individual's experiences with, and knowledge of, the nature, teaching and learning of mathematics, including perceptions of oneself and others. It is often thought of as one's relationship with mathematics.

- My role is to listen and facilitate the telling of your story, rather than search for answers to specific questions. You are the expert when it comes to your own mathematical experience, and I hope to learn about those experiences from you.
- Opening question: I'd like to talk about your experiences year-by-year. Think about your experience of mathematics in first year in DCU. What are the dominant features/memories/events that come to mind about that time?

Cover the story from 1st-4th year, probing any topics the participant wishes to talk about.

Part A: Questions about DCU

- How did your experience of mathematics differ in n-th and (n-1)-st year?
- Was that different from what you expected?
- Are there any other memories you have of mathematics in X year?
- Were there any influential people or key events?
- How did mathematics feature in your other subjects?

Part B: Key events graph

- You've spent the first part of this interview telling me your story about your time in DCU, and particularly your relationship with mathematics in that period. What are the key events that influenced this story?
- Interviewer will present the list of key events mentioned in the discussion, along with some suggested additions. Participants will decide whether to include these suggestions.
- Now let's place the key events in chronological order.
- Ask the participant about the name/meaning of upper and lower limits in the key events graph.
- Which of these events would you describe as positive? Negative? Anything in between?
- This graph is one way to represent your journey in terms of key events. Knowing what you know about your own journey, have you any thoughts about how to interpret this graph?

Part C: Clarification questions from Data1

Mathematical identity can change over time and I'm interested in studying this change. As you read your response, I'd like to know your reaction to what you wrote three years ago.

- What did you mean by that part? I wasn't sure if you meant X or Y.
- Would you like to say a bit more about that?

Part D: Compare current and former self

- What strikes you about what you wrote three years ago? What stands out to you as you read it now?
- Is that still important to you now? [I like to work like X, has that position changed?]

- Does that experience still affect you strongly? [influence of experiences may fade over time]
- Do you still see it/yourself/mathematics that way? [perceptions of oneself and the nature of M]
- In the future, how will your mathematical identity inform your work/study?

Notes for the Interviewer

Files to have open:

- NVivo Data1 and Data2 responses.
- Questions document.
- Data1 response in word for sharing with participant.
- Key events graph and word doc for key events list.
- Modules list.

Process:

- Start meeting.
- Stop automatic transcription in zoom.
- Check share portion of the screen for part B.
- Admit and greet participant.
- I haven't starting recording yet. I'll check with you before I start that.
- Read protocol.
- Start audio recording in zoom (set to record audio only) and transcription in otter.

H.2 Transcription Conventions

The transcription conventions for the focus group and interview data are shown in Table H.1. Conventions 7 and 8 were created by the researcher, as was the notation for convention 9. Although Labov (1982, p. 227) formalised the idea of a "reportable event," I did not find an established transcription notation for this.

Table H.1
Conventions used for data transcription.

#	Symbol	Represents	Example	Rationale
1	(...n)	Pause of n seconds where n is a positive whole number (Oliver et al., 2005, p. 1276).	Ciarán_ENG: Yeah (...4) FH: Ok, so (...2) give me one second (...3) Yeah you did say...	To identify questions which are unclear or topics which require more thinking time to answer.
2	-	Word cut off sharply (Linde, 1993; Oliver et al., 2005).	Ciarán_ENG: I don't even real- like to me it doesn't even feel like maths ...	To show changes in thinking as one asks or answers questions. Applies to interviewer and participant.
3	,	Pause of less than one second. Additionally added/removed for readability in the text (Linde, 1993, pp. xii–xiv).	Ciarán_ENG: So it was literally, write everything down as fast as you can on the page that she writes, and then when the class is over, go to the library and look at it...	To indicate short pauses or to make longer sentences more readable, particularly those which are re-moulded as one speaks.

4	[]	Overlapping talk (Linde, 1993, pp. xii–xiv; Sparkes & Smith, 2008, p. 146).	Dónal_SC1: So, in a way, that was a negative already [but...] FH: [Sure.] It sort of ends up as an overall positive ...	To include everything that was said by the participant and interviewer in the course of the interview.
5	[ID123]	Participant name replaced with ID number (Sparkes & Smith, 2008, p. 146).	It's like [ID123] said earlier ...	To preserve anonymity in the focus group.
6	((italics))	Involuntary vocalization (Oliver et al., 2005, p. 1276).	FH: ... what do you think are the key events that influenced the story that you you've just given? (...6) Breandán_SC1: ((exhales)) Ehm ((laughs)).	To convey jokes or funny moment ((laughs)) or moments of frustration ((exhales)).
7	...	Sentence that trails off or gets interrupted, but will be continued shortly in the transcript.	Dónal_SC1: Right and that's... FH: Yeah. Dónal_SC1: ...you know, interesting in a way.	To enhance readability of sentences which are interrupted and then continued.
8	???	Audio is unclear.	Aodhán_SC2: Yeah, I [???] yeah I forgot about that ...	To denote missing parts of the audio transcript.

9	Yeah!	Expression of surprise, type B response, a "reportable event" (Labov, 1982, p. 227).	Éabha_ENG: I think there was a scary side to some of the engineering maths lecturers. FH: Right!	Identify moments of surprise on the part of the interviewer. Unexpected comments by the participant.

H.3 Narrative Processes

Author	Year	Story	Description	Argument	Theorising	Augmentation
Schutze	1970/ 2008		Deals with social frames of the characters (routines, institutions, social units).	Why were the social frames established and why did they influence the flow of events?		
Labov	1972	Clauses, temporal juncture.				
Labov	1982	A sequence of narrative clauses separated by one or more temporal junctures, which allows the listener to infer the reported temporal order of past events from the temporal sequence of clauses in the report of those events.				
Mishler	1991	Clauses, temporal ordering, Labov.				
Rosenthal	1993	Narrations refer to single sequences of events from the past. Sequences of actual or fictitious occurrences, which are related to one another through a series of temporal or causal links.	The decisive feature distinguishing them from narrative is that descriptions present static structures" Schutze & Kallmeyer.	Abstracted elements occuring outside the storytelling sequence.	Declarations of the general idea. They show the narrator's general orientation at the moment.	
Riessman	1993	Beginning, middle and end. Recognisable boundaries. Can be organised temporally or thematically.				
McCormack	2000	Recognisable boundaries. Linked events/actions organised chronologically or thematically. "And then what happened?" Together they give the story a point, a reason why it was told.	Offer little interpretation or explanation. Add detail to the picture built up through other processes.	Augmentation which is outside the story. Abstracted elements. They add meaning.	Reflecting, giving an opinion or trying to work out something (work out the “why”).	Adding information to stories already told. Includes when change occurred.
O'Kane & Pamphilon	2015		Details about people or places that help the listener to get a more complete picture of the story.	Abstracted elements outside the story.	Teller asks themselves why they behaved a particular way.	Additional comments to their story to help with plot development.

H.4 Story Elements

Author	Year	Orientation	Abstract	Linked events	Evaluation	Coda
Labov	1972	Time, place, persons, and their activity or the situation. Sketching the kind of thing that was going on before the first event of the narrative occurred. Can be at the start or at strategic point later on. "Who, when, what, where?"	One or two caluses summarizing the whole story. "What was this about?"	"Then what happened" or "What finally happened?"	Indicate the point of the narrative. Pointless stories are met with "So what?" whereas "He did?" indicates the reportable quality of the events of a narrative. "So what?" Narrator steps out of the narrative to explain what was in their mind at the time. Displaced Orientation.	Signals the narrative has finished. May also contain general observations or effects of the events. Bridges the gap between the moment of time at the end of the narrative and the present. None of the events that followed were important. End of "turn" at speaking.
Labov	1982	the time, the place, the participants in the action, and their general behavior before or at the time of the first action.	Substance of the narrative as viewed by the narrator. Normally the same as the point of the narrative. Linked with previous utterance.	Complicating action: Sequence of narrative clauses: temporal sequence of clauses. Three possible tenses. Indicative mood verbs.	Comparing events that did occur and what did not. The main point or focus of the narrative.	
Mishler	1991	Time, place, people.	Summarizing the point of the story.		How the teller feels about what was happening. Selecting one event in the narrative for emphasis.	Returns the speakers to the present situation.
Riessman	1993				How the teller wants the story to be understood. What is the point of the story?	
McCormack	2000	Who, what, when, where?	Summarises the point.	Responses to "And then what happened?" Organised chronologically or thematically.	Why was the story told? Highlights the point.	Brings story to a close.
O'Kane & Pamphilon	2015	Who, what, when, where?	Summarises the point.	Organised thematically or chronologically.	Conveys the teller's emotions and attitudes to the narration.	Brings story to a close.

H.5 Interview Analysis Extract

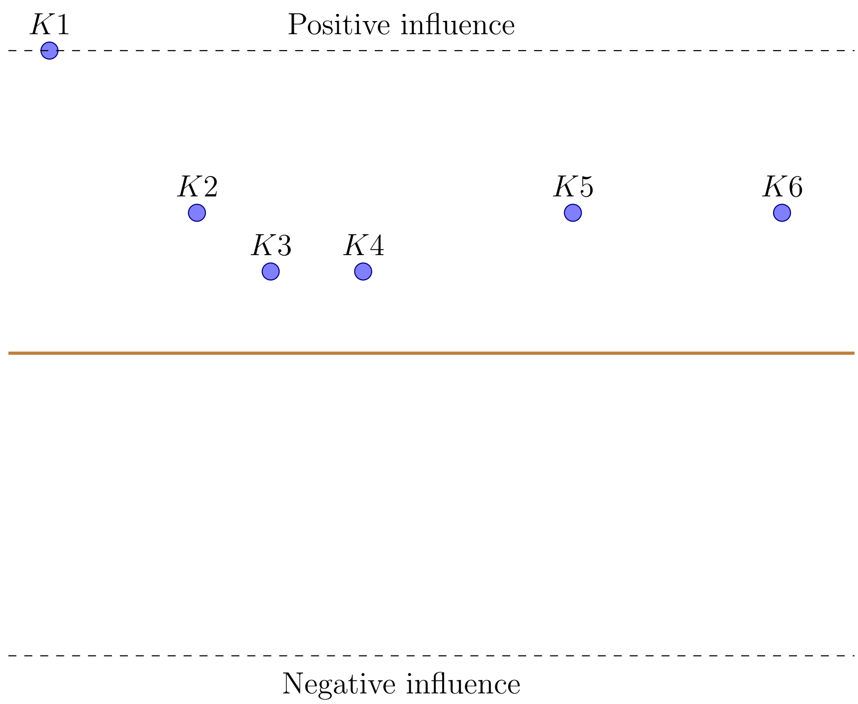
ID031	Order	Narrative Process	Story	Argument	Description	Theorising	Augmentation	Year
	94	Theorising				Three plus three is six, you know, se		**Primary**
	60	Argument		[We didn't have] a lot of support when I was in school, and we did				**Post-primary**
Story 8	63	Story	**I had an awful secondary school teacher so they brought in a new one. The**					**Post-primary**
	71	Augmentation					Fionnan Howard 35	**Post-primary**
	89	Augmentation					**as I said, very clear**	**Post-primary**
	58	Description			And he was just sitting in his room as he always was			**Post-primary**
Story 9	64	Story	My LC teacher knew what he was talking about, but he didn't have any way					**Post-primary**
	96	Theorising				Yeah, cool. I'm just reading the que		**Post-primary**
Story 7	57	Story	So I sort of said I was going to be a teacher back in the last day of sixth year					**Present (about the self)**
	2	Theorising				Ehm, it was (...1) probably the best		**PLC**
	3	Argument		And then, they covered a lot there and the maths was very (...1) it				**PLC**
	4	Theorising				No, no, the PLC was great, and the		**PLC**
	70	Theorising				let's start with the PLC because tha		**PLC**
	90	Augmentation					Hmm, see I (...1) tha	**PLC**
Story 1	6	Story	College was just too early in the mornings for me to get up. I was getting up					**1**
	7	Augmentation					And how were you t	**1**
	8	Theorising				So that's four hours a day, right and		**1**
	10	Description			Ehm I did the exams and ehm I (...1) basically, as all			**1**
	11	Theorising				And I was like, I had to, I really have		**1**
	13	Theorising				It was, more like I was (...1) unsure		**1**
	73	Theorising				031 36:48So that was positive in a		**1**
	84	Augmentation					Only kind of one ne	**1**
	79	Augmentation					Of course, you will a	**1**
	92	Theorising				Fionnan Howard 52:44What do yo		**1**
Story 4	20	Story	**...we were being judged on a module we have yet to be doing ... to be held**					**1**
Story 5	24	Story	**In chemistry too we were judged based on modules we hadn't done yet.**					**3**

Findings Order	Short Description	Analytic Point	Form Code	Key event
0	In primary school I loved technical maths: numbers with pen and paper rather than writing.			
1	We didn't have much s	Pointing to factors out of their control that hindered them		
1	I had an awful seconda	A lot of uncertainty a	Then and now	
1	My OL maths teacher didn't check our work		Then and now	K1
1	I had lots of teachers in secondary school bu		Then and now	K1
2	My LC teachers inspire	Decided to try teach	Reasons to study this course	
2	My LC maths teacher was a key figure			K1
2	Looking back, I wouldn't say my physics teacher who taught r			K1
2	I've overcome doubts about becoming a tea		Reasons to study this course	
3	Best decision of my life	PLC was a positive e	Then and now	K2
3	Mix of orindary and higher level. We did the questions straigh			K2
3	I did pretty well with it. The teacher was great to work with.			K2
3	I grew into myself during the PLC		Then and now	K2
3	I don't remember why I said the PLC was all		Then and now	K2
4	Missed lectures because it was difficult to travel			
4	It took two hours each way to and from college on the bus.			
4	Twenty hours per weel	Didn't engage with lectures enough in first year		
5	I crammed for first year exams and couldn't		I took charge	
5	I had done ok at maths	Decided to repeat an	Then and now	
5	I repeated the year so I	Repeated because I	I took charge	
6	Speaking to PT was the	Appreciated receivin	Then and now	K4
6	One positive key event	Failing exam lead to	I took charge	K4
6	I don't remember the tutorials for calculus, problem solving is more like a hobby, it hasn't been useful for anything.			
6	I don't remember much about first year calculus but the end of the module was tough.			
6	The maths course structure was bad			
6	The chemistry course structure was bad			

H.6 Key Events Graphs

Figure H.1

Key events graph for Aodhán_SC2.



$K1$. LC mathematics teacher.

$K2$. Monty Hall problem in Probability and Statistics module.

$K3$. Matrices inorganic chemistry, outside of mathematics module.

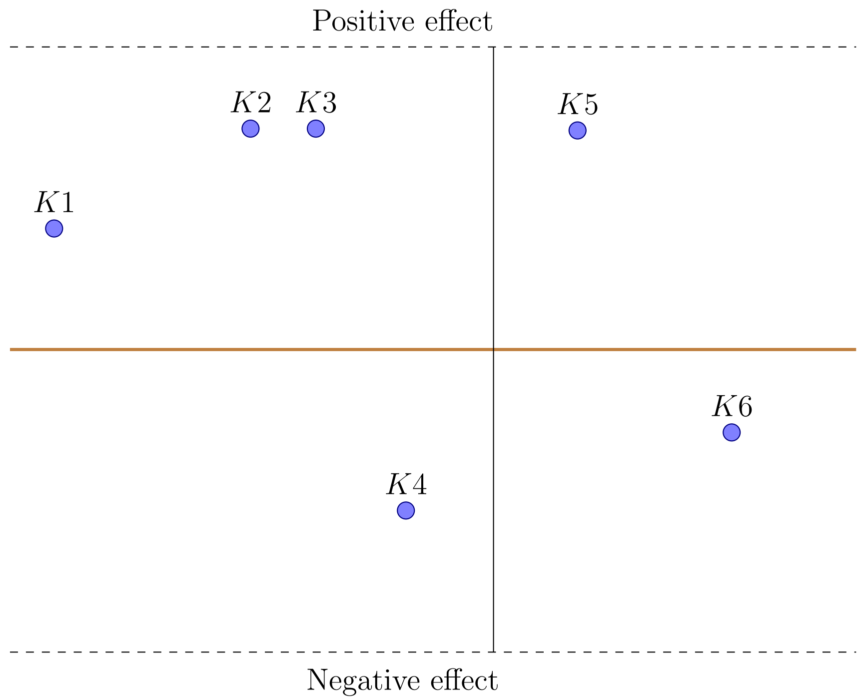
$K4$. Friends, grinds.

$K5$. Work placement (INTRA).

$K6$. Fourth year, units, thinking logically.

Figure H.2

Key events graph for Breandán_SC1.



$K1$. Depth of first year calculus.

$K2$. Teaching placement.

$K3$. Mindsets in “the mathematical experience” module.

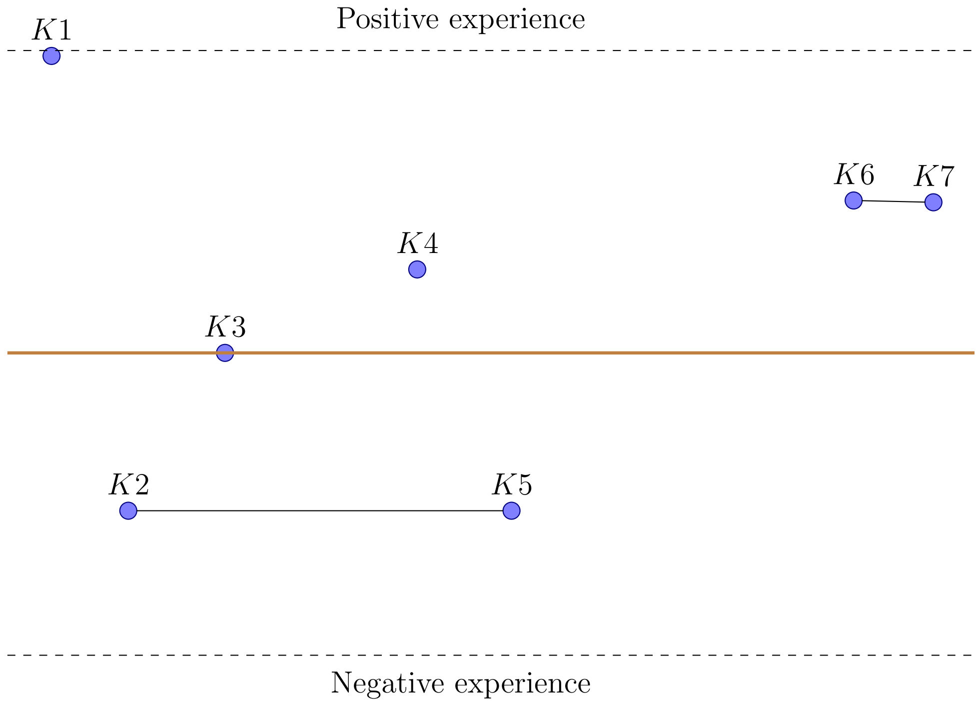
$K4$. Linear algebra module.

$K5$. Teaching placement.

$K6$. Abstract algebra.

Figure H.3

Key events graph for Ciarán_ENG.



$K1$. Lecturer A, easier style of lecturing.

$K2$. Lecturer B, the bad kind of lecturing.

$K3$. Classmates arguing about solutions in the library.

$K4$. Change to mechatronic engineering.

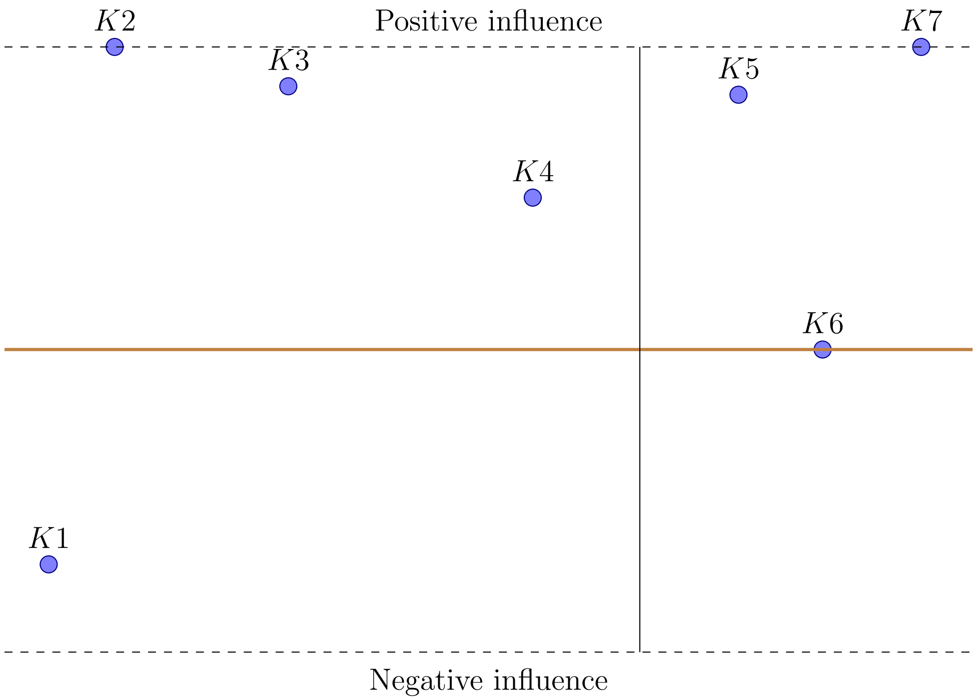
$K5$. Lecturer B, the bad kind of lecturing.

$K6$. Modules linking together in third and fourth year.

$K7$. Fourth year, applying mathematics.

Figure H.4

Key events graph for Dónal_SC1.



$K1$. Leaving Certificate mathematics teacher, change to ordinary-level.

$K2$. Further education programme.

$K3$. Working with friend in computer science.

$K4$. Meeting first year calculus lecturer over the summer.

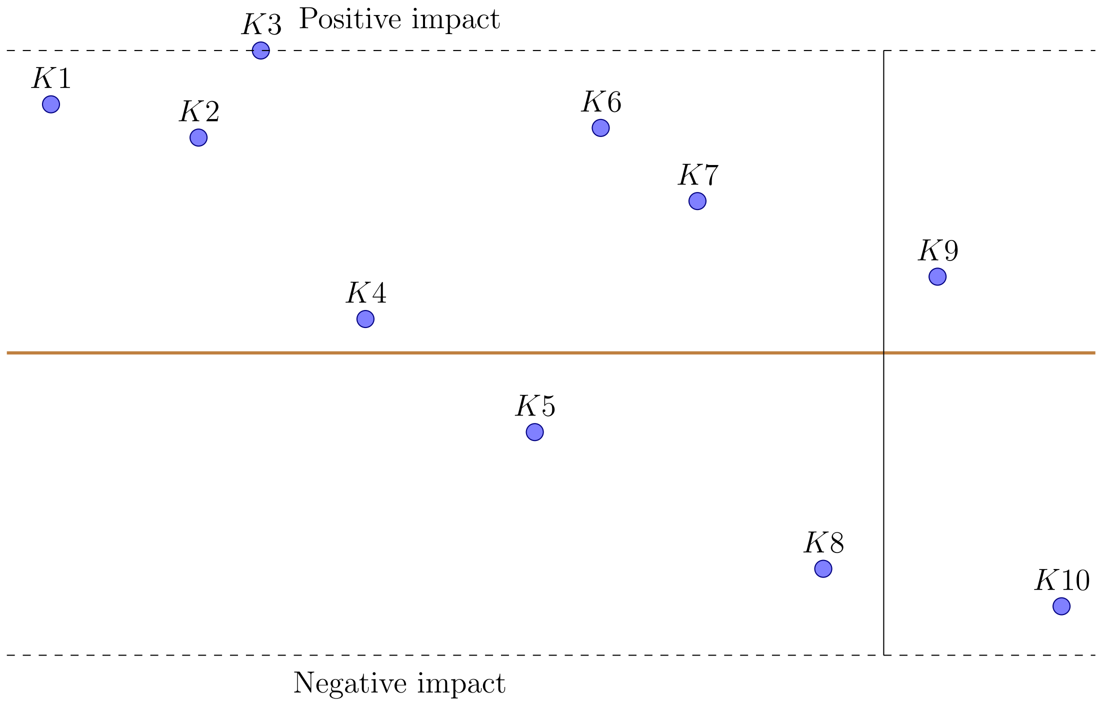
$K5$. Probability and statistics module.

$K6$. Linear algebra module.

$K7$. MLC.

Figure H.5

Key events graph for Éabha_ENG.



$K1$. Problem solving, fixing the bike.

$K2$. Junior Certificate mathematics teacher.

$K3$. Physics for Leaving Certificate.

$K4$. Seeing labs in the open day before choosing engineering.

$K5$. Face paced mathematics in first year.

$K6$. Lecturer A in second year.

$K7$. Internship in first and second year.

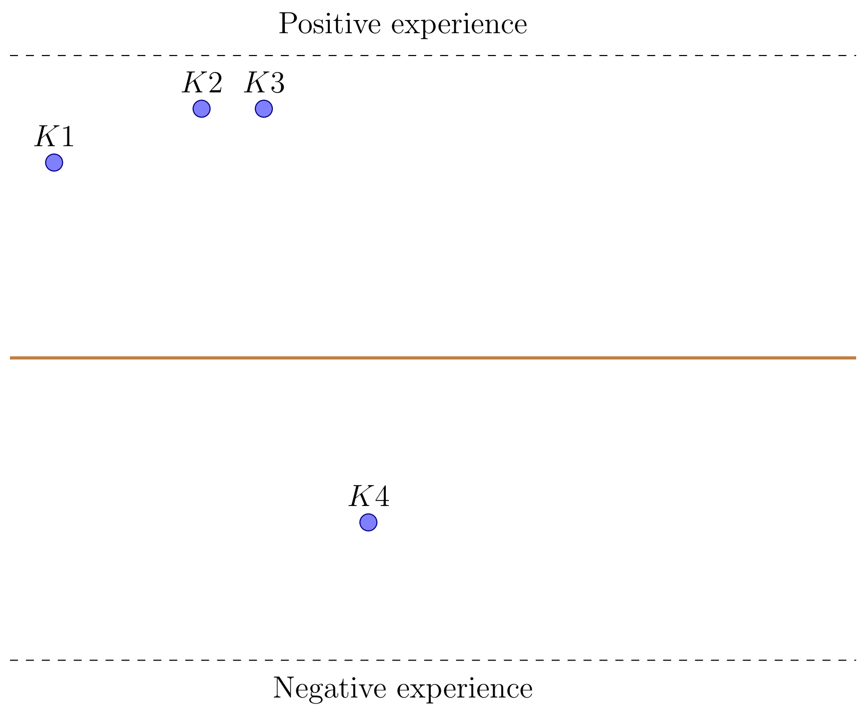
$K8$. Lecturer C.

$K9$. Data analytics with Lecturer A.

$K10$. COVID-19, working from home.

Figure H.6

Key events graph for Fiach_SC1.



$K1$. MLC

$K2$. Working in groups with classmates.

$K3$. Generally good lecturers.

$K4$. COVID-19, working from home.

H.7 Relevant Modules for Personal Narratives from Interviews

Engineering students all took modules entitled Engineering Mathematics I/II in first year, and Engineering Mathematics III/IV in second year (taught by lecturers in the Faculty of Engineering and Computings), while both Engineering interview participants (Ciarán_ENG, Éabha_ENG) also discussed a third year Data Analytics module, along with their cancelled work placements (INTRA[1]) and final year projects. Science I and Science II students took separate year-long Calculus modules in first year (entitled Calculus and its Applications, and Calculus for Teachers, respectively) and in second year, they shared a module in Probability and Statistics. Aodhán_SC2 further discussed his work placement (INTRA) in third year, and his final year project. Science I interview participants all studied the Science Education programme (Breandán_SC1, Dónal_SC1, Fiach_SC1), and, therefore, took modules related to mathematics and its teaching. They included second year modules in Linear Algebra, teaching preparation (including a short teaching placement), and a module entitled The Mathematical Experience, which involved mathematical language, mindsets, and problem solving. Science Education participants discussed a third-year module in Teaching and Assessing Junior Cycle Mathematics, while in fourth year, those who specialised in mathematics (Breandán_SC1, Fiach_SC1) took an equivalent module with regard to Senior Cycle, and also discussed an Abstract Algebra module. These participants undertook teaching placements in school in third and fourth year, which were conducted online because of the COVID-19 pandemic.

Table H.2, Table H.3, and Table H.4 show the modules mentioned by the six narrative interview participants, presented according to their grouping (Science II, Engineering, and Science I respectively). This is not a complete list of the modules they studied, but a condensed list of those which were relevant for the personal narratives presented in chapter 7. The modules are denoted with a different short code to indicate whether they were delivered by the School of Mathematical Sciences (MS), Faculty of Engineering and Computing (EM), School of Physical Sciences (PS), or work placement (IN).

[1] INTRA is part of DCU's flagship programme which facilitates students' workplace-based learning. It takes the form of internships in industry for science and engineering students, and teaching placement in schools for PSTs (Dublin City University, n.d.).

Table H.2
Modules relevant to Aodhán_SC2's personal narrative in section 7.2.1.

Year	Module
1	Calculus and its applications (MS)
2	Probability and statistics (MS)
3	Work placement (IN)
4	Final year project (PS)

Table H.3
Modules relevant to the personal narratives for Ciarán_ENG (section 7.2.3) and Éabha_ENG (section 7.2.5).

Year	Module
1	Engineering mathematics I/II (EM)
2	Engineering mathematics III/IV (EM)
3	Data analytics (EM)
	Work placement (IN)
4	Final-year project (EM)

Table H.4
Modules relevant to Fiach_SC1's interview (section 7.1.2) and the personal narratives for Breandán_SC1 (section 7.2.2) and Dónal_SC1 (section 7.2.4).

Year	**Module**	Fiach_SC1	Breandán_SC1	Dónal_SC1
1	Calculus for teachers (MS)	✓	✓	✓
2	Linear algebra (MS)	✓	✓	✓
	Probability and statistics (MS)	✓	✓	✓
	The mathematical experience (MS)	✓	✓	✓
	Microteaching and Teaching Preparation	✓	✓	✓
3	Teaching and assessing Junior Certificate mathematics (MS)	✓	✓	✓
	Teaching Placement	✓	✓	✓
	Quantum Physics (PS)			✓
4	Teaching and assessing Leaving Certificate mathematics (MS)	✓	✓	
	Abstract algebra (MS)	✓	✓	
	Teaching Placement	✓	✓	✓

H.8 Example of Story Titles

The following is a list of the story titles (the evaluation element of each story) which resulted from analysis of Aodhán_SC2's interview (see section 7.2.1). This narrative process only gave a partial view of the personal narrative which was developed by including other processes wherein the participant discussed his mathematical identity more fully.

1. **Story 1:** I think it kind of (...2) like that's more, you would kind of need to know that stuff going forward because probability and statistics, they would come up like, a good bit in a lot of other things.
2. **Story 2:** There wasn't much maths involved in the INTRA placement.
3. **Story 3:** And it was the only time I'd seen that brought up ehm (...2) outside of actual maths itself. That's when we kind of realised oh this kind of, it does overlap now. I see why they taught us that.
4. **Story 4:** I do like maths but I don't think I'm necessarily like very good at it.
5. **Story 5:** After the Gaeltacht and TY, I was able to go back (...1) and I just felt like I could raise my hand in class and actually ask the question and not feel like an idiot doing it.

H.9 Evaluation of the Narrative Interview Process

As discussed in section 3.2, the emergent design of this study meant that each data collection stage influenced subsequent stages. Since Data3 was the final data collection, and the narrative interview method consisted of several different parts (see section 4.5), this appendix presents an evaluation of the method.

Eliciting Stories

The narrative interview method proved to be a productive means of eliciting stories from participants. Most of these stories arose in part A of the interview, where the participants were asked to talk about their experience of mathematics in university, starting with first year and moving year-by-year up to the present. It was beneficial to establish the conversational narrator-listener roles, since several stories exhibited elements that arose because of questioning from the interviewer, which teased out important detail. For example, in the following story, the coda (conclusion to the story) was followed up with a question by the interviewer, which prompted the participant to share an evaluation of the experience ("coda" and "evaluation" are story elements that were defined in section 5.3.1):

Ciarán_ENG: Just that whole way of teaching for me wasn't, like I couldn't learn from that.

FH: And what was it that made it so inaccessible to you?

Ciarán_ENG: [They] never uploaded, like the lecture notes that we covered.

FH: Ok, so there's nothing online.

Ciarán_ENG: So it was literally, write everything down as fast as you can on the page that she writes, and then when the class is over, go to the library and look at it.

The Influence of Key Events Graphs on the Data

The key events graphs worked well as a tool to stimulate the discussion, and part B of the interview represented on average 16.5 minutes or 31% of the total interview time. They also allowed verification of the timeline of the participants' journeys by checking which modules and events occurred, and in what order. As was hoped beforehand, participants called on their knowledge of graphs in mathematics to interpret and comment on their own key events graphs:

FH: Is this the kind of graph you would have expected to get from plotting your key events, or is there anything surprising about it?

Breandán_SC1: Ehm (...5) I suppose it kind of would be. Started off with great enthusiasm, got into teaching, very positive and then got into like, went, going through [module], thinking about maths totally changed.

By following on from the chronological discussion of their time in DCU, part B of the interview allowed important space for participants to reflect on the experiences that had been discussed, and reconsider the importance of the various elements of their journey (one participant considered the graph in silence for 51 seconds before beginning to discuss it). In particular, participants' comments demonstrated how the graph allowed them to step back from the finer points of their journeys, and consider the big picture:

> Yeah, because I think you're learning new things, but you're also relearning things that you forgot before ... But even now you'd be learning, it's maths in a different way (Aodhán_SC2).
>
> Then, kind of around, third year placement, first semester third year, kinda change in, thinking about what we were doing, away from "maths because it's maths" to thinking how it can help [in the classroom] (Breandán_SC1).

Participants were allowed space to foreground some events over others, and clarify potential researcher misinterpretations. For example, Ciarán_ENG explained how he collaborated with his classmates on assignments, and they would debate whose answer was correct. In the first part of the interview, I interpreted this as a positive experience, and shared my own memory of enjoying hearty disagreements with my mathematics classmates during my undergraduate studies. However, when placing this key event on the graph, it emerged that Ciarán_ENG saw a negative side to this that I had not probed earlier in the interview: "It definitely pushed me a little bit harder. But then, like just becoming frustrated when people are telling you that you're wrong after spending a couple of hours working on something" (Ciarán_ENG).

Analysis of Key Events Graphs

The key events graphs stimulated conversation but were not reliable to be subjected to analysis or comparison. As described in section 4.5.1, participants were invited to use their own terminology to ascribe meaning to the vertical axes of the graph. Among their descriptions of the axes were the influence or effect of the events, and whether they represented positive or negative experiences. I questioned the reliability of the vertical placement of the events, because some events had both positive and negative elements:

> Could you nearly do two for that? One good and one bad (Ciarán_ENG).
>
> It's more, as everything, it's always gray right, you can't say anything's wholly positive (Dónal_SC1).

I also questioned the horizontal placement of events, since some events unfolded gradually over the course of a module, a year, or longer:

> It's hard to call [attending the MLC] a key event right because it was just a constant thing (Dónal_SC1).
>
> Yeah friends is really over the whole four years, realistically, because we're still doing it like. I'm still kind of cooperating with some of my peers for assignments (Ciarán_ENG).

The inherent subjectivity and uncertainty in the graphs was reflected in the participants' opinions about how well the graphs represented their journeys. However, this can been seen as an advantage, since it inspired participants to characterise their overall journeys, and correct the misinterpretations that they felt the graphs might communicate. The following examples illustrate the variety of the responses:

Dónal_SC1: It would sort of be like this (gestures upwards to the right), but, which would mean I really ended off on a positive, but I really didn't.

Éabha_ENG: Yeah, that it's towards the end that looks negative.

FH: Do you think that's how it was?

Éabha_ENG: Ehm (...1) no.

Ciarán_ENG: It's really like, up and down, isn't it? So it shows there are, good and bad things, I suppose.

FH: Do you think your experience was up and down?

Ciarán_ENG: Yeah, 100%.

Thus, the graphs themselves were not used for analysing the interviews, but participants' interpretations of their key events graphs were incorporated into their personal narratives, which are discussed in section 7.2. I suggest that the key events graphs, which are included in appendix H.6, should not be consulted before reading the corresponding personal narratives.

Participants' Reading their Questionnaire Response

As mentioned in section 4.5.1, in Part C of the interview the participants were invited to read their response from Data1 since the pilot interviewee indicated that she did not remember much of what she had written. This was a vital discovery for the trustworthiness of this stage of data collection, because the participants in the interviews expressed similarly weak memories of their questionnaire responses:

> What's on this now? (laughs) (Breandán_SC1).
>
> I can't remember it at all (Ciarán_ENG).
>
> Yeah I can read that, that's mine right? Just all this paragraph? (Dónal_SC1)

This may have aided the final part of the interview, since participants reacted to their response as if reading it for the first time, making it easier for them to separate their past mathematical identity from their current feelings, and consider the differences.

Appendix I

List of Important Terms

Term	See page	Term	See page
Coding saturation	63	Participant identifiers	53
Contextual codes	83	Personal narrative	99
Deductive coding	81	Praxeologies	18
Grinds	115	Procedural Fluency	15
Groupings	49	Project Maths	4
Group narrative	56	Saturation	62
Inductive coding	81	Senior Cycle	3
Junior Cycle	2	Storying stories	99
Key events graph	73	Theoretical saturation	63
Meaning saturation	63	Transferability	60
Member checking	62	Trustworthiness	60
Naturalist transcription	93	Validity	62

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