DILEMMAS EXPERIENCED IN LECTURING UNDERGRADUATE CALCULUS

Sinéad Breen[1], Ann O'Shea[2]

[1]CASTEL, Dublin City University, Ireland; [2]Maynooth University, Ireland

We consider a set of accounts written by two university lecturers describing incidents that took place during their first-year Calculus modules. Analysis of these accounts revealed that the lecturers had to make some difficult decisions while teaching. These situations sometimes involved choices between two or more alternatives each of which had disadvantages. We labelled these choices 'dilemmas'. Here we present and discuss the three most common types of dilemma evident from our data: namely, balancing good practice in teaching with students' feeling of discomfort; balancing the needs of students with different abilities; balancing time constraints and active participation by students.

INTRODUCTION

This study arises from a project in which five university mathematics lecturers attempted to follow the Discipline of Noticing (Mason, 2002b) in order to develop their own teaching. They wrote brief-but-vivid accounts of incidents in their classrooms and shared them with each other. Although lectures might be perceived as scripted and non-dynamic, previous analysis of our set of accounts (O'Shea, Breen & Meehan, 2017) revealed that lecturers experience a range of in-the-moment decision points in class. Further analysis has recently caused us to label some of these decision points as dilemmas because of the difficult nature of the choices involved. In this paper we will use as data the accounts written by two of these lecturers (referred to as Lecturer Y and Lecturer Z) while they were teaching modules on Differential Calculus to large first-year groups.

LITERATURE REVIEW

Teaching dilemmas have been a subject of research for some time. Lampert (1985) defined a dilemma as 'a problem forcing a choice between two equally undesirable alternatives…even though choosing would bring problematic consequences' and 'an argument between opposing tendencies within oneself in which neither side can come out the winner' (p. 182). Other authors have used similar definitions, for example Scager, Akkeman, Pilot & Wubbels (2017) considered dilemmas faced by teachers in university settings and considered dilemmas to be 'conflicts in which there are multiple, equally viable alternatives, each of which has advantages and disadvantages'(p. 319). They contend that a dilemma by its nature poses problems for an instructor not just because of the possible negative outcomes arising from any action but also because of the difficulty in trying to take the consequences of the possible actions into account. Schoenfeld (2008) asserted that it is natural that teachers face dilemmas because of the need to 'resolve the inevitable tensions that result from trying to achieve many things and honor many constraints at once' (p. 81). Tripp (1993) noted that dealing with teaching dilemmas and in-the-moment decision-making in general is stressful for teachers and that teachers need to use their professional judgement to make choices (p. 49).

Even though making these choices can be stressful, Lampert (1985) put forward the view that dealing with and reflecting on dilemmas can be useful to teachers and a means to professional growth. In her paper, she outlined two teaching situations which necessitated difficult choices and showed how teachers can use their knowledge of themselves and their goals to overcome problems. She posed a number of questions concerning further study in this area, including the question of how often dilemmas arise in classrooms and how teachers manage them.

At elementary school level, Ball (1993) examined the challenge of creating classroom practises to engage students in authentic mathematical tasks (e.g. formulating and solving problems, experimenting, conjecturing). She believes that trying to teach in such an 'intellectually honest' manner gives rise to dilemmas because of the competing aims of such an approach and the uncertainties inherent in addressing them. She described three types of dilemma: representing the content; respecting the children as mathematical thinkers; and creating and using community. Teachers typically face the dilemmas of their work alone and so Ball advocates forums for professional exchange in which teachers explore one another's practice as a resource for improving teaching and learning.

Very little research has been undertaken into dilemmas arising in mathematics classrooms at university. However, in his *Guide for University and College Lecturers*, Mason (2002a) described some tensions in teaching mathematics and, while he cautioned that there are no universal solutions to such tensions, he suggested they be thought of as sources of energy rather than problems as the latter induce anguish and frustration. In order to discuss the tensions he believes to be recurrent, he clustered them under three main headings: student and tutor agenda and expectation; doing and construing, knowing and understanding; being subtle and being explicit.

At all levels of education, students need to be challenged to stimulate their learning. However, as Scager et al. (2017) point out, challenging students can conflict with other teacher responsibilities, creating dilemmas for teachers. At university, classes are often large and comprise students of widely differing abilities, and choosing to serve one group of students can have adverse consequences for the learning of others. Scager et al. (2017) conducted a study involving twelve university lecturers from different disciplines reflecting on how challenges for their students were managed. Seven recurrent dilemmas were idenitified, the two most frequent of these being maximising challenge versus maintaining psychological safety of students and maximising challenge versus keeping all students aboard.

Speer and Wagner (2009) considered the tension that arises in the context of whole-class discussions in mathematics lectures between encouraging student ideas and participation and using students' suggestions in a mathematically productive manner. They made use of the terms *social scaffolding* (ways of supporting discourse and participation) and *analytic scaffolding* (ways of supporting mathematical progress) which were previously defined by Williams and Baxter (1996). Their analysis showed that providing both types of scaffolding at once is a very difficult task and requires instructors to draw on their pedagogical content knowledge (to recognise the pedagogical potential of a contribution to the discussion) and their specialised content knowledge (to interpret and evaluate the contributions). They call for

further work to study the experience of mathematics lecturers and in particular to analyse situations in which pedagogical and specialised content knowledge could be developed.

We take Lamperts's (1985) and Mason's (2002a) view that reflecting on problematic teaching situations or dilemmas can provide opportunities for teachers to develop their knowledge in this way. As a first step, it is important to have information about the types of tensions that lecturers face in the course of teaching a module. We will endeavour to provide some information on the question: What dilemmas do lecturers encounter when teaching large groups of undergraduate calculus students?

METHODOLOGY

In the Discipline of Noticing project mentioned earlier, the group of lecturers attempted to follow the practices outlined by Mason (2002b). In order to ensure that an incident noticed while teaching is available for further reflection, Mason recommends it should be recorded in a 'brief-but-vivid' account. Such an account is

> one which readers readily find relates to their experience. Brevity is obtained by omitting details which divert attention away from the main issue. … Vividness is obtained by sticking as much as possible to descriptions of behaviour which others, had they been present, would have readily agreed to having seen, heard or felt. (p. 57).

The aim is to give an 'account-of' an incident, describing it as objectively as possible, rather than 'accounting-for' (offering interpretation, explanation, value-judgement or criticism). When the purpose of an account is to describe an emotion, then it should do so as physiologically and impartially as possible.

We will consider here only the accounts written by two of the lecturers in one academic year which relate to large group teaching. Both lecturers wrote accounts relating to Differential Calculus modules for first-year students; these modules were aimed at non-specialist students. Lecturer Y's module ran for the entire academic year and about 50 students were enrolled, while Lecturer Z's class consisted of about 150 students and the module ran for one semester only.

The two lecturers wrote 58 accounts in total relating to the modules in question here, and each account usually consisted of between 100 and 150 words. The data analysis process started with us reading and rereading both sets of accounts highlighting what we saw as dilemmas. We adapted the definition of the term *dilemma* given by Lampert (1985); for us a dilemma is a situation in which a difficult choice has to be made by the instructor between two or more undesirable alternatives. Following a general inductive approach (Thomas, 2006), we studied accounts which showed the lecturers' indecision or dissatisfaction stemming from having to make a choice where all courses of action had disadvantages. We met to discuss the accounts selected in this manner and agreed on the final identification of dilemmas. We then compared the issues described, and created categories of dilemmas. In this way the categories emerged from our data, and only later did we consult the research literature for comparison.

We found three main categories of dilemmas in our analysis of the set of accounts. Examples of these categories were evident in both lecturers' accounts and therefore we considered that

these problematic situations or choices were general and indicative of the difficulties that other Calculus lecturers might face. Other types of dilemmas occurred less frequently in the accounts and so we will not consider them here.

EXAMPLES OF DILEMMAS

The first category of dilemmas which emerged contains accounts where there is a clash between belief about good practice in teaching and concerns about students feeling uncomfortable (either socially or academically). This category has two important subcategories: creating cognitive conflict or encouraging participation versus not wanting to embarrass students or undermine their confidence (Category 1a); and fostering agency versus providing scaffolding (Category 1b). The second category concerns dilemmas arising from the wish to balance the needs of students with strong mathematical backgrounds versus those with weaker mathematical backgrounds (Category 2). The accounts in this category concern decisions about spending class time on revision or on basic mathematical skills versus keeping all students in the lecture hall engaged. The third category is about trying to balance time constraints with the wish to encourage student participation and/or develop understanding (Category 3). We will explain these categories further below and give examples of accounts which illustrate the choices facing lecturers in these situations.

Category 1a: Balancing good practice in teaching with students' feelings of confidence or embarrassment

The accounts which showed evidence of a dilemma arising from a conflict between the lecturers' views about good teaching practice and their wish not to embarrass or undermine students' confidence occurred in the context of asking questions of the whole class. Both lecturers wrote accounts about incidents where they deliberately introduced cognitive conflict for students with the aim of getting students to recognise the conflict and to develop deeper understanding of a particular concept. However, both lecturers worried about the implications of such a strategy for student confidence and willingness to participate. Both lecturers spoke about asking whole-class questions to identify cognitive conflicts and worried about embarrassing students in public if they were seen to give the 'wrong' answer. On the other hand, they felt that by not causing the conflict in class some students might not be aware of a conflict in their views and/or be able to resolve it by themselves. For example, Lecturer Z wrote:

> Account A: I asked a question that I knew would probably generate a wrong answer. I did this so as to point out the pitfall and misunderstanding. When the wrong answer was given I said 'I'm glad you said that' and explained some more. However, I felt bad that I had more or less deliberately caused someone to give a wrong answer. (Lecturer Z)

Other accounts that dealt with similar situations included lecturers wanting to ask questions to generate a debate but not liking to call on individual students in case they would be embarrassed and lecturers worrying about the effect of being wrong on student confidence.

Category 1b: Balancing student independence with providing scaffolding

The second type of dilemma in this category concerned the tension between the desire to foster agency and independence for students and wanting to provide adequate scaffolding for their learning. This type of situation usually arose when lecturers asked students to work on tasks in lectures or tutorials. Sometimes students looked to the lecturer for validation instead of relying on their own understanding. The lecturer was then faced with a choice of whether to provide the validation (and make the students feel comfortable) or to refuse to do this. A similar situation can occur when a lecturer asks a whole-class question and no one offers a suggestion – the lecturer is faced with a choice between leaving the question unanswered (and possibly causing discomfort) or answering it herself and reducing the opportunity for learning. In the account below, the lecturer noticed that students had problems with an unfamiliar task and provided scaffolding to help them with it. However, she felt that in doing this, she might have compromised the effectiveness of the task:

> Account B: Due to the students' difficulties in yesterday's tutorials in relation to drawing graphs meeting a number of criteria, I changed my plan for today's lecture. The students were given the graphs of functions with various points of discontinuity (but no formulaic description of the functions) and asked to determine whether particular statements were true or false. Asking individuals for their answers indicated that they could correctly determine the truth or otherwise of the statements. In undertaking this exercise, I was a little uneasy that I was perpetuating a type of helplessness by making an unfamiliar problem assigned as homework more manageable for them. (Lecturer Y)

Category 2: Balancing the needs of students with different backrounds

The second category of dilemmas that emerged from our analysis arose from teaching students with a range of mathematical backgrounds in large class settings. The lecturers faced difficult choices when trying to balance the needs of different groups of students. As we see in the account below, the lecturers sometimes felt the need to review material for students who were less prepared but found it difficult to do this without losing the interest of others. However, they worried that if they moved on then they risked losing many of the students.

> Account C: I started the class by drawing the graphs of sin, cos and tan on the board. We had covered trig functions and domains and ranges last week. My intention was to talk a little about periodicity and then move on to inverse functions. However, I asked the class to tell me the domain and range of the three trig functions and the answers were not great. […] I spent half the class trying to address these issues. I felt that some students were definitely getting bored. (Lecturer Z)

In other accounts related to this category the lecturers spoke of noticing that some students in the class were bored while others were struggling with the material. Most first year university students have already studied quite a lot of calculus at school although some have not developed a deep understanding of the subject. Lecturers face a choice between reviewing topics that some students find difficult and moving on to new material without spending time working on the foundations. One possible solution is to approach these topics from a fresh perspective, however this also is a difficult task and Lecturer Y remarked that she experienced 'a tension between maintaining [students'] interest and motivation and undermining their prior knowledge'. It is common for first-year undergraduate mathematics courses in Ireland to

include students with very different levels of mathematical preparation. For example, students who had studied mathematics at Ordinary Level in the State Examinations (Leaving Certificate) often find themselves in the same module as those who have studied at Higher Level. In contrast, this would be rare in secondary school classrooms.

Category 3: Balancing time constraints and active participation by students

The final category of dilemmas that we identified in our data concerned the tension between wanting to spend time helping students to develop understanding and needing to be cognisant of time constraints. The lecturers spoke about wanting to encourage interaction in their classes but realised that this takes quite a lot of class-time. They had to balance this aim against the need to cover the syllabus and finish the course within a tight timeframe. In the account below, the lecturer speaks about the time implications of inquiry-based learning:

> Account D: I tried to use a 'guided-discovery' approach to facilitate students' realization that the graph of a function and its inverse are mirror images of each other in the line y=x. However, each step of this took a lot longer than I envisaged. Moreover, I wasn't convinced at the end that the students would retain this particular piece of information longer or understand it better for having discovered it themselves as a class community. (Lecturer Y)

Other accounts in this category concerned conflicts in lecturers' priorities, that is between employing progressive teaching practices and adhering to the syllabus. There was a recognition that some teaching practices take more time than 'lecturing' and that time-pressure places a heavy burden on lecturers.

DISCUSSION

Although it was not our intention when writing accounts of our experiences of teaching first-year Calculus to document the dilemmas inherent in this practice, dilemmas were recorded in 23 of the 58 accounts in question (i.e. 40%). This serves to support Schoenfeld's (2008) view that dilemmas are 'natural' and illustrates their prevalence in day-to-day teaching practice. Moreover, it underlines the importance of examining them with a view to improving practice.

Scager et al. (2017) explored the dilemmas which arise in presenting challenges to students, and subsequently managing these challenges, across a number of disciplines at university level. We found echoes of the two most frequent dilemmas documented by Scager and her colleagues in our study, despite the fact that mathematics was not included in the disciplines they examined. Account A above (introducing a cognitive conflict for students) is an example of a 'maximising challenge versus maintaining psychological safety of students' dilemma. Scager et al. explain how a student's psychological safety could be threatened by being asked difficult questions or by having critical feedback on their work openly communicated to them. Students need to be challenged to move outside of their comfort zones but yet a teacher wants to avoid a student feeling embarrassed or inadequate in order to preserve the student's freedom to contribute to class discussions and activities. On the other hand, Account C clearly describes a situation in which the teacher experiences a dilemma in relation to 'maximising challenge versus keeping all students aboard'. In a class in which there are students with a wide range of different mathematical backgrounds, a teacher tries to avoid setting the

challenge too high in order to preserve the self-confidence of some students while simultaneously avoiding demotivating other students with an insufficient level of challenge.

Aside from providing appropriate challenge for students, other dilemmas arise when trying to teach in an authentic or intellectually honest way, respecting the integrity of the discipline of mathematics. The account given above of trying to use a guided discovery approach (Account D) resonates with Ball's (1993) discussion of the dilemmas integral in creating and using a community in which ideas can be developed and critiqued. Ball questions whether the efforts involved in establishing such a community always involve the best use of the (often limited) time. She suggests that constructing a classroom pedagogy to model authentic mathematical practice precisely would be not only inappropriate but also irresponsible because mathematicians have the luxury to focus on a small number of problems while a teacher is usually bound to cover an entire curriculum and develop the associated skills. A teacher must also facilitate the learning of all the learners in her care, in the same room, at the same time. The dilemmas evoked by these constraints are also felt at university level.

Although both lecturers in our study recognised the value in generating debate and cognitive conflict they were both uneasy with the possible consequences for student affect. The setting of a first-year university lecture exacerbates this problem, since the class-sizes are normally large and the lecturers usually do not have an opportunity to get to know the students well. Since students are often reluctant to answer (or even ask) questions in this setting, even when explicitly encouraged to do so (Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011), lecturers may be excessively cautious not to damage student engagement. The large class size may also result students having an increased sense of anonymity and may discourage participation from that perspective. Thus, the context of a first-year university lecture may mean the dilemmas encountered in categories 1a, 1b and 3 occur more frequently or are felt more acutely than at primary or secondary school level. More reflection on how to provide social scaffolding in this situation is needed. The situation may be further aggravated by the subject matter, as students often appear to struggle with the mathematical vocabulary needed to participate in a whole class discussion in a mathematics lecture.

Despite Speer and Wagner's (2009) contention that teachers are often more successful at using social scaffolding than analytic scaffolding, the issue of a lecturer having problems providing analytic scaffolding did not appear in the subset of accounts analysed here. Speer and Wagner explain how providing both types of scaffolding at once requires instructors to draw on both their pedagogical content knowledge and their specialised content knowledge. It may be that the specialised content knowledge required to interpret and evaluate a student's contribution is unproblematic in the context of first-year Calculus for non-specialist students.

The dilemmas described here arose from a broader project involving five mathematics lecturers engaging with the Discipline of Noticing (Breen, McCluskey, Meehan, O'Donovan & O'Shea, 2014). Accounts written by each of the lecturers were shared with all and meetings were held periodically to discuss any matters relating to the project. Often these meetings became a forum for the discussion of a dilemma related in one of the accounts and allowed us, a group of teachers, to explore one another's practice and how a lecturer might act when faced with such a dilemma. We found this collaborative aspect of the project to be very beneficial in

terms of professional development. As Ball (1993) pointed out, teachers regularly face the dilemmas of their work alone and so forums for professional exchange in which these dilemmas are discussed can provide a necessary opportunity for improving teaching and learning. Scager et al. (2017) also believe that expertise can be developed through collaborative reflection. They assert that reflecting on dilemmas in particular, because the nature of dilemmas allows for 'the evocation of reflection and argumentation, encouraging teachers to talk about choices and considerations' (p. 333), can lead to professional growth.

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